VE527: Assignment #5

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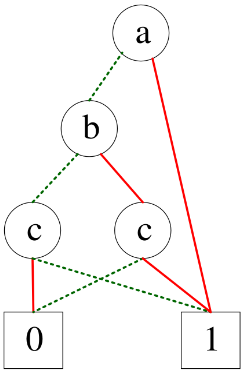
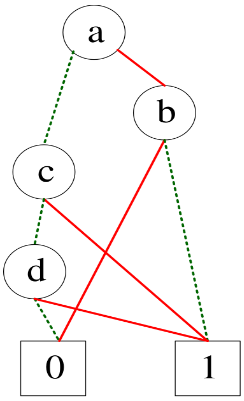
1. (10%) What is the function $F(a, b, c)$ for the following ROBDD?



Answer:
There are 3 paths from a to 0: a, $\bar{a}bc$, and $\bar{a}\bar{b}\bar{c}$. So the function is:

$$F(a, b, c) = a + \bar{a}bc + \bar{a}\bar{b}\bar{c} = a + \bar{b}\bar{c} + bc$$

2. (10%) How many input patterns (a, b, c, d), where $a, b, c, d \in 0, 1$, will satisfy the following BDD (i.e., let the function of this BDD be 1)?



Answer:
There are 10 input patterns. They are (1, 0, 0, 0), (1, 0, 0, 1), (1, 0, 1, 0), (1, 0, 1, 1), (0, 0, 1, 0), (0, 0, 1, 1), (0, 1, 1, 0), (0, 1, 1, 1), (0, 0, 0, 1), (0, 1, 0, 1).

3. (40%) A simple comparator takes two 2-bit unsigned binary numbers a_1a_0 and b_1b_0 and compares their maginitude. Its output $Z = 1$ if and only if a_1a_0 is GREATER THAN

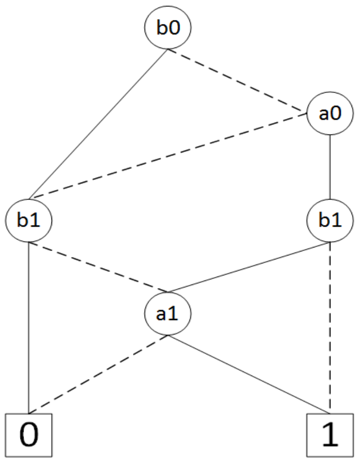
b_1b_0. For example, since $a_1a_0 = 11 > b_1b_0 = 01$, the output $Z = 1$. However, for $a_1a_0 = 01$ and $b_1b_0 = 01$, since they are equal, the output $Z = 0$.
Draw the ROBDD for the output Z of the 2-bit comparator. Suppose the variable ordering is $b_0 < a_0 < b_1 < a_1$ (i.e., b_0 is the closest to the root and a_1 is the farest away from the root). What does the ROBDD look like for the variable ordering $a_1 < b_1 < a_0 < b_0$?
Answer:
All possible patterns for $Z = 1$ are listed below:

a_1a_0	b_1b_0 that is smaller than a_1a_0
00	$\backslash$
01	00
10	$00, 01$
11	$00, 01, 10$

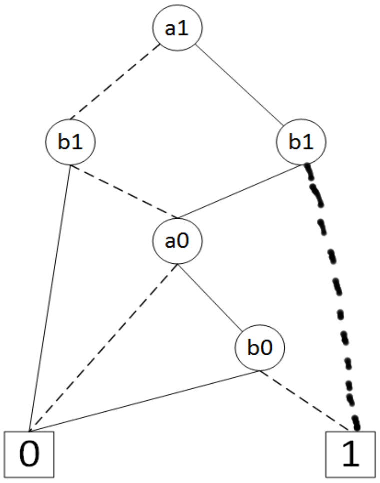
The K-Map is shown below:

a_1a_0 b_1b_0	00	01	11	10
00	0	1	1	1
01	0	0	1	1
11	0	0	0	0
10	0	0	1	0

So $Z = a_1\bar{b_1} + a_1a_0\bar{b_0} + a_0\bar{b_1}\bar{b_0}$.
In the ordering of $b_0 < a_0 < b_1 < a_1$, I decompose Z recursively:
b_0:
$Z_{b_0} = a_1\bar{b_1}$, $Z_{\bar{b_0}} = a_1\bar{b_1} + a_1a_0 + a_0\bar{b_1}$;
a_0:
Z_{b_0} is not relevant to a_0;
$Z_{\bar{b_0}a_0} = a_1\bar{b_1} + a_1 + \bar{b_1} = a_1 + \bar{b_1}$, $Z_{\bar{b_0}\bar{a_0}} = a_1\bar{b_1}$;
b_1:
$Z_{b_0b_1} = 0$, $Z_{b_0\bar{b_1}} = a_1$;
$Z_{\bar{b_0}a_0b_1} = a_1$, $Z_{\bar{b_0}a_0\bar{b_1}} = 1$;
$Z_{\bar{b_0}\bar{a_0}b_1} = 0$, $Z_{\bar{b_0}\bar{a_0}\bar{b_1}} = a_1$.



In the ordering of $a_1 < b_1 < a_0 < b_0$, I decompose Z recursively:
a_1:
$Z_{a_1} = \bar{b_1} + a_0\bar{b_0} + a_0\bar{b_1}\bar{b_0} = \bar{b_1} + a_0\bar{b_0}$, $Z_{\bar{a_1}} = a_0\bar{b_1}\bar{b_0}$;
b_1:
$Z_{a_1b_1} = a_0\bar{b_0}$, $Z_{a_1\bar{b_1}} = 1$;
$Z_{\bar{a_1}b_1} = 0$, $Z_{\bar{a_1}\bar{b_1}} = a_0\bar{b_0}$;
a_0:
$Z_{a_1b_1a_0} = \bar{b_0}$, $Z_{a_1b_1\bar{a_0}} = 0$;
$Z_{\bar{a_1}\bar{b_1}a_0} = \bar{b_0}$, $Z_{\bar{a_1}\bar{b_1}\bar{a_0}} = 0$.



4. (18%) Consider the function f expressed in the CNF form:

$$f = \omega_1\omega_2 \ldots \omega_8$$

where $\omega_1 = a_1+a_2$, $\omega_2 = a_1+a_2+a_3+\bar{a_4}$, $\omega_3 = \bar{a_2}+a_4$, $\omega_4 = a_1+\bar{a_2}+\bar{a_4}$, $\omega_5 = \bar{a_1}+a_2+a_4$, $\omega_6 = \bar{a_1} + a_2 + a_3$, $\omega_7 = \bar{a_1} + \bar{a_2} + \bar{a_4} + \bar{a_5}$, $\omega_8 = \bar{a_1} + a_2 + \bar{a_3} + a_4 + \bar{a_5}$.
Assume that we have decide $a_4 = 0$. Make this assignment and then run Boolean constraint propagation (BCP) by applying the unit clause rule. You should continue BCP until you cannot make any additional progress on the formula. What is your conclusion after BCP and simplification? If the CNF is still unresolved, show the final CNF.
Answer:
Applying $a_4 = 0$:
$\omega_1 = a_1 + a_2$,
$\omega_2 = 1$,
$\omega_3 = \bar{a_2}$,
$\omega_4 = 1$,
$\omega_5 = \bar{a_1} + a_2$;
$\omega_6 = \bar{a_1} + a_2 + a_3$,
$\omega_7 = 1$,
$\omega_8 = \bar{a_1} + a_2 + \bar{a_3} + \bar{a_5}$.
Implicate $a_2 = 0$:
$\omega_1 = a_1$,
$\omega_2 = 1$,
$\omega_3 = 1$,

