

## 15.415.1x Sample Exam

### Grade Sheet

1.	_____	/	10
2.	_____	/	12
3.	_____	/	11
4.	_____	/	16
6.	_____	/	18
6.	_____	/	18
7.	_____	/	15
8.	_____	/	20
Total	_____	/	120

1. (10 points) True or false? **False. Discount rates are equal to the opportunity cost of capital. There is no meaningful relation between that and the variance of cash flow forecasts.**
- (a) (2 points) When evaluating investment projects, for the projects with higher variance of sales forecasts, firms should apply higher discount rates. **T F**
- (b) (2 points) Consider a firm maximizing its market value. When selecting among two mutually exclusive projects with the same initial investment, this firm **may optimally select the project with the lower** internal rate of return. **F T**
- (c) (2 points) When computing the NPV of potential new investments, a firm should discount expected future cash flows attributed to the project **using the firm's cost of capital** as the discount rate. **T F** **project-specific**
- (d) (2 points) If the yield curve is flat, then yield to maturity on a risk-free coupon bond is equal to the expected return (in annualized terms) the investor would collect by holding the bond to its maturity. **T F** **Investors need to reinvest the coupon payments**
- (e) (2 points) Consider a nominal risk-free cash flow. When computing the present value in the presence of inflation risk, it is generally wrong to discount the expected real cash flows at the real risk-free interest rates. **F T**
- Real cash flow is risky due to random inflation, and therefore it should be discounted at the appropriate risk-adjusted rate of return, which in general is not the same as the real interest rate.**
2. (12 points) Consider a state-space model with two periods and three states at time  $t = 1$ : 1, 2, and 3. All three states are equally likely. Primitive state-contingent claims on each state are traded in the market, and their time-0 prices are:

$$\phi_1 = 0.4; \phi_2 = 0.3; \phi_3 = 0.2.$$

In addition to the primitive claims, the risk-free asset is traded.

- (a) (4 points) Based on absence of arbitrage, what is the risk-free interest rate in this market? **11.11%**
- (b) (4 points) Consider a cash flow  $C_1$  equal to \$1, \$2, and \$4, in states 1, 2, and 3, respectively. What is the expected value of this cash flow (as of time 0)? **2.333**
- (c) (4 points) Compute the time-0 market value of the cash flow  $C_1$ . **1.8**
3. (11 points) Alice is taking out a bank loan to pay for a new addition to her house. She is comparing two options: a 10-year loan with an annual APR of 6%, compounded monthly; and a 5-year loan, with an annual APR of 6.7%, compounded monthly. The market interest rate is **4% (EAR)**, and is the same for all maturities. Alice needs to borrow \$50,000. **monthly rate != 4%/12**
- (a) (3 points) Compute the EAR on each of the two loans. **6.168% 6.910%**
- (b) (4 points) Compute the monthly payments on each of the two loans. **555.1025097 982.9982063**
- (c) (3 points) Compute the **present value** of payments on each of the two loans. **the discount rate for your payments is your opportunity rate, not the bank interest rate**
- (d) (1 point) Is the ten-year loan a better deal, judging by the present value of the payments that Alice would need to make between now and the maturity of the loan? "Yes" or "no." **No**
- 54827.57195, 53375.88733**
4. (17 points) Consider a frictionless market. Several Treasury bonds (with face values of \$100) are traded in the market. Their coupon rates and yield-to-maturity are given in the following table:

Bond name	Maturity	Coupon rate	Yield to maturity
A	1-year	0%	3%
B	2-year	6%	4%
C	3-year	3%	5%

The coupons are paid annually. Now is year 0.

- (a) (2 points) What is the 1-year spot interest rate? **3%**
- (b) (2 points) What is the 2-year spot interest rate? **4.0297%**
- (c) (2 points) What is the 3-year spot interest rate? **5.0419%**
- (d) (3 points) Suppose that a new Treasury bond is introduced to the market. It is a zero-coupon bond with 3 years to maturity, and it trades at the 4% yield to maturity. What is the no-arbitrage price of the new bond? **88.8996**
- (e) (7 points) Describe explicitly the arbitrage trading strategy involving bonds A, B, C, and the new bond, which pays \$1 at time  $t = 0$  and nothing afterwards.

-0.010490628  
-0.010490628  
0.370668861  
-0.381788927

5. (18 points) You are advising a local municipal treasury on a bridge construction project. The project requires an upfront investment (at time/year 0) of \$10M, with an additional investment in year 1 of \$5M. The bridge will become operational two years from now, and will start generating toll revenue. Specifically, the bridge produces no cash flows in year 1, and produces a perpetual stream of cash flows of \$1M per year in subsequent years. Assume that all cash flows are risk-free.

The treasury is financing this project with a ten-year zero-coupon bond. The current term structure of risk-free interest rates is flat at 2%. Assume that the treasury is able to finance this project at the risk-free interest rate.

