

# A Glimpse into Advances of Mesh Representation Learning

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# Outline

- 1 Motivation
- 2 Common Industrial Approaches
  - Edge Collapse
  - Remeshing
- 3 Recent Advances
  - Discrete Riemannian Geometry
  - Mesh Representation Learning
- 4 Conclusion

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# 3D Object Representations

- Modern CG aims to render 3D objects in real-time [Akenine-Möller et al., 2018].
- 3D Objects has many representations:



	Volumetric	Point Cloud	Polygonal Mesh
Memory Efficiency	Poor	Not good	Good
Textures	Not good	No	Yes

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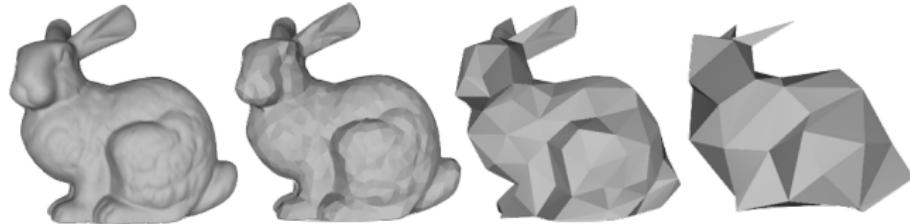


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- Most of the objects are represented via **triangulated meshes**.

# Mesh Simplification 101

**Mesh Simplification** is one of the key phase in 3D surface reconstruction (modeling).



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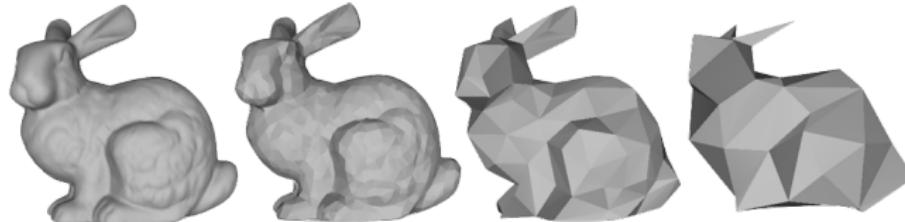
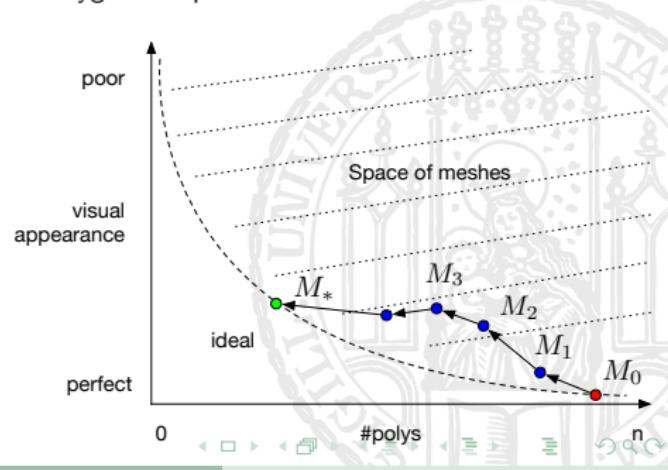


Figure: High Polygonal and Low Polygonal Representation

- Two common techniques:
  - Reduction: polygon reduction
  - Remeshing: topological reconstruction
- Process:  $M_0$  (Initial Mesh)  $\rightarrow M_1$   $\rightarrow M_2 \rightarrow \dots \rightarrow M_*$  (Ideal Mesh)
- Factors: #polygons, viewport, ...



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- **AR/VR/MR:** reduce huge memory and computation consumption for mobile applications
- **3D Print:** reduce the complexity of a object to speed up the print
- **Biometric Authentication:** efficient model and recognize 3D faces
- and more...

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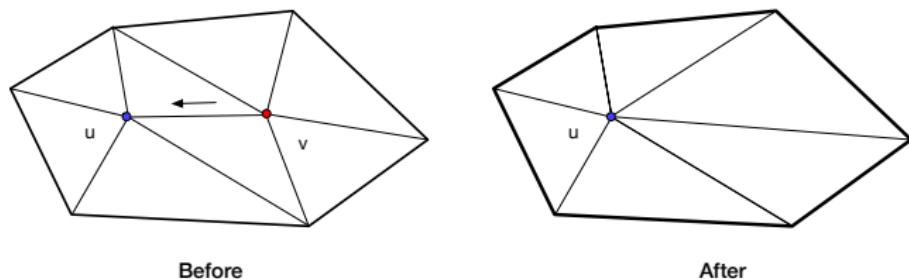
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Mesh Energy  $E(\mathcal{M})$  [Hoppe et al., 1993]:

$$E(\mathcal{M}) = E_{\text{dist}}(K, V) + E_{\text{rep}}(K) + E_{\text{spring}}(K, V)$$

