Advanced Macro II
The reference answer to Mid-term Exam

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Note: The answer is just used for reference.

Solve a model of a decentralized economy with monopolistic competition

The representative household

Consider the following model where a representative household solves

$$\max_{C_t,N_t,B_t} E_0\left\{\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\sigma}}{1-\sigma}-\frac{N_t^{1+\varphi}}{1+\varphi}\right)\right\} \tag{1}$$

s.t.

$$P_tC_t+Q_tB_t=B_{t-1}+W_tN_t+T_t \tag{2}$$

where $\beta\,(0<\beta<1)\,,\sigma$ and φ are parameters. B_t is the holding of a one-period bond, $Q_t=e^{-i_t}$ is the bond price. W_t is wage for labour supply N_t,and T_t is possible transfer from firms owned by the households and /or government tax/subsidy. P_t is a price index for a composite consumption index C_t,which contains two types of consumption(say goods and services): C_{1t} and C_{2t},and the composite index is defined as

$$C_t\equiv\left(C_{1t}^{1-\frac{1}{\epsilon}}+C_{2t}^{1-\frac{1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$$

Two sectors of monopolistic firms

There are two sectors for producing the two types of goods, and each sector is occupied by a monopolistic firm. Although the technology is the same,i.e,

$$Y_{it}=A_tN_{it},i=1,2.$$

The two types of goods are differentiated and not perfectly substitutable for a finite ε.The firms seek to maximize profit and transfer it back to households owners. Both firms are subject to a productivity shock ε_t in the technology process

$$a_t=(1-\rho_a)\rho+\rho_a a_{t-1}+\varepsilon_t^a,\varepsilon_t^a\overset{iid}{\sim}\left(0,\sigma_a^2\right)$$

where $a_t=\log A_t,0<\rho_a<1,\rho=-\log(\beta)$.

Government

The government has a central bank which reacts to inflation as follows

$$i_t=\rho+\phi_\pi\pi_t$$

where $\pi_t\equiv\log P_t-\log P_{t-1}$.

Questions:

1. (4 points) Describe the economy briefly. Comment on the preference, endowment, technology, and information.

   The economic environment:

   1)Preference:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \right\}$$

   2)Technology:

   The two sector of monopolistic firms produce with the same technology, as follows

$$\begin{aligned} Y_{it} &= A_t N_{it}, i = 1, 2. \\ a_t &= (1-\rho_a)\rho + \rho_a a_{t-1} + \varepsilon_t^a, \varepsilon_t^a \overset{iid}{\sim} \left(0, \sigma_a^2\right) \\ a_t &= \log A_t, 0 < \rho_a < 1, \rho = -\log(\beta). \end{aligned}$$

   3)Endowment:

   The representative household is endowed with capital $k_{-1} > 0$ at $t = 0$.

   4)Information: decision made based on all information I_t up to time t.

2. Find the first order necessary conditions(FONCs) for household in two steps.

   (a) (4 points) Withon period: derive the demand for C_{it}.It amounts to maximizing the consumption basket given a certain expenditure Z_t.

$$\max_{(C_{1t}, C_{2t})} C_t \equiv \left(C_{1t}^{1-\frac{1}{\epsilon}} + C_{2t}^{1-\frac{1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}, \epsilon > 1,$$

$$s.t. : P_{1t}C_{1t} + P_{2t}C_{2t} = Z_t$$

You can define the price index as

$$P_t \equiv \left(P_{1t}^{1-\epsilon} + P_{2t}^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}.$$

(Note: if you cannot make the derivation, take the solution of demand function for later analysis, which is $C_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} C_t$.)

ANS:

The Lagrangean function is

$$L(C_{1t}, C_{2t}) = \left(C_{1t}^{1-\frac{1}{\epsilon}} + C_{2t}^{1-\frac{1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} - \lambda_t \left(P_{1t}C_{1t} + P_{2t}C_{2t} - Z_t \right)$$

