Week 3 – Duration and convexity-based strategies for risk management

MIT Sloan School of Management

Outline

- Duration and convexity: basic and generalized measures
  - Motivation, basic definitions and extensions
  - Application to bonds, forwards, futures and swaps

- Duration-based risk-management strategies
  - delta and gamma hedging in the spot (cash) market
  - delta and gamma hedging with forwards, futures and swaps

Overview and objectives

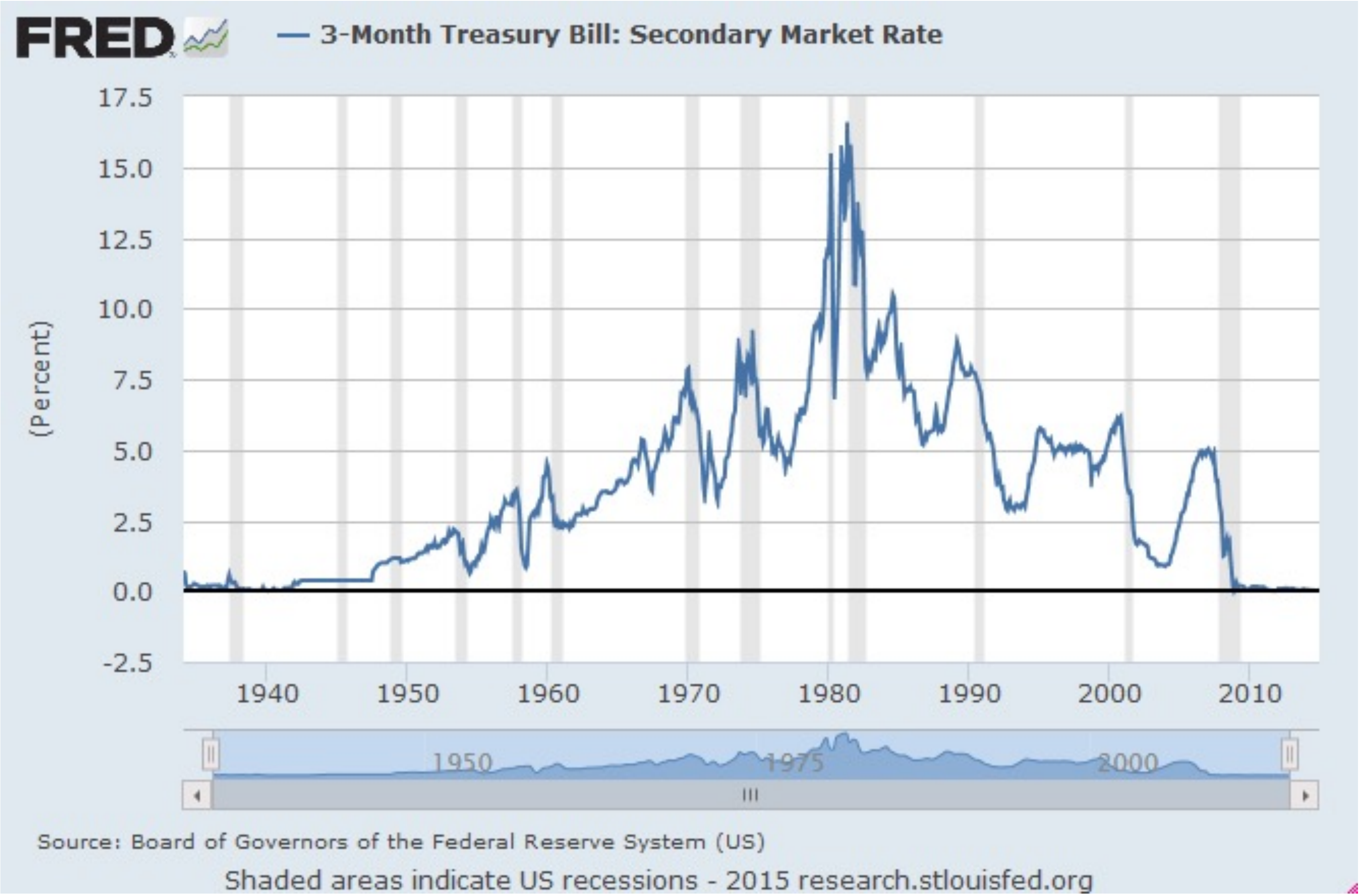
delta gamma

Duration and **convexity** are two fundamental concepts for measuring, hedging, and speculating on interest rate risk

