Advanced Microeconomics II
Quiz 2

WISE, Xiamen University

Spring 2011

1. (Gibbons 1.2) Players 1 and 2 are bargaining over how to split five dollars. Both players simultaneously name shares they would like to have, s_1 and s_2, where $0 \leq s_i \leq 5, i = 1, 2$. If $s_1 + s_2 \leq 5$, then the players receive the shares they named; if $s_1 + s_2 > 5$, then both players receive zero.

   (a) Consider a symmetric mixed strategy equilibrium, where each player randomizes over two numbers a and b.

      i. (3 points) Show that to be a mixed strategy equilibrium $a + b \leq 5$.

Solution: Note that if $a = b = 5$ then we have a Nash equilibrium, thereby disproving the statement.
If $a < b$ and $a + b > 5$ then Note that $a < b$. Otherwise, we have pure strategy equilibrium that are have been identified above. Suppose $a + b > 1$, then $E(U_i(b)) = 0$. Since every action in the support of each player's equilibrium mixed strategy yields that player the same payoff, then $U_i = 0$. If $b < 5$ a player could choose $s = 5 - b > 0$ which guarantees a strictly positive payoff. If $b = 5$ then there exists ϵ such that $F(b - \epsilon) > 0$ (otherwise the player is playing the pure strategy b). Hence, a player could choose $s = \epsilon$ which guarantees a strictly positive payoff of $\epsilon F(b - \epsilon)$.

      ii. (3 points) Show that to be a mixed strategy equilibrium $a + b \geq 5$.

Solution: Suppose $a + b < 5$, then $E(U_i(a)) = a$ and $b < 1$. Since every action in the support of each player's equilibrium mixed strategy yields that player the same payoff, then $U_i = a$. But if player i chooses $5 - b$, they guarantee a payoff of $5 - b > a$.

      iii. (4 points) Derive such a mixed strategy equilibrium where $a = 1$ and $b = 4$.

Solution: Denote α as the probability of choosing 1. To be an equilibrium the player must be indifferent between choosing between the two numbers. Hence,

$$1 = 4\alpha \Rightarrow \alpha = \frac{1}{4}.$$

Note that any other action gives a payoff strictly less than a_1.
If you assume that the strategy is over $[a, b]$, then $E(U_i(a)) = a = E(U_i(b)) = bF(a)$, $F(a) = \frac{a}{b}$. To be an equilibrium strategy,

$$a \equiv sF(1-s) \quad \text{for all } s \in [a, b].$$

Differentiating gives $F(1-s) - f(1-s)s = 0$, which implies $F(s) = \frac{c}{(1-s)}$, where c is the constant of integration. To solve the constant of integration which use the properties of a probability distribution function, i.e. $F(b) = 1$, which implies $c = 1 - b = a$. So the symmetric mixed strategy equilibrium is

$$F(s_i) = \frac{1}{1-s_i}$$

where $i \in \{1, 2\}$.