Advanced Macroeconomics II
Gali (2008), Chapter 4

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Chapter 4. Monetary Policy Design in the Basic New Keynesian Model

How to conduct monetary policy in the basic New Keynesian model?

- With the reference of the efficient allocation under
  - monopolistic competition and flexible prices
  - with a subsidy to correct the distortion of monopolistic competition.
- When prices are sticky, the efficient allocation can be obtained by means of a policy that
  - fully stabilizes the price level.

Monetary Policy Design in the Basic New Keynesian Model

Outline of the chapter

- Objective of the optimal monetary policy (M.P.)
- Optimal monetary policy
  - Examples
  - Feasibility problem: unobservable natural rate of interest
- Rules of M.P. that central banks can follow in practice
  - Simple rules
  - Welfare evaluation: a Taylor rule vs. a constant money growth rule.

4.1 The efficient allocation

The efficient allocation can be determined by a social planner's problem

$$\max U(C_t, N_t)$$

s.t.

$$C_t(i) = A_t F\left(N_t\left(i\right)\right)$$

for all $i \in [0,1]$ with

$$\begin{aligned} C_t &\equiv \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}} \\ N_t &= \int_0^1 N_t(i) di. \end{aligned}$$

The efficient allocation

The associated optimality conditions are

$$C_t(i) = C_t, \text{ all } i \in [0,1] \tag{1}$$
$$N_t(i) = N_t, \text{ all } i \in [0,1] \tag{2}$$
$$-\frac{U_{N,t}}{U_{C,t}} = MPN_t \tag{3}$$

where $MPN_t \equiv (1-\alpha)A_t N_t^{-\alpha}$ denotes the economy's average marginal product of labor.

In the case of the symmetric allocation considered above, MPN_t also happens to coincide with the marginal product for each individual firm.

The efficient allocation

These optimality conditions can be met because of

- all goods entering the utility function symmetrically,
- concavity of utility,
- identical technologies to produce all goods.

The symmetry implies that

- the marginal rate of substitution between consumption and work hours is equal to the corresponding marginal rate of transformation, which in turn corresponds to the marginal product of labor.

4.2 Sources of suboptimality in the basic New Keynesian model

The basic New Keynesian model developed in Chapter 3 is characterized by two distortions,

- The presence of market power in goods markets, exercised by monopolistically competitive firms.
  - Unrelated to price stickiness.
- Infrequent adjustment of prices by firms.

The implications of each of the distortion are worth considering separately.

4.2.1 Distortions unrelated to sticky prices: monopolistic competition

To isolate the role of monopolistic competition let us assume flexible prices. Under an isoelastic demand function (with price-elasticity ε), the optimal price-setting rule is given by

$$P_t = \mu \frac{W_t}{MPN_t}$$

where $\mu \equiv \frac{\varepsilon}{\varepsilon - 1} > 1$ is the (gross) optimal markup chosen by firms and $\frac{W_t}{MPN_t}$ is the marginal cost. Accordingly,

$$-\frac{U_{N,t}}{U_{C,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mu} < MPN_t$$

- Optimal condition (3) of the social planner is violated.
- An inefficiently low level of employment and output.

Distortions unrelated to sticky prices: monopolistic competition

A subsidy policy to eliminate the distortion

- Let τ denote the rate at which the cost of employment is subsidized.
- Assume that the outlays associated with the subsidy are financed by means of lump-sum taxes, so that no distortion on incentive of households.
- Under flexible prices, $P_t = \mu \frac{(1-\tau)W_t}{MPN_t}$.
- Accordingly,
$$-\frac{U_{N,t}}{U_{C,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mu(1-\tau)}.$$
- Hence, the optimal allocation can be attained if $\mu(1-\tau) = 1$, or equivalently, by setting $\tau = 1/\varepsilon$.

4.2.2 Distortions associated with the presence of staggered price setting

Price stickiness: a source of inefficiency on two grounds.

1. **Average markup varies over time.**

$$\mu_t = \frac{P_t}{(1-\tau)\left(W_t / MPN_t\right)} = \frac{P_t \mu}{W_t / MPN_t}$$

$$-\frac{U_{N,t}}{U_{C,t}} = \frac{W_t}{P_t} = MPN_t \frac{\mu}{\mu_t}$$

The efficiency condition (3) is violated if $\mu_t \neq \mu$.

2. **Staggered price setting.**

$$\begin{aligned} P_t(i) &\neq P_t(j) \\ C_t(i) &\neq C_t(j) \\ N_t(i) &\neq N_t(j) \end{aligned}$$

Violations on efficiency conditions (1) and (2).

