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Constraining M_ν with the Bispectrum III: Compressing the Bispectrum

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ABSTRACT

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1. INTRODUCTION

Table 1. The Quijote suite includes 15,000 standard N-body simulations at the fiducial cosmology to accurately estimate the covariance matrices. It also includes sets of 500 simulations at 13 different cosmologies, where only one parameter is varied from the fiducial value (underlined), to estimate derivatives of observables along the cosmological parameters.

Name	M_ν	Ω_m	Ω_b	h	n_s	σ_8	ICs	realizations
Fiducial	0.0	0.3175	0.049	0.6711	0.9624	0.834	2LPT	15,000
Fiducial ZA	0.0	0.3175	0.049	0.6711	0.9624	0.834	Zel'dovich	500
M_ν^+	$\underline{0.1}$ eV	0.3175	0.049	0.6711	0.9624	0.834	Zel'dovich	500
M_ν^{++}	$\underline{0.2}$ eV	0.3175	0.049	0.6711	0.9624	0.834	Zel'dovich	500
M_ν^{+++}	$\underline{0.4}$ eV	0.3175	0.049	0.6711	0.9624	0.834	Zel'dovich	500
Ω_m^+	0.0	$\underline{0.3275}$	0.049	0.6711	0.9624	0.834	2LPT	500
Ω_m^-	0.0	$\underline{0.3075}$	0.049	0.6711	0.9624	0.834	2LPT	500
Ω_b^+	0.0	0.3175	$\underline{0.051}$	0.6711	0.9624	0.834	2LPT	500
Ω_b^-	0.0	0.3175	$\underline{0.047}$	0.6711	0.9624	0.834	2LPT	500
h^+	0.0	0.3175	0.049	$\underline{0.6911}$	0.9624	0.834	2LPT	500
h^-	0.0	0.3175	0.049	$\underline{0.6511}$	0.9624	0.834	2LPT	500
n_s^+	0.0	0.3175	0.049	0.6711	$\underline{0.9824}$	0.834	2LPT	500
n_s^-	0.0	0.3175	0.049	0.6711	$\underline{0.9424}$	0.834	2LPT	500
σ_8^+	0.0	0.3175	0.049	0.6711	0.9624	$\underline{0.849}$	2LPT	500
σ_8^-	0.0	0.3175	0.049	0.6711	0.9624	$\underline{0.819}$	2LPT	500

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2. THE QUIJOTE SIMULATION SUITE

We use a subset of simulations from the Quijote suite, a set of 43,000 N-body simulations that spans over 7000 cosmological models and contains, at a single redshift, over 8.5 trillion particles. The Quijote suite was designed to quantify the information content of cosmological observables and also to train machine learning algorithms. Hence, the suite includes enough realizations to accurately estimate the covariance matrices of high-dimensional observables such as the bispectrum as well as the derivatives of these observables with respect to cosmological parameters. For the derivatives, the suite includes sets of simulations run at different cosmologies where only one parameter is varied from the fiducial cosmology ($\Omega_{\rm m}$=0.3175, $\Omega_{\rm b}$=0.049, h=0.6711, n_s=0.9624, σ_8=0.834, and M_ν=0.0 eV). Along $\Omega_{\rm m}$, $\Omega_{\rm b}$, h, n_s, and σ_8, the fiducial cosmology is adjusted by either a small step above or below the fiducial value: $\{\Omega_{\rm m}^+, \Omega_{\rm m}^-, \Omega_{\rm b}^+, \Omega_{\rm b}^-, h^+, h^-, n_s^+, n_s^-, \sigma_8^+, \sigma_8^-\}$. Along M_ν, because $M_\nu \geq 0.0$ eV and the derivative of certain observable with respect to M_ν is noisy, Quijote includes sets of simulations for $\{M_\nu^+, M_\nu^{++}, M_\nu^{+++}\} = \{0.1, 0.2, 0.4~\text{eV}\}$. At each of these 14 cosmologies, Quijote includes sets of both standard N-body and paired-fixed simulations.

