Advanced Macroeconomics II
Gali (2008), Chapter 2

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Assumptions

- Perfect competition
- Fully flexible prices

Implications

- Limited role for money: only an accounting unit
- When monetary policy is specified in terms of an interest rate rule, no reference is made to the quantity of money in equilibrium, unless using a money demand function.
- A money demand function can be derived from certain utility function of the household, but often omitted.

A Classical Monetary Model

The economic environment

Preference: household optimizing expected utility
Production: firms optimizing profit
Technology: exogenous process
Information: rational expectation

In equilibrium – a dichatomy

- Real variables are uniquely determined independently of monetary policy.
- Determination of price level and other nominal variables under alternative monetary policy rules

2.1 Households

$$E_0 \Sigma_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{1}$$

where U is continuous and twice differentiable.

$$s.t. P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t - T_t \tag{2}$$

B_t: quantity of one-period, nominally riskless discount bonds purchased at t and maturing at $t+1$.
Q_t: price of bonds.
T_t: lump-sum taxes or dividends.
No-Ponzi condition:

$$\lim_{T \to \infty} E_t \{B_T\} \geq 0 \tag{3}$$

Households

FONCs for optimal consumption and labor

$$
\begin{aligned}
\mathcal{L} \quad &: \quad \underset{\{C_t, N_t, B_t\}}{E_0} \Sigma_{t=0}^{\infty} \beta^t \left\{ U(C_t, N_t) \right. \\
&\quad \left. + \lambda_t \left[B_{t-1} + W_t N_t - T_t - P_t C_t - Q_t B_t \right] \right\} \\
\frac{\partial \mathcal{L}}{\partial C_t} \quad &: \quad U_{C,t} - \lambda_t P_t \overset{!}{=} 0 \\
\frac{\partial \mathcal{L}}{\partial N_t} \quad &: \quad U_{N,t} + \lambda_t W_t \overset{!}{=} 0 \\
\frac{\partial \mathcal{L}}{\partial B_t} \quad &: \quad Q_t \overset{!}{=} \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \right] \\
\frac{\partial \mathcal{L}}{\partial \lambda_t} \quad &: \quad P_t C_t + Q_t B_t \overset{!}{=} B_{t-1} + W_t N_t - T_t
\end{aligned}
$$

Households

FONCs for optimal consumption and labor

Combine the FONCs of C_t and N_t

$$-\frac{U_{N,t}}{U_{C,t}} = \frac{W_t}{P_t} \tag{4}$$

Combine the FONCs of C_t and B_t

$$Q_t = \beta E_t \left[\frac{U_{C,t+1}}{U_{C,t}} \frac{P_t}{P_{t+1}} \right] \tag{5}$$

Through the combination, λ_t is deleted so that only three equations remain.

Household

An equivalent variational argument on the optimal consumption path

- Intra-period variation: increase in dC_t and dN_t.

$$\begin{aligned} \text{Utility} &: \quad U_{C,t}dC_t + U_{N,t}dN_t = 0 \\ \text{Budget constraint} &: \quad P_t dC_t + W_t dN_t = 0 \end{aligned}$$

  Equation (4) follows.

- Inter-period variation: reallocation of consumptions between t and $t+1$.

$$\begin{aligned} U_{C,t}dC_t + \beta E_t\left[U_{C,t+1}dC_{t+1}\right] &= 0 \\ \frac{P_t}{Q_t}dC_t + P_{t+1}dC_{t+1} &= 0 \end{aligned}$$

  Equation (5) follows.

Household

FONCs with a specific utility function

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

The consumer's optimality conditions become

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi} \quad (6)$$

$$Q_t = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \quad (7)$$

Also,

- the budget constraint holds with binding equality;
- the demand for real money balance is postulated,

$$M_t / P_t = f(Y_t, i_t)$$

2.2 Firms

Maximize profit subject to the production technology

$$\max_{(Y_t,\, N_t)} P_t Y_t - W_t N_t$$

$$s.t. \quad Y_t = A_t N_t^{1-\alpha}$$

FONC:

$$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha}$$

2.3 Competitive equilibrium

Market clearing

$$Y_t = C_t$$

- Implicitly we have assumed labor market clearing.
- Then these market equilibrium will guarantee the equilibrium in the bond holding.

