Monetary Policy, Inflation, and the Business Cycle

Chapter 4.
Monetary Policy Design in the Basic New Keynesian Model

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The present chapter addresses the question of how monetary policy should be conducted, using as a reference framework the basic new Keynesian model developed in chapter 3. We start by characterizing that model's efficient allocation. The latter is shown to correspond to the equilibrium allocation of the decentralized economy under monopolistic competition and flexible prices, once an appropriately chosen subsidy is in place. As it will be shown, when prices are sticky, that allocation can be attained by means of a policy that fully stabilizes the price level.

After determining the objectives of the optimal monetary policy, we turn to issues pertaining to its implementation. We provide examples of interest rules that implement the optimal policy, i.e. optimal interest rate rules. But we also argue that none of those rules seems a likely candidate to guide monetary policy in practice, for they all require that the central bank responds contemporaneously to changes in a variable–the natural rate of interest– which is not observable in actual economies. That observation motivates the introduction of rules that a central bank could arguably follow in practice (which we label as "simple rules"), and the development of a criterion to evaluate the relative desirability of those rules, based on their implied welfare losses. We provide an illustration of that approach to policy evaluation by analyzing the properties of two such simple rules: a Taylor rule and a constant money growth rule.

1 The Efficient Allocation

The efficient allocation associated with the model economy described in chapter 3 can be determined by solving the problem facing a benevolent social planner seeking to maximize the representative household's welfare, given technology and preferences. Thus, each period the optimal allocation must maximize the household's utility

$$U\left(C_t, N_t\right)$$

where $C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}}\ di\right)^{\frac{\epsilon}{\epsilon-1}}$ subject to the resource constraints

$$C_t(i) = A_t\ N_t(i)^{1-\alpha}$$

for all $i \in [0,1]$ and

$$N_t = \int_0^1 N_t(i)\ di$$

The associated optimality conditions are

$$C_t(i) = C_t, \ \ all \ i \in [0,1] \tag{1}$$

$$N_t(i) = N_t, \ \ all \ i \in [0,1] \tag{2}$$

$$-\frac{U_{n,t}}{U_{c,t}} = MPN_t \tag{3}$$

where $MPN_t \equiv (1-\alpha) \ A_t N_t^{-\alpha}$ denotes the economy's average marginal product of labor (which in the case of the symmetric allocation considered above, also happens to coincide with the marginal product for each individual firm).

Thus we see that it is optimal to produce and consume the same quantity of all goods, and to allocate the same amount of labor to all firms. That result is a consequence of all goods entering the utility function symmetrically, combined with concavity of utility and identical technologies to produce all goods. Once that symmetric allocation is imposed, the remaining condition defining the efficient allocation (equation (3)), equates the marginal rate of substitution between consumption and work hours to the corresponding marginal rate of transformation (which in turn corresponds to the marginal product of labor). Note also that the latter condition coincides with the one determining the equilibrium allocation of the classical monetary model (with perfect competition and fully flexible prices) analyzed in chapter 2.

Next we discuss the factors that make the equilibrium allocation in our baseline model suboptimal.

2 Sources of Suboptimality in the Basic New Keynesian Model

The basic new Keynesian model developed in chapter 3 is characterized by two distortions, whose implications are worth considering separately. The first distortion is the presence of market power in goods markets, exercised by monopolistically competitive firms. That distortion is unrelated to the presence of sticky prices, i.e. it would be effective even under the assumption of flexible prices. The second distortion results from our assumption of infrequent adjustment of prices by firms. Next we discuss both types of distortions and their implications for the efficiency of equilibrium allocations.

2.1 Distortions Unrelated to Sticky Prices: Monopolistic Competition

The fact the each firm perceives the demand for its differentiated product to be imperfectly-elastic endows it with some market power and leads to pricing-above-marginal cost policies. To isolate the role of monopolistic competition let us suppose for the time being that prices are fully flexible, i.e. each firm can adjust freely the price of its good each period. In that case, and under our assumptions, the profit maximizing price is identical across firms. In particular, under an isoelastic demand function (with price-elasticity ϵ), that optimal price-setting rule is given by:

$$P_t = \mathcal{M} \ \frac{W_t}{MPN_t}$$

where $\mathcal{M} \equiv \frac{\epsilon}{\epsilon - 1} > 1$ is the (gross) optimal markup chosen by firms and $\frac{W_t}{MPN_t}$ is the marginal cost. Accordingly,

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} < MPN_t$$

where the first equality follows from the optimality conditions of the household. Hence, we see that the presence of a non-trivial price markup implies that condition (3) characterizing the efficient allocation is violated. Since, in equilibrium, the marginal rate of substitution, $-U_{n,t}/U_{c,t}$, and the marginal product of labor are, respectively, increasing and decreasing (or non-increasing) in hours, the presence of a markup distortion leads to an inefficiently low level of employment and output.

The above inefficiency resulting from the presence of market power can be eliminated through the suitable choice of an employment subsidy. Let τ denote the rate at which the cost of employment is subsidized, and assume that the outlays associated with the subsidy are financed by means of lump-sum taxes. Then, under flexible prices, we have $P_t = \mathcal{M} \ \frac{(1-\tau)W_t}{MPN_t}$. Accordingly,

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}(1-\tau)}$$

Hence, the optimal allocation can be attained if $\mathcal{M}(1-\tau) = 1$ or, equivalently, by setting $\tau = \frac{1}{\epsilon}$. In much of the analysis below we assume that such an optimal subsidy is in place. By construction, the equilibrium under flexible prices is efficient in that case.

2.2 Distortions Associated with the Presence of Staggered Price Setting

The assumed constraints on the frequency of price adjustment constitute a source of inefficiency on two different grounds. First, the fact that firms do not adjust their prices continuously implies that the economy's average markup will vary over time in response to shocks, and will generally differ from the constant frictionless markup $\mathcal{M}$. Formally, and denoting the economy's average markup as $\mathcal{M}_t$ (defined as the ratio of average price to average marginal cost), we have:

$$\mathcal{M}_t = \frac{P_t}{(1-\tau)(W_t/MPN_t)} = \frac{P_t\mathcal{M}}{W_t/MPN_t}$$

where the second equality follows from the assumption that the subsidy in place exactly offsets the monopolistic competition distortion, which allows us to isolate the role of sticky prices. In that case we have

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = MPN_t \, \frac{\mathcal{M}}{\mathcal{M}_t}$$

which violates efficiency condition (3) to the extent that $\mathcal{M}_t \neq \mathcal{M}$. The efficiency of the equilibrium allocation can only be restored if policy manages to stabilize the economy's average markup at its frictionless level.

