Advanced Macroeconomics II
Problem Set 7—Detrending and Calibration

ANS 1 *Imported the RBC data to matlab*

See the script.

ANS 2 *Take log of each original data and plot them in a graph like Figure 3.1 on page 33 of Dave & Dejong (2007).*

See Figure 1.

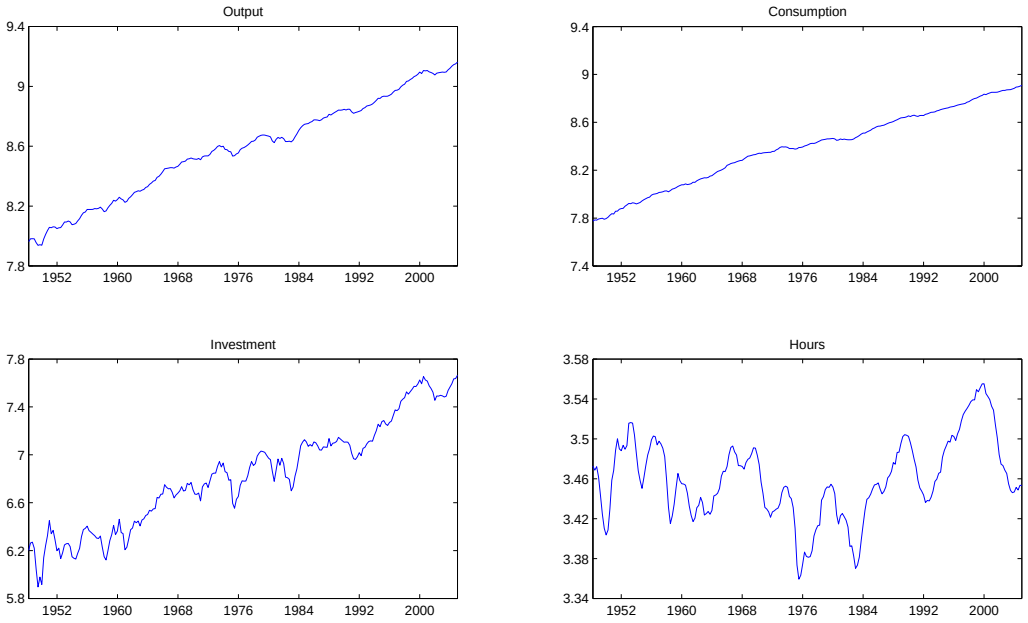

Figure 1: Logged Trajectories of Business Cycle Data

ANS 3 *Remove a linear trend*

By OLS, we can calculate that $\hat{\alpha}_0^1 = 7.9672$, $\hat{\alpha}_0^2 = 7.7748$, $\hat{\alpha}_0^3 = 6.2150$, $\hat{\alpha}_1^1 = \hat{\alpha}_1^2 = \hat{\alpha}_1^3 = 0.0054$. Note that the first entry was assume to take $t = 0$ (if the first entry corresponds to $t = 1$, then minus all the $\hat{\alpha}_0^j$ by $\hat{\alpha}_1$). See Figure 2.

ANS 4 *Remove trend by dierencing.*

Note that the trend is hardly distinguishable from the original data. By OLS, $\hat{\gamma} = 0.0056$. See Figure 3.

ANS 5 *Remove a linear trend*

See Figure 4.

ANS 6 *Comparing detrended investment by the above three methods.*

See Figure 5 and 6. The correlations are:

corr	Differencing	H-P filtered
Linear	0.11342	0.21061
Differencing		0.27697

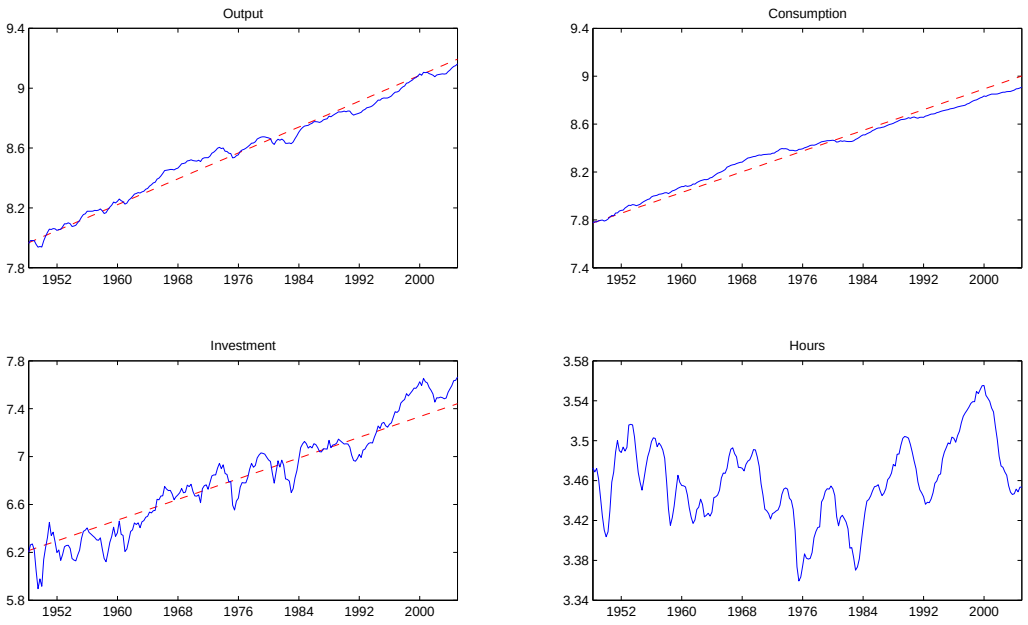

Figure 2: Logged Trajectories and a Common Linear Trend

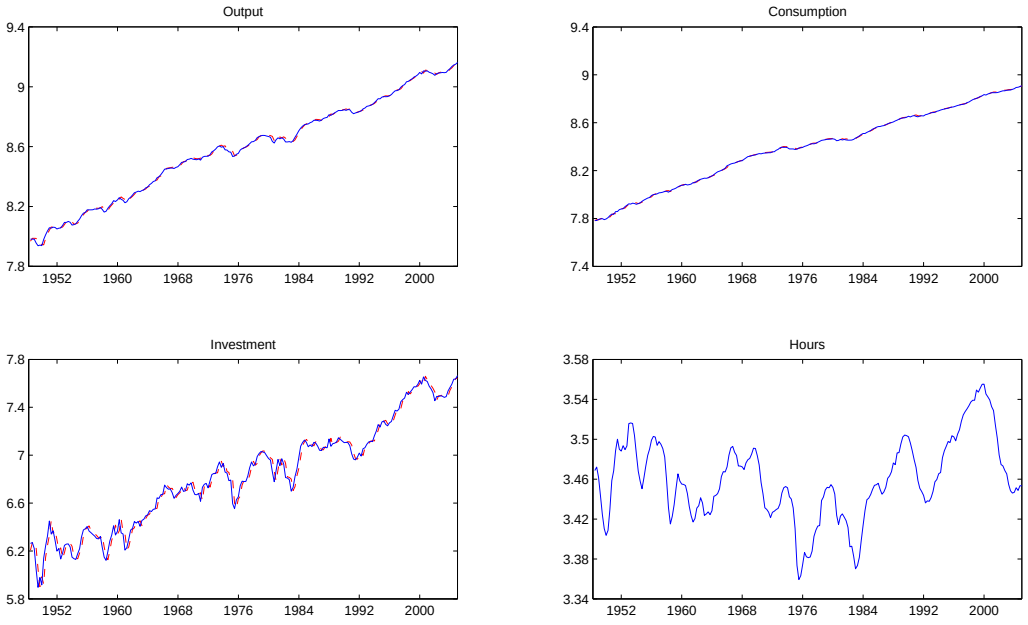

Figure 3: Logged Trajectories and a Common Growth Rate (Differencing Trends)

