Lecture Notes on

ADVANCED ECONOMETRICS

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Preface

Modern economies are full of uncertainties and risk. Economics studies resource allocations in an uncertain market environment. As a generally applicable quantitative analytic tool for uncertain events, probability and statistics have been playing an important role in economic research. Econometrics is statistical analysis of economic and financial data. It has become an integral part of training in modern economics and business. This book develops a coherent set of econometric theory and methods for economic models. It is written for an advanced econometrics course for doctoral students in economics, business, management, statistics, applied mathematics, and related fields. It can also be used as a reference book on econometric theory by scholars who may be interested in both theoretical and applied econometrics.

The book is organized in a coherent manner. Chapter 1 is a general introduction to econometrics. It first describes the two most important features of modern economics, namely mathematical modeling and empirical validation, and then discusses the role of econometrics as a methodology in empirical studies. A few motivating economic examples are given to illustrate how econometrics can be used in empirical studies. Finally, it points out the limitations of econometrics and economics due to the fact that an economy is not a repeatedly controlled experiment. Assumptions and careful interpretations are needed when conducting empirical studies in economics and finance.

Chapter 2 introduces a general regression analysis. Regression analysis is modeling, estimation, inference, and specification analysis of the conditional mean of economic variables of interest given a set of explanatory variables. It is most widely applied in economics. Among other things, this chapter interprets the mean squared error and its optimizer, which lays down the probability-theoretic foundation for least squares estimation. In particular, it provides an interpretation for the least squares estimator and its relationship with the true parameter value of a correctly specified regression model.

Chapter 3 introduces the classical linear regression analysis. A set of classical assumptions are given and discussed, and conventional statistical procedures for estimation, inference, and hypothesis testing are introduced. The roles of conditional homoskedasticity, serial uncorrelated, and normality of the disturbance of a linear regression model are analyzed in a finite sample econometric theory. We also discuss the generalized least squares estimation as an efficient estimation method of a linear regression model when the variance-covariance matrix is known up to a constant. In particular, the generalized least squares estimation is embedded as an ordinary least squares estimation of a suitably transformed regression model via conditional variance scaling and autocorrelation filtering.

The subsequent chapters 4–7 are the generalizations of classical linear regression analysis when various classical assumptions fail. Chapter 4 first relaxes the normality and conditional homoskedasticity assumptions, two key conditions assumed in classical linear regression modeling. A large sample theoretic approach is taken. For simplicity, it is assumed that the observed data are generated from an independent and identically distributed random sample. It is shown that while the finite distributional theory is no longer valid, the classical statistical procedures are still approximately applicable when the sample size is large, provided conditional homoskedasticity holds. In contrast, if the data display conditional heteroskedasticity, classical statistical procedures are not applicable even for large samples, and heteroskedasticity-robust procedures will be called for. Tests for existence of conditional heteroskedasticity in a linear regression framework are introduced.

Chapter 5 extends the linear regression theory to time series data. First, it introduces a variety of basic concepts in time series analysis. Then it shows that the large sample theory for i.i.d. random samples carries over to stationary ergodic time series data if the regression error follows a martingale difference sequence. We introduce tests for serial correlation, and tests for conditional heteroskedasticity and autoregressive conditional heteroskedasticity in a time series regression framework. We also discuss the impact of autoregressive conditional heteroskedasticity on inferences of static time series regressions and dynamic time series regressions.

Chapter 6 extends the large sample theory to a very general case where there exist conditional heteroskedasticity and autocorrelation. In this case, the classical regression theory cannot be used, and a long-run variance-covariance estimator is called for to validate statistical inferences in a time series regression framework.

Chapter 7 is the instrumental variable estimation for linear regression models, where the regression error is correlated with the regressors. This can arise due to measurement errors, simultaneous equation biases, and various other reasons. Two stage-least squares estimation method and related statistical inference procedures are fully exploited. We describe tests for endogeneity.

Chapter 8 introduces the generalized method of moments, which is a popular estimation method for possibly nonlinear econometric models characterized as a set of moment conditions. Indeed, most economic theories, such as rational expectations, can be formulated by a moment condition. The generalized method of moments is particularly suitable to estimate model parameters contained in the moment conditions for which the conditional distribution is usually not available.

Chapter 9 introduces the maximum likelihood estimation and the quasi-maximum likelihood estimation methods for conditional probability models and other nonlinear econometric models. We exploit the important implications of correctly specification of a conditional distribution model, especially the analogy between the martingale difference sequence property of the score function and serial uncorrelatedness, and the analogy between the conditional information equality and conditional homoskedasticity. These links can provide a great help in understanding the large sample properties of the maximum likelihood estimator and the quasi-maximum likelihood estimator.

Chapter 10 concludes the book by summarizing the main econometric theory and methods covered in this book, and pointing out directions for further build-up in econometrics.

This book has several important features. It covers, in a progressive manner, various econometrics models and related methods from conditional means to possibly nonlinear conditional moments to the entire conditional distributions, and this is achieved in a unified and coherent framework. We also provide a brief review of asymptotic analytic tools and show how they are used to develop the econometric theory in each chapter. By going through this book progressively, readers will learn how to do asymptotic analysis for econometric models. Such skills are useful not only for those students who intend to work on theoretical econometrics, but also for those who intend to work on applied subjects in economics because with such analytic skills, readers will be able to understand more specialized or more advanced econometrics textbooks.

This book is based on my lecture notes taught at Cornell University, Renmin University of China, Shandong University, Shanghai Jiao Tong University, Tsinghua University, and Xiamen University, where the graduate students provide detailed comments on my lecture notes.

CHAPTER 1 INTRODUCTION TO ECONOMETRICS

Key Words: Data generating process, Econometrics, Probability law, Quantitative analysis, Statistics.

Abstract: Econometrics has become an integral part of training in modern economics and business. Together with microeconomics and macroeconomics, econometrics has been taught as one of the three core courses in most undergraduate and graduate economic programs in North America. This chapter discusses the philosophy and methodology of econometrics in economic research, the roles and limitations of econometrics, and the differences between econometrics and mathematical economics as well as mathematical statistics. A variety of illustrative econometric examples are given, which cover various fields of economics and finance.

