Solution to Option Exercise

May 20, 2012

1. The lower bound is

$$28 - 25\exp\left(-0.08 \times \frac{1}{3}\right) = \$3.66$$

2. The lower bound is

$$15\exp\left(-0.06 \times \frac{1}{12}\right) - 12 = \$2.93$$

3. In this case $c = 1$, $T = \frac{1}{4}$, $S_0 = 19$, $X = 20$, and $r = 0.04$. From put-call parity

$$\begin{aligned} p &= c + X\exp(-rT) - S_0 \\ &= 1 + 20\exp\left(-0.04 \times \frac{1}{4}\right) - 19 = 1.80 \end{aligned}$$

so that the European put price is \$1.80.

4. The present value of the strike price is

$$60\exp\left(-0.12 \times \frac{4}{12}\right) = \$57.65.$$

The present value of the dividend is

$$0.80\exp\left(-0.12 \times \frac{1}{12}\right) = 0.79.$$

Because

$$5 < 64 - 57.65 - 0.79$$

the condition of the lower bound for Europeam call option is violated. An arbitrageur should buy the option and short the stock. This generates $64 - 5 = \$59$. The arbitrageur invests \$0.79 of this at 12% for one month to pay the dividend of \$0.80 in one month. The remaining \$58.21 is invested for four months at 12%. Regardless of what happens a profit will materialize.

(a) If the stock price declines below \$60 in four months, the arbitrageur loses the \$5 spent on the option but gains on the short position. The arbitrageur shorts when the stock price is \$64, has to pay dividends with a present value of \$0.79, and closes out the short position when the stock price is \$60 or less. Because \$57.65 is the present value of \$60, the short position generates at least $64 - 57.65 - 0.79 = \$5.56$ in present value terms. The present value of the arbitrageur's gain is therefore at least $5.56 - 5.00 = \$0.56$.

(b) If the stock price is above \$60 at the expiration of the option, the option is exercised. The arbitrageur buys the stock for \$60 in four months and closes out the short position. The present value of the \$60 paid for the stock is \$57.65 and as before the dividend has a present value of \$0.79. The gain from the short position and the exercise of the option is therefore exactly $64 - 57.65 - 0.79 = \$5.56$. The arbitrageur's gain in present value terms is exactly $5.56 - 5.00 = \$0.56$.

5. If the call is worth \$3, put-call parity shows that the put should be worth

$$3 + 20\exp\left(-0.10 \times \frac{3}{12}\right) + \exp\left(-0.1 \times \frac{1}{12}\right) - 19 = 4.50$$

This is greater than \$3. The put is therefore undervalued relative to the call. The correct arbitrage strategy is to buy the put, buy the stock, and short the call. This costs \$19.

(a) If the stock price in three months is greater than \$20, the call is exercised.

(b) If it is less than \$20, the put is exercised.

(c) In either case the arbitrageur sells the stock for \$20 and collects the \$1 dividend in one month. The present value of the gain to the arbitrageur is

$$-3 - 19 + 20\exp\left(-0.10 \times \frac{3}{12}\right) + \exp\left(-0.10 \times \frac{1}{12}\right) = 1.50$$

6. The Solution

(a) Delaying exercise delays the payment of the strike price. This means that the option holder is able to earn interest on the strike price for a longer period of time.

(b) Delaying exercise also provides insurance against the stock price falling below the strike price by the expiration date. Assume that the option holder has an amount of cash

X and that interest rates are zero. Exercising early means that the option holder's position will be worth S_τ at expiration. Delaying exercise means that it will be worth $\max(X, S_\tau)$ at expiration.

7. An American put when held in conjunction with the underlying stock provides insurance. It guarantees that the stock can be sold for the strike price, X. If the put is exercised early, the insurance ceases. However, the option holder receives the strike price immediately and is able to earn interest on it between the time of the early exercise and the expiration date.

8. The Solution

   (a) An American call option can be exercised at any time. If it is exercised its holder gets the intrinsic value. It follows that an American call option must be worth at least its intrinsic value.

   (b) A European call option can be worth less than its intrinsic value. Consider, for example, the situation where a stock is expected to provide a very high dividend during the life of an option. The price of the stock will decline as a result of the dividend. Because the European option can be exercised only after the dividend has been paid, its value may be less than the intrinsic value today.

9. Consider a portfolio that is long one option with strike price X_1, long one option with strike price X_3, and short two options with strike price X_2 . The value of the portfolio can be worked out in four different situations

$$
\begin{aligned}
S_\tau \leq X_1 &: \quad \text{Portfolio Value } = 0 \\
X_1 \leq S_\tau \leq X_2 &: \quad \text{Portfolio Value } = S_\tau - X_1 \\
X_2 \leq S_\tau \leq X_3 &: \quad \text{Portfolio Value } = S_\tau - X_1 - 2(S_\tau - X_2) = X_2 - X_1 - (S_\tau - X_2) \geq 0 \\
S_\tau > X_3 &: \quad \text{Portfolio Value } = S_\tau - X_1 - 2(S_\tau - X_2) + S_\tau - X_3 \\
&\qquad = X_2 - X_1 - (X_3 - X_2) = 0
\end{aligned}
$$

The value is always either positive or zero at the expiration of the option. In the absence of arbitrage possibilities it must be positive or zero today. This means that

$$c_1 + c_3 - 2c_2 \geq 0$$

or

$$c_2 \leq 0.5(c_1 + c_3)$$