

Praktikum

# Geometry Processing

## 1 Introduction

Ludwig-Maximilians-Universität München

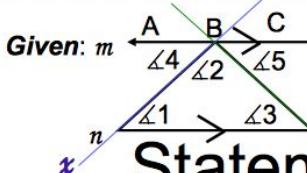
# Session 1: Introduction

- Motivation
- Recap: Graphics Pipelines
- Geometry Representations
- Blender Basics
- Summary

# This course is **not** about ...

Statement	Reason
<ol style="list-style-type: none"><li>1) Lines <u><math>m</math></u> and <u><math>n</math></u> are <u>parallel</u></li><li>2) <math>\angle ABC</math> is a <u>Straight</u> angle.</li><li>3) <u><math>m\angle ABC = 180^\circ</math></u></li><li>4) <math>m\angle 4 + m\angle 2 + m\angle 5 = m\angle ABC</math></li><li>5) <math>m\angle 4 + m\angle 2 + m\angle 5 = 180^\circ</math></li><li>6) <u><math>x</math> is <u>transversal</u> forming <math>\angle 1</math> &amp; <math>\angle 4</math></u> <u><math>y</math> is <u>transversal</u> forming <math>\angle 3</math> &amp; <math>\angle 5</math></u></li><li>7) <math>\angle 1</math> &amp; <math>\angle 4</math> are <u>alternate</u> Int. <math>\angle</math>s</li><li>8) <math>\angle 3</math> &amp; <math>\angle 5</math> are Alternate Int. <math>\angle</math>s</li><li>9) <math>\angle 1 \cong \angle 4</math> &amp; <math>\angle 3 \cong \angle 5</math></li><li>10) <math>m\angle 1 = m\angle 4</math> &amp; <math>m\angle 3 = m\angle 5</math></li><li>11) <u><math>m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ</math></u></li></ol>	<ol style="list-style-type: none"><li>1) <u>Given</u></li><li>2) <u>Definition</u> of Straight Angle</li><li>3) If Straight Angle, then 180</li><li>4) Angle Addition Postulate</li><li>5) Substitution <u>Property</u> of <u>Equality</u></li><li>6) Definition of Transversal(s)</li><li>7) Definition of Alt Interior Angles.</li><li>8) Definition of Alt Interior Angles</li><li>9) If <u>parallel</u> transversal then <u>congruent</u> Alt. Int. <math>\angle</math></li><li>10) Definition of <u>congruent</u> Angles</li><li>11) Substitution Property of =</li></ol> <p><i>QED</i></p>

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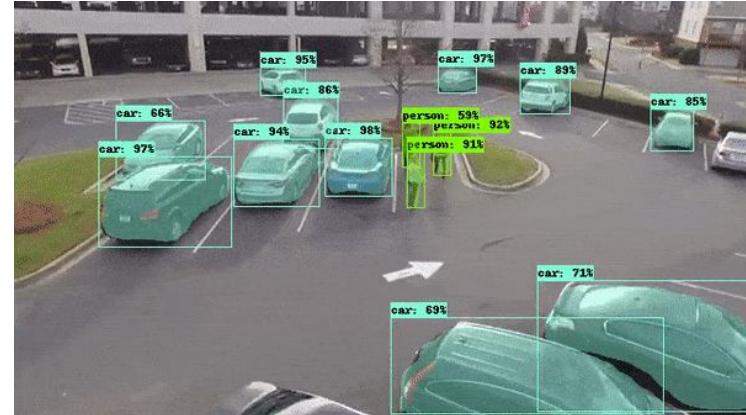
	
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*QED*

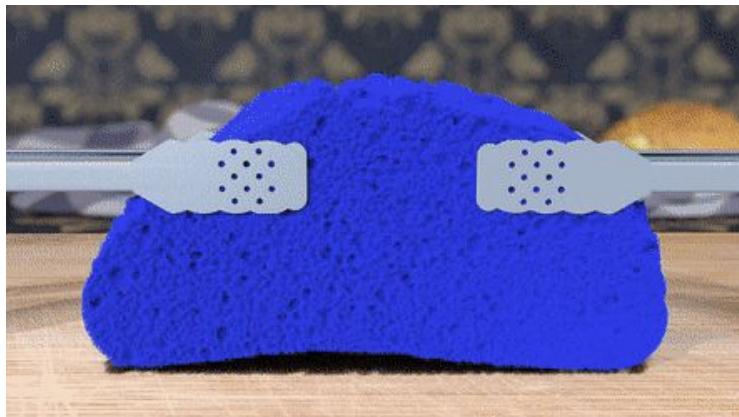
# This course is *not* about ...



[Yan et al. 2015]



[He et al. 2018]

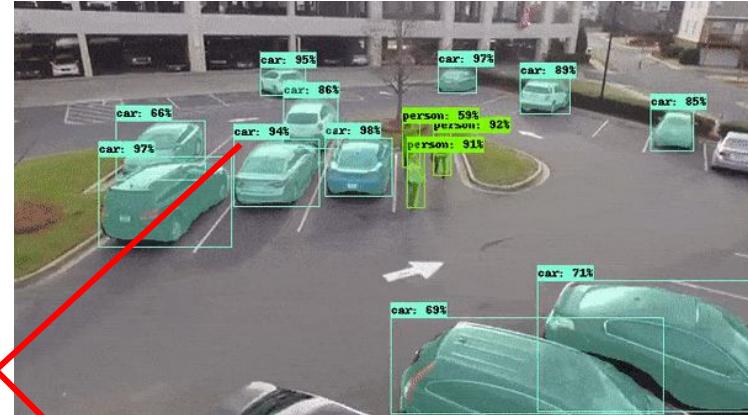


[Wolper et al. 2019]

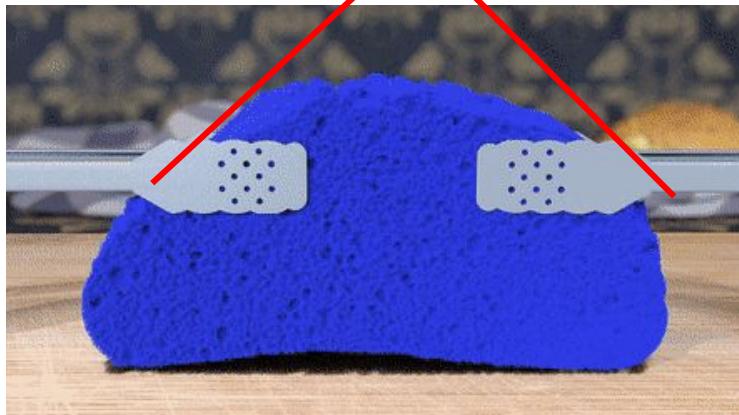
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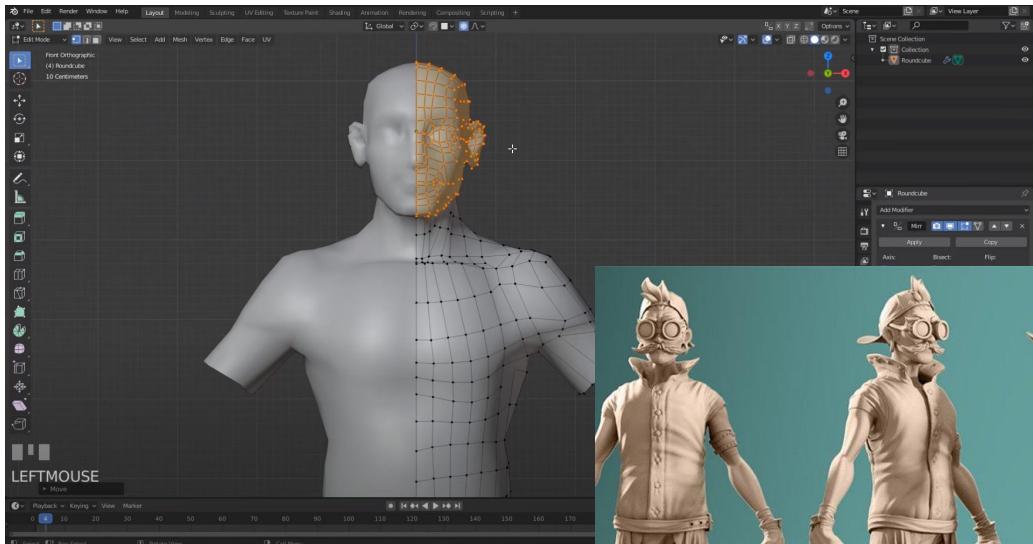


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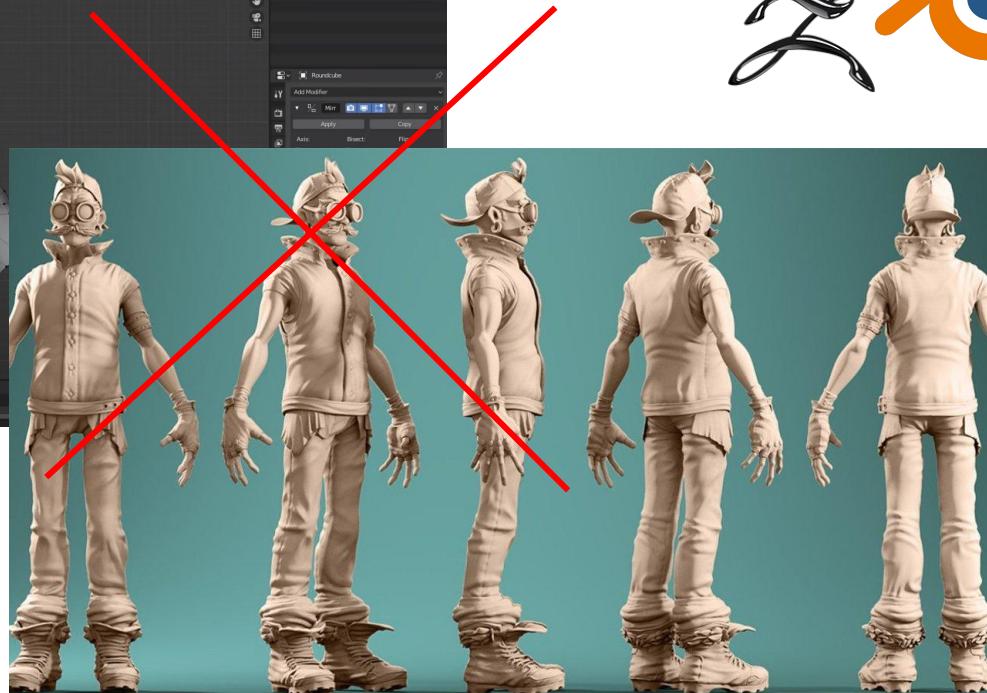
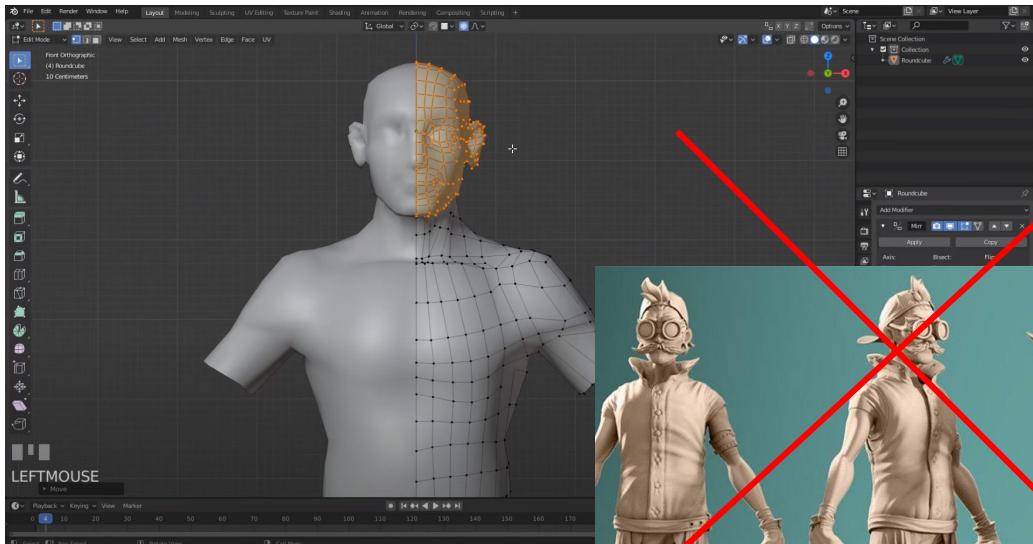


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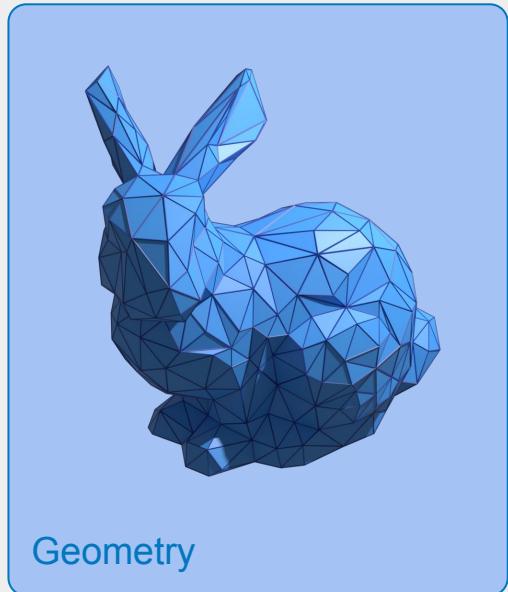
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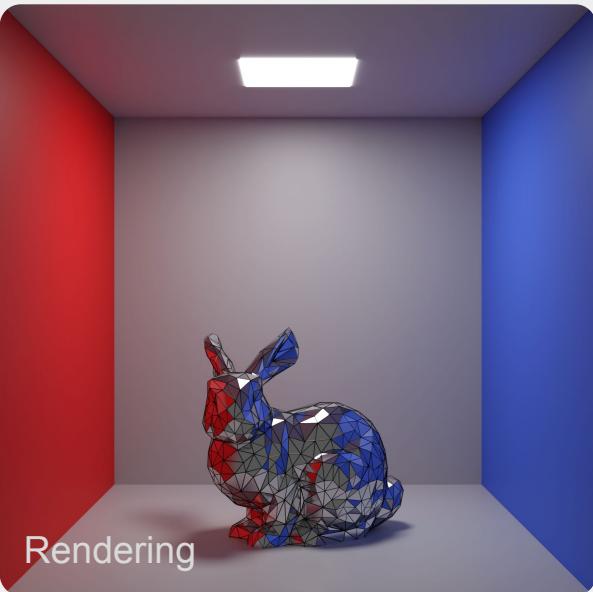
# This course is also *not* about ...



