Advanced Microeconomics II
Quiz 1

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1. Two people are engaged in a joint project. If each person i puts in the effort x_i, a nonnegative number equal to at most 1, which costs her $c(x_i)$, the outcome of the project is worth $f(x_1, x_2)$. The worth of the project is split equally between the two people, regardless of their effort levels.

   (a) (10 points) Formulate this situation as a strategic game.

   Solution:

   $$N = \{1, 2\}; \quad A_i = [0, 1] \text{ for all } i \in N;$$
   $$u_i(a_1, a_2) = f(a_1, a_2)/2 - c(a_i).$$

   (b) Assume $f(x_1, x_2) = 4x_1x_2$ and $c(x_i) = x_i$ for $i = 1, 2$.

   i. (10 points) Write the best-response function for player 1 as a function of player 2's action.

   Solution: First derive the best-response function for each player. For player 1 the optimal action solves

   $$\max_{a_1} 2a_1a_2 - a_1.$$

   The first-order derivative of utility with respect to a_1 is $2a_2 - 1$. Hence,

   $$B(a_2) = \begin{cases} 0 & \text{if } a_2 < 1/2 \\ [0, 1] & \text{if } a_2 = 1/2. \\ 1 & \text{if } a_2 > 1/2. \end{cases}$$

   ii. (10 points) Find the pure strategy Nash equilibria of the game.

   Solution: Using symmetry,

   $$B(a_1) = \begin{cases} 0 & \text{if } a_1 < 1/2 \\ [0, 1] & \text{if } a_1 = 1/2 \\ 1 & \text{if } a_1 > 1/2. \end{cases}$$

   Hence, there are three pure strategy Nash equilibria: $\{(0, 0), (1/2, 1/2), (1, 1)\}$.

   iii. (10 points) Find a symmetric mixed strategy Nash equilibrium of the game.

Solution: There are many possible answers for this question. Any strategy where $\mathrm{E}(a_i) = 1/2$ for every player i is a mixed-strategy Nash equilibrium. For example, $\alpha_1(0) = \alpha_1(1) = \alpha_2(0) = \alpha_2(1) = 1/2$ is an additional mixed strategy Nash equilibrium .