Zero-sum Games and Nash Equilibrium

- Let $v^* = \max_x \min_y u_1(x, y) = \min_y \max_x u_1(x, y)$.
- Since x^* is a maxminimizer for player 1, $u_1(x^*, y) \geq v^*$ for all $y \in A_2$.
  In particular, $u_1(x^*, y^*) \geq v^*$.
- From the lemma, $\max_y \min_x u_2(x, y) = -v^*$.
  y^* is a maxminimizer for player 2 $\Rightarrow u_2(x, y^*) \geq -v^*$ for all $x \in A_1$
  $\Rightarrow u_1(x, y^*) \leq v^*$ for all $x \in A_1$.
  In particular, $u_2(x^*, y^*) \geq -v^* \Rightarrow u_1(x^*, y^*) \leq v^*$.
- $v^* = u_1(x^*, y^*) \geq u_1(x, y^*)$ for all $x \in A_1$.
- Repeat for player 2.

Notes:

- A way to find Nash equilibria in strictly competitive games.
- Equilibria are interchangeable.
- $u_1(x^*, y^*)$ is the value of the game for player 1.