# This course is a direct extend to the CG1 for geometry



Geometry



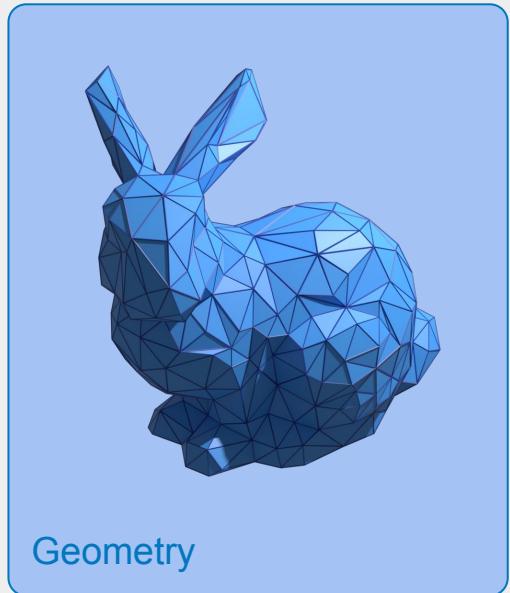
Rendering



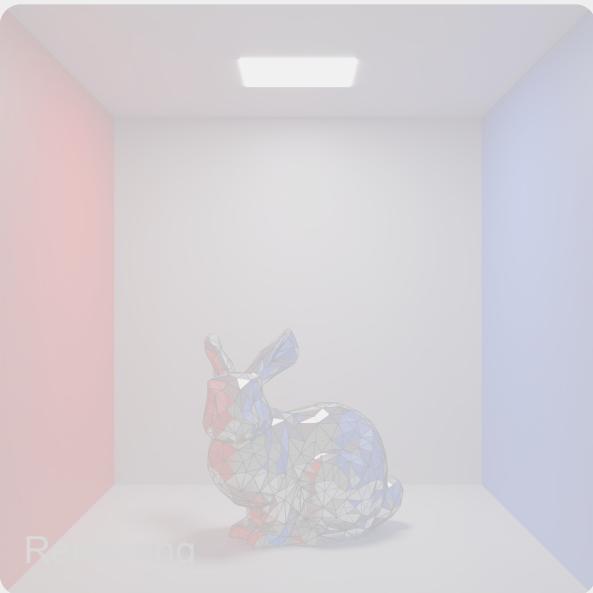
Animation

Computer Graphics

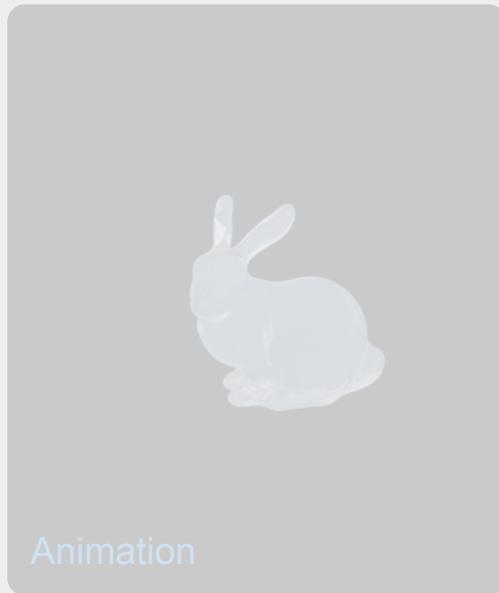
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Geometry



Rendering



Animation

Computer Graphics

# We will Focus on How to Deal with 3D Geometries *Algorithmically*



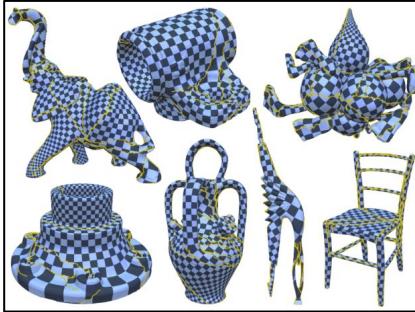
Curvature;

# We will Focus on How to Deal with 3D Geometries *Algorithmically*



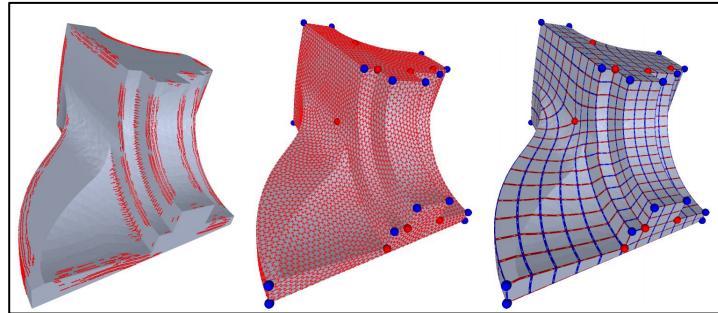
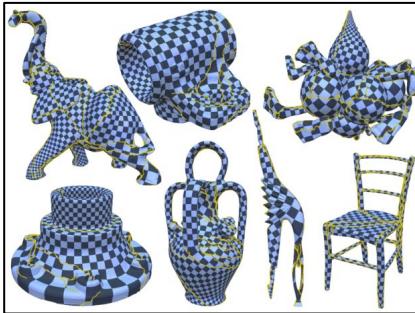
Curvature; Smoothing;

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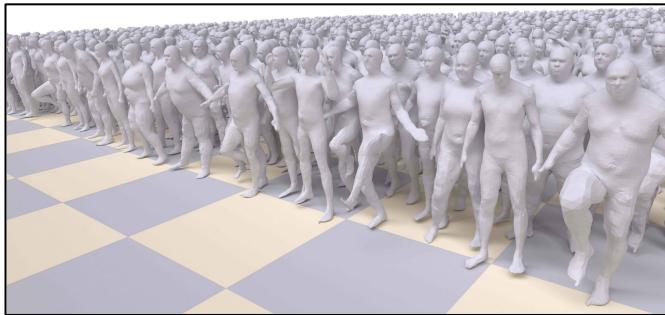
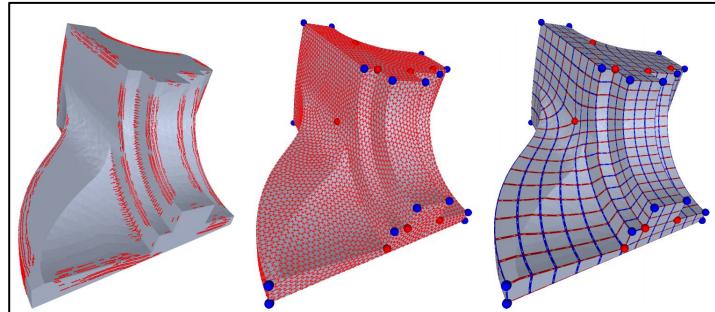
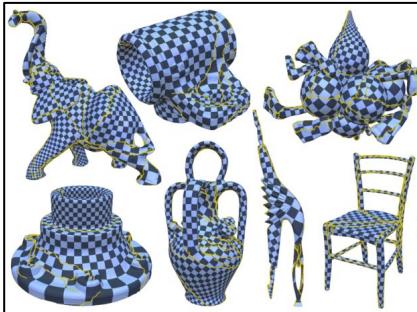
Curvature; Smoothing; Parameterization;

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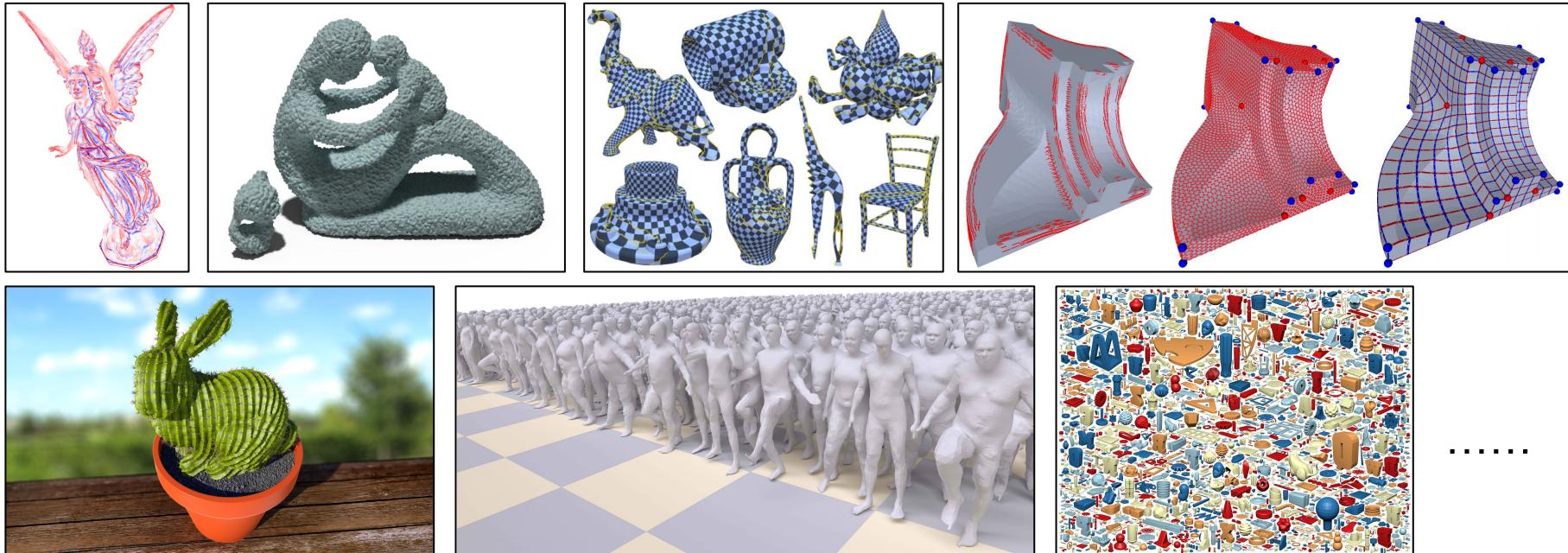
Curvature; Smoothing; Parameterization; Remeshing;

# We will Focus on How to Deal with 3D Geometries *Algorithmically*



Curvature; Smoothing; Parameterization; Remeshing; Deformation;

# We will Focus on How to Deal with 3D Geometries *Algorithmically*



Curvature; Smoothing; Parameterization; Remeshing; Deformation; Shape Analysis; ...

# This Course remains Interdisciplinary ...

$$\mathcal{H}(\mathcal{M}, \mathcal{M}') = \sqrt{\frac{1}{|\mathcal{S}|} \iint_{v \in \mathcal{S}} d(p, \mathcal{S}')^2 d\mathcal{S}}$$

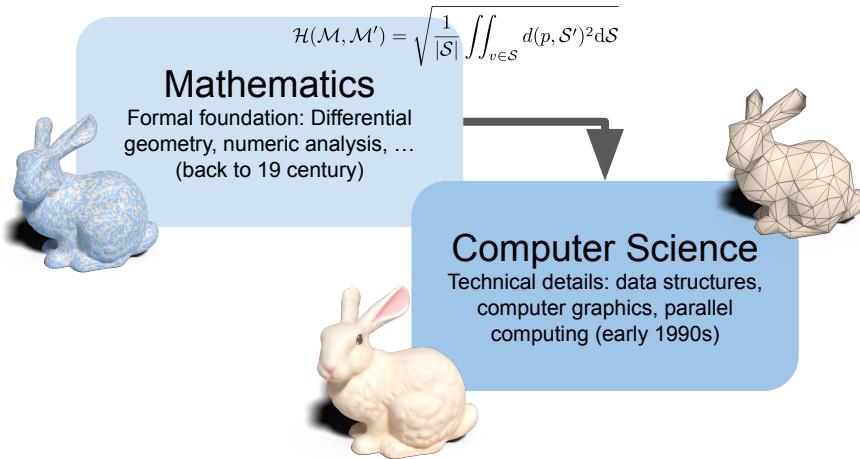
## Mathematics

Formal foundation: **Differential geometry**, numeric analysis, ...  
(back to 19 century)



