Week 1 – Forward Contracts

MIT Sloan School of Management

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Outline

- Forward contract basics
  - Definition
  - Profit/Loss
  - A forward currency hedge and payoff diagrams
  - Review of risk-free rates and the yield curve
  - Interpretation of forward contracts as highly levered positions
- Pricing formulas
  - Financial assets: stocks, bonds, currencies
  - Commodities
- Key concepts for hedging and speculating

Forward contract basics

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Definition: Forward Contract

A **forward contract** is an agreement between two counterparties to trade a pre-specified amount of goods or securities at a pre-specified future date, T, for a pre-specified price, F_0.

Notes:

- It is free to enter into a forward contract initially: the contract is an agreement to exchange goods or securities for money in the future, and not today
  - In practice you may have to post collateral, but typically you earn a return on the collateral
    to ensure that both sides meet their obligations
  - A "pre-paid forward" is an exception. In this case the payment is made upfront.
- The pre-specified price F_0 is set to ensure that the value of the forward contract is zero for both counterparties at the inception of the contract
- **Terminology**: The counterparty who agrees to **buy (sell)** the goods or securities has **long (short)** position
- **Other features**: Forwards are traded "over-the-counter". The contract may specify physical delivery or cash settlement.

Profit/Loss from a forward contract

Let S_t be the spot price of one unit of the good or security at *t*, and *N* be the size of the contract (# of units)

- *Definition:* The "spot price" is the current market price

The Profit/Loss (P/L) at the contract maturity *T* for each counterparty is

- **P/L long = N x $(S_T - F_0)$**
- **P/L short = N x $(F_0 - S_T)$**

Questions for you to think about:

- What are the details of the transactions that makes this true for physical settlement?
- What are the details of the transactions that make this true for cash settlement?
- Does it matter if the short already owns the asset?
- Does it matter whether or not the long wants to own the asset?
- How can a zero-sum contract benefit both counterparties?
- What is the P/L if you sell the contract at time $t < T$

Forward contract timing of cash flows

t=0	t = T
Long contract	Payoff = S_T-F_0
Short contract	Payoff = F_0-S_T

No initial cash flows but collateral may be required

Forward contracts are always "zero-sum" in their payoffs

Example 2.1: Hedging with a forward currency contract

A US firm has sold a piece of equipment to a German client and now it has a receivable of EUR 5 million in T = 6 months.

Let S_t = USD/EUR exchange rate at t. Assume the current rate S_0 = 1.2673

Unhedged dollar payoff at T = EUR 5 million x S_T

What is the risk?

Exchange rate risk: Euro can depreciate vs. the dollar (S_T declines)

Example 2.1 (cont.)

Hedging strategy: enter into a forward contract with a bank to exchange euros for dollars at T = 6 months at an exchange rate F, say F = 1.28, decided today

