**Advanced Macro II**
the reference answer to **quiz1**

Note: The answer is just for reference.
**Loglinearize the following equations:**
setting $X_t = \overline{X}e^{\widehat{x}_t}$.

1. $X_t(1-Q_t) = \frac{Z_t(V+X_t)}{V-Y_t}$

   **ANS:**

   the steady state: $\overline{X}(1-\overline{Q}) = \frac{\overline{Z}(V+\overline{X})}{V-\overline{Y}}$

   loglinearize:

$$
\begin{aligned}
\overline{X}e^{\widehat{x}_t}(1-\overline{Q}e^{\widehat{q}_t}) &= \frac{\overline{Z}e^{\widehat{z}_t}(V+\overline{X}e^{\widehat{x}_t})}{V-\overline{Y}e^{\widehat{y}_t}} \\
\overline{X}e^{\widehat{x}_t}(1-\overline{Q}e^{\widehat{q}_t})(V-\overline{Y}e^{\widehat{y}_t}) &= \overline{Z}e^{\widehat{z}_t}(V+\overline{X}e^{\widehat{x}_t}) \\
\overline{X}e^{\widehat{x}_t}(V-\overline{Y}e^{\widehat{y}_t}-V\overline{Q}e^{\widehat{q}_t}+\overline{Q}e^{\widehat{q}_t}\overline{Y}e^{\widehat{y}_t}) &= V\overline{Z}e^{\widehat{z}_t}+\overline{Z}e^{\widehat{z}_t}\overline{X}e^{\widehat{x}_t}) \\
V\overline{X}e^{\widehat{x}_t}-\overline{XY}e^{\widehat{x}_t+\widehat{y}_t}-\overline{X}V\overline{Q}e^{\widehat{x}_t+\widehat{q}_t}+\overline{XQY}e^{\widehat{x}_t+\widehat{q}_t+\widehat{y}_t} &= V\overline{Z}e^{\widehat{z}_t}+\overline{ZX}e^{\widehat{z}_t+\widehat{x}_t} \\
V\overline{X}\widehat{x}_t-\overline{XY}(\widehat{x}_t+\widehat{y}_t)-V\overline{XQ}(\widehat{x}_t+\widehat{q}_t)-\overline{XQY}(\widehat{x}_t+\widehat{q}_t+\widehat{y}_t) &= V\overline{Z}(\widehat{z}_t)+\overline{ZX}(\widehat{z}_t+\widehat{x}_t)
\end{aligned}
$$

   By rearranging the coefficients of the variables, we can get that
   $(V\overline{X}-\overline{XY}-V\overline{XQ}+\overline{XQY}-\overline{ZX})\widehat{x}_t-(\overline{XY}-\overline{XQY})\widehat{y}_t-(V\overline{XQ}-\overline{XQY})\widehat{q}_t = (V\overline{Z}+\overline{ZX})\widehat{z}_t$
   $\widehat{x}_t + \frac{\overline{XY}(1-\overline{Q})}{V\overline{Z}}\widehat{y}_t + \frac{\overline{XQ}(\overline{Y}-V)}{V\overline{Z}}\widehat{q}_t = \frac{(V\overline{Z}+\overline{ZX})}{V\overline{Z}}\widehat{z}_t$

2. $\frac{W_t}{P_T} = \alpha\frac{A_T}{N_T^{1-\alpha}}$

   **ANS:**

   the steady state: $\frac{\overline{W}}{\overline{P}} = \alpha\frac{\overline{A}}{\overline{N}^{1-\alpha}}$

   loglinearize:

$$
\begin{aligned}
\frac{W_t}{P_T} &= \alpha\frac{A_T}{N_T^{1-\alpha}} \\
\frac{\overline{W}e^{\widehat{w}_t}}{\overline{P}e^{\widehat{p}_t}} &= \alpha\frac{\overline{A}e^{\widehat{a}_t}}{\overline{N}^{1-\alpha}e^{(1-\alpha)\widehat{n}_t}} \\
\frac{\overline{W}}{\overline{P}}e^{\widehat{w}_t-\widehat{p}_t} &= \alpha\frac{\overline{A}}{\overline{N}^{1-\alpha}}e^{\widehat{a}_t-(1-\alpha)\widehat{n}_t} \\
\widehat{w}_t-\widehat{p}_t &= \widehat{a}_t-(1-\alpha)\widehat{n}_t
\end{aligned}
$$

3. $\lambda_t = \beta E_t[\lambda_{t+1}(1+r_t)]$ and $\widehat{r}_t = \log\frac{1+r_t}{1+\overline{r}}$

