Kakutani's Fixed Point Theorem

Lemma

Let X be a compact convex subset of $\mathbf{R^n}$ and let $f : X \to X$ be a set-valued function for which

- *for all $x \in X$ the set $f(x)$ is nonempty and convex*
- *the graph of f is closed (i.e. for all sequences $\{x_n\}$ and $\{y_n\}$ such that $y_n \in f(x_n)$ for all n, $x_n \to x$, $y_n \to y$, we have $y \in f(x)$).*

Then there exists $x^* \in X$ such that $x^* \in f(x^*)$.