

Advanced Macroeconomics II  
Problem Set 4  
April 23, 2014

The due date for this assignment is Wednesday, April 30. It needs to be delivered by 2:30pm before the lecture starts. You can form a group of up to three persons. Remember to sign your name and student ID on the cover page of your group homework.

**Solve a model of a decentralized economy with monopolistic competition (100 points)**

**The representative household**

Consider the following model, where a representative household solves

$$\max_{C_t, N_t, B_t} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \right\} \quad (1)$$

s.t.

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \quad (2)$$

where  $\beta (0 < \beta < 1)$ ,  $\sigma$  and  $\varphi$  are parameters.  $B_t$  is the holding of a one-period bond,  $Q_t = e^{-i_t}$  is the bond price.  $W_t$  is wage for labor supply  $N_t$ , and  $T_t$  is possible transfer from firms owned by the households and/or government tax/subsidy.  $P_t$  is a price index for a composite consumption index  $C_t$ , which contains two types of consumption (say goods and services):  $C_{1t}$  and  $C_{2t}$ , and the composite index is defined as

$$C_t \equiv (C_{1t}^{\frac{\varepsilon-1}{\varepsilon}} + C_{2t}^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}.$$

**Two sectors of monopolistic firms**

There are two sectors for producing the two types of goods, and each sector is occupied by a monopolistic firm. Although the technology is the same, i.e.,

$$Y_{it} = A_t N_{it}, \quad i = 1, 2.$$

The two types of goods are differentiated and not perfectly substitutable for a finite  $\varepsilon$ . The firms seek to maximize profit and transfer it back to households owners. Both firms are subject to a productivity shock  $\epsilon_t$  in the technology process

$$a_t = (1 - \rho_a)\rho + \rho_a a_{t-1} + \epsilon_t^a, \quad \epsilon_t^a \overset{i.i.d.}{\sim} (0, \sigma_a^2)$$

where  $a_t = \log A_t$ ,  $0 < \rho_a < 1$ ,  $\rho = -\log(\beta)$ .

**Government**

The government has a central bank which reacts to inflation as follows

$$i_t = \rho + \phi_\pi \pi_t,$$

where  $\pi_t \equiv \log P_t - \log P_{t-1}$ .

## Questions:

1. (4 points) Describe the economy briefly. Comment on the preference, endowment, technology, and information.
2. Find the first order necessary conditions (FONCs) for households in two steps.

(a) (4 points) Within period: derive the demand for  $C_{it}$ . It amounts to maximizing the consumption basket given a certain expenditure  $Z_t$ .

$$\begin{aligned} \max_{(C_{1t}, C_{2t})} C_t &\equiv (C_{1t}^{\frac{\varepsilon-1}{\varepsilon}} + C_{2t}^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1, \\ \text{s.t.} \quad &P_{1t}C_{1t} + P_{2t}C_{2t} = Z_t \end{aligned}$$

You can define the price index as

$$P_t \equiv (P_{1t}^{1-\varepsilon} + P_{2t}^{1-\varepsilon})^{\frac{1}{1-\varepsilon}}.$$

(Note: if you cannot make the derivation, take the solution of demand function for later analysis, which is  $C_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} C_t$ .)

- (b) (4 points) Intertemporal choice: derive the FONCs of households for  $C_t$ ,  $N_t$ ,  $B_t$  and the Lagrange multiplier.
3. (4 points) Find the first order necessary conditions (FONCs) for firms. Notes: 1) The labor market is competitive; 2). Due to the monopolistic power, firms seek to maximize their profits by setting optimal prices, which affect market demand ( $C_{it}$ ), output ( $Y_{it}$ ) and input ( $N_{it}$ ) in turn; 3). Due to the symmetry of the two sectors, the equilibrium labor input is the same across sectors, i.e.  $N_{1t} = N_{2t}$ , and so is the price setting, i.e.,  $P_{1t} = P_{2t}$ ; 4) The aggregate output is also a composite index, i.e.  $Y_t \equiv (Y_{1t}^{\frac{\varepsilon-1}{\varepsilon}} + Y_{2t}^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}$ , while the labor is homogenous such that  $N_t = N_{1t} + N_{2t}$ .
4. (3 points) Derive the aggregate price as a function of  $W_t$  and  $A_t$  according to above mentioned symmetry and the price index.
5. (2 points) Discuss the role of  $\varepsilon$  in affecting the price setting and profit.
6. (3 points) Write down all equations necessary to describe the equilibrium, including the FONCs, market clearing conditions (in each good,  $Y_{it} = C_{it}$ , and the aggregate level  $Y_t = C_t$ ), the government policy rule, and a Fisherian equation.
7. (2 points) In this economy, is monetary policy affecting the equilibrium of real variables?
8. (10 points) Compute the steady states of the variables given parameters  $\sigma, \varphi, \beta, \varepsilon, \rho_a, \phi_\pi$ , and assuming zero steady-state inflation  $\bar{\pi} = 0$ . Note: you can define real wage as  $W^r = W/P$ .

9. (3 points) Discuss the role of  $\varepsilon$  in affecting the steady states of variables. Other things being equal, what value should  $\varepsilon$  assume to achieve the best possible steady state consumption?
10. (3 points) Could the government design a policy to address the distortion induced by a finite  $\varepsilon$ ? In this case, what kind of policy will work? Please explain it.
11. (30 points) Log-linearize the equations collected in Question 6. Define log-deviation of variable  $x_t$  as  $\hat{x}_t = \log(X_t/\bar{X})$ . Write down all necessary log-linearized equations characterizing the equilibrium in the notation of log-deviation term  $\hat{x}_t$ .
12. (10 points) Simplify these equations, and to obtain the solutions of  $\hat{y}_t$  and  $\hat{\pi}_t$  as functions of the exogenous shocks  $\hat{a}_t$ .
13. Parameter set values calibration.
  - (a) Case 1:  $\beta = 0.99, \sigma = \varphi = 1, \varepsilon = 2, \rho_a = 0.95, \phi_\pi = 1.5$ .
  - (b) Case 2:  $\beta = 0.99, \sigma = \varphi = 1, \varepsilon = 4, \rho_a = 0.95, \phi_\pi = 1.5$ .
  - (c) Case 3:  $\beta = 0.99, \sigma = \varphi = 1, \varepsilon = 2, \rho_a = 0.95, \phi_\pi = 3$ .
14. (9 points) For each case of parameter set: calculate  $\bar{Y}$ ; calculate the coefficients of the recursive equilibrium law of motion for  $\hat{y}_t$  and  $\hat{\pi}_t$ .
15. (4 points) Make comparison on Case 1 and 2, to which extent does  $\varepsilon$  affect the steady state output and why?
16. (5 points) Make comparison on Case 1 and 3, to which extent does  $\phi_\pi$  affect the variance of inflation and why?