Advanced Macroeconomics II
Mid-term Exam

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This is a **closed-book** exam. Total time is *90 minutes*. Please make sure to follow instructions and label all answers carefully. Total points available to the exam: 36 points.

A Simple Heterogeneous Agents Model

Assume the following stylized economy. There is an island inhabited by two households - consisting of former economists who settled down there. They don't have to work, since every period there comes a ship bringing expected income to both households. The households also own some armenian paintings a which they can use as an asset to lend or borrow. Formally, we have

$$\max_{c_{t,j},\quad a_{t,j}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{c_{t,j}^{1-\eta} - 1}{1-\eta} \right) \right\},\ j = I, II \tag{1}$$

s.t.

$$c_{t,j} + q_t a_{t,j} = y_{t,j} + a_{t-1,j} \tag{2}$$

where j stands for household I or II, q_t stands for the price of the painting, and $0 < \beta < 1$ and $\eta > 0$ are parameters. Market clearing implies

$$c_{t,I} + c_{t,II} = y_{t,I} + y_{t,II}. \tag{3}$$

Suppose the households have the following exogenous log income:

$$\ln y_{t,I} = \ln \bar{y}_I + \epsilon_t, \text{ with } \epsilon_t \sim N(0, \sigma^2) \tag{4}$$

and

$$\ln y_{t,II} = \ln \bar{y}_{II}. \tag{5}$$

1. Describe the economy briefly. Comment on the preference, endowment, technology, and information. (1 point)

   ANS:

   The economic environment:

   1)Preference: There are two household with idendtical utility function

   $$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{c_{t,j}^{1-\eta} - 1}{1-\eta} \right) \right\},\ j = I, II$$

   2)Technology: For the two household, the budget constraint is the same

   $$c_{t,j} + q_t a_{t,j} = y_{t,j} + a_{t-1,j},\ j = I, II$$

The log income is exogenous,

$$\begin{aligned}\ln y_{t,I} &= \ln \bar{y}_I + \epsilon_t, \text{with } \epsilon_t \sim N(0,\sigma^2) \\ \ln y_{t,II} &= \ln \bar{y}_{II}\end{aligned}$$

3)Endowment:

The households initially own some armenian paintings a.

And in each period, the log exogenous income $\ln y_{t,I}$ and $\ln y_{t,II}$ are endowed with the two household respectively.

4)Information: decision made based on all information I_t up to time t.

2. Classify the model variables according to endogenous state variable(s), exogenous state variable(s) and other endogenous variables. (1 point)

   ANS:

   Endogenous(predetermined) state variable(s): $a_{t,I}$, $a_{t,II}$,

   Exogenous state variable(s): $y_{t,I}$, $y_{t,II}$, ϵ_t

   Other endogenous variables: $c_{t,I}$, $c_{t,II}$, q_t

3. Find the first-order necessary conditions for the two households. (3 points)

   ANS:

   For each household j, the optimal problem is:

$$\max_{c_{t,j},\quad a_{t,j}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{c_{t,j}^{1-\eta} - 1}{1-\eta} \right) \right\}$$

$$c_{t,j} + q_t a_{t,j} = y_{t,j} + a_{t-1,j}$$

   The Lagrangian:

$$L_j = \max E \underset{t=0}{\infty} \beta^t \left\{ \frac{c_{t,j}^{1-\eta} - 1}{1-\eta} - \lambda_{t,j} \left[c_{t,j} + q_t a_{t,j} - y_{t,j} - a_{t-1,j} \right] \right\}$$

$$\frac{\partial L_j}{\partial c_{t,j}} : \quad c_{t,j}^{-\eta} - \lambda_{t,j} \stackrel{!}{=} 0 \tag{6}$$

$$\frac{\partial L_j}{\partial a_{t,j}} : \quad E_t \left\{ -\lambda_{t,j} q_t + \beta \lambda_{t+1,j} \right\} \stackrel{!}{=} 0 \tag{7}$$

$$\frac{\partial L_j}{\partial \lambda_{t,j}} : \quad c_{t,j} + q_t a_{t,j} = y_{t,j} + a_{t-1,j} \tag{8}$$

4. Substitute out the Lagrange multiplier on the agents' constraints. Write down all equations that are left and necessary to describe an equilibrium.

