



# 15.415x Foundations of Modern Finance

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## **Lecture 20: Payout & Risk Management**

# Key concepts

- Payout overview
- Payout empirics
- MM and irrelevance of payout policy
- Payout beyond MM: Taxes, information asymmetry, agency costs...
- Corporate risk management
- MM and irrelevance of risk management
- Risk management beyond MM
- Hedging mechanics for different risks

# Key concepts

## ■ Payout overview

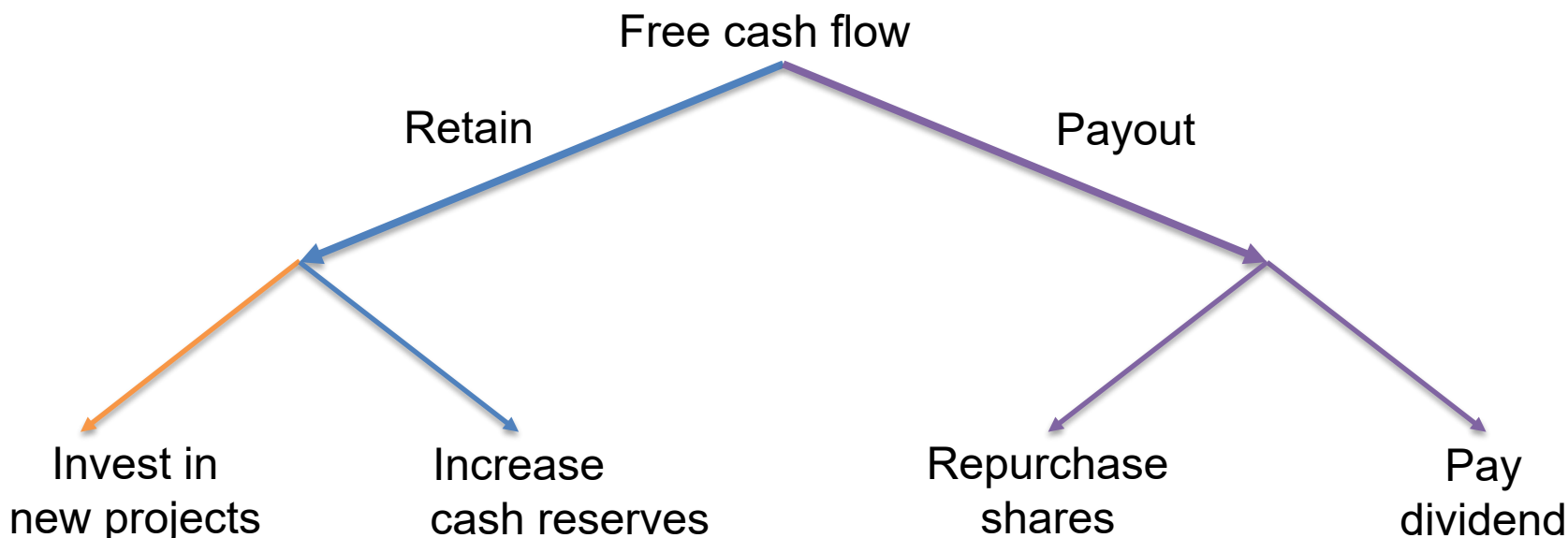
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# Payout: An overview

- Payout policy is one of the two major components of a firm's financing decisions (the other being its capital structure).
  - Dividend cuts can lead to severe declines in the stock price.
  - Maintaining a stable payout policy can prevent a firm from undertaking profitable investments.
- Corporate finance.
  - Can a firm add value by changing its payout policy?
  - If so, what factors determine a firm's optimal payout policy?
- Market reaction.
  - If a firm deviates significantly from its optimal payout policy, traditional investors may sell it.
  - In some cases, activist investors may try to change the firm's payout policy.

# Distribution to shareholders

**Payout policy:** Alternative ways to distribute free cash flow to equity holders.



- The tradeoff between retaining and paying out free cash flow depends on the efficiency with which cash is used within the firm.
  - Financing positive-NPV projects (now or in the near future) will clearly be a good use of cash.
  - Excessive cash can create problems (e.g., misuse).

# Payout methods

## ■ Dividends

- Regular dividend,
- Special dividend.

## ■ Dividend mechanics

- **Declaration date:** The date on which the board of directors authorizes the payment of a dividend.
- **Record date:** When a firm pays a dividend, only shareholders on record on this date receive the dividend.
- **Ex-dividend date:** A date, two days prior to a dividend's record date (due to T+2 settlement), on or after which anyone buying the stock will not be eligible for the dividend.
- **Cum-dividend date:** The trading date before the ex-dividend date.
- **Distribution date:** A date, generally within a month after the record date, on which a firm mails dividend checks to its registered stockholders.

# Payout methods

## ■ Share repurchases (buybacks)

- A firm can use cash to buy its own shares outstanding.
- The firm can either retire the repurchased shares or keep them as treasury stock, available for re-issuance in the future.

## ■ Share repurchase mechanics

- **Open market repurchase:** Buying shares in the open market (about 95% of all repurchases).
- **Tender offer:** A public offer to all existing shareholders to buy back a given amount of outstanding shares at a pre-specified price (typically 10-20% premium over current market price) over a pre-specified period (typically 20 days).
- **Targeted repurchase:** Direct negotiation with a major shareholder.

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# Payout empirics

- Dividends are widespread (most Fortune 500 firms pay dividends).
- Dividends are substantial (50-60% of after-tax profit).
- Cash dividend is a major form (until recently).
- Share repurchase has become more popular.
- High tax bracket individuals receive most dividends.
- General payout policy (Lintner, 1956):
  - Payout ratios are quite stable over long-run;
  - Managers worry more about changes rather than levels of dividends;
  - Dividend policies precede other decisions (use outside funds to pay dividends if needed).
- Stock price increases (decreases) when dividends are initiated (omitted).
  - 3.7% at initiation (Asquith and Mullins, 1983)
  - -7.0% at omission (Michaeli, Thaler and Womack, 1995).

