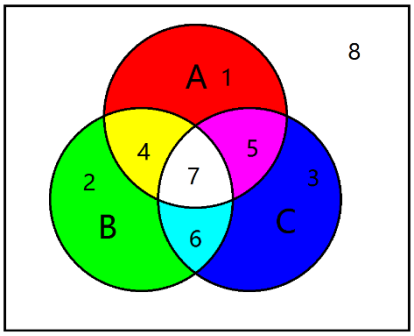
Homework 1.1

True or False?



$A - B = A \cap \overline{B}$

$(A \cup B) - C = A \cup (B - C)$

$(A \cup B) \cap C = A \cup (B \cap C)$

$A - (\overline{A} \cap B) = A$

$A \cap \overline{B} \cap C = (A - B) \cap C$

$A - B = \{1, 4, 5, 7\} - \{2, 4, 6, 7\} = \{1, 5\}$

$A \cap \overline{B} = \{1, 4, 5, 7\} \cap \{1, 3, 5, 8\} = \{1, 5\}$

True.

$(A \cup B) - C = (\{1, 4, 5, 7\} \cup \{2, 4, 6, 7\}) - \{3, 5, 6, 7\} = \{1, 2, 4\}$

$A \cup (B - C) = \{1, 4, 5, 7\} \cup (\{2, 4, 6, 7\} - \{3, 5, 6, 7\}) = \{1, 2, 4, 5, 7\}$

False.

$(A \cup B) \cap C = (\{1, 4, 5, 7\} \cup \{2, 4, 6, 7\}) \cap \{3, 5, 6, 7\} = \{5, 6, 7\}$

$A \cup (B \cap C) = \{1, 4, 5, 7\} \cup (\{2, 4, 6, 7\} \cap \{3, 5, 6, 7\}) = \{1, 4, 5, 6, 7\}$

False.

$A - (\overline{A} \cap B) = \{1, 4, 5, 7\} - (\{2, 3, 6, 8\} \cap \{2, 4, 6, 7\}) = \{1, 4, 5, 7\}$

$A = \{1, 4, 5, 7\}$

True.

$A \cap \overline{B} \cap C = \{1, 4, 5, 7\} \cap \{1, 3, 5, 8\} \cap \{3, 5, 6, 7\} = \{5\}$

$(A - B) \cap C = (\{1, 4, 5, 7\} - \{2, 4, 6, 7\}) \cap \{3, 5, 6, 7\} = \{5\}$

True.

Homework 1.2

Juan is playing the following game: he rolls two dice. If they sum up to 7 he loses a dollar. If they sum up to 2, he wins 2 dollars. Otherwise, he doesn't win nor lose.

After playing this game for a long time, what shall happen? Why?

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$\mathrm{P}(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}$$

$$\mathrm{P}(\text{sum} = 2) = \frac{1}{36}$$

$$\mathrm{P}(\text{others}) = \frac{29}{36}$$

$$\text{Money} = -1 \times \frac{1}{6} + 2 \times \frac{1}{36} + 0 \times \frac{29}{36} = -\frac{1}{9} \approx -0.11$$

The result is the average he will lose is 0.11 dollar per roll.

Homework 1.3

Jerry and Susan have a joint bank account.

Jerry goes to the bank 20% of the days.

Susan goes there 30% of the days.

Together they are at the bank 8% of the days.

a. Susan was at the bank last Monday. What's the probability that Jerry was there too?

b. Last Friday, Susan wasn't at the bank. What's the probability that Jerry was there?

c. Last Wednesday at least one of them was at the bank. What is the probability that both of them were there?

	Jerry at bank	Jerry not at bank	
Susan at bank	0.08	0.22	0.3
Susan not at bank	0.12	0.58	0.7
	0.2	0.8	1

a.

$$\mathrm{P(J|S)} = \frac{\mathrm{P(JS)}}{\mathrm{P(S)}} = \frac{0.08}{0.3} = \frac{4}{15} \approx 26.67\%$$

b.

$$\mathrm{P(J|\bar{S})} = \frac{\mathrm{P(J\bar{S})}}{\mathrm{P(\bar{S})}} = \frac{0.12}{0.7} = \frac{6}{35} \approx 17.14\%$$

c.

$$\mathrm{P\big(JS|(J\ or\ S)\big)} = \frac{\mathrm{P(JS)}}{\mathrm{P(J\ or\ S)}} = \frac{0.08}{0.42} = \frac{4}{21} \approx 19.05\%$$

Homework 1.4

Harold and Sharon are studying for a test.

Harold's chances of getting a "B" are 80%. Sharon's chances of getting a "B" are 90%.

