Solution to P.S.3

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1. solution

- Assuming the individual's absolute risk aversion is increasing in wealth. As before, let r^h denote a realization of $\tilde{r}$ such that it exceeds r_f, and let W^h be the corresponding level of $\tilde{W}$. Then for $A \geq 0$, we have $W^h \geq W_0(1 + r_f)$.

- If the absolute risk aversion is increasing in wealth, this implies

$$R(W^h) \geq R(W_0(1 + r_f)) \tag{1}$$

where, as before, $R(W) = -U^{"}(W)/U^{'}(W)$. Multiplying both terms of (1) by $-U^{'}(W^h)(r^h - r_f)$, which is negative, then the inequality sign changes

$$U^{''}(W^h)(r^h - r_f) \leq -U^{'}(W^h)(r^h - r_f)R(W_0(1 + r_f)) \tag{2}$$

- similarly, let r^ldenote a realization of $\tilde{r}$ that is lower than r_f, and define W^l to be the corresponding level of $\tilde{W}$. By similar approach, one could easily get

$$U^{''}(W^l)(r^l - r_f) \leq -U^{'}(W^l)(r^l - r_f)R(W_0(1 + r_f)) \tag{3}$$

- Notice the inequalities (2) and (3) are of the same form, i.e. the inequality holds whether the realization is $\tilde{r} = r^h$ or $\tilde{r} = r^l$. There fore, if we take expectations over all realizations, we could obtain

$$E[U^{''}(\tilde{W})(\tilde{r} - r_f)] \leq -E[U^{'}(\tilde{W})(\tilde{r} - r_f)]R(W_0(1 + r_f)) \qquad (4)$$

- Since the first term on the right-hand-side is the first order condition, inequality (4) reduces to $E[U^{''}(\tilde{W})(\tilde{r} - r_f)] \leq 0$. Thus we have

$$\frac{dA}{dW_0} = \frac{(1 + r_f)E[U''(\tilde{W})(\tilde{r} - r_f)]}{-E[U''(\tilde{W})(\tilde{r} - r_f)^2]} \leq 0$$

$\square$

2. for the bonus question, one can refer to the textbook, Page 31 and 32, the method is almost the same as that in the textbook.