→ The firm is short the Euro forward

Dollar P/L of forward contract at T = 5 mil x $(F - S_T)$

profit or loss

Total payoff at T

= payoff from original position T + payoff of forward contract at T

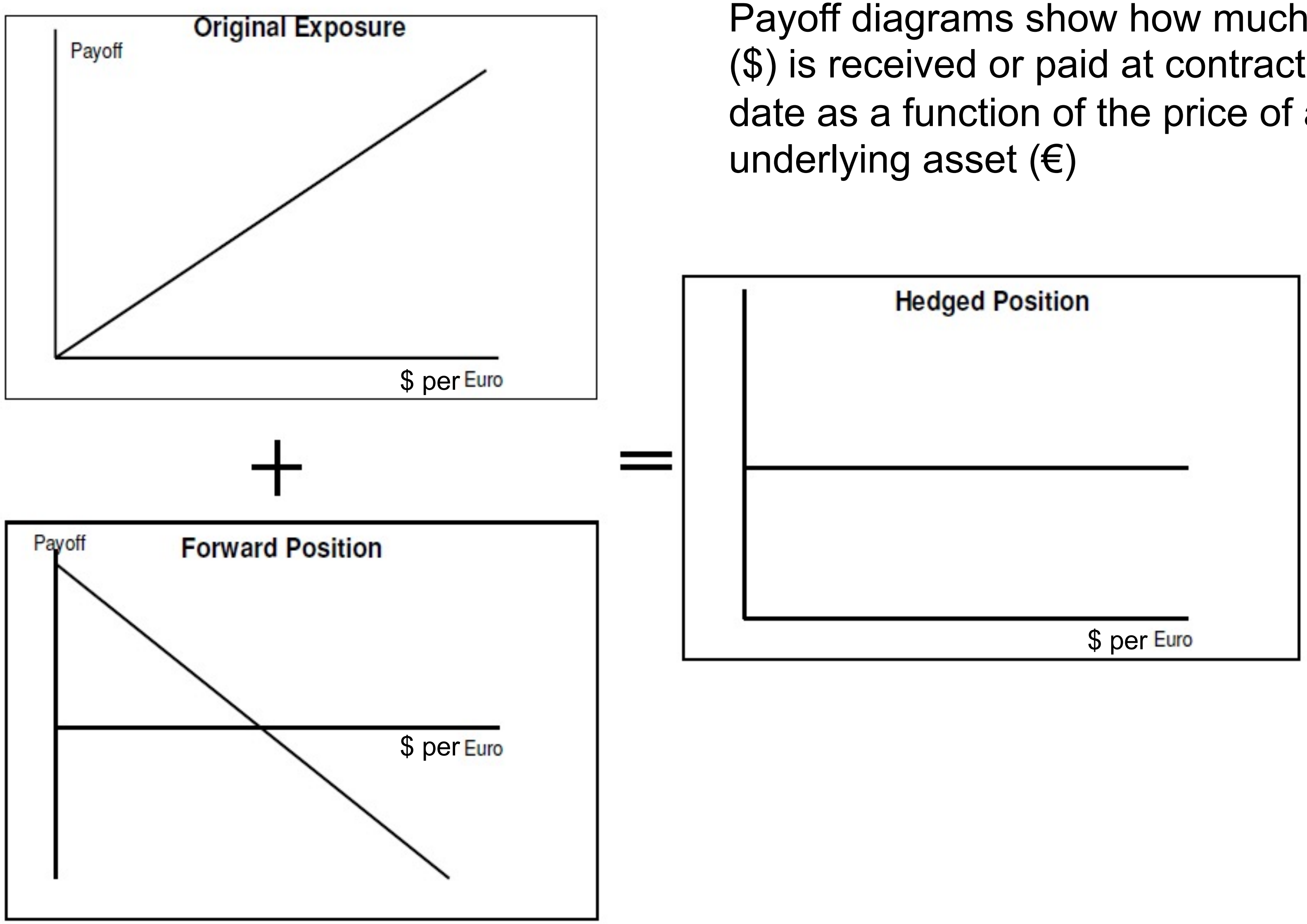
= $(N \times S_T) + N \times (F - S_T)$

= 5 mil x 1.28 = $6.4 million

The hedge locks in the dollar payoff at T to be N x F, regardless of the exchange rate movement

Payoff diagrams for hedging example

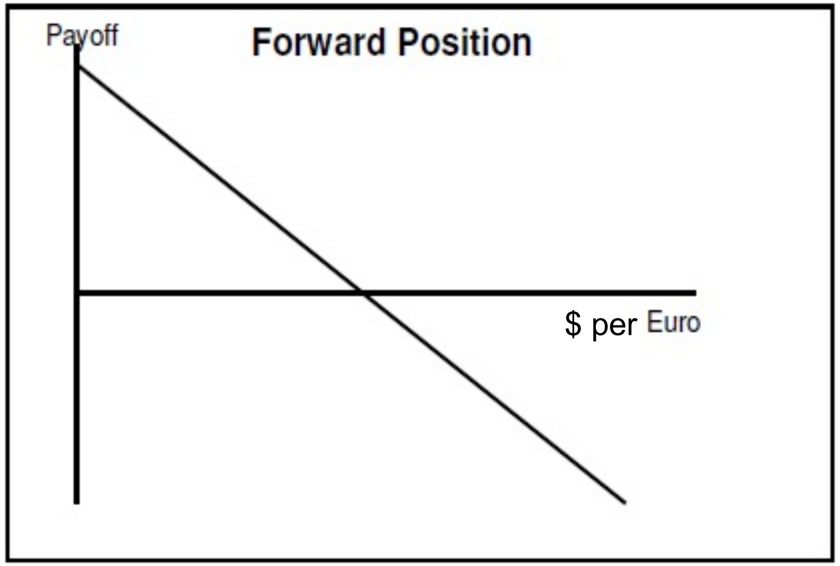
Payoff diagrams show how much money ($) is received or paid at contract expiration date as a function of the price of an underlying asset (€)



