



15.415x Foundations of Modern Finance

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Lecture 19: Investment and Financing

Key concepts

- Interaction between investment and financing
- Leverage without tax shield (review)
- Leverage with tax shield: APV and WACC with tax shield
- Implementing APV
- Implementing WACC
- APV versus WACC

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Interaction btw investment and financing

So far in analyzing financing, we considered:

- How to choose capital structure, taking investment decisions as given.

Now we consider:

- How financing may affect investment decisions.
 - Debt capacity of a project's assets may bring tax shield, which adds value.

Main question:

- How to value a project, taking into account how it can be financed.

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Leverage and taxes

In order to find out the proper valuation/discount rates for future cash flows, we need to properly specify their evolution over time.

Let:

- X_t – CF from the firm's assets at time t (independent of leverage),
- $V_{U,t}$ – value of firm without leverage at t ,
- $V_{L,t}$ – value of the firm with leverage at t ,
- D_t – value of its debt,
- E_t – value of its equity,
- r_A – required rate of return on the firm's assets/unlevered firm,
- r_L – required rate of return on the levered firm,
- r_D – required rate of return (interest) on debt,
- r_E – required rate of return on equity,
- τ – corporate tax rate.

Leverage without tax shield

without the tax shield from debt

In the absence of taxes impact from leverage, MM implies:

- A firm's asset can be viewed as a portfolio of its debt and equity:

$$V = D + E$$

- Asset return equals weighted average of debt and equity returns:

$$r_A = \frac{D}{D + E} r_D + \frac{E}{D + E} r_E = WACC$$

- The value of the firm is given by:

$$V = D + E = \sum_{s=1}^{\infty} \frac{(1 - \tau) X_s}{(1 + r_A)^s} = V_U = \sum_{s=1}^{\infty} \frac{(1 - \tau) X_s}{(1 + WACC)^s} = V_L$$

Leverage without tax shield

- MM II: Cost of equity with leverage (D/E) is:

$$r_E = r_A + \frac{D}{E}(r_A - r_D)$$

Without the tax benefits of debt (and other frictions),

- r_A is independent of D/E (leverage), given only by the firm's asset.
- r_E increases with D/E (assuming riskless debt),
- r_D may also increase with D/E as debt becomes risky.

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Leverage with tax shield - APV

As in the case with tax shield, MM implies:

- The value of a firm/project with leverage consists of the following:

$$V_L = E + D = V_U + PVTS - PVDC = APV$$

For simplicity, we will ignore the cost of default (PVDC) for the remainder of our discussion here, assuming debt is mostly riskless.

- We can then view the firm/project with leverage in two equivalent ways:

$$V_L = E + D = V_U + PVTS = APV$$

- 1) A portfolio of debt and equity, or
- 2) A portfolio of two assets:
 - a) an asset with payoff identical to the project (after tax),
 - b) an asset with payoff identical to the tax shield.

two cash flows, one from the project or asset and the other from tax shield

Leverage with tax shield - WACC

In order to find the proper valuation/discount rate(s) for future cash flows, we need to properly specify their evolution over time.

Assume: **Leverage ratio remains constant over time**. That is:

$$\frac{D_t}{V_{L,t}} = w_D, \quad \frac{E_t}{V_{L,t}} = w_E$$

First, we have:

$$r_L = \frac{D_t}{V_{L,t}} r_D + \frac{E_t}{V_{L,t}} r_E = w_D r_D + w_E r_E$$

Next, we have:

$$\begin{aligned} (1 + r_L) V_{L,t} &= (1 - \tau) X_{t+1} + \tau r_D D_t + V_{L,t+1} \\ &= (1 - \tau) X_{t+1} + \tau r_D \frac{D_t}{V_{L,t}} V_{L,t} + V_{L,t+1} \end{aligned}$$

We can rewrite the above as: after tax cash flow, tax credit, and the resale value.

$$(1 + r_L - w_D \tau r_D) V_{L,t} = (1 - \tau) X_{t+1} + V_{L,t+1}$$

Leverage with tax shield - WACC

From

$$(1 + r_L - w_D \tau r_D) V_{L,t} = (1 - \tau) X_{t+1} + V_{L,t+1}$$

define:

$$\begin{aligned} WACC &= r_L - w_D \tau r_D = w_D r_D + w_E r_E - w_D \tau r_D \\ &= w_D (1 - \tau) r_D + w_E r_E \end{aligned}$$

We then have:

$$(1 + WACC) V_{L,t} = (1 - \tau) X_{t+1} + V_{L,t+1}$$

or

$$V_{L,t} = \frac{(1 - \tau) X_t + V_{L,t+1}}{1 + WACC} = \sum_{s=1}^{\infty} \frac{(1 - \tau) X_{t+s}}{(1 + WACC)^s}$$

WACC with taxes - WACC

Thus, with leverage and taxes, we can value a firm/project in two equivalent ways (**assuming constant leverage ratio**):

1. NPV adjusted for the impact of leverage, Adjusted Present Value (APV):

$$V_L = E + D = V_U + PVTS = \sum_{s=1}^{\infty} \frac{(1 - \tau) X_s}{(1 + r_A)^s} + PVTS$$

(Still ignore the cost of default, assuming debt mostly riskless.)

