Advanced Macroeconomics II
Problem Set 6

Question 1. Optimal allocation of consumption expenditure. (5 points)
Suppose a household consumes a basket of all goods Note that we have defined the following

$$C_t = \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$$

where ε is the elasticity of substitution between any two given varieties i and j. Now solve the minimization problem of consumption expenditure Z_t given a unit of consumption basket

$$\min_{C(i)} Z_t = \int_0^1 P_t(i) C_t(i) di$$

$$s.t. \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} = 1$$

1. Set up a Lagrangean equation to solve for the FONC of $C_t(i)$.(1 point)(Note: when taking FONC, the $C_t(i)$ is for a specific good i in the interval [0,1]).
ANS:
The Lagrangean function is

$$L(C_t(i)) = \int_0^1 P_t(i) C_t(i) di - \lambda_t \left[\left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} - 1\right]$$

FONC for $C_t(i)$ is

$$\begin{aligned} \frac{\partial L(C_t(i))}{\partial C_t(i)} &= P_t(i) - \lambda_t \left[\frac{\epsilon}{\epsilon - 1}\left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}-1} (1 - \frac{1}{\epsilon}) C_t(i)^{-\frac{1}{\epsilon}}\right] \\ &= P_t(i) - \lambda_t C_t^{\frac{1}{\epsilon}} C_t(i)^{-\frac{1}{\epsilon}} = 0 \end{aligned}$$

$$P_t(i) = \lambda_t C_t^{\frac{1}{\epsilon}} C_t(i)^{-\frac{1}{\epsilon}}$$

2. What is the relationship between consumption of goods i and j with respect to their prices $P_t(i)$ and $P_t(j)$? (1 point)
ANS:
From the equation

$$P_t(i) = \lambda_t C_t^{\frac{1}{\epsilon}} C_t(i)^{-\frac{1}{\epsilon}}$$

divided by $P_t(j)$, we can get that

$$\begin{aligned} \frac{P_t(i)}{P_t(j)} &= \left(\frac{C_t(i)}{C_t(j)}\right)^{-\frac{1}{\epsilon}} \\ C_t(i) &= \left(\frac{P_t(i)}{P_t(j)}\right)^{-\varepsilon} C_t(j) \end{aligned}$$

3. Substitute the result from the last step back to the expenditure function and the unit consumption basket. Prove that under unit consumption,

$$Z_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$$

Since this is the expenditure for one unit of composition consumption, we can define it as a price index for the consumption basket,i.e.$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$.(3 points)

ANS:

Firstly, substitute $C_t(i) = \left(\frac{P_t(i)}{P_t(j)}\right)^{-\varepsilon} C_t(j)$ into the unit consumption basket

$$\begin{aligned} C_t &= \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} \\ &= \left[\int_0^1 \left(\frac{P_t(i)}{P_t(j)}\right)^{1-\varepsilon} C_t(j)^{1-\frac{1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} \\ &= C_t(j)P_t(j)^{\varepsilon}\left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{\frac{\epsilon}{\epsilon-1}} = 1 \end{aligned}$$

Under unit consumption, we can get that

$$C_t(j)P_t(j)^{\varepsilon} = \left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{-\frac{\epsilon}{\epsilon-1}}$$

Then substitute $C_t(i) = \left(\frac{P_t(i)}{P_t(j)}\right)^{-\varepsilon} C_t(j)$ into the expenditure function and combine with the above equation,

$$\begin{aligned} Z_t &= \int_0^1 P_t(i)\left(\frac{P_t(i)}{P_t(j)}\right)^{-\varepsilon} C_t(j) di \\ &= C_t(j)P_t(j)^{\varepsilon}\int_0^1 P_t(i)^{1-\epsilon} di \\ &= \left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}} \end{aligned}$$

Question 2. A Basic New Keyesian Model (20 points)

$$\begin{aligned} \hat{\pi}_t &= \beta E_t\{\hat{\pi}_{t+1}\} + \kappa\tilde{y}_t && (1) \\ \tilde{y}_t &= -\tfrac{1}{\sigma}\left(\hat{\imath}_t - E_t\{\hat{\pi}_{t+1}\} - \hat{r}_t^n\right) + E_t\{\tilde{y}_{t+1}\} && (2) \\ \hat{\imath}_t &= \phi_\pi\hat{\pi}_t + \phi_y\tilde{y}_t + v_t && (3) \\ \hat{r}_t^n &= \sigma\psi_{ya}^n E_t\{\Delta a_{t+1}\} && (4) \\ v_t &= \rho_v v_{t-1} + \epsilon_t^v && (5) \\ a_t &= \rho_a a_{t-1} + \epsilon_t^a && (6) \end{aligned}$$

We have discussed in detail this basic New Keyesian model and the notations here are the same as in class. We derived the solution of this model with undetermined coefficients method with monetary policy shock.

1. Suppose we now only have technology shock, while the monetary shock remains zero. Please derive the solutions to output gap, inflation, output, employment, nominal interest rate, real interest rate. You should present the intermediate steps of your derivation in order to get the full credit. (3 points).

ANS:

Firstly, combine equation (2) and (3), then the DIS becomes

$$\tilde{y}_t = -\tfrac{1}{\sigma}\left(\phi_\pi\hat{\pi}_t + \phi_y\tilde{y}_t - E_t\{\hat{\pi}_{t+1}\} - (\hat{r}_t^n - v_t)\right) + E_t\{\tilde{y}_{t+1}\}$$

Rewrite the NKPC and DIS into a system of difference equations of $\begin{bmatrix} \tilde{y}_t & \hat{\pi}_t \end{bmatrix}'$

$$\left(1 + \tfrac{\phi_y}{\sigma}\right)\tilde{y}_t + \tfrac{\phi_\pi}{\sigma}\hat{\pi}_t = E_t\{\tilde{y}_{t+1}\} + \tfrac{1}{\sigma}E_t\{\hat{\pi}_{t+1}\} + \tfrac{1}{\sigma}\left(\hat{r}_t^n - v_t\right)$$

