Monetary Policy, Inflation, and the Business Cycle

Chapter 5. *Monetary Policy Tradeoffs: Discretion vs Commitment* *

Jordi Galí †
CREI and UPF

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† *Correspondence*: Centre de Recerca en Economia Internacional (CREI), Ramon Trias Fargas 25; 08005 Barcelona (Spain). E-mail: `jordi.gali@upf.edu`

1 Monetary Policy Tradeoffs

In the previous chapter we analyzed the optimal monetary policy problem in the context of a baseline model in which the presence of staggered price setting was the only relevant distortion that the central bank had to confront. We showed that a policy that seeks to replicate the flexible price equilibrium allocation is both feasible and optimal in that context. That policy requires that the central bank responds to shocks so that the price level is fully stabilized. The rationale for such a policy is easy to summarize: with zero inflation, output equals its natural level which in turn, under the assumptions made in chapter 4, is also the efficient level. Thus, in the environment analyzed in the previous chapter, the central bank does not face a meaningful policy tradeoff, and "strict inflation targeting" emerges as the optimal policy.

We view the analysis of such an environment and its implications for the design of monetary policy as useful from a pedagogical point of view, but not as a realistic one. The reason is that, in practice, central banks view themselves as facing significant tradeoffs, at least in the short run. As a result, even central banks that call themselves "inflation targeters" do not claim to be seeking to stabilize inflation completely in the short run, independently of the consequences that this would entail for the evolution of real variables like output and employment. Instead, the presence of short run tradeoffs have led inflation targeting central banks to pursue a policy that allows for a partial accomodation of inflationary pressures in the short run, in order to avoid too large instability of output and employment, while remaining committed to a medium term inflation target. A policy of that kind is often referred to in the literature as "flexible inflation targeting."[1]

In the present chapter we introduce a policy tradeoff, which we model in a simple fashion, and revisit the problem of optimal monetary policy. As shown below, the existence of such a policy tradeoff, combined with the forward-looking nature of inflation, makes it desirable for the central bank to be able to commit to a state-contingent policy plan (as opposed to pursuing a policy characterized by sequential, or period-by-period optimization).

[1]The term flexible inflation targeting was coined by Lars Svensson, to refer to the kind of optimal monetary policies that result from the minimization of of a central bank loss function that attaches a non-zero penalty to output gap fluctuations, in addition to inflation fluctuations, whenever there is a tradeoff between the stabilization of both variables.

2 The Monetary Policy Problem: the Case of an Efficient Steady State

When nominal rigidities coexist with *real* imperfections, the flexible price equilibrium allocation is generally inefficient. In that case, it is no longer optimal for the central bank to seek to replicate that allocation. On the other hand, any deviation of economic activity from its natural (i.e. flexible price) level generates variations in inflation, with the consequent relative price distortions.

A special case of interest arises when the possible inefficiencies associated with the flexible price equilibrium do not affect the steady state, which remains efficient. The present section analyzes the optimal monetary policy problem under that assumption. In contrast with the analysis in chapter 4, however, here we allow for *short run* deviations between the natural and efficients levels of output. More precisely, we assume that the gap between the two follows a stationary process, with a zero mean. Implicitly, we are assuming the presence of some real imperfections that generate a time-varying gap between output and its efficient counterpart, even in the absence of price rigidities.

In that case, and as shown in the appendix, the welfare losses experienced by the representative household are, up to a second order approximation, proportional to

$$E_0\{\sum_{t=0}^{\infty}\beta^t \ \left(\pi_t^2 + \alpha_x \ x_t^2\right)\} \tag{1}$$

where $x_t \equiv y_t - y_t^e$ denotes the *welfare-relevant output gap*, i.e. the deviation between (log) output y_t and its efficient level y_t^e. As before $\pi_t \equiv p_t - p_{t-1}$ denotes the rate of inflation between periods $t-1$ and t. Coefficient α_x represents the weight of output gap fluctuations (relative to inflation) in the loss function, and is given by $\alpha_x = \frac{\kappa}{\epsilon}$ where κ is the coefficient on x_t in the new Keynesian Phillips curve (NKPC), and ϵ is the elasticity of substitution between goods. More generally, and stepping beyond the welfare-theoretic justification for (1), one can interpret α_x as the weight attached by the central bank to deviations of output from its efficient level (relative to price stability) in its own loss function, which does not necessarily have to coincide with the household's.

A structural equation relating inflation and the welfare-relevant output gap can be derived by using the identity $\widetilde{y}_t \equiv x_t + (y_t^e - y_t^n)$, to substitute for the output gap $\widetilde{y}_t$ in the NKPC relationship derived in chapter 3. This yields the following structural equation for inflation

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \kappa \; x_t + u_t \tag{2}$$

where $u_t \equiv \kappa(y_t^e - y_t^n)$.

Hence, the central bank will seek to minimize (1) subject to the sequence of constraints given by (2). Two features of that problem are worth stressing. First, note that, under our assumptions, the disturbance u_t is exogenous with respect to monetary policy, since the latter can influence neither the natural nor the efficient level of output. As a result, the central bank will take the current and anticipated values of u_t as given when solving its policy problem.

Secondly, and most importantly, time variations in the gap between the efficient and natural levels of output–reflected in fluctuations in u_t– generate a tradeoff for the monetary authority, since they make it impossible to attain simultaneously zero inflation and an efficient level of activity. This is a key difference from the model analyzed in the previous chapter, where we had $y_t^n = y_t^e$ for all t, thus implying $u_t = 0$ for all t. In the appendix we discuss several potential sources of variation in the gap between the efficient and natural levels of output, including exogenous changes in desired price or wage markups, as well as fluctuations in labor income taxes. Nevertheless, at least for the purposes of the analysis in the present chapter, knowledge of the specific source of that gap is not important.

Following much of the literature, we refer to disturbance u_t in (2) as a cost-push shock. Also, and for the remainder of this chapter, we assume that u_t follows the exogenous AR(1) process:

$$u_t = \rho_u \; u_{t-1} + \varepsilon_t^u \tag{3}$$

where $\rho_u \in [0, 1)$, and $\{\varepsilon_t^u\}$ is a white noise process with constant variance σ_u^2.

While (2) is the only constraint needed in order to determine the equilibrium path for output and inflation under the optimal policy, implementation of that policy requires that we make use of an additional condition linking those variables with the monetary policy instrument, i.e. the interest rate. That condition can be obtained by rewriting the dynamic IS equation first derived in chapter 3 in terms of the welfare-relevant output gap,

$$x_t = -\frac{1}{\sigma} \left(i_t - E_t\{\pi_{t+1}\} - r_t^e\right) + E_t\{x_{t+1}\} \tag{4}$$

where $r_t^e \equiv \rho + \sigma E_t\{\Delta y_{t+1}^e\}$ is the interest rate that supports the efficient allocation, and which is invariant to monetary policy. Henceforth, we refer to r_t^e as the *efficient* interest rate.