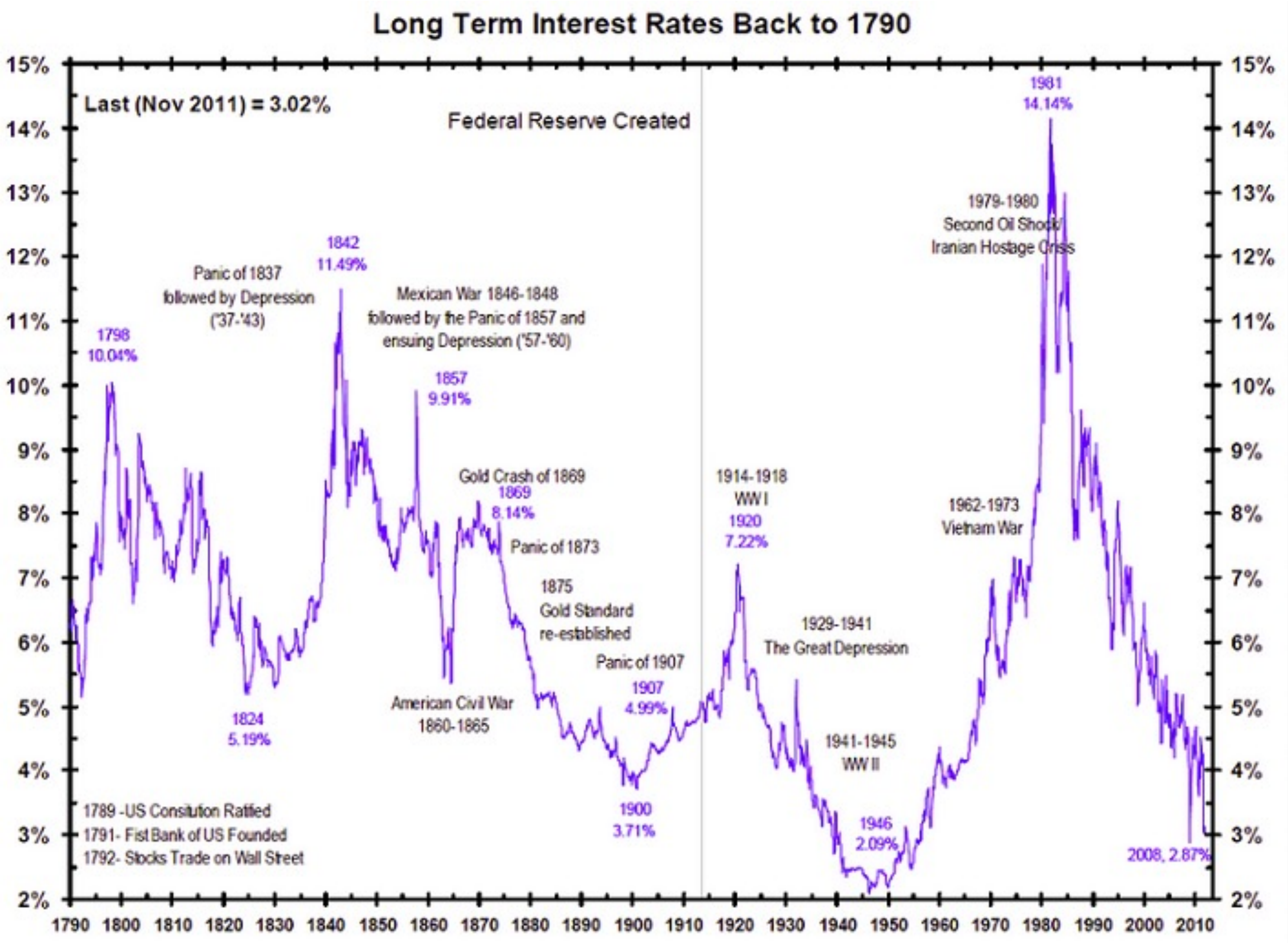
You will become familiar with:

- Some basic and advanced definitions of duration and convexity
- Basic mathematical formulas and spreadsheet calculations
- Graphical interpretations
- Applications in risk management

Short term yields have historically been volatile



Long term yields are also volatile, but less so than short-term yields when compared over similar horizons



Yield volatility => price volatility

- Interest rate or yield volatility translates in to price volatility for fixed income securities
  - Recall that bond prices and yields are inversely related
  - For a zero coupon bond

face value

$$\underset{\text{price}}{P} = \frac{F}{(1+r)^N}$$

- Financial institutions like commercial and investment banks, and fixed income portfolio managers, are highly exposed to interest rate volatility

price risk is the main concern

- These institutions manage interest rate risk in a variety of ways that include forward, future and swap contracts, and dynamic hedging strategies

Duration and convexity: basic and generalized measures

Finance at MIT

Where ingenuity drives results

Recall the formula that relates a bond's price to its yield:

A bond's yield (YTM) answers the question: *"What is the constant rate of return that makes the bond price equal to the present value of promised future payments?"*

or

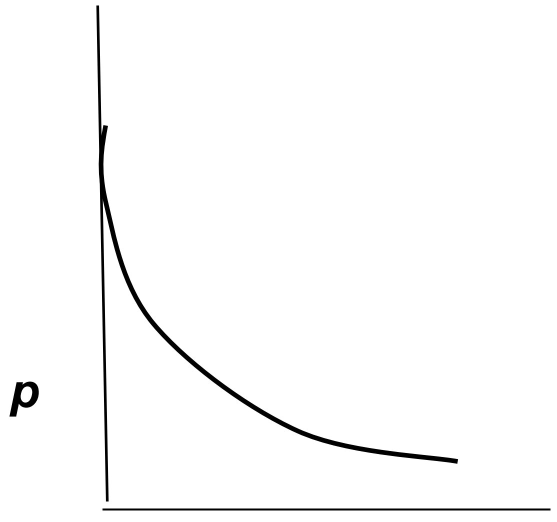
$$p = \frac{C_1}{(1+y)} + \frac{C_2}{(1+y)^2} + \ldots + \frac{C_n}{(1+y)^n}$$

$$p = \sum_{i=1}^{n} \frac{Ci}{(1+y)^i}$$

p = price

C_i = cash flow at end of period i

y = yield to maturity (per period)



Given the price and cash flows, you can solve for the yield using a financial calculator or Excel.

- =pv(rate, nper, pmt, fv) gives price for coupon bond
- =rate(nper,pmt,pv,fv) gives YTM for a coupon bond

Remember that bond prices depend on *all* the relevant yields in the spot yield curve; the YTM abstracts from the shape of the spot yield curve.

The big question: Approximately how much will a security's price change for a small change in its yield?

