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Constraining M_ν with the Bispectrum III: Compressing the Bispectrum

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ABSTRACT

Keywords: cosmology: —

1. INTRODUCTION

intro goes here

2. SIMULATIONS

Very brief description of the simulations. Just highlight the numbers

3. RESULTS

4. SUMMARY

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APPENDIX

A. FISHER MATRIX FOR MODIFIED T-DISTRIBUTION LIKELIHOODS

The standard approach for Fisher matrix forecasts and parameter inference in LSS assumes that the p-dimensional likelihood is Gaussian and uses the Hartlap et al. (2007) factor,

$$f_{\rm Hartlap} = \frac{N-p-2}{N-1} \tag{A1}$$

to account for the bias in the inverse covariance matrix $\widehat{\boldsymbol{C}}^{-1}$ estimated from N mocks. In addition to breaking down on large scales where Central Limit Theorem no longer holds (Hahn et al. 2019), this assumption also breaks down when the covariance matrix $\boldsymbol{C}$ is estimated from a finite number of mocks (Sellentin & Heavens 2016). In fact, Sellentin & Heavens (2016) show that the likelihood is no longer Gaussian but a modified t-distribution:

$$p(y\,|\,\mu(\theta), \widehat{\boldsymbol{C}}, N) = \frac{c_p}{|\widehat{\boldsymbol{C}}|^{1/2}} \left(1 + \frac{(y-\mu(\theta))^T\, \widehat{\boldsymbol{C}}^{-1}\,(y-\mu(\theta))}{1-N}\right)^{-N/2}. \tag{A2}$$

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$$c_p = \frac{\Gamma(\frac{N}{2})}{[\pi(N-1)]^{p/2}\ \Gamma(\frac{N-p}{2})} \tag{A3}$$

where Γ is the Gamma function, y is the data, μ is our model, and $\widehat{\boldsymbol{C}}$ is the estimated covariance matrix. Adopting the wrong likelihood will yield incorrect posterior distributions, with biased parameter estimates and incorrect errors even when the bias of the inverse covariance matrix is accounted for (Sellentin & Heavens 2016). Therefore, we derive below the Fisher matrix for the modified t-distribution likelihood. We follow the derivations from Lange et al. (1989) and refer readers to it for details.

For simplicity, let $\ell(\theta)$ be the log-likelihood, $z = \widehat{\boldsymbol{C}}^{-1/2}(y-\mu)$, and

$$g(s) = c_p \left(1 + \frac{s}{1-N}\right)^{-N/2} \tag{A4}$$

so that Eq. A2 can be written as $p(y\,|\,\mu(\theta), \widehat{\boldsymbol{C}}, N) = |\widehat{\boldsymbol{C}}|^{-1/2} g(||z||^2)$. Then the derivative of the log-likelihood is

$$\frac{\partial \ell}{\partial \theta_i} = \left(\frac{1}{g}\frac{\partial g}{\partial s}\right)\left(-2\frac{\partial \mu}{\partial \theta_i}^T \widehat{\boldsymbol{C}}^{-1}(y-\mu)\right) \tag{A5}$$

We can write the Fisher matrix as

$$F_{ij} = -\left\langle \frac{\partial^2 \ell}{\partial \theta_i \partial \theta_j} \right\rangle = \left\langle \frac{\partial \ell}{\partial \theta_i}\frac{\partial \ell}{\partial \theta_j} \right\rangle \tag{A6}$$

$$= 4\left\langle \left(\frac{1}{g}\frac{\partial g}{\partial s}\right)^2 \left(z^T \widehat{\boldsymbol{C}}^{-1/2} \frac{\partial \mu}{\partial \theta_i}\frac{\partial \mu^T}{\partial \theta_j} \widehat{\boldsymbol{C}}^{-1/2} z\right) \right\rangle. \tag{A7}$$