4.3 Optimal monetary policy in the basic New Keynesian model

Assumptions for simplicity

- An optimal subsidy in place to offset the market power distortion
- The economy starts without relative price distortion, i.e., $P_{-1}(i) = P_{-1}$ for all $i \in [0, 1]$.

Requirement of the optimal policy

- to stabilize marginal costs at a level consistent with firms' desired markup, given the prices in place.

Consequence of the optimal policy

- $P_t^* = P_{t-1}$ and, hence, $P_t = P_{t-1}$ for $t = 0, 1, 2, ...$, so that conditions (1) and (2) hold.
- $\mu_t = \mu$ for all t, so that condition (3) holds.

Optimal monetary policy in the basic New Keynesian model

Using the notation for the log-linearized model in Chapter 3, the optimal policy requires that for all t,

$$\begin{aligned} \tilde{y}_t &= 0 \\ \hat{\pi}_t &= 0 \end{aligned}$$

The dynamic IS equation then implies

$$\hat{\imath}_t = \hat{r}_t^n$$

for all t.

Optimal monetary policy in the basic New Keynesian model

Two features of the optimal policy

1. Stablizing output is not desirable in and of itself. As usually

$$\hat{y}_t \neq \tilde{y}_t,$$

   and $\tilde{y}_t = 0$ implies $y_t = y_t^n$ for all t, where the natural level of output is subject to technology shocks.

2. Price stability emerges as a feature of the optimal policy even though, a priori, the policy maker does not attach any weight to such an objective.

4.3.1 Implementation: Optimal Interest Rate Rules

The non-policy block:

$$\tilde{y}_t = -\frac{1}{\sigma}\left(\hat{\imath}_t - E_t\left\{\hat{\pi}_{t+1}\right\} - \hat{r}_t^n\right) + E_t\left\{\tilde{y}_{t+1}\right\}. \tag{4}$$

$$\hat{\pi}_t = \beta E_t\left\{\hat{\pi}_{t+1}\right\} + \kappa\tilde{y}_t \tag{5}$$

The policy block:

- An exogenous interest rate rule
- An interest rate rule with an endogenous component
- A forward-looking interest rate rule

4.3.1.1 An exogenous interest rate rule

The candidate interest rate rule

$$\hat{\imath}_t = \hat{r}_t^n \tag{6}$$

for all t. – Setting the goal as the policy.
Substituting (6) into (4) and rearranging terms

$$\begin{bmatrix} \tilde{y}_t \\ \hat{\pi}_t \end{bmatrix} = A_0 \begin{bmatrix} E_t\left\{\tilde{y}_{t+1}\right\} \\ E_t\left\{\hat{\pi}_{t+1}\right\} \end{bmatrix} \tag{7}$$

where

$$A_0 \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix}.$$

An exogenous interest rate rule

- Note that $\tilde{y}_t = \hat{\pi}_t = 0$ for all t – the outcome associated with the optimal policy – is one solution to (7), but not unique.
- Given that both $\tilde{y}_t$ and $\hat{\pi}_t$ are nonpredetermined, the existence of an eigenvalue outside the unit circle implies the existence of a multiplicity of equilibria.
- No guarantee on the realisation of $\tilde{y}_t = \hat{\pi}_t = 0$ for all t.

4.3.1.2 An interest rate rule with an endogenous component

$$\hat{\imath}_t = \hat{r}_t^n + \phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t \tag{8}$$

where ϕ_π and ϕ_y are non-negative.
Substituting (8) into (4) and rearranging terms

$$\begin{bmatrix} \tilde{y}_t \\ \hat{\pi}_t \end{bmatrix} = A_T \begin{bmatrix} E_t \{\tilde{y}_{t+1}\} \\ E_t \{\hat{\pi}_{t+1}\} \end{bmatrix} \tag{9}$$

where

$$A_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \kappa\sigma & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}$$

and $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$.

An interest rate rule with an endogenous component

- The desired outcome ($\tilde{y}_t = \hat{\pi}_t = 0$ for all t) is always a solution.
- For uniqueness of solution, both eigenvalues of matrix A_T should lie within the unit circle, which implies

$$\kappa\,(\phi_\pi - 1) + (1-\beta)\phi_y > 0. \tag{10}$$

- Figure 4.1 illustrates graphically the regions of parameter space for (ϕ_π, ϕ_y) required by (10).
- If condition (10) is satisfied, $\tilde{y}_t = \hat{\pi}_t = 0$ and $\hat{\imath}_t = \hat{r}_t^n$ for all t will hold ex-post.

