von Neumann-Morgenstern Utility

Definition

A simple lottery L is a list $L = (p_1, \ldots, p_{|C|})$ with $p_c \geq 0$ for all c and $\sum_c p_c = 1$.

Denote by $\mathcal{L}$ the set of all simple lotteries over the set of outcomes C.

Definition

A von Neumann-Morgenstern utility function is a function $U : \mathcal{L} \to R$ such that there exists an assignment of numbers $(u_1, \ldots, u_{|C|})$ to the $|C|$ outcomes such that for every L we have

$$U(L) = u_1 p_1 + \ldots + u_{|C|} p_{|C|}$$

and that for any two lotteries L, $L' \in \mathcal{L}$, $U(L) \geq U(L')$ iff $L \succeq L'$.

Implied assumptions?