# Survey evidence

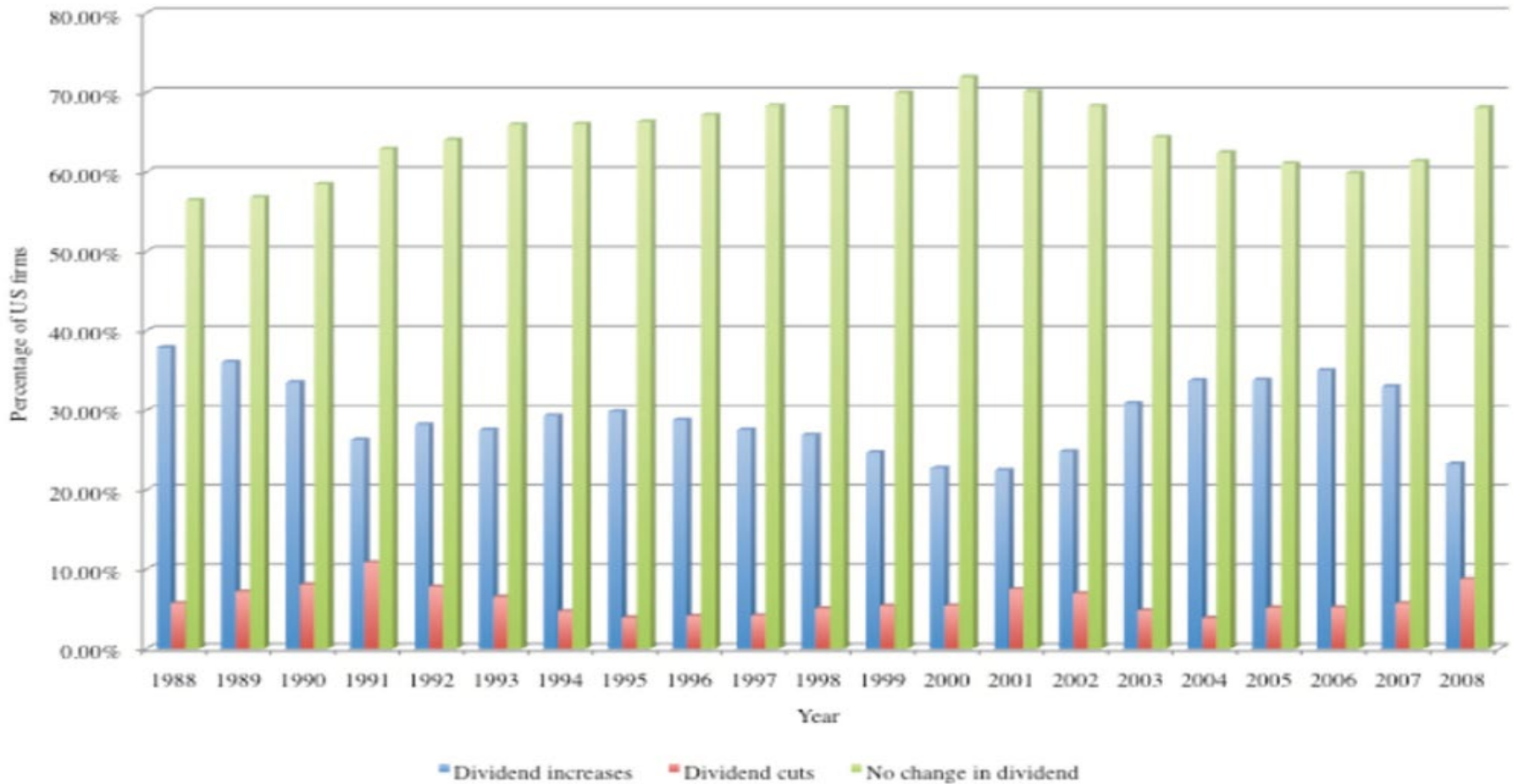
- Survey of financial executives about dividend policy finds similar evidence (Brav, Graham, Harvey and Michaely, 2005).
- Executives' response to the following question: "Do these statements describe factors that affect your company's dividend decisions?"

Policy Statement	% agree or strongly agree
1. We try to avoid reducing dividends per share.	93.8%
2. We try to maintain a smooth dividend from year to year.	89.6%
3. We consider the level of dividends per share that we have paid in recent quarters.	88.2%
4. We are reluctant to make dividend changes that might have to be reversed in the future.	77.9%
5. We consider the change or growth in dividends per share.	66.7%
6. We consider the cost of raising external capital to be smaller than the cost of cutting dividends.	42.8%

# Dividend smoothing

Dividend policy of US firms (1988-2008): 65% no change, 30% increases, 5% decreases.