1.1 Introduction

Econometrics has become an integrated part of teaching and research in modern economics and business. The importance of econometrics has been increasingly recognized over the past several decades. In this chapter, we will discuss the philosophy and methodology of econometrics in economic research. First, we will discuss the quatitative feature of modern economics, and the differences between econometrics and mathematical economics as well as mathematical statistics. Then we will focus on the important roles of econometrics as a fundamental methodology in economic research via a variety of illustrative economic examples including the consumption function, marginal propensity to consume and multipliers, rational expectations models and dynamic asset pricing, the constant return to scale and regulations, evaluation of effects of economic reforms in a transitional economy, the efficient market hypothesis, modeling uncertainty and volatility, and duration analysis in labor economics and finance. These examples range from econometric analysis of the conditional mean to the conditional variance to the conditional distribution of economic variables of interest. we will also discuss the limitations of econometrics, due to the nonexperimental nature of economic data and the time-varying nature of econometric structures.

1.2 Quantitative Features of Modern Economics

Modern market economies are full of uncertainties and risk. When economic agents make a decision, the outcome is usually unknown in advance and economic agents will take this uncertainty into account in their decision-making. Modern economics is a study on scarce resource allocations in an uncertain market environment. Generally speaking, modern economics can be roughly classified into four categories: macroeconomics, microeconomics, financial economics, and econometrics. Of them, macroeconomics, microeconomics and econometrics now constitute the core courses for most economic doctoral programs in North America, while financial economics is now mainly being taught in business and management schools.

Most doctoral programs in economics in the U.S. emphasize quantitative analysis. Quantitative analysis consists of mathematical modeling and empirical studies. To understand the roles of quantitative analysis, it may be useful to first describe the general process of modern economic research. Like most natural science, the general methodology of modern economic research can be roughly summarized as follows:

- Step 1: Data collection and summary of empirical stylized facts. The so-called stylized facts are often summarized from observed economic data. For example, in microeconomics, a well-known stylized fact is the Engel's curve, which characterizes that the share of a consumer's expenditure on a commodity out of her or his total income will eventually decline as the income increases; in macroeconomics, a well-known stylized fact is the Phillips Curve, which characterizes a negative correlation between the inflation rate and the unemployment rate in an aggregate economy; and in finance, a well-known stylized fact about financial markets is volatility clustering, that is, a high volatility today tends to be followed by another high volatility tomorrow, a low volatility today tends to be followed by another low volatility tomorrow, and both alternate over time. The empirical stylized facts often serve as a starting point for economic research. For example, the development of unit root and cointegration econometrics was mainly motivated by the empirical study of Nelson and Plossor (1982) who found that most macroeconomic time series are unit root processes.

- Step 2: Development of economic theories/models. With the empirical stylized facts in mind, economists then develop an economic theory or model in order to explain them. This usually calls for specifying a mathematical model of economic theory. In fact, the objective of economic modelling is not merely to explain the stylized facts, but to understand the mechanism governing the economy and to forecast the future evolution of the economy.

- Step 3: Empirical verification of economic models. Economic theory only suggests a qualitative economic relationship. It does not offer any concrete functional form. In the process of transforming a mathematical model into a testable empirical econometric model, one often has to assume some functional form, up to some unknown model parameters. One needs to estimate unknown model parameters based on the observed data, and check whether the econometric model is adequate. An adequate model should be at least consistent with the empirical stylized facts.

- Step 4: Applications. After an econometric model passes the empirical evaluation, it can then be used to test economic theory or hypotheses, to forecast future evolution of the economy, and to make policy recommendations.

For an excellent example highlighting these four steps, see Gujarati (2006, Section 1.3) on labor force participation. We note that not every economist or every research paper has to complete these four steps. In fact, it is not uncommon that each economist may only work on research belonging to certain stage in his/her entire academic lifetime.

From the general methodology of economic research, we see that modern economics has two important features: one is mathematical modeling for economic theory, and the other is empirical analysis for economic phenomena. These two features arise from the effort of several generations of economists to make economics a "science". To be a science, any theory must fulfill two criteria: one is logical consistency and coherency in theory itself, and the other is consistency between theory and stylized facts. Mathematics and econometrics serve to help fulfill these two criteria respectively. This has been the main objective of the econometric society. The setup of the Nobel Memorial Prize in economics in 1969 may be viewed as the recognition of economics as a science in the academic profession.

1.3 Mathematical Modelling

We first discuss the role of mathematical modeling in economics. Why do we need mathematics and mathematical models in economics? It should be pointed out that there are many ways or tools (e.g., graphical methods, verbal discussions, mathematical models) to describe economic theory. Mathematics is just one of them. To ensure logical consistency of the theory, it is not necessary to use mathematics. Chinese medicine is an excellent example of science without using mathematical modeling. However, mathematics is well-known as the most rigorous logical language. Any theory, when it can be represented by the mathematical language, will ensure logical consistency and coherency of economic theory, thus indicating that it has achieved a rather sophisticated level. Indeed, as Karl Marx pointed out, the use of mathematics is an indication of the mature development of a science.

It has been a long history to use mathematics in economics. In his *Mathematical Principles of the Wealth Theory*, Cournot (1838) was among the earliest to use mathematics in economic analysis. Although the *marginal revolution*, which provides a cornerstone for modern economics, was not proposed using mathematics, it was quickly found in the economic profession that the marginal concepts, such as marginal utility, marginal productivity, and marginal cost, correspond to the derivative concepts in calculus. Walras (1874), a mathematical economist, heavily used mathematics to develop his general equilibrium theory. The game theory, which was proposed by Von Neumann and Morgenstern (1944) and now becomes a core in modern microeconomics, originated from a branch in mathematics.

Why does economics need mathematics? Briefly speaking, mathematics plays a number of important roles in economics. First, the mathematical language can summarize the essence of a theory in a very concise manner. For example, macroeconomics studies relationships between aggregate economic variables (e.g., GDP, consumption, unemployment, inflation, interest rate, exchange rate, etc.) A very important macroeconomic theory was proposed by Keynes (1936). The classical Keynesian theory can be summarized by two simple mathematical equations:

$$\begin{aligned} \text{National Income identity:} \qquad & Y = C + I + G, \\ \text{Consumption function:} \qquad & C = \alpha + \beta Y, \end{aligned}$$

where Y is income, C is consumption, I is private investment, G is government spending, α is the "survival level" consumption, and β is the marginal propensity to consume. Substituting the consumption function into the income identity, arranging terms, and taking a partial derivative, we can obtain the multiplier effect of (e.g.) government spending

$$\frac{\partial Y}{\partial G} = \frac{1}{1-\beta}.$$

Thus, the Keynesian theory can be effectively summarized by two mathematical equations.

