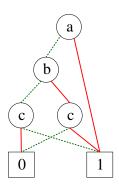
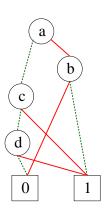
## VE527 Computer-Aided Design of Integrated Circuits Written Assignment Five

Out: Nov. 16, 2018; Due: Nov. 26, 2018; 6 Problems

1. (10%) What is the function F(a, b, c) for the following ROBDD?



2. (10%) How many input patterns (a, b, c, d), where  $a, b, c, d \in \{0,1\}$ , will satisfy the following BDD (i.e., let the function of this BDD be 1)?



3. (40%) A simple comparator takes two 2-bit unsigned binary numbers  $a_1a_0$  and  $b_1b_0$  and compares their magnitude. Its output Z=1 if and only if  $a_1a_0$  is GREATER THAN  $b_1b_0$ . For example, since  $a_1a_0=11>b_1b_0=01$ , the output Z=1. However, for  $a_1a_0=01$  and  $b_1b_0=01$ , since they are equal, the output Z=0.

Draw the ROBDD for the output Z of the 2-bit comparator. Suppose the variable ordering is  $b_0 < a_0 < b_1 < a_1$  (i.e.,  $b_0$  is the closest to the root and  $a_1$  is the farthest away from the root).

What does the ROBDD look like for the variable ordering  $a_1 < b_1 < a_0 < b_0$ ?

4. (18%) Consider the function f expressed in the CNF form:

$$f = \omega_1 \omega_2 \cdots \omega_8$$

where  $\omega_1 = a_1 + a_2$ ,  $\omega_2 = a_1 + a_2 + a_3 + \overline{a}_4$ ,  $\omega_3 = \overline{a}_2 + a_4$ ,  $\omega_4 = a_1 + \overline{a}_2 + \overline{a}_4$ ,  $\omega_5 = \overline{a}_1 + a_2 + a_4$ ,  $\omega_6 = \overline{a}_1 + a_2 + a_3$ ,  $\omega_7 = \overline{a}_1 + \overline{a}_2 + \overline{a}_4 + \overline{a}_5$ ,  $\omega_8 = \overline{a}_1 + a_2 + \overline{a}_3 + a_4 + \overline{a}_5$ . Assume that we have decided  $a_4 = 0$ . Make this assignment and then run Boolean constraint propagation (BCP) by applying the unit clause rule. You should continue BCP <u>until</u> you cannot make any additional progress on the formula. What is your conclusion after BCP and simplification? If the CNF is still unresolved, show the final CNF.

- 5. (10%) BDDs operating on general Boolean logic, and SAT solvers operating on CNF clause lists, are both techniques that we can use to work with complex logic. But they each have different capabilities. Which of the following statements are true about each respective technology? Select the correct ones.
  - a) We can use a BDD to find ALL satisfying assignments of a Boolean function.
  - b) We can use SAT to find a satisfying assignment of a Boolean function.
  - c) SAT solvers and BDD packages may not always work for any Boolean function.
  - d) SAT solvers and BDD packages will always work for any Boolean function.
  - e) We can use a BDD to find a satisfying assignment of a Boolean function.
  - f) A SAT solver allows us to perform many manipulations on a set of Boolean functions, such as AND, OR, NOT, XOR, XNOR, etc.
  - g) A BDD package allows us to perform many manipulations on a set of Boolean functions, such as AND, OR, NOT, XOR, XNOR, etc.
  - h) Suppose we have two gate level hardware implementations of some function, call them F and G. We want to check if the hardware for F produces identical outputs as the hardware for G. We can do this using BDDs, but we cannot do this using a SAT solver.
  - i) Suppose we have two gate level hardware implementations of some function, call them F and G. We want to check if the hardware for F produces identical outputs as the hardware for G. We can do this using a SAT solver, but we cannot do this using BDDs.

6. (12%) As we talked in lecture, we can apply SAT to check if two different gate-level implementations F and G of a logic function are identical. In this problem, we will try a trivial example. We want to compare whether F(x,y) = x is identical to G(x,y) = x + xy. The figure below shows the gate-level circuit on which we can ask the SAT question to see whether F and G are identical. We have also labeled two internal nodes as P and P0 as shown below. Write the CNF formula that is satisfiable if and only if the gate-level circuit below is satisfiable. **Hint**: apply the gate consistency function discussed in class.

