Advanced Microeconomics II

Sequential Equilibrium

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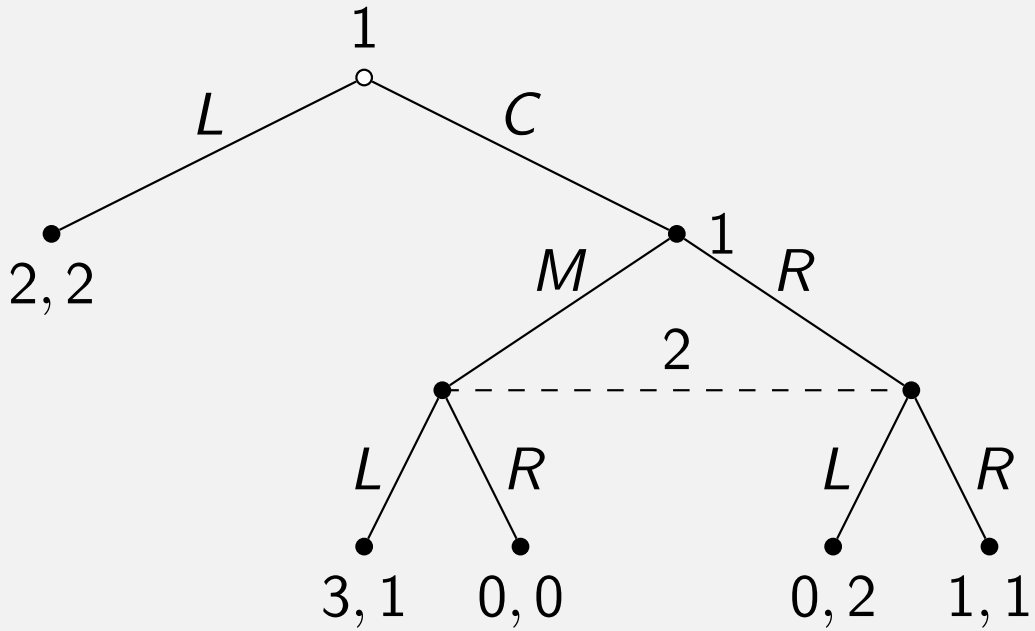
Nash Equilibrium



- Two types of Nash equilibrium;
  - $\beta_1(\varnothing)(L) = 0, \beta_1(C)(M) = 1, \beta_2(I_2)(L) = 1.$
  - $\beta_1(\varnothing)(L) = 1, 0 \leq \beta_1(C)(M) \leq 1, 0 \leq \beta_2(I_2)(L) \leq 2/3.$
- To extend the concept of SPE we need to specify how beliefs are formed for information sets off the equilibrium path.
- Equilibrium will consist of strategies and beliefs.

Assessment

Definition

An assessment in an extensive game is a pair (β, μ) where β is a profile of behavioural strategies and μ is a function that assigns to every information set a probability measure on the set of histories in the information set.

Definition

The outcome $O(\beta, \mu|I)$ of (β, μ) conditional on I is the distribution over terminal histories determined by β and μ conditional on I being reached, as follows. Let $h^* = (a^1, \ldots, a^K)$ be a terminal history. Then