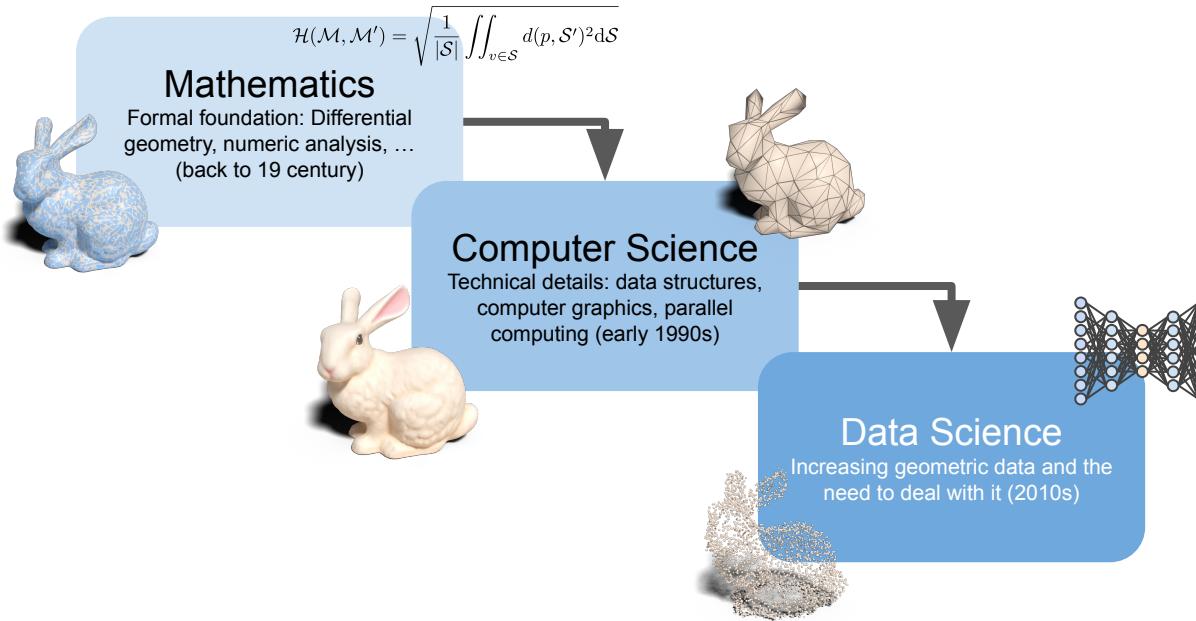
## Geometry Processing

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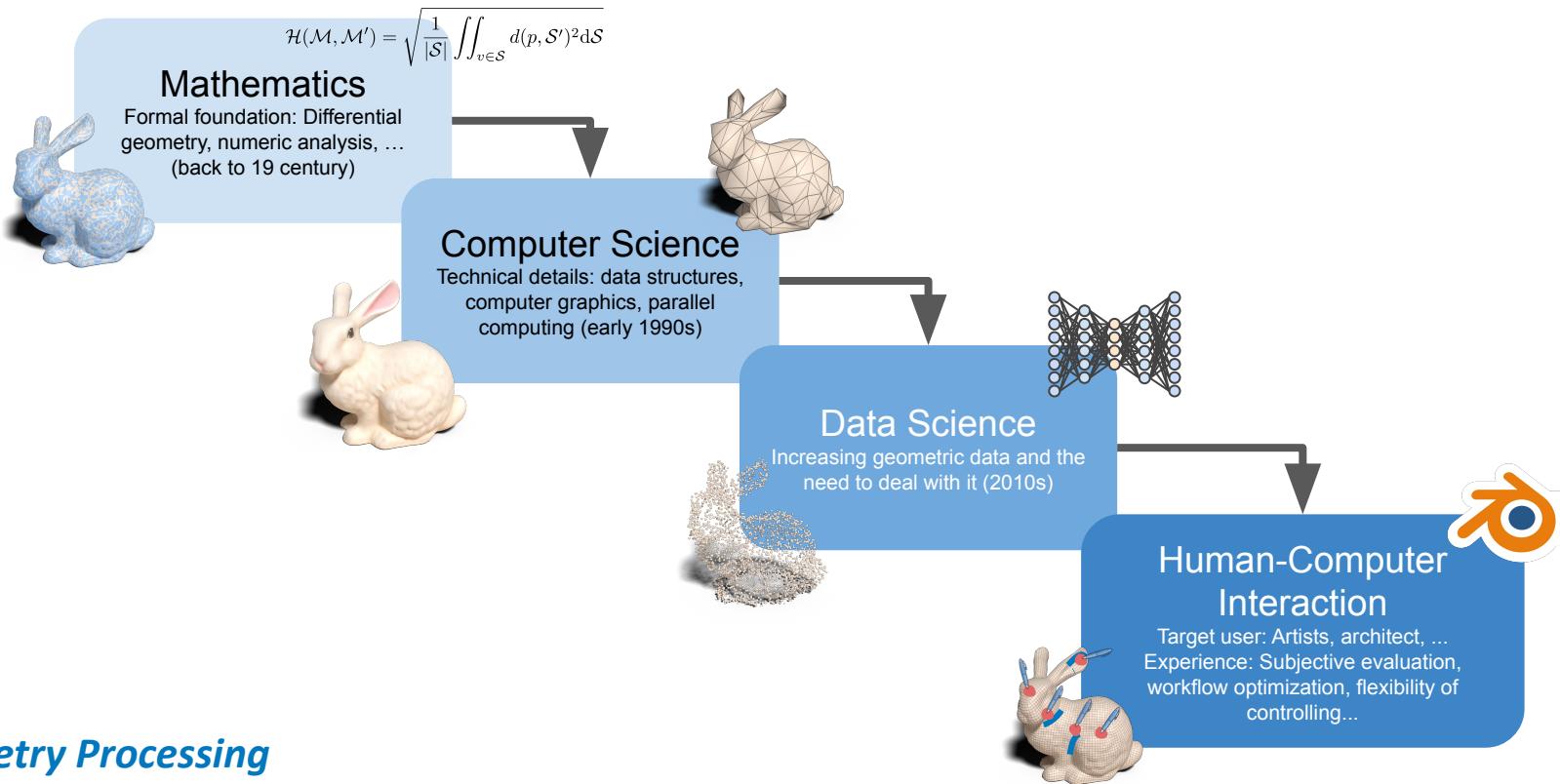
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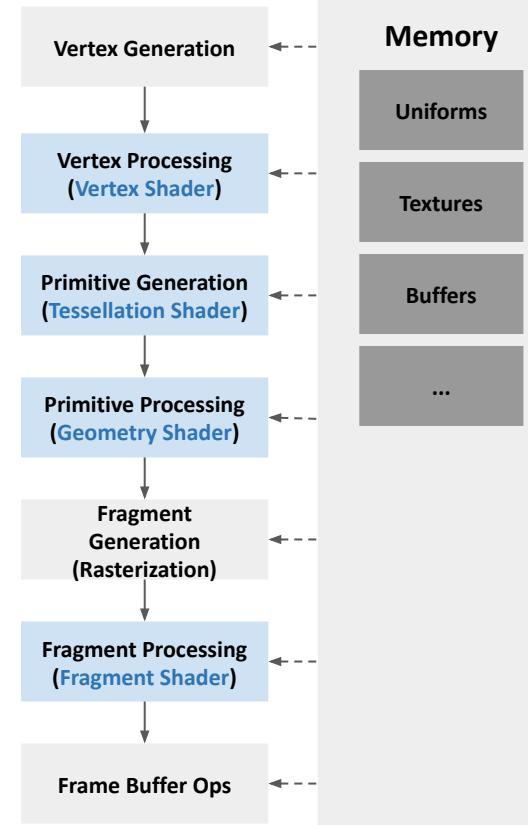
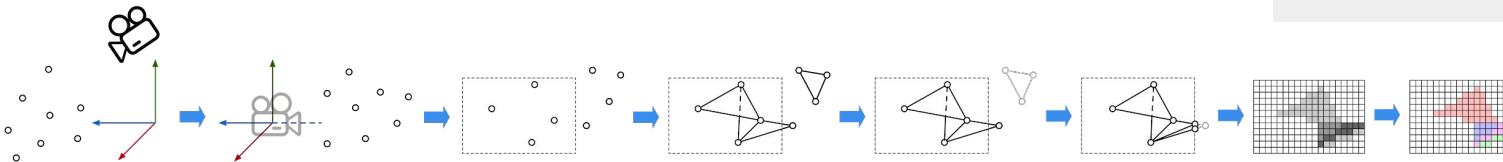


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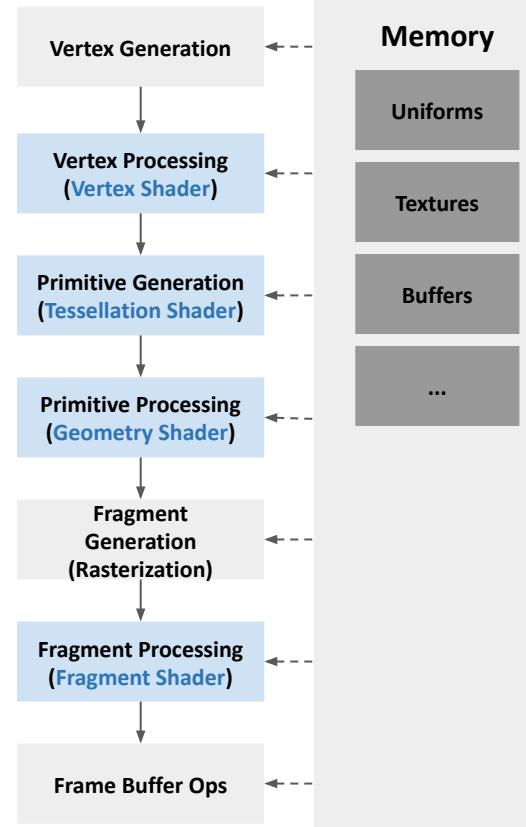
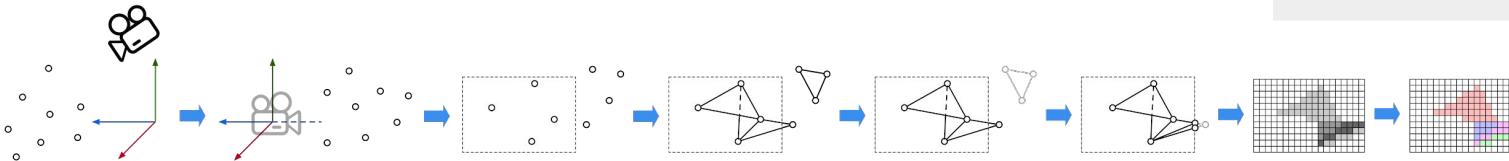
# Rasterization Pipeline

```
init frame buffer  
init depth buffer  
for each triangle t in scene {  
    tp = project(t)  
    for each pixel p in frame buffer {  
        if tp covers p {  
            if z passes depth test at p {  
                update z buffer and frame buffer  
            }  
        }  
    }  
}  
flush frame buffer to monitor
```



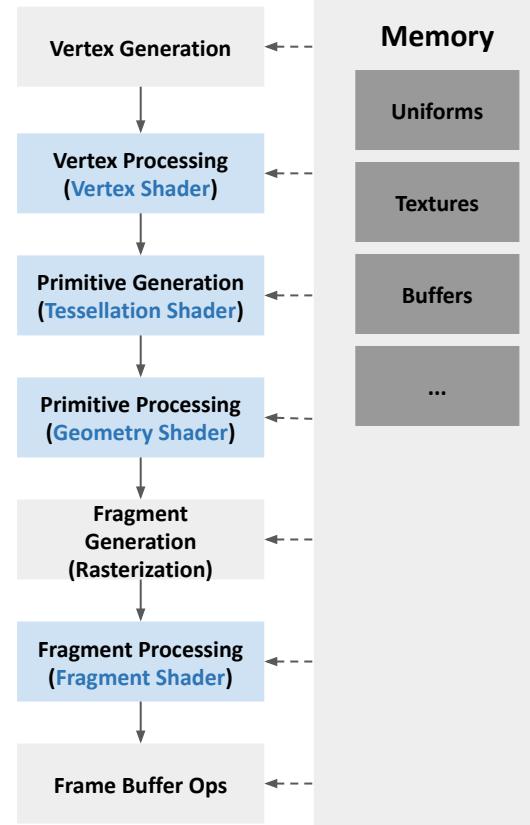
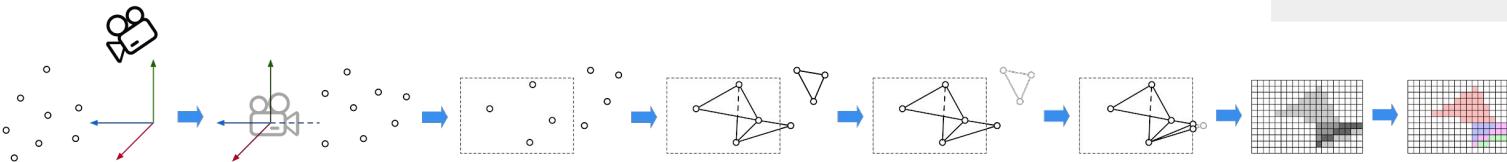
# Rasterization Pipeline

```
init frame buffer  
init depth buffer  
for each triangle t in scene {  
    tp = project(t) // MVP  
    for each pixel p in frame buffer {  
        if tp covers p {  
            if z passes depth test at p {  
                update z buffer and frame buffer  
            }  
        }  
    }  
}  
flush frame buffer to monitor
```