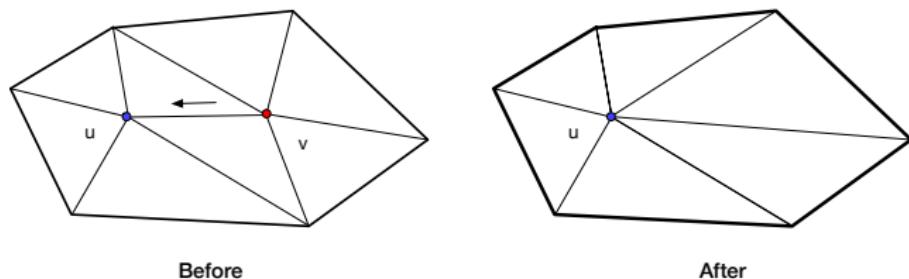
- $E_{\text{dist}}$ : distance energy sum of squared distances from  $X$  to the mesh
- $E_{\text{rep}}$ : representation energy penalizes meshes with a large number of vertices
- $E_{\text{spring}}$ : spring energy penalizes sharp dihedral angles in the mesh

# Edge Collapse



Minimize the (curvature) cost of collapse:

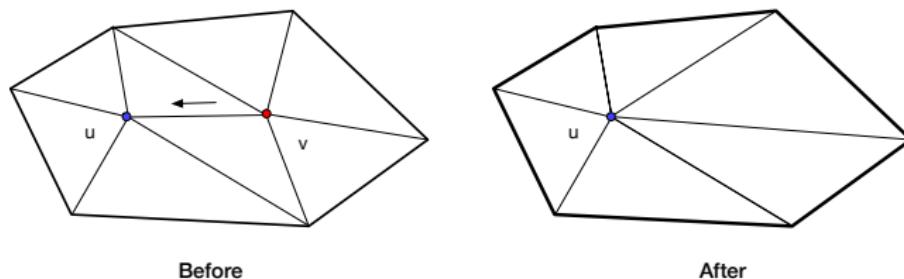
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$$\text{cost}(u, v) = \underbrace{\|u - v\|}_{\text{distance}} \times \max_{f \in T_u} \left\{ \min_{n \in T_{uv}} \left\{ \underbrace{1 - f \cdot \text{normal} \cdot n \cdot \text{normal}}_{\text{curvature}} \right\} \right\}$$

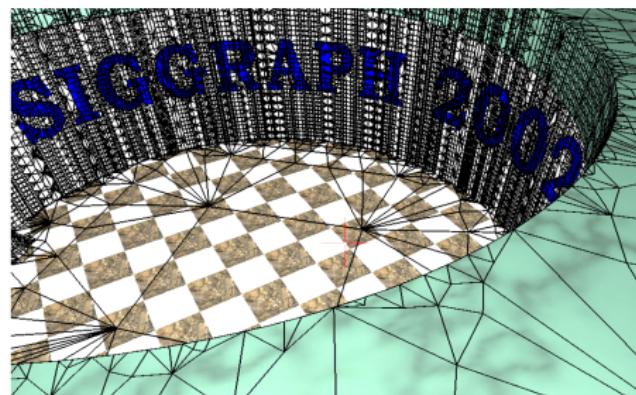
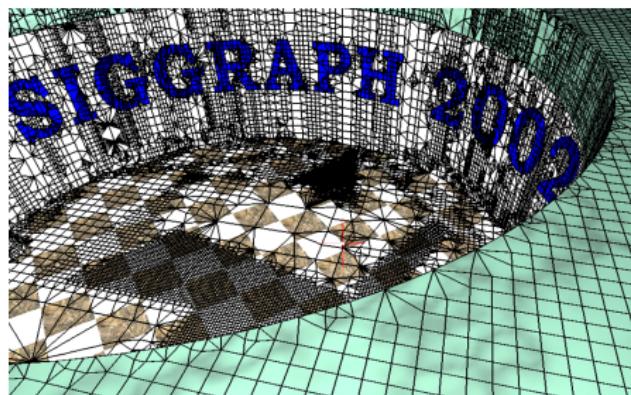
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Find a approximation of existing mesh. Key ideas:

- ① reduce or increase polygons
- ② construct octree using quadradic errors and simplify the tree
- ③ recursively above steps



# Industrial Solutions

- Open source 😊:
  - **Blender**: Decimate Modifier<sup>1</sup>, Remesh Modifier<sup>2</sup>
  - **Meshlab**: Quadric Edge Collapse Decimation<sup>1</sup>

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A proficient designer takes almost 1 week:

---

**input** : Mesh  $M_0$ , Criterial (e.g. #polys, ...)

**output**: Simplified mesh  $M_*$

**begin**

**repeat**

**if** has good visual appearance **then**  
    | polygon reduction

**else**

    | remeshing

**end**

**until** converge to target criterial;

**end**

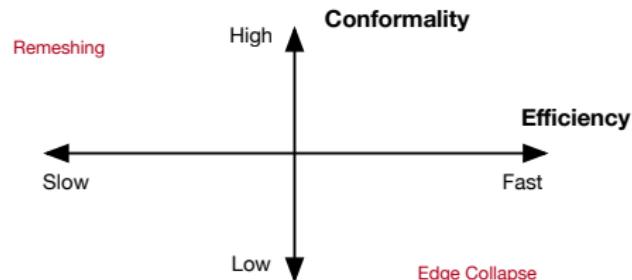
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# Comparison of Edge Collapse and Remeshing

Practical experience:



More: Topology-preserving,  
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distance-preserving, ...