A security's price "*p*" is function of its yield "*y*" and other factors:

- Other factors include maturity, coupon, embedded options, default risk, market conditions, etc.

$$p = f(y;\ other\ factors)$$

Duration measures are related to the first partial derivative: $\partial f/\partial y$

Convexity measures are related to the second derivative: $\partial^2 f/\partial y^2$

Three main factors affect a bond's price sensitivity to yield changes ($\partial P/\partial Y$):

- Remaining maturity
- Coupon rate
- Level of interest rates

Duration increases with maturity and it decreases with coupon rates and the level of market interest rates.

- Duration and convexity take into account all 3 factors.
- Credit risk and attached options also affect $\partial P/\partial Y$. We'll incorporate these effects using the more general concept of "effective duration."

Example 3-1:

The Effect of Maturity and Coupon on Price Sensitivity

Four bonds; each priced to yield 9% (b.e.b), semiannual payments. Two have 9% coupon, two have 5% coupon; two mature in 5 years, two mature in 20 years

[Prices at 9% yield are \$100, \$100, \$84.175, and \$63.1968]
init

Instantaneous Percentage Price Changes

New Yld (%)	BP change	9% 5 yr	9% 20 yr	5% 5 yr	5% 20 yr
6.00	-300	12.80	34.67	13.73	39.95
8.00	-100	4.06	9.90	4.35	11.26
8.90	-10	0.40	0.93	0.42	1.05
9.01	1	-0.04	-0.092	-0.042	-.104
9.50	50	-1.95	-4.44	-2.09	-5.01
10.00	100	-3.86	-8.58	-4.13	-9.64
12.00	300	-11.0	-22.6	-11.9	-25.1

11.26>4.35
11.26>9.90

Link to Excel calculations

Duration Basics

Duration measures the first order **bond price sensitivity to interest rate changes**.

- Higher duration means higher price sensitivity to interest rate changes (more price volatility).
- More precisely, it is the elasticity of a bond price with respect to its yield

Basic duration measures take into account yield, coupon, and maturity.

dP/P: percentage of change in price

dY/(1+Y/k): percentage of change in yield

$$\boxed{D = \left(\frac{-dP}{dY} \cdot \frac{(1+Y/k)}{P} \right)}$$

"Y" is an annual percentage rate APR

(Note: *k* is the number of compounding periods per year.
For continuous compounding *k*=∞)

- For Macaulay duration, *dP/dY* is based on the standard formula relating bond price *to promised* cash flows. As such, it is only accurate for risk-free bonds with no embedded options. See *Appendix Slides* for derivation.

default or prepayment options

Duration is a property of a security or a portfolio at a point in time.

It changes over time.

The formula for Macauley duration can be written as:

$$D = \sum_{t=1}^{T} \frac{\dfrac{C_t}{(1+\frac{y}{k})^t}}{P_B} \times \frac{t}{k}$$

D = Macauley duration
C_t = period t cash flow
T = total number of periods
y = yield as an APR
k = assumed compounding periods in a year
P_B = bond price (or present value of cash flows)

That formula implies that Macaulay duration is a **weighted average arrival time of cash flows**, where the weights are the fraction of present value represented by that cash flow:

$$D = \frac{\left(\frac{1}{k}\right) \times PVCF_1}{PVTCF} + \frac{\left(\frac{2}{k}\right) \times PVCF_2}{PVTCF} + \ldots + \frac{\left(\frac{T}{k}\right) \times PVCF_T}{PVTCF}$$

$PVCF_i$ = present value of i^{th} cash flow
PVTCF = present value of total cash flows
(Note in this representation, duration can be described in terms of years although it is intrinsically unitless)

Macauley Duration vs. Maturity for Bonds in Example 3-1

Coupon	Maturity	Macaulay Duration
9%	5	4.13
9%	20	9.61
5%	5	4.43
5%	20	10.87

Verify using calculator!

Properties of Macauley Duration

$$\boxed{D = \frac{\left(\frac{1}{k}\right) \times PVCF_1}{PVTCF} + \frac{\left(\frac{2}{k}\right) \times PVCF_2}{PVTCF} + \ldots + \frac{\left(\frac{T}{k}\right) \times PVCF_T}{PVTCF}}$$

- Duration of an option-free coupon bond is less than or equal to its time to maturity
- Duration of a zero coupon option-free bond is equal to its time to maturity
- The higher the coupon rate the shorter the duration
- As market yield increases, duration decreases

PV of the more distant payments gets smaller
relative to the payments arriving sooner

Using Duration to Estimate Price Volatility; and Defining Modified Duration

Recall that $\boxed{D = \left(\frac{-dP_B}{dY} \cdot \frac{(1+Y/k)}{P_B} \right)}$ Rearrange to get:

$$\frac{dP_B}{P_B} = -D \frac{dy}{(1+\frac{y}{k})}$$

where D is Macauley duration, y is an APR, and k is the number of compounding periods in a year. Rewrite this as:

$$\boldsymbol{\frac{dP_B}{P_B} = -D_M \times dy}$$

$$P_B = \sum_{t=1}^{T} \frac{C_t}{(1+\frac{y}{k})^t}$$

$$\frac{dP_B}{dyP_B} = \frac{1}{P_B} \sum_{t=1}^{T} \frac{(-t)C_t}{(1+\frac{y}{k})^{t+1}} \frac{1}{k}$$

$$= \frac{-1}{1+\frac{y}{k}} \sum_{t=1}^{T} \frac{\frac{C_t}{(1+\frac{y}{k})^t}}{P_B} \frac{t}{k}$$

$$= \frac{-1}{1+\frac{y}{k}} * D$$

$$= -D_M$$

<u>Definition</u>: D_M is called the "**modified duration**." It is defined as:

$$D_M = \frac{D}{1+\frac{y}{k}}$$

In words, the percentage price change is approximated by:
(modified duration) times (the change in the yield).