The initial conditions for all the simulations were generated at $z = 127$ using second-order perturbation theory for simulations with massless neutrinos ($M_\nu = 0.0$ eV) and the Zel'dovich approximation for massive neutrinos ($M_\nu > 0.0$ eV). The initial conditions with massive neutrinos take their scale-dependent growth factors/rates into account using the **?** method, while for the massless neutrino case we use the traditional scale-independent rescaling. From the initial conditions, the simulations follow the gravitational evolution of 512^3 dark matter particles, and 512^3 neutrino particles for massive neutrino models, to $z = 0$ using Gadget-III TreePM+SPH code (**?**). Simulations with massive neutrinos are run using the "particle method", where neutrinos are described as a collisionless and pressureless fluid and therefore modeled as particles, same as CDM (**??**). Halos are then identified using the Friends-of-Friends algorithm (FoF; **?**) with linking length $b = 0.2$ on the CDM + baryon distribution. We limit the halo catalogs to halos with masses above $M_{\rm lim} = 3.2 \times 10^{13} h^{-1} M_\odot$. For the fiducial cosmology, the halo catalogs have $\sim$156,000 halos ($\bar{n} \sim 1.56 \times 10^{-4}~h^3{\rm Gpc}^{-3}$) with $\bar{n}P_0(k = 0.1) \sim 3.23$. We refer readers to Villaescusa-Navarro et al. (in preparation) and Hahn et al. (2019) for further details on the Quijote simulations.

3. FISHER MATRIX FOR MODIFIED T-DISTRIBUTION LIKELIHOODS

The standard approach for Fisher matrix forecasts and parameter inference in LSS assumes that the p-dimensional likelihood is Gaussian and uses the Hartlap et al. (2007) factor,

$$f_{\rm Hartlap} = \frac{N - p - 2}{N - 1} \tag{1}$$

to account for the bias in the inverse covariance matrix $\widehat{\boldsymbol{C}}^{-1}$ estimated from N mocks. In addition to breaking down on large scales where Central Limit Theorem no longer holds (Hahn et al. 2019), this assumption also breaks down when the covariance matrix $\boldsymbol{C}$ is estimated from a finite number

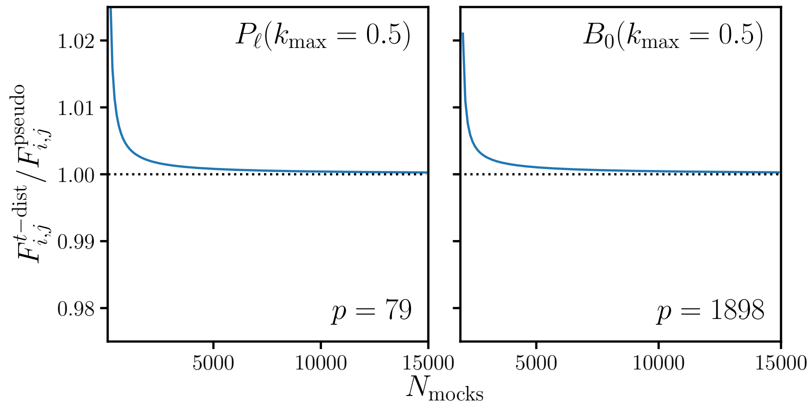

Figure 1. Ratio of the Fisher matrix for a modified t-distribution, $F_{ij}^{t\text{-dist}}$, over the Fisher matrix of the Gaussian pseudo-likelihood, $F_{ij}^{\rm pseudo}$ for the power spectrum multipole (P_ℓ; left) and the bispectrum (B_0; right). The likelihoods for P_ℓ and B_0 up to $k_{\rm max}$ have 89 and 1898 dimensions respectively.

