

# Constraining $\sum m_\nu$ with the Bispectrum I: Breaking Parameter Degeneracies

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## ABSTRACT

Some intro sentences about neutrinos. We investigate whether the bispectrum, blah  
We present the first  $N$ -body simulation based Fisher matrix forecast of cosmological  
parameters () using the bispectrum.

*Keywords:* cosmology: —

## 1. INTRODUCTION

very brief intro on neutrinos

Brief intro on the impact of massive active neutrinos on the matter powerspectrum and how that's  
detectable with CMB and LSS

Quick summary of current constraints and where they come from. Talk about the CMB-LSS lever  
arm. The degeneracy between  $A_s$  and  $\tau$  and how that's a bottleneck short thing about how  $\tau$  is  
hard to constrain.

Fortunately the imprint of neutrinos on the matter distribution leaves imprints on clustering. So  
with clustering measurements alone we can derive constraints on  $\sum m_\nu$  and at the very least tighten  
constraints.

Brief summary of previous works that look at the powerspectrum. Then Discuss the shortcomings  
of the powerspectrum only analysis– Not good enough.

However, we don't have to settle for just two point statistics, three-point statistics such as the  
bispectrum and 3PCF...

In Section blah

**CH:** List plans for paper 2

## 2. HADES AND QUIJOTE SIMULATION SUITES

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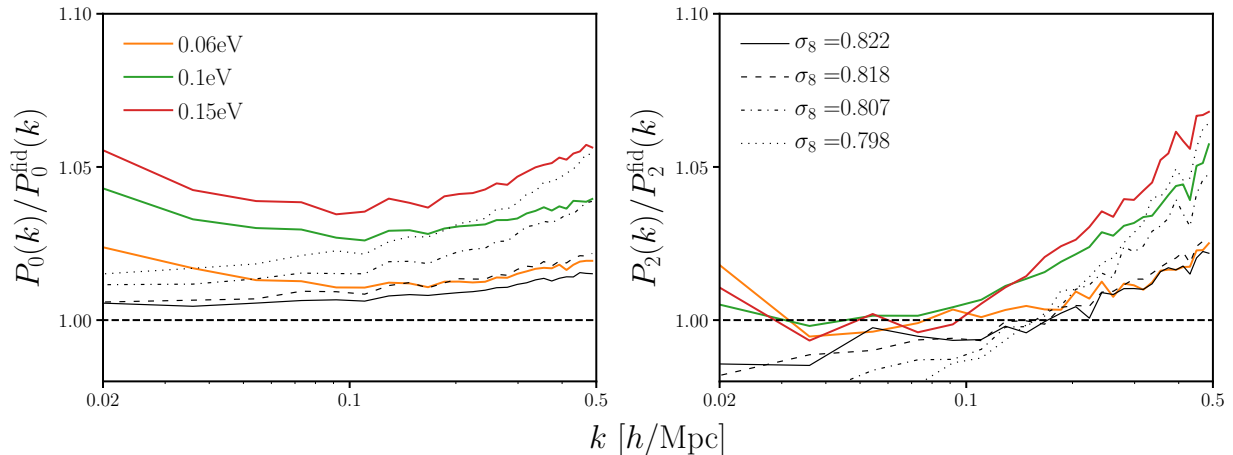
We use a subset of the HADES<sup>1</sup> and Quijote simulation suites. Below, we briefly describe these simulations; a brief summary of the simulations can be found in Table 1. The HADES simulations start from Zel’dovich approximated initial conditions generated at  $z = 99$  using the Zennaro et al. (2017) rescaling method and follow the gravitational evolution of  $N_{\text{cdm}} = 512^3$  CDM, plus  $N_\nu = 512^3$  neutrino particles (for massive neutrino models), to  $z = 0$ . They are run using the GADGET-III TreePM+SPH code (Springel 2005) in a periodic  $(1h^{-1}\text{Gpc})^3$  box. All of the HADES simulations share the following cosmological parameter values, which are in good agreement with Planck constraints Ade et al. (2016):  $\Omega_{\text{m}}=0.3175$ ,  $\Omega_{\text{b}}=0.049$ ,  $\Omega_{\Lambda}=0.6825$ ,  $n_s=0.9624$ ,  $h=0.6711$ , and  $k_{\text{pivot}} = 0.05 h\text{Mpc}^{-1}$ .

The HADES suite includes models with degenerate massive neutrinos of different masses:  $\sum m_\nu = 0.06, 0.10$ , and  $0.15$  eV. These massive neutrino models are run using the “particle method”, where neutrinos are described as a collisionless and pressureless fluid and therefore modeled as particles, same as CDM (Brandbyge et al. 2008; Viel et al. 2010). HADES also includes models with massless neutrino and different values of  $\sigma_8$  to examine the  $\sum m_\nu - \sigma_8$  degeneracy. The  $\sigma_8$  values were chosen to match either  $\sigma_8^m$  or  $\sigma_8^c - \sigma_8$  computed with respect to total matter (CDM + baryons +  $\nu$ ) or CDM + baryons — of the massive neutrino models:  $\sigma_8 = 0.822, 0.818, 0.807$ , and  $0.798$ . Each model has 100 independent realizations and we focus on the snapshots saved at  $z = 0$ . Halos closely trace the CDM+baryon field rather than the total matter field and neutrinos have negligible contribution to halo masses (*e.g.* Ichiki & Takada 2012; Castorina et al. 2014; LoVerde 2014; Villaescusa-Navarro et al. 2014). Hence, dark matter halos are identified in each realization using the Friends-of-Friends algorithm (FoF; Davis et al. 1985) with linking length  $b = 0.2$  on the CDM + baryon distribution; only halos with masses  $> 3.2 \times 10^{13} h^{-1} M_\odot$  are included. For further details on the HADES simulations, we refer readers to Villaescusa-Navarro et al. (2018).

In addition to HADES, we use simulations from the Quijote simulation suite, a set of 23,000  $N$ -body simulations that in total contain more than 3.3 trillion ( $3.3 \times 10^{12}$ ) particles over a volume of  $23000(h^{-1}\text{Gpc})^3$ . These simulations were constructed to quantify the information content of different cosmological observables using Fisher matrix forecasting (*e.g.* Section 4.2). They are therefore designed to accurately calculate the covariance matrices of observables and the derivatives of observables with respect to cosmological parameters. The suite considers 6 cosmological parameters:  $\Omega_{\text{m}}$ ,  $\Omega_{\text{b}}$ ,  $h$ ,  $n_s$ ,  $\sigma_8$ , and  $\sum m_\nu$ .

To calculate covariance matrices, Quijote includes 15,000  $N$ -body simulations run at a fiducial cosmology ( $\Omega_{\text{m}}=0.3175$ ,  $\Omega_{\text{b}}=0.049$ ,  $h=0.6711$ ,  $n_s=0.9624$ ,  $\sigma_8=0.834$ , and  $\sum m_\nu=0.0$  eV). It also includes sets of 500  $N$ -body simulations run at different cosmologies where only one parameter is varied from the fiducial cosmology at a time for the derivatives. Along  $\Omega_{\text{m}}$ ,  $\Omega_{\text{b}}$ ,  $h$ ,  $n_s$ , and  $\sigma_8$ , the fiducial cosmology is adjusted by either a small step above and below the fiducial value. Along  $\sum m_\nu$ , because the derivative of certain observable with respect to  $\sum m_\nu$  is noisy, Quijote includes sets of 500 simulations for  $\sum m_\nu = 0.1, 0.2$ , and  $0.4$  eV. In Table 1, we list the cosmologies included in the Quijote suite.

<sup>1</sup> <https://franciscovillaescusa.github.io/hades.html>



**Figure 1.** Impact of  $\sum m_\nu$  and  $\sigma_8$  on the redshift-space halo power spectrum monopole and quadrupole measured using the HADES simulation suite.  $\sum m_\nu$  and  $\sigma_8$  produce almost identical effects on halo clustering on small scales ( $k > 0.1 h/\text{Mpc}$ ). This degeneracy can be partially broken through the quadrupole; however,  $\sum m_\nu$  and  $\sigma_8$  produce almost the same effect on two-point clustering — within a few percent.

The initial conditions for all Quijote simulations were generated at  $z = 127$  using 2LPT for simulations with massless neutrinos and the Zel’dovich approximation for massive neutrinos. Like HADES, the initial conditions of simulations with massive neutrinos take their scale-dependent growth factors/rates into account using the Zennaro et al. (2017) method. From the initial conditions, all of the simulations follow the gravitational evolution of  $512^3$  dark matter particles, and  $512^3$  neutrino particles (for massive neutrino models), to  $z = 0$  using GADGET-III TreePM+SPH code (same as HADES). The simulations run at the fiducial cosmology for covariance matrix estimation are standard  $N$ -body simulations. However, the rest are paired fixed simulations, which greatly reduce cosmic variance without introducing bias for a large set of statistics (Angulo & Pontzen 2016; Pontzen et al. 2016; Villaescusa-Navarro et al. 2018). We confirm that the paired fixed simulations do not introduce any bias for the redshift-space halo bispectrum (the observable we consider in this paper). For further details on the Quijote simulations, we refer readers to Villaescusa-Navarro et al. (in preparation).

