Recitation 2

Question 1

Suppose there are three economic states next year. Suppose that state 1 occurs with probability 0.3, state 2 occurs with probability 0.4, and state 3 occurs with probability 0.3. There is a riskless bond that pays is $100 in all three states, that is, [$100,$100,$100]. This bond is currently traded at $96. There is a risky stock that pays the following amounts in each state: [$120,$75,$130]. The stock is currently traded at $80.

(a) What is the return on the bond?

(b) What is the expected return of the stock?

Solutions:

(a) Expected payoff:

$$E(B) = 0.3 \times \$100 + 0.4 \times \$100 + 0.3 \times \$100 = \$100.$$

Expected return:

$$\bar{r} = \frac{E(B) - P}{P} = \frac{100 - 95}{95} = 4.17\%.$$

(b) Expected payoff:

$$E(S) = 0.3 \times \$120 + 0.4 \times \$75 + 0.3 \times \$130 = \$105.$$

Expected return:

$$\bar{r} = \frac{E(S) - P}{P} = \frac{105 - 80}{80} = 31.25\%.$$

Question 2

Suppose there are two economic states next year. The prices for the two states are given by $(\phi_1, \phi_2) = (0.35, 0.55)$. The payoff of a riskless bond in one year is $100 in both states, that is, [$100,$100]. The payoff of a share of stock in one year is [$150,$40].

(a) What is the bond price today?

(b) What is the stock price today?

Solutions:

(a) The price of the bond is:

$$P_B = 0.35 \times \$100 + 0.55 \times \$100 = \$90.$$

sum of state prices should be less than 1, it is the discount rate of riskless bond

(b) The price of the stock is:

$$P_S = 0.35 \times \$150 + 0.55 \times \$40 = \$74.5.$$

Question 3

Let's consider a state space model that has three states. There are three assets traded in the market.

A riskless bond that pays \$100 in each of the three states is currently traded at \$95. Stock 1 pays off [\$250,\$0,\$500] and is currently traded at \$150, and Stock 2 pays off [\$0,\$0,\$500], and is traded at \$100.

What are the state prices?

Solutions:

The pricing equation is:

$$P = \phi_1 X_1 + \phi_2 X_2 + \phi_3 X_3.$$

Then for the three assets, we have:

$$95 = \phi_1 \times 100 + \phi_2 \times 100 + \phi_3 \times 100,$$

$$150 = \phi_1 \times 250 + \phi_2 \times 0 + \phi_3 \times 500,$$
$$100 = \phi_3 \times 500.$$

Solving this system of equations gives us the state prices: $(\phi_1, \phi_2, \phi_3) = (0.2, 0.55, 0.2)$.

Question 4

Let's consider a state space model that has three states. Current state prices are [0.2,0.55,0.2]. All three states are equally likely.

(a) What would be the price and expected return of Asset X that pays of [100,80,100]?

(b) What would be the price and expected return of Asset Y that pays of [100,100,80]?

(c) Economically, what explains the difference in prices and expected returns on assets X and Y?

Solutions:

(a) The price of asset X is computed as

$$P_X = 0.2 \times \$100 + 0.55 \times \$80 + 0.2 \times 100 = \$84.$$

To compute expected return, we first need to compute the expected payoff from Asset X. The expected payoff equals:

$$E[X] = 1/3 \times \$80 + 1/3 \times \$100 = \$93.33.$$

The expected return on X is computed as

$$\bar{r} = \frac{E[X] - P_X}{P_X} = \frac{93.33 - 84}{84} = 11.11\%.$$

Therefore, the expected rate of return from investing in asset X is 11.11%.

(b) The price of asset Y is computed as:

$$P_Y = 0.2 \times 100 + 0.55 \times 100 + 0.2 \times 80 = \$91.$$

To compute expected return, we first need to compute the expected payoff from Asset Y. The expected payoff equals:

$$E[Y] = 1/3 \times \$100 + 1/3 \times \$100 + 1/3 \times \$80 = \$93.33.$$

The expected return on Y is computed as

$$\bar{r} = \frac{E[Y] - P_Y}{P_Y} = \frac{93.33 - 91}{91} = 2.56\%.$$

(c) The price of state 2 is much higher than the price of states 1 and 3. This means that investors are willing to pay a higher price to receive \$1 in state 2. Put differently, investors value money in state 2 more than they do in states 1 and 3.