of mocks (Sellentin & Heavens 2016). In fact, Sellentin & Heavens (2016) show that the likelihood is no longer Gaussian but a modified t-distribution:

$$p(y\,|\,\mu(\theta), \widehat{\boldsymbol{C}}, N) = \frac{c_p}{|\widehat{\boldsymbol{C}}|^{1/2}} \left(1 + \frac{(y-\mu(\theta))^T\, \widehat{\boldsymbol{C}}^{-1}\, (y-\mu(\theta))}{1-N}\right)^{-N/2}. \tag{2}$$

where

$$c_p = \frac{\Gamma(\frac{N}{2})}{[\pi(N-1)]^{p/2}\ \Gamma(\frac{N-p}{2})}, \tag{3}$$

Γ is the Gamma function, y is the data, μ is our model, and $\widehat{\boldsymbol{C}}$ is the estimated covariance matrix. Adopting the wrong likelihood, even when the bias of the inverse covariance matrix is accounted for, will yield incorrect posterior distributions with biased parameter estimates and incorrect errors (Sellentin & Heavens 2016). Therefore, we derive below the Fisher matrix for the modified t-distribution likelihood. We follow the derivations from Lange et al. (1989) and refer readers to it for details.

Let $\ell(\theta)$ be the log-likelihood, $z = \widehat{\boldsymbol{C}}^{-1/2}(y-\mu)$, and

$$g(s) = c_p \left(1 + \frac{s}{N-1}\right)^{-N/2} \tag{4}$$

so that Eq. 2 can be written as $p(y\,|\,\mu(\theta), \widehat{\boldsymbol{C}}, N) = |\widehat{\boldsymbol{C}}|^{-1/2} g(||z||^2)$. Then the derivative of the log-likelihood is

$$\frac{\partial \ell}{\partial \theta_i} = \left(\frac{1}{g}\frac{\partial g}{\partial s}\right)\left(-2\frac{\partial \mu}{\partial \theta_i}^T \widehat{\boldsymbol{C}}^{-1}(y-\mu)\right) \tag{5}$$

We can write the Fisher matrix as

$$F_{ij} = -\left\langle \frac{\partial^2 \ell}{\partial\theta_i \partial\theta_j}\right\rangle = \left\langle \frac{\partial \ell}{\partial\theta_i}\frac{\partial \ell}{\partial\theta_j}\right\rangle \tag{6}$$

$$= 4\left\langle \left(\frac{1}{g}\frac{\partial g}{\partial s}\right)^2 \left(z^T \widehat{\boldsymbol{C}}^{-1/2}\frac{\partial \mu}{\partial\theta_i}\frac{\partial \mu^T}{\partial\theta_j}\widehat{\boldsymbol{C}}^{-1/2} z\right)\right\rangle. \tag{7}$$

Using Lemma 1 from Lange et al. (1989), we get

$$= 4\left\langle \left(\frac{1}{g}\frac{\partial g}{\partial s}\right)^2 ||z||^2 \left(\frac{z^T}{||z||}\widehat{\boldsymbol{C}}^{-1/2}\frac{\partial \mu}{\partial \theta_i}\frac{\partial \mu^T}{\partial \theta_j}\widehat{\boldsymbol{C}}^{-1/2}\frac{z}{||z||}\right)\right\rangle \tag{8}$$

$$= 4\left\langle ||z||^2 \left(\frac{1}{g}\frac{\partial g}{\partial s}\right)^2 \right\rangle \frac{1}{p}\mathrm{Tr}\left(\widehat{\boldsymbol{C}}^{-1/2}\frac{\partial \mu}{\partial \theta_i}\frac{\partial \mu^T}{\partial \theta_j}\widehat{\boldsymbol{C}}^{-1/2}\right) \tag{9}$$

$$= 4\left\langle ||z||^2 \left(\frac{1}{g}\frac{\partial g}{\partial s}\right)^2 \right\rangle \frac{1}{p}\frac{\partial \mu^T}{\partial \theta_i}\widehat{\boldsymbol{C}}^{-1}\frac{\partial \mu}{\partial \theta_j} \tag{10}$$

