Advanced Macroeconomics II
Gali (2008), Chapter 5

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Chapter 5. Monetary Policy Tradeoffs: Discretion versus Commitment

In Chapter 4, we discussed optimal monetary policy under the condition that:

- The staggered price setting was the only relevant distortion;
- Distortion from monopolistic competition was corrected by a wage subsidy;
- Policy that seeks to replicate the flexible price equilibrium allocation is both feasible and optimal.

Monetary Policy Tradeoffs: Discretion versus Commitment

Under the conditions in Chapter 4,

- The efficient allocation can be obtained by means of a policy that fully stabilizes the price level.
- Thus, the zero inflation output equals its natural level, which is also the efficient level.
- There is no tradeoff between stabilizing output gap and inflation, and "**strict inflation targeting**" emerges as the optimal policy.

Monetary Policy Tradeoffs: Discretion versus Commitment

Under more realistic environment and flexible inflation targeting

In reality, there could be real imperfections other than staggered price.

- Central banks may face short run tradeoffs between stabilizing inflation and real variables like output and employment.
- The existence of such a tradeoff makes it desirable for the central bank to be able to commit to a state-contingent policy plan

5.1 The Monetary Policy Problem: The Case of an Efficient Steady State

When nominal regidities coexist with real imperfections, the flexible price equilibrium allocation is generally inefficient.
A special case of interest arises when the possible inefficiencies associated with the flexible price equilibrium do not affect the steady state, which remains efficient.

$$\bar{y}^n = \bar{y} = \bar{y}^e$$

But short run deviations may exist between the natural and efficient levels of output, even under flexible price.

$$y_t^n \neq y_t^e$$

In general, $y_t \neq y_t^n \neq y_t^e$.

The Monetary Policy Problem: The Case of an Efficient Steady State

As shown in appendix 5.1, the welfare losses are, up to a second-order approximation, proportional to

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \alpha_x x_t^2 \right) \right\} \tag{1}$$

Definition

Welfare-relevant output gap is defined as

$$x_t \equiv y_t - y_t^e$$

i.e., the deviation between (log) output y_t and its efficient level y_t^e.

- $\pi_t \equiv p_t - p_{t-1}$ denotes the rate of inflation between periods $t-1$ and t.
- $\alpha_x = \frac{\kappa}{\varepsilon}$, the weight of output gap fluctuations (relative to inflation) in the loss function.

The Monetary Policy Problem: The Case of an Efficient Steady State

$$\begin{aligned} \tilde{y}_t &\equiv y_t - y_t^n \\ &= x_t + (y_t^e - y_t^n) \end{aligned}$$

Substituting the output gap $\tilde{y}_t$ in the NKPC relationship yields

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa x_t + u_t \tag{2}$$

where $u_t \equiv \kappa(y_t^e - y_t^n)$.
Hence, the central bank will seek to minimize (1) subject to the sequence of constraints given by (2).

The Monetary Policy Problem: The Case of an Efficient Steady State

- The disturbance u_t is exogenous with respect to monetary policy.
  - The central bank will take the current and anticipated values of u_t as given when solving its policy problem.
- Time variations in the gap between the efficient and natural levels of output $(y_t^e - y_t^n)$ – reflected in fluctuations in u_t – generate a tradeoff for the monetary authority.
  - u_t makes it impossible to attain simultaneously zero inflation and an efficient level of activity.
  - Sources of the *cost-push shock* u_t – exogenous changes in desired price or wage markups, or fluctuations in labor income taxes.

The Monetary Policy Problem: The Case of an Efficient Steady State

- Assume an exogenous AR(1) process of u_t as

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u \tag{3}$$

where $\rho_u \in [0, 1)$, and $\{\varepsilon_t^u\}$ is a white-noise process with constant variance σ_u^2.

- The NKPC Equation (2) is the only constraint needed in order to determine the equilibrium path for output and inflation under the optimal policy.
- The DIS equation is needed to implement the policy

$$x_t = -\frac{1}{\sigma}\left(\hat{\imath}_t - E_t\left\{\hat{\pi}_{t+1}\right\} - r_t^e\right) + E_t\left\{x_{t+1}\right\} \tag{4}$$

where $r_t^e \equiv \rho + \sigma E_t\left\{\Delta y_{t+1}^e\right\}$ is the *efficient interest rate* that supports the efficient allocation.

