Chapter2:a classical monetary model

Assumption:

Perfect competition;

Fully flexible prices.

Conclusion:

Monetary policy is neutral.(n_t, y_t,r_t,w_t are all determined by a_t, independent of monetary policy)

By solving the real equilibrium, we get the variables in terms of exogenous variable a_t,

$$y_t = \psi_{ya} a_t + \vartheta_y$$
$$n_t = \psi_{na} a_t + \vartheta_n$$
$$w_t^r = \psi_{wa} a_t + \vartheta_w$$

Monetary policy and price level determination:

(1)an exogenous path for nominal interest rate: the nominal interest rate as an exogenous stationary process $\{i_t\}$.

With the existence of sunspot shocks,there is indeterminacy in price level and other nominal variables such as money supply or wage.

(2)A simple inflation-based interest rate rule: $i_t = \rho + \phi_\pi \pi_t$

Case 1： $\phi_\pi > 1$

We have **Taylor principle**:central banks need to adjust nominal interest rates more than one for one in response to changes in inflation, for the price level to be uniquely determined.

Case 2: $\phi_\pi \leq 1$

Again,with sunspot shocks, inflation and price level are indetermined.

(3)an exogenous path for the money supply:it does not respond to other economic variables.

Price level can be uniquely determined.

It contrast with the empirical findings:sluggish response of price level to monetary policy and **liquidity effect**.

(4)optimal monetary policy

Among all the possible paths, no one is better than any other.

Money in the utility function:

(1)separable utility: Neither $U_{C,t}$ nor $U_{N,t}$ depend on the level of real balance.

In this economy, it is as in the cashless economy, that monetary policy does not affect any real variables.

(2)Nonseparable utility:

In the particular case of $\nu = \sigma$,money is neutral.

In the case where $\nu \neq \sigma$,monetary policy is not neutral.

Different monetary policy rules $\Longrightarrow$ Impacts on nominal interest rates $\Longrightarrow$ Real balances $\Longrightarrow$ Labor supply and output.

(3)Optimal monetary policy:

Friedman rule: central banks keep the short term nominal rate constant at a zero level.

Chapter3:the basic new Keynesian model

Assumption: two departures from the classical monetary economy.
Imperfect competition in the goods market, Price rigidity.

Households: differentiated goods.

Step1:optimal (static) expenditure allocation;

$$\max_{C_t(i)} C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$s.t. \quad \int^1 P_t(i) C_t(i) di \equiv Z_t$$

Step2:intertemporal problem.

$$\max E_t \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

$$s.t. \int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t$$

Firms:

(4)price setting under monopolistic competition and flexible prices:there exists markup and inefficiently low level of employment and output,

(5)Price setting under sticky prices: $\hat{p}_t = \theta \hat{p}_{t-1} + (1-\theta) \hat{p}_t^*$.

Optimal price setting:

$$\hat{p}_t^* = (1-\beta\theta)(\Theta \widehat{mc}_t + \hat{p}_t) + \beta\theta E_t \hat{p}_{t+1}^*; \quad \hat{\pi}_t = \beta E_t \{\hat{\pi}_{t+1}\} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \Theta \cdot \widehat{mc}_t$$

Output gap: $\tilde{y}_t = \hat{y}_t - \hat{y}_t^n$,which is resulting from the sticky price.

NKPC: $\hat{\pi}_t = \beta E_t \{\hat{\pi}_{t+1}\} + \kappa \tilde{y}_t$,it determines inflation given a path for the output gap;

DIS: $\tilde{y}_t = -\frac{1}{\sigma}(\hat{\imath}_t - E_t\{\hat{\pi}_{t+1}\} - \hat{r}_t^n) + E_t\{\tilde{y}_{t+1}\}$ it determines output gap given a path for exogenous natural rate and the actual real rate.

