

Zero-sum Games and Nash Equilibrium

Proposition

Let $G = \{\{1, 2\}, (A_i), (u_i)\}$ be a zero-sum strategic game.

- 1 If (x^*, y^*) is a Nash equilibrium of G then x^* is a maximinimizer for player 1 and y^* is a maximinimizer for player 2.
- 2 If (x^*, y^*) is a Nash equilibrium of G then $\max_x \min_y u_1(x, y) = \min_y \max_x u_1(x, y)$, and thus all Nash equilibria of G yield the same payoffs.
- 3 If $\max_x \min_y u_1(x, y) = \min_y \max_x u_1(x, y)$, x^* is a maximinimizer for player 1, and y^* is a maximinimizer for player 2, then (x^*, y^*) is a Nash equilibrium of G .