Using Lemma 1 from Lange et al. (1989), we get

$$= 4\left\langle \left(\frac{1}{g}\frac{\partial g}{\partial s}\right)^2 ||z||^2 \left(\frac{z^T}{||z||} \widehat{\boldsymbol{C}}^{-1/2} \frac{\partial \mu}{\partial \theta_i}\frac{\partial \mu^T}{\partial \theta_j} \widehat{\boldsymbol{C}}^{-1/2} \frac{z}{||z||}\right) \right\rangle \tag{A8}$$

$$= 4\left\langle ||z||^2 \left(\frac{1}{g}\frac{\partial g}{\partial s}\right)^2 \right\rangle \frac{1}{p} \mathrm{Tr}\left(\widehat{\boldsymbol{C}}^{-1/2} \frac{\partial \mu}{\partial \theta_i}\frac{\partial \mu^T}{\partial \theta_j} \widehat{\boldsymbol{C}}^{-1/2}\right) \tag{A9}$$

$$= 4\left\langle ||z||^2 \left(\frac{1}{g}\frac{\partial g}{\partial s}\right)^2 \right\rangle \frac{1}{p} \frac{\partial \mu^T}{\partial \theta_i} \widehat{\boldsymbol{C}}^{-1} \frac{\partial \mu}{\partial \theta_j} \tag{A10}$$

Since

$$\frac{1}{g}\frac{\partial g}{\partial s} = -\frac{N}{2}\left(\frac{1}{N-1}\right)\left(1 + \frac{s}{N-1}\right)^{-1} \tag{A11}$$

we can expand

$$\left\langle ||z||^2 \left(\frac{1}{g}\frac{\partial g}{\partial s}\right)^2 \right\rangle = \left\langle ||z||^2 \frac{N^2}{4}\left(\frac{1}{N-1}\right)^2 \left(1+\frac{s}{N-1}\right)^{-2} \right\rangle \tag{A12}$$

$$= \frac{N^2}{4(N-1)}\left\langle \frac{||z||^2}{N-1}\left(1+\frac{||z||^2}{N-1}\right)^{-2}\right\rangle \tag{A13}$$

$$= \frac{N^2}{4(N-1)}\frac{p(N-1)}{(N+p+1)(N+p-1)} \tag{A14}$$

$$= \frac{N^2 p}{4(N+p+1)(N+p-1)}. \tag{A15}$$

Plugging the expression back into Eq. A8, we get the Fisher matrix for the modified t-distribution:

$$F_{ij}^{t\text{-dist}} = \frac{N^2}{(N+p+1)(N+p-1)}\frac{\partial \mu^T}{\partial \theta_i}\widehat{\boldsymbol{C}}^{-1}\frac{\partial \mu}{\partial \theta_j} = f_{t\text{-dist}}\widehat{F_{ij}}. \tag{A16}$$

In contrast, the Fisher matrix for the Gaussian pseudo-likelihood is

$$F_{ij}^{\text{pseudo}} = \frac{N-p-2}{N-1}\frac{\partial \mu^T}{\partial \theta_i}\widehat{\boldsymbol{C}}^{-1}\frac{\partial \mu}{\partial \theta_j} = f_{\text{Hartlap}}\widehat{F_{ij}}. \tag{A17}$$

For a p=47 dimensional likelihood (P_ℓ likelihood for $k_{\max} = 0.5$) $f_{t\text{-dist}} > f_{\text{Hartlap}}$ for $N \leq 81$ and $f_{t\text{-dist}} < f_{\text{Hartlap}}$ for $N > 81$. For a p=428 dimionsional likelihood (B_0 likelihood for $k_{\max} = 0.5$), $f_{t\text{-dist}} > f_{\text{Hartlap}}$ for $N \leq 697$ and $f_{t\text{-dist}} < f_{\text{Hartlap}}$ for $N > 697$. As the number of mocks increases, both $f_{t\text{-dist}}$ and f_{Hartlap} converge to 1.

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