Recitation 10

December 1, 2020

Question 1

Now is Year 0. You are considering opening a new concept restaurant. Last year (i.e., in Year -1), you have hired a consulting company to help you evaluate your business idea. The consulting company charged you $125,000, and turned in the following operating projections for the next 10 years.

Annual revenue is expected to be $270,000 and will start in Year 1. Annual expenses will also start in Year 1, and are expected to be $130,000. In Year 0, you will make capital investments, which primarily include restaurant equipment, totaling $650,000. This equipment will be depreciated using straight-line depreciation method over the next ten years.

The appropriate cost of capital is 10% and current tax rate is 21%. Your goal is to determine whether you should open the restaurant.

Solutions:

In this question, we are asked to evaluate an investment decision. The NPV implies

$$NPV > 0 \Longrightarrow \text{Invest}$$

NPV is defined as

$$NPV = CF_0 + \frac{CF_1}{1+r_1} + \frac{CF_2}{(1+r_2)^2} + ... \frac{CF_T}{(1+r_T)^T}$$

where $\{CF_0, CF_1, ...CF_T\}$are **expected** cash flows.

Cash flows are defined as

$$CF = (1-\tau) \times \text{Operating Profit} - \text{Capital Expenditures} + \tau \times \text{Depreciation} - \text{Change in Working Capital}$$

In this question, we are not given any information on working capital requirements for the restaurant. **Assuming that changes in working capital are zero**, cash flows can be computed as:

$$CF = (1-\tau) \times \text{Operating Profit} - \text{Capital Expenditures} + \tau \times \text{Depreciation}$$

Then what is the total present value of all cash flows? Let's first consider the Year 1 cash flow.

Since there is no capital expenditure in Year 1, Year 1 after-tax cash flow is

$$CF_1 = (1-21\%) \times \$140,000 + 21\% \times \$65,000 = \$124,150.$$

The PV of Year 1 cash flow is

$$\frac{CF_1}{(1+r)^1} = \frac{\$124,150}{1+10\%}$$

Similarly, Year t cash flow is the same as Year 1 cash flow

$$CF_t = (1-21\%) \times \$140,000 + 21\% \times \$65,000 = \$124,250$$

use excel function to calculate NPV at year 0=:
npv(discount rate, year 1 value, year 2 value, …)

The PV of Year t cash flow is

$$\frac{CF_t}{(1+r)^t} = \frac{\$124,250}{(1+10\%)^t}$$

The the total PV of cash flows from Year 1 to Year 10 is use math equation

$$\frac{\$124,150}{1+10\%} + \frac{\$124,150}{(1+10\%)^2} + ... \frac{\$124,150}{(1+10\%)^{10}} = \frac{\$124,250}{10\%} \times \left(1 - \frac{1}{(1+10\%)^{10}}\right) = \$763,462$$

Remember the cost of the project (Year 0 expenditure) is \$650,000, so the NPV is

$$NPV = \$763,462 - \$650,000 = \$113,462$$

Question 2

Cyber Vision, a computer vision startup in Cambridge, Massachusetts, considers opening a new division to explore viability of a new technology. To evaluate this decision, Cyber Vision first needs to forecast working capital needs for the new division. Its CFO developed the following projections (in thousands of dollars):

	Year			
	0	1	2	3
(1) Accounts receivable	\$0	\$50	\$60	\$75
(2) Inventory	\$120	\$150	\$200	\$200
(3) Accounts payable	\$40	\$40	\$80	\$100

- Calculate the cash flows associated with changes in working capital for the first three years of this investment.

Solutions:

Remember

Working Capital (WC,or Net Working Capital, or NWC) = Inventory+Accounts receivable− Accounts payable

So first we can calculate the working capital for Year 0,1,2,3, by

$$\text{WC} = (1) + (2) - (3)$$

Thus we have

	Year			
	0	1	2	3
(1) Accounts receivable	\$0	\$50	\$60	\$75
(2) Inventory	\$120	\$150	\$200	\$200
(3) Accounts payable	\$40	\$40	\$80	\$100
(4) Working Capital=(1)+(2)-(3)	\$80	\$160	\$180	\$175

To calculate cash flows, we need to calculate the changes of WC. For the first year, the change of WC is

$$\$80 - 0 = \$80$$

where 0 is the WC in Year -1. Similarly, we can get all changes of WC for Year 1,2 and 3.

