

Problem Set 2

February 26, 2012

The Arrow's approach is as follows, suppose that an asset (lottery), $\tilde{\epsilon}$, has the following payoffs and probabilities (this could be generalized to other types of fair payoffs):

$$\tilde{\epsilon} = \begin{cases} +\epsilon & \text{with probability } \frac{1}{2} \\ -\epsilon & \text{with probability } \frac{1}{2} \end{cases} \quad (1.34)$$

where $\epsilon \geq 0$. Note that, as before, $E[\tilde{\epsilon}] = 0$.

Now consider the following question.

- By how much should we change the expected value (return) of the asset, by changing the probability of winning, in order to make the individual indifferent between taking and not taking the risk?

$$\tilde{\epsilon} = \begin{cases} +\epsilon & \text{with probability } p \\ -\epsilon & \text{with probability } (1 - p) \end{cases}$$

- If p is the probability of winning, we can define the risk premium as

$$\begin{aligned} \theta &= \text{prob}(\tilde{\epsilon} = +\epsilon) - \text{prob}(\tilde{\epsilon} = -\epsilon) \\ &= p - (1 - p) \\ &= 2p - 1 \end{aligned} \quad (1.35)$$

Therefore, from (1.35) we have

$$\tilde{\epsilon} = \begin{cases} +\epsilon & \text{with probability } p = \frac{1}{2}(1 + \theta) \\ -\epsilon & \text{with probability } (1 - p) = \frac{1}{2}(1 - \theta) \end{cases}$$

Please try to interpret the Arrow risk premium θ in terms of expected rate of return in excess of the risk-free rate of return.

The Problem 2 is due to Mar, 7th.