Equilibrium dynamics under monetary policy rules:

Under an interest rate rule:

$$\hat{\imath}_t = \phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t + v_t$$

$$\begin{bmatrix} \tilde{y}_t \\ \hat{\pi}_t \end{bmatrix} = A_T \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\hat{\pi}_{t+1}\} \end{bmatrix} + B_T (\hat{r}_t^n - v_t)$$

To get unique solution, $\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$.

Therefore, we can analyze the effects of a monetary policy shock or a technology shock.

Under an exogenous money supply: $\Delta \hat{m}_t$

$$A_{M,0} \begin{bmatrix} \tilde{y}_t \\ \hat{\pi}_t \\ \hat{\imath}_{t-1} \end{bmatrix} = A_{M,1} \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\hat{\pi}_{t+1}\} \\ \hat{\imath}_t \end{bmatrix} + B_M \begin{bmatrix} \hat{r}_t^n \\ \hat{y}_t^n \\ \Delta \hat{m}_t \end{bmatrix}$$

To get a stationary solution, $A_M \equiv A_{M,0}^{-1} A_{M,1}$

Therefore, we can analyze the effects of a monetary policy shock or a technology shock.

Chapter 4. Monetary Policy Design in the Basic New Keynesian Model

Assumption

(1)With the reference of the efficient allocation under

- monopolistic competition and ‡flexible prices
- with a subsidy to correct the distortion of monopolistic competition.

(2)When prices are sticky, the efficient allocation can be obtained by means of a policy that fully stabilizes the price level.

Objective of the optimal monetary policy

optimal condition

$$C_t(i) = C_t, \text{ all } i \in [0,1] \qquad (1)$$
$$N_t(i) = N_t, \text{ all } i \in [0,1] \qquad (2)$$
$$-\frac{U_{N,t}}{U_{C,t}} = MPN_t \qquad (3)$$

$$MPN_t \equiv (1-\alpha)A_t N_t^{-\alpha}$$

	distortion	subsidy policy
Distortions unrelated to sticky prices: monopolistic competition	$P_t = \mu \frac{W_t}{MPN_t} \quad \mu \equiv \frac{\varepsilon}{\varepsilon-1} > 1$ $-\frac{U_{N,t}}{U_{C,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mu} < MPN_t$	$P_t = \mu \frac{(1-\tau)W_t}{MPN_t}$. $-\frac{U_{N,t}}{U_{C,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mu(1-\tau)}$. if $\mu(1-\tau) = 1$, or setting $\tau = 1/\varepsilon$.
Distortions associated with the presence of staggered price setting	Average markup varies over time $\mu_t = \frac{P_t}{(1-\tau)(W_t/MPN_t)} = \frac{P_t\mu}{W_t/MPN_t}$ $-\frac{U_{N,t}}{U_{C,t}} = \frac{W_t}{P_t} = MPN_t\frac{\mu}{\mu_t}$ if $\mu_t \neq \mu$.	
	Staggered price setting $P_t(i) \neq P_t(j)$ $C_t(i) \neq C_t(j)$ $N_t(i) \neq N_t(j)$	

Optimal monetary policy

(1)Two features of the optimal policy

Stablizing output is not desirable in and of itself. As usually $\hat{y}_t \neq \tilde{y}_t$,

and $\tilde{y}_t = 0$ implies $y_t = y_t^n$ for all t, where the natural level of output is subject to technology shocks.

Price stability emerges as a feature of the optimal policy even though, a priori, the policy maker does not attach any weight to such an objective.