### 3. BISPECTRUM

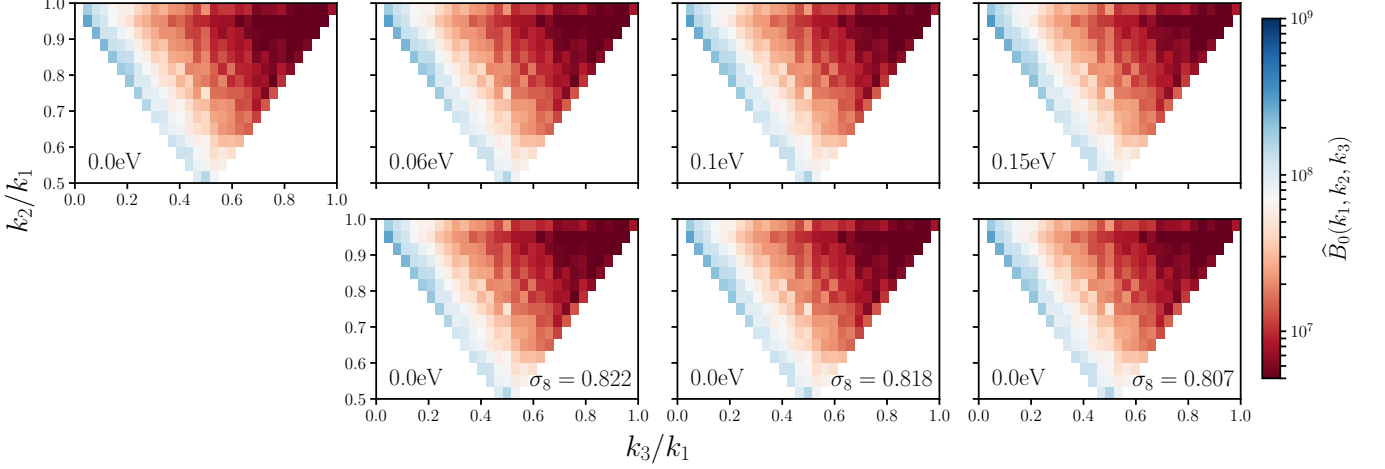
We’re interested in breaking parameter degeneracies that limit the constraining power on  $\sum m_\nu$  of two-point clustering analyses using three-point clustering statistics — *i.e.* the bispectrum. In this section, we describe the bispectrum estimator used throughout the paper. We focus on the bispectrum monopole ( $\ell = 0$ ) and use an estimator that exploits Fast Fourier Transforms (FFTs). Our estimator is similar to the estimators described in Scoccimarro (2015); Sefusatti et al. (2016); we also follow their formalism in our description below. Although Sefusatti et al. (2016) and Scoccimarro (2015) respectively describe estimators in redshift- and real-space, since we focus on the bispectrum monopole, we note that there is no difference.

To measure the bispectrum of our halo catalogs, we begin by interpolating the halo positions to a grid,  $\delta(\mathbf{x})$  and Fourier transforming the grid to get  $\delta(\mathbf{k})$ . We use a fourth-order interpolation

**Table 1.** Specifications of the HADES and Quijote simulation suites.

Name	$\sum m_\nu$ (eV)	$\Omega_m$	$\Omega_b$	$h$	$n_s$	$\sigma_8^m$	$\sigma_8^c$	$m_{\text{cdm}}$ ( $10^{10}h^{-1}M_\odot$ )	$m_\nu$ ( $10^{10}h^{-1}M_\odot$ )	realizations
HADES suite										
Fiducial	0.0	0.3175	0.049	0.6711	0.9624	0.833	0.833	65.66	0	100
	0.06	0.3175	0.049	0.6711	0.9624	0.819	0.822	65.36	29.57	100
	0.10	0.3175	0.049	0.6711	0.9624	0.809	0.815	65.16	49.28	100
	0.15	0.3175	0.049	0.6711	0.9624	0.798	0.806	64.92	73.95	100
	0.0	0.3175	0.049	0.6711	0.9624	0.822	0.822	65.66	0	100
	0.0	0.3175	0.049	0.6711	0.9624	0.818	0.818	65.66	0	100
	0.0	0.3175	0.049	0.6711	0.9624	0.807	0.807	65.66	0	100
	0.0	0.3175	0.049	0.6711	0.9624	0.798	0.798	65.66	0	100
Quijote suite										
Fiducial	0.0	0.3175	0.049	0.6711	0.9624	0.834	0.834			15,000
$\sum m_\nu^+$	<u>0.1</u>	0.3175	0.049	0.6711	0.9624	0.834	0.834			500
$\sum m_\nu^{++}$	<u>0.2</u>	0.3175	0.049	0.6711	0.9624	0.834	0.834			500
$\sum m_\nu^{+++}$	<u>0.4</u>	0.3175	0.049	0.6711	0.9624	0.834	0.834			500
$\Omega_m^+$	0.0	<u>0.3275</u>	0.049	0.6711	0.9624	0.834	0.834			500
$\Omega_m^-$	0.0	<u>0.3075</u>	0.049	0.6711	0.9624	0.834	0.834			500
$\Omega_b^+$	0.0	0.3175	<u>0.050</u>	0.6711	0.9624	0.834	0.834			500
$\Omega_b^-$	0.0	0.3175	<u>0.048</u>	0.6711	0.9624	0.834	0.834			500
$h^+$	0.0	0.3175	0.049	<u>0.6911</u>	0.9624	0.834	0.834			500
$h^-$	0.0	0.3175	0.049	<u>0.6511</u>	0.9624	0.834	0.834			500
$n_s^+$	0.0	0.3175	0.049	0.6711	<u>0.9824</u>	0.834	0.834			500
$n_s^-$	0.0	0.3175	0.049	0.6711	<u>0.9424</u>	0.834	0.834			500
$\sigma_8^+$	0.0	0.3175	0.049	0.6711	0.9624	<u>0.849</u>	<u>0.849</u>			500
$\sigma_8^-$	0.0	0.3175	0.049	0.6711	0.9624	<u>0.819</u>	<u>0.819</u>			500

**Top:** The HADES suite includes sets of 100  $N$ -body simulations with degenerate massive neutrinos of  $\sum m_\nu = 0.06, 0.10$ , and  $0.15$  eV as well as sets of simulations with massless neutrino and  $\sigma_8 = 0.822, 0.818, 0.807$ , and  $0.798$  to examine the  $\sum m_\nu - \sigma_8$  degeneracy. **Bottom:** The Quijote suite includes 15,000  $N$ -body simulations at the fiducial cosmology to accurately estimate the covariance matrices. It also includes sets of 500 paired fixed simulations at different cosmologies, where only one parameter is varied from the fiducial value (underlined), to estimate derivatives of observables along the cosmological parameters.



**Figure 2.** The redshift-space halo bispectrum,  $\hat{B}_0(k_1, k_2, k_3)$ , as a function of triangle configuration shape for  $\sum m_\nu = 0.0, 0.06, 0.10$ , and  $0.15$  eV (upper panels) and  $\sigma_8 = 0.822, 0.818$ , and  $0.807$  (lower panels). The HADES simulations of the top and bottom panels in the three right-most columns, have matching  $\sigma_8$  values (Section 2). We describe the triangle configuration shape by the ratio of the triangle sides:  $k_3/k_1$  and  $k_2/k_1$ . The upper left bin contains squeezed triangles ( $k_1 = k_2 \gg k_3$ ); the upper right bin contains equilateral triangles ( $k_1 = k_2 = k_3$ ); and the bottom center bin contains folded triangles ( $k_1 = 2k_2 = 2k_3$ ). We include all triangle configurations with  $k_1, k_2, k_3 \leq k_{\text{max}} = 0.5$   $h/\text{Mpc}$ . and use the  $\hat{B}_0$  estimator in Section 3.

to interlaced grids, which has advantageous anti-aliasing properties (Hockney & Eastwood 1981; Sefusatti et al. 2016) that allow unbiased measurements up to the Nyquist frequency. Then using  $\delta(\mathbf{k})$ , we measure the bispectrum monopole as

$$\hat{B}_{\ell=0}(k_1, k_2, k_3) = \frac{1}{V_B} \int_{k_1} d^3 q_1 \int_{k_2} d^3 q_2 \int_{k_3} d^3 q_3 \delta_D(\mathbf{q}_{123}) \delta(\mathbf{q}_1) \delta(\mathbf{q}_2) \delta(\mathbf{q}_3) - B_{\ell=0}^{\text{SN}} \quad (1)$$

$\delta_D$  above is a Dirac delta function and hence  $\delta_D(\mathbf{q}_{123}) = \delta_D(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3)$  ensures that the  $\mathbf{q}_i$  triplet actually form a closed triangle. Each of the integrals above represent an integral over a spherical shell in  $k$ -space with radius  $\delta k$  centered at  $\mathbf{k}_i$  — *i.e.*