The Monetary Policy Problem: The Case of an Efficient Steady State

- The forward-looking nature of constraint (2) in the policy problem requires the specification of the extent to which the central bank can credibly commit in advance to future policy actions.

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa x_t + u_t$$

- Two alternatives: Commitment $\succ$ Discretion.

5.1.1 Optimal Discretionary Policy

Optimal policy under discretion

Sequential optimization. The central bank makes whatever decision is optimal each period without committing itself to any future actions. In each period, the central bank's problem:

$$\min_{(x_t,\ \pi_t)} \pi_t^2 + \alpha_x x_t^2$$

s.t.

$$\pi_t = \kappa x_t + v_t$$

where the term $v_t \equiv \beta E_t \{\pi_{t+1}\} + u_t$,

- u_t is exogenous
- $E_t \{\pi_{t+1}\}$ is a function of expectations about future output gaps. Both independent to the current policy.

Optimal Discretionary Policy

The optimality condition.

$$L : \pi_t^2 + \alpha_x x_t^2 - \lambda \left(\pi_t - \kappa x_t - v_t \right)$$

$$\frac{\partial L}{\partial \pi_t} \quad : \quad 2\pi_t = \lambda \tag{5}$$

$$\frac{\partial L}{\partial x_t} \quad : \quad 2\alpha_x x_t = -\lambda\kappa$$

$$\Longrightarrow \quad x_t = -\frac{\kappa}{\alpha_x}\pi_t \tag{6}$$

for $t = 0, 1, 2, \ldots$

- This is a relation between target variables that the discretionary central bank will seek to maintain at all times and it is in that sense that it may be labeled a "**targeting rule**."
- With the objective of dampening the rise in inflation due to cost-push shock, the central bank must respond by driving output below its efficient level, thus creating a negative output gap. – **Policy**

Optimal Discretionary Policy

Using (6) to substitute for x_t in (2) yields the following difference equation for inflation

$$\pi_t = \frac{\alpha_x \beta}{\alpha_x + \kappa^2} E_t \{\pi_{t+1}\} + \frac{\alpha_x}{\alpha_x + \kappa} u_t .$$

Iterating forward,

$$\pi_t = \alpha_x \Psi u_t \tag{7}$$

where $\Psi \equiv \frac{1}{\kappa^2 + \alpha_x (1 - \beta \rho_u)}$.
Combining (6) and (7),

$$x_t = -\kappa \Psi u_t . \tag{8}$$

Optimal Discretionary Policy

The central bank lets the output gap and inflation deviate from their targets in proportion to the current value of the cost-push shock.

$$\begin{aligned} \pi_t &= \alpha_x \Psi u_t \\ x_t &= -\kappa \Psi u_t \end{aligned}$$

- Figure 5.1, purely transitory ($\rho_u = 0$) cost-push shock.
- Figure 5.2, positively autocorrelated ($\rho_u = 0.5$) cost-push shock.

Optimal Discretionary Policy

- The central bank finds it optimal to accommodate partly the inflationary pressures resulting from the cost-push shock by letting inflation rise.
- However, that the increase in inflation is smaller than the increase that would be obtained if the output gap remained unchanged, in which case
$$\pi_t = \frac{1}{1 - \beta\rho_u} u_t$$
- The impact on inflation is dampened by the negative response of the output gap.
- The implied response of inflation leads naturally to a permanent change in the price level.

Optimal Discretionary Policy: Implementation

- Analysis shows that for optimal discretionary policy, the following condition should hold:
$$x_t = -\frac{\kappa}{\alpha_x}\pi_t$$
- However, the central bank cannot set either variable. A policy rule is needed to guarantee the desired outcome.
- Before deriving the rule, let's first determine the equilibrium interest rate under the optimal policy as a funtion of u_t.

Optimal Discretionary Policy: Implementation

The resulting equilibrium interest rate

Combining the following equations:

$$\pi_t = \alpha_x \Psi u_t$$

$$x_t = -\kappa \Psi u_t. \tag{9}$$

$$x_t = -\frac{1}{\sigma}\left(\hat{\imath}_t - E_t\left\{\hat{\pi}_{t+1}\right\} - r_t^e\right) + E_t\left\{x_{t+1}\right\}$$

yields

$$\hat{\imath}_t = r_t^e + \Psi_i u_t \tag{10}$$

where $\Psi_i \equiv \Psi\left[\kappa\sigma(1-\rho_u) + \alpha_x \rho_u\right]$.
Again, this should be the result of optimal M.P., not the optimal M.P. per se, as discussed in Chapter 3.