(2) Optimal Interest Rate Rules

	Analysis	Conclusion
An exogenous interest rate rule	$\hat{\imath}_t = \hat{r}_t^n$ $\begin{bmatrix} \tilde{y}_t \\ \hat{\pi}_t \end{bmatrix} = A_0 \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\hat{\pi}_{t+1}\} \end{bmatrix}$ $A_0 \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix}$	solutions are not unique yt_tuta and pit_hat are nonpredetermined, the existence of an eigenvalue outside the unit circle implies the existence of a multiplicity of equilibria. No garantee on the realisation of yt_tuta=pit_hatt = 0 for all t.
An interest rate rule with an endogenous component	$\hat{\imath}_t = \hat{r}_t^n + \phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t$ $\begin{bmatrix} \tilde{y}_t \\ \hat{\pi}_t \end{bmatrix} = A_T \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\hat{\pi}_{t+1}\} \end{bmatrix}$ $A_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \kappa\sigma & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}$ $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$	The desired outcome (yt_tuta=pit_hat = 0 for all t) is always a solution. $\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0.$ make sure the uniqueness of solution.
A forward-looking interest rate rule	$\hat{\imath}_t = \hat{r}_t^n + \phi_\pi E_t\{\hat{\pi}_{t+1}\} + \phi_y E_t\{\tilde{y}_{t+1}\}$ $\begin{bmatrix} \tilde{y}_t \\ \hat{\pi}_t \end{bmatrix} = A_F \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\hat{\pi}_{t+1}\} \end{bmatrix}$ $A_F \equiv \begin{bmatrix} 1 - \sigma^{-1}\phi_y & -\sigma^{-1}\phi_\pi \\ \kappa\left(1 - \sigma^{-1}\phi_y\right) & \beta - \kappa\sigma^{-1}\phi_\pi \end{bmatrix}$	$\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$ $\kappa(\phi_\pi - 1) + (1+\beta)\phi_y < 2\sigma(1+\beta)$ central bank reacts neither "too strongly" nor "too weakly" to yt_tuta and pit_hat

Rules of M.P. that central bank can follow in practice

Two simple monetary policy rules

(1) A Taylor-type interest rate rule

$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t$, $i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t$, $v_t \equiv \phi_y \hat{y}_t^n$

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = A_T \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + B_T(\hat{r}_t^n - v_t)$$

A simple Taylor-type rule that responds aggressively to movements in inflation can approximate arbitrarily well the optimal policy.

(2) A constant money growth rule

$\Delta m_t = 0$, $\hat{l}_t = \tilde{y}_t + \hat{y}_t^n - \eta \hat{\imath}_t - \zeta_t$;

$$A_{M,0} \begin{bmatrix} \tilde{y}_t \\ \pi_t \\ \hat{l}_{t-1}^{+} \end{bmatrix} = A_{M,1} \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ \hat{l}_t^{+} \end{bmatrix} + B_M \begin{bmatrix} \hat{r}_t^n \\ \hat{y}_t^n \\ \Delta\zeta_t \end{bmatrix}$$

Chapter 5 monetary policy tradeoffs

Assumption:

In this chapter, we relax the assumption of efficient output, and turn to a more realistic environment and flexible inflation targeting. In reality, there could be real imperfections other han staggered price.

The case of an efficient steady state

The possible inefficiencies associated with the flexible price equilibrium do not affect the steady state,which remains efficient, i.e. $\bar{y}^n = \bar{y} = \bar{y}^e$.

Conclusion:

The welfare losses: $E_0\left\{\sum_{t=0}^{\infty}\beta^t(\pi_t^2+\alpha_x x_t^2)\right\}$,where x_t denotes the deviation between output and its efficient level.

NKPC relationship yields: $\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t$ where the disturbance is exogenous w.r.t monetary policy. Time variations in the gap between the efficient and natural levels generate a tradeoff for the monetary policy.Its forward-looking nature requires that we specify the extent the central bank can credibly commit in advance to future policy actions.

Assume an exogenous AR(1) process of u_t as $u_t = \rho_u u_{t-1} + \varepsilon_t^u$.

