Behavioural Finance

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Prospect Theory

- In 1979, Kahneman and Tversky developed prospect theory to provide realistic model of investor behaviour:
    - Gain or loss is measured relative to reference level
    - Loss aversion: investor is more sensitive to loss vs gain (of same magnitude) $\implies$ more sensitive to downside risk
- Motivated by results of controlled experiments in gambling that didn't match predictions of expected utility theory
- In 2001, Barberis, Huang, and Santos developed asset-pricing model that combines power utility of consumption with prospect theory applied to gain or loss from recent investments
- "Quasi-behavioral" model where rational investors aim to maximise expected utility from non-standard preferences

Economic Environment

- Riskless bond provides (constant) risk-free rate of R_f
- Risky stock represents equity claim on perishable output, and provides (random) return of R_{t+1} over next time period
- In equilibrium, aggregate consumption and aggregate dividend both evolve as lognormal random walk with drift:

$$\ln\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right) = \ln\left(\frac{\overline{D}_{t+1}}{\overline{D}_t}\right) = \mu + \sigma\epsilon_{t+1}$$

- Here $\epsilon_t \sim N(0,1)$ is independent and identically distributed (i.i.d.) random variable that captures economic fluctuations
- Simplified setting without investment across multiple risky assets, but can still interpret as investment in market portfolio

Investor Preferences

- Infinitely-lived investor receives time-separable utility from individual consumption as well as recent financial gain or loss:

$$E\left[\sum_{t=0}^{\infty}\left(\delta^t \frac{C_t^{1-\gamma}}{1-\gamma} + \delta^{t+1} b_t V(X_{t+1})\right)\right]$$

- Here $\delta = e^{-\rho} \in (0,1)$ is subjective discount factor and $\gamma > 0$ is coefficient of relative risk aversion for consumption shocks
- Also X_t represents recent financial gain or loss, and $V(X_t)$ represents utility from recent financial gain or loss
- Then b_t is (time-varying) scale factor to ensure that amount of utility from consumption remains similar to amount of utility from recent gain or loss, over different time intervals

Prospect Theory

- Let w_t be (dollar) value of investment in stock at time t, and assume that investor keeps track of year-to-year fluctuations
- Recent financial gain or loss is measured relative to reference level based on risk-free rate:

$$X_{t+1} = w_t \left(R_{t+1} - R_f \right)$$

- Loss aversion makes investor more sensitive to shortfall in financial gain (or outright financial loss), so $\lambda > 1$:

$$V(X_{t+1}) = \begin{cases} X_{t+1}, & X_{t+1} \geq 0 \\ \lambda X_{t+1}, & X_{t+1} < 0 \end{cases}$$

Prospect Theory

- Utility from recent financial gain or loss is piecewise-linear, so define scale-invariant utility function for financial gain or loss:

$$V(X_{t+1}) = V(w_t\,(R_{t+1} - R_f)) = w_t v(R_{t+1})\,,$$
$$v(R_{t+1}) = \begin{cases} R_{t+1} - R_f\,, & R_{t+1} \geq R_f \\ \lambda\,(R_{t+1} - R_f)\,, & R_{t+1} < R_f \end{cases}$$

- Use marginal utility of aggregate consumption as scale factor:

$$b_t = b_0 \overline{C}_t^{-\gamma}$$

- Here $b_0 \geq 0$ determines extent to which utility from recent financial gain or loss contributes to investor's lifetime utility

Optimisation Problem

- Investor's consumption and asset allocation problem:

$$\max_{\{C_t, w_t\}} E\left[\sum_{t=0}^{\infty}\left(\delta^t \frac{C_t^{1-\gamma}}{1-\gamma} + \delta^{t+1} b_0 \overline{C}_t^{-\gamma} w_t v(R_{t+1})\right)\right]$$

- Subject to investor's intertemporal budget constraint:

$$W_{t+1} = (W_t - C_t)\, R_f + w_t\, (R_{t+1} - R_f)$$

- Assume that market is complete, and that representative investor has same utility function as individual investors
- Representative investor optimally consumes (per capita) aggregate consumption and invests in market portfolio

Optimal Consumption

- First-order condition for optimal individual consumption:

$$\delta R_f E_t\left[\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma}\right] = 1$$

