

Suppressed Variance Simulations for the Bispectrum

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ABSTRACT

Keywords: cosmology: —

1. INTRODUCTION

Pontzen et al. (2016) Angulo & Pontzen (2016) ?

$$\delta(\mathbf{x}) = \frac{\rho(\mathbf{x}) - \bar{\rho}}{\bar{\rho}} \quad (1)$$

$$\delta(\mathbf{k}) = \frac{1}{(2\pi)^3} \int d^3\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} \delta(\mathbf{x}) = A e^{i\theta} \quad (2)$$

For Gaussian random field, θ is uniformly sampled from 0 to 2π and A is sampled from Rayleigh distribution:

$$p(A)dA = \frac{A}{\sigma^2} e^{-A^2/2\sigma^2} dA \quad (3)$$

where $\sigma^2 = V P(k)/(16\pi^3)$. The mean of this distribution is

$$\langle A \rangle = \int_0^\infty \frac{A^2}{\sigma^2} e^{-A^2/2\sigma^2} dA = \sqrt{\frac{V P(k)}{32\pi^2}}. \quad (4)$$

Also,

$$\langle \delta(\mathbf{k}) \delta^*(\mathbf{k}) \rangle = \langle A^2 \rangle = \int_0^\infty \frac{A^3}{\sigma^2} e^{-A^2/2\sigma^2} dA = \frac{V P(k)}{(2\pi)^3}. \quad (5)$$

A paired Gaussian field is where you have two fields δ_1 and δ_2 where $\delta_2(k) = A e^{i(\theta+\pi)} = -\delta_1(k)$.

A fixed field is when the amplitude is fixed,

$$A = \sqrt{\frac{V P(k)}{(2\pi)^3}}, \quad (6)$$

such that the power spectrum is the same.

Paired fixed is when you do both.

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Table 1. The QUIJOTE suite includes 15,000 standard N -body simulations at the fiducial cosmology to accurately estimate the covariance matrices. It also includes sets of 500 simulations at 13 different cosmologies, where only one parameter is varied from the fiducial value (underlined), to estimate derivatives of observables along the cosmological parameters. At every cosmology, the QUIJOTE suite also includes 250 pairs of paired-fixed simulations.

Name	M_ν	Ω_m	Ω_b	h	n_s	σ_8	ICs	standard realizations	paired-fixed pairs
Fiducial	0.0	0.3175	0.049	0.6711	0.9624	0.834	2LPT	15,000	250
Fiducial ZA	0.0	0.3175	0.049	0.6711	0.9624	0.834	Zel’dovich	500	250
M_ν^+	<u>0.1</u> eV	0.3175	0.049	0.6711	0.9624	0.834	Zel’dovich	500	250
M_ν^{++}	<u>0.2</u> eV	0.3175	0.049	0.6711	0.9624	0.834	Zel’dovich	500	250
M_ν^{+++}	<u>0.4</u> eV	0.3175	0.049	0.6711	0.9624	0.834	Zel’dovich	500	250
Ω_m^+	0.0	<u>0.3275</u>	0.049	0.6711	0.9624	0.834	2LPT	500	250
Ω_m^-	0.0	<u>0.3075</u>	0.049	0.6711	0.9624	0.834	2LPT	500	250
Ω_b^+	0.0	0.3175	<u>0.051</u>	0.6711	0.9624	0.834	2LPT	500	250
Ω_b^-	0.0	0.3175	<u>0.047</u>	0.6711	0.9624	0.834	2LPT	500	250
h^+	0.0	0.3175	0.049	<u>0.6911</u>	0.9624	0.834	2LPT	500	250
h^-	0.0	0.3175	0.049	<u>0.6511</u>	0.9624	0.834	2LPT	500	250
n_s^+	0.0	0.3175	0.049	0.6711	<u>0.9824</u>	0.834	2LPT	500	250
n_s^-	0.0	0.3175	0.049	0.6711	<u>0.9424</u>	0.834	2LPT	500	250
σ_8^+	0.0	0.3175	0.049	0.6711	0.9624	<u>0.849</u>	2LPT	500	250
σ_8^-	0.0	0.3175	0.049	0.6711	0.9624	<u>0.819</u>	2LPT	500	250