Second, complicated logical analysis in economics can be greatly simplified by using mathematics. In introductory economics, economic analysis can be done by verbal descriptions or graphical methods. These methods are very intuitive and easy to grasp. One example is the partial equilibrium analysis where a market equilibrium can be characterized by the intersection of the demand curve and the supply curve. However, in many cases, economic analysis cannot be done easily by verbal languages or graphical methods. One example is the general equilibrium theory first proposed by Walras (1874). This theory addresses a fundamental problem in economics, namely whether the market force can achieve an equilibrium for a competitive market economy where there exist many markets and when there exist mutual interactions between different markets. Suppose there are n goods, with demand $D_i(P)$, supply $S_i(P)$ for good i, where $P = (P_1, P_2, ..., P_n)'$ is a price vector for n goods. Then the general equilibrium analysis addresses whether there exists an equilibrium price vector P^* such that all markets are clear simultaneously:

$$D_i(P^*) = S_i(P^*) \text{ for all } i \in \{1, ..., n\}.$$

Conceptually simple, it is rather challenging to give a definite answer because both the demand and supply functions could be highly nonlinear. Indeed, Walras was unable to establish this theory formally. It was satisfactorily solved by Arrow and Debreu many years later, when they used the fixed point theorem in mathematics to prove the existence of an equilibrium price vector. The power and magic of mathematics was clearly demonstrated in the development of the general equilibrium theory.

Third, mathematical modeling is a necessary path to empirical verification of an economic theory. Most economic and financial phenomena are in form of data (indeed we are in a digital era!). We need "digitalize" economic theory so as to link the economic theory to data. In particular, one needs to formulate economic theory into a testable mathematical model whose functional form or important structural model parameters will be estimated from observed data.

1.4 Empirical Validation

We now turn to discuss the second feature of modern economics: empirical analysis of an economic theory. Why is empirical analysis of an economic theory important? The use of mathematics, although it can ensure logical consistency of a theory itself, cannot ensure that economics is a science. An economic theory would be useless from a practical point of view if the underlying assumptions are incorrect or unrealistic. This is the case even if the mathematical treatment is free of errors and elegant. As pointed out earlier, to be a science, an economic theory must be consistent with reality. That is, it must be able to explain historical stylized facts and predict future economic phenomena.

How to check a theory or model empirically? Or how to validate an economic theory? In practice, it is rather difficult or even impossible to check whether the underlying assumptions of an economic theory or model are correct. Nevertheless, one can confront the implications of an economic theory with the observed data to check if they are consistent. In the early stage of economics, empirical validation was often conducted by case studies or indirect verifications. For example, in his well-known *Wealth of Nations*, Adam Smith (1776) explained the advantage of specialization using a case study example. Such a method is still useful nowadays, but is no longer sufficient for modern economic analysis, because economic phenomena are much more complicated while data may be limited. For rigorous empirical analysis, we need to use econometrics. Econometrics is the field of economics that concerns itself with the application of mathematical statistics and the tools of statistical inference to the empirical measurement of relationships postulated by economic theory. It was founded as a scientific discipline around 1930 as marked by the founding of the econometric society and the creation of the most influential economic journal—*Econometrica* in 1933.

Econometrics has witnessed a rather rapid development in the past several decades, for a number of reasons. First, there is a need for empirical verification of economic theory, and for forecasting using economic models. Second, there are more and more high-quality economic data available. Third, advance in computing technology has made the cost of computation cheaper and cheaper over time. The speed of computing grows faster than the speed of data accumulation.

Although not explicitly stated in most of the econometric literature, modern econometrics is essentially built upon the following fundamental axioms:

- Any economy can be viewed as a stochastic process governed by some probability law.
- Economic phenomenon, as often summarized in form of data, can be reviewed as a realization of this stochastic data generating process.

There is no way to verify these axioms. They are the philosophic views of econometricians toward an economy. Not every economist or even econometrician agrees with this view. For example, some economists view an economy as a deterministic chaotic process which can generate seemingly random numbers. However, most economists and econometricians (e.g., Granger and Terasvirta 1993, Lucas 1977) view that there are a lot of uncertainty in an economy, and they are best described by stochastic factors rather than deterministic systems. For instance, the multiplier-accelerator model of Samuelson (1939) is characterized by a deterministic second-order difference equation for aggregate output. Over a certain range of parameters, this equation produces deterministic cycles with a constant period of business cycles. Without doubt this model sheds deep insight into macroeconomic fluctuations. Nevertheless, a stochastic framework will provide a more realistic basis for analysis of periodicity in economics, because the observed periods of business cycles never occur evenly in any economy. Frisch (1933) demonstrates that a structural propagation mechanism can convert uncorrelated stochastic impulses into cyclical outputs with uneven, stochastic periodicity. Indeed, although not all uncertainties can be well characterized by probability theory, probability is the best quantitative analytic tool to describe uncertainties. The probability law of this stochastic economic system, which characterizes the evolution of the economy, can be viewed as the "law of economic motions." Accordingly, the tools and methods of mathematical statistics will provide the operating principles.

One important implication of the fundamental axioms is that one should not hope to determine precise, deterministic economic relationships, as do the models of demand, production, and aggregate consumption in standard micro- and macro-economic textbooks. No model could encompass the myriad essentially random aspects of economic life (i.e., no precise point forecast is possible, using a statistical terminology). Instead, one can only postulate some stochastic economic relationships. The purpose of econometrics is to infer the probability law of the economic system using observed data. Economic theory usually takes a form of imposing certain restrictions on the probability law. Thus, one can test economic theory or economic hypotheses by checking the validity of these restrictions.

It should be emphasized that the role of mathematics is different from the role of econometrics. The main task of mathematical economics is to express economic theory in the mathematical form of equations (or models) without regard to measurability or empirical verification of economic theory. Mathematics can check whether the reasoning process of an economic theory is correct and sometime can give surprising results and conclusions. However, it cannot check whether an economic theory can explain reality. To check whether a theory is consistent with reality, one needs econometrics. Econometrics is a fundamental methodology in the process of economic analysis. Like the development of a natural science, the development of economic theory is a process of refuting the existing theories which cannot explain newly arising empirical stylized facts and developing new theories which can explain them. Econometrics rather than mathematics plays a crucial role in this process. There is no absolutely correct and universally applicable economic theory. Any economic theory can only explain the reality at certain stage, and therefore, is a "relative truth" in the sense that it is consistent with historical data available at that time. An economic theory may not be rejected due to limited data information. It is possible that more than one economic theory or model coexist simultaneously, because data does not contain sufficient information to distinguish the true one (if any) from false ones. When new data become available, a theory that can explain the historical data well may not explain the new data well and thus will be refuted. In many cases, new econometric methods can lead to new discovery and call for new development of economic theory.