The forward-looking nature of constraint (2) in the policy problem, requires that we specify the extent to which the central bank can credibly commit in advance to future policy actions. As will be clear below, the reason is that by committing to some future policies the central bank is able to influence expectations in a way that improves its short-run tradeoffs. The following two sections characterize the optimal monetary policy under two alternative (and extreme) assumptions regarding the central bank's ability to commit to future policies.

2.1 Optimal Discretionary Policy

We start by considering the case in which the central bank treats the problem described above as one of sequential optimization, i.e. it makes whatever decision is optimal each period without committing itself to any future actions. That case if often referred to in the literature as optimal policy under discretion.

More especifically, each period the monetary authority is assumed to choose (x_t, π_t) in order to minimize the period losses

$$\pi_t^2 + \alpha_x \, x_t^2$$

subject to the constraint

$$\pi_t = \kappa \, x_t + v_t$$

where the term $v_t \equiv \beta \, E_t\{\pi_{t+1}\} + u_t$ is taken as given.by the monetary authority, since u_t is exogenous and $E_t\{\pi_{t+1}\}$ is a function of expectations about *future* output gaps (as well as future u_t's) which, by assumption, cannot be currently influenced by the policymaker.[2]

[2] To be precise, the term $E_t\{\pi_{t+1}\}$ can be treated as given by the central bank because there are no endogenous state variables (e.g. past inflation) affecting current inflation. Otherwise the central bank would have to take into account the influence that its current actions, through their impact on those state variables, would have on future inflation.

The optimality condition for the problem above is given by

$$x_t = -\frac{\kappa}{\alpha_x}\ \pi_t \tag{5}$$

for $t = 0, 1, 2, ...$The previous condition has a simple interpretation: in the face of inflationary pressures resulting from a cost-push shock the central bank must respond by driving output below its efficient level–thus creating a negative output gap–, with the objective of dampening the rise in inflation. The central bank carries out such a "leaning against the wind" policy up to the point where condition (5) is satisfied. Thus, one can view (5) as a relation between target variables that the discretionary central bank will seek to maintain at all times and it is in that sense that may be labeled a "targeting rule."[3]

Using (5) to substitute for x_t in (2), yields the following difference equation for inflation:

$$\pi_t = \frac{\alpha_x \beta}{\alpha_x + \kappa^2}\ E_t\{\pi_{t+1}\} + \frac{\alpha_x}{\alpha_x + \kappa^2}\ u_t$$

Iterating the previous equation forward we obtain an expression for equilibrium inflation under the optimal discretionary policy:

$$\pi_t = \alpha_x \Psi\ u_t \tag{6}$$

where $\Psi \equiv \frac{1}{\kappa^2 + \alpha_x(1-\beta\rho_u)}$. Combining (5) and (6) we get an analogous expression for the output gap.

$$x_t = -\kappa \Psi\ u_t \tag{7}$$

Thus, under the optimal discretionary policy, the central bank lets the output gap and inflation deviate from their targets in proportion to the current value of the cost-push shock. This is illustrated graphically by the circled lines in Figures 5.1 and 5.2, which represent the responses under the optimal discretionary policy of the output gap, inflation and the price level to a one-percent increase in u_t. In Figure 5.1 the cost-push shock is assumed to be purely transitory ($\rho_u = 0$), whereas in Figure 5.2 we assume it has a positive autocorrelation ($\rho_u = 0.5$). The remaning parameters are set at the values assumed in the baseline calibration of chapter 3.

[3]See, e.g. Svensson (1998) and Svensson and Woodford (1999) for a discussion of "targeting" vs "instrument" rules as alternative approaches to implementation of inflation targeting policies.

The path of the cost-push shock u_t after a one percent innovation is displayed in the bottom-right plot of Figures 5.1 and 5.2 . In both cases we see that the central bank finds it optimal to accommodate partly the inflationary pressures resulting from the cost-push shock, and thus let inflation increase. Note, however, that the increase in inflation is smaller than the one that would obtain if the output gap remained unchanged. In the latter case it is easy to check that inflation would be given by

$$\pi_t = \frac{1}{1 - \beta\rho_u} \; u_t$$

thus implying a larger response of inflation (in absolute value) at all horizons in response to the cost-push shock. Instead, under the optimal discretionary policy, the impact on inflation is dampened by the negative response of the output gap, also displayed in both figures. Finally, we see that the implied response of inflation leads naturally to a permanent change in the price level, whose size is increasing in the persistence of the shock.

The analysis above implicitly assumes that the monetary authority can choose its desired level of inflation and the output gap at each point in time. Of course, in practice a central bank cannot set either variable directly. One possible approach to implementing that policy is to adopt an interest rate rule that guarantees that the desired outcome is attained. Before we derive the form that such a rule may take it is convenient to determine the *equilibrium* interest rate under the optimal discretionary policy as a function of the exogenous driving forces. Thus, combining (6) and (7) with (4) we obtain:

$$i_t = r_t^e + \Psi_i \; u_t \tag{8}$$

where $\Psi_i \equiv \Psi \; [\kappa\sigma(1 - \rho_u) + \alpha_x\rho_u]$

Applying the arguments of chapter 3, it is easy to see that (8) cannot be viewed as a desirable interest rate rule, for it does not guarantee a unique equilibrium and, hence, the attainment of the desired outcome. In particular, if we use "rule" (8) to eliminate the nominal rate in (4), the resulting equilibrium dynamics are represented by the system:

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \mathbf{A_O} \begin{bmatrix} E_t\{x_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B_O} \; u_t \tag{9}$$

where

$$\mathbf{A_O} \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix} \quad ; \quad \mathbf{B_O} \equiv \begin{bmatrix} -\frac{\Psi_i}{\sigma} \\ 1 - \frac{\kappa \Psi_i}{\sigma} \end{bmatrix}$$

As argued in chapter 4, matrix $\mathbf{A_O}$ has always one eigenvalue outside the unit circle, thus implying that (9) has a multiplicity of solutions, only one of which corresponds to the desired outcome given by (6) and (7).

Consider instead the rule

$$i_t = r_t^e + \phi_\pi \ \pi_t \tag{10}$$

where $\phi_\pi \equiv (1-\rho_u)\frac{\kappa\sigma}{\alpha_x} + \rho_u$, and which can be obtained by combining (6) and (8), in a way that makes the nominal rate a function of inflation, an endogenous variable. It is easy to check that the previous rule is always consistent with the desired outcome of the policy problem under consideration here. Furthermore, using the arguments of chapter 4, we know that a rule of the form (10) leads to a determinate equilibrium (corresponding to the desired outcome) if and only if the inflation coefficient is greater than one or, equivalently, if and only if $\kappa\sigma > \alpha_x$ a condition that may or may not be satisfied.