Example 2.2: Speculating with forwards

Directional bets

- Think that euro will depreciate versus dollar?
  - Short forwards or futures on dollar/euro exchange rate
  - It costs essentially nothing
  - Each tick in exchange rate movement => large profits (or losses)



Levered position: return on capital very high (or low)

A derivatives position functions as a hedge when it offsets the risk of another position in an investor's portfolio. It's speculation when there's no offsetting exposure.

Speculators serve an important function in derivatives markets by helping to make it possible for hedgers to protect themselves even when there aren't enough other hedgers with an opposite exposure to take the other sides of the contract.

Review: risk-free rates and the yield curve

default– risk free bonds still entail significant risk because the bond's market price will change over time as market interest rates change. Hedging bond price risk is a very popular use of derivatives contracts

The price of a bond is the present value of its promised payments

- The Treasury spot yield curve provides a reference market rate for discounting cash flows of different maturities that are free from default risk
- Rates along the Treasury yield curve make up the term structure of “risk-free” rates
- Risky or illiquid bond cash flows are discounted at rates that include a premium

Notation: The market rate to discount a cash flow arriving at time t back to time 0, stated on a continuous basis, is r_t

Price of a zero coupon bond with face value Z paid at time T is given by

$$P = e^{-r_T T} Z$$ continuous compounding

Recall that: $\lim_{n\to\infty}(1 + r/n)^n = e^r$

Implied forward prices and forward rates

Current price of bond is maturing at T_2 is:

$$P_{B,0} = e^{-r_{T_2} T_2} Z$$

We can prove that the forward price for delivery at T_1 is:

$$F = e^{r_{T_1} T_1} P_{B,0}$$

Substituting for bond price =>

$$F = e^{-r_{T_2} T_2} Z e^{r_{T_1} T_1} = Z e^{(r_{T_1} T_1 - r_{T_2} T_2)}$$

The **forward price implies a "forward rate"** which is the return locked in on the bond delivered in the forward contract.

The implied forward rate $f(0,T_1,T_2)$ solves:

$$F = Z e^{-f(0,T_1,T_2)(T_2 - T_1)}$$

Note on notation: $f(0,T_1,T_2)$ denotes the forward rate between time T_1 and T_2, as of time 0.

$$\boxed{f(0,T_1,T_2) = \frac{T_2 r_{T_2} - T_1 r_{T_1}}{T_2 - T_1}}$$

Risk-free borrowing and lending

- The standard pricing formulas for forwards that we will derive depend on the assumption that market participants can borrow and lend at a risk-free rate

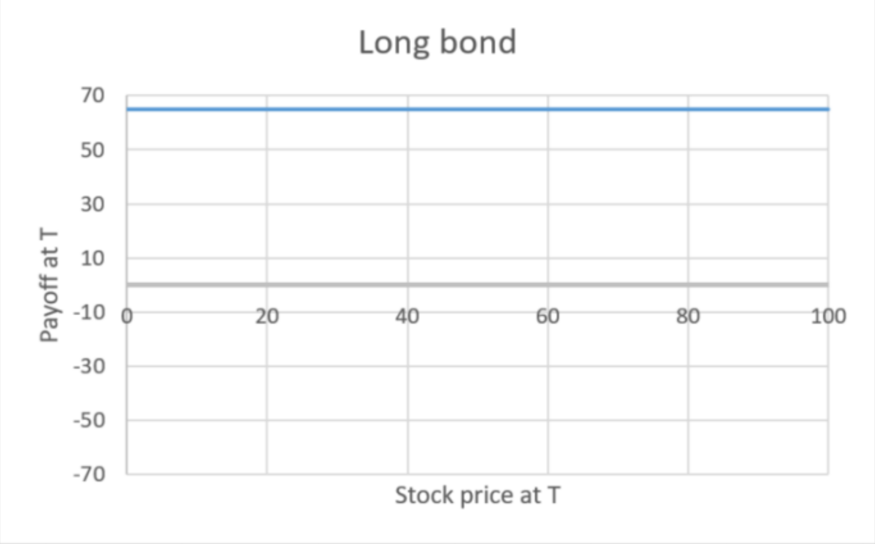
- In practice this can be accomplished by
  - Buying and selling Treasury securities
  - Collateralized borrowing and lending, e.g., using repurchase agreements (RPs or repos)