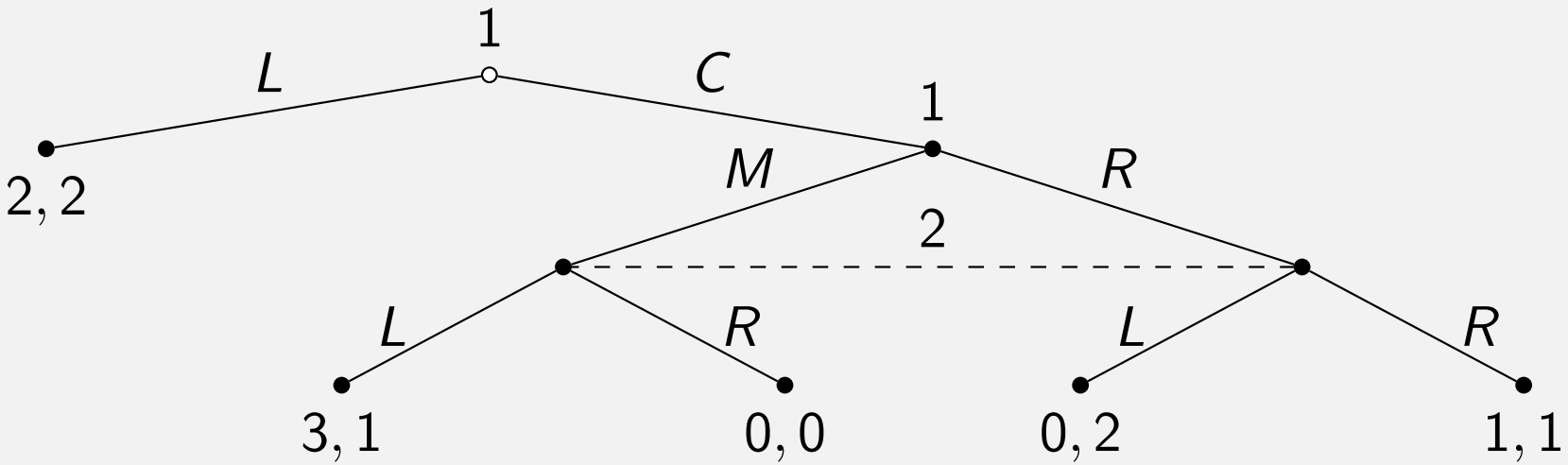
- $O(\beta, \mu|I)(h^*) = 0$ if there is no subhistory of h^* in I.
- $O(\beta, \mu|I)(h^*) = \mu(I)(h) \cdot \Pi_{k=L}^{K-1} \beta_{P(a^1,\ldots,a^k)}(a^1, \ldots, a^k)(a^{k+1})$ if the subhistory $h = (a^1, \ldots, a^L)$ of h^* is in I, where $L < K$.

Sequential Rationality

Definition

Let $\Gamma = \{N, H, P, f_c, (\mathcal{I}_i), (\succeq_i)\}$ be a finite extensive game with perfect recall. An assessment (β, μ) is sequentially rational if for every player $i \in N$ and every information set $I_i \in \mathcal{I}_i$ we have

$$O(\beta, \mu | I_i) \succeq_i O((\beta_i', \beta_{-i}), \mu | I_i) \text{ for every strategy } \beta_i' \text{ of player } i.$$

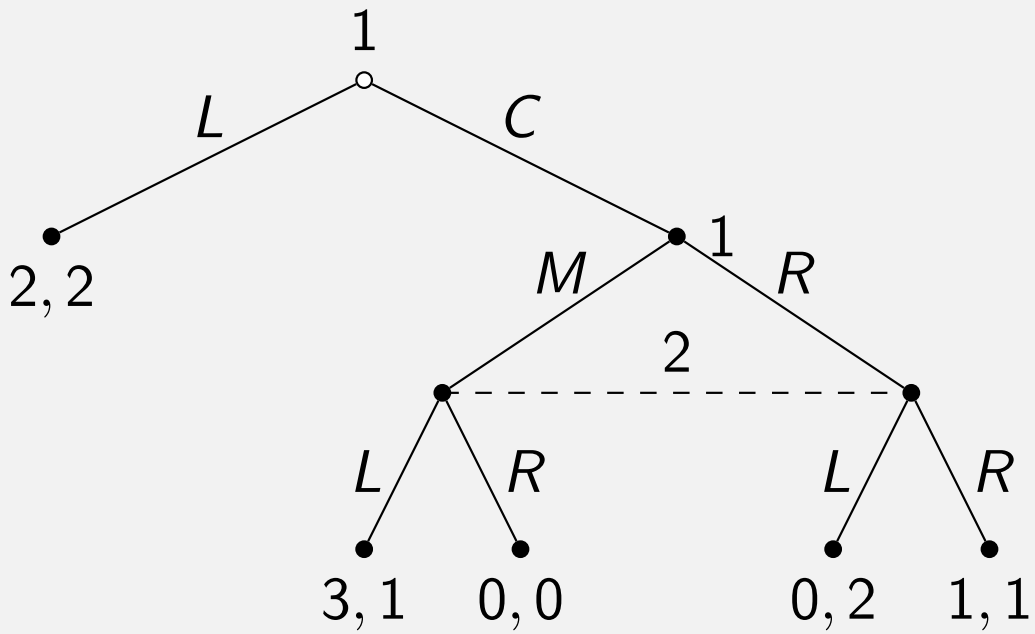


- For what belief can $\beta_2(I_2)(L) = 0$ be part of a sequentially rational assessment?