In the context of our model, one can always derive a rule that guarantees equilibrium uniqueness (independently of parameter values) by appending to the expression for the equilibrium nominal rate under the optimal discretionary policy (given by (8)) a term proportional to the deviation between inflation and the equilibrium value of the latter under that policy, with the coefficient of proportionality being greater than one (in order to satisfy the Taylor principle). Formally,

$$\begin{aligned} i_t &= r_t^e + \Psi_i \ u_t + \phi_\pi \ (\pi_t - \alpha_x \Psi \ u_t) \\ &= r_t^e + \Theta_i \ u_t + \phi_\pi \ \pi_t \end{aligned} \tag{11}$$

where $\Theta_i \equiv \Psi \ [\kappa\sigma(1-\rho_u) - \alpha_x(\phi_\pi - \rho_u)]$ and for an arbitrary inflation coefficient satisfying $\phi_\pi > 1$.

In practice, interest rate rules like (10) and (11) are not easy to implement, for the reasons spelled out in chapter 4: they require knowledge of the model's parameters, and real-time observation of variations in the cost-push shock and the efficient interest rate. Those difficulties have led some authors to emphasize "targeting rules" like (5) as practical guides for monetary policy, as opposed to "instrument rules" like (10) and (11). Under a targeting rule, the central bank would adjusts its instrument until a certain optimal relation between target variables is satisfied. In our example, however, following such a targeting rule requires that the efficient level of output y_t^e be observed in real time, in order to determine the output gap x_t.

2.2 Optimal Policy under Commitment

After having analyzed the optimal policy under discretion, we turn to the case of a central bank which is assumed to be able to commit, with full credibility, to a *policy plan*. In the constext of our model such a plan consists of a specification of the desired levels of inflation and the output gap at all possible dates and states of nature, current and future. More specifically, the monetary authority is assumed to choose a state-contingent sequence $\{x_t, \pi_t\}_{t=0}^{\infty}$ that minimizes

$$\frac{1}{2}\ E_0 \sum_{t=0}^{\infty} \beta^t\ (\pi_t^2 + \alpha_x\ x_t^2)$$

subject to the sequence of constraints:

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ x_t + u_t$$

and where, as in the previous section, $\{u_t\}$ follows the exogenous process (3).

In order to solve the previous problem it is useful to write down the associated Lagrangian, which is given by:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t\ \left[\frac{1}{2}\left(\pi_t^2 + \alpha_x\ x_t^2\right) + \gamma_t\ (\pi_t - \kappa\ x_t - \beta\ \pi_{t+1})\right]$$

where $\{\gamma_t\}$ is a sequence of Lagrange multipliers, and where the law of iterated expectations has been used to eliminate the conditional expectation that appeared in each constraint.

Differentiating the Lagrangian with respect x_t and π_t we obtain the optimality conditions

$$\alpha_x\ x_t - \kappa\ \gamma_t = 0$$

$$\pi_t + \gamma_t - \gamma_{t-1} = 0$$

which must hold for $t = 0, 1, 2, ...$and where $\gamma_{-1} = 0$, since the inflation equation corresponding to period -1 is not an effective constraint for the central bank choosing its optimal plan in period 0.

Combining the two optimality conditions to eliminate the Lagrange multipliers yields

$$x_0 = -\frac{\kappa}{\alpha_x}\ \pi_0 \tag{12}$$

and

$$x_t = x_{t-1} - \frac{\kappa}{\alpha_x}\ \pi_t \tag{13}$$

for $t = 1, 2, 3, ...$

Note that (12) and (13) can be jointly represented by the single equation in "levels":

$$x_t = -\frac{\kappa}{\alpha_x}\ \widehat{p}_t \tag{14}$$

for $t = 0, 1, 2, ...$where $\widehat{p}_t \equiv p_t - p_{-1}$ is the (log) deviation between the price level and an "implicit target" given by the price level prevailing one period before the central bank chooses its optimal plan. Thus, (14) can be viewed as a "targeting rule" which the central bank must follow period by period in order to implement the optimal policy under commitment.

It is worth pointing out the difference between (14) and the corresponding targeting rule for the discretionary case, given by (5). Thus, the optimal discretionary policy requires that the central bank keeps output below (above) its efficient level as long as inflation is positive (negative). By way of contrast, under the optimal policy with commitment the central bank sets the sign and size of the output gap in proportion to the deviations of the price *level* from its implicit target. As we discuss next, this has important consequences for the economy's equilibrium response to a cost push shock.

By combining optimality condition (14) with (2) (after rewriting the latter in terms of the price level) we can derive the stochastic difference equation satisfied by $\widehat{p}_t$ under the optimal policy:

$$\widehat{p}_t = a\ \widehat{p}_{t-1} + a\beta\ E_t\{\widehat{p}_{t+1}\} + a\ u_t$$

for $t = 0, 1, 2, ...$where $a \equiv \frac{\alpha_x}{\alpha_x(1+\beta)+\kappa^2}$.

The stationary solution to the previous difference equation is given by:

$$\widehat{p}_t = \delta\ \widehat{p}_{t-1} + \frac{\delta}{(1-\delta\beta\rho_u)}\ u_t \tag{15}$$

for $t = 0, 1, 2, ...$where $\delta \equiv \frac{1-\sqrt{1-4\beta a^2}}{2a\beta} \in (0, 1)$. We can then use (14) to derive the equilibrium process for the output gap:

$$x_t = \delta\ x_{t-1} - \frac{\kappa\delta}{\alpha_x(1-\delta\beta\rho_u)}\ u_t \tag{16}$$

for $t = 1, 2, 3, ...$, with the response at the time of the shock ($t = 0$) being given by

$$x_0 = -\frac{\kappa\delta}{\alpha_x(1-\delta\beta\rho_u)}\ u_0$$

The lines with crosses in Figure 5.1 show the equilibrium responses of the output gap, inflation, and the price level to a one percent transitory cost-push shock. Analogous responses for the case of a persistence cost-push shock are displayed in Figure 5.2 . In both cases those responses are shown side by side with the responses implied by the optimal discretionary policy (represented by the circled lines described earlier), thus facilitating comparison of the two regimes' outcomes.