Firstly, we take the FONC for C_{1t} and C_{2t}, as

$$\begin{aligned}
\frac{\partial L(C_{1t}, C_{2t})}{\partial C_{1t}} &= C_t^{\frac{1}{\epsilon-1}} C_{1t}^{-\frac{1}{\epsilon}} - \lambda_t P_{1t} = 0 \\
C_{1t} &= \left(\lambda_t P_{1t} C_t^{\frac{1}{1-\epsilon}}\right)^{-\varepsilon} \\
C_{2t} &= \left(\lambda_t P_{2t} C_t^{\frac{1}{1-\epsilon}}\right)^{-\varepsilon} \\
\text{taking ratio, } C_{1t} &= \left(\frac{P_{1t}}{P_{2t}}\right)^{-\varepsilon} C_{2t}
\end{aligned}$$

substitute the above equation into C_t,

$$\begin{aligned}
C_t &= \left(C_{1t}^{1-\frac{1}{\epsilon}} + C_{2t}^{1-\frac{1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \\
&= \frac{\left(P_{1t}^{1-\epsilon} + P_{2t}^{1-\epsilon}\right)^{\frac{\varepsilon}{\epsilon-1}}}{P_{2t}^{-\epsilon}} C_{2t} \\
\text{with } P_t &\equiv \left(P_{1t}^{1-\epsilon} + P_{2t}^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \\
C_{2t} &= \left(\frac{P_{2t}}{P_t}\right)^{-\varepsilon} C_t \\
\text{Therefore, } C_{it} &= \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} C_t
\end{aligned}$$

1. (b) (4 points) Intertemporal choice: derive the FONCs of households for C_t, N_t, B_t and the Lagrange multiplier.

ANS:

The representative household's Intertemporal choice problem:

$$\max_{C_t, N_t, B_t} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \right\}$$

$$s.t. P_t C_t + Q_t B_t = B_{t-1} + W_t N_t + T_t$$

The Lagrangian function is:

$$L = \max_{(C_t, N_t, B_t)_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \lambda_t \left(B_{t-1} + W_t N_t + T_t - P_t C_t - Q_t B_t\right) \right] \right\}$$

The FONCs:

$$\frac{\partial L}{\partial C_t} : 0 \stackrel{!}{=} C_t^{-\sigma} - \lambda_t P_t$$

$$\frac{\partial L}{\partial N_t} : 0 \stackrel{!}{=} -N_t^{\varphi} + \lambda_t W_t$$

$$\begin{aligned}\frac{\partial L}{\partial B_t} &: \quad 0 \stackrel{!}{=} \beta^t\left(-\lambda_t Q_t\right)+\beta^{t+1} E_t\left(\lambda_{t+1}\right) \\ &\Rightarrow \quad 0 \stackrel{!}{=} -\lambda_t Q_t+\beta E_t\left(\lambda_{t+1}\right)\end{aligned}$$

$$\frac{\partial L}{\partial \lambda_t}: 0 \stackrel{!}{=} B_{t-1}+W_t N_t+T_t-P_t C_t-Q_t B_t$$

3. (4 points) Find the first order necessary conditions (FONCs) for firms. Notes: 1)The labor market is competitive; 2) Due to the monopolistic power, firms seek to maximize their profits by setting optimal prices, which affect market demand(C_{it}), output(Y_{it}) and input(N_{it}) in turn; 3) Due to the symmetry of the two sectors, the equilibrium labor input is the same across sectors, i.e. $N_{1t}=N_{2t}$,and so is the price setting, i.e.,$P_{1t}=P_{2t}$; 4) The aggregate output is also a composite index, i.e. $Y_t \equiv \left(Y_{1t}^{1-\frac{1}{\epsilon}}+Y_{2t}^{1-\frac{1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$, while the labor is homogenous such that $N_t=N_{1t}+N_{2t}$.

   ANS:

   Each firm with some monopolistic power seeks to maximize their profits:

$$\begin{aligned}\max_{P_{it}} \Pi_{it} &= P_{it} Y_{it}-W_t N_{it} \\ \text{where } Y_{it} &= C_{it}=\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} C_t, \\ N_{it} &= \frac{Y_{it}}{A_t}=\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} \frac{C_t}{A_t} \\ \text{equivalently, } \max_{P_{it}} \Pi_{it} &= P_{it}\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} C_t-W_t\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} \frac{C_t}{A_t}\end{aligned}$$

   Taking FOC with respect to P_{it}:

$$\begin{aligned}\frac{\partial \Pi_{it}}{\partial P_{it}} &: \quad 0 \stackrel{!}{=} (1-\varepsilon)\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} C_t+\varepsilon W_t\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon-1} \frac{C_t}{P_t A_t} \\ &\Rightarrow \quad P_{it}=\frac{\varepsilon}{\varepsilon-1} \frac{W_t}{A_t}\end{aligned}$$

4. (3 points) Derive the aggregate price as a function of W_t and A_t according to above mentioned symmetry and the price index.