   (Hint: You should obtain 5 equations (including 2 FONCs for each consumer, and a market clearing condition) plus 2 for the income processes.)

ANS:

$$
\begin{aligned}
E_t\left[-c_{t,I}^{-\eta}q_t + \beta c_{t+1,I}^{-\eta}\right] &= 0 \\
E_t\left[-c_{t,II}^{-\eta}q_t + \beta c_{t+1,II}^{-\eta}\right] &= 0 \\
c_{t,I} + q_t a_{t,I} &= y_{t,I} + a_{t-1,I} \\
c_{t,II} + q_t a_{t,II} &= y_{t,I} + a_{t-1,II} \\
c_{t,I} + c_{t,II} &= y_{t,I} + y_{t,II} \\
\ln y_{t,I} &= \ln \bar{y}_I + \epsilon_t \\
\ln y_{t,II} &= \ln \bar{y}_{II}
\end{aligned}
$$

5. Solve for the steady state, i.e provide formulas for $\bar{q}$, $\bar{a}_I$, $\bar{a}_{II}$ and $\bar{c}_{II}$ given $\bar{c}_I$, $\bar{y}_I$, $\bar{y}_{II}$ and all parameters.

   ANS:

   1) $\bar{c}_{II} = \bar{y}_I + \bar{y}_{II} - \bar{c}_I$

   2) $\bar{q} = \beta$

   3)

$$
\begin{aligned}
\bar{c}_I + \bar{q}\bar{a}_I &= \bar{y}_I + \bar{a}_I \\
\bar{c}_I - \bar{y}_I &= \bar{a}_I(1-\bar{q}) \\
\bar{a}_I &= (\bar{c}_I - \bar{y}_I)/(1-\beta)
\end{aligned}
$$

   4)

$$
\begin{aligned}
\bar{c}_{II} + \bar{q}\bar{a}_{II} &= \bar{y}_{II} + \bar{a}_{II} \\
\bar{a}_{II} &= (\bar{c}_{II} - \bar{y}_{II})/(1-\beta)
\end{aligned}
$$

6. Show that in the steady state the market clearing condition for the asset market $\bar{a}_I + \bar{a}_{II} = 0$ is already given implicitly. Do you know why it must hold in general?

   ANS:

   Combine 3) and 4) and Equation (3), $\bar{a}_I + \bar{a}_{II} = 0$ can be verified. This is due to Walras' law that if there exist N markets, when N-1 markets clear, the remaining one must clear.

7. Log-linearize the equations in terms of log deviation $\hat{x}_t = \ln(x_t/\bar{x})$. Write down all necessary log-linearized equations characterizing the equilibrium. What is $\hat{y}_{t,II}$?

   ANS:

   1)

$$
\begin{aligned}
E_t\left[-c_{t,I}^{-\eta}q_t + \beta c_{t+1,I}^{-\eta}\right] &= 0 \\
q_t &= E_t\left[\beta\left(\frac{c_{t+1,I}}{c_{t,I}}\right)^{-\eta}\right] \\
\bar{q}e^{\hat{q}_t} &= E_t\left[\beta e^{-\eta\left(\hat{c}_{t+1,I} - \hat{c}_{t,I}\right)}\right] \\
\bar{q}\left(1+\hat{q}_t\right) &\approx \beta\left[1 - \eta\left(E_t\hat{c}_{t+1,I} - \hat{c}_{t,I}\right)\right]
\end{aligned}
$$

using the s.s. $\bar{q} = \beta$, we have

$$\begin{aligned} \hat{q}_t &= -\eta \left(E_t \hat{c}_{t+1,I} - \hat{c}_{t,I} \right) \\ \hat{q}_t &= -\eta E_t \hat{c}_{t+1,I} + \eta \hat{c}_{t,I} \end{aligned}$$