Optimal Discretionary Policy: Implementation

An inflation targeting rule

Consider an inflation-targeting rule

$$i_t = r_t^e + \phi_\pi \pi_t \tag{11}$$

where $\phi_\pi \equiv (1-\rho_u)\frac{\kappa\sigma}{\alpha_x} + \rho_u$, which can be obtained by combining (7) and (10).

Using the arguments of Chapter 4, a determinate equilibrium exists if and only if

$$\phi_\pi > 1,$$

equivalently,

$$\kappa\sigma > \alpha_x,$$

which may not be satisfied.

Optimal Discretionary Policy: Implementation

An inflation targeting rule ensuring unique determinacy

To derive a rule that guarantees equilibrium uniqueness independent of parameter values, appending to the equilibrium nominal rate in Equation (10) a proportion to inflation deviation from its equilibrium value under the policy,

$$
\begin{aligned}
i_t &= r_t^e + \Psi_i u_t + \phi_\pi(\pi_t - \alpha_x \Psi u_t) \\
&= r_t^e + \Theta_i u_t + \phi_\pi \pi_t
\end{aligned} \tag{12}
$$

where $\Theta_i \equiv \Psi\left[\kappa\sigma(1-\rho_u) - \alpha_x(\phi_\pi - \rho_u)\right]$ for an arbitrary inflation coefficient satisfying $\phi_\pi > 1$.

Optimal Discretionary Policy: Implementation

Targeting rules vs. instrument rules

- The feasibility of implementing rules like (11) and (12) is questionable, as discussed in Chapter 4.
- Hence "targeting rules" like (6) is regarded as more practical guides than "instrument rules" like (11) and (12).
- Under the targeting rule (6), the central bank would adjust its instrument until a certain optimal relation between target variables is satisfied, in this case

$$x_t = -\frac{\kappa}{\alpha_x}\pi_t.$$

- However, such a targeting rule requires the knowledge of $y_t^e \rightarrow x_t$, which is also infeasible.

Optimal Policy under Commitment

Assume the central bank is able to commit, with full credibility, to a *policy plan*. That is,

$$\min_{\{x_t,\ \pi_t\}_{t=0}^{\infty}} \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \alpha_x x_t^2)$$

subject to the sequence of constraints

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa x_t + u_t$$

where $\{u_t\}$ follows the exogenous process $u_t = \rho_u u_{t-1} + \varepsilon_t^u$.

Optimal Policy under Commitment

The Lagrangian equation

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} (\pi_t^2 + \alpha_x x_t^2) + \gamma_t \left(\pi_t - \kappa x_t - \beta \pi_{t+1} \right) \right]$$

FONCs:

$$\begin{aligned} \alpha_x x_t - \kappa \gamma_t &= 0 \\ \pi_t + \gamma_t - \gamma_{t-1} &= 0 \end{aligned}$$

that must hold for $t = 0, 1, 2, \ldots$and where $\gamma_{-1} = 0$, as the constraint in period -1 is not effective for optimal plan in period 0.

Optimal Policy under Commitment

The Lagrangian equation

Combining the two optimality conditions to eliminate the Lagrange multipliers yields

$$x_0 = -\frac{\kappa}{\alpha_x}\pi_0 \qquad (13)$$

$$x_t = x_{t-1} - \frac{\kappa}{\alpha_x}\pi_t \qquad (14)$$

for $t = 1, 2, 3, \ldots$
These two equations can be represented jointly by a single equation in price "levels"

$$x_t = -\frac{\kappa}{\alpha_x}\hat{p}_t \qquad (15)$$

for $t = 0, 1, 2, \ldots$where $\hat{p}_t \equiv p_t - p_{-1}$ is the (log) deviation between the price level and an "implicit target" given by the price level prevailing one period before the central bank chooses it optimal plan.

Optimal Policy under Commitment

Combine equation (15) with

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa x_t + u_t$$

$$\hat{p}_t = a\hat{p}_{t-1} + a\beta E_t \{\hat{p}_{t+1}\} + au_t$$

for $t = 0, 1, 2, \ldots$where $a \equiv \frac{\alpha_x}{\alpha_x(1+\beta)+\kappa^2}$.
By forward iteration,

$$\hat{p}_t = \delta\hat{p}_{t-1} + \frac{\delta}{1-\delta\beta\rho_u} u_t \tag{16}$$

for $t = 0, 1, 2, \ldots$where $\delta \equiv \frac{1-\sqrt{1-4\beta a^2}}{2a\beta} \in (0, 1)$.