   ANS:

$$\begin{aligned}P_t &\equiv \left(P_{1t}^{1-\epsilon}+P_{2t}^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \\ P_{it} &= \frac{\varepsilon}{\varepsilon-1} \frac{W_t}{A_t} \\ P_t &\equiv \left(P_{1t}^{1-\epsilon}+P_{2t}^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}} \\ P_t &= 2^{\frac{1}{1-\epsilon}} \frac{\varepsilon}{\varepsilon-1} \frac{W_t}{A_t}\end{aligned}$$

5. (2 points) Discuss the role of ε in affecting the price setting and profit.

ANS:

$$\begin{aligned}
P_{it} &= \frac{\varepsilon}{\varepsilon-1}\frac{W_t}{A_t} = \left(1+\frac{1}{\varepsilon-1}\right)\frac{W_t}{A_t} \\
\frac{\partial P_{it}}{\partial \varepsilon} &= -\frac{1}{(\varepsilon-1)^2}\frac{W_t}{A_t} < 0 \\
&\Rightarrow \varepsilon\uparrow, P_{it}\downarrow
\end{aligned}$$

$$\begin{aligned}
P_t &= 2^{\frac{1}{1-\epsilon}}\frac{\varepsilon}{\varepsilon-1}\frac{W_t}{A_t} \\
\frac{\partial P_t}{\partial \varepsilon} &= 2^{\frac{1}{1-\epsilon}}\frac{\ln 2-1}{(\varepsilon-1)^2}\left(\frac{\varepsilon-\frac{1}{1-\ln 2}}{\varepsilon-1}\right)\frac{W_t}{A_t} \\
\text{since } \varepsilon &> 1, \\
\text{if } 1 &< \varepsilon < \frac{1}{1-\ln 2}, \frac{\partial P_t}{\partial \varepsilon} > 0, \varepsilon\uparrow, P_t\uparrow \\
\text{if } \varepsilon &> \frac{1}{1-\ln 2}, \frac{\partial P_t}{\partial \varepsilon} < 0, \varepsilon\uparrow, P_t\downarrow \\
\text{if } \varepsilon &= \frac{1}{1-\ln 2}, P_t \text{ is maximized with respect to } \varepsilon
\end{aligned}$$

$$\begin{aligned}
\Pi_{it} &= P_{it}Y_{it} - W_tN_{it} = \frac{1}{\varepsilon-1}2^{\frac{\varepsilon}{1-\varepsilon}}\frac{W_t}{A_t}C_t \\
\frac{\partial \Pi_{it}}{\partial \varepsilon} &= \frac{\ln 2+1-\varepsilon}{(\varepsilon-1)^3}2^{\frac{\varepsilon}{1-\varepsilon}}\frac{W_tC_t}{A_t} \\
\text{if } 1 &< \varepsilon < \ln 2+1, \frac{\partial \Pi_{it}}{\partial \varepsilon} > 0, \varepsilon\uparrow, \Pi_{it}\uparrow \\
\text{if } \varepsilon &> \ln 2+1, \frac{\partial \Pi_{it}}{\partial \varepsilon} < 0, \varepsilon\uparrow, \Pi_{it}\downarrow \\
\text{if } \varepsilon &= \ln 2+1, \Pi_{it} \text{ is maximized with respect to } \varepsilon
\end{aligned}$$

Generally speaking, when ε decreases, the profits increase.

6. (3 points) Write down all equations necessary to describe the equilibrium, including the FONCs, market clearing conditions (in each good, $Y_{it} = C_{it}$,and the aggregate level $Y_t = C_t$), the government policy rule, and a Fisherian equation.

