Advanced Microeconomics II
Quiz 4

WISE, Xiamen University

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1. Consider the following auction process. There are 2 people bidding for an item. The bidder's valuations are private information and are drawn from independent uniform distributions on $[0, 1]$. The person who makes the highest bid wins the item. If both players make the same bid then they each win the item with probability $1/2$. Each player must pay whatever they bid. Bids are restricted to be non-negative.

   (a) (5 points) Give the Normal Form representation of this static Bayesian game.

   Solution: $N = \{1, 2\}$, $A_i = [0, +\infty), i \in N$, $T_i = [0, 1], i \in N$, $p(t)$ is uniformly distrubted over $[0, 1]^2$.

   $$u_i(a_1, a_2, t_1, t_2) = \begin{cases} t_i - a_i & \text{if } a_i > a_j \\ \frac{t_i}{2} - a_i & \text{if } a_i = a_j \\ -a_i & \text{if } a_i < a_j, \end{cases}$$

   where j is the other bidder.

   (b) (5 points) Find a symmetric Bayesian equilibrium for this game.

   Solution: Let b be a strictly increasing bidding function for both players. A bid of a_i wins with probability $\Pr(a_i > b(t_j)) = \Pr(b^{-1}(a_i) > t_j) = b^{-1}(a_i)$ since t_j is uniformly distributed between 0 and 1. Given a value t_i, the optimal bid solves

   $$\max_{a_i} t_i b^{-1}(a_i) - a_i$$

   which implies that in equilibrium

   $$t_i \frac{\partial b^{-1}(a_i)}{\partial a_i} = 1.$$

   Furthermore, in equilibrium $a_i = b(t_i)$. Hence, $t_i = b'(t_i)$ or $b(t_i) = t^2/2 + c$. Noting that bidder whose value for the object is zero must be zero implies that $b(t_i) = t^2/2$.