2)

$$\hat{q}_t = -\eta E_t \hat{c}_{t+1,II} + \eta \hat{c}_{t,II}$$

3)

$$\begin{aligned} c_{t,I} + q_t a_{t,I} &= y_{t,I} + a_{t-1,I} \\ \bar{c}_I e^{\hat{c}_{t,I}} + \bar{q}\bar{a}_I e^{\widehat{a}_{t,I} + \widehat{q}_{t,I}} &= \overline{y}_I e^{\widehat{y}_{t,I}} + \bar{a}_I e^{\widehat{a}_{t-1,I}} \\ \bar{c}_I \hat{c}_{t,I} + \bar{q}\bar{a}_I \widehat{a}_{t,I} + \bar{q}\bar{a}_I \widehat{q}_{t,I} &= \overline{y}_I \widehat{y}_{t,I} + \bar{a}_I \widehat{a}_{t-1,I} \end{aligned}$$

To summarise:

$$\begin{aligned} \hat{q}_t &= -\eta E_t \hat{c}_{t+1,I} + \eta \hat{c}_{t,I} \\ \hat{q}_t &= -\eta E_t \hat{c}_{t+1,II} + \eta \hat{c}_{t,II} \\ \bar{c}_I \hat{c}_{t,I} + \bar{q}\bar{a}_I \widehat{a}_{t,I} + \bar{q}\bar{a}_I \widehat{q}_{t,I} &= \overline{y}_I \widehat{y}_{t,I} + \bar{a}_I \widehat{a}_{t-1,I} \\ \bar{c}_{II} \hat{c}_{t,II} + \bar{q}\bar{a}_I \widehat{a}_{t,II} + \bar{q}\bar{a}_I \widehat{q}_{t,II} &= \overline{y}_{II} \widehat{y}_{t,II} + \bar{a}_{II} \widehat{a}_{t-1,II} \\ \bar{c}_I \hat{c}_{t,I} + \bar{c}_{II} \hat{c}_{t,II} &= \overline{y}_I \widehat{y}_{t,I} + \overline{y}_{II} \widehat{y}_{t,II} \\ \widehat{y}_{t,I} &= \epsilon_t \\ \widehat{y}_{t,II} &= 0 \end{aligned}$$

8. Simplify the equations until you have two equations in $\widehat{a}_{t,I}$, $\hat{c}_{t,I}$ and $\widehat{y}_{t,I}$ of the form of equations (3.45) and (3.46) in the toolkit paper. (Hint: You can make use of the log-linearized version of the asset market clearing condition given above.) What are $a_1, a_2, a_3, a_4, a_5, a_6$?

$$\begin{aligned} -\hat{a}_{t,I} + a_1 \hat{a}_{t-1,I} + a_2 \hat{c}_{t,I} + a_3 \hat{y}_{t,I} &= 0 \\ E_t \left[-\hat{c}_{t,I} + a_4 \hat{a}_{t,I} + a_5 \hat{c}_{t+1,I} + a_6 \hat{y}_{t,I} \right] &= 0 \end{aligned}$$

ANS:

$$\begin{aligned} \hat{q}_t &= -\eta E_t \hat{c}_{t+1,I} + \eta \hat{c}_{t,I} \\ \hat{q}_t &= -\eta E_t \hat{c}_{t+1,II} + \eta \hat{c}_{t,II} \\ \bar{c}_I \hat{c}_{t,I} + \bar{q}\bar{a}_I \hat{q}_t + \bar{q}\bar{a}_I \hat{a}_{t,I} &\approx \bar{y}_I \hat{y}_{t,I} + \bar{a}_I \hat{a}_{t-1,I} \\ \bar{c}_{II} \hat{c}_{t,II} + \bar{q}\bar{a}_{II} \hat{q}_t + \bar{q}\bar{a}_{II} \hat{a}_{t,II} &\approx \bar{a}_{II} \hat{a}_{t-1,II} \\ \bar{c}_I \hat{c}_{t,I} + \bar{c}_{II} \hat{c}_{t,II} &= \bar{y}_I \hat{y}_{t,I} \\ \hat{y}_{t,I} &= \epsilon_t \end{aligned}$$

Asset market clearing condition $a_{t,I} + a_{t,II} = 0$ implies

$$\begin{aligned} \bar{a}_I + \bar{a}_{II} &= 0 \\ \bar{a}_I \hat{a}_{t,I} + \bar{a}_{II} \hat{a}_{t,II} &= 0 \end{aligned}$$

such that

$$\begin{aligned}
\bar{a}_{II} &= -\bar{a}_I \\
\bar{a}_I \hat{a}_{t,I} - \bar{a}_I \hat{a}_{t,II} &= 0 \\
\hat{a}_{t,I} &= \hat{a}_{t,II}
\end{aligned}$$