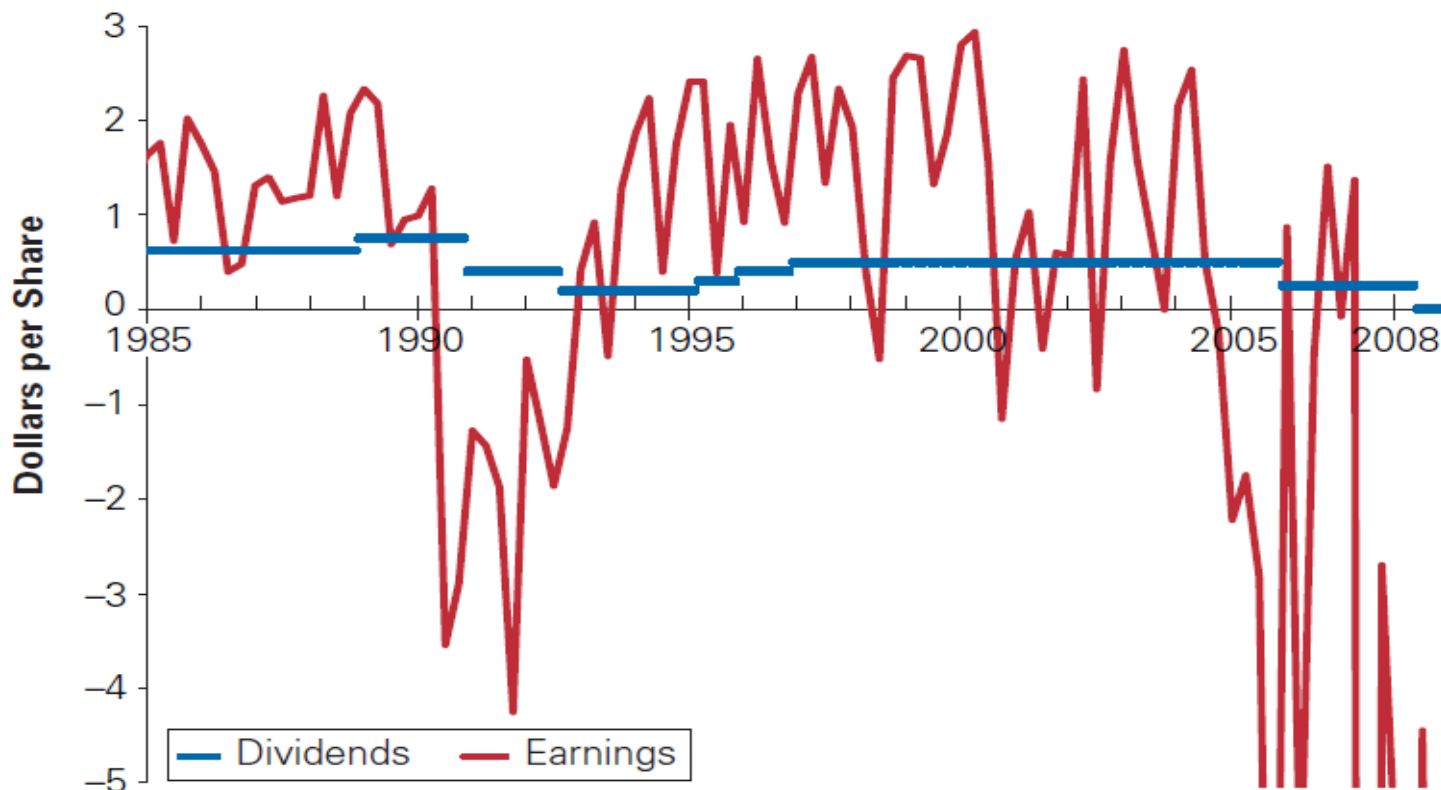
the majority of firms keep their dividends stable.



# Dividend smoothing

What's striking is that GM's earnings are very volatile, while its dividend has been relatively smooth.

- For example, General Motors has changed its regular dividend only 8 times over a 23-year period, while its earnings varied widely.

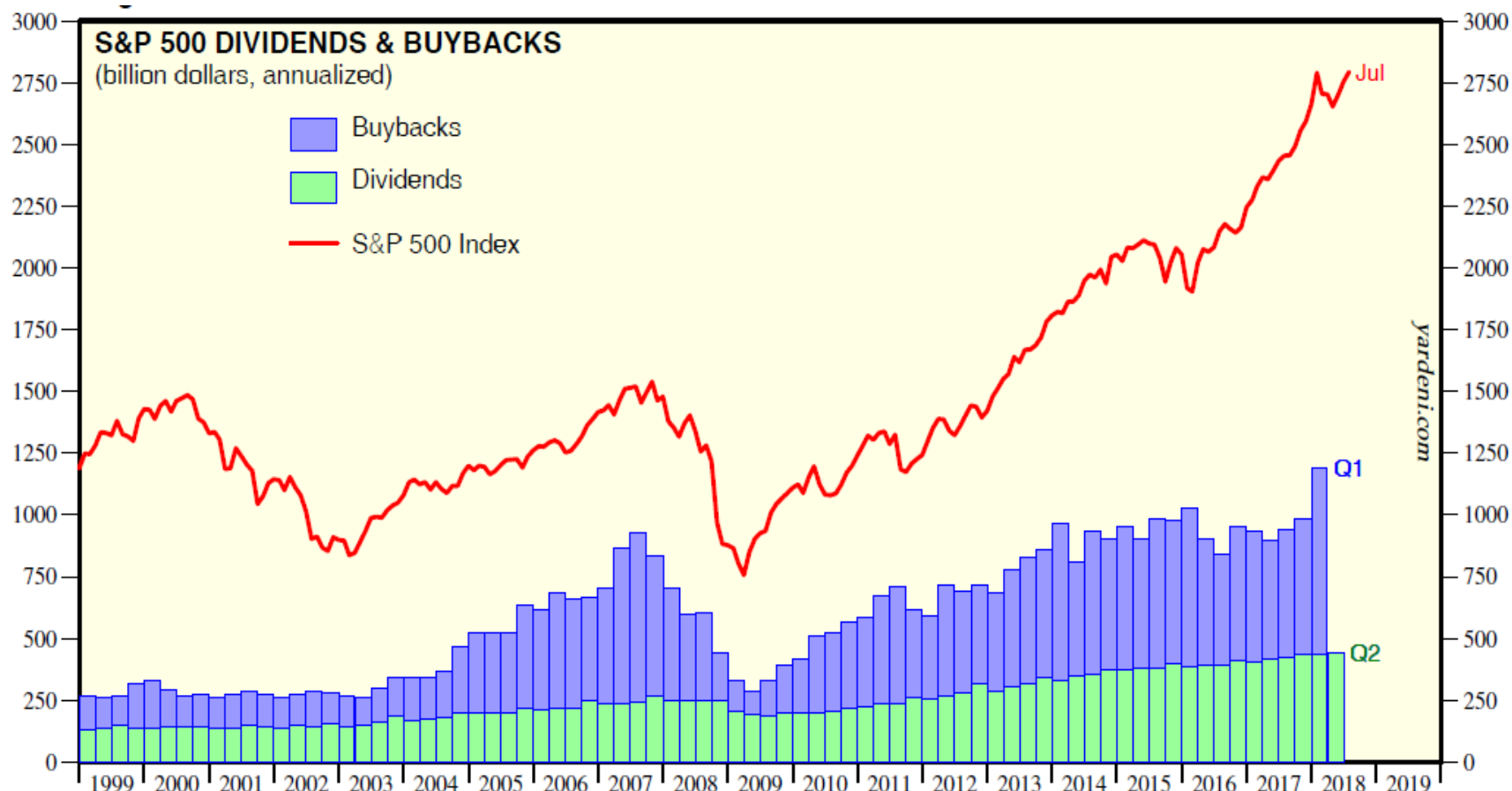


- Question: Why do firms tend to behave this way?

