Recitation 3

Fall 2020

Question 1

Suppose that you have an outstanding loan that requires ten annual payments of \$2,300. Now is Year 0 and the next payment on the loan is due next year, i.e., in Year 1. The last payment is due in Year 10. There is a bank account that offers 5% annual interest rate. You would like to make a deposit into this account today (i.e., in Year 0) that would allow you to repay this loan.

(a) What is the amount you need to deposit?

(b) Demonstrate that the amount you deposited is enough to repay the loan.

Solutions:

(a) The amount that you need to deposit is the present value of an annuity with 10 payments and 5% interest rate.

The present value of this annuity is

$$\frac{\$2,300}{5\%}\left(1-\frac{1}{(1+5\%)^{10}}\right)=\$17,760.$$

(b) The table below demonstrates that \$17,760 invested at 5% is enough to service the loan. Refer to the video for details.

	Begining balance	Interest	Payment	End-of-year balance
Year 1	\$17,760.00	\$888.00	\$2,300.00	\$16,348.00
Year 2	\$16,348.00	\$817.40	\$2,300.00	\$14,865.40
Year 3	\$14,865.00	\$743.30	\$2,300.00	\$13,308.70
Year 4	\$13,308.00	\$665.40	\$2,300.00	\$11,674.10
Year 5	\$11,674.00	\$583.70	\$2,300.00	\$9,957.80
Year 6	\$9,957.80	\$497.90	\$2,300.00	\$8,155.70
Year 7	\$8,155.70	\$407.80	\$2,300.00	\$6,263.50
Year 8	\$6,263.50	\$313.20	\$2,300.00	\$4,276.70
Year 9	\$4,276.70	\$213.80	\$2,300.00	\$2,190.50
Year 10	\$2,190.5	\$109.50	\$2,300.00	\$0

when you do calculations in excel, you should better use cells to specify number in equation, then you can drag and extend the similar calculation

=C10/(1+A1)^B10

auto increase row number in column C and B, do not increase row number in column A

Question 2

brute-force: use excel

Suppose that you have an outstanding loan that requires ten annual payments of $2,300. Now is Year 0 and the next payment on the loan is due in 6 years from now, i.e., in Year 6. The last payment is due in Year 15. There is a bank account that offers 5% annual interest rate. You would like to make a deposit into this account today (i.e., in Year 0) that would allow you to repay this loan. What is the amount you need to deposit?

Solutions:

This question is similar to Question 1, with the only difference that repayment starts in Year 6 and ends in Year 15.

This stream of payments is sometimes referred to as a delayed annuity.

Let's consider a new timeline, on this new timeline, the first payment starts in year 1 and ends in year 10, and hence looks like a regular annuity. We can find the value of this new annuity at t=0 in this new timeline by using the annuity formula:

$$\frac{2300}{5\%} \times \left(1 - \frac{1}{(1+5\%)^{10}}\right).$$

This value equals to $17,760. To find the present value of this amount, notice that the value of this annuity $17,760 is as of t = 5 in our original, or actual, timeline. Therefore, to find the present value of it, we need to discount it to present. The present value of this amount would equal to:

$$\frac{\$17,760}{(1+5\%)^5} = \$13,915.$$

Similar to what we have done in Question 1, we can demonstrate that depositing this amount today is enough to repay the loan.

Question 3

Now is Year 0. A biotechnology company has developed a new nano-robotic surgical device. The patent on the device will last 15 years. You expect that the profit from the device will be $5 million in Year 1. From Year 1 until Year 15, annual profit is expected to grow at a rate of 4% per year. After the patent expires, the emergence of new devices and competition with other biotech firms will drive profits from this surgical device to zero. What is the present value of the new device if the discount rate is 9% per year?

Solutions:

This is a 15-year growing annuity. The present value is:

$$PV = \frac{\$5}{9\% - 4\%} \times \left[1 - \left(\frac{1+4\%}{1+9\%}\right)^{15}\right] = \$50.56 \text{ million}$$

Question 4

Suppose that you are a CFO of a company that owns a commercial real estate property that is currently unused and is unoccupied. Assuming that the appropriate discount rate is 7%, you would like to estimate the value of this property under the following two assumptions:

(a) You can start renting it out immediately, i.e., in Year 0. You expect the annual rental profit to be $650,000 and to continue in perpetuity. The first payment will occur in Year 1.

