Advanced Microeconomics II

Bargaining Games

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Bargaining Games

Bargaining occurs whenever

- individuals (players) have the possibility of concluding a mutually beneficial agreement,
- there is a conflict of interests about which agreement to conclude, and
- no agreement may be imposed on any individual without his approval.

Bargaining theory explores the relationship between the outcome of bargaining and the characteristics of the situation. Characteristics include:

- Player preferences, e.g. patience.
- Institutional features, e.g. who sets the agenda, costs of negotiation.

Bargaining Game Model

- Two players need to reach agreement about how best to split a pie, of value 1
  - The set of possible agreements $X = \{(x_1, x_2) : x_1 + x_2 = 1\}$
- Each player prefers more pie. For each player i, $x \succeq_i x'$ if and only if $x_i \geq x'_i$.
- If agreement cannot be reached then both players receive nothing, D.
  - $(0,1) \sim_1 D$ and $(1,0) \sim_2 D$.
- How players reach agreement depends on the structure of the bargaining process.

Bargaining Games - Static

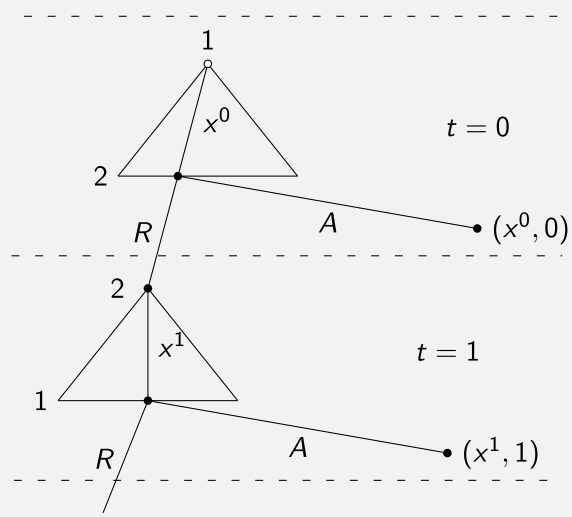
- Dictator Game
  - Let player 1 decide the division of the pie.
    - What are the set of Nash equilibria.
    - What are the set of subgame perfect equilibria.
- Ultimatum Game
  - Let player 1 make a 'one-time offer' to player 2.
  - Player 2 can then choose accept, A, or reject, R.
    - What are the set of Nash equilibria.
    - What are the set of subgame perfect equilibria.

Bargaining Game of Alternating Offers

At time 0, player 1 proposes a split, player 2 accepts or rejects.

- If player 2 accepts, the game ends.
- If player 2 rejects, he makes a proposal at time 1 which player 1 can accept or reject
  - If player 1 accepts, the game ends.
  - If player 1 rejects, he makes a proposal at time 2 which player 2 can accept or reject
    - If player 2 accepts, the game ends.
    - If player 2 rejects, he makes a proposal at time 3 which player 1 can accept or reject
    - ...
- Outcomes are denoted by the split of the pie $x = (x_1, x_2)$ and the time of agreement t.
- If $(x, t) \succeq_1 (y, t)$ then $(y, t) \succeq_2 (x, t)$.
- Time is valuable, agreement (x, t) generates a payoff to player i of $\delta_i^t x_i$.

Alternating Offers Game Tree



Nash Equilibria

Any x^* is a Nash equilibrium.

- The player who makes the offer always offers x^*.
- Player 2 accepts any offer $x : x_2 \geq x_2^*$
- Player 1 accepts any offer $x : x_1 \geq x_1^*$

These strategies are stationary. Offer and acceptance rules don't depend on h.

- What is the outcome?

Subgame Perfect Equilibria?

Any (x^*, t) is a Nash equilibrium.

- The player who makes the offer demands the whole pie if $k \leq t - 1$.
- Both players reject any demand for every period $k \leq t - 1$.
- The player who makes the offer always offers x^* if $k \geq t$.
- Player 2 accepts any offer $x : x_2 \geq x_2^*$ if $k \geq t$.
- Player 1 accepts any offer $x : x_1 \geq x_1^*$ if $k \geq t$.

These strategies are not stationary. Are they SPE strategies?

This game satisfies the one deviation principle. Can we show a profitable one-shot deviation?

- Consider the outcome (x^*, t), where t is odd (player 2 made the offer).
- Player 2 rejecting any demand at $t - 1$ is not part of an SPE.
  - At $t - 1$ player 2 must accept any offer $x_2 > \delta_2 x_2^*$.

