Advanced Macro II
The reference answer to homework 1

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Note: The answer is just used for reference.

A simple real business cycle model

Consider the following model where a representative household solves

$$\max_{C_t, K_t} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [\ln(C_t) - AN_t] \right\} \tag{1}$$

$$C_t + K_t = Z_t K_{t-1}^{\theta} N_t^{1-\theta} + (1-\delta) K_{t-1} \tag{2}$$

where C_t, N_t, K_t denote consumption,labor and capital. Z_t is a stochasitc process for technology with $\varepsilon_t \sim N(0, \sigma_\epsilon^2)$, and $0 \le \rho < 1. A, \beta, \theta$ are parameters with $0 < \beta < 1, 0 < \theta < 1$.It might be useful to define the expression for output as:

$$Y_t = Z_t K_{t-1}^{\theta} N_t^{1-\theta} \tag{3}$$

1. Describe the economy briefly. Comment on the preference, endowment, technology and information. (1 point)

ANS:

The economic environment:

1)Preference:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [\ln(C_t) - AN_t] \right\}$$

2)Technology:

$$\begin{aligned} C_t + K_t &= Z_t K_{t-1}^{\theta} N_t^{1-\theta} + (1-\delta) K_{t-1} \\ \ln Z_t &= (1-\rho) \ln \overline{Z} + \rho \ln Z_{t-1} + \varepsilon_t \end{aligned}$$

where Z_t is a stochasitc process for technology with $\varepsilon_t \sim N(0, \sigma_\epsilon^2)$,and $0 \le \rho < 1$.

3)Endowment: $K_{-1} > 0$

4)Information: decision made based on all information I_t up to time t.

2. Find the first order necessary conditions of the representative agent. (2 points)

ANS:

The social planner's problem:

$$\max_{C_t, K_t} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [\ln(C_t) - AN_t] \right\}$$

$$\begin{aligned} C_t + K_t &= Z_t K_{t-1}^{\theta} N_t^{1-\theta} + (1-\delta) K_{t-1} \\ \ln Z_t &= (1-\rho) \ln \overline{Z} + \rho \ln Z_{t-1} + \varepsilon_t \\ \varepsilon_t &\overset{iid}{\sim} N(0, \sigma_\epsilon^2) \end{aligned}$$

The Lagrangian:

$$L = \max_{(C_t, K_t)_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \left[\ln(C_t) - AN_t - \lambda_t \left(C_t + K_t - Z_t K_{t-1}^{\theta} N_t^{1-\theta} - (1-\delta) K_{t-1} \right) \right] \right\}$$

The necessary FOCs:

$$\frac{\partial L}{\partial C_t} : 0 \stackrel{!}{=} \frac{1}{C_t} - \lambda_t$$

$$\begin{aligned} \frac{\partial L}{\partial K_t} \quad : \quad & 0 \stackrel{!}{=} \beta^t[-\lambda_t] + \beta^{t+1} E_t \left[(-\lambda_{t+1})[-\theta Z_{t+1} K_t^{\theta-1} N_{t+1}^{1-\theta} - (1-\delta)] \right] \\ \Rightarrow \quad & 0 \stackrel{!}{=} -\lambda_t + \beta E_t \left[\lambda_{t+1} \left(\theta Z_{t+1} K_t^{\theta-1} N_{t+1}^{1-\theta} + (1-\delta) \right) \right] \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial N_t} \quad : \quad & 0 \stackrel{!}{=} -A - \lambda_t \left[-(1-\theta) Z_t K_{t-1}^{\theta} N_t^{-\theta} \right] \\ \Rightarrow \quad & 0 \stackrel{!}{=} \ -A + (1-\theta) \lambda_t Z_t K_{t-1}^{\theta} N_t^{-\theta} \end{aligned}$$

$$\frac{\partial L}{\partial \lambda_t} : 0 \stackrel{!}{=} C_t + K_t - Z_t K_{t-1}^{\theta} N_t^{1-\theta} - (1-\delta) K_{t-1}$$

