

15.435x Sample Final Exam

Part 1: Shorter Questions

S1. (5 points) Which of the following can be estimated for an American option by constructing a single binomial tree (indicate all that are true):

- (a) delta
- (b) gamma
- (c) vega
- (d) theta
- (e) rho

S2. (5 points) What is a first-to-default credit default swap? Does its value **increase** or decrease as the default correlation between the companies in the basket increases? Briefly explain your answer.

S3. (6 points) Here is a statement to the shareholders of Abacus Inc. explaining the value per share they will receive when their company is acquired three months from now. If the price per share of Xylon Industries on the acquisition date is:

- (a) \$84.22 or greater, then you will receive .5462 shares of Xylon
- (b) less than \$84.22 but greater than \$76.20, then you will receive the number of shares of Xylon having a total value of \$46.00
- (c) \$76.20 or less, then you will receive .6037 shares of Xylon

Neither company pays dividends. How would you express the current value of a share of Abacus in terms of call options on Xylon?

S4. (5 points) Consider a one-year European put option on a 10-year bond. Assume that the current value of the bond is \$125, the strike price on the put is \$110, the one-year risk-free interest rate is 10% per annum, the bond's forward price volatility is 8% per annum, and the present value of the coupons to be paid during the life of the option is \$10. According to Black's model, what is the value of the put option?

S5. (5 points) According to Merton's model for pricing default risk, which of the following will **decrease** the price charged upfront for a credit guarantee on a zero-coupon defaultable corporate bond, all else equal? (*Indicate all that apply, there may be more than one correct answer.*)

- (a) A higher risk-free interest rate
- (b) Higher asset volatility
- (c) A higher face value of the bond
- (d) A longer bond maturity

S6: (5 points) Consider an MBS that splits all the income from a pool of mortgages between two tranches, an IO that receives all the interest payments, and a PO that receives all the principal payments. Imagine that there is an unanticipated upward parallel shift in the yield curve, e.g., rates at all maturities go up by 1%. Which of the following statements are generally true? (There may be more than one correct answer.)

- (a) Prepayment risk becomes more severe
- (b) The sum of the value of the IO and PO tranches goes down
- (c) The value of the IO increases relative to the value of the PO, i.e., $\text{Price}(\text{IO})/\text{Price}(\text{PO})$ goes up
- (d) The duration of the PO increases

Part 2: Longer Questions

L1. New Century Lending has a balance sheet consisting of high-quality auto and consumer loans, financed with a combination of bank loans, bonds, and equity. Regulators require it to hold equity equal to at least 5% of its assets, which creates an incentive to hedge its exposure to interest rate risk. Its market value balance sheet (\$ millions) stands at:

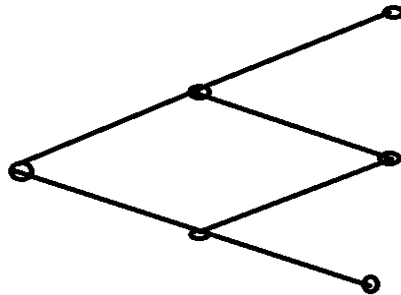
<u>Assets</u>		<u>Liabilities</u>	
Short-term loans	\$3,758	bank loans	\$7,801
(avg. modified duration = 1 yr)		(avg. modified duration = .4 yrs)	
Fixed-rate loans	\$6,242	Long-term debt	\$1,699
(avg. modified duration = 6.0 yrs)		(avg. modified duration = 7.8 yrs)	
		Owners' equity	\$500

- (a) (5 points) Calculate the dollar duration of New Century's assets **and** of its debt liabilities, treating each side of the balance sheet as a portfolio. [41210, 16372.6](#)
- (b) (8 points) Using the fact that the value of equity is the difference between the values of assets and liabilities, based on your above calculations, write down an approximation formula for how much the dollar value of equity will change for a parallel shift in the yield curve " Δy " (i.e., the dollar duration of equity). Use that formula to estimate approximately how much the dollar value of equity will change if the yield curve makes a parallel upward shift of 1%. [- 248.374](#)
- (c) (3 points) If New Century wanted to use an interest rate swap to reduce the sensitivity of its equity to changes in interest rates by delta hedging, would it be the fixed or floating rate payor? [fixed payor](#)
- (d) (5 points) Imagine that New Century wants to set up a delta hedge that minimizes the sensitivity of its equity to interest rate changes, using an interest rate swap. What would be the hedge ratio for that swap position?

[24837.4](#)

L2. The current price of a non-dividend paying stock is \$80. The risk-free rate is 3% per annum (continuous compounding), and the volatility is 16% per annum.

(a) (6 points) Construct a two-step stock price tree for a one-year horizon, using the approximation for a log-normal stock price process. Each step represents 6 months. Fill in the stock price for the nodes on the tree in this diagram:



(b) (12 points) Use the tree to estimate the value of (i) a one-year European put option with a strike price of \$77; (2) a one-year European put option with a strike price of \$83; and (iii) a long bear spread constructed with those two options.
 2.72953408
 5.417311906
 2.687777825

(c) (5 points) Imagine that the company is expected to pay a dividend of \$0.80 in 6 months. Holding the volatility, interest rate, strike prices, and time horizon constant, what would this do to your estimate of the value of the bear spread (circle one, do not explain):

- (i) increase value
- (ii) decrease value
- (iii) no effect on value
- (iv) indeterminate

(d) (10 points) Using the Black-Scholes-Merton model, and with everything as in part (a) (i.e., no dividends), what is the theoretical value of the one-year European put option with a strike price of \$83? What is the delta of the put option?
 \$5.3966
 - 0.4851

(e) (5 points) There are also forward contracts trading on the stock. What is the forward price of the stock for delivery in six months?
 81.20904517