15.435x: Recitation 10

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Outline

1. IO & PO
2. Securitization
3. Duration & Convexity

Overview

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2. Securitization
3. Duration & Convexity

IO & PO

Recall from the Week 10 lecture that **principal-only (PO)** and **interest-only (IO)** securities can created from mortgage-backed securities (MBS). Investors in IO's receive only the interest payments from the underlying pool of mortgages, while investors in PO's receive only the principal payments.

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Assume you are provided with the following information on the estimated prices and prepayment rates for two mortgage-backed securities, Security A and Security B, based on the same pool of underlying mortgages:

Yield	**Percent PSA**	**Price A**	**Price B**
6%	450%	$92	$103
7%	375%	$96.5	$100
8%	360%	$98	$98

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(a) Which security is an IO? Which is a PO? Why?

IO & PO

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Solution: First, what does the "Percent PSA" column mean? Recall that the standard 100% PSA model assumes that the annual prepayment rate increases linearly from 0% immediately after origination to 6% after 30 months. Higher values of percent PSA indicate faster prepayment. As evidenced in the table above, prepayments generally increase as interest rates decline.

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Notice that the price of Security A falls when the percent PSA is larger, and prepayments increase. On the other hand, the price of Security B increases when the percent PSA is larger. Since investors in PO's prefer faster repayment of principal, Security B is the PO. Security A is the IO, because as prepayments increase, fewer interest payments are made to IO investors.

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(b) Calculate the effective duration and effective convexity for each security, assuming yields are currently at 7%.

IO & PO

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Solution: Recall from the Week 8 recitation that we can calculate effective duration as follows:

modified duration that takes into consideration the influence of optionality

$$D_{\text{eff}} = \frac{1}{P_{\text{initial}}} \times \frac{P_{\text{rates fall}} - P_{\text{rates rise}}}{2s} \tag{1}$$

where s is the amount that interest rates rise or fall.

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where s is the amount that interest rates rise or fall. Thus, the effective duration of Security A is:

$$D_{\text{eff}}^{\text{A}} = \frac{1}{96.5} \times \frac{92 - 98}{2(0.01)} = -3.1,$$

and the effective duration of Security B is:

$$D_{\text{eff}}^{\text{B}} = \frac{1}{100} \times \frac{103 - 98}{2(0.01)} = 2.5.$$

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As we discussed in the Week 10 lecture, it's typical for IO's to have a negative effective duration over a range of interest rates and PO's to have a positive effective duration.

IO & PO

Solution (Cont.): Effective convexity can be calculated as:

$$C_{eff} = \frac{1}{P_{\text{initial}}} \times \frac{(P_{\text{rates fall}} - P_{\text{initial}}) - (P_{\text{initial}} - P_{\text{rates rise}})}{s^2}. \qquad (2)$$

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Thus, the effective convexity of Security A is:

$$C^{A}_{eff} = \frac{1}{96.5} \times \frac{(92 - 96.5) - (96.5 - 98)}{(0.01)^2} = -310.9,$$

and the effective convexity of Security B is:

$$C^{B}_{eff} = \frac{1}{100} \times \frac{(103 - 100) - (100 - 98)}{(0.01)^2} = 100.$$

bonds having positive effective convexity, but it's this optionality, the prepayment risk, that is causing the IO to have negative effective convexity for the given range of interest rates.

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In this case, Security A, the IO, actually has *negative* effective convexity, whereas Security B, the PO, has positive effective convexity.

Overview

Securitization

The following bonds are available in the market (all with LIBOR risk and pricing):

	3.75% Coupon Bond	**Zero-Coupon Bond**
Yield (b.e.b.)	3.716%	3.150%
Maturity	5 years	1 year
Modified duration	4.483 years	0.969 years
Convexity	25.29	1.879

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(1+3.716%/2)^2– 1=3.75%

on a bond–equivalent basis

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1/(1+3.15%)
=0.969461949

(a) Consider a plain-vanilla interest rate swap with an annual fixed-rate payment of 3.75%, an annual floating-rate payment set to LIBOR, and a maturity of 5 years. What is the modified duration of the swap from the perspective of the floating rate payor?

Securitization

Solution: The floating rate payor is effectively long a fixed-rate bond and short a floating-rate bond, with equal long and short amounts in present value. Since the modified duration of that long-short portfolio is simply the difference between the modified durations of its component parts, the duration of the swap from the perspective of the floating rate payor is $4.483 - 0.969 = 3.514$.