2. THE QUIJOTE SIMULATION SUITE

We use a subset of simulations from the QUIJOTE suite, a set of 43,000 N -body simulations that spans over 7000 cosmological models and contains, at a single redshift, over 8.5 trillion particles. The QUIJOTE suite was designed to quantify the information content of cosmological observables and also to train machine learning algorithms. Hence, the suite includes enough realizations to accurately estimate the covariance matrices of high-dimensional observables such as the bispectrum as well as the derivatives of these observables with respect to cosmological parameters. For the derivatives, the suite includes sets of simulations run at different cosmologies where only one parameter is varied from the fiducial cosmology ($\Omega_m=0.3175$, $\Omega_b=0.049$, $h=0.6711$, $n_s=0.9624$, $\sigma_8=0.834$, and $M_\nu=0.0$ eV). Along Ω_m , Ω_b , h , n_s , and σ_8 , the fiducial cosmology is adjusted by either a small step above or below the fiducial value: $\{\Omega_m^+, \Omega_m^-, \Omega_b^+, \Omega_b^-, h^+, h^-, n_s^+, n_s^-, \sigma_8^+, \sigma_8^-\}$. Along M_ν , because $M_\nu \geq 0.0$ eV and the derivative of certain observable with respect to M_ν is noisy, QUIJOTE includes sets of simulations for $\{M_\nu^+, M_\nu^{++}, M_\nu^{+++}\} = \{0.1, 0.2, 0.4 \text{ eV}\}$. At each of these 14 cosmologies, QUIJOTE includes sets of standard N -body and suppressed variance simulations.

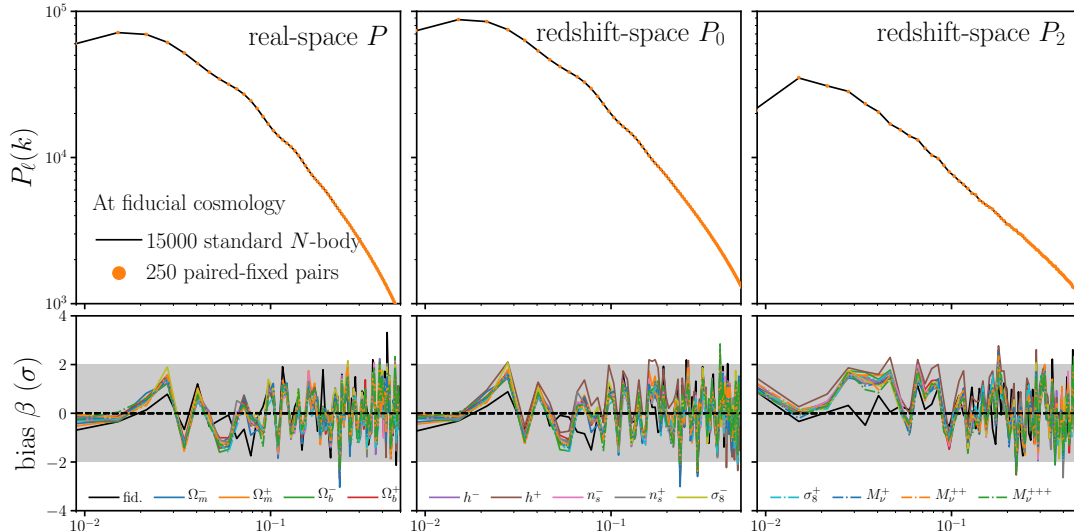


Figure 1. Comparison of the power spectra of standard N -body simulations and paired-fixed simulations. We compare the real-space power spectrum, redshift-space power spectrum monopole, and quadrupole in the left, center, and right columns, respectively. In the top panels, we compare the average power spectra of the standard simulations (black line) to the paired-fixed simulations (orange scatter) at the fiducial cosmology. In the bottom panels, we compare the bias, β of the paired-fixed simulations for the power spectra for all 14 cosmologies (Table 1). We mark bias within $\pm 2\sigma$ within the shaded region. At the fiducial cosmology we use 15,000 standard N -body and 250 pairs of paired-fixed simulations. For the other cosmologies, we use 500 standard N -body and 250 pairs of paired-fixed simulations. Consistent with previous results, *we find no significant bias in the real and redshift-space power spectra of paired-fixed simulations.*

The initial conditions for all the simulations were generated at $z = 127$ using second-order perturbation theory for simulations with massless neutrinos ($M_\nu = 0.0$ eV) and the Zel’dovich approximation for massive neutrinos ($M_\nu > 0.0$ eV). The initial conditions with massive neutrinos take their scale-dependent growth factors/rates into account using the Zennaro et al. (2017) method, while for the massless neutrino case we use the traditional scale-independent rescaling.

From the initial conditions, the simulations follow the gravitational evolution of 512^3 dark matter particles, and 512^3 neutrino particles for massive neutrino models, to $z = 0$ using GADGET-III TreePM+SPH code (Springel 2005). Simulations with massive neutrinos are run using the “particle method”, where neutrinos are described as a collisionless and pressureless fluid and therefore modeled as particles, same as CDM (Brandbyge et al. 2008; Viel et al. 2010). Halos are then identified using the Friends-of-Friends algorithm (FoF; Davis et al. 1985) with linking length $b = 0.2$ on the CDM + baryon distribution. We limit the halo catalogs to halos with masses above $M_{\text{lim}} = 3.2 \times 10^{13} h^{-1} M_\odot$. For the fiducial cosmology, the halo catalogs have $\sim 156,000$ halos ($\bar{n} \sim 1.56 \times 10^{-4} h^3 \text{Gpc}^{-3}$) with $\bar{n}P_0(k = 0.1) \sim 3.23$. We refer readers to Villaescusa-Navarro et al. (in preparation) and Hahn et al. (2019) for further details on the QUIJOTE simulations.