A look at the case of a transitory cost-push shock illustrates the difference most clearly. In the case of discretionary policy, both the output gap and inflation return to their zero initial value once the shock has vanished (i.e. one period after the shock). By contrast, and as implied by (15) and (16), under the optimal policy with commitment the deviations in the output gap and inflation from target persist well beyond the life of the shock, i.e. they display endogenous or intrinsic persistence. Given that a zero inflation, zero output gap outcome is feasible once the shock has vanished, why does the central bank find it optimal to maintain a persistently negative output gap and inflation? The reason is simple: by committing to such a response, the central bank manages to improve the output gap/inflation tradeoff in the period when the shock occurs. In the case illustrated in Figure 5.1 it lowers the initial impact of the cost-push shock on inflation (relative to the discretionary case), while incurring smaller output gap losses in the same period. This is possible because of the forward-looking nature of inflation, which can be highlighted by iterating (2) forward to yield:

$$\pi_t = \kappa\ x_t + \kappa\sum_{k=1}^{\infty}\beta^k\ E_t\{x_{t+k}\} + u_t$$

Hence, we see that the central can offset the inflationary impact of a cost push shock by lowering the current output gap x_t, but also by committing to lower future output gaps (or, equivalently, future reductions in the price level). If credible, such "promises" will bring about a downward adjustment in the sequence of expectations $E_t\{x_{t+k}\}$ for $k = 1, 2, 3, ...$As a result, and in response to a positive realization of the cost-push shock u_t, the central bank may achieve any given level of current inflation π_t with a smaller decline in the current output gap x_t. That is the sense in which the output gap/inflation tradeoff is improved by the possibility of commitment. Given the convexity of the loss function in inflation and output gap deviations, the dampening of those deviations in the period of the shock brings about an improvement in overall welfare relative to the case of discretion, since the implied benefits are not offset by the (relatively small) losses generated by the deviations in subsequent periods (and which are absent in the discretionary case).

Figure 5.2 displays analogous impulse responses under the assumption that $\rho_u = 0.8$. Note that in this case the economy reverts back to the initial position only asymptotically, even under the optimal discretionary policy (since the inflationary pressures generated by the shock remains effective at all horizons, albeit with a declining influence). Yet, some of the key qualitative features emphasized above are still present: in particular, the optimal policy with commitment manages once again to attain both lower inflation and a smaller output gap (in absolute value) at the time of the shock, relative to the optimal discretionary policy. Note also that under the optimal policy with commitment the price level reverts back to its original level, albeit at a slower rate than in the case of a transitory shock. As a result inflation displays some positive short run autocorrelation, illustrating the fact that the strong negative short run autocorrelation observed in the case of a purely transitory shock is not a necessary implication of the policy with commitment.

In all cases, a feature of the economy's response under discretionary policy is the attempt to stabilize the output gap in the medium term more than the optimal policy under commitment calls for, without internalizing the benefits in terms of short-term stability that result from allowing larger deviations of the output gap at future horizons. This characterisctic, which is most clearly illustrated by the example of a purely transitory cost-push shock represented in Figure 5.1, is often referred to as the *stabilization bias* associated with the discretionary policy.[4]

As in the case of discretion, one might be interested in deriving an interest rate rule that would bring about the paths of output gap and inflation implied by the optimal policy under commitment. Next we derive such a rule for the

[4] That stabilization bias must be distinguished from the inflation bias which arises when the zero inflation steady state is associated with an inefficiently low level of activity. The stabilization bias obtains independently of the degree of inefficiency of the steady state, as discussed below.

special case of serially uncorrelated cost push shocks ($\rho_u = 0$). In that case, we can combine (4), (15) and (16) to obtain the process describing the equilibrium nominal rate under the optimal policy with commitment:

$$
\begin{aligned}
i_t &= r_t^e - (1-\delta)\left(1 - \frac{\sigma\kappa}{\alpha_x}\right)\ \widehat{p}_t \\
&= r_t^e - (1-\delta)\left(1 - \frac{\sigma\kappa}{\alpha_x}\right)\sum_{k=0}^{t}\delta^{k+1}\ u_{t-k}
\end{aligned}
$$

Thus, one possible rule that would brIng about the desired allocation as the unique equilibrium is given by

$$
i_t = r_t^e - \left(\phi + (1-\delta)\left(1 - \frac{\sigma\kappa}{\alpha_x}\right)\right)\sum_{k=0}^{t}\delta^{k+1}\ u_{t-k} + \phi_p\ \widehat{p}_t
$$

for any $\phi_p > 0$. Note that under the previous formulation the central bank stands ready responds to any deviation of the price level from the path prescribed by (15), though this will not be necessary in equilibrium.[5]

3 The Monetary Policy Problem: the Case of a Distorted Steady State

Next we consider the case in which the presence of uncorrected real imperfections generate a permanent gap between the natural and the efficient levels of output, which is reflected in an inefficient steady state. We measure the size of the steady state distortion by a parameter Φ representing the wedge between the marginal product of labor and the marginal rate of substitution between consumption and hours, both evaluated at the steady state. Formally, Φ is defined by

$$
-\frac{U_n}{U_c} = MPN\ (1-\Phi)
$$

Below we assume $\Phi > 0$, which implies that the steady state levels of output and employment are below their respective efficient levels. The presence of firms' market power in the goods market as assumed in the basic

[5] An interest rate rule that displays a positive response to the price level can be shown to genrate a unique equilibrium in the basic new Keynesian model. See exercise 5 in chapter 4.

model of chapter 3 constitutes an example of the kind of distortion which, if uncorrected through an appropriate subsidy, would generate an inefficiently low level of activity. In that case, and as implied by the analysis of chapter 4, we have $\Phi \equiv 1 - \frac{1}{\mathcal{M}} > 0$, where $\mathcal{M}$ is the steady state gross markup.

Under the assumption of a "small" steady state distortion (i.e. when Φ has the same order of magnitude as fluctuations in the output gap or inflation), and as shown in the Appendix to the present chapter, the component of the welfare losses experienced by the representative household that can be affected by policy is approximately proportional, in a neighborhood of the zero inflation steady state, to the expression

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \alpha_x \, \widehat{x}_t^2 \right) - \Lambda \, \widehat{x}_t \right] \tag{17}$$

where $\Lambda \equiv \Phi \frac{\lambda}{\epsilon} > 0$ and $\widehat{x}_t = x_t - x$ represents the deviation of the welfare-relevant output gap from its value $x < 0$ in the zero inflation steady state. Note that the linear term $\widehat{x}_t$ captures the fact that any marginal increase in output has a positive effect on welfare (thus increasing welfare losses), since output is assumed to be below its efficient level.