Econometrics is not simply an application of a general theory of mathematical statistics. Although mathematical statistics provides many of the operating tools used in econometrics, econometrics often needs special methods because of the unique nature of economic data, and the unique nature of economic problems at hand. One example is the generalized method of moment estimation (Hansen 1982), which was proposed by econometricians aiming to estimate rational expectations models which only impose certain conditional moment restrictions characterized by the Euler equation and the conditional distribution of economic processes is unknown (thus, the classical maximum likelihood estimation cannot be used). The development of unit root and cointegration (e.g., Engle and Granger 1987, Phillips 1987), which is a core in modern time series econometrics, has been mainly motivated from Nelson and Plosser's (1982) empirical documentation that most macroeconomic time series display unit root behaviors. Thus, it is necessary to provide an econometric theory for unit root and cointegrated systems because the standard statistical inference theory is no longer applicable. The emergence of financial econometrics is also due to the fact that financial time series display some unique features such as persistent volatility clustering, heavy tails, infrequent but large jumps, and serially uncorrelated but not independent asset returns. Financial applications, such as financial risk management, hedging and derivatives pricing, often call for modeling for volatilities and the entire conditional probability distributions of asset returns. The features of financial data and the objectives of financial applications make the use of standard time series analysis quite limited, and therefore, call for the development of financial econometrics. Labor economics is another example which shows how labor economics and econometrics have benefited from each other. Labor economics has advanced quickly over the last few decades because of availability of high-quality labor data and rigorous empirical verification of hypotheses and theories on labor economics. On the other hand, microeconometrics, particularly panel data econometrics, has also advanced quickly due to the increasing availability of microeconomic data and the need to develop econometric theory to accommondate the features of microeconomic data (e.g., censoring and endogeneity).

In the first issue of *Econometrica*, the founder of the econometric society, Fisher (1933), nicely summarizes the objective of the econometric society and main features of econometrics: "Its main object shall be to promote studies that aim at a unification of the theoretical-quantitative and the empirical-quantitative approach to economic problems and that are penetrated by constructive and rigorous thinking similar to that which has come to dominate the natural sciences.

But there are several aspects of the quantitative approach to economics, and no single one of these aspects taken by itself, should be confounded with econometrics. Thus, econometrics is by no means the same as economic statistics. Nor is it identical with what we call general economic theory, although a considerable portion of this theory has a definitely quantitative character. Nor should econometrics be taken as synonymous [sic] with the application of mathematics to economics. Experience has shown that each of these three viewpoints, that of statistics, economic theory, and mathematics, is a necessary, but not by itself a sufficient, condition for a real understanding of the quantitative relations in modern economic life. It is the unification of all three that is powerful. And it is this unification that constitutes econometrics."

1.5 Illustrative Examples

Specifically, econometrics can play the following roles in economics:

- Examine how well an economic theory can explain historical economic data (particularly the important stylized facts);
- Test validity of economic theories and economic hypotheses;
- Predict the future evolution of the economy.

To appreciate the roles of modern econometrics in economic analysis, we now discuss a number of illustrative econometric examples in various fields of economics and finance.

0.1 The Keynes Model, the Multiplier and Policy Recommendation

The simplest Keynes model can be described by the system of equations

$$\begin{cases} Y_t = C_t + I_t + G_t, \\ C_t = \alpha + \beta Y_t + \varepsilon_t, \end{cases}$$

where Y_t is aggregate income, C_t is private consumption, I_t is private investment, G_t is government spending, and ε_t is consumption shock. The parameters α and β can have appealing economic interpretations: α is survival level consumption, and β is the marginal propensity to consume. The multiplier of the income with respect to government spending is

$$\frac{\partial Y_t}{\partial G_t} = \frac{1}{1-\beta},$$

which depends on the marginal propensity to consume β.

To assess the effect of fiscal policies on the economy, it is important to know the magnitude of β. For example, suppose the Chinese government wants to maintain a steady growth rate (e.g., an annual 8%) for its economy by active fiscal policy. It has to figure out how many government bonds to be issued each year. Insufficient government spending will jeopardize the goal of achieving the desired growth rate, but excessive government spending will cause budget deficit in the long run. The Chinese government has to balance these conflicting effects and this crucially depends on the knowledge of the value of β. Economic theory can only suggest a positive qualitative relationship between income and consumption. It never tells exactly what β should be for a given economy. It is conceivable that β differs from country to country, because cultural factors may have impact on the consumption behavior of an economy. It is also conceivable that β will depend on the stage of economic development in an economy. Fortunately, econometrics offers a feasible way to estimate β from observed data. In fact, economic theory even does not suggest a specific functional form for the consumption function. The linear functional form for the consumption is assumed for convenience, not implied by economic theory. Econometrics can provide a consistent estimation procedure for the unknown consumption function. This is called the nonparametric method (see, e.g., Hardle 1990, Pagan and Ullah 1999).

0.2 Rational Expectations and Dynamic Asset Pricing Models

Suppose a representative agent has a constant relative risk aversion utility

$$U = \sum_{t=0}^{n} \beta^t u(C_t) = \sum_{t=0}^{n} \beta^t \frac{C_t^\gamma - 1}{\gamma},$$

where $\beta > 0$ is the agent's time discount factor, $\gamma \geq 0$ is the risk aversion parameter, $u(\cdot)$ is the agent's utility function in each time period, and C_t is consumption during period t. Let the information available to the agent at time t be represented by the σ-algebra I_t —in the sense that any variable whose value is known at time t is presumed to be I_t-measurable, and let $R_t = P_t/P_{t-1}$ be the gross return to an asset acquired at time $t-1$ at a price of P_{t-1}. The agent's optimization problem is to choose a sequence of consumptions $\{C_t\}$ over time to

$$\max_{\{C_t\}} E(U)$$

subject to the intertemporal budget constraint

$$C_t + P_t q_t \leq W_t + P_t q_{t-1},$$

where q_t is the quantity of the asset purchased at time t and W_t is the agent's period t income. Define the marginal rate of intertemporal substitution

$$\text{MRS}_{t+1}(\theta) = \frac{\frac{\partial u(C_{t+1})}{\partial C_{t+1}}}{\frac{\partial u(C_t)}{\partial C_t}} = \left(\frac{C_{t+1}}{C_t}\right)^{\gamma-1},$$

where model parameter vector $\theta = (\beta, \gamma)'$. Then the first order condition of the agent's optimization problem can be characterized by

$$E\left[\beta \text{MRS}_{t+1}(\theta) R_{t+1} | I_t\right] = 1.$$

That is, the marginal rate of intertemporal substitution discounts gross returns to unity. This FOC is usually called the Euler equation of the economic system (see Hansen and Singelton 1982 for more discussion).

