

# Advanced Microeconomics II

## Problem Set 1

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1. Consider two heterogeneous products produced by different firms, firm 1 and firm 2. Inverse demand for the products is given by

$$p_1 = a - b(q_1 + \theta q_2); \quad p_2 = a - b(q_2 + \theta q_1),$$

where  $0 < \theta < 1$ . Firms simultaneously choose prices and both firms produce their products at constant marginal cost  $c > 0$ .

- (a) Find the prices that maximize joint firm profits.
  - (b) Write down the normal form of this game.
  - (c) Find the set of Nash equilibria (restrict attention to positive prices and quantities for both firms).
  - (d) Now allow each firm to offer a rebate to consumers if the other firm charges a lower price equal to  $d$  times the difference in price, i.e., if firm 1 charges a price  $p_1$  and firm 2 charges a higher price  $p_2$ , then the net prices faced by consumers would be  $p_1$  for good 1 and  $p_2 - d(p_2 - p_1)$  for good 2.
    - i. Write down the normal form of this game.
    - ii. Find the set of Nash equilibria (again, restrict attention to positive prices and quantities for both firms).
2. The residents of Nirvana live on Main Street, which is one kilometre long and the only road in town. Two firms decide to set up general stores. Each can locate at any point along the road. The two decide independently where to locate and they must remain there forever (both can occupy the same location). Each store will attract the customers who are closest to it and the stores will share equally customers who are equidistant between the two. Each customer contributes \$1.00 in profits each day to the general store it visits.
    - (a) Write the strategic form of this game.
    - (b) Find the set of Nash equilibria for this game.
    - (c) Suppose there are three general stores, each independently choosing a location point along the road. Show that there is no pure-strategy Nash equilibrium.
  3. In this game,  $n$  players each choose a number between 1 and  $X > 1$ . Suppose  $x_i$  is the choice of player  $i$  and the lowest number is  $y$ . Then player  $i$  wins  $X - 1 + 2y - x_i$ .
    - (a) Write down the strategic form for this game.
    - (b) Find the set of Nash equilibria for this game.
  4. Consider the following two player game. Each player  $i$ , independently and simultaneously, chooses a number  $a_i$  between -1 and 1. Player 1 receives  $a_1 \times a_2$ , player 2 receives  $-a_1 \times a_2$ .

- (a) Show that this is a strictly competitive game.
- (b) Prove that the game has a Nash equilibrium.
- (c) Fully characterize one Nash equilibrium of the game. (You may repeat the answer to [b](#) if sufficient.) Is this the unique Nash equilibrium?
- (d) What is the value of the game?