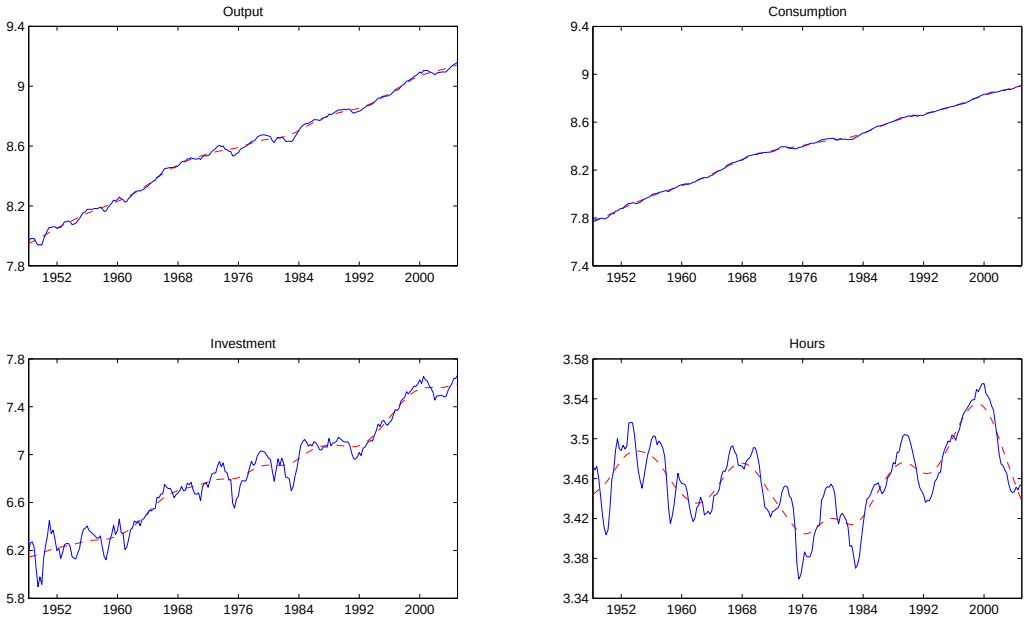

Figure 4: Logged Trajectories and H-P Trend

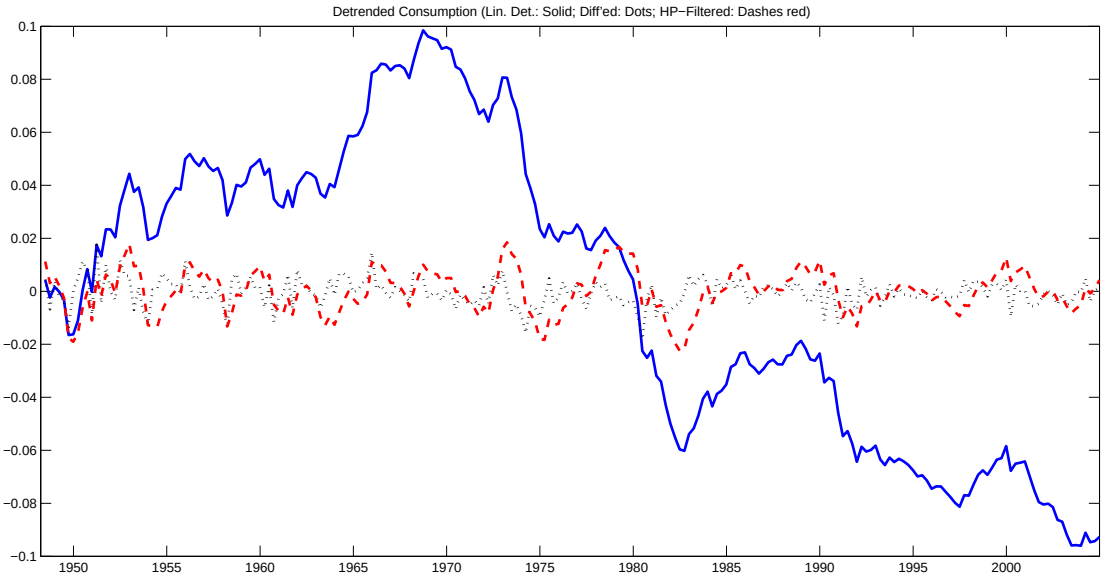

Figure 5: The detrended consumption by the three methods

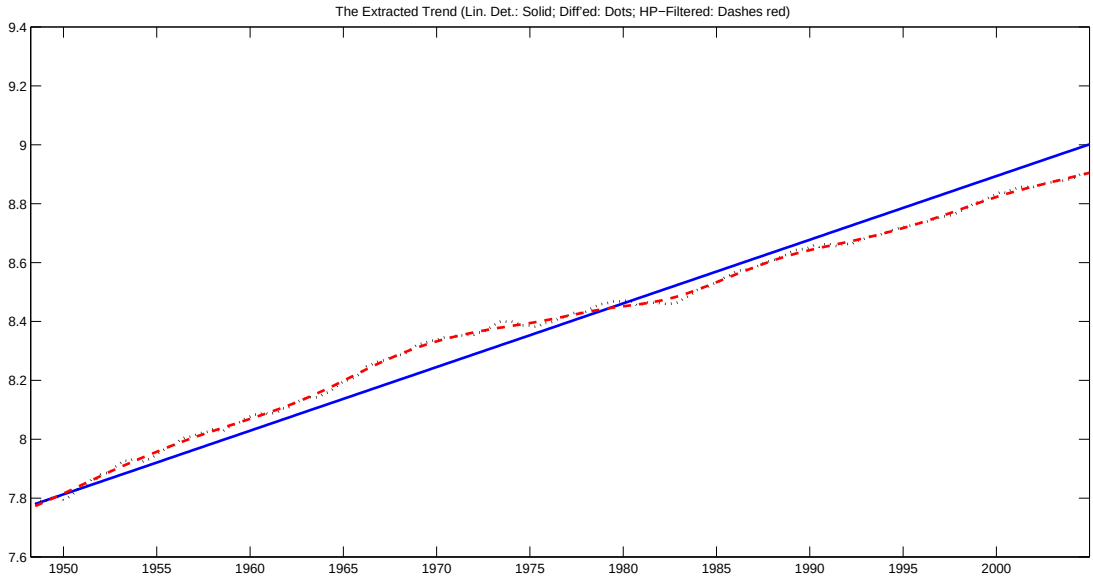

Figure 6: The extracted trend by the three methods

ps7.m

```
clear

%----------------------- ANS 1 --------------------------------
% data description --- http://www.econ.pitt.edu/dbook/code/ycih_readme.txt
% data downloaded from --- http://www.econ.pitt.edu/dbook/code/ycih.txt
% load the dataset into MATLAB
load ycih.txt;

% copy the four columns into four series respectively
% all variables are in per capita terms
Y = ycih(:,1); % output
C = ycih(:,2); % consumption
I = ycih(:,3); % investment
H = ycih(:,4); % hours

% generate the TIME vector
time = [1948.24:0.25:2005]';

%----------------------- ANS 2 --------------------------------
% taking logs of all series
y = log(Y);
c = log(C);
i = log(I);
h = log(H);

% Plot the Business Cycle Data
% Business Cycle Data in DeJong (2007), p. 33
figure(1)

subplot(2,2,1)
plot(time,y);
xlim([min(time) max(time)])
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```
ylim([7.8 9.4])
set(gca,'xtick',[1952:8:2000])
set(gca,'ytick',[7.8:0.4:9.4])
title('Output');

subplot(2,2,2)
plot(time,c);
xlim([min(time) max(time)])
ylim([7.4 9.4])
set(gca,'xtick',[1952:8:2000])
set(gca,'ytick',[7.4:0.4:9.4])
title('Consumption');

subplot(2,2,3)
plot(time,i);
xlim([min(time) max(time)])
ylim([5.8 7.8])
set(gca,'xtick',[1952:8:2000])
set(gca,'ytick',[5.8:0.4:7.8])
title('Investment');

subplot(2,2,4)
plot(time,h);
xlim([min(time) max(time)])
ylim([3.34 3.58])
set(gca,'xtick',[1952:8:2000])
set(gca,'ytick',[3.34:0.04:3.58])
title('Hours');

%----------------------- ANS 3 -------------------------------
% Remove the Trend
% Fit a linear model log y(t) = alpha_0 + alpha_1 * t + u (t)
% u(t) is covariance stationary stochastic process
% output, consumption and investment share a common trend
nnA = length(y);
TTA = [0:nnA-1]';
% explained variables
YYA = [y; c; i];