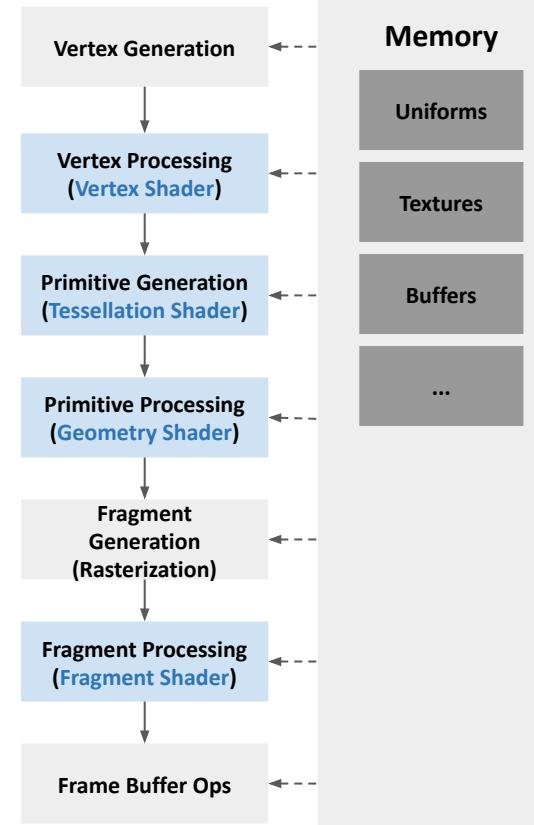
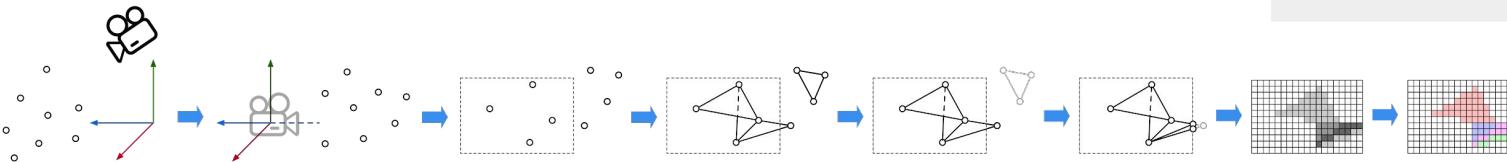
# Rasterization Pipeline

```
init frame buffer  
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for each triangle t in scene {  
    tp = project(t) // MVP  
    for each pixel p in frame buffer {  
        if tp covers p { // culling  
            if z passes depth test at p {  
                update z buffer and frame buffer  
            }  
        }  
    }  
}  
flush frame buffer to monitor
```



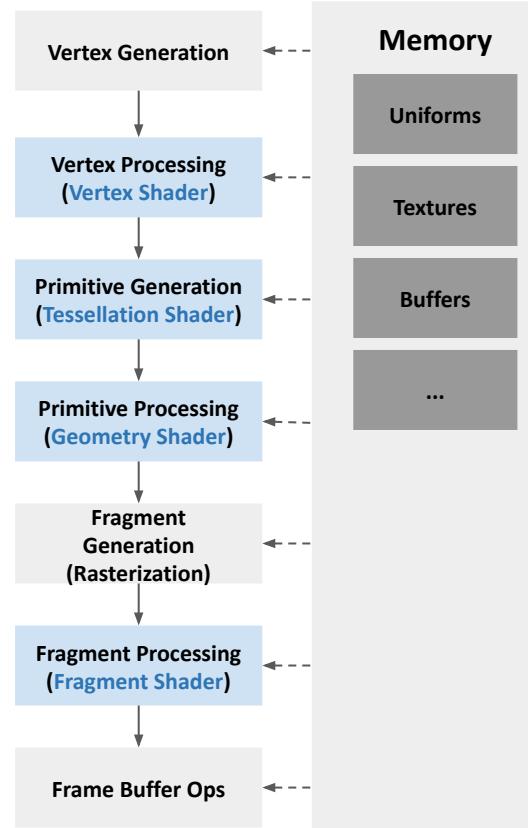
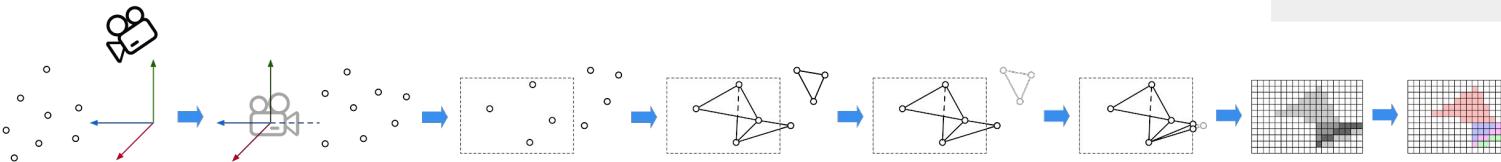
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                update z buffer and frame buffer  
            }  
        }  
    }  
}  
flush frame buffer to monitor
```



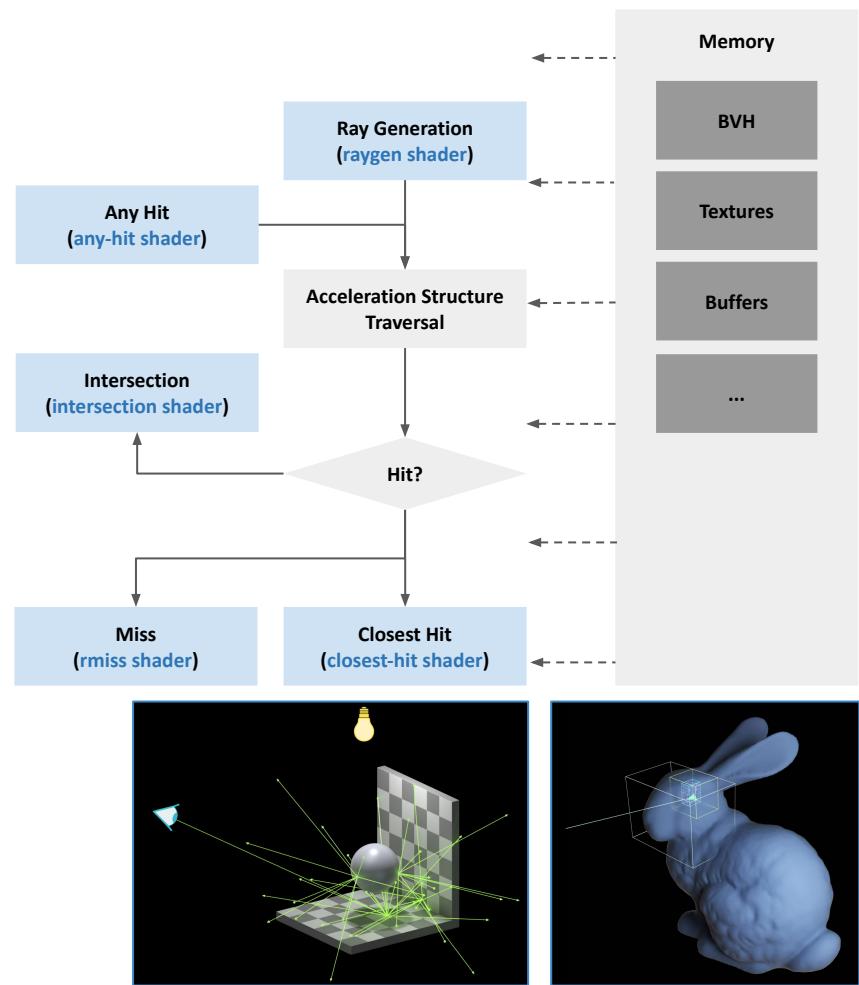
# Rasterization Pipeline

```
init frame buffer  
init depth buffer  
for each triangle t in scene {  
    tp = project(t) // MVP  
    for each pixel p in frame buffer {  
        if tp covers p { // culling  
            if z passes depth test at p { // depth-test  
                update z buffer and frame buffer // interpolation & update  
            }  
        }  
    }  
}  
flush frame buffer to monitor
```



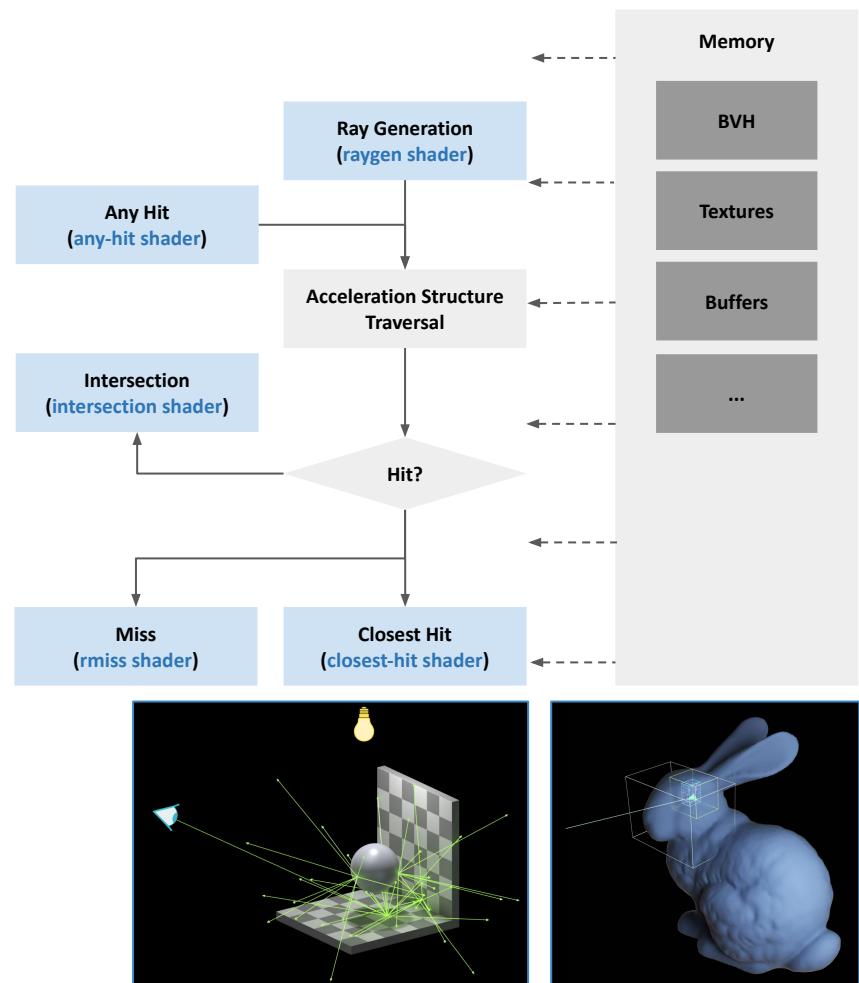
# Ray Tracing Pipeline

```
init frame buffer
for each pixel p in frame buffer {
    construct a ray from p
    for ray bounces is not over {
        for each triangle t in the scene {
            if ray hit t at x {
                keep x if closest and update the ray
                break
            }
        }
    }
    update frame buffer
}
flush frame buffer to monitor
```



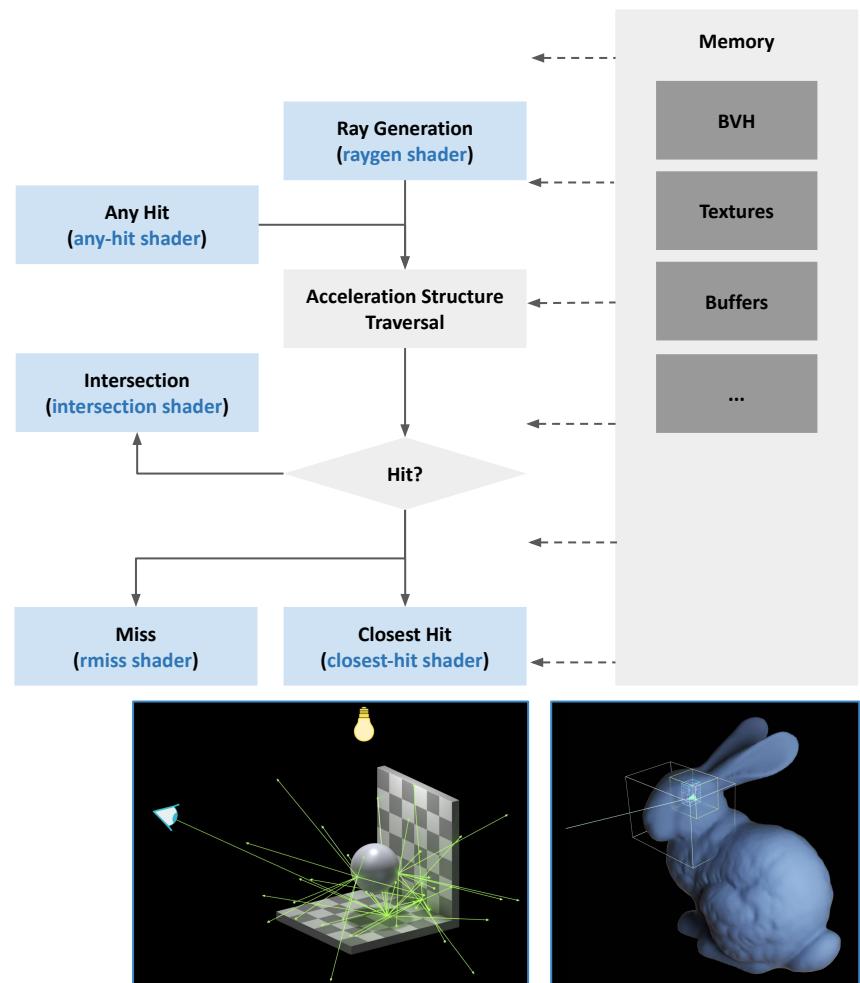
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    update frame buffer
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flush frame buffer to monitor
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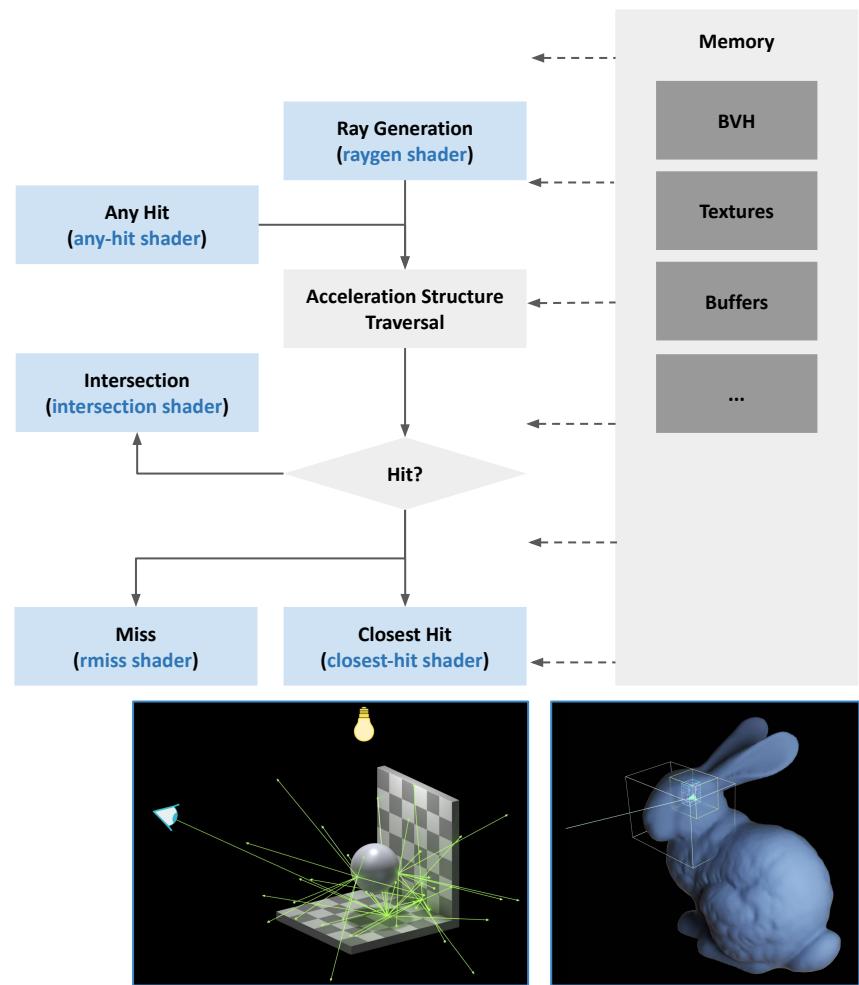
# Ray Tracing Pipeline

