Solutions

1. (a) False. Discount rates are equal to the opportunity cost of capital. There is no meaningful relation between that and the variance of cash flow forecasts.

(b) True. A firm should select the project with a higher NPV. Even with the same initial investment, project rankings implied by IRR do not necessarily correspond to the project rankings implied by the NPV rule.

(c) False. Cash flows of a project should be discounted at the project-specific discount rate (rate of return on projects with the same characteristics).

(d) False. Investors need to reinvest the coupon payments at the prevailing interest rates, which are not known in advance, and which are not equal in expectation to the bond's yield to maturity. The fact that the yield curve is flat does not imply that expected future interest rates are all the same (the expectation's hypothesis need not hold).

(e) True. Real cash flow is risky due to random inflation, and therefore it should be discounted at the appropriate risk-adjusted rate of return, which in general is not the same as the real interest rate.

2. (a) The risk-free bond pays \$1 in each state. Therefore, the price of the bond is

$$B = \phi_1 + \phi_2 + \phi_3 = \$0.4 + \$0.3 + \$0.2 = \$0.9$$

The risk free interest rate then is

$$r = \frac{\$1}{\$0.9} - 1 = 11\%.$$

(b) The time-0 expected value of the cash flow is

$$\$1 \times \frac{1}{3} + \$2 \times \frac{1}{3} + \$4 \times \frac{1}{3} = \$2.33$$

(c) The time-0 market value of the cash flow equals the sum of the state prices multiplied by the corresponding state-contingent payoffs"

$$\begin{aligned} P =& \$1 \times \phi_1 + \$2 \times \phi_2 + \$4 \times \phi_3 \\ =& \$1 \times 0.4 + \$2 \times 0.3 + \$4 \times 0.2 = \$1.8 \end{aligned}$$

3. (a) With monthly compounding, the EAR on each loan is given by

$$EAR = \left(1 + \frac{APR}{12}\right)^{12} - 1$$

We find

$$EAR^{10} = 6.17\%, \quad EAR^{5} = 6.91\%$$

(b) We use the following equation to solve for monthly payments at mortgage rate y:

$$\$50,000 = \sum_{t=1}^{T} \frac{M}{(1+y)^t} = \frac{M}{y}\left(1 - \frac{1}{(1+y)^T}\right),$$

which implies

$$M = \frac{\$50,000r}{\left(1 - \frac{1}{(1+y)^T}\right)},$$

where $y = APR/12$ and T is the maturity of the loan in months.

$$M^{10Y} = \frac{50,000 \times .005}{(1 - \frac{1}{(1.005)^{120}})} \approx \$555.10$$

$$M^{5Y} = \frac{50,000 \times .00558}{(1 - \frac{1}{(1.00558)^{60}})} \approx \$983.00$$

(c)

$$r^{monthly} = (1.04)^{1/12} - 1 = 0.003$$

$$NPV^{10Y} = \frac{555.10}{0.00327}\left[1 - \left(\frac{1}{1.00327}\right)^{120}\right] = \$55,023.41$$

$$NPV^{5Y} = \frac{983}{0.00327}\left[1 - \left(\frac{1}{1.00327}\right)^{60}\right] = \$53,475.51.$$

(d) No, the five-year contract is a better deal. For the same amount of money upfront \$50,000, the NPV of payments under the five-year contract is lower

4. (a) Bond A is a zero coupon bond; the yield to maturity is the spot rate: $r_1 = 3\%$.

(b) Bond B's spot rate is $r_2 = 4.03\%$. Solve for r_2 given r_1:

$$\frac{6}{(1+r_1)} + \frac{106}{(1+r_2)^2} = \frac{6}{(1.04)} + \frac{106}{(1.04)^2}.$$

(c) Bond C's spot rate is $r_3 = 5.04$. Solve for r_3 given r_1, r_2 :

$$\frac{3}{(1+r_1)} + \frac{3}{(1+r_2)^2} + \frac{103}{(1+r_3)^3} = \frac{3}{(1.05)} + \frac{3}{(1.05)^2} + \frac{103}{(1.05)^3}.$$

(d) Denote the new bond as bond D. The market price of bond D is

$$P_D = \frac{100}{1.04^3} = \$88.90$$

The no-arbitrage price of the three zero coupon bond is

$$P_D^{no-arbitrage} = \frac{100}{1.0504^3} = \$86.2804.$$

(e) We construct a portfolio of bonds A, B, C, and D that costs $-\$1$ at time $t = 0$ and pays nothing in later periods. First, compute prices of all the bonds from their yields to maturity:

$$P_A = 97.0874;\ P_B = 103.7722;\ P_C = 94.5535;\ P_D = 88.8996.$$

Next, we solve a system of linear equations relating portfolio positions x_A, x_B, x_C, and x_D to the payoffs in different periods:

$$\begin{bmatrix} P_A & P_B & P_C & P_D \\ 100 & 6 & 3 & 0 \\ 0 & 106 & 3 & 0 \\ 0 & 0 & 103 & 100 \end{bmatrix} \begin{bmatrix} x_A \\ x_B \\ x_C \\ x_D \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