In addition to the above inefficiency, which implies either too low or too high a level of aggregate employment and output, the presence of staggered price setting is a source of a second type of inefficiency. The latter has to do with the fact that the relative prices of different goods will vary in a way unwarranted by changes in preferences or technologies, as a result of the lack of synchronization in price adjustments. Thus, we will generally have $P_t(i) \neq P_t(j)$ for any pair of goods (i, j) whose prices do not happen to have been adjusted in the same period. Such *relative price distortions* will lead, in turn, to different quantities of the different goods being produced and consumed, i.e. $C_t(i) \neq C_t(j)$, and, as a result, $N_t(i) \neq N_t(j)$ for some (i, j). That outcome violates efficiency conditions (1) and (2). Attaining the efficiency allocation requires that the quantities produced and consumed of all goods are equalized (and, hence, that so are their prices and marginal costs). Accordingly, markups should be identical across firms and goods at all times, in addition to being constant (and equal to the frictionless markup) on average.

Next we characterize the policy that will attain those objectives.

3 Optimal Monetary Policy in the Basic New Keynesian Model

In addition to assuming an optimal subsidy in place that exactly offsets the market power distortion, and in order to keep the analysis simple, we restrict ourselves to the case in which there are no inherited relative price distortions, i.e. we assume that $P_{-1}(i) = P_{-1}$ for all $i \in [0,1]$.[1] Under those assumptions, the efficient allocation can be attained by a policy that stabilizes marginal costs at a level consistent with firms' desired markup, *given the prices in place*. If that policy is expected to be in place indefinitely, no firm has an incentive to adjust its price, since it is currently charging its optimal markup and expects to keep doing so in the future without having to change its price. As a result, $P_t^* = P_{t-1}$ and, hence, $P_t = P_{t-1}$ for $t = 0, 1, 2, ...$ In other words, the aggregate price level is fully stabilized and no relative price distortions emerge. In addition, $\mathcal{M}_t = \mathcal{M}$ for all t, and output and employment match their counterparts in the flexible price equilibrium allocation (which, in turn, corresponds to the efficient allocation, given the subsidy in place)

Using the notation for the log-linearized model introduced in the previous chapter, the optimal policy requires that for all t,

$$\widetilde{y}_t = 0$$

$$\pi_t = 0$$

i.e., the output gap is closed at all times, which (as implied by the new Keynesian Phillips curve) leads to zero inflation. The dynamic IS equation then implies

$$i_t = r_t^n$$

for all t, i.e. the equilibrium nominal interest rate (which equals the real rate, given zero inflation) must be equal to the natural interest rate.

[1] The case of a non-degenerate initial distribution of prices is analyzed in Yun (2005). In the latter case the optimal monetary policy converges to the one described here after a transition period.

Two features of the optimal policy are worth emphasizing. First, stabilizing output is not desirable in itself. Instead, output should vary one for one with the natural level of output, i.e. $y_t = y_t^n$ for all t. There is no reason, in principle, why the natural level of output should be constant or follow a smooth trend, since all kinds of real shocks will be a source of variations in its level. In that context, policies that stress output stability (possibly about a smooth trend) may generate potentially large deviations of output from its natural level and, thus, be suboptimal. This point is illustrated in section 4 below, in the context of a quantitative analysis of a simple policy rule.

Secondly, price stability emerges as a feature of the optimal policy even though, *a priori*, the policymaker does *not* attach any weight to such an objective. Instead, price stability is closely associated with the attainment of the efficient allocation (which is a more immediate policy objective). But the only way to replicate the (efficient) flexible price allocation when prices are sticky is by making all firms happy with their existing prices, so that the assumed constraints on the adjustment of those prices are effectively non-binding. Aggregate price stability then follows as a consequence of no firm willing to adjust its price.

3.1 Implementation: Optimal Interest Rate Rules

Next we consider some candidate rules for implementing the optimal policy. All of them are consistent with the desired equilibrium outcome. Some, however, are *also* consistent with other suboptimal outcomes. In all cases, and in order to analyze its equilibrium implications, we embed the candidate rule considered in the two equations describing the non-policy block of the basic new Keynesian model introduced in chapter 3. Those two key equations are shown here again for convenience:

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - r_t^n\right) \tag{4}$$

$$\pi_t = \beta\, E_t\{\pi_{t+1}\} + \kappa\, \widetilde{y}_t \tag{5}$$

3.1.1 An Exogenous Interest Rate Rule

Consider the candidate interest rate rule

$$i_t = r_t^n \tag{6}$$

for all t . This is a rule that instructs the central bank to adjust the nominal rate one for one with variations in the natural rate (and only in response to variations in the latter). Such a rule would seem a natural candidate to implement the optimal policy since (6) was shown earlier to be always satisfied in an equilibrium that attains the optimal allocation.

Substituting (6) into (4) and rearranging terms we can represent the equilibrium conditions under rule (6) by means of the system:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A_O} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} \tag{7}$$

where

$$\mathbf{A_O} \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix}$$

Note that $\widetilde{y}_t = \pi_t = 0$ for all t –the outcome associated with the optimal policy– is *a* solution to (7). That solution, however, is *not* unique: it can be shown that one of the two (real) eigenvalues of $\mathbf{A_O}$ always lies in the interval $(0, 1)$, while the second is strictly greater than unity. Given that both $\widetilde{y}_t$ and π_t are non-predetermined, the existence of an eigenvalue outside the unit circle implies the the existence of a multiplicity of equilibria in addition to $\widetilde{y}_t = \pi_t = 0$ for all t.[2] In that case nothing guarantees that the latter allocation will be precisely the one that will emerge as an equilibrium. That shortcoming leads us to consider alternative rules to (6).

3.1.2 An Interest Rate Rule with an Endogenous Component

Let us consider next the following interest rate rule

$$i_t = r_t^n + \phi_\pi \ \pi_t + \phi_y \ \widetilde{y}_t \tag{8}$$

where ϕ_π and ϕ_y are non-negative coefficients determined by the central bank, and describing the strength of the interest rate response to deviations of inflation or the output gap from their target levels.

As above, we can substitute the nominal rate out using the assumed interest rate rule, and represent the equilibrium dynamics by means of a system of difference equations of the form

[2] See, e.g. Blanchard and Kahn (1980).

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A_T} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} \tag{9}$$

where

$$\mathbf{A_T} \equiv \Omega \begin{bmatrix} \sigma & 1-\beta\phi_\pi \\ \sigma\kappa & \kappa+\beta(\sigma+\phi_y) \end{bmatrix}$$

and $\Omega \equiv \frac{1}{\sigma+\phi_y+\kappa\phi_\pi}$.

Once again, the desired outcome ($\widetilde{y}_t = \pi_t = 0$ for all t) is always a solution to the dynamical system (9) and, hence, an equilibrium of the economy under rule (8). Yet, in order for that outcome to be the only (stationary) equilibrium both eigenvalues of matrix $\mathbf{A_T}$ should lie within the unit circle. The size of those eigenvalues now depends on the policy coefficients (ϕ_π, ϕ_y), in addition to the non-policy parameters. If we restrict ourselves to non-negative values for (ϕ_π, ϕ_y), a necessary and sufficient condition for $\mathbf{A_T}$ to have two eigenvalues within the unit circle and, hence, for the equilibrium to be unique, is given by[3]

$$\kappa\ (\phi_\pi - 1) + (1-\beta)\ \phi_y > 0 \tag{10}$$

that is (and roughly speaking): the monetary authority should respond to deviations of inflation and the output gap from their target levels by adjusting the nominal rate with "sufficient strength." Figure 4.1 illustrates graphically the regions of parameter space for (ϕ_π, ϕ_y) associated with determinate and indeterminate equilibria, as implied by condition (10).