Delete $\hat{a}_{t,II}$ using $\hat{a}_{t,II} = \hat{a}_{t,I}$

$$\begin{aligned}
\hat{q}_t &= -\eta E_t \hat{c}_{t+1,I} + \eta \hat{c}_{t,I} \\
\hat{q}_t &= -\eta E_t \hat{c}_{t+1,II} + \eta \hat{c}_{t,II} \\
\bar{c}_I \hat{c}_{t,I} + \bar{q}\bar{a}_I \hat{q}_t + \bar{q}\bar{a}_I \hat{a}_{t,I} &\approx \bar{y}_I \hat{y}_{t,I} + \bar{a}_I \hat{a}_{t-1,I} \\
\bar{c}_{II} \hat{c}_{t,II} + \bar{q}\bar{a}_{II} \hat{q}_t + \bar{q}\bar{a}_{II} \hat{a}_{t,II} &\approx \bar{a}_{II} \hat{a}_{t-1,II} \\
\bar{c}_I \hat{c}_{t,I} + \bar{c}_{II} \hat{c}_{t,II} &= \bar{y}_I \hat{y}_{t,I} \\
\hat{y}_{t,I} &= \epsilon_t
\end{aligned}$$

1) Combine 1st and 2nd

$$\begin{aligned}
-\eta E_t \hat{c}_{t+1,I} + \eta \hat{c}_{t,I} + \eta E_t \hat{c}_{t+1,II} - \eta \hat{c}_{t,II} &= 0 \\
-E_t \hat{c}_{t+1,I} + \hat{c}_{t,I} + E_t \hat{c}_{t+1,II} - \hat{c}_{t,II} &= 0 \\
-\bar{c}_{II} E_t \hat{c}_{t+1,I} + \bar{c}_{II} \hat{c}_{t,I} + \bar{c}_{II} E_t \hat{c}_{t+1,II} - \bar{c}_{II} \hat{c}_{t,II} &= 0
\end{aligned}$$

from the 5th

$$\begin{aligned}
\bar{c}_{II} \hat{c}_{t,II} &= \bar{y}_I \hat{y}_{t,I} - \bar{c}_I \hat{c}_{t,I} \\
\bar{c}_{II} E_t \hat{c}_{t+1,II} &= \bar{y}_I E_t \hat{y}_{t+1,I} - \bar{c}_I E_t \hat{c}_{t+1,I} \\
&= -\bar{c}_I E_t \hat{c}_{t+1,I}
\end{aligned}$$

and insert them above

$$\begin{aligned}
-\bar{c}_{II} E_t \hat{c}_{t+1,I} + \bar{c}_{II} \hat{c}_{t,I} - \bar{c}_I E_t \hat{c}_{t+1,I} + \bar{c}_I \hat{c}_{t,I} - \bar{y}_I \hat{y}_{t,I} &= 0 \\
-(\bar{c}_I + \bar{c}_{II}) E_t \hat{c}_{t+1,I} + (\bar{c}_I + \bar{c}_{II}) \hat{c}_{t,I} - \bar{y}_I \hat{y}_{t,I} &= 0 \\
E_t \hat{c}_{t+1,I} - \hat{c}_{t,I} + \frac{\bar{y}_I}{(\bar{c}_I + \bar{c}_{II})} \hat{y}_{t,I} &= 0 \\
E_t \left[-\hat{c}_{t,I} + \hat{c}_{t+1,I} + \frac{\bar{y}_I}{(\bar{c}_I + \bar{c}_{II})} \hat{y}_{t,I} \right] &= 0
\end{aligned}$$

So,

$$\begin{aligned}
a_4 &= 0 \\
a_5 &= 1 \\
a_6 &= \frac{\bar{y}_I}{(\bar{c}_I + \bar{c}_{II})}
\end{aligned}$$