Stationarity of Game

- The game is stationary. In every even period the game looks the same. In every odd period the game looks the same.
  - Terminal node payoffs in the subgame starting at time t can be rescaled to match terminal node payoffs in the original game.
  - Define the continuation payoffs of a strategy profile starting at t as the utilities in time-t units of the outcome induced by that profile.
  - E.g., the continuation payoff of player 1 in period 2 of a profile that gives player 1 the whole pie in period 3 is δ_1, whereas this outcome has utility δ_1^3 in time-0 units.
- The set of subgame perfect continuation payoffs is the same in each even period and the same in each odd period.

Subgame Perfect Equilibria

Look for a stationary SPE?

- In each even period player 1 offers x^*, player 2 accepts any offer $x_2 \geq x_2^*$.
- In each odd period player 2 offers y^*, player 1 accepts any offer $y_1 \geq y_1^*$.
- In an odd period player 2 can get y_2^*, so to accept x^*, $x_2^* \geq \delta_2 y_2^*$.
  - Since player 2 must accept any $x_2 \geq \delta_2 y_2^*$ player 1 must offer $x_2^* = \delta_2 y_2^*$.
- In an even period player 1 can get x_1^*, so to accept y^*, $y_1^* \geq \delta_1 x_1^*$.
  - Since player 1 must accept any $y_1 \geq \delta_1 x_1^*$ player 2 must offer $y_1^* = \delta_1 x_1^*$.

Solving gives

$$x^* = \left(\frac{1-\delta_2}{1-\delta_1\delta_2}, \frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}\right) \quad y^* = \left(\frac{\delta_1(1-\delta_2)}{1-\delta_1\delta_2}, \frac{1-\delta_1}{1-\delta_1\delta_2}\right)$$

Uniqueness of Payoffs

There are 2 classes of subgames, G_1 - player 1 makes an offer, and G_2 - player 2 makes an offer.

- Denote $M_i(G_k)$ and $m_i(G_k)$ as the best and worst subgame perfect continuation payoffs for player i in G_k, $i, k \in \{1, 2\}$.
  - $M_2(G_2; \delta_2, \delta_1) = M_1(G_1; \delta_1, \delta_2)$.
  - $m_2(G_2; \delta_2, \delta_1) = m_1(G_1; \delta_1, \delta_2)$.
- Claim: $M_1(G_1) = m_1(G_1) = x_1^*, M_2(G_2) = m_2(G_2) = y_2^*$.
  - $m_1(G_1) \geq 1 - \delta_2 M_2(G_2)$ (Why?)
  - $m_1(G_2) \geq \delta_1 m_1(G_1) \Rightarrow M_2(G_2) \leq 1 - \delta_1 m_1(G_1)$ (Why?)
  - $M_2(G_2) \leq \frac{1-\delta_1}{1-\delta_1\delta_2}, m_1(G_1) \geq \frac{1-\delta_2}{1-\delta_1\delta_2}$
  - $M_1(G_1) \leq \frac{1-\delta_2}{1-\delta_1\delta_2} \leq m_1(G_1), M_2(G_2) \leq \frac{1-\delta_1}{1-\delta_1\delta_2} \leq m_2(G_2)$.
- Claim: $M_1(G_2) = m_1(G_2) = y_1^*, M_2(G_1) = m_2(G_1) = x_2^*$.
  - $y_1^* \leq M_1(G_2) \leq 1 - m_2(G_2), x_2^* \leq M_2(G_1) \leq 1 - m_1(G_1)$.

Payoffs are unique and sum to one.

Uniqueness of Strategy

Since payoffs sum to one and are unique the equilibrium strategy is unique.

- In every SPE of G_1 player 1's initial proposal is x^*, which is immediately accepted by player 2.
  - If agreement was reached later than the initial period, payoffs would not sum to one.
- In every SPE of G_1 player 2 accepts the proposal if $x_2 \geq x_2^*$ and rejects if $x_2 < x_2^*$.
  - A rejection by player 2 leads to G_2 and a payoff of y_2^*.
  - Player 2 should accept if $x_2 > \delta_2 y_2^* = x_2^*$ and reject if $x_2 < \delta_2 y_2^* = x_2^*$.
  - Player 2 must accept if $x_2 = x_2^*$ since otherwise no best response for player 1 exists.
- Similar analysis for the SPE of G_2 shows that SPE strategies are unique.

