Monetary Policy, Inflation, and the Business Cycle

Chapter 3

The Basic New Keynesian Model

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In the present chapter we describe the key elements of the baseline model that will be used as a reference framework in the remainder of the book. In doing so we depart from the assumptions of the classical monetary economy discussed in chapter 2 in two ways. First, we introduce imperfect competition in the goods market, by assuming that each firm produces a differentiated good for which it sets the price (instead of taking the price as given). Second, we impose some constraints on the price adjustment mechanism, by assuming that only a fraction of firms can reset their prices in any given period. In particular, and following much of the literature, we adopt a model of staggered price setting due to Calvo (1983), and characterized by with random price durations.[1] We refer to the resulting framework as the *basic new Keynesian model*. As discussed in chapter 1, that model has become in recent years the workhorse for the analysis of monetary policy, fluctuations and welfare.

The introduction of differentiated goods requires that the household problem be modified slightly relative to the one considered in the previous chapter. We first discuss that modification, before turning to the firms' optimal price setting problem and the implied inflation dynamics.

1 Households

Once again we assume a representative infinitely-lived household, seeking to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \; U\left(C_t, N_t\right)$$

where C_t is now a consumption index given by

$$C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} \; di \right)^{\frac{\epsilon}{\epsilon-1}}$$

with $C_t(i)$ representing the quantity of good i consumed by the household in period t. Note that we assume the existence of a continuum of goods represented by the interval $[0, 1]$. The period budget constraint now takes the form

$$\int_0^1 P_t(i) \; C_t(i) \; di + Q_t \; B_t \leq B_{t-1} + W_t \; N_t + T_t$$

[1] The resulting inflation dynamics can also be derived under the assumption of quadratic costs of price adjustment. See, e.g. Rotemberg (1982).

for $t = 0, 1, 2...$, where $P_t(i)$ is the price of good i, and where the remaining variables are defined as in the previous chapter: N_t denotes hours of work (or the measure of household members employed), W_t is the nominal wage, B_t represents purchases of one-period bonds (at a price Q_t), and T_t is a lump-sum component of income (which may include, among other items, dividends from ownership of firms). The above sequence of period budget constraints is supplemented with a solvency condition of the form $\lim_{T\to\infty} E_t\{B_T\} \geq 0$.

In addition to the consumption/savings and labor supply decision analyzed in the previous chapter, the household now must decide how to allocate its consumption expenditures among the different goods. This requires that the consumption index C_t be maximized for any given level of expenditures $\int_0^1 P_t(i)\ C_t(i)\ di$. As shown in the appendix, the solution to that problem yields the set of demand equations

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t \tag{1}$$

for all $i \in [0,1]$, where $P_t \equiv \left[\int_0^1 P_t(i)^{1-\epsilon}\ di\right]^{\frac{1}{1-\epsilon}}$ is an aggregate price index. Furthermore, and conditional on such optimal behavior, we have

$$\int_0^1 P_t(i)\ C_t(i)\ di = P_t\ C_t$$

i.e., we can write total consumption expenditures as the product of the price index times the quantity index. Plugging the previous expression into the budget constraint yields

$$P_t\ C_t + Q_t\ B_t \leq B_{t-1} + W_t\ N_t + T_t$$

which is formally identical to the constraint facing households in the single good economy analyzed in chapter 2. Hence, the optimal consumption/savings and labor supply decisions are identical to the ones derived therein, and described by the conditions

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta\ E_t\left\{\frac{U_{c,t+1}}{U_{c,t}}\ \frac{P_t}{P_{t+1}}\right\}$$

Under the assumption of a period utility given by $U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$, and as shown in the previous chapter, the resulting log-linear versions of the above optimality conditions take the form

$$w_t - p_t = \sigma \; c_t + \varphi \; n_t \tag{2}$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} \; (i_t - E_t\{\pi_{t+1}\} - \rho) \tag{3}$$

where $i_t \equiv -\log Q_t$ is the short-term nominal rate and $\rho \equiv -\log \beta$ is the discount rate, and where lower case letter are used to denote the logs of the original variables. As before, the previous conditions will be supplemented, when necessary, with an ad-hoc log-linear money demand equation of the form:

$$m_t - p_t = y_t - \eta \; i_t \tag{4}$$

2 Firms

We assume a continuum of firms indexed by $i \in [0, 1]$. Each firm produces a differentiated good, but they all use an identical technology, represented by the production function

$$Y_t(i) = A_t \; N_t(i)^{1-\alpha} \tag{5}$$

where A_t represents the level of technology, assumed to be common to all firms and to evolve exogenously over time.

All firms face an identical isoelastic demand schedule given by (1), and take the aggregate price level P_t and aggregate consumption index C_t as given.

Following the formalism proposed in Calvo (1983), each firm may reset its price only with probability $1-\theta$ in any given period, independently of the time elapsed since the last adjustment. Thus, each period a measure $1-\theta$ of producers reset their prices, while a fraction θ keep their prices unchanged. As a result, the average duration of a price is given by $(1-\theta)^{-1}$. In this context, θ becomes a natural index of price stickiness.

2.1 Aggregate Price Dynamics

As shown in the appendix, the above environment implies that the aggregate price dynamics are described by the equation

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\ \left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \tag{6}$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate between $t-1$ and t, and P_t^* is the price set in period t by firms reoptimizing their price in that period. Notice that, as shown below, all firms will choose the same price since they face an identical problem. It follows from (6) that in a steady state with zero inflation ($\Pi = 1$) we must have $P_t^* = P_{t-1} = P_t$, for all t. Furthermore, a log-linear approximation to the aggregate price index around that steady state yields

$$\pi_t = (1-\theta)\ (p_t^* - p_{t-1}) \tag{7}$$

The previous equation makes clear that, in the present setup, inflation results from the fact that firms reoptimizing in any given period choose a price that differs from the economy's average price in the previous period. Hence, and in order to understand the evolution of inflation over time, one needs to analyze the factors underlying firms' price setting decisions, a question to which we turn next.

2.2 Optimal Price Setting

A firm reoptimizing in period t will choose the price P_t^* that maximizes the current market value of the profits generated while that price remains effective. Formally, it solves the following problem:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k\ E_t \left\{ Q_{t,t+k}\ \left(P_t^*\ Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon}\ C_{t+k} \tag{8}$$

for $k = 0, 1, 2, ...$where $Q_{t,t+k} \equiv \beta^k\ (C_{t+k}/C_t)^{-\sigma}\ (P_t/P_{t+k})$ is the stochastic discount factor for nominal payoffs, $\Psi_t(\cdot)$ is the cost function, and $Y_{t+k|t}$ denotes output in period $t+k$ for a firm that last reset its price in period t.

The first order condition associated with the problem above takes the form:

$$\sum_{k=0}^{\infty} \theta^k \; E_t \left\{ Q_{t,t+k} \; Y_{t+k|t} \; \left(P_t^* - \mathcal{M} \; \psi_{t+k|t} \right) \right\} = 0 \tag{9}$$

where $\psi_{t+k|t} \equiv \Psi'_{t+k}(Y_{t+k|t})$ denotes the (nominal) marginal cost in period $t+k$ for a firm which last reset its price in period t, and $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$.

Note that in the limiting case of no price rigidities ($\theta = 0$) the previous condition collapses to the familiar optimal price setting condition under flexible prices

$$P_t^* = \mathcal{M} \; \psi_{t|t}$$

which allows us to interpret $\mathcal{M}$ as the desired markup in the absence of constraints on the frequency of price adjustment. Henceforth, we refer to $\mathcal{M}$ as the desired or frictionless markup.

Next we log-linearize the optimal price setting condition (9) around the zero inflation steady state. Before doing so, however, it is useful to rewrite it in terms of variables that have a well defined value in that steady state. In particular, dividing by P_{t-1} and letting $\Pi_{t,t+k} \equiv P_{t+k}/P_t$, we can write

$$\sum_{k=0}^{\infty} \theta^k \; E_t \left\{ Q_{t,t+k} Y_{t+k|t} \; \left(\frac{P_t^*}{P_{t-1}} - \mathcal{M} \; MC_{t+k|t} \; \Pi_{t-1,t+k} \right) \right\} = 0 \tag{10}$$

where $MC_{t+k|t} \equiv \psi_{t+k|t}/P_{t+k}$ is the real marginal cost in period $t+k$ for a firm whose price was last set in period t.