2)Solve for $\hat{q}_t$

Using

$$\begin{aligned}
\hat{c}_{t,II} &= \frac{\bar{y}_I}{\bar{c}_{II}}\hat{y}_{t,I} - \frac{\bar{c}_I}{\bar{c}_{II}}\hat{c}_{t,I} \\
E_t\hat{c}_{t+1,II} &= -\frac{\bar{c}_I}{\bar{c}_{II}}E_t\hat{c}_{t+1,I}
\end{aligned}$$

$$\begin{aligned}
\hat{q}_t &= -\eta E_t\hat{c}_{t+1,II} + \eta\hat{c}_{t,II} \\
&= \eta\frac{\bar{c}_I}{\bar{c}_{II}}E_t\hat{c}_{t+1,I} + \eta\frac{\bar{y}_I}{\bar{c}_{II}}\hat{y}_{t,I} - \eta\frac{\bar{c}_I}{\bar{c}_{II}}\hat{c}_{t,I} \\
&= \eta\frac{\bar{c}_I}{\bar{c}_{II}}\left(E_t\hat{c}_{t+1,I} - \hat{c}_{t,I}\right) + \eta\frac{\bar{y}_I}{\bar{c}_{II}}\hat{y}_{t,I} \\
&= -\frac{\bar{c}_I}{\bar{c}_{II}}\hat{q}_t + \eta\frac{\bar{y}_I}{\bar{c}_{II}}\hat{y}_{t,I} \\
\hat{q}_t &= \frac{\eta}{(1+\frac{\bar{c}_I}{\bar{c}_{II}})}\frac{\bar{y}_I}{\bar{c}_{II}}\hat{y}_{t,I} \\
\hat{q}_t &= \eta\frac{\bar{y}_I}{(\bar{c}_I + \bar{c}_{II})}\hat{y}_{t,I}
\end{aligned}$$

Insert it into the 3rd equation:

$$\begin{aligned}
\bar{c}_I\hat{c}_{t,I} + \bar{q}\bar{a}_I\hat{q}_t + \bar{q}\bar{a}_I\hat{a}_{t,I} &\approx \bar{y}_I\hat{y}_{t,I} + \bar{a}_I\hat{a}_{t-1,I} \\
\bar{c}_I\hat{c}_{t,I} + \bar{q}\bar{a}_I\eta\frac{\bar{y}_I}{(\bar{c}_I + \bar{c}_{II})}\hat{y}_{t,I} + \bar{q}\bar{a}_I\hat{a}_{t,I} &\approx \bar{y}_I\hat{y}_{t,I} + \bar{a}_I\hat{a}_{t-1,I} \\
\frac{\bar{c}_I}{\bar{q}\bar{a}_I}\hat{c}_{t,I} + \eta\frac{\bar{y}_I}{(\bar{c}_I + \bar{c}_{II})}\hat{y}_{t,I} + \hat{a}_{t,I} &\approx \frac{\bar{y}_I}{\bar{q}\bar{a}_I}\hat{y}_{t,I} + \frac{1}{\bar{q}}\hat{a}_{t-1,I}
\end{aligned}$$

$$0 = -\hat{a}_{t,I} + \frac{1}{\bar{q}}\hat{a}_{t-1,I} - \frac{\bar{c}_I}{\bar{q}\bar{a}_I}\hat{c}_{t,I} + \left(\frac{\bar{y}_I}{\bar{q}\bar{a}_I} - \eta\frac{\bar{y}_I}{(\bar{c}_I + \bar{c}_{II})}\right)\hat{y}_{t,I}$$

So,

$$\begin{aligned}
a_1 &= \frac{1}{\bar{q}} \\
a_2 &= -\frac{\bar{c}_I}{\bar{q}\bar{a}_I} \\
a_3 &= \frac{\bar{y}_I}{\bar{q}\bar{a}_I} - \eta\frac{\bar{y}_I}{(\bar{c}_I + \bar{c}_{II})}
\end{aligned}$$

9. Guess a suitable recursive law of motion for your two model variables(2 points).

   ANS:

$$\begin{aligned}
\hat{a}_{t,I} &= v_{a_I a_I}\hat{a}_{t-1,I} + v_{a_I y_I}\hat{y}_{t,I} \\
\hat{c}_{t,I} &= v_{c_I a_I}\hat{a}_{t-1,I} + v_{c_I y_I}\hat{y}_{t,I}
\end{aligned}$$