# Patterns in dividends and buybacks

buybacks have increased a lot and is overtaking dividend as a more important mean of payout

- Interestingly, the level of stock repurchases is related to the state of the economy, but the level dividends is not (or not as much).



Source: Standard & Poor's.

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# MM irrelevancy on dividends

## ■ Question:

- Firm has total value of \$50, with \$48 in fixed assets and \$2 in cash.
- Would you prefer to leave the \$2 cash in the firm or put it in your pocket?

## ■ What does MM say? It does not matter.

In the MM world,

- No transaction costs or taxes,
- No frictions in the financial market.

Holding constant the firm's investment policy, dividend policy does not affect stockholder wealth (or total value of the firm).

# Dividends vs. repurchases

**Example.** Consider Genron Corporation.

- It has 10 million shares outstanding.
- Genron expects a perpetual future free cash flow of \$48 million per year.
- Genron has \$20 million excess cash reserve.
- Genron has no debt. Its cost of capital is 12%.
- The board is deciding how to pay out the excess cash to shareholders.

Consider the following payout options:

1. Pay out excess cash as cash dividend,
  2. Pay out excess cash by share repurchase.
- Excess cash is  $20/10 = \$2/\text{share}$ .
  - Expected future dividend (starting next year) is \$4.80 per share forever.
  - Share price before payout:

$$P = 2 + \frac{4.80}{0.12} = 42$$



# Pay cash dividend

## Policy 1: Cash dividend

- Ex-dividend, shareholders will only receive future dividends. Thus,

not eligible for dividend: exclude dividend

$$P_{ex} = PV(\text{Future Dividends}) = \frac{4.80}{0.12} = 40$$

- The cum-dividend price will be:

eligible for dividend: include dividend

$$P_{cum} = \text{Current Dividend} + PV(\text{Future Dividends}) = 2 + \frac{4.80}{0.12} = 2 + 40 = 42$$

	Cum-Dividend	Ex-Dividend
Cash	20	0
Other assets	400	400
Total market value	420	400
Shares (millions)	10	10
Share price	\$42	\$40

- Conclusion: In a perfect financial market, the share price drops by the same amount after a dividend is paid.   
paying a dividend does not change the firm's value

# Share repurchase (no dividend)

## Policy 2: Share repurchase (with no dividend)

- Genron uses the \$20 million to repurchase its shares on the open market.
  - With initial share price of \$42, Genron will repurchase 476,000 shares.
    - $\$20 \text{ million} \div \$42 \text{ per share} = 0.476 \text{ million shares.}$
  - This will leave 9.524 million shares outstanding.
    - $10 \text{ million} - 0.476 \text{ million} = 9.524 \text{ million.}$
- The net effect is that the share price remains unchanged.

	Before Repurchase	After Repurchase
Cash	20	0
Other assets	400	400
Total market value of assets	420	400
Shares (millions)	10	9.524
Share price	\$42	\$42

## Share repurchase (cont'd)

- Genron's future dividends

- It should not be surprising that the repurchase had no effect on the stock price.
- After the repurchase, the future dividend would rise to \$5.04/share.
  - \$48 million ÷ 9.524 million shares = \$5.04 per share
  - Genron's share price is:

$$P_{Repurchase} = \frac{5.04}{0.12} = 42$$

- Conclusion: In a perfect financial market, an open market share repurchase has no effect on the current stock price, which is the same as the cum-dividend price if a dividend were paid instead.

Thus, in the MM world,

- Shifting dividend/payout over time does not change firm value: Higher current dividend reduces future dividends.
- Form of payout, e.g., dividend or repurchase, does not change firm value.

# Modigliani – Miller on payout

- **MM Payout Policy Irrelevance:** In a financial market with no imperfections, holding fixed its investment policy (hence its free cash flow), a firm's payout policy is irrelevant and does not affect its initial share price.
  - Paying dividends or buying back shares is a zero NPV transaction
- Paying dividends is a zero NPV transaction.
  - Firm value before dividend = Firm value dividend + Dividend.
- Investor preferences do not matter:
  - In a perfect financial market, investors are indifferent between the firm distributing funds via dividends or share repurchases.
  - By reinvesting dividends or selling shares, they can replicate either payout method on their own.

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# Dividends beyond MM

- The MM insight about dividend irrelevance helps us to avoid fallacies and illusions about payout policy.

**Example.** “Bird-in-the-hand” Hypothesis: Since dividends are safer than future payments, paying dividends puts money in the hands of shareholders and hence increases the value of the firm. What do you think?

No difference. But if it's about potential misuse of cash by the managers, which is a friction, then you may prefer dividend

- It also gets us to ask the right question: How does a change in payout policy affect the size of the pie?
  - Taxes
  - Signaling asymmetric information,
  - Agency problems...

# Tax disadvantage of dividends

Tax code on dividends and capital gain varies substantially across countries and over time.

## Taxes on dividends and capital gains

- Shareholders must pay taxes on the dividends they receive, and they must also pay capital gains taxes when they sell their shares.
- Dividends are typically taxed at a higher rate than capital gains. In fact, long-term investors can defer capital gains tax forever by not selling.

