R-1.6

The order of simple functions: $\log n < \log^2 n < \sqrt{(n)} < n < n \log n < n^2 < n^3 < 2^n$.

The order of this question: $1/n < 2^{100} < \log\log n < \sqrt{(\log n)} < \log^2 n < n^{0.01} < \underline{\lfloor \sqrt{(n)} \rfloor < 3n^{0.5}} < \underline{2^{\log n} < 5n} < \underline{n \log_4 n < 6n \log n} < \lfloor 2n \log^2 n \rfloor < 4n^{3/2} < 4^{\log n} < n^2 \log n < n^3 < 2^n < \underline{4^n < 2^{2^{\wedge}n}}$.

R-1.23

Proof:

By the definition of big-Omega, we need to find a constant $c > 0$ and an integer constant $n_0 \geq 1$ such that $f(n) \geq c \cdot g(n)$ for $n \geq n_0$. And we can choose $c = 1$ and $n_0 = 2$, so that $\log n \geq 1$ and $n^3 \geq n^3 \log n$. So $n^3 \log n$ is $\Omega(n^3)$.

C-1.5

Proof:

Base case: if $n = 0$, $T(0) = 2^{0+1} - 1 = 1$; if $n = 1$, $T(1) = T(0) + 2^1 = 3 = 2^{1+1} - 1$

Induction step: we can assume that if $n = k - 1$, $T(k - 1) = 2^k - 1$ is correct.

If $n = k$, $T(k) = T(k - 1) + 2^n = 2^k - 1 + 2^k = 2^{k+1} - 1$.

So the formula is correct.

C-1.10

There is a formula: $1^2 + 2^2 + \ldots + n^2 = 1/6 \times n \times (n + 1) \times (2n + 1) = n^3/3 + n^2/2 + n/6$.

Now we can prove it.

Base case: if $n = 1$, $1^2 = 1/3 + 1/2 + 1/6 = 1$.

Induction step: we can assume that if $n = k - 1$, $1^2 + \ldots + (k - 1)^2 = (k - 1)^3/3 + (k - 1)^2/2 + (k - 1)/6 = k^3/3 - k^2/2 + k/6$ is correct.

If $n = k$, $1^2 + \ldots + (k - 1)^2 + k^2 = k^3/3 - k^2/2 + k/6 + k^2 = k^3/3 + k^2/2 + k/6$.

So the formula is correct.

$1^2 + 2^2 + \ldots + n^2 = n^3/3 + n^2/2 + n/6 \leq c\, n^3$

By the definition of big-O, we need to find a constant $c > 0$ and an integer constant $n_0 \geq 1$. And we can choose $c = 1$ and $n_0 = 1$, so that for $n \geq n_0$, $\sum i^2 \leq n^3$. And $\sum i^2$ is $O(n^3)$.

C-1.22

$\sum \lceil \log_2 i \rceil$

$= \lceil \log_2 1 \rceil + \lceil \log_2 2 \rceil + \ldots + \lceil \log_2 n \rceil$

$\leq \log_2 1 + 1 + \log_2 2 + 1 + \ldots + \log_2 n + 1$

$= \log_2 1 + \log_2 2 + \ldots + \log_2 n + n$

$\leq \log n + \log n + \ldots + \log n + n$

$= n \log n + n$

$\leq c\, n \log n$

By the definition of big-O, we need to find a constant $c > 0$ and an integer constant $n_0 \geq 1$. And we can choose $c = 2$ and $n_0 = 2$, so that for $n \geq n_0$, $\sum \lceil \log_2 i \rceil \leq n \log n$. And $\sum \lceil \log_2 i \rceil$ is $O(n \log n)$.