<u>Example 3-2</u>: Using Modified Duration to Approximate Price Changes

20 yr, 5% coupon bond (semiannual payments), P=63.1968, to yield 9% (b.e.b.).

$D = 10.87$ years (see table above)

$D_M = 10.87/(1.045) = 10.40$

Say yields increase from 9% to 9.10%. ***What is the predicted price change?***

-10.40(.0010) = -1.04%
The actual change in price is -1.03%

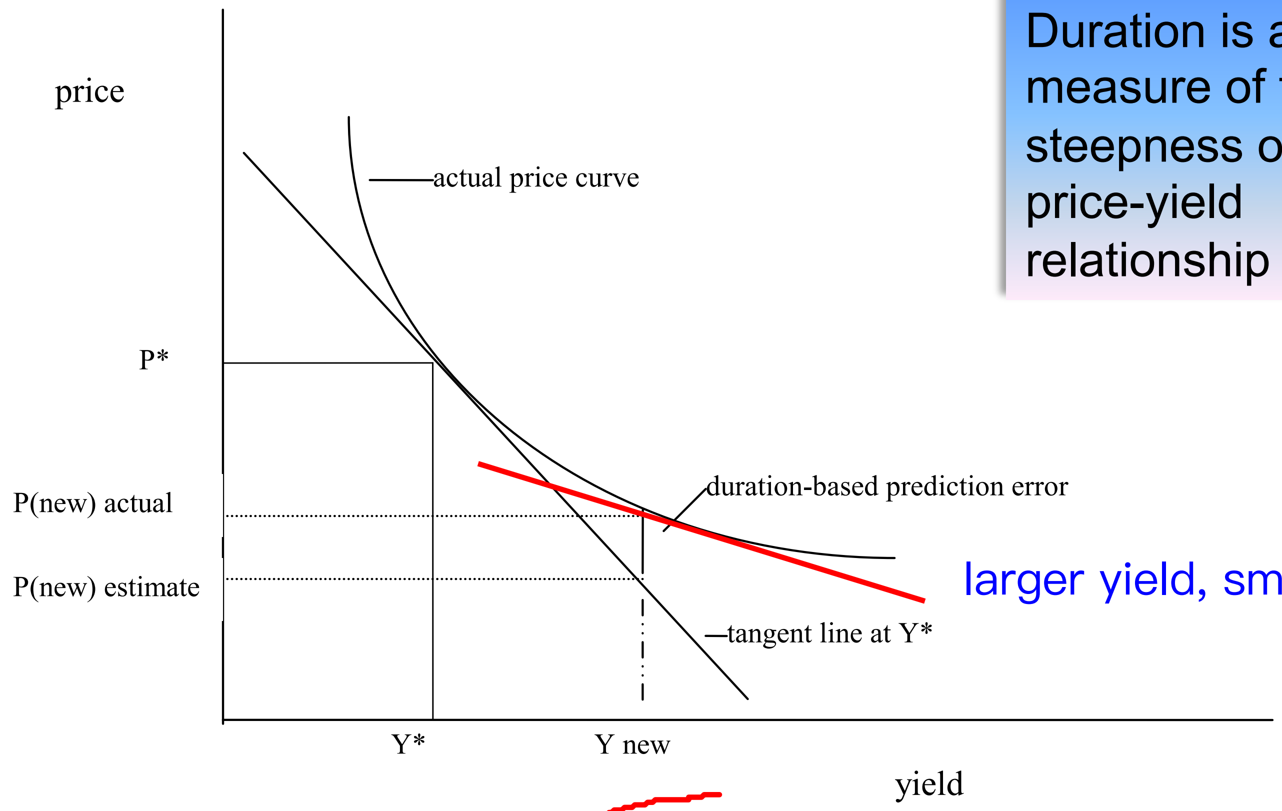
EXCEL example of actual change.

Say yields increase from 9% to 11%. ***What is the predicted price change?***

-10.40(.020) = -20.80%
The actual change in price is -17.94%

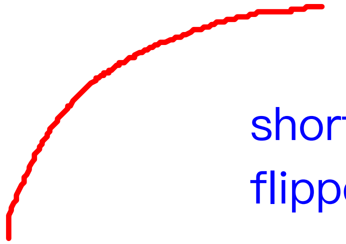
A general conclusion is that duration only gives an accurate estimate of price change for small yield changes.

Graphical Interpretation of Duration



Duration is a measure of the steepness of the price-yield relationship

larger yield, smaller duration



short position flips the sign of all of the cash flows. That flipped the actual price curve over the x– axis

Other basic duration measures

1. Dollar Duration

We have been using duration to estimate percentage price changes.
It also can be used to estimate dollar price changes:

$$dP_B = -D_M \times P_B \times dy$$

Definition: D_d is dollar duration. It is defined as: $\boldsymbol{D_d = D_M \times P_B}$

- Thus, $dP_B = -D_d \times dy$
- Dollar duration is useful in hedging strategies and for understanding risk of zero NPV portfolios.

2. Portfolio Duration

One can prove that

(1) The **modified duration of a bond portfolio** is the **value-weighted average modified duration of bonds in the portfolio**

(2) The **dollar duration of a portfolio** is the **sum of the dollar durations of the bonds in the portfolio**

- Added flexibility in duration-targeting comes from the fact that a short position contributes negative duration.
- For a zero-value portfolio, only dollar duration is defined.

