Zero-sum Games and Nash Equilibrium

Lemma

Let $\{\{1,2\},(A_i),(u_i)\}$ *be a zero-sum strategic game. Then* $\max_{y\in A_2}\min_{x\in A_1} u_2(x,y) = -\min_{y\in A_2}\max_{x\in A_1} u_1(x,y)$. *Further,* $y \in A_2$ *solves the problem* $\max_{y\in A_2}\min_{x\in A_1} u_2(x,y)$ *if and only if it solves the problem* $\min_{y\in A_2}\max_{x\in A_1} u_1(x,y)$.

- For any function f, $-\min_z f(z) = \max_z -f(z)$ and $\arg\min_z f(z) = \arg\max_z -f(z)$.
- $-\min_{x\in A_1} u_2(x,y) = \max_{x\in A_1} -u_2(x,y) = \max_{x\in A_1} u_1(x,y)$.
- $\max_{y\in A_2}\min_{x\in A_1} u_2(x,y) = -\min_{y\in A_2}\max_{x\in A_1} u_1(x,y)$.