How to estimate this model? How to test validity of a rational expectations model? Here, the traditional popular maximum likelihood estimation method cannot be used, because one does not know the conditional distribution of economic variables of interest. Nevertheless, econometricians have developed a consistent estimation method based on the conditional moment condition or the Euler equation, which does not require knowledge of the conditional distribution of the data generating process. This method is called the generalized method of moments (see Hansen 1982).

In the empirical literature, it was documented that the empirical estimates of risk aversion parameter γ are often too small to justify the substantial difference between the observed returns on stock markets and bond markets (e.g., Mehra and Prescott 1985). This is the well-known equity premium puzzle. To resolve this puzzle, effort has been devoted to the development of new economic models with time-varying, large risk aversion. An example is Campbell and Cochrance's (1999) consumption-based capital asset pricing model. This story confirms our earlier statement that econometric analysis calls for new economic theory after documenting the inadequacy of the existing model.

0.3 The Production Function and the Hypothesis on Constant Return to Scale

Suppose that for some industry, there are two inputs—labor L_i and capital stock K_i, and one output Y_i, where i is the index for firm i. The production function of firm i is a mapping from inputs (L_i, K_i) to output Y_i :

$$Y_i = \exp(\varepsilon_i) F(L_i, K_i),$$

where ε_i is a stochastic factor (e.g., the uncertain weather condition if Y_i is an agricultural product). An important economic hypothesis is that the production technology displays a constant return to scale (CRS), which is defined as follows:

$$\lambda F(L_i, K_i) = F(\lambda L_i, \lambda K_i) \text{ for all } \lambda > 0.$$

CRS is a necessary condition for the existence of a long-run equilibrium of a competitive market economy. If CRS does not hold for some industry, and the technology displays the increasing return to scale (IRS), the industry will lead to natural monopoly. Government regulation is then necessary to protect consumers' welfare. Therefore, testing CRS versus IRS has important policy implication, namely whether regulation is necessary.

A conventional approach to testing CRS is to assume that the production function is a Cob-Douglas function:

$$F(L_i, K_i) = A \exp(\varepsilon_i) L_i^{\alpha} K_i^{\beta}.$$

Then CRS becomes a mathematical restriction on parameters (α, β) :

$$\mathbf{H}_0 : \alpha + \beta = 1.$$

If $a + \beta > 1$, the production technology displays IRS.

In statistics, a popular procedure to test one-dimensional parameter restriction is Student's t-test. Unfortunately, this procedure is not suitable for many cross-sectional economic data, which usually display conditional heteroskedasticity (e.g., a larger firm has a larger output variation). One needs to use a robust, heteroskedasticity-consistent test procedure, originally proposed in White (1980).

It should be emphasized that CRS is equivalent to the statistical hypothesis $\mathbf{H}_0 : \alpha + \beta = 1$ under the assumption that the production technology is a Cob-Douglas function. This additional condition is not part of the CRS hypothesis and is called an auxiliary assumption. If the auxiliary assumption is incorrect, the statistical hypothesis $\mathbf{H}_0 : \alpha + \beta = 1$ will not be equivalent to CRS. Correct model specification is essential here for a valid conclusion and interpretation for the econometric inference.

0.4 Effect of Economic Reforms on a Transitional Economy

We now consider an extended Cob-Dauglas production function (after taking a logarithmic operation)

$$\ln Y_{it} = \ln A_{it} + \alpha \ln L_{it} + \beta \ln K_{it} + \gamma \text{Bonus}_{it} + \delta \text{Contract}_{it} + \varepsilon_{it},$$

where i is the index for firm $i \in \{1, ..., N\}$, and t is the index for year $t \in \{1, ..., T\}$, Bonus_{it} is the proportion of bonus out of total wage bill, and Contract_{it} is the proportion of workers who have signed a fixed-term contract. This is an example of the so-called panel data model (see, e.g., Hsiao 2003).

Paying bonus and signing fixed-term contracts were two innovative incentive reforms in the Chinese state-owned enterprises in the 1980s, compared to the fixed wage and life-time employment systems in the pre-reform era. Economic theory predicts that the introduction of the bonus and contract systems provides stronger incentives for workers to work harder, thus increasing the productivity of a firm (see Groves, Hong, McMillan and Naughton 1994).

To examine the effects of these incentive reforms, we consider the null statistical hypothesis

$$\mathbf{H}_0: \ \gamma = \delta = 0.$$

It appears that conventional t-tests or F-tests would serve our purpose here, if we can assume conditional homoskedasticity. Unfortunately, this cannot be used because there may well exist the other way of causation from Y_{it} to Bonus_{it} : a productive firm may pay its workers higher bonuses regardless of their efforts. This will cause correlation between the bonuses and the error term u_{it}, rendering the OLS estimator inconsistent and invalidating the conventional t-tests or F-tests. Fortunately, econometricians have developed an important estimation procedure called Instrumental Variables estimation, which can effectively filter out the impact of the causation from output to bonus and obtain a consistent estimator for the bonus parameter. Related hypothesis test procedures can be used to check whether bonus and contract reforms can increase firm productivity.

In evaluating the effect of economic reforms, we have turned an economic hypothesis—that introducing bonuses and contract systems has no effect on productivity—into a statistical hypothesis $\mathbf{H}_0 : \delta = \gamma = 0$. When the hypothesis $\mathbf{H}_0 : \delta = \gamma = 0$ is not rejected, we should not conclude that the reforms have no effect. This is because the extended production function model, where the reforms are specified additively, is only one of many ways to check the effect of the reforms. For example, one could also specify the model such that the reforms affect the marginal produtivities of labor and capital (i.e., the coefficients of labor and capital). Thus, when the hypothesis $\mathbf{H}_0 : \delta = \gamma = 0$ is not rejected, we can only say that we do not find evidence against the economic hypothesis that the reforms have no effect. We should not conclude that the reforms have no effect.