Interestingly, and somewhat paradoxically, if condition (10) is satisfied, both the output gap and inflation will be zero and, hence, $i_t = r_t^n$ for all t will hold ex-post. Thus, and in contrast with the case considered above (in which the equilibrium outcome $i_t = r_t^n$ was also taken to be the policy rule), it is the presence of a "threat" of a strong response by the monetary authority to an eventual deviation of the output gap and inflation from target that suffices to rule out any such deviation in equilibrium.

Some economic intuition for the form of condition (10) can be obtained by considering the eventual implications of rule (8) for the nominal rate, were a permanent increase in inflation of size $d\pi$ to occur (and assuming no permanent changes in the natural rate):

[3] See Bullard and Mitra (2002) for a proof.

$$
\begin{aligned}
di &= \phi_\pi \, d\pi + \phi_y \, d\widetilde{y} \\
&= \left(\phi_\pi + \frac{\phi_y \, (1-\beta)}{\kappa} \right) \, d\pi \qquad (11)
\end{aligned}
$$

where the second equality makes use of the long-term relationship between inflation and the output gap implied by (5). Note that condition (10) is equivalent to the term in brackets in (11) being greater than one. Thus, the equilibrium will be unique under interest rate rule (8) whenever ϕ_π and ϕ_y are sufficiently large to guarantee that the real rate eventually rises in the face of an increase in inflation (thus tending to counteract that increase and acting as a stabilizing force). The previous property is often referred to as the *Taylor principle* and, to the extent that it prevents the emergence of multiple equilibria, it is naturally viewed as a desirable feature of any interest rate rule.[4]

3.1.3 A Forward-Looking Interest Rate Rule

In order to illustrate the existence of a multiplicity of policy rules capable of implementing the optimal policy, let us consider the following forward-looking rule

$$
i_t = r_t^n + \phi_\pi \, E_t\{\pi_{t+1}\} + \phi_y \, E_t\{\widetilde{y}_{t+1}\} \qquad (12)
$$

which has the monetary authority adjust the nominal rate in response to variations in *expected* inflation and the *expected* output gap (as opposed to their current values, as assumed in (8)).

Under (12) the implied dynamics are described by the system

$$
\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A_F} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}
$$

where

$$
\mathbf{A_F} \equiv \begin{bmatrix} 1 - \sigma^{-1}\phi_y & -\sigma^{-1}\phi_\pi \\ \kappa(1 - \sigma^{-1}\phi_y) & \beta - \kappa\sigma^{-1}\phi_\pi \end{bmatrix}
$$

[4] See Woodford (2000) for a discussion.

In this case the conditions for a unique equilibrium (i.e. for both eigenvalues of $\mathbf{A_F}$ lying within the unit circle) are twofold and given by[5]

$$\kappa\ (\phi_\pi - 1) + (1 - \beta)\ \phi_y \ > \ 0 \tag{13}$$

$$\kappa\ (\phi_\pi - 1) + (1 + \beta)\ \phi_y \ < \ 2\sigma(1 + \beta) \tag{14}$$

Figure 4.2 represents the determinacy/indeterminacy regions in (ϕ_π, ϕ_y) space, under the baseline calibration for the remaining parameters. Note that in contrast with the "contemporaneous" rule considered in the previous subsection, determinacy of equilibrium under the present forward-looking rule requires that the central bank reacts neither "too strongly" nor "too weakly" to deviations of inflation and/or the output gap from target. Yet, the figure suggests that the kind of overreaction that would be conducive to indeterminacy would require rather extreme values of the inflation and/or output gap coefficients, well above those characterizing empirical interest rate rules.

3.2 Practical Shortcomings of Optimal Policy Rules

In the previous subsection we have provided two examples of interest rate rules that implement the optimal policy, thus guaranteeing that the efficient allocation is attained as the unique equilibrium outcome. While such optimal interest rate rules appear to take a relatively simple form, there exists an important reason why they are unlikely to provide useful practical guidance for the conduct of monetary policy. The reason is that they both require that the policy rate is adjusted one-for-one with the natural rate of interest, thus implicitly assuming observability of the latter variable. That assumption is plainly unrealistic since determination of the natural rate and its movements requires an exact knowledge of (i) the economy's "true model," (ii) the values taken by all its parameters, and (iii) the realized value (observed in real time) of all the shocks impinging on the economy.

Note that a similar requirement would have to be met if, as implied by (8) and (12), the central bank should also adjust the nominal rate in response to deviations of output from the natural level of output, since the latter is also unobservable. That requirement, however, is not nearly as binding as the

[5] Bullard and Mitra (2002) list a third condition, given by the inequality $\phi_y < \sigma(1 + \beta^{-1})$, as necessary for uniqueness. But it can be easily checked that the latter condition is implied by the two conditions (13) and (14).

unobservability of the natural rate of interest, for nothing prevents the central bank from implementing the optimal policy by means of a rule that does not require a systematic response to changes in the output gap. Formally, ϕ_y in (8) or (12) could be set to zero, with uniqueness of equilibrium being still guaranteed by the choice of an inflation coefficient greater than unity (and no greater than $1 + 2\sigma(1 + \beta)\kappa^{-1}$ in the case of the forward-looking rule).

The practical shortcomings of optimal interest rate rules discussed above have led many authors to propose a variety of "simple rules"–understood as rules that a central bank could arguably adopt in practice–and to analyze their properties.[6] In that context, an interest rate rule is generally considered "simple" if it makes the policy instrument a function of observable variables only, and does not require any precise knowledge of the exact model or the values taken by its parameters. The desirability of any given simple rule is thus given to a large extent by its robustness, i.e. its ability to yield a good performance across different models and parameter configurations.

In the following section we analyze two such simple rules –a simple Taylor-type rule, and a constant money growth rule– and assess their performance in the context of our baseline new Keynesian model.

4 Two Simple Monetary Policy Rules

In this section we provide an illustration of how the basic new Keynesian model developed in the previous chapter can be used to assess the performance of two policy rules. A formal evaluation of the performance of a simple rule (relative, say, to the optimal rule or to an alternative simple rule) requires the use of some quantitative criterion. Following the seminal work of Rotemberg and Woodford (1999) much of the literature has adopted a welfare-based criterion, relying on a second-order approximation to the utility losses experienced by the representative consumer as a consequence of deviations from the efficient allocation. As shown in the appendix, under the assumptions made in the present chapter (which guarantee the optimality of the flexible price equilibrium), that approximation yields the following

[6] The volume edited by John Taylor (1999) contains several important contributions in that regard.

welfare loss function

$$\mathbb{W} = \frac{1}{2}\, E_0 \sum_{t=0}^{\infty} \beta^t \left(\left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) \widetilde{y}_t^2 + \frac{\epsilon}{\lambda}\, \pi_t^2 \right)$$

where welfare losses are expressed in terms of the equivalent permanent consumption decline, measured as a fraction of steady state consumption.

