Week 2

1. How do to calculate duration of forward contract & floating leg of swap with & w/o coupon?

   Answer For duration of forward contract on a bond, please refer to Week 2 Lecture slides 35-36. Meanwhile, the duration of a floating leg of a swap is equal to the duration of a floating rate bond with the same maturity and frequency.

2. **Problem Set Q4** Please explain.

   Answer Our goal is to determine the fixed swap rate in a 1-year semiannual interest rate swap in Switzerland.

   Firstly, remember that the initial value of a swap must be zero. Since the floating leg is always priced at par, the fixed leg must be too.

   Now, suppose your fixed swap rate is denoted c. Then, given a swap with notional of \$1, your cash flow will be:

   - c at $t = 6$ months
   - $1 + c$ at $t = 1$ year

   The present value formula for the fixed leg then becomes

   $$P_{fixed} = \frac{c}{(1 + r_{.5})^{.5}} + \frac{1 + c}{1 + r_1}$$

   where $P_{fixed} = 1$ because the leg is priced at par. Now, we need to find the interest rates $r_{.5}$ and r_1. We can compute this from the numbers given, but a smarter way would be to realize that the price of a zero-coupon bond with face value of 1 maturing at time t, $P_{B,t}$, is equal to the discount rate at time t. (Remember that $P_{B,t} = \frac{1}{(1+r_t)^t}$). Therefore, the next step is to calculate the price of a 6-month and a 1-year zero coupon bond in Switzerland. This is quite straightforward, as shown in the solutions to this question.

   Going back to the equation above,

   $$\begin{aligned} P_{fixed} &= \frac{c}{(1 + r_{.5})^{.5}} + \frac{1 + c}{1 + r_1} \\ 1 &= cP_{B,.5} + (1 + c)P_{B,1} \\ c &= \frac{1 - P_{B,1}}{P_{B,.5} + P_{B,1}} \end{aligned}$$

   Finally, the result for c is still semiannual here, so we annualize by multiplying by 2 to get the 1-year semiannual swap rate.

Week 3

1. "coupon rate being 3.25%", I went through the problem and can't find where it hint the coupon rate, same confusion had me in week 10 problem set about coupon rate that is not mentioned.

   Answer Both questions you mentioned are about interest rate swaps. In a swap, you have a fixed leg and floating leg. If you only look at the fixed leg, the cash flow is similar to a coupon bond, where the fixed swap rate is the coupon rate. Similarly, the floating leg is similar to a floating-rate bond. This is why in most questions involving swaps, we treat a swap as a portfolio of long coupon bond and short floating bond (or short coupon bond and long floating bond), depending on which side of the swap you are at.

Week 8

1. **Problem Set Q2c1** Please explain.

   Answer Firstly, to summarize:

   - The bond has both embedded call and put optionalities, with the bondholder reserving rights to both.
   - At any point in time, a bondholder could either exercise the call, exercise the put, or not exercise at all.
   - At any node, the sequence of events is: coupon gets paid first, then the bondholder makes a decision on whether to exercise. This means two things. First, regardless of whether the bondholder exercises, she would still get the coupon at the time of exercise. Secondly, in making decisions about exercising, the bondholder will look at the ex-coupon bond price.

   Also, we denote Π as the value of position, V^B as ex-coupon bond price, V^C as value of embedded call, V^P as value of embedded put, and C as coupon payment. I have also ignored putting in some subscripts on my variables. With this, we can start dissecting into the question.

   - As with binomial tree pricing approaches, we start from the end nodes (at maturity). Π_3 is quite straightforward as it is simply the face value plus coupon.
   - Going backward in time, we calculate V^B using risk-neutral probabilities and interest rates at each node, $V^B = \frac{q\Pi_H+(1-q)\Pi_L}{1+r}$. Why does this give us V^B? Think about what happens within a node. When a bond arrives at time $t = 2$, the price is initially Π. After the coupon is paid out, the price becomes $V^B + \max(V^C, V^P)$. If the option is exercised, then the tree would have stopped branching out of that node. Therefore, to go from node 2 to node 3, the bond must have been still outstanding, i.e., optionality not exercised.

Note that this is irrespective of whether the actual optimal decision is to exercise or not —the price movement dictated by the binomial tree specifications would always correspond to going from V^B at one time step to Π at the next time step.