Optimal Policy under Commitment

Combine (16) and (15), the equilibrium process for the output gap

$$x_t = \delta x_{t-1} - \frac{\kappa\delta}{\alpha_x(1 - \delta\beta\rho_u)} u_t \tag{17}$$

for $t = 1, 2, 3, \ldots$with the response at the time of the shock $(t = 0)$ being given by

$$x_0 = -\frac{\kappa\delta}{\alpha_x(1 - \delta\beta\rho_u)} u_0.$$

Optimal Policy under Commitment

Figure 5.1: Impulse responses to a 1% transitory cost-push shock.

- **Discretionary policy**: both the output gap and inflation return to their zero initial value once the shock has vanished.
- **Commitment**: deviations in the output gap and inflation from target persist well beyond the life of the shock.
  - Improvement in the output gap/inflation tradeoff initially.
  - Forward-looking nature of inflation. Iterating the NKPC forward

$$\pi_t = \kappa x_t + \kappa \sum_{k=1}^{\infty} \beta^k E_t \{x_{t+k}\} + u_t.$$

  - In response to u_t, the central bank may lower future output gap with credible promises. Thus, given π_t, current x_t may decline less.
  - Due to convexity of loss function, the dampening of deviations in the period of shock improves welfare.

Optimal Policy under Commitment

Figure 5.2: Impulse responses to a persistant cost-push shock.

- The economy reverts back to the initial position slowly.
- Under commitment, initial responses of inflation and output gap are both lower.
- Under commitment, price level reverts back to its original level. Inflation displays positive short-run autocorrelation.
- *Stabilization bias* associated with the discretionary policy: attempts to stabilize the output gap in the medium term more than the optimal policy under commitment.

Optimal Policy under Commitment

Optimal policy rule

Assume transitory shock ($\rho_u = 0$), combining (4), (16) and (17) yields the equilibrium nominal rate under the optimal policy with commitment

$$\begin{aligned} i_t &= r_t^e - (1-\delta)\left(1 - \frac{\sigma\kappa}{\alpha_x}\right)\hat{p}_t \\ &= r_t^e - (1-\delta)\left(1 - \frac{\sigma\kappa}{\alpha_x}\right)\sum_{k=0}^{t}\delta^{k+1}\tilde{u}_{t-k}. \end{aligned}$$

Thus, one possible rule that would bring about the desired allocation as the unique equilibrium is given by

$$i_t = r_t^e - \left[\phi_p + (1-\delta)\left(1 - \frac{\sigma\kappa}{\alpha_x}\right)\right]\sum_{k=0}^{t}\delta_{t-k}^{k+1}\tilde{u}_{t-k} + \phi_p\hat{p}_t$$

for any $\phi_p > 0$.

5.2 The Monetary Policy Problem: The Case of a Distorted Steady State

The presence of uncorrected real imperfections generate a permanent gap between the natural and the efficient levels of output.

$$-\frac{U_N}{U_C} = MPN(1 - \Phi).$$

where $\Phi > 0$ measures the size of the steady state distortion. Without an appropriate subsidy, $\Phi \equiv 1 - \frac{1}{\mu}$. Output is inefficiently low.

The Monetary Policy Problem: The Case of a Distorted Steady State

Under a "small" steady state distortion, the welfare losses

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \alpha_x \hat{x}_t^2 \right) - \Lambda \hat{x}_t \right] \tag{18}$$

where $\Lambda \equiv \Phi \frac{\lambda}{\varepsilon} > 0$ and $\hat{x}_t = x_t - x$ represents the deviation of the welfare-relevant output gap from its value $x < 0$ in the zero inflation steady state.

$$\begin{aligned} x_t &\equiv y_t - y_t^e \\ x &\equiv y - y^e = y^n - y^e \end{aligned}$$

The Monetary Policy Problem: The Case of a Distorted Steady State

The inflation equation

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa \hat{x}_t + u_t \tag{19}$$

The central bank will seek to minimize (18) subject to the sequence of constraints given by (19) for $t = 0, 1, 2, \ldots$

5.2.1 Optimal Discretionary Policy

$$\min_{(x_t,\pi_t)} \frac{1}{2}\left(\pi_t^2 + \alpha_x \hat{x}_t^2\right) - \Lambda \hat{x}_t$$

s.t.

$$\pi_t = \kappa \hat{x}_t + v_t$$

where $v_t \equiv \beta E_t\left\{\pi_{t+1}\right\} + u_t$ is taken as given by the policymaker.
The FONC:

$$\hat{x}_t = \frac{\Lambda}{\alpha_x} - \frac{\kappa}{\alpha_x}\pi_t. \tag{20}$$

- a more expansionary policy than that given in the absence of a steady state distortion.