3. Write down the model in six equations,including output and the langrange multiplier λ_t (0.6 point)

ANS:

Firstly,collect all the necessary coditions and exogenous process:

$$\lambda_t = \frac{1}{C_t}$$

$$Y_t = Z_t K_{t-1}^{\theta} N_t^{1-\theta}$$

$$C_t = Y_t + (1-\delta) K_{t-1} - K_t$$

$$1 = E_t \left[\beta \frac{C_t}{C_{t+1}} \left(\theta \frac{Y_{t+1}}{K_t} + (1-\delta) \right) \right] or 1 = E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \left(\theta \frac{Y_{t+1}}{K_t} + (1-\delta) \right) \right]$$

$$A = (1-\theta) \frac{1}{C_t} \frac{Y_t}{N_t} or A = (1-\theta) \lambda_t \frac{Y_t}{N_t}$$

$$\ln Z_t = (1-\rho) \ln \overline{Z} + \rho \ln Z_{t-1} + \varepsilon_t, \varepsilon_t \stackrel{iid}{\sim} N(0, \sigma_{\epsilon}^2)$$

4. Solve for the steady state, i.e. provide formulas for $\overline{C}, \overline{\lambda}, \overline{K}, \overline{N}$ and $\overline{Y}$,

given $\overline{Z}$ and other parameters. (Show the steps that you solve these values subsequently.) (1 point)

ANS:

To find the s.s of economy, drop time indices:

$\overline{\lambda} = \frac{1}{\overline{C}}$ (1)

$\overline{Y} = \overline{Z}\overline{K}^{\theta}\overline{N}^{1-\theta}$ (2)

$\overline{C} = \overline{Y} + (1-\delta)\overline{K} - \overline{K}$ (3)

$1 = \beta\left(\theta\frac{\overline{Y}}{\overline{K}} + (1-\delta)\right)$ (4)

$A = (1-\theta)\frac{1}{\overline{C}}\frac{\overline{Y}}{\overline{N}}$ (5)

Five variables need to be solved for $\left(\overline{C}, \overline{\lambda}, \overline{K}, \overline{N}, \overline{Y}\right)$ given the parameters $\left(\overline{Z}, \theta, \delta, A, \rho\right)$.

firstly from (4), $\frac{\overline{Y}}{\overline{K}} = \frac{1-\beta(1-\delta)}{\beta\theta}$

so $\overline{Y} = \frac{1-\beta+\beta\delta}{\beta\theta}\overline{K}$, substituting into (2) and (3) respectively,

$\overline{N} = \left(\frac{\beta\theta\overline{Z}}{1-\beta+\beta\delta}\right)^{\frac{1}{\theta-1}}\overline{K}$

$\overline{C} = \overline{Y} - \delta\overline{K} = \left(\frac{1-\beta+\beta\delta}{\beta\theta} - \delta\right)\overline{K} = \frac{1-\beta+\beta\delta-\beta\delta\theta}{\beta\theta}\overline{K}$

Then we solve (5) with the above equations, we can get that

$\overline{N} = \frac{(1-\theta)(1-\beta+\beta\delta)}{A(1-\beta+\beta\delta-\beta\delta\theta)}$

$\overline{K} = \left(\frac{\beta\theta\overline{Z}}{1-\beta+\beta\delta}\right)^{\frac{1}{1-\theta}}\overline{N}$

$\overline{Y} = \frac{1-\beta+\beta\delta}{\beta\theta}\overline{K}$,

$\overline{C} = \overline{Y} - \delta\overline{K}$

$\overline{\lambda} = \frac{1}{\overline{C}}$

5. Now suppose $\overline{N}$ is given, while A needs to be calculated together with $\overline{C}, \overline{\lambda}, \overline{K}, \overline{Y}$.solve for the steady state again. (1 point)

ANS:

If $\overline{N}$ is given,firstly from (4), $\overline{Y} = \frac{1-\beta+\beta\delta}{\beta\theta}\overline{K}$,

substituting into (2), we can get that

$\overline{K} = \left(\frac{\beta\theta\overline{Z}}{1-\beta+\beta\delta}\right)^{\frac{1}{1-\theta}}\overline{N}$

$\overline{Y} = \frac{1-\beta+\beta\delta}{\beta\theta}\overline{K}$

$\overline{C} = \overline{Y} - \delta\overline{K}$

$\overline{\lambda} = \frac{1}{\overline{C}}$

$\overline{A} = (1-\theta)\frac{\overline{Y}}{\overline{C}\overline{N}}$

6. Log-linearize the equations. Define log-deviation of variable X_t as $\widehat{x}_t = \log X_t/\overline{X}$.Write down all necessary log-linearized equations characterizing the equilibrium in the notation of log-deviation term $\widehat{x}_t$.(2 points)

ANS:

Using $X_t = \overline{X}e^{\widehat{x}_t}ff0c$we loglinearize as follows:

Consumption:

$$
\begin{aligned}
\lambda_t &= \frac{1}{C_t} \\
\overline{\lambda} e^{\widehat{\lambda}_t} &= \frac{1}{\overline{C} e^{\widehat{c}_t}}
\end{aligned}
$$

$$
\widehat{\lambda}_t = -\widehat{c}_t
$$

Budget constraint:

$$
\begin{aligned}
C_t &= Y_t + (1-\delta)K_{t-1} - K_t \\
\overline{C} e^{\widehat{c}_t} &= \overline{Y} e^{\widehat{y}_t} + (1-\delta)\overline{K} e^{\widehat{k}_{t-1}} - \overline{K} e^{\widehat{k}_t} \\
\overline{C}\widehat{c}_t &= \overline{Y}\widehat{y}_t + (1-\delta)\overline{K}\widehat{k}_{t-1} - \overline{K}\widehat{k}_t
\end{aligned}
$$

$$
\widehat{c}_t = \frac{\overline{Y}}{\overline{C}}\widehat{y}_t + (1-\delta)\frac{\overline{K}}{\overline{C}}\widehat{k}_{t-1} - \frac{\overline{K}}{\overline{C}}\widehat{k}_t
$$

Euler equation:

$$
\begin{aligned}
1 &= E_t\left[\beta\frac{C_t}{C_{t+1}}\left(\theta\frac{Y_{t+1}}{K_t} + (1-\delta)\right)\right] \\
1 &= E_t\left[\beta\frac{\overline{C} e^{\widehat{c}_t}}{\overline{C} e^{\widehat{c}_{t+1}}}\left(\theta\frac{\overline{Y} e^{\widehat{y}_{t+1}}}{\overline{K} e^{\widehat{k}_t}} + (1-\delta)\right)\right] \\
1 &= E_t\left[\theta\beta\frac{\overline{Y}}{\overline{K}} e^{\widehat{c}_t - \widehat{c}_{t+1} + \widehat{y}_{t+1} - \widehat{k}_t} + (1-\delta)\beta e^{\widehat{c}_t - \widehat{c}_{t+1}}\right] \\
0 &= E_t\left[\theta\beta\frac{\overline{Y}}{\overline{K}}\left(\widehat{c}_t - \widehat{c}_{t+1} + \widehat{y}_{t+1} - \widehat{k}_t\right) + (1-\delta)\beta\left(\widehat{c}_t - \widehat{c}_{t+1}\right)\right] \\
with\ 1 &= \beta\left(\theta\frac{\overline{Y}}{\overline{K}} + (1-\delta)\right)
\end{aligned}
$$

$$
\widehat{c}_t - \theta\beta\frac{\overline{Y}}{\overline{K}}\widehat{k}_t = E_t\left[\widehat{c}_{t+1} - \theta\beta\frac{\overline{Y}}{\overline{K}}\widehat{y}_{t+1}\right]
$$