```
init frame buffer
for each pixel p in frame buffer {
    construct a ray from p          // ray generation
    for ray bounces is not over { // russian roulette
        for each triangle t in the scene {
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            }
        }
    }
    update frame buffer
}
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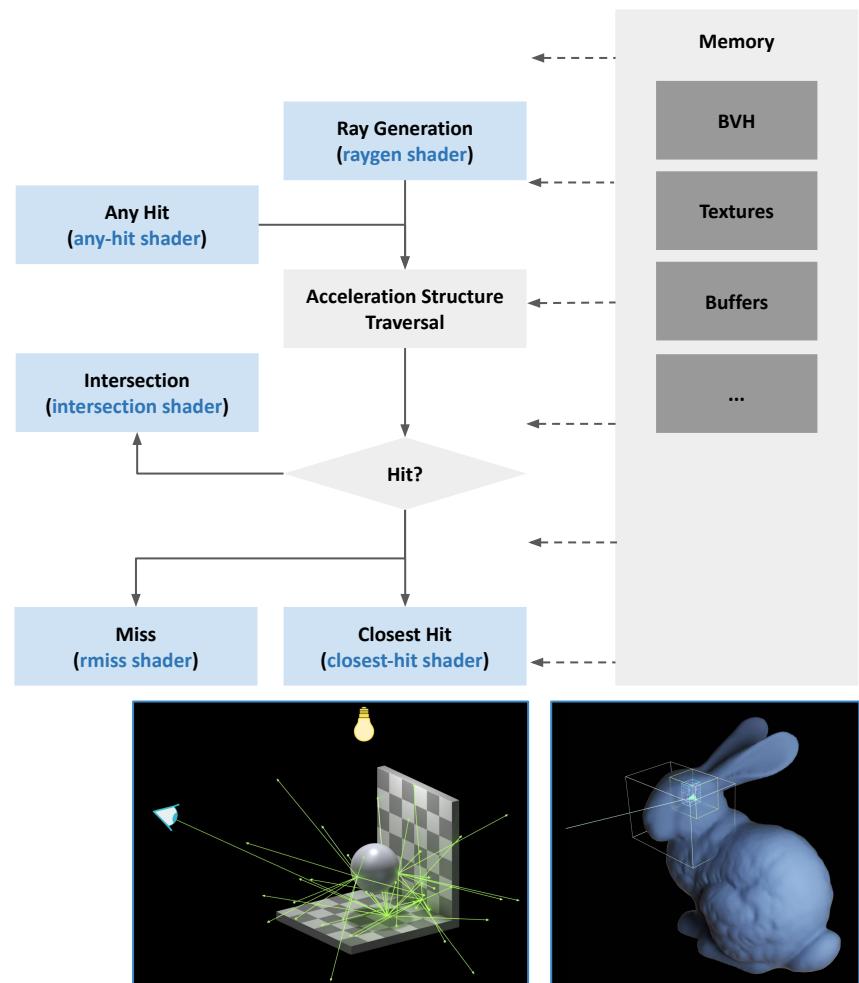
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init frame buffer
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    for ray bounces is not over { // russian roulette
        for each triangle t in the scene { // BVH
            if ray hit t at x {
                keep x if closest and update the ray
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            }
        }
    }
    update frame buffer
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            if ray hit t at x { // ray casting
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- How to deal with normals/uv's if a mesh is modified?
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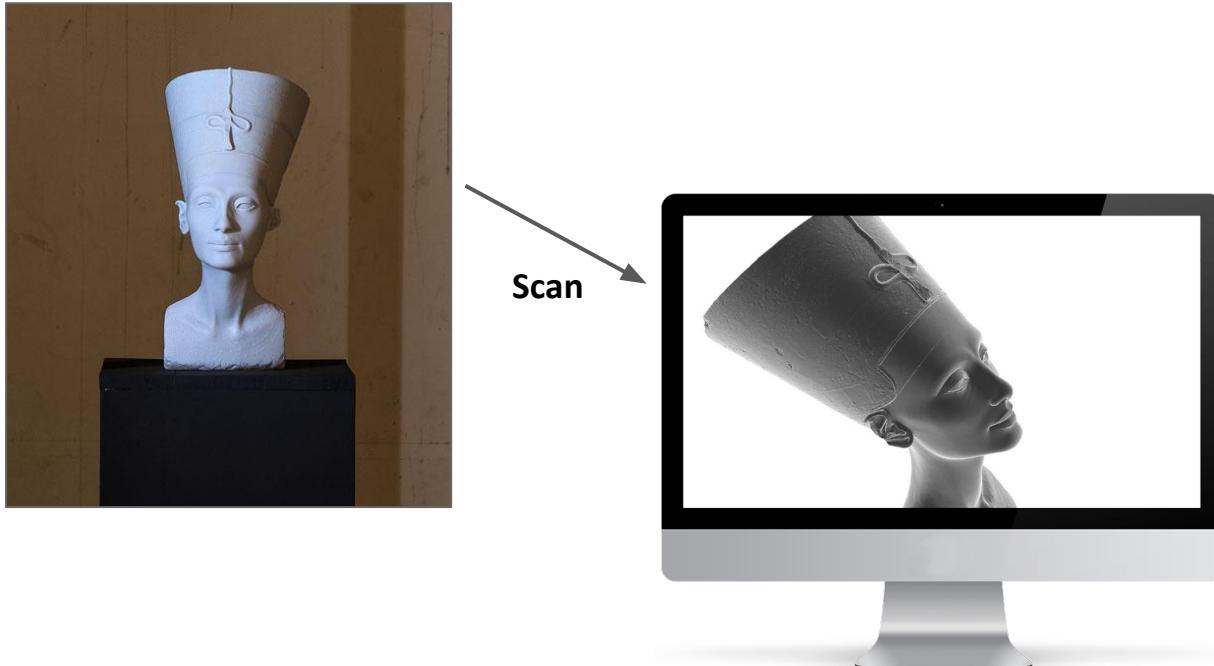
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Let's restart from the very beginning...

# Geometry Processing Pipeline

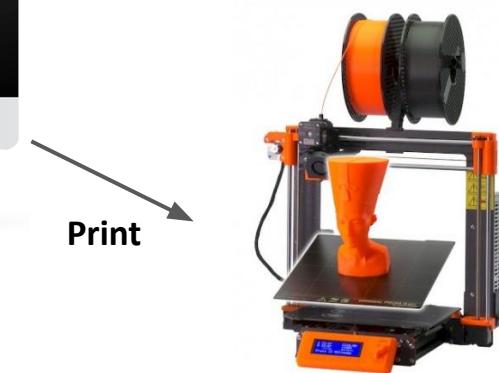
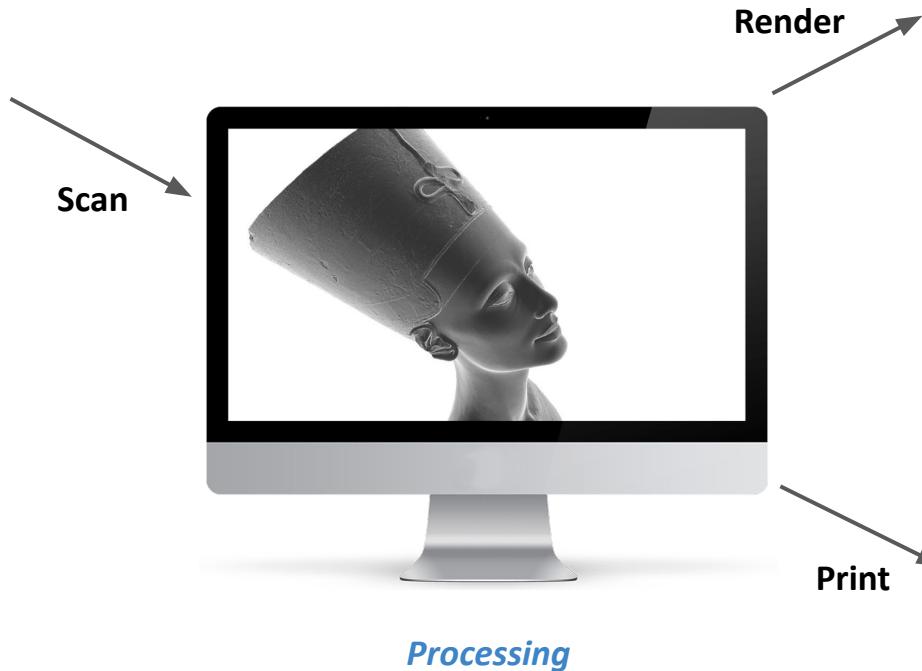


# Geometry Processing Pipeline



*Processing*

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# Representations of Geometry Objects

Point cloud

Voxels

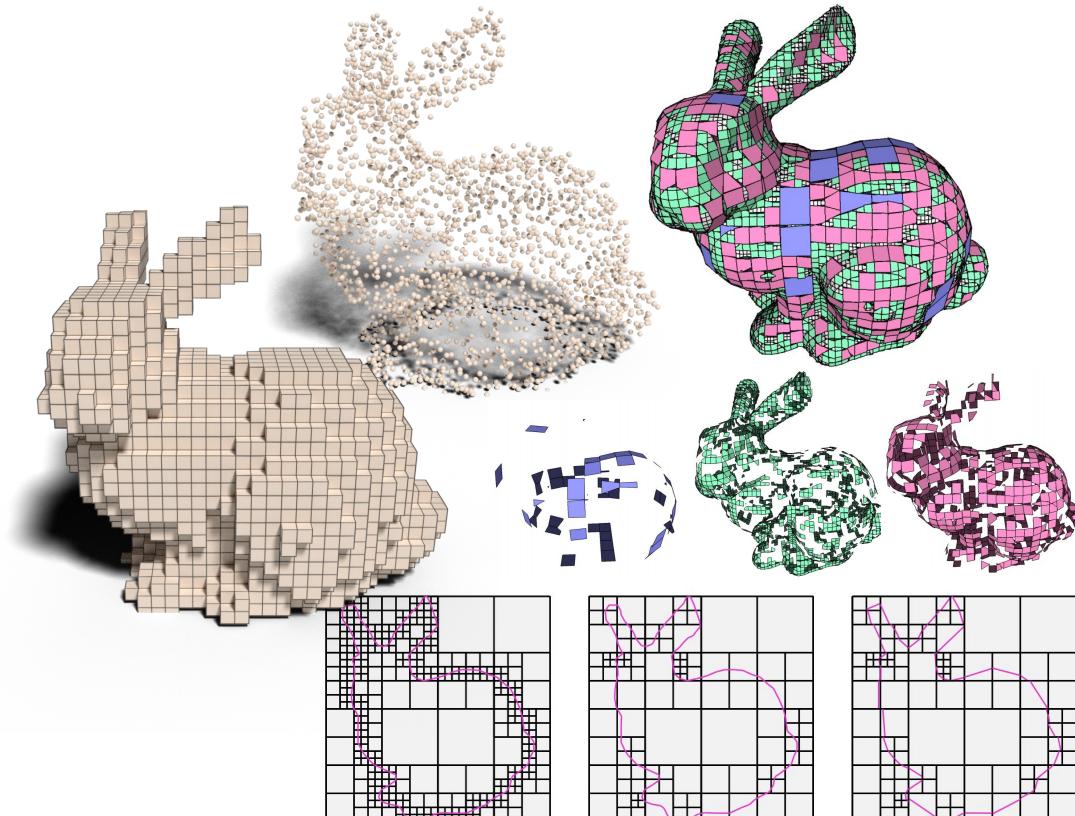
Patches

Implicit

Explicit

Parametric

...