ANS:

FONCs:

$$
\begin{aligned}
\frac{W_t}{P_t} &= N_t^{\varphi} C_t^{\sigma} \\
Q_t &= \beta E_t \left(\frac{P_t C_t^{\sigma}}{P_{t+1} C_{t+1}^{\sigma}} \right) \\
P_t C_t + Q_t B_t &= B_{t-1} + W_t N_t + T_t \\
a_t &= (1-\rho_a)\rho + \rho_a a_{t-1} + \varepsilon_t^a \\
a_t &= \log A_t, \rho = -\log(\beta) \\
Y_{it} &= A_t N_{it}, i = 1, 2.
\end{aligned}
$$

Market clearing conditions:

$$
\begin{aligned}
P_{it} &= \frac{\varepsilon}{\varepsilon - 1} \frac{W_t}{A_t}, i = 1, 2 \\
Y_{it} &= C_{it}, Y_t = C_t \\
N_{1t} &= N_{2t}, P_{1t} = P_{2t} \\
N_t &= N_{1t} + N_{2t} \\
Y_t &\equiv \left(Y_{1t}^{1-\frac{1}{\epsilon}} + Y_{2t}^{1-\frac{1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}
\end{aligned}
$$

Money policy:

$$
\begin{aligned}
i_t &= \rho + \phi_\pi \pi_t \\
\pi_t &\equiv \log P_t - \log P_{t-1}
\end{aligned}
$$

Fisher equation:

$$
i_t = r_t + E_t(\pi_{t+1})
$$

7. (2 points) In this economy, is monetary policy affecting the equilibrium of real variables?

   No.

8. (10 points) Compute the steady states of the variables given parameters $\sigma, \varphi, \beta, \varepsilon, \rho_a, \phi_\pi$, and assuming zero steady-state inflation $\overline{\pi} = 0$.Note: you can define real wage as $W^r = \frac{W}{P}$.

ANS:

$$\begin{aligned}
\frac{W}{P} &= \overline{N}^{\varphi}\overline{C}^{\sigma} \\
\overline{Q} &= \beta \\
\overline{P}_i &= \frac{\varepsilon}{\varepsilon-1}\frac{W}{A}, i = 1, 2 \\
\overline{Y}_i &= \overline{C}_i, \overline{Y} = \overline{C} \\
\overline{P} &= 2^{\frac{1}{1-\varepsilon}}\frac{\varepsilon}{\varepsilon-1}\frac{W}{A} \\
\overline{N}_1 &= \overline{N}_2, \overline{P}_1 = \overline{P}_2 \\
\overline{i} &= \overline{r} = \rho, \overline{\pi} = 0 \\
\overline{a} &= \log A = \rho = -\log(\beta), A = \frac{1}{\beta}
\end{aligned}$$

Solving the above equations, yielding

$$\begin{aligned}
\frac{W}{P} &= 2^{\frac{1}{\varepsilon-1}}\frac{\varepsilon-1}{\varepsilon}A \\
\overline{N} &= \left(2^{\frac{1-\sigma}{\varepsilon-1}}\frac{\varepsilon-1}{\varepsilon}A^{1-\sigma}\right)^{\frac{1}{\varphi+\sigma}}, \overline{N}_i = \frac{1}{2}\overline{N} \\
\overline{Y} &= 2^{\frac{1}{\varepsilon-1}}A\overline{N}, \overline{C}_i = \overline{Y}_i = A\overline{N}_i, \\
\overline{Y} &= \overline{C} = \left(\frac{W}{P}\overline{N}^{-\varphi}\right)^{\frac{1}{\sigma}} = \left(2^{\frac{1+\varphi}{\varepsilon-1}}\frac{\varepsilon-1}{\varepsilon}A^{1+\varphi}\right)^{\frac{1}{\varphi+\sigma}}
\end{aligned}$$

9. (3 points) Discuss the role of ε in affecting the steady states of variables. Other things being equal,what value should ε assume to achieve the best possible steady state consumption?