If the bonds have different yields, this means that the duration of each bond in the portfolio will be based on a different yield.

<u>Example 3-3</u>: Two-bond portfolio. $P(1) = \$8{,}000$, $D_M(1) = 4.3$ years. $P(2) = \$12{,}000$, $D_M(2) = 3.6$ years.

$$D_M(\text{portfolio}) = (8/20)(4.3) + (12/20)(3.6) = 3.88$$

Later we will see that portfolio duration can be changed using forward, futures and swap contracts.

Generalized duration measures

These measures increase the accuracy of sensitivity estimates, which in turn makes risk assessment and hedging strategies more robust

- For example, partial durations are inputs into statistical analyses, such as value-at-risk

“Effective Duration” is the most important generalized measure of price sensitivity

For risk-free securities, effective duration and modified duration are the same thing

For securities with uncertain cash flows (like MBS or securities with credit risk), effective duration is defined as the **true price sensitivity** of the security to changes in yield.

there isn't a general formula you can use to calculate it

It may be based on a theoretical model, or it may be estimated empirically.

The standard duration formulas are inaccurate when cash flows are highly uncertain. In such cases effective duration is the better measure.

Generalized duration measures, continued

Traditional duration measures price sensitivity to small changes in the general level of interest rates.

But other factors also influence bond prices.

Generalized measures of duration can be used to describe the total % change in bond price as the sum of the partial effects of multiple factors in a linear model:

$$\frac{dP}{P}=\frac{1}{P}\left[\frac{\partial P}{\partial f_1}\Delta f_1+\frac{\partial P}{\partial f_2}\Delta f_2+\ldots+\frac{\partial P}{\partial f_n}\Delta f_n\right]$$

where f_i is the ith factor that influences price.

inflation, stock prices

$\frac{\partial P}{\partial f_i}\Delta f_i$ is the sensitivity of price to the ith factor times a unit change in the ith factor. This is sometimes called a **"partial duration."**

When the factors of interest are rates along the yield curve, the resulting partial durations are called **"key rate durations."**

Convexity

- Convexity measures the degree of inward curvature of the price-yield relationship. It is based on the second derivative of a security price with respect to yield
- As for duration, there are basic and generalized convexity measures
- It is used to improve upon duration-based approximations and hedging strategies
- A long position in non-callable bonds always has positive convexity
- Positive convexity is a desirable property for a long position; negative convexity is a bad thinghigher convexity means that prices increase more when yields fall and decrease less when yields rise.
  - Positive convexity means that duration underestimates the price increase resulting from a drop in yields, and overestimates the price decrease from an increase in yields.

Calculating Convexity

Convexity is found by taking the second derivative of the bond price function and then dividing by the price. An explicit expression for convexity of an option-free bond is:

$$C_0 = \sum_{t=1}^{T} \frac{t(t+1)X_t}{(1+y/k)^{t+2}k^2} \,/\, P_B$$

it's quite helpful to have a mental shortcut to know that for a zero– coupon bond, convexity is approximately its maturity squared.

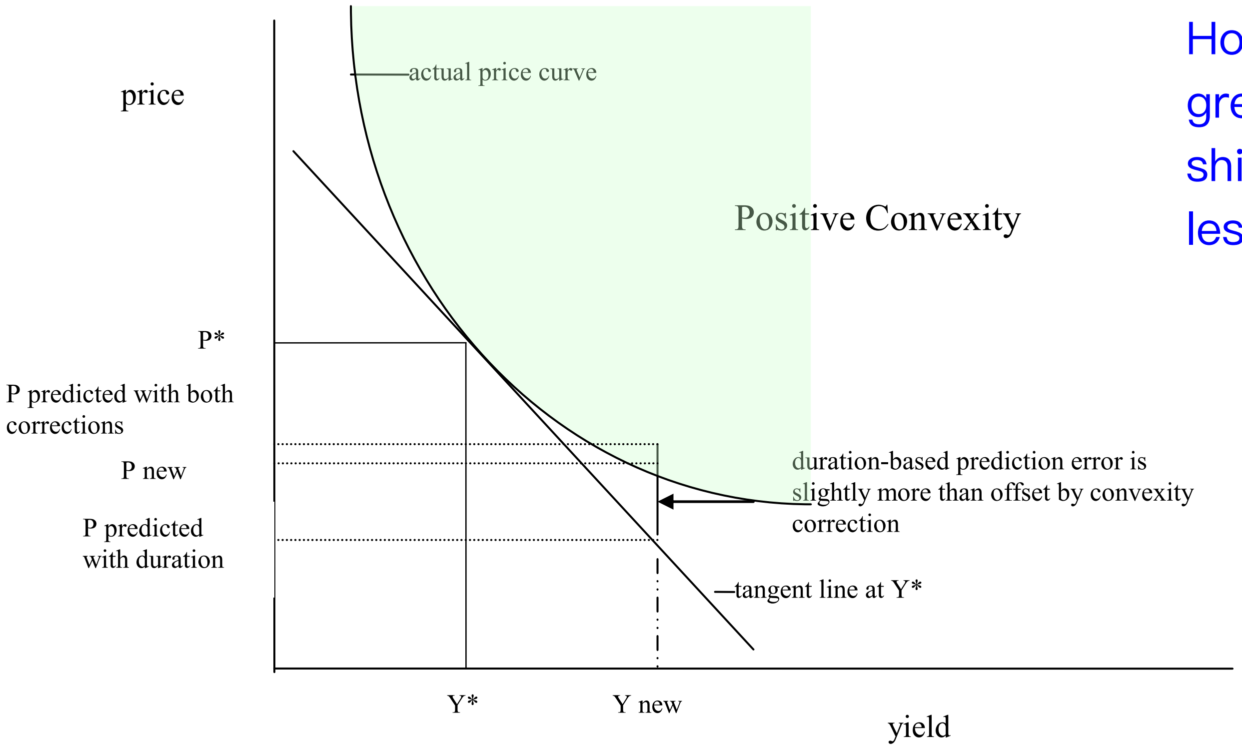
T = number of periods (= maturity in years $\times$ k)
P_B = bond price (present value of cash flows)
X_t = time t cash flow
y = quoted annual percentage rate (so y/k = effective yield per period)
k = number of compounding periods per year

Note that convexity is measured in terms of years squared. The units have no intuitive interpretation.