- Still within time $t = 2$, at each node, we determine the optimal decision out of the 3 choices. If the optimal decision is to exercise the put, we add V^P to V^B, else if call is optimal, we add V^C to V^B, otherwise we keep V^B as is.
- Finally, we add back the coupon to get the value of position, Π at time 2 for every node.
- We repeat steps 2-4 for the nodes in time 1.

Week 9

1. **Problem Set Q1** Can you show your method for solving the question, including the use of Excel Solver?

   Answer To recap, the question asks you to compute the market value of assets, V_0 and implied volatility of assets, σ_V. We will solve for these variables according to the Merton model (see Week 9 Lecture Slide 21). The procedure is as follows:

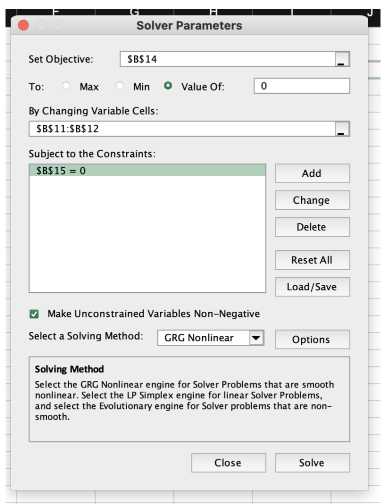
   - First, we populate the given values, as well as compute d_1 and d_2. Also, initialize some value of V_0 and σ_V to some random values. Here, I just input $V_0 = 10$ millions and $\sigma_V = 25\%$.

	A	B	C
1	E0	7.6	(millions)
2	sigma_e	33.16%	
3	r	1.51%	
4	exp_return	5%	
5	F	5	(millions)
6	T	2	
7			
8	d1	2.222711481	
9	d2	1.869158091	
10			
11	V0	10	(millions)
12	sigma_v	25%	

   - We define the Solver objectives. These objectives should be based on the two equations in slide 21, such that the result should be 0. Here, "obj1" is $E_0 - V_0 N(d_1) + Fe^{-rT}N(d_2)$, and "obj2" is $\sigma_E - \frac{V_0 N(d_1)}{E}\sigma_V$.

A	B	C
E0	7.6	(millions)
sigma_e	33.16%	
r	1.51%	
exp_return	5%	
F	5	(millions)
T	2	
d1	2.222711481	
d2	1.869158091	
V0	10	(millions)
sigma_v	25%	
obj1	2.433012904	
obj2	0.70%	

- Next, we set as the Solver as follows: set "obj1" as the objective, to "Value Of 0". Set "By Changing Variable Cells" to the cells containing V_0 and σ_V. Set "obj2 = 0" as a constraint.



- Verify that after running the solver, the objective values are very close to 0.

	A	B	C
1	E0	7.6	(millions)
2	sigma_e	33.16%	
3	r	1.51%	
4	exp_return	5%	
5	F	5	(millions)
6	T	2	
7			
8	d1	3.435019318	
9	d2	3.148688111	
10			
11	V0	12.45096783	(millions)
12	sigma_v	20%	
13			
14	obj1	2.75033E-10	
15	obj2	0.00%	
16			

2. **Problem Set Q2c** Where does the 100 multiplier come from?

   Answer From the face value invested divided 1,000, i.e., 100,000/1,000 = 100.

3. **Recitation Problem 1** For Recitation Week 9 (Problem 1) the equation used for Pricing had 107 (FV of bond) in the numerator: $\frac{107(1-d)+107dg}{1+r}$, multiplying both $(1-d)$ & dg. But for The Problem Set 2(a), the equation changed and the Future value only multiplies $(1-d)$ and not gd: $\frac{1000(1-d)+gd}{1+r}$. Why does 1000 not multiply g*d in the problem set but it does in recitation?

   Answer In the problem set, the question asks for recovery amount, instead of recovery rate. We don't know g in the problem set question —this is essentially what we are trying to solve for, except we want the recovery amount $1000g$ instead of the recovery rate g.

4. Is there a Merton model variation that includes dividends?

   Answer The original Merton model assumes that the firm pays no dividends. There could be variations/extensions to Merton model to account for dividends, but these are often proprietary, and as such not discussed in this course.