Write in a matrix form

$$\underbrace{\begin{bmatrix} 1+\frac{\phi_y}{\sigma} & \frac{\phi_\pi}{\sigma} \\ -\kappa & 1 \end{bmatrix}}_{M} \begin{bmatrix} \tilde{y}_t \\ \hat{\pi}_t \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{bmatrix} \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\hat{\pi}_{t+1}\} \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma} \\ 0 \end{bmatrix} (\hat{r}_t^n - v_t)$$

$$\begin{bmatrix} \tilde{y}_t \\ \hat{\pi}_t \end{bmatrix} = M^{-1} \begin{bmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{bmatrix} \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\hat{\pi}_{t+1}\} \end{bmatrix} + M^{-1} \begin{bmatrix} \frac{1}{\sigma} \\ 0 \end{bmatrix} (\hat{r}_t^n - v_t)$$

$$M^{-1} = \frac{\underbrace{\begin{bmatrix} 1 & -\frac{\phi_\pi}{\sigma} \\ \kappa & 1+\frac{\phi_y}{\sigma} \end{bmatrix}}}{\det(M)} = \begin{bmatrix} \sigma & -\phi_\pi \\ \kappa\sigma & \sigma+\phi_y \end{bmatrix} \bullet \underbrace{\frac{1}{\sigma+\phi_y+\kappa\phi_\pi}}_{\Omega}$$

$$A_T \equiv M^{-1} \begin{bmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{bmatrix} = \Omega \begin{bmatrix} \sigma & 1-\beta\phi_\pi \\ \kappa\sigma & \kappa+\beta(\sigma+\phi_y) \end{bmatrix}$$

$$B_T \equiv M^{-1} \begin{bmatrix} \frac{1}{\sigma} \\ 0 \end{bmatrix} = \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

The system can be written as

$$\begin{bmatrix} \tilde{y}_t \\ \hat{\pi}_t \end{bmatrix} = A_T \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\hat{\pi}_{t+1}\} \end{bmatrix} + B_T(\hat{r}_t^n - v_t)$$

with the constraint

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$$

Assume a VAR(1) process of a_t,

$$a_t = \rho_a a_{t-1} + \epsilon_t^a, \rho_a \in [0,1).$$

In the DIS equation,

$$\begin{aligned} \hat{r}_t^n &\equiv \sigma E_t\{\triangle\widehat{y}_{t+1}^n\} = \sigma\psi_{ya}^n E_t\{\triangle a_{t+1}\} \\ &= -\sigma\psi_{ya}^n(1-\rho_a)a_t \\ E_t\{\hat{r}_{t+1}^n\} &= \rho_a \hat{r}_t^n \end{aligned}$$

Setting the monetary shock $v_t = 0, \forall t$.
Use the undertemined coefficient method w.r.t $\hat{r}_t^n$,

$$\begin{aligned} \tilde{y}_t &= \psi_{yr}\hat{r}_t^n \\ \hat{\pi}_t &= \psi_{\pi r}\hat{r}_t^n \end{aligned}$$

Insert the solution into

$$\left(1+\frac{\phi_y}{\sigma}\right)\tilde{y}_t + \frac{\phi_\pi}{\sigma}\hat{\pi}_t = E_t\{\tilde{y}_{t+1}\} + \frac{1}{\sigma}E_t\{\hat{\pi}_{t+1}\} + \frac{1}{\sigma}(\hat{r}_t^n - v_t)$$

$$\left(1+\frac{\phi_y}{\sigma}\right)\psi_{yr} + \frac{\phi_\pi}{\sigma}\psi_{\pi r} = \rho_a\psi_{yr} + \frac{\rho_a}{\sigma}\psi_{\pi r} + \frac{1}{\sigma}$$

$$\left[\sigma(1-\rho_a)+\phi_y\right]\psi_{yr}=\left(\rho_a-\phi_\pi\right)\psi_{\pi r}+1 \tag{7}$$

For the NKPC equation,

$$\begin{aligned}
\hat{\pi}_t &= \beta E_t\left\{\hat{\pi}_{t+1}\right\}+\kappa\tilde{y}_t \\
\psi_{\pi r} &= \beta\rho_a\psi_{\pi r}+\kappa\psi_{yr}
\end{aligned}$$

$$\psi_{yr}=\frac{1-\beta\rho_a}{\kappa}\psi_{\pi r} \tag{8}$$

Insert equation (8) into (7)

$$\left[\sigma(1-\rho_a)+\phi_y\right]\frac{1-\beta\rho_a}{\kappa}\psi_{\pi r}=\left(\rho_a-\phi_\pi\right)\psi_{\pi r}+1$$

$$\psi_{\pi r}=\kappa\cdot\underbrace{\frac{1}{\left(1-\beta\rho_a\right)\left[\sigma(1-\rho_a)+\phi_y\right]+\kappa\left(\phi_\pi-\rho_a\right)}}_{\Lambda_a>0}$$

Therefore we can get the inflation and output gap equations,

$$\begin{aligned}
\hat{\pi}_t &= \kappa\Lambda_a\cdot\hat{r}_t^n \\
&= -\sigma\psi_{ya}^n(1-\rho_a)\kappa\Lambda_a a_t
\end{aligned}$$

$$\begin{aligned}
\tilde{y}_t &= \left(1-\beta\rho_a\right)\Lambda_a\hat{r}_t^n \\
&= -\sigma\psi_{ya}^n(1-\rho_a)\left(1-\beta\rho_a\right)\Lambda_a a_t
\end{aligned}$$

where

$$\Lambda_a\equiv\frac{1}{\left(1-\beta\rho_a\right)\left[\sigma(1-\rho_a)+\phi_y\right]+\kappa\left(\phi_\pi-\rho_a\right)}>0$$