0.5 The Efficient Market Hypothesis and Predictability of Financial Returns

Let Y_t be the stock return in period t, and let $I_{t-1} = \{Y_{t-1}, Y_{t-2}, ...\}$ be the information set containing the history of past stock returns. The weak form of efficient market hypothesis (EMH) states that it is impossible to predict future stock returns using the history of past stock returns:

$$E(Y_t|I_{t-1}) = E(Y_t).$$

The LHS, the so-called conditional mean of Y_t given I_{t-1}, is the expected return that can be obtained when one is fully using the information available at time $t-1$. The RHS, the unconditional mean of Y_t, is the expected market average return in the long-run; it is the expected return of a buy-and-hold trading strategy. When EMH holds, the past information of stock returns has no predictive power for future stock returns. An important implication of EMH is that mutual fund managers will have no informational advantage over layman investors.

One simple way to test EMH is to consider the following autoregression

$$Y_t = \alpha_0 + \sum_{j=1}^{p} \alpha_j Y_{t-j} + \varepsilon_t,$$

where p is a pre-selected number of lags, and ε_t is a random disturbance. EMH implies

$$\mathbf{H}_0 : \alpha_1 = \alpha_2 = \cdots = \alpha_p = 0.$$

Any nonzero coefficient $\alpha_j, 1 \leq j \leq p$, is evidence against EMH. Thus, to test EMH, one can test whether the α_j are jointly zero. The classical F-test in a linear regression model can be used to test the hypothesis $\mathbf{H}_0$ when $\text{var}(\varepsilon_t|I_{t-1}) = \sigma^2$, i.e., when there exists conditional homoskedasticity. However, EMH may coexist with volatility clustering (i.e., $\text{var}(\varepsilon_t|I_{t-1})$ may be time-varying), which is one of the most important empirical stylized facts of financial markets (see Chen and Hong (2003) for more discussion). This implies that the standard F-test statistic cannot be used here, even asymptotically. Similarly, the popular Box and Pierce's (1970) portmanteau Q test, which is based on the sum of the first p squared sample autocorrelations, also cannot be used, because its asymptotic χ^2 distribution is invalid in presence of autoregressive conditional heteroskedasticity. One has to use procedures that are robust to conditional heteroskedasticity.

Like the discussion in Subsection 5.4, when one rejects the null hypothesis $\mathbf{H}_0$ that the α_j are jointly zero, we have evidence against EMH. Furthermore, the linear AR(p) model has predictive ability for asset returns. However, when one fails to reject the hypothesis $\mathbf{H}_0$ that the α_j are jointly zero, one can only conclude that we do not find evidence against EMH. One cannot conclude that EMH holds. The reason is, again, that the linear AR(p) model is one of many possibilities to check EMH (see, e.g., Hong and Lee 2005, for more discussion).

0.6 Volatility Clustering and ARCH Models

Since the 1970s, oil crisis, the floating foreign exchanges system, and the high interest rate policy in the U.S. have stimulated a lot of uncertainty in the world economy. Economic agents have to incorporate these uncertainties in their decision-making. How to measure uncertainty has become an important issue.

In economics, volatility is a key instrument for measuring uncertainty and risk in finance. This concept is important to investigate information flows and volatility spillover, financial contagions between financial markets, options pricing, and calculation of Value at Risk.

Volatility can be measured by the conditional variance of asset return Y_t given the information available at time $t-1$:

$$\sigma_t^2 \equiv var(Y_t|I_{t-1}) = E\left[(Y_t - E(Y_t|I_{t-1}))^2|I_{t-1}\right].$$

An example of the conditional variance is the AutoRegressive Conditional Heteroskedasticity (ARCH) model, originally proposed by Engle (1982). An ARCH(q) model assumes that

$$\begin{cases} Y_t = \mu_t + \varepsilon_t, \\ \varepsilon_t = \sigma_t z_t, \\ \mu_t = E(Y_t|I_{t-1}), \\ \sigma_t^2 = \alpha + \sum_{j=1}^q \beta_j \varepsilon_{t-j}^2, \qquad \alpha > 0, \beta > 0, \\ \{z_t\} \sim i.i.d.(0,1). \end{cases}$$

This model can explain a well-known stylized fact in financial markets—volatility clustering: a high volatility tends to be followed by another high volatility, and a small volatility tends to be followed by another small volatility. It can also explain the non-Gaussian heavy tail of asset returns. More sophisticated volatility models, such as Bollerslev's (1986) Generalized ARCH or GARCH model, have been developed in time series econometrics.

In practice, an important issue is how to estimate a volatility model. Here, the models for the conditional mean μ_t and the conditional variance σ_t^2 are assumed to be correctly specified, but the conditional distribution of Y_t is unknown, because the distribution of the standardized innovation $\{z_t\}$ is unknown. Thus, the popular maximum likelihood estimation (MLE) method cannot be used. Nevertheless, one can assume that $\{z_t\}$ is i.i.d.$N(0,1)$ or follows other plausible distribution. Under this assumption, we can obtain a conditional distribution of Y_t given I_{t-1} and estimate model parameters using the MLE procedure. Although $\{z_t\}$ is not necessarily i.i.d.$N(0,1)$ and we know this, the estimator obtained this way is still consistent for the true model parameters. However, the asymptotic variance of this estimator is larger than that of the MLE (i.e., when the true distribution of z_t is known), due to the effect of not knowing the true distribution of z_t. This method is called the quasi-MLE, or QMLE (see, e.g., White 1994). Inference procedures based on the QMLE are different from those based on the MLE. For example, the popular likelihood ratio test cannot be used. The difference comes from the fact that the asymptotic variance of the QMLE is different from that of the MLE, just like the fact that the asymptotic variance of the OLS estimator under conditional heteroskedasticity is different from that of the OLS under conditional homoskedasticity. Incorrect calculation of the asymptotic variance estimator for the QMLE will lead to misleading inference and conclusion (see White 1982, 1994 for more discussion).

0.7 Modeling Economic Durations

Suppose we are interested in the time it takes for an unemployed person to find a job, the time that elapses between two trades or two price changes, the length of a strike, the length before a cancer patient dies, and the length before a financial crisis (e.g., credit default risk) comes out. Such analysis is called duration analysis.

In practice, the main interest often lies in the question of how long a duration will continue, given that it has not finished yet. The so-called hazard rate measures the chance that the duration will end now, given that it has not ended before. This hazard rate therefore can be interpreted as the chance to find a job, to trade, to end a strike, etc.

