Week 9 – Credit risk

MIT Sloan School of Management

Finance at MIT

Where ingenuity drives results

Outline

- Statistical approach to default risk
  - Decomposing the credit spread
- Structural approach to default risk
  - Simple binomial example
  - Merton model and extensions
- Credit derivatives

Corporate debt

- Divided into two broad credit quality categories by rating agencies
  1. Investment grade (IG)
  2. Non-investment grade, high yield (HY), or “junk” speculative
     - Much higher historical default rates
     - Original issue vs. “fallen angels” originally investment grade and were downgraded as issuers ran into trouble
     - Strong historical return performance
- Default risk manifests itself as:
  - Downgrade risk While default events are rare, rating changes are much more frequent.
  - Event risk
    - E.g., legal changes, surprise actions by managers
  - Liquidity risk
    - Riskier bonds tend to be less liquidity

Default and recovery rates:
key statistics for assessing credit risk

- Default rate
- Probability of a default event
  - Usually stated as an annual rate
  - What is a default event depends on who is defining it; rating agencies provide definitions
- Recovery rate
  - Amount expected to be recovered as a fraction of what is owed

    usually principal plus any accrued interest
  - Sometimes expressed in present value terms
    - Can be based on price of a defaulted bond relative to principal

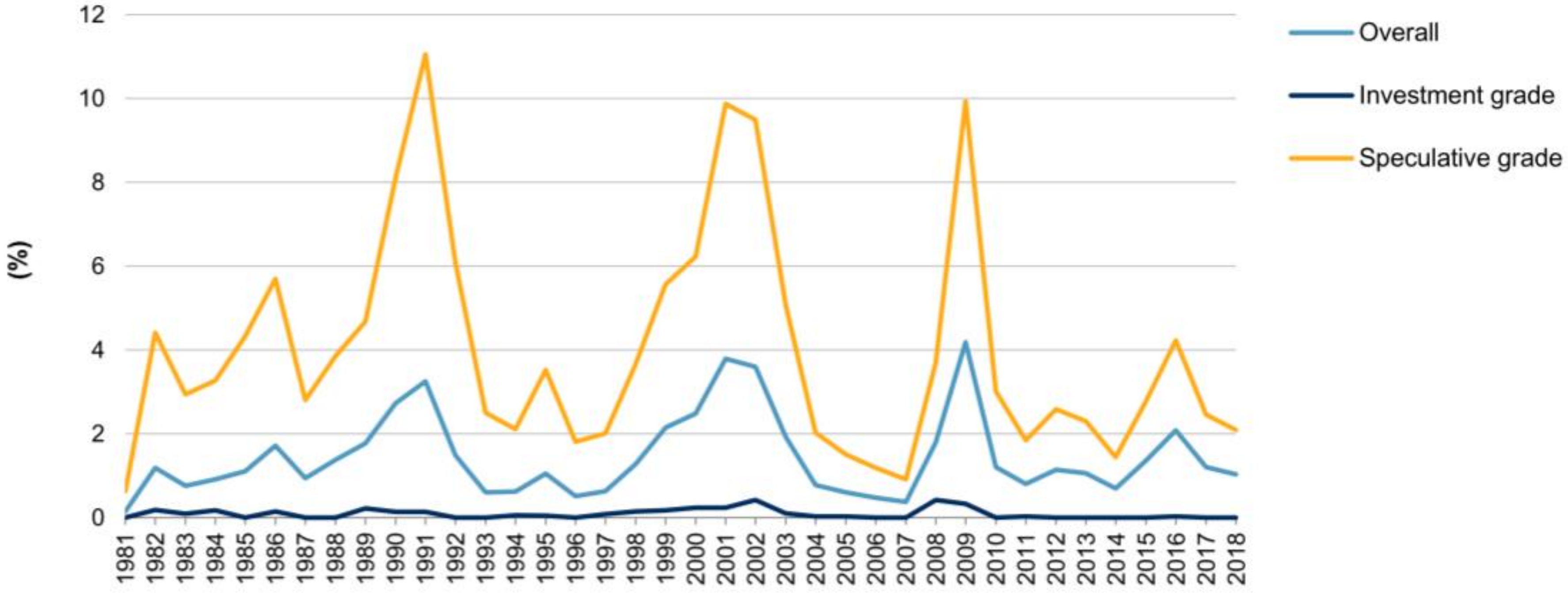
    One approach is to estimate the present value of the cash that will be eventually recovered, such as when the remaining assets of the firm are liquidated. Alternatively, sometimes the market price of the bond right after default is announced is used as a proxy for the present value of recoveries.
  - recovery rate = (1 – loss rate)
  - Loss rate is also called "loss given default"
- Term structure of default and recovery
  - Expected rates may vary over the life of a security

Default rates vary enormously:
over time, by credit rating, and with the business cycle

the positive covariance of default rates with the economic downturns imparts market risk to corporate debt, hence risky corporate debt has a positive beta and the cost of that market risk is reflected in bond prices

