Problem Set 8

April 23, 2012

Given

$$P_t = E_t \left[\sum_{j=1}^{\infty} \delta^j \frac{u_C\left(C_{t+j}^*\right)}{u_C\left(C_t^*\right)} d_{t+j} \right] = E_t \left[\sum_{j=1}^{\infty} \delta^j \left(\frac{C_{t+j}^*}{C_t^*} \right)^{\gamma - 1} d_{t+j} \right],$$

$$\ln\left(C_{t+1}^*/C_t^*\right) = \mu_c + \sigma_c \eta_{t+1},$$

$$\ln\left(d_{t+1}/d_t\right) = \mu_d + \sigma_d \varepsilon_{t+1},$$

and

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right).$$

Show that

$$P_t = d_t \frac{\delta e^{\alpha}}{1 - \delta e^{\alpha}},$$

where

$$\alpha \equiv \mu_d - (1-\gamma)\mu_c + \frac{1}{2}\left[(1-\gamma)^2 \sigma_c^2 + \sigma_d^2 \right] - (1-\gamma)\rho\sigma_c\sigma_d.$$

See textbook for the context.

This problem set is due to May 9th.