$\omega_4 = 1$,
$\omega_5 = \bar{a_1}$;
$\omega_6 = \bar{a_1} + a_3$,
$\omega_7 = 1$,
$\omega_8 = \bar{a_1} + \bar{a_3} + \bar{a_5}$.
Implicate $a_1 = 1$ from $\omega_1 = a_1$:
$\omega_1 = 1$,
$\omega_2 = 1$,
$\omega_3 = 1$,
$\omega_4 = 1$,
$\omega_5 = 0$;
$\omega_6 = a_3$,
$\omega_7 = 1$,
$\omega_8 = \bar{a_3} + \bar{a_5}$.
$\omega 5$ is conficting.
The conclusion is that there does not exist a SAT assignment for $a_4 = 0$, then $a_4 = 1$ should be tried.
5. (10%) BDDs operating on general Boolean logic, and SAT solvers operating on CNF clause lists, are both techniques that we can use to work with complex logic. But they each have different capabilities. Which of the following statements are true about each respective technology? Select the correct ones.

a) We can use a BDD to find ALL satisfying assignemnts of a Boolean function.

b) We can use SAT to find a satisfying assignment of a Boolean function.

c) SAT solvers and BDD packages may not always work for any Boolean function.

d) SAT solvers and BDD packages will always work for any Boolean function.

e) We can use a BDD to find a satisfying assignment of a Boolean function.

f) A SAT solver allows us to perform many manipulations on a set of Boolean functions, such as AND, OR, NOT, XOR, XNOR, ect.

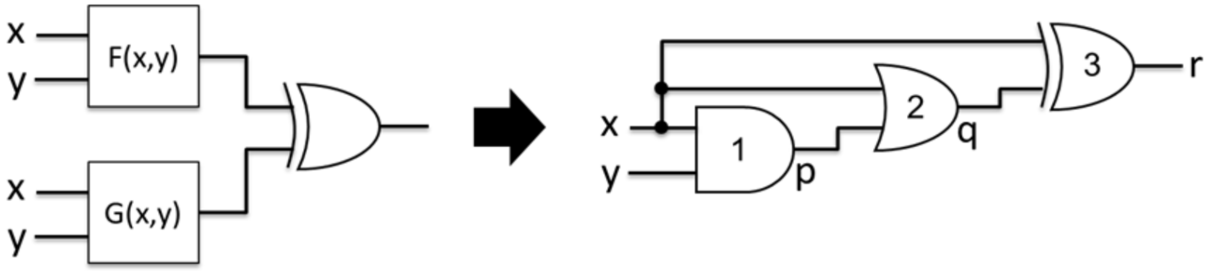
g) A BDD package allows us to perform many manipulations on a set of Boolean functions, such as AND, OR, NOT, XOR, XNOR, ect.

h) Suppose we have two gate level hardware implementations of some functions, call them F and G. We want to check if the hardware for F produces identical outputs as the hardware for G. We can do this using BDDs, but we cannot do this using a SAT solver.

i) Suppose we have two gate level hardware implementations of some functions, call them F and G. We want to check if the hardware for F produces identical outputs as the hardware for G. We can do this using a SAT solver, but we cannot do this using BDDs.

Answer:
a), b), c), e), g) are correct.

6. (12%) As we talked in lecture, we can apply SAT to check if two different gate-level implementations F and G of a logic function are identical. In this problem, we will try a trivial example. We want to compare wheter $F(x, y) = x$ is identical to $G(x, y) = x + xy$. The figure below shows the gate-level circuit on which we can ask the SAT question to see whether F and G are identical. We have also leabled two internal nodes as p and q as shown below. Write the CNF formula that is satisfiable if and only if the gate-level circuit below is satisfiable.



Hint: apply the gate consistency function discucssed in class.
Answer:
$\Phi_p = (x + \bar{p})(y + \bar{p})(\bar{x} + \bar{y} + p)$,
$\Phi_q = (\bar{x} + q)(\bar{p} + q)(x + p + \bar{q})$,
$\Phi_r = (\bar{x} + \bar{q} + \bar{r})(x + q + \bar{r})(x + \bar{q} + r)(\bar{x} + q + r)$.
$\Psi = r\Phi_p\Phi_q\Phi_r = r(x + \bar{p})(y + \bar{p})(\bar{x} + \bar{y} + p)(\bar{x} + q)(\bar{p} + q)(x + p + \bar{q})(\bar{x} + \bar{q} + \bar{r})(x + q + \bar{r})(x + \bar{q} + r)(\bar{x} + q + r)$.