A repurchase agreement (repo) is a form of short–term borrowing for dealers in government securities. In the case of a repo, a dealer sells government securities to investors, usually on an overnight basis, and buys them back the following day at a slightly higher price. That small difference in price is the implicit overnight interest rate

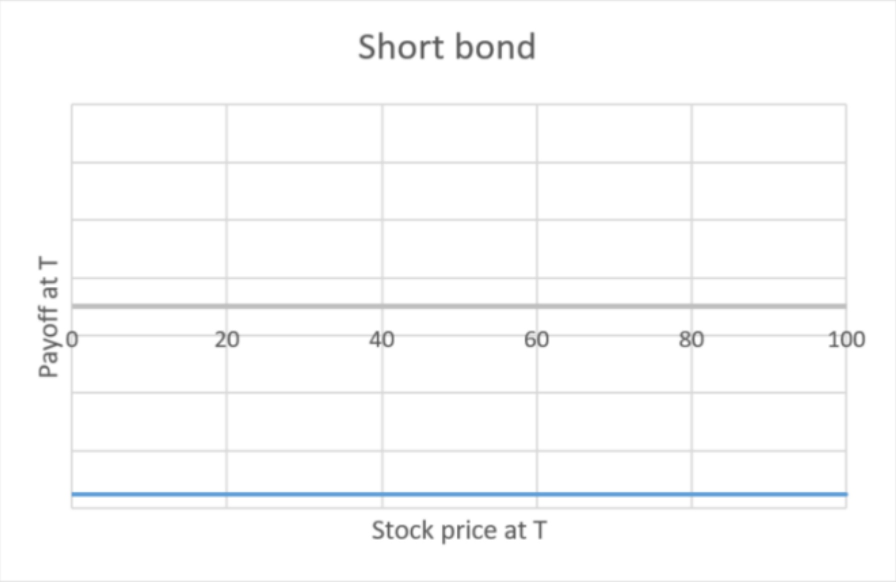
For the party selling the security and agreeing to repurchase it in the future, it is a repo; for the party on the other end of the transaction, buying the security and agreeing to sell in the future, it is a reverse repurchase agreement

Using payoff diagrams to understand forward contracts as highly levered positions

Stock spot price T	Long bond Payoff T
0	65
10	65
20	65
30	65
40	65
50	65
60	65
70	65
80	65
90	65
100	65

