Solution to Quiz 1

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An expected utility maximizing individual has constant relative risk-aversion utility, $U(W) = W^{\gamma}/\gamma$ with relative risk-aversion coefficient of $\gamma = -1$. The individual currently owns a product that has a probability p of failing, an event that would result in a loss of wealth that has a present value equal to L. With probability $1 - p$ the product will not fail and no loss will result. The individual is considering whether to purchase an extended warranty on this product. The warranty costs C and would insure the individual against loss if the product fails. Assuming that the cost of the warranty exceeds the expected loss from the product's failure, determine the individual's level of wealth at which she would be just indifferent between purchasing or not purchasing the warranty.

Solution: Suppose the wealth level that makes the individual indifferent between purchasing or not purchasing the warranty is W_0.

1. If the individual purchases the warranty, which would insure the individual against loss if the product fails, he will always own the wealth W_0 with probability 1. Thus the vNM utility in this case is

$$U(W_0 - C)$$

2. If he does not, the wealth would be

$$\tilde{W} = \begin{cases} W_0 - L & p \\ W_0 & 1-p \end{cases}$$

thus the vNM utility is

$$E[U(\tilde{W})] = pU(W_0 - L) + (1-p)U(W_0)$$

3. The individual is indifferent between purchasing or not purchasing, i.e.

$$U(W_0 - C) = E[U(\tilde{W})] = pU(W_0 - L) + (1-p)U(W_0) \tag{1}$$

substituting $U(W) = \frac{W^\gamma}{\gamma}$, and $\gamma = -1$ into 1, and solve for W_0, we could get

$$W_0 = \frac{(1-p)LC}{C - pL}$$

In conclusion, when the wealth level is $W_0 = \frac{(1-p)LC}{C-pL}$, the individual is indifferent. ■