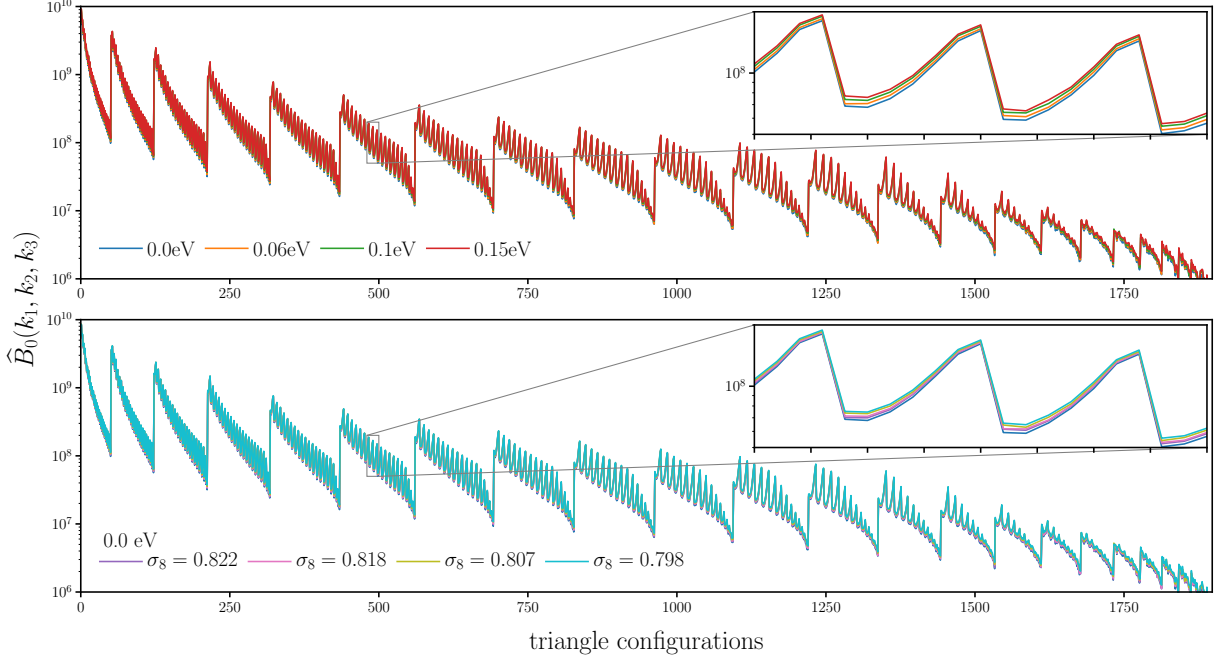
$$\int_{k_i} d^3 q \equiv \int_{k_i - \delta k/2}^{k_i + \delta k/2} dq q^2 \int d\Omega. \quad (2)$$

$V_B$  is a normalization factor proportional to the number of triplets  $\mathbf{q}_1$ ,  $\mathbf{q}_2$ , and  $\mathbf{q}_3$  that can be found in the triangle bin defined by  $k_1$ ,  $k_2$ , and  $k_3$  with width  $\delta k$ :

$$V_B = \int_{k_1} d^3 q_1 \int_{k_2} d^3 q_2 \int_{k_3} d^3 q_3 \delta_D(\mathbf{q}_{123}) \quad (3)$$

Lastly,  $B_{\ell=0}^{\text{SN}}$  is the correction for the Poisson shot noise, which contributes due to the self-correlation of individual objects:

$$B_{\ell=0}^{\text{SN}}(k_1, k_2, k_3) = \frac{1}{\bar{n}} (P_0(k_1) + P_0(k_2) + P_0(k_3)) + \frac{1}{\bar{n}^2}. \quad (4)$$



**Figure 3.** The redshift-space halo bispectrum,  $\hat{B}_0(k_1, k_2, k_3)$ , as a function of triangle configurations for  $\sum m_\nu = 0.0, 0.06, 0.10$ , and  $0.15$  eV (top panel) and  $\sum m_\nu = 0.0$  eV,  $\sigma_8 = 0.822, 0.818, 0.807$ , and  $0.798$  (lower panel). We include all possible triangle configurations with  $k_1, k_2, k_3 \leq k_{\max} = 0.5$   $h/\text{Mpc}$  where we order the configurations by looping through  $k_3$  in the inner most loop and  $k_1$  in the outer most loop satisfying  $k_1 \leq k_2 \leq k_3$ . In the insets of the panels we zoom into triangle configurations with  $k_1 = 0.113$ ,  $0.226 \leq k_2 \leq 0.283$ , and  $0.283 \leq k_3 \leq 0.377$   $h/\text{Mpc}$ .

$\bar{n}$  is the number density of objects (halos) and  $P_0$  is the powerspectrum monopole.

In order to evaluate the integrals in Eq. 1, we take advantage of the plane-wave representation of the Dirac delta function and rewrite the equation as

$$\hat{B}_{\ell=0}(k_1, k_2, k_3) = \frac{1}{V_B} \int \frac{d^3x}{(2\pi)^3} \int_{k_1} d^3q_1 \int_{k_2} d^3q_2 \int_{k_3} d^3q_3 \delta(\mathbf{q}_1) \delta(\mathbf{q}_2) \delta(\mathbf{q}_3) e^{i\mathbf{q}_{123} \cdot \mathbf{x}} - B_{\ell=0}^{\text{SN}} \quad (5)$$

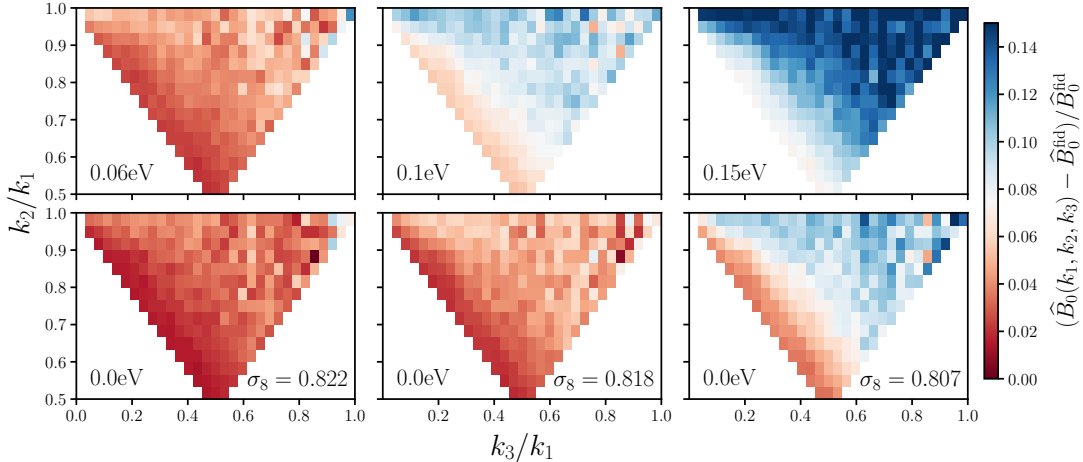
$$= \frac{1}{V_B} \int \frac{d^3x}{(2\pi)^3} \prod_{i=1}^3 I_{k_i}(\mathbf{x}) - B_{\ell=0}^{\text{SN}} \quad (6)$$

where

$$I_{k_i}(\mathbf{x}) = \int_k d^3q \delta(\mathbf{q}) e^{i\mathbf{q} \cdot \mathbf{x}}. \quad (7)$$

At this point, we measure  $\hat{B}_{\ell=0}(k_1, k_2, k_3)$  by calculating the  $I_{k_i}$ s with inverse FFTs and summing over in real space.<sup>2</sup> For  $\hat{B}_{\ell=0}$  measurements throughout the paper, we use  $\delta(\mathbf{x})$  grids with  $N_{\text{grid}} = 360$  and triangle configurations defined by  $k_1, k_2, k_3$  bins of width  $\Delta k = 3k_f = 0.01885$   $h/\text{Mpc}$ .

<sup>2</sup> The code that we use to evaluate  $\hat{B}_{\ell=0}$  is publicly available at <https://github.com/changhoonhahn/pySpectrum>



**Figure 4.** The shape dependence of the  $\sum m_\nu$  and  $\sigma_8$  imprint on the redshift-space halo bispectrum,  $\Delta\hat{B}_0/\hat{B}_0^{\text{fid}}$ . We align the  $\sum m_\nu = 0.06, 0.10$ , and  $0.15$  eV HADES models in the upper panels with  $\sum m_\nu = 0.0$  eV  $\sigma_8 = 0.822, 0.818$ , and  $0.807$  models on the bottom such that the top and bottom panels in each column have matching  $\sigma_8^{\mathcal{E}}$ , which produce mostly degenerate imprints on the redshift-space power spectrum. The difference between the top and bottom panels highlight that  $\sum m_\nu$  leaves a distinct imprint on elongated and isosceles triangles (bins along the bottom left and bottom right edges, respectively) from  $\sigma_8$ . *The imprint of  $\sum m_\nu$  has an overall distinct shape dependence on the bispectrum that cannot be replicated by varying  $\sigma_8$ .*