Chart 1

Global Default Rates: Investment Grade Versus Speculative Grade

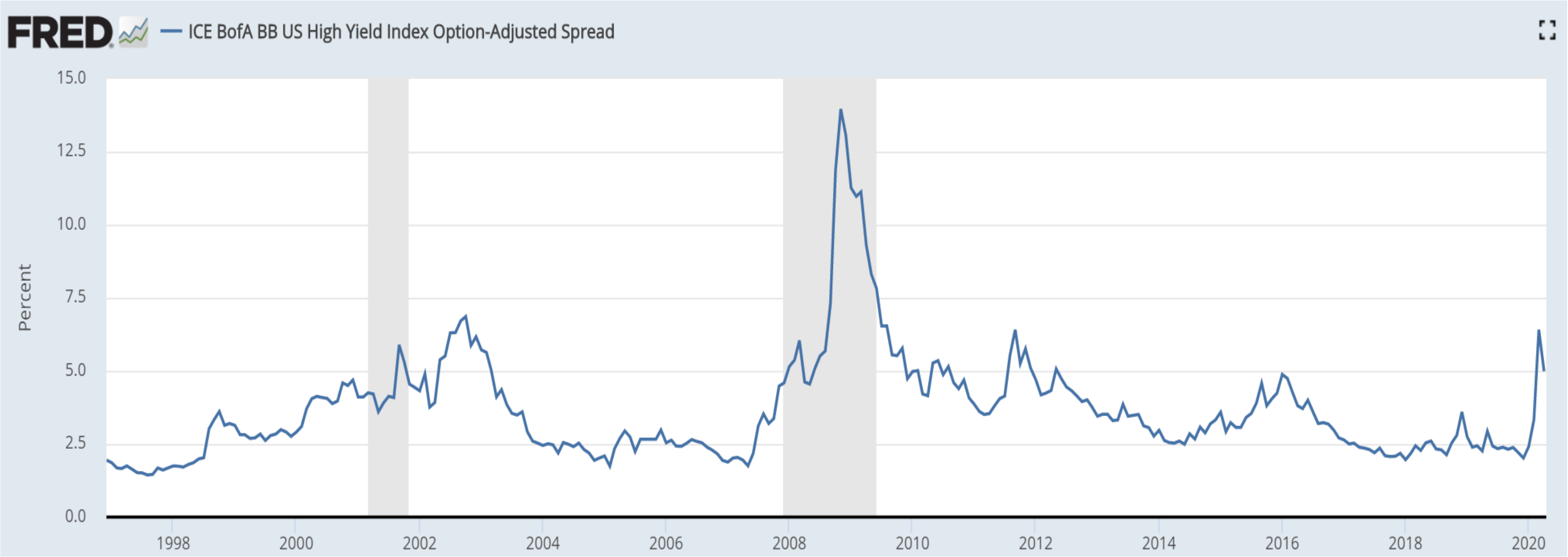


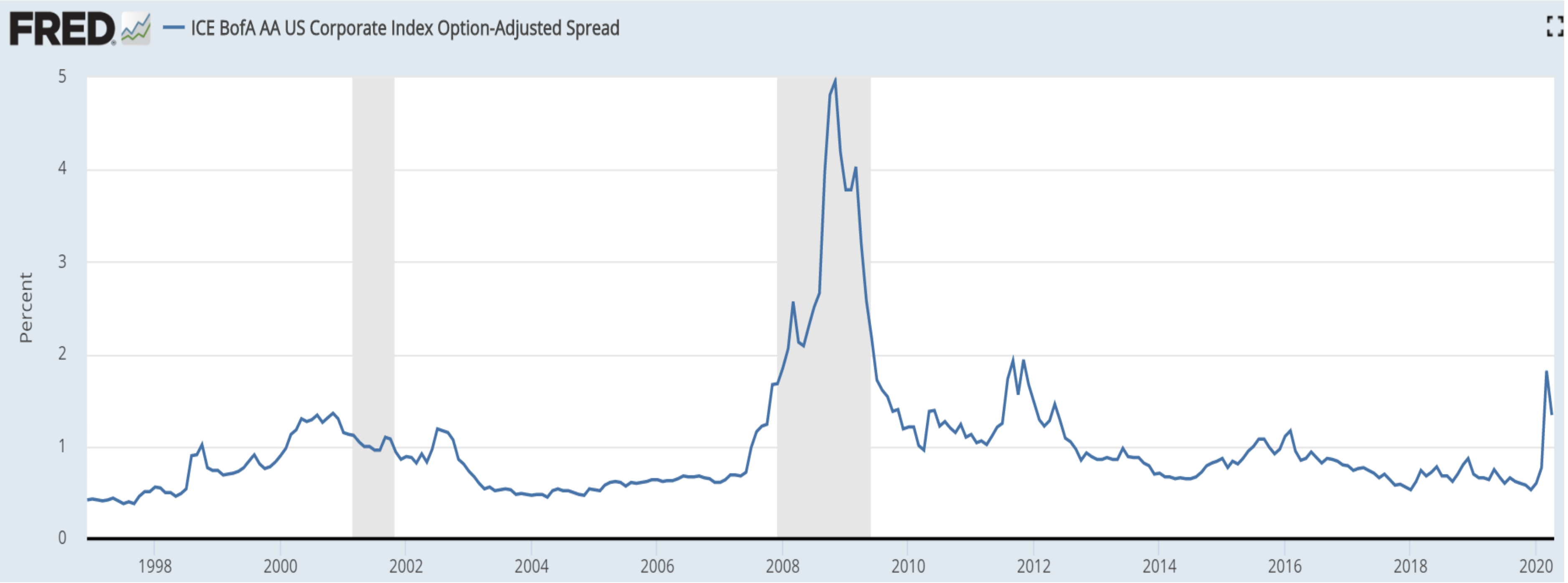
Sources: S&P Global Fixed Income Research and S&P Global Market Intelligence's CreditPro®.

Credit spreads on risky debt

- The "credit spread" (or yield spread) is the difference in the yield to maturity on a risky bond and on a Treasury bond of similar maturity
  - If it is an "options-adjusted spread" it assumes no other embedded options (e.g., no prepayment or call option)

- For a given credit rating, observed yield spreads vary with maturity
  - The yield spread usually increases with maturity over moderate horizons greater risk of adverse event
  - Sometimes credit spreads are decreasing at very long maturities only high quality borrowers able to issue very long maturity debt
  - These patterns partially reflect compositional differences in firms borrowing at different maturities





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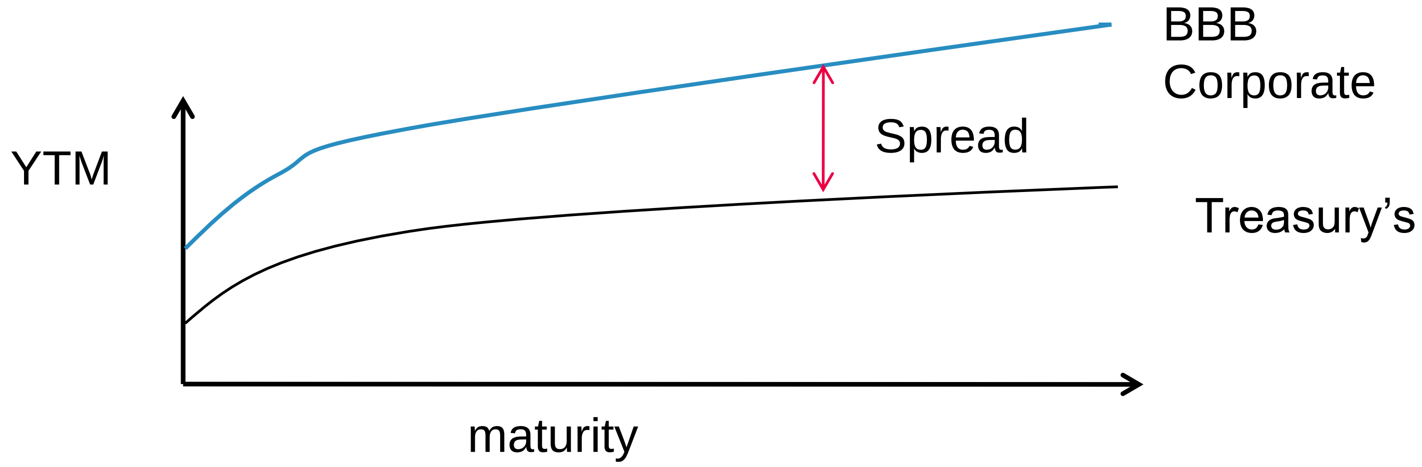
Decomposition of Credit Spreads

- What determines the YTM spread between Treasury's and defaultable securities of the same maturity?

- Conceptually the spread has several components:
  - Expected losses
  - Premium for market risk (β)
  - "Liquidity premium" The residual after accounting for all identifiable effects is described as liquidity premium
  - Non credit features (e.g. tax treatment, embedded options)

Prob*(Default) x E*(Loss given default)

("*" denotes a risk-neutral variable)



A simple valuation model for risky bonds

- This simple model illustrates that **expected cash flows are not the promised cash flows**, and the **yield to maturity is not the expected return**.

- In the model, the difference between the expected return, r, and the YTM, y, is influenced by the default rate, the recovery rate, and the bond maturity.

- The model is implemented in the spreadsheet default.xls (on webpage)

A simple valuation model for risky bonds

- Consider a risky **T**-period coupon bond, with coupon rate, **c**, and face value **F=1**.
- Assumptions
- A constant default rate each period, **d**
  - The probability of no default from time 0 to time t is $\mathbf{(1\text{-}d)^{t-1}}$
- A constant recovery rate, **g**
- An expected return **r** equal to the risk-free rate plus a premium for market risk,

  e.g., $\mathbf{r = r_f + \beta(E(r_m) - r_f)}$

- Investors discount **expected** cash flows at **r** to determine the price of the bond

Basis for numerator in bond Sharpe ratio

A simple valuation model for risky bonds

- Default rate d
- Recovery rate g
- Expected return r
- Coupon rate c
- Maturity T
- face value 1;

$$P = \sum_{i=1}^{T} \left((1-d)^{i-1} \frac{(dg(1+c)+(1-d)c)}{(1+r)^i} \right) + \frac{(1-d)^T}{(1+r)^T}$$

if the bond is already in default, then the expected payment at that time is zero

P = price per $1 face value

$(1-d)^{i-1}$ is the probability that the bond is still outstanding at time i

$\frac{(dg(1+c)+(1-d)c)}{(1+r)^i}$ is the pv of the expected coupon plus recovery at time i

$\frac{(1-d)^T}{(1+r)^T}$ is the pv of the expected $1 principal payment at time T

you'll see that for bonds with high default or low recovery rates, the yield–to– maturity will be significantly higher than the expected return.That's because part of the yield spread is compensation for expected losses.

The yield to maturity takes promised payments as certain:

$$P = \sum_{i=1}^{T} \left(\frac{c}{(1+y)^i} \right) + \frac{1}{(1+y)^T}$$

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Structural models of credit risk

