Digital Image Processing Learning

Chapter 4

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Contents

Total Frame

The 2-D Fourier transform and its application to digital image.
The result of Fourier transform:magitude image and phase image.

2-D Sample Theory

Sampling used to digitize images.
The sampling intervals need to be

$$\frac{1}{\Delta T} > 2\mu_{max} \tag{1}$$

and

$$\frac{1}{\Delta Z} > 2v_{max} \tag{2}$$

$\frac{1}{\Delta T}, \frac{1}{\Delta Z}$ are the separations between samples and μ_{max}, v_{max} are the highest frequency content of the function in both the $\mu-$ and v direction.
If the function is under-sampled the periods overlap.Aliasing would result.

Zooming and shrinking

Zooming may be viewed as over-sampling,shrinking may be viewed as under-sampling.

original image

Zooming

zoom

Nothing changed.

Shrinking

shrink

We could see the aliasing clearly.

2-D Discrete Fourier Transform and Its Inverse

$$F(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\,e^{-j2\pi(ux/M+vy/N)} \tag{3}$$

where $f(x,y)$ is a dgital image of size $M*N$.
$u=0,1,2,\ldots,M-1$ and $v=0,1,2,\ldots,N-1$
The corresponding relationship between frequency domain and spatial domain.

$$f(x,y)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)\,e^{j2\pi(ux/M+vy/N)} \tag{4}$$

$x=0,1,2,\ldots,M-1$ and $y=0,1,2,\ldots,N-1$

Properties of the 2-D Discrete Fourier Transform

- Relationships Between Spatial and Frequency Intervals

$$\Delta u = \frac{1}{M\Delta T} \tag{5}$$

$$\Delta v = \frac{1}{N\Delta Z} \tag{6}$$

- Translation and Rotation

$$f(x,y)e^{j2\pi(u_0x/M+v_0y/N)} \Leftrightarrow F(u-u_0, v-v_0) \tag{7}$$

$$f(x-x_0, y-y_0) \Leftrightarrow F(u,v)e^{-j2\pi(x_0u/M+y_0v/N)} \tag{8}$$

Rotating $f(x,y)$ by an angle θ_0 rotates $F(u,v)$ by the same angle.Rotating $F(u,v)$ rotates $f(x,y)$ by the same angle.

- Periodicity

Properties of the 2-D Discrete Fourier Transform

- Symmetry Properties
  About the center point of a sequence.

$$w_e(x, y) = w_e(M - x, N - y) \tag{9}$$

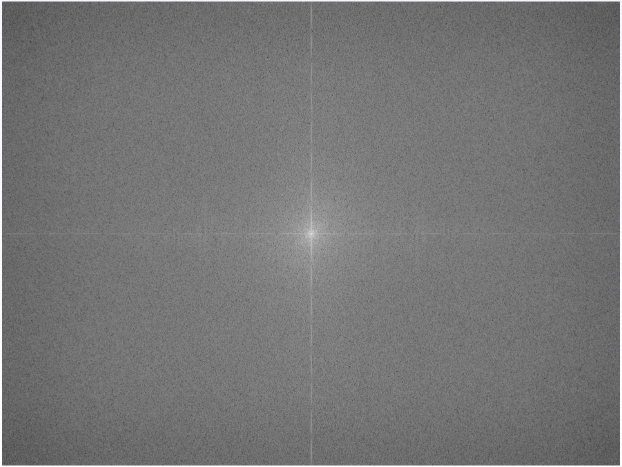
$$w_o(x, y) = -w_o(M - x, N - y) \tag{10}$$

- Spectrum and Phase Angle
  While the magnitude of the 2-D DFT is an array Whose components determine the intensities in the image,the corresponding phase carry much of the information about where discernable objects are located in the image.

- The 2-D Convolution Theorem
  Append zeros to solve wraparound error problem.
  $f(x, y)$ and $h(x, y)$ are two image arrays of sizes $A \times B$ and $C \times D$
  The resulting padded images are both of size $P \times Q$.Where $P \geqslant A + C - 1$ and $Q \geqslant B + D - 1$

