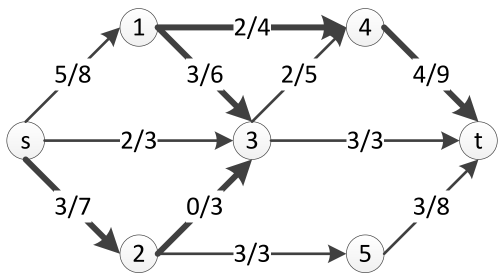
R-8.2

Answer the following questions on the flow network N and flow f shown Figure 8.6a:

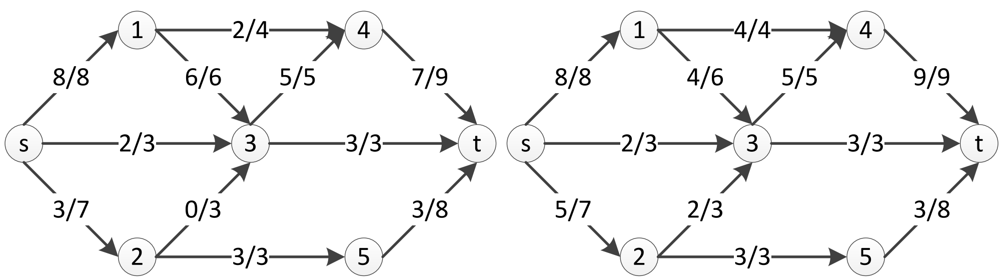
- What are the forward edges of augmenting path π? What are the backward edges?
- How many augmenting paths are there with respect to flow f? For each such path, list the sequence of vertices of the path and the residual capacity of the path.
- What is the value of a maximum flow in N?



Forward edge: s-v2, v2-v3, v1-v4, v4-t. Backward edge: v3-v1.

Augmenting path: there are 6 paths, s-v2-v3-v1-v4-t (residual capacity: 2), s-v3-v1-v4-t (residual capacity: 1), s-v2-v3-v4-t (residual capacity: 3), s-v1-v4-t (residual capacity: 2), s-v3-v4-t (residual capacity: 1), and s-v1-v3-v4-t (residual capacity: 3).

The value of a maximum flow: after adding value in the path (s-v1-v3-v4-t) and (s-v2-v3-v1-v4-t), there is no more augmenting path in the graph. As a result, the maximum flow is 15.



C-8.3

Let N be a flow network with n vertices and m edges. Show how to compute an augmenting path with the largest residual capacity in $O((n+m)\log n)$ time.

The residual capacity is the smallest capacity of one of the edge. We need to find this augmenting path using maximum spanning tree algorithm. We can design an algorithm like Kruskal. For each step, we pick each edge which has the greatest residual capacity. In the final spanning tree, the path from s to t is the result. Due to the theorem for Kruskal, the running time is $O\big((n + m)\log n\big)$ which n is the number of vertex and m is the number of edge.

C-8.6

Given a flow network N and a maximum flow f for N, suppose that the capacity of an edge e of N is decreased by one, and let N' be the resulting network. Give an algorithm for computing a maximum flow in network N' by modifying f.

If the flow of this edge f (v1, v2) is smaller than the capacity of this edge c (v1, v2), decreasing the capacity of this edge on one will not have an effect on the value of the flow f and we can just return the flow f as the result. If the flow of this edge f (v1, v2) is equal to the capacity of this edge c (v1, v2), decreasing the capacity of this edge on one will have an obvious effect on the value of the flow f and this will cause the decrease of the value of flow f. As a result, we need to find an augmenting path in this graph.

We need to find a single path p1 from s to v1 and a single path p2 from v2 to t by using DFS, which both are in the flow. We can use p2, edge e (v2, v1) and p1 to create a new path from t to s, and each edge has the value 1, and the directions of each edge are from t to s. We add this path to flow f, which means we decrease this flow by one, and then we can get the flow f' in the network N'. The running time is $O(n + m)$ which n is the number of vertex and m is the number of edge. (Use $O(n + m)$ time to find the each path)