The probability of at least one of them getting a "B" is 91%.

a. What is the probability that only Harold gets a "B"?

b. What is the probability that only Sharon gets a "B"?

c. What is the probability that both won't get a "B"?

	Harold gets B	Harold not get B	
Sharon gets B	0.79	0.11	0.9
Sharon not get B	0.01	0.09	0.1
	0.8	0.2	1

* $P(\bar{H}\bar{S}) = 1 - 0.91 = 0.09$

a.

$P(H\bar{S}) = 1\%$

b.

$P(\bar{H}S) = 11\%$

c.

$P(\bar{H}\bar{S}) = 9\%$

Homework 1.5

Jerry and Susan have a joint bank account.

Jerry goes to the bank 20% of the days.

Susan goes there 30% of the days.

Together they are at the bank 8% of the days.

Are the events “Jerry is at the bank” and “Susan is at the bank” independent?

$P(J) \times P(S) = 0.2 \times 0.3 = 0.06$

$P(JS) = 0.08$

$0.06 \neq 0.08$

No. They are not independent.

Homework 1.6

You roll 2 dice.

a. Are the events “the sum is 6” and “the second die shows 5” independent?

b. Are the events “the sum is 7” and “the first die shows 5” independent?

a	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

a.

$$P(\text{sum} = 6) = \frac{5}{36}$$

$$P(\text{sec} = 5) = \frac{6}{36}$$

$$P(\text{sum} = 6 \& \text{sec} = 5) = \frac{1}{36}$$

$$\frac{5}{36} \times \frac{6}{36} = \frac{5}{216} \neq \frac{1}{36}$$

No. “The sum is 6” and “the second die shows 5” are not independent.

b.

$$P(\text{sum} = 7) = \frac{6}{36}$$

$$P(\text{fir} = 5) = \frac{6}{36}$$

$$P(\text{sum} = 7 \& \text{fir} = 5) = \frac{1}{36}$$

$$\frac{6}{36} \times \frac{6}{36} = \frac{1}{36} = \frac{1}{36}$$

Yes. “The sum is 7” and “the first die shows 5” are independent.

Homework 1.7

An oil company is considering drilling in either TX, AK and NJ. The company may operate in only one state. There is 60% chance the company will choose TX and 10% chance – NJ.

There is 30% chance of finding oil in TX, 20% - in AK, and 10% - in NJ.

1. What's the probability of finding oil?

2. The company decided to drill and found oil. What is the probability that they drilled in TX?

	TX	AK	NJ	
OIL	0.18	0.06	0.01	0.25
NO OIL	0.42	0.24	0.09	0.75
	0.6	0.3	0.1	1

1.

$P(TX\&OIL) = P(TX) \times P(OIL|TX) = 0.6 \times 0.3 = 0.18$

$P(AK\&OIL) = P(AK) \times P(OIL|AK) = 0.3 \times 0.2 = 0.06$

$P(NJ\&OIL) = P(NJ) \times P(OIL|NJ) = 0.1 \times 0.1 = 0.01$

$P(OIL) = P(TX\&OIL) + P(AK\&OIL) + P(NJ\&OIL) = 0.18 + 0.06 + 0.01 = 0.25$

2.

$$P(TX|OIL) = \frac{P(TX\&OIL)}{P(OIL)} = \frac{0.18}{0.25} = 0.72$$

Homework 1.8

Example 1:

A company is considering an investment.

The outcomes can be as follows:

Success: 20%

Average: 50%

Failure: 30%

The company decides to hire a specialist. His advice is either YES or NO.

From past experience, the company knows that:

p(YES|success) = 0.9

p(YES|Average) = 0.2

p(YES|Failure) = 0.1

The specialist said YES. What is the probability for success?

	SUCCESS	AVERAGE	FAILURE	
YES	0.18	0.1	0.03	0.31
NO	0.02	0.4	0.27	0.69
	0.2	0.5	0.3	1

$$P(\text{YES\&SUCCESS}) = P(\text{SUCCESS}) \times P(\text{YES}|\text{SUCCESS}) = 0.2 \times 0.9 = 0.18$$

$$P(\text{YES\&AVERAGE}) = P(\text{AVERAGE}) \times P(\text{YES}|\text{AVERAGE}) = 0.5 \times 0.2 = 0.1$$

$$P(\text{YES\&FAILURE}) = P(\text{FAILURE}) \times P(\text{YES}|\text{FAILURE}) = 0.3 \times 0.1 = 0.03$$

$$P(\text{SUCCESS}|\text{YES}) = \frac{P(\text{SUCCESS\&YES})}{P(\text{YES})} = \frac{0.18}{0.18 + 0.1 + 0.03} = \frac{18}{31} \approx 58.06\%$$