$$\begin{aligned}
\widehat{y_t} &= \widehat{y_t^n}+\tilde{y}_t=\psi_{ya}^n a_t-\sigma\psi_{ya}^n(1-\rho_a)\left(1-\beta\rho_a\right)\Lambda_a a_t \\
&= \psi_{ya}^n\left[1-\sigma(1-\rho_a)\left(1-\beta\rho_a\right)\Lambda_a\right]a_t
\end{aligned}$$

$$\begin{aligned}
(1-\alpha)\,\widehat{n}_t &= \widehat{y}_t-a_t \\
&= \left[\left(\psi_{ya}^n-1\right)-\sigma\psi_{ya}^n(1-\rho_a)\left(1-\beta\rho_a\right)\Lambda_a\right]a_t \\
\widehat{n}_t &= \frac{1}{1-\alpha}\left[\left(\psi_{ya}^n-1\right)-\sigma\psi_{ya}^n(1-\rho_a)\left(1-\beta\rho_a\right)\Lambda_a\right]a_t
\end{aligned}$$

$$\begin{aligned}
\hat{\imath}_t &= \phi_\pi\hat{\pi}_t+\phi_y\tilde{y}_t+v_t=\phi_\pi\kappa\Lambda_a\hat{r}_t^n+\phi_y\left(1-\beta\rho_a\right)\Lambda_a\hat{r}_t^n \\
&= -\left[\phi_\pi\kappa+\phi_y\left(1-\beta\rho_a\right)\right]\sigma\psi_{ya}^n(1-\rho_a)\Lambda_a a_t
\end{aligned}$$

$$\begin{aligned}
\widehat{r}_t &= \widehat{\imath}_t-E_t\left\{\hat{\pi}_{t+1}\right\}=\phi_\pi\kappa\Lambda_a\hat{r}_t^n+\phi_y\left(1-\beta\rho_a\right)\Lambda_a\hat{r}_t^n-\rho_a\kappa\Lambda_a\hat{r}_t^n \\
&= -\left[\kappa\left(\phi_\pi-\rho_a\right)+\phi_y\left(1-\beta\rho_a\right)\right]\sigma\psi_{ya}^n(1-\rho_a)\Lambda_a a_t \\
&= -\left[1-\sigma(1-\rho_a)\left(1-\beta\rho_a\right)\Lambda_a\right]\sigma(1-\rho_a)\psi_{ya}^n a_t
\end{aligned}$$

To sum up:
When assume an AR(1) process of technology shock a_t,

$$
\begin{aligned}
\tilde{y}_t &= -\sigma\psi_{ya}^n(1-\rho_a)(1-\beta\rho_a)\Lambda_a a_t \\
\hat{\pi}_t &= -\sigma\psi_{ya}^n(1-\rho_a)\kappa\Lambda_a a_t \\
\widehat{y}_t &= \psi_{ya}^n\left[1-\sigma(1-\rho_a)(1-\beta\rho_a)\Lambda_a\right]a_t \\
\widehat{n}_t &= \frac{1}{1-\alpha}\left[\left(\psi_{ya}^n-1\right)-\sigma\psi_{ya}^n(1-\rho_a)(1-\beta\rho_a)\Lambda_a\right]a_t \\
\hat{\imath}_t &= -\left[\phi_\pi\kappa+\phi_y(1-\beta\rho_a)\right]\sigma\psi_{ya}^n(1-\rho_a)\Lambda_a a_t \\
\widehat{r}_t &= -\left[1-\sigma(1-\rho_a)(1-\beta\rho_a)\Lambda_a\right]\sigma(1-\rho_a)\psi_{ya}^n a_t \\
\Lambda_a &\equiv \frac{1}{(1-\beta\rho_a)\left[\sigma(1-\rho_a)+\phi_y\right]+\kappa(\phi_\pi-\rho_a)} > 0
\end{aligned}
$$

2. Comment on the responses of each shock. Notice what are the differences with respect to a classical monetary model. (3 points).
ANS:
For a positive technology shock ($\Delta a_t > 0$)

1. As long as $\rho_a < 1$, inflation and output gap will decline.
2. The signs of the response of output, employment, real interest rate, and nominal interest rate are in general, ambiguous.

For the positive monetary shock,
Positive Monetary Shock
We have derived in class that if there is only monetary shock, then

$$
\begin{aligned}
\tilde{y}_t &= -(1-\beta\rho_v)\Lambda_v v_t \\
\hat{\pi}_t &= -\kappa\Lambda_v v_t
\end{aligned}
$$

where

$$\Lambda_v = \frac{1}{(1-\beta\rho_v)\{\sigma(1-\rho_v)+\phi_y\}+\kappa(\phi_\pi-\rho_y)}$$

if $\kappa(\phi_\pi-1)+(1-\beta)\phi_y > 0$, then Λ_v is greater than zero. As $a_t \equiv 0$,

1. Real interest rate: $\hat{r}_t = \sigma(1-\rho_v)(1-\beta\rho_v)\Lambda_v v_t$, yet $\hat{r}_t^n = 0$.
2. Nominal interest rate: $\hat{\imath}_t = \{\sigma(1-\rho_v)(1-\beta\rho_v)-\rho_v\kappa\}\Lambda_v v_t$
3. Output: since $y_t^n = 0$, output equals output gap.
4. Employment: $n_t = \frac{1}{1-\alpha}y_t = -\frac{(1-\beta\rho_v)\Lambda_v}{1-\alpha}v_t$

Since the sign of Λ_v is ambiguous, we are not sure about the responses of each variables when there's a positive monetary shock.
In classic monetary model, money is neutral; but in this basic New Keynesian model, there is monetary shock—which means that when $v_t \neq 0$, real variables will be affected by price level.