Output:

$$
\begin{aligned}
Y_t &= Z_t K_{t-1}^{\theta} N_t^{1-\theta} \\
\overline{Y} e^{\widehat{y}_t} &= \overline{Z} e^{\widehat{z}_t}\overline{K}^{\theta} e^{\theta\widehat{k}_{t-1}}\overline{N}^{1-\theta} e^{(1-\theta)\widehat{n}_t}
\end{aligned}
$$

$$
\widehat{y}_t = \widehat{z}_t + \theta\widehat{k}_{t-1} + (1-\theta)\widehat{n}_t
$$

Labor equation:

$$
\begin{aligned}
A &= (1-\theta)\frac{1}{C_t}\frac{Y_t}{N_t} \\
A &= (1-\theta)\frac{1}{\overline{C}e^{\widehat{c}_t}}\frac{\overline{Y}e^{\widehat{y}_t}}{\overline{N}e^{\widehat{n}_t}}
\end{aligned}
$$

$$\widehat{y}_t = \widehat{c}_t + \widehat{n}_t$$

Technology process

$$\ln \overline{Z}e^{\widehat{z}_t} = (1-\rho)\ln \overline{Z} + \rho \ln \overline{Z}e^{\widehat{z}_t} + \varepsilon_t$$

$$\widehat{z}_t = \rho\widehat{z}_{t-1} + \varepsilon_t.$$

Collect all the four L.L. equations, which form a system of linear and homogeneous equations.

$$\widehat{\lambda}_t = -\widehat{c}_t$$

$$\widehat{c}_t = \frac{\overline{Y}}{\overline{C}}\widehat{y}_t + (1-\delta)\frac{\overline{K}}{\overline{C}}\widehat{k}_{t-1} - \frac{\overline{K}}{\overline{C}}\widehat{k}_t$$

$$\widehat{c}_t - \theta\beta\frac{\overline{Y}}{\overline{K}}\widehat{k}_t = E_t\left[\widehat{c}_{t+1} - \theta\beta\frac{\overline{Y}}{\overline{K}}\widehat{y}_{t+1}\right]$$

$$\widehat{y}_t = \widehat{z}_t + \theta\widehat{k}_{t-1} + (1-\theta)\widehat{n}_t$$

$$\widehat{y}_t = \widehat{c}_t + \widehat{n}_t$$

$$\widehat{z}_t = \rho\widehat{z}_{t-1} + \varepsilon_t$$

7. Sort all your variables in the above equations, classify what are exogenous state variable, endogenous state variable and other endogenous variables. (0.4 point)
ANS:
The exogenous state variable: $\widehat{z}_t$
The endogenous state variable: $\widehat{k}_{t-1}$
The other endogenous variables: $\widehat{c}_t, \widehat{y}_t, \widehat{n}_t$

8. Now reduce the above equations into three equations with three variables$\widehat{z}_t$, $\widehat{k}_t$and λ_t.Following the instructions step by step: 1) Substitute out $\widehat{c}_t$ by λ_t in the log-linearized budget constraint, then combine the linearized FOC of labor and the production function to reduce $\widehat{n}_t$ and possibly $\widehat{y}_t$ to get the form of equation (5) below. 2) Substitute $\widehat{n}_t$ (or $\widehat{y}_t$) depending on your early steps) in the Euler equation to obtain a function of the form (6) below. Please state clearly your steps to get the solutions. Now you can obtain the following two-dimensional stochastic dirence equation together with exogenous technology process

$$0 = -\widehat{k}_t + a_1\widehat{k}_{t-1} + a_2\widehat{\lambda}_t + a_3\widehat{z}_t \tag{4}$$

$$0 = E_t\left[-\widehat{\lambda}_t + a_4\widehat{k}_t + a_5\widehat{\lambda}_{t+1} + a_6\widehat{z}_{t+1}\right] \tag{5}$$

$$\widehat{z}_t = \rho\widehat{z}_{t-1} + \varepsilon_t \tag{6}$$

what are $a_1, a_2, a_3, a_4, a_5, a_6$in terms of the original parameters? (3 points)

ANS:

Collect all the focs:

$\widehat{c}_t = \frac{\overline{Y}}{\overline{C}}\widehat{y}_t + (1-\delta)\frac{\overline{K}}{\overline{C}}\widehat{k}_{t-1} - \frac{\overline{K}}{\overline{C}}\widehat{k}_t$ (1)

$\widehat{y}_t = \widehat{z}_t + \theta\widehat{k}_{t-1} + (1-\theta)\widehat{n}_t$ (2)

$\widehat{y}_t = \widehat{c}_t + \widehat{n}_t$ (3)

$\widehat{\lambda}_t = -\widehat{c}_t$ (4)

$\widehat{c}_t - \theta\beta\frac{\overline{Y}}{\overline{K}}\widehat{k}_t = E_t\left[\widehat{c}_{t+1} - \theta\beta\frac{\overline{Y}}{\overline{K}}\widehat{y}_{t+1}\right]$ (5)

Firstly, we combine (2) and (3),

$\widehat{y}_t = \frac{1}{\theta}\widehat{z}_t + \widehat{k}_{t-1} - \frac{(1-\theta)}{\theta}\widehat{c}_t$ (6)

substitute (6) into (1) and combining (4),

$0 = -\widehat{k}_t + \left[\frac{\overline{Y}}{\overline{K}} + (1-\delta)\right]\widehat{k}_{t-1} + \left[\frac{\overline{C}}{\overline{K}} + \frac{\overline{Y}}{\overline{K}}\frac{(1-\theta)}{\theta}\right]\widehat{\lambda}_t + \frac{\overline{Y}}{\overline{K}}\frac{1}{\theta}\widehat{z}_t$

also, substitute (6) into (5) and combining (4),

$0 = E_t\left[-\widehat{\lambda}_t + \left[1 + (1-\theta)\beta\frac{\overline{Y}}{\overline{K}}\right]\widehat{\lambda}_{t+1} + \beta\frac{\overline{Y}}{\overline{K}}\widehat{z}_{t+1}\right]$

So we can conclude that:

$a_1 = \frac{\overline{Y}}{\overline{K}} + (1-\delta)$

$a_2 = \frac{\overline{C}}{\overline{K}} + \frac{\overline{Y}}{\overline{K}}\frac{(1-\theta)}{\theta}$

$a_3 = \frac{\overline{Y}}{\overline{K}}\frac{1}{\theta}$

$a_4 = 0$

$a_5 = 1 + (1-\theta)\beta\frac{\overline{Y}}{\overline{K}}$

$a_6 = \beta\frac{\overline{Y}}{\overline{K}}$

9. According to your results in step 4, calculate steady state values of $\overline{C}, \overline{\lambda}, \overline{K}, \overline{N}$ and $\overline{Y}$,and calculatea a_1 to a_6 given the following parameter values:

$\beta = 0.98, \rho = 0.95, \theta = 0.4, \delta = 0.025, \overline{z} = 1, A = 3.44, \sigma_\epsilon^2 = 0.712.$

Assume that your model parameters are calibrated from quarterly data. (1 point)

ANS:

$\overline{N} = \frac{(1-\theta)(1-\beta+\beta\delta)}{A(1-\beta+\beta\delta-\beta\delta\theta)}$ =0.2237

$\overline{K} = \left(\frac{\beta\theta\overline{Z}}{1-\beta+\beta\delta}\right)^{\frac{1}{1-\theta}}\overline{N}$ =8.4043

$\overline{Y} = \frac{1-\beta+\beta\delta}{\beta\theta}\overline{K}$ =0.9541

$\overline{C} = \overline{Y} - \delta\overline{K}$ =0.7439

$\overline{\lambda} = \frac{1}{\overline{C}}$ =1.3442

$a_1 = \frac{\overline{Y}}{\overline{K}} + (1-\delta) = 1.0885$

$a_2 = \frac{\overline{C}}{\overline{K}} + \frac{\overline{Y}}{\overline{K}}\frac{(1-\theta)}{\theta}$ =0.2588

