Advanced Microeconomics II
Quiz 2

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1. Suppose that two players choose locations a_1 and a_2 in the unit interval ($[0,1]$); each wishes to be as close as possible to the other, the payoff of each player being $-|a_1 - a_2|$.

   (a) Formulate this situation as a strategic game.

   Solution: $N = \{1, 2\}$, $A_i = [0, 1]$, $u_i(a_i, a_j) = -|a_1 - a_2|$.

   (b) What are the set of rationalizable strategies for each player?

   Solution: a_i is the unique best response to the action $a_j = a_i$. Hence, A_i is the set of rationalizable strategies for player i.

   (c) Is the game dominance solvable? If so what is the predicted outcome of the game?

   Solution: No strategy is weakly dominated - a_i is the unique best response to the action $a_j = a_i$. Hence, the game is not dominance solvable.

   (d) What are the set of Nash equilibria for the game (including mixed strategy Nash equilibria).

   Solution: The best-response function of each player coincides, hence the set of pure strategy equilibria are $\{(a_i, a_j) : a_i = a_j\}$.

   Furthermore, if we consider two actions a and b then it is clear that $\{(\alpha_i, \alpha_j) : \alpha_i(a) = \alpha_i(b) = \alpha_j(a) = \alpha_j(b) = 1/2\}$ are (non-pure) mixed-strategy Nash equilibrium strategy profiles.

   I claim that these are the only Nash equilibria of the game. Let α_j be the mixed strategy of player a_j. Then $BR(\alpha_j)$ is the set of pure actions that solve

   $$max_{a_i} E(-|a_1 - a_2|).$$

   This is equivalent to minimizing the sum of least absolute deviations, to which the solution is the median of α_j , i.e. any a_i such that $\Pr(a_j \leq a_i) \geq 1/2$ and $\Pr(a_j \geq a_i) \geq 1/2$ (for a non-calculus proof refer to "A Simple Noncalculus Proof That the Median Minimizes the Sum of the Absolute Deviations" by Neil C Schwertman, A J Gilks, J Cameron, American Statistician (1990)).

   Hence, the set of best responses is restricted to either a unique action or an interval that includes only two elements of the support of α_j, one at each end of the interval. Since this is true for both players, the maximum and minimum of the support of each player's mixed strategy must be the same. Otherwise one of the players would be playing an action that was not an element of his best-response. For the same reason, each player must play at most two actions with positive probability. Only if those actions have equal probabilty is each action a best response to the other player's strategy.