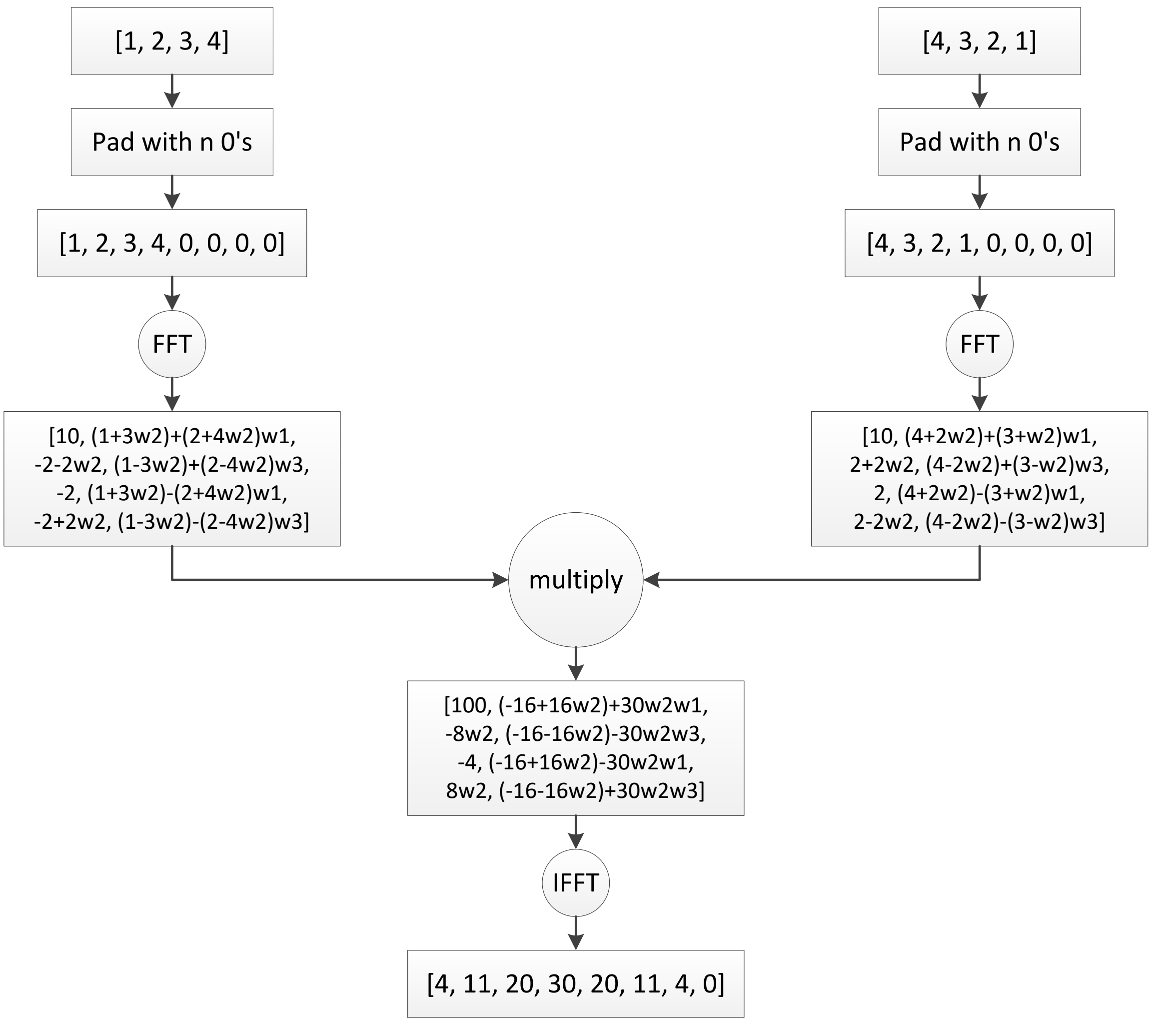
R-10.18

Use the FFT and inverse FFT to compute the convolution of a = [1, 2, 3, 4] and b = [4, 3, 2, 1]. Show the output of each component as Figure 10.15.