Suppose T_i is the duration from a population with the probability density function $f(t)$ and probability distribution function $F(t)$. Then the survival function is

$$S(t) = P(T_i > t) = 1 - F(t),$$

and the hazard rate

$$\lambda(t) = \lim_{\delta \to 0^+} \frac{P(t < T_i \leq t + \delta | T_i > t)}{\delta} = \frac{f(t)}{S(t)}.$$

Intuitively, the hazard rate $\lambda(t)$ is the instantaneous probability that an event of interest will end at time t given that it has lasted for period t. Note that the specification of $\lambda(t)$ is equivalent to a specification of the probability density $f(t)$. But $\lambda(t)$ is more interpretable from an economic point of view.

The hazard rate may not be the same for all individuals. To control heterogeneity across individuals, we assume that the individual-specific hazard rate depends on some individual characteristics X_i via the form

$$\lambda_i(t) = \exp(X_i'\beta)\lambda(t).$$

This is called the proportional hazard model, originally proposed by Cox (1972). The parameter

$$\beta = \frac{\partial}{\partial X_i} \ln \lambda_i(t) = \frac{1}{\lambda_i(t)} \frac{\partial}{\partial X_i} \lambda_i(t)$$

can be interpreted as the marginal relative effect of X_i on the hazard rate of individual i. Inference of β will allow one to examine how individual characteristics affect the duration of interest. For example, suppose T_i is the unemployment duration for individual i, then the inference of β will allow us to examine how individual characteristics, such as age, education, gender, and etc, can affect the unemployment duration. This will provide important policy implication on labor markets.

Because one can obtain the conditional probability density function of Y_i given X_i

$$f_i(t) = \lambda_i(t)S_i(t),$$

where the survival function $S_i(t) = \exp[-\int_0^t \lambda_i(s)ds]$, we can estimate β by the maximum likelihood estimation method.

For an excellent survey on duration analysis in labor economics, see Kiefer (1988), and for a complete and detailed account, see Lancaster (1990). Duration analysis has been also widely used in credit risk modeling in the recent financial literature.

The above examples, although not exhaustive, illustrate how econometric models and tools can be used in economic analysis. As noted earlier, an economy can be completely characterized by the probability law governing the economy. In practice, which attributes (e.g., conditional moments) of the probability law should be used depends on the nature of the economic problem at hand. In other words, different economic problems will require modeling different attributes of the probability law and thus require different econometric models and methods. In particular, it is not necessary to specify a model for the entire conditional distribution function for all economic applications. This can be seen clearly from the above examples.

1.6 Limitations of Econometric Analysis

Although the general methodology of economic research is very similar to that of natural science, in general, economics and finance have not reached the mature stage that natural science (e.g., physics) has achieved. In particular, the prediction in economics and finance is not as precise as natural science (see, e.g., Granger 2001, for an assessment of macroeconomic forecasting practice).

Why?

Like any other statistical analysis, econometrics is the analysis of the "average behavior" of a large number of realizations, or the outcomes of a large number of random experiments with the same or similar features. However, economic data are not produced by a large number of repeated random experiments, due to the fact that an economy is not a controlled experiment. Most economic data are nonexperimental in their nature. This imposes some limitations on econometric analysis.

First, as a simplification of reality, economic theory or model can only capture the main or most important factors, but the observed data is the joint outcome of many factors together, and some of them are unknown and unaccounted for. These unknown factors are well present but their influences are ignored in economic modeling. This is unlike natural science, where one can remove secondary factors via controlled experiments. In the realm of economics, we are only passive observers; most data collected in economics are nonexperimental in that the data collecting agency may not have direct control over the data. The recently emerging field of experimental economics can help somehow, because it studies the behavior of economic agents under controlled experiments (see, e.g., Samuelson 2005). In other words, experimental economics controls the data generating process so that data is produced by the factors under study. Nevertheless, the scope of experimental economics is limited. One can hardly imagine how an economy with 1.3 billions of people can be experimented. For example, can we repeat the economic reforms in China and former Eastern European Socialist countries?

Second, an economy is an irreversible or non-repeatable system. A consequence of this is that data observed are a single realization of economic variables. For example, we consider the annual Chinese GDP growth rate $\{Y_t\}$ over the past several years:

$$\begin{array}{ccccccccc} Y_{1997} & Y_{1998} & Y_{1999} & Y_{2000} & Y_{2001} & Y_{2002} & Y_{2003} & Y_{2004} & Y_{2005} \\ 9.3\% & 7.8\% & 7.6\% & 8.4\% & 8.3\% & 9.1\% & 10.0\% & 10.1\% & 9.9\% \end{array}.$$

GDP growths in different years should be viewed as different random variables, and each variable Y_t only has one realization! There is no way to conduct statistical analysis if one random variable only has a single realization. As noted earlier, statistical analysis studies the "average" behavior of a large number of realizations from the same data generating process. To conduct statistical analysis of economic data, economists and econometricians often assume some time-invarying "common features" of an economic system so as to use time series data or cross-sectional data of different economic variables. These common features are usually termed as "stationarity" or "homogeneity" of the economic system. With these assumptions, one can consider that the observed data are generated from the same population or populations with similar characters. Economists and econometricians assume that the conditions needed to employ the tools of statistical inference hold, but this is rather difficult, if not impossible, to check in practice.

Third, economic relationships are often changing over time for an economy. Regime shifts and structural changes are rather a rule than an exception, due to technology shocks and changes in preferences, population structure and institution arrangements. An unstable economic relationship makes it difficult for out-of-sample forecasts and policy-making. With a structural break, an economic model that was performing well in the past may not forecast well in the future. Over the past several decades, econometricians have made some progress to copy with the time-varying feature of an economic system. Chow's (1960) test, for example, can be used to check whether there exist structural breaks. Engle's (1982) volatility model can be used to forecast time-varying volatility using historical asset returns. Nevertheless, the time-varying feature of an economic system always imposes a challenge for economic forecasts. This is quite different from natural sciences, where the structure and relationships are more or less stable over time.

Fourth, data quality. The success of any econometric study hinges on the quantity as well as the quality of data. However, economic data may be subject to various defects. The data may be badly measured or may correspond only vaguely to the economic variables defined in the model. Some of the economic variables may be inherently unmeasurable, and some relevant variables may be missing from the model. Moreover, sample selection bias will also cause a problem. In China, there may have been a tendency to over-report or estimate the GDP growth rates given the existing institutional promotion mechanism for local government officials. Of course, the advances in computer technology, the development of statistical sampling theory and practice can help improve the quality of economic data. For example, the use of scanning machines makes every transaction data available.

The above features of economic data and economic systems together unavoidably impose some limitations for econometrics to achieve the same mature stage as the natural science.