ANS:

$$\begin{aligned}
\ln\overline{C} &= \frac{1}{\varphi+\sigma}\left(\frac{1+\varphi}{\varepsilon-1}\ln 2 + \ln(\varepsilon-1) - \ln\varepsilon + (1+\varphi)\ln A\right) \\
\frac{\partial\ln\overline{C}}{\partial\varepsilon} &= \frac{1}{\varphi+\sigma}\left(-\frac{(1+\varphi)\ln 2}{(\varepsilon-1)^2} + \frac{1}{\varepsilon-1} - \frac{1}{\varepsilon}\right) \\
&= \frac{1}{\varphi+\sigma}\frac{[1-(1+\varphi)\ln 2]\left[\varepsilon - \frac{1}{1-(1+\varphi)\ln 2}\right]}{(\varepsilon-1)^2\varepsilon}
\end{aligned}$$

Therefore,

when $\varphi > \frac{1}{\ln 2} - 1, \frac{\partial\ln\overline{C}}{\partial\varepsilon} > 0, \Rightarrow \varepsilon\uparrow, \overline{C}\uparrow$

when $0 < \varphi < \frac{1}{\ln 2} - 1,$

if $\varepsilon > \frac{1}{1-(1+\varphi)\ln 2}, \frac{\partial\ln\overline{C}}{\partial\varepsilon} > 0, \Rightarrow \varepsilon\uparrow, \overline{C}\uparrow$

if $1 < \varepsilon < \frac{1}{1-(1+\varphi)\ln 2}, \frac{\partial\ln\overline{C}}{\partial\varepsilon} < 0, \Rightarrow \varepsilon\uparrow, \overline{C}\downarrow$

if $\varepsilon = \frac{1}{1-(1+\varphi)\ln 2}$,steady state consumption is optimal.

10. (3 points) Could the government design a policy to address the distortion induced by a finite ε? In this case, what kind of policy will work? Please explain it.

    Yes. subsidy. The government can use subsidy for firm to make the price setting under the complete competition, $P_{it} = \frac{W_t}{A_t}$. Under the condition, the distortion induced by a finite ε will disappear.

11. (30 points) Log-linearize the equations collected in Question 6. Define log-deviation of variable x_t as $\widehat{x}_t = \log(X_t/\overline{X})$.Write down all necessary log-linearized equations characterizing the equilibrium in the notation of log-deviation term $\widehat{x}_t$.

    ANS:

    FONCs:

$$\begin{aligned}
\widehat{w}_t - \widehat{p}_t &= \varphi \widehat{n}_t + \sigma \widehat{c}_t \\
\widehat{q}_t &= E_t \left(\widehat{p}_t + \sigma \widehat{c}_t - \widehat{p}_{t+1} - \sigma \widehat{c}_{t+1} \right) \\
\overline{PC} \left(\widehat{p}_t + \widehat{c}_t \right) + \overline{QB} \left(\widehat{q}_t + \widehat{b}_t \right) &= \overline{B} \widehat{b}_t + \overline{WN} \left(\widehat{w}_t + \widehat{n}_t \right) + \overline{T} \widehat{t}_t \\
\widehat{a}_t &= \rho_a \widehat{a}_{t-1} + \varepsilon_t^a \\
\widehat{y}_{it} &= \widehat{a}_t + \widehat{n}_{it}, i = 1, 2.
\end{aligned}$$

    Market clearing conditions:

$$\begin{aligned}
\widehat{p}_{it} &= \widehat{w}_t - \widehat{a}_t, i = 1, 2 \\
\widehat{y}_{it} &= \widehat{c}_{it}, \widehat{y}_t = \widehat{c}_t \\
\widehat{n}_{it} &= \widehat{n}_t, \widehat{p}_{it} = \widehat{p}_t, \widehat{c}_{it} = \widehat{c}_t
\end{aligned}$$

    Money policy:

$$\begin{aligned}
\text{Since } i_t &= \rho + \phi_\pi \pi_t, \overline{i} = \overline{r} = \rho, \\
\widehat{i}_t &= \phi_\pi \widehat{\pi}_t \\
\widehat{\pi}_t &= \pi_t = \widehat{p}_t - \widehat{p}_{t-1} \\
\text{Since } Q_t &= e^{-i_t}, \overline{Q} = \beta, \rho = -\log(\beta) \\
\widehat{q}_t &= -i_t - \log \beta = -i_t + \rho = -\phi_\pi \widehat{\pi}_t
\end{aligned}$$

    Fisher equation:

$$\widehat{i}_t = \widehat{r}_t + E_t \left(\widehat{\pi}_{t+1} \right)$$

12. (10 points) Simplify these equations, and to obtain the solution of $\widehat{y}_t$ and $\widehat{\pi}_t$ as functions of the exogenous shocks $\widehat{a}_t$.