## Long-Term Capital Gains Versus Dividend Tax Rates in the US, 1971–2018

Year	Capital Gains	Dividends
1971-1978	35%	70%
1979-1981	28%	70%
1982-1986	20%	50%
1987	28%	39%
1988-1990	28%	28%
1991-1992	28%	31%
1992-1996	28%	40%
1997-2000	20%	40%
2001-2002	20%	39%
2003-2012	15%	15%
2013*-	20%	20%

marginal tax rate on dividends (marginal means the highest tax bracket)

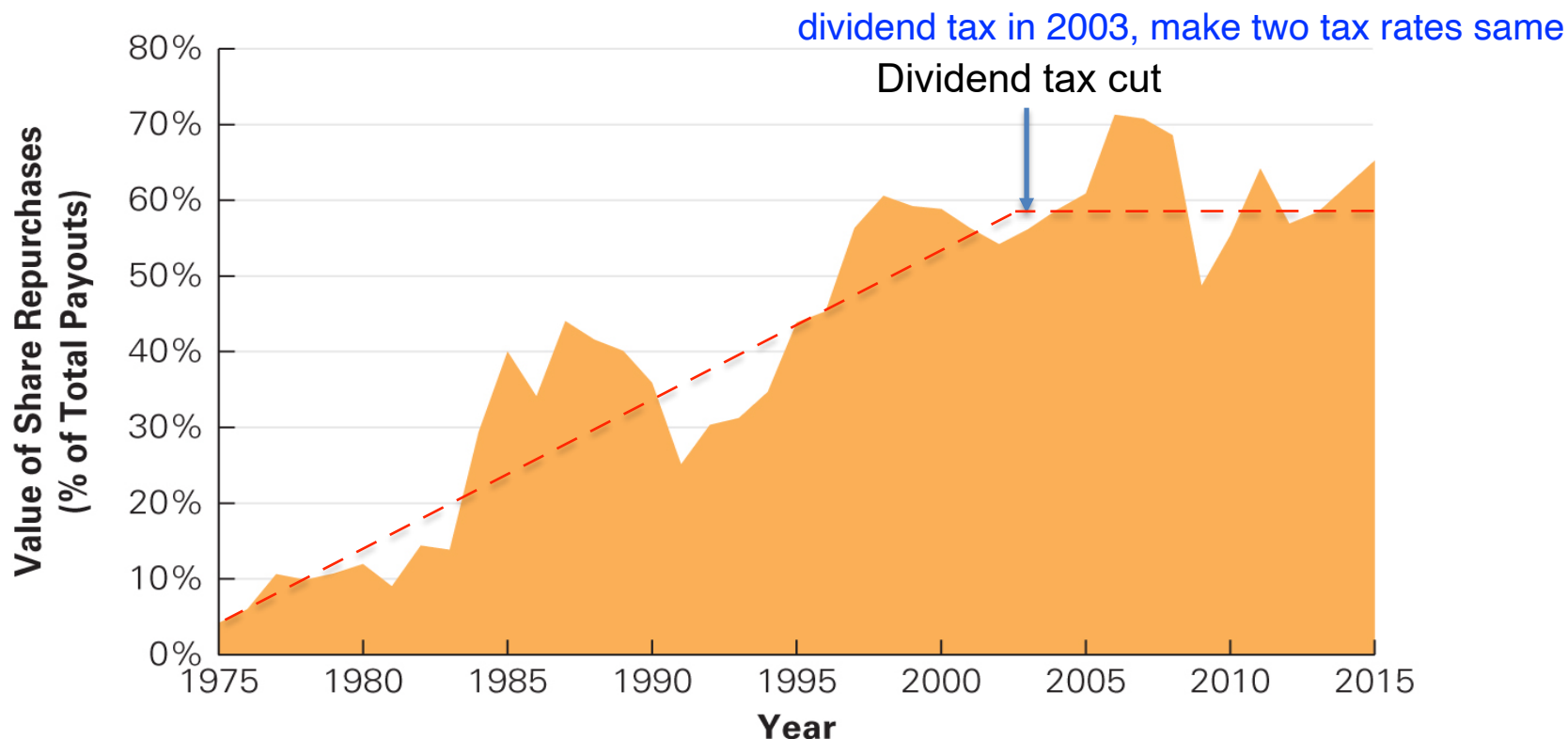
## Dividend policy with taxes

- When the tax rate on dividends is greater than the tax rate on capital gains, shareholders will pay lower taxes if a firm uses share repurchases rather than dividends.
  - This tax savings will increase the value of a firm that uses share repurchases rather than dividends.
- The optimal dividend policy when the dividend tax rate exceeds the capital gain tax rate is to pay no dividends at all.
  - The payment of dividends has declined on average over the last 30 years while the use of repurchases has increased.



# Dividend policy with taxes

The changing composition of shareholder payouts:



- **Dividend Puzzle:** Why firms continue to issue dividends despite their tax disadvantage?

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# Why pay dividends

Why pay dividends?

- At the corporate level, dividends are not tax exempt (interests are).
- At the personal level, dividends are taxed as interest (before 2003).

Thus,

- Why pay dividends when interest payments are tax exempt?
- Why pay cash instead of repurchasing equity?
- Possible explanations:
  - Transaction costs (selling small amounts of stocks is costly)
  - The signaling hypothesis
  - Free-cash flow hypothesis...
- Still much needs to be understood.

# Dividends as signals

There is asymmetric information between the market and firm's managers.

- Firms can signal good prospects through dividends:
  - Firms know more about their own value (prospects).
  - Firms with good prospects can afford to pay dividends.
  - Firms with bad prospects cannot afford to mimic the good firms.
    - If future earnings are low, firms need to raise outside funds and incur transaction/information costs or forego positive NPV projects.
- The financial market does interpret unanticipated dividend changes as signals about firms' future prospects.
  - Grullon, Michaely and Swaminathan (2002): -3.7% for large dividend cuts; +1.3% for large dividend increases.
- Caveats:
  - Why not signal through repurchases?  
given that firms don't change their dividends often, how can dividend policy be an effective signal?
  - How about signal about risk?  
can dividend be signal about risk?