2. NPV discounted by WACC (which adjusts for tax shield):

$$V_L = \sum_{s=1}^{\infty} \frac{(1 - \tau) X_s}{(1 + WACC)^s}$$

where the weighted average cost of capital is given by:

$$WACC = w_D (1 - \tau) r_D + w_E r_E = \frac{D}{D + E} (1 - \tau) r_D + \frac{E}{D + E} r_E$$

$RA = w_D r_D + w_E r_E > WACC$

less discount rate lead to more value (PVTS)

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The first step is to find the right discount rate for the after-tax cash flow from the asset.

Implementing APV

1. Find a traded firm with the same business risk:
 - Debt to equity ratio D/E ,
 - Equity return r_E (by CAPM or APT),
 - Debt return r_D ,
 - Tax rate τ .
2. Uncover r_A (the discount rate without leverage).
3. Apply r_A to the after-tax cash flow of the project to get V_U .
4. Compute PV of debt tax shield.
5. Compute APV.

Implementing APV: Uncovering r_A

A key step in this approach is uncovering r_A from data on a levered firm.

In order to do so, we need to make several simplifying assumptions:

- Assume that pre-tax payout of the firm **grows geometrically**:

$$X_t = X_{t-1} g_t, \quad g_t \text{ are IID growth rates with mean } g.$$

IID: independent identical distribution

- **Time periods are short.**
- The discount rate on total payout r_A **is constant**.
- The firm rebalances its debt each period to maintain a **constant debt/equity ratio**.

The value of the firm's assets is then given by:

$$V_{U,t} = \sum_{s=1}^{\infty} \frac{(1 - \tau) X_t g^s}{(1 + r_A)^s} = \frac{(1 - \tau) X_t}{r_A - g}$$

which evolves proportionally with X_t .

Implementing APV: Uncovering r_A

Now consider the value of firm's assets and liabilities:

$$D_t + E_t = V_{U,t} + PVTS_t$$

where $PVTS_t$ is the present value of the firm's tax shield.

- From this, we relate average returns on assets and liabilities:

$$r_L = \frac{D_t}{V_{L,t}} r_D + \frac{E_t}{V_{L,t}} r_E = \frac{V_{U,t}}{V_{U,t} + PVTS_t} r_A + \frac{PVTS_t}{V_{U,t} + PVTS_t} r_{TS}$$

- The critical observation at this point is that, because of the constant debt/equity ratio, the risk of PVTS is virtually the same as the risk of the firm's asset value $V_{U,t}$ (if the periods are short). That is,

$$r_{TS} = r_A \quad \text{thus } r_L = r_A$$

- The same logic implies that their market betas satisfy:

$$\beta_{TS} = \beta_A$$

If we assume constant debt amount, then $r_{TS} = r_D$

Implementing APV: Uncovering r_A

- We then conclude that:

$$r_L = \frac{D_t}{V_{L,t}} r_D + \frac{E_t}{V_{L,t}} r_E = w_D r_D + w_E r_E = r_A$$

- The required rate of return on equity, as stated by **MM II with taxes**, is:

$$r_E = r_A + \frac{D}{E} (r_A - r_D)$$

- Under CAPM, we have:

$$\beta_L = w_D \beta_D + w_E \beta_E = \beta_A$$

- Note that this is the same formula we obtained previously under the assumption of no taxes.
- Here, there are corporate taxes. Yet, the formulas still holds because the risk of the tax shield generated by debt, under **stated assumptions**, has the same risk as firm's assets. **constant debt to equity ratio and continuous rebalancing.**

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Implementing APV: Example

Example. Want to buy SuperSoft, a software company:

- Currently privately held and 100% equity financed.
- Projected annual pre-tax cash flow \$1.5M perpetually.
- Effective corporate tax rate is 30%.
- SuperSoft's assets can sustain a debt level up to 20% of its asset value without significant distress cost. unlevered constant leverage ratio
- The (risk-free) interest rate SuperSoft pays on its debt is $r_F = 5\%$.
- There are two publicly traded software companies, 1 and 2:

	r_E (%)	r_D (%)	D/E	τ
Firm 1	24.31	6.95	0.5	0.34
Firm 2	29.07	9.88	1.2	0.30

Implementing APV: Example

Example (cont'd).