In the zero inflation steady state we must have $P_t^*/P_{t-1} = 1$ and $\Pi_{t-1,t+k} = 1$ Furthermore, constancy of the price level implies that $P_t^* = P_{t+k}$ in that steady state, from which it follows that $Y_{t+k|t} = Y$ and $MC_{t+k|t} = MC$, since all firms will be producing the same quantity of output. In addition, $Q_{t,t+k} = \beta^k$ must hold in that steady state. Accordingly, we must have $MC = 1/\mathcal{M}$. A first-order Taylor expansion of (10) around the zro inflation steady state yields:

$$p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{\widehat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})\} \tag{11}$$

where $\widehat{mc}_{t+k|t} \equiv mc_{t+k|t} - mc$ denotes the log deviation of marginal cost from its steady state value $mc = -\mu$, and where $\mu \equiv \log \mathcal{M}$ is the log of the desired gross markup (which, for $\mathcal{M}$ close to one, is approximately equal to the net markup $\mathcal{M} - 1$).

In order to gain some intuition about the factors determining firms' price setting decision it is useful to rewrite the (11) as follows:

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{mc_{t+k|t} + p_{t+k}\}$$

Hence, firms resetting their prices will choose a price that corresponds to the desired markup over a weighted average of their current and expected (nominal) marginal costs, with the weights being proportional to the probability of the price remaining effective at each horizon, θ^k.

3 Equilibrium

Market clearing in the goods market requires

$$Y_t(i) = C_t(i)$$

for all $i \in [0, 1]$ and all t. Letting aggregate output be defined as $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} \, di\right)^{\frac{\epsilon}{\epsilon-1}}$ it follows that

$$Y_t = C_t$$

must hold for all t. One can combine the above goods market clearing condition with the consumer's Euler equation to yield the equilibrium condition.

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} \, (i_t - E_t\{\pi_{t+1}\} - \rho) \tag{12}$$

Market clearing in the labor market requires

$$N_t = \int_0^1 N_t(i) \; di$$

Using (5) we have

$$\begin{aligned} N_t &= \int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}} \; di \\ &= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} \; di \end{aligned}$$

where the second equality follows from (1) and goods market clearing. Taking logs,

$$(1-\alpha)\ n_t = y_t - a_t + d_t$$

where $d_t \equiv (1-\alpha)\log \int_0^1 (P_t(i)/P_t)^{-\frac{\epsilon}{1-\alpha}}\ di$ is a measure of price (and, hence, output) dispersion across firms. In the appendix it is shown that, in a neighborhood of the zero inflation steady state, d_t is equal to zero up to a first order approximation. Hence one can write the following approximate relation between aggregate output, employment and technology:

$$y_t = a_t + (1-\alpha)\ n_t \tag{13}$$

Next we derive an expression for an individual firm's marginal cost in terms of the economy's average real marginal cost. The latter is defined by

$$\begin{aligned} mc_t &= (w_t - p_t) - mpn_t \\ &= (w_t - p_t) - (a_t - \alpha n_t) - \log(1-\alpha) \\ &= (w_t - p_t) - \frac{1}{1-\alpha}\ (a_t - \alpha y_t) - \log(1-\alpha) \end{aligned}$$

for all t, where the second equality defines the economy's average marginal product of labor, mpn_t, in a way consistent with (13). Using the fact that

$$\begin{aligned} mc_{t+k|t} &= (w_{t+k} - p_{t+k}) - mpn_{t+k|t} \\ &= (w_{t+k} - p_{t+k}) - \frac{1}{1-\alpha}\ (a_{t+k} - \alpha y_{t+k|t}) - \log(1-\alpha) \end{aligned}$$

we have

$$\begin{aligned} mc_{t+k|t} &= mc_{t+k} + \frac{\alpha}{1-\alpha}\ (y_{t+k|t} - y_{t+k}) \\ &= mc_{t+k} - \frac{\alpha\epsilon}{1-\alpha}\ (p_t^* - p_{t+k}) \end{aligned} \tag{14}$$

where the second equality follows from the demand shedule (1) combined with the market clearing condition $c_t = y_t$. Notice that under the assumption of constant returns to scale ($\alpha = 0$) we have $mc_{t+k|t} = mc_{t+k}$, i.e. marginal cost is independent of the level of production and, hence, it is common across firms.

Substituting (14) into (11) and rearranging terms we obtain

$$\begin{aligned} p_t^* - p_{t-1} &= (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t \{\Theta \; \widehat{mc}_{t+k} + (p_{t+k} - p_{t-1})\} \\ &= (1-\beta\theta)\Theta \; \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{\widehat{mc}_{t+k}\} + \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{\pi_{t+k}\} \end{aligned}$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon} \leq 1$. Notice that the above discounted sum can be rewritten more compactly as the difference equation

$$p_t^* - p_{t-1} = \beta\theta \; E_t\{p_{t+1}^* - p_t\} + (1-\beta\theta)\Theta \; \widehat{mc}_t + \pi_t \tag{15}$$

Finally, combining (7) and (15) yields the inflation equation:

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \lambda \; \widehat{mc}_t \tag{16}$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \; \Theta$$

is strictly decreasing in the index of price stickiness θ, in the measure of decreasing returns α, and in the demand elasticity ϵ.

Solving (16) forward, we can express inflation as the discounted sum of current and expected future deviations of real marginal costs from steady state:

$$\pi_t = \lambda \sum_{k=0}^{\infty} \beta^k \; E_t\{\widehat{mc}_{t+k}\}$$

Equivalently, and defining the average markup in the economy as $\mu_t = -mc_t$, we see that inflation will be high when firms expect average markups to be below their steady state (i.e. desired) level μ, for in that case firms that have the opportunity to reset prices will choose a price above the economy's average price level, in order to realign their markup closer to its desired level.

It is worth emphasizing here that the mechanism underlying fluctuations in the aggregate price level and inflation laid out above has little in common with the one at work in the classical monetary economy. Thus, in the present model, inflation results from the aggregate consequences of purposeful price-setting decisions by firms, which adjust their prices in light of current and anticipated cost conditions. By contrast, in the classical monetary economy analyzed in chapter 2 inflation is a consequence of the changes in the aggregate price level that, given the monetary policy rule in place, are required in order to support an equilibrium allocation that is independent of the evolution of nominal variables, with no account given of the mechanism (other than an invisible hand) that will bring about those price level changes.

Next we derive a relation between the economy's real marginal cost and a measure of aggregate economic activity. Notice that independently of the nature of price setting, average real marginal cost can be expressed as

$$\begin{aligned} mc_t &= (w_t - p_t) - mpn_t \\ &= (\sigma \ y_t + \varphi \ n_t) - (y_t - n_t) - \log(1-\alpha) \\ &= \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) \ y_t - \frac{1+\varphi}{1-\alpha} \ a_t - \log(1-\alpha) \end{aligned} \tag{17}$$

where derivation of the second and third equalities make use of the household's optimality condition (2) and the (approximate) aggregate production relation (13).

Furthermore, and as shown at the end of section 2.2, under *flexible prices* the real marginal cost is constant and given by $mc = -\mu$. Defining the *natural* level of output, denoted by y_t^n, as the equilibrium level of output under flexible prices we have:

$$mc = \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) \ y_t^n - \frac{1+\varphi}{1-\alpha} \ a_t - \log(1-\alpha) \tag{18}$$

thus implying

$$y_t^n = \psi_{ya}^n \ a_t + \vartheta_y^n \tag{19}$$

where $\vartheta_y^n \equiv -\frac{(1-\alpha) \ (\mu - \log(1-\alpha))}{\sigma(1-\alpha)+\varphi+\alpha} > 0$ and $\psi_{ya}^n \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$. Notice that when $\mu = 0$ (perfect competition) the natural level of output corresponds to the equilibrium level of output in the classical economy, as derived in chapter 2. The presence of market power by firms has the effect of lowering that output level uniformly over time, without affecting its sensitivity to changes in technology.

Subtracting (18) from (17) we obtain

$$\widehat{mc}_t = \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) \ (y_t - y_t^n) \tag{20}$$

i.e., the log deviation of real marginal cost from steady state is proportional to the log deviation of output from its flexible price counterpart. Following convention, we henceforth refer to that deviation as the *output gap,* and denote it by $\widetilde{y}_t \equiv y_t - y_t^n$.

By combining (20) with (16) one can obtain an equation relating inflation to its one period ahead forecast and the output gap:

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \kappa \; \widetilde{y}_t \tag{21}$$

where $\kappa \equiv \lambda \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$. Equation (21) is often referred to as the *new Keynesian Phillips curve* (or NKPC, for short), and constitutes one of the key building blocks of the basic new Keynesian model.

The second key equation describing the equilibrium of the new Keynasian model can be obtained by rewriting (12) in terms of the output gap as follows

$$\widetilde{y}_t = -\frac{1}{\sigma} \; (i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\} \tag{22}$$

where r_t^n is the *natural rate of interest*, given by

$$\begin{aligned} r_t^n &\equiv \rho + \sigma \; E_t\{\Delta y_{t+1}^n\} \\ &= \rho + \sigma \psi_{ya}^n \; E_t\{\Delta a_{t+1}\} \end{aligned} \tag{23}$$

Henceforth we will refer to (22) as the *dynamic IS equation* (or DIS, for short). Under the assumption that the effects of nominal rigidities vanish asymptotically, we will have $\lim_{T \longrightarrow \infty} E_t\{\widetilde{y}_{t+T}\} = 0$. In that case one can solve equation (22) forward to yield the expression

$$\widetilde{y}_t = -\frac{1}{\sigma} \; \sum_{k=0}^{\infty} \; (r_{t+k} - r_{t+k}^n) \tag{24}$$

where $r_t \equiv i_t - E_t\{\pi_{t+1}\}$ is the expected real return on a one period bond (i.e. the real interest rate). The previous expression emphasizes the fact that the output gap is proportional to the sum of current and anticipated deviations between the real interest rate and its natural counterpart.