Weak Perfect Bayesian Equilibrium

Definition

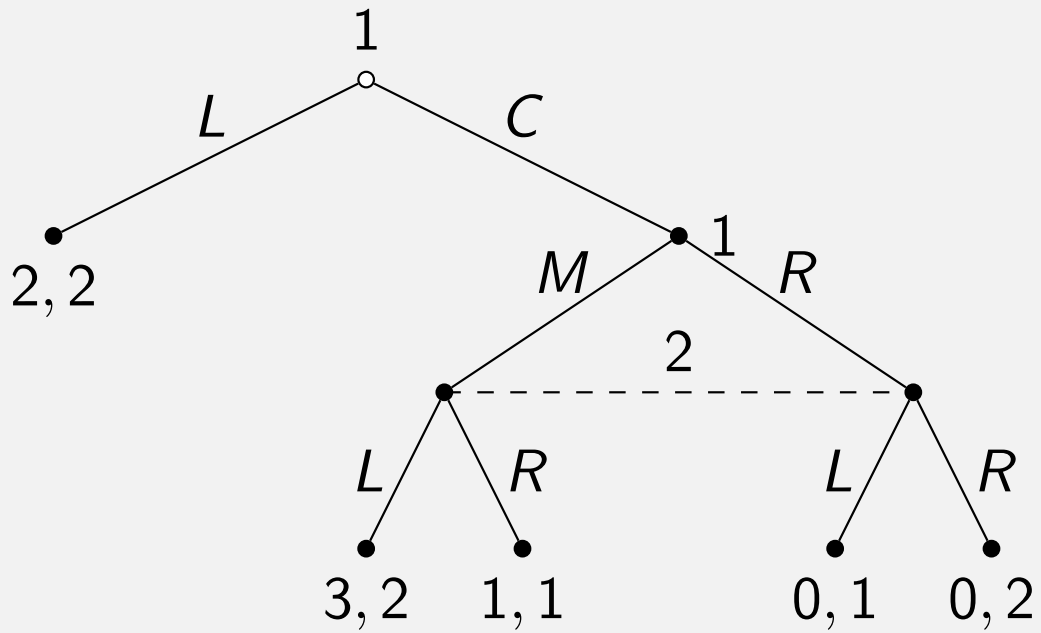
Let $\Gamma = \{N, H, P, f_c, (\mathcal{I}_i), (\succeq_i)\}$ be a finite extensive game with perfect recall. An assessment (β, μ) is a weak perfect Bayesian equilibrium of Γ if it is sequentially rational and at every information set reached with positive probability μ is derived from β using Bayes' rule.



- Weak Perfect Bayesian Equilibrium?