3. Solve the model with both shocks using the toolkit software. You can download it (version4.1) from http://www2.wiwi.hu-berlin.de/institute/wpol/html/toolkit.htm.
Cast the above system of equations into the following three blocks of equations

$$
\begin{aligned}
0 &= Ax_t + Bx_{t-1} + Cy_t + Dz_t & (9)\\
0 &= E_t\left[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t\right] & (10)\\
z_{t+1} &= Nz_t + \epsilon_{t+1}; E_t\left[\epsilon_{t+1}\right] = 0 & (11)
\end{aligned}
$$

1) What is the order of your equations, and how do you define vector of x_t, y_t, z_t?(1 point)
ANS:
Suppose there is an endogenous state vector $\mathbf{x}_t$ of size $m \times 1$, a list of other endogenous variables (jumps variables) $\mathbf{y}_t$, of size $n \times 1$, and a list of exogenous stochastic processes $\mathbf{z}_t$, of size $k \times 1$. It is assumed that C is of size $l \times n$, $l \geq m$ and of rank n, that F is of size $(m+n-l) \times n$, and that N has only stable eigenvalues. The case $l < n$ can be treated such that one can redeclare some other endogenous variables to be state variables instead, i.e. to raise m and lower n, until $l = n$.
The order of the difference equations is one. For the system in (1) to (6), to meet above requirement, artificially set
(1) x_t: $\tilde{y}_t$, $\hat{\pi}_t$
(2) y_t: $\hat{\imath}_t$, $\hat{r}_t^n$
(3) z_t: v_t, a_t.
2)What are the corresponding matrices $A, B, C, D, F, G, H, J, K, L, M$ and N?(3 points).
So we can now cast the system of equations (1) through (6) into the following three blocks of equations:

$$
\begin{aligned}
\begin{pmatrix} 0 \\ 0 \end{pmatrix} &= \begin{pmatrix} \phi_y & \phi_\pi \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{y}_t \\ \hat{\pi}_t \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{\imath}_t \\ \hat{r}_t^n \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & \sigma\psi_{ya}(1-\rho_a) \end{pmatrix} \begin{pmatrix} v_t \\ a_t \end{pmatrix} \\
\begin{pmatrix} 0 \\ 0 \end{pmatrix} &= \begin{pmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{pmatrix} \begin{pmatrix} E_t[\tilde{y}_{t+1}] \\ E_t[\hat{\pi}_{t+1}] \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ \kappa & -1 \end{pmatrix} \begin{pmatrix} \tilde{y}_t \\ \hat{\pi}_t \end{pmatrix} + \begin{pmatrix} -\frac{1}{\sigma} & \frac{1}{\sigma} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{\imath}_t \\ \hat{r}_t^n \end{pmatrix} \\
\begin{pmatrix} v_{t+1} \\ a_{t+1} \end{pmatrix} &= \begin{pmatrix} \rho_v & 0 \\ 0 & \rho_a \end{pmatrix} \begin{pmatrix} v_t \\ a_t \end{pmatrix} + \begin{pmatrix} \epsilon_{t+1}^v \\ \epsilon_{t+1}^a \end{pmatrix}
\end{aligned}
$$

so we conclude that

1. $A = \begin{pmatrix} \phi_y & \phi_\pi \\ 0 & 0 \end{pmatrix}$, $C = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 0 \\ 0 & \sigma\psi_{ya}(1-\rho_a) \end{pmatrix}$, $F = \begin{pmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{pmatrix}$, $G = \begin{pmatrix} -1 & 0 \\ \kappa & -1 \end{pmatrix}$, $K = \begin{pmatrix} -\frac{1}{\sigma} & \frac{1}{\sigma} \\ 0 & 0 \end{pmatrix}$, $N = \begin{pmatrix} \rho_v & 0 \\ 0 & \rho_a \end{pmatrix}$

   $B, H, J, L, M = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

3)Modify an example m file in the toolkit directory to fit your model with the following parameter values: $\beta = 0.99, \sigma = 1, \varphi = 1, \alpha = 1/3, \varepsilon = 6, \theta = 0.8, \phi_\pi = 1.5, \phi_y = 0.5, \rho_v = 0.5,$
$\rho_a = 0.95$.
What are the intermediate parameter values such as ψ_{ya}^n and κ?(2 points).
ANS:
By the deductions in Gali (2008, Chapter 3), we have

$$
\begin{aligned}
\psi_{ya}^n &= \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} \\
\kappa &= \lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)
\end{aligned}
$$

given the value of the parameters, $\psi_{ya}^n = 1$, $\kappa = 0.0390$.
4) Plot impulse responses of output gap, inflation, nominal rate, real rate to a one standard deviation shock of monetary policy. Assume $var(\varepsilon_t^v) = \sigma_v = 0.25$.Check your results with our derivation in the class and see whether the signs and scales are correct. (1 point)
ANS:
See Figure 1. Real interest rate responds only to technological shocks.
5) Plot impulse responses of the above variables to a positive technology shock. Assume $var(\varepsilon_t^a) = \sigma_a = 1$. Check your results with your derivation and see whether the signs and scales are correct. (1 point)
ANS:
See Figure 2.
Vary the parameters to the baseline model parameters given in step 3) to see the differences made by some crucial parameters.