Equations (21) and (22), together with an equilibrium process for the natural rate r_t^n (which in general will depend on all the real exogenous forces in the model), constitute the non-policy block of the basic new Keynesian model. That block has a simple recursive structure: the NKPC determines inflation given a path for the output gap, whereas the DIS equation determines the output gap given a path for the (exogenous) natural rate *and* the actual real rate. In order to close the model, we need to supplement those two equations with one or more equations determining how the nominal interest rate i_t evolves over time, i.e. with a description of how monetary policy is conducted. Thus, and in contrast with the classical model analyzed in chapter 2, when prices are sticky the equilibrium path of real variables cannot be determined independently of monetary policy. In other words: monetary policy is non-neutral.

In order to illustrate the workings of the basic model just developed, we consider next two alternative specifications of monetary policy and analyze some of their equilibrium implications.

4 Equilibrium Dynamics under Alternative Monetary Policy Rules

4.1 Equilibrium under an Interest Rate Rule

We first analyze the equilibrium under a simple interest rate rule of the form:

$$i_t = \rho + \phi_\pi \, \pi_t + \phi_y \, \widetilde{y}_t + v_t \tag{25}$$

where v_t is an exogenous (possibly stochastic) component with zero mean. We assume ϕ_π and ϕ_y are non-negative coefficients, chosen by the monetary authority. Note that the choice of the intercept ρ makes the rule consistent with a zero inflation steady state.

Combining (21), (22), and (25) we can represent the equilibrium conditions by means of the following system of difference equations.

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T \, (\widehat{r}_t^n - v_t) \tag{26}$$

where $\widehat{r}_t^n \equiv r_t^n - \rho$, and

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} \qquad ; \qquad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

with $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$.

Given that both the output gap and inflation are non-predetermined variables, the solution to (26) is locally unique if and only if $\mathbf{A}_T$ has both eigenvalues within the unit circle.[2] Under the assumption of non-negative coefficients (ϕ_π, ϕ_y) it can be shown that a necessary and sufficient condition for uniqueness is given by:[3]

$$\kappa\ (\phi_\pi - 1) + (1-\beta)\ \phi_y > 0 \tag{27}$$

which we assume to hold, unless stated otherwise. An economic interpretation to the previous condition will be offered in chapter 4.

Next we examine the economy's equilibrium response to two exogenous shocks–a monetary policy shock and a technology shock–when the central bank follows the interest rate rule (25).

4.1.1 The Effects of a Monetary Policy Shock

Let us assume that the exogenous component of the interest rate, v_t, follows an AR(1) process

$$v_t = \rho_v\ v_{t-1} + \varepsilon_t^v$$

where $\rho_v \in [0,1)$. Note that a positive (negative) realization of ε_t^v should be interpreted as a contractionary (expansionary) monetary policy shock, leading to a rise (decline) in the nominal interest rate, *given* inflation and the output gap.

Since the natural rate of interest is not affected by monetary shocks we set $\widehat{r}_t^n = 0$, for all t for the pursposes of the present exercise. Next we guess that the solution takes the form $\widetilde{y}_t = \psi_{yv}\ v_t$ and $\pi_t = \psi_{\pi v}\ v_t$, where ψ_{yv} and $\psi_{\pi v}$ are coefficients to be determined. Imposing the guessed solution on (22) and (21) and using the method of undetermined coefficients, we find:

$$\widetilde{y}_t = -(1-\beta\rho_v)\Lambda_v\ v_t$$

and

$$\pi_t = -\kappa\Lambda_v\ v_t$$

where $\Lambda_v \equiv \frac{1}{(1-\beta\rho_v)[\sigma(1-\rho_v)+\phi_y]+\kappa(\phi_\pi-\rho_v)}$. It can be easily shown that as long as (27) is satisfied we have $\Lambda_v > 0$. Hence, an exogenous increase in the interest rate leads to a persistent decline in the output gap and inflation.

[2] See, e.g., Blanchard and Kahn (1980)

[3] See Bullard and Mitra (2002) for a proof.

Since the natural level of output is unaffected by the monetary policy shock, the response of output matches that of the output gap.

One can use (22) to obtain an expression for the real interest rate

$$\widehat{r}_t = \sigma(1-\rho_v)(1-\beta\rho_v)\Lambda_v \; v_t$$

which is thus shown to increase unambiguosly in response to an exogenous increase in the nominal rate.

The response of the nominal interest rate combines both the direct effect of v_t and the variation induced by lower output gap and inflation. It is given by:

$$\widehat{i}_t = \widehat{r}_t + E_t\{\pi_{t+1}\} = [\sigma(1-\rho_v)(1-\beta\rho_v) - \rho_v\kappa] \; \Lambda_v \; v_t$$

Note that if the persistence of the monetary policy shock, ρ_v, is sufficiently high, the nominal rate will decline in response to a rise in v_t . This is a result of the downward adjustment in the nominal rate induced by the decline in inflation and the output gap more than offsetting the direct effect of a higher v_t. In that case, and despite the lower nominal rate, the policy shock still has a contractionary effect on output, since the latter is inversely related to the real rate, which goes up unambiguously.

Finally, one can use (4) to determine the change in the money supply required to bring about the desired change in the interest rate. In particular, the response of m_t on impact is given by:

$$\begin{aligned}\frac{dm_t}{d\varepsilon_t^v} &= \frac{dp_t}{d\varepsilon_t^v} + \frac{dy_t}{d\varepsilon_t^v} - \eta\frac{di_t}{d\varepsilon_t^v} \\ &= -\Lambda_v \; [(1-\beta\rho_v)(1+\eta\sigma(1-\rho_v)) + \kappa(1-\eta\rho_v)]\end{aligned}$$

Hence, we see that the sign of the change in the money supply that supports the exogenous policy intervention is, in principle, ambiguous. Even though the money supply needs to be tightened to raise the nominal rate *given output and prices*, the decline in the latter induced by the policy shocks combined with the possibility of an induced nominal rate decline make it impossible to rule out a countercyclical movement in money in response to an interest rate shock. Note, however, that $di_t/d\varepsilon_t^v > 0$ is a sufficient condition for a contraction in the money supply, as well as for the presence of a liquidity effect (i.e. a negative short-run comovement of the nominal rate and the money supply in response to an exogenous monetary policy shock).

The previous analysis can be used to quantify the effects of a monetary policy shock, given numerical values for the model's parameters. Next we briefly present a baseline calibration of the model, which takes the relevant period to correspond to a quarter.

In the baseline calibration of the model's preference parameters we assume $\beta = 0.99$, which implies a steady state real return on financial assets of about four percent. We also assume $\sigma = 1$ (log utility) and $\varphi = 1$ (a unitary Frisch elasticity of labor supply), values commonly found in the business cycle literature. We set the interest semi-elasticity of money demand, η, to equal 4.[4] In addition we assume $\theta = 2/3$, which implies an average price duration of three quarters, a value consistent with the empirical evidence.[5] As to the interest rate rule coefficients we assume $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$, which are roughly consistent with observed variations in the Federal Funds rate over the Greenspan era.[6] Finally, we set $\rho_v = 0.5$, a value associated with a moderately persistent shock.

Figure 3.1 illustrates the dynamic effects of an expansionary monetary policy shock. The shock corresponds to an increase of 25 basis points in ε_t^v , which, in the absence of a further change induced by the response of inflation or the output gap, would imply an increase of 100 basis points in the annualized nominal rate, on impact. The responses of inflation and the two interest rates shown in the figure are expressed in annual terms (i.e. they are obtained by multiplying by 4 the responses of π_t, i_t and r_t in the model).

In a way consistent with the analytical results above we see that the policy shock generates an increase in the real rate, and a decrease in inflation and output (whose response corresponds to that of the output gap, since the natural level of output is not affected by the monetary policy shock). Note than under our baseline calibration the nominal rate goes up, though by

[4] The calibration of η is based on the estimates of an OLS regression of (log) M2 inverse velocity on the three month Treasury Bill rate (quarterly rate, per unit), using quarterly data over the period 1960:1-1988:1. We focus on that period, since it is characterized by a highly stable relationship between velocity and the nominal rate, which is consistent with the model.