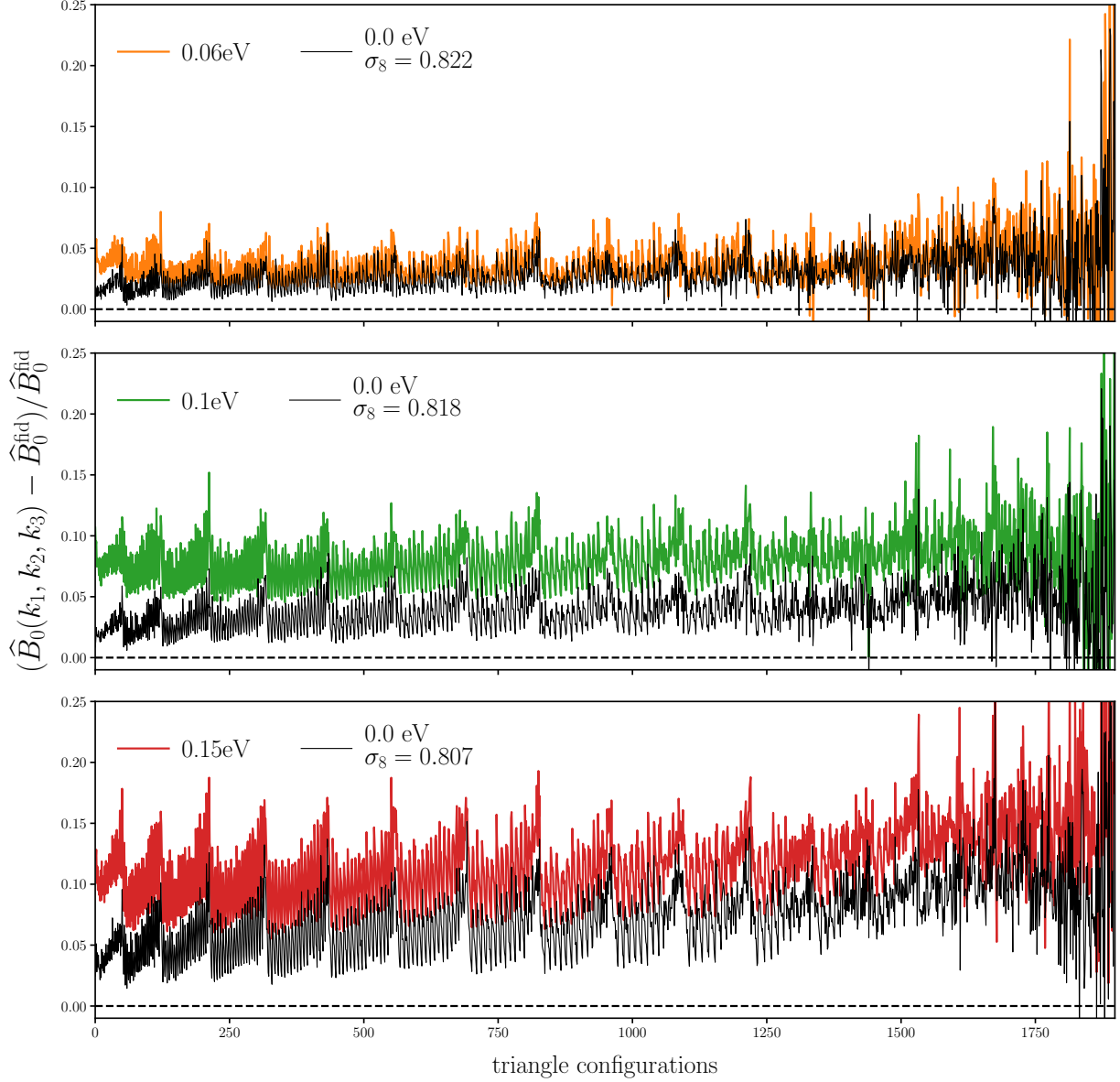
We present the redshift-space halo bispectrum of the HADES simulations measured using the estimator above in two ways: one that emphasizes the triangle shape dependence (Figure 2) and the other that emphasizes the amplitude (Figure 3). In Figure 2, we plot  $\hat{B}_0(k_1, k_2, k_3)$  as a function of  $k_2/k_1$  and  $k_3/k_1$ , which describe the triangle configuration shape. In each panel, the colormap in each  $(k_2/k_1, k_3/k_1)$  bin is the weighted average  $\hat{B}_0$  amplitude of all triangle configurations in the bin. The upper left bins contain squeezed triangles ( $k_1 = k_2 \gg k_3$ ); the upper right bins contain equilateral triangles ( $k_1 = k_2 = k_3$ ); and the bottom center bins contain folded triangles ( $k_1 = 2k_2 = 2k_3$ ). We include all possible triangle configurations with  $k_1, k_2, k_3 < k_{\text{max}} = 0.5$  h/Mpc. The  $\hat{B}_0$  in the upper panels are HADES models with  $\sum m_\nu = 0.0$  (fiducial),  $0.06, 0.10$ , and  $0.15$  eV;  $\hat{B}_0$  in the lower panels are HADES models with  $\sum m_\nu = 0.0$  eV and  $\sigma_8 = 0.822, 0.818$ , and  $0.807$ . The top and bottom panels of the three right-most columns have matching  $\sigma_8$  values (Section 2).

Next, in Figure 3, we plot  $\hat{B}_0(k_1, k_2, k_3)$  for all possible triangle configurations with  $k_1, k_2, k_3 < k_{\text{max}} = 0.5$  h/Mpc where we order the configurations by looping through  $k_3$  in the inner most loop and  $k_1$  in the outer most loop with  $k_1 \leq k_2 \leq k_3$ . In the top panel, we present  $\hat{B}_0$  of HADES models with  $\sum m_\nu = 0.0, 0.06, 0.10$ , and  $0.15$  eV; in the lower panel, we present  $\hat{B}_0$  of HADES models with  $\sum m_\nu = 0.0$  eV and  $\sigma_8 = 0.822, 0.818$ , and  $0.807$ . We zoom into triangle configurations with  $k_1 = 0.113$ ,  $0.226 \leq k_2 \leq 0.283$ , and  $0.283 \leq k_3 \leq 0.377$  h/Mpc in the insets of the panels.

## 4. RESULTS

### 4.1. Breaking the $\sum m_\nu - \sigma_8$ degeneracy





**Figure 5.** The impact of  $\sum m_\nu$  and  $\sigma_8$  on the redshift-space halo bispectrum,  $\Delta\hat{B}_0/\hat{B}_0^{\text{fid}}$ , for all 1898 triangle configurations with  $k_1, k_2, k_3 \leq 0.5h/\text{Mpc}$ . We compare  $\Delta\hat{B}_0/\hat{B}_0^{\text{fid}}$  of the  $\sum m_\nu = 0.06$  (top), 0.10 (middle), and 0.15 eV (bottom) HADES models to  $\Delta\hat{B}_0/\hat{B}_0^{\text{fid}}$  of  $\sum m_\nu = 0.0$  eV  $\sigma_8 = 0.822, 0.818$ , and 0.807 models. The impact of  $\sum m_\nu$  on the bispectrum has a significantly different amplitude than the impact of  $\sigma_8$ . For instance,  $\sum m_\nu = 0.15$  eV (red) has a  $\sim 5\%$  stronger impact on the bispectrum than  $\sum m_\nu = 0.0$  eV  $\sigma_8 = 0.798$  (black) even though their powerspectrums only differ by  $< 1\%$  (Figure 1). Combined with the different shape-dependence (Figure 4), the distinct imprint of  $\sum m_\nu$  on the bispectrum illustrate that the bispectrum can break the degeneracy between  $\sum m_\nu$  and  $\sigma_8$  that degrade constraints from two-point analyses.

One major bottleneck of constraining  $\sum m_\nu$  with the power spectrum alone is the strong  $\sum m_\nu - \sigma_8$  degeneracy. The imprint of  $\sum m_\nu$  and  $\sigma_8$  on the power spectrum are degenerate and for models with the same  $\sigma_8^c$ , the power spectrum only differ by  $< 1\%$  (see Figure 1 and Villaescusa-Navarro

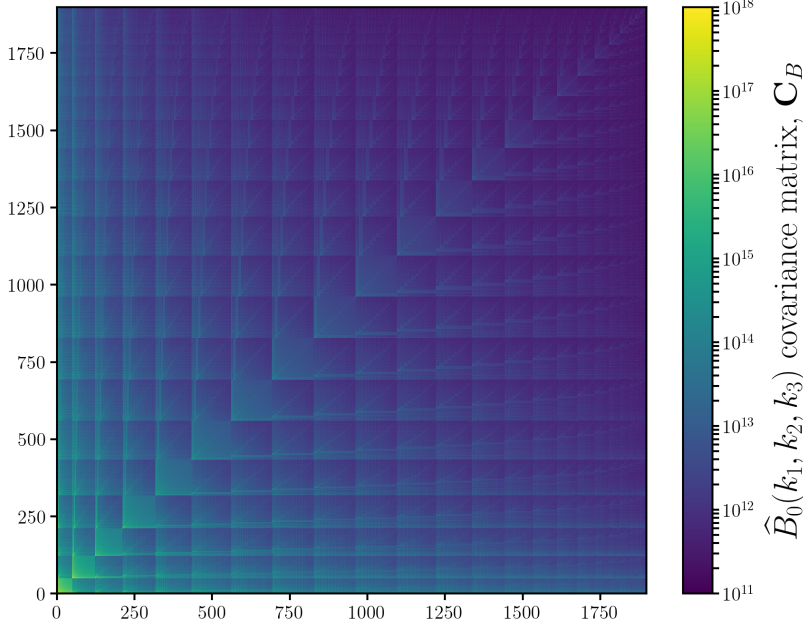


et al. 2018). The HADES suite, which has simulations with  $\sum m_\nu = 0.0, 0.06, 0.10$ , and  $0.15$  eV as well as  $\sum m_\nu = 0.0$  eV simulations with matching  $\sigma_8^c - \sigma_8 = 0.822, 0.818$ , and  $0.807$ , provide an ideal set of simulations to separate the impact of  $\sum m_\nu > 0.0$  eV and examine the degeneracy between  $\sum m_\nu$  and  $\sigma_8$  (Section 2 and Table 1). Hence, by measuring bispectrum of these simulations (Figure 2 and 3), we can determine whether the bispectrum helps break the  $\sum m_\nu - \sigma_8$  degeneracy. Below, we present our comparison of the HADES bispectrum and illustrate that the bispectrum can significantly improve  $\sum m_\nu$  constraints by breaking the  $\sum m_\nu - \sigma_8$  degeneracy.