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(b) Frida Mae specializes in financing non-callable, commercial fixed-rate mortgages. The company's balance sheet (in millions of dollars) looks like:

Assets		Liabilities
short-term mortgages (avg. modified duration = 0.5 yrs)	\$280	bank loans \$600 (avg. modified duration = 1.0 yr)
long-term mortgages (avg. modified duration = 6.0 yrs)	\$350	owners equity \$30

Securitization

Is Frida adversely exposed to an increase or decrease in interest rates? If Frida wants to use the swap described in Part (a) to reduce the duration gap between its assets and liabilities, would it enter the swap as a fixed rate payor or a floating rate payor? Explain.

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Solution: The modified duration of Frida Mae's assets is equal to:

$$\frac{280}{630} \times 0.5 + \frac{350}{630} \times 6 = 3.556 \text{ years.}$$

The modified duration of Frida Mae's liabilities is clearly 1 year.

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Since the modified duration of Frida Mae's assets is greater than that of its liabilities, Frida Mae is exposed to an increase in interest rates, as higher interest rates will cause Frida Mae's net worth (owner's equity) to fall. Frida Mae could hedge this duration gap by entering the swap as a fixed-rate payor (negative duration), which is effectively long a floating-rate bond and short a fixed-rate one.

Securitization

(c) If Frida enters into the interest rate swap with a notional principal of \$370 million (on the side of the transaction recommended in Part (b)), estimate what happens to the value of owners equity if interest rates rise at all maturities by 1%.

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Solution: We can calculate the change in the value of owners equity using the following formula:

$$dE = d(STM) + d(LTM) - d(\text{Loans}) + d(\text{Swap}), \tag{3}$$

where $d(\cdot)$ denotes a change in value, STM denotes short-term mortgages, and LTM denotes long-term mortgages.

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Recall that we can use modified duration to approximate the price change of a security for a small change in interest rates as follows:

$$dP \cong -D_{\text{mod}}(P)(dy).$$

Securitization

Solution (Cont.): Therefore, each component of Equation (3) can be approximated by the negative of its modified duration multiplied by its initial value P and the change in interest rates dy:

$$dE \cong -0.5(280)(.01) - 6(350)(.01) + 1(600)(.01) + 3.514(370)(.01) = -3.398.$$

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(d) As an alternative to the swap, Frida Mae's CFO considers securitizing some of the long-term mortgages and using the proceeds to pay down its bank loans. There is no recourse on the securitized loans. Show what happens to the balance sheet if \$200 million of the long-term mortgages are securitized. After the securitization, would owner's equity be adversely affected by a rise or fall in interest rates?

Securitization

Solution: Frida Mae's balance sheet after securitization would be as follows:

After Securitization Balance Sheet

Assets		Liabilities
short-term mortgages (avg. modified duration = 0.5 yrs)	$280	bank loans $400 (avg. modified duration = 1.0 yr)
long-term mortgages (avg. modified duration = 6.0 yrs)	$150	owners equity $30

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Notice that the value of long-term mortgages on the asset side of the balance sheet falls from \$350 million to \$150 million, and the value of bank loans on the liability side decreases from \$600 million to \$400 million.

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Solution (Cont.): Now, using Equation (3), the change in the value of owners equity for a 1% increase in interest rates would be:

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In this example, when using securitization, the value of owners equity falls by \$6.4 million, more than when using the swap in Part (c).

In all, the securitization of a large amount of long-term mortgages still leaves Frida Mae with exposure to rising interest rates, since the average duration of its assets is longer than that of the liabilities funding them.

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Duration & Convexity

Consider a 5-year 3.5% coupon bond with no embedded options, priced at par. For each of the following changes to the bond's structure or to the market environment—just one at a time—what happens to the effective duration and effective convexity of the bond?

- Effective duration: (i) No change; (ii) Smaller; (iii) Bigger; (iv) Indeterminate.
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(a) The yield curve makes a parallel shift downward.

Solution: Effective duration: (iii) Bigger; Effective convexity: (iii) Bigger.

- This has the effect of making more distant payments relative more valuable. That puts more weight on those terms in the calculation of duration and convexity.

Duration & Convexity

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Duration & Convexity

$$P_{risky} = \frac{dgF + (1-d)F}{1+y} = (dg + (1-d)) * \frac{F}{1+y}$$

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less yield, higher price, larger decrease in bond price due to default risk

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With default risk, the bond price would decrease across all yields compared to the risk–free bond. However, the impact of default risk on bond price is stronger when yield is low. Visually, this means that the price/yield curve will uncurl (as the decrease in price due to default risk is larger in the low yield regime)

- Higher default probability reduces the present value of more distant payments relative to the total present value of cash flows, which pulls down both duration and convexity.

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Solution: Yes! As discussed in the Week 10 lecture, investors are promised full payment of promised principal and any accrued interest because Fannie Mae and Freddie Mac provide **credit guarantees**. However, default affects the pattern of realized cash flows, because at the time of default, the full amount owed is repaid to the investors.

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Thus, the possibility of default affects the duration of the MBS and its exposure to interest rate risk.