3. RESULTS

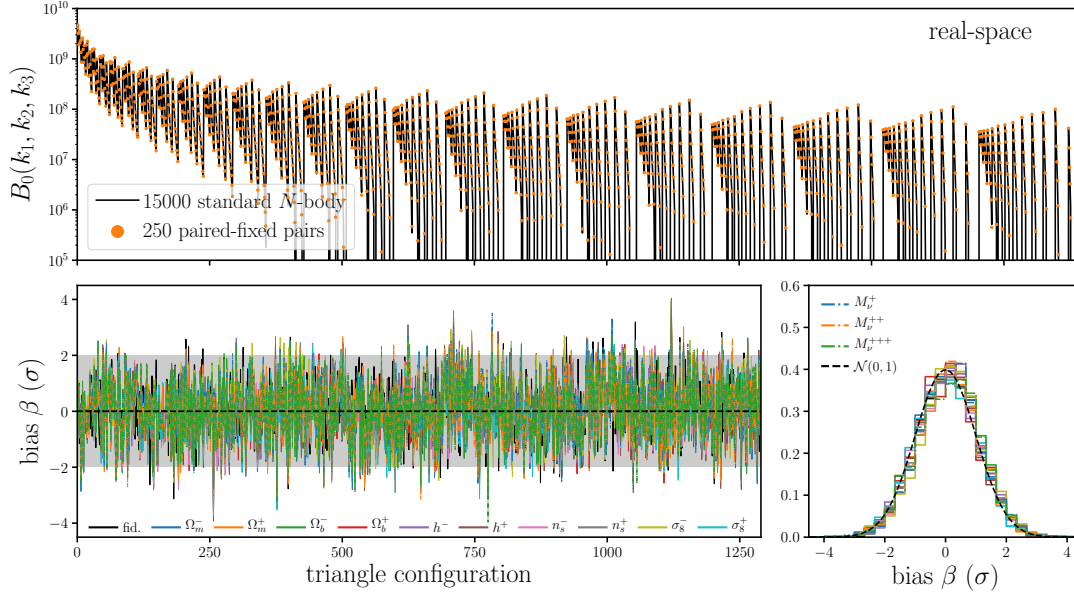


Figure 2. Comparison of the real-space bispectrum of standard N -body simulations and paired-fixed simulations. In the top panel, we compare the average bispectrum of the standard simulations (black line) to the paired-fixed simulations (orange scatter) at the fiducial cosmology. We include all 1898 triangles out to $k_{\max} = 0.5$. In the bottom panels, we compare the bias, β of the paired-fixed simulations for the bispectrum for all 14 cosmologies, as a function of triangle configuration (left) and their distribution (right). The β distributions are in good agreement with a Gaussian distribution, $\mathcal{N}(0, 1)$ (blackdashed). Hence, we find no significant bias in the real-space bispectrum of paired-fixed simulations.

$$X_{\text{pf},i} = \frac{1}{2} [X_{\text{pf},i}^{(1)} + X_{\text{pf},i}^{(2)}] \quad (7)$$

The main metric we use to compare some observable X of standard N -body simulations to the paired-fixed simulations is bias, which we estimate as follows:

$$\beta_X = \frac{\overline{X}_{\text{std}} - \overline{X}_{\text{pf}}}{\sqrt{\sigma_{\text{std}}^2 + \sigma_{\text{pf}}^2}} \quad (8)$$