We begin by examining the triangle shape dependent imprint of  $\sum m_\nu$  on the redshift-space halo bispectrum versus  $\sigma_8$  alone. In Figure 4, we present the fractional residual,  $(\Delta\hat{B}_0 = \hat{B}_0 - \hat{B}_0^{\text{fid}})/\hat{B}_0^{\text{fid}}$ , as a function of  $k_2/k_1$  and  $k_3/k_1$  for  $\sum m_\nu = 0.06, 0.10$ , and  $0.15$  eV in the upper panels and  $0.0$  eV  $\sigma_8 = 0.822, 0.818$ , and  $0.807$  in the bottom panels. The simulations in the top and bottom panels of each column have matching  $\sigma_8^c$ . Overall as  $\sum m_\nu$  increases, the bispectrum increases for all triangle shapes (top panels). This increase is due to halo bias (Villaescusa-Navarro et al. 2018, ; see also Figure 1). We impose a fixed  $M_{\text{lim}}$  on our halos so lower values of  $\sigma_8$  translate to a larger halo bias, which boosts the amplitude of the bispectrum. Within the overall increase in amplitude, however, equilateral triangles (upper left) have the largest increase. For  $\sum m_\nu = 0.15$  eV, the bispectrum is  $\sim 15\%$  higher than  $\hat{B}_0^{\text{fid}}$  for equilateral triangles. Meanwhile, the bispectrum increases by  $\sim 8\%$  for folded triangles for  $0.15$  eV (lower center). The noticeable difference in  $\Delta\hat{B}_0/\hat{B}_0^{\text{fid}}$  between equilateral and squeezed triangles (upper left) is roughly consistent with the comparison in Figure 7 of Ruggeri et al. (2018). They, however, fix  $A_s$  in their simulations and measure the real-space halo bispectrum so we refrain from any detailed comparisons.

As  $\sigma_8$  increases, with  $\sum m_\nu = 0.0$  eV fixed, the bispectrum increases overall for all triangle shapes (bottom panels). However, the comparison of the top and bottom panels in each column reveals significant differences in  $\Delta\hat{B}_0/\hat{B}_0^{\text{fid}}$  for  $\sum m_\nu$  versus  $\sigma_8$  alone. Between  $\sum m_\nu = 0.15$  eV and  $0.0$  eV  $\sigma_8 = 0.807$ , there is an overall  $\gtrsim 5\%$  difference. In addition, the shape dependence of the  $\Delta\hat{B}_0/\hat{B}_0^{\text{fid}}$  increase is different for  $\sum m_\nu$  than  $\sigma_8$ . This is particularly clear in the differences between  $0.1$  eV (top center panel) and  $0.0$  eV and  $\sigma_8 = 0.807$  (bottom right panel): near equilateral triangles in the two panels have similar  $\Delta\hat{B}_0/\hat{B}_0^{\text{fid}}$  while triangle shapes near the lower left edge from the squeezed to folded triangles have significantly different  $\Delta\hat{B}_0/\hat{B}_0^{\text{fid}}$ . Hence,  $\sum m_\nu$  leaves an imprint on the bispectrum with a distinct triangle shape dependence than  $\sigma_8$  alone. In other words, unlike the power spectrum, the triangle shape dependent impact of  $\sum m_\nu$  on the bispectrum cannot be replicated by varying  $\sigma_8$ .

We next examine the amplitude of the  $\sum m_\nu$  imprint on the redshift-space halo bispectrum versus  $\sigma_8$  alone for all triangle configurations. We present  $\Delta\hat{B}_0/\hat{B}_0^{\text{fid}}$  for all 1898 possible triangle configurations with  $k_1, k_2, k_3 < k_{\text{max}} = 0.5 h/\text{Mpc}$  in Figure 5. We compare  $\Delta\hat{B}_0/\hat{B}_0^{\text{fid}}$  of the  $\sum m_\nu = 0.06, 0.10$ , and  $0.15$  eV HADES models to the  $\Delta\hat{B}_0/\hat{B}_0^{\text{fid}}$  of  $\sum m_\nu = 0.0$  eV  $\sigma_8 = 0.822, 0.818$ , and  $0.807$  models in the top, middle, and bottom panels, respectively. The comparison confirms the difference in overall amplitude of varying  $\sum m_\nu$  and  $\sigma_8$  (Figure 4). For instance,  $\sum m_\nu = 0.15$  eV (red) has a  $\sim 5\%$  stronger impact on the bispectrum than  $\sum m_\nu = 0.0$  eV  $\sigma_8 = 0.798$  (black) even though their power spectra differ by  $< 1\%$  (Figure 1).



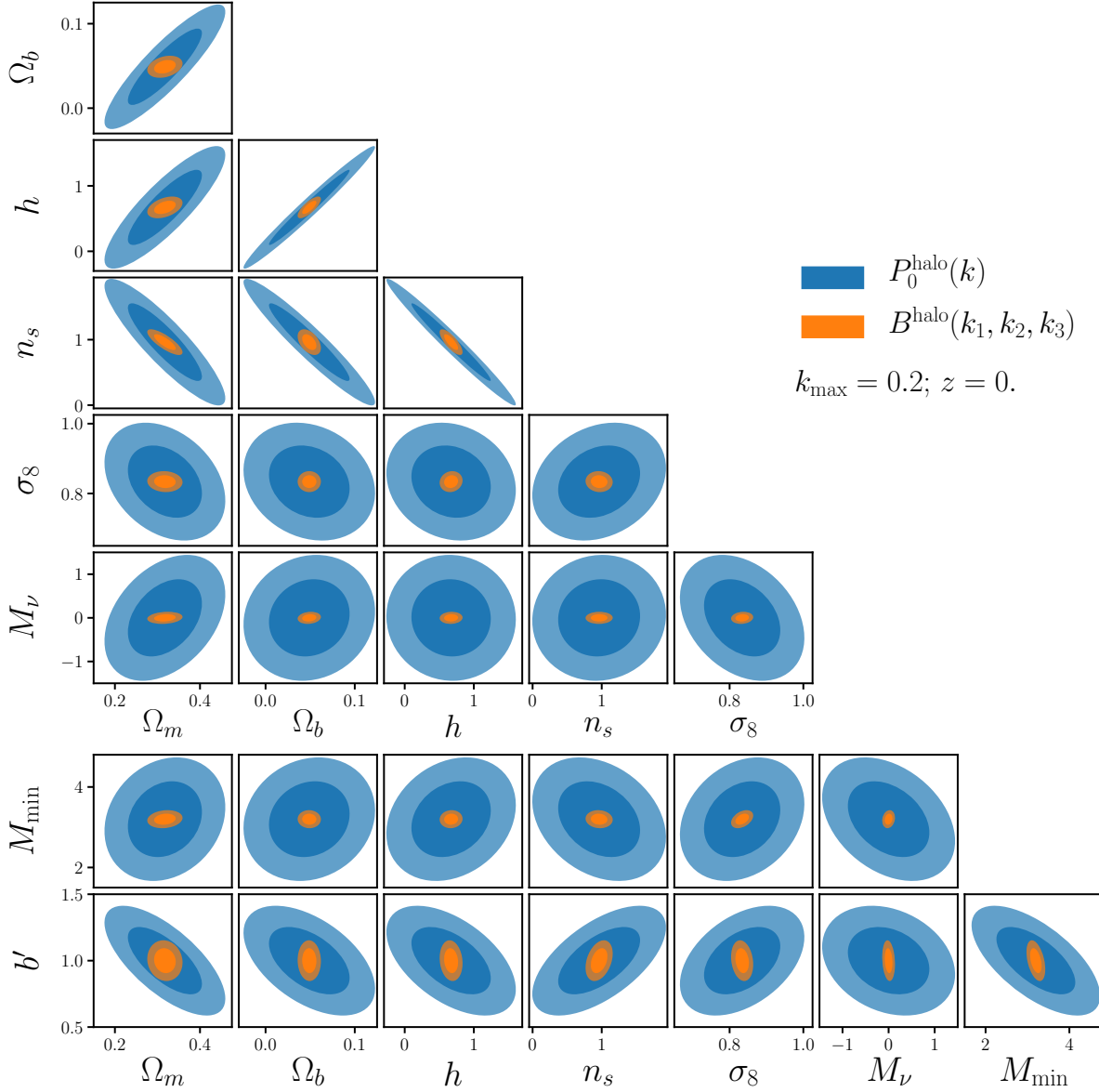
**Figure 6.** Covariance matrix of the redshift-space halo bispectrum estimated using the 15,000 realizations of the Quijote simulation suite at the fiducial cosmology:  $\Omega_m=0.3175$ ,  $\Omega_b=0.049$ ,  $h=0.6711$ ,  $n_s=0.9624$ ,  $\sigma_8=0.834$ , and  $\sum m_\nu=0.0$  eV. We include all possible triangle configurations with  $k_1, k_2, k_3 < k_{\max} = 0.5 h/\text{Mpc}$  and order the configurations (bins) in the same way as Figures 3 and 5. We use the covariance matrix above for the Fisher matrix forecasts presented in Section 4.2.