In the above system of equations, the first row corresponds to $t = 0$, the second to $t = 1$, etc. We find:

$$X_A = -0.0105,\ X_B = -0.0105,\ X_C = 0.3707,\ X_D = -0.3818$$

5. (a) There are two components to this project. The net present value of the investment, and the net present value of cash flows; the latter of which can be represented as a perpetuity. The sum of these is

$$NPV = -10 + \frac{-5}{(1.02)} + \frac{1}{(1.02)}\frac{1M}{.02} = \$34.12M$$

(b) The net present value of the construction costs is $(10 + \frac{5}{1.02}) \times 10^6$. The price of X ten-year zero coupon bonds with face values of \$1,000 is $\frac{1{,}000X}{(1.02)^{10}}$. We want to solve

$$(10 + \frac{5}{1.02}) \times 10^6 = 14.90 = \frac{1,000X}{(1.02)^{10}}$$
$$\Longrightarrow X = 18,165.41 \text{ bonds}$$

(c) Since we have a zero-coupon bond, the duration is equal to the maturity of the bond: $D = 10$. The modified duration of the bond is

$$MD = \frac{D}{1+r} = \frac{10}{1.02} = 9.8.$$

(d) Recall that you can abstract away from how the project is financed. The net present value of the new project is

$$NPV^{old} = -10 - \frac{5}{1.02} + \frac{1}{(1.02)}\frac{1}{.02} = \$34.12M$$
$$NPV^{new} = -\ 10 - \frac{5}{1.03} + \frac{1}{(1.03)}\frac{1}{.03} = \$17.51M$$

The change is then $\Delta NPV = 17.51 - 34.12 = -\$16.61M$.

(e) Using the delta-based approximation, the change in the value of the bonds outstanding is:

$$\Delta P = -P \times \frac{D}{(1+y)}\Delta y = -14.90 \times \frac{10}{1.02} \times .01 = -\$1.46M.$$

6. All the dollar figures below are in millions.

(a) Note that $Inv_0 = E_0$, there is no pay-out at time $t = 0$. Then, $E_1 = E_0 + Inv_0 \times 0.2 = \$100 + \$100 \times 0.2 = \$120$.

(b) At $t = 1$ there is no pay-out, so $Inv_0 = E_0$. Then, $E_2 = E_1 + Inv_1 \times 0.2 = \$120 + \$120 \times 0.2 = \$144$.

(c) All investments made starting in year 2 are zero-NPV, because each \$1 of investment generates \$0.10 in perpetuity, and the cost of capital for new investments of XYZ is 10%.

To compute the market value of XYZ, excluding time-0 cash flows, note that the value of the firm without any investments after time $t = 0$ is

$$V_{0,\,\text{no investment}} = \frac{\$120}{0.10} = \$1,200.$$

Firm's investments in year 1 generate the NPV of

$$NPV_1 = -\$120 + \frac{\$24}{0.10} = \$120.$$

The time-0 value of XYZ is then

$$V_0 = V_{0,\,\text{no investment}} + \frac{NPV_1}{1 + 0.10} = \$1,200 + \frac{\$120}{1.1} = \$1309.091.$$

(d) The present value of growth opportunities of XYZ is the difference between the firm value with and without growth:

$$PVGO_0 = V_0 - V_{0,\,\text{no investment}} = \$1,309.09 - \$1,200 = \$109.09.$$

7. (a) $\bar{r} = 0.02 + 0.2 \times 0.2 + 0.1 \times 0.3 = 9\%$

(b) Volatility of stock 1 is $\sigma_1 = \sqrt{0.2^2 + 0.1^2 + 0.2^2} = 0.3$; Volatility of stock 2 is $\sigma_2 = \sqrt{0.1^2 + 0.4^2 + 0.25^2} = 0.482183$, then the correlation between stock 1 and stock 2 is

$$corr_{1,2} = \frac{0.2 \times 0.1 + 0.1 \times 0.4}{\sigma_1 \sigma_2} = 0.414781$$

(c) $\bar{r}_1 = 0.02 + 0.2 \times 0.2 + 0.1 \times 0.3 = 9\%$; $\bar{r}_2 = 0.02 + 0.1 \times 0.2 + 0.4 \times 0.3 = 16\%$. Then w_1 satisfies

$$w_1 \times 9\% + (1 - w_1) \times 16\% = 2\% \Rightarrow w_1 = 2; w_2 = 1 - w_1 = -1$$

8. (a) The following data summarizes the calculation of the free cash flows and the NPV of the investment:

Year:	0	1	2	3
CAPX	-500			
Material Saved		100	100	100
Labor Saved		83.2	83.2	83.2
Installation Cost	-25			
Equipment Sale				25
SUM:	-525	183.2	183.2	208.2
PV:	-525	166.55	151.40	156.42

The cash flow in year 0 is -\$525.

(b) The cash flow in year 1 is \$183.2.

(c) The cash flow in year 3 is \$208.2.

(d) The NPV is -\$50.63.

(e) Depreciation per year $= \frac{500}{3} * \tau = 158.33 * .35 = 58.33$. The following table updates the cash flow numbers to take into account the corporate tax.

Year:	0	1	2	3
CAPX	-500			
Material Saved		100(1-.35)	100 (1-.35)	100 (1-.35)
Labor Saved		83.2(1-.35)	83.2(1-.35)	83.2 (1-.35)
Installation Cost	-25(1-.35)			
Equipment Sale				25(1-.35)
Depreciation		58.33	558.33	58.33
SUM:	-516.25	177.41	177.41	193.66
PV:	-516.25	161.28	146.62	145.50

The cash flow in year 0 is -\$516.25.

(f) The cash flow in year 1 is \$177.41.2.

(g) The cash flow in year 3 is \$193.66.

(h) The NPV is -\$62.84.