R-5.4

Characterize each of the following recurrence equations using the master method (assuming that $T(n) = c$ for $n < d$, for constants $c > 0$ and $d \geq 1$).

a. $T(n) = 2T(n/2) + \log n$
b. $T(n) = 8T(n/2) + n^2$
c. $T(n) = 16T(n/2) + (n \log n)^4$
d. $T(n) = 7T(n/3) + n$
e. $T(n) = 9T(n/3) + n^3 \log n$

a. $T(n) = 2T(n/2) + \log n$

$\mathrm{a} = 2, \mathrm{b} = 2, \log_b a = 1, \mathrm{f(n)} = \log n$, case 1

$T(n) = O(n)$

b. $T(n) = 8T(n/2) + n^2$

$\mathrm{a} = 8, \mathrm{b} = 2, \log_b a = 3, \mathrm{f(n)} = n^2$, case 1

$T(n) = O(n^3)$

c. $T(n) = 16T(n/2) + (n \log n)^4$

$\mathrm{a} = 16, \mathrm{b} = 2, \log_b a = 4, \mathrm{f(n)} = (n \log n)^4$, case 2

$T(n) = O(n^4 (\log n)^5)$

d. $T(n) = 7T(n/3) + n$

$\mathrm{a} = 7, \mathrm{b} = 3, \log_b a = \log_3 7\,, \mathrm{f(n)} = n$, case 1

$T(n) = O(n^{\log_3 7})$

e. $T(n) = 9T(n/3) + n^3 \log n$

$\mathrm{a} = 9, \mathrm{b} = 3, \log_b a = 2, \mathrm{f(n)} = n^3 \log n$, case 3

$T(n) = O(n^3 \log n)$

R-5.12

Sally is hosting an Internet auction to sell n widgets. She receives m bids, each of the form "I want k_i widgets for d_i dollars," for i $= 1, 2, \dots,$ m. Characterize her optimization problem as a knapsack problem. Under what conditions is this a 0-1 versus fractional problem?

We can assume that the weight of the sack is n, the weight of each bid is k_i and the value of each bid is d_i. If the bidder cannot accept that the weight of the bid is less than k_i, it is a 0-1 knapsack problem. If the bidder can accept that the weight of the bid is less than k_i, it is a fractional knapsack problem.

C-5.1

A native Australian named Anatjari wishes to cross a desert carrying only a single water bottle. He has a map that marks all the watering holes along the way. Assuming he can walk k miles on one bottle of water, design an efficient algorithm for determining where Anatjari should refill his bottle in order to make as few stops as possible. Argue why your algorithm is correct.

He can use the greedy algorithm. First, he can draw a line between every pair of water holes that the distance between them is no farther than k miles, which means he can go from one point to the other point on one bottle of water. Then, he can create a path that includes the fewest lines from his position to the water hole that the distance between this water hole and the other side is less than k miles.

C-5.5

In the art gallery guarding problem we are given a line L that represents a long hallway in an art gallery. We are also given a set X $= \{x_0, x_1, \dots, x_{n-1}\}$ of real numbers that specify the positions of paintings in this hallway. Suppose that a single guard can protect all the paintings within distance at most 1 of his or her position (on both sides). Design an algorithm for finding a placement of guards that uses the minimum number of guards to guard all the paintings with positions in X.

We can use the greedy algorithm. We can assume a distance whose length is 2. We start from x_0 and cover the whole points that the distance between these points and x_0 is shorter than 2. Next we find the next point x_i that is not covered and start from x_i and cover the whole rest points that the distance between these points and x_i is shorter than 2. Then we repeat the above step until we have covered the whole points in the set X.

C-5.9

How can we modify the dynamic programming algorithm from simply computing the best benefit value for the 0-1 knapsack problem to computing the assignment that gives this benefit?

In this algorithm for 0-1 knapsack problem, we can add a new function to store the index of the item when we are updating the value. At last, when we get the final answer for the best benefit value, we can also get the whole indexes for this benefit and know which items should put in the knapsack.