The DIS needed to implement the policy: $x_t = -\frac{1}{\sigma}(\hat{i}_t - E_t\{\hat{\pi}_{t+1}\} - r_t^e) + E_t\{x_{t+1}\}$

Monetary policy: two alternatives discretion<commitment

<table>
<tr><th></th><th colspan="2">Optimal discretional policy</th><th>Optimal commitment policy</th></tr>
<tr><td>Problem</td><td colspan="2">$\min_{(x_t,\ \pi_t)} \pi_t^2 + \alpha_x x_t^2$
S.t. $\pi_t = \kappa x_t + v_t$
$v_t \equiv \beta E_t\{\pi_{t+1}\} + u_t$</td><td>$\min_{\{x_t,\ \pi_t\}_{t=0}^{\infty}} \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t(\pi_t^2+\alpha_x x_t^2)$
S.t.
$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t$</td></tr>
<tr><td>Optimality condition</td><td colspan="2">$\frac{\partial L}{\partial \pi_t}$: $2\pi_t = \lambda$
$\frac{\partial L}{\partial x_t}$: $2\alpha_x x_t = -\lambda\kappa$
Which yield:
$x_t = -\frac{\kappa}{\alpha_x}\pi_t$
(targeting rule)</td><td>$\alpha_x x_t - \kappa\gamma_t = 0$
$\pi_t + \gamma_t - \gamma_{t-1} = 0$
Which yield:
$x_0 = -\frac{\kappa}{\alpha_x}\pi_0$
$x_t = x_{t-1} - \frac{\kappa}{\alpha_x}\pi_t$
In price level: $x_t = -\frac{\kappa}{\alpha_x}\hat{p}_t$</td></tr>
<tr><td>Inflation/price level</td><td>$\pi_t = \alpha_x \Psi u_t$</td><td rowspan="2">Optimal to let the inflation rise (permanent change in price) while the output gap changes.</td><td>$\hat{p}_t = \delta\hat{p}_{t-1} + \frac{\delta}{1-\delta\beta\rho_u}u_t$</td></tr>
<tr><td>Output gap</td><td>$x_t = -\kappa\Psi u_t$.</td><td>$x_t = \delta x_{t-1} - \frac{\kappa\delta}{\alpha_x(1-\delta\beta\rho_u)}u_t$
$x_0 = -\frac{\kappa\delta}{\alpha_x(1-\delta\beta\rho_u)}u_0$.</td></tr>
</table>

Interest rate	$i_t = r_t^e + \Psi_i u_t$		Assume transitory shock: $i_t = r_t^e - (1-\delta)\left(1 - \frac{\sigma\kappa}{\alpha_x}\right)\hat{p}_t$ $= r_t^e - (1-\delta)\left(1 - \frac{\sigma\kappa}{\alpha_x}\right)\sum_{k=0}^{t}\delta^{k+1}u_{t-k}$
Implementation	$i_t = r_t^e + \phi_\pi \pi_t$ (targeting rule)which requires $\kappa\sigma > \alpha_x$,and may not be satisfied. $i_t = r_t^e + \Psi_i u_t + \phi_\pi(\pi_t - \alpha_x \Psi u_t)$ $= r_t^e + \Theta_i u_t + \phi_\pi \pi_t$ (instrument rule) Its feasibility of implementing rules is questionable. Targeting rules is regarded as more practical guides.		One possible rule that would bring about the desired allocation as the unique equilibrium: $i_t = r_t^e - \left[\phi_p + (1-\delta)\left(1 - \frac{\sigma\kappa}{\alpha_x}\right)\right]$ $* \sum_{k=0}^{t}\delta^{k+1}u_{t-k} + \phi_p \hat{p}_t$ For any $\phi_p > 0$.

Where $\Psi \equiv \frac{1}{\kappa^2 + \alpha_x(1-\beta\rho_u)}$; $\Psi_i \equiv \Psi\left[\kappa\sigma(1-\rho_u) + \alpha_x \rho_u\right]$; $\phi_\pi \equiv (1-\rho_u)\frac{\kappa\sigma}{\alpha_x} + \rho_u$;

$\Theta_i \equiv \Psi\left[\kappa\sigma(1-\rho_u) - \alpha_x(\phi_\pi - \rho_u)\right]$; $a \equiv \frac{\alpha_x}{\alpha_x(1+\beta)+\kappa^2}$; $\delta \equiv \frac{1-\sqrt{1-4\beta a^2}}{2a\beta} \in (0,1)$;

And $\hat{p}_t \equiv p_t - p_{-1}$ (the deviation between the price level and an implicit target)

Examples:

Figure 5.1: Impulse responses to a 1% transitory cost-push shock.

