Advanced Microeconomics II
Final Exam

WISE, Xiamen University

Spring 2010

14:00-17:00 June 10, 2010

1. Write precisely the definitions of the following concepts:
   (a) (5 points) an extensive game with perfect information.
   (b) (5 points) a subgame perfect equilibrium of an extensive game with perfect information.
   (c) (5 points) a behavioural strategy of player i in an extensive game $\{N, H, P, f_c, (\mathcal{I}_i), (\succeq_i)\}$.
   (d) (5 points) sequential rationality of an assessment (β, μ) of an extensive game with perfect recall $\{N, H, P, f_c, (\mathcal{I}_i), (\succeq_i)\}$.

2. In a first price sealed bid auction the highest bidder wins the object and pays his bid. Consider a first price sealed bid auction in which bidder $i \in \{1, 2\}$ observes only his own valuation v_i. This valuation is distributed uniformly and independently on $[0, 1]$ for each bidder. Every bidder is risk neutral.
   (a) (10 points) Describe precisely this first price auction as a Bayesian game.
   (b) (10 points) Calculate a symmetric pure strategy Nash equilibrium in which bidder i's bid is a linear function of his own valuation v_i.

3. (a) (10 points) Let $G = \{\{1, 2\}, (A_i), (u_i)\}$ be described by the following payoff matrix.

	C	D
C	$8, 8$	$1, 4$
D	$4, 1$	$2, 2$

   Construct a profile of strategies of the 2-period repeated game of G for which the outcome in each period is (D, D) but which is not a Nash equilibrium of the 2-period repeated game of G. (Be explicit about the profitable deviation that exists.)

   (b) (10 points) Prove the following proposition. If the strategic game G has a unique Nash equilibrium payoff profile then for any value of T the action profile chosen after any history in any subgame perfect equilibrium of the T-period repeated game of G is a Nash equilibrium of G.

4. Consider the extension of Rubinstein's infinite horizon alternating offer bargaining model from two players to three players. Every player has the same discount factor δ.

   In period 1, player 1 makes an offer which consists of (p_1, p_2, p_3) which represents the share of each player. Then player 2 decides to accept or to reject the offer within the same period. If player 2 accepts the offer, then player 3 is asked to accept or reject the offer within the same period. If both player 2 and player 3 accept the offer, then the bargaining is over and each player takes p_i. If either player 2 or player 3 rejects, the first person who rejects the present offer will initiate the next round.

In period t, player i initiates the offer, which specifies the shares of every player, Then player $i+1 (\text{mod } 3)$ either accepts or rejects the offer. If player $i+1 (\text{mod } 3)$ accepts the offer, then player $i+2 (\text{mod } 3)$ either accepts or rejects the offer. If both players accept the offer, then the bargaining is over and player i receives a payoff of $\delta^{t-1} p_i$. If $j \neq i$ is the first player who rejects the offer, then the next round starts with player j's offer.

(a) (10 points) Calculate a stationary subgame perfect equilibrium.

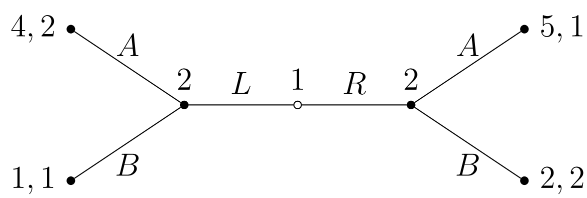
(b) (10 points) Can you sustain $(0.5, 0.5, 0)$ which is agreed upon in the initial round as an outcome of a subgame perfect equilibrium if δ is sufficiently close to to 1. Explain your answer.

5. Consider the Cournot duopoly model with linear demand $p = 1 - q_1 - q_2$, where q_i is the quantity produced by player i. Assume that each firm's marginal cost is 0, and the production capacity is larger than 1. Suppose that this game is infinitely repeated. Each player discounts the future by δ.

   (a) (10 points) The cartel outcome is the pair of outcomes that maximize the sum of the one-period profits of the two firms. Calculate the smallest $\delta > 0$ that can sustain the cartel outcome by a subgame perfect equilibrium in which the punishment against any deviation is to play one shot Nash equilibrium indefinitely.

   (b) (10 points) Suppose that $\delta = 1/2$. Calculate the best possible collusive outcome that can be sustained by a subgame perfect equilibrium with the same punishment scheme as given in (a). (By "the best possible", I mean the sum of the profits of the two firms is largest possible among all subgame perfect equilibria with the given punishment scheme.)

6. Consider the extensive form game below.



   (a) (2 points) Find the set of subgame perfect Nash equilibrium of this game.

   (b) (2 points) Now suppose that player 2 cannot observe player 1's move. Find the set of subgame perfect Nash equilibrium of this game.

   (c) Now suppose that player 2 observes player 1's move correctly with probability $p \in (0, 1)$ and incorrectly with probability $1 - p$ (e.g. if player 1 plays L, player 2 observes L with probability p and observes R with probability $1-p$). Suppose that player 2's propensity to observe incorrectly (i.e. given by the value of p) is common knowledge to the two players.

      i. (4 points) Draw the extensive form game tree.

      ii. (4 points) Find the set of pure strategy weak perfect Bayesian equilibria.

      iii. (5 points) Find the set of mixed strategy weak perfect Bayesian equilibria.

      iv. (3 points) Find the set of sequential equilibria (be explicit about consistency of beliefs).