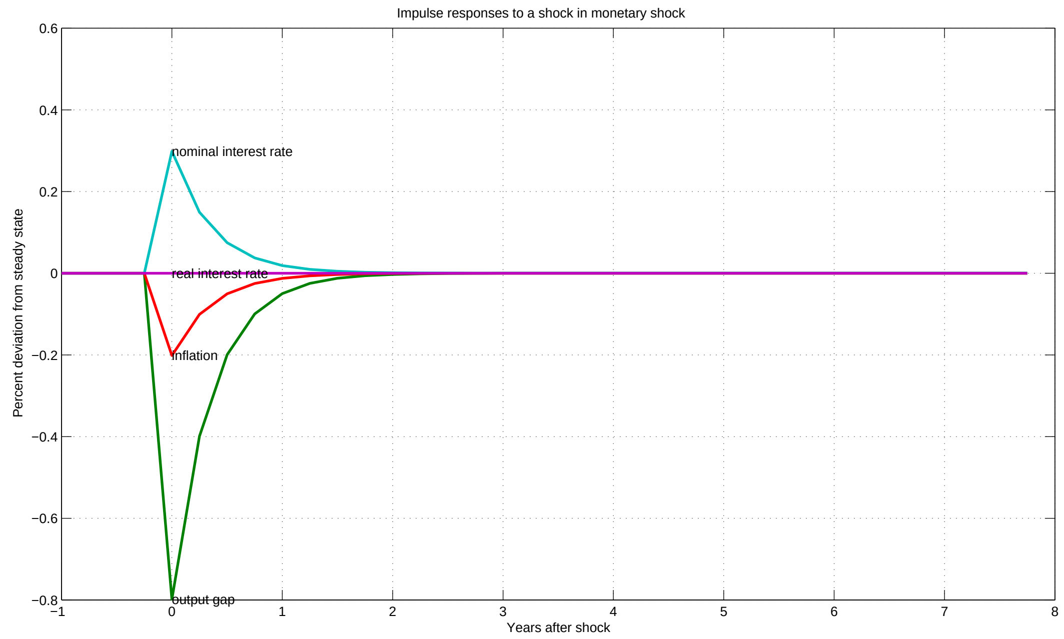

Figure 1: Impulse responses of output gap, inflation, nominal rate, real rate to a one-standard deviation shock of monetary policy.

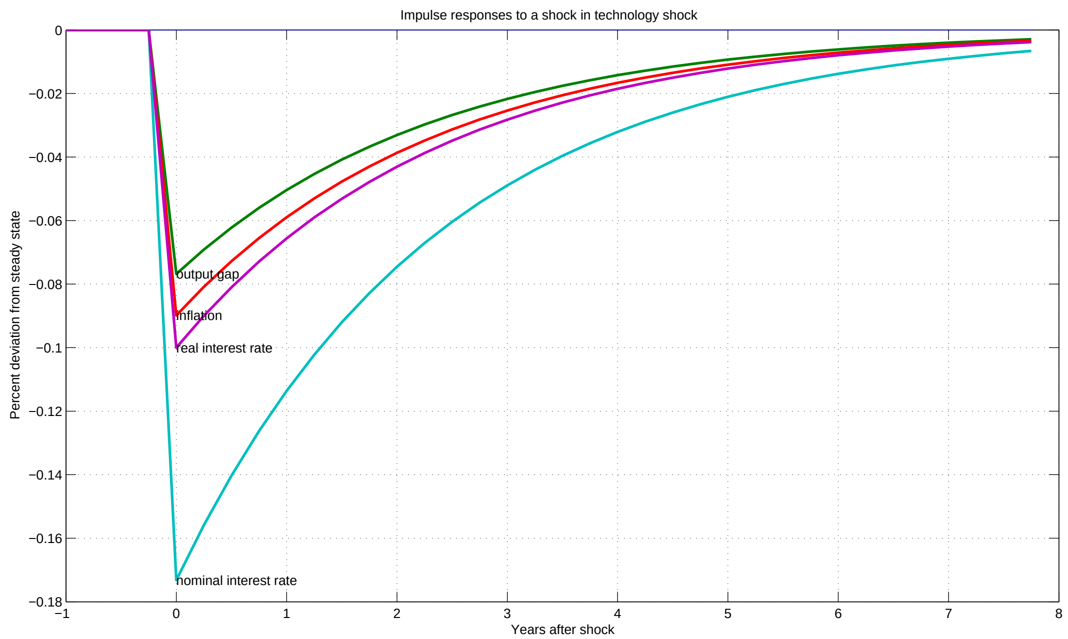

Figure 2: Impulse responses of Impulse responses of output gap, inflation, nominal rate, real rate to a one-standard deviation shock technology shock.

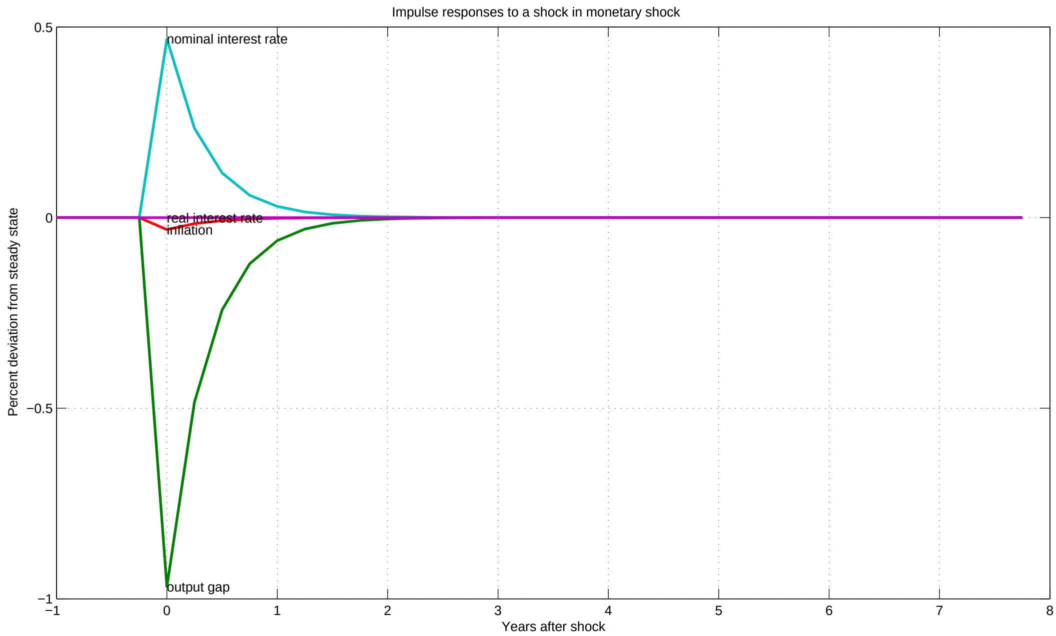

Figure 3: Impulse responses of output gap, in.ation, nominal rate, real rate to a one standard deviation shock of monetary policy ($\epsilon = 60$)

6) Set $\varepsilon = 20$ and plot the impulse responses to the two shocks again, what are the main differences and comment. (2 points)

ANS:

See figure 3 and 4. We can see that when the elasticities of substitution (ϵ) increases, (1) for a one-standard-deviation change in monetary shock, nominal interest rate has sharp increase, and output gap declines, while the real interest rate and inflation remains almost the original level; (2) for a one-standard-deviation change in technological shock, output gap narrows, but the nominal interest rate experiences a very large decrease.