The comparison in the panels of Figure 5 also reveal a difference in the configuration dependence in  $\Delta\hat{B}_0/\hat{B}_0^{\text{fid}}$  between  $\sum m_\nu$  versus  $\sigma_8$ . The triangle configurations are ordered by looping through  $k_3$  in the inner most loop and  $k_1$  in the outer most loop such that  $k_1 \leq k_2 \leq k_3$ . In this ordering,  $k_1$  increases from left to right.  $\Delta\hat{B}_0/\hat{B}_0^{\text{fid}}$  of  $\sum m_\nu$  expectedly increases with  $k_1$ : for small  $k_1$  (on large scales), neutrinos behave like CDM and therefore the impact is reduced. However,  $\Delta\hat{B}_0/\hat{B}_0^{\text{fid}}$  of  $\sum m_\nu$  has a smaller  $k_1$  dependence than  $\Delta\hat{B}_0/\hat{B}_0^{\text{fid}}$  of  $\sigma_8$ . Combined with the different shape-dependence (Figure 4), the distinct imprint of  $\sum m_\nu$  on the redshift-space halo bispectrum illustrates that the bispectrum can break the degeneracy between  $\sum m_\nu$  and  $\sigma_8$ . Moreover it illustrates that by including the bispectrum, we can more precisely constrain  $\sum m_\nu$  than with the power spectrum alone.

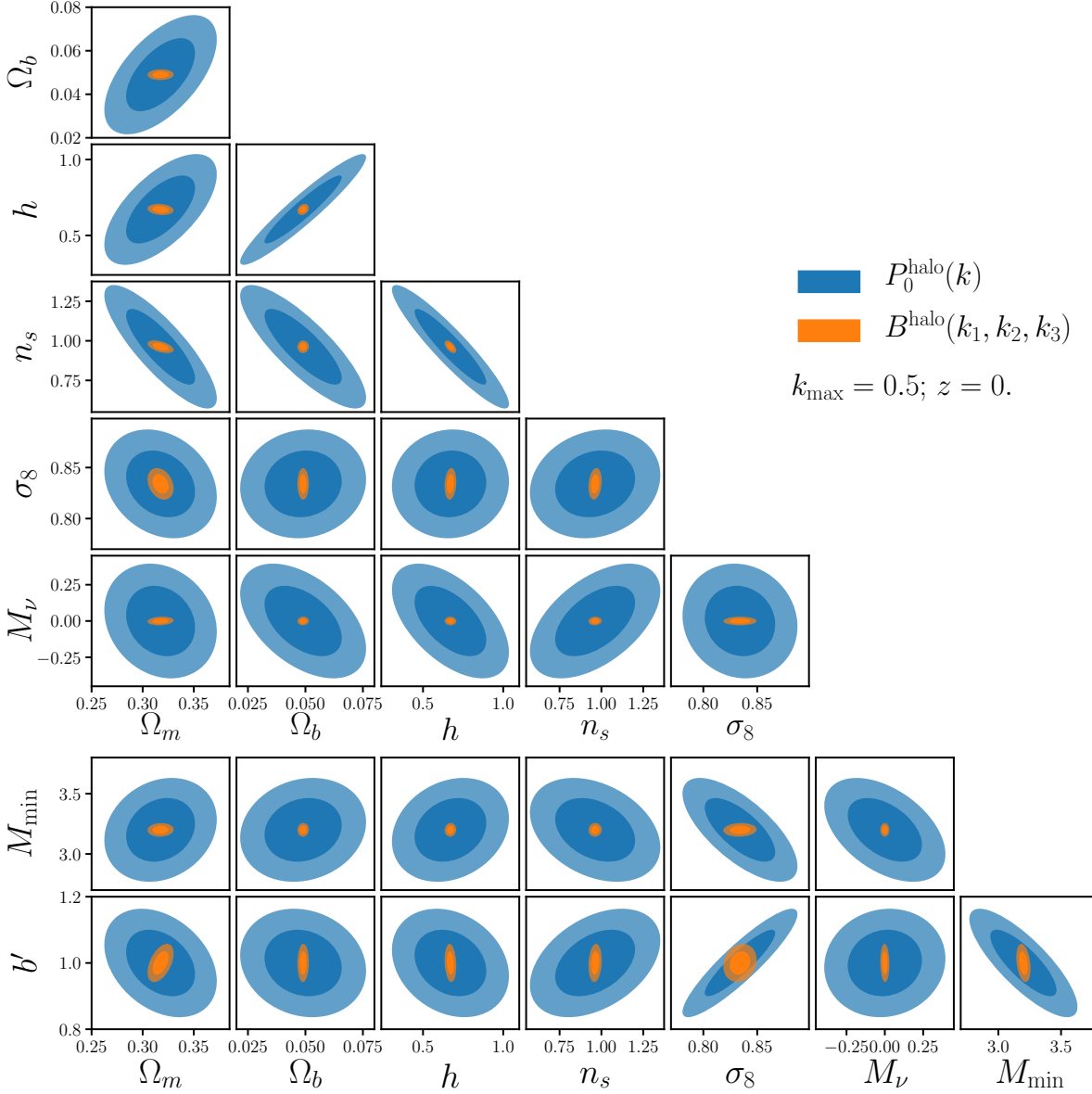
#### 4.2. $\sum m_\nu$ and other Cosmological Parameter Forecasts

We demonstrate in the previous section with the HADES simulations, that the bispectrum helps break the  $\sum m_\nu$ - $\sigma_8$  degeneracy, a major challenge in precisely constraining  $\sum m_\nu$  with the power spectrum. This establishes the bispectrum as a promising probe for  $\sum m_\nu$ . However, we are ultimately interested in determining the constraining power of the bispectrum for an analysis that include cosmological parameters beyond  $\sum m_\nu$  and  $\sigma_8$ —*i.e.*  $\Omega_m$ ,  $\Omega_b$ ,  $h$ , and  $n_s$ . The Quijote suite of simulations is *specifically* designed to answer this question using Fisher matrix forecast.

First, the Quijote suite includes 15,000 realizations run at a fiducial cosmology:  $\sum m_\nu=0.0\text{eV}$ ,  $\Omega_m=0.3175$ ,  $\Omega_b=0.049$ ,  $n_s=0.9624$ ,  $h=0.6711$ , and  $\sigma_8 = 0.834$  (see Table 1. This allows us to robustly



**Figure 7.** Fisher matrix constraints for  $\sum m_\nu$  and other cosmological parameters for the redshift-space halo bispectrum monopole (orange). For comparison, we include Fisher parameter constraints for the redshift-space halo powerspectrum monopole in blue. The contours mark the 68% and 95% confidence intervals. We set  $k_{\text{max}} = 0.2 \, h/\text{Mpc}$  for both power spectrum and bispectrum. We include in our forecasts  $b'$  and  $M_{\text{min}}$ , a free amplitude scaling factor and halo mass limit, respectively. They serve as a simplistic bias model and we marginalize over them so that our constraints do not include extra constraining power from the difference in bias/number density in the different Quijote cosmologies. The bispectrum *substantially* improves constraints on all of the cosmological parameters over the power spectrum. For  $\sum m_\nu$ , the bispectrum improves the constraint from  $\sigma_{\sum m_\nu} = 1.163$  to  $0.112 \, \text{eV}$  — an order of magnitude improvement over the power spectrum.



