Advanced Macroeconomics II

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Removing Trends and Isolating Cycles

Chapter 3, Structural Macroeconometrics, DeJong and Dave, 2007, Princeton University Press.

To bring models to data

- To establish correspondence between what is being characterized by the model and what is being measured in the data.
  - e.g. Models with only private sector, should not use aggregate GDP.
- Removing trend
  - Model solutions are typically in terms of stationary variables.
  - Ideal to build both trend and cyclical behavior into the model, and eliminate trends from the model and actual data in parallel fashion.
- Isolating cycle
  - Even after removing trend, it may be necessary to separate higher frenquency fluctuations out of the business cycle (6 - 40 quarters).

Removing Trends and Isolating Cycles

Isolation of cycles

The isolation of cycles is related to aligning models with appropriate data of relevant frequencies.

- Economic growth: average of long time span (e.g., over half-decade intervals)
- Aggregate asset pricing behavior: annual data to mitigate seasonal fluctuations.
- Business cycles: typically with quarterly data.
  - Tradeoff between introducing seasonal fluctuations and loss of information.
  - Alternative of time aggregation is necessary to isolate cycles.

Removing Trends and Isolating Cycles

Data set

US business cycle data. Figure 3.1 (Refer to the first slides for updated data plots.)

- Quarterly 1948:I - 2004:IV, seasonally adjusted.
- Consumption of nondurables and services. (Both seasonally adjusted and non-adjusted.)
- Gross private domestic investment
- Output as the sum of consumption and investment
- Hours of labor in nonfarm section.

3.1 Removing Trends

Three leading approaches

The common goal:
- to tranform the data into mean-zero covariance stationary stochastic processes (CSSPs).
- to work with logged versions of data before transformation

$$\frac{\partial}{\partial t} \log y_t = \frac{\frac{\partial}{\partial t} y_t}{y_t} \equiv \frac{\dot{y}_t}{y_t} \equiv g_{y_t} \tag{1}$$

- Roughly constant growth rate
  - Detrending
  - Differencing
- Time-varying trend
  - HP filter

3.1 Removing Trends

3.1.1 Detrending

$$y_t = y_0 e^{g_{y_t} \cdot t} e^{u_t}, \quad u_t \sim CSSP. \tag{2}$$

$$\begin{aligned} \log y_t &= \log y_0 + g_y t + u_t \\ \log y_t &= (\log y_0 - g_y n) + g_y (t+n) + u_t \end{aligned} \tag{3}$$

Trend removal amounts to

- fitting a linear trend to $\log y_t$ using OLS regression,

$$\log y_t = \alpha_0 + \alpha_1 t + u_t$$

- and subtracting the estimated trend

$$\tilde{y}_t = \log y_t - \hat{\alpha}_0 + \hat{\alpha}_1 t = \hat{u}_t \tag{4}$$

- *Trend stationary.*

3.1.1 Detrending

Common trend

In working with a set of m variables sharing a common trend component, symmetry implies the removal of a common trend of all variables.

- Defining α_1^j as the trend coefficient for variable j,
- Imposing linear restrictions in the OLS

$$\alpha_1^1 - \alpha_1^j = 0, \quad j = 2, ..., m.$$

3.1.2 Differencing

$$
\begin{aligned}
y_t &= y_0 e^{\varepsilon_t} && (5)\\
\varepsilon_t &= \gamma + \varepsilon_{t-1} + u_t, \quad u_t \sim CSSP. && (6)
\end{aligned}
$$

Iterate Equation (6) forward,

$$
\varepsilon_t = \gamma t + \sum_{j=0}^{t-1} u_{t-j} + \varepsilon_0 \tag{7}
$$

Comparison:

$$
\begin{aligned}
Detrending &: \quad y_0 e^{g_{y_t} \cdot t} e^{u_t}\\
Differencing &: \quad y_0 e^{\gamma \cdot t} e^{\sum_{j=0}^{t-1} u_{t-j} + \varepsilon_0}
\end{aligned}
$$

Differencing

From Equation (5),

$$\begin{aligned}\log y_t &= \log y_0 + \varepsilon_t \\ &= \log y_{t-1} + \gamma + u_t\end{aligned} \tag{8}$$

The first log difference is

$$\begin{aligned}\log y_t - \log y_{t-1} &\equiv (1-L)\log y_t \\ &= \varepsilon_t - \varepsilon_{t-1} \\ &= \gamma + u_t\end{aligned} \tag{9}$$

Difference stationary.

Differencing

- Trend estimation: OLS
- Subtracting the estimated trend

$$\tilde{y}_t = \log y_t - \log y_{t-1} - \hat{\gamma} = \hat{u}_t \tag{10}$$

- Imposition of common growth rate amounts to a restricted OLS

$$\gamma^1 - \gamma^j = 0, \ j = 2, ..., m. \tag{11}$$

Comparison between detrending and differencing

- Assumptions hinge on the representation of Equations (3) or (9).
- The empirical distinction is proven to be difficult to resolve.
- A remedy is to work with both specifications and evaluate the sensitivity of results.

With the US data set,

- The choice of either specification is problematic
- Structural break in mid 1970s results in spurious persistence.

3.1.2 Hodrick-Prescott (H-P) filter

Decomposing

$$\log y_t = g_t + c_t \tag{12}$$

g_t: growth component
c_t: cyclical component
The H-P filter estimates g_t and c_t in order to minimize

$$\sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=3}^{T} \left[(1 - L)^2 g_t \right]^2 \tag{13}$$

taking λ as given.

Hodrick-Prescott (H-P) filter

The minimization amounts to find out $\hat{g}_t$ or $\hat{c}_t$, and trend removal is accomplished by

$$\tilde{y}_t = \log y_t - \hat{g}_t = \hat{c}_t \tag{14}$$

The parameter λ determines the importance of having a smoothly evolving component:

- $\lambda = 0$: smoothness receives no value, and all variation in $\log y_t$ will be assigned to the trend component.
- As $\lambda \to \infty$, the trend is assigned to be maximally smooth, i.e. linear.
- In practice, λ is specified to strike a balance between the two extremes.
  - $\lambda = 1600$ for quarterly business cycle data.
  - $\lambda = 14400$ for monthly data.

Hodrick-Prescott (H-P) filter

Pros and cons

- It is convenient to apply H-P filter to time series for removing trend.
- The trend is data driven by each time series, absent of a common-trend restriction.

Comparison of the three resulting cyclical components of ouput

The results are different from the three methods.
Standard deviation:

Linearly detrended	0.046
Differenced	0.010
H-P filtered	0.018

Correlation:

ρ	L	D
D	0.12	
HP	0.49	0.27