Quiz 2 solution
Econometrics II Spring, 2013

1. Show that $R^2 = \hat{\rho}^2_{Y\hat{Y}}$.
Proof:

$$\begin{aligned} R^2 &= \frac{\sum(\hat{Y}_t - \bar{Y})^2}{\sum(Y_t - \bar{Y})^2} \\ &= \frac{\sum(\hat{Y}_t - \bar{Y})^2 \sum(\hat{Y}_t - \bar{Y})^2}{\sum(Y_t - \bar{Y})^2 \sum(\hat{Y}_t - \bar{Y})^2} \end{aligned}$$

and note that

$$\begin{aligned} \sum(\hat{Y}_t - \bar{Y})^2 &= \sum(\hat{Y}_t - \bar{Y})(\hat{Y}_t - \bar{Y}) \\ &= \sum(\hat{Y}_t - \bar{Y})[(\hat{Y}_t - Y_t) + (Y_t - \bar{Y})] \\ &= \sum(\hat{Y}_t - \bar{Y})(Y_t - \bar{Y}) - \sum(\hat{Y}_t - \bar{Y})e_t \\ &= \sum(\hat{Y}_t - \bar{Y})(Y_t - \bar{Y}) \end{aligned}$$

Thus

$$R^2 = \frac{[\sum(\hat{Y}_t - \bar{Y})(Y_t - \bar{Y})]^2}{\sum(Y_t - \bar{Y})^2 \sum(\hat{Y}_t - \bar{Y})^2} = \hat{\rho}^2_{Y\hat{Y}}$$

2. Show that $\overline{R}^2 = 1 - [\frac{n-1}{n-K}(1 - R^2)]$.
Proof:

$$\begin{aligned} \frac{n-1}{n-K}(1 - R^2) &= \frac{n-1}{n-K}\frac{e^{'}e}{(Y - \bar{Y})'(Y - \bar{Y})} \\ &= \frac{e^{'}e/(n-K)}{(Y - \bar{Y})'(Y - \bar{Y})/(n-1)} \end{aligned}$$

Therefore

$$\overline{R}^2 = 1 - [\frac{n-1}{n-K}(1 - R^2)]$$