Draft version August 19, 2019
Typeset using LaTeX **preprint** style in AASTeX62

Constraining M_ν with the Bispectrum III: Compressing the Bispectrum

ChangHoon Hahn[1,2,*] and Francisco Villaescusa-Navarro[3]

[1] *Lawrence Berkeley National Laboratory, 1 Cyclotron Rd, Berkeley CA 94720, USA*
[2] *Berkeley Center for Cosmological Physics, University of California, Berkeley, CA 94720, USA*
[3] *Center for Computational Astrophysics, Flatiron Institute, 162 5th Avenue, New York, NY 10010, USA*

(Dated: `DRAFT --- b2baa8e --- 2019-08-19 --- NOT READY FOR DISTRIBUTION`)

ABSTRACT

Keywords: cosmology: —

1. INTRODUCTION

intro goes here

2. SIMULATIONS

Very brief description of the simulations. Just highlight the numbers

3. RESULTS

4. SUMMARY

ACKNOWLEDGEMENTS

It's a pleasure to thank ... for valuable discussions and comments.

APPENDIX

A. FISHER MATRIX FOR MODIFIED T-DISTRIBUTION LIKELIHOOD

The standard approach for Fisher matrix forecasts and parameter inference in LSS assumes that the p-dimensional likelihood is a Gaussian and uses the **?** factor,

$$f_{\rm Hartlap} = \frac{N-p-2}{N-1} \tag{A1}$$

to account for the bias in the inverse covariance matrix estimated from N mocks. In addition to breaking down on large scales where Central Limit Theorem no longer holds (**?**), this assumption also breaks down when the covariance matrix is estimated from a finite number of mocks (**?**). In fact, **?** show that the likelihood is a modified t-distribution:

$$p(y|\mu(\theta), \Psi, N) = |\Psi|^{-1/2} c_p \left(1 + \frac{(y-\mu(\theta))^T \Psi^{-1} (y - \mu(\theta))}{1-N}\right)^{-N/2}. \tag{A2}$$

* hahn.changhoon@gmail.com

$$c_p = \frac{\Gamma(\frac{N}{2})}{[\pi(N-1)]^{p/2}\Gamma(\frac{N-p}{2})} \tag{A3}$$

where Γ is the Gamma function, μ is our model, and Ψ is the covariance matrix. Adopting the wrong sampling distribution will yield incorrect posterior distributions, with biased parameter estimates and incorrect errors even when the bias of the inverse covariance matrix is accounted for (**?**). We therefore, derive the Fisher matrix for the modified t-distribution likelihood following the calculations from **?**.

For simplicity, let $\ell(\theta)$ be the log-likelihood, $z = \Psi^{-1/2}(y-\mu)$, and

$$g(s,v) = c_p\left(1+\frac{s}{v}\right)^{-N/2} \tag{A4}$$

so that Eq. A2 can be written as $p(y|\mu(\theta),\Psi,N) = |\Psi|^{-1/2}g(||z||^2, 1-N)$. Then the derivative of the log-likelihood is

$$\frac{\partial\ell}{\partial\theta_i} = \left(\frac{1}{g}\frac{\partial g}{\partial s}\right)\left(-2\frac{\partial\mu}{\partial\theta_i}^T\Psi^{-1}(y-\mu)\right) \tag{A5}$$

Using this derivative we can write the Fisher matrix as

$$F_{ij} = \left\langle \frac{\partial\ell}{\partial\theta_i}\frac{\partial\ell}{\partial\theta_j}\right\rangle \tag{A6}$$

$$= 4\left\langle \left(\frac{1}{g}\frac{\partial g}{\partial s}\right)^2\left(z^T\Psi^{-1/2}\frac{\partial\mu}{\partial\theta_i}\frac{\partial\mu^T}{\partial\theta_j}\Psi^{-1/2}z\right)\right\rangle \tag{A7}$$

Using Lemma 1 from **?**, we can express this as

$$= 4\left\langle \left(\frac{1}{g}\frac{\partial g}{\partial s}\right)^2 ||z||^2\left(\frac{z^T}{||z||}\Psi^{-1/2}\frac{\partial\mu}{\partial\theta_i}\frac{\partial\mu^T}{\partial\theta_j}\Psi^{-1/2}\frac{z}{||z||}\right)\right\rangle \tag{A8}$$

$$= 4\left\langle ||z||^2\left(\frac{1}{g}\frac{\partial g}{\partial s}\right)^2\right\rangle\frac{1}{p}\mathrm{Tr}\left(\Psi^{-1/2}\frac{\partial\mu}{\partial\theta_i}\frac{\partial\mu^T}{\partial\theta_j}\Psi^{-1/2}\right) \tag{A9}$$

$$= 4\left\langle ||z||^2\left(\frac{1}{g}\frac{\partial g}{\partial s}\right)^2\right\rangle\frac{1}{p}\frac{\partial\mu^T}{\partial\theta_i}\Psi^{-1}\frac{\partial\mu}{\partial\theta_j} \tag{A10}$$

Since,

$$\left(\frac{1}{g}\frac{\partial g}{\partial s}\right) = -\frac{N}{2}\left(\frac{1}{N-1}\right)\left(1+\frac{s}{N-1}\right)^{-1} \tag{A11}$$

we can expand,

$$\left\langle ||z||^2\left(\frac{1}{g}\frac{\partial g}{\partial s}\right)^2\right\rangle = \left\langle ||z||^2\frac{N^2}{4}\left(\frac{1}{N-1}\right)^2\left(1+\frac{s}{N-1}\right)^{-2}\right\rangle \tag{A12}$$

$$= \frac{N^2}{4(N-1)}\left\langle\frac{||z||^2}{N-1}\left(1+\frac{||z||^2}{N-1}\right)^{-2}\right\rangle \tag{A13}$$

$$= \frac{N^2}{4(N-1)}\frac{p(N-1)}{(N+p+1)(N+p-1)} \tag{A14}$$

$$= \frac{N^2 p}{4(N+p+1)(N+p-1)}. \tag{A15}$$

Plugging the expression back into Eq. A8,

$$F_{ij} = \frac{N^2}{(N+p+1)(N+p-1)} \frac{\partial \mu^T}{\partial \theta_i} \Psi^{-1} \frac{\partial \mu}{\partial \theta_j} \tag{A16}$$

REFERENCES