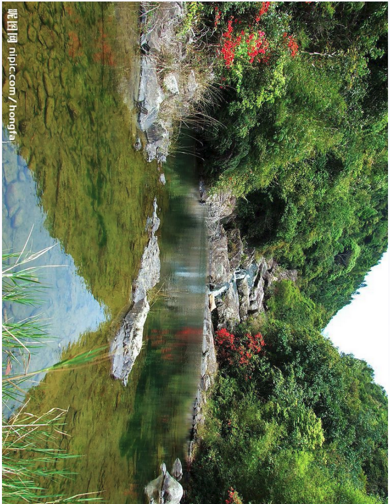
Original image

DFT image



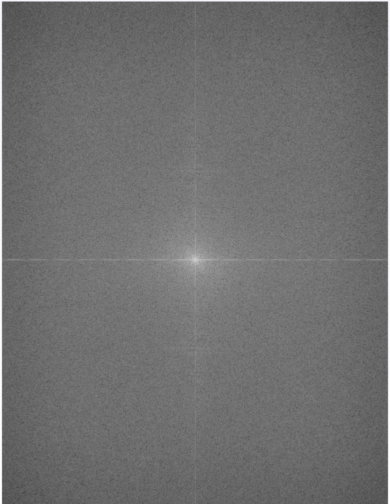
Rotating

Original image



Rotating

Rotating image



Summary of Fourier Transform

- The DFT of an image will be a frequency spectrum,whose magnitude is connected with its light and the phase is connected with its location about the original position.The spectrum rotates by the same angle of a rotated image.
- The low frequencies correspond to the slowly varying intensity components of an image while the high frequencies correspond to faster intensity changes.

Basic

Modifying the the Fourier transform to achieve a specific object,computing the inverse DFT to get back to the image domain.
Equations:

$$g(x,y) = \mathscr{F}^{-1}[H(u,v)F(u,v)] \tag{11}$$

Steps for Filtering

1. An input image $f(x, y)$ of size $M \times N$,obtain the padding parameters $P = 2M, Q = 2N$.
2. Form $f_p(x, y)$ of size $P \times Q$ by appending zeros to $f(x, y)$
3. Multiply $f_p(x, y)$ by $(-1)^{x+y}$ to center its transform
4. Compute the DFT from step 3
5. Generate a real,symmetric filter function,$H(u, v)$ of size $P \times Q$ with center at coordinates$(P/2, Q/2)$. $G(u, v) = H(u, v)F(u, v)$
6. $g_p(x, y) = real[\mathscr{F}^{-1}[G(u, v)]](-1)^{x+y}$, the real part is selected in order to ignore parasitic complex components resulting from computational inaccuracies.
7. Obtain the final processed result,$g(x, y)$,by extracting the $M \times N$ region from the top,left quadrant of $g_p(x, y)$

Image smoothing.

Origin image

Ideal Lowpass Filters

ILPF image $D_0 = 100$

Butterworth Lowpass FIlters

BLPF image $D_0 = 100, n = 2$

Gaussian Lowpass FIlters

GLPF image $D_0 = 100$

FIlters Summery

ILPF:ringing.
BLPF:good compromise between effective lowpass filtering and acceptable ringing.
GLPF:no ringing,less smoothing than the BLPF.

Highpass Filters-Butterworth Highpass Filters

BHPF image

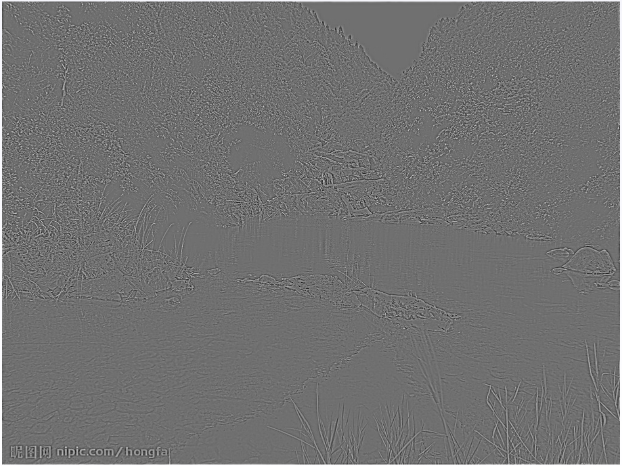
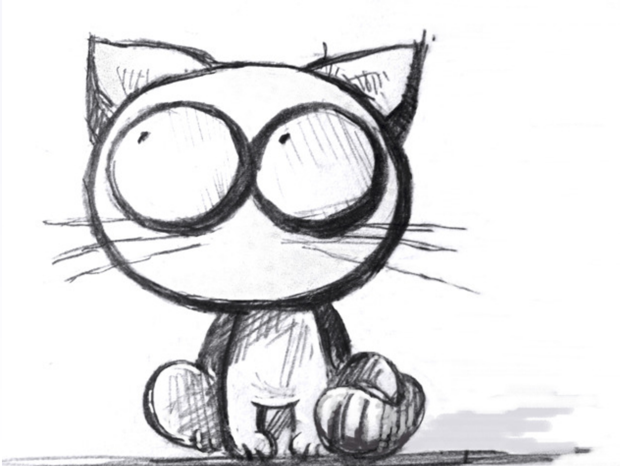