State 2, for example, can be an economic recession. In recessions, many economic agents may receive lower income. For example, workers may receive lower wages, firms may earn lower profits. Therefore, a security that pays money in bad states of the world typically has higher prices because money in bad states is more valuable.

Assets X and Y give the same payout in state 1. However, asset X pays \$80 in state 2 and \$100 in state 3, while asset Y pays \$100 in state 2 and

\$80 in state 1. Therefore, Y pays \$20 more in a bad state compared to X, but pays \$20 less than X in a good state. Intuitively, this implies that the price of Y should be higher than the price of X.

And because Y pays more when money is more valuable, investors are willing to accept lower rate of return from investing in it compared to asset X. As we have seen above, the expected return on X is 11.11% while on asset Y it is only 2.56%.

Question 5

(a) Your sister has offered to give you \$10,000 today or \$11,500 in 1 year from now. Suppose that the interest rate is 8%. Which option you prefer?

(b) Now suppose that your sister has offered to give you \$10,000 today or \$18,000 in 10 year from now. Suppose that the interest rate is 8%. Which option you prefer?

Solutions:

(a) To make a decision regarding which option is better, we need to compare the value of two options at the same moment in time. Let's compare these two options as of today.

The present value of \$10,000 to be received today is simply \$10,000.

The present value of \$11,500 to be received one year from today if the interest rate is 8% is PV = \$11,500/(1+8%)=\$10,648.

Therefore, \$11,500 one year from now is better.

Intuitively, 8% here represents your opportunity cost of capital. This means you can earn 8% annual rate of return elsewhere in the market from investing into assets that have similar risk as your sister's promise. If you take \$10,000 and invest into these comparable assets that return 8%, you will only get \$10,800 one year from now, which is lower than \$11,500 offered by your sister.

(b) Let's compute the present value of the second option:

$$PV = \$18,000/(1+8\%)^{10} = \$8,337.$$

Therefore, the present value of \$18,000 received 10 years from now discounted at 8% is lower than the receiving \$10,000 today.

You should take \$10,000 today.

Question 6

Suppose you have purchased an apartment in New York City 20 years ago. The value of this apartment has tripled since then.

(a) What is the average annual rate of return you earned on this purchase?

(b) Suppose that instead of buying the apartment, you have invested the same amount in the stock market. Assume that the average annual return on the market was 10.2% during this time period. For reference, we have shown in the lecture that large stocks in the U.S. had an average return of 11.9% between 1929 and 2018. By how much would your wealth have grown over this time period?

(c) How much of the wealth growth in part (b) is due to return earned on previous return?

Solutions:

(a) Let's assume that we bought the apartment for X dollars. Now its value is 3X dollars. We have had this apartment for twenty years. We can write down the following equation that equates the purchasing price of the apartment with its current price:

$$
\begin{aligned}
&3X = X \times (1+r)^{20} \\
\Leftrightarrow\ &3 = (1+r)^{20} \\
\Leftrightarrow\ &1+r = 3^{1/20} \\
\Leftrightarrow\ &r = 5.65\%.
\end{aligned}
$$

(b) Suppose that we invested X dollars 20 years ago. The question asks us what would be the value of this investment today. An alternative way of thinking about this is – what would be the future value of X dollars in 20 years if the annual interest rate is 10.2%. The future value is

$$X \times (1+r)^{20} = X \times (1+10.2)^{20} = 7X.$$

Therefore, our investment would have grown 7 times.

Note that in part (a) we have shown that an initial investment grows 3 times over 20 years if the interest rate is 5.65%. Here, we have shown that it grows 7 times over the same time period if interest rate is 10.2%.

Therefore, small differences in annual interest rates may result in large differences in wealth accumulated over long horizons.

(c) To answer this question, let's first compute what would have been our wealth if we earned return only on the principal amount of our investment.

Each year, we would have earned $X \times 10.2\%$ and over 20 years this would have been $20 \times X \times 10.2\% = 2 \times X$. The difference between 7X computed in part (b) and 2X computed here is the return earned on previous return. Put differently, most of the growth in wealth in part (b) is due to return earned on previous return.

Often, this is colloquially referred to as the power of compounding. Over long horizons, most of the growth in wealth comes from return earned on previously earned return.