10. Solve for the coefficients of the recursive equilibrium law of motion by plugging in the recursive equilibrium law of motion into the loglinearized equations and compare coefficients. Which of the two roots for $v_{a_I a_I}$ do you use and why? (16 points)

ANS:

$$\begin{aligned}
-\hat{a}_{t,I} + a_1\hat{a}_{t-1,I} + a_2\hat{c}_{t,I} + a_3\hat{y}_{t,I} &= 0 \\
-v_{a_I a_I}\hat{a}_{t-1,I} - v_{a_I y_I}\hat{y}_{t,I} + a_1\hat{a}_{t-1,I} + a_2 v_{c_I a_I}\hat{a}_{t-1,I} + a_2 v_{c_I y_I}\hat{y}_{t,I} + a_3\hat{y}_{t,I} &= 0 \\
\left(-v_{a_I a_I} + a_1 + a_2 v_{c_I a_I}\right)\hat{a}_{t-1,I} + \left(-v_{a_I y_I} + a_3 + a_2 v_{c_I y_I}\right)\hat{y}_{t,I} &= 0
\end{aligned}$$

we can get $\begin{cases} -v_{a_I a_I} + a_1 + a_2 v_{c_I a_I} = 0 \\ -v_{a_I y_I} + a_3 + a_2 v_{c_I y_I} = 0 \end{cases}$

$$\begin{cases} v_{c_I a_I} = \frac{v_{a_I a_I} - a_1}{a_2} \\ -v_{a_I y_I} + a_3 + a_2 v_{c_I y_I} = 0 \end{cases}$$

$$\begin{aligned}
E_t\left[-\hat{c}_{t,I} + a_4\hat{a}_{t,I} + a_5\hat{c}_{t+1,I} + a_6\hat{y}_{t,I}\right] &= 0 \\
E_t\left[-v_{c_I a_I}\hat{a}_{t-1,I} - v_{c_I y_I}\hat{y}_{t,I} + a_4\hat{a}_{t,I} + a_5 v_{c_I a_I}\hat{a}_{t,I} + a_5 v_{c_I y_I}\hat{y}_{t+1,I} + a_6\hat{y}_{t,I}\right] &= 0 \\
E_t\left[-v_{c_I a_I}\hat{a}_{t-1,I} - v_{c_I y_I}\hat{y}_{t,I} + \left(a_4 + a_5 v_{c_I a_I}\right)\left(v_{a_I a_I}\hat{a}_{t-1,I} + v_{a_I y_I}\hat{y}_{t,I}\right) + a_5 v_{c_I y_I}\epsilon_{t+1} + a_6\hat{y}_{t,I}\right] &= 0 \\
\left(a_4 v_{a_I a_I} + a_5 v_{c_I a_I} v_{a_I a_I} - v_{c_I a_I}\right)\hat{a}_{t-1,I} + \left(a_4 v_{a_I a_I} + a_5 v_{c_I a_I} v_{a_I y_I} - v_{c_I y_I} + a_6\right)\hat{y}_{t,I} &= 0
\end{aligned}$$

we can get $\begin{cases} a_5 v_{c_I a_I} v_{a_I a_I} - v_{c_I a_I} = 0 \\ a_5 v_{c_I a_I} v_{a_I y_I} - v_{c_I y_I} + a_6 = 0 \end{cases}$

combined with the above equation

$v_{a_I a_I} = 1$ or $v_{a_I a_I} = a_1$(omitted, since $a_1 = \frac{1}{\bar{q}} > 1$ is explosive)

$$\begin{cases} v_{a_I a_I} = 1 \\ v_{c_I a_I} = \frac{v_{a_I a_I} - a_1}{a_2} \\ v_{c_I y_I} = \frac{a_6 + a_3 a_5 v_{c_I a_I}}{1 - a_2 a_5 v_{c_I a_I}} \\ v_{a_I y_I} = a_3 + a_2 v_{c_I y_I} \end{cases}$$