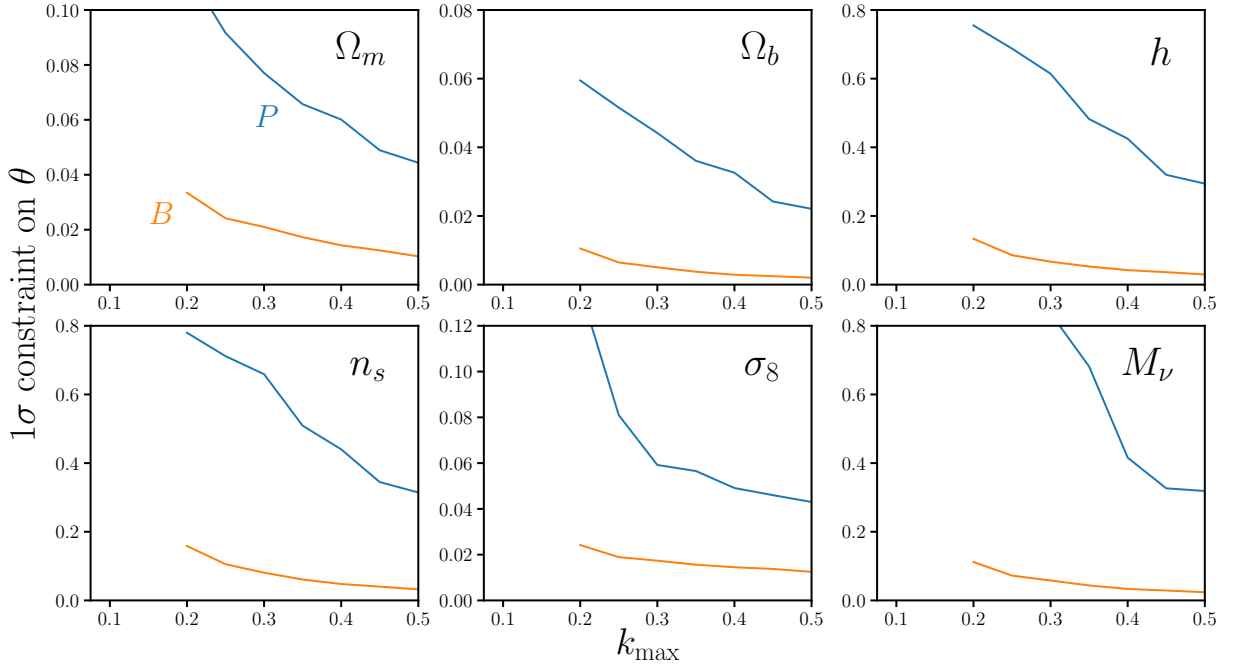
**Figure 8.** Same as Figure 7 but for  $k_{\text{max}} = 0.5 \, h/\text{Mpc}$ . Again, the bispectrum substantially improves constraints on all cosmological parameters over the power spectrum. For  $\sum m_\nu$ , the bispectrum improves  $\sigma_{\sum m_\nu}$  from 0.319 to 0.0239 eV — over an order of magnitude improvement over the power spectrum.

estimate the covariance matrix of the bispectrum,  $\mathbf{C}$ , which has  $\sim 1,800$  triangle configurations (Figure 6). Second, the Quijote suite includes 500 fixed-pair realizations evaluated at 13 different cosmologies, each a small step away from the fiducial cosmology parameter values along one parameter (Section 2 and Table 1). These realizations allow us to precisely estimate the derivatives of the bispectrum with respect to each of the cosmological parameters.

Since their introduction to cosmology over two decades ago, Fisher Information matrices have been ubiquitously used to forecast the constraining power of future experiments (*e.g.* Jungman et al.

**Table 2.** Cosmological parameter constraints from the redshift-space halo power spectrum (top) and bispectrum (bottom) for different values of  $k_{\max}$ .

	$k_{\max}$ ( $h/\text{Mpc}$ )	$\sum m_\nu$ (eV)	$\Omega_m$	$\Omega_b$	$h$	$n_s$	$\sigma_8$	$b'$	$M_{\min}$ ( $10^{13}h^{-1}M_\odot$ )
		0.0	0.3175	0.049	0.6711	0.9624	0.834	1.	3.2
$P_0$	0.2	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
	0.3	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
	0.4	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
	0.5	$\pm 0.319$	$\pm 0.0444$	$\pm 0.0221$	$\pm 0.294$	$\pm 0.315$	$\pm 0.0430$	$\pm 0.35$	$\pm 0.13$
$B_0$	0.2	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
	0.3	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
	0.4	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
	0.5	$\pm 0.0239$	$\pm 0.0103$	$\pm 0.00202$	$\pm 0.0295$	$\pm 0.0321$	$\pm 0.0125$	$\pm 0.046$	$\pm 0.047$



**Figure 9.** Marginalized  $1\sigma$  constraints of  $\Omega_m$ ,  $\Omega_b$ ,  $h$ ,  $n_s$ ,  $\sigma_8$ , and  $\sum m_\nu$  as a function of  $k_{\max}$  for the redshift-space halo bispectrum (orange) and power spectrum (blue). Though not included in the figure, we marginalize over  $b'$  and  $M_{\min}$  in our forecast. **CH: more**

1996; Tegmark et al. 1997; Dodelson 2003; Heavens 2009; Verde 2010). Defined as

$$F_{ij} = -\left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial \theta_i \partial \theta_j} \right\rangle, \quad (8)$$

where  $\mathcal{L}$  is the likelihood, the Fisher matrix for the bispectrum can be written as

$$F_{ij} = \frac{1}{2} \text{Tr} \left[ \mathbf{C}^{-1} \frac{\partial \mathbf{C}}{\partial \theta_i} \mathbf{C}^{-1} \frac{\partial \mathbf{C}}{\partial \theta_j} + \mathbf{C}^{-1} \left( \frac{\partial \bar{B}_0}{\partial \theta_i} \frac{\partial \bar{B}_0}{\partial \theta_j}^T + \frac{\partial \bar{B}_0}{\partial \theta_i}^T \frac{\partial \bar{B}_0}{\partial \theta_j} \right) \right]. \quad (9)$$

Since we assume that the  $B_0$  likelihood is Gaussian, including the first term in Eq. 9 runs the risk of incorrectly including information from the covariance already included in the mean (Carron 2013). We, therefore, conservatively neglect the first term and calculate the Fisher matrix,

$$F_{ij} = \frac{1}{2} \text{Tr} \left[ \mathbf{C}^{-1} \left( \frac{\partial \bar{B}_0}{\partial \theta_i} \frac{\partial \bar{B}_0}{\partial \theta_j}^T + \frac{\partial \bar{B}_0}{\partial \theta_i}^T \frac{\partial \bar{B}_0}{\partial \theta_j} \right) \right], \quad (10)$$

directly with  $\mathbf{C}$  and  $\partial B_0/\partial \theta_i$  along each cosmological parameter from the Quijote simulations. For  $\Omega_m$ ,  $\Omega_b$ ,  $h$ ,  $n_s$ , and  $\sigma_8$ , we estimate

$$\frac{\partial \bar{B}_0}{\partial \theta_i} \approx \frac{\bar{B}_0(\theta_i^+) - \bar{B}_0(\theta_i^-)}{\theta_i^+ - \theta_i^-}. \quad (11)$$

Meanwhile, for  $\sum m_\nu$ , where the fiducial value is 0.0 eV and we cannot have negative  $\sum m_\nu$ , we use the Quijote simulations at  $\sum m_\nu^+$ ,  $\sum m_\nu^{++}$ ,  $\sum m_\nu^{+++} = 0.1, 0.2, 0.4$  eV (Table 1) to estimate:

$$\frac{\partial \bar{B}_0}{\partial \sum m_\nu} \approx \frac{-21\bar{B}_0(\sum m_\nu^{\text{fid}}) + 32\bar{B}_0(\sum m_\nu^+) - 12\bar{B}_0(\sum m_\nu^{++}) + \bar{B}_0(\sum m_\nu^{+++})}{1.2}. \quad (12)$$

By using these  $N$ -body simulations, instead of analytic methods (*e.g.* perturbation theory), we exploit the accuracy of numerical simulations in the nonlinear regime and rely on fewer assumptions and approximations. **CH: something about quantity the information content of the bispectrum in the nonlinear regime unprecedented.** We discuss subtleties of the bispectrum derivatives in Appendix B and discuss tests of convergence in Appendix C.

We present the constraints on  $\sum m_\nu$  and other cosmological parameters  $\{\Omega_m, \Omega_b, h, n_s, \sigma_8\}$  derived from the redshift-space halo bispectrum Fisher matrix (Eq. 10) in Figures 7 and 8. We include Fisher constraints for the redshift-space halo power spectrum monopole in blue for comparison. The contours mark the 68% and 95% confidence intervals. For both the power spectrum and bispectrum, we set  $k_{\text{max}} = 0.5 h/\text{Mpc}$ . We also include in our Fisher constraints, parameters  $b'$ , a free amplitude scaling factor, and  $M_{\text{min}}$ , the halo mass limit. These parameters serve as a simplistic bias model and by marginalizing over them we ensure that our Fisher constraints do not include extra constraining power from the difference in bias/number density in the different Quijote cosmologies used to calculate  $\partial \bar{B}_0/\partial \theta_i$ .  $b'$  is a multiplicative factor so  $\partial \bar{B}_0/\partial b' = \bar{B}_0$ . Meanwhile, we numerically estimate  $\partial \bar{B}_0/\partial M_{\text{min}}$  using  $\bar{B}_0$  evaluated at  $M_{\text{min}}=3.3$  and  $3.1 \times 10^{13} h^{-1} M_\odot$ , with all other parameters fixed. In Table 2, we list the parameter constraints for  $P_0(k < k_{\text{max}} = 0.5 h/\text{Mpc})$  and  $B_0(k_1, k_2, k_3 < k_{\text{max}} = 0.5 h/\text{Mpc})$ .