Competitive equilibrium

Equilibrium conditions so far

$$
\begin{aligned}
\frac{W_t}{P_t} &= C_t^{\sigma} N_t^{\varphi} \\
Q_t &= \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \\
P_t C_t + Q_t B_t &= B_{t-1} + W_t N_t \\
Y_t &= A_t N_t^{1-\alpha} \\
\frac{W_t}{P_t} &= (1-\alpha) A_t N_t^{-\alpha} \\
Y_t &= C_t
\end{aligned}
$$

Seven unknowns: W_t, P_t, C_t, N_t, $Q_t(i_t)$, B_t, Y_t, one exogenous: A_t. But only six equations.

Need monetary policy to close the system.

Competitive equilibrium
The real side

Denote real wage as $W_t^r \equiv W_t / P_t$, take only equations with the real variables.

$$
\begin{aligned}
W_t^r &= C_t^\sigma N_t^\varphi \\
Y_t &= A_t N_t^{1-\alpha} \\
W_t^r &= (1-\alpha) A_t N_t^{-\alpha} \\
Y_t &= C_t
\end{aligned}
$$

Four unknowns: W_t^r, C_t, N_t, Y_t; four equations.

Real equilibrium

- Due to the simple functional forms, no assumption needs to be made on the steady state and trend to get the loglinear terms.
- Taking logs and denote $x_t \equiv \log X_t$

$$
\begin{aligned}
w_t^r &= \sigma c_t + \varphi n_t \\
y_t &= a_t + (1-\alpha) n_t \\
w_t^r &= a_t - \alpha n_t + \log(1-\alpha) \\
y_t &= c_t
\end{aligned}
$$

Real equilibrium

Solve the real equilibrium, in terms of exogenous variable a_t,

$$\begin{aligned} y_t &= \psi_{ya} a_t + \vartheta_y \\ n_t &= \psi_{na} a_t + \vartheta_n \\ w_t^r &= \psi_{wa} a_t + \vartheta_w \end{aligned}$$

where

$$\begin{aligned} \psi_{ya} &\equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} > 0, & \vartheta_y &\equiv \frac{(1-\alpha)\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha} \\ \psi_{na} &\equiv \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha} (>=< 0?), & \vartheta_n &\equiv \frac{\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha} \\ \psi_{wa} &\equiv \frac{\varphi+\sigma}{\sigma(1-\alpha)+\varphi+\alpha} > 0, & \vartheta_w &\equiv \frac{[\sigma(1-\alpha)+\varphi]\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha} \end{aligned}$$

Real equilibrium

The real interest rate

From the Euler equation

$$Q_t = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right],$$

denote $Q_t \equiv \exp(-i_t)$ and $\beta \equiv \exp(-\rho)$, and take log

$$c_t = E_t \left\{ c_{t+1} \right\} - \frac{1}{\sigma} \left[i_t - E_t \left\{ \pi_{t+1} \right\} - \rho \right].$$

Define real interest rate as

$$r_t \equiv i_t - E_t \left\{ \pi_{t+1} \right\},$$

$$\begin{aligned} r_t &= \rho + \sigma E_t \left\{ \Delta y_{t+1} \right\} \\ &= \rho + \sigma \psi_{ya} E_t \left\{ \Delta a_{t+1} \right\}. \end{aligned}$$

Real interest rate decreases for transitory shock.

2.4 Monetary Policy and Price Level Determination

Fisherian equation

$$i_t \approx E_t \{\pi_{t+1}\} + r_t$$

The norminal interest rate adjusts one for one with expected inflation, given a real rate determined by real factors.

2.4.1 An Exogenous Path for the Nominal Interest Rate

- The nominal interest rate as an exogenous stationary process $\{i_t\}$.
- Assume $E(i_t) = \rho$ with zero inflation and no secular growth as steady state.
- As a special case, $i_t = i = \rho$.
- Rewrite the Fisherian equation

$$E_t\{\pi_{t+1}\} = i_t - r_t$$

$$p_{t+1} = p_t + i_t - r_t + \xi_{t+1}$$

As long as $E_t\{\xi_{t+1}\} = 0$ for all t, which may not be related to economic fundamentals, the inflation or price level determination is consistent with the equilibrium.