	Year			
	0	1	2	3
(1) Accounts receivable	\$0	\$50	\$60	\$75
(2) Inventory	\$120	\$150	\$200	\$200
(3) Accounts payable	\$40	\$40	\$80	\$100
(4) Working Capital=(1)+(2)-(3)	\$80	\$160	\$180	\$175
(5) Changes in WC	\$80	\$80	\$20	(\$5)

By the following formula

$$CF = (1 - \tau) \times \text{Operating Profit} - \text{Capital Expenditures} + \tau \times \text{Depreciation} - \text{Change in Working Capital}$$

ceteris paribus, the change of CF is the opposite of change in WC for each year.

	Year			
	0	1	2	3
(1) Accounts receivable	\$0	\$50	\$60	\$75
(2) Inventory	\$120	\$150	\$200	\$200
(3) Accounts payable	\$40	\$40	\$80	\$100
(4) Working Capital=(1)+(2)-(3)	\$80	\$160	\$180	\$175
(5) Changes in WC	\$80	\$80	\$20	(\$5)
(6) Cash flow	(\$80)	(\$80)	(\$20)	\$5

Question 3

Let's reconsider the restaurant project we studied in Question 1.

Suppose that to run the restaurant you need \$190,000 each year in net working capital. This working capital consists of food and beverage stock needed to produce and serve meals.

You are required to make this investment in Year 0. In Year 10, you expect to close the restaurant, at which point the net working capital will go down to \$0. Does this change your decision on whether you should open this restaurant?

Solutions:

The only difference is the change in WC in the cash flow calculation. From Year -1 to Year0, the change in WC is $\$190,000 - 0 = \$190,000$, so the additional cash flow in Year 0 is the opposite of change in WC, which is $-\$190,000$. The WC is $\$190,000$ from Year 0 to Year 9, and will change to \$0 in Year 10. So the change in WC is zero from Year 1 to Year 9, thus the additional cash flow is zero from Year 1 to Year 9. The change in WC in Year 10 is $\$0 - \$190,000 = -\$190,000$, thus the additional cash flow in Year 10 is $\$190,000$.

So how does the WC calculation change the PV of the project? Basically we have the additional $-\$190,000$ in Year 0 and the additional $\$190,000$ in Year 10, then the new PV is

$$\$113,462 - \$190,000 + \frac{\$190,000}{(1+10\%)^{10}} = -\$3285.$$

(Note: the accurate answer should be $-\$3284.31$, here we get $-\$3285$ because of rounding errors in intermediate steps. In the exam, please avoid rounding numbers in intermediate steps.)

A complete analysis:

Consulting fees	\$ 125,000										
Annual revenue	\$ 270,000										
Annual expenses	\$ 130,000										
CAPEX	\$ 650,000										
Net working capital requirements	\$ 190,000										
Tax rate	21%										
Cost of capital	10%										
						Year					
	0	1	2	3	4	5	6	7	8	9	10
(1) Revenue		\$270,000	\$270,000	\$270,000	\$270,000	\$270,000	\$270,000	\$270,000	\$270,000	\$270,000	\$ 270,000
(2) Expenses		\$130,000	\$130,000	\$130,000	\$130,000	\$130,000	\$130,000	\$130,000	\$130,000	\$130,000	\$ 130,000
(3) Operating profit (1)-(2)		\$140,000	\$140,000	\$140,000	\$140,000	\$140,000	\$140,000	\$140,000	\$140,000	\$140,000	\$ 140,000
(4) Depreciation		\$ 65,000	\$ 65,000	\$ 65,000	\$ 65,000	\$ 65,000	\$ 65,000	\$ 65,000	\$ 65,000	\$ 65,000	\$ 65,000
(5) Capital expenditure	\$ 650,000										
(6) Net working requirements	\$ 190,000	\$190,000	\$190,000	\$190,000	\$190,000	\$190,000	\$190,000	\$190,000	\$190,000	\$190,000	\$ -
(7) Increase in NWC	\$ 190,000	\$ -	\$ -	\$ -	\$ -	\$ -	\$ -	\$ -	\$ -	\$ -	\$ (190,000)
(8) After-tax cash flow	\$ (840,000)	\$124,250	\$124,250	\$124,250	\$124,250	\$124,250	\$124,250	\$124,250	\$124,250	\$124,250	\$ 314,250
NPV	\$(3,284.31)										