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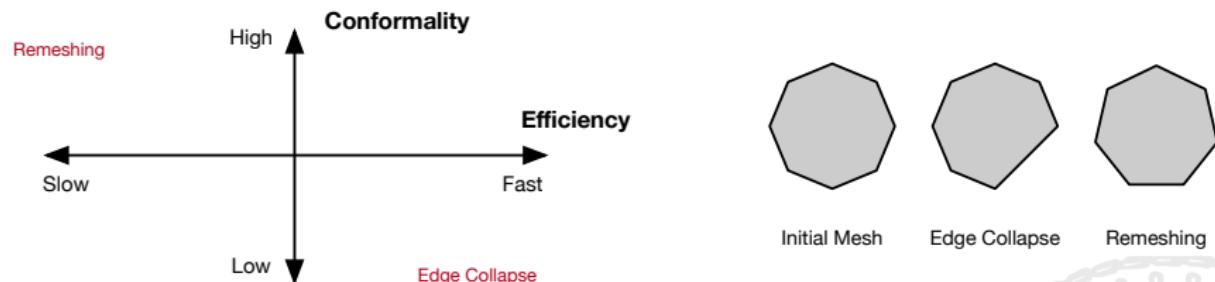


Figure: An octagon mesh simplification: comparing remeshing and edge collapse

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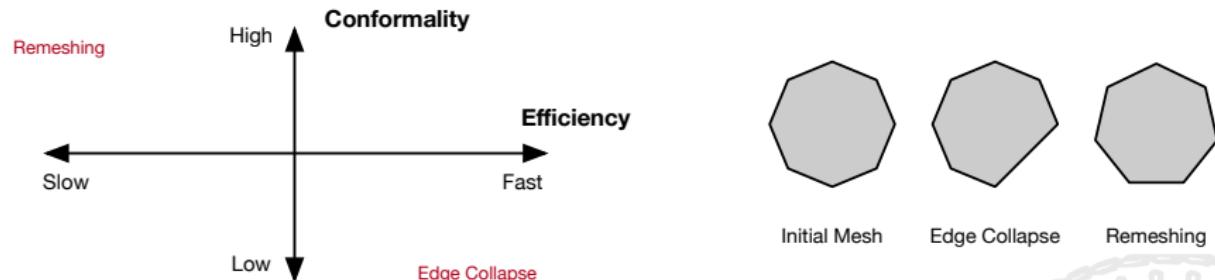


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More: Topology-preserving,  
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Is there a way to facilitate them both?

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# Bernhard Riemann



Figure: Bernhard Riemann (1826 – 1866)

Euclidean Parallel postulate (simplified): In a plane, given a line and a point not on it, at most one line parallel to the given line can be drawn through the point.

On the Hypotheses which lie at the Bases of Geometry.

Bernhard Riemann

Translated by William Kingdon Clifford

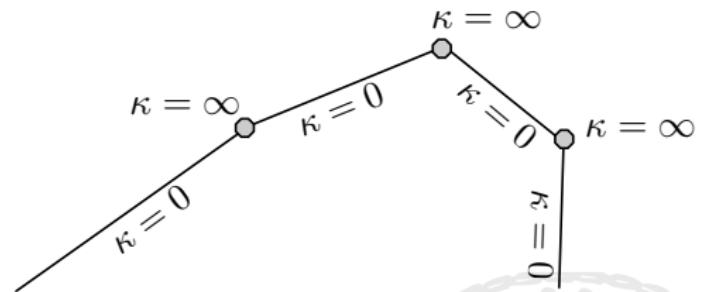
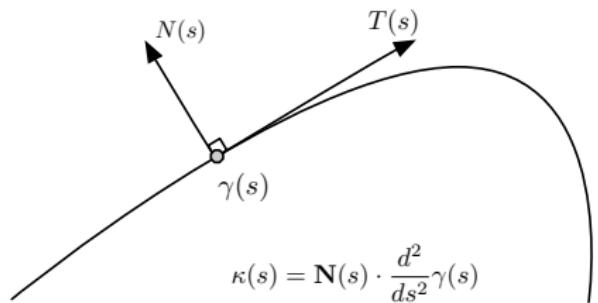
[*Nature*, Vol. VIII. Nos. 183, 184, pp. 14–17, 36, 37.]