The average welfare loss per period is thus given by the following linear combination of the variances of the output gap and inflation

$$\mathbb{L} = \frac{1}{2} \left[\left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) var(\widetilde{y}_t) + \frac{\epsilon}{\lambda}\, var(\pi_t) \right]$$

Note that the relative weight of output gap fluctuations in the loss function is increasing in σ, φ and α. The reason is that larger values of those "curvature" parameters amplify the effect of any given deviation of output from its natural level on the size of the gap between the marginal rate of substitution and the marginal product of labor, which is a measure of the economy's aggregate inefficiency. On the other hand, the weight of inflation fluctuations is increasing in the elasticity of substitution among goods ϵ –since the latter amplifies the welfare losses caused by any given price dispersion– and the degree of price stickiness θ (which is inversely related to λ), which amplifies the degree of price dispersion resulting from any given deviation from zero inflation.

Given a policy rule and a calibration of the model's parameters, one can determine the implied variance of inflation and the output gap and the corresponding welfare losses associated with that rule (relative to the optimal allocation). That procedure is illustrated next through the analysis of two simple rules.

4.1 A Taylor-type Interest Rate Rule

Let us first consider the following interest rule, in the spirit of Taylor (1993)

$$i_t = \rho + \phi_\pi\, \pi_t + \phi_y\, \widehat{y}_t \tag{15}$$

where $\widehat{y}_t \equiv \log(Y_t/Y)$ denotes the log deviation of output from its steady state and where $\phi_\pi > 0$ and $\phi_y > 0$ are assumed to satisfy the determinacy condition (10). Again, the choice of intercept $\rho \equiv -\log\beta$ is consistent with a zero inflation steady state.

Note that we can rewrite (15) in terms of the output gap as

$$i_t = \rho + \phi_\pi \ \pi_t + \phi_y \ \widetilde{y}_t + v_t \tag{16}$$

where $v_t \equiv \phi_y \ \widehat{y}_t^n$. The resulting equilibrium dynamics are thus identical to those of the interest rate rule analyzed in chapter 3, with v_t now re-interpreted as a driving force proportional to the deviations of natural output from steady state, instead of an exogenous monetary policy shock. Note that the variance of the "shock" v_t is no longer exogenous, but increasing in ϕ_y, the coefficient determining the response of the monetary authority to fluctuations in output. Formally, the equilibrium dynamics are described by the system

$$\left[\begin{array}{c} \widetilde{y}_t \\ \pi_t \end{array} \right] = \mathbf{A_T} \ \left[\begin{array}{c} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{array} \right] + \mathbf{B_T} \ (\widehat{r}_t^n - v_t)$$

where $\mathbf{A_T}$ and $\mathbf{B_T}$ are defined as in chapter 3. Assuming that variations in the technology parameter a_t represent the only driving force in the economy, and are described by a stationary AR(1) process with autoregressive coefficient ρ_a, we have:

$$\begin{aligned} \widehat{r}_t^n - v_t &= -\sigma\psi_{ya}^n(1-\rho_a) \ a_t - \phi_y\psi_{ya}^n \ a_t \\ &= -\psi_{ya}^n \ [\sigma(1-\rho_a) + \phi_y] \ a_t \end{aligned}$$

where, as in chapter 3, $\psi_{ya}^n \equiv \frac{1+\varphi}{\sigma+\varphi+\alpha(1-\sigma)} > 0$. From the analysis in the previous chapter, we know that the variance of the output gap and inflation under a rule of the form (16) is proportional to that of $\mathbf{B}_T \ (\widehat{r}_t^n - v_t)$, which is strictly increasing in ϕ_y. Hence, a policy seeking to stabilize output by responding aggressively to deviations in that variable from steady state (or trend) is bound to lower the representative consumer's utility, by increasing the variance of the output gap and inflation.[7]

The left panel of Table 4.1 displays some statistics for four different calibrations of rule (15), corresponding to alternative configurations for ϕ_π and ϕ_y. The first column corresponds to the calibration proposed by Taylor (1993) as a good approximation to the interest rate policy of the Fed during

[7]Notice that in this simple example the optimal allocation can be attained by setting $\phi_y = -\sigma(1-\rho_a)$. In that case, our simple rule is equivalent to the optimal rule $i_t = r_t^n + \phi_\pi \pi_t$.

the Greenspan years.[8] The second and third rules assume no response to output fluctuations, and with a very aggressive anti-inflation stance in the case of the third rule ($\phi_\pi = 5$). Finally, the fourth rule assumes a strong output-stabilization motive ($\phi_y = 1$). The remaining parameters are calibrated at their baseline values, as introduced in the previous chapter.

For each version of the Taylor rule, Table 4.1 shows the implied standard deviations of the output gap and (annualized) inflation, both expressed in percent terms, as well as the welfare losses resulting from the associated deviations from the efficient allocation, expressed as a fraction of steady state consumption. Several results stand out. First, in a way consistent with the analysis above, versions of the rule that involve a systematic response to output variations generate larger fluctuations in the output gap and inflation and, hence, larger welfare losses. Those losses are moderate (0.3 percent of steady state consumption) under Taylor's original calibration, but they become substantial (close to 2 percent of steady state consumption) when the output coefficient ϕ_y is set to unity. Secondly, the smallest welfare losses are attained when the monetary authority responds to changes in inflation only. Furthermore, those losses (as well as the underlying fluctuations in the output gap and inflation) become smaller as the strength of that response increases. Hence, and at least in the context of the basic new Keynesian model considered here, a simple Taylor-type rule that responds aggressively to movements in inflation can approximate arbitrarily well the optimal policy.

4.2 A Constant Money Growth Rule

Next we consider a simple rule consisting of a constant growth rate for the money supply, a rule generally associated with Friedman (1960). Without loss of generality, we assume a zero rate of growth of the money supply, which is consistent with zero inflation in the steady state (given the absence of secular growth). Formally,

$$\Delta m_t = 0$$

for all t.

[8] Taylor's proposed coefficient values were 1.5 for inflation and 0.5 for output, based on a specification with *annualized* inflation and interest rates. Our choice of $\phi_y = 0.5/4$ is consistent with Taylor's proposed calibration since both i_t and π_t in our model are expressed in quarterly rates.