Similarly, we can write the inflation equation in terms of $\widehat{x}_t$ as

$$\pi_t = \beta \, E_t\{\pi_{t+1}\} + \kappa \, \widehat{x}_t + u_t \tag{18}$$

where now $u_t \equiv \kappa(\widehat{y}_t^e - \widehat{y}_t^n)$. Thus the monetary authority will seek to minimize (17) subject to the sequence of constraints given by (18) for $t = 0, 1, 2, ...$

Note that under the assumption of a "small" steady state distortion made above, the linear term $\Lambda \, \widehat{x}_t$ is already of second order, thus giving the central bank's problem the convenient linear-quadratic format.[6]

As in the previous section, we characterize the solution to the central bank's problem under discretion, before turning to the optimal policy with commitment.

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[6] In the presence of a large distortion, the presence of a linear term in (17) would require the use of a second order approximation to the equilibrium condition connecting outpup gap and inflation.

3.1 Optimal Discretionary Policy

In the absence of a commitment technology, the monetary authority chooses (x_t, π_t) in order to minimize the period losses

$$\frac{1}{2}\ (\pi_t^2 + \alpha_x\ \widehat{x}_t^2) - \Lambda\ \widehat{x}_t$$

subject to the constraint

$$\pi_t = \kappa\ \widehat{x}_t + v_t$$

where, once again, $v_t \equiv \beta\ E_t\{\pi_{t+1}\} + u_t$ is taken as given by the policymaker.

The associated optimality condition is

$$\widehat{x}_t = \frac{\Lambda}{\alpha_x} - \frac{\kappa}{\alpha_x}\ \pi_t \tag{19}$$

Note that (19) implies, for any given level of inflation, a more expansionary policy than in the absence of a steady state distortion. This is a consequence of the desire by the central bank to partly correct for the inefficiently low average level of activity.

Plugging (19) into (18) and solving the resulting difference equation yields the following expression for equilibrium inflation:

$$\pi_t = \frac{\Lambda\kappa}{\kappa^2 + \alpha_x(1-\beta)} + \alpha_x\Psi\ u_t \tag{20}$$

Combining (20) and (19) yields the corresponding expression for the equilibrium output gap:

$$\widehat{x}_t = \frac{\Lambda(1-\beta)}{\kappa^2 + \alpha_x(1-\beta)} - \kappa\Psi\ u_t$$

Thus, we see that the presence of a distorted steady state does not affect the response of the output gap and inflation to shocks under the optimal policy. It has, however, an effect on the average levels of inflation and the output gap around which the economy fluctuates. In particular, when the natural level of output and employment are inefficiently low ($\Lambda > 0$) the optimal discretionary policy leads to positive average inflation, as a consequence of the central bank's incentive to push output above its natural steady state level.[7] That incentive increases with the degree of inefficiency of the natural steady state, which explains the fact that the average inflation is increasing in Λ (and hence in Φ), giving rise to the *classical inflation bias* phenomenon.

[7] Notice that in the steady state $\widehat{x} = y - y^n$

3.2 Optimal Policy under Commitment

As in the case of an efficient steady state, we solve for the optimal policy under commitment by setting up the Lagrangean corresponding to the central bank's problem, which in this case is given by

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}(\pi_t^2 + \alpha_x \, \widehat{x}_t^2) - \Lambda \, \widehat{x}_t + \gamma_t \, (\pi_t - \kappa \, \widehat{x}_t - \beta \, \pi_{t+1}) \right]$$

where $\{\gamma_t\}$ are the Lagrange multipliers associated with the sequence of constraints (18), for $t = 0, 1, 2, ...$

The corresponding optimality conditions are given by

$$\alpha_x \, \widehat{x}_t - \kappa \, \gamma_t - \Lambda = 0$$

$$\pi_t + \gamma_t - \gamma_{t-1} = 0$$

which must hold for $t = 0, 1, 2, ...$and where $\gamma_{-1} = 0$. The previous conditions can be combined to yield the following difference equation for the (log) price level:

$$\widehat{p}_t = a \, \widehat{p}_{t-1} + a\beta \, E_t\{\widehat{p}_{t+1}\} + \alpha\kappa\Lambda + a \, u_t$$

for $t = 0, 1, 2, ...$where, as above, $\widehat{p}_t \equiv p_t - p_{-1}$, and $a \equiv \frac{\alpha_x}{\alpha_x(1+\beta)+\kappa^2}$.

The stationary solution to the previous difference equation describes the evolution of the equilibrium price level under the optimal policy with commitment. It takes the following form:

$$\widehat{p}_t = \delta \, \widehat{p}_{t-1} + \frac{\delta}{1 - \delta\beta\rho_u} \, u_t + \frac{\delta\kappa\Lambda}{1 - \delta\beta}$$

where $\delta \equiv \frac{1-\sqrt{1-4\beta a^2}}{2a\beta} \in (0, 1)$. The corresponding path for the output gap is given by:

$$\widehat{x}_t = \delta \, \widehat{x}_{t-1} - \frac{\kappa\delta}{\alpha_x(1 - \delta\beta\rho_u)} \, u_t + \Lambda \left[1 - \delta \left(1 + \frac{\kappa^2}{\alpha_x(1 - \delta\beta)} \right) \right]$$

Thus, as it was the case under the discretionary policy, the response to a cost-push shock under the optimal policy with commitment is not affected by the presence of a distorted steady state. Hence the impulse responses displayed in Figures 5.1 and 5.2 illustrating the economy's response to a cost-push shock under discretion and under commitment, remain valid in the present context. In particular, the optimal policy under discretion is characterized by an identical stabilization bias.

In the presence of a distorted steady state, an additional difference arises between the discretionary and commitment policies, unrelated to the response to shocks: it has to do with the deterministic component of inflation and its evolution over time. As shown above, in the case of discretion that component takes the form a constant positive mean, resulting from the period-by-period incentive to close the gap between output and its efficient level, which results in inflation. In the case of commitment, however, we see that the price level converges asymptotically to a constant, given by $\lim_{T\to\infty} p_T = p_{-1} + \frac{\delta\kappa\Lambda}{(1-\delta\beta)(1-\delta)}$. Hence, after displaying a positive value at the beginning of the optimal plan's implementation, the deterministic component of inflation (around which actual inflation fluctuates in response to shocks) declines gradually over time, following the path $\frac{\delta^{t+1}\kappa\Lambda}{1-\delta\beta}$. Hence, under the optimal plan the economy eventually converges to an equilibrium characterized by zero average inflation, and in that sense observationally equivalent to that of an economy with an efficient steady steady state. The desirability of such a policy is justified by the benefits arising from its anticipation by the public, which improves the short-run tradeoff facing the central bank, allowing it to raise output above its natural level (with the consequent welfare improvement) with more subdued effects on inflation (since the public anticipates a gradual return of output to its natural level) . Thus, the central bank's ability to commit avoids (at least asymptotically) the inflation bias that characterizes the outcome of the discretionary policy.