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APPENDIX

REFERENCES

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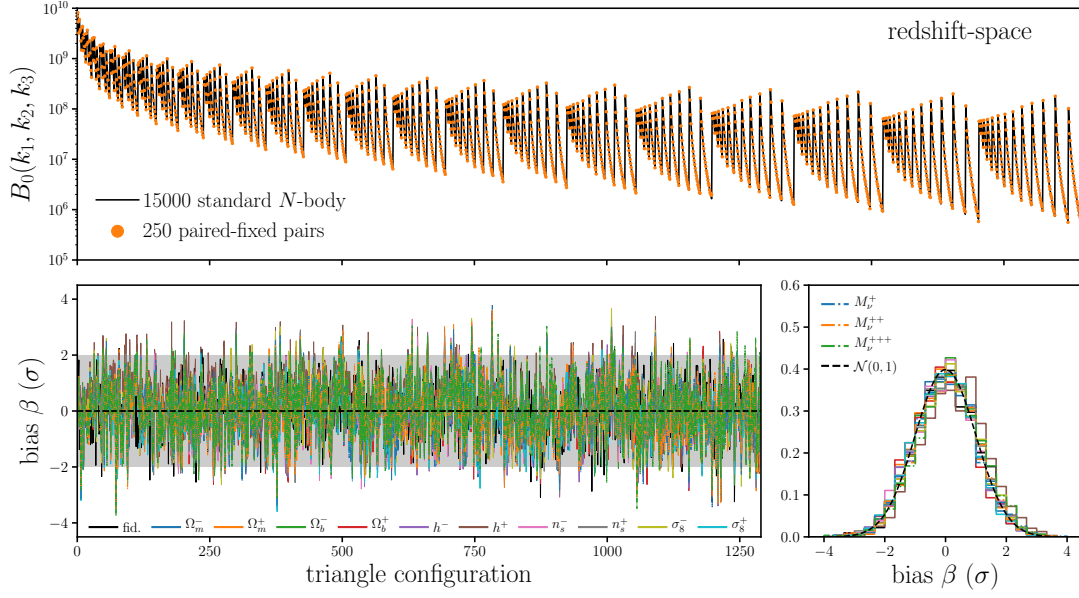


Figure 3. Same as Figure 2 but for the redshift-space bispectrum. The β distributions of the different cosmologies are mostly in good agreement with $\mathcal{N}(0, 1)$ (lower right panel). A few cosmologies, h^+ , M_ν^{+++} , have noticeable discrepancies; however, these discrepancies are within 1σ . Hence, we also find no significant bias in the redshift-space bispectrum of paired-fixed simulations.

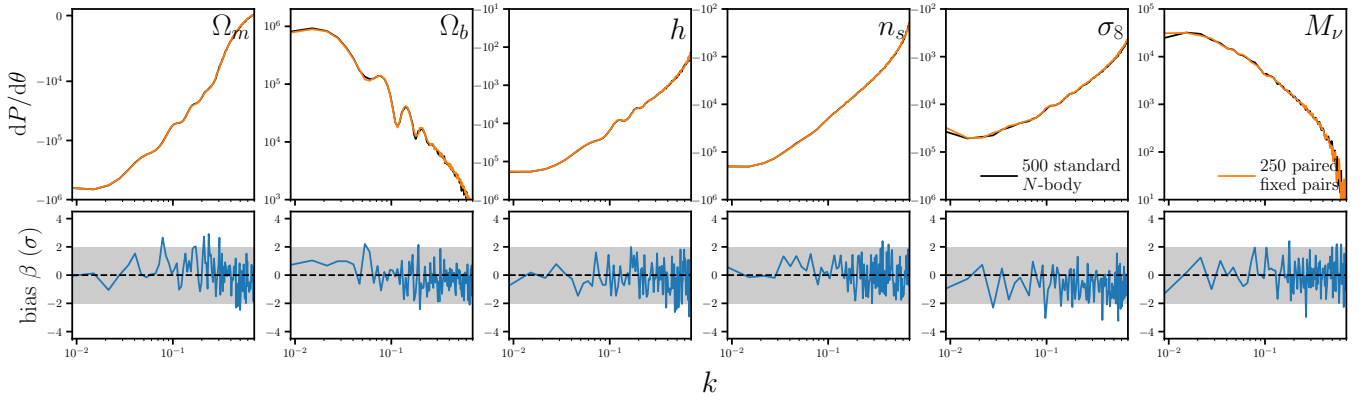


Figure 4. Comparison of the real-space power spectrum derivatives with respect to Ω_m , Ω_b , h , n_s , σ_8 , and M_ν derived from standard N -body simulations versus paired-fixed simulations. In the top panels, we compare the derivatives from the standard simulations (black) to the paired-fixed simulations (orange). In the bottom panels, we compare the bias, β of each power spectrum derivative. The derivatives with respect to Ω_m and σ_8 have noticeably non-zero biases; however these deviations are within 1σ . Therefore, we find no significant bias in the real-space power spectrum derivatives calculated from paired-fixed simulations.