*Plan of the Investigation.*

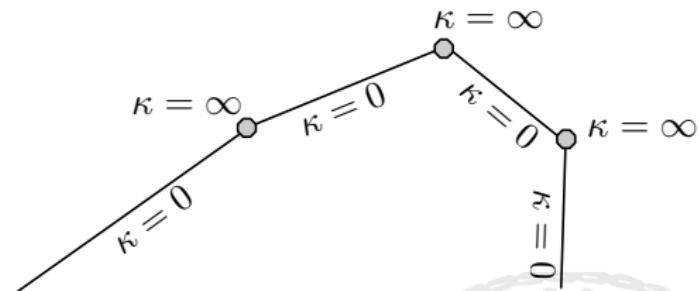
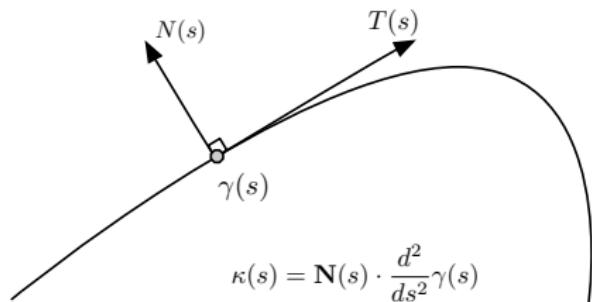
It is known that geometry assumes, as things given, both the notion of space and the first principles of constructions in space. She gives definitions of them which are merely nominal, while the true determinations appear in the form of axioms. The relation of these assumptions remains consequently in darkness; we neither perceive whether and how far their connection is necessary, nor *a priori*, whether it is possible.

From Euclid to Legendre (to name the most famous of modern reforming geometers) this darkness was cleared up neither by mathematicians nor by such philosophers as concerned themselves with it. The reason of this is doubtless that the general notion of multiply extended magnitudes (in which space-magnitudes are included) remained entirely unworked. I have in the first place, therefore, set myself the task of constructing the notion of a multiply extended magnitude out of general notions of magnitude. It will follow from this that a multiply extended magnitude is capable of different measure-relations, and consequently that space is only a particular case of a triply extended magnitude. But hence flows as a necessary consequence that the propositions of geometry cannot be derived from general notions of magnitude, but that the properties which distinguish space from other conceivable triply extended magnitudes are only to be deduced from experience. Thus arises the problem, to discover the simplest matters of fact from which the measure-relations of space may be determined; a problem which from the nature of the case is not completely determinate, since there may be several systems of matters of fact which suffice to determine the measure-relations of space—the most important system for our present purpose being that which Euclid has laid down as a foundation. These matters of fact are—like all

# Revisited: Curvature From Continus to Discrete



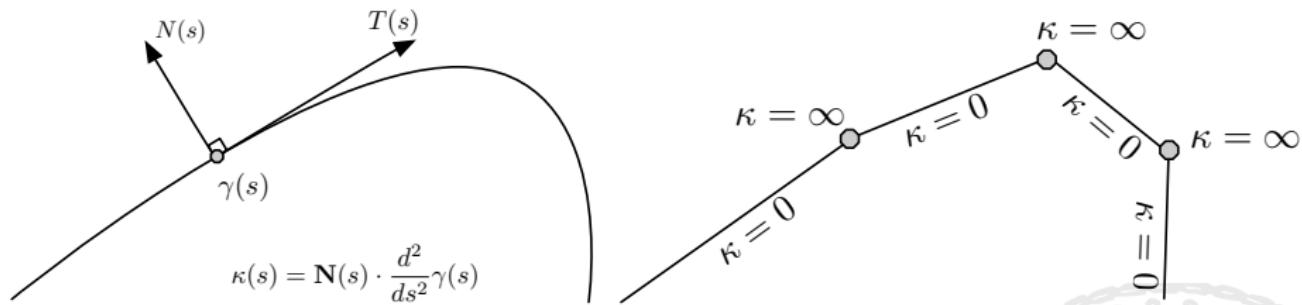
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Theorem (Whitney-Graustein 1937, human readable version)

*Not all properties of a smooth object can be preserved exactly at the discrete level.*

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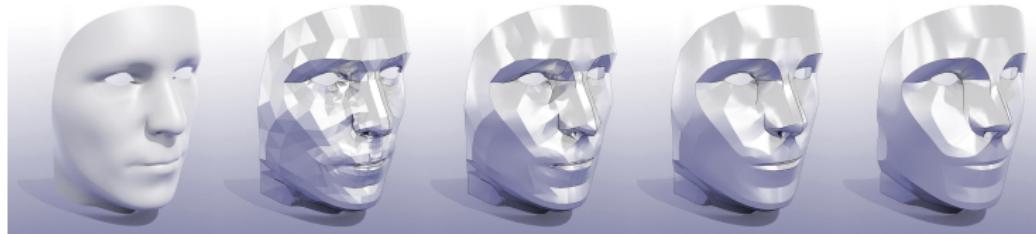
*Not all properties of a smooth object can be preserved exactly at the discrete level.*

→ **Discrete differential geometry** defines different discrete versions of definitions to preserve various critical geometric properties.

