Problem Set 2

Advanced Macroeconomics II, WISE, Xiamen University

March 16, 2009

The due date for this assignment is Monday March 23. It needs to be delivered by 8 am before the lecture starts. The total score is 10 points.

Solving a real business cycle model with adjustment cost to investment.

Consider the following model where a representative household solves

$$\max_{c_t,k_t} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \ln(c_t) \right\} \tag{1}$$

s.t.

$$c_t + k_t = z_t k_{t-1}^{\theta} \bar{n}^{1-\theta} - \frac{\phi}{2}(k_t - k_{t-1})^2 \tag{2}$$

$$\ln z_t = (1-\rho)\ln \bar{z} + \rho \ln z_{t-1} + \varepsilon_t \tag{3}$$

where c_t, n_t, k_t denote consumption, labor and capital. z_t is a stochastic process for technology with $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, and $0 \le \rho < 1$. Labor is constantly supplied and normalized to be equal to one: $\bar{n} = 1$. $\bar{z}$ is the steady state value of z_t. β, θ, and ϕ are parameters with $.0 < \beta < 1$, $0 < \theta < 1$, and $\phi \ge 0$. It might be useful to define the expression for output as:

$$y_t = z_t k_{t-1}^{\theta} \bar{n}^{1-\theta} \tag{4}$$

1. Describe the economy briefly. Comment on the preference, endowment, technology, and information. (0.4 point)
2. Find the first order necessary conditions (FONCs) of the representative agent. (1.5 points)
3. Write down the model in five equations, including output and the lagrange multiplier. (0.5 point)
4. Solve for the steady state, i.e. provide formulas for $\bar{c}$, $\bar{\lambda}$, $\bar{k}$, and $\bar{y}$ given $\bar{z}$ and other parameters. (Show the steps that you solve these values subsequently.) (0.8 points)
5. Log-linearize the equations. Define log-deviation of variable x_t as $\tilde{x}_t = \log(x_t/\bar{x})$. Write down all necessary log-linearized equations characterizing the equilibrium in the notation of log-deviation term $\tilde{x}_t$. (Note: you can use $\theta\beta\bar{z}\bar{k}^{\theta-1} = 1$ to simplify one equation.) (1.5 points)
6. Classify the exogenous state as $\tilde{z}_t$, endogenous state as $\tilde{k}_t$ and other endogenous variables as $\tilde{\lambda}_t$, $\tilde{c}_t$, and $\tilde{y}_t$. Substitute out $\tilde{c}_t$ by $\tilde{\lambda}_t$ in the log-linearized budget constraint, and combine this equation

with the log-linearized Euler equation, so that you can obtain the following two-dimensional stochastic difference equation together with exogenous technology process:

$$
\begin{aligned}
0 &= -\tilde{k}_t + a_1 \tilde{k}_{t-1} + a_2 \tilde{\lambda}_t + a_3 \tilde{z}_t && (5) \\
0 &= E_t \left[a_4 \tilde{k}_{t+1} + a_5 \tilde{k}_t + a_6 \tilde{k}_{t-1} + \tilde{\lambda}_{t+1} - \tilde{\lambda}_t + \tilde{z}_{t+1} \right] && (6) \\
\tilde{z}_t &= \rho \tilde{z}_{t-1} + \varepsilon_t && (7)
\end{aligned}
$$

what are a_1, a_2, a_3, a_4, a_5, and a_6 in terms of the original parameters? (Note: $\tilde{c}_t$, and $\tilde{y}_t$ can be solved immediately according to the log-linearized equation, once the above system is solved out. So for the moment we leave them out.) (1 points)

7. Calculate steady state values of $\bar{c}$, $\bar{\lambda}$, $\bar{k}$, and $\bar{y}$, and calculate a_1 to a_6 given the following parameter values: $\phi = 8$, $\beta = 0.99$, $\rho = 0.95$, $\theta = 1/3$, $\bar{z} = 1$, $\sigma_\varepsilon^2 = 0.712$. Assume that your model parameters are calibrated from quarterly data. (0.8 points)

8. Guess the recursive law of motion of the above system in step 6 as:

$$
\begin{aligned}
\tilde{\lambda}_t &= \eta_{\lambda k} \tilde{k}_{t-1} + \eta_{\lambda z} \tilde{z}_t && (8) \\
\tilde{k}_t &= \eta_{kk} \tilde{k}_{t-1} + \eta_{kz} \tilde{z}_t && (9)
\end{aligned}
$$

and exploit $E_t [\tilde{z}_{t+1}] = \rho \tilde{z}_t$. Using the undetermined coefficient method, insert these expression into (5) and (6) and collect terms on $\tilde{k}_{t-1}$ and $\tilde{z}_t$, to get two transformed equations. (Note that you may insert (8) and (9) twice into (6) to eventually reduce the equation to a function of only $\tilde{k}_{t-1}$ and $\tilde{z}_t$.) (0.4. points)

9. By comparing the coefficients on $\tilde{k}_{t-1}$, you can get a characteristic quadratic equation η_{kk} as $a\eta_{kk}^2 + b\eta_{kk} + c = 0$, where a, b, and c are determined by a_1, to a_6. What are a, b, and c? Solve this equation, what are the roots? Which root should you choose and why? Use your chosen value of η_{kk} to calculate $\eta_{\lambda k}$. (1.2 points)

10. By comparing the coefficients on $\tilde{z}_t$, solve for η_{kz} and $\eta_{\lambda z}$ (0.2 points)

11. Assume $\tilde{z}_0 = 0$, $\varepsilon_1 = 1$, and $\varepsilon_t = 0$ for $t = 1, ..., \infty$. Calculate $\tilde{z}_t$ recursively and plot the impulse response of $\tilde{z}_t$ for $t = 1, ..., 8$. (0.2 points)

12. Assume $\tilde{k}_0 = 0$, given the above technology shock, calculate $\tilde{k}_t$ recursively and plot the impulse response of $\tilde{k}_t$ for $t = 1, ..., 8$. (0.4 points)

13. Assume now $\phi = 50$. Recalculate the solution and plot impulse response of $\tilde{k}_t$ for the same technology shock as given in step 11. Compare the plot in step 12, give an economic explanation for the difference between scenario I and II. (1.1 points)