Homework 5

Consider the following dynamic linear model

$$y_t = Y_t \gamma + X_t \beta + u_t$$

$$u_t = U_t \alpha + e_t \quad e_t \sim I.I.D.N\left(0, \sigma^2\right)$$

where $Y_t = \{y_{t-1}, \cdots, y_{t-m}\}$, $X_t = \{x_{1t}, \cdots, x_{st}\}$, $U_t = \{u_{t-1}, \cdots, u_{t-p}\}$ for $t = 1, \cdots, n$.
We want to test for autocorrelation $H_0 : \quad \alpha = 0$. All of observations can be written in compact matrix form as follows:

$$y = Y\gamma + X\beta + u$$

$$u = U\alpha + e$$

Let $Z = (Y : X)$ and $\delta' = (\gamma' : \beta')$, the OLS estimates of the parameters in the model restricted by H_0 are $\hat{\delta} = (Z'Z)^{-1} Z'y$ with residuals $\hat{u} = y - Z\hat{\delta}$.

1. Show that the estimate of the information matrix can be given by $\hat{I} = \frac{1}{n\hat{\sigma}^2} \begin{bmatrix} \hat{U}'\hat{U} & \hat{U}'Y & \hat{U}'X \\ Y'\hat{U} & Y'Y & Y'X \\ X'\hat{U} & X'Y & X'X \end{bmatrix} \equiv \frac{1}{n\hat{\sigma}^2} \begin{bmatrix} \hat{U}'\hat{U} & \hat{U}'Z \\ Z'\hat{U} & Z'Z \end{bmatrix}$ where $\hat{\sigma}^2 = \frac{1}{n}\hat{u}'\hat{u}$.

2. Show that the LM statistic is given by

$$\hat{u}'\hat{U}\left\{\hat{U}'\hat{U} - \hat{U}'Z(Z'Z)^{-1}Z'\hat{U}\right\}^{-1}\hat{U}'\hat{u}/\hat{\sigma}^2$$

3. Show that the above LM statistic can be interpreted as nR^2 where R^2 is the usual coefficient of determination in the regression of $\hat{u}$ on $\hat{U}$ and Z.
(Hints: note that $Z'\hat{u} = 0$, the orthogonality of least squares regressors and residuals.)