Optimal Discretionary Policy

Plugging (20) into (19) and solving the difference equation,

$$\pi_t = \frac{\Lambda\kappa}{\kappa^2 + \alpha_x(1-\beta)} + \alpha_x \Psi u_t. \tag{21}$$

Combining (21) and (20) yields the equilibrium output gap

$$\hat{x}_t = \frac{\Lambda(1-\beta)}{\kappa^2 + \alpha_x(1-\beta)} - \kappa\Psi u_t.$$

Optimal Discretionary Policy

The distorted steady state

- does not affect the responses of the output gap and inflation to shocks under the optimal policy.
- affects the average levels of inflation and the output gap around which the economy fluctuates.

The inefficiently low natural level of output and employment (for $\Lambda > 0$)

- leads to positive average inflation as a consequence of the central bank's incentive to push output above its natural steady state level.
- increases the incentive of keeping inflation higher than zero as Λ (and hence Φ) increases. *Classical inflation bias.*

5.2.2 Optimal Policy under Commitment

Under credible commitment, the central bank solves a dynamic problem

$$L = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \alpha_x \hat{x}_t^2 \right) - \Lambda \hat{x}_t + \gamma_t \left(\pi_t - \kappa \hat{x}_t - \beta \pi_{t+1} \right) \right]$$

The FONCs:

$$\begin{aligned} \alpha_x \hat{x}_t - \kappa \gamma_t - \Lambda &= 0 \\ \pi_t + \gamma_t - \gamma_{t-1} &= 0 \end{aligned}$$

which must hold for $t = 0, 1, 2, \ldots$ and where $\gamma_{-1} = 0$.

Optimal Policy under Commitment

The FONCs can be combined to yield the difference equation for the log price level

$$\hat{p}_t = a\hat{p}_{t-1} + a\beta E_t\left\{\hat{p}_{t+1}\right\} + \alpha_x \kappa \Lambda + a u_t$$

for $t = 0, 1, 2, \ldots$where, as above, $\hat{p}_t \equiv p_t - p_{-1}$ and $a \equiv \frac{\alpha_x}{\alpha_x(1+\beta)+\kappa^2}$.
Iterating forward,

$$\hat{p}_t = \delta \hat{p}_{t-1} + \frac{\delta}{1-\delta\beta\rho_u} u_t + \frac{\delta\kappa\Lambda}{1-\delta\beta}$$

where $\delta \equiv \frac{1-\sqrt{1-4\beta a^2}}{2a\beta} \in (0,1)$.
The corresponding path for the output gap is given by

$$\hat{x}_t = \delta \hat{x}_{t-1} - \frac{\kappa\delta}{\alpha_x(1-\delta\beta\rho_u)} u_t + \Lambda\left[1 - \delta\left(1 + \frac{\kappa^2}{\alpha_x(1-\delta\beta)}\right)\right].$$

Optimal Policy under Commitment

- As with the discretionary policy, an identical stabilization bias exists.
- Also, the response to a cost-push shock is not affected by the presence of a distorted steady state.
- In the presence of a distorted steady state, an additional difference arises between the discretionary and commitment policies.
  - Discretion: a constant positive mean in both variables.
  - Commitment: the price level converges asymptotically to a constant, given by $\lim_{T\to\infty} p_T = p_{-1} + \frac{\delta\kappa\Lambda}{(1-\delta\beta)(1-\delta)}$, resulting in zero average inflation in equilibrium.

Optimal Policy under Commitment

The existence of zero average inflation is observationally equivalent to that of an economy with an efficient steady state.

- Anticipation of public can improve the short run tradeoff facing the central bank.
  - output can be raised above its natural level with welfare improvement
  - more subdued effects on inflation.
- Commitment avoids the inflation bias under discretionary policy.