QF620 Assignment 4

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$V_T = \sqrt{S_T}$

(a)

so using martingale valuation framework, we have

$$S_T = S_0 e^{(r-\frac{\sigma^2}{2})T+\sigma W_T^*}$$

$$\begin{aligned}
\frac{V_0}{B_0} &= \mathbb{E}\left[\frac{V_T}{B_T}\right] \\
V_0 &= e^{-rT}\mathbb{E}[V_T] \\
&= e^{-rT}\mathbb{E}\left[\sqrt{S_0 e^{(r-\frac{\sigma^2}{2})T+\sigma W_T^*}}\right] \\
&= e^{-rT}\sqrt{S_0 e^{(r-\frac{1}{2}\sigma^2)T}}\,\mathbb{E}\left[e^{\frac{\sigma}{2}W_T^*}\right] \\
&= \sqrt{S_0}e^{(-r-\frac{\sigma^2}{4})\frac{T}{2}}
\end{aligned}$$

(b)

Using the Carr and Madan static replication formula, we have

$$\begin{aligned}
h(S_T) &= \sqrt{S_T} \\
h^{'}(S_T) = -\frac{1}{2S_T^{\frac{1}{2}}}, \quad h^{''}(S_T) &= -\frac{1}{4S_T^{\frac{3}{2}}}
\end{aligned}$$

$$\begin{aligned}
V_0 &= e^{-rT}\sqrt{S_0 e^{rT}} - \int_0^F \frac{1}{4K^{\frac{3}{2}}}P(K)\,dK - \int_F^\infty \frac{1}{4K^{\frac{3}{2}}}C(K)\,dK \\
&= \sqrt{\frac{S_0}{e^{rT}}} - \int_0^F \frac{1}{4K^{\frac{3}{2}}}P(K)\,dK - \int_F^\infty \frac{1}{4K^{\frac{3}{2}}}C(K)\,dK
\end{aligned}$$