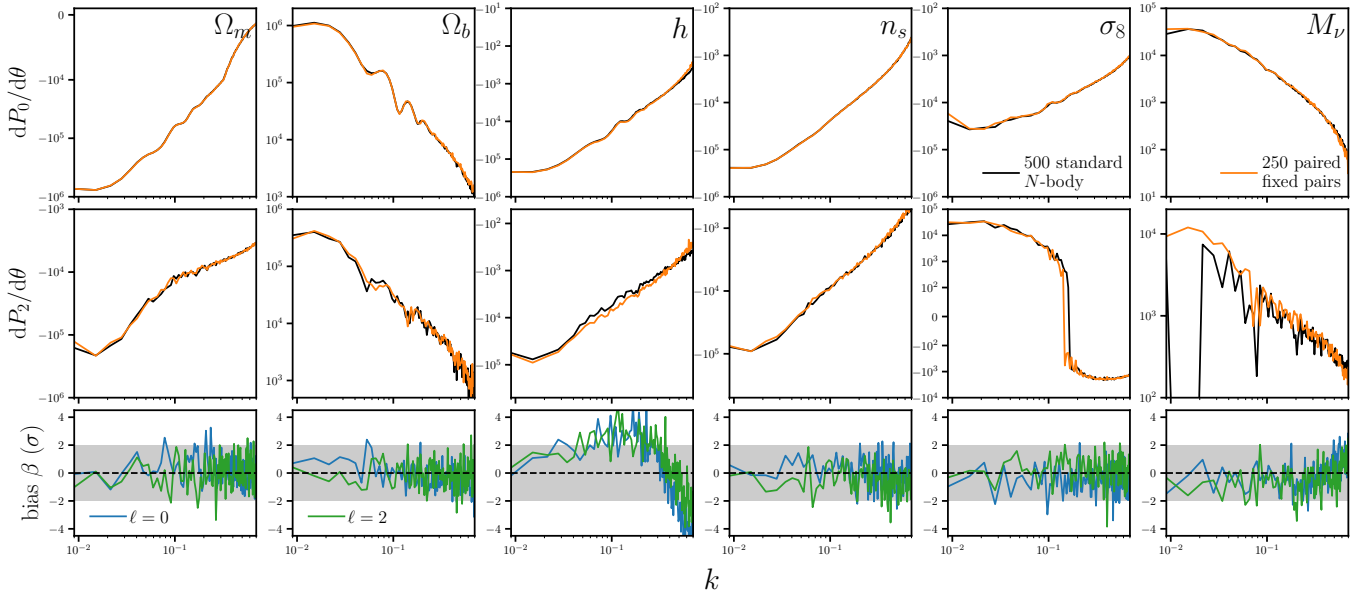


Figure 5. Same as Figure 4 but for the derivatives of the power spectrum monopole (top panels) and quadrupole (center panels) with respect to Ω_m , Ω_b , h , n_s , σ_8 , and M_ν . The bottom panel reveals significantly larger biases for $dP_0/d\theta$ and $dP_2/d\theta$ than the real-space $dP/d\theta$, especially for h , σ_8 and M_ν . In fact, the bias for both dP_0/dh and dP_2/dh exceed 2σ in the range $k > 0.04 h/\text{Mpc}$. **CH:** some nuanced discussion about how this still is not a cause for concern because this is the bias for a 500 Gpc volume so it's well within the tolerance.

