

# VE527 Computer-Aided Design of Integrated Circuits

## Written Assignment Four

Out: Nov. 3<sup>rd</sup>, 2018; Due: Nov 12<sup>th</sup>, 2018; 7 Problems

1. (12%) Given the Boolean function  $F(x, y, z, w) = (xy + \bar{x}z) \oplus (wz)$ , obtain its cofactors  $F_x$  and  $F_{\bar{y}z}$ .

2. (12%) In class, we have shown one form of Shannon expansion:

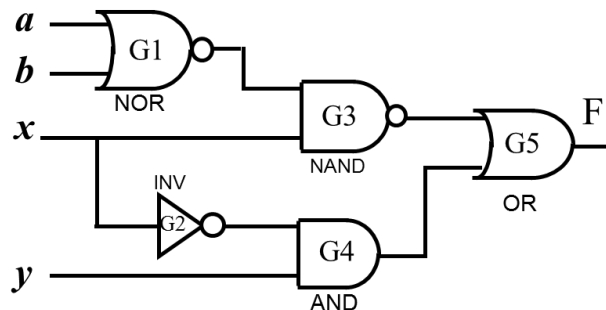
$$F(x_1, \dots, x_i, \dots, x_n) = x_i \cdot F(x_i = 1) + \bar{x}_i \cdot F(x_i = 0)$$

The above form can be thought of as a “sum of products” form. Actually, there is also a “product of sums” form of the Shannon expansion:

$$F(x_1, \dots, x_i, \dots, x_n) = (\bar{x}_i + F(x_i = 1)) \cdot (x_i + F(x_i = 0))$$

Prove the above “product of sums” expression.

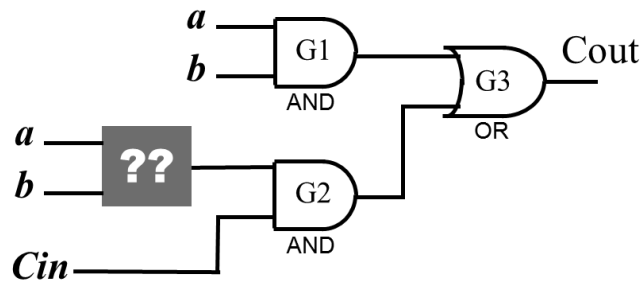
3. (16%) Consider the small logic network shown below. Obtain the Boolean differences  $\frac{\partial F}{\partial x}$  and  $\frac{\partial F}{\partial y}$ .



4. (10%) For the logic network from Problem 3, obtain the universal quantification  $(\forall x F)$  and the existential quantification  $(\exists x F)$ .

5. (24%) Network repair.

The carry output of a 1-bit adder has the Boolean function  $c_{out} = ab + (a + b)c_{in}$ , where  $a$  and  $b$  are the 1-bit numbers we want to add, and  $c_{in}$  is the input carry bit. The figure below shows an implementation of the above function. However, the implementation is not correct. We suspect that the gate with the “??” label is incorrect. Use the logic network repair procedure discussed in the lecture to fix the suspicious gate. What could this gate be?



6. (10%) Suppose that we have the following cube-list at one node of our URP tautology recursion, and we need to decide on the splitting variable to use to cofactor and recurse. Which variable will you pick, and why?

$x$	$y$	$z$	$w$
11	01	01	11
01	11	10	01
01	10	11	11
10	11	10	10
01	11	01	10

7. (16%) For the cube-list from Problem 6, suppose that we choose the splitting variable as  $w$ . What are the resulting cube-lists for its positive and negative cofactors, respectively?