# Developable Surface [Stein et al., 2018]

Developable surface captures two key properties of smooth developable surfaces:

- Flattenability
- Presence of straight ruling lines



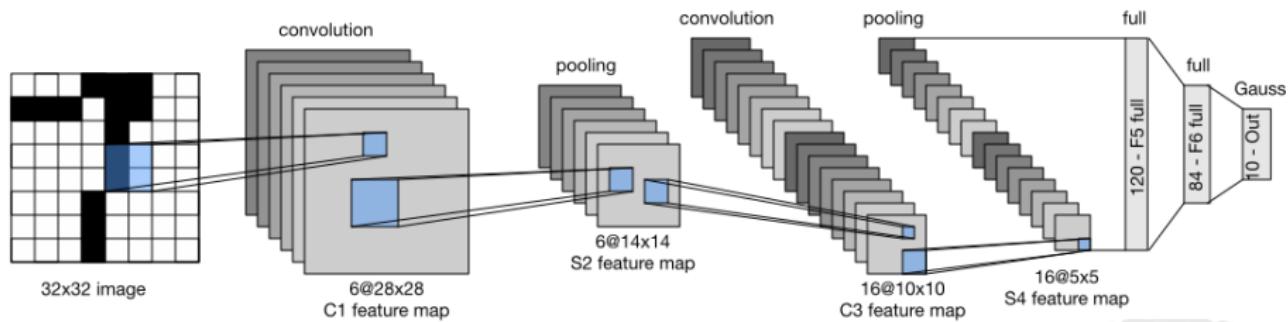
**Figure:** A given mesh evolves toward a shape comprised of flattenable pieces separated by highly regular seam curves.

# Key Properties of Riemannian Manifolds

- Flattenability
  - Presence of straight ruling lines
  - **Locality!**
  - ...

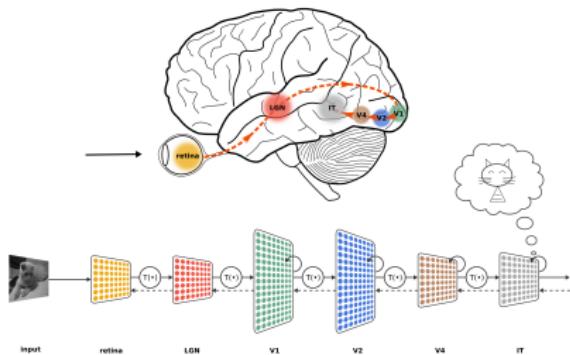
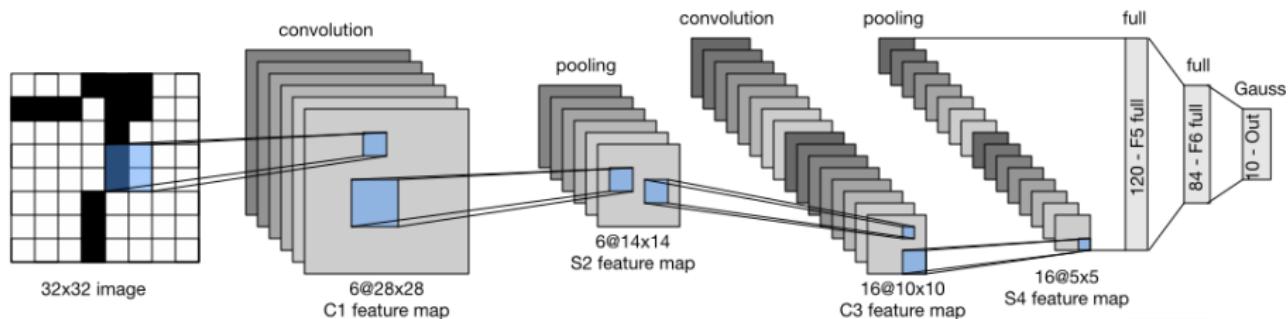
# Convolutional Neural Network (CNN)

LeNet-5 [LeCun et al, 1998]:



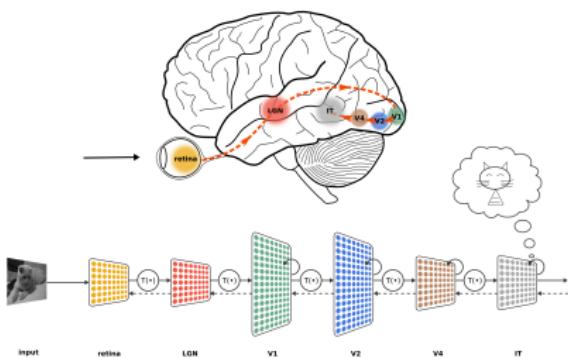
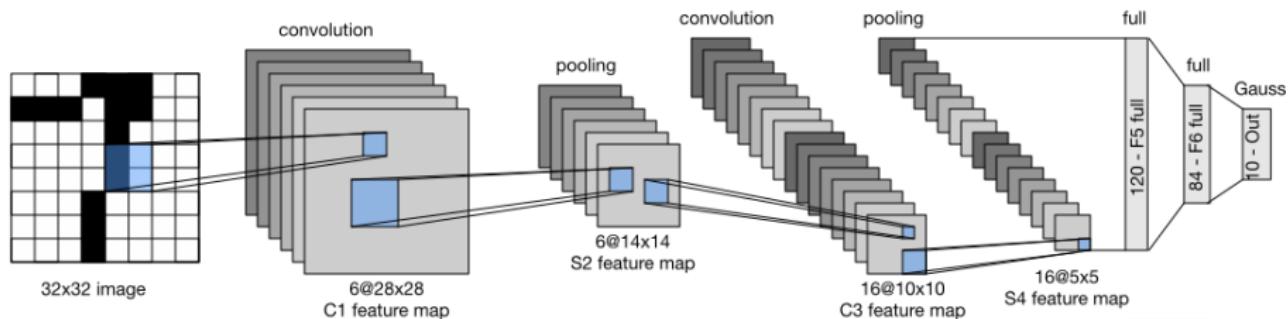
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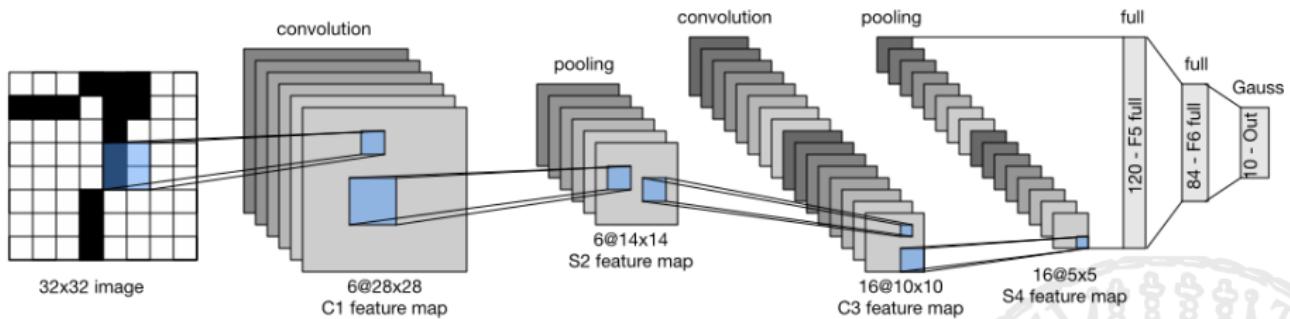


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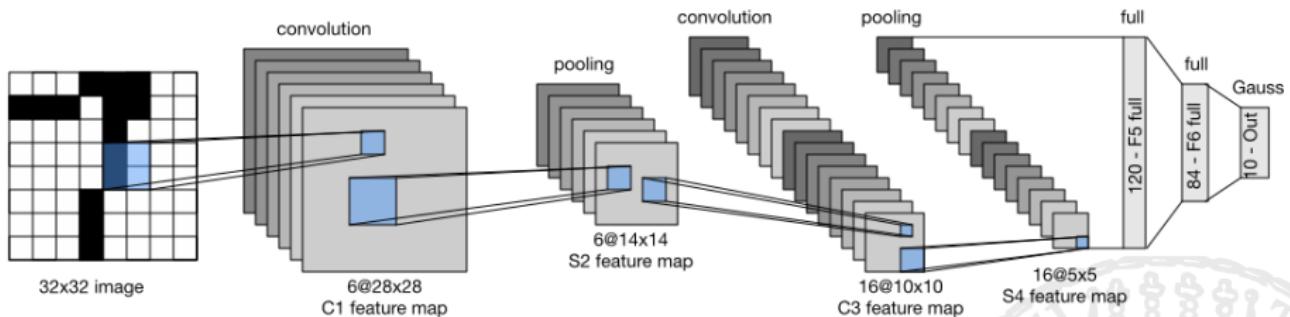
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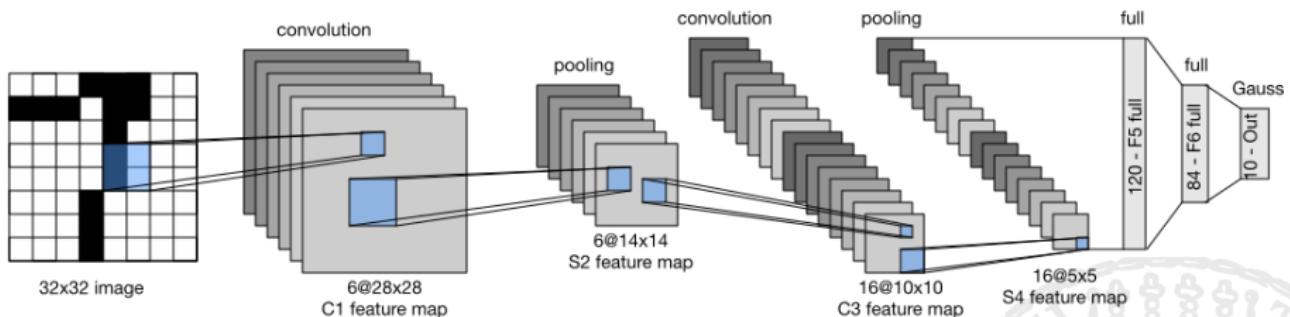


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- 😊 Convolutional (Translation invariance)

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- 😊 Filters localized in space (Deformation Stability)
- ...