7) Set $\theta = 0$ and plot the impulse responses to the two shocks again, what are the main differences and comment. (2 points)

ANS:

See figure 5 and 6. When $\theta = 0$, the price is purely flexible, so inflation is inresponsive to both monetary and technology shock.

8) Set $\phi_\pi = 0.8$ and $\phi_\pi = 8$ and plot the impulse responses to the two shocks again, what are the main differences and comment. (2 points)

ANS:

See figure 7, 8, 9 and 10. When ϕ_π decreases, the nominal interest rate becomes more responsive to both monetary and technology shock; when ϕ_π increases, output gap will be more responsive to monetary shock but less responsive to technological shock.

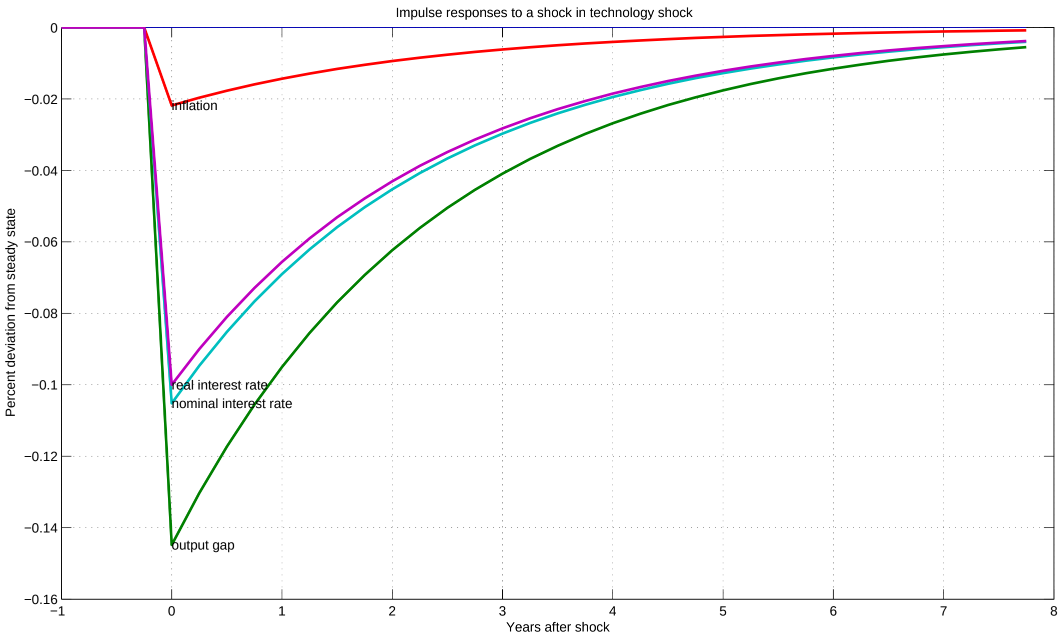

Figure 4: Plot impulse responses of output gap, in.ation, nominal rate, real rate to a one standard deviation shock of technology policy ($\epsilon = 60$)

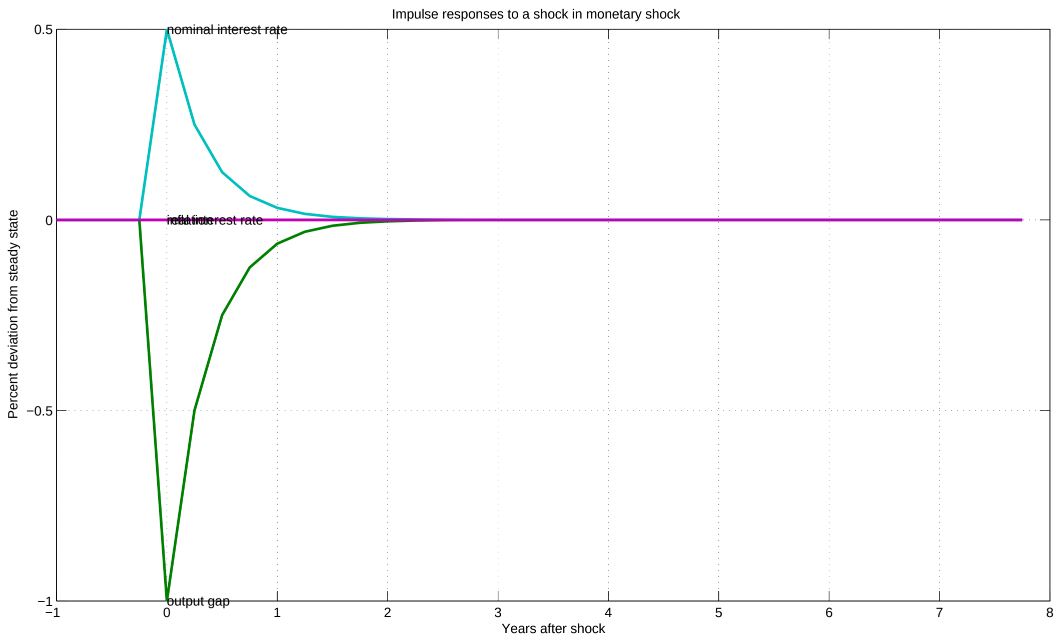

Figure 5: Impulse responses of output gap, in.ation, nominal rate, real rate to a one standard deviation shock of monetary policy ($\theta = 0$)