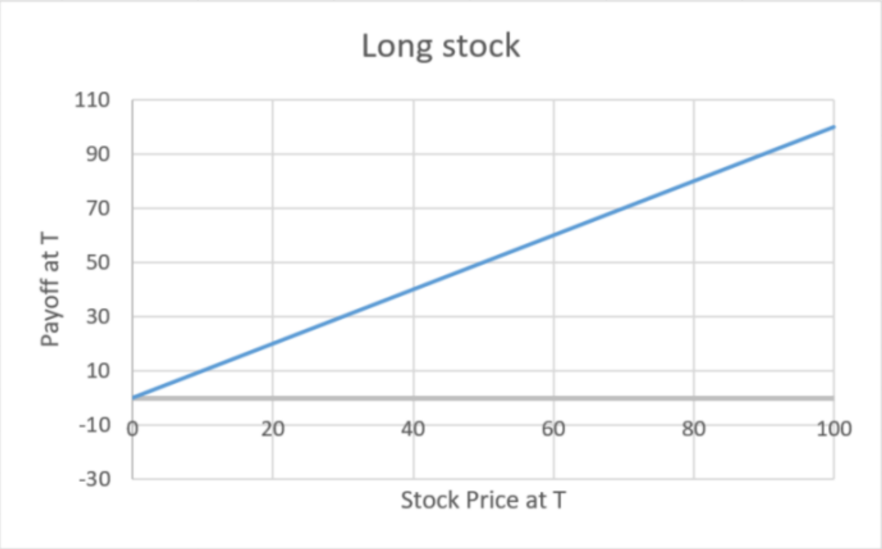


Stock spot price T	Short bond Payoff T
0	-65
10	-65
20	-65
30	-65
40	-65
50	-65
60	-65
70	-65
80	-65
90	-65
100	-65



Using pay off diagrams to understand forwards as highly levered positions

Stock spot price T	Long stock Payoff T
0	0
10	10
20	20
30	30
40	40
50	50
60	60
70	70
80	80
90	90
100	100

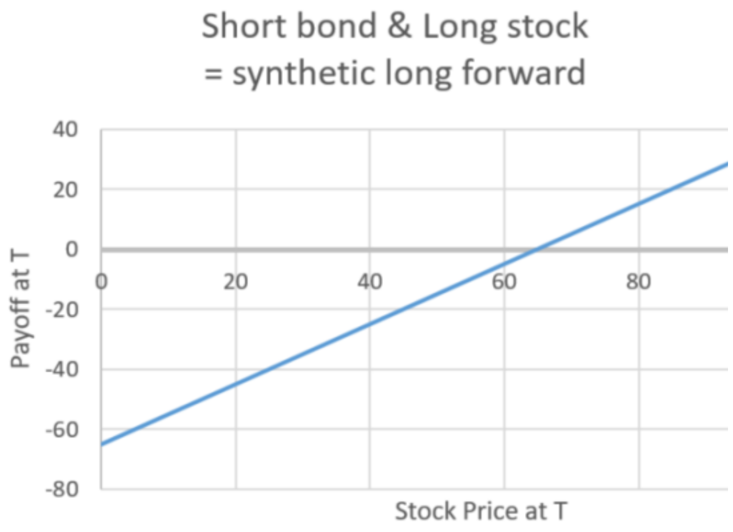


Stock spot price T	Short stock Payoff T
0	0
10	-10
20	-20
30	-30
40	-40
50	-50
60	-60
70	-70
80	-80
90	-90
100	-100

Using pay off diagrams to understand forwards as highly levered positions

Stock spot price T	Long stock forward Payoff T
0	-65
10	-55
20	-45
30	-35
40	-25
50	-15
60	-5
70	5
80	15
90	25
100	35

Stock spot price T	Short bond & Long stock Payoff T
0	-65
10	-55
20	-45
30	-35
40	-25
50	-15
60	-5
70	5
80	15
90	25
100	35



Takeaways from payoff diagram example

shorting a bond

Long forward positions are equivalent to borrowing and going long in the underlying asset

replicating portfolio

- Allows high long risk exposure to underlying with no money down (except collateral)
- Implicit leverage allows some people to borrow who otherwise couldn't

Forward short positions are equivalent to lending and going short the underlying

- Allows high short risk exposure to underlying with no money down (except collateral)
- Provides access to short exposure even when underlying is unavailable to borrow and short

Pricing formulas and their derivations

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Some definitions: No arbitrage

Definition*: An **arbitrage opportunity** is a trading strategy that either

(1) Yields a positive profit today, and zero cash flows in the future; or

(2) Costs nothing today and yields a positive profit in the future

- The value of many derivative securities is estimated by assuming that no arbitrage opportunities exist
- In well functioning markets, no arbitrage opportunities can persist
- If they did, arbitrageurs would take giant positions to profit from them, quickly eliminating them

Note that "no arbitrage" does not imply there are no market frictions

*This is a technical definition. The term "arbitrage" is used more loosely by traders.

Some definitions: No arbitrage

The Law of One Price:
Securities with identical payoffs must have the same price

Otherwise, an arbitrage opportunity arises..