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# Geodesic CNN [Masci et al., 2015]



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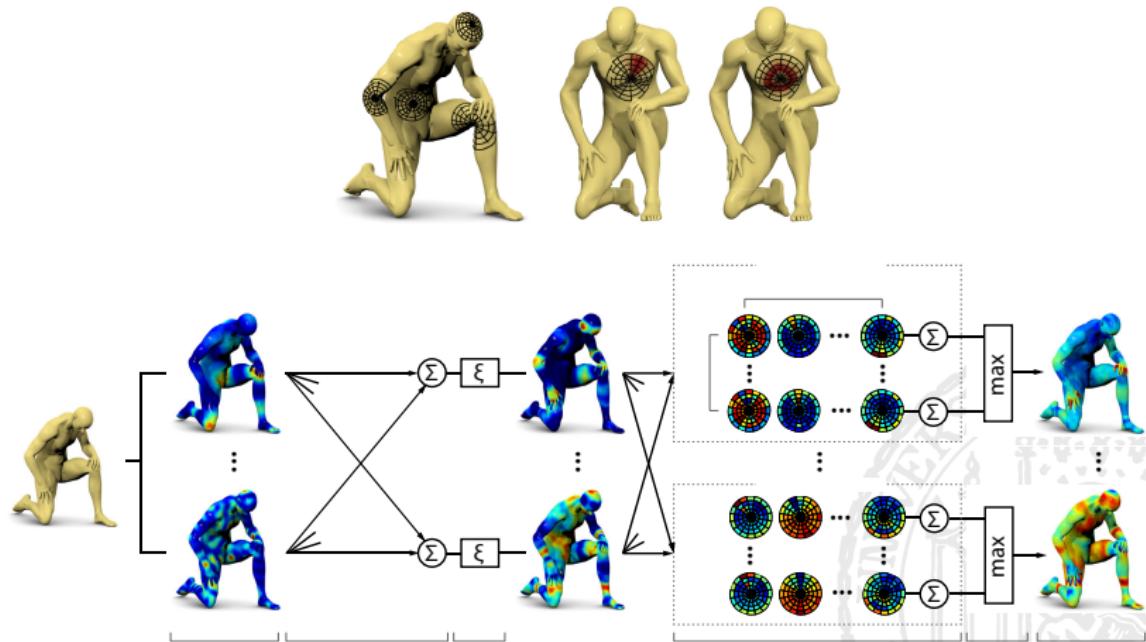
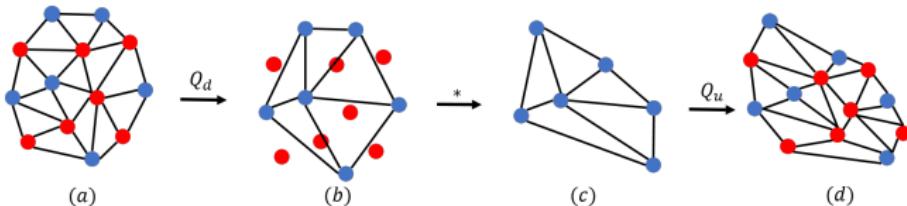


Figure: A Toy Geodesic CNN Architecture

# Conv. Mesh Autoencoders [Ranjan et al., 2018]



**Figure:** Mesh Sampling Op: (a) down-sampled (b) store the barycentric coordinates (c) transformed using convolutional operations (d) add contracted vertices at the barycentric locations.

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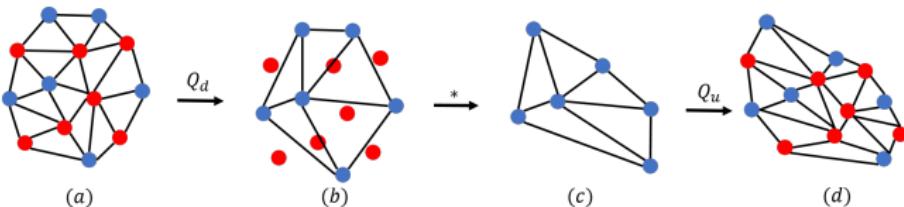


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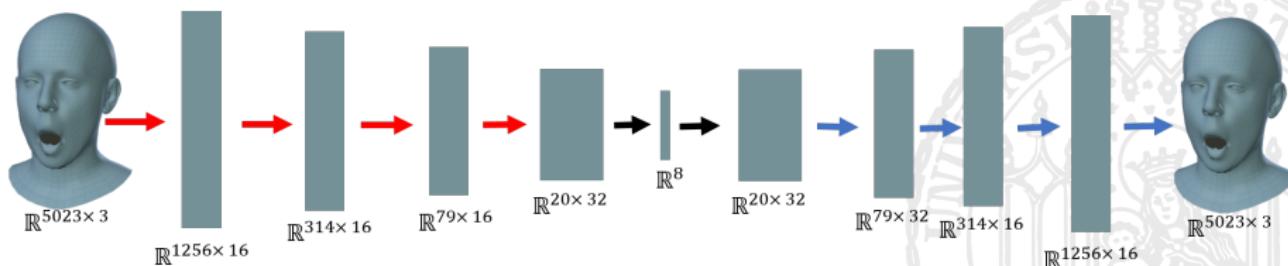


Figure: Convolutional Mesh Autoencoder: The red and blue arrows indicate down-sampling and up-sampling layers respectively.