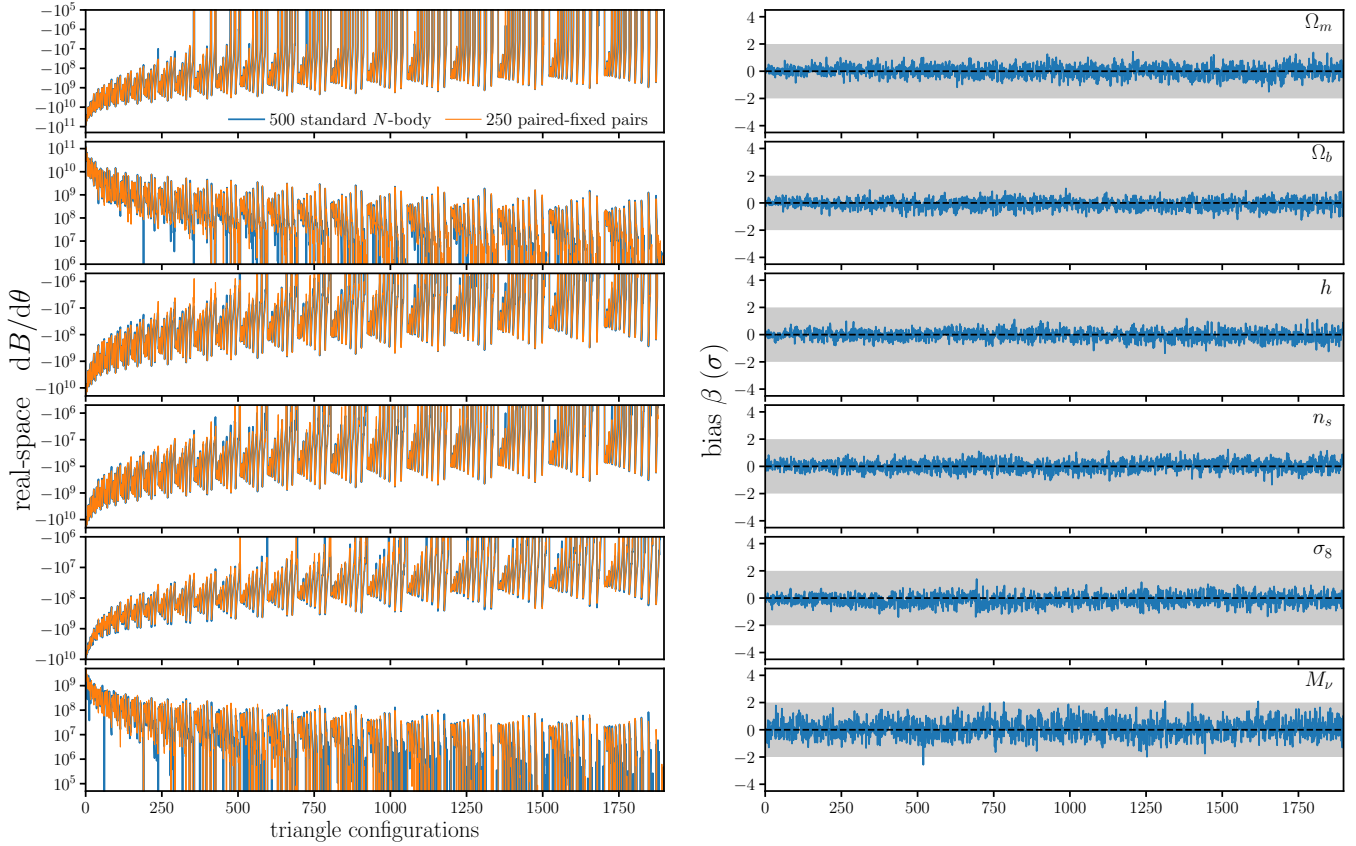


Figure 6. Comparison of the real-space bispectrum derivatives with respect to Ω_m , Ω_b , h , n_s , σ_8 , and M_ν derived from standard N -body simulations versus paired-fixed simulations. In the left panels, we compare the derivatives from the standard simulations (black) to the paired-fixed simulations (orange). In the right panels, we compare the bias, β of the derivatives for each parameter. *We find no significant bias in the real-space bispectrum derivatives calculated from paired-fixed simulations.*

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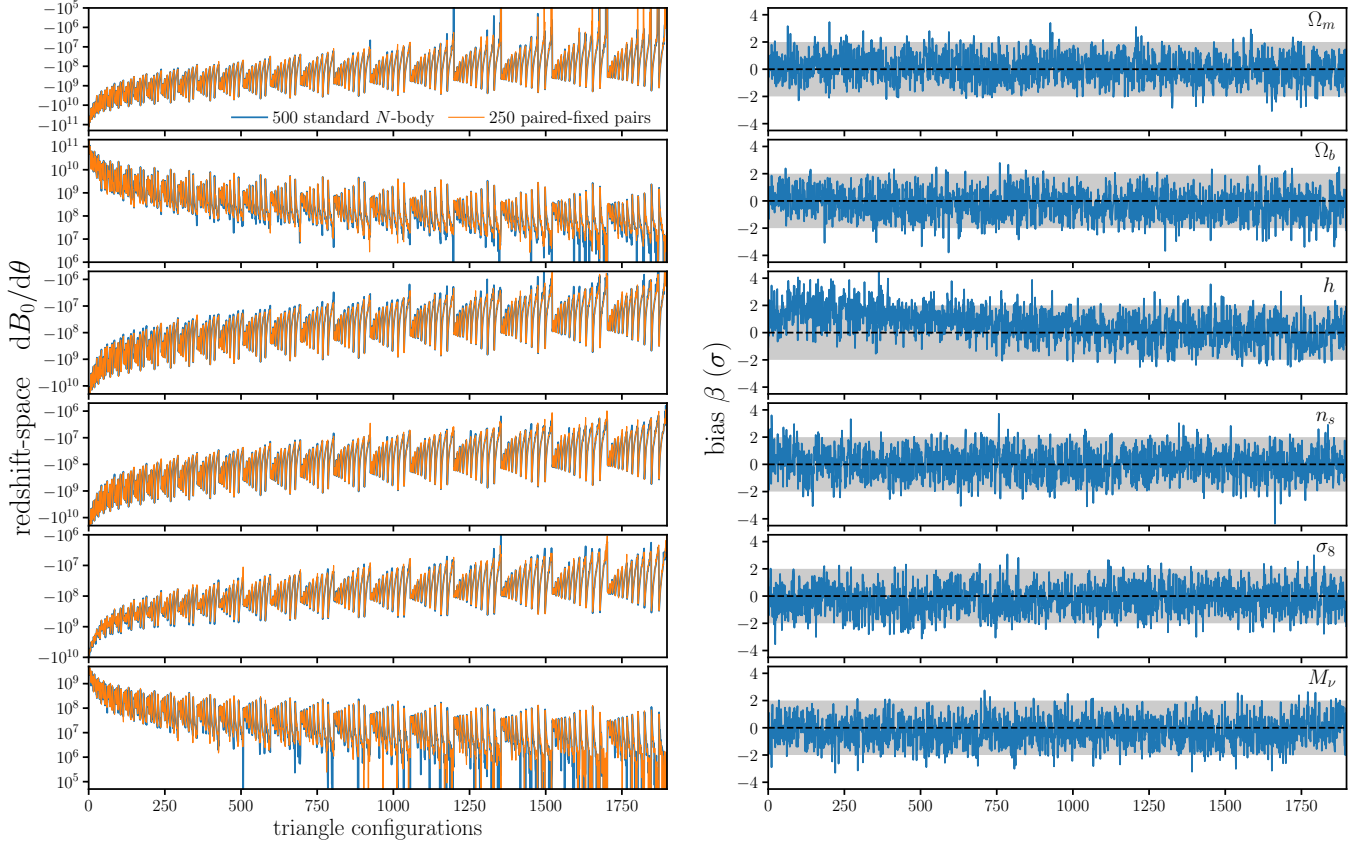