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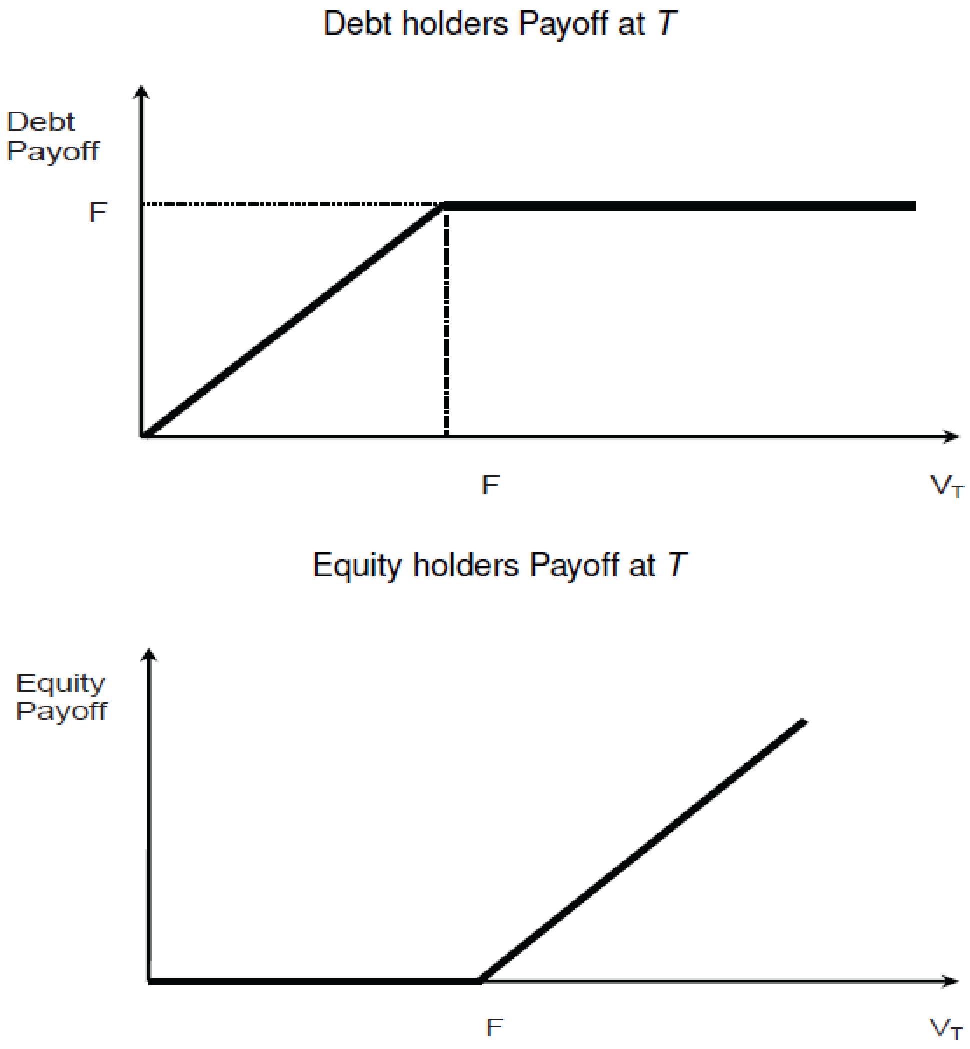
Risky corporate debt and equity payoff diagrams

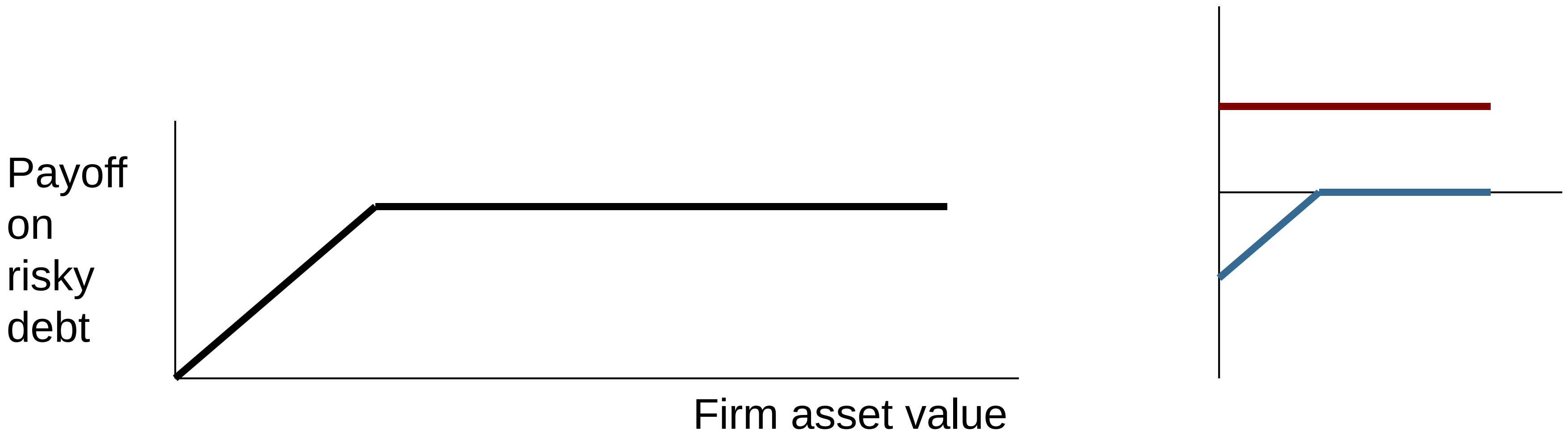


Notes:

1. Sum of debt and equity is asset value
2. Underlying in these diagrams is firm asset value (not equity)

Decomposing payoff on risky debt

- A risky zero coupon corporate bond is equivalent to a portfolio that includes:
  - a long position in a risk-free zero coupon bond
  - a short position in a put option on the assets of the firm, with a strike price equal to the face value of the bond



- A loan guarantee is equivalent to a put option **on firm assets.** The guarantor writes the put option in exchange for a fee (premium).

Valuing loan guarantees as put options

Example

Binomial guarantee pricing:

Today XYZ Co. has a market value of $100 million, and next year the company’s assets will take on one of two values: $140 million or $70 million:

$140
$100
$70

XYZ also has guaranteed debt with a face value of $90 million (covering principal and interest), coming due next year.

What is the value of the guarantee (from perspective of guarantor)**?**

The payments of the guarantor will be $0 if the assets are worth $140 or -$20 if the assets are worth $70.

Then the payoffs for the guarantor look like:

$0
P
-$20

This is a written put option on the company’s assets with a strike price of $90.

Valuing loan guarantees as put options

Assume the price of a 1-year risk-free bond is \$95 per \$100 face. Its value is represented by:

\$100

\$95

\$100

The value of the option can be replicated using the information on asset value and the risk-free bond:

We require that the payments match in the good and bad state of the world:

$X100 + Y140 = 0$

$X100 + Y70 = -20$

These two linear equations in two unknowns can be solved for X and Y to yield:

We replicate the payoff of the guarantee by buying a fraction X of the risk– free bond and a fraction Y of the risky firm assets.

$X = -.4$
$Y = .2857$

The price of this portfolio, based on the \$95 price of the bond and the \$100 current asset value, is -.4(\$95) + .2857(\$100) **= -\$9.43**

(The guarantor has a highly levered position in the assets of the firm!)

long position

The Merton Model

- Today is $t = 0$ and consider a firm with current assets $V_0 = E_0 + D_0$
- Assume the firm's (market value) return on assets is log-normally distributed:

not its stock price

$$V_T = V_0 \times e^{\left(\mu - \frac{1}{2}\sigma^2\right)T + \sigma\sqrt{T}\epsilon}$$

a draw from a standard normal density function, epsilon

- The assets for financed with equity and debt. The debt is a zero coupon bond with face value F and maturity T

- There are two possible outcomes for debt holders at maturity T:
  1. If $V_T > F \implies$ the firm can sell some of its assets and pay the debt holders
     - Debt holders get F; Equity holders get the residual $V_T - F$
  2. If $V_T < F \implies$ the firm will be unable to pay its debt, and therefore defaults
     - The debt holders take possession of the assets of the firm $\implies$ their payoff is V_T; Equity holders get nothing

The Merton Model: Valuing equity as a call option

- The payoff to equity holders is then the one of a call option

$$\max\left(V_T - F, 0\right)$$

- If we denote E_0 the value of equity today,

$$E_0 = \text{Call}\left(V_0, F, r, T, \sigma\right)$$

where

$$\text{Call}\left(V_0, F, r, T, \sigma\right) = V_0\, N(d_1) - F\, e^{-rT}\, N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{V_0}{F}\right) + (r + \sigma^2/2)\, T}{\sigma\sqrt{T}}; \quad d_2 = d_1 - \sigma\sqrt{T}$$

Notice the ***volatility in this formula is for assets****, not equity*

The Merton model: Volatility of levered equity

the volatility of a firm's equity can be estimated based on historical stock price data or from implied volatilities of options that are traded on the firm.

