

R-12.12

What is the worst-case depth of a k-d tree defined on n points in the plane? What about in higher dimensions?

In 2-dimension, because with every cut procedure, all the points are always divided into two equal parts, and k-d tree is a binary tree, the depth of the tree is $O(\log n)$.

In high-dimension, because of the same reason as 2-dimension, the depth of the tree is $O(\log n)$.

C-12.4

Design a static data structure (which does not support insertions and deletions) that stores a two-dimensional set S of n points and can answer queries of the form `countAllInRange(a, b, c, d)`, in $O(\log^2 n)$ time, which return the number of points in S with x-coordinates in the range $[a, b]$ and y-coordinates in the range $[c, d]$. What is the space used by this structure?

For this algorithm, we can use the range tree. In the searching procedure, we need to count the number of the point which satisfies the condition. The running time is $O(\log^2 n)$, which is the same to the range tree. And the space used by this structure is $O(n \log n)$.