The bispectrum substantially improves constraints on all of the parameters over the power spectrum (Figure 7 and Table 2). More precisely, the bispectrum improves the marginalized  $\Omega_m$ ,  $\Omega_b$ ,  $h$ ,  $n_s$ , and  $\sigma_8$  constraints by factors of  $\sim 4, 11, 10, 10$ , and 3 with respect to their power spectrum

constraints. For  $\sum m_\nu$ , the bispectrum improves the constraint from  $\sigma_{\sum m_\nu} = 0.319$  to  $0.0239 \text{ eV}$  — over an order of magnitude improvement over the power spectrum.

Even at lower  $k_{\text{max}} < 0.5 \text{ h/Mpc}$ , the bispectrum significantly improves cosmological parameter constraints. We compare the marginalized  $1\sigma$  constraints of  $\Omega_m$ ,  $\Omega_b$ ,  $h$ ,  $n_s$ ,  $\sigma_8$ , and  $\sum m_\nu$  as a function of  $k_{\text{max}}$  for  $B_0$  (orange) and  $P_0$  (blue) in Figure 9. **CH: write this once we have to updated powerspectrum calculations.**

discussing the results

- emphasize that this is only for a 1 Gpc box so our constraints will be so much better
- how does the bispectrum do so much better? — *i.e.* triangles that contribute most fisher information
- redshift-space helps us out (refer to appendix)

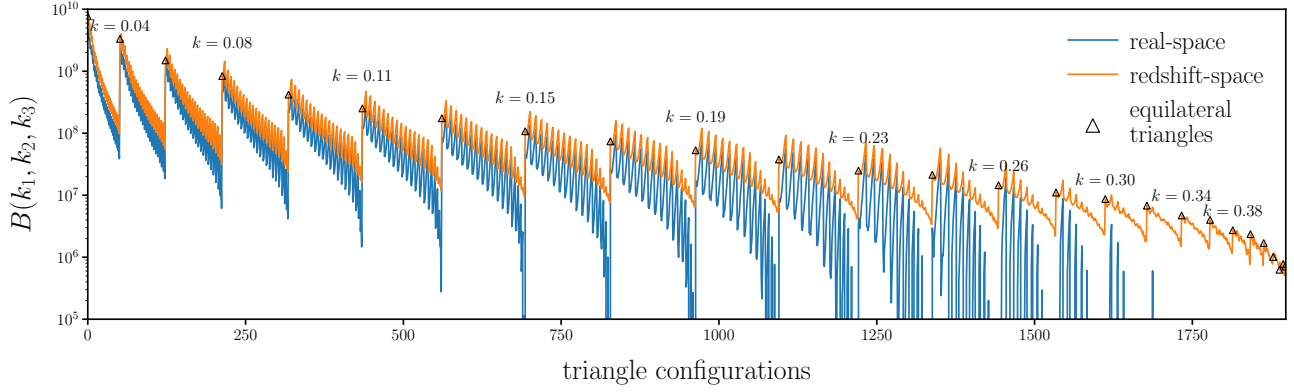
The results above definitively show that the bispectrum has significant constraining power in the weakly nonlinear regime ( $k > 0.1 \text{ h/Mpc}$ ) beyond the power spectrum. Our results also demonstrated the potential of the bispectrum in constraining  $\sum m_\nu$  (an order of magnitude improvement over  $P_0$ ). Below, we underline a few caveats of the results we present above. First, the parameter constraints were derived using the Fisher matrix. This assumes

- mention the caveats of fisher forecasts
- emphasize the lack of bananas in the contours
- convergence concerns (refer to appendix)
- $\sum m_\nu$  dependence of derivatives along  $\sum m_\nu$ . (refer to appendix)
- take Fisher forecasts with a grain of salt

Another caveat is that our parameter constraints were derived using the power spectrum and bispectrum of halo in a periodic box. We do not consider a realistic survey geometry or radial selection function. A realistic selection function will smooth out the triangle configuration dependence and consequently degrade the constraining power of the bispectrum. In Sefusatti & Scoccimarro (2005), for instance, they find that the signal-to-noise of the bispectrum is significantly reduced once survey geometry is included in their forecast. Survey geometry, however, also degrade the signal-to-noise of their power spectrum forecasts. Hence, with the order of magnitude improve in the  $\sum m_\nu$  constraining power of the bispectrum, even with survey geometry, including the bispectrum will improve  $\sum m_\nu$  constraints.

Although we focus on the halo bispectrum and power spectrum in this paper, constraints on  $\sum m_\nu$  will ultimately be derived from the distribution of galaxies. Besides the cosmological parameters, bias and nuisance parameters that allow us to marginalize over the galaxy—halo connection need to be incorporated to forecast  $\sum m_\nu$  and other cosmological parameter constraints for the galaxy bispectrum. Although we include a *naive* bias model through  $b'$  and  $M_{\text{min}}$ , this is insufficient to describe how galaxies occupy halos. A more realistic bias model such as a halo occupation distribution





**Figure 10.** Comparison of the fiducial HADES simulations real and redshift-space halo bispectrum for triangle configurations with  $k_1, k_2, k_3 \leq k_{\max} = 0.5h/\text{Mpc}$  (blue and orange respectively). We mark equilateral triangle configurations (empty triangle marker) along with their side lengths  $k$ .

(HOD) model involve extra parameters that describe the distribution of central and satellite galaxies in halos (*e.g.* Zheng et al. 2005; Leauthaud et al. 2012; Tinker et al. 2013; Zentner et al. 2016; Vakili & Hahn 2019). **CH:** maybe something about Uros and Nick’s model involving a lot of parameters. Marginalizing over these extra parameters, will likely reduce the constraining power at high  $k$ . Even if the constraining power at high  $k$  is reduced, the bispectrum still offers significant improvements over the power spectrum at  $k_{\max} \sim 0.2$ . Jointly analyzing power spectrum and bispectrum will help constrain these extra bias parameters. Furthermore, we again emphasize that the constraints we present in this paper is for a  $1h^{-1}\text{Gpc}$  box. In Hahn et al. (in preparation), we will include a realistic HOD model and quantify the information content and constraining power of a joint galaxy power spectrum and bispectrum analysis.

## 5. SUMMARY

### ACKNOWLEDGEMENTS

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## APPENDIX

### A. REDSHIFT-SPACE BISPECTRUM

### B. FISHER FORECAST DERIVATIVES

here’s how we take derivatives

### C. TESTING CONVERGENCE

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- |   |   |
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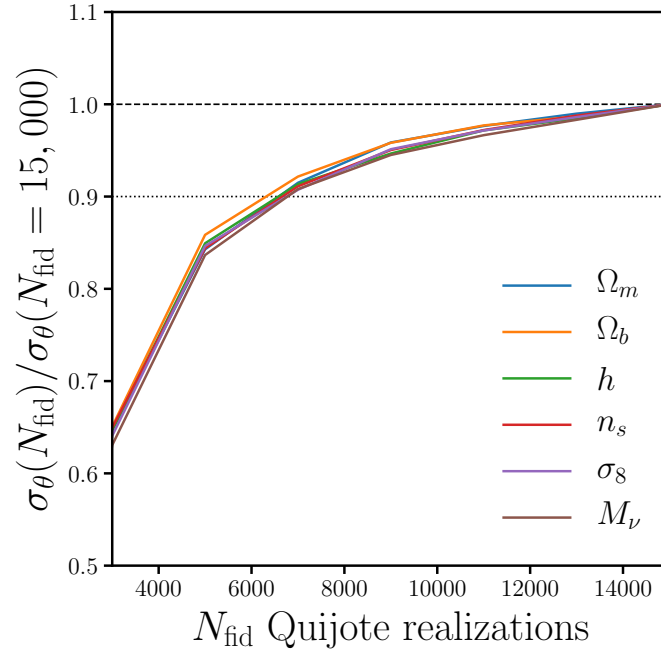


Figure 11.

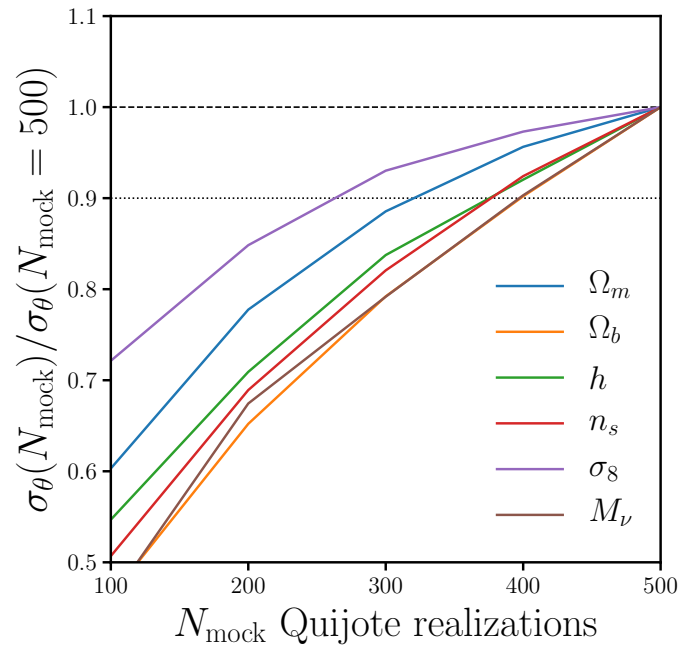


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