- Optimality condition must also apply to representative investor, who consumes aggregate consumption:

$$\delta R_f E_t\left[\left(\frac{\overline{C}_{t+1}}{\overline{C}_t}\right)^{-\gamma}\right] = 1 \implies R_f = e^{\rho+\gamma\mu-\frac{1}{2}\gamma^2\sigma^2}$$

- Same risk-free rate as model with lognormal consumption growth and investors with constant relative risk aversion

Optimal Asset Allocation

- First-order condition for optimal asset allocation:

$$\delta b_0 \left(\frac{\overline{C}_t}{C_t^*} \right)^{-\gamma} E_t[v(R_{t+1})] + \delta E_t \left[\left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\gamma} R_{t+1} \right] = 1$$

- Also applies to representative investor, who consumes aggregate consumption and invests in market portfolio:

$$\delta b_0 E_t[v(R_{t+1})] + \delta E_t \left[\left(\frac{\overline{C}_{t+1}}{\overline{C}_t} \right)^{-\gamma} R_{t+1} \right] = 1$$

- In equilibrium, market return must satisfy optimality condition

Market Return

- Assume that market portfolio has constant price-dividend ratio: $P_t/D_t = k$ for all $t = 1, 2, \ldots$, and let $\kappa = (1+k)/k$
- Then market return will have i.i.d. probability distribution:

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \kappa \frac{D_{t+1}}{D_t} = \kappa \frac{\overline{C}_{t+1}}{\overline{C}_t}$$

- Substitute into optimality condition for representative investor:

$$\delta b_0 E_t \left[v\left(\kappa \frac{\overline{C}_{t+1}}{\overline{C}_t} \right) \right] + \delta \kappa E_t \left[\left(\frac{\overline{C}_{t+1}}{\overline{C}_t} \right)^{1-\gamma} \right] = 1$$

Market Return

- Use distribution of consumption growth to get equilibrium condition for price-dividend ratio of market portfolio:

$$
\begin{aligned}
&\delta b_0 E_t\left[v\left(\kappa e^{\mu+\sigma\epsilon_{t+1}}\right)\right] + \delta\kappa E_t\left[e^{(1-\gamma)(\mu+\sigma\epsilon_{t+1})}\right] = 1 \\
\implies &\delta b_0 E_t\left[v\left(\kappa e^{\mu+\sigma\epsilon_{t+1}}\right)\right] + \delta\kappa e^{(1-\gamma)\mu+\frac{1}{2}(1-\gamma)^2\sigma^2} = 1
\end{aligned}
$$

- No analytical solution, so use numerical approach to find equilibrium price-dividend ratio of market portfolio
- Use price-dividend ratio of market portfolio and distribution of aggregate consumption growth to find expected market return:

$$
E_t[R_{t+1}] = \kappa E_t\left(e^{\mu+\sigma\epsilon_{t+1}}\right) = \kappa e^{\mu+\frac{1}{2}\sigma^2}
$$

Empirical Results

- Set $\lambda = 2.25$, based on results of controlled laboratory experiments by Kahneman and Tversky
- Set $\mu = 1.84\%$ and $\sigma = 3.79\%$, based on annual per capita aggregate consumption for US economy from 1889 to 1995
- Set $\gamma = 0.9$ and $\delta = 0.98 \implies$ annual risk-free rate of 3.7%
- Annual equity premium of 0.06% for $b_0 = 0$, increasing to 0.91% for $b_0 = 2$, and converging to 1.2% as $b_0 \to \infty$
- Equity premium increases as utility from recent financial gain or loss makes bigger contribution to investor's lifetime utility
- But equity premium is still too small to match historical data, for reasonable levels of risk aversion and loss aversion

House Money Effect

- House money effect: investor becomes more (or less) willing to gamble after prior gains (or losses) $\implies$ prior outcomes affect investor's degree of risk aversion (and loss aversion)
- Barberis, Huang, and Santos extended their model to allow investor to keep track of accumulated financial gain or loss over entire lifetime (relative to appropriate reference level)
- No loss aversion if investor has accumulated financial gain
- But investor becomes even more loss averse with accumulated financial loss: λ rises by one for every 2% shortfall in value of investment in stock (relative to appropriate reference level)
- Equity premium of 4.1% per year for $b_0 = 2$, after adding house money effect to existing model with prospect theory