(b) You will spend the first 4 years redeveloping the property. The present value of redevelopment costs is $1,000,000. Starting in Year 5, you will receive annual rental profit of $950,000, which will continue in perpetuity.

What is the value of the property under scenarios (a) and (b)?

Solutions:

(a) The present value of this perpetuity is:

$$PV = \frac{\$650,000}{7\%} = \$9,285,714.$$

(b) The present value of rental profit is

$$\frac{1}{(1+7\%)^4} \frac{\$9,285,714}{7\%} = \$10,353,578.$$

Taking costs into account, the value of the property under this scenario is:

$$\$10,353,578 - \$1,000,000 = \$9,353,578.$$

Question 5

Jeff Bezos recently made an announcement that he would donate $10 billion to combat climate change. Suppose that he plans to establish a foundation, to which he will pledge the entire amount of $10 billion. Every year, his foundation will distribute funds to organizations that fight climate change. Let's assume that the foundation will invest funds into financial assets that will on average generate nominal annual rate of return of 8%.

(a) If Bezos foundation wants to be able to distribute funds in perpetuity and if they plan to distribute the same dollar amount each year, what would be the annual distribution amount? Assume that the first distribution happens one year from now.

(b) Now suppose that Jeff Bezos wants to start fighting the climate change immediately, and therefore wants to start distributing funds from the foundation immediately. What would be the annual distribution amount?

(c) Now suppose that the expected future annual rate of inflation is 2%. If Jeff Bezos wants to distribute an equal annual amount in today's dollars to guarantee purchasing power of distributed funds, how much would the annual distribution amount be? Assume that the first distribution happens one year from now.

Solutions:

(a) Notice that the distribution stream from the foundation represents perpetuity. Suppose that the annual distribution equals to C dollars. Then the present value of this stream of payments is:

$$PV = \frac{C}{1+r} + \frac{C}{(1+r)^2} + .. = \frac{C}{r}.$$

The present value equals to the amount of money Jeff Bezos pledged to the foundation, which is \$10 billion. Given the interest rate of 8%, we can solve this equation for the annual payment amount:

$$C = PV \times r = \$10 \times 8\% = \$0.8 \text{ billion}$$

(b) Suppose that the annual distribution equals to C dollars. Then the present value of this stream of payments is:

$$PV = C + \frac{C}{1+r} + \frac{C}{(1+r)^2} + .. = \frac{C}{r} + C.$$

The present value equals to the amount of money Jeff Bezos pledged to the foundation, which is \$10 billion. Given the interest rate of 8%, we can solve this equation for the annual payment amount:

$$C = PV \times \frac{r}{1+r} = \$10 \text{ billion} \times \frac{8\%}{1+8\%} = \$740,740,740.$$

(c) Suppose that the annual distribution from the fund equals to C dollars. Let's denote by i the annual rate of inflation. Future distributions from the fund need to grow at the same rate as the inflation to guarantee their purchasing power. In year 1 for example, distribution will equal $C \times (1+i)$

More generally, in year t, distribution amount will equal $C \times (1+i)^t$. Then the present value of this stream is

$$
\begin{aligned}
PV &= \frac{C \times (1+i)}{1+r} + \frac{C \times (1+i)^2}{(1+r)^2} + \frac{C \times (1+i)^3}{(1+r)^3} + \ldots. \\
&= (1+i) \boxed{\left[\frac{C}{1+r} + \frac{C \times (1+i)}{(1+r)^2} + \frac{C \times (1+i)^2}{(1+r)^3} + \ldots. \right]} \\
&= (1+i) \frac{C}{r-i}
\end{aligned}
$$

using Growing Perpetuity formula

The present value equals to the amount of money Jeff Bezos pledged to the foundation, which is \$10 billion. Given the interest rate of 8%, we can solve this equation for the payment amount C:

$$10 \text{ billion} = (1+2\%) \frac{C}{8\% - 2\%} \Rightarrow C = \$588.2 \text{ million}$$

Note that this value is significantly lower than the constant distribution value that we found in part (a), which was \$800 million. Intuitively, we have to start paying lower amount so that we have enough funds to guarantee future adjustment for inflation.

