Stochastic_Discount_Factor

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# 1 QF600 - Homework 5

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## 1.1 Stochastic Discount Factor

[1]:
```
import pandas as pd
import warnings
import numpy as np
import matplotlib.pyplot as plt

warnings.filterwarnings("ignore", category=UserWarning, module="openpyxl")
warnings.filterwarnings("ignore", category=UserWarning, module="matplotlib")
```

Suppose that consumption growth has lognormal distribution with the possibility of rare disasters:

$$\ln \tilde{g} = 0.02 + 0.02\tilde{\epsilon} + \tilde{\nu}$$

Here is a standard normal random variable, while is an independent random variable that has value of either zero (with probability of 98.3%) or ln(0.65) (with probability of 1.7%). Simulate with (at least) 10^4 random draws from standard normal distribution, and simulate with (at least) 10^4 random draws from standard uniform distribution.

[2]:
```
def estimate_pricing_kernel(size, p_nu, phi, gamma)->np.array:
    epsilon = np.random.standard_normal(size)
    nu = np.random.choice([1,0], size=size, p=[p_nu,1-p_nu]) * np.log(phi)


    ln_g = .02 + .02 * epsilon + nu
    est_M = 0.99*np.power(np.exp(ln_g),-gamma)

    mean_M = np.mean(est_M)
    sig_M = np.std(est_M)
    sig_M_over_mean_M = sig_M / mean_M

    return mean_M, sig_M, sig_M_over_mean_M
```

Use the simulated distribution of consumption growth to find the simulated distribution of the pricing kernel for power utility:

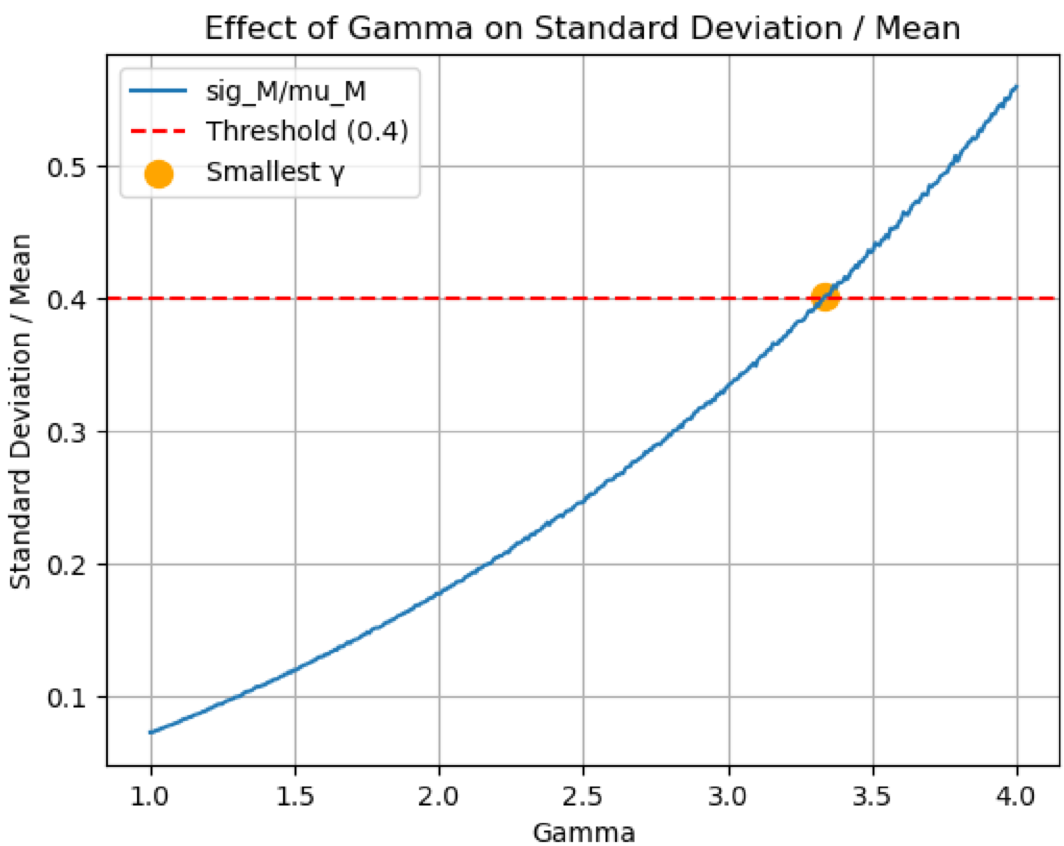
$$\tilde{M} = 0.99\tilde{g}^{-\gamma}$$

Repeat this process for values of   in the range from 1 to 4, in increments of 0.1 (or less). (You can reuse the same simulated distribution of consumption growth for all values of  ) - Calculate the mean ( M) and standard deviation ( M) of pricing kernel for each value of  , and plot the volatility ratio ( M/ M) on the vertical axis vs   on the horizontal axis. - Find the smallest value of   (in your data) for which  M/ M > 0.4.

[3]:
```
size         = 10**6
p_nu         = .017
phi          = .65
arr_gamma    = np.linspace(1,4,400)

# Generate results for gamma values from 1 to 4
results_df = pd.DataFrame(columns=['Gamma', 'Mean_M', 'Sig_M',␣
 ↪'Sig_M_over_Mean_M'])

for gamma in arr_gamma:
    mean_M, sig_M, sig_M_over_mean_M = estimate_pricing_kernel(size, p_nu, phi,␣
 ↪gamma)
    # Create a temporary DataFrame for the current result
    temp_df = pd.DataFrame({
        'Gamma': [gamma],
        'Mean_M': [mean_M],
        'Sig_M': [sig_M],
        'Sig_M_over_Mean_M': [sig_M_over_mean_M]
    })
    # Concatenate the temporary DataFrame to results_df
    results_df = pd.concat([results_df, temp_df], ignore_index=True)

# Find the smallest gamma where Sig_M_over_Mean_M > 0.4
threshold = 0.4
condition_met = results_df[results_df['Sig_M_over_Mean_M'] > threshold]\
smallest_gamma = condition_met['Gamma'].iloc[0] if not condition_met.empty else␣
 ↪None

# Display the smallest gamma
print(f"The smallest value of gamma for which sig_M/mu_M > 0.4 is:␣
 ↪{smallest_gamma}")

# Plotting the last output against gamma
plt.plot(results_df['Gamma'], results_df['Sig_M_over_Mean_M'], label='sig_M/
 ↪mu_M')
plt.axhline(y=threshold, color='r', linestyle='--', label='Threshold (0.4)')
if smallest_gamma is not None:
```

```
    plt.scatter(smallest_gamma, results_df['Sig_M_over_Mean_M'].
 ↪iloc[condition_met.index[0]], color='orange', s=100, label='Smallest ')
plt.xlabel('Gamma')
plt.ylabel('Standard Deviation / Mean')
plt.title('Effect of Gamma on Standard Deviation / Mean')
plt.legend()
plt.grid()
plt.show()
```

```
The smallest value of gamma for which sig_M/mu_M > 0.4 is: 3.338345864661654
```




> Explain (in words, without using mathematical equations or formulas) the economic significance of this result.

The results show that investors must account for potential economic disruptions caused by rare disasters, which greatly increase consumption growth changes. Considering Consumption CAPM and H-J bound there is an equity premium puzzle where an unreasonably high degree of (relative) risk aversion is given by gamma greater than 20. However by including rare disasters, it is demonstrated that a moderate level of risk aversion can explain the higher-than-expected returns on equities where gamma is more reasonably around 3.3. Also, it underscores the importance of considering historical data in light of these disruptions, as excluding outliers them can lead to misleading interpretations of stability.