[5] See, in particular, the estimates in Galí, Gertler and López-Salido (2001) and Sbordone (2002), based on aggregate data. Using the price of individual goods, Bils and Klenow (2004) uncover a mean duration slightly shorter (7 months). After controling for sales, Nakamura and Steinsson (2007) found an average price duration close to one year.

[6] See, e.g., Taylor (1999). Note that empirical interest rate rules are generally estimated using inflation and interest rate data expressed in annual rates. Conversion to quarterly rates requires that the output gap coefficient be divided by 4. As discussed later, the output gap measure used in empirical interest rate rules does not necessarily match the concept of output gap in the model.

less than its exogenous component–as a result of the downward adjustment induced by the decline in inflation and the output gap. In order to bring about the observed interest rate response, the central bank must engineer a reduction in the money supply. The calibrated model thus displays a liquidity effect. Note also that the response of the real rate is larger than that of the nominal rate as a result of the decrease in expected inflation.

Overall, the dynamic responses to a monetary policy shock shown in Figure 3.1 are similar, at least in a qualitative sense, to those estimated using structural VAR methods, as described in chapter 1. Nevertheless, and as emphasized in Christiano, Eichenbaum and Evans (2005), amomg others, matching some of the quantitative features of the empirical impulse responses requires that the basic new Keynesian model be enriched in a variety of dimensions.

4.1.2 The Effects of a Technology Shock

In order to determine the economy 's response to a technology shock we must first specify a process for the technology parameter $\{a_t\}$, and derive the implied process for the natural rate. We assume the following AR(1) process for $\{a_t\}$,

$$a_t = \rho_a \; a_{t-1} + \varepsilon_t^a \tag{28}$$

where $\rho_a \in [0, 1)$ and $\{\varepsilon_t^a\}$ is a zero mean white noise process. Given (23), the implied natural rate, expressed in terms of deviations from steady state, is given by

$$\widehat{r}_t^n = -\sigma \psi_{ya}^n \; (1 - \rho_a) \; a_t$$

Setting $v_t = 0$, for all t (i.e., turning off monetary shocks), and guessing that output gap and inflation are proportional to $\widehat{r}_t^n$, we can apply the method of undetermined coefficients in a way analogous to previous subsection, or just exploit the fact that $\widehat{r}_t^n$ enters the equilibrium conditions in a way symmetric to v_t, but with the opposite sign, to obtain

$$\begin{aligned} \widetilde{y}_t &= (1 - \beta\rho_a)\Lambda_a \; \widehat{r}_t^n \\ &= -\sigma\psi_{ya}^n (1 - \rho_a)(1 - \beta\rho_a)\Lambda_a \; a_t \end{aligned}$$

and

$$\begin{aligned} \pi_t &= \kappa\Lambda_a \; \widehat{r}_t^n \\ &= -\sigma\psi_{ya}^n (1 - \rho_a)\kappa\Lambda_a \; a_t \end{aligned}$$

where $\Lambda_a \equiv \frac{1}{(1-\beta\rho_a)[\sigma(1-\rho_a)+\phi_y]+\kappa(\phi_\pi-\rho_a)} > 0$

Hence, and as long as $\rho_a < 1$, a positive technology shock leads to a persistent decline in both inflation and the output gap. The implied equilibrium responses of output and employment are given by

$$\begin{aligned} y_t &= y_t^n + \widetilde{y}_t \\ &= \psi_{ya}^n \left(1 - \sigma(1-\rho_a)(1-\beta\rho_a)\Lambda_a\right) a_t \end{aligned}$$

and

$$\begin{aligned} (1-\alpha)\, n_t &= y_t - a_t \\ &= \left[(\psi_{ya}^n - 1) - \sigma\psi_{ya}^n(1-\rho_a)(1-\beta\rho_a)\Lambda_a\right] a_t \end{aligned}$$

Hence, we see that the sign of the response of output and employment to a positive technology shock is in general ambiguous, depending on the configuration of parameter values, including the interest rate rule coefficients. In our baseline calibration we have $\sigma = 1$ which in turn implies $\psi_{ya}^n = 1$. In that case, a technological improvement leads to a persistent employment decline. Such a response of employment is consistent with much of the recent empirical evidence on the effects of technology shocks.[7]

Figure 3.2 shows the responses of a number of variables to a favorable technology shock, as implied by our baseline calibration and under the assumption of $\rho_a = 0.9$. Notice that the improvement in technology is partly accommodated by the central bank, which lowers nominal and real rates, while increasing the quantity of money in circulation. That policy, however, is not sufficient to close a negative output gap, which is responsible for the decline in inflation. Under the baseline calibration output increases (though less than its natural counterpart), and employment declines, in a way consistent with the evidence mentioned above.

4.2 Equilibrium under an Exogenous Money Supply

Next we analyze the equilibrium dynamics of the basic new Keynesian model under an exogenous path for the growth rate of the money supply, Δm_t. As a preliminary step, it is useful to rewrite the money market equilibrium condition in terms of the output gap, as follows:

$$\widetilde{y}_t - \eta\, i_t = l_t - y_t^n \tag{29}$$

[7] See Galí and Rabanal (2005) for a survey of that empirical evidence.

where $l_t \equiv m_t - p_t$. Substituting the latter equation into (22) yields

$$(1+\sigma\eta)\ \widetilde{y}_t = \sigma\eta\ E_t\{\widetilde{y}_{t+1}\} + l_t + \eta\ E_t\{\pi_{t+1}\} + \eta\ \widehat{r}_t^n - y_t^n \tag{30}$$

Note also that real balances are related to inflation and money growth through the identity

$$l_{t-1} = l_t + \pi_t - \Delta m_t \tag{31}$$

Hence, the equilibrium dynamics for real balances, output gap and inflation are described by equations (30), and (31), together with the NKPC equation (21). They can be summarized compactly by the system

$$\mathbf{A_{M,0}} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \\ l_{t-1} \end{bmatrix} = \mathbf{A_{M,1}} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ l_t \end{bmatrix} + \mathbf{B_M} \begin{bmatrix} \widehat{r}_t^n \\ y_t^n \\ \Delta m_t \end{bmatrix} \tag{32}$$

where

$$\mathbf{A_{M,0}} \equiv \begin{bmatrix} 1+\sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad ; \quad \mathbf{A_{M,1}} \equiv \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B_M} \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

The system above has one predetermined variable (l_{t-1}) and two non-predetermined variables ($\widetilde{y}_t$ and π_t). Accordingly, a stationary solution will exist and be unique if and only if $\mathbf{A_M} \equiv \mathbf{A_{M,0}^{-1}A_{M,1}}$ has two eigenvalues inside and one outside (or on) the unit circle. The latter condition can be shown to be always satisfied so, in contrast with the interest rate rule discussed above, the equilibrium is always determined under an exogenous path for the money supply.[8]

Next we examine the equilibrium responses of the economy to a monetary policy shock and a technology shock.

4.2.1 The Effects of a Monetary Policy Shock

In order to illustrate how the the economy responds to an exogenous shock to the money supply, we assume that Δm_t follows the AR(1) process

$$\Delta m_t = \rho_m\ \Delta m_{t-1} + \varepsilon_t^m \tag{33}$$

[8]Since $\mathbf{A_M}$ is upper triangular its eigenvalues are given by its diagonal elements which can be shown to be $\sigma\eta/(1+\sigma\eta)$, β, and -1. Hence existence and uniqueness of a stationary solution is guaranteed under any rule implying an exogenous path for the money supply.

where $\rho_m \in [0,1)$ and $\{\varepsilon_t^m\}$ is white noise.

The economy's response to a monetary policy shock can be obtained by determining the stationary solution to the dynamical system consisting of (32) and (33) and tracing the effects of a shock to ε_t^m (while setting $\widehat{r}_t^n = y_t^n = 0$, for all t).[9] In doing so, we assume $\rho_m = 0.5$, a value roughly consistent with the first-order autocorrelation of money growth in postwar U.S. data.

Figure 3.3 displays the dynamic responses of several variables of interest to an expansionary monetary policy shock, which takes the form of positive realization of ε_t^m of size 0.25. That impulse corresponds to a one percent increase, on impact, in the annualized rate of money growth, as shown in the Figure. The sluggishness in the adjustment of prices implies that real balances rise in response to the increase in the money supply. As a result, clearing of the money market requires either a rise in output and/or a decline in the nominal rate. Under the calibration considered here, output increases by about a third of a percentage point on impact, after which it slowly reverts back to its initial level. The nominal rate, however, shows a slight increase. Hence, and in contrast with the case of an interest rate rule considered above, a liquidity effect does not emerge here. Note however that the rise in the nominal rate does not prevent the real rate from declining persistently (due to higher expected inflation), leading in turn to an expansion in aggregate demand and output (as implied by (24)) and, as a result, a persistent rise in inflation (which follows from (21)).

