Advanced Microeconomics II
Problem Set 1

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1. Two Xiamen teenagers Wang and Li are playing Chicken. Wang drives his motorbike south down a one-lane road, and Li drives his motorbike north along the same road. Each has two strategies: Stay or Swerve. If one player chooses Swerve he looses face; if both Swerve, they both lose face. However, if both choose Stay, they are both killed. The payoff matrix for Chicken looks like this:

		Li	
		Stay	*Swerve*
Wang	*Stay*	$-3, -3$	$2, 0$
	Swerve	$0, 2$	$1, 1$

   (a) Find all pure strategy equilibria.

   (b) Find all mixed strategy equilibria.

   (c) What is the probability that both teenagers will survive?

2. There are two firms, each with zero marginal costs and fixed costs of k, who compete over price. There are two types of consumers, informed consumers know the lowest price being charged and uninformed consumers simply choose a store at random. Suppose that there are I informed consumers and $2U$ uninformed consumers. Hence, each store will get U uninformed consumers each period for certain and will get the informed consumers only if they happen to have the lowest price. The reservation price of each consumer is r.

   (a) Formulate this as a strategic game.

   (b) Find the symmetric mixed-strategy Nash equilibrium of this game.

   (c) As the ratio of uninformed consumers in the market increases what happens to the distribution of firm prices? Interpret this result.

3. Consider the strategic game $G = \{N, (A_i), (u_i)\}$. For each $i \in N$, let A_i be a nonempty compact convex subset of Euclidean space and the utility function u_i be continuous and quasi-concave on A_i.

   (a) Prove that $B(a) = \times_{i \in N} B_i(a_{-i})$ is convex, where $B_i(a_{-i})$ is the best response function of player i, i.e. show that if $b \in B(a)$ and $c \in B(a)$ then for any $\lambda \in [0, 1]$, $\lambda b + (1-\lambda)c \in B(a)$.

   (b) Let A_i be finite for each $i \in N$. Prove that for each player i, the U_i associated with the mixed extension of G is quasi-concave over $\times_{j \in N} \Delta(A_j)$.

4. Two people can perform a task if, and only if, they both exert effort. They are both better off if they both exert effort and perform the task than if neither exerts effort (and nothing is accomplished); the worst outcome for each person is that she exerts effort and the other does not (in which case again nothing is accomplished). Specifically, the players' preferences

are represented by the expected value of the payoff functions in the following table, where c is a positive number less than 1 that can be interpreted as the cost of exerting effort.

	No effort	*Effort*
No effort	$0, 0$	$0, -c$
Effort	$-c, 0$	$1-c, 1-c$

(a) Find all the mixed strategy Nash equilibria of this game.

(b) How do the equilibria change as c increases? Explain the reasons for the changes.

5. Each of n people chooses whether or not to contribute a fixed amount toward the provision of a public good. The good is provided if and only if at least k people contribute, where $2 \leq k \leq n$; if it is not provided, contributions are not refunded. Each person ranks outcomes from best to worst as follows:

   (i) any outcome in which the good is provided and she does not contribute,

   (ii) any outcome in which the good is provided and she contributes,

   (iii) any outcome in which the good is not provided and she does not contribute,

   (iv) any outcome in which the good is not provided and she contributes.

   (a) Formulate this situation as a strategic game.

   (b) Find its Nash equilibria.