Limitations of WPBE

WPBE may not be an SPE

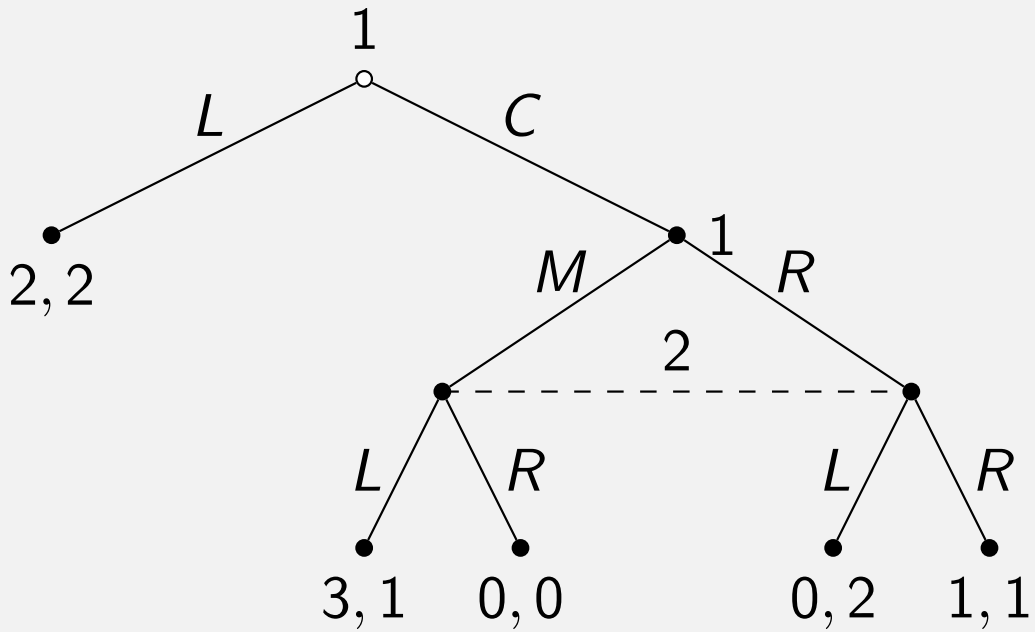


- Weak Perfect Bayesian Equilibrium?
- Subgame Perfect Equilibrium?