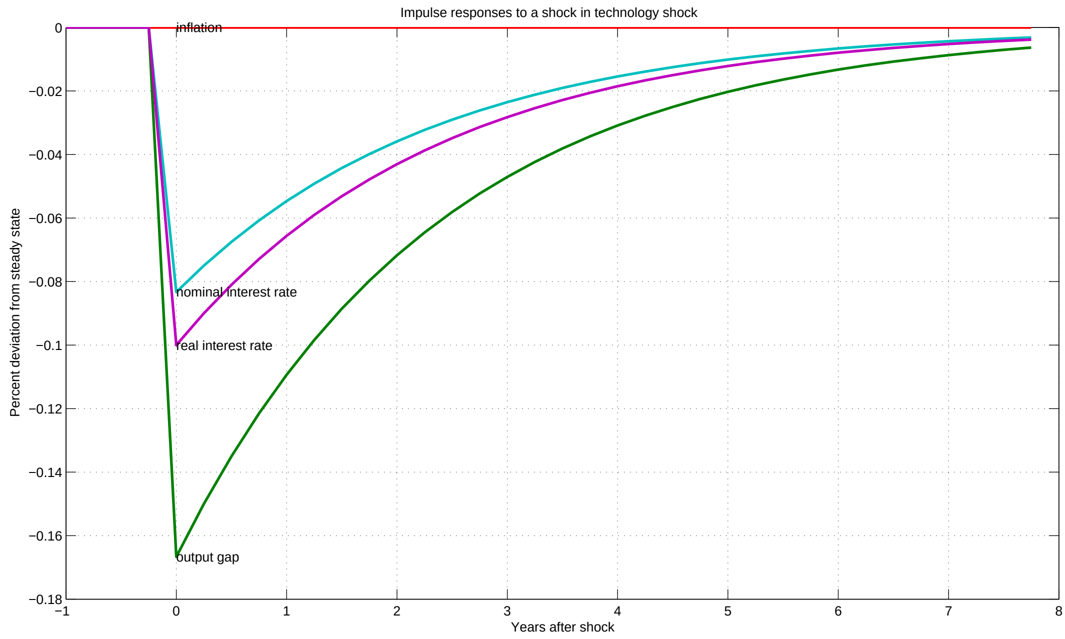

Figure 6: Plot impulse responses of output gap, in.ation, nominal rate, real rate to a one standard deviation shock of technology policy ($\theta = 0$)

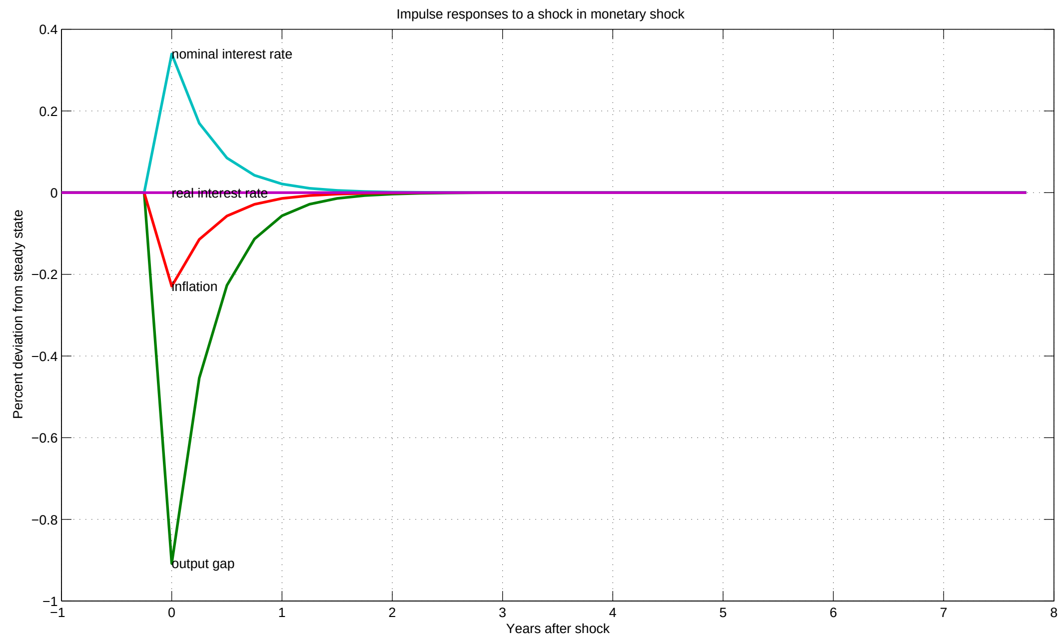

Figure 7: Plot impulse responses of output gap, in.ation, nominal rate, real rate to a one standard deviation shock of monetary policy ($\phi_\pi = 0.9$)