- What is the volatility of levered equity?
- We know the relation between the volatility of a call option and its underlying volatility is given by its vega. Applying the same formula implies:

not easy to directly infer volatility of a firm's assets because assets generally aren't traded

$$\text{Volatility of Equity Returns} = \underbrace{\sigma_E}_{\text{equity volatility}} = \left(\frac{VN(d_1)}{\boxed{VN(d_1) - Ke^{-rT}N(d_2)}} \right) \times \underbrace{\sigma}_{\text{asset volatility}}$$

BSM value of a call option on assets is the value of equity

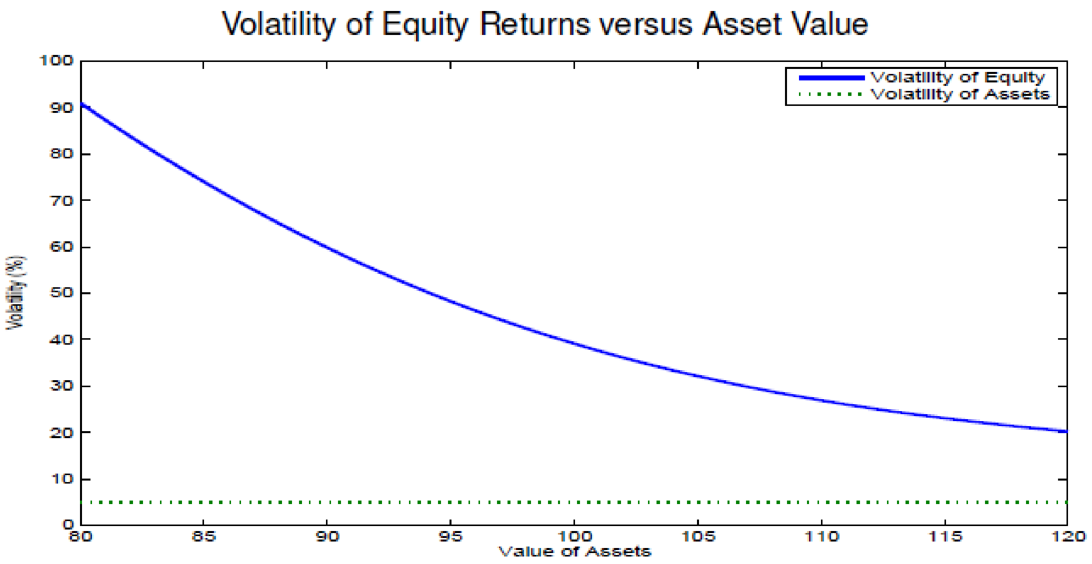
- The term in parenthesis can be much larger than 1, implying that equity return volatility can be many times higher than the volatility of the underlying assets

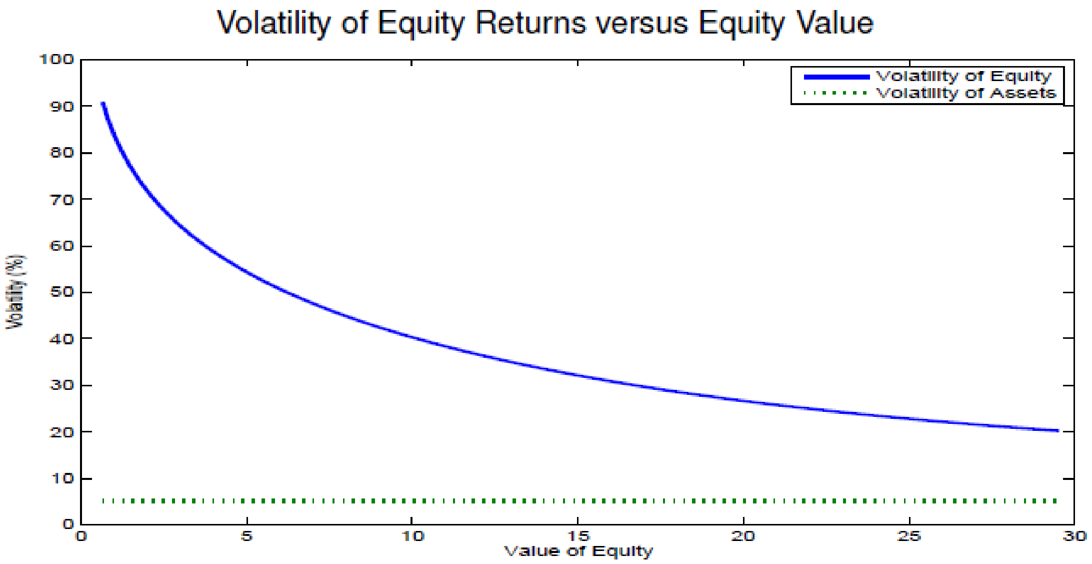
- This can also be written as $\sigma_e = \frac{V \times N(d_1)}{E} \sigma$

could estimate asset volatility if we had historical data on both the market value of a firm's equity and the market value of its debt, but it's often difficult to find data on the market price of a firm's debt because either it trades infrequently or it's privately placed and that information isn't made public.

- As the value of equity relative to assets falls, its volatility increases
- This is known as the “leverage effect” on equity volatility, first noted by Fischer Black
- *Important implication*: As a firm becomes distressed, analyzing its expected returns using the CAPM becomes highly problematic because its beta not well-approximated by a constant

The Merton model: Volatility of levered equity





under the assumptions of the model, asset volatility is constant over time, and therefore independent of the value of assets or of equity

The Merton Model: Inferring asset value and volatility

assuming we can observe or estimate the market value of equity and its volatility.

- We have two non-linear equations in two unobservable variables, V_0 and σ

initial market value of assets and the volatility of assets

$$E = \text{Call}(V_0, F, r, T, \sigma) = V_0\, N(d_1) - F\, e^{-rT}\, N(d_2)$$

risk–free

$$d_1 = \frac{\ln\left(\frac{V_0}{F}\right) + (r + \sigma^2/2)\, T}{\sigma\sqrt{T}}; \quad d_2 = d_1 - \sigma\sqrt{T}$$

time to maturity

total bond face value

And

volatility of equity $$\sigma_e = \frac{V_0 \times N(d_1)}{E}\sigma$$

equity

- Solve simultaneously given other parameters to find V_0 and σ

– define another cell =MAX(ABS(A), ABS(B)) and apply solver on the new cell. This is preferable than the first approach, but since you are adding more nonlinearities into the objective function, you may need to decrease the Solver tolerance and increase the maximum number of iterations (Solver > Options > ...)

– the most preferable approach, which for me yielded the official answers, is setting one of the objectives as a constraint to the Solver, and then applying the Solver to the other objective.

The Merton model: Value of debt

What is the value of defaultable debt in the model?

- The payoff to debt holders is

$$\min(V_T, F) = V_T - \max(V_T - F, 0)$$

- The value today of this payoff is then

$$D_0 = V_0 - E_0 = \underbrace{V_0}_{\text{asset}} - \underbrace{Call(V_0, F, r, T, \sigma)}_{\text{equity}}$$

- Note that this expression also comes immediately from the balance sheet identity

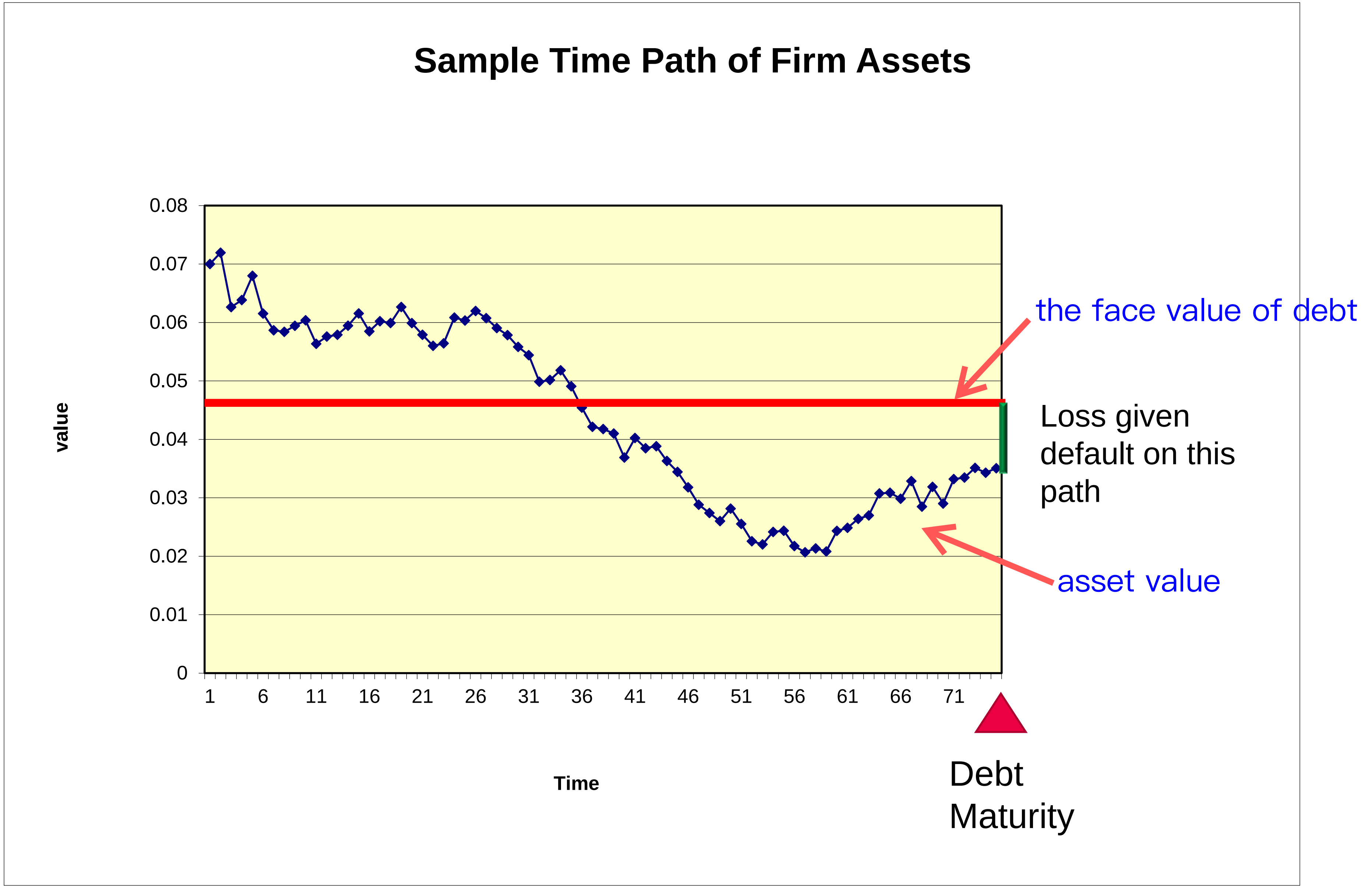
$$\text{Assets of a Firm} = \text{Debt} + \text{Equity}$$

