Problem Set 2
Advanced Macroeconomics II
WISE, Xiamen University

March 12, 2014

The due date for this assignment is Wednesday, March 19, before the class starts at 2:30pm. You can form a group of up to three persons.

Solving a model with stochastic labor supply.

Consider the following model, where a representative household solves

$$\max_{c_t,k_t,n_t} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\ln c_t - m_t n_t\right) \right\} \tag{1}$$

s.t.

$$c_t + k_t = Ae^{z_t} k_{t-1}^{\theta} n_t^{1-\theta} + (1-\delta)k_{t-1} \tag{2}$$

and

$$y_t = Ae^{z_t} k_{t-1}^{\theta} n_t^{1-\theta} \tag{3}$$

$$R_t = \theta \frac{y_t}{k_{t-1}} + 1 - \delta \tag{4}$$

$$\ln m_t = (1-\rho)\ln \bar{m} + \rho \ln m_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \quad 0 \le \rho < 1 \tag{5}$$

$$z_t = \phi z_{t-1} + v_t, \quad v_t \sim N(0, \sigma_v^2) \quad 0 \le \phi < 1 \tag{6}$$

where c_t, n_t, k_t denote consumption, labor and capital. m_t is a stochastic process for "labor preference". A, θ, β, δ, ρ and ϕ are paramters with $0 < \beta < 1$, $0 < \theta < 1$, $|\rho| < 1, |\phi| < 1$and $\bar{m}$ is the steady state value of m_t.

1. Describe the economy briefly. Comment on the preference, endowment, technology, and information.

2. If m_t rises, what happens to the marginal utility or disutility?

3. Find the first order necessary conditions (FONCs) of the representative agent for c_t, k_t, and n_t.

4. Write down the model in eight equations, including FONCs, budget constraint, output, interest rate, and processes defining shocks.

5. Solve for the steady states, i.e. provide formulas for $\bar{c}$, $\bar{\lambda}$, $\bar{k}$, $\bar{n}$, $\bar{R}$ and $\bar{y}$, given $\bar{m}$ and other parameters. (Show the steps that you solve these values subsequently, if you cannot get that far, it would be useful to derive $\frac{\bar{y}}{\bar{k}}$., $\frac{\bar{c}}{\bar{k}}$, and $\frac{\bar{n}}{\bar{k}}$)

6. Log-linearize the eight equations. Define $\tilde{x}_t \equiv \log\left(x_t/\bar{x}\right)$. Write down all necessary log-linearized equations characterizing the equilibrium in the notation of log-deviation term $\tilde{x}_t$. (Note: z_t is already in the deviation term for technology factor in the sense of $z_t = \log((Ae^{z_t})/(Ae^0))$.)

7. Classify the exogenous state as z_t and $\hat{m}_t$, endogenous state as $\tilde{k}_t$ and the rest as other endogenous variables. Find algebraic manipulations such that you are left with four equations for the four variables $\tilde{k}_t$, $\hat{\lambda}_t$, z_t and $\hat{m}_t$.

$$\begin{aligned} 0 &= -\tilde{k}_t + \alpha_1 \tilde{k}_{t-1} + \alpha_2 \tilde{\lambda}_t + \alpha_3 \hat{m}_t + \alpha_7 z_t \\ 0 &= E_t \left[-\tilde{\lambda}_t + \alpha_4 \tilde{k}_t + \alpha_5 \tilde{\lambda}_{t+1} + \alpha_6 \hat{m}_{t+1} + \alpha_8 \tilde{z}_{t+1} \right] \\ \tilde{z}_t &= ? \\ \hat{m}_t &= ? \end{aligned} \tag{7a}$$

what are a_1, a_2, ..., till α_8 in terms of the original parameters?

8. Given the following parameter values: $\phi = 0.9$, $\beta = 0.99$, $\rho = 0.96$, $\theta = 0.4$, $\delta = 0.025, \bar{m} = 3.5, A = 0.3$, calculate steady state values and a_1 to a_8 Assume that your model parameters are calibrated from quarterly data.

9. For questions 9 - 11, focus on the shocks of labor preference, so you can ignore the part of technology shock. Guess the recursive law of motion of the above system as:

$$\begin{aligned} \tilde{\lambda}_t &= \eta_{\lambda k} \tilde{k}_{t-1} + \eta_{\lambda m} \hat{m}_t \\ \tilde{k}_t &= \eta_{kk} \tilde{k}_{t-1} + \eta_{km} \hat{m}_t \end{aligned}$$

and exploit $E_t \left[\hat{m}_{t+1} \right] = \rho \hat{m}_t$.

10. By comparing the coefficients on $\tilde{k}_{t-1}$, you can get a characteristic quadratic equation η_{kk} as $a\eta_{kk}^2 + b\eta_{kk} + c = 0$, where a, b, and c are determined by a_1 to a_6. By solving this equation, what are the roots you get? Which root should you choose and why? Use the corresponding η_{kk} to calculate $\eta_{\lambda k}$.

11. By comparing the coefficients on $\hat{m}_t$, solve for η_{km} and $\eta_{\lambda m}$.

12. Now use the Toolkit.4.1 to solve the model. Classify the 8 equations you obtain from Step 6 into the three blocks of equations, what are the endogenous state variables, endogenous other variables ane exogenous variables?

13. Assume a one time shock to $\hat{m}_0$, plot the impulse responses of $\hat{m}_t, \hat{c}_t, \hat{k}_t, \hat{n}_t$ and $\hat{y}_t$.

14. Interpret the results of the previous exercise. Do the results fit the interpretation of the "labor preference shifter" you gave in the second question? Explain briefly.

15. Assume a one time shock to z_0, plot the impulse responses of $z_t, \hat{c}_t, \hat{k}_t, \hat{n}_t$ and $\hat{y}_t$.