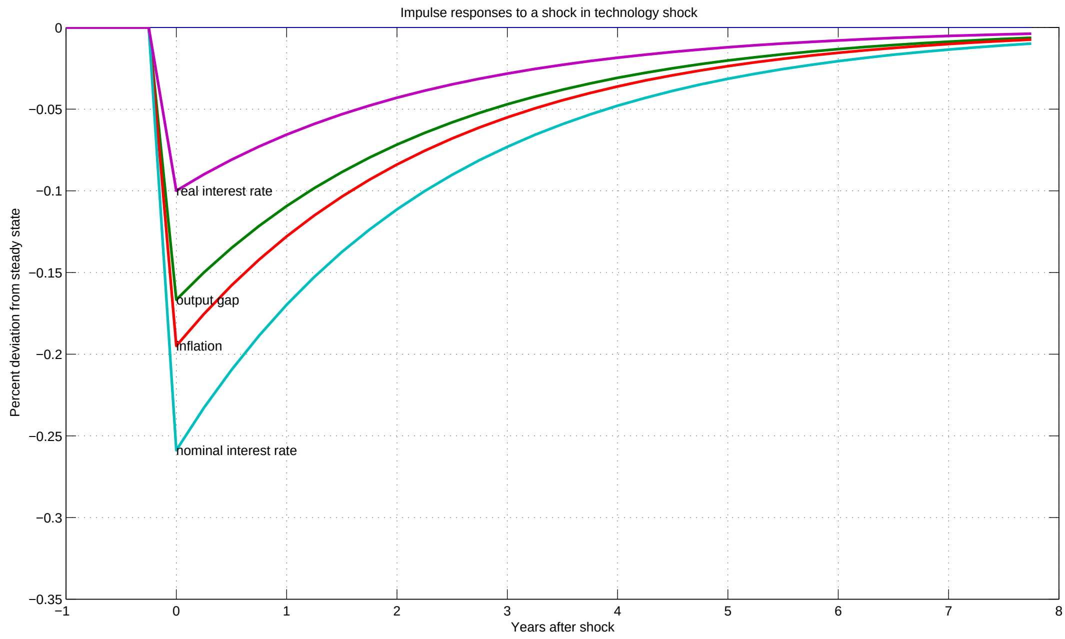

Figure 8: Plot impulse responses of output gap, in.ation, nominal rate, real rate to a one standard deviation shock of technology policy ($\phi_\pi = 0.9$)

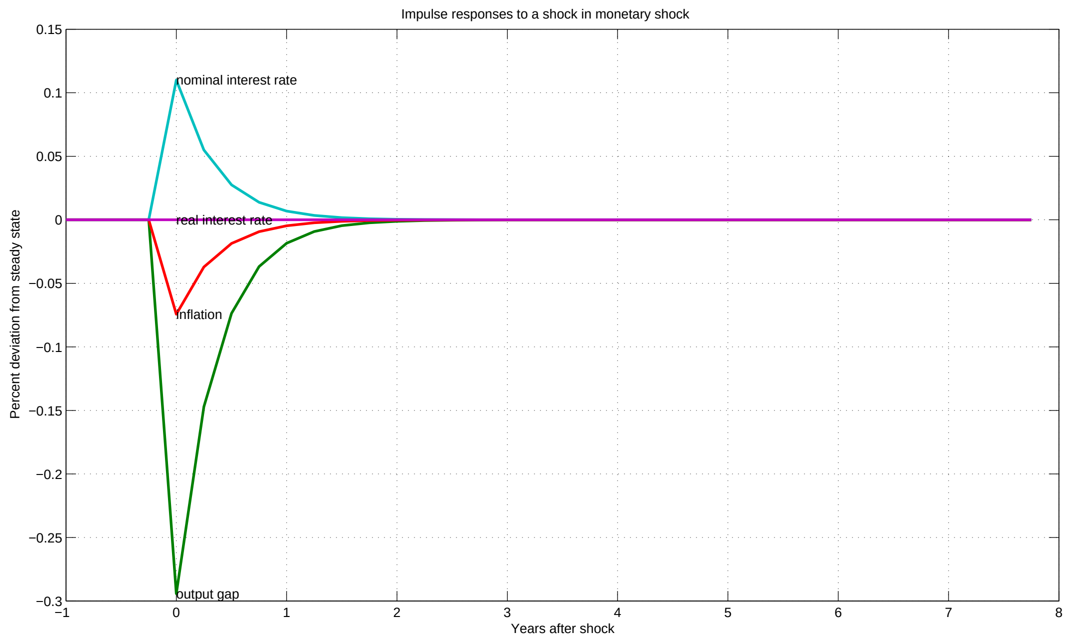

Figure 9: Plot impulse responses of output gap, in.ation, nominal rate, real rate to a one standard deviation shock of monetary policy ($\phi_\pi = 10$)

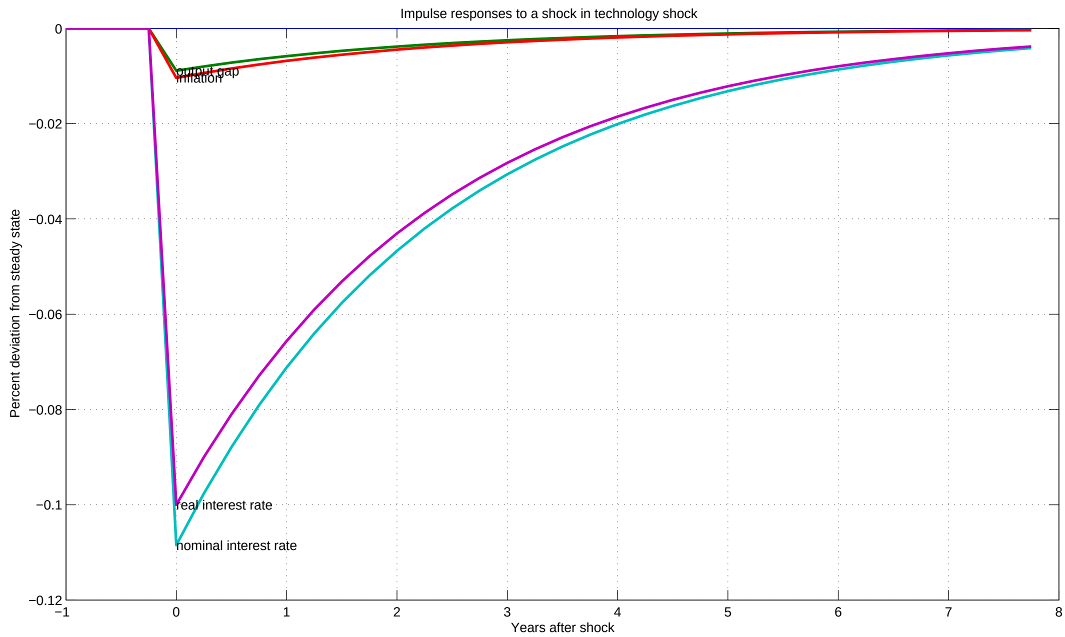

Figure 10: Plot impulse responses of output gap, in.ation, nominal rate, real rate to a one standard deviation shock of technology policy (ϕ_π = 10)