- These nonfundamental factors: sunspot shocks.
- Equilibrium fluctuation caused: indeterminate equilibrium.
- The price level indeterminancy also causes indeterminancy in other norminal variables such as money supply or wage.

$$
\begin{aligned}
m_t &= p_t + y_t - \eta i_t \\
w_t &= w_t^r + p_t
\end{aligned}
$$

2.4.2. A simple Inflation-Based Interest Rate Rule

$$i_t = \rho + \phi_\pi \pi_t$$

where $\phi_\pi \geq 0$.
Combine it with the Fisherian equation,

$$\phi_\pi \pi_t = E_t \{\pi_{t+1}\} + \hat{r}_t$$

where $\hat{r}_t \equiv r_t - \rho$.

A simple Inflation-Based Interest Rate Rule

Case 1.

$\phi_\pi > 1$
Iterate forward:

$$\pi_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t \{\hat{r}_{t+k}\}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

where $\rho_a \in [0, 1]$. Then

$$\hat{r}_t = -\sigma\psi_{ya}(1 - \rho_a)a_t,$$

$$\pi_t = -\frac{\sigma\psi_{ya}(1 - \rho_a)}{\phi_\pi - \rho_a} a_t.$$

The larger is ϕ_π, the smaller is the variation of inflation caused by a_t.

A simple Inflation-Based Interest Rate Rule

Case 2.

$\phi_\pi \leq 1$

$$\pi_{t+1} = \phi_\pi \pi_t - \hat{r}_t + \xi_{t+1}$$

Again, with $\{\xi_t\}$, sunspot shocks, inflation and price level are indetermined.

A simple Inflation-Based Interest Rate Rule

Determinancy: Taylor principle

Central bank needs to adjust nominal interest rates more than one for one in response to changes in inflation, for the price level to be uniquely determined.

Although we have made a special case in a classical monetary economy, the result can be generalized into the New Keyesian framework.

2.4.3 An Exogenous Path for the Money Supply

Money supply as the policy instrument

Monetary policy instruments

- Interest rate
- Money supply

We have examined the monetary policy using interest rate, where money demand is

$$m_t - p_t = y_t - \eta i_t. \tag{8}$$

With money supply equals demand, m_t is determined given y_t, i_t and p_t. Now suppose money is the policy instrument, i_t can be determined from the money demand function given m_t, p_t and y_t.

An Exogenous Path for the Money Supply

Money supply as the policy instrument

Assuming money supply has an exogenous path, i.e. it does not respond to other economic variables.

Insert the equation (8) into the Fisherian equation to eliminate i_t,

$$\begin{aligned} m_t - p_t &= y_t - \eta E_t\{\pi_{t+1}\} - \eta r_t \\ (1+\eta)p_t &= \eta E_t\{p_{t+1}\} + m_t - y_t + \eta r_t \end{aligned}$$

$$p_t = \frac{\eta}{1+\eta} E_t\{p_{t+1}\} + \frac{1}{1+\eta} m_t + \underbrace{\frac{1}{1+\eta}(\eta r_t - y_t)}_{u_t}$$

where $u_t \equiv (1+\eta)^{-1}(\eta r_t - y_t)$ evolves independently of $\{m_t\}$.

With $\eta > 0$, solve forward.

An Exogenous Path for the Money Supply

Price level determination

$$
\begin{aligned}
p_t &= \frac{\eta}{1+\eta} E_t \left\{ \frac{\eta}{1+\eta} E_t \{p_{t+2}\} + \frac{1}{1+\eta} m_{t+1} + u_{t+1} \right\} \\
&\quad + \frac{1}{1+\eta} m_t + u_t \\
&\quad \ldots \\
&= \frac{1}{1+\eta} \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t \{m_{t+k}\} + \underbrace{\sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t \{u_{t+k}\}}_{u'_t}
\end{aligned}
$$

An Exogenous Path for the Money Supply

Price level determination

Now use

$$m_t = \frac{1}{1+\eta} m_t + \frac{\eta}{1+\eta} m_t$$

$$p_t = \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t \left\{ m_{t+k} - \frac{\eta}{1+\eta} m_{t+k} \right\} + u_t'$$

$$p_t = m_t + \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t \left\{ \Delta m_{t+k} \right\} + u_t' \tag{9}$$

- An arbitrary exogenous path for the money supply always determines the price level uniquely.

An Exogenous Path for the Money Supply

Interest rate determination

Given m_t and p_t, interest rate can be determined by the money demand function.

$$\begin{aligned} i_t &= \eta^{-1}\left[y_t - (m_t - p_t)\right] \\ &= \eta^{-1}\sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k E_t\left\{\Delta m_{t+k}\right\} + u_t'' \end{aligned} \tag{10}$$

where $u_t'' \equiv \eta^{-1}(u_t' + y_t)$, independent of monetary policy.