Once again, the assumption of a monetary rule requires that equilibrium conditions (4) and (5) be supplemented with a money market clearing condition. Here we take the latter to be of the form

$$l_t = y_t - \eta \; i_t - \zeta_t$$

where $l_t \equiv m_t - p_t$ denotes (log) real balances, and ζ_t is an exogenous money demand shock following the process

$$\Delta\zeta_t = \rho_\zeta \; \Delta\zeta_{t-1} + \varepsilon_t^\zeta$$

It is convenient to rewrite the money market equilibrium condition in terms of deviations from steady state as follows:

$$\widehat{l}_t = \widetilde{y}_t + \widehat{y}_t^n - \eta \; \widehat{i}_t - \zeta_t$$

Letting $l_t^+ \equiv l_t + \zeta_t$ denote (log) real balances adjusted by the exogenous component of money demand, we have

$$\widehat{i}_t = \frac{1}{\eta} \; (\widetilde{y}_t + \widehat{y}_t^n - \widehat{l}_t^+)$$

In addition, using the definition of l_t^+ together with the assumed rule $\Delta m_t = 0$, we have

$$\widehat{l}_{t-1}^+ = \widehat{l}_t^+ + \pi_t - \Delta\zeta_t$$

Combining the previous two equations with (4) and (5) to substitute out the nominal rate, the equilibrium dynamics under a constant money growth rule can be summarized by the system

$$\mathbf{A_{M,0}} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \\ \widehat{l}_{t-1}^+ \end{bmatrix} = \mathbf{A_{M,1}} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ \widehat{l}_t^+ \end{bmatrix} + \mathbf{B_M} \begin{bmatrix} \widehat{r}_t^n \\ \widehat{y}_t^n \\ \Delta\zeta_t \end{bmatrix}$$

where $\mathbf{A_{M,0}}$, $\mathbf{A_{M,1}}$ and $\mathbf{B_M}$ are defined as in the previous chapter.

The right hand panel of Table 4.1 reports the standard deviation of the output gap and inflation, as well as the implied welfare losses, under a constant money growth rule. Two cases are considered, depending on whether money demand is assumed to be subject or not to exogenous disturbances. In both cases the natural output and the natural rate of interest vary in response to technology shocks (according to the baseline calibration of the

latter introduced in the previous chapter). When money demand shocks are allowed for, the corresponding process for $\Delta\zeta$ is calibrated by estimating an AR(1) process for the (first-differenced) residual of a money demand function for the period 1989:I-2004:IV-a period characterized by substantial instability in the demand for money–computed using an interest rate-semielasticity $\eta = 4$ (see discussion in the previous chapter). The estimated standard deviation for the residual of the AR(1) process is $\sigma_\zeta = 0.0063$ while the estimated AR(1) coefficient is $\rho_\zeta = 0.6$.

Notice that in the absence of money demand shocks, a constant money growth rule delivers a performance comparable, in terms of welfare losses, to a Taylor rule with coefficients $\phi_\pi = 1.5$ and $\phi_y = 0$. Yet, when the calibrated money demand shock is introduced the performance of a constant money growth rule deteriorates considerably, with the volatility of both the output gap and inflation rising to a level associated with welfare losses above those of the baseline Taylor rule. Thus, and not surprisingly, the degree of stability of money demand is a key element in determining the desirability of a rule that focuses on the control of a monetary aggregate.

5 Notes on the Literature

An early detailed discussion of the case for price stability in the basic new Keynesian model can be found in Goodfriend and King (1997). Svensson (1997) contains an analysis of the desirability of inflation targeting strategies, using a not-fully-microfounded model.

When deriving the optimal policy we have assumed no inherited dispersion of prices across firms. A rigorous analysis of the optimal monetary policy in the case of an initial non-degenerate price distribution can be found in Yun (2005).

Taylor (1993) introduced the simple formula commonly known as the Taylor rule, as providing a good approximation to Fed policy in the early Greenspan years. Judd and Rudebusch (1998) and Clarida, Galí, and Gertler (2000) estimate alternative versions of the Taylor rule, and examined its (in)stability over the postwar period. Taylor (1999) uses the rule calibrated for the Greenspan years as a benchmark for the evaluation of monetary policy during other episodes over the postwar period. Orphanides (2003) argues that the bulk of the deviations from the baseline Taylor rule observed in the pre-Volcker era may have been the result of large biases in real time measures

of the output gap.

Key contributions to the literature on the properties of alternative simple rules can be found in the papers contained in the volume edited by Taylor (1999). In particular, the paper by Rotemberg and Woodford (1999) derives a second order approximation to the utility of the representative consumer. Chapter 6 in Woodford (2003) provides a detailed discussion of welfare-based evaluations of policy rules.

Appendix. A Second Order Approximation to Household's Welfare: the Case of an Undistorted Steady State

In the present appendix we derive a second order approximation to the utility of the representative consumer when the economy remains in a neighborhood of an efficient steady state, in a way consistent with the assumptions made in the present chapter. The generalization to the case of a distorted steady state is left for chapter 5.

We start by deriving a second order approximation of utility around a given steady state allocation. Below we make frequent use of the following second order approximation of relative deviations in terms of log deviations:

$$\frac{Z_t - Z}{Z} \simeq \widehat{z}_t + \frac{1}{2}\,\widehat{z}_t^2$$

where $\widehat{z}_t \equiv z_t - z$ is the log deviation from steady state for a generic variable z_t. All along we assume that utility is separable in consumption and hours (i.e., $U_{cn} = 0$). In order to lighten the notation we define $U_t \equiv U(C_t, N_t)$, $U_t^n \equiv U(C_t^n, N_t^n)$, and $U \equiv U(C, N)$.

The second order Taylor expansion of U_t around a steady state (C, N) yields

$$U_t - U \simeq U_c C\,\left(\frac{C_t - C}{C}\right) + U_n N\,\left(\frac{N_t - N}{N}\right) + \frac{1}{2}U_{cc}C^2\,\left(\frac{C_t - C}{C}\right)^2 + \frac{1}{2}U_{nn}N^2\,\left(\frac{N_t - N}{N}\right)^2$$

In terms of log deviations,

$$U_t - U \simeq U_c C\,\left(\widehat{y}_t + \frac{1-\sigma}{2}\,\widehat{y}_t^2\right) + U_n N\,\left(\widehat{n}_t + \frac{1+\varphi}{2}\,\widehat{n}_t^2\right)$$

where $\sigma \equiv -\frac{U_{cc}}{U_c}C$ and $\varphi \equiv \frac{U_{nn}}{U_n}N$, and where we have made use of the market clearing condition $\widehat{c}_t = \widehat{y}_t$.

The next step consists in rewriting $\widehat{n}_t$ in terms of output. Using the fact that $N_t = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di$, we have

$$(1-\alpha)\,\widehat{n}_t = \widehat{y}_t - a_t + d_t$$

where $d_t \equiv (1-\alpha)\log \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di$. The following lemma shows that d_t is proportional to the cross-sectional variance of relative prices.

Lemma 1: in a neighborhood of a symmetric steady state, and up to a second order approximation, we have $d_t = \frac{\epsilon}{2}\ var_i\{p_t(i)\}$.