Question 4

You are considering a project that requires immediate investment (in Year 0) of \$37,000. You expect that it will generate \$20,000 in Year 1, \$15,000 in Year 2, \$10,000 in Year 3, and \$2,500 in Year 4.

(a). What is the payback period of this project?

(b). If you require a payback period of two years, will you take this project?

(c). What is the NPV of this project if the cost of capital is 12%? According to the NPV rule, should you take the project?

Solutions:

(a). By definition, the payback period is the amount of time it takes to pay back the initial investment. In our case, the initial investment is \$37,000. In the first year, the project returns \$20,000, which is less than the initial investment. During the first two years, the project returns: \$20,000+\$15,000 = \$35,000 < \$37,000 During the first three years, the project returns: \$20,000+\$15,000+\$10,000 = \$45,000 > \$37,000 Therefore, the payback period is 3 years.

(b). If you follow a payback period rule, then you only take projects that pay back within 2 years of initial investment. Since this project's payback period is 3 years, you will not take this project.

(c). If cost of capital is 12%, the NPV of this project is:

$$NPV = -\$37,000 + \frac{\$20,000}{1+12\%} + \frac{\$15,000}{(1+12\%)^2} + \frac{\$10,000}{(1+12\%)^3} + \frac{\$2,500}{(1+12\%)^4} = \$1,521.65$$

Since the NPV of this project is positive, we should take this project.

The important lesson here is that payback period rule can lead to wrong investment decisions. This happens because:

(a). Payback period rule relies on ad hoc decision criterion. (Why we take projects with payback period 2 years and less, and not 3 years?)

(b). Ignores cash flows after payback period. (This project returns \$10,000 in Year 3 and \$2,500 in Year 4.)

(c). Ignores project's cost of capital and time value of money.

Question 5

Consider a project that requires an investment of $30,000 in Year 0. This project is expected to generate cash flows of $4,600 over the next 10 years.

(a). What is the internal rate of return (IRR) of this project?

(b). If the cost of capital for this project is 9%, should you take this project?

Solutions:

(a). If the discount rate is r, the NPV of this project is:

$$NPV = -\$30,000 + \frac{\$4,600}{r} \times \left(1 - \frac{1}{(1+r)^{10}}\right)$$

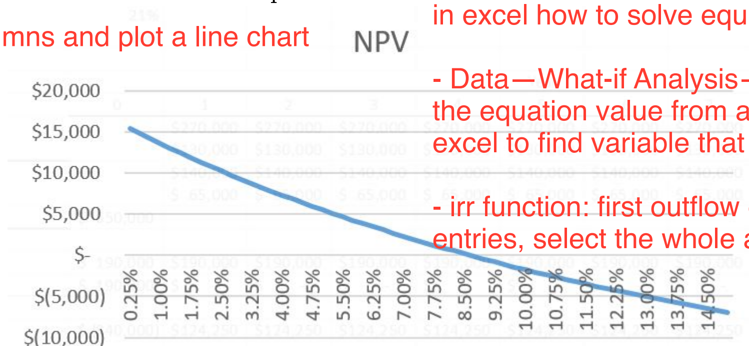
Let's use Excel to compute this NPV for different discount rates:

in excel how to solve equation:

select two columns and plot a line chart

- Data—What-if Analysis—Goal Seek, just calculate the equation value from a initial variable and then let excel to find variable that make equation 0

- irr function: first outflow entry, then several inflow entries, select the whole array



We can solve IRR numerically by solving

calculated 19% in excel is not accurate!
you should improve data display precision!

$$NPV = -\$30,000 + \frac{\$4,600}{r} \times \left(1 - \frac{1}{(1+r)^{10}}\right) = 0$$

The solution is $IRR \approx 8.64\%$.