- **Discretionary policy**: both the output gap and inflation return to their zero initial value once the shock has vanished.
- **Commitment**: deviations in the output gap and inflation from target persist well beyond the life of the shock.
  - Improvement in the output gap/inflation tradeoff initially.
  - Forward-looking nature of inflation. Iterating the NKPC forward

$$\pi_t = \kappa x_t + \kappa \sum_{k=1}^{\infty} \beta^k E_t\{x_{t+k}\} + u_t.$$

  - In response to u_t, the central bank may lower future output gap with credible promises. Thus, given π_t, current x_t may decline less.
  - Due to convexity of loss function, the dampening of deviations in the period of shock improves welfare.

Figure 5.2: Impulse responses to a persistant cost-push shock.

- The economy reverts back to the initial position slowly.
- Under commitment, initial responses of inflation and output gap are both lower.
- Under commitment, price level reverts back to its original level. Inflation displays positive short-run autocorrelation.
- *Stabilization bias* associated with the discretionary policy: attempts to stabilize the output gap in the medium term more than the optimal policy under commitment.

The case of a distorted steady state

The presence of uncorrected real imperfections generate a permanent gap between the natural and efficient levels of output. $-\frac{U_N}{U_C} = MPN(1-\Phi)$.

Conclusion:

Welfare losses: $E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}(\pi_t^2 + \alpha_x \hat{x}_t^2) - \Lambda \hat{x}_t \right]$; where $\hat{x}_t = x_t - x$ represents the deviation of the welfare-relevant output gap from its value in the zero inflation steady state.

	Optimal discretional policy	Optimal commitment policy
Problem	$\min_{(x_t,\pi_t)} \frac{1}{2}(\pi_t^2 + \alpha_x \hat{x}_t^2) - \Lambda \hat{x}_t$ s.t. $\pi_t = \kappa \hat{x}_t + \upsilon_t$	$L = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}(\pi_t^2 + \alpha_x \hat{x}_t^2) - \Lambda \hat{x}_t \right.$ $\left. + \gamma_t (\pi_t - \kappa \hat{x}_t - \beta \pi_{t+1}) \right]$
Optimality condition	$\hat{x}_t = \frac{\Lambda}{\alpha_x} - \frac{\kappa}{\alpha_x}\pi_t$ (a more expansionary policy than given in the absence of a steady state distortion)	$\alpha_x \hat{x}_t - \kappa \gamma_t - \Lambda = 0$ $\pi_t + \gamma_t - \gamma_{t-1} = 0$
Inflation/price level	$\pi_t = \frac{\Lambda \kappa}{\kappa^2 + \alpha_x (1-\beta)} + \alpha_x \Psi u_t$.	$\hat{p}_t = \delta \hat{p}_{t-1} + \frac{\delta}{1-\delta\beta\rho_u} u_t + \frac{\delta\kappa\Lambda}{1-\delta\beta}$
Output gap	$\hat{x}_t = \frac{\Lambda(1-\beta)}{\kappa^2 + \alpha_x(1-\beta)} - \kappa \Psi u_t$.	$\hat{x}_t = \delta \hat{x}_{t-1} - \frac{\kappa\delta}{\alpha_x(1-\delta\beta\rho_u)} u_t + \Lambda \left[1 - \delta \left(1 + \frac{\kappa^2}{\alpha_x(1-\delta\beta)} \right) \right]$
Implementation	(1)A positive average inflation resulting from the central bank's incentive to push output above its natural steady state level (2)Lead to the **classical inflation bias**	In zero average inflation in equilibrium,the price level converges to a constant. Commitment avoids the inflation bias under discretionary policy.