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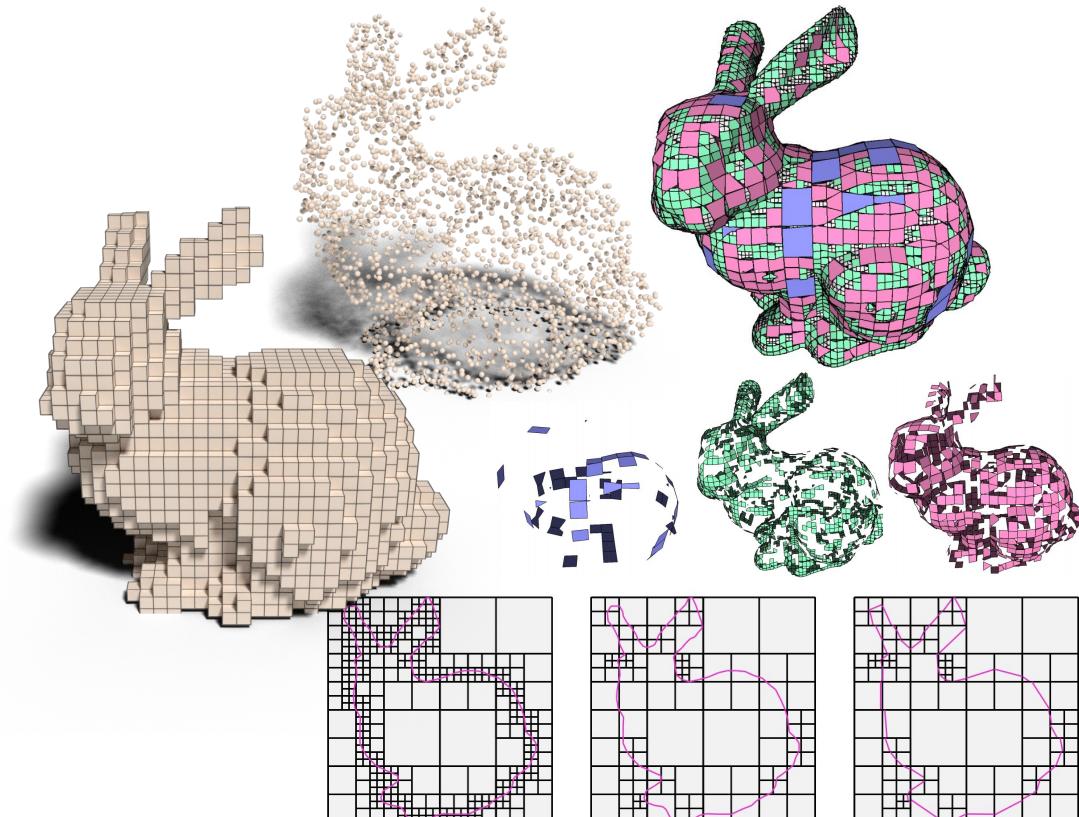
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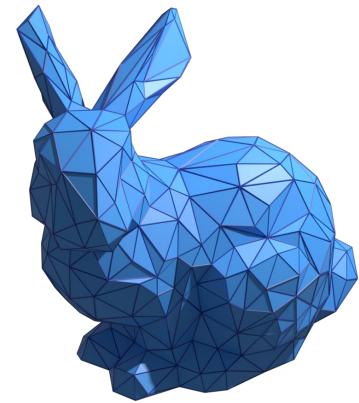
**Mesh-based Surface**

...



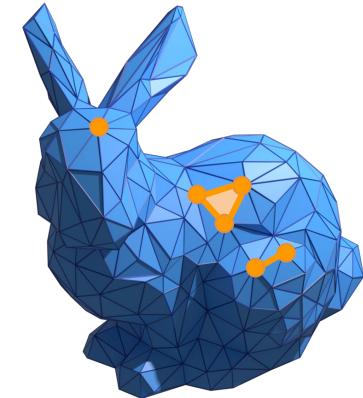
# *Polygonal Mesh*

- A collection of polygons: a segment of a piecewise linear surface representation



# Polygonal Mesh

- A collection of polygons: a segment of a piecewise linear surface representation
- *Geometrical* component
  - Vertices  $\mathcal{V} = \{v_1, v_2, \dots, v_V\}, v_i \in \mathbb{R}^3$
- *Topological* components
  - Faces  $\mathcal{F} = \{f_1, f_2, \dots, f_F\}$
  - Edges  $\mathcal{E} = \{e_1, e_2, \dots, e_E\}$
- A polygonal mesh can be formulated as  $\mathcal{M} = (\mathcal{V}, \mathcal{F}, \mathcal{E})$



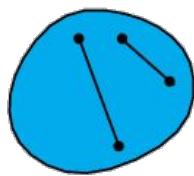
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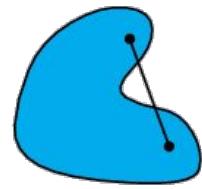
*Q: Why are meshes different from graphs?*



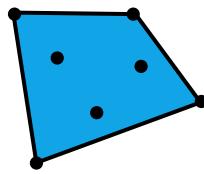
# Terminologies



Convex



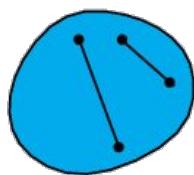
Non-convex



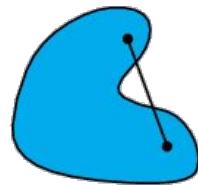
Convex Hull

- *Convex and Convex Hull*

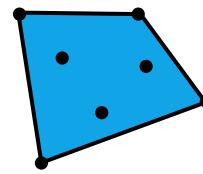
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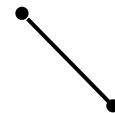
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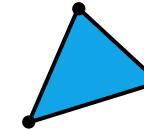
Convex Hull



0-simplex



1-simplex



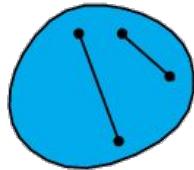
2-simplex

- Convex and Convex Hull
- $k$ -Simplex: the convex hull of  $k+1$  affine-independent vertices
  - e.g. tetrahedra is a 3-simplex

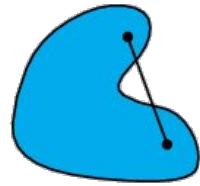


3-simplex

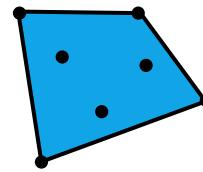
# Terminologies



Convex



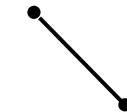
Non-convex



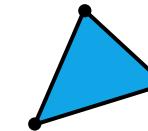
Convex Hull



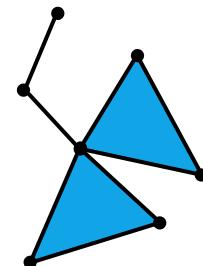
0-simplex



1-simplex



2-simplex



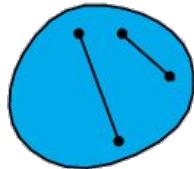
simplicial complex

- Convex and Convex Hull
- *k*-Simplex: the convex hull of  $k+1$  affine-independent vertices
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- Face: any simplices of a subset of vertices

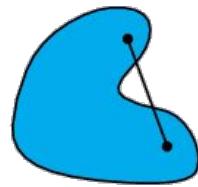


3-simplex

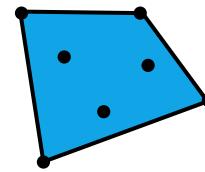
# Terminologies



Convex



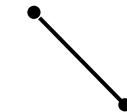
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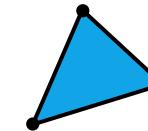
Convex Hull



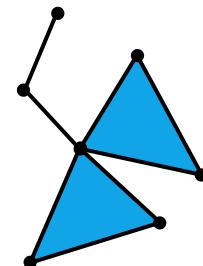
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1-simplex

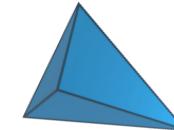


2-simplex



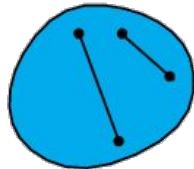
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- *Simplicial complex*: a collection of simplices
  - e.g. Graph is simplicial **1-complexes**, triangle meshes are simplicial **2-complexes**

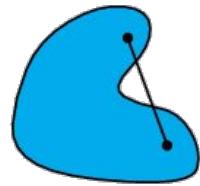


3-simplex

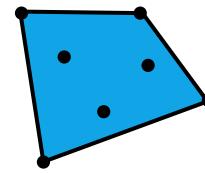
# Terminologies



Convex



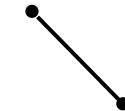
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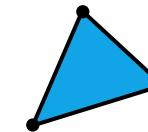
Convex Hull



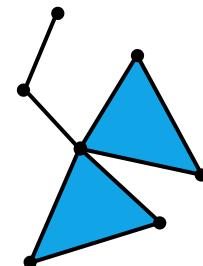
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1-simplex



2-simplex

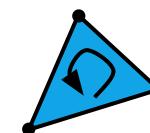
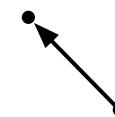
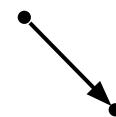


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- Simplicial complex: a collection of simplices
  - e.g. Graph is simplicial **1-complexes**, triangle meshes are simplicial **2-complexes**
- Simplex can have *orientation*

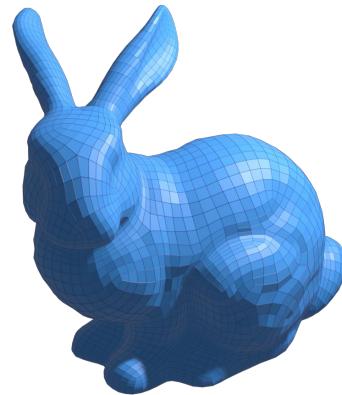
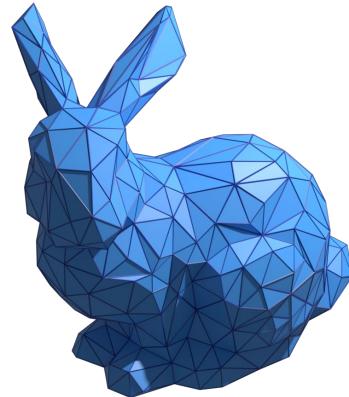


3-simplex



# Types of Polygon Meshes

- Triangle Meshes
- Quadrilateral meshes
- ...



Q: What do they have in common?

# Topological Invariant: Euler-Poincaré Formula

$$F - E + V = \boxed{2(1 - g)}$$

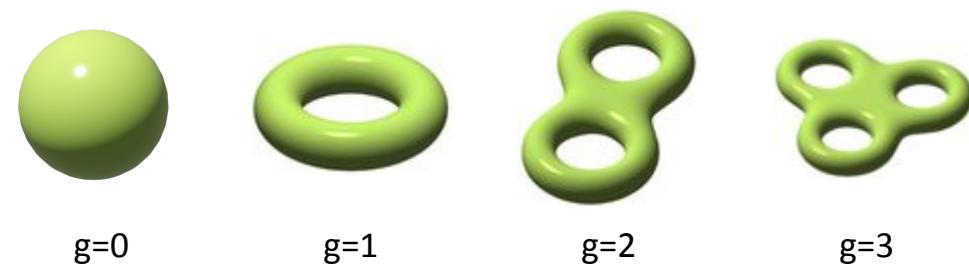
genus: #holes  
Euler characteristic

Euler characteristic allows to check topological property at constant time

e.g. the euler characteristic of a convex polyhedron is 2

Corollary (why?):

- Triangle Mesh
  - $F \approx 2V, E \approx 3V$
  - avg. vertex degree = 6
- Quad Mesh
  - $F \approx V, E \approx 2V$
  - avg. vertex degree = 4



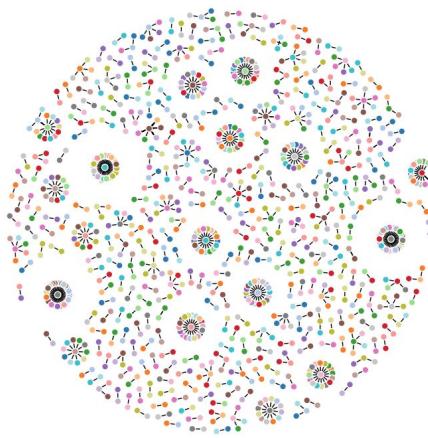
# Why Polygons?

- Polygon mesh is a good surface compromise of "physical" solid
  - Think about approximation error (recall Taylor formula from calculus)
- Arbitrary topology
- Flexibility for piecewise smooth surfaces
- Flexibility for adaptive refinement (subdivision)
- Render efficiency
- ...

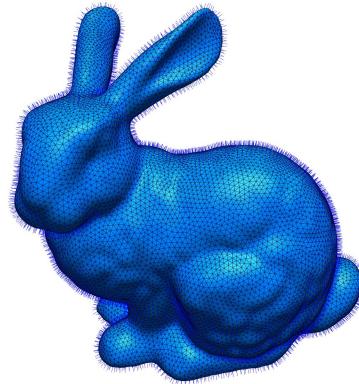
**What's an appropriate data structure  
represent polygon-based  
surface geometry?**

# Mesh Data Structure: Critical Information

- Position ( $x, y, z$ )

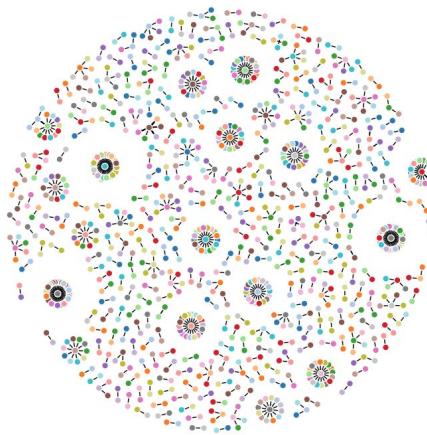


v.s.