Iterated Conditional Dominance - First Round

- When player 2 makes an offer, player 1 must accept $y_1 > y_1^1 = \delta_1$.
- When player 1 makes an offer, player 2 must accept $x_2 > x_2^1 = \delta_2$.

Implications

- Player 2 never offers $y_1 > \delta_1$.
- Player 2 rejects any $x_2 < \delta_2(1 - \delta_1)$.
- Player 1 never offers $x_2 > \delta_2$.
- Player 1 rejects any $y_1 < \delta_1(1 - \delta_2)$.

Claim

- Player 1 must accept $y_1 > y_1^2 = \delta_1 - \delta_1\delta_2 + \delta_1^2\delta_2$.
- Player 2 must accept $x_2 > x_2^2 = \delta_2 - \delta_1\delta_2 + \delta_1\delta_2^2$.

Check for Player 1

Claim

- Player 1 must accept $y_1 > y_1^2 = \delta_1 - \delta_1\delta_2 + \delta_1^2\delta_2$.
- Recall
  - Player 2 never offers $y_1 > \delta_1$.
  - Player 2 rejects any $x_2 < \delta_2(1 - \delta_1)$.
- If Player 1 rejects y_1 what are the possible outcomes:
  - Agreement is never reached; worth zero.
  - A player 1 proposal is accepted; worth at most $\delta_1 - \delta_1\delta_2 + \delta_1^2\delta_2$.
  - A player 2 proposal is accepted; worth at most δ_1^3.

Iterated Conditional Dominance in General

Assume

- When player 2 makes an offer, player 1 accepts any $y_1 > y_1^k$.
- When player 1 makes an offer, player 2 accepts any $x_2 > x_2^k$.

Implications

- Player 2 never offers $y_1 > y_1^k$.
- Player 2 rejects any $x_2 < \delta_2(1 - y_1^k)$.
- Player 1 never offers $x_2 > x_2^k$.
- Player 1 rejects any $y_1 < \delta_1(1 - x_2^k)$.

Claim

- Player 1 must accept $y_1 > y_1^{k+1} = \delta_1(1 - \delta_2) + \delta_1\delta_2 y_1^k$.
- Player 2 must accept $x_2 > x_2^{k+1} = \delta_2(1 - \delta_1) + \delta_1\delta_2 x_2^k$.

Check for Player 2

Claim

- Player 2 must accept $x_2 > x_2^{k+1} = \delta_2(1 - \delta_1) + \delta_1\delta_2 x_2^k$.
- Recall
  - Player 1 never offers $x_2 > x_2^k$.
  - Player 1 rejects any $y_1 < \delta_1(1 - x_2^k)$.
- If Player 2 rejects x_2 what are the possible outcomes:
  - Agreement is never reached; worth zero.
  - A player 2 proposal is accepted; worth at most $\delta_2(1 - \delta_1(1 - x_2^k))$.
  - A player 1 proposal is accepted; worth at most $\delta_2^2 x_2^k$.
- The sequence x_2^k is monotone and bounded with limit $\frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}$.

Equilibrium Properties

- Efficiency: agreement is reached immediately.
- Stationarity: SPE strategies are stationary (not history dependent).
- Comparative Statics of Impatience: The more impatient a player the lower his payoff.
  - As $\delta_1 \to 0, x_1^* \to 1 - \delta_2$.
  - As $\delta_2 \to 0, x_2^* \to 0$.
- First Mover Advantage:
  - Other things equal, player 1 gets larger share of the pie.
    - If $\delta_1 = \delta_2 = \delta$ then payoffs are $(\frac{1}{1+\delta}, \frac{\delta}{1+\delta})$.
  - If time periods are very short the advantage disappears.
    - Set $\delta = e^{-\Delta r}$. What are the SPE payoffs as $\Delta \to 0$.

Variants

Robust extensions

- Outside Option: An outside option b for player 2 can affect the outcome.
  - Only if $b > \delta_2 y_2^*$.
- Risk of Breakdown: Equivalent to increasing player impatience.

Non-robust extensions

- Importance of Procedure:
  - If player 1 always makes the offer, he gets all the pie.
  - If players make joint offers, any division is possible.
- More Than Two Players: Multiplicity of equilibria for sufficiently patient players.
- Discrete pie divisions: Multiplicity of equilibria for sufficiently few divisions.

Predictive Ability