- Exploiting put call parity, we can express the value of debt alternatively, and more intuitively, as

$$D_0 = Fe^{-r\times T} - Put(V_0, F, r, T, \sigma)$$

- The value of debt is equal to the risk free debt minus a put option. The put option represents the (risk adjusted) expected losses due to default (when assets in the firm are insufficient to pay the debt at T)

Note: The put option also represents the value of a guarantee on that debt.



Sample Time Path of Firm Assets
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
value
1
6
11
16
21
26
31
36
41
46
51
56
61
66
71
Time
the face value of debt
Loss given default on this path
asset value
Debt Maturity

The Merton model: Credit spreads

- We can then use the Merton's model to compute a corporate bond credit spread
- From the definition of yield to maturity y for a corporate bond, we have:

$$D_0 = e^{-y \times T} \times F \quad \Longrightarrow \quad Fe^{-r \times T} - Put(V_0, F, r, T, \sigma) = e^{-y \times T} F$$

which implies

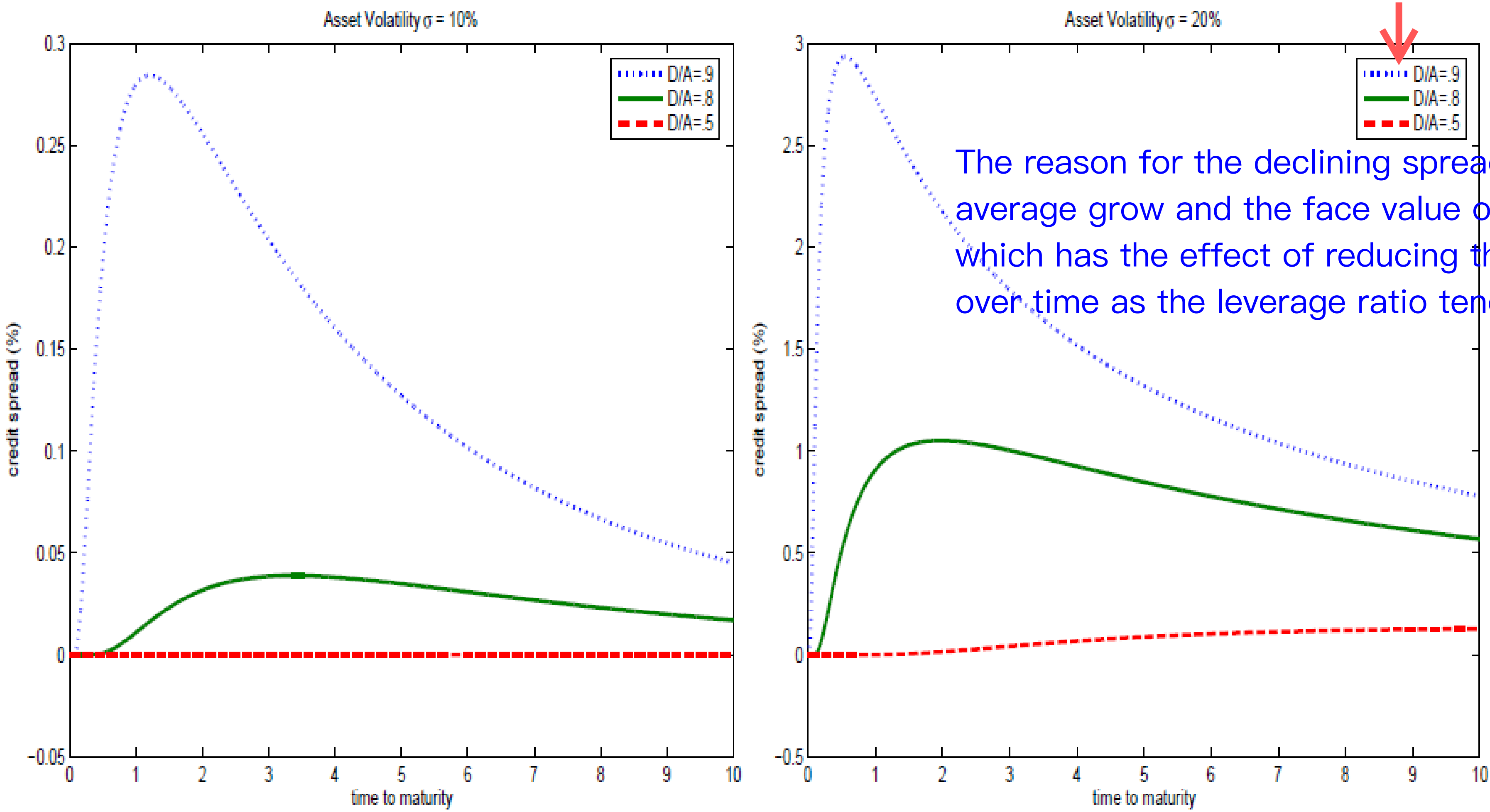
$$e^{-r \times T} - Put\left(\frac{V_0}{F}, 1, r, T, \sigma\right) = e^{-y \times T}$$

$$1 - e^{r \times T} \times Put\left(\frac{V_0}{F}, 1, r, T, \sigma\right) = e^{-(y-r) \times T}$$

$$\text{Credit Spread} = y - r = -\frac{1}{T} log\left[1 - e^{r \times T} Put\left(\frac{V_0}{F}, 1, r, T, \sigma\right)\right]$$

The Merton model: Credit spread over time

leverage ratio



The reason for the declining spreads is that assets on average grow and the face value of the debt is fixed, which has the effect of reducing the likelihood of default over time as the leverage ratio tends to shrink.

reason: empirical credit spread reflects more than just the direct cost of default losses.

- Issues: (A) They are small; (B) They converge to zero at $T \to 0$

credit spread is close to 0 when the bond is nearing maturity. reason: the price of the asset is assumed to follow a continuous path,and there's essentially no chance over a short horizon of a price move that is adverse enough for the possibility of loss to become significant.

