Advanced Microeconomics II
Problem Set 4

WISE, Xiamen University

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1. A husband and wife must simultaneously choose whether to attend the football game (F) or the movie theatre (M). With probability θ preferences are as stated in game A. With probability $1-\theta$ preferences are as stated in game B. The husband knows which game is being played but the wife does not.

		Husband	
		M	F
Wife	M	$2,1$	$0,0$
	F	$0,0$	$1,2$

A

		Husband	
		M	F
Wife	M	$2,2$	$0,0$
	F	$0,0$	$1,1$

B

   (a) Write down the normal form representation of this static Bayesian game

   (b) Find the set of pure strategy Nash equilibria for this Bayesian Game.

   (c) Find the (non-pure) mixed strategy Nash equilibria for this Bayesian Game.

2. A buyer and a seller are bargaining over an object. The rules of bargaining are that they simultaneously announce prices. If $p_b \geq p_s$, then trade occurs at price $p = \frac{p_b+p_s}{2}$; if $p_b < p_s$, then no trade occurs. The buyer's valuation for the good is v_b, the seller's is v_s. These valuations are private information and are drawn from independent uniform distribution on $[0, 1]$. If there is no trade, both players' utility are 0; if the buyer gets the good for price p, the buyer's utility is $v_b - p$ and the seller's utility is $p - v_s$.

   (a) Construct a 'one-price' Bayesian Nash equilibrium of this game: an equilibrium in which trade occurs at a single price if it occurs at all.

   (b) Compare the efficiency of the equilbrium constructed in (a) and the 'linear' Bayesian Nash equilibrium constructed in class.

   (c) Use the Revelation Principle to construct a Bayesian game with an incentive-compatible equilibrium with the same outcome as the equilibrium in (a).

3. Each of two players receives a ticket on which there is a number in some finite subset S of the interval $[0, 1]$. The number on a player's ticket is the size of a prize that he may receive. The two prizes are identically and independently distributed, with distribution function F. Each player is asked independently and simultaneously whether he wants to exchange his prize for the other player's prize. If both players agree then the prizes are exchanged; otherwise each player receives his own prize. Each player's objective is to maximize his expected payoff.

   (a) Model this situation as a Bayesian game.

   (b) Construct a Bayesian Nash equilibrium where the probability of exhcange is zero.

   (c) Construct a Bayesian Nash equilibrium where the probability of exchange is positive.

4. Players 1 and 2 must decide whether or not to carry an umbrella when leaving home. They know that there is a 50-50 chance of rain. Each player's payoff is -5 if he doesn't carry an umbrella and it rains, -2 if he carries an umbrella and it rains, -1 if he carries an umbrella and it is sunny, and 1 if he doesn't carry an umbrella and it is sunny. Player 1 learns the weather before leaving home; player 2 does not, but he can observe player 1's action before choosing his own. Give the extensive and strategic forms of the game. Is it dominance solvable?

5. There are two firms, firm 1 is the incumbent and firm 2. At the start of the game chance, c, chooses the type of player 1 from one of two possible states. With probability λ firm 1 is "rational", R, and with probability $1-\lambda$ firm 1 is "crazy", C. The firms then interact in the market for two periods. In the first period, firm 1 takes one of two possible actions, fight F or accommodate, A. In the second period, firms simultaneously choose actions. Player 1 again chooses whether to fight or accommodate, while player 2 chooses one of two possible actions, stay, S, or exit, E.

   In each period, firm profits are realized and firms discount the second period profits by the common discount factor δ. If both firms operate in the market then a rational firm 1 makes A_R if it accommodates and F_R if it fights, while a crazy firm 1 makes A_C if it accommodates and F_C if it fights. If only firm 1 operates in the market it makes monopoly profit M_1. Player 2 makes A_2 if he stays and player 1 accommodates, F_2 if if he stays and player one fights and 0 if he exits. Assume that $M_1 > A_R > F_R$, $F_C > M_1 > A_C$ and $A_2 > 0 > F_2$.

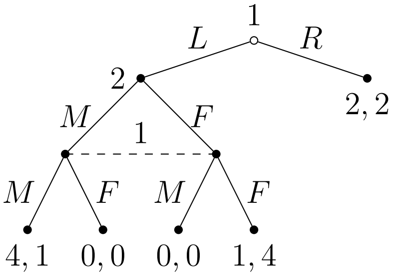
   (a) Write down this problem as an extensive game of incomplete information.

   (b) Find parameter values for the payoffs for which there exists separating perfect Bayesian equilibria in this game.

   (c) Find parameter values for the payoffs for which there exist pooling perfect Bayesian equilibria in this game.

   (d) Find parameter values for the payoffs for which there exist hybrid perfect Bayesian equilibria in this game.

6. Consider the following extensive game:



   (a) Solve for the set of Nash equilibria.

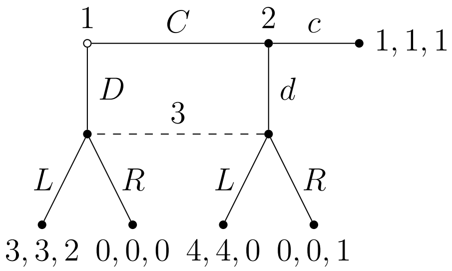
   (b) Solve for the set of subgame perfect Nash equilibria.

   (c) Solve for the set of weak perfect Bayesian equilibria.

   (d) Solve for the set of perfect Bayesian equilibria.

(e) Solve for the set of sequential equilibria.

7. Consider the following extensive game:



(a) Solve for the set of Nash equilibria.

(b) Solve for the set of subgame perfect Nash equilibria.

(c) Solve for the set of weak perfect Bayesian equilibria.

(d) Solve for the set of perfect Bayesian equilibria.

(e) Solve for the set of sequential equilibria.