# Disambiguation: The Markov Process and the Martingale

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Items 1-6 follow the settings of the standard lattice model.

1. $M_n = \max_{1 \le k \le n} S_n$ is neither a Markov process ($S_n$ needed for $\mathbb{E}_n \left[ f \left( M_{n+1} \right) \right]$) nor a martingale ($\mathbb{E}_n \left[ M_{n+1} \right] = p \left( M_n \vee uS_n \right) + qM_n > M_n$).

2. $(M_n, S_n)$ is a Markov process since $\mathbb{E}_n \left[ f \left( M_{n+1}, S_{n+1} \right) \right] = pf \left( M_n \vee uS_n, uS_n \right) + qf \left( M_n, dS_n \right) = g \left( M_n, S_n \right)$.

3. The (discounted) price of any risky asset under the actual probability is not a martingale but Markovian.

4. The discounted stock price $\tilde{S}_n = \frac{S_n}{(1+r)^n}$ under the risk-neutral probability measure. $\mathbb{E}_n \left[ f \left( \tilde{S}_{n+1} \right) \right] = \tilde{p} f \left( \frac{u\tilde{S}_n}{1+r} \right) + \tilde{q} f \left( \frac{q\tilde{S}_n}{1+r} \right) = g \left( \tilde{S}_n \right)$.

5. The discounted wealth process $W_n$ under the risk-neutral probability measure where $\Delta_n$ is an adapted process. It is a martingale but not a Markov process.

6. The discounted price of the look-back option $\tilde{V}_n$ under the risk-neutral probability measure. It is a non-Markovian martingale ($M_n$ and $S_n$ needed).

7. $X_{n+1} = X_n + \varepsilon_{n+1} X_n$, where $\mathbb{E} \left[ \varepsilon_{n+1} \middle| X_n \right] = 0$ and the conditional distribution of $\varepsilon_{n+1}$, $\rho(\cdot)$, is independent of $X_1, \ldots, X_n$. $X_n$ is a Markovian martingale. $\mathbb{E}_n \left[ f \left( X_{n+1} \right) \right] = \int f \left( X_n + zX_n \right) \rho(z)\, dz = g \left( X_n \right)$. Note if the conditional distribution of $\varepsilon_{n+1}$, $\rho(\cdot)$, requires knowledge of $\{X_1, \ldots, X_{n-1}\}$ it is no longer Markovian. The statement is compulsory.

To construct a non-Markovian martingale one should consider a martingale whose evolutionary path is dependent on its historical information and/or other information (e.g. from another process) that can not be inferred from its prevailing value.

One is requested to state all primises following standard mathematical or financial representations. Otherwise the answer will be deemed invalid.

FOR FURTHER INFORMATION, PLEASE CONTACT PROF.TSAI.