- i_t can be uniquely determined under exognous money supply.

An Exogenous Path for the Money Supply

Implication on monetary policy

- Assume money growth follows an AR(1) process.

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$$

- Assume away real shocks, and for simplicity, $y_t = r_t = 0$. Thus, also u_t' and u_t'' are zeros.
- Equation (9) becomes

$$\begin{aligned} p_t &= m_t + \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k \rho_m^k \Delta m_t \\ &= m_t + \frac{\eta \rho_m}{1+\eta(1-\rho_m)} \Delta m_t \\ &= m_{t-1} + \underbrace{[1 + \frac{\eta \rho_m}{1+\eta(1-\rho_m)}]}_{>1} \Delta m_t \end{aligned}$$

An Exogenous Path for the Money Supply

Implication on monetary policy

Problem

1. $\rho_m > 0 \Longrightarrow \frac{\partial p_t}{\partial \Delta m_t} > 1$. *This is in contrast with empirical finding of sluggish response of price level to monetary policy shocks.*

An Exogenous Path for the Money Supply

Implication on monetary policy

With similar derivation of the interest rate

$$\begin{aligned} i_t &= \eta^{-1} \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k \rho_m^k \Delta m_t \\ &= \underbrace{\frac{\rho_m}{1+\eta(1-\rho_m)}}_{>0} \Delta m_t \end{aligned} \tag{11}$$

Problem

2. $\rho_m > 0 \Longrightarrow \frac{\partial i_t}{\partial \Delta m_t} > 0$. *In contrast to the liquidity effect observed empirically.*

Optimal Monetary Policy

- Real variables are independent of monetary policy.
- Nominal variables are influenced by monetary policy.
- However, as only real variables of consumption and hours worked enter the household's utility, no policy is better than any other.
- The welfare effect can be generated with money in the utility function. However, the overall predictions are not in line with empirical evidence.

2.5 Money in the Utility Function

- In the previous "cashless economies", money is introduced ad-hocly without explicit justification.
- No explanation on why agents hold money which is inferior to bond as it does not bring return.
- To generate a demand for money, the real balance has to be built in the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}, N_t\right). \tag{12}$$

- Money holdings in the budget constraint

$$P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t - T_t \tag{13}$$

Money in the Utility Function

Define wealth at the beginning of each period before making economic decisions

$$A_t \equiv B_{t-1} + M_{t-1}$$

The budget constraint is now

$$P_t C_t + Q_t A_{t+1} + (1 - Q_t) M_t \leq A_t + W_t N_t - T_t \tag{14}$$

The solvency (no-Ponzi) constraint

$$\lim_{T \to \infty} E_t \{A_T\} \geq 0 \;\; \forall \; t.$$

Money in the Utility Function

Implications

- All financial assets A_t yield a gross nominal return $Q_t^{-1} = \exp\{i_t\}$.
- The agents purchase the utility-yielding "services" of money balance at a unit price, roughly the nominal interest rate.

$$\begin{aligned} 1 - Q_t &= 1 - \exp\{-i_t\} \\ &\approx i_t \end{aligned}$$

- $1 - Q_t$: the opportunity cost of holding one's financial wealth in terms of monetary assets, instead of interest-bearing bonds.

Money in the Utility Function

FONCs

Lagrange:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U\left(C_t, \frac{M_t}{P_t}, N_t \right) \right.$$
$$\left. -\lambda_t \left[P_t C_t + Q_t A_{t+1} + (1 - Q_t) M_t - A_t - W_t N_t + T_t \right] \right\}$$

$$\frac{\partial L}{\partial \frac{M_t}{P_t}} : U_{m,t} - \lambda_t (1 - Q_t) \overset{!}{=} 0$$

$$\frac{U_{m,t}}{U_{C,t}} = 1 - \exp(-i_t) \tag{15}$$

Whether the introduction of money into the utility function has consequence on the real equilibrium depends on the utility function, i.e., whether money is separable or nonseparable in the utility function.

2.5.1. An Example with Separable Utility

Household's problem

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{(M_t/P_t)^{1-v}}{1-v} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

Separability: Neither $U_{C,t}$ nor $U_{N,t}$ depend on the level of real balance.