Buy Low / Sell High

Buy the security with the low price and simultaneously sell (short) the one with a high price

- This yields an upfront profit

The arbitrageur is hedged, as future cash flows cancel out exactly

Some definitions: Short sales

Exploiting an arbitrage opportunity often requires taking offsetting long and short positions

Traders can take a short position either by

1. selling an asset they already own
2. borrowing an asset and then selling it
   - The trader must later buy the asset and return it to the lender
   - Shorting involves borrowing costs
   - Not all assets are available to short

If the stock you borrow pays a dividend, you must pay the dividend to the person or firm making the loan

When shorting becomes expensive or isn't possible, derivative prices can diverge from what is predicted using no– arbitrage conditions.

Which forwards and futures are priced by no-arbitrage?

Forwards for **pure investment assets can be priced using** relatively simple **no-arbitrage** arguments

- For instance, in Example 2.1, we will see how the pre-specified forward exchange rate F_0 = 1.28 USD/EUR was determined

We will see that a “cash and carry” strategy can be used to derive forward prices for pure investment assets via no-arbitrage reasoning

Forwards and futures for **non-investment assets** (e.g., agricultural commodities, energy, metals) **cannot be valued simply by no-arbitrage conditions**

Inventory of pricing formulas for financial forwards

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- Stocks
  - Stock with known dividend D at time $t < T$: $F_0 = [P_{S,0} - D(e^{-rt})](e^{rT})$ discrete flow
  - Stock with known dividend yield q: $F_0 = P_{S,0}\, e^{(r-q)T}$ continuous flow
    - r is the risk-free rate that matches cash flow maturity
    - $P_{S,t}$ is the stock price at time t
- Bonds
  - Bond with coupon C at time $t < T$: $F_0 = [P_{B,0} - C(e^{-rt})](e^{rT})$
    - r is the risk-free rate that matches cash flow maturity
    - $P_{B,t}$ is the bond price at time t

how to get \$1 at T:

- convert $\frac{1}{S_0}e^{-r_\$ T}$ of euro now at conversion rate S_0 to get $e^{-r_\$ T}$ dollar, which will exponentially grow to 1 dollar
- save $\frac{1}{F_0}e^{-r_€ T}$ of euro now and grow exponentially to $\frac{1}{F_0}$ euro at time T, convert to dollar at forward rate F_0 and produce 1 dollar

- Currencies
  - E.g., pay euros for dollars: $F_0 = S_0(e^{(r_\$ - r_€)T})$
    - $r_\$$ ($r_€$) is the USD (EUR) risk-free rate
    - S_t is the exchange rate (USD per EUR) at time t

Recap of example 2.1: Hedging with a forward currency contract

A US firm has sold a piece of equipment to a German client and now it has a receivable of EUR 5 million in T = 6 months.

Risk is that Euro can depreciate vs. the dollar (S_T declines)

Hedge with short forward contract in Euros

Let S_t = USD/EUR exchange rate at t. Assume the current rate S_0 = 1.2673

Unhedged dollar payoff at T = EUR 5 million x S_T

Also assume 6-month USD interest rate is 5%, & 6-month EUR interest rate is 3%

Recap example 2.1 (cont.)

Hedging strategy: enter into a forward contract with a bank to exchange euros for dollars at T = 6 months at an exchange rate *F*, say *F* = 1.28, decided today

How is the prespecified exchange rate *F* = 1.28 USD/EUR determined?

<u>Practice problem:</u> Verify that if interest rates and the spot exchange rate are as stated in this example, that the no-arbitrage forward exchange rate is 1.28.

$$F_0 = S_0(e^{(r_\$ - r_€)T})$$

Deriving forward prices from no-arbitrage conditions:
Cash and carry for a non-dividend paying stock

t=0	t = T
short stock forward	Receive F Deliver share of stock
buy stock for $-P_{S,0}$ borrow $P_{S,0}$	Use for delivery in forward Repay borrowing $-P_{S,0}(e^{rT})$
Net cash flow = 0	P/L = F - $P_{S,0}(e^{rT})$ <=0: no arbitrage

Conclusion: $F \leq P_{S,0}(er^{T})$

Deriving forward prices from no-arbitrage conditions:

Reverse cash and carry for a non-dividend paying stock

$t=0$	$t = T$
long stock forward	Pay $-F$ receive share of stock
short stock for $P_{S,0}$ lend $-P_{S,0}$	Use stock received to cover short Receive loan repayment $P_{S,0}(e^{rT})$
Net cash flow = 0	P/L = $P_{S,0}(e^{rT}) - F$ <=0: no arbitrage

Conclusion: $F \geq P_{S,0}(er^{T})$

Forward price with no arbitrage

If F – $P_{S,0}(e^{rT}) > 0$ then cash and carry is arbitrage opportunity

If $P_{S,0}(e^{rT}) - F > 0$ then reverse cash & carry is arbitrage opportunity

Hence, the forward price for a stock with no dividends between 0 and T must be:

$$\boldsymbol{F_0 = P_{S,0}(e^{rT})}$$

Forward prices for commodities

Some assets that are not purely financial---like oil, metals and some agricultural commodities--are storable and held as investments

Storing commodities has costs and benefits

- Storage cost “U” (lump sum payments)
- Storage cost “u” (percentage of spot price)
- Convenience yield “y” (percentage of spot price)

Forward price $F_{0,T}$ at contract initiation comes from no-arbitrage condition that again can be demonstrated using cash-and-carry and reverse cash-and-carry logic.

- Adjustments for costs and benefits are like adjustments for stock dividends and bond coupons

Storage costs are like negative dividends and convenience yields are like positive dividends.

Forward prices for commodities

Forward price with lump-sum storage cost U

$$F_{0,T} = (S_0 + PV(U)) \times e^{rT}$$

Forward price with proportional storage cost u

$$F_{0,T} = S_0 \times e^{(r+u)T}$$

Forward price with convenience yield y

$$F_{0,T} = S_0 \times e^{(r-y)T}$$

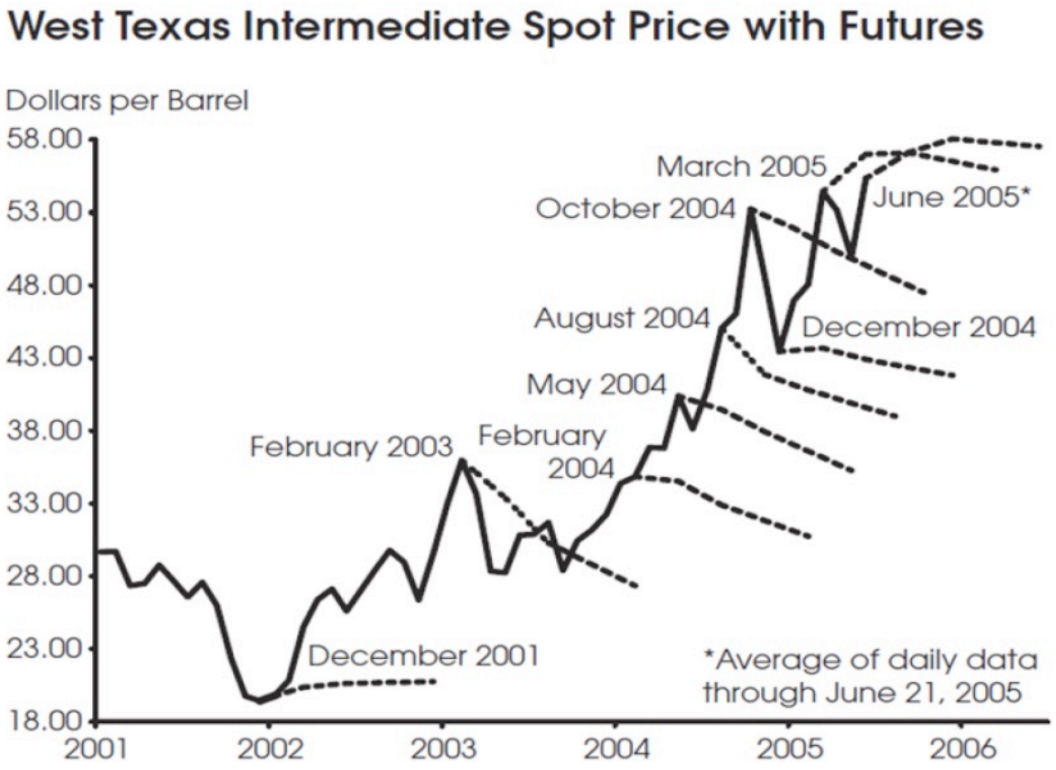
Forward price with proportional storage cost and convenience yield

$$F_{0,T} = S_0 \times e^{(r+u-y)T}$$

Describing the shape of the forward curve

Contango is a pattern of forward prices that increases with contract maturity

Backwardation is a pattern of forward prices over time that decreases with contract maturity



