QF620 Stochastic Modelling in Finance
Assignment 1/4
Due Date: 25-Sep-2024

1. Let $X \sim N(\mu, \sigma^2)$ be a random variable following normal distribution. Use the Moment Generating Function to evaluate the following expectation:
   (a) $\mathbb{E}\left[e^{X}\right]$.
   (b) $\mathbb{E}\left[e^{2X}\right]$.

2. Let $W_t \sim N(0, t)$ be a random variable following normal distribution. Use the Moment Generating Function to evaluate the following expectation:
   (a) $\mathbb{E}\left[e^{W_t}\right]$.
   (b) $\mathbb{E}\left[e^{\sigma W_t}\right]$, where σ is a constant.

3. Consider a Cox-Ross-Rubinstein binomial tree with $S_0 = \$5$, $u = 2$, $r = 4\%$. We would like to value a vanilla option with a strike price of $K = \$10$, maturing on the second time step, i.e. $t = 2$. Calculate the option price if the exercise style and payoff are:
   (a) European put option.
   (b) American put option.
   (c) European call option.
   (d) American call option.

4. Let W_t denote a standard Brownian motion. Calculate the following probabilities:
   (a) $\mathbb{P}(W_2 < 0 | W_1 > 0)$
   (b) $\mathbb{P}(W_1 \times W_2 < 0)$
   (c) $\mathbb{P}(W_1 < 0 \cap W_2 < 0)$

5. Let W_t denote a standard Brownian motion. Let $s < t$, determine the variance of
   (a) $V\Big[(W_t - W_s)\Big]$.
   (b) $V\Big[(W_t - W_s)^2\Big]$.

6. Let W_t denote a standard Brownian motion. Evaluate the following expectation

$$\mathbb{E}[|W_{t+\Delta t} - W_t|],$$

   where $|\cdot|$ denote absolute value.