(b). We have found that the IRR of this project is 8.64%. IRR rule is: Invest if $IRR > COC$. Since in our case $IRR = 8.64\%$, we should not take this project. We can also compute the NPV of this project:

$$NPV = -\$30,000 + \frac{\$4,600}{9\%} \times \left(1 - \frac{1}{(1+9\%)^{10}}\right) = -\$479$$

NPV of this project is negative, therefore we should not take it. Both IRR and NPV investment decision rules agree in this case.

Question 6

Rio Tinto is considering a development of a bauxite mine. Development costs are estimated to be $24 million in Year 0. The mine will generate $4 million in cash flows over the next 10 years, starting in Year 1 and ending in Year 10. After that, Rio Tinto is planning to close the mine and to bring the site in compliance with environmental laws. The estimated costs of closing and compliance are expected to be $0.35 million per year, starting in Year 11 and continuing in perpetuity. Assume that the cost of capital is 8%.

(a). Find the IRR of this project. Should you invest in this project based on IRR rule?

(b). Find the NPV of this project. Should you invest in this project based on NPV rule?

Solutions:

(a). Suppose the discount rate is r, the NPV of the project is

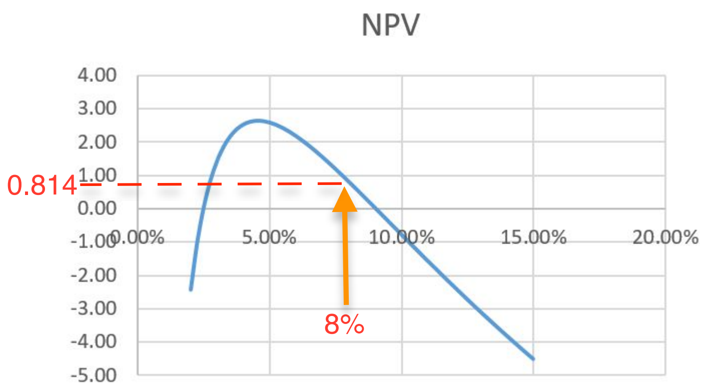
$$NPV(r) = -24 + \frac{4}{r} \times \left(1 - \frac{1}{(1+r)^{10}}\right) - \frac{1}{(1+r)^{10}} \times \frac{0.35}{r} (\text{million})$$

The first term -24 is the development cost. The second term $\frac{4}{r} \times \left(1 - \frac{1}{(1+r)^{10}}\right)$ is the PV of cash flows from Year 1 to Year 10. The last term $-\frac{1}{(1+r)^{10}} \times \frac{0.35}{r}$ is the present value of all costs of closing and compliance after Year 10.

To find IRR, we need to solve the equation irr formula is not usable: perpetuity infinite array

$$NPV(r) = -24 + \frac{4}{r} \times \left(1 - \frac{1}{(1+r)^{10}}\right) - \frac{1}{(1+r)^{10}} \times \frac{0.35}{r} = 0$$

We have two solutions of the above equation: $IRR_1 = 2.48\%$, $IRR_2 = 9.04\%$. Below is the graph of $NPV(r)$ function. In the excel, if our initial guess is 1% and we get the first root $IRR_1 = 2.48\%$, then by IRR rule, we shouldn't take the project. if initial guess is 6%, then we get the second root



(b). With cost of capital 8%, the NPV of the project is

$$NPV = -24 + \frac{4}{8\%} \times \left(1 - \frac{1}{(1+8\%)^{10}}\right) - \frac{1}{(1+8\%)^{10}} \times \frac{0.35}{8\%} = \$0.814(\text{million})$$

The NPV of opening this mine is \$814,000, therefore Rio Tinto should go ahead with the project.