

# Kuhn's Theorem

## Proposition

*Every finite extensive game with perfect information,  $\Gamma = \{N, H, P, (\succeq_i)\}$ , has a subgame perfect equilibrium.*

- If  $I(\Gamma(h)) = 0$  define  $R(h) = h$ .
- Let  $R(h)$  be defined for all  $h \in H$  with  $I(\Gamma(h)) \leq k$  for some  $k \geq 0$ .
- Let  $h^*$  be a history for which  $I(\Gamma(h^*)) = k + 1$ ; let  $i = P(h^*)$ .
- $I(\Gamma(h^*)) = k + 1 \Rightarrow I(\Gamma(h^*, a)) \leq k$  for all  $a \in A(h^*)$ .
  - ▶ Define  $s_i(h^*)$  to be a  $\succeq_i$ -maximizer of  $R(h^*, a)$  over  $a \in A(h^*)$
  - ▶ Define  $R(h^*) = R(h^*, s_i(h^*))$ .
- This process defines a strategy profile  $s$  in  $\Gamma$ ; by the one-shot deviation property,  $s$  is a subgame perfect equilibrium of  $\Gamma$ .