15.435x Sample Final Exam

Part 1: Shorter Questions

S1. (5 points) Which of the following can be estimated for an American option by constructing a single binomial tree (indicate all that are true):

(a) delta
(b) gamma
(c) vega
(d) theta
(e) rho

Answer: Delta, gamma, and theta can be determined from a single binomial tree. Vega is determined by making a small change to the volatility and recomputing the option price using a new tree. Rho is calculated by making a small change to the interest rate and recomputing the option price using a new tree.

S2. (5 points) What is a first-to-default credit default swap? Does its value increase or decrease as the default correlation between the companies in the basket increases? Briefly explain your answer.

Answer: In a first-to-default basket CDS there are a number of reference entities. When the first one defaults there is a payoff (calculated in the usual way for a CDS) and the basket CDS terminates.

The value of a first-to-default basket CDS decreases as the correlation between the reference entities in the basket increases. This is because the probability of a default is high when the correlation is zero and decreases as the correlation increases. In the limit when the correlation is one, there is in effect only one company and the probability of a default is quite low.

S3. (6 points) Here is a statement to the shareholders of Abacus Inc. explaining the value per share they will receive when their company is acquired three months from now. If the price per share of Xylon Industries on the acquisition date is:

(a) \$84.22 or greater, then you will receive .5462 shares of Xylon

(b) less than \$84.22 but greater than \$76.20, then you will receive the number of shares of Xylon having a total value of \$46.00

(c) \$76.20 or less, then you will receive .6037 shares of Xylon

Neither company pays dividends. How would you express the current value of a share of Abacus in terms of call options on Xylon?

Answer: One share of Abacus gives the exact same payoffs as the following portfolio:

-Long 0.6037 calls with K=0

-Short 0.6037 calls with K=76.2

-Long 0.5461 calls with K=84.2

S4. (5 points) Consider a one-year European put option on a 10-year bond. Assume that the current value of the bond is $125, the strike price on the put is $110, the one-year risk-free interest rate is 10% per annum, the bond's forward price volatility is 8% per annum, and the present value of the coupons to be paid during the life of the option is $10. According to Black's model, what is the value of the put option?

European futures options and European spot options are equivalent when futures contract matures at the same time as the option

Answer: In this case,

$F_0 = (125 - 10)e^{0.1\times1} = 127.09$, $K = 110$, $P(0,T) = e^{-0.1\times1}$, $\sigma_B = 0.08$, and $T = 1.0$.

$$d_1 = \frac{\ln(127.09/110) + (0.08^2/2)}{0.08} = 1.8456$$
$$d_2 = d_1 - 0.08 = 1.7656$$

From Black's model, the value of the put option is

$$110e^{-0.1\times1}N(-1.7656) - 127.09e^{-0.1\times1}N(-1.8456) = 0.12$$

or $0.12.

S5. (5 points) According to Merton's model for pricing default risk, which of the following will **decrease** the price charged upfront for a credit guarantee on a zero-coupon defaultable corporate bond, all else equal? (*Indicate all that apply, there may be more than one correct answer*)

(a) A higher risk-free interest rate

(b) Higher asset volatility

(c) A higher face value of the bond

(d) A longer bond maturity

Answer: (a), (d)

The higher risk-free rate decreases the present value of any eventual payoff. With a longer bond maturity, the positive average drift in the value of the assets reduces the probability that there will be insufficient funds to cover the liability.

S6: (5 points) Consider an MBS that splits all the income from a pool of mortgages between two tranches, an IO that receives all the interest payments, and a PO that receives all the principal payments. Imagine that there is an unanticipated upward parallel shift in the yield curve, e.g., rates at all maturities go up by 1%. Which of the following statements are generally true? (There may be more than one correct answer.)

(a) Prepayment risk becomes more severe
(b) The sum of the value of the IO and PO tranches goes down
(c) The value of the IO increases relative to the value of the PO, i.e., Price(IO)/Price(PO) goes up
(d) The duration of the PO increases

Answer: (b), (c)

For whole mortgages, as for most other fixed income securities, the value falls when interest rates rise because of discounting. Prepayment risk goes down when rates rise, which is relatively good for the IO because it means the income stream will be higher for longer. The decrease in prepayment risk is bad for the PO because they have to wait longer for the principal to be returned. Hence, the relative value of the IO will increase.

Part 2: Longer Questions

L1. New Century Lending has a balance sheet consisting of high-quality auto and consumer loans, financed with a combination of bank loans, bonds, and equity. Regulators require it to hold equity equal to at least 5% of its assets, which creates an incentive to hedge its exposure to interest rate risk. Its market value balance sheet ($ millions) stands at:

Assets		Liabilities	
Short-term loans	$3,758	bank loans	$7,801
(avg. modified duration = 1 yr)		(avg. modified duration = .4 yrs)	
Fixed-rate loans	$6,242	Long-term debt	$1,699
(avg. modified duration = 6.0 yrs)		(avg. modified duration = 7.8 yrs)	
		Owners' equity	$500

(a) (5 points) Calculate the dollar duration of New Century's assets **and** of its debt liabilities, treating each side of the balance sheet as a portfolio.

(b) (8 points) Using the fact that the value of equity is the difference between the values of assets and liabilities, based on your above calculations, write down an approximation formula for how much the dollar value of equity will change for a parallel shift in the yield curve "Δy" (i.e., the dollar duration of equity). Use that formula to estimate approximately how much the dollar value of equity will change if the yield curve makes a parallel upward shift of 1%.

(c) (3 points) If New Century wanted to use an interest rate swap to reduce the sensitivity of its equity to changes in interest rates by delta hedging, would it be the fixed or floating rate payor?

