Recitation 2

Problem 1

Company A, a British manufacturer, wishes to borrow U.S. dollars at a fixed rate of interest. Company B, a U.S. multinational, wishes to borrow sterling at a fixed rate of interest. They have been quoted the following interest rates per annum (adjusted for differential tax effects):

	Pounds	Dollars
Company A	11.0%	7.0%
Company B	10.6%	6.2%

Design a swap that will net a financial institution, acting as an intermediary, 10 basis points per annum and will produce a gain of 15 basis points per annum for each of the two companies, relative to what they would pay borrowing directly in their preferred currencies.

Solution: The first thing to note is that Company A has a *relative* advantage when borrowing in pounds. Although it costs Company A 40 bps more to borrow in pounds relative to Company B, it costs Company A 80 bps more to borrow in dollars. Symmetrically, Company B has a relative advantage when borrowing in dollars. So, there is an opportunity for both companies to lower their borrowing costs using a (fixed-for-fixed) currency swap. How exactly should the swap be structured?

Let's assume that both Company A and Company B want to borrow the same principal amount, which we will denote P in dollars and P^* in pounds. Company A initially borrows P^* in pounds at an interest rate of 11%. In a swap, the intermediary exchanges P^* for P at the spot exchange rate between pounds and dollars. Since we're on an annual basis, the swap can be structured so that every year Company A pays a fixed interest rate on the principal P to the intermediary in dollars and receives a fixed interest rate on the principal P^* in pounds.

To finance its loan, Company A needs to receive a fixed interest rate of 11% in pounds. Since Company A wants to borrow U.S. dollars, it will pay an interest rate that is 15 bp less than the 7% interest rate it could have paid without the swap, or 6.85%.

What about Company B? Company B initially borrows P in dollars at an interest rate of 6.2%. In the currency swap, the intermediary exchanges P for P^* at the spot exchange rate. Each year, Company B pays a fixed interest rate on the principal P^* to the intermediary in pounds and receives a fixed interest rate on the principal P in dollars.

To finance its loan, Company B needs to receive a fixed interest rate of 6.2% in dollars. Since Company B wants to borrow in pounds, it will pay an interest rate that is 15 bp less than the 10.6% interest rate it could have paid without the swap, or 10.45%.

At the terminal date of the currency swap, the principal is exchanged again. The intermediary

earns a spread of 10 bp on the swap. The spread is partly compensation for the **exchange rate risk** faced by the intermediary, but the intermediary could always hedge this risk using forward contracts. The more important risk is **counterparty risk**, the risk that either Company A or Company B fails to meet its obligations.

The diagram below summarizes the cash flows of the swap:

Note: At initiation, the principal amounts P and P^* flow in the opposite direction of the arrows. Interest payments flow in the same direction as the arrows during the life of the swap, and the principal amounts also flow in the same direction as the arrows at the terminal date of the swap.

Problem 2

Imagine that a commercial builder takes a short position in a mortgage futures contract, locking in a borrowing rate linked to the price of a mortgage-backed security (MBS). The builder, however, has no plans to borrow using a mortgage. Do you think that shorting the futures contract is speculating or hedging in this case?

Solution: First, let's review the mechanics of a mortgage futures contract. A mortgage futures contract is similar to the familiar commodities futures contract, except that the "commodity" is an MBS, which is backed by the cash flows from a pool of mortgages. A mortgage futures contract locks in a pre-specified price for a MBS—equivalently, a borrowing rate—in the future.

A long position in the mortgage futures contract will *increase* in value if the price of the MBS increases or, equivalently, if interest rates decrease. (Recall that prices and yields on fixed-income securities move inversely.) On the other hand, a short position will *decrease* in value if the price of the MBS increases or if interest rates decrease. Since the builder is taking a short position in the mortgage futures contract, he will profit if interest rates increase. Typically, higher interest rates discourage real estate investment.

So, it is reasonable to think that the builder is shorting the mortgage futures contract to protect himself against a decline in the demand for commercial real estate construction. This would be consistent with a hedging motive.

Problem 3

A currency swap has a remaining life of 15 months. It involves exchanging interest at 10% on £20 million for interest at 6% on $30 million once a year. The term structure of risk-free interest rates in the UK is flat at 7%, and the term structure of risk-free rates in the U.S. is flat at 4% (both with annual compounding). The current exchange rate is 1.55 dollars per pound sterling.

(a) What is the value of the swap to the party paying sterling?

Solution: The swap involves exchanging the sterling interest of $20 \times 0.10 = £2$ million for the dollar interest of $30 \times 0.06 = \$1.8$ million. The principal amounts are also exchanged at the swap's terminal date.

Notice that the cash flows of the swap are identical to those of a coupon bond. The value of the sterling bond underlying the swap is:

$$\frac{2}{(1.07)^{3/12}} + \frac{22}{(1.07)^{15/12}} = £22.182 \text{ million.}$$

Similarly, the value of the dollar bond underlying the swap is:

$$\frac{1.8}{(1.04)^{3/12}} + \frac{31.8}{(1.04)^{15/12}} = \$32.061 \text{ million.}$$

Thus, the value of the swap to the party paying sterling—and receiving dollars—is $32.061 - (22.182 \times 1.55) = -\$2.321$ million.

(b) What is the value to the party paying dollars?

Solution: Since the cash flows are reversed, the value of the swap to the party paying dollars is the negative of the value to the party paying sterling, or \$2.321 million. An alternative way to see this is to view the swap as a portfolio of forward contracts.

To be consistent with Part (a), we will value the swap to the party paying sterling. First, we can find the continuously compounded interest rates in sterling and dollars that correspond to the given annual rates to be $\ln(1.07) \approx 6.766\%$ per annum and $\ln(1.04) \approx 3.922\%$ per annum, respectively.

Using these continuously compounded interest rates, we can find the 3-month and 15-month forward exchange rates as $1.55e^{(0.03922-0.06766)\times 3/12} = 1.539$ and $1.55e^{(0.03922-0.06766)\times 15/12} = 1.4959$.

The values of the two forward contracts corresponding to the exchange of interest for the party paying sterling are:

$$(1.8 - 2 \times 1.5390)e^{-0.03922\times 3/12} = -\$1.2656 \text{ million}$$

and

$$(1.8 - 2 \times 1.4959)e^{-0.03922\times 15/12} = -\$1.1347 \text{ million.}$$

Finally, the value of the forward contract corresponding to the exchange of principals is:

$$(30 - 20 \times 1.4959)e^{-0.03922\times 15/12} = \$0.0787 \text{ million.}$$

The total value of the swap to the party paying sterling is the sum of these three forward contracts, $-\$1.2656 - \$1.1347 + \$0.0787 = -\$2.322$ million, which, allowing for rounding errors, is the same as that given by the bond valuation method in Part (a)!