The Merton model: Relative pricing of junior and senior debt

- The Merton model can be used to find the price of bonds with different priorities
- For example, suppose that a firm issues two bonds: one senior and one junior (also called subordinated), with face value F_S and F_J
- At maturity we have the following “waterfall” of payoffs

the face value of the senior debt **Payoffs** the face value of the junior debt

	$0 < V_T < F_S$	$F_S < V_T < F_S + F_J$	$F_S + F_J < V_T$
Senior	V_T	F_S	F_S
Junior	0	$V_T - F_S$	F_J
Equity	0	0	$V_T - (F_S + F_J)$

Merton model: Relative pricing of junior and senior debt

- That is, senior and junior debt and equity must satisfy

at maturity V = V_T

$$
\begin{aligned}
\text{Payoff of Senior Debt} &= V - \max(V - F_S, 0) \\
\text{Payoff of Junior Debt} &= \max(V - F_S, 0) - \max(V - (F_S + F_J), 0) \\
\text{Payoff of Equity} &= \max(V - (F_S + F_J), 0)
\end{aligned}
$$

risk–free rate

- We have then have

current V = V_0 sigma of V

$$
\begin{aligned}
D_{S,0} &= V - BSC(V, F_S, r, T, \sigma) \quad \text{Black– Scholes Call} \\
D_{J,0} &= BSC(V, F_S, r, T, \sigma) - BSC(V, F_S + F_J, r, T, \sigma) \\
E_0 &= BSC(V, F_S + F_J, r, T, \sigma)
\end{aligned}
$$

The payoffs on more complicated structures can also be valued using Monte Carlo simulation of the risk-neutral asset process

The Merton model: Extensions

Merton's analytical formulas rest on strong assumptions that are clearly violated in practice

- Many extensions of this basic model exist, including:
- Early bankruptcy
  - American put option: there is a lower bound V_b to assets so that if $V(t) < V_b$ the firm is bankrupt
- Stationary leverage

stationary leverage incorporates the idea that debt tends to be rolled over rather than retired and that as asset values change, leverage may be gradually adjusted towards some target leverage ratio.

  - Merton model indicates often counterfactual decline in leverage over time
- Unobservable firm value

incorporates the possibility of surprise bankruptcies

  - Investors can only rely on noisy accounting information to estimate V(t): the default barrier could be closer than you think
  - Can be modeled by incorporating jumps in asset process
- Stochastic interest rates
  - interest rates follow processes like the ones we saw last week

Example: KMV model and distance to default

- A generalization of the Merton model is the KMV model, used commercially by Moody’s
- Inputs are stock prices, **book value of liabilities**, stock volatility, interest rates.
- To get from book liabilities to **market liabilities** and therefore initial market assets, solve two simultaneous non-linear equations:

Equity value = F[**asset value, asset vol**, cap str, r]

Equity volatility = F[**asset value, asset vol**, cap str, r]

The main feature distinguishing it from the classic Merton model is that rather than using the face value of debt at maturity as a strike price, there's a default barrier (default point) that's based on a firm's observed capital structure.

  - Unknowns are asset value and asset vol

- “**Default point**” defined by a rule for assets relative to liabilities
  - An abstraction, proxy for the complicated question of when a firm really is likely to default
  - E.g., assets fall to < 70% of short-term + ½ long-term book liabilities
- These quantities are used as inputs to find **“distance to default**.”

Example: KMV model and distance to default

- The likelihood and severity of default depends on asset volatility and leverage
- The "**Distance to Default**" measure is defined by:

$$\text{DTD} = \frac{[(\text{mkt value assets}) - (\text{default point})]}{[(\text{mkt value assets})(\text{asset volatility})]}$$

- Interpretation: the number of standard deviations the firm is away from default.
- KMV has a proprietary algorithm to map DTD into the "**Expected Frequency of Default**" (EDF)
- Simple implementations of the model tend to under-estimate default probabilities over short horizons because of the assumption that assets are following a continuous log– normal process
- Can address by modeling shocks as having a **jump component**
  - Jumps often modeled as Poisson process
  - Also can address with a stochastic default barrier

Valuing loan guarantees on a binomial tree

- When the default barrier is more complicated and default can occur at any time, a binomial pricing approach is a natural choice.
- For example, consider the case of America West Airlines (AWA), that received a loan guarantee after 9/11 from the U.S. government.
  - In 2002, AWA was the eighth largest passenger airline in the U.S. Softening economic conditions had already severely reduced airline revenues. Following 9/11, Moody's downgraded AWA from B1 in April 2001 to Ca on November 21, 2001.
  - After the terrorist attacks, Congress enacted the Air Transportation Safety and System Stabilization Act, allowing airlines to apply for credit guarantees.
  - AWA received final approval from the Air Transportation Stabilization Board (ATSB) for a **government loan guarantee of $380 million in January 2002**.
  - It paid fees and gave the government warrants in compensation.
- What was that assistance worth on net?

Valuing loan guarantees as put options

- This example is based on analysis in “Estimating the Value of Subsidies for Federal Loans and Loan Guarantees,“ A Congressional Budget Office Study available on its website
- Key inputs:
  - Expected return on equity (based on estimated equity beta)
  - Volatility equity return (estimated from historical data)
  - risk-free rate
  - default/prepayment rules
  - loan maturity
  - Loan coupon rate
  - Rest of firm capital structure

Valuing Loan Guarantees as Put Options

- Stage 1 estimation: use equity data to find asset stats
  - volatility of firm asset value
  - current value of firm assets

Estimated first stage using Merton model

  - average return (physical) on firm assets
    - Derived from implied asset beta; used for risk assessment, e.g., VaR

$$dA_t / A_t = \boxed{(r_A - \delta)} dt + \sigma_A dZ_t$$

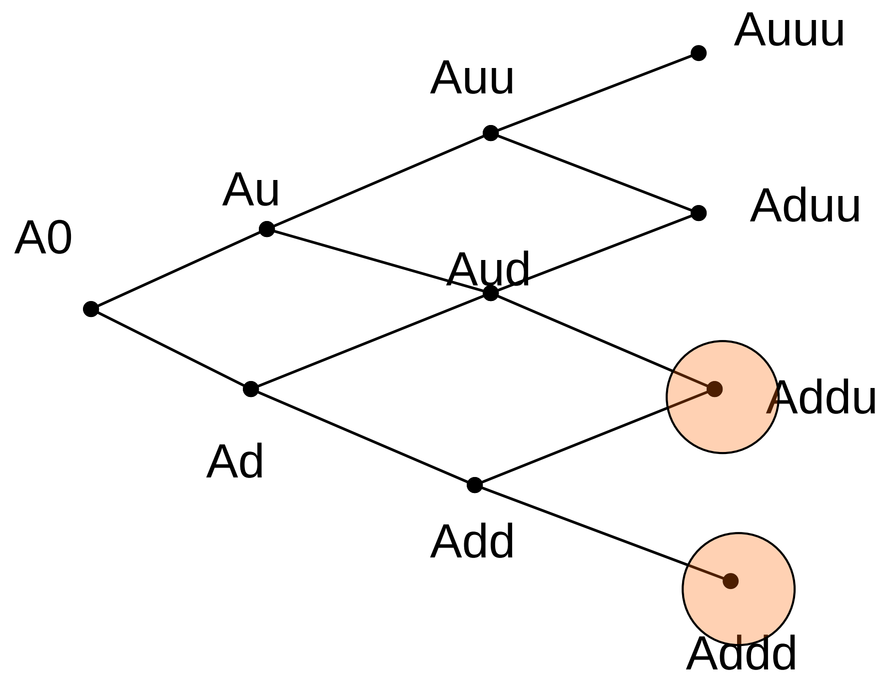
risk– adjusted return on assets minus any payouts.

  - risk-neutral return on firm assets
    - Used for pricing guarantees

$$dA_t / A_t = \boxed{(r_f - \delta)} dt + \sigma_A dZ_t$$

risk– free rate minus an assumed payout rate

Pricing



$$A_u = A_0(1 + (r_f - \delta)\Delta t + \sigma_A \sqrt{\Delta t})$$

$$A_d = A_0(1 + (r_f - \delta)\Delta t - \sigma_A \sqrt{\Delta t})$$

Denotes nodes where a loss occurs according to trigger rule.

Loss(t) = F – A(t)

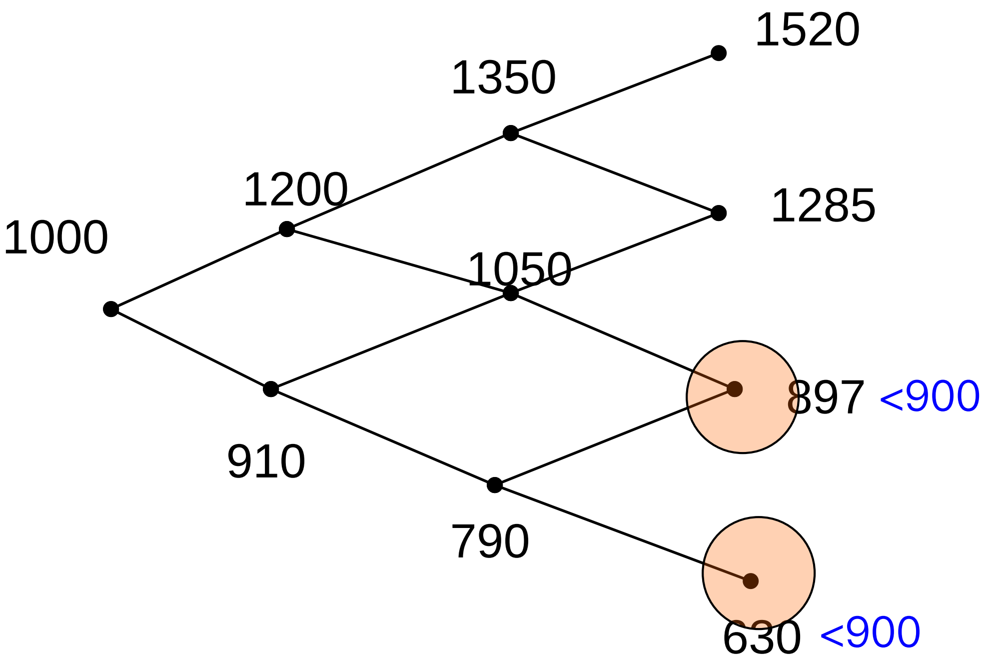
where F is payoff value of guaranteed debt

Losses are weighted by probability of occurrence and discounted to the present.