# Dividends as discipline

Free cash flow hypothesis:

- Excess cash can lead to agency problems.
  - Empire building managers take on projects not in the best interest of shareholders
  - Perks...
- Paying out excess cash prevents wasting money on bad projects.
  - Dividends are a way to get money back to investors. out of manager's hands
- **Caveat: Dividends are discretionary.** not a strong mechanism to get money out.
  - Interest payments on debt are a stronger commitment;
  - Nevertheless investors expect dividends to be continued. smooth.

reduce surplus cash flow to mitigate agency problems:

- increase the debt/equity ratio: the pressure of making money to pay for debt interests and principals, If default, the managers will lose their jobs and reputations
- payout policy: share repurchase and dividend payout

agency costs in the event of financial distress can be mitigated by issuing more senior debt

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firms face all kinds of risks from all the items on the asset side, which is business risk, and the liability side, which is financial risk.

## Risk management

A firm's action in dealing with its risks primarily through financial means is also referred to as corporate risk management.

Firms face many types of risks:

- Factor risks     market risk
  - Commodity prices     The factors include commodity prices, interest rates, exchange rates, other market factors such as the stock market, macroeconomy, inflation.
  - Interest rates, FX and other market factors
- Idiosyncratic risks...

Three methods to manage different types of risks:

- Factor risks
  - Hedging – trading financial contracts/assets.
- Idiosyncratic risks
  - Diversification – trading real assets.
  - Insurance – trading insurance/financial contracts.

# Risk management

Our analysis focus on

A firm's risk management policy refers to its transactions in the financial market to modify (hedge) its exposures to factor risks.

- This is a narrow definition of risk management. More generally, it should also include idiosyncratic risks. Nonetheless, the basic approach developed here also applies to the general situation.
- Hedging uses products in the financial market, involving little frictions and costs, hence is relatively easy to implement.

Two questions:

1. Should a firm hedge? (Can hedging increase firm value?)
2. If it decides to hedge, how?

We discuss them sequentially.



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# MM and risk management

- Hedging involves merely transactions in the financial market (at market prices).
- Hedging has no impact on firm value if conditions for MM hold (i.e., in a perfect financial market):
  - No taxes,
  - No transactions costs <sup>or constraint</sup> in the financial market,
  - Equal access to the financial market for stakeholders and managers,
  - Same information for stakeholders and managers,
  - No costs of financial distress.

Logic of MM: Managers cannot increase firm value by making financial transactions that stakeholders can make themselves. (Transactions in the financial market are zero NPV activities.) <sup>in absence of superior information,</sup>

<sup>stakeholders can diversify idiosyncratic risks more easily than the firm by holding a diversified portfolio.</sup>

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# Risk management beyond MM

Hedge only if it increases firm value. This can happen if

- Hedging reduces taxes.
- Managers have better hedging opportunities than stakeholders:
  - Transactions costs and constraints
  - Capital requirements...
- Managers have better information.
- Hedging reduces costs of financial distress.
- Hedging reduces agency costs: [information asymmetry](#)
  - Between managers and shareholders,
  - Between shareholders and debtholders...

[Earnings and firm cash flows are summary measures capturing the effects of both managerial decisions and ability and of exposures to macroeconomic factors of which shareholders may not be fully informed. Thus, by hedging, managers can reduce the noise in these measures contributed by factors outside of management's control, and in turn lessen the asymmetric information in the market regarding managerial ability and firm value.](#)

# Hedging and default cost: 415inc

**Example.** Recall the 415inc example (see Lecture 18 for details).

- Asset value is **100 now** and can be 120 or 100 next year with equal probability. *which is 100% equity financed.*
- Risk-free rate is 5%. Risk-neutral probability of the up state is  $q = 1/4$ .  
*par value = nominal value = face value*
- Intends to issue debt with par value of 80 and a promised yield of  $y$ .
- Tax rate 30%, distress cost is  $DC = (1/320)(\text{default amount})^2$ .
- Optimal promised yield is  $y = 0.45$ .
- Default in the down state, with interest payment  $100/80 - 1 = 25\%$ .
- With no hedging, 415inc's cash flow, tax shield, and distress costs are:

	Cash flow	Tax shield	Distress costs
Up state	\$120	$(0.3)(80)(0.45) = \$10.8$	\$0
Down state	\$100	$(0.3)(20) = \$6$	$(1/320)(80)^2(0.45 - 0.25)^2 = \$0.8$

$$PVT S - PVDC = \frac{(1/4)(10.8) + (3/4)(6 - 0.8)}{1.05} = 6.29$$

- Firm value is  $100 + 6.29 = \$106.29$ .

when it comes to the question about whether to hedge, we need to first identify frictions that have negative effects on firm value, and then consider if reducing certain risks by hedging can help to mitigate these effects.

## Hedging and default risk: 415inc

- Suppose 415inc enters into a financial hedge, which pays \$1 in the low state, and -\$3 in the high state.
- Recall that the risk-neutral probability of the high state is  $q = 1/4$ .
- The time-0 PV of this cash flow is \$0 (this could be a forward or a swap).
  - $F = [(1/4)(-3) + (3/4)(1)]/1.05 = 0$
- With the hedge, interest in low state is  $101/80 - 1 = 26.25\%$ .
- With the hedge, 415inc's cash flow, tax shield, and distress costs are:

	Cash flow	Tax shield	Distress costs
High state	\$117	$(0.3)(80)(0.45) = \$10.8$	\$0
Low state	\$101	$(0.3)(21) = \$6.3$	$(1/320)(80)^2(0.45 - 0.2625)^2 = \$0.7031$

$$PVTS - PVDC = \frac{(1/4)(10.8) + (3/4)(6.3 - 0.7031)}{1.05} = 6.57$$

- Firm value is \$106.57 with the hedge, and \$106.29 without.

still below the promised 45%. The default still occurs, but it increases the total interest payment in a down state which creates more tax shield and decreases the default amount,

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# Hedging basics

- A firm's asset and liability are both exposed to market/factor risks.