4 Notes on the Literature

The present chapter follows closely Clarida, Galí and Gertler (1999), where the optimal monetary policy in the context of the basic new Keynesian model augmented with an ad-hoc cost-push shocks is analyzed, and where the outcomes under discretion and commitment are compared. That paper also contains a discussion of the classical inflation bias, whose ultimate source is modeled as a positive target for the output gap in the policymaker's loss function. The original treatment of the inflation bias and the gains from commitment, in the context of a new classical model with a Lucas supply curve, can be found in Kydland and Prescott (1980) and Barro and Gordon

(1983).

Woodford (2003a) discusses a source of monetary policy tradeoffs different from cost-push shocks: that created by the presence of transactions friction which leads to an indirect utility function in which real balances are one of the arguments, as in the model at the end of chapter 2. In that context, and in addition to variations in inflation and the output gap, variations in the nominal rate (which acts as a tax on money holdings) are a source of welfare losses. As a result, a policy that fully stabilizes the output gap and inflation by making the interest rate move one-for-one with the natural rate, while feasible, it is no longer optimal since it implies excessive interest rate volatility. The optimal policy, as shown by Woodford, smoothens the fluctuations in the nominal rate, at the cost of some variations in inflation and output gap.

The approximation to welfare in the presence of "small" steady state distortions presented here follows the analysis in Woodford (2003b). The analysis of optimal policy in the presence of "large" steady state distortions lies beyond the scope of the present book. The main difficulty in that case arises from the presence of a linear term in the second-order approximation to the welfare loss function. In that context, the use of a log-linear (i.e. first order) approximation to the equilibrium conditions to describe the evolution of endogenous variables leads to second-order terms potentially relevant to welfare being ignored (e.g. the ones associated with the steady state effects of different degrees of volatility).

Several approaches to overcoming that problem are found in the literature. A first approach consists in solving for the evolution of the endogenous variables using a second-order (or higher) approximation to the equilibrium conditions under a given policy rule, and evaluating the latter using the original second-order approximation to the welfare losses. An application of that approach to the monetary policy problem can be found in Schmitt-Grohé and Uribe (2004), among others.

The second approach, due to Benigno and Woodford (2005), makes use of a second-order approximation to the structural equations of the model in order to replace the linear terms appearing in the welfare loss function, and rewriting those losses as a function of quadratic terms only. The resulting quadratic loss function can then be minimized subject to the constraints provided by log-linearized equilibrium conditions. That approach allows one to preserve the convenient structure and properties of linear-quadratic problems, including the linearity of their implied optimal policy rules.

A third approach, illustrated in Khan, King and Wolman (2003), requires that the optimal policy be determined in a first stage using the exact structural equations and utility function, and log-linearizing the resulting equilibrium conditions (embedding the optimal policy) in order to characterize the optimal responses to shocks.

Appendix 1: A Second Order Approximation to Welfare Losses in the Case of a Small Steady State Distortion

As shown in the appendix to chapter 4, a second order Taylor expansion to period t utility, combined with a goods market clearing condition, yields:

$$U_t - U = U_c C \left(\widehat{y}_t + \frac{1-\sigma}{2} \, \widehat{y}_t^2 \right) + \frac{U_n N}{1-\alpha} \left(\widehat{y}_t + \frac{\epsilon}{2\Theta} \, var_i\{p_t(i)\} + \frac{1+\varphi}{2(1-\alpha)} \, (\widehat{y}_t - a_t)^2 \right) + t.i.p..$$

where $t.i.p.$ stands for "terms independent of policy".

Let Φ denote the size of the steady state distortion, implicitly defined by $-\frac{U_n}{U_c} = MPN \ (1-\Phi)$. Using the fact that $MPN = (1-\alpha)(Y/N)$ we have

$$\frac{U_t - U}{U_c C} = \widehat{y}_t + \frac{1-\sigma}{2} \, \widehat{y}_t^2 - (1-\Phi) \left(\widehat{y}_t + \frac{\epsilon}{2\Theta} \, var_i\{p_t(i)\} + \frac{1+\varphi}{2(1-\alpha)} \, (\widehat{y}_t - a_t)^2 \right) + t.i.p.$$

Under the "small distortion" assumption (so that the product of Φ with a second order term can be ignored as negligible) we can write:

$$\begin{aligned}
\frac{U_t - U}{U_c C} &= \Phi \, \widehat{y}_t - \frac{1}{2} \left[\frac{\epsilon}{\Theta} \, var_i\{p_t(i)\} - (1-\sigma) \, \widehat{y}_t^2 + \frac{1+\varphi}{1-\alpha} \, (\widehat{y}_t - a_t)^2 \right] + t.i.p. \\
&= \Phi \, \widetilde{y}_t - \frac{1}{2} \left[\frac{\epsilon}{\Theta} \, var_i\{p_t(i)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha} \right) \widehat{y}_t^2 - 2 \left(\frac{1+\varphi}{1-\alpha} \right) \widehat{y}_t a_t \right] + t.i.p. \\
&= \Phi \, \widetilde{y}_t - \frac{1}{2} \left[\frac{\epsilon}{\Theta} \, var_i\{p_t(i)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha} \right) (\widehat{y}_t^2 - 2 \widehat{y}_t \, \widehat{y}_t^e) \right] + t.i.p. \\
&= \Phi \, \widehat{x}_t - \frac{1}{2} \left[\frac{\epsilon}{\Theta} \, var_i\{p_t(i)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha} \right) \widehat{x}_t^2 \right] + t.i.p.
\end{aligned}$$

where $\widehat{y}_t^e \equiv y_t^e - y^e$, and where we have used the fact that $\widehat{y}_t^e = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} \ a_t$ .and $\widehat{y}_t - \widehat{y}_t^e = x_t - (y - y^e) = x_t - x \equiv \widehat{x}_t$.

Accordingly, we can write a second order approximation to the consumer's welfare losses (up to additive terms independent of policy), and expressed as a fraction of steady state consumption, as:

$$\mathbb{W} = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{U_t - U}{U_c C} \right) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi \, \widehat{x}_t - \frac{1}{2} \left(\frac{\epsilon}{\Theta} \, var_i\{p_t(i)\} + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha} \right) \widehat{x}_t^2 \right) \right] + t.i.p.$$

Using Lemma 2 in the appendix of chapter 4, we can rewrite the welfare losses as

$$\mathbb{W} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi \, \widehat{x}_t - \frac{1}{2} \left(\frac{\epsilon}{\lambda} \right) \pi_t^2 + \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \widehat{x}_t^2 \right] + t.i.p.$$

Note that in the particular case of an efficient steady state we have $\Phi = 0$ and $\widehat{x}_t = x_t$. Moreover, if as in Chapter 4 the model satisfies $y_t^n = y_t^e$ for all t , the we have $\widehat{x}_t = x_t = \widetilde{y}_t$., with the implied loss function taking the form used in that Chapter.