    ANS:

$$\begin{aligned}
\widehat{a}_t &= \widehat{w}_t - \widehat{p}_t = \varphi \widehat{n}_t + \sigma \widehat{c}_t, \\
\widehat{c}_t &= \widehat{y}_t = \widehat{a}_t + \widehat{n}_t \\
\widehat{y}_t &= \frac{\varphi + 1}{\varphi + \sigma} \widehat{a}_t
\end{aligned}$$

For the inflation rate,

$$\begin{aligned}
\widehat{q}_t &= E_t\left(\widehat{p}_t + \sigma\widehat{c}_t - \widehat{p}_{t+1} - \sigma\widehat{c}_{t+1}\right) \\
\widehat{\pi}_t &= \widehat{p}_t - \widehat{p}_{t-1}, \widehat{q}_t = -\phi_\pi\widehat{\pi}_t \\
\phi_\pi\widehat{\pi}_t &= E_t\left(\frac{\sigma(\varphi+1)}{\varphi+\sigma}\left(\widehat{a}_{t+1} - \widehat{a}_t\right) + \widehat{\pi}_{t+1}\right)
\end{aligned}$$

Now guess the equation $\widehat{\pi}_{t+1} = \phi_{\pi a}\widehat{a}_{t+1}$,substitue into the above equation, we can get

$$\begin{aligned}
\phi_{\pi a} &= \frac{\sigma(\varphi+1)(\rho_a-1)}{(\varphi+\sigma)(\phi_\pi-\rho_a)} \\
\widehat{\pi}_t &= \frac{\sigma(\varphi+1)(\rho_a-1)}{(\varphi+\sigma)(\phi_\pi-\rho_a)}\widehat{a}_t
\end{aligned}$$

13. Parameter set values calibration.
    (a) Case 1: $\beta = 0.99, \sigma = \varphi = 1, \varepsilon = 2, \rho_a = 0.95, \phi_\pi = 1.5$.
    (b) Case 2: $\beta = 0.99, \sigma = \varphi = 1, \varepsilon = 4, \rho_a = 0.95, \phi_\pi = 1.5$.
    (c) Case 3: $\beta = 0.99, \sigma = \varphi = 1, \varepsilon = 2, \rho_a = 0.95, \phi_\pi = 3$.

14. (9 points) For each case of parameter set: calculate $\overline{Y}$;calculate the coefficients of the recursive equilibrium law of motion for $\widehat{y}_t$ and $\widehat{\pi}_t$.

    ANS:

$$\begin{aligned}
\overline{Y} &= \left(2^{\frac{1+\varphi}{\varepsilon-1}}\frac{\varepsilon-1}{\varepsilon}\left(\frac{1}{\beta}\right)^{1+\varphi}\right)^{\frac{1}{\varphi+\sigma}} \\
\phi_{ya} &= \frac{\varphi+1}{\varphi+\sigma}, \phi_{\pi a} = \frac{\sigma(\varphi+1)(\rho_a-1)}{(\varphi+\sigma)(\phi_\pi-\rho_a)}
\end{aligned}$$

	$\overline{Y}$	ϕ_{ya}	$\phi_{\pi a}$
case 1	1.4285	1	-0.0909
case 2	1.1021	1	-0.0909
case 3	1.4285	1	-0.0244

15. (4 points) Make comparision on Case 1 and 2, to which extent does ε affect the steady state output and why?

    when ε is doubled from 2 to 4, steady state output $\overline{Y}$ decrease from 1.4285 to 1.1021,dropping by 22.85%.

    Since $P_{it} = \frac{\varepsilon}{\varepsilon-1}\frac{W_t}{A_t}$, the mark-up for the monopolic power, $\frac{\varepsilon}{\varepsilon-1}$ decrease when the elasticity of goods ε gets larger, the the price drops which induce the firm to produce less.

16. (5 points) Make comparision on Case 1 and 3, to which extent does ϕ_π affect the variance of inflation and why?

    when ϕ_π is doubled from 1.5 to 3, the variance of inflation $\phi_{\pi a}^2$ drops from 83 basis points to 6 basis points(supposing $\sigma_a^2 = 1$). when ϕ_π is large, the government responds to the inflation largely, the monetary policy is very strict with the controlling of inflation, therefore, the variance of inflation is small.