Perfect Bayesian Equilibrium for Extensive Games

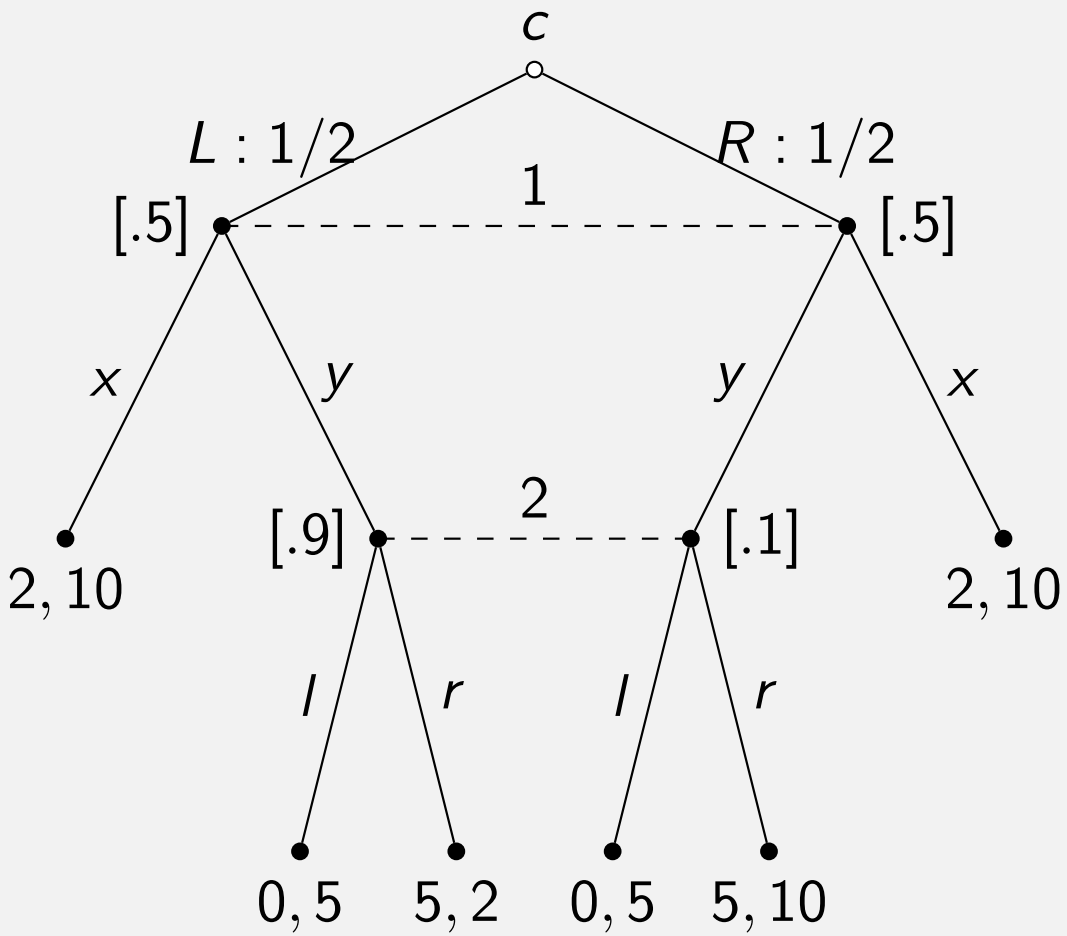
Definition

Let $\Gamma = \{N, H, P, f_c, (\mathcal{I}_i), (\succeq_i)\}$ be a finite extensive game with perfect recall. An assessment (β, μ) is a perfect Bayesian equilibrium of Γ if it is a weak perfect Bayesian equilibrium of every subgame of Γ.



- Perfect Bayesian Equilibrium?

Limitations of Perfect Bayesian Equilibrium



- Perfect Bayesian Equilibrium: Beliefs are inconsistent with any plausible explanation of reaching I_2.