It is worth noting here that the absence of a liquidity effect is not a necessary feature of the exogenous money supply regime considered here, but instead a consequence of the calibration used. To see this note that one can combine equations (4) and (22), to obtain the difference equation

$$i_t = \frac{\eta}{1+\eta}\ E_t\{i_{t+1}\} + \frac{\rho_m}{1+\eta}\ \Delta m_t + \frac{\sigma-1}{1+\eta}\ E_t\{\Delta y_{t+1}\}$$

whose forward solution yields:

$$i_t = \frac{\rho_m}{1+\eta(1-\rho_m)}\ \Delta m_t + \frac{\sigma-1}{1+\eta}\sum_{k=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k\ E_t\{\Delta y_{t+1+k}\}$$

Note that when $\sigma = 1$, as in the baseline calibration underlying Figure 3.3, the nominal rate always comoves positively with money growth. Nevertheless, and given that quite generally the summation term will be negative

[9] See e.g. Blanchard and Kahn (1980) a description of a solution method.

(since for most calibrations output tends to adjust monotonically to its original level after the initial increase), a liquidity effect emerges given values of σ sufficiently above one combined with sufficiently low (absolute) values of ρ_m.[10]

4.2.2 The Effects of a Technology Shock

Finally, we turn to the analysis of the effects of a technology shock under a monetary policy regime characterized by an exogenous money supply. Once again, we assume the technology parameter a_t follows the stationary process given by (28). That assumption combined with (19) and (23) is used to determine the implied path of $\widehat{r}_t^n$ and y_t^n as a function of a_t, as needed to solve (32). In a way consistent with the assumption of exogenous money, I set $\Delta m_t = 0$ for all t for the purpose of the present exercise.

Figure 3.4 displays the dynamic responses to a one percent increase in the technology. A comparison with the responses shown in Figure 3.2 (corresponding to the analogous exercise under an interest rate rule) reveals many similarities: in both cases the output gap (and, hence, inflation) display a negative response to the technology improvement, as a result of output failing to increase as much as its natural level. Note, however, that in the case of exogenous money the gap between output and its natural level is much larger, which explains also the larger decline in employment. This is due to the upward response of the real rate implied by the unchanged money supply, which contrasts with its decline (in response to the negative response of inflation and the output gap) under the interest rate rule. Since the natural real rate also declines in response to the positive technology shock (in order to support the transitory increase in output and consumption), the response of interest rates generated under the exogenous money regime becomes overly contractionary, as illustrated in Figure 3.4.

The previous simulations have served several goals. First, they have helped us illustrate the workings of the new Keynesian model, i.e. how the model can be used to answer some specific questions about the behavior of the economy under different assumption. Secondly, we have seen how, under a plausible calibration, the simulated responses to monetary and technology shocks display notable similarities (at least qualitative) with the empirical evidence on the effects of those shocks. Thirdly, the previous analysis has

[10] See Galí (2003) for a detailed analysis.

made clear that monetary policy in the new Keynesian model can have large and persistent effects on both real and nominal variables. The latter feature leads one to raise a natural question, which is the focus of the next chapter: how should monetary policy be conducted?

5 Notes on the Literature

Early examples of microfounded monetary models with monopolistic competition and sticky prices can be found in Akerlof and Yellen (1985), Mankiw (1985), Blanchard and Kiyotaki (1987) and Ball and Romer (1990).

An early version and analysis of the baseline new Keynesian model can be found in Yun (1996), which used a discrete-time version of the staggered price-setting model originally developed in Calvo (1983). King and Wolman (1996) provides a detailed analysis of the steady state and dynamic properties of that model. King and Watson (1996).compare its predictions regarding the cyclical properties.of money, interest rates, and prices with those of flexible price models. Woodford (1996) incorporates a fiscal sector in the model and analyzes its properties under a non-Ricardian fiscal policy regime.

An inflation equation identical to the new Keynesian Phillips curve can be derived under the assumption of quadratic costs of price adjustment, as shown in Rotemberg (1982). Hairault and Portier (1993) developed and analyzed an early version of a monetary model with quadratic costs of price adjustment and compared its second moment predictions with those of the French and U.S. economies.

Two main alternatives to the Calvo random price duration model can be found in the literature. The first one is given by staggered price setting models with a deterministic price duration, originally proposed by Taylor (1980) in the context of a non microfounded model. A microfounded version of the Taylor model can be found in Chari, Kehoe and McGrattan (2000) who analyzed the output effects of exogenous monetary policy shocks. An alternative price-setting structure is given by state dependent models, in which which the timing of price adjustments is influenced by the state of the economy. A quantitative analysis of a state dependent pricing model can be found in Dotsey, King and Wolman (1999) and, more recently, in Golosov and Lucas (2003) and Gertler and Leahy (2006).

The empirical performance of new Keynesian Phillips curve has been the object of numerous criticisms. An early critical assessment can be found in

Fuhrer and Moore (1986). Mankiw and Reis (2002) give a quantitative review of the perceived shortcomings of the NKPC and propose an alternative price setting structure, based on the assumption of sticky information. Galí and Gertler (1999), Sbordone (2002) and Galí, Gertler and López-Salido (2002) provide favorable evidence of the empirical fit the equation relating inflation to marginal costs, and discuss the difficulties in estimating or testing the NKPC given the unobservability of the output gap.

Rotemberg and Woodford (1999) and Christiano, Eichenbaum and Evans (2005) provide empirical evidence on the effects monetary policy shocks, and discuss a number of modifications of the baseline new Keynesian model aimed at improving the model's ability to match the estimated impulse responses.

Evidence on the effects of technology shocks and its implications for the relevance of alternative models can be found in Galí (1999) and Basu, Fernald and Kimball (2004), among others. Recent evidence as well as alternative interpretations are surveyed in Galí and Rabanal (2005).

Appendix

Optimal Allocation of Consumption Expenditures

The problem of maximization of C_t for any *given* expenditure level $\int_0^1 P_t(i)\, C_t(i)\, di \equiv Z_t$ can be formalized by means of the Lagrangean

$$\mathcal{L} = \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} \, di \right]^{\frac{\epsilon}{\epsilon-1}} - \lambda \, \left(\int_0^1 P_t(i) \; C_t(i) \; di - Z_t \right)$$

The associated first order conditions are:

$$C_t(i)^{-\frac{1}{\epsilon}} \; C_t{}^{\frac{1}{\epsilon}} = \lambda \; P_t(i)$$

for all $i \in [0,1]$. Thus, for any two goods (i,j) we have:

$$C_t(i) = C_t(j) \; \left(\frac{P_t(i)}{P_t(j)} \right)^{-\epsilon}$$

which can be substituted into the expression for consumption expenditures to yield

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} \; \frac{Z_t}{P_t}$$

for all $i \in [0,1]$. The latter condition can then be substituted into the definition of C_t to obtain

$$\int_0^1 P_t(i) \; C_t(i) \; di = P_t \; C_t$$

Combining the two previous equations we obtain the demand schedule:

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} \; C_t$$

Aggregate Price Level Dynamics

Let $S(t) \subset [0,1]$ represent the set of firms not re-optimizing their posted price in period t. Using the definition of the aggregate price level and the fact that all firms resetting prices will choose an identical price P_t^* we have

$$
\begin{aligned}
P_t &= \left[\int_{S(t)} P_{t-1}(i)^{1-\epsilon} \, di + (1-\theta) \, (P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \\
&= \left[\theta \, (P_{t-1})^{1-\epsilon} + (1-\theta) \, (P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}
\end{aligned}
$$

where the second equality follows from the fact that the distribution of prices among firms not adjusting in period t corresponds to the distribution of effective prices in period $t-1$, though with total mass reduced to θ.

Dividing both sides by P_{t-1} ,

$$
\Pi_t^{1-\epsilon} = \theta + (1-\theta) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon} \tag{34}
$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$. Notice that in a steady state with zero inflation $P_t^* = P_{t-1} = P_t$, for all t.

Log-linearization of (34) around $\Pi_t = 1$ and $\frac{P_t^*}{P_{t-1}} = 1$ yields:

$$
\pi_t = (1-\theta) \, (p_t^* - p_{t-1}) \tag{35}
$$

Price Dispersion

From the definition of the price index:

$$
\begin{aligned}
1 &= \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{1-\epsilon} di \\
&= \int_0^1 \exp\{(1-\epsilon)(p_t(i) - p_t)\} \, di \\
&\simeq 1 + (1-\epsilon) \int_0^1 (p_t(i) - p_t) \, di + \frac{(1-\epsilon)^2}{2} \int_0^1 (p_t(i) - p_t)^2 \, di
\end{aligned}
$$

where the approximation results from a second-order Taylor expansion around the zero inflation steady state. Thus, and up to second order, we have

$$
p_t \simeq E_i\{p_t(i)\} + \frac{(1-\epsilon)}{2} \int_0^1 (p_t(i) - p_t)^2 \, di
$$

where $E_i\{p_t(i)\} \equiv \int_0^1 p_t(i)\ di$ is the cross-sectional mean of (log) prices.