- Real equilibrium as captured before is not affected.
- The introduction of money, however, allows a money demand equation to be derived as

$$m_t - p_t = y_t - \eta i_t + const.$$

An Example with Separable Utility

Money demand equation

From the optimality condition in Equation (15),

$$
\begin{aligned}
\frac{(M_t / P_t)^{-v}}{C_t^{-\sigma}} &= 1 - \exp\{-i_t\} \\
\frac{M_t}{P_t} &= C_t^{\frac{\sigma}{v}} \left[1 - \exp\{-i_t\}\right]^{-\frac{1}{v}}
\end{aligned}
$$

Taking log,

$$
\begin{aligned}
m_t - p_t &= \frac{\sigma}{v} c_t - \frac{1}{v} \log\left[1 - \exp\{-i_t\}\right] \\
&\approx \frac{\sigma}{v} c_t - \underbrace{\frac{1}{v\left[\exp(i) - 1\right]}}_{\eta} i_t + const.
\end{aligned}
$$

An Example with Separable Utility

Money demand equation

Ignore the constant, assume $\sigma = v$ and market clearing such that $c_t = y_t$,

$$m_t - p_t = y_t - \eta i_t.$$

- In this economy, it is as in the cashless economy, that monetary policy does not affect real variables.

2.5.2 An Example with Nonseparable Utility

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{X_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

X_t: a composite index of consumption and real balances

$$\begin{aligned} X_t &\equiv \left[(1-\theta)\, C_t^{1-v} + \theta(\frac{M_t}{P_t})^{1-v}\right]^{\frac{1}{1-v}}, \forall v \neq 1 \\ &\equiv C_t^{1-\theta}(\frac{M_t}{P_t})^{\theta}, \forall v = 1 \end{aligned}$$

v: (inverse) elasticity of substitution between C_t and $\frac{M_t}{P_t}$.
θ: relative weight of real balances in utility.

An Example with Nonseparable Utility

$$
\begin{aligned}
X_t &\equiv \left[(1-\theta)\, C_t^{1-v} + \theta \left(\frac{M_t}{P_t}\right)^{1-v} \right]^{\frac{1}{1-v}} \\
U_{c,t} &= (1-\theta) X_t^{v-\sigma} C_t^{-v} \\
U_{m,t} &= \theta X_t^{v-\sigma} \left(\frac{M_t}{P_t}\right)^{-v} \\
U_{n,t} &= -N_t^{\varphi}
\end{aligned}
$$

An Example with Nonseparable Utility

FONCs

$$
\begin{aligned}
-\frac{U_{N,t}}{U_{C,t}} &= \frac{W_t}{P_t} && (16)\\
Q_t &= \beta E_t\left[\frac{U_{C,t+1}}{U_{C,t}}\frac{P_t}{P_{t+1}}\right] && (17)\\
\frac{U_{m,t}}{U_{C,t}} &= 1-\exp(-i_t) && (18)
\end{aligned}
$$

The first two hold as before without money in the utility function.

An Example with Nonseparable Utility

FONCs specific

$$\frac{W_t}{P_t} = N_t^{\varphi} X_t^{\sigma - v} C_t^{v} (1-\theta)^{-1} \tag{19}$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-v} \left(\frac{X_{t+1}}{X_t} \right)^{v-\sigma} \frac{P_t}{P_{t+1}} \right\} \tag{20}$$

$$\frac{M_t}{P_t} = C_t \left(1 - e^{-i_t}\right)^{-\frac{1}{v}} \left(\frac{\theta}{1-\theta} \right)^{\frac{1}{v}} \tag{21}$$

In the particular case of $v = \sigma$,

- the first two optimality conditions match those obtained in the case of the separable utility, and thus
- lead to the same equilibrium implications derived in section before. Money is neutral.

An Example with Nonseparable Utility

In the general case of

$v \neq \sigma$

- Both labor supply equation (19) and the Euler equation (20) are influenced by the level of real balances through the dependence of the index X_t.
- The level of real balances depends, in turn, on the nominal interest rate, as implied in equation (21).
- Monetary policy (MP) is not neutral.
- Different monetary policy rules $\Longrightarrow$ Impacts on nominal interest rates $\Longrightarrow$ Real balances $\Longrightarrow$ Labor supply and output.

