Recitation 1

Problem 1

Southern California Edison (SCE) will need to buy 200,000 barrels of oil in 10 days, and it is worried about fuel costs rising. Suppose SCE goes long 200 oil forward contracts of the April contract (it is March), each for 1,000 barrels of oil, at the current forward price of \$50 per barrel. Suppose forward prices change each day as follows:

Day 1	\$49.75
Day 2	\$47.25
Day 3	\$48.50
Day 4	\$49.25
Day 5	\$49.50
Day 6	\$50.75
Day 7	\$51.25
Day 8	\$50.50
Day 9	\$51.75
Day 10	\$52.50

(a) What is the mark-to-market profit or loss (in dollars) that SCE will have on each date?

Solution: Define F_T to be the forward price for Day T and N to be the number of barrels of oil in each forward contract. The mark-to-market profit/loss (P/L) on each forward contract on Day T is equal to $N \times (F_T - F_{T-1})$ for a long position and $N \times (F_{T-1} - F_T)$ for a short position.

Let F_0 be the current forward price of \$50. Since SCE has a long position in 200 forward contracts with a size of $N = 1,000$ barrels for each contract, SCE's P/L on Day 1 is:

$$200 \times [N \times (F_1 - F_0)] = 200 \times [1,000 \times (49.75 - 50)] = -\$50,000.$$

In other words, SCE makes a mark-to-market **loss** of \$50,000 on Day 1.

Similarly, on Day 2, SCE's P/L is:

$$200 \times [N \times (F_2 - F_1)] = 200 \times [1,000 \times (47.25 - 49.75)] = -\$500,000.$$

Iterating the calculation above for Days 3-10, SCE's daily P/L is displayed in the table below:

Day 1	-\$50,000
Day 2	-\$500,000
Day 3	\$250,000
Day 4	\$150,000
Day 5	\$50,000
Day 6	\$250,000
Day 7	\$100,000
Day 8	-\$150,000
Day 9	\$250,000
Day 10	\$150,000

(b) What is SCE's total profit or loss after 10 days?

Solution: Recall that SCE's P/L on its long position in each forward contract on Day 1 is $N \times (F_1 - F_0)$ and on Day 2 is $N \times (F_2 - F_1)$. Summing these two quantities, we see that SCE's P/L *after 2 days* is:

$$[N \times (F_1 - F_0)] + [N \times (F_2 - F_1)] = N \times (F_2 - F_0).$$

Iterating this logic forward to Day 10, we see that SCE's P/L on each forward contract *after 10 days* is given by $N \times (F_{10} - F_0)$. Thus, SCE's total P/L on its long position in 200 forward contracts after 10 days is:

$$200 \times [N \times (F_{10} - F_0)] = 200 \times [1,000 \times (52.50 - 50)] = \$500,000.$$

(c) Explain why this may not be a perfect hedge for SCE.

Solution: A couple reasons why this strategy may not be a perfect hedge for SCE include:

1. The interest earned on the margin account was not included in the calculations and will affect SCE's payoff.

2. Not all oil is the same! Oil prices differ based on quality and location, so the reference price in the forward contract may not be perfectly correlated with the cash purchase price that SCE faces.

Problem 2

A stock is expected to pay a dividend of \$1 per share in two months and in five months. The stock price is \$50, and the risk-free rate of interest is 8% per annum with continuous compounding for all maturities. An investor has just taken a short position in a six-month forward contract on the stock.

(a) What are the forward price and the initial value of the forward contract?

Solution: We'll solve this problem in three steps.

1. Using the given risk-free rate, find the present value of the stock's expected dividends.

Let t denote the time in years and r be the continuously-compounded, constant risk-free rate expressed in decimal form. As we saw in lecture, we can discount any cash flow arriving at time t

back to the present using the expression $PV = D \times e^{-r\times t}$ on a continuous basis, where D is the cash flow (in dollars) and P is its present value.

discount at risk–free rate: value it at risk–neutral probability

Thus, the present value of the $1 dividends we expect to receive in two and five months is:

$$I = 1 \times e^{-0.08\times 2/12} + 1 \times e^{-0.08\times 5/12} = \$1.954.$$

2. Find the forward price of the stock using the equation we learned in lecture.

Recall from lecture that we can calculate the forward price of a stock as:

$$F_0 = (P_{s,0} - I)e^{r\times T},$$

where F_0 is the initial forward price, $P_{s,0}$ is the current price of the stock, and T is the time to maturity of the forward contract in years.

Plugging in $P_{s,0} = \$50$, $I = \$1.954$, and $T = 0.5$, we get that the forward price is:

$$F_0 = (50 - 1.954)e^{0.08\times 0.5} = \$50.01.$$

3. Find the initial value of the short position in the forward contract.

This not so hard, really! As discussed in lecture, in the absence of arbitrage, **all forward contracts must have a net present value of 0 at inception**. Why?