In addition,

$$\begin{aligned}
\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di &= \int_0^1 \exp\left\{-\frac{\epsilon}{1-\alpha}\ (p_t(i)-p_t)\right\}\ di \\
&\simeq 1 - \frac{\epsilon}{1-\alpha}\int_0^1 (p_t(i)-p_t)\ di + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)^2 \int_0^1 (p_t(i)-p_t)^2\ di \\
&\simeq 1 + \frac{1}{2}\frac{\epsilon(1-\epsilon)}{1-\alpha}\int_0^1 (p_t(i)-p_t)^2\ di + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)^2 \int_0^1 (p_t(i)-p_t)^2\ di \\
&= 1 + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)\frac{1}{\Theta}\int_0^1 (p_t(i)-p_t)^2\ di \\
&\simeq 1 + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)\frac{1}{\Theta}\ var_i\{p_t(i)\} > 1
\end{aligned}$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$, and where the last equality follows from the observation that, up to second order,

$$\begin{aligned}
\int_0^1 (p_t(i)-p_t)^2\ di &\simeq \int_0^1 (p_t(i)-E_i\{p_t(i)\})^2\ di \\
&\equiv var_i\{p_t(i)\}
\end{aligned}$$

Finally, using the definition of d_t we obtain

$$d_t \equiv (1-\alpha)\log\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di \simeq \frac{1}{2}\frac{\epsilon}{\Theta}\ var_i\{p_t(i)\}$$

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Exercises

1. Interpreting Discrete-Time Records of Data on Price Adjustment Frequency

Suppose firms operate in continuous time, with the *pdf* for the duration of the price of an individual good being $f(t) = \phi \exp(-\phi\ t)$, where $t \in \mathbb{R}^+$ is expressed in month units.

a) Show that the implied instantaneous probability of a price change is constant over time and given by ϕ.

b) What is the *mean* duration of a price? What is the *median* duration? What is the relationship between the two?

c) Suppose that the prices of individual goods are recorded once a month (say, on the first day, for simplicity). Let λ_t denote the fraction of items in a given goods category whose price in month t is different from that recorded in month $t-1$ (note: of course, the price may have changed more than once since the previous record). How would you go about estimating parameter ϕ?

d) Given information on monthly frequencies of price adjustment, how would you go about calibrating parameter θ in a quarterly Calvo model?

2. Introducing Government Purchases in the Basic New Keynesian Model

Assume that the government purchases quantity $G_t(i)$ of good i, for all $i \in [0,1]$. Let $G_t \equiv \left[\int_0^1 G_t(i)^{1-\frac{1}{\epsilon}}\ di\right]^{\frac{\epsilon}{\epsilon-1}}$ denote an index of public consumption, which the government seeks to maximize for any level of expenditures $\int_0^1 P_t(i)\ G_t(i)\ di$. We assume government expenditures are financed by means of lump-sum taxes.

a) Derive an expression for total demand facing firm i.

b) Derive a log-linear aggregate goods market clearing condition that is valid around a steady state with a constant public consumption share $S_G \equiv \frac{G}{Y}$.

c) Derive the corresponding expression for average real marginal cost as a function of aggregate output, government purchases, and technology, and provide some intuition for the effect of government purchases.

d) How is the equilibrium relationship linking interest rates to current and expected output affected by the presence of government purchases?

3. Government Purchases and Sticky Prices

Consider a model economy with the following equilibrium conditions. The household's log-linearized Euler equation takes the form:

$$c_t = -\frac{1}{\sigma}\ (i_t - E_t\{\pi_{t+1}\} - \rho) + E_t\{c_{t+1}\}$$

where c_t is consumption, i_t is the nominal rate, and $\pi_{t+1} \equiv p_{t+1} - p_t$ is the rate of inflation between t and $t+1$ (as in the text, lower case letters denote the logs of the original variable). The household's log-linearized labor supply is given by:

$$w_t - p_t = \sigma\ c_t + \varphi\ n_t$$

where w_t denotes the nominal wage, p_t.is the price level, and n_t is employment.

Firms' technology is given by:

$$y_t = n_t$$

The time between price adjustments is random, which gives rise to an inflation equation:

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t$$

where $\widetilde{y}_t \equiv y_t - y_t^n$ is the output gap.(with y_t^n representing the natural level of output). We assume that in the absence of constraints on price adjustment firms would set a price equal to a constant markup over marginal cost given by μ (in logs).

Suppose that the government purchases a fraction τ_t of the output of each good, with τ_t varying exogenously. Government purchases are financed through lump-sum taxes.(remark: we ignore the possibility of capital accumulation or the existence of an external sector).

a) Derive a log-linear version of the goods market clearing condition, of the form $y_t = c_t + g_t$., where $g_t \equiv -\log(1 - \tau_t)$.

b) Derive an expression for (log) real marginal cost mc_t as a function of y_t and g_t .

c) Determine the behavior of the natural level of output y_t^n as a function of g_t and discuss the mechanism through which a fiscal expansion leads to an increase in output when prices are flexible.

d) Assume that $\{g_t\}$ follows a simple AR(1) process with autoregressive coefficient $\rho_g \in [0, 1)$. Derive the DIS equation:

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}\,(i_t - E_t\{\pi_{t+1}\} - r_t^n)$$

together for an expression for the natural rate r_t^n as a function of g_t.

4. Indexation and the New Keynesian Phillips Curve

Consider the Calvo model of staggered price setting with the following modification: in the periods between price re-optimizations firms adjust mechanically their prices according to some indexation rule. Formally, a firm that re-optimizes its price in period t (an event which occurs with probability $1-\theta$) sets a price P_t^* in that period. In subsequent periods (i.e., until it re-optimizes prices again) its price is adjusted according to one of the following two alternative rules:

Rule #1: full indexation to steady state inflation Π :

$$P_{t+k|t} = P_{t+k-1|t}\ \Pi$$

Rule #2: partial indexation to past inflation (assuming zero inflation in the steady state)

$$P_{t+k|t} = P_{t+k-1|t}\ (\Pi_{t+k-1})^{\omega}$$

for $k = 1, 2, 3, ...$and

$$P_{t,t} = P_t^*$$

and where $P_{t+k|t}$ denotes the price effective in period $t+k$ for a firm that last re-optimized its price in period t, $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the aggregate gross inflation rate, and $\omega \in [0,1]$ is an exogenous parameter that measures the degree of indexation (notice that when $\omega = 0$ we are back to the standard Calvo model, with the price remaining constant between re-optimization period).

Suppose that all firms have access to the same constant returns to scale technology and face a demand schedule with a constant price elasticity ϵ.

The objective function for a firm re-optimizing its price in period t (i.e., choosing P_t^*) is given by

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k\ E_t\left\{Q_{t,t+k}\ [P_{t+k|t}\ Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})]\right\}$$

subject to a sequence of demand contraints, and the rules of indexation described above. $Y_{t+k|t}$ denotes the output in period $t+k$ of a firm that last re-optimized its price in period t, $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$ is the usual stochastic discount factor for nominal payoffs, Ψ is the cost function, and θ is the probability of not being able to re-optimize the price in any given period. For each indexation rule:

a. Using the definition of the price level index $P_t \equiv \left[\int_0^1 P_t(i)^{1-\epsilon}\, di\right]^{\frac{1}{1-\epsilon}}$ derive a log-linear expression for the evolution of inflation π_t as a function of the average price adjustment term $p_t^* - p_{t-1}$.

b. Derive the first order condition for the firm 's problem, which determines the optimal price P_t^*.

c. Log-linearize the first-order condition around the corresponding steady state and derive an expression for p_t^* (i.e., the approximate log-linear price setting rule).

d. Combine the results of (a) and (c) to derive an inflation equation of the form:

$$\widehat{\pi}_t = \beta\ E_t\{\widehat{\pi}_{t+1}\} + \lambda\ \widehat{mc}_t$$

where $\widehat{\pi}_t \equiv \pi_t - \pi$ in the case of rule #1, and

$$\pi_t = \gamma_b\ \pi_{t-1} + \gamma_f\ E_t\{\pi_{t+1}\} + \lambda\ \widehat{mc}_t$$

in the case of rule #2 .

5. Optimal Price Setting and Equilibrium Dynamics in the Taylor Model

We assume a continuum of firms indexed by $i \in [0,1]$. Each firm produces a differentiated good, with a technology

$$Y_t(i) = A_t\ N_t(i)$$

where A_t represents the level of technology, and $a_t \equiv \log A_t$ evolves exogenously according to some stationary stochastic process.