11. Calibrate the model with the following values: β= 0.95, η= 1, $\ln\bar{y}_I = 2$, $\ln\bar{y}_{II} = 3$, $\frac{\bar{c}_I}{\bar{y}_I} = 0.3$, i.e. the consumption share for household I is set equal to 30 percent. Calculate $\bar{a}_I$, $\bar{a}_{II}$, $\bar{c}_I$, $\bar{c}_{II}$ and $\bar{q}$. Calculate also the discounted sums of the two steady state utilities. Compute a_1, ..., a_6. Calculate the coefficients of the recursive equilibrium law of motion for $\hat{c}_{t,I}$, $\hat{c}_{t,II}$ and $\hat{q}_t$. (21 points)

ANS:

$\bar{a}_I$	$\bar{a}_{II}$	$\bar{c}_I$	$\bar{c}_{II}$	$\bar{q}$
-103.4468	103.4468	2.2167	25.2579	0.9500

At steady state, the utility is

$$\begin{aligned}
U &= E_0\left\{\sum_{t=0}^{\infty}\beta^t\left(\frac{\bar{c}_j^{1-\eta} - 1}{1-\eta}\right)\right\},\ j = I, II \\
U &= \frac{1}{1-\beta}\left(\frac{\bar{c}_j^{1-\eta} - 1}{1-\eta}\right), j = I, II
\end{aligned}$$

Since $\eta = 1$,the discounted sum of steady state utility is

$$\begin{aligned} U &= \frac{1}{1-\beta}\left(\ln \bar{c}_j\right) \\ U_I &= 15.9205 \\ U_{II} &= 64.5828 \end{aligned}$$

a_1	a_2	a_3	a_4	a_5	a_6
1.0526	0.0226	−0.3441	0	1	0.2689

$$\begin{aligned} \hat{c}_{t,I} &= v_{c_I a_I}\hat{a}_{t-1,I} + v_{c_I y_I}\hat{y}_{t,I} \\ \hat{c}_{t,I} &= -2.3333\hat{a}_{t-1,I} + 1.0183\hat{y}_{t,I} \end{aligned}$$

$$\begin{aligned} \hat{c}_{t,II} &= \frac{\bar{y}_I}{\bar{c}_{II}}\hat{y}_{t,I} - \frac{\bar{c}_I}{\bar{c}_{II}}\hat{c}_{t,I} \\ \hat{c}_{t,II} &= \frac{\bar{y}_I}{\bar{c}_{II}}\hat{y}_{t,I} - \frac{\bar{c}_I}{\bar{c}_{II}}v_{c_I a_I}\hat{a}_{t-1,I} - \frac{\bar{c}_I}{\bar{c}_{II}}v_{c_I y_I}\hat{y}_{t,I} \\ \hat{c}_{t,II} &= -\frac{\bar{c}_I v_{c_I a_I}}{\bar{c}_{II}}\hat{a}_{t-1,I} + \frac{\bar{y}_I - \bar{c}_I v_{c_I y_I}}{\bar{c}_{II}}\hat{y}_{t,I} \\ \hat{c}_{t,II} &= v_{c_{II} a_{II}}\hat{a}_{t-1,I} + v_{c_{II} y_{II}}\hat{y}_{t,I} \\ v_{c_{II} a_{II}} &= -\frac{\bar{c}_I v_{c_I a_I}}{\bar{c}_{II}}, v_{c_{II} y_{II}} = \frac{\bar{y}_I - \bar{c}_I v_{c_I y_I}}{\bar{c}_{II}} \\ \hat{c}_{t,II} &= 0.2048\hat{a}_{t-1,I} + 0.2032\hat{y}_{t,I} \\ \hat{q}_t &= \eta\frac{\bar{y}_I}{(\bar{c}_I + \bar{c}_{II})}\hat{y}_{t,I} = 0.2689\hat{y}_{t,I} \end{aligned}$$

12. Assume$\epsilon_t = 1$ for $t = 0, \epsilon_t = 0$ for $t > 0$. Show your results for $c_{t,I}$, $c_{t,II}$, $\ln y_{t,I}$ and $\ln y_{t,II}$ graphically for $t \in \{0,1,2,3,4,5,6,7\}$. (8 points)