Appendix 2: Sources of Cost Push Shocks

The present appendix describes two possible sources of cost-push shocks, variations in desired price markups and exogenous variations in wage markups.

Variations in desired price markups.

Assume that the elasticity of substitution among goods varies over time, according to some stationary stochastic process $\{\epsilon_t\}$. Let the associated desired markup be given by $\mu_t^n \equiv \frac{\epsilon_t}{\epsilon_t - 1}$ .The log-linearized price setting rule is then given by:

$$\begin{aligned} p_t^* &= (1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta)^k \; E_t\{\mu_{t+k}^n + mc_{t+k} + p_{t+k}\} \\ &= (1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta)^k E_t\{\widetilde{mc}_{t+k} + p_{t+k}\} \end{aligned}$$

where $\widetilde{mc}_t \equiv mc_t + \mu_t^n$ The resulting inflation equation then becomes

$$\begin{aligned} \pi_t &= \beta E_t\{\pi_{t+1}\} + \lambda \; \widetilde{mc}_t \\ &= \beta E_t\{\pi_{t+1}\} + \lambda \; \widehat{mc}_t + \lambda(\mu_t^n - \mu) \\ &= \beta E_t\{\pi_{t+1}\} + \kappa \; (y_t - \overline{y}_t^n) + \lambda(\mu_t^n - \mu) \end{aligned}$$

where $\overline{y}_t^n$ denotes the equilibrium level of output under flexible prices *and* a constant price markup μ. Letting $x_t \equiv y_t - \overline{y}_t^n$ and $u_t \equiv \lambda(\mu_t^n - \mu)$ we obtained the formulation used in the main text.

Exogenous Variations in Wage Markups

In that case we still have $\pi_t = \beta E_t\{\pi_{t+1}\} + \lambda \; \widehat{mc}_t$, though now

$$\begin{aligned} mc_t &= w_t - a_t \\ &= \mu_{w,t} + mrs_t - a_t \\ &= \mu_{w,t} + (\sigma + \varphi) \; y_t - (1+\varphi) \; a_t \end{aligned}$$

where $\mu_{w,t}$ represents a time-varying, exogenous wage markup. Under flexible prices and a constant wage markup (at its steady state level μ_w), we have

$$mc = \mu_w + (\sigma + \varphi) \; \overline{y}_t^n - (1+\varphi) \; a_t$$

where $\overline{y}_t^n$ denotes the equilibrium level of output under a constant price and wage markup.

The difference between the two previous expression is thus given by

$$\widehat{mc}_t = (\sigma + \varphi)\ (y_t - \overline{y}_t^n) + (\mu_{w,t} - \mu_w)$$

which can be plugged into the inflation equation to yield

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t$$

where $x_t \equiv y_t - \overline{y}_t^n$ and $u_t \equiv \lambda(\mu_{w,t} - \mu_w)$.

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Exercises

1. An Optimal Taylor Rule

Consider an economy with Calvo-type staggered price setting whose equilibrium dynamics are described by the system:

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma}\ (i_t - E_t\{\pi_{t+1}\} - \rho) + \varepsilon_t$$

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ x_t + u_t$$

where $\{\varepsilon_t\}$ and $\{u_t\}$ are *i.i.d.*, mutually uncorrelated, demand and supply disturbances, with variances given by σ_ε^2 and σ_u^2 respectively

Assume that the monetary authority adopts a simple Taylor rule of the form

$$i_t = \rho + \phi_\pi\ \pi_t$$

a) Solve for the equilibrium processes for the output gap and inflation, as a function of the exogenous supply and demand shocks.

b) Determine the value of the inflation coefficient ϕ_π which minimizes the central bank's loss function:

$$\alpha_x\ var(x_t) + var(\pi_t)$$

c) Discuss and provide intuition for the dependence of the optimal inflation coefficient on the weight α_x and the variance ratio $\frac{var(\varepsilon)}{var(u)}$. What assumptions on parameter values would warrant an aggressive response to inflation, implemented thorugh a large ϕ_π? Explain.

2. Optimal Markovian Policy

Consider an economy where inflation is described by the augmented NKPC

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ x_t + u_t$$

where $\{u_t\}$ is an exogenous cost-push shock following a stationary AR(1) process

$$u_t = \rho_u\ u_{t-1} + \varepsilon_t^u$$

In period 0, the central bank chooses once and its policy among the class of Markovian policies of the form $x_t = \psi_x u_t$ and $\pi_t = \psi_\pi u_t$ for all t, in order to minimize the loss function

$$E_0 \sum_{t=0}^{\infty} \beta^t \ \left(\pi_t^2 + \alpha_x \ x_t^2\right)$$

subject to the sequence of constraints describing the evolution of inflation.

a) Determine the optimal values of ψ_x and ψ_π.

b) Compare the resulting optimal policy to the optimal discretionary policy analyzed in the chapter. Which one is more desirable from a welfare point of view? Explain

c) Compare the resulting optimal policy to the optimal policy under commitment analyzed in the chapter. Which one is more desirable from a welfare point of view? Explain.

3. Optimal Monetary Policy in the Presence of Transaction Frictions *(based on Woodford (2003a))*

As shown in Woodford (2003a), in the presence of real balances as a source of indirect utility in an otherwise standard NK model, a second order approximation to the representative household's welfare is proportional to:

$$-\frac{1}{2} \ E_0 \sum_{t=0}^{\infty} \beta^t \ \left(\pi_t^2 + \alpha_x \ x_t^2 + \alpha_i \ i_t^2\right)$$

Consider the problem of choosing the state-contingent policy $\{x_t, \pi_t\}_{t=0}^{\infty}$ that maximizes welfare subject to the sequence of constraints:

$$\pi_t = \beta \ E_t\{\pi_{t+1}\} + \kappa \ x_t$$

$$x_t = -\frac{1}{\sigma} \ (i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{x_{t+1}\}$$

for $t = 0, 1, 2, ...$where the natural rate r_t^n is assumed to follow an exogenous process.

a) Determine the optimality conditions for the problem described above

b) Show that the implied optimal policy can be implemented by means of an interest rate rule of the form

$$i_t = (1 + \frac{\kappa}{\sigma\beta}) \ i_{t-1} + \frac{1}{\beta} \ \Delta i_{t-1} + \frac{\kappa}{\alpha_i \sigma} \ \pi_t + \frac{\alpha_x}{\alpha_i \sigma} \ \Delta x_t$$

which is independent of r_t^n and its properties.