Since

$$\frac{1}{g}\frac{\partial g}{\partial s} = -\frac{N}{2}\left(\frac{1}{N-1}\right)\left(1+\frac{s}{N-1}\right)^{-1} \tag{11}$$

we can expand

$$\left\langle ||z||^2 \left(\frac{1}{g}\frac{\partial g}{\partial s}\right)^2 \right\rangle = \left\langle ||z||^2 \frac{N^2}{4}\left(\frac{1}{N-1}\right)^2\left(1+\frac{s}{N-1}\right)^{-2}\right\rangle \tag{12}$$

$$= \frac{N^2}{4(N-1)}\left\langle \frac{||z||^2}{N-1}\left(1+\frac{||z||^2}{N-1}\right)^{-2}\right\rangle \tag{13}$$

$$= \frac{N^2}{4(N-1)}\int_0^\infty A_p\left(\frac{s^2}{N-1}\right)\left(1+\frac{s^2}{N-1}\right)^{-2} c_p\left(1+\frac{s^2}{N-1}\right)^{-\frac{N}{2}} s^{p-1}\mathrm{d}s \tag{14}$$

where A_p is the surface area of the unit sphere in $\mathbb{R}$. Evaluating the integral we get

$$= \frac{N^2}{4(N-1)}\left[\frac{2\pi^{\frac{p}{2}}c_p}{(N-1)\Gamma(\frac{p}{2})}\frac{(N-1)^{\frac{p}{2}+1}}{2}B\left(\frac{p}{2},\frac{N-p+2}{2}\right)\right]. \tag{15}$$

where B is the beta function. Expanding this expression we get

$$= \frac{p(N-p)N}{4(N-1)(N+2)} \tag{16}$$

Plugging the expression back into Eq. 10, we get the Fisher matrix for the modified t-distribution:

$$F_{ij}^{t\text{-dist}} = \frac{N(N-p)}{(N-1)(N+2)}\frac{\partial \mu^T}{\partial \theta_i}\widehat{\boldsymbol{C}}^{-1}\frac{\partial \mu}{\partial \theta_j} = f_{t\text{-dist}}\widehat{F_{ij}}. \tag{17}$$

In contrast, the Fisher matrix for the Gaussian pseudo-likelihood is

$$F_{ij}^{\text{pseudo}} = \frac{N-p-2}{N-1}\frac{\partial \mu^T}{\partial \theta_i}\widehat{\boldsymbol{C}}^{-1}\frac{\partial \mu}{\partial \theta_j} = f_{\text{Hartlap}}\widehat{F_{ij}}. \tag{18}$$

For a p=79 dimensional likelihood (P_ℓ likelihood for $k_{\max} = 0.5$) $f_{t\text{-dist}} > f_{\text{Hartlap}}$ for $N \leq 81$ and $f_{t\text{-dist}} < f_{\text{Hartlap}}$ for $N > 81$. For a p=428 dimionsional likelihood (B_0 likelihood for $k_{\max} = 0.5$), $f_{t\text{-dist}} > f_{\text{Hartlap}}$ for $N \leq 697$ and $f_{t\text{-dist}} < f_{\text{Hartlap}}$ for $N > 697$. As the number of mocks increases, both $f_{t\text{-dist}}$ and f_{Hartlap} converge to 1. We note that although the likelihood $p(y \,|\, \mu(\theta), \widehat{\boldsymbol{C}}, N)$ is a function of N, we do not explicitly marginalize over it. This is because as Lange et al. (1989) proves, the Fisher matrix for a t-disttribution is block diagonal — *i.e.* $F_{\theta_i,N} = 0$ and $F_{\theta_j,N} = 0$. Hence, N does not impact our Fisher forecasts constraints for cosmological parameters.