   **ANS:**

   the steady state:

$$
\begin{aligned}
\overline{\lambda} &= \beta E_t[\overline{\lambda}(1+\overline{r})] \\
\beta(1+\overline{r}) &= 1
\end{aligned}
$$

since

$$\begin{aligned}\widehat{r}_t &= \log\frac{1+r_t}{1+\overline{r}}\\ 1+r_t &= (1+\overline{r})e^{\widehat{r}_t}\end{aligned}$$

loglinearize:

$$\begin{aligned}\lambda_t &= \beta E_t[\lambda_{t+1}(1+r_t)]\\ \overline{\lambda}e^{\widehat{\lambda}_t} &= \beta E_t[\overline{\lambda}e^{\widehat{\lambda}_{t+1}}(1+\overline{r})e^{\widehat{r}_t}]\\ \widehat{\lambda}_t &= E_t[\widehat{\lambda}_{t+1}]+\widehat{r}_t\end{aligned}$$

4. $Y_t = A_t[\alpha K_{t-1}^{\rho}+(1-\alpha)N_{t-1}^{\rho}]^{1/\rho}$

   **ANS:**

   the steady state:

   $\overline{Y} = \overline{A}[\alpha\overline{K}^{\rho}+(1-\alpha)\overline{N}^{\rho}]^{1/\rho}$

   loglinearize:

$$\begin{aligned}Y_t &= A_t[\alpha K_{t-1}^{\rho}+(1-\alpha)N_{t-1}^{\rho}]^{1/\rho}\\ \overline{Y}^{\rho}e^{\rho\widehat{y}_t} &= \overline{A}^{\rho}e^{\rho\widehat{a}_t}[\alpha\overline{K}^{\rho}e^{\rho\widehat{k}_{t-1}}+(1-\alpha)\overline{N}^{\rho}e^{\rho\widehat{n}_{t-1}}]\\ \frac{\overline{Y}^{\rho}}{\overline{A}^{\rho}}e^{\rho\widehat{y}_t-\rho\widehat{a}_t} &= \alpha\overline{K}^{\rho}e^{\rho\widehat{k}_{t-1}}+(1-\alpha)\overline{N}^{\rho}e^{\rho\widehat{n}_{t-1}}\\ \frac{\overline{Y}^{\rho}}{\overline{A}^{\rho}}(\widehat{y}_t-\widehat{a}_t) &= \alpha\overline{K}^{\rho}\widehat{k}_{t-1}+(1-\alpha)\overline{N}^{\rho}\widehat{n}_{t-1}\\ \widehat{y}_t &= \widehat{a}_t+\alpha\frac{\overline{A}^{\rho}\overline{K}^{\rho}}{\overline{Y}^{\rho}}\widehat{k}_{t-1}+(1-\alpha)\frac{\overline{A}^{\rho}\overline{N}^{\rho}}{\overline{Y}^{\rho}}\widehat{n}_{t-1}\end{aligned}$$

5. $C_t + K_t = Z_t K_{t-1}^{\theta}\overline{N}^{1-\theta} - \frac{\Phi}{2}(K_t - K_{t-1})^2$

   **ANS:**

   the steady state:

$$\begin{aligned}\overline{C}+\overline{K} &= \overline{ZK}^{\theta}\overline{N}^{1-\theta}-\frac{\Phi}{2}(\overline{K}-\overline{K})^2\\ \overline{C}+\overline{K} &= \overline{ZK}^{\theta}\overline{N}^{1-\theta}\end{aligned}$$

   loglinearize:

$$\begin{aligned}C_t+K_t &= Z_tK_{t-1}^{\theta}\overline{N}^{1-\theta}-\frac{\Phi}{2}(K_t-K_{t-1})^2\\ C_t+K_t &= Z_tK_{t-1}^{\theta}\overline{N}^{1-\theta}-\frac{\Phi}{2}(K_t^2-2K_tK_{t-1}+K_{t-1}^2)\\ \overline{C}e^{\widehat{c}_t}+\overline{K}e^{\widehat{k}_t} &= \overline{Z}e^{\widehat{z}_t}\overline{K}^{\theta}e^{\theta\widehat{k}_{t-1}}\overline{N}^{1-\theta}-\frac{\Phi}{2}(\overline{K}^2e^{2\widehat{k}_t}-2\overline{K}^2e^{\widehat{k}_t+\widehat{k}_{t-1}}+\overline{K}^2e^{2\widehat{k}_{t-1}})\\ \overline{C}\widehat{c}_t+\overline{K}\widehat{k}_t &= \overline{ZK}^{\theta}\overline{N}^{1-\theta}(\widehat{z}_t+\theta\widehat{k}_{t-1})-\frac{\Phi\overline{K}^2}{2}[2\widehat{k}_t-2(\widehat{k}_t+\widehat{k}_{t-1})+2\widehat{k}_{t-1}]\\ \frac{\overline{C}}{\overline{C}+\overline{K}}\widehat{c}_t+\frac{\overline{K}}{\overline{C}+\overline{K}}\widehat{k}_t &= \widehat{z}_t+\theta\widehat{k}_{t-1}\end{aligned}$$