The solid line is the spot price at each date on the x– axis. The forward curve, starting at the selected date, is also plotted as a dashed line.

What about commodities that cannot be stored?

- May be no storage or very limited storage life
  - Electricity
  - Lettuce, strawberries, live concerts…

forward prices are not tied to current spot prices by no–arbitrage conditions

- For non-storable commodities, forward prices can have information about future spot prices because no-arbitrage conditions don't hold
  - Approach to pricing is to model stochastic future spot prices supply and demand
  - Also must infer discount rates
risk–adjusted

Key takeaways on forwards

- For stocks, bonds, currencies, metals, stored agricultural commodities, etc., there is no new information in forward prices over what can be learned from spot prices!

- Why?
  - Forward price *F* has no uncertainty associated with it, a forward transition involves a certain payment for delivery of a specific asset.
  - The forward price is tied down by no-arbitrage conditions that depend only on the underlying spot price, interest rates, and associated cash flows between 0 and T (dividends, coupons, storage costs, convenience yield)

- Implication: Don't try to predict future spot prices using forward prices!
  - Discussion question: Is the expected future price of a non-dividend paying stock higher or lower than its forward price?

expected return on a stock includes a risk premium that's compensation for market risk. And hence, the expected return on Tesla is higher than the risk– free rate. By contrast, the forward price is based on an expected return that's equal to the risk– free rate. Hence, the forward price of a stock tends to give a downward– biased estimate of the future price of the stock.

Key concepts for hedging and speculating

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Valuing a forward contract over time

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We have seen that the initial forward price is determined by no-arbitrage, and the initial value of the contract, f_0, is 0.

How does the value of a forward contract evolve over time?

This is determined by another no-arbitrage relationship:

- Suppose that $K = F_0$ is the original delivery price and F_t is the forward price for a contract that would be negotiated today for the same underlying and delivery date.
- By considering the difference between a contract with delivery price K and a contract with delivery price F_t we can deduce that the value of a long forward contract, $f_{t,T}$, at time t is

$$(F_t - K)e^{-r(T-t)}$$

- the value of a short forward contract is

negative

$$(K - F_t)e^{-r(T-t)}$$

A forward contract can be exited by taking an offsetting forward position. Alternatively, you can sell the contract to another trader for its current value.

Basis risk and cross hedging

Definition: **Basis** is the difference between the spot and forward price of a security or commodity.

No-arbitrage implies basis goes to zero on contract expiration date.

When a hedge is set up that will be closed out prior to contract expiration, this basis risk can be quite significant, and means that the hedge may not be very effective.

cross–hedging example: an electric utility might hedge production costs using contracts on West Texas crude oil, even though the oil that it uses in production comes from a different source

Definition: **Cross-hedging** involves using a contract type to hedge which differs from the security or commodity being hedged.

- Limited contract types => traders often have to cross-hedge

Why do a cross hedge? Certain forward contracts, such as those traded on futures exchanges, are more available and more liquid than others, making them less expensive to transact in. The disadvantage of cross hedging, as we saw,is that the basis risk tends to be greater, reducing the quality of the hedge.

Hedge ratios

<ins>Definition</ins>: The **hedge ratio** is the relative number of forward contracts to units of the asset being hedged that maximizes the effectiveness of the hedge.

Goal is to have $N_s \times E(dS) = N_F \times E(dF)$ so that same total dollar expected price change

$$\Rightarrow N_s / N_F = E(dF) / E(dS)$$

- N_F = number forward units
- N_S = number spot units
- $E(.)$ = expected value
- dF = change in value of forward contract
- dS = change in value of spot security or commodity

Note: If long in spot then short in forwards, and vice versa.