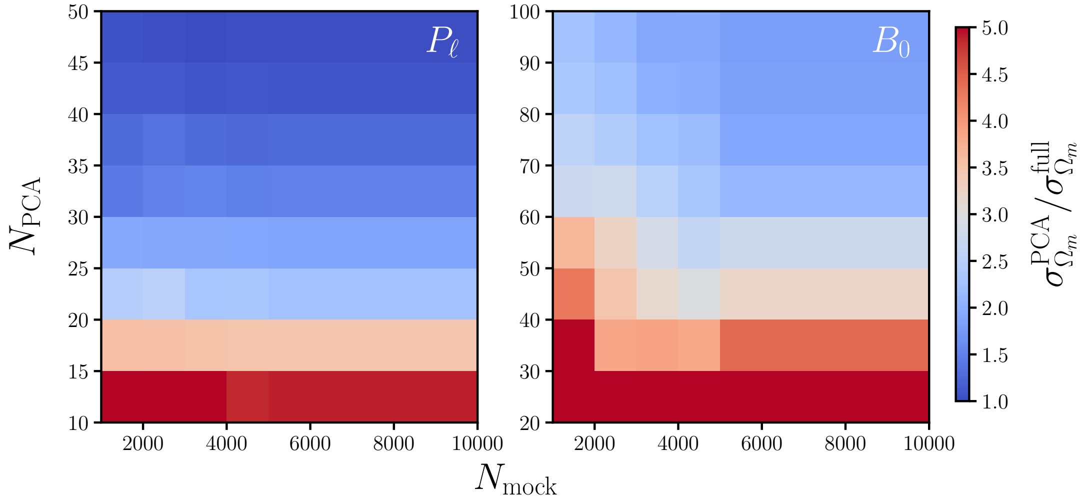

Figure 2.

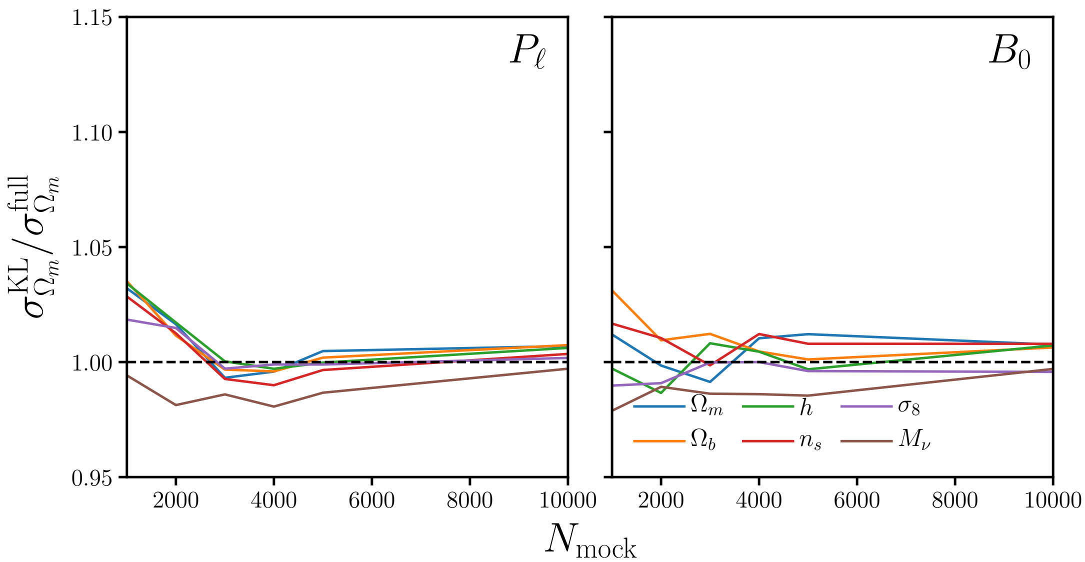

Figure 3.