- (a) (3 points) What is the NPV of this project? **34.117**
- (b) (3 points) Suppose that the treasury wants to issue enough bonds to cover the present value of construction costs of this project. Let the face value of each bond be \$1,000. What is the total number of bonds that need to be issued? **18165.40704**
- (c) (5 points) Compute the modified duration of the bond issued by the treasury. **9.803921569**
- (d) (5 points) Suppose the treasury goes ahead with your suggestion in (b) and starts the project. Right after its start (at time 0), the yield curve unexpectedly **rises by 1%** across all maturities. What is the resulting change in the NPV of the project, following the change in interest rates? **84.05940594-34.117**
- (e) (2 points) Using the duration-based approximation, what would be the change in the value of the **outstanding bonds** following a 1% rise in interest rates? **80.42630391**

= 2% + 1%  
!= 1%

6. (18 points) A private equity investment fund has firm XYZ in its portfolio. Your task is to estimate the value of this firm, which does not trade publicly. XYZ is 100% equity financed. It is now year 0, and you have the following data on XYZ:

Full-year earnings over year 0	\$100M
Payout ratio in year 0	0%
Cost of capital	10%

Based on your market analysis, you forecast that without any new investments, XYZ is expected to generate \$100M in earnings per year in perpetuity. Investments made in year 0 and 1 are expected to generate \$0.20 per year in perpetuity for each \$1 invested, starting in a year following the investment. Starting in year 2, new investments are expected to generate \$0.10 per year in perpetuity for each \$1 invested, starting in a year following the investment.

The payout ratio of XYZ will stay at zero in year 1, rising permanently to 60% afterward. Assume the cost of capital in the above table applies to all future cash flows generated by XYZ (including its future investments and earnings), and will remain constant.

including -120 in your calculation NPV = (-120 + (120\*20%)/10%)/(1+10%)

value(invest) = value(no-invest) + NPV of valuable invest (ROI > COC)

- (a) (2 points) Compute the expected earnings of XYZ in year 1. 120
- (b) (3 points) Compute the expected earnings of XYZ in year 2. 144
- (c) (10 points) Compute the market value of XYZ as of year 0. 1309.090909
- (d) (3 points) What is the net present value of growth opportunities (PVGO) of XYZ as of year 0? Do not include the net present value of year-0 investment into the PVGO. 209.0909091 exclude year 0, calculate from year 1
7. (15 points) Suppose that asset returns are described by a 2-factor APT model, which applies exactly to all assets:

$$\tilde{r}_i = \bar{r}_i + b_{i1}\tilde{f}_1 + b_{i2}\tilde{f}_2 + \tilde{u}_i, \quad i = 1, 2, \dots$$

where both factors have unit variance and are uncorrelated with each other.

The risk premia associated with factors 1 and 2 are 20% and 30%, respectively. The risk-free rate is 2%.

- (a) (3 points) We are contemplating investing in a stock, which has the following factor loadings:  $b_1 = 0.20$  and  $b_2 = 0.10$ . According to APT, what should be the expected return on this stock? 9%
- (b) (6 points) Consider two stocks, 1 and 2, with the following parameters. For stock 1:  $b_{11} = 0.2$ ,  $b_{12} = 0.1$ ,  $SD(\tilde{u}_1) = 0.20$ . For stock 2:  $b_{21} = 0.1$ ,  $b_{12} = 0.4$ ,  $SD(\tilde{u}_2) = 0.25$ . Compute the correlation between returns on stocks 1 and 2. 0.414780678
- (c) (6 points) Construct a portfolio with stocks 1 and 2 above, with weights  $w_1$  and  $w_2$ , with the expected return equal to the risk-free rate. What is  $w_1$ ? 2% 2
8. (20 points) A semiconductor company is considering a purchase of a silicon measurement system costing \$500,000. By reducing wasted Silicon, the system is expected to save approximately \$100,000 per year in raw material costs. The system costs \$25,000 to install. The device would reduce approximately 40 hours of work per week provided by an outsourcing service charged at \$40 per hour. The company is expecting to upgrade the entire plant in 3 years at which time they expect to dispose of the equipment. They estimate that they would receive \$25,000 for the equipment. The equipment is to be depreciated on a straight line basis over 3 years. The discount rate of the firm is 10%.

portfolio with short position: 2, -1

CAPX will generate profit since the following year

CAPX and Installation Cost can be used to reduce profit thus reduce tax (tax credit). CAPX is depreciated over the following 3 years, thus is not used to reduce tax in the first year. Installation cost is used in the first year. one dollar saved is one dollar earned

**previous investment and all the consequences  
should be ignored by current decision**

- (a) (2 points) Assume no taxes. What is the total cash flow generated by this project in year 0? **-525000**
- (b) (2 points) Assume no taxes. What is the total cash flow generated by this project in year 1? **183200**
- (c) (2 points) Assume no taxes. What is the total cash flow generated by this project in year 3? **208200**
- (d) (2 points) Assume no taxes. What is the Net Present Value of the measurement system? **-50625.84523**
- (e) (3 points) Suppose now the corporate tax rate is 35%. What is the total cash flow generated by this project in year 0? **-525000**
- (f) (3 points) Suppose now the corporate tax rate is 35%. What is the total cash flow generated by this project in year 1? **177413.3333**
- (g) (3 points) Suppose now the corporate tax rate is 35%. What is the total cash flow generated by this project in year 3? **193663.3333**
- (h) (3 points) Suppose now the corporate tax rate is 35%. What is the Net Present Value of the measurement system? **-71590.43326**