Advanced Microeconomics II
Quiz 3

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1. In the following two-person bargaining game only decimal divisions are possible, i.e. the set of possible divisions are $\{(1,0),(.9,.1),\ldots,(.1,.9),(0,1)\}$. Consider the following stationary strategies: For each player i, player i always proposes $x=(x_1,x_2)$ and accepts a proposal y if and only if $y_i \geq x_i$. (Note that the offer x is the same for every player.)
   (a) For what divisions of the pie are such stationary strategies sub-game perfect equilibria if $\delta_1=\delta_2=.9$?
   (b) For what divisions of the pie are such stationary strategies sub-game perfect equilibria if $\delta_1=\delta_2=.85$?
   (c) For what divisions of the pie are such stationary strategies sub-game perfect equilibria if $\delta_1=\delta_2=.8$?

Solution: For any stationary strategy of this nature to be an equilibrium it is sufficient to check that no player prefers to accept a smaller share of the pie in a period where he accepts or rejects than to wait for his equilibrium share when he makes the offer. That is, if we denote x_i' as the next smallest possible division of the pie that player i could receive then we require that

$$\delta x_i \geq x_i' \text{ for all } i.$$

As δ increases, the possible divisions of the pie that are associated with stationary sub-game perfect equilibria increases. The threshold values of δ_1 and δ_2 indicated in tha table below.

Division	δ_1	δ_2	$\max\{\delta_1,\delta_2\}$
(1,0)	0.9	0	0.9
(0.9,0.1)	0.889	0	0.889
(0.8,0.2)	0.875	0.5	0.875
(0.7,0.3)	0.857	0.667	0.857
(0.6,0.4)	0.833	0.75	0.833
(0.5,0.5)	0.8	0.8	0.8
(0.4,0.6)	0.75	0.833	0.833
(0.3,0.7)	0.667	0.857	0.857
(0.2,0.8)	0.5	0.875	0.875
(0.1,0.9)	0	0.889	0.889
(0,1)	0	0.9	0.9

As can be seen from the table: when $\delta_1=\delta_2=0.9$, all divisions of the pie are associated with a stationary sub-game perfect equilibrium; when $\delta_1=\delta_2=0.85$, only $(0.6,0.4)$, $(0.5,0.5)$ and $(0.4,0.6)$ are associated with a stationary sub-game perfect equilibrium; when $\delta_1=\delta_2=0.8$, only $(0.5,0.5)$ is associated with a stationary sub-game perfect equilibrium.