Proof: Let $\widehat{p}_t(i) \equiv p_t(i) - p_t$. Notice that,

$$\begin{aligned}\left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} &= \exp\left[(1-\epsilon)\ \widehat{p}_t(i)\right] \\ &= 1 + (1-\epsilon)\ \widehat{p}_t(i) + \frac{(1-\epsilon)^2}{2}\ \widehat{p}_t(i)^2\end{aligned}$$

Note that from the definition of P_t, we have $1 = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} di$. A second order approximation to this expression thus implies

$$E_i\{\widehat{p}_t(i)\} = \frac{(\epsilon-1)}{2}\ E_i\{\widehat{p}_t(i)^2\}$$

In addition, a second order approximation to $\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}}$ yields:

$$\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} = 1 - \frac{\epsilon}{1-\alpha}\ \widehat{p}_t(i) + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)^2\ \widehat{p}_t(i)^2$$

Combining the two previous results, it follows that

$$\begin{aligned}\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di &= 1 + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)\frac{1}{\Theta}\ E_i\{\widehat{p}_t(i)^2\} \\ &= 1 + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)\frac{1}{\Theta}\ var_i\{p_t(i)\}\end{aligned}$$

$\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$, and where the last equality follows from the observation that, up to second order,

$$\begin{aligned}\int_0^1 (p_t(i) - p_t)^2\ di &\simeq \int_0^1 (p_t(i) - E_i\{p_t(i)\})^2\ di \\ &\equiv var_i\{p_t(i)\}\end{aligned}$$

Finally, using the definition of d_t and up to a second order approximation we have

$$d_t \equiv (1-\alpha)\log\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di \simeq \frac{\epsilon}{2\Theta}\ var_i\{p_t(i)\}$$

QED.

Now we can rewrite period t utility as:

$$U_t - U = U_c C\ \left(\widehat{y}_t + \frac{1-\sigma}{2}\ \widehat{y}_t^2\right) + \frac{U_n N}{1-\alpha}\ \left(\widehat{y}_t + \frac{\epsilon}{2\Theta}\ var_i\{p_t(i)\} + \frac{1+\varphi}{2(1-\alpha)}\ (\widehat{y}_t - a_t)^2\right) + t.i.p.$$

where $t.i.p.$ stands for "terms independent of policy".

Efficiency of the steady state implies $-\frac{U_n}{U_c} = MPN$. Thus, and using the fact that $MPN = (1-\alpha)(Y/N)$ and $Y = C$ we can write

$$\begin{aligned}
\frac{U_t - U}{U_c C} &\simeq -\frac{1}{2}\ \left[\frac{\epsilon}{\Theta}\ var_i\{p_t(i)\} - (1-\sigma)\ \widehat{y}_t^2 + \frac{1+\varphi}{1-\alpha}\ (\widehat{y}_t - a_t)^2\right] + t.i.p. \\
&= -\frac{1}{2}\ \left[\frac{\epsilon}{\Theta}\ var_i\{p_t(i)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\ \widehat{y}_t^2 - 2\left(\frac{1+\varphi}{1-\alpha}\right)\ \widehat{y}_t a_t\right] + t.i.p. \\
&= -\frac{1}{2}\ \left[\frac{\epsilon}{\Theta}\ var_i\{p_t(i)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\ (\widehat{y}_t^2 - 2\widehat{y}_t\ \widehat{y}_t^n)\right] + t.i.p. \\
&= -\frac{1}{2}\ \left[\frac{\epsilon}{\Theta}\ var_i\{p_t(i)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\ \widetilde{y}_t^2\right] + t.i.p.
\end{aligned}$$

where $\widehat{y}_t^n \equiv y_t^n - y^n$, and where we have used the fact that $\widehat{y}_t^n = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}\ a_t$ .and $\widehat{y}_t - \widehat{y}_t^n = \widetilde{y}_t$.

Accordingly, we can write a second order approximation to the consumer's welfare losses, expressed as a fraction of steady state consumption (and up to additive terms independent of policy) as:

$$\begin{aligned}
\mathbb{W} &= E_0 \sum_{t=0}^{\infty} \beta^t\ \left(\frac{U_t - U}{U_c C}\right) \\
&= -\frac{1}{2}\ E_0 \sum_{t=0}^{\infty} \beta^t\ \left(\frac{\epsilon}{\Theta}\ var_i\{p_t(i)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\ \widetilde{y}_t^2\right)
\end{aligned}$$

The final step consists in rewriting the terms involving the price dispersion variable as a function of inflation. In order to do so we make use of the following lemma

Lemma 2: $\sum_{t=0}^{\infty} \beta^t \ var_i\{p_t(i)\} = \frac{\theta}{(1-\beta\theta)(1-\theta)} \sum_{t=0}^{\infty} \beta^t \ \pi_t^2$
Proof: Woodford (2003, chapter 6)

Using the fact that $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \ \Theta$ we can combine the previous lemma with the expression for the welfare losses above to obtain

$$\mathbb{W} = -\frac{1}{2} \ E_0 \sum_{t=0}^{\infty} \beta^t \ \left[\left(\frac{\epsilon}{\lambda} \right) \ \pi_t^2 + \left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) \ \widetilde{y}_t^2 \right]$$

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Exercises

1. Inflation Targeting with Noisy Data

Consider a model economy whose output gap and inflation dynamics are described by the system:

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \kappa \; \widetilde{y}_t \tag{17}$$

$$\widetilde{y}_t = -\frac{1}{\sigma} \; (i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\} \tag{18}$$

where all variables are defined as in the text. The natural rate r_t^n is assumed to follow the exogenous process

$$r_t^n - \rho = \rho_r \; (r_{t-1}^n - \rho) + \varepsilon_t$$

where $\{\varepsilon_t\}$ is a white noise process and $\rho_r \; \in [0, 1)$.

Suppose that inflation is measured with some *i.i.d.* error ξ_t , i.e., $\pi_t^o = \pi_t + \xi_t$ where π_t^o denotes measured inflation. Assume that the central bank follows the rule

$$i_t = \rho + \phi_\pi \; \pi_t^o \tag{19}$$

a) Solve for the equilibrium processes for inflation and the output gap under the rule (19) (hint: you may want to start analyzing the simple case of $\rho_r = 0$).

b) Describe the behavior of inflation, the output gap, and the nominal rate when ϕ_π approaches infinity.

c) Determine the size of the inflation coefficient that minimizes the variance of actual inflation.

2. Monetary Policy and the Effects of Technology Shocks

Consider a new Keynesian economy with equilibrium conditions:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} \; (i_t - E_t\{\pi_{t+1}\} - \rho) \tag{20}$$

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \kappa \; (y_t - y_t^n) \tag{21}$$

where all variables are defined as in the text.

Monetary policy is described by a simple rule of the form

$$i_t = \rho + \phi_\pi \ \pi_t$$

where $\phi_\pi > 1$. Labor productivity is given by

$$y_t - n_t = a_t$$

where a_t is an exogenous technology parameter which evolves according to

$$a_t = \rho_a \ a_{t-1} + \varepsilon_t$$

where $\rho_a \in [0, 1)$ and $\{\varepsilon_t\}$ is an i.i.d. process.

The underlying RBC model is assumed to imply a natural level of output proportional to technology

$$y_t^n = \psi_y \ a_t$$

where $\psi_y > 1$.

a) Describe in words where (20) and (21) come from.

b) Determine the equilibrium response of output, employment, and inflation to a technology shock. (*hint*: guess that each endogenous variable will be proportional to the contemporaneous value of technology).

c) Describe how those responses depend on the value of ϕ_π and κ. Provide some intuition. What happens when $\phi_\pi \longrightarrow \infty$? What happens as we change the degree of price rigidities?

d) Analyze the joint response of employment and output to a technology shock and discuss briefly the implications for our assessment of the role of technology as a source of business cycles.