5. **Problem Set Q3b** Please explain. Remember that Asset = Debt + Equity and this holds all the time. The initiation of a dividend payment does not create value for the firm, nor does it destroy value (to clarify further, asset value would only decrease once dividend has been paid). As much the value of assets does not change. Meanwhile, because the firm pays dividend, asset value is expected to decline over time. This increases the risk of the firm defaulting on their debts, which reduces the value of debts.

   Answer Since asset value has not changed, but debt value has gone down, then the value of equity must go up.

Week 10

1. How to calculate risk neutral probabilities used in finding price and yield in securitization?

   Answer Please refer to Week 10 Lecture Slide 18. Risk neutral probabilities are computed from the bond price, risk-free rate, and recovery rate. You have 4 variables: bond price, risk-free rate, default probability and recovery rate. If you are given 3 of them, you can calculate the fourth variable.

2. Do we need to calculate the cash flows that the prepayment model generates? I understand the logic that if the interest rate decreases, the prepayment size changes and it is lower. But I am having trouble understanding how the last values (100,104,50,58) were calculated.

   Assume that your prepayment model generates th
   following cash flows from an MBS, corresponding
   the four equally likely interest rate paths:

6.0%	*6.5%*	*7.0%*	*(up, up)*
$100	$100	$100	
6.0%	*6.5%*	*6.0%*	*(up, down)*
$100	$100	$104	
6.0%	*5.5%*	*6.0%*	*(down, up)*
$100	$150	$50	
6.0%	*5.5%*	*5.0%*	*(down, down)*
$100	$150	$58	

   each path is equally likely (because of the assump
   that pr(up) = pr(down) = .5).

   Answer When you are told what prepayment model to use, you can always calculate the effect of prepayment on the mortgage outstanding principal. This will most likely not come up in the final exam, as it has not been explored in depth throughout the course.

   Looking at the figure above, the prepayment model exemplified here seems arbitrary. While you are familiar with the PSA model, you can imagine that there are other models that incorporate other factors —yield being one of them. I would not worry too much about where these numbers come from.

Calculator

1. In combined calculators file, the last few tabs for CirDemo, will we need to use these tabs? Because the lectures didn't mention about this excel model.

Answer The CirDemo sheets correspond to Week 8 lecture slides 33 onwards. While this was covered in lectures, it was not covered in recitations nor problem sets. Therefore, you will very likely not need to use CirDemo in the final exam.

However, just to explain what the sheets do, the Input tab is where you specify the Cox-Ingersoll-Ross (CIR) model parameters. The Output tab performs Monte-Carlo simulation to simulate a path of short rates, while the TermStructure tab computes the implied yield curve given the CIR parameters.

2. Finally, we received some excel sheets which are not explained how and when to use. Could you explain how and purpose of using the sheets.

   Answer Please refer to the live recitation videos, where I explained the sheets. Aside from CirDemo and TreeFitter_Tree2, you should be familiar with the rest of the sheets as you have used them in the past problem sets.

3. How can i enable my VBA calculator to work?

   Answer If you are using Microsoft, please refer to this thread. When you open the Excel file, you should get a prompt with two options: "Disable Macros" or "Enable Macros". Click on "Enable Macros" and it should work.

4. Is the binomial tree model useful to calculate other types of options (e.g. american or exotics) to make this faster in exam or is it only useful for standard options?

   Answer The binomial tree model is designed for European options. To price American and exotic options, you would need to modify the Excel formulas accordingly.

5. Please explain what are the number in TreeFitter_Tree2 represents and how to use them

   Answer This spreadsheet is relevant when you are calibrating a binomial tree using callable bonds data. Please refer to the live recitation videos for more details.

6. In the tree module. we input Option premia (which from my understanding, is the present value of embedded option in the bond). In this case, why do we need to input the strike price since the PV of embedded option is already indicated?

   Answer In short, you need both option premia and the strike price to calibrate a binomial tree model using callable bonds data. Furthermore, the call option strike price is not always constant. Inputting the strike prices gives you more flexibility in setting up the inputs for the calibration procedure.

7. Is the Assumed Volatility (2*sigma) in the file still calculated as = exp(2*sigma)-1?

   Answer You can use either, but for exam purposes use $e^{2\sigma} - 1$.

1 Exam-related questions

1. For multiple part questions, will the answers be conditional on earlier questions?

Answer If you get the first part of a question wrong and that wrong value is used in the second part, the automatic grader will mark you down at first. For the final exam, we will review the grades more thoroughly to check for "errors carried forward", which should make the answers conditional on earlier questions.

