Solution to Quiz 2

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solution

- Let σ_a^2 be the variance of the return on an arbitrary portfolio and let $\sigma_{a,mv}$ be the covariance of this portfolio's return with that of the minimum variance portfolio. Then the variance of the composite portfolio consisting of proportions x and $1-x$ in the minimum variance and arbitrary portfolios, respectively, is

$$x^2\sigma_{mv}^2 + (1-x)^2\sigma_a^2 + 2x(1-x)\sigma_{a,mv}$$

- If we minimize this composite portfolio's variance with respect to x, we could obtain the F.O.C

$$2x\sigma_{mv}^2 - 2(1-x)\sigma_a^2 + 2(1-2x)\sigma_{a,mv} = 0$$

- Since the minimum variance portfolio has the smallest variance of all portfolios, it must be the case that $x = 1$ is the solution to the F.O.C. Thus, we could get

$$\sigma_{mv}^2 = \sigma_{a,mv}$$