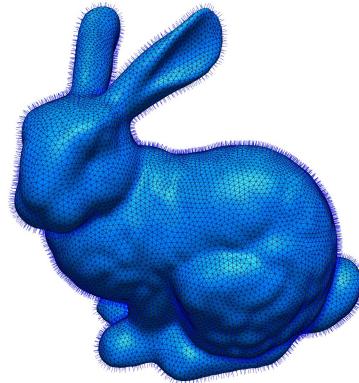


# Mesh Data Structure: Critical Information

- Position ( $x, y, z$ )
- *Attributes, e.g. per-vertex/face normals, UV coordinates, per-vertex/face colors*

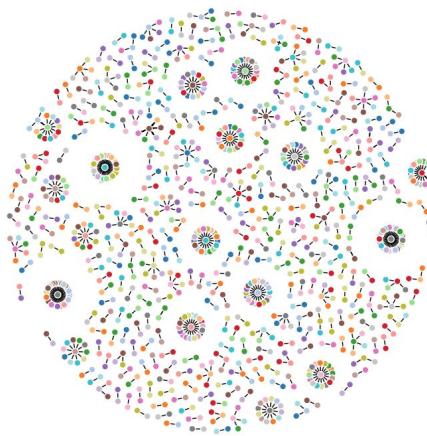


v.s.

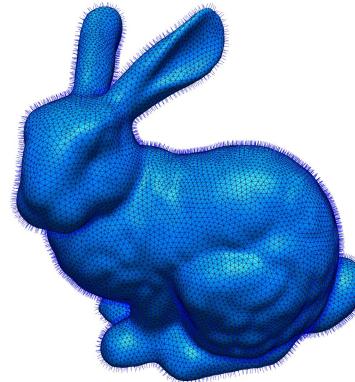


# Mesh Data Structure: Geometric Information

- Position ( $x, y, z$ )
- Attributes, e.g. per-vertex/face normals, UV coordinates, per-vertex/face colors
- **Connectivity**



v.s.

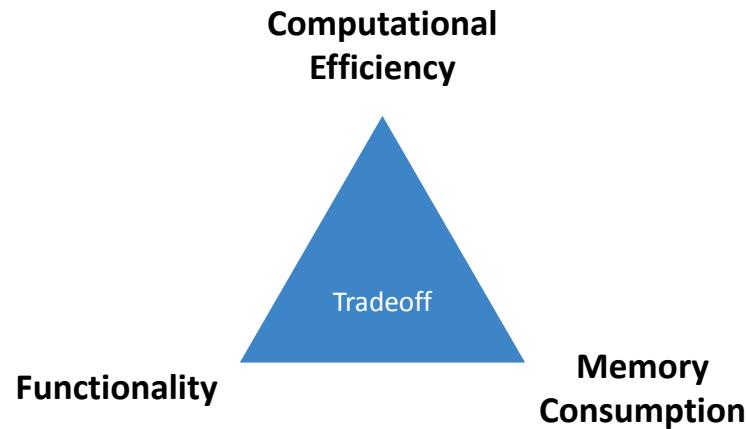


# Mesh Data Structure: Optimized on-demand

- Optimize for storage, e.g. persistent on a cache/disk, compressed for consumption
- Optimize for runtime rendering tasks, e.g. Vertex buffer, BVH, fetch efficiency
- Optimize for queries, e.g. What are the vertices and faces of a given face?
- Optimize for manipulation, e.g. Add, remove, collapse, reconstruct a edge/face?
- ...

# Mesh Data Structure: Evaluation Criteria

- Preprocessing time, e.g. Time complexity to build a structure from another format
- Query time, e.g. Time complexity of search adjacency faces of a given edge
- Operation time, e.g. Time complexity of removing an edge/face
- Space complexity, e.g. Storage consumption of a structure



# Why connectivity is important and how to represent it?

- Connectivity conveys the understanding of *local* information of a vertex
- With connectivity, one can avoid expensive searches
- Different types of connectivity
  - No connectivity: Face set
  - Vertex-based connectivity: Shared vertex
  - Face-based connectivity: Shared face
  - Edge-based connectivity: Shared edge
  - Halfedge-based connectivity

# Why connectivity is important and how to represent it?

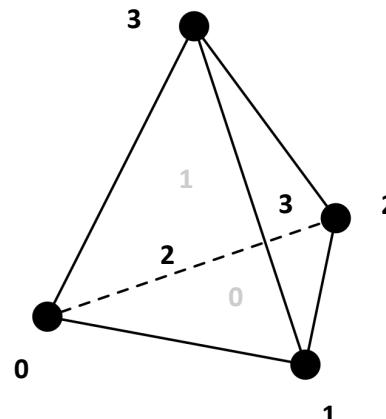
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  - Edge-based connectivity: Shared edge
  - Halfedge-based connectivity
- We will revisit more about why local operations are so important later

# Shared Vertex (.obj, .off formats)

**Basic Idea:** isolate vertex positions, store an *adjacency list* for vertices

- Storage cost
  - 1 floating number = 4 bytes
  - 1 vertex =  $4 \times 3 = 12$  bytes
  - 1 face =  $4 \times 2 = 12$  bytes
  - total:  $\#v \times 12 + \#f \times 12 \approx \#v \times 12 \times 3 = 36$  bytes/vertex
- Pros
  - Still simple, and with small redundancy
- Cons
  - No access to neighbors (why?)

Vertices	Triangles
x1,y1,z1	i11,i12,i13
x2,y2,z3	i21,i22,i23
...	...
xv,yv,zv	...
...	...
...	...
in1,in2,in3	...



Adjacency list for vertices:

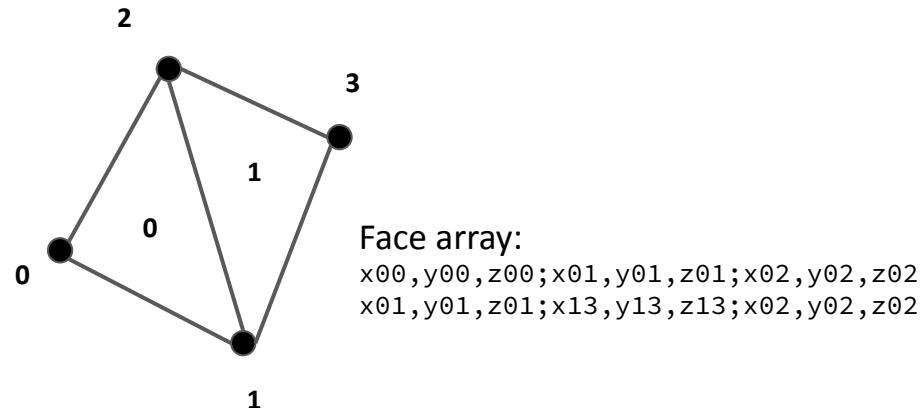
0: 0 2 1
1: 0 3 2
2: 3 0 1
3: 3 1 2

# Face Set (.stl format)

**Basic Idea:** each row stores the vertices of the face (*array*)

- Storage cost
  - 1 floating number = 4 bytes
  - 1 face =  $4 \times 9 = 36$  bytes
  - 1 vertex  $\approx 2 \times 36 = 72$  bytes (why?)
  - total: 72 bytes/vertex
- Pros
  - Very simple
- Cons
  - No connectivity
  - Redundancy (why?)

Triangles		
x11,y11,z11	x12,y12,z12	x13,y13,z13
x21,y21,z21	x22,y22,z22	x23,y23,z23
...	...	...
xn1,yn1,zn1	xn2,yn2,zn2	xn3,yn3,zn3



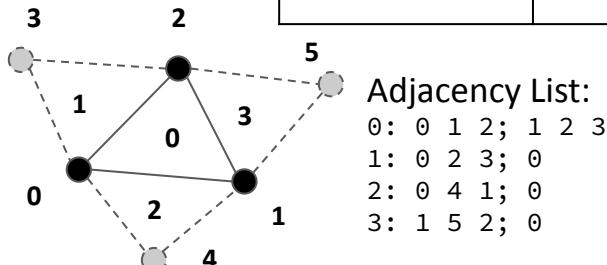
# Face-based Connectivity

**Basic Idea:** store an *adjacency list* for faces and neighboring faces

- Vertex contains
  - Position
  - Associated face
- A face contains
  - Vertices
  - Face neighbors
- Pros
  - Constant time to access all neighboring faces
- Cons
  - No edge information

Vertices	
x1,y1,z1	f1
x2,y2,z3	f2
...	...
xv,yv,zv	fn

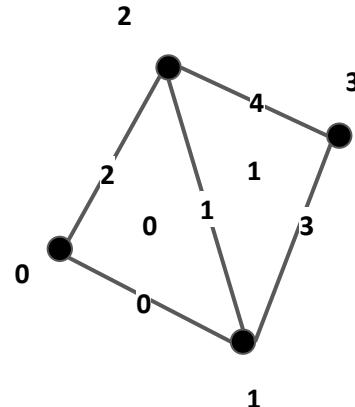
Triangles	
i11,i12,i13	f11,f12,f13
i21,i22,i23	f21,f22,f23
...	
...	
...	
...	
in1,in2,in3	



# Edge-based Connectivity

**Basic Idea:** store an *adjacency list* for edges

- Vertex contains
  - Position
  - 1 adjacent edge index
- A edge contains
  - vertex indices
  - neighboring face indices
  - edges
- A face contains 1 edge index
- Pros: Constant time to access all neighboring faces and edges
- Cons: No edges orientation (why matters?)



Adjacency List:  
0: 1 2  
1: 0 3 2 4  
2: 0 1  
3: 4 1  
4: 3 1

# Incidence Matrix

**Basic idea:** Store *all* neighbor informations via incidence matrices

Vertex-Edge Matrix

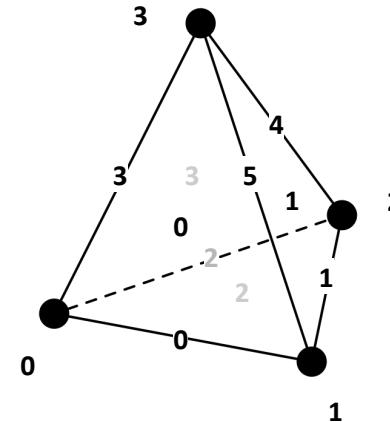
	0	1	2	3
0:	1	1	0	0
1:	0	1	1	0
2:	1	0	1	0
3:	1	0	0	1
4:	0	0	1	1
5:	0	1	0	1

Edge-Face Matrix

	0	1	2	3	4	5
0:	1	0	0	1	0	1
1:	0	1	0	0	1	1
2:	1	1	1	0	0	0
3:	0	0	1	1	1	0

For large meshes, most of the elements will be zero

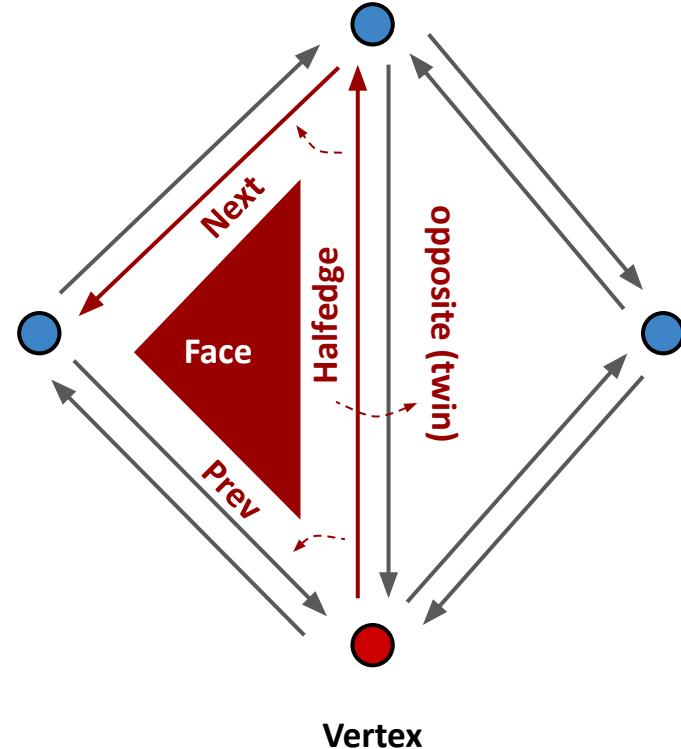
⇒ Use sparse matrix for tasks at scale



# Data Structure: *Halfedge*

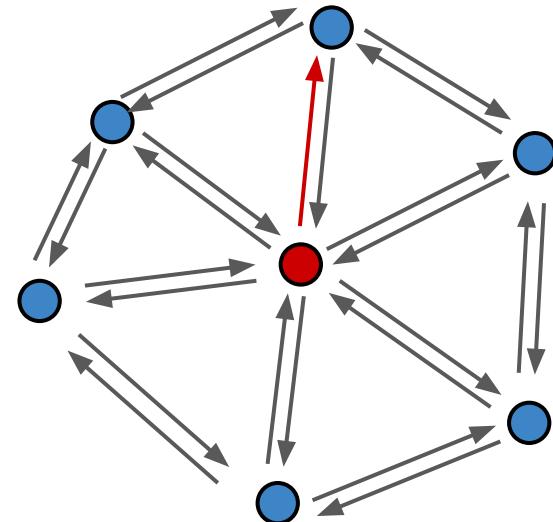
**Basic idea:** each edge gets split into two half edges

