VE527: Assignment #4

Name: Chang Meng
Student ID:118370910019

November 12, 2018

1. (12%) Given the Boolean function $F(x, y, z, w) = (xy + \overline{x}z) \oplus (wz)$, obtain its cofactors F_x and $F_{\overline{y}z}$.
Answer:
Let $x = 1$,

$$F_x = (y + 0) \oplus (wz) = y \oplus (wz)$$

Let $y = 0$ and $z = 1$,

$$F_{\overline{y}z} = (0 + \overline{x}) \oplus w = \overline{x} \oplus w$$

2. (12%) In class, we have shown one form of Shannon expansion:

$$F(x_1, \ldots, x_i, \ldots, x_n) = x_i \cdot F(x_i = 1) + \overline{x_i} \cdot F(x_i = 0)$$

The above form can be thought of as a "sum of products" form. Actually, there is also a "product of sums" of the Shannon expansion:

$$F(x_1, \ldots, x_i, \ldots, x_n) = (\overline{x_i} + F(x_i = 1)) \cdot (x_i + F(x_i = 0))$$

Prove the above "product of sums" expression.
Answer:
If $x_i = 0$, we have:

$$F(x_1, \ldots, x_i = 0, \ldots, x_n) = x_i \cdot F(x_i = 1) + \overline{x_i} \cdot F(x_i = 0) = 0 + F(x_i = 0) = F(x_i = 0)$$

$$(\overline{x_i} + F(x_i = 1)) \cdot (x_i + F(x_i = 0)) = 1 \cdot F(x_i = 0) = F(x_i = 0)$$

If $x_i = 1$, we have:

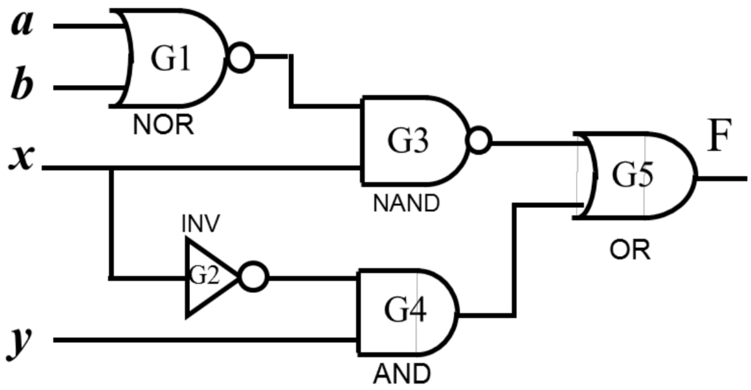
$$F(x_1, \ldots, x_i = 1, \ldots, x_n) = x_i \cdot F(x_i = 1) + \overline{x_i} \cdot F(x_i = 0) = Fx_i = 1 + 0 = F(x_i = 1)$$

$$(\overline{x_i} + F(x_i = 1)) \cdot (x_i + F(x_i = 0)) = F(x_i = 1) \cdot 1 = F(x_i = 1)$$

Since the expressions are equal for both $x_i = 0$ and $x_i = 1$,

$$F(x_1, \ldots, x_i, \ldots, x_n) = (\overline{x_i} + F(x_i = 1)) \cdot (x_i + F(x_i = 0))$$

3. (16%) Consider the small logic network shown below. Obtain the Boolean differences $\frac{\partial F}{\partial x}$ and $\frac{\partial F}{\partial y}$.



Answer:
The boolean function of F is:

$$F = \overline{(\overline{a+b})x} + \overline{x}y = a + b + \overline{x} + \overline{x}y = a + b + \overline{x}$$

So we have:

$$\frac{\partial F}{\partial x} = (a+b+1) \oplus (a+b+0) = 1 \oplus (a+b) = \overline{a+b}$$

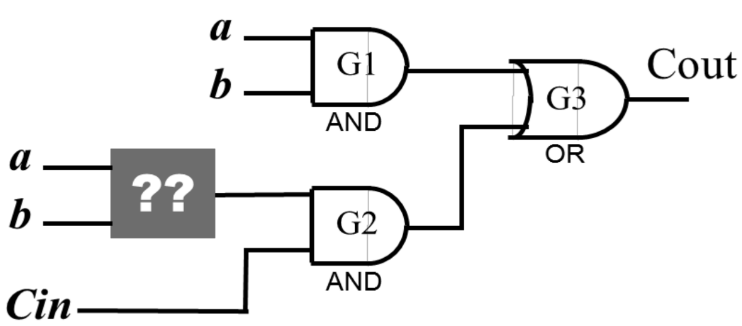
$$\frac{\partial F}{\partial y} = 0, \text{ because } F \text{ is not relevant to } y.$$

4. (10%) For the logic network from Problem 3, obtain the universal quantification ($\forall x F$) and the exisitential quantification ($\exists x F$).
Answer:

$$\forall x F = F_x \cdot F_{\overline{x}} = (a+b) \cdot 1 = a + b$$

$$\exists x F = F_x + F_{\overline{x}} = a + b + 1 = 1$$

5. (24%) Network repair.
The carry output of a 1-bit adder has the Boolean function $c_{out} = ab + (a+b)c_{in}$, where a and b are the 1-bit numbers we want to add, and c_{cin} is the input carry bit. The figure below shows an implementation of the above function. However, the implementation is not correct. We suspect that the gate the the "??" label is incorrect. Use the logic network repair procedure discussed in the lecture to fix the suspicious gate. What could this gate be?



