Correlated Equilibrium and Nash Equilibrium

Proposition

For every mixed strategy Nash equilibrium α of a finite strategic game $\{N, (A_i), (u_i)\}$ there is a correlated equilibrium $\{(\Omega, \pi), (\mathcal{P}_i), (\sigma_i)\}$ in which for each player $i \in N$ the distribution on A_i induced by σ_i is α_i.

- Set $\Omega = A$ and $\pi(a) = \Pi_{j \in N} \alpha_j(a_j)$.
- For each $i \in N$ and $b_i \in A_i$ set $P_i(b_i) = \{a \in A : a_i = b_i\}$ and let $\mathcal{P}_i$ consist of the $|A_i|$ sets $P_i(b_i)$.
- Define $\sigma_i(a) = a_i$ for each $a \in A$.
- This is a correlated equilibrium since for each player i, for each $a_i \in A_i$

$$\sum_{\{b \in A : b_i = a_i\}} \pi(b) u_i(a_i, b_{-i}) \geq \sum_{\{b \in A : b_i = a_i\}} \pi(b) u_i(b_i', b_{-i}) \text{ for any } b_i' \in A_i.$$

- The distribution on A_i induced by σ_i is α_i.