Each period a fraction $\frac{1}{N}$ of firms reset their prices, which then remain effective for N periods. Hence a firm i setting a new price P_t^* in period t will seek to maximize

$$\sum_{k=0}^{N-1} E_t\left\{Q_{t,t+k}\ \left(P_t^*\ Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})\right)\right\}$$

subject to

$$Y_{t+k|t} = (P_t^*/P_{t+k})^{-\epsilon} \; C_{t+k}$$

where $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right)$ is the usual stochastic discount factor for nominal payoffs.

a) Show that P_t^* must satisfy the first order condition:

$$\sum_{k=0}^{N-1} E_t \left\{ Q_{t,t+k} \; Y_{t+k|t}^d \; \left[P_t^* - \mathcal{M} \; \psi_{t+k} \right] \right\} = 0$$

where $\psi_t \equiv \Psi_t'$ is the nominal marginal cost and $\mathcal{M} \equiv \frac{\epsilon}{\epsilon - 1}$

b) Derive the following log-linearized optimal price setting rule (around a zero inflation steady state):

$$p_t^* = \mu + \sum_{k=0}^{N-1} \omega_k \; E_t \left\{ \psi_{t+k} \right\}$$

where $\omega_k \equiv \frac{\beta^k (1-\beta)}{1-\beta^N}$ and $\mu \equiv \log \mathcal{M}$. Show that in the limiting case of $\beta = 1$ (no discounting) we can rewrite the above equation as

$$p_t^* = \mu + \frac{1}{N} \sum_{k=0}^{N-1} E_t \left\{ \psi_{t+k} \right\}$$

How does the previous price setting rule differ from the one generated by the Calvo model?

c) Recalling the expression for the aggregate price index $P_t \equiv \left[\int_0^1 P_t(i)^{1-\epsilon} \; di \right]^{\frac{1}{1-\epsilon}}$, show that around a zero inflation steady state the (log) price level will satisfy:

$$p_t = \left(\frac{1}{N} \right) \sum_{k=0}^{N-1} \; p_{t-k}^*$$

d) Consider the particular case of $N = 2$ and $\beta = 1$, and assume that the consumer's marginal rate of substitution between labor and consumption is given by $\sigma c_t + \varphi n_t$. Assume also that all output is consumed. Show that in this case we can write:

$$p_t^* = \frac{1}{2} \; p_{t-1}^* + \frac{1}{2} \; E_t\{p_{t+1}^*\} + \delta \; (\widetilde{y}_t + E_t\{\widetilde{y}_{t+1}\})$$

where $\delta \equiv \sigma + \varphi$.

e) Assume that money demand takes the simple form $m_t - p_t = y_t$ and that both m_t and a_t follow (independent) random walks, with innovations ε_t^m and ε_t^a, respectively. Derive a closed-form expression for the output gap, employment, and the price level as a function of the exogenous shocks.

f) Discuss the influence of δ on the persistence of the effects of a monetary shock, and provide some intuition for that result.

6. The Mankiw-Reis Model: Inflation Dynamics under Predetermined Prices

Suppose that each period a fraction of firms $1-\theta$ gets to choose *a path of future prices* for their respective goods (a "price plan"), while the remaining fraction θ keep their current price plans. We let $\{P_{t,t+k}\}_{k=0}^{\infty}$ denote the price plan chosen by firms that get to revise that plan in period t. Firm's technology is given by $Y_t(i) = \sqrt{A_t}\, N_t(i)$. Consumer's period utility is given assumed to take the form $U(C_t, N_t) = C_t - \frac{N_t^2}{2}$, where $C_t \equiv \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$. The demand for real balances is assumed to be given by $\frac{M_t}{P_t} = C_t$. All output is consumed.

a) Let $P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$ denote the aggregate price index. Show that, up to a first order approximation, we will have:

$$p_t = (1-\theta) \sum_{j=0}^{\infty} \theta^j \; p_{t-j,t} \tag{36}$$

b) A firm i, revising its price plan in period t will seek to maximize

$$\sum_{k=0}^{\infty} \theta^k \; E_t \left\{ Q_{t,t+k} Y_{t+k}(i) \left(P_{t,t+k} - \frac{W_{t+k}}{\sqrt{A_{t+k}}} \right) \right\}$$

Derive the first order condition associated with that problem, and show that it implies the following approximate log-linear rule for the price plan:

$$p_{t,t+k} = \mu + E_t\{\psi_{t+k}\} \tag{37}$$

for $k = 0, 1, 2, ...$where $\psi_t = w_t - \frac{1}{2} a_t$ is the nominal marginal cost.

c) Using the optimality conditions for the consumer's problem, and the labor market clearing condition show that the *natural* level of output satisfies

$y_t^n = -\mu + a_t$, and (ii) the (log) real marginal cost (in deviation from its perfect foresight steady state value) equals the output gap, i.e.

$$\widehat{mc}_t = \widetilde{y}_t$$

for all t, where $\widetilde{y}_t \equiv y_t - y_t^n$.

d) Using (36) and (37) show how one can derive the following equation for inflation:

$$\pi_t = \frac{1-\theta}{\theta}\,\widetilde{y}_t + \frac{1-\theta}{\theta}\sum_{j=1}^{\infty}\theta^j\ E_{t-j}\{\Delta\widetilde{y}_t + \pi_t\} \tag{38}$$

e) Suppose that the money supply follows a random walk process $m_t = m_{t-1} + u_t$, where $m_t \equiv \log M_t$ and $\{u_t\}$ is white noise. Determine the dynamic response of output, employment, and inflation to a money supply shock. Compare the implied response to the one we would obtain under the standard new Keynesian Phillips curve, where $\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t$. (hint: use the fact that in equilibrium $y_t = m_t - p_t$ to substitute for $\widetilde{y}_t$ in (38), in order to obtain a difference equation for the (log) price level)

f) Suppose that technology is described by the random walk process $a_t = a_{t-1} + \varepsilon_t$, where where $a_t \equiv \log A_t$, and $\{\varepsilon_t\}$ is white noise. Determine the dynamic response of output, the output gap, employment, and inflation to a technology shock. Compare the implied response to the one we would obtain under the standard new Keynesian Phillips curve, where $\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t$. (hint: same as above).

Figure 3.1: Effects of a Monetary Policy Shock (Interest Rate Rule))

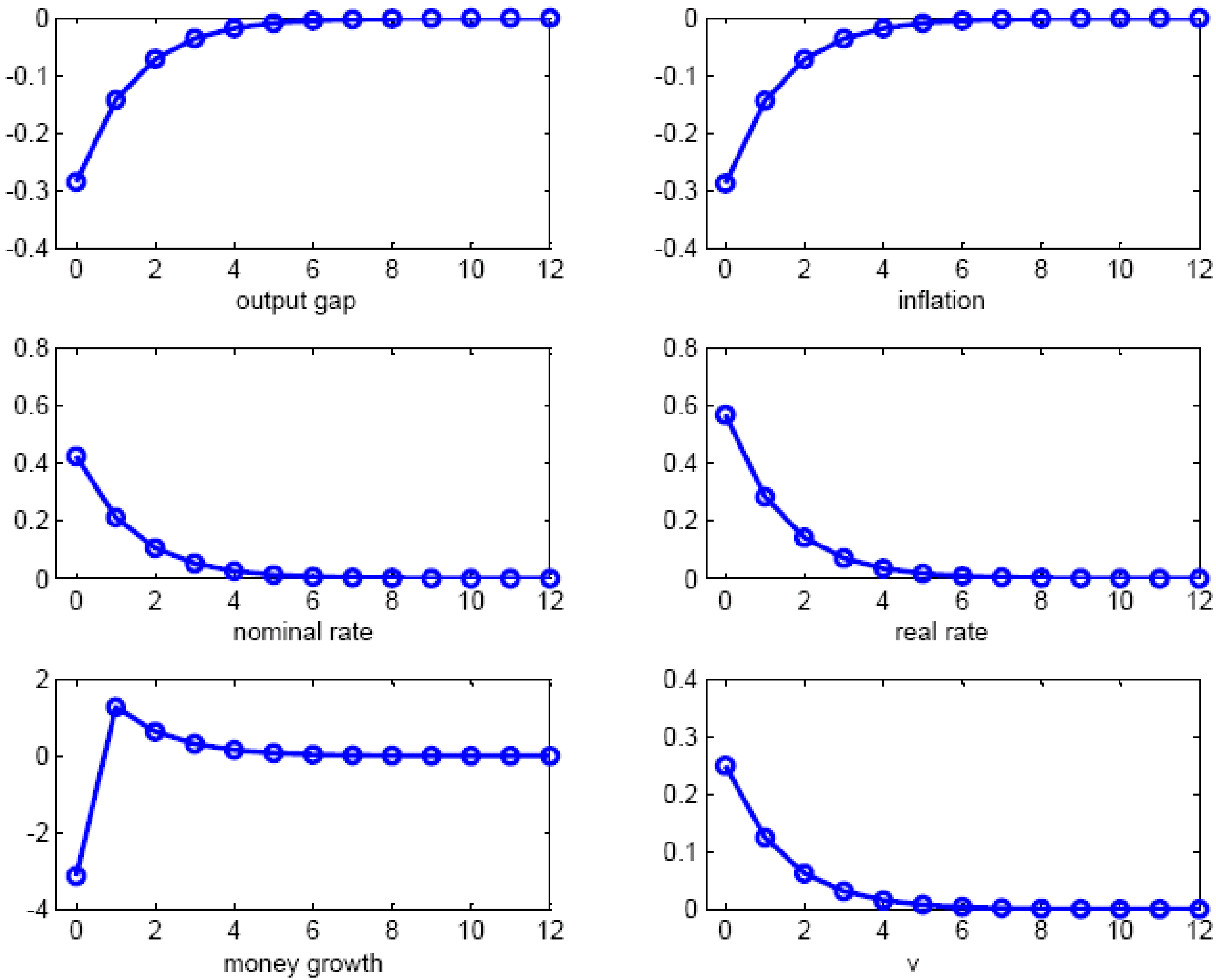

Figure 3.2: Effects of a Technology Shock (Interest Rate Rule)