Consistency and Sequential Equilibrium

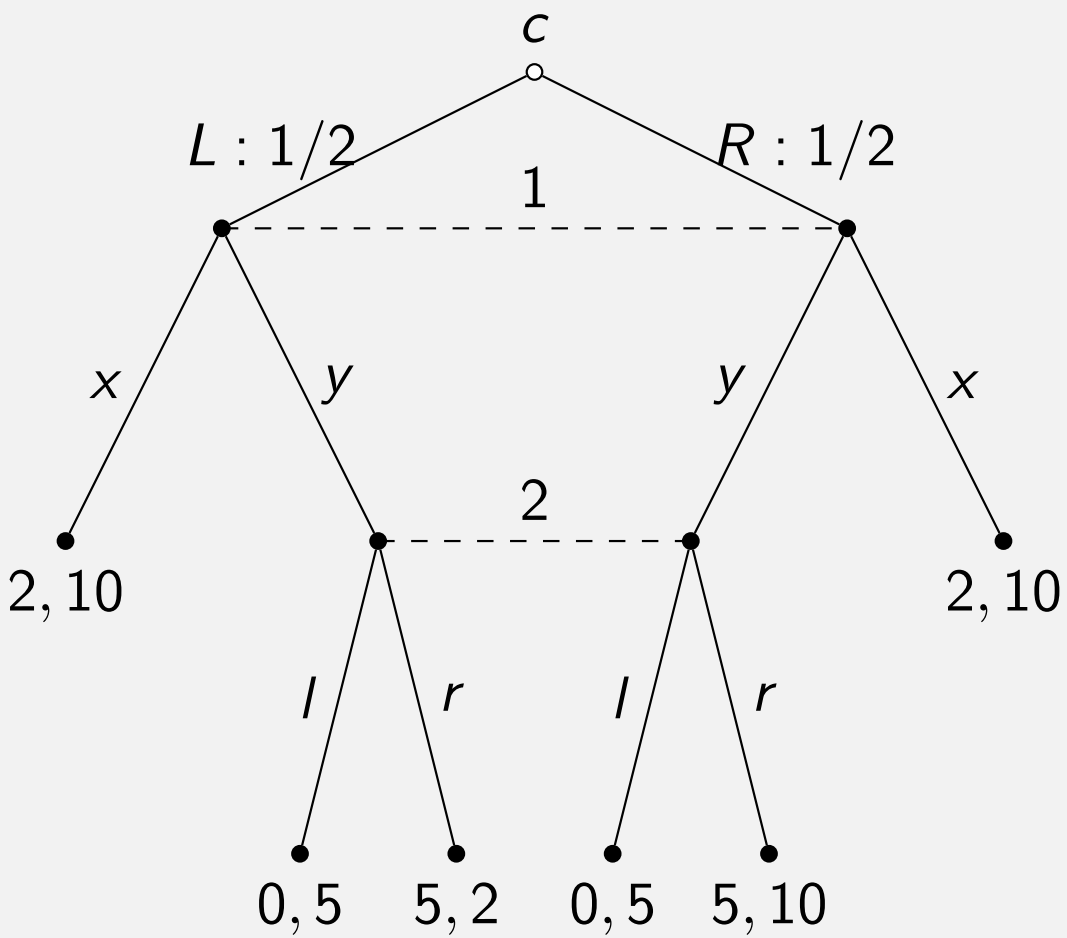
Definition

Let $\Gamma = \{N, H, P, f_c, (\mathcal{I}_i), (\succeq_i)\}$ be a finite extensive game with perfect recall. An assessment (β, μ) is consistent if there is a sequence $((\beta^n, \mu^n))_{n=1}^{\infty}$ of assessments that converge to (β, μ) in Euclidean space and has the properties that each strategy profile β^n is completely mixed and that each belief system μ^n is derived from β^n using Bayes' rule.

Definition

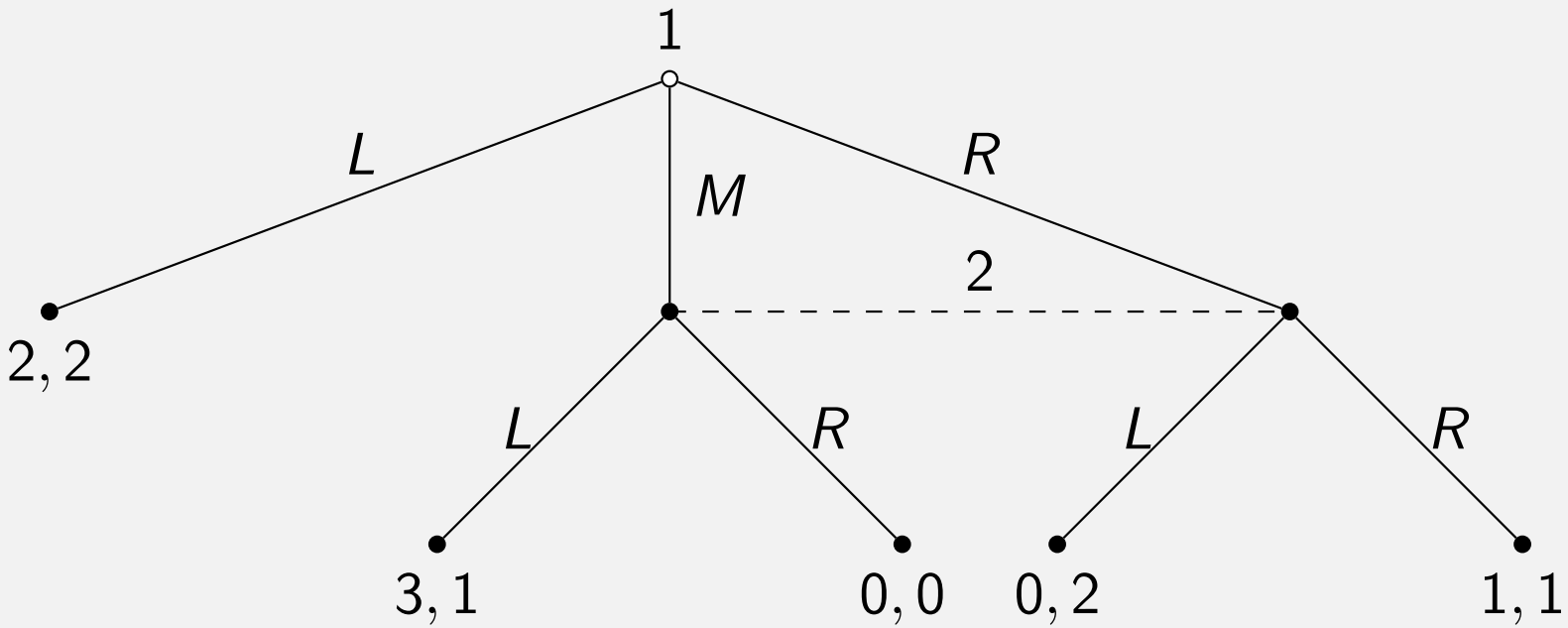
An assessment is a sequential equilibrium of a finite extensive game with perfect recall if it is sequentially rational and consistent.

