Advanced Microeconomics II
Problem Set 4

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1. Prove the following proposition.

   Proposition 1. *Let w be a strictly enforceable feasible payoff profile of $G = \{N, (A_i), (u_i)\}$. For all $\epsilon > 0$ there exists $\underline{\delta} < 1$ such that if $\delta > \underline{\delta}$ then the δ-discounted infinitely repeated game of G has a Nash equilibrium whose payoff profile w' satisfies $|w' - w| < \epsilon$.*

2. Consider the following stage game.

	A	D
A	$2, 2$	$0, 1$
D	$5, 4$	$1, 0$

   (a) Construct a pair of strategies that generate the average per-period payoffs of $(3.5, 3)$, and are a Nash equilibrium but are not a subgame perfect equilibrium when players use the limit of means criterion to evaluate payoffs.

   (b) Construct a pair of strategies that generate average per-period payoffs of $(3.5, 3)$, and are a subgame perfect equilibrium when players use the limit of means criterion to evaluate payoffs but not a subgame perfect equilibrium when players use the overtaking criterion to evaluate payoffs.

   (c) Construct a pair of strategies that generate the average per-period payoffs of $(3.5, 3)$, and are a subgame perfect equilibrium when players use overtaking criterion to evaluate payoffs.

3. A buyer and a seller are bargaining over an object. The rules of bargaining are that they simultaneously announce prices. If $p_b \geq p_s$, then trade occurs at price $p = \frac{p_b + p_s}{2}$; if $p_b < p_s$, then no trade occurs. The buyer's valuation for the good is v_b, the seller's is v_s. These valuations are private information and are drawn from independent uniform distributions on $[0, 1]$. If there is no trade, both players' utility are 0; if the buyer gets the good for price p, the buyer's utility is $v_b - p$ and the seller's utility is $p - v_s$.

   (a) Construct a 'one-price' Bayesian Nash equilibrium of this game: an equilibrium in which trade occurs at a single price if it occurs at all.

   (b) Compare the efficiency of the equilibrium constructed in (a) and the 'linear' Bayesian Nash equilibrium constructed in class.

   (c) Use the Revelation Principle to construct a Bayesian game with an incentive-compatible equilibrium with the same outcome as the equilibrium in (a).

4. In this question we model differences in players knowledge as a Bayesian game. There are two players and three possible states of the world, i.e. $\Omega = \{\alpha, \beta, \gamma\}$. The prior probability of each state is $p(\alpha) = 1/5, p(\beta) = 3/5, p(\gamma) = 1/5$. Each player has two types. In any state player 1 either knows the state is α or knows the state is β or γ, while player 2 either knows the state is α or β or knows the state is γ.

The payoffs for each action profile and state are shown in the following three payoff matrices, one for each state.

	L	R
L	$2,2$	$0,0$
R	$3,0$	$1,1$

State α

	L	R
L	$2,2$	$0,0$
R	$0,0$	$1,1$

State β

	L	R
L	$2,2$	$0,0$
R	$0,0$	$1,1$

State γ

(a) Write down this problem as a Bayesian game of incomplete information.

(b) Solve for the set of Nash equilibria.