Answer:

We see the suspicious gate as a 4-to-1 MUX, the output of MUX f is:

$$f = d_0\overline{a}\overline{b} + d_1\overline{a}b + d_2a\overline{b} + d_3ab$$

Then we can calculate the output G:

$$G1 = ab$$

$$G2 = fc_{in} = c_{in}(d_0\overline{a}\overline{b} + d_1\overline{a}b + d_2a\overline{b} + d_3ab)$$
$$G = G1 + G2 = ab + c_{in}(d_0\overline{a}\overline{b} + d_1\overline{a}b + d_2a\overline{b} + d_3ab)$$

By xnor G with the given c_{out} function, we get:

$$z = G\overline{\oplus}c_{out} = (ab + c_{in}(d_0\overline{a}\overline{b} + d_1\overline{a}b + d_2a\overline{b} + d_3ab))\overline{\oplus}(ab + (a+b)c_{in})$$

To repair it, we should find values of d_0, d_1, d_2, d_3 so that:

$$(\forall abc_{in}\ z)(d_0, d_1, d_2, d_3) = z_{\overline{a}\overline{b}\overline{c}_{in}} \cdot z_{\overline{a}\overline{b}c_{in}} \cdot z_{\overline{a}b\overline{c}_{in}} \cdot z_{\overline{a}bc_{in}} \cdot z_{a\overline{b}\overline{c}_{in}} \cdot z_{a\overline{b}c_{in}} \cdot z_{ab\overline{c}_{in}} \cdot z_{abc_{in}} = 1$$

So values of every z's cofactor are 1.

$$z_{\overline{a}\overline{b}\overline{c}_{in}} = G_{\overline{a}\overline{b}\overline{c}_{in}}\overline{\oplus}c_{out,\overline{a}\overline{b}\overline{c}_{in}} = 1$$

$$z_{\overline{a}\overline{b}c_{in}} = G_{\overline{a}\overline{b}c_{in}}\overline{\oplus}c_{out,\overline{a}\overline{b}c_{in}} = \overline{d_0} = 1, d_0 = 0$$
$$z_{\overline{a}b\overline{c}_{in}} = G_{\overline{a}b\overline{c}_{in}}\overline{\oplus}c_{out,\overline{a}b\overline{c}_{in}} = 1$$
$$z_{\overline{a}bc_{in}} = G_{\overline{a}bc_{in}}\overline{\oplus}c_{out,\overline{a}bc_{in}} = d_1 = 1$$
$$z_{a\overline{b}\overline{c}_{in}} = G_{a\overline{b}\overline{c}_{in}}\overline{\oplus}c_{out,a\overline{b}\overline{c}_{in}} = 1$$
$$z_{a\overline{b}c_{in}} = G_{a\overline{b}c_{in}}\overline{\oplus}c_{out,a\overline{b}c_{in}} = d_2 = 1$$
$$z_{ab\overline{c}_{in}} = G_{ab\overline{c}_{in}}\overline{\oplus}c_{out,ab\overline{c}_{in}} = 1$$
$$z_{abc_{in}} = G_{abc_{in}}\overline{\oplus}c_{out,abc_{in}} = 1$$

We have $d_0 = 0, d_1 = 1, d_2 = 1, d_3 = X$, the gate could be OR, XOR.

6. (10%) Suppose we have the following cube-list at one node of our URP tautology recursion, and we need to decide on splitting variable to use to cofactor and recurse. Which variable will you pick, and why?

x	y	z	w
11	01	01	11
01	11	10	01
01	10	11	11
10	11	10	10
01	11	01	10

Answer:
x, y, z, w has 4, 2, 4, 3 product dependent terms separately, we should pick one from x or z.
The value of $|\#true\_var - \#compl\_var|$ of x, z is 2, 0 separately. So we pick z for balance.

7. (16%) For the cube-list from Problem 6, suppose we choose the splitting variable as w. What are the resulting cube-list for its positive and negative cofactors, respectively?
Answer:
Positive cofactor:

x	y	z	w
11	01	01	11
01	11	10	11
01	10	11	11

Negative cofactor:

x	y	z	w
11	01	01	11
01	10	11	11
10	11	10	11
01	11	01	11