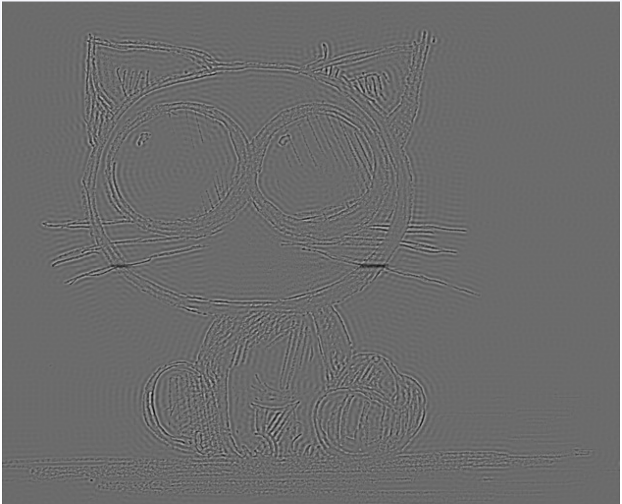
Image Sharpening.

Origin image



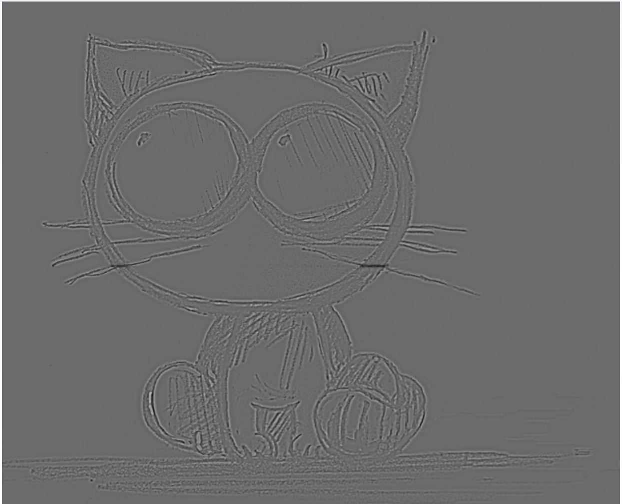
Ideal Highpass Filters

IHPF image



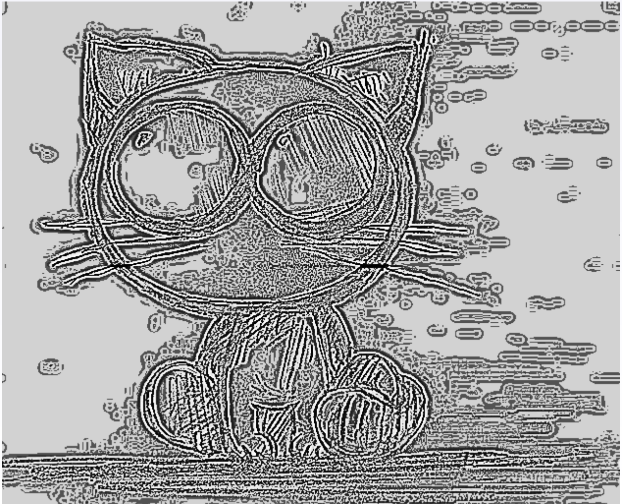
Butterworth Highpass Filters

BHPF image



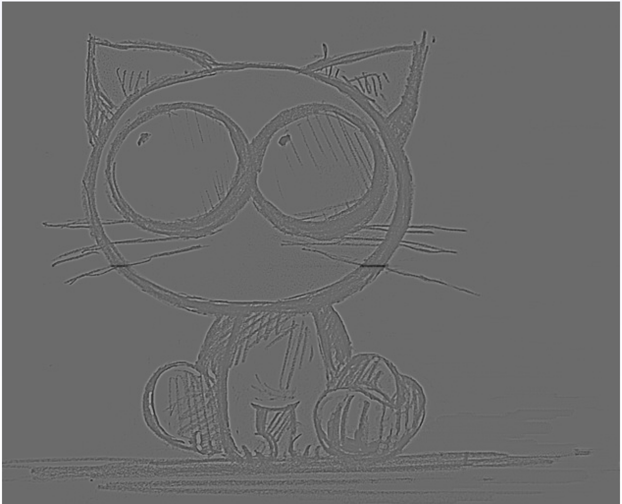
Equalization

Equal image



Gaussian Highpass Filters

GHPF image



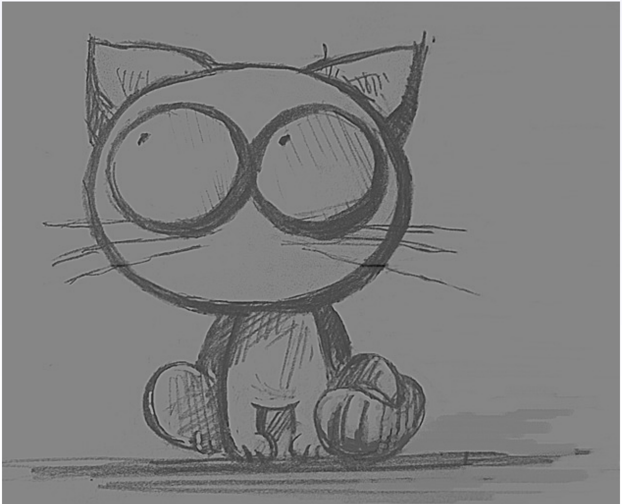
High-Frequency-Emphasis Filters

$$g(x, y) = \mathscr{F}^{-1}\{[k_1 + k_2 * H_{HP}(u, v)]F(u, v)\} \tag{12}$$

$k_1 \geqslant 0$ gives controls of the offset from the origin