Dollar Convexity = C_0P_B

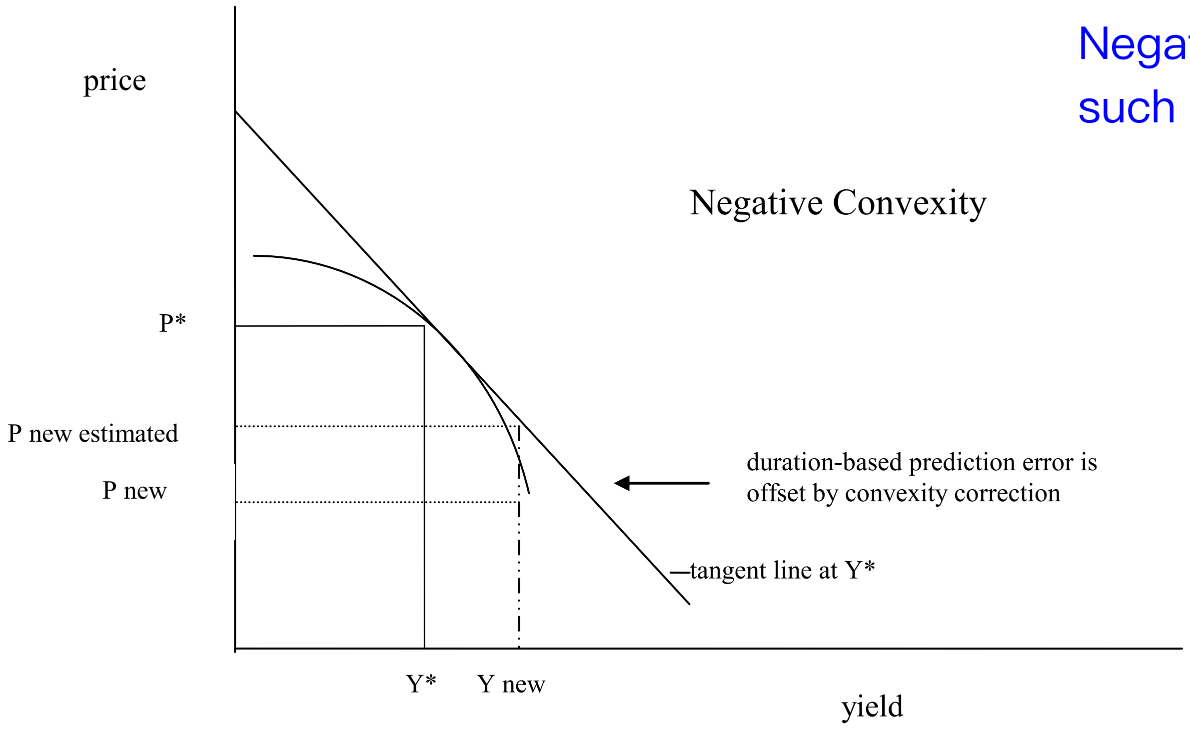
only true for parallel shifts in the yield curve.

Graphical Interpretation of Convexity



Holding duration constant, you can also see that the greater convexity is a good thing. Because for any shifts in yields,the price is higher than in the case of less convexity.

Graphical Interpretation of Negative Convexity



Negative Convexity: short bond position or long positions such as for callable bonds over some range of yields.

That's because as rates fall, the bond becomes more likely to be called before maturity, which puts an upper bound on its price appreciation.

undesirable: worse for any given change in yields, holding all else equal including duration.

Using Convexity to Improve Price Sensitivity Estimates

The prediction equation with duration and convexity is:

$$\frac{dP_B}{P_B} = -D_M(dy) + \tfrac{1}{2} C_0 (dy)^2$$

(This is a 2nd order Taylor's Series expansion of the price-yield function.)

<u>Example 3-4:</u> Approximating price change with duration and convexity

Estimate the percent price change of a 6% coupon bond with 25 years to maturity, selling to yield 9% if there is a 200 basis point increase in required yield.

$D_M = 10.62$
$C_0 = 182.92$

$$\frac{dP_B}{P_B} = -10.62(.02) + \tfrac{1}{2} 182.92(.02)^2 = -21.24\% + 3.66\% = -17.58\%$$

Contrast to the actual change = -18.03%.

Using convexity gets much closer than the estimate using duration alone!

Duration and Convexity-Based Hedging Strategies

- Delta Hedging (duration-based hedging)
- Gamma Hedging (duration and convexity-based hedging)
- Delta & gamma hedging with forwards, futures and swaps

Using Duration and Convexity to Structure a Hedge œDelta and Gamma Hedging

How much does a bond price change with a change in interest rates?