4. Inflation Persistence and Monetary Policy *(based on Steinsson (2003))*

As shown in Steinsson (2003), in the presence of partial price indexation by firms the second order approximation to the the household's welfare losses takes the form:

$$\frac{1}{2}\ E_0 \sum_{t=0}^{\infty} \beta^t [\alpha_x\ x_t^2 + (\pi_t - \gamma \pi_{t-1})^2]$$

where γ denotes the degree of price indexation to past inflation. The equation describing the evolution of inflation is now given by:

$$\pi_t - \gamma \pi_{t-1} = \kappa\ x_t + \beta\ E_t\{(\pi_{t+1} - \gamma \pi_t)\} + u_t$$

where u_t represents an exogenous $i.i.d.$ cost-push shock.

a) Determine the optimal policy under discretion, i.e. under the assumption that the monetary authority seeks to minimize each period the short-term losses $\alpha_x\ x_t^2 + (\pi_t - \gamma \pi_{t-1})^2$

b) Determine the optimal policy under commitment

c) Discuss how the degree of indexation γ affects the optimal responses to a transitory cost-push shock under the previous two scenarios.

5. Monetary Policy, Optimal Steady State Inflation and the Zero Lower Bound

Consider a new Keynesian model with equilibrium conditions given by

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma}\ (i_t - E_t\{\pi_{t+1}\} - \rho) + \varepsilon_t$$

and

$$\pi_t - \pi = \beta\ E_t\{(\pi_{t+1} - \pi)\} + \kappa\ x_t + u_t$$

where x_t is the (welfare-relevant) output gap, π_t denotes inflation, i_t is the nominal rate, and π is steady state inflation. The disturbances ε_t and u_t represent demand and cost-push shocks, and are assumed to follow independent and serially uncorrelated normal distributions with zero mean and variances σ_ε^2 and σ_u^2 respectively.

Assume that the loss function for the monetary authority is given by

$$\Theta \ \pi + E_0 \sum_{t=0}^{\infty} \beta^t \ \left[\alpha_x \ x_t^2 + (\pi_t - \pi)^2\right]$$

where the first term is assumed to capture the costs of steady state inflation.

(a) Derive the optimal policy under discretion (i.e., the time-consistent policy, resulting from period-by-period maximization) –including the choice of steady state inflation π–, subject to the constraint that the interest rate hits the zero-bound constraint with only with a 5 percent probability.

(b) Derive an interest rate rule that would implement the optimal allocation derived in (a) as the unique equilibrium.

Figure 5.1: Optimal Responses to a Transitory Cost-Push Shock

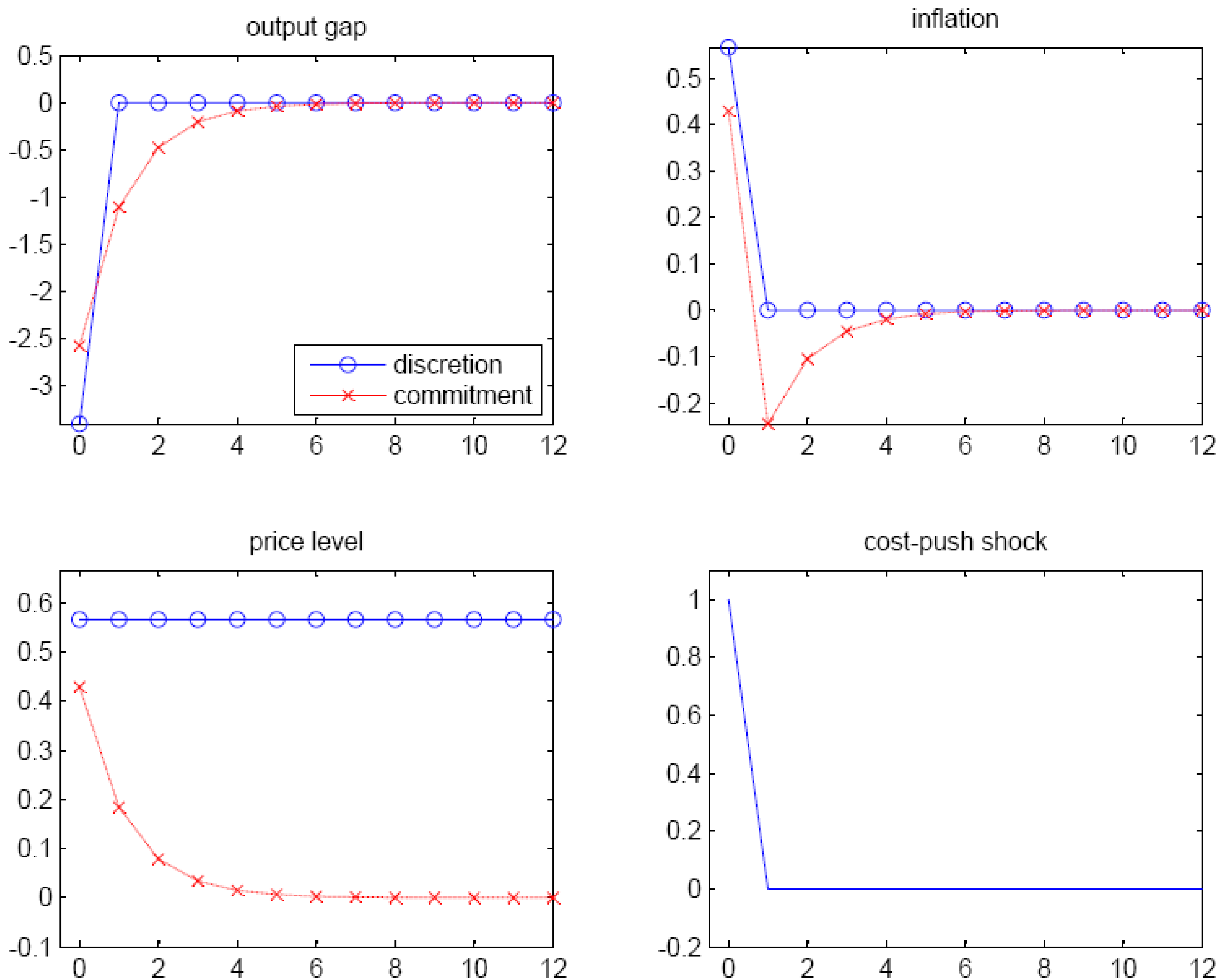

Figure 5.2 : Optimal Responses to a Persistent Cost Push Shock