4.3.1.3 A forward-looking interest rate rule

$$\hat{\imath}_t = \hat{r}_t^n + \phi_\pi E_t \left\{ \hat{\pi}_{t+1} \right\} + \phi_y E_t \left\{ \tilde{y}_{t+1} \right\} \tag{11}$$

Under (11) the implied dynamics are described by the system

$$\begin{bmatrix} \tilde{y}_t \\ \hat{\pi}_t \end{bmatrix} = A_F \begin{bmatrix} E_t \left\{ \tilde{y}_{t+1} \right\} \\ E_t \left\{ \hat{\pi}_{t+1} \right\} \end{bmatrix}$$

where

$$A_F \equiv \begin{bmatrix} 1 - \sigma^{-1}\phi_y & -\sigma^{-1}\phi_\pi \\ \kappa \left(1 - \sigma^{-1}\phi_y \right) & \beta - \kappa\sigma^{-1}\phi_\pi \end{bmatrix}.$$

A forward-looking interest rate rule

The conditions for a unique equilibrium (i.e., for both eigenvalues of A_F lying within the unit circle) are twofold and given by

$$\begin{aligned} \kappa\left(\phi_{\pi}-1\right)+(1-\beta)\phi_{y} &> 0 && (12)\\ \kappa\left(\phi_{\pi}-1\right)+(1+\beta)\phi_{y} &< 2\sigma(1+\beta) && (13) \end{aligned}$$

Determinacy of equilibrium requires that the central bank reacts neither "too strongly" nor "too weakly" to $\tilde{y}_t$ and $\hat{\pi}_t$. (Figure 4.2)

4.3.2 Practical shortcomings of optimal policy rules

- **Unrealistic:** Both require that the policy rate be adjusted one-for-one with the natural rate of interest, thus implicitly assuming observability of the latter variable. Implication on knowledge of
  - the economy's "true model",
  - the values taken by all its parameters, and
  - the realized value (observed in real time) of all the shocks.
- Unobservability of output gap is not binding.
- Simple rules are needed in practice.

4.4 Two simple monetary policy rules

- Design of practical rules for monetary policy.
- Welfare-based evaluation criterion, following Rotemberg and Woodford (1999)
  - A second-order approximation to the utility losses due to a consequence of deviations from the efficient allocation
  - The welfare loss function in terms variations of output gap and inflation

$$W = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \tilde{y}_t^2 + \frac{\varepsilon}{\lambda} \pi_t^2 \right]$$

  - The loss can be expressed as a permanent consumption decline w.r.t. steady state consumption.

Two simple monetary policy rules

Crutial parameters to affect the welfare losses.

Per period loss:

$$L = \frac{1}{2}\left[\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) var(\tilde{y}_t) + \frac{\varepsilon}{\lambda} var(\pi_t)\right]$$

- Loss increasing in σ, φ and α.
  - Larger values of those "curvature" parameters amplify the effect of any given deviation of output from its natural level.
  - Aggregate inefficiency increases due to the extent of violation to

$$-\frac{U_{N,t}}{U_{C,t}} = MPN_t$$

- Loss increasing in ε, θ (which is inversely related to λ)
  - Higher ε amplifies the welfare losses caused by any given price dispersion.
  - Higher θ amplifies the degree of price dispersion.

Two simple monetary policy rules

Evaluation of rules

Given a policy rule and a calibration of the model's parameters, to calculate

- the implied variance of inflation and the output gap
- the corresponding welfare losses associated with that rule.

4.4.1 A Taylor-type interest rate rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t \tag{14}$$

where $\phi_\pi > 0$ and $\phi_y > 0$ are assumed to satisfy the determinacy condition.
With $\rho \equiv -\log \beta$, it can be equivalently expressed around zero steady state of inflation

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t$$

In terms of output gap

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t \tag{15}$$

where $v_t \equiv \phi_y \hat{y}_t^n$, as a driving force proportional to the deviations of natural output from steady state, instead of an exogenous monetary policy shock.

A Taylor-type interest rate rule

The equilibrium dynamics:

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = A_T \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + B_T(\hat{r}_t^n - v_t)$$

where A_T and B_T are defined as in chapter 3.

- Assuming that technology shock a_t represents the only driving force in the economy, which follows a stationary AR(1) process with autoregressive coefficient ρ_a.