This is approximated by:

$$dP = -D_m(P)dy + .5(P)C_0(dy)^2$$

Definitions

D_m = modified duration (also can use effective duration)
P = price
C_0 = convexity
Hedge Ratio = $-D_m(P)$ (= ‑dollar duration“)
Gamma = $(P)C_0$ (= ‑dollar convexity“)

A delta neutral portfolio equates the hedge ratio of assets and liabilities:

$$P_{asset}D_{m,asset} = P_{liability}D_{m,liability}$$

A gamma neutral portfolio is delta neutral, and also equates the gammas of assets and liabilities.

Example 3-5: Hedging Dealer Portfolio Risk

A dealer in corporate bonds finds herself with an inventory of $1mm in a 5 year 6.9% bonds (semiannual payments) at the end of the trading day, priced at par. The bonds are illiquid, so selling them would entail a loss. Holding them overnight is risky, since their price might fall if rates rise.

An alternative to selling the corporate bonds is to short more liquid Treasury bonds. The following bonds are available:

1. 10 yr 8% Treasury, p = $1,109.0 per $1,000 face
2. 3 yr 6.3% Treasury, p = $1,008.1 per $1,000 face

a. How much of the 10 year bond would she need to short to hedge? How much of the 3 year bond?

b. If yields rise by 1% overnight on all the bonds, show the result of the transactions the next day when the short position is closed out.

Plan to answer (a):

1. Find modified duration of the bond to be hedged
2. Find modified duration of the bonds to be shorted
3. Use this to find hedge ratios.

For 5 year 6.9% bond, y = 6.9% b.e.b., D_m = 4.1688

For 10 year 8% bond, y=6.5% b.e.b, D_m=7.005

For 3 year 6.3% bond y = 6.00% b.e.b., D_m = 2.700

Amount of 10 year bond to sell, x, solves: x(7.005) = \$1mm(4.1688). x = \$593,861.5

Amount of 3 year bond to sell, y, solves: y(2.7) = \$1mm(4.1688) y = \$1.54072 mm

because of its shorter duration, necessitates a much larger short position.That could involve additional transactions costs or make it harder to find a counterparty.

.

Part b: If yields rise by 1% overnight on all the bonds, show the result of the transactions the next day when the short position is closed out.

For 5 yr, yield to 7.9%, => price to $959.344/$1000. Loss on long position = $1mm(1-.959344) = $40,656

For 10 yr, yield to 7.5% => price to 1034.74/$1000. 1034.74/1109 = .933. (1-.933)(593,861.5) = $39,765.7 gain.

For 3 yr, yield to 7% => price to 981.35/$1000. 981.35/1,008.1 = .97346. (1-.97346)(1,540,720) = $40,891 gain.

Note: You can also find price change by first finding face value and then rediscounting cash flows at new rate.

a delta hedge will have better results when the convexity of the long position is greater in absolute value than the convexity of the short position.

more convexity is desirable when long and undesirable when short

Example 3-5 extended to gamma hedging

"What if the dealer wants the added protection of doing a gamma neutral hedge?"

- Let bond 1 be the bond to be hedged (5 year 6.9% corporate)
- Let bond 2 be the 10 year 8% Treasury
- Let bond 3 be the 3 year 6.3% Treasury

Investment must be both delta neutral and gamma neutral. This requires matching deltas and gammas, and requires investments in both bonds.

- P1 = $1 million by assumption. D1 = 4.1688, C1 = 21.038
- P2 = ?, D2 = 7.005, C2 = 62.98
- P3 = ?, D3 = 2.700, C3 = 8.939

frequently adjusting a delta hedge, which is also known as dynamic hedging, is a substitute for putting on a more robust gamma hedge that can be adjusted less frequently for the same amount of protection.

Match hedge ratios:

- $1m(4.1688) = P2(7.005) + P3(2.700)

Match gamma:

- $1m(21.038) = P2(62.98) + P3(8.939)

2 linear equations in two unknowns. Solve for P2 and P3. (Full solution is posted on the class web page.)

<u>Practice exercise</u>: Redo calculation of what happens when rates move (as in example in notes), and verify that hedge is better than with delta hedge.

it's more effective to gamma hedge, why not always do that instead of just delta hedging? The answer is that it's more complicated and usually more expensive to gamma hedge. So if a delta hedge provides sufficient protection, it's going to be the easier and cheaper approach

Duration and convexity for forwards, futures and swaps

- Interest rate forwards, futures and swaps are often used in place of cash market delta and gamma hedging strategies
  - Advantages may include lower transactions costs and more liquidity

- The logic of delta and gamma hedging is the same using these derivatives as it is using cash instruments: The goal is to take a position whose gains or losses will offset the gains or losses on the position being hedged.

- To implement an interest rate risk-management strategy with interest rate derivatives, we need to know how to calculate their duration and convexity

Duration of a forward or futures contract for a bond

- Recall that the dollar duration of a bond is defined as $D_m(P)$, and that $dP/dy = -D_m(P)$

modified or effective duration

- **Definition:** The **dollar duration of a forward bond contract is equal to the duration of the security specified in the forward contract multiplied by the prepaid forward price of the security.**

purchase price

- **Definition**: The **prepaid forward price** is the present value of the forward price.

Forward contracts for bonds can be replicated in the cash market with offsetting long and short positions. You can also find the dollar duration of a forward contract by finding the dollar duration of the **replicating portfolio of spot market positions**.

  - An example of finding the duration of the replicating portfolio is in the appendix to these slides

Using futures in a duration-based hedge

The challenge: There are relatively few futures contracts available. Hence, the duration of the obligation being hedged typically differs from the duration of the futures contract.

The solution: Adjust the number of futures contracts bought or sold to equate the sensitivity to yield change. This uses the idea of delta hedging using a hedge ratio.