**Example.** An airliner's profit is exposed to oil price risk: a hike in oil price will have a negative impact on its profit.

- The firm can take use market instruments to reduce these risks.

**Example (cont'd).** An airliner can reduce its exposure to oil price risk by buying oil forwards or futures.

- Such an operation is called **hedging**: To take on additional positions whose gains or losses can at least partially offset the gains or losses of the original position.



# Hedging basics

Let

- $V_{original}$  - Value of the original position (unhedged),
- $V_{hedging}$  - Value of the **hedging position**, **hedging instrument** or **hedging vehicle**
- $V_{net}$  - Value of the **hedged position**.

Then,

$$\Delta V_{net} = \Delta V_{original} + (\text{hedge ratio}) \times \Delta V_{hedging}$$

The hedge is **perfect** if: **perfect hedge: the risk can be completely avoided or hedged away.**

1.  $\Delta V_{original}$  and  $\Delta V_{hedging}$  are perfectly correlated, and
2. Hedge ratio is appropriately chosen.

Otherwise, the hedging is imperfect.

We will consider a few common risks firms face and how they can be hedged.

# Hedging commodity price risk

**Example.** Suppose that you, the manager of an oil exploration firm, have just struck oil. You expect to have 1 million barrels of oil in 5 months. You are unsure of the future price of oil and would like to hedge the oil price risk.

Using forwards: Hedge your position by selling forward 1 million barrels of oil.

Let  $S(t)$  be the spot oil price at  $t$  (in months). Then, for 1 barrel of oil:

$F$  is the current forward price for oil for maturity in five months.

Position	Value in 5 months (per barrel)
Long position in oil	$S(5)$
Short position in forwards	$F - S(5)$
Net payoff	$F$

- The value of the original position is perfectly correlated with the hedging instrument (oil forward with matching maturity).
- Shorting the forwards with hedge ratio  $\delta = 1$  yields a perfect hedge, locking in the value of the original position at the current forward price  $F$ .

# Hedging with forwards & futures

and counterparty risk.

One drawback from using forwards to hedge is that they are illiquid.

- If after 1 month you discover that there is no oil, then you no longer need the forward contract. In fact, holding just the forward contract you are now exposed to the risk of oil-price changes.
- In this case, you would want to unwind your position by buying back the contract. Given the illiquidity of forward contracts, this can be difficult and expensive.
- To avoid problems with illiquidity of forwards, one may use futures contracts.

**Example (cont'd).** In the above example, you can sell 1 million barrels worth of futures. Suppose that the size of each futures contract is 1,000 barrels. The number of contracts you want to short is:

$$\frac{1,000,000}{1,000} = 1,000$$

The futures position can be easily unwound if needed.

futures contracts can be easily bought and sold in the market.

# Hedging with forwards & futures

Since futures contracts are standardized, they may not perfectly match your hedging need. The following mismatches may arise when hedging with futures:

- Maturity, futures contracts typically have short maturities in months while our exposure can be for much longer term
- Contract size, Since we cannot trade in fractional contracts
- Underlying asset. oil from Texas--this is the underlying asset for oil futures, NYMEX. It is not the same as oil from the Middle East

Thus, a perfect hedge is available only when:

- 1) the maturity of futures matches that of the cash flow to be hedged, match timing
- 2) the contract has the same size as the position to be hedged,
- 3) the cash flow being hedged is linearly related to the futures.

In the event of a mismatch between the position to be hedged and the futures contract, the hedge may not be perfect.

Forwards have the advantage of being fully customized, so it is easier to match the original exposure

although hedging partially reduces the risk of the original position, it may also pick up additional risks from the hedging position.

## Liquidity risk with futures

Hedging with futures may create liquidity risk due to mark to market.

- Suppose we have an oil well, with market value  $A_t$  affected by oil price shocks.
- To reduce risk, we can take an offsetting position in oil futures.
- As the price of oil falls,  $A_t$  declines, but the **hedge offsets** these losses, that is good!  
short position in futures will make money
- As the price of oil rises,  $A_t$  increases, but we lose money on the hedge:
  - In terms of market value, we are not exposed to oil price risk;
  - Marking to market on the futures position creates an immediate cash outflow, while the oil well will deliver higher cash flows over time going forward. The mismatch in the timing of these two cash flows gives rise to a liquidity need.
- **Liquidity risk**: need to come up with funds to meet margin requirements now, either from cash reserves or borrowing against future cash flows.
- What if the firm is financially constrained?

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When interest rates change over time, the discount rates on most assets also change, which will then change the values of most assets.

## Managing interest rate risk

Given a fixed-income position, we will be exposed to interest rate risk as interest rates move over time. To hedge this risk, we can take another fixed-income position to offset the interest rate risk of the original position. Thus, the interest rate risk of the total position (the portfolio) is reduced.

Assume a flat term structure for interest rates. Consider a bond portfolio consisting of  $n_A$  units of bond A and  $n_B$  units of bond B, and

Bond	Price	Duration	Modified Duration
A	$B_A$	$D_A$	$MD_A$
B	$B_B$	$D_B$	$MD_B$

Here, modified duration quantifies the interest rate risk of the two bonds.

# Managing interest rate risk

- The value of the portfolio is then:

$$V_P = V_A + V_B = n_A B_A + n_B B_B$$

- When interest rate changes,

$$\begin{aligned}\Delta V_P &= \Delta V_A + \Delta V_B = n_A \Delta B_A + n_B \Delta B_B \\ &\approx - (V_A MD_A + V_B MD_B)(\Delta y)\end{aligned}$$

- Thus,

$$MD_P = \frac{V_A}{V_A + V_B} MD_A + \frac{V_B}{V_A + V_B} MD_B$$

as for stock market, you can hedge by short SP500 futures to make combined beta zero, but it is still not perfect, you can hedge only the systematic market risk. You still cannot hedge the idiosyncratic stock component and any other possible sources of systematic risks.



