Advanced Microeconomics II
Problem Set 3

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1. Armies 1 and 2 are fighting over an island initially held by a battalion of army 2. Army 1 has K battalions and army 2 has L. Whenever the island is occupied by one army the opposing army can launch an attack. The outcome of the attack is that the occupying battalion and one of the attacking battalions are destroyed; the attacking army wins and, so long as it has battalions left, occupies the island with one battalion. The commander of each army is interested in maximizing the number of surviving battalions but also regards the occupation of the island as worth more than one battalion but less than two. (If after an attack, neither army has any battalions left, then the payoff to each commander is 0.) Analyze this situation as an extensive game and, using the notion of subgame perfect equilibrium, predict the winner as a function of K and L.

2. How many Nash equilibria are there in the 2 period Chain Store game?

3. Consider the split the pie game with T periods.
   (a) Find the SPE when $T = 2$.
   (b) Find the SPE when $T = 3$.
   (c) Find the SPE when T is even.
   (d) Find the SPE when T is odd.
   (e) What are the limit payoffs when $T \to \infty$?

4. Consider the extension of Rubinstein's infinite horizon alternating offer bargaining model from two players to three players. Every player has the same discount factor δ.

   In period 1, player 1 makes an offer which consists of (p_1, p_2, p_3) which represents the share of each player. Then player 2 decides to accept or to reject the offer within the same period. If player 2 accepts the offer, then player 3 is asked to accept or reject the offer within the same period. If both player 2 and player 3 accept the offer, then the bargaining is over and each player takes p_i. If either player 2 or player 3 rejects, the first person who rejects the present offer will initiate the next round.

   In period t, player i initiates the offer, which specifies the shares of every player, Then player $i+1(\text{mod } 3)$ either accepts or rejects the offer. If player $i+1(\text{mod } 3)$ accepts the offer, then player $i+2(\text{mod } 3)$ either accepts or rejects the offer. If both players accept the offer, then the bargaining is over and player i receives a payoff of $\delta^{t-1}p_i$. If $j \neq i$ is the first player who rejects the offer, then the next round starts with player j's offer.

   (a) (10 points) Calculate a stationary subgame perfect equilibrium.
   (b) (10 points) Can you sustain $(0.5, 0.5, 0)$ which is agreed upon in the initial round as an outcome of a subgame perfect equilibrium if δ is sufficiently close to to 1. Explain your answer.

5. Prove the following proposition.

   Proposition 1. *Let w be a strictly enforceable feasible payoff profile of $G = \{N, (A_i), (u_i)\}$. For all $\epsilon > 0$ there exists $\underline{\delta} < 1$ such that if $\delta > \underline{\delta}$ then the δ-discounted infinitely repeated game of G has a Nash equilibrium whose payoff profile w' satisfies $|w' - w| < \epsilon$.*

6. Consider the following stage game.

	A	D
A	$2, 2$	$0, 1$
D	$5, 4$	$1, 0$

   (a) Construct a pair of strategies that generate the average per-period payoffs of $(3.5, 3)$, and are a Nash equilibrium but are not a subgame perfect equilibrium when players use the limit of means criterion to evaluate payoffs.

   (b) Construct a pair of strategies that generate average per-period payoffs of $(3.5, 3)$, and are a subgame perfect equilibrium when players use the limit of means criterion to evaluate payoffs but not a subgame perfect equilibrium when players use the overtaking criterion to evaluate payoffs.

   (c) Construct a pair of strategies that generate the average per-period payoffs of $(3.5, 3)$, and are a subgame perfect equilibrium when players use overtaking criterion to evaluate payoffs.

7. Consider a game in which the following strategic game is repeated twice:

		Player 2		
		b_1	b_2	b_3
	a_1	$10, 10$	$2, 12$	$0, 13$
Player 1	a_2	$12, 2$	$5, 5$	$0, 0$
	a_3	$13, 0$	$0, 0$	$1, 1$

The players observe the actions chosen in the first play of the game prior to the second play.

(a) What are the pure strategy sub-game perfect Nash equilibrium payoffs of this game?

(b) What are the pure strategy subgame perfect Nash equilibria of this game? In particular, how many pure strategy sub-game perfect Nash equilibria are there?