A Taylor-type interest rate rule

The following equality holds:

$$\begin{aligned}\hat{r}_t^n - v_t &= -\sigma\psi_{ya}^n(1-\rho_a)a_t - \phi_y\psi_{ya}^n a_t \\ &= -\psi_{ya}^n\left[\sigma(1-\rho_a)+\phi_y\right]a_t\end{aligned}$$

where, as in chapter 3, $\psi_{ya}^n \equiv \frac{1+\varphi}{\sigma+\varphi+\alpha(1-\sigma)} > 0$.

- The variance of the output gap and inflation is proportional to that of $B_T(\hat{r}_t^n - v_t)$, which is strictly increasing in ϕ_y.
- A policy seeking to stabilize output aggressively is bound to lower the representative consumer's utility.

A Taylor-type interest rate rule

Table 4.1 Evaluation of Simple Monetary Policy Rules

	Taylor Rule			
ϕ_π	1.5	1.5	5	1.5
ϕ_y	0.125	0	0	1
$(\sigma_\zeta, \rho_\zeta)$	–	–	–	–
$\sigma(\tilde{y})$	0.55	0.28	0.04	1.40
$\sigma(\pi)$	2.60	1.33	0.21	6.55
Welfare loss	0.30	0.08	0.002	1.92

A Taylor-type interest rate rule

A simple Taylor-type rule that responds aggressively to movements in inflation can approximate arbitrarily well the optimal policy.

A constant money growth rule

A constant growth rate for the money supply associated with Friedman (1960) consistent with zero inflation in the steady state,

$$\Delta m_t = 0$$

for all t.
A money market clearing condition to augment with DIS and NKPC,

$$l_t = y_t - \eta i_t - \zeta_t$$

where $l_t \equiv m_t - p_t$ denotes (log) real balances and ζ_t is an exogenous money demand shock following the process

$$\Delta\zeta_t = \rho_\zeta \Delta\zeta_{t-1} + \varepsilon_t^\zeta$$

where $\rho_\zeta \in [0, 1)$.

A constant money growth rule

Rewrite the money market equilibrium condition in terms of deviations

$$\hat{l}_t = \tilde{y}_t + \hat{y}_t^n - \eta \hat{\imath}_t - \zeta_t.$$

Letting $l_t^+ \equiv l_t + \zeta_t$ denote (log) real balances adjusted by the exogenous component of money demand,

$$\hat{\imath}_t = \frac{1}{\eta}\left(\tilde{y}_t + \hat{y}_t^n - \hat{l}_t^+\right).$$

In addition, using the definition of l_t^+ together with the assumed rule $\Delta m_t = 0$,

$$\hat{l}_{t-1}^+ = \hat{l}_t^+ + \pi_t - \Delta\zeta_t.$$

A constant money growth rule

The equilibrium dynamics

$$A_{M,0}\begin{bmatrix}\tilde{y}_t\\ \pi_t\\ \hat{l}_{t-1}^{+}\end{bmatrix} = A_{M,1}\begin{bmatrix}E_t\left\{\tilde{y}_{t+1}\right\}\\ E_t\left\{\pi_{t+1}\right\}\\ \hat{l}_t^{+}\end{bmatrix} + B_M\begin{bmatrix}\hat{r}_t^n\\ \hat{y}_t^n\\ \Delta\zeta_t\end{bmatrix}$$

where $A_{M,0}$, $A_{M,1}$ and B_M are defined as in chapter 3.

A constant money growth rule

Table 4.1 Evaluation of Simple Monetary Policy Rules

	Taylor Rule				Constant Δm	
ϕ_π	1.5	1.5	5	1.5	–	–
ϕ_y	0.125	0	0	1	–	–
$(\sigma_\zeta, \rho_\zeta)$	–	–	–	–	(0, 0)	(0.0063,0.6)
$\sigma(\tilde{y})$	0.55	0.28	0.04	1.40	1.02	1.62
$\sigma(\pi)$	2.60	1.33	0.21	6.55	1.25	2.77
Welfare loss	0.30	0.08	0.002	1.92	0.08	0.38

A constant money growth rule

- The natural output and the natural rate of interest vary in response to technology shocks.
- Without money demand shocks, a constant money growth rule delivers a performance comparable, in terms of welfare losses, to a Taylor rule with coefficient $\phi_\pi = 1.5$ and $\phi_y = 0$.
- When money demand shocks are allowed for, the corresponding process for $\Delta\zeta_t$ is calibrated with $\sigma_\zeta = 0.0063$, $\rho_\zeta = 0.6$.
  - the performance of a constant money growth rule deteriorates considerably.
- The degree of stability of money demand is a key element in determining the desirability of a money growth rule.