# Managing interest rate risk

**Example.** Suppose that you are long in 4-year bonds and you want to use 3-year bonds to hedge the interest rate risk. The data on these bonds are:

Bond	YTM	Duration	Modified duration
3-year	0.10	2.75	2.50
4-year	0.10	3.52	3.20

*yield curve is flat*

To hedge the long position in 4-year bonds, we need to sell (short) 3-year bonds.

How much to sell?

# Managing interest rate risk

- For each dollar worth of the 4-year bond, short  $\delta$  dollars worth of the 3-year bond such that that total portfolio has zero volatility:

$$MD_4 - \delta MD_3 = 0$$

- $\delta$  is the **hedge ratio**, given by:

$$\delta = \frac{MD_4}{MD_3} = \frac{3.20}{2.50} = 1.28$$

- For the hedged portfolio:

Position	Value change when yields $\uparrow$ 0.1%
Long \$1,000 4-year bond	$-(1000)(3.20)(0.001) = -3.20$
Short \$1,280 3-year bond	$(1280)(2.50)(0.001) = +3.20$
Net	0

imperfect hedge: we are hedging only local (i.e., small), parallel shifts in the interest rate.

# Key concepts

- Payout overview
- Payout empirics
- MM and irrelevance of payout policy
- Payout beyond MM: Taxes, information asymmetry, agency costs...
- Corporate risk management
- MM and irrelevance of risk management
- Risk management beyond MM
- Hedging mechanics for different risks (3)

# Managing interest rate risk

**Example.** (Asset-liability management) An insurance company issues a guaranteed investment contract (GIC, which are basically zeros) for  $\text{par value}$  \$10,000. It has a five-year maturity and a guaranteed interest rate of 8%. Thus, the insurance company pays  $(10,000)(1.08)^5$  in five years. Suppose that insurance company chooses to fund this obligation by  $\text{par value}$  \$10,000 of 8% coupon bonds with a six-year maturity, selling at par.

- After issuing the contract, the insurance company incurs a fixed-income liability (in the form of 5-year zeros) with market value of  $L_0 = \$10,000$ .
- It funds this liability with 6-year coupon bond (an asset) with 8% coupon value and market value of  $A_0 = \$10,000$ .
- If interest rate stays at 8% in the next five years, the value of the asset will perfectly match the value of the liability:  $A_5 = (10,000)(1.08)^5 = L_5$ .
- What if interest rate changes?

# Managing interest rate risk

## Example (cont'd).

- The duration of the liability:  $D_L = 5$  (years).
- The duration of the asset:  $D_A = 4.9927$  (years).
- Since asset and liability have almost the same duration, the liability is close to being fully funded for small interest rate changes:

$$\Delta A - \Delta L \approx -(10,000) \frac{4.9927 - 5}{1.08} (\Delta r) \approx (10,000)(0.006759)(\Delta r) \\ \approx 0 \text{ if } \Delta r \text{ is small.}$$

- What about large interest rate changes?

Interest rate	Asset	Liability
8%	\$10,000.00	\$10,000.00
9%	\$ 9,551.41	\$ 9,549.62

- The mismatch between asset and liability becomes more substantial for larger interest rate changes as they have different convexities.

# Managing interest rate risk

## Example (cont'd).

- The convexity of the liability:  $CX_L = 12.86$ .
- The convexity of the asset:  $CX_A = 14.02$ .

$$\Delta A - \Delta L \approx (10,000)(0.006759)(1\%) + (10,000)(14.02 - 12.86)(1\%)^2 = 1.84$$

we can properly choose the asset to greatly reduce interest rate risk by matching their modified durations

for large interest rate movements, however, matching modified duration still leaves an interest rate risk of the second order. To further reduce the second-order risk, we may need to consider further matching convexities between the asset and liability.

# Hedging with forwards & futures

**Example.** We have \$1 billion invested in government bonds and are concerned with volatile interest rates over the next three months. Use the 3-month T-bond forwards to protect investment value.

- Modified duration of the bond portfolio is 6.80.
- Forwards on 10-year Treasury bonds are actively traded.
  - The T-bond to be delivered has a modified duration of 9.20,
  - The T-bond pays no coupons over the next 3 months,
  - Let the T-bond price be  $B_{10}$ ,
  - Each contract delivers \$100,000 face value of the T-bonds,
  - The forward price is  $F = \$93.50$  (for face value of \$100),
  - Forward price for the total contract is \$93,500,
  - The 3-month interest rate is  $r = 1\%$  (not annualized),
  - We then have:  $F = (1 + r)B_{10} = (1.01) B_{10}$ . **By no arbitrage**  $PV_0(\tilde{S}_T - F_T) = 0$
- Should we short or long the forwards. (Why?) **short, original position is long**
- How many contracts to short? Match modified duration.

# Hedging with forwards & futures

## Example (cont'd).

- If yield curves shifts by  $\Delta y$ , the value of the bond portfolio changes by:

$$\Delta V_P \approx -(6.80) V_P \times \Delta y$$

- The current price of the 10-year T-bond changes by:

$$\Delta B_{10} \approx -MD_{10} B_{10} \times \Delta y = -(9.20) B_{10} \times \Delta y$$

- Forward price changes by:

$$\Delta F \approx \text{redacted} = -(1.01)(9.20) B_{10} \times \Delta y = -(9.20) F \times \Delta y$$

- Gains/losses on the forward contract are realized 3 months from now.

- Matching the gain/lost in present value terms, we have:

$$-(\# \text{ of contracts}) \frac{1}{1.01} (\$93,500)(9.20) \times \Delta y = -(\$1 \text{ billion})(6.80) \times \Delta y$$

- Thus, number of contracts to short:

$$\# \text{ of contracts} = (1.01)(\$1 \text{ billion}/\$93,500)(6.80/9.20) = 7984.19$$

Need to rebalance as durations of the bond portfolio and the hedge change.



# Key concepts

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- Hedging basics
- Hedging mechanics for different risks