3. Interest Rate vs Money Supply Rules

Consider an economy described by the equilibrium conditions:

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma} \ (i_t - E_t\{\pi_{t+1}\} - r_t^n)$$

$$\pi_t = \beta \ E_t\{\pi_{t+1}\} + \kappa \ \widetilde{y}_t$$

$$m_t - p_t = y_t - \eta \ i_t$$

where all variables are defined as in the text. Both y_t^n and r_t^n evolve exogenously, independently of monetary policy.

The central bank seeks to minimize a loss function of the form

$$\alpha \ var(\widetilde{y}_t) + var(\pi_t)$$

a) Show how the optimal policy could be implemented by means of an interest rate rule

b) Show that a rule requiring a constant money supply will generally be suboptimal. Explain. (hint: derive the path of money under the optimal policy)

c) Derive a money supply rule that would implement the optimal policy.

4. Optimal Monetary Policy with Price-Setting in Advance

Consider an economy where the representative consumer maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \ U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

subject to a sequence of dynamic budget constraints

$$P_t \ C_t + M_t + Q_t B_t \leq M_{t-1} + B_{t-1} + W_t \ N_t + T_t$$

and where all variables are defined as in the main text.

Assume that period utility is given by:

$$U(C_t, \frac{M_t}{P_t}, N_t) = \log C_t + \log \frac{M_t}{P_t} - \frac{{N_t}^{1+\varphi}}{1+\varphi} \tag{22}$$

Firms are monopolistically competitive, each producing a differentiated good whose demand is given by $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t$. Each firm has access to the linear production function

$$Y_t(i) = A_t \ N_t(i) \tag{23}$$

where productivity evolves according to:

$$\frac{A_t}{A_{t-1}} = (1+\gamma_a) \ \exp\{\varepsilon_t\}$$

with $\{\varepsilon_t\}$ being an i.i.d., normally distributed process with mean 0 and variance σ_ε^2.

The money supply varies exogenously according to the process

$$\frac{M_t}{M_{t-1}} = (1+\gamma_m)\exp\{u_t\} \tag{24}$$

where $\{u_t\}$ is an i.i.d., normally distributed process with mean 0 and variance σ_u^2.

Finally, we assume that all output is consumed, so that in equilibrium $Y_t = C_t$ for all t.

a) Derive the optimality conditions for the problem facing the representative consumer.

b) Assume that firms are monopolistically competitive, each producing a differentiated good. Each period, after observing the shocks, firms set the price of their good in order to maximize current profit

$$Y_t(i)\left(P_t(i) - \frac{W_t}{A_t}\right)$$

subject to the demand schedule above. Derive the optimality condition associated with the firm's problem.

c) Show that the equilibrium levels of aggregate employment, output, and inflation are given by

$$N_t = \left(1 - \frac{1}{\epsilon}\right)^{\frac{1}{1+\varphi}} \equiv \Theta$$

$$Y_t = \Theta\ A_t$$

$$\pi_t = (\gamma_m - \gamma_a) + u_t - \varepsilon_t$$

d) Discuss how utility depends on the two parameters describing monetary policy, γ_m and σ_u^2 (recall that the nominal interest rate is constrained to be non-negative, i.e., $i_t \geq 0$ for all t). Show that the optimal policy must satisfy the Friedman rule and discuss alternative ways of supporting that rule in equilibrium.

e) Next let us assume that each period firms have to set the price in advance, i.e., before the realization of the shocks. In that case they will choose a price in order to maximize the discounted profit

$$E_{t-1}\left\{Q_{t-1,t}\ Y_t(i)\left(P_t(i) - \frac{W_t}{A_t}\right)\right\}$$

subject to the demand schedule $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t$, where $Q_{t-1,t} \equiv \beta \frac{C_{t-1}}{C_t} \frac{P_{t-1}}{P_t}$ is the stochastic discount factor. Derive the first order condition of the firm's problem and solve (exactly) for the equilibrium levels of employment, output and real balances.

f) Evaluate expected utility at the equilibrium values of output, real balances and employment:

g) Consider the class of money supply rules of the form (24) such that $u_t = \phi_\varepsilon \ \varepsilon_t + \phi_v \ \nu_t$, where $\{\nu_t\}$ is a normally distributed i.i.d. process with zero mean and unit variance, and independent of $\{\varepsilon_t\}$ at all leads and lags. Notice that within that family of rules, monetary policy is fully described by three parameters: γ_m, ϕ_ε, and ϕ_v. Determine the values of those parameters that maximize expected utility, subject to the constraint of a non-negative nominal interest rate. Show that the resulting equilibrium under the optimal policy replicates the flexible price equilibrium analyzed above.

5. A Price Level Based Interest Rate Rule

Consider an economy described by the equilibrium conditions:

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}\ (i_t - E_t\{\pi_{t+1}\} - r_t^n)$$