Question 7

Suppose you own an apartment building. During the last fiscal year, which closed today, your rental revenue was \$650,000. Over the next year, you expect to grow rental revenue by 3% in real terms. Inflation is expected to be 2.5%. Assume that the appropriate real discount rate is 6%.

(a) What is your expected rental revenue next year in real terms?

(b) What is the present value of your real rental revenue that you expect to receive next year?

(c) What is your expected rental revenue next year in nominal terms?

(d) What is the present value of your nominal rental revenue that you expect to receive next year?

Now suppose that real annual growth rate of 3% will continue over the next 4 years. The inflation is also expected to remain 2.5% over the next 4 years. Assume that the appropriate real discount rate is 6%.

(e) What is your expected rental revenue in Year 1, 2, 3, and 4 in real terms?

(f) What is the present value of your real rental revenue that you expect to receive over the next 4 years.

(g) What is your expected rental revenue in Year 1, 2, 3, and 4 in nominal terms?

(h) What is the present value of your nominal rental revenue that you expect to receive over the next 4 years.

Solutions:

Part I

(a) The expected rental revenue next year (in real terms) is

$$\$650,000 \times (1 + 3\%) = \$669,500.$$

(b) The present value is

$$PV = \frac{\$669,500}{1+6\%} = \$631,604.$$

1+g_n = (1+g_r)(1+i)
nominal growth = real growth * inflation

(c) The expected rental revenue next year in nominal terms is

$$\$650,000 \times (1+3\%) \times (1+2.5\%) = \$686,237.5.$$

Here remember 2.5% is the inflation rate.

The present values should not be affected, whether we're dealing with nominal or real units.

(d) The present value is

$$PV = \frac{\$686237.5}{(1+6\%)(1+2.5\%)} = \$631,604.$$

1+r_n = (1+r_r) (1+i)
nominal discount rate= real discount rate * inflation

Part II

(e) The current revenue is \$650,000. Then the expected revenue in Year 1 is

$$R_1 = \$650000 \times (1+3\%) = \$669,500.$$

Similarly, the expected revenue in Year 2 is

$$R_2 = \$650000 \times (1+3\%)^2 = \$689,585.$$

In Year 3,

$$R_3 = \$650,000 \times (1+3\%)^3 = \$710,272.55.$$

In Year 4,

$$R_4 = \$650,000 \times (1+3\%)^4 = \$731,580.73.$$

(f) The present value is

$$\begin{aligned} PV &= \sum_{i=1}^{4} \frac{R_i}{(1+r)^i} \\ &= \frac{\$669,500}{(1+6\%)} + \frac{\$689,585}{(1+6\%)^2} + \frac{\$710,272.55}{(1+6\%)^3} + \frac{\$731,580.73}{(1+6\%)^4} \\ &= \$2,421,171. \end{aligned}$$

(g) The nominal revenue in Year 1 is $R_{N,1} = \$650,000 \times (1+g)$, where g is the nominal growth rate $g = (1+3\%) \times (1+2.5\%) - 1 = 5.575\%$.

Then

$$R_{N,1} = \$650,000 \times (1+5.575\%) = \$686,237.5.$$

Similarly,

$$R_{N,2} = \$650,000 \times (1+5.575\%)^2 = \$724,495.24.$$

$$R_{N,3} = \$650,000 \times (1+5.575\%)^3 = \$764,885.85.$$

$$R_{N,4} = \$650,000 \times (1+5.575\%)^4 = \$807,528.24.$$

(h) We need to first find the nominal discount rate $r_{nominal}$. It satisfies the following equation:

$$\begin{aligned}
1 + r_{nominal} &= (1 + r_{real})\,(1 + i) \\
&= (1 + 6\%)\,(1 + 2.5\%) \\
&= 1.0865
\end{aligned}$$

Hence, $r_{nominal} = 8.65\%$.

The present value is

$$\begin{aligned}
PV &= \sum_{i=1}^{4} \frac{R_{N,i}}{(1 + r_{nominal})^i} \\
&= \frac{\$686,237.5}{(1 + 8.65\%)} + \frac{\$724,495.24}{(1 + 8.65\%)^2} + \frac{\$764,885.85}{(1 + 8.65\%)^3} + \frac{\$807,528.24}{(1 + 8.65\%)^4} \\
&= \$2,421,171.
\end{aligned}$$