Advanced Microeconomics II

WISE, Xiamen University

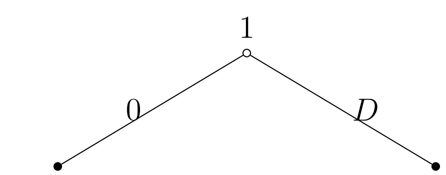
Spring 2011 Final

1. Clearly define the following concepts.
   (a) (5 points) A desirable good.
   (b) (5 points) A maxminimizing strategy for player 1 in a two-player strategic game.
   (c) (5 points) The subgame that follows the history h of the extensive game with perfect information $\Gamma = \{N, H, P, (\succeq_i)\}$.
   (d) (5 points) A Bayesian extensive game with observable actions.

2. (10 points) Prove the following: Every feasible enforceable payoff profile of $G = \{N, (A_i), (u_i)\}$ is a Nash equilibrium payoff profile of the limit of means infinitely repeated game of G.

3. For each proof clearly state what has been proven.
   (a) (5 points) *Proof.* Define $B : A \to A$ by $B(a) = \times_{i \in N} B_i(a_{-i})$ (where B_i is the best response function of player i). For every $i \in N$ the set $B_i(a_{-i})$ is nonempty since $\succeq_i$ is continuous and A_i is compact, and is convex since $\succeq_i$ is quasi-concave on A_i; B has a closed graph since each $\succeq_i$ is continuous. Thus by Kakutani's theorem B has a fixed point. ■
   (b) (5 points) *Proof.* Define the probability that player i plays the pure strategy s_i, which specifies an action $s_i(I_i)$ for every information set $I_i \in \mathcal{I}_i$, as $\Pi_{I_i \in \mathcal{I}_i} \beta_i(I_i)(s_i(I_i))$. ■

4. Consider the extensive form game shown in the following figure. Two individuals are going to play Battle of the Sexes with monetary payoffs as in the left-hand table in the figure. Before doing so, player 1 can discard a dollar (take the action D) or refrain from doing so (take the action 0); her move is observed by player 2.



	M	F
M	3, 1	0, 0
F	0, 0	1, 3

	M	F
M	2, 1	−1, 0
F	−1, 0	0, 3

   (a) (10 points) Find the set of subgame perfect equilibria for this game.
   (b) (5 points) Find a Nash equilibrium that is not subgame perfect or prove that one does not exist.

5. (10 points) Consider the following stage game of a discounting-infinitely repeated game where both players have the same discount factor.

		Player 2	
		A	B
Player 1	A	$5,5$	$0,10$
	B	$10,0$	$1,1$

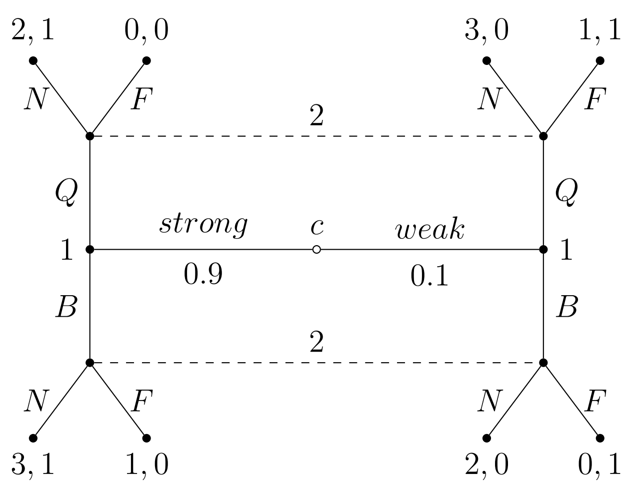
What is the minimum discount factor required to sustain average per-period payoffs of $(5,5)$.

6. Consider a 2-consumer economy in which consumer i's preferences are given by $u_i(x_i, g) = x_i + \theta_i g - g^2/2$, where x_i is consumer i's consumption of the private good, g is the quantity of the public good, and θ_i is i's marginal valuation of the public good, which is private information for each consumer. What is publicly known, however, is that for each player, $\theta_i \sim U[3,4]$ independently across the two players. The total amount of the public good)g is determined by the sum of the individual contributions, i.e., $g = g_1 + g_2$. Each consumer is endowed with 5 units of the private good that can be converted into the public good in a 1-for-1 fashion. That is, each consumer faces a budget constraint of the form $x_i + g_i \leq 5$.

   (a) (5 points) Given the realizations of θ_1 and θ_2, what is the Pareto-efficient amount of the public good?

   (b) (10 points) Now suppose the public good is provided by private contributions and consider a Bayesian Nash equilibrium where each consumer only knows his own θ_i. What is the ex ante expected level of the public good provided in such an equilibrium? Is there a unique equilibrium? What is the actual equilibrium amount of the public good given the realizations of θ_1 and θ_2? How does it compare to the Pareto-efficient amount?

7. Consider the signaling game shown in the following figure, in which there are two types of player 1, *strong* and *weak*; the probabilities of these types are 0.9 and 0.2 respectively, the set of messages is $\{B, Q\}$ (the consumption of beer or quiche for breakfast), and player 2 has two actions, $F(ight)$ or $N(ot)$.



   (a) (10 points) Construct a pooling equilibrium of this game or prove that one does not exist.

   (b) (10 points) Construct a separating equilibrium of this game or prove that one does not exist.