2. In the final exam, there seems to be multiple (indefinite) attempts to answer the question. In this case, we still don't know the answer is right or wrong but we can always change the previous answers. Is my understanding correct?

   Answer Yes.

3. There are not many additional questions in the sample exam. Is it likely that questions may be asked in exam from the reading material/questions in the textbook that we may not have covered? It seems there are not a lot of extra questions apart from problem sets and recitations.

   Answer My suggestion is to focus on having a sound conceptual understanding of the course materials. The questions are all designed to be doable given only materials in this course. However, you should be prepared for questions that test your critical thinking and mastery of the concepts, rather than memorization of formulas and procedures.

2 General

1. What does "if the yield curve makes a parallel upward shift of 1%" mean?

   Answer In this course, we often make two simplifications: a flat yield curve and a parallel shift. Yield curve being flat means that the yield is the same across the term structure, e.g., same yield between a 2-year bond and a 10-year bond.

   A parallel shift means that the yield moves by the same amount across the entire term structure. Upward shift means yield goes up, and downward shift means yield goes down.

2. Can you show a brief example of finding risk-neutral probabilities given a binomial tree of rates?

   Answer This is actually not possible for interest rate trees. Binomial trees of interest rates are calibrated assuming that the risk-neutral up probability is 0.5. If you see an interest rate tree, you can safely assume that the risk-neutral probability of an up movement is 0.5.

3. In a portfolio of securities with different durations and convexities: can you always compute portfolio duration and convexity as weighted averages of the component quantities?

   Answer You should be able to, except for some cases like swap at initialization, where the present value of the portfolio is 0.

4. I couldn't find any references to what is meant by a 'shock term', and they appear in a number of our formulas, could you please explain why it is called a 'shock term'?

   Answer Shock term means a random variable. It is defined so as shock term is something that one cannot predict.

5. Can you explain NORM.S.DIST function (why True, why don't use just NORM.DIST)?

   Answer A (univariate) normal distribution is parameterized by mean and standard deviation. A standard normal distribution is a special kind of normal distribution with mean of 0 and standard deviation of 1.

   When you are calculating, say, $N(d_1)$ in BSM formula, you are calculating the cumulative distribution function (CDF) of a standard normal distribution.

   In Excel, the ".S" is used when you are using standard normal distribution. The function syntax is =NORM.S.DIST(z, cumulative). The first input, z, is the z-score you want to compute the CDF for (d_1 in the example above). The second input is TRUE if you want to calculate CDF, and FALSE if you wnat to calculate PDF (probability density function).

   Meanwhile, NORM.DIST allows you to specify the mean and standard deviation. The syntax is =NORM.DIST(x,mean,std,cumulative), where x is equivalent to z-score in NORM.S.DIST, mean and std are the distribution parameters, and cumulative is TRUE for cdf and FALSE for pdf.

   Therefore, to calculate $N(d_1)$, you can either use $\bar{\text{N}}$ORM.S.DIST(d1, TRUE) or $\bar{\text{N}}$ORM.DIST(d1, 0, 1, TRUE). Both should give you the same answer.

6. Please kindly share the articles you mentioned about to gain more intuitive understanding of the risk neutral probabilities

   Answer For an intuitive understanding, you can check out Investopedia or Wikipedia. You can also find a more rigorous derivation of risk-neutral measure in the first few chapters of "Stochastic Calculus for Finance I: The Binomial Asset Pricing Model" by Steven Shreve.

7. Any chance for one more explanation of how yield on bond-equivalent-basis relates to yield-to-maturity? Just in some questions we had to convert the beb rate to a different rate prior to using in a formula.

   Answer They are fundamentally different. Yield-to-maturity is the value of y that solves the equation $P = \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t}$ (assuming discrete compounding), where CF_t is the cash flow at time t and P is the current bond price.

   Meanwhile, bond-equivalent basis is one yield basis convention, which is a method of quoting bond yield. In particular, bond equivalent yield is an annualized semiannual basis. If a bond pays annually and you have solved for y using the formula above, then your YTM is quoted annually. Whereas if the bond pays semiannually, using the equation above, your YTM will be quoted in semiannual compounding, from which you can annualize by multiplying by 2 to quote in bond-equivalent basis.