Recall that: $$\mathbf{dP \approx -PD_m dY}$$

futures are an attractive tool for hedging because of their high liquidity, low transactions costs, rapid speed of execution, and safety from counter party risk.

- P = bond price; dP = change in bond price; Y = yield (APR); dY = change in yield; D_m = modified or effective duration

Similarly, if F is the contract price for an interest rate futures contract, then

$$\mathbf{dF \approx -FD_F dY}$$

- where D_F is the modified duration of the futures contract, and F is the prepaid forward price of the security in the futures contract.

For the contract to serve as a hedge, we want dF = dP, which implies **equating the hedge ratios** in a long and short position**:**

$$\mathbf{PD_m = FD_F}$$

A potential advantage of hedging with swaps is the wide variety of durations and convexities that are available.

Swap duration

- Recall that for a fixed rate receiver, a swap is like having a portfolio that is long a fixed rate bond and short a floating rate bond.
  - Initially the value of the long and short are equal to the notional principal F

- The effective duration of a (pure) floating rate bond is the time until the next reset, divided by (1 + Y/k)
  - Y is the APR; k is the number of compounding periods in a year.
  - This is because the price of a floating rate bond between reset dates varies with short-term interest rates, but the price at the next reset date is fixed at par.

- The modified (and also effective) duration of the fixed rate bond can be calculated in the usual way.

It follows that for the fixed rate receiver, **the dollar duration of a swap, -dP/dY is:**

$$+P(\text{fixed}) \times D_m\ (\text{fixed}) - P(\text{floating}) \times D_{\text{effective}}\ (\text{floating})$$

Example 3-6: Calculating the duration of a swap

Consider a new 5-year interest rate swap, offering a fixed rate of 6% (s.a.), and a floating rate of 6-mo LIBOR, with notional principal of $1m. Assume current 6-mo LIBOR is also 6%.

- What is the dollar duration for the fixed rate receiver?
- What is the dollar duration for the floating rate receiver?

Using the duration calculator, a 5-year fixed rate bond with a 6% (s.a.) coupon selling at par has a modified duration of 4.265 years. The effective duration of the floating rate side is .5/(1.03) = .485 years. The difference is 3.78 years.

- **The dollar duration of the swap is 3.78($1m)**

The floating rate receiver's position is the negative of the fixed rate receiver's position.

- **The dollar duration of the swap is -3.78($1m)**

Putting it all together

- Interest rate risk is often managed effectively and flexibly using duration and convexity-based strategies.
- These strategies can be implemented in the cash market, or with derivatives. The choice between different options depends on cost, availability and risk tolerance.
- To practice implementing a delta hedge with forwards, the recitation will revisit Example 3-5 of hedging a security dealer’s overnight exposure to interest rates using (1) a bond futures contract; and (2) using in interest rate swap.

Appendix: Optional derivations

Finance at MIT

Where ingenuity drives results

Deriving Macaulay duration

Bond price $P_B = \sum_{t=1}^{T} \frac{C_t}{(1+\frac{y}{k})^t}$.

C_t is the promised cash flow in period t, T is the total number of periods (= k times maturity in years), y is the annual percentage rate, and k is the number of compounding periods in a year.

To find change in bond price for a small change in yield differentiate:

$$\frac{dP_B}{dy} = \sum_{t=1}^{T} (\frac{-t}{k}) C_t (1+\frac{y}{k})^{-t-1}$$

This gives the dollar price change per small change in annual yield.

To get the percent price change, divide by P_B.

To get the Macaulay duration (measured in years) multiply the percentage price change by *-(1+y/k)*:

$$D = \sum_{t=1}^{T} (\frac{t}{k}) C_t (1+\frac{y}{k})^{-t} / P_B$$

Deriving dollar duration of a forward contract

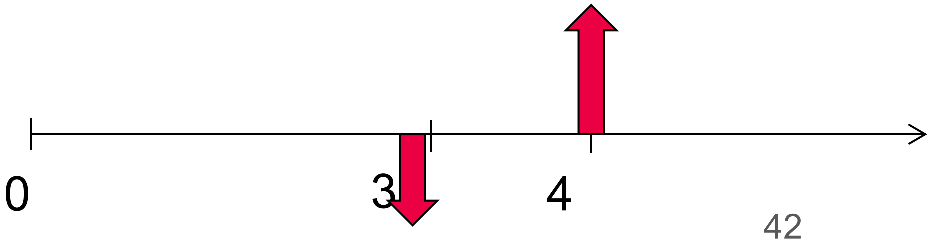
A forward bond contract is equivalent to a portfolio consisting of a long and a short position in zero coupon bonds of equal value.

Using this fact, we can derive an expression for the **yield-sensitivity of the present value of the forward contract**.

E.g., a long forward contract in a one-year bond to be delivered 3 years in the future:

- Equivalent to a long position in a 4-year zero coupon bond, and a short position of equal value in a 3-year zero coupon bond.
- Assume contract is such that $P_0(3) = P_0(4) = \$100$;
- $V_0=P_0(4)-P_0(3) = 0$; $D_m(4) = 4/(1+y/k)$; $D_m(3) = 3/(1+y/k)$
  - $dV_0/dy = dP_0(4)/dy - dP_0(3)/dy = -D_m(4)P_0(4) + D_m(3)P_0(3)$

$$= -(D_m(4) - D_m(3))100 = -(4-3)\times 100/(1+y/k)$$



Note that the modified duration of the security in the contract is $1/(1+y/k)$, and the prepaid forward price, $P_0(3)$, is \$100.