(d) (5 points) Imagine that New Century wants to set up a delta hedge that minimizes the sensitivity of its equity to interest rate changes, using an interest rate swap. What would be the hedge ratio for that swap position?

Answers to L1:

(a) Dm(Assets) = 37.58%*1+62.42%*6 = 4.12, Dm(Liabilities) = (7801/9500)*0.4 + 1699/9500)*7.8 = 1.72. Thus, the dollar duration of assets = 4.12*10000 = 41,250 and the dollar duration of liabilities = 1.72*9500 = 16,340.

(b) Dollar duration of equity = 41,250 – 13,340 = 2,480.

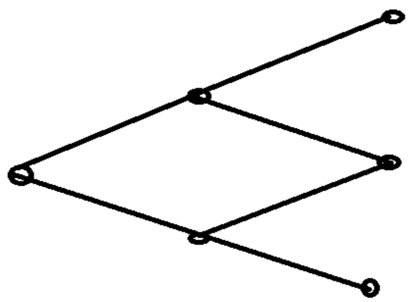
The dollar value of equity would change in the following way: -2,480*Δy. So, for a change of -1%, the change in the value of equity would be: -2,480*(-1%) = 248.

(c) Fixed payor. You want to benefit from yields going up, so you want to receive floating and pay fixed.

(d) They would equate the hedge ratio to the dollar duration of the equity, so the hedge ratio is 2,480.

L2. The current price of a non-dividend paying stock is $80. The risk-free rate is 3% per annum (continuous compounding), and the volatility is 16% per annum.

(a) (6 points) Construct a two-step stock price tree for a one-year horizon, using the approximation for a log-normal stock price process. Each step represents 6 months. Fill in the stock price for the nodes on the tree in this diagram:



Answer: We solve for u, d, and the risk neutral probability q* by:

$$u = e^{\sigma\sqrt{h}} = \exp\left(0.16 * \sqrt{6/12}\right) = 1.1198$$

$$d = \frac{1}{u} = 0.893$$

$$q^* = (e^{rh} - d)/(u - d) = (\exp(0.03 * 6/12) - 0.893)/(1.1197 - 0.893) = 0.5384$$

The t = 0 node is $80. The nodes at t = 1 are:

S_{1u}= $80 * 1.1197 = $89.5828

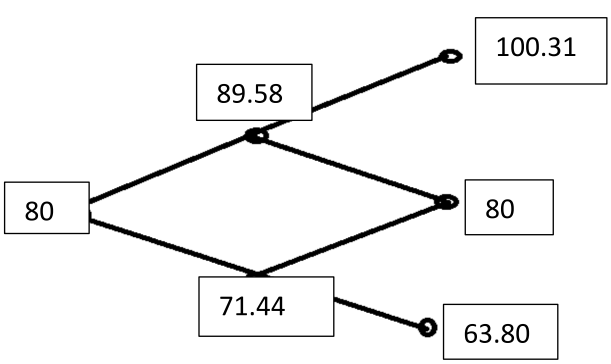
S_{1d} = $80 * 0.893= $71.442

The nodes at t = 2 are:

S_{2uu} = $80 * 1.1197^2 = $100.3136

S_{2ud} = S_{2du} = $80 * 1.1197 * 0.893 = $80

S_{2dd} = $80 * 0.893^2 = $63.80



(b) (12 points) Use the tree to estimate the value of (i) a one-year European put option with a strike price of \$77; (2) a one-year European put option with a strike price of \$83; and (iii) a long bear spread constructed with those two options.

Answer: By risk neutral pricing, and with h= 0.5:
(i) P(K=77) = e^{-r*2h} $[\,(77-63.8)(1-q^*)^2] = \$2.7295$
(ii) P(K=83) = e^{-r*2h} $[\,2*(83-80)*q^*(1-q^*)+\ (83-63.8)(1-q^*)^2]$ = \$5.4173
(iii) A bear spread is long a higher priced put and short a lower priced put. Hence with these two options:

Bear Spread = P(K=83) – P(K=77) = \$2.6878

(c) (5 points) Imagine that the company is expected to pay a dividend of \$0.80 in 6 months. Holding the volatility, interest rate, strike prices, and time horizon constant, what would this do to your estimate of the value of the bear spread (circle one, do not explain):

(i) increase value
(ii) decrease value
(iii) no effect on value
(iv) indeterminate

Answer: (i) Increase value. Dividends lower the stock price by the amount of the dividend, which increases the value of both the long put and the short put. Redoing the calculations, the effect is to increase the value of the long put by more than it increases the value of the short put, increasing the value of the spread.

(d) (10 points) Using the Black-Scholes-Merton model, and with everything as in part (a) (i.e., no dividends), what is the theoretical value of the one-year European put option with a strike price of \$83? What is the delta of the put option?

Answer:

$$\mathrm{d1}=\frac{\ln\left(\frac{\mathrm{S}}{\mathrm{K}}\right)+\left(r+\frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}=\frac{ln\left(\frac{80}{83}\right)+(3\%+0.5\times 0.16^2)\times 1}{16\%\times 1}$$

$$\mathrm{d2}=\mathrm{d1}-\sigma\sqrt{T-t}=d1-0.16$$

We have d1 = 0.0374, d2 = -0.1226. Plugging these into the BSM formula, we have:

Put price = \$5.3966

Delta = -N(-d1) = -N(-0.0374) = -0.485

(e) (5 points) There are also forward contracts trading on the stock. What is the forward price of the stock for delivery in six months?

Answer:

$$\mathrm{F}\ =\ \mathrm{S}_0\ \exp(\mathrm{rT})=80\exp\left(0.03*\frac{6}{12}\right)=\$81.209$$