% explanatory variales XX - constant and time
% XX is 3nn by 4 matrix
XX1 = kron(eye(3), ones(nnA,1));
XX2 = kron(ones(3,1),TTA);
XXA = cat(2, XX1, XX2);
% BETA is 5 by 1 coefficient vector
BETAA = inv(XXA'*XXA)*XXA'*YYA;
YYAHAT = XXA*inv(XXA'*XXA)*XXA'*YYA;
UUA = YYA - YYAHAT;

% Show the fitted coefficients
disp('alpha_0^1 is '), disp(BETAA(1))
disp('alpha_0^2 is '), disp(BETAA(2))
disp('alpha_0^3 is '), disp(BETAA(3))
disp('alpha_1^1 = alpha_1^2 = alpha_1^3 is '), disp(BETAA(4))

% Obtained the detrended series
ydr = YYAHAT(1:nnA,:);
cdr = YYAHAT((nnA+1):(2*nnA),:);
```

```
idr = YYAHAT((2*nnA+1):(3*nnA),:);

% Plot the detrended Business Cycle Data
figure(2)

subplot(2,2,1)
plot(time,ydr,'--r');
hold on
plot(time,y);
hold off
xlim([min(time) max(time)])
ylim([7.8 9.4])
set(gca,'xtick',[1952:8:2000])
set(gca,'ytick',[7.8:0.4:9.4])
title('Output');

subplot(2,2,2)
plot(time,cdr,'--r');
hold on
plot(time,c);
hold off
xlim([min(time) max(time)])
ylim([7.4 9.4])
set(gca,'xtick',[1952:8:2000])
set(gca,'ytick',[7.4:0.4:9.4])
title('Consumption');

subplot(2,2,3)
plot(time,idr,'--r');
hold on
plot(time, i);
hold off
xlim([min(time) max(time)])
ylim([5.8 7.8])
set(gca,'xtick',[1952:8:2000])
set(gca,'ytick',[5.8:0.4:7.8])
title('Investment');

subplot(2,2,4)
plot(time, h);
xlim([min(time) max(time)])
ylim([3.34 3.58])
set(gca,'xtick',[1952:8:2000])
set(gca,'ytick',[3.34:0.04:3.58])
title('Hours');

%----------------------- ANS 4 -------------------------------
% Data are generated by
% log y(t) = log y(0) + epsilon (t)
% epsilon (t) = gamma + epsilon (t-1) + u(t)
% Doing differencing
ydif = diff(y);
cdif = diff(c);
idif = diff(i);

nnB = length(ydif);
YYB = [ydif; cdif; idif];
```

```
147 XXB = ones(3*nnB, 1);
148 BETAB = inv(XXB'*XXB)*XXB'*YYB;
149 YYBHAT = XXB*inv(XXB'*XXB)*XXB'*YYB;
150 UUB = YYB - YYBHAT;
151
152 disp('gamma is'), disp(BETAB)
153
154 ydif1 = y(1:end-1) + BETAB(1);
155 cdif1 = c(1:end-1) + BETAB(1);
156 idif1 = i(1:end-1) + BETAB(1);
157 % Estimate the common trend according to the