An Example with Nonseparable Utility

To derive the equilibrium: log linearize

Take log on equation (21), again

$$m_t - p_t = y_t - \eta i_t$$

An Example with Nonseparable Utility

To derive the equilibrium: log linearize

Take log on equation (19),

$$w_t - p_t = \sigma c_t + \varphi n_t + \underbrace{(v - \sigma)(c_t - x_t)}_{extra} - \log(1 - \theta)$$

Loglinearize around the steady state of X_t,

$$c_t - x_t = \chi\left[c_t - (m_t - p_t)\right]$$

where

$$\chi \equiv \frac{\theta(\bar{M}/\bar{P})^{1-v}}{\bar{X}^{1-v}} = \frac{\theta^{\frac{1}{v}}(1-\beta)^{1-\frac{1}{v}}}{(1-\theta)^{\frac{1}{v}} + \theta^{\frac{1}{v}}(1-\beta)^{1-\frac{1}{v}}}.$$

An Example with Nonseparable Utility

To derive the equilibrium: log linearize

$$\begin{aligned} w_t - p_t &= \sigma c_t + \varphi n_t + \chi(v-\sigma)\left[c_t - (m_t - p_t)\right] - \log(1-\theta) \quad (22) \\ &= \sigma c_t + \varphi n_t + \chi\eta(v-\sigma) i_t - \log(1-\theta) \\ &= \sigma c_t + \varphi n_t + \omega i_t - \log(1-\theta) \quad (23) \end{aligned}$$

with $\omega = \chi\eta(v-\sigma)$.

An Example with Nonseparable Utility

Examine the coefficient

Around the steady state of equation (21), the inverse consumption velocity can be defined as

$$k_m \equiv \frac{\bar{M}/\bar{P}}{\bar{C}} = \left[\frac{\theta}{(1-\beta)(1-\theta)}\right]^{\frac{1}{v}}.$$

We can rewrite χ as

$$\chi = \frac{k_m(1-\beta)}{1+k_m(1-\beta)}$$

Then to transform ω

$$\begin{aligned}
\omega &\equiv \chi\eta(v-\sigma) \\
&= \frac{k_m(1-\beta)}{1+k_m(1-\beta)}\frac{1}{v\left[\exp(i)-1\right]}(v-\sigma) \\
&= \frac{k_m\beta(1-\frac{\sigma}{v})}{1+k_m(1-\beta)}
\end{aligned}$$

An Example with Nonseparable Utility

Examine the coefficient

$$w_t - p_t = \sigma c_t + \varphi n_t + \omega i_t - \log(1-\theta)$$

$$\omega \equiv \frac{k_m \beta (1 - \frac{\sigma}{v})}{1 + k_m(1-\beta)}$$

- For plausible value of η and σ, $v \simeq \frac{1}{i\eta}$ (from $\eta = \frac{1}{v[\exp(i)-1]}$), $v \gg \sigma$, then $\omega > 0$.
- $i_t \uparrow$, $m_t - p_t \downarrow$, $x_t \downarrow$. For any give c_t, $U_{c,t} \downarrow$, $U_{cm} > 0$.
- At any given wage, $n_t \downarrow$.

An Example with Nonseparable Utility

To derive the equilibrium: log linearize Euler Equ.

Log-linearize the Euler equation (20)

$$
\begin{aligned}
c_t &= E_t\left\{c_{t+1}\right\} - \frac{1}{\sigma}\left[i_t - E_t\left\{\pi_{t+1}\right\}\right. \\
&\quad \left. -(v-\sigma)E_t\left\{(c_{t+1}-x_{t+1})-(c_t-x_t)\right\} - \rho\right] \\
&= E_t\left\{c_{t+1}\right\} - \frac{1}{\sigma}\left[i_t - E_t\left\{\pi_{t+1}\right\}\right. \\
&\quad \left. -\chi(v-\sigma)E_t\left\{\Delta c_{t+1} - \Delta\left(m_{t+1}-p_{t+1}\right)\right\} - \rho\right] \\
&= E_t\left\{c_{t+1}\right\} - \frac{1}{\sigma}\left[i_t - E_t\left\{\pi_{t+1}\right\} - \omega E_t\left\{\Delta i_{t+1}\right\} - \rho\right]
\end{aligned}
$$

So, for $v > \sigma$ such that $\omega > 0$, $E_t\left\{\Delta i_{t+1}\right\} \uparrow$, $c_t \uparrow$.