Sequential Equilibrium - Example 1



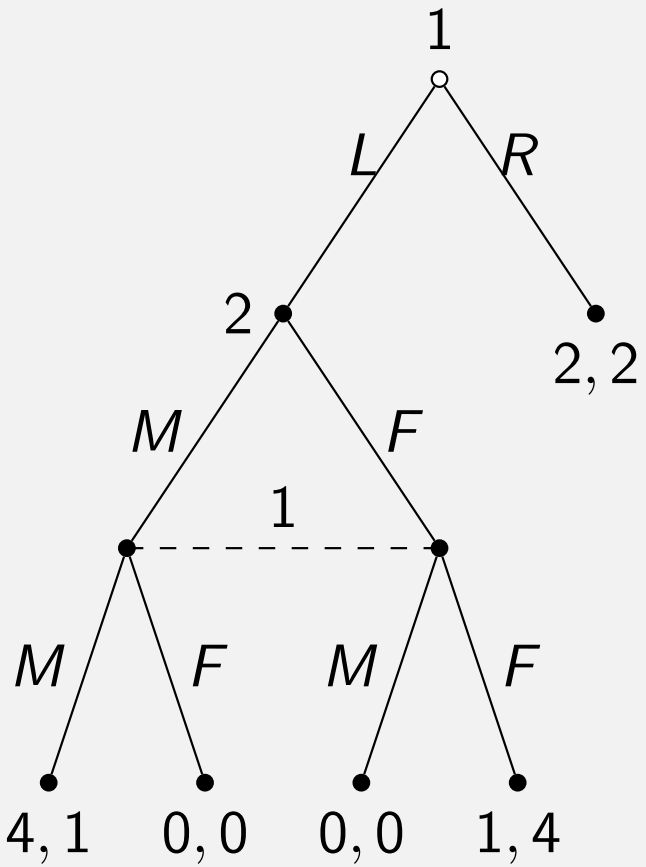
- Weak PBE?
- Sequential Equilibrium?

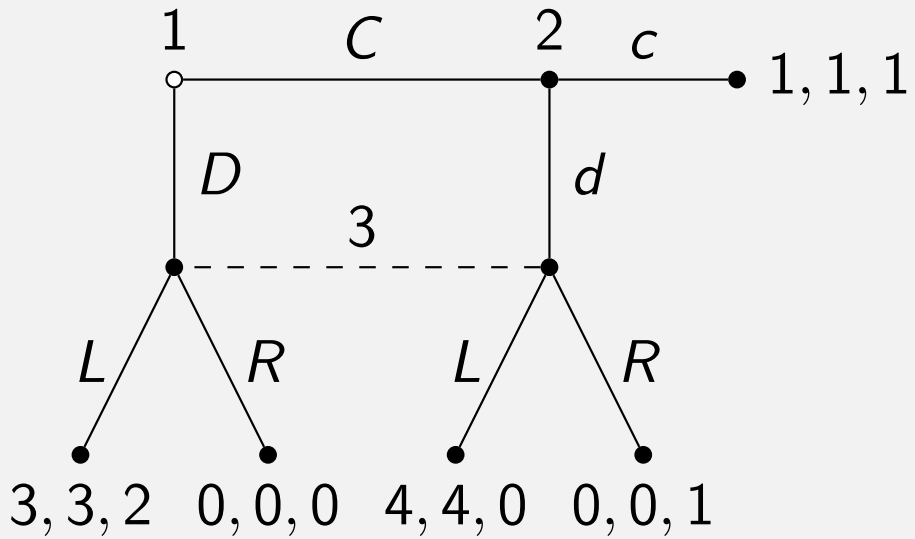
Sequential Equilibria - Example 2



- Sequential equilibria?

Homework





Establish the set of NE, Weak PBE and Sequential Equilibria.