# Advanced Macroeconomics II
# Problem Set 3
March 24, 2014

The due date for this assignment is Wednesday, April 2. It needs to be delivered by 2:30pm before the lecture starts. You can form a group of up to three persons. Remember to sign your name and student ID on the cover page of your group homework.

**Solve a model of a decentralized economy with taxation (100 points)**

Consider the following model, where a **representative household** solves

$$\max_{c_t, k_t, n_t} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{1-\eta} - 1}{1-\eta} - A n_t \right) \right\} \tag{1}$$

s.t.

$$c_t + k_t = \left[ (1-\tau_k) d_t + (1-\delta) \right] k_{t-1} + (1-\tau_n) w_t n_t + g_t \tag{2}$$

where $\beta (0 < \beta < 1)$, $\eta$, $A$ and $\delta$ are parameters. The household rents out $k_{t-1}$ units of capital to the representative firm and earns the dividend $d_t$ on each unit of capital. She faces a dividend tax rate of $\tau_k$ and labor income tax rate of $\tau_n$. She receives a transfer, $g_t$, from the government which she takes as given.

**The representative firm** produces output $y_t$ using a Cobb-Douglas production function with capital and labor as inputs. $z_t$ is an exogenous stochastic process with $E_{t-1}[\epsilon_t] = 0$ and $Var[\epsilon_t] = \sigma^2$. The firm pays to the representative agent the dividend $d_t$ on rented capital and the real wage $w_t$ to labor. $\bar{\gamma}$ is a parameter of steady state productivity.

$$\max_{k_{t-1}, n_t} \bar{\gamma} e^{z_t} k_{t-1}^{\theta} n_t^{1-\theta} - d_t k_{t-1} - w_t n_t \tag{3}$$

$$z_t = \rho z_{t-1} + \epsilon_t$$

where $\rho < 1$.

**The government** collects capital income taxes and labor income taxes and transfer $g_t$ units to the agent.

$$g_t = \tau_k d_t k_{t-1} + \tau_n w_t n_t \tag{4}$$

It might be useful to define expressions for output and the gross return on capital:

$$\begin{aligned} y_t &= \bar{\gamma} e^{z_t} k_{t-1}^{\theta} n_t^{1-\theta} && (5) \\ R_t &= \theta \frac{y_t}{k_{t-1}} + 1 - \delta && (6) \end{aligned}$$

where $0 < \theta < 1$.

**Questions:**

1. Describe the economy briefly. Comment on the preference, endowment, technology, and information. (3 points)

2. Classify the model variables according to endogenous state variable(s), exogenous state variable(s) and other endogenous variables. (3 points)

3. Find the first order necessary conditions (FONCs) for

   (a) the representative household. (4 points)

   (b) the firm. (2 points)

4. Substitute out the Lagrange multiplier on the agents constraint. Furthermore substitute the government's budget into the individuals budget to get rid of $g_t$. Finally plug in the firm's first order conditions into the ones of the agent to get rid of $d_t$ and $w_t$. Write down all equations that are left and necessary to describe an equilibrium. [*Hint: You should obtain 6 equations.*] (6 points)

5. According to the set of equations what is (are) the difference(s) between your model and the Hansen RBC model without the government sector? (2 points)

6. Solve for the steady state, i.e. provide formulas for $\bar{k}$, $\bar{y}$, $\bar{R}$, $\bar{c}$ and $A$, given $\bar{n}$ and all other parameters. (5 points)

7. Log-linearize the equations. Define log-deviation of variable $x_t$ as $\hat{x}_t = \log(x_t/\bar{x})$. Write down all necessary log-linearized equations characterizing the equilibrium in the notation of log-deviation term $\hat{x}_t$. (Note: $z_t$ is already in the deviation term for technology factor in the sense of $z_t = \log[(\bar{\gamma}e^{z_t})/(\bar{\gamma}e^0)]$.) (24 points)