$a_3 = \frac{\overline{Y}}{\overline{K}}\frac{1}{\theta} = 0.2838$
$a_4 = 0$
$a_5 = 1 + (1-\theta)\beta\frac{\overline{Y}}{\overline{K}}$ =1.0668
$a_6 = \beta\frac{\overline{Y}}{\overline{K}}$ =0.1113

10. According to your results in step 5, calculate steady state values of $A, \overline{C}, \overline{\lambda}, \overline{K}$ and $\overline{Y}$,and calculatea a_1 to a_6 given the following parameter values:
$\beta = 0.98, \rho = 0.95, \theta = 0.4, \delta = 0.025, \overline{z} = 1, \overline{N} = 1/3, \sigma_\epsilon^2 = 0.712$. In what follows, take the parameter values given here.(1 point)

ANS:

$\overline{K} = \left(\frac{\beta\theta\overline{Z}}{1-\beta+\beta\delta}\right)^{\frac{1}{1-\theta}}\overline{N}$ =12.5244
$\overline{Y} = \frac{1-\beta+\beta\delta}{\beta\theta}\overline{K}$ =1.4218
$\overline{C} = \overline{Y} - \delta\overline{K}$ =1.1087
$\overline{\lambda} = \frac{1}{\overline{C}} = 0.9020$
$\overline{A} = (1-\theta)\frac{\overline{Y}}{\overline{CN}}$ =2.3084
$a_1 = \frac{\overline{Y}}{\overline{K}} + (1-\delta)$ =1.0885
$a_2 = \frac{\overline{C}}{\overline{K}} + \frac{\overline{Y}}{\overline{K}}\frac{(1-\theta)}{\theta}$ =0.2588
$a_3 = \frac{\overline{Y}}{\overline{K}}\frac{1}{\theta}$ =0.2838
$a_4 = 0$
$a_5 = 1 + (1-\theta)\beta\frac{\overline{Y}}{\overline{K}}$ =1.0668
$a_6 = \beta\frac{\overline{Y}}{\overline{K}} = 0.1113$

11. Guess the recursive law of motion of the above system in step 8 as:

$$\widehat{\lambda}_t = \eta_{\lambda k}\widehat{k}_{t-1} + \eta_{\lambda z}\widehat{z}_t \quad (7)$$

$$\widehat{k}_t = \eta_{kk}\widehat{k}_{t-1} + \eta_{kz}\widehat{z}_t \quad (8)$$

and exploit $E_t[\widehat{z}_{t+1}] = \rho\widehat{z}_t$.Using the undetermined coeffcient method, insert these expressions into (5) and (6) and collect terms on $\widehat{k}_{t-1}$ and $\widehat{z}_t$, to get two transformed equations. (Note that you may insert (8) and (9) twice into (6) to eventually reduce the equation to a function of only $\widehat{k}_{t-1}$ and $\widehat{z}_t$. (0.4 point)

ANS:

$$0 = [-\eta_{kk} + a_1 + a_2\eta_{\lambda k}]\,\widehat{k}_{t-1} + [-\eta_{kz} + a_3 + a_2\eta_{\lambda z}]\,\widehat{z}_t$$

$$0 = [-\eta_{\lambda k} + a_5\eta_{\lambda k}\eta_{kk}]\,\widehat{k}_{t-1} + [-\eta_{\lambda z} + a_5\eta_{\lambda z}\rho + a_5\eta_{\lambda k}\eta_{kz} + a_6\rho]\,\widehat{z}_t$$

12. By comparing the coeffcients on $\widehat{k}_{t-1}$,you can get a characteristic quadratic equation η_{kk} as $a\eta_{kk}^2 + b\eta_{kk} + c = 0$,where a, b, and c are determined by a_1 to a_6. What are a,b, and c? Solve this equation, what are the roots? Which root should you choose and why? Use your chosen value of η_{kk} to calculate $\eta_{\lambda k}$.(1.2 points)

ANS:

Comparing the coeffcients on $\widehat{k}_{t-1}$,

$$\begin{cases} -\eta_{kk} + a_1 + a_2\eta_{\lambda k} = 0 \\ -\eta_{\lambda k} + a_5\eta_{\lambda k}\eta_{kk} = 0 \end{cases}$$
$$\Rightarrow$$
$$\begin{cases} \eta_{\lambda k} = (\eta_{kk} - a_1)/a_2 \\ (a_5\eta_{kk} - 1)(\eta_{kk} - a_1) = 0 \\ a_5\eta_{kk}^2 - (a_5 a_1 + 1)\eta_{kk} + a_1 = 0 \end{cases}$$
$$\Rightarrow a = a_5, b = -a_5 a_1 - 1, c = a_1;$$

$$\begin{cases} \eta_{kk} = 1/a_5 = 0.9374 \\ \eta_{\lambda k} = (\eta_{kk} - a_1)/a_2 = -0.5838 \end{cases}$$

13. By comparing the coefficients on $\widehat{z}_t$,solve for $\eta_{\lambda z}$ and η_{kz}.(0.4 points).

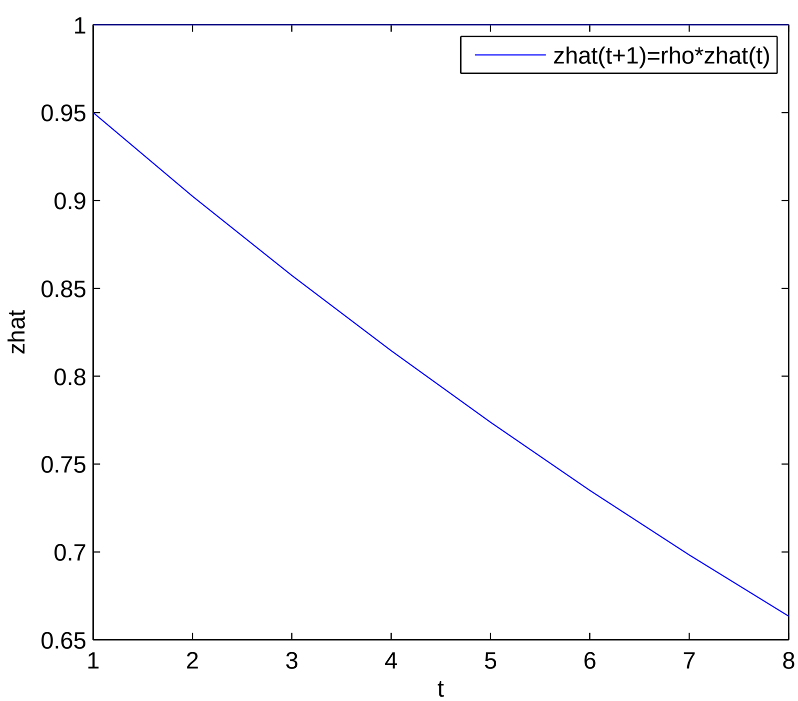
ANS:

Comparing the coeffcients on $\widehat{z}_t$,

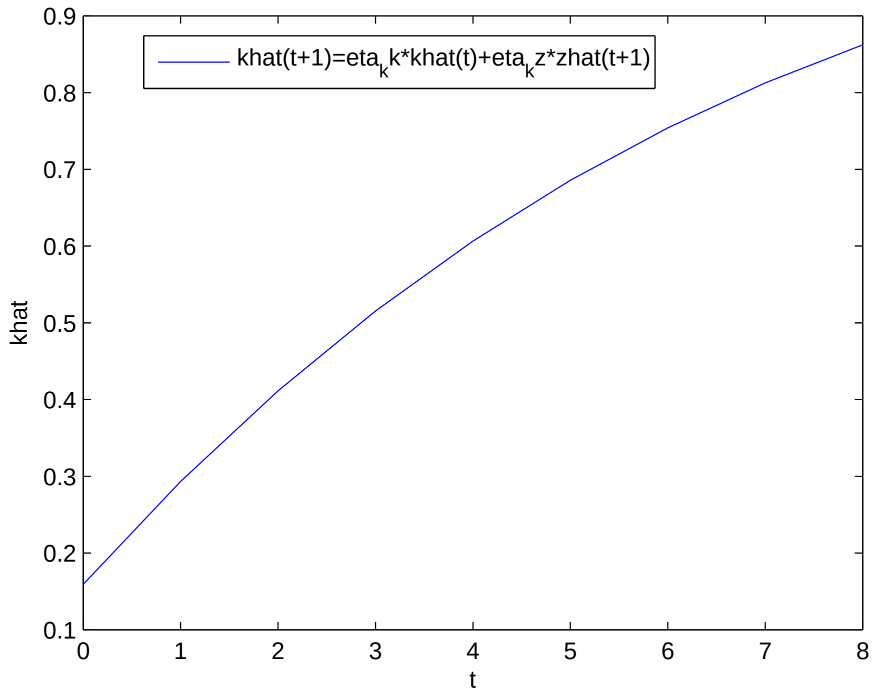
$$\begin{cases} -\eta_{kz} + a_3 + a_2\eta_{\lambda z} = 0 \\ -\eta_{\lambda z} + a_5\eta_{\lambda z}\rho + a_5\eta_{\lambda k}\eta_{kz} + a_6\rho = 0 \end{cases}$$
$$\Rightarrow$$
$$\begin{cases} \eta_{kz} = a_3 + a_2\eta_{\lambda z} \\ -\eta_{\lambda z} + a_5\eta_{\lambda z}\rho + a_5\eta_{\lambda k}\eta_{kz} + a_6\rho = 0 \end{cases}$$
$$\Rightarrow$$
$$\begin{cases} \eta_{kz} = a_3 + a_2\eta_{\lambda z} \\ -\eta_{\lambda z} + a_5\eta_{\lambda z}\rho + a_5\eta_{\lambda k}\eta_{kz} + a_6\rho = 0 \end{cases}$$
$$\Rightarrow$$
$$\begin{cases} \eta_{\lambda z} = \frac{a_3 a_5 \eta_{\lambda k} + a_6\rho}{1 - a_5\rho - a_2 a_5 \eta_{\lambda k}} = -0.4809 \\ \eta_{kz} = a_3 + a_2\eta_{\lambda z} = 0.1593 \end{cases}$$

14. Assume $\widehat{z}_{-1} = 0, \varepsilon_0 = 1, and\ \varepsilon_t = 0$ for $t = 1, 2, ...\infty$. Calculate $\widehat{z}_t$ recursively and plot the impluse response of $\widehat{z}_t$ for $t = 1, 2, ...8$.(0.4 points)

ANS:



15. Assume $\widehat{k}_{-1} = 0$, given the above technology shock,calculate $\widehat{k}_t$ recursively and plot the impluse response of $\widehat{k}_t$ for $t = 1, 2, ...8$.(0.6 points)
ANS:



16. According to your loglinearized system in step 6, solve the law of motion for $\widehat{n}_t = \eta_{nk}\widehat{k}_{t-1} + \eta_{nz}\widehat{z}_t$ (10)
what are the values of η_{nk} and η_{nz} given the parameter values? Notice the difference in their signs, and discuss the economic interpretation. (2 points)
ANS:

$$\begin{aligned}
\widehat{n}_t &= \frac{\theta + \eta_{\lambda k}}{\theta}\widehat{k}_{t-1} + \frac{1 + \eta_{\lambda z}}{\theta}\widehat{z}_t \\
\eta_{nk} &= \frac{\theta + \eta_{\lambda k}}{\theta} = -0.4596 \\
\eta_{nz} &= \frac{1 + \eta_{\lambda z}}{\theta} = 1.2977
\end{aligned}$$

η_{nk} is negative, meaning that the increase in capital may squeeze out the input of labor. In the contrast, the sign of η_{nz} is postive, indicating that any positve technological impluse implies positive labor input.