An Example with Nonseparable Utility

Equilibrium

Combine FONC of real wage and production

$$
\begin{aligned}
w_t - p_t &= a_t - \alpha n_t \\
y_t &= a_t + (1-\alpha) n_t
\end{aligned}
$$

and obtain

$$w_t - p_t = y_t - n_t.$$

Then Equation (23) becomes

$$\sigma c_t + \varphi n_t + \omega i_t + \log(1-\theta) = y_t - n_t.$$

Using $y_t = c_t$, we get

$$y_t = \psi_{ya} a_t - \psi_{yi} i_t \tag{24}$$

by ignoring the constant.

An Example with Nonseparable Utility

Equilibrium

$$\psi_{yi} \equiv \frac{\omega(1-\alpha)}{\sigma+\varphi+\alpha(1-\sigma)}$$

Implications: when $\omega \neq 0$,

- $i_t \longrightarrow y_t$, money is non-neutral
- However, equation (24) cannot be determined without the description of a monetary policy.

An Example with Nonseparable Utility

Equilibrium

In order to pin down the equilibrium conditions, it needs to be combined with other conditions and the monetary policy rule.

Using $y_t = c_t$, one such condition is

$$y_t = E_t\left\{y_{t+1}\right\} - \frac{1}{\sigma}\left[i_t - E_t\left\{\pi_{t+1}\right\} - \omega E_t\left\{\Delta i_{t+1}\right\} - \rho\right] \qquad (25)$$

An Example with Nonseparable Utility
Equilibrium

Combine (24), (25) and

$$i_t = \rho + \phi_\pi \pi_t + v_t \tag{26}$$

where

$$\begin{aligned} v_t &= \rho_v v_{t-1} + \varepsilon_t^v \\ a_t &= \rho_a a_{t-1} + \varepsilon_t^a \end{aligned}$$

Inserting (26) into (24) and (25) yields...

An Example with Nonseparable Utility

Equilibrium

$$\pi_t = -\frac{\sigma(1-\rho_a)\psi_{ya}}{\phi_\pi(1+\omega\psi)(1-\Theta\rho_a)}a_t - \frac{1+(1-\rho_v)\omega\psi}{\phi_\pi(1+\omega\psi)(1-\Theta\rho_v)}v_t$$

$$i_t = -\frac{\sigma(1-\rho_a)\psi_{ya}}{(1+\omega\psi)(1-\Theta\rho_a)}a_t - \frac{\rho_v}{\phi_\pi(1+\omega\psi)(1-\Theta\rho_v)}v_t$$

$$y_t = \psi_{ya}\left(1+\frac{\sigma(1-\rho_a)\psi_{yi}}{(1+\omega\psi)(1-\Theta\rho_a)}\right)a_t + \frac{\rho_v\psi_{yi}}{\phi_\pi(1+\omega\psi)(1-\Theta\rho_v)}v_t$$

where

$$\Theta \equiv \frac{1+\omega\psi\phi_\pi}{(1+\omega\psi)\phi_\pi}$$

$$\psi \equiv \frac{\alpha+\varphi}{\sigma(1-\alpha)+\alpha+\varphi}$$

An Example with Nonseparable Utility

Impact of monetary policy on the economy's equilibrium

Conditional on an exogenous MP shock ε_t^v to v_t,

$$\frac{dy_t}{di_t} = \frac{dy_t/dv_t}{di_t/dv_t} = -\psi_{yi}$$

Assume $\sigma = \varphi = 1$, $\alpha = 1/3$ (baseline calibration),

$$\psi_{yi} = \omega/3.$$

With $\omega = \frac{k_m \beta(1-\frac{\sigma}{\nu})}{1+k_m(1-\beta)} \approx k_m$, ν large, $\beta \approx 1$, so

$$\psi_{yi} \approx k_m/3.$$

An Example with Nonseparable Utility

Impact of monetary policy on the economy's equilibrium

Calibration using post war US data.

- Using M1, $k_m \approx 0.3$, $\psi_{yi} \approx 0.1$.
  - i_t: increase by 1% annually, or 0.25% quarterly, y_t decrease by 0.025%, very small.
- Using M2, $k_m \approx 3$, $\psi_{yi} \approx 1$.
  - i_t: increase by 1% annually, or 0.25% quarterly, y_t decrease by 0.25%, closer to the estimated output effects of a MP shock.