158 % restricted constraint
159 % Plot the differenced Business Cycle Data
160 figure(3)
161
162 subplot(2,2,1)
163 plot(time(2:end),ydif1,'--r');
164 hold on
165 plot(time(2:end),y(2:end));
166 hold off
167 xlim([min(time) max(time)])
168 ylim([7.8, 9.4])
169 set(gca,'xtick',[1952:8:2000])
170 set(gca,'ytick',[7.8:0.4:9.4])
171 title('Output');
172
173 subplot(2,2,2)
174 plot(time(2:end),cdif1,'--r');
175 hold on
176 plot(time(2:end),c(2:end));
177 hold off
178 xlim([min(time) max(time)])
179 ylim([7.4 9.4])
180 set(gca,'xtick',[1952:8:2000])
181 set(gca,'ytick',[7.4:0.4:9.4])
182 title('Consumption');
183
184 subplot(2,2,3)
185 plot(time(2:end),idif1,'--r');
186 hold on
187 plot(time(2:end), i(2:end));
188 hold off
189 xlim([min(time) max(time)])
190 ylim([5.8 7.8])
191 set(gca,'xtick',[1952:8:2000])
192 set(gca,'ytick',[5.8:0.4:7.8])
193 title('Investment');
194
195 subplot(2,2,4)
196 plot(time(2:end), h(2:end));
197 xlim([min(time) max(time)])
198 ylim([3.34 3.58])
199 set(gca,'xtick',[1952:8:2000])
200 set(gca,'ytick',[3.34:0.04:3.58])
201 title('Hours');
202
203 %---------------------- ANS 5 -------------------------------
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```
% Remove trend by H-P filter.
% [y_hpcycle; y_hptrend] = HP_filter_fun(y; lambda)
lambda = 1600; % setting lambda
[y_hpcycle, y_hptrend] = HP_filter_fun(y, lambda);
[c_hpcycle, c_hptrend] = HP_filter_fun(c, lambda);
[i_hpcycle, i_hptrend] = HP_filter_fun(i, lambda);
[h_hpcycle, h_hptrend] = HP_filter_fun(h, lambda);

% Plot the H-P trends
figure(4)

subplot(2,2,1)
%plot(time,y_hpcycle,'--r');
%hold on
plot(time, y_hptrend,'--r');
hold on
plot(time,y);
hold off
xlim([min(time) max(time)])
ylim([7.8, 9.4])
set(gca,'xtick',[1952:8:2000])
set(gca,'ytick',[7.8:0.4:9.4])
title('Output');

subplot(2,2,2)
%plot(time,c_hpcycle,'--r');
%hold on
plot(time,c_hptrend,'--r');
hold on
plot(time, c);
hold off
xlim([min(time) max(time)])
ylim([7.4 9.4])
set(gca,'xtick',[1952:8:2000])
set(gca,'ytick',[7.4:0.4:9.4])
title('Consumption');

subplot(2,2,3)