Probabilities are "risk-neutral", so discounting is at risk-free rate.

Pricing

e.g., 0 coupon bond F = 900, T=3; and r_f=.05, p*(up) =.5



loan guarantee had payouts on the lower branches of the tree

Denotes nodes where a loss occurs.

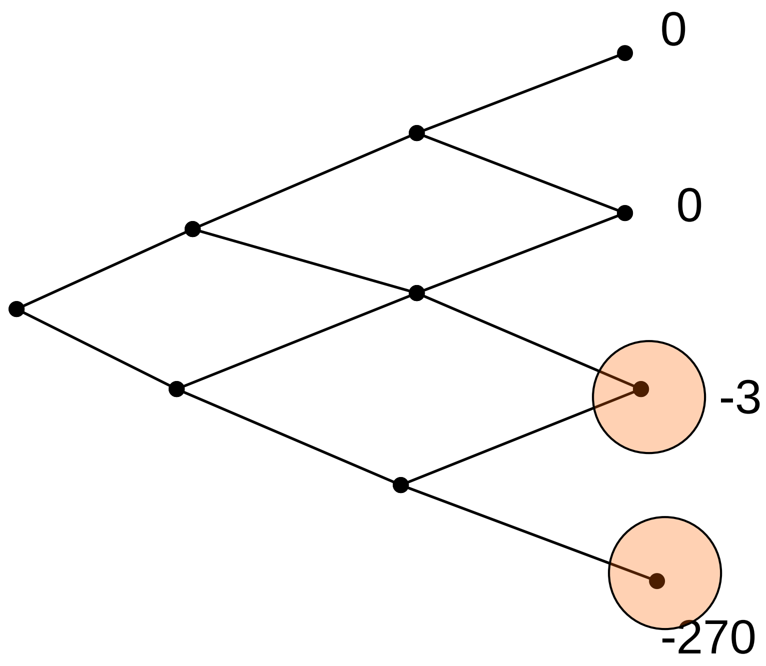
Loss(t) = F – A(t)

Losses are weighted by probability of occurrence and discounted to the present.

Probabilities p* are “risk-neutral”, so discounting is at risk-free rate.

Pricing
0 coupon bond F = 900, T=3; and r_f=.05, p(up) =.5

payoff by the gov on the guarantee



t=0 t=1 t=2

30.12634	0.680272	0
	62.58503	1.428571
		130

Losses are weighted by probability of occurrence and discounted to the present.

Probabilities are “risk-neutral”, so discounting is at risk-free rate.

PV(losses) = pr(3d)(-270)/$(1.05)^3$ + pr(2d1u)(-3)/$(1.05)^3$ =

$-30.126

Risky bond value = 900/$(1.05)^3$ – 30.13

= $777.46 - $30.13 = $747.33

Valuing loan guarantees as put options

The warrants gave the government the right to purchase 18.8 million shares of stock for $3 per share after 10 years.

Terms

	AWA
Warrants	10 years; $3/share; 18.8 million shares
Guarantee Fees	8% per year
Loan Guarantee	7 years; LIBOR+0.4%
Loan size	$380 million

No prepayment penalties.

with the guarantee, the borrowing rate is still higher than the risk– free rate. That can be attributed to the low liquidity of the debt and also that investors may believe that an unforeseen event could cause the guarantee to be revoked

Valuing loan guarantees as put options

Note: The same model can be used to value guarantee fees and warrants, whose payoffs are also contingent on the firm's asset value.

Results

Table 1: Net Market Value Subsidies (millions of dollars)

	AWA
Warrants	50.2
Guarantee Fees Paid	56.6
Loan Guarantee	(150.5)
Net Profit (Loss) to the Government	(43.7)

just an example

Credit derivatives

Finance at MIT
Where ingenuity drives results

Credit derivatives

- Includes the many types of contracts whose payoffs depend on credit events:
  - Loan and bond guarantees
  - Credit default swaps (CDS)
  - Structured credit products (e.g., CLOs, MBS)
  - Total rate of return swaps
  - Credit-linked notes
- Trading is mostly over-the-counter
- Used primarily by financial institutions to manage credit risk exposures
- Exponential growth prior to 2008 financial crisis. After that sharp falloff, and now slower growth
  - Excessive CDS exposure brought down insurance giant AIG, which was bailed out

Credit derivatives

- Credit derivatives generally involve two counterparties
  - Protection buyer
  - Protection seller

- There is also an underlying “reference entity” whose behavior determines the cash flows on the credit derivatives
- E.g., a skipped payment on a debt obligation by Boeing would trigger a payment on Boeing CDS

Question: Under what conditions is the protection buyer hedging? Under what conditions is the protection buyer speculating?

If the protection buyer has no other exposure to Boeing, it would be speculation that Boeing would be unlikely to default over the period covered by the protection

Credit Default Swaps (CDS)

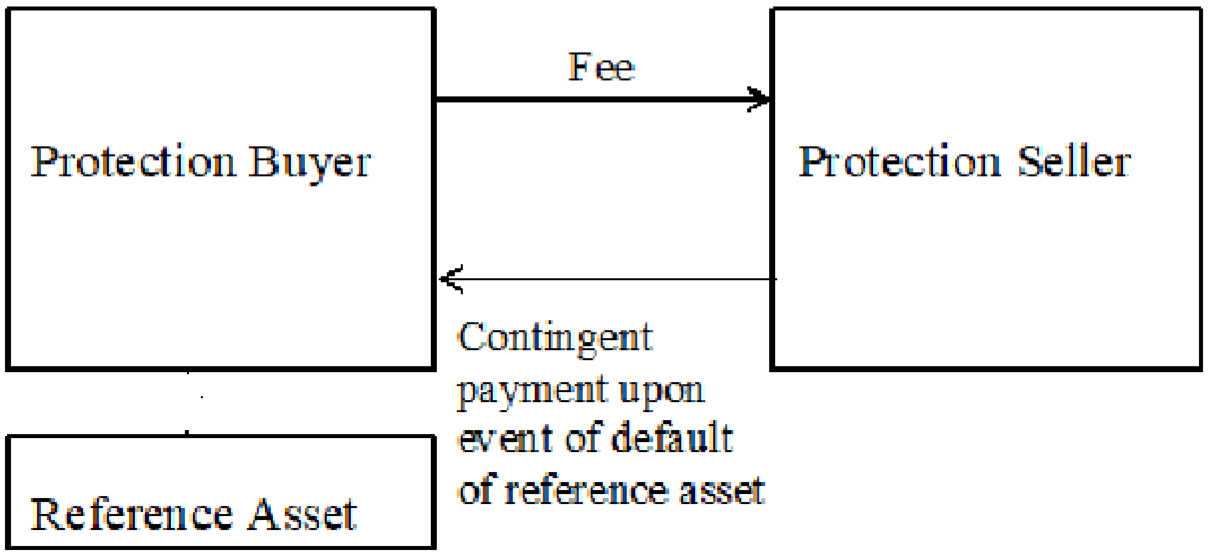
If insurance fee is paid over time, it's a "swap"

If insurance fee is paid up front, it's an "option" guarantee or option

This type of contract can be written on anything

- e.g., loan, bond, sovereign risk, credit exposure on derivative contract

fee is a percentage of the notional principle upon which the insurance is based



CDS structures

- **Fixed maturity**
- **Fee paid until maturity or default**
- **Various triggering events:**
  - Bankruptcy
  - Credit event upon merger
  - Downgrade
  - Failure to pay
  - Repudiation
  - Restructuring
  - Payment acceleration
  - Must be verifiable public announcement of the event

the list of trigger events in the CDS contract may not include all eventualities that would cause a firm to experience financial distress,the contract might offer less than 100% protection

Otherwise whether payments are made can involve a lot of legal complications.