An Example with Nonseparable Utility

However, the deficiency

$$
\begin{aligned}
\frac{d\pi_t}{di_t} &= \frac{d\pi_t \,/\, dv_t}{di_t \,/\, dv_t} = (1 + (1 - \rho_v)\omega\psi)\rho_v^{-1} > 0 \\
\frac{dr_t}{di_t} &= 1 - \frac{dE_t \{\pi_{t+1}\} \,/\, dv_t}{di_t \,/\, dv_t} = -(1 - \rho_v)\omega\psi < 0
\end{aligned}
$$

In response to a MP shock that raises the nominal interest rate and lowers output,

- inflation tends to increase,
- the real rate tends to go down (as a result of the dominant effect of higher expected inflation)

Contradicting to empirical observations.

An Example with Nonseparable Utility

Deficiency in Long-run vs. short-run tradeoff

- Empirically, we find money and inflation are non-neutral in the short run, but are neutral in the long run when price can adjust.
- However, this model implies that the long-run and short-run effects are the same.

2.5.3 Optimal Monetary Policy in a Classical Economy with Money in the Utility Function

- Optimal monetary policy in terms of welfare optimization.
- In a classical monetary model without money in the utility f-n., since i_t does not affect any real variables, which affect the consumer's utility f-n, no policy is better than another.
- In a classical economy with money in the utility f-n., since i_t affects real money balance, it in turn affects consumer's utility.

A Social Planner's Problem

Assume a social planner takes the role of the central banker, and seeks to maximize the utility of the representative household. The intertemporal problem:

- For household: they can reallocate their own consumption over time through financial market,
- For the economy as a whole: there are no mechanisms to provide intertemporal links.

A Social Planner's Problem

The social planner would solve a sequence of static problems of the form:

$$\max U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

$$s.t. C_t = A_t N_t^{1-\alpha} \tag{27}$$

Lagrange:

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) - \lambda_t \left(C_t - A_t N_t^{1-\alpha}\right) \tag{28}$$

FONCs:

$$-\frac{U_{n,t}}{U_{C,t}} = (1-\alpha) A_t N_t^{-\alpha} \tag{29}$$

$$U_{m,t} = 0. \tag{30}$$

A Social Planner's Problem
Implications

Equation (29) requires that marginal rate of substitution btw. hours of work and consumption equal marginal product of labor, as with firm and household's problem.

- Firm: real wage = marginal product of labor
- Household: real wage = marginal rate of substitution btw. hours of work and consumption.
- Equ. (29) independent of monetary policy.

A Social Planner's Problem

Implications

- Equation (30) equates the marginal utility of real balances to the "social" marginal cost of producing real balances, which is implicitly assumed to be zero.

$$U_{m,t} = 0$$

- The agents purchase the utility-yielding "services" of money balance at a unit price, roughly the nominal interest rate.

$$\frac{U_{m,t}}{U_{C,t}} = 1 - \exp\{-i_t\} \approx i_t$$

- The social and individual optimal coincides if and only if $i_t = 0 \ \forall t$. This is a policy known as "*Friedman rule*".
- Under the rule, consumption, output, employment and real money balance are uniquely determined by equations (27), (29) and (30).

Implementation of the Friedman rule

- For the long run: From the Fisherian equation, $i = \pi + r = 0 \Longrightarrow \pi = -r = -\rho < 0$, i.e., moderate deflation.
- For the short run: to keep $i_t = 0\ \forall t$, a policy rule of $i_t = 0$ will not work for determinacy, even though indeterminacy should not have any welfare consequences.

Implementation of the Friedman rule

To avoid indeterminacy, a central bank can follow a rule as

$$i_t = \phi(r_{t-1} + \pi_t), \text{ with } \phi > 1.$$

Combine with the Fisherian equation

$$\begin{aligned} i_t &= E_t\{\pi_{t+1}\} + r_t, \\ E_t\{i_{t+1}\} &= \phi E_t(r_t + \pi_{t+1}) = \phi i_t. \end{aligned}$$

The only stationary solution is $i_t = 0$ for all t.

- Think about $E_t\{i_{t+s}\} = \phi^s i_t$ as being finite when s is large, then i_t has to approach to zero.
- $\pi_t = -r_{t-1}$ is guaranteed, i.e., inflation is fully determined.

Implementation of the Friedman rule

To generalize, any policy settings to guarantee that current inflation moves inversely, and one for one with r_{t-1}, will imply $i_t = 0$ and an efficient amount of real balances.