

Advanced Microeconomics II

Quiz 4

WISE, Xiamen University

Spring 2012

1. A worker knows her talent $\theta \in \{1, 2\}$, while her employer does not. A worker has productivity 1 with probability $1/2$ and productivity 2 with probability $1/2$. The value of the worker to the employer is θ , but the employer pays the worker a wage w that is equal to the expectation of θ (there is a competitive labour market). The timing of the game is as follows:

1. The worker chooses an amount of education $e \in [0, \infty)$.
2. Employer makes an offer $w \in [0, \infty)$ to the worker.

Payoffs: The worker's payoff is $w - e/\theta$ and the employer's payoff is $-(w - \theta)^2$.

- (a) Construct a separating equilibrium of this game. Clearly specify the beliefs of the firm both on and off the equilibrium path that support the behavioural strategies of your equilibrium.

Solution: Recall that in this model, it is optimal for the firm to pay the worker his expected productivity. To be a separating equilibrium we require that the low productivity type worker has no incentive to imitate the high type and the high type has no incentive to imitate the low type. Furthermore, along the equilibrium path, firms know the workers productivity, hence, low productivity type workers will obtain no education in equilibrium. Their payoff will be 1. High productivity type workers will obtain the education level e and receive a wage payment of 2. To be a separating equilibrium we require that signals are incentive compatible (IC), that is,

$$\text{IC of low productivity type : } 1 \geq 2 - e.$$

$$\text{IC of high productivity type : } 2 - e/2 \geq 1.$$

These two inequalities imply that $1 \leq e \leq 2$. Note that to support the behavioural strategies it is sufficient that, off the equilibrium path, the firm believes that the worker is a low productivity type with probability 1.

- (b) Amongst the separating equilibria, what is the range of possible education levels for high types?

Solution: Amongst the set of separating equilibria identified above, the range of possible education levels for high types is $[1, 2]$.

- (c) Construct a hybrid equilibrium where the expected worker productivity is 1.25 when the observed education level is zero.

Solution: Since expected productivity when the observed education is zero is below the ex-ante expected productivity of 1.5, this hybrid equilibrium involves randomization between 0 and e by high types, while low types obtain zero education. To randomize between 0 and e , high types must be indifferent between the two payoffs. Hence,

$$1.25 = 2 - e/2 \Rightarrow e = 1.5.$$

To make sure that expected productivity is 1.25 high types must choose zero education with the appropriate probability, that is,

$$1.25 = \frac{(.5)\lambda(2) + (.5)(1)}{(.5)\lambda + (.5)} \Rightarrow \lambda = \frac{1}{3}.$$

Again, to support the behavioural strategies it is sufficient that off the equilibrium path ($e \notin \{0, 0.75\}$) the firm believes that the worker is a low productivity type with probability 1.