For easily understanding, I use the following expression for the eighth root of unity.

$$w_8^n = \{w_8^0, w_8^1, w_8^2, w_8^3, w_8^4, w_8^5, w_8^6, w_8^7\} = \{1, w1, w2, w3, w4, w5, w6, w7\}$$



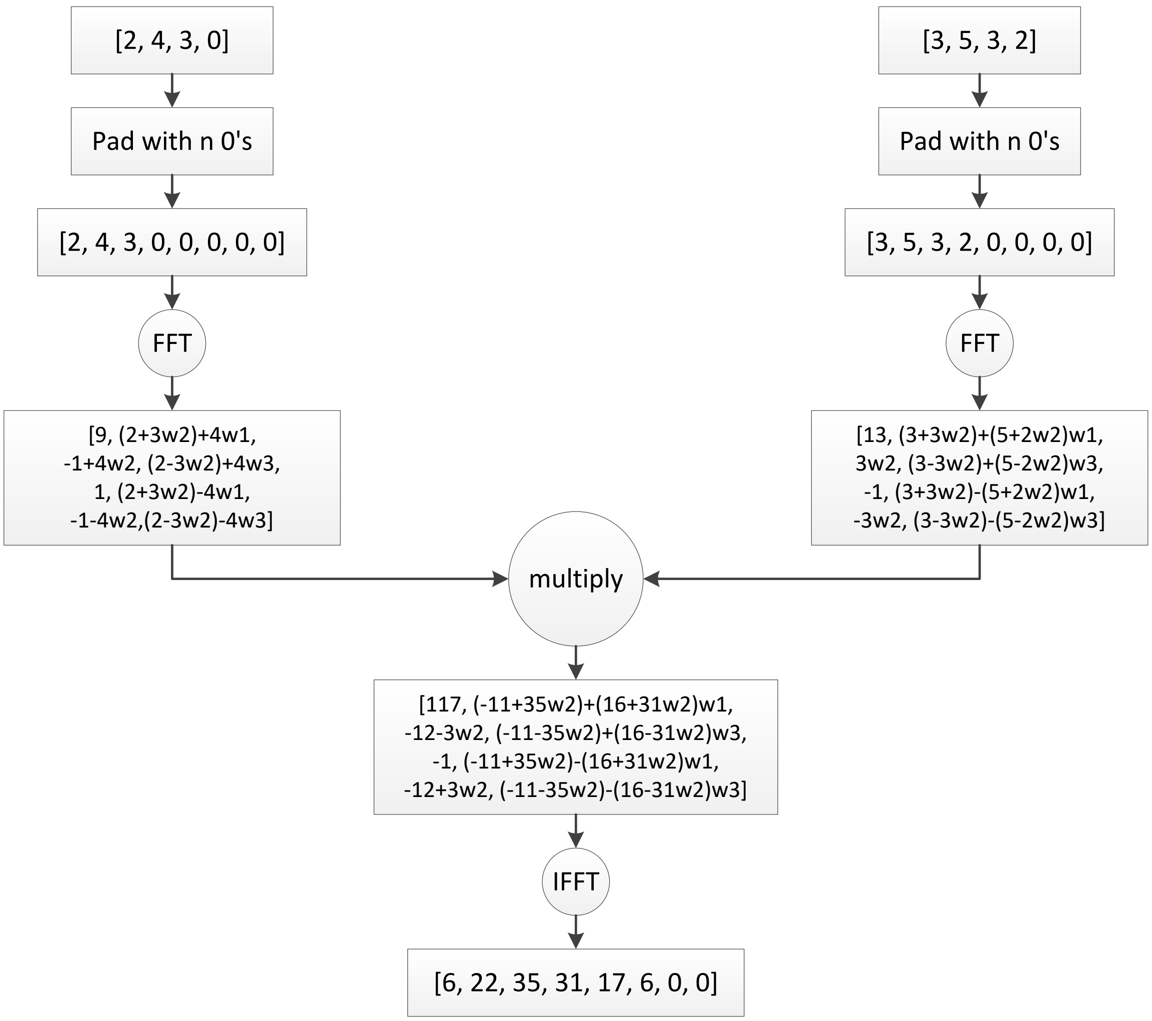
The result is [4, 11, 20, 30, 20, 11, 4, 0].