- Using MM II with taxes, we obtain the required rate of return on assets for firm 1 and 2:

$$r_A = w_E * r_E + w_D * r_D$$

$$r_A^1 = 18.51\%, \quad r_A^2 = 18.60\%$$

- Take the average: we have $r_A = 18.56\%$.
- The value for SuperSoft, if unlevered, is (in millions):

$$V_U = \frac{(1 - 0.3)(1.5)}{0.1856} = \$5.66 \quad \text{Asset, Unlevered}$$

- The value if levered (in millions):

$$V_L = V_U + PVTS = 5.66 + (0.3)(0.2)(0.05)(5.66)/(0.1856) = \$5.75$$

tax rate: 0.3

discount rate: r_A

asset: 5.66

debt level: 0.2

debt return: 0.05

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Implementing WACC

Example (cont'd).

- $V_U = 5.66$ is the value of SuperSoft's assets. The safe debt level is then:

$$D = (0.2) V_U$$

- Since the debt is safe, the (instantaneous) interest rate is the risk-free rate $r_F = 5\%$. The present value of tax shield is:

$$PVT S = \tau (r_F D) / r_A = (0.3) \left(\frac{0.05}{0.1856} \right) (0.2) V_U = (0.016) V_U$$

- The value of equity with leverage is: PVT S belongs to Equity

$$E = V_U + PVT S - D = [1 + 0.016 - 0.2] V_U = (0.816) V_U$$

- We then have:

$$w_D = \frac{0.2}{0.2 + 0.816} = \frac{0.2}{1.016}, \quad w_E = \frac{0.816}{1.016}$$

$$r_E = r_A + \frac{D}{E} (r_A - r_D) = 18.56\% + (0.2/0.816)(13.56\%) = 21.88\%$$

include PVT S

un-levered (asset) return: same as WACC, except (1-0.3)
by MM theorem, repurchase its debt to become an all-equity
financed firm will change return of equity, but not return of asset

Implementing WACC

Example (cont'd).

- We can compute the WACC:

weight in WACC includes PVTs

$$WACC = \frac{(0.2)(1 - 0.3)}{1.016} (5\%) + \frac{0.816}{1.016} (21.88\%) = 18.26\%$$

- Using WACC, the value of the levered firm is:

$$V_L = \frac{(1 - 0.3)(1.5)}{WACC} = \frac{(1 - 0.3)(1.5)}{0.1826} = \$5.75$$

We obtain the same answer as APV.

- With WACC, we can apply to similar projects with the same **financing**
(assuming it is optimal). project level, not firm level leverage level

We can use the data on firms with similar assets but different leverage levels.
What we have to do is to un-lever to obtain the required rate of return on asset.

WACC with taxes

Selected Industry Capital Structures, Betas, and WACCs

Industry	Debt ratio (%)	Equity beta	Asset beta	WACC (%)
Electric and Gas	43.2	0.58	0.33	8.1%
Food production	22.90	0.85	0.66	11.0%
Paper and plastic	30.40	1.03	0.72	11.4%
Equipment	19.10	1.02	0.83	12.4%
Retailers	21.70	1.19	0.93	13.2%
Chemicals	17.30	1.34	1.11	14.7%
Computer software	3.50	1.33	1.28	16.2%
Average of all industries	21.50	1.04	0.82	12.3%
Assumptions: Risk-free rate 6%; market risk premium 8%; cost of debt 7.5%; tax rate 35%				

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WACC vs. APV

in the presence of frictions, such as taxes and distress cost, investment and financing become connected

Pros of WACC:

- Widely used.
- Fewer computations needed (important before computers).
- More straightforward, easier to understand and explain (?).

Cons of WACC:

- Mixes up effects of assets and liabilities. Errors/approximations in the effect of liabilities contaminate valuation.
- Not very flexible:
 - Non-constant debt ratios?
 - Cost of hybrid securities (e.g., convertibles)?
 - Other effects of financing (e.g., costs of distress)?
 - Personal taxes?

WACC vs. APV

Advantages of APV: it's clear on where value comes from--asset, tax shield, and cost of distress. There's no mix between different effects.

- No contamination.in errors and approximations.
- Clearer: Easier to track down where value comes from.
- More flexible: Just add other effects as separate terms.

Cons of APV:

- Almost nobody uses it. But...both conceptually and operationally, it is the right approach, and should be followed whenever possible.

Overall:

- For complex, changing or highly leveraged capital structure (e.g., LBO), APV is much better. leveraged buyout
- Under the right circumstances, it doesn't matter much which method to use (if applied properly).

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