%plot(time,i_hpcycle,'--r');
%hold on
plot(time,i_hptrend,'--r');
hold on
plot(time, i);
hold off
xlim([min(time) max(time)])
ylim([5.8 7.8])
set(gca,'xtick',[1952:8:2000])
set(gca,'ytick',[5.8:0.4:7.8])
title('Investment');

subplot(2,2,4)
plot(time, h_hptrend,'--r');
hold on
plot(time, h)
hold off
xlim([min(time) max(time)])
ylim([3.34 3.58])
```

```
set(gca,'xtick',[1952:8:2000])
set(gca,'ytick',[3.34:0.04:3.58])
title('Hours');

 %---------------------- ANS 6 -------------------------------
 % Plot the cyclical components of consumption (logged consumption minus the
 % estimated trend) from the three methods
 % (1) time trend
 % (2) differencing
 % (3) H-P filter

 % Calculate the detrended consumption

 c_timedetrend = UUA(nnA+1:nnA*2);
 c_differenced = UUB(nnB+1:nnB*2);
 c_HP = c - c_hptrend;

 % Plot detrended investment
 figure(5)
 plot(time(2:end), c_timedetrend(2:end),'LineWidth',2);
 hold on
 plot(time(2:end), c_differenced,':k','LineWidth',2);
 hold on
 plot(time(2:end), c_HP(2:end),'--r','LineWidth',2);
 xlim([min(time), max(time)])
 hold off
 title('Detrended Consumption (Lin. Det.: Solid; Diff''ed: Dots; HP-Filtered: Dashes red)')

% Calculate the trends
 c_timetrend = BETAA(2)+BETAA(4)*TTA;
 c_difftrend = c(1:end-1) + BETAB;
 c_HPtread = c_hptrend;

 % Plot the trends extracted
 figure(6)
 plot(time(2:end), c_timetrend(2:end),'LineWidth',2);
 hold on
 plot(time(2:end), c_difftrend, ':k','LineWidth',2);
 hold on
 plot(time(2:end), c_HPtread(2:end),'--r', 'LineWidth',2 );
 xlim([min(time), max(time)])
 hold off
 title('The Extracted Trend (Lin. Det.: Solid; Diff''ed: Dots; HP-Filtered: Dashes red)')

 % Calculate the correlation
 corr1=corr(c_timedetrend(2:end), c_differenced);
 corr2=corr(c_differenced, c_HP(2:end));
 corr3=corr(c_timedetrend(2:end), c_HP(2:end));


display(char(['corr','         Differenced','         H-P filtered']))
display(char(['Linear', '          ',num2str(corr1), '             ',num2str(corr3)]))
display(char(['Differenced','                          ',num2str(corr2)]))
```