Rationalizable Strategies - Equivalence

Lemma

The two definitions of rationalizable are equivalent.

- $(\Rightarrow)$ Set $Z_i = \{a_i\} \cup (\cup_{t=1}^{\infty} X_i^t)$ and $Z_j = (\cup_{t=1}^{\infty} X_j^t)$ for each $j \in N \backslash \{i\}$.
- $(\Leftarrow)$ Define $\mu_i^1 = \mu_i(a_i)$ and $\mu_j^t(a_j) = \mu_j(a_j)$ for each $j \in N$ and each integer $t \geq 2$.
  - Let $X_i^1 = \emptyset$ and for each $j \in N \backslash \{i\}$ let X_j^1 be the set of all $a_j' \in A_j$ such that there is some a_{-i} in the support of μ_i^1 for which $a_j = a_j'$
  - for each $t \geq 2$ and each $j \in N$ let X_j^t be the set of all $a_j' \in A_j$ such that there is some player $k \in N \backslash \{j\}$, some action $a_k \in X_k^{t-1}$, and some a_{-k} in the support of $\mu_k^t(a_k)$ for which $a_j' = a_j$.