Show the multiplication of a matrix by a vector in a "Map-Reduce" environment when a Matrix is too large to fin into a single machine.

$$A_{m\times n} \times C_{n\times 1} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \times \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} (a_{1i} \times c_i) \\ \sum_{i=1}^{n} (a_{2i} \times c_i) \\ \vdots \\ \sum_{i=1}^{n} (a_{mi} \times c_i) \end{bmatrix}$$

First, we can divide the matrix A into m × n numbers, and each will be stored as $(i,\ j,\ a_{ij})$ like:

$(1, 1, a_{11}), (2, 1, a_{21}), \dots, (m, 1, a_{m1})$

$(1, 2, a_{12}), (2, 2, a_{22}), \dots, (m, 2, a_{m2})$

…

$(1, n, a_{1n}), (2, n, a_{2n}), \dots, (m, n, a_{mn})$

And we can divide the matrix C into n numbers, and each will be stored as $(j,\ c_j)$ like:

$(1, c_1), (2, c_2), \dots, (n, c_n)$

Next, if $(i,\ j,\ a_{ij})$ and $(j,\ c_j)$ have the same j, they can be sent to machine j. The map task will multiply each a_{ij} with c_j and we can get a series of $(i,\ a_{ij} \times c_j)$ like:

$$\begin{array}{ccccc} \backslash & j=1 & j=2 & & j=n \\ \begin{array}{c} i=1 \\ i=2 \\ \vdots \\ i=m \end{array} & \begin{bmatrix} a_{11}\times c_1 \\ a_{21}\times c_1 \\ \vdots \\ a_{m1}\times c_1 \end{bmatrix} & \begin{bmatrix} a_{12}\times c_2 \\ a_{22}\times c_2 \\ \vdots \\ a_{m2}\times c_2 \end{bmatrix} & \dots & \begin{bmatrix} a_{1n}\times c_n \\ a_{2n}\times c_n \\ \vdots \\ a_{mn}\times c_n \end{bmatrix} \end{array}$$

Finally, the reduce task will add every $a_{ij} \times c_j$ together if it has the same i and we get $(i, \sum_{j=1}^{n}(a_{ij} \times c_j))$ like:

$$\begin{array}{ccccc} i=1 & & i=2 & & i=m \\ \sum_{j=1}^{n}(a_{1j} \times c_j) & & \sum_{j=1}^{n}(a_{2j} \times c_j) & \cdots & \sum_{j=1}^{n}(a_{mj} \times c_j) \end{array}$$

The number i means the row of $\sum_{j=1}^{n}(a_{ij} \times c_j)$ in the final matrix and we can get the answer.

$$\begin{bmatrix} \sum_{j=1}^{n} \left(a_{1j} \times c_j\right) \\ \sum_{j=1}^{n} \left(a_{2j} \times c_j\right) \\ \vdots \\ \sum_{j=1}^{n} \left(a_{mj} \times c_j\right) \end{bmatrix}$$