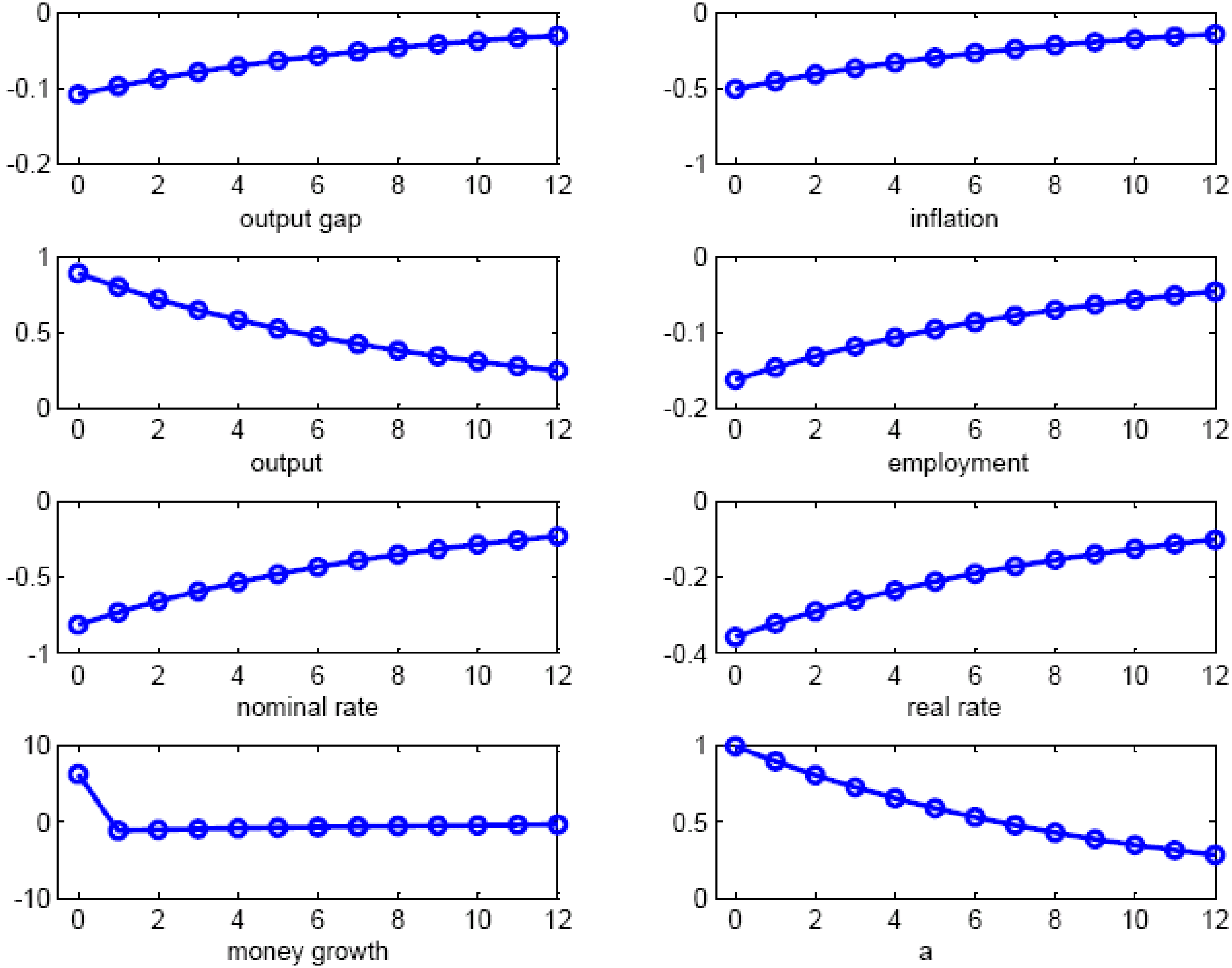

Figure 3.3: Effects of a Monetary Policy Shock (Money Growth Rule)

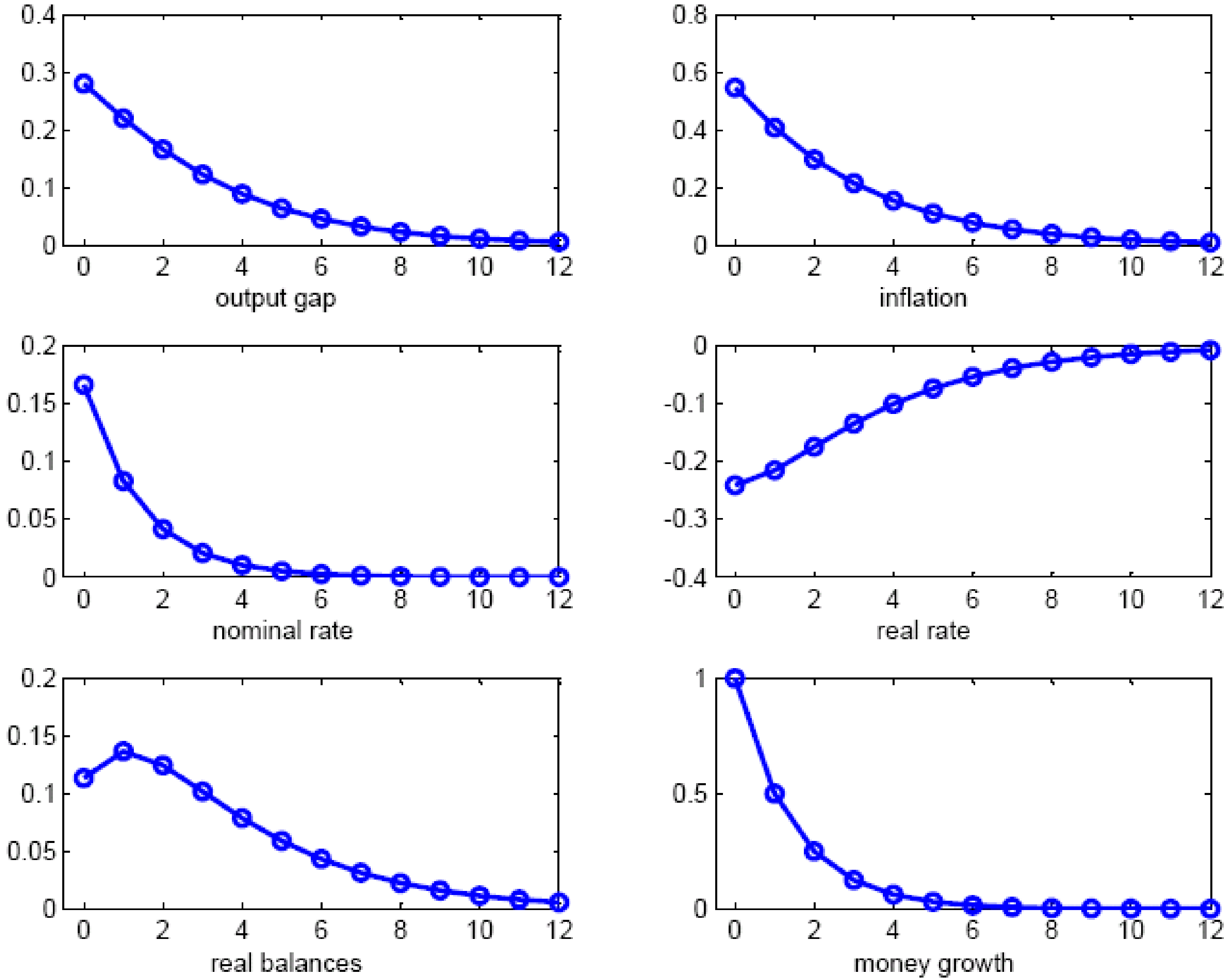

Figure 3.4: Effects of a Technology Shock (Money Growth Rule)