# Conv. Mesh Autoencoder (cont.) [Ranjan et al., 2018]

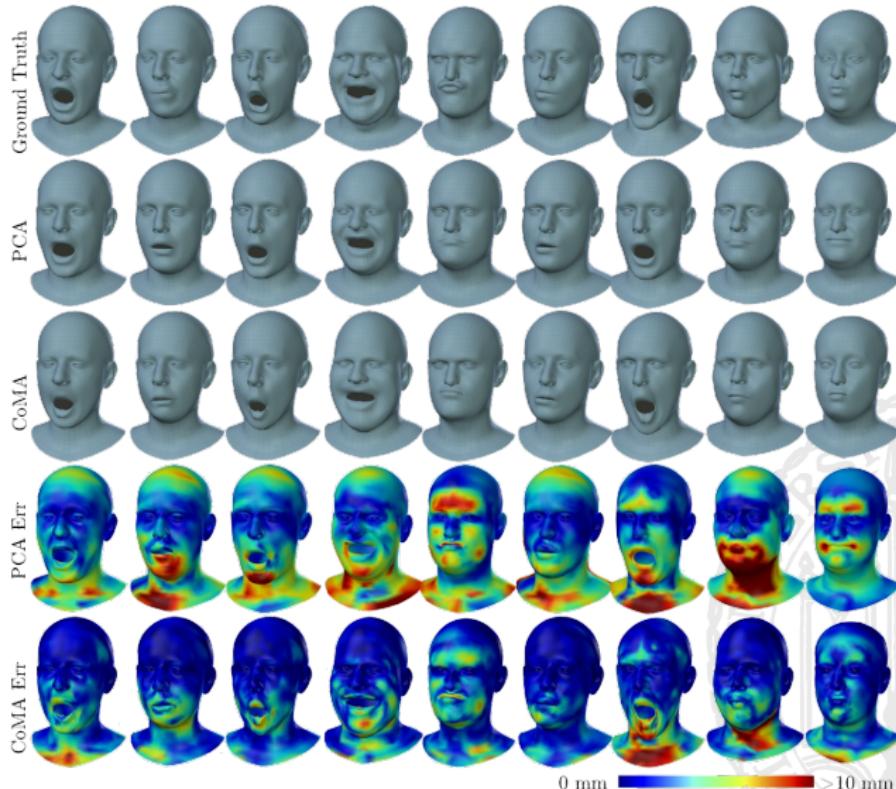


Figure: Comparison with PCA: Qualitative results for interpolation experiment

## More Variants

- SurfaceNet [Kostrikov et al., 2017]
  - MeshVAE [Tan et al., 2018]
  - MeshCNN [Hanocka et al., 2019]
  - MeshGAN [Cheng et al., 2019]
  - Volumetric Representation Approaches
  - Multi-view Representation Approaches
  - ...

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# Further Readings I

-  Akenine-Möller, T., Haines, E., Hoffman, N., Pesce, A., Iwanicki, M., and Hillaire, S. (2018).  
Real-Time Rendering (Fourth Edition).  
Taylor & Francis CRC Press.
-  Bronstein, M. M., Bruna, J., Lecun, Y., Szlam, A., and Vandergheynst, P. (2017).  
Geometric Deep Learning: Going beyond Euclidean data.  
IEEE Signal Processing Magazine, 34(4):18–42.
-  Hoppe, H., DeRose, T., Duchamp, T., McDonald, J., and Stuetzle, W. (1993).  
Mesh optimization.  
In Proceedings of the 20th annual conference on Computer graphics and interactive techniques - SIGGRAPH '93, volume 86, pages 19–26, New York, New York, USA. ACM Press.

## Further Readings II

- Ju, T., Losasso, F., Schaefer, S., and Warren, J. (2002).  
Dual contouring of hermite data.  
*21(3):339–346.*
  - Masci, J., Boscaini, D., Bronstein, M. M., and Vandergheynst, P. (2015).  
Geodesic Convolutional Neural Networks on Riemannian Manifolds.  
In 2015 IEEE International Conference on Computer Vision Workshop (ICCVW), pages 832–840. IEEE.
  - Stein, O., Grinspun, E., and Crane, K. (2018).  
Developability of triangle meshes.  
*ACM Transactions on Graphics (TOG)*, 37(4):1–14.



Thank you!