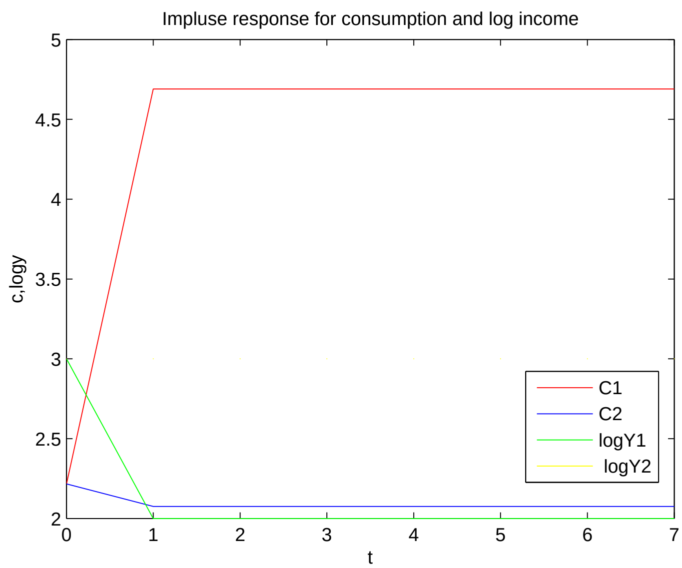

Figure 5-1:Impluse response for consumption and log income.

13. Do you observe a “permanent effect of a singular shock”? (1 points)
    Yes.

14. What happens if you change income, i.e. $\ln \bar{y}_I = 5$ and $\ln \bar{y}_{II} = 8$? (5 points)

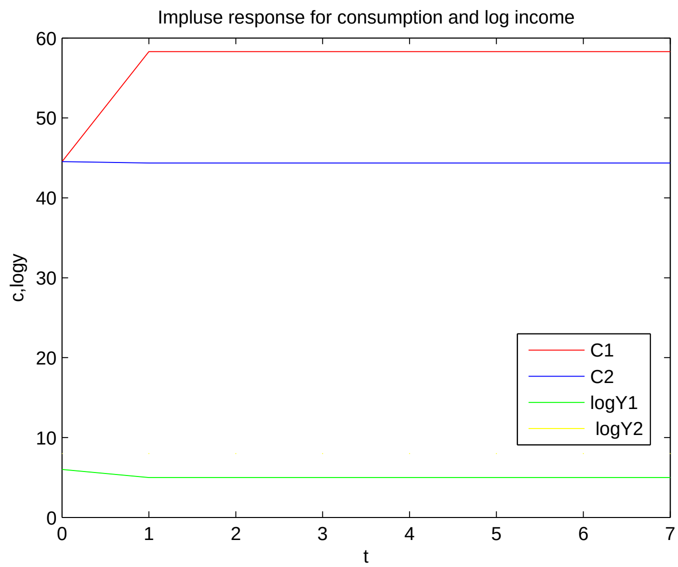

Figure 5-2:Impluse response for consumption and log income.

15. What happens if you change the ranking of the households' income, i.e. $\ln \bar{y}_{II} = 8$ and $\ln \bar{y}_I = 5$? (5 points)

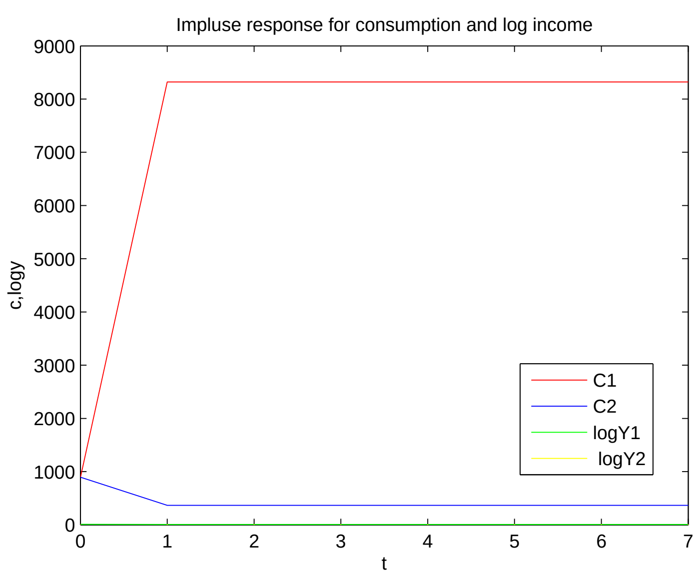

Figure 5-3:Impluse response for consumption and log income.