R-10.19

Use the convolution theorem to compute the product of the polynomials $p(x) = 3x^2 + 4x + 2$ and $q(x) = 2x^3 + 3x^2 + 5x + 3$.

For easily understanding, I use the following expression for the eighth root of unity.

$$w_8^n = \{w_8^0, w_8^1, w_8^2, w_8^3, w_8^4, w_8^5, w_8^6, w_8^7\} = \{1, w1, w2, w3, w4, w5, w6, w7\}$$



The result is $p(x) \cdot q(x) = 6 + 22x + 35x^2 + 31x^3 + 17x^4 + 6x^5$.

C-10.9

Given degree-n polynomials p(x) and q(x), describe an $O(n \log n)$-time method for multiplying the derivative of p(x) by the derivative of q(x).

First, we can use the standard calculus rule to compute the derivative of p(x) and q(x) and get p'(x) and q'(x). Second, we can use the FFT to compute the product of p'(x) and q'(x).

R-12.7

Provide a complete proof of Theorem 12.12.

Theorem 12.12:

Let S be a set of points in the plane, and let a be a point of S that is a vertex of the convex hull H of S. The next vertex of H, going counterclockwise from a, is the point p, such that triplet (a, p, q) makes a left turn with every other point q of S.

Because point a and p are the vertices of the convex hull, all the other points of S are contained on one side of the line through a and p, and all the angle are smaller than π.

And because p is the next vertex of H, going counterclockwise from a, all the point q are on the left side.

Totally, triplet (a, p, q) makes a left turn.

C-12.12

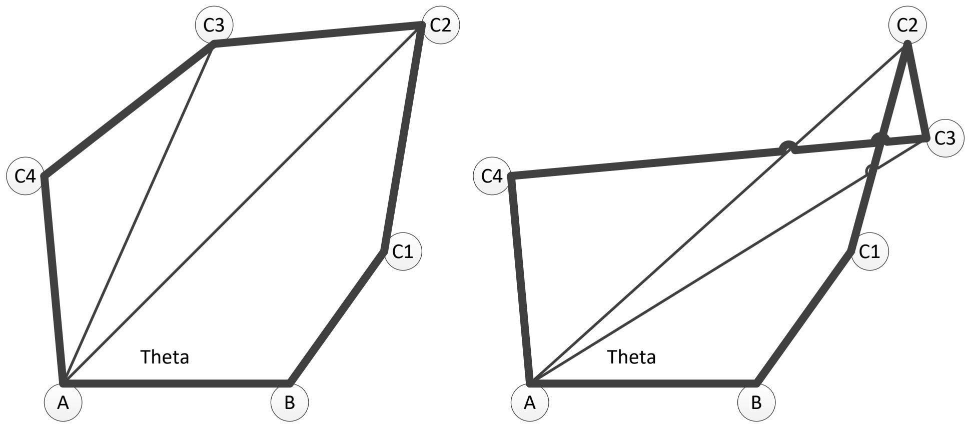
Design an $O(n)$-time algorithm to test whether a given n-vertex polygon is convex. You should not assume that P is nonintersecting.

Algorithm:

To test if a polygon is convex, we must test this polygon twice.

First, we must test if each vertex is convex. We can traverse this polygon in a counterclockwise order, and test if each turn is a left turn.

Second, we must test if each line is not intersect. We need to scan this polygon. We can choose one vertex A as the apex of an angle. Next, we can traverse this polygon in a counterclockwise order. We can choose the first vertex B and AB is as one line of this angle. For each vertex C in this polygon, AC is as the other line of this angle, and we will compute the angle in degree. With this traverse, this angle would increase. If there is a decrease of this angle, there must be intersect lines in the polygon. For example, in the following graph 1, ∠BAC3 > ∠BAC2, so there might not be intersect lines in the polygon. And in graph 2, ∠BAC3 < ∠BAC2, so there must be intersect lines in the polygon.



Running time:

For the each step, it will take $O(n)$-time to traverse the polygon. As a result, the total running time is $O(n)$.