High-Frequency-Emphasis Filters

$$k_1 = 1, k_2 = 4$$

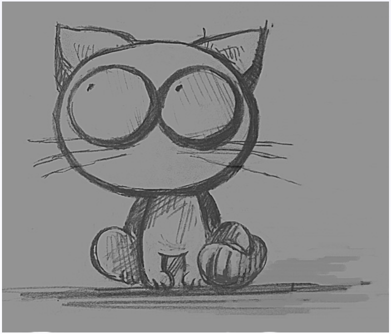


High-Frequency-Emphasis Filters

$$k_1 = 1, k_2 = 4$$

Homomorphic Filters

Homo image

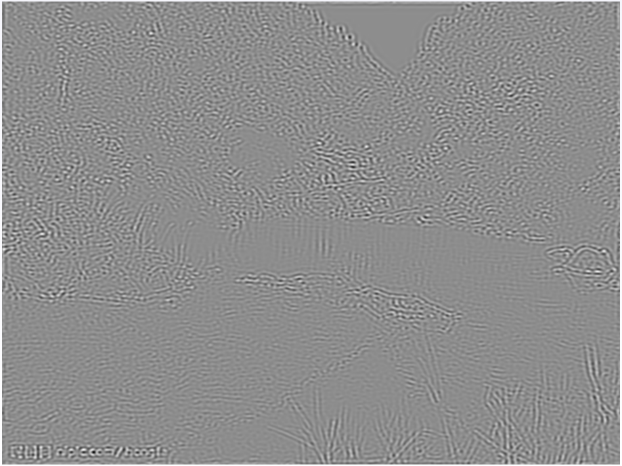


A procedure for improving the appearance of an image by simultaneous intensity range compres-sion(ln) and contrast enhancement(filter).

Origin image

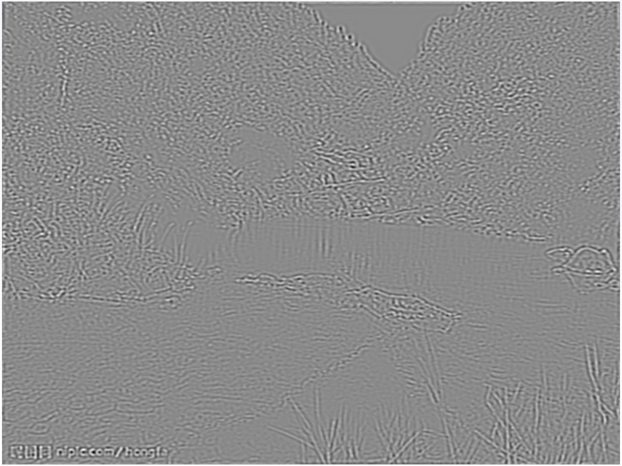
Bandpass Filters

$$D_0 = 100, w = 40$$



Bandpass Filters

$$D_0 = 100, w = 80$$



Bandpass Filters

$$D_0 = 5, w = 140$$