- **Various alternative settlement rules; contracts can differ:**

protection buyer receive

- Cash settlement = face value – market value at trigger event
  - Market value determined by average of dealer quotes

protection seller

- Physical delivery: deliver defaulted bond for face value (may be multiple deliverables, and hence cheapest to deliver option)

receives what's owed

- Digital settlement: fixed payment in event of trigger event
- **Contracts usually governed by ISDA rules**

International Swap Dealers Association

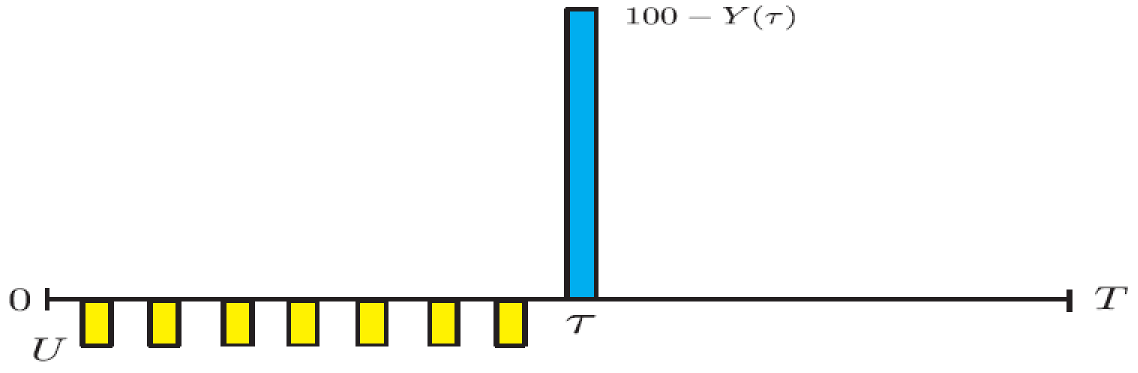
CDS structures

- Single name and indices
  - “Single name” means debt is from one company
  - Indices
    - Payoff based on defaults on a pool of bonds
    - CDX (e.g., iTraxx)
  - Structured products based on index give rise to
    - synthetic CDOs synthetic collateralized debt obligations
    - Nth to default bonds
    - Other repackaging of risk

- To read about credit indices, you can download “Credit Indices: A Primer” from IHS Markit’s website

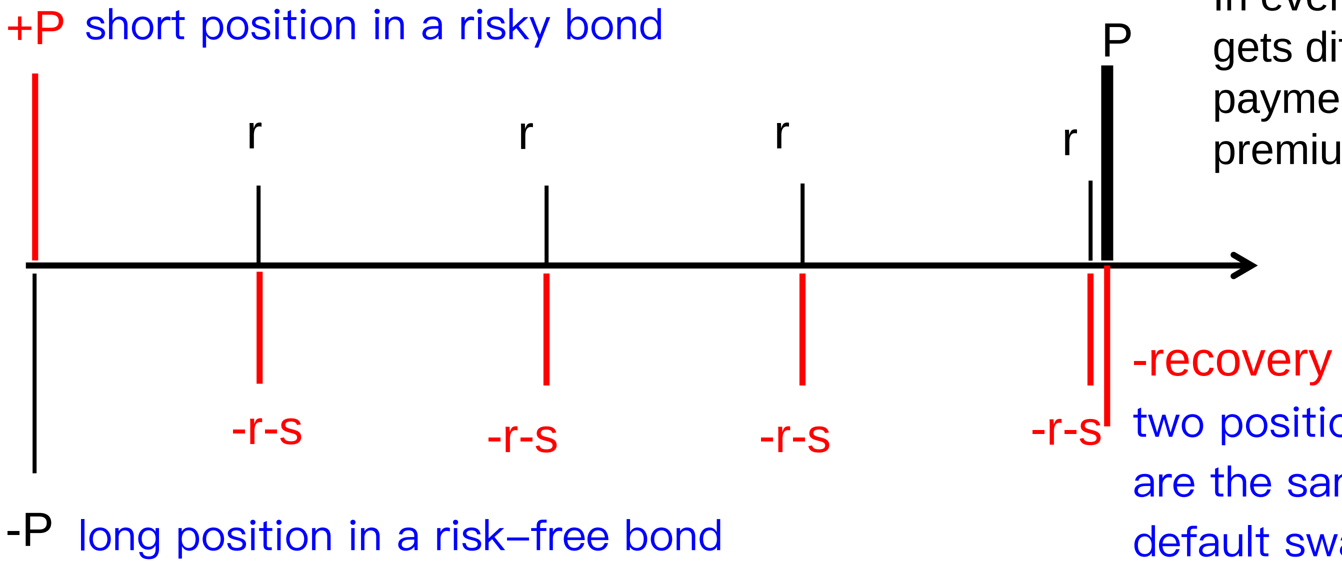
Single-name credit default swap

- This is essentially an insurance contract.
- Protection buyer pays periodic premium *U* on a notional amount of protection for a period of length *T*, say on \$100 face value of the underlying entity's debt
- In the event the underlying entity defaults on the debt at time τ and the recovery amount is *Y(τ)* per \$100 face value, the protection buyer is made whole
- If the contract calls for cash settlement the protection buyer gets 100 - *Y(τ)*
- If the contract specifies delivery the protection buyer exchanges \$100 face value of the bond which is worth *Y(τ)* for \$100.



CDS pricing

- Bottom line: The credit spread is approximately the fair CDS premium. no arbitrage
  - As for loan guarantee, writing protection is like being long the risky bond and short a risk-free bond
  - Conversely, buying a CDS is like shorting the risky bond and buying a risk-free bond (see picture)
  - Simple characterization is only precisely true in special cases; more complicated model is needed to account for different liquidity, counterparty risk, fixed vs. floating



In event of default, protection buyer gets difference between promised payment P and recovery net of premium

two positions result in net cash flows that are the same as long position in a credit default swap on the bond with a premium rate equal to s

CDS Pricing

- Approach 1: A CDS is like a credit guarantee. The present value of the insurance can be estimated using options pricing methods (e.g., structural models like the extensions of the Merton model discussed earlier). preferred because it's more accurate

- Approach 2: Price delivery-settled swap by "no arbitrage" with reference to underlying securities:* (corresponds to graph on previous slide)

a number of unrealistic assumptions

  - Assume CDS written on floating rate corporate bond "C" with spread S over risk-free floating rate bond, originally priced at par.
  - Ignoring transactions costs, the same protection is obtained by the CDS buyer by shorting the risky bond C, and investing in a par value default-free floating rate note.
  - Hold portfolio through maturity or credit event.
  - Net spread paid is S until termination.
  - In event of credit event, liquidate portfolio and get face value of risk-free bond – value of defaulted bond. Same is CDS payoff!
    - This assumes credit event occurs on a coupon reset date, so risk-free bond is priced at par
  - **It follows that S, the credit spread, is the fair premium rate on the swap.**

Multi-name CDS

- Suppose an investor holding a portfolio of defaultable bonds is worried about default
- The investor can

  (1) Purchase a CDS for each bond in the portfolio; or

  (2) Purchase insurance on the portfolio itself

Question: Which is more expensive?

Asian options that have a payoff based on average outcomes are less expensive than otherwise similar options because averaging or diversification narrows the variance of outcomes
A portfolio of bonds diversifies credit risk relative to individual bond holdings and buying a CDS based on the default losses on a portfolio is typically then much less expensive

Multi-name CDS

An even less expensive form of protection is to limit it to events that involve some large number of defaults

- **Example**: Nth-to-default basket default swaps providing a payoff only after the n– th default occurs in a basket of reference securities.
  - Not all firms in a portfolio will default at the same time, particularly if the credits are diversified across country, industry, etc.
  - It is popular to purchase protection that just pays off after some number of defaults have occurred
  - This is much cheaper than buying protection for each individual credit
- The basket spread (premium) depends on:
  - Number of credits: more credits => credit event more likely => more costly to insure
  - Credit quality and recovery rates
  - Default correlation across underlying reference entities (important!)

Summary

- This week we looked at how statistical and structural models can be used to price credit risk
- Statistical models derive default and recovery rates from data on borrower characteristics, including leverage ratios, credit ratings, tangible collateral, etc.
- Structural models infer default and recovery rates based on the stochastic structure of a borrower’s assets and a default barrier
- Both approaches predict the effect of credit risk on the value of credit-sensitive securities
  - For defaultable bonds, both should produce similar answers if properly calibrated
  - The structural approach is more flexible and better suited for pricing more complex credit derivatives
- In practice, hybrids of the two approaches are often used and give better results.