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t$$

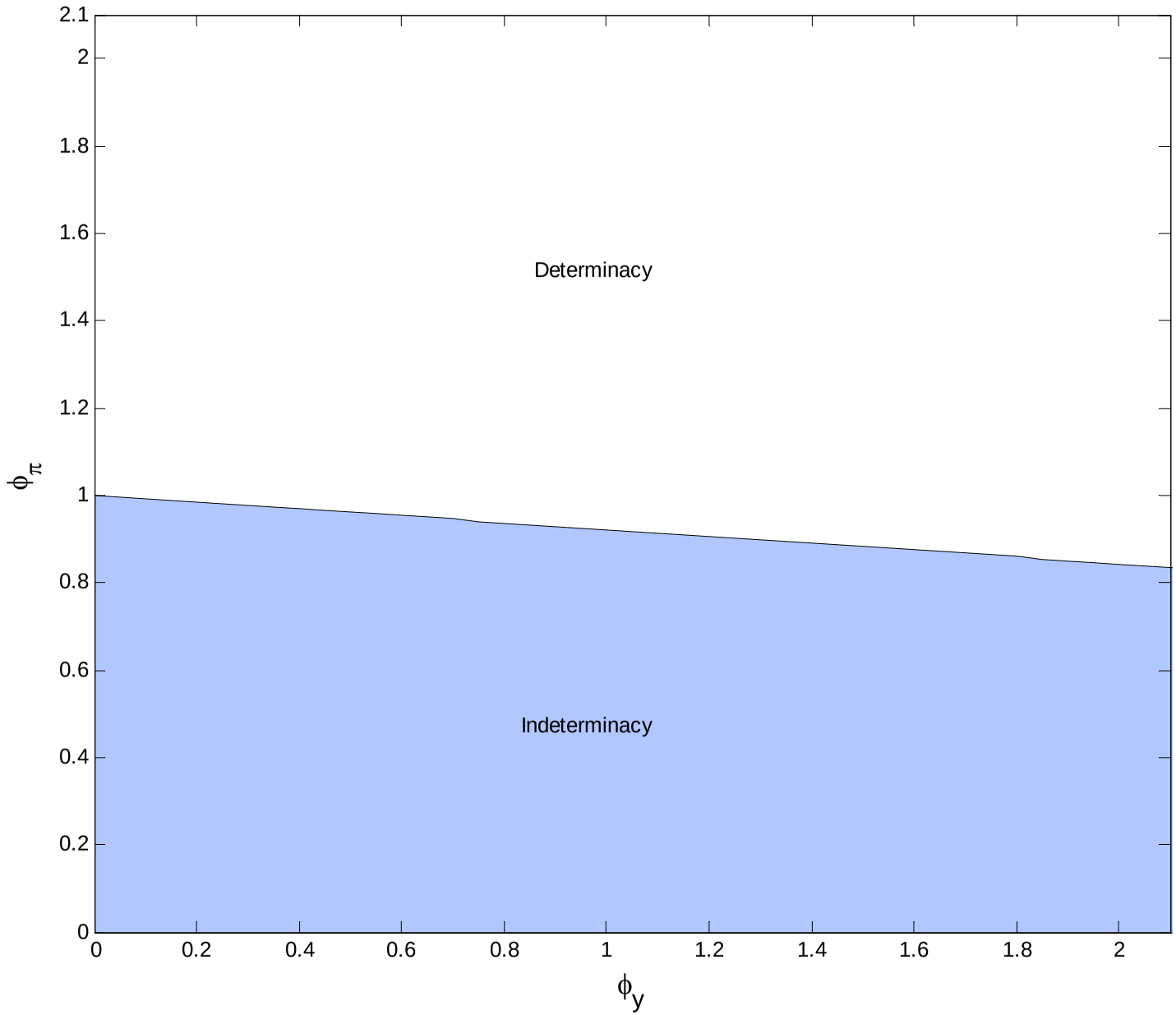
Show that the interest rate rule

$$i_t = r_t^n + \phi_p\ \widehat{p}_t$$

where $\widehat{p}_t \equiv p_t - p^*$, where p^* is a price level target generates a unique stationary equilibrium if and only if $\phi_p > 0$.

Table 4.1: Evaluation of Simple Monetary Policy Rules						
	Taylor Rule				*Constant Money Growth*	
ϕ_π	1.5	1.5	5	1.5	-	-
ϕ_y	0.125	0	0	1	-	-
$(\sigma_\zeta, \rho_\zeta)$	-	-	-	-	$(0, 0)$	$(0.0063, 0.6)$
$\sigma(\widetilde{y})$	0.55	0.28	0.04	1.40	1.02	1.62
$\sigma(\pi)$	2.60	1.33	0.21	6.55	1.25	2.77
$welfare\ loss$	0.30	0.08	0.002	1.92	0.08	0.38

Figure 4.1


Determinacy
Indeterminacy
ϕ_π
ϕ_y
2.1
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
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0.8
1
1.2
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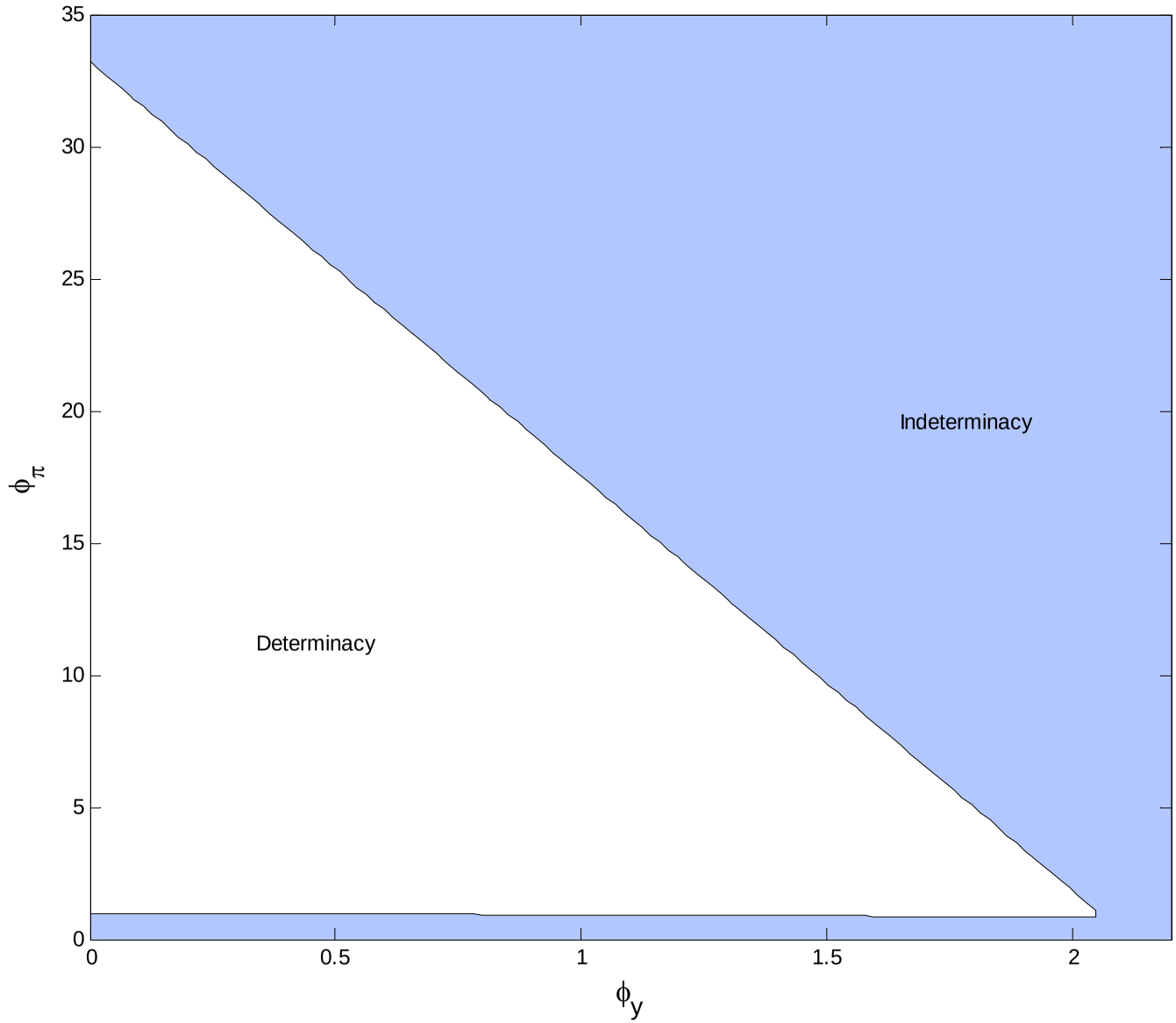
1.6
1.8
2

Figure 4.2


35
30
25
20
15
10
5
0
0
0.5
1
1.5
2
ϕ_π
ϕ_y
Indeterminacy
Determinacy