Let $P_{s,0}$ be the current price of the stock underlying the forward contract. The cash flows from a forward short position are equivalent to those obtained by simultaneously lending $-P_{s,0}$ dollars today and shorting the stock at $P_{s,0}$.

Clearly, the net cash flow today from this "reverse cash-and-carry" strategy is 0, so by the Law of One Price, the net present value of the short position in the forward contract is also 0.

(b) Three months later, the price of the stock is $48 and the risk-free rate is still 8% per annum. What are the forward price and the value of the short position in the forward contract?

Solution: Let's go through our three steps again.

1. Using the given risk-free rate, find the present value of the stock's expected dividends.

Since we originally expected to receive $1 dividends in two and five months, three months later we now expect to receive a single dividend in two months. The risk-free rate is still 8%, so the present value of this dividend is:

$$I = 1 \times e^{-0.08\times 2/12} = \$0.9868.$$

2. Find the forward price of the stock using the equation we learned in lecture.

Suppose F_t is the forward price for a contract negotiated at time t (in years), $P_{s,t}$ is the stock price at time t, and T is the time to maturity of the contract (also in years). In this general form, our equation for the forward price of a stock is now:

$$F_t = (P_{s,t} - I)e^{r\times T}.$$

Plugging in $P_{s,t} = \$48$, $I = \$0.9868$, and $T = 0.25$, we can find the forward price of the stock three months later (i.e., at $t = 0.25$):

$$F_{0.25} = (48 - 0.9868)e^{0.08\times 3/12} = \$47.96.$$

3. Find the value of the short position in the forward contract.

In lecture, we saw that the value of a short position in a forward contract at time t is given by:

$$f_t = (F_0 - F_t)e^{-r\times(T-t)}.$$

Thus, we can find the value of a short position in the forward contract three months later by plugging in $F_0 = \$50.01$, $F_{0.25} = \$47.96$, $T = 0.5$, and $t = 0.25$, into the equation above:

$$f_{0.25} = (50.01 - 47.96)e^{-0.08\times(0.5-0.25)} = \$2.01.$$

Problem 3

A company enters into a forward contract with a bank to sell a foreign currency for K_1 at time T_1. The exchange rate at time T_1 proves to be $S_1 > K_1$. The company asks the bank if it can roll the contract forward until time $T_2 > T_1$ rather than settle at time T_1. The bank agrees to a new delivery price, K_2. Explain how K_2 should be calculated.

Solution: As a convention, assume that the exchange rate is quoted as the number of units of domestic currency per unit of foreign currency. So, if the exchange rate goes up, then the foreign currency has *appreciated*—i.e., its price in terms of the domestic currency has increased. Also, note that the bank has a long position in the forward contract, and hence has agreed to *buy* the foreign currency in the future.

First, let's find the value of the forward contract to the bank at T_1. Since the spot exchange rate S_1 at T_1 is greater than the forward exchange rate K_1, the bank can purchase the foreign currency at a lower price using the forward contract than on the spot market. Thus, the value of the forward contract to the bank is equal to the cost savings from purchasing the foreign currency with the forward contract, which is $S_1 - K_1$.

value of forward contract in domestic at T1

Now, how will the bank optimally choose the new delivery price K_2 at T_2? To be as well off as before, the bank requires the value of the new, rolled-forward contract at T_1 to be equal to $S_1 - K_1$. This implies that

As for bank:

bank pay K2 domestic currency at T2

T1 value in domestic

bank get 1 foreign currency at T2

T1 value in foreign:

T1 value in domestic:

$$S_1 \times \boxed{e^{-r_f(T_2-T_1)}} - K_2 \times e^{-r(T_2-T_1)} = S_1 - K_1,$$

where r and r_f are the domestic and foreign risk-free rates, respectively, observed at T_1 and applicable to the period between T_1 and T_2. The first term on the LHS, $S_1 \times e^{-r_f(T_2-T_1)}$, is the present value of investing S_1 units of the foreign currency at the foreign risk-free rate of r_f from T_1 to T_2. The second term on the LHS, $K_2 \times e^{-r(T_2-T_1)}$, is the present value of investing K_2 units of the foreign currency at the foreign risk-free rate of r_f from T_1 to T_2.

bank get 1 foreign currency at T1

T1 value in domestic: S1

bank pay K1 domestic currency at T1

T1 value in domestic: K1

Solving for K_2, we see that:

$$K_2 = S_1 \times e^{(r-r_f)(T_2-T_1)} - (S_1 - K_1) \times e^{r(T_2-T_1)}.$$

So, there are two components to K_2: (i) the forward price at time T_1 and (ii) an adjustment to the forward price equal to the bank's gain on the first part of the contract compounded forward at the domestic risk-free rate.