- Vertex
  - Position
  - 1 Halfedge
- Halfedge
  - 1 Vertex
  - 1 Face
  - Prev; Next; Opposite (Twin)
- Face
  - 1 Halfedge



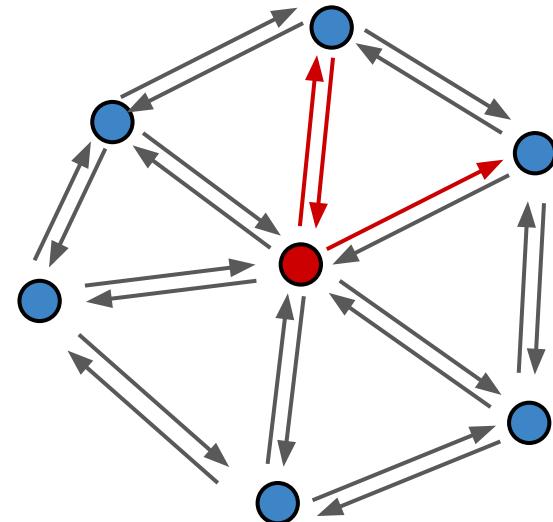
# Example 1: Access All Adjacency Edges with Halfedge

```
/**  
 * numAdjacentEdges returns the number of adjacency edges of the given vertex  
 * @param v is an vertex from a haledge-based mesh  
 */  
function numAdjacentEdges(v: Vertex) {  
    const e0 = v.halfedge  
    let edge_indices = [e0.index]  
    for (let e = e0.opposite.next; e != e0; e = e.opposite.next) {  
        edge_indices.push(e.index)  
    }  
    return edge_indices.length  
}
```



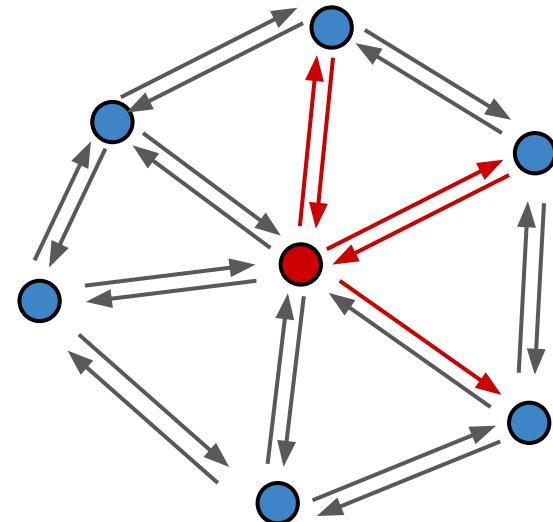
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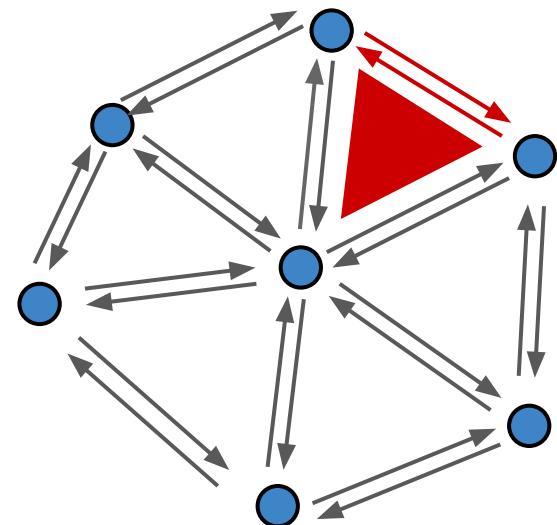
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    }  
    return edge_indices.length  
}
```



# Example 2: Check if an edge is on the boundary of a mesh

```
class Halfedge {  
    ...  
  
    onBoundary() {  
        let connectedFaces = 0  
        if (this.face != null) {  
            connectedFaces++  
        }  
        if (this.opposite.face != null) {  
            connectedFaces++  
        }  
        if (connectedFaces != 1) {  
            return false  
        }  
        return true  
    }  
}
```



# See the benefits of halfedge?

- Key benefits: makes traversal easy
  - Easy for editing a mesh: constant time access of local neighbors
- Cons?

# Manifold v.s. Non-Manifold

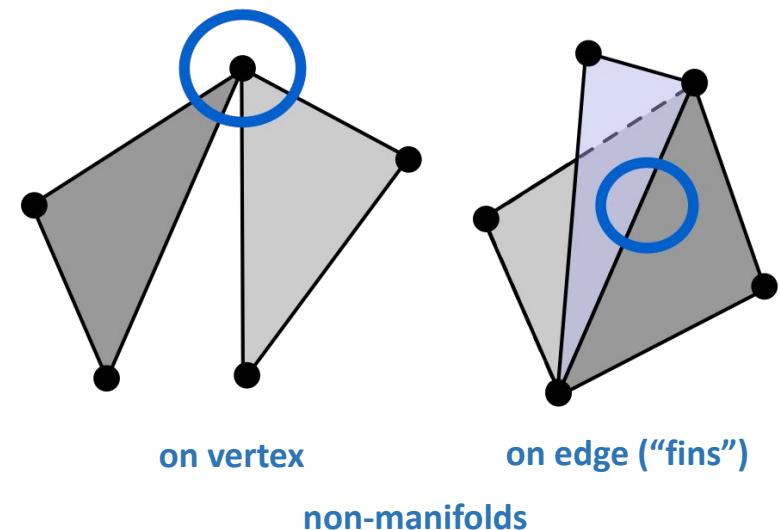
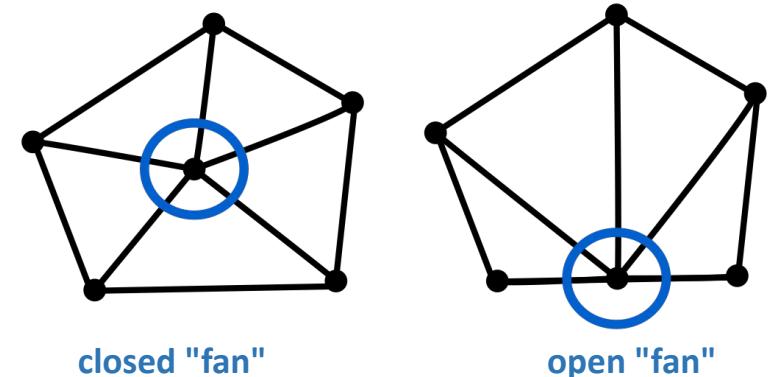
Manifold:

1. Each edge is incident to one or two faces, and
2. faces incident to a vertex from a closed or open fan.

Non-manifold:

1. every edges is constrained in only two polygons (no "fins")
2. the polygons containing each vertex makes a single "fan"

*Q: Can halfedge structure deal with non-manifolds?*



# Alternatives to Halfedge

Winged edge

Corner table

Quadedge

...

Similar to halfedge and each stores local neighborhood information

Tradeoffs in general:

- + Convenient and better access time for individual elements, easy for traversal of locals
- Additional storage; cache incoherent; ...

# More Sophisticated Mesh Data Structures

**BMesh** from Blender

**FbxMesh** from Autodesk

**FDynamicMesh** from Unreal Engine

...

These data structures are not only for geometry processing purpose but also impacts the subsequent workflows, such as animation, rendering, etc.

# Why Mesh Representation Is Still an Issue Today?

- Mesh is still the most *efficient/compact/structured* way to represent a solid geometric object
- Mesh structure has a *fundamental impact* on the way we implement an algorithm (why?)

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- Approaching the end of Moore's law: The needs for a concurrent-safe mesh structure
- We will have time to revisit these questions in later sessions.
- Be careful & patient 😊
- Let's now go for something maybe more funny...

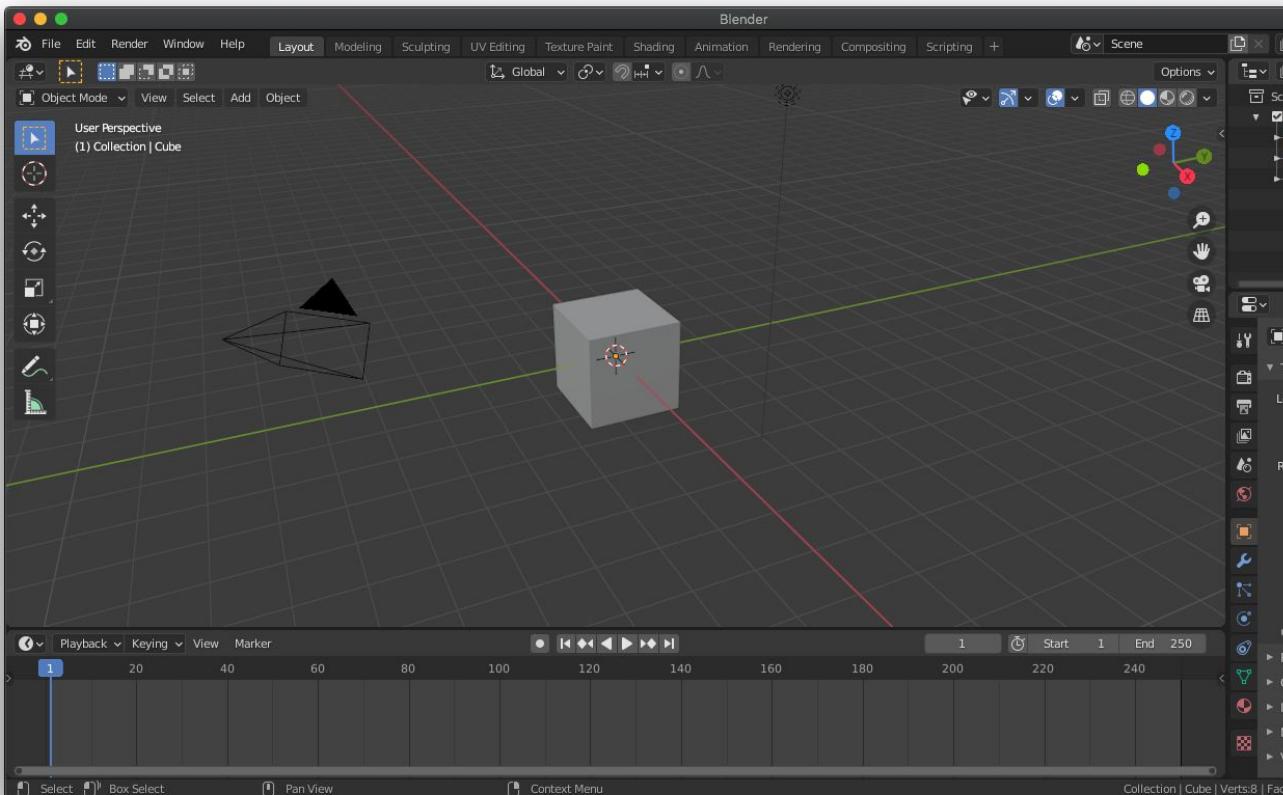
# Session 1: Introduction

- Motivation
- Recap: Graphics Pipelines
- Geometry Representations
- Blender Basics
- Summary

# Blender

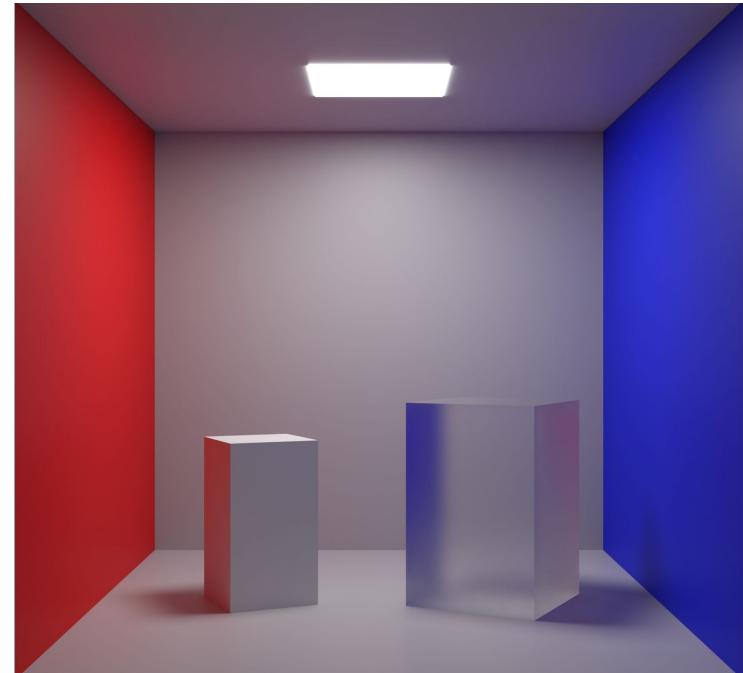
Why not XYZ?

Because of *open source*



# Practice: Reproduce The Cornell Box in Blender

- What are the geometry objects do we need for the Cornell box?
- What are their material properties?



# Session 1: Introduction

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# Summary

- Geometry representation has a long term impact on the geometry processing pipeline
- Mesh is a good compromise and the connectivity is critical for local operations
- No-free Lunch! ⇒ Think about the task, then choose (or design) the right structure
- Understanding the modeling process helps understand more processing workflows

	Adjacency List Based Structure	Incident Matrices Based Structure	Halfedge Based Structure
Constant-time neighborhood access?	No	Yes	Yes
Easy to remove elements?	No	No	Yes
Deal with non-manifold geometry?	Yes	Yes	No