Nash Equilibrium Existence Proof

- Use Kakutani's fixed point theorem. We need
  - $f : A \to A$ such that
  - for all $a \in A$ the set $f(a)$ is nonempty and convex
  - the graph of f is closed (i.e. for all sequences $\{x_n\}$ and $\{y_n\}$ such that $y_n \in f(x_n)$ for all n, $x_n \to x$, $y_n \to y$, we have $y \in f(x)$).
- Use $B(a) = \times_{i \in N} B_i(a_{-i})$.
  - $B : A \to A$.
  - For all $a \in A$ the set $B(a)$ is nonempty (Why?).
  - For all $a \in A$ the set $B(a)$ is convex (Why?).
  - B has a closed graph (Why?).
- Thus B has a fixed point, which is a Nash equilibrium.