8. Simplify these equations until you have two equations in $\hat{k}_t$, $\hat{c}_t$ and $z_t$ of the following form of equations

$$0 = -\hat{k}_t + a_1\hat{k}_{t-1} + a_2\hat{c}_t + a_3\hat{z}_t \tag{7}$$

$$0 = E_t\left[-\hat{c}_t + a_4\hat{k}_t + +a_5\hat{c}_{t+1} + a_6\hat{z}_{t+1}\right], \tag{8}$$

   what are $a_1$,...,$a_6$ in terms of the original parameters? (6 points)

9. Guess a suitable recursive law of motion for your two model variables. (2 points)

10. Solve for the coefficients of the recursive equilibrium law of motion by plugging the recursive law of motion into the loglinearized equations and compare coefficients. Which of the two roots for $\eta_{kk}$ do you use and why? (7 points)

    - Calibration:

    (a) Parameter set 1: Assume that $\beta = 0.99$, $\theta = 0.36$, $\delta = 0.025$, $\rho = 0.95$, $\bar{n} = 1/3$, $\eta = 1$, $\bar{\gamma} = 1$, $\tau_k = 0.2$, $\tau_n = 0.3$.

(b) Parameter set 2: Assume that $\beta = 0.99$, $\theta = 0.36$, $\delta = 0.025$, $\rho = 0.95$, $\bar{n} = 1/3$, $\eta = 1$, $\bar{\gamma} = 1$, $\tau_k = 0$, $\tau_n = 0$.

11. Parameter set 1: calculate $\bar{k}$, $\bar{y}$, $\bar{R}$, $\bar{c}$ and $A$. Calculate also the discounted sum of steady state utility. Compute $a_1$,...,$a_6$. Calculate the coefficients of the recursive equilibrium law of motion for $\hat{c}_t$, $\hat{k}_t$ and $\hat{R}_t$. (4 points)

12. Parameter set 2: calculate $\bar{k}$, $\bar{y}$, $\bar{R}$, $\bar{c}$ and $A$. Calculate also the discounted sum of steady state utility. Compute $a_1$,...,$a_6$. Calculate the coefficients of the recursive equilibrium law of motion for $\hat{c}_t$, $\hat{k}_t$ and $\hat{R}_t$. (4 points)

13. Suppose $\hat{k}_t = \hat{c}_t = \hat{R}_t = 0$ and $\epsilon_t = 0$ for $t \leq -1$. At $t = 0$, $\epsilon_t = 1$ (interpret that as one percent) and for $t \geq 1$, $\epsilon_t = 0$. Graph the impulse responses for $\hat{k}_t$, $\hat{c}_t$ and $\hat{R}_t$ to a productivity shock for $t = -1, 0, 1, 2, ..., 15$ for each parameter set. (4 points)

14. Suppose $\hat{k}_t = \hat{c}_t = \hat{R}_t = 0$ and for $t \leq -1$. At $t = 0$, $\hat{k}_t = 1$ (interpret that as one percent of a capital shock). $\epsilon_t = 0$ for all time periods. Graph the impulse responses for $\hat{k}_t$, $\hat{c}_t$ and $\hat{R}_t$ to the capital shock for $t = -1, 0, 1, 2, ..., 15$ for each parameter set. (4 points)

15. Compare the impulse responses of parameter set 2 to the ones of parameter set 1 for the Hansen RBC model. Are there any differences? (4 points)

16. Give a brief economic reason for the qualitative and quantitative differences of impulse responses for parameter set 1 and 2. (4 points)

17. Interprete a period as a quarter. Calculate for each parameter set how many quarters does it approximately take for this economy to close half of the gap between its initial capital shock and its steady state? (4 points)

18. (4 points) Take parameter set 1. What happens to the path of consumption, $\hat{c}_t$, in case of a technology shock as in question (13) if

    (a) $\eta = 0.1$? Why?

    (b) $\eta = 5$? Why?

19. Take parameter set 1. What happens to the path of capital, $\hat{k}_t$, in case of a technology shock as in question (13) if $\delta = 0.9$? Why? (4 points)