1.7 Conclusion

In this Chapter, we have discussed the philosophy and methodology of econometrics in economic research, and the differences between econometrics and mathematical economics and mathematical statistics. I first discussed two most important features of modern economics, namely mathematical modeling and empirical analysis. This is due to the effort of several generations of economists to make economics as a science. As the methodology for empirical analysis in economics, econometrics is an interdisciplinary field. It uses the insights from economic theory, uses statistics to develop methods, and uses computers to estimate models. I then discussed the roles of econometrics and its differences from mathematics, via a variety of illustrative examples in economics and finance. Finally, we pointed out some limitations of econometric analysis, due to the fact that any economy is not a controlled experiment. It should be emphasized that these limitations are not only the limitations of econometrics, but of economics as a whole.

References

Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroskedastcity", *Journal of Econometrics* 31, 307-327.

Box, G.E.P. and D.A. Pierce (1970), "Distribution of Residual Autorrelations in Autoregressive Moving Average Time Series Models," *Journal of the American Statistical Association* 65, 1509-1526.

Campbell, J.Y. and J. Cochrance (1999), "“By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior” *Journal of Political Economy* 107, 205-251.

Chen, D. and Y. Hong (2003), "Has Chinese Stock Market Become Efficient? Evidence from a New Approach," *China Economic Quarterly* 1 (2), 249-268.

Chow, G. C. (1960), "Tests of Equality Between Sets of Coefficients in Two Linear Regressions," *Econometrica* 28, 591-605.

Cournot, A. (1838), *Researches into the Mathematical Properties of the Theory of Wealth*, trans. Nathaniel T. Bacon, with an essay and an biography by Irving Fisher. McMillan: New York, 2nd edition, 1927.

Cox, D. R. (1972), "Regression Models and Life Tables (with Discussion)," *Journal of the Royal Statistical Society, Series B,* 34, 187-220,

Engle, R. (1982), "Autoregressive Conditional Hetersokedasticity with Estimates of the Variance of United Kingdom Inflation,” *Econometrica* 50, 987-2008.

Engle, R. and C.W.J. Granger (1987), "Cointegration and Error-Corretion Representation, Estimation and Testing,” *Econometrica* 55, 251-276.

Fisher, I. (1933), "Report of the Meeting," *Econometrica* 1, 92-93

Frisch, R. (1933), “Propagation Problems and Impulse Problems in Dynamic Economics.” In *Economic Essays in Honour of Gustav Cassel.* London: Allen and Unwin, 1933.

Granger, C.J.W. (2001), "Overview of Nonlinear Macroeconometric Empirical Models," *Journal of Macroeconomic Dynamics* 5, 466-481.

Granger, C.J.W. and T. Terasvirta (1993), *Modeling Nonlinear Economic Relationships*, Oxford University Press: Oxford.

Groves, T., Hong, Y., McMillan, J. and B. Naughton (1994), "Incentives in Chinese State-owned Enterprises," *Quarterly Journal of Economics* CIX, 183-209.

Gujarati, D.N. (2006), *Essentials of Econometrics*, 3rd Edition, McGraw-Hill: Boston.

Hansen, L.P. (1982), "Large Sample Properties of Generalized Method of Moments Estimators,” *Econometrica* 50, 1029-1054.

Hansen, L.P. and K. Singleton (1982), "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models," *Econometrica* 50, 1269-1286.

Hardle, W. (1990), *Applied Nonparametric Regression.* Cambridge University Press: Cambridge.

Hong, Y. and Y.J. Lee (2005), "Generalized Spectral Testing for Conditional Mean Models in Time Series with Conditional Heteroskedasticity of Unknown Form," *Review of Economic Studies* 72, 499-451.

Hsiao, C. (2003), *Panel Data Analysis, 2nd Edition*, Cambridge University Press: Cambridge.

Keynes, J. M. (1936), *The General Theory of Employment, Interest and Money*, McMillan Cambridge University Press: Cambridge, U.K.

Kiefer, N. (1988), "Economic Duration Data and Hazard Functions," *Journal of Economic Literature* 26, 646-679.

Lancaster, T. (1990), *The Econometric Analysis of Transition Data*, Cambridge University Press: Cambridge, U.K.

Lucas, R. (1977), "Understanding Business Cycles," in *Stabilization of the Domestic and International Economy*, Karl Brunner and Allan Meltzer (eds.), Carnegie-Rochester Conference Series on Public Policy, Vol. 5. North-Holland: Amsterdam.

Mehra, R. and E. Prescott (1985), "The Equity Premium: A Puzzle," *Journal of Monetary Economics* 15, 145-161.

Nelson, C.R. and C. I. Plosser (1982), "Trends and Random Walks in Macroeconomic Time Series: Some Evidence and Implications," *Journal of Monetary Economics* 10, 139-162.

Pagan, A. and A. Ullah (1999), *Nonparametric Econometrics*, Cambridge University Press: Cambridge.

Phillips, P.C. (1987), "Time Series Regression with a Unit Root," *Econometrica* 55, 277-301.

Samuelson, L. (2005), "Economic Theory and Experimental Economics," *Journal of Economic Literature* XLIII, 65-107.

Samuelson, P. (1939), "Interactions Between the Multiplier Analysis and the Principle of Acceleration," *Review of Economic Studies* 21, 75-78.

Smith, A. (1776), *An Inquiry into the Nature and Causes of the Wealth of Nations*, edited, with an Introduction, Notes, Marginal Summary and an Enlarged Index, by Edwin Cannan; with an Introduction by Max Lerner. New York :The Modern library, 1937.

Von Neumann, J. and O. Morgenstern (1944), *Theory of Games and Economic Behavior*, Princeton University Press: Princeton.

Walras, L. (1874), *Elements of Pure Economics, or, The Theory of Social Wealth,* translated by William Jaffe. Fairfield, PA; Kelley,1977.

White, H. (1980), "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heterokedasticity," *Econometrica* 48, 817-838.

White, H. (1982), "Maximum Likelihood Estimation of Misspecified Models," *Econometrica* 50, 1-26.

White, H. (1994), *Estimation, Inference and Specification Analysis.* Cambridge University Press: Cambridge.

EXERCISES

1.1. Discuss the differences of the roles of mathematics and econometrics in economic research.

1.2. What are the fundamental axioms of econometrics? Discuss their roles and implications.

1.3. What are the limitations of econometric analysis? Discuss possible ways to alleviate the impact of these limits.

1.4. How do you perceive the roles of econometrics in decision-making in economics and business?