4. RESULTS

5. SUMMARY

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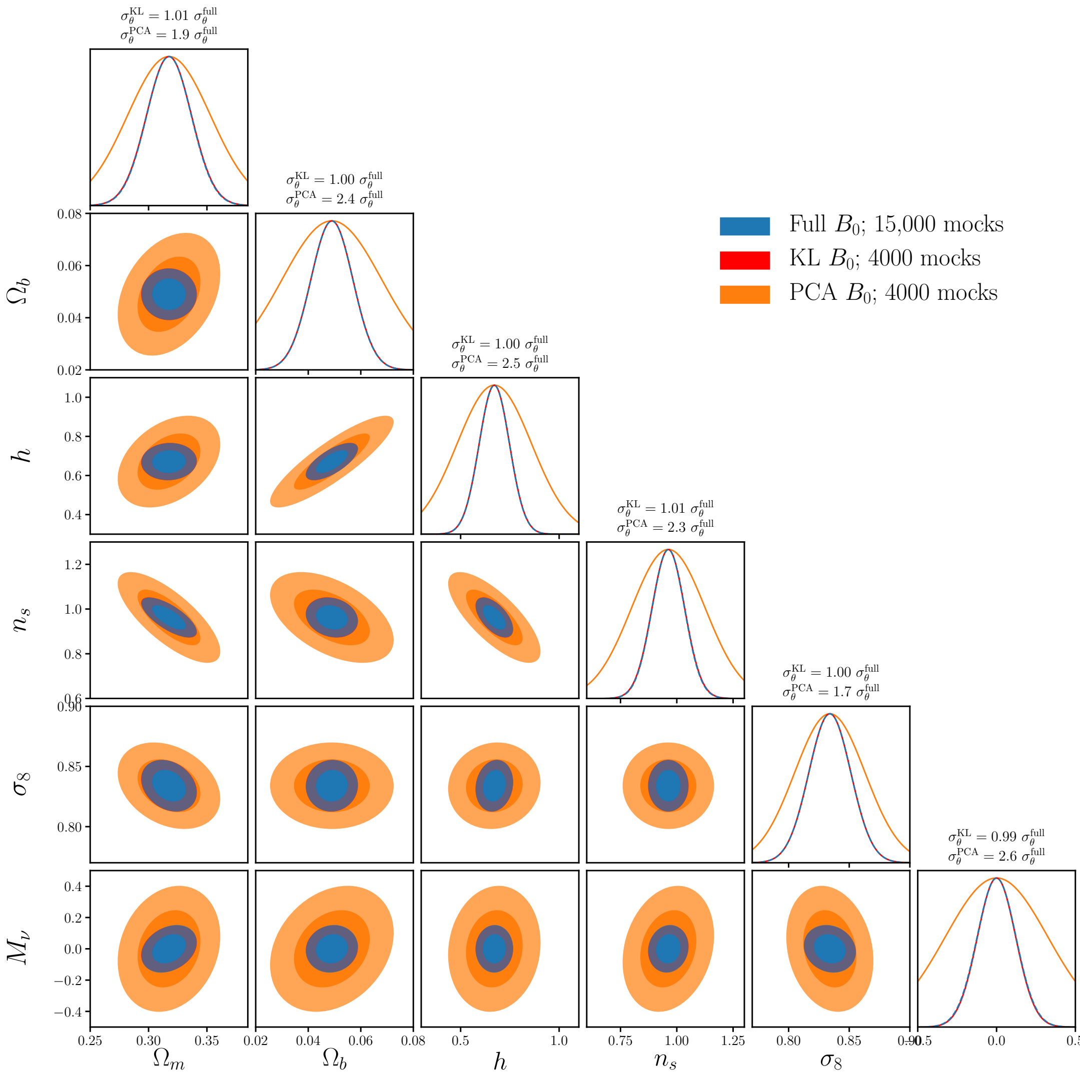

Figure 4.

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