Figure 7. Same as Figure 6 but for the derivatives of the redshift-space bispectrum monopole with respect to the cosmological parameters. Similar to the power spectrum derivatives, we find significantly larger biases for redshift-space derivatives than in real-space. The bias for dB_0/dh exceeds 2σ for triangle configurations with $0.1 \lesssim k_1, k_2, k_3 \lesssim 0.3 h/\text{Mpc}$. **CH:** again some nuanced discussion about how this still is not a cause for concern because this is the bias for a 500 Gpc volume so it's well within the tolerance.

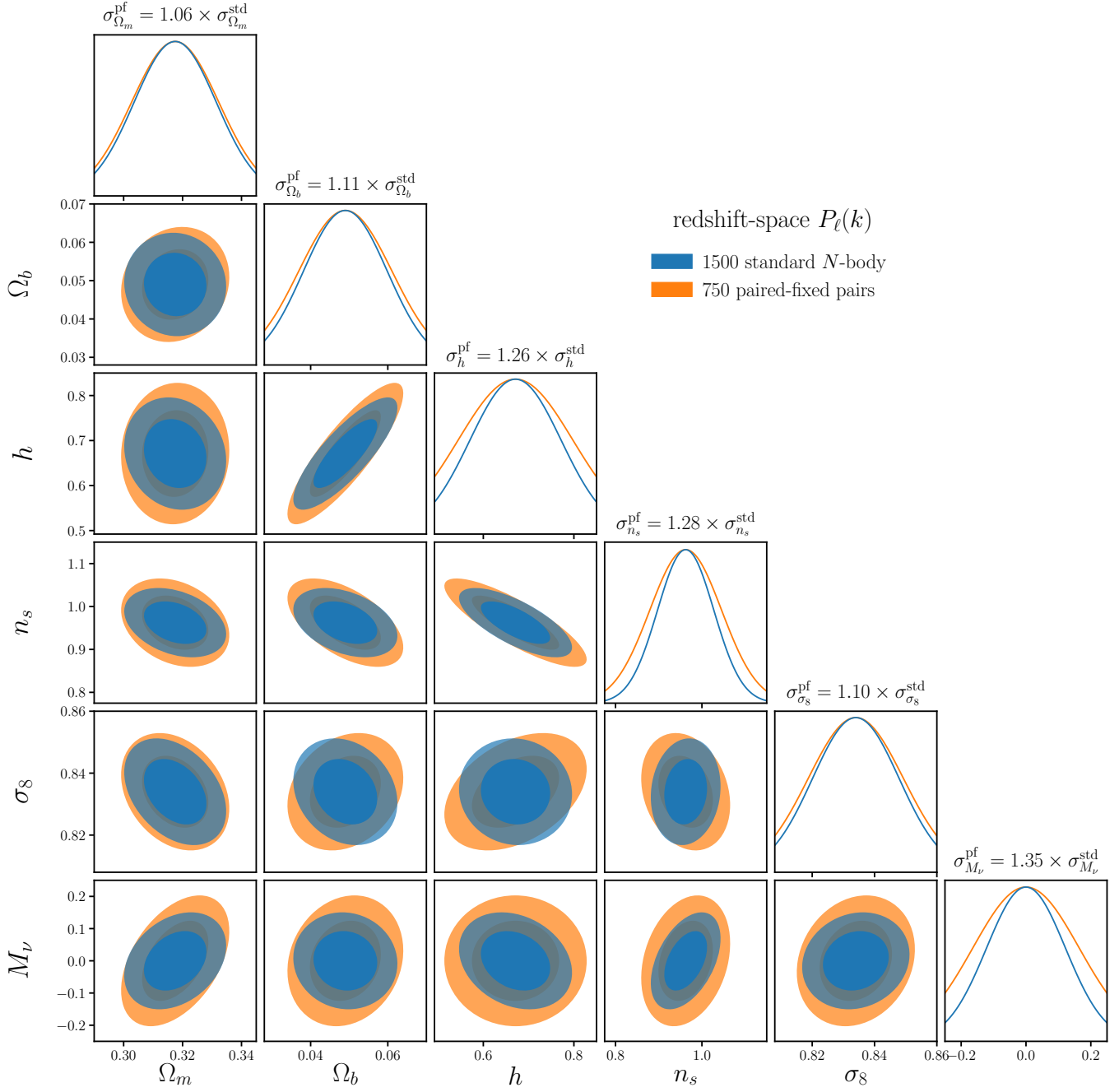


Figure 8. CH: to be updated

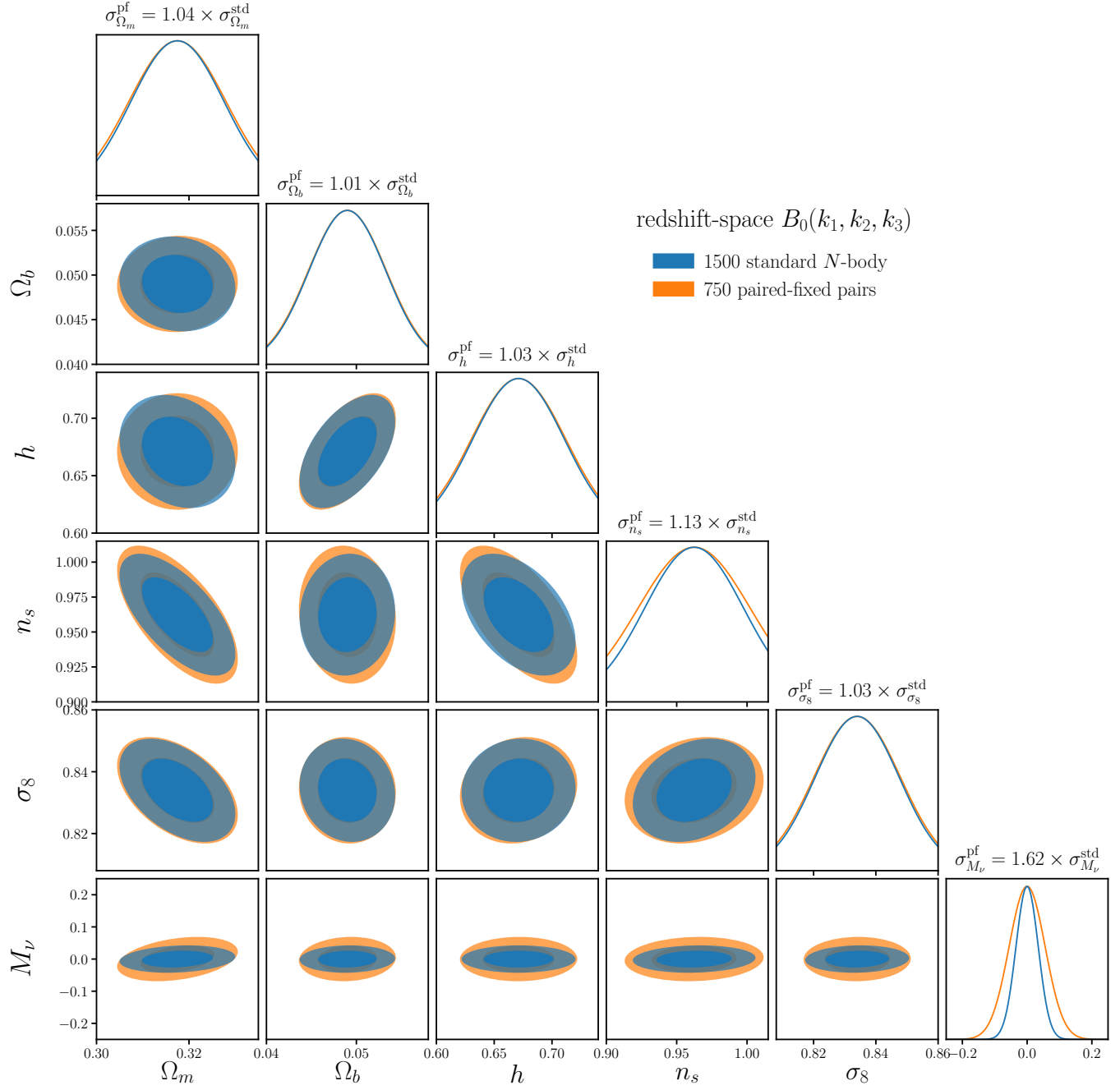


Figure 9. CH: to be updated