Question 6

(a) A credit card company charges you 20% annual percentage rate (APR), compounded monthly. What is the effective annual rate (EAR) that you are being charged?

(b) Suppose that you work for a credit card company, and you would like to earn an effective annual rate (EAR) of 25% on your credit card loans. You are free to choose a compounding interval, with the only constraint that the shortest period has to be one day. Assuming that you want to present the lowest APR to your customers, what is the optimal frequency of compounding? What is the APR at the optimal compounding frequency.

Solutions:

(a) The effective annual interest rate is:

$$EAR = \left(1 + \frac{20\%}{12}\right)^{12} - 1 = 21.94\%.$$

(b) Daily compounding is optimal. Assume there are 365 days in a year. Then

$$\left(1 + \frac{APR}{365}\right)^{365} = 1 + 25\% \Rightarrow APR = 22.32\%.$$

Question 7

When the Federal Reserve Bank introduced its Quantitative Easing (QE) program shortly after the financial crisis, it intervened in long-term bond markets with the objective of reducing long-term interest rates (e.g., mortgage borrowing costs). Let's take the perspective of a homeowner who currently has a \$300,000 mortgage balance, 20 years remaining on the life of the loan and a fixed interest rate of 6% EAR. Suppose that, following the QE bond purchases, the borrower has the option to refinance the loan at a fixed interest rate of 4.5% APR, monthly compounded. Both loans have monthly payments.

(a) Suppose that the household replaces the old loan with a new one, with the same principal and maturity, but at the lower 4.5% monthly compounded APR. (In other words, the homeowner borrows the \$300,000 from a new bank and uses it to pay back the original loan with the old bank.) Compute the monthly interest rate on the old and new loans, respectively, and the change in the monthly payment which the borrower would enjoy following refinancing the loan.

(b) Suppose that the household is comfortable with its ability to make the existing payments and instead chooses to extract equity from their home, which provides a source of cash that is potentially available for other purchases, such as a new car. As such, they decide to do a "cash-out refinance," in which the homeowner prepays their existing loan and replaces it with a new one with the same maturity but with a different amount of principal, a 4.5% monthly compounded APR, and the same payment as the original loan. (In other words, the homeowner borrows an amount greater than \$300k from a new bank and uses \$300k to pay back the original loan with the old bank and keeps the remainder as cash that is available for other purchases.) You may assume that the value of the house is high enough that the bank will be willing to let you take out a larger mortgage if desired. How much equity is household able to extract (i.e., what is the difference between the amount borrowed on the new loan and the balance on the old one) from following this strategy?

Solutions:

(a) Monthly interest rate on the old loan is:

$$(1 + 6\%)^{1/12} - 1 = 0.487\%.$$

Monthly interest rate on the new loan is:

$$\left(1 + \frac{4.5\%}{12}\right) - 1 = 0.375\%.$$

Current payment is:

$$\frac{\$300,000}{\frac{1}{0.487\%}\left(1-\frac{1}{(1+0.487\%)^{240}}\right)}=\$2121.88.$$

New payment is:

$$\frac{\$300,000}{\frac{1}{0.375\%}\left(1-\frac{1}{(1+0.375\%)^{240}}\right)}=\$1897.95.$$

The household can save \$2,121.88-\$1,897.95=\$223.93 per month, or about 10.6%.

(b) The present value of the old monthly payment at the new mortgage rate is:

$$PV=\frac{\$2,121.88}{\frac{1}{0.375\%}\left(1-\frac{1}{(1+0.375\%)^{240}}\right)}=\$335,395.29$$

Therefore, the household can maintain the existing payments and extract \$35,395.29 from the equity of the home, which would be available to spend or invest in other ways.