Simulation Homework

Advanced Econometrics I

Consider the following model

$$y_i = \beta_0 + x_{1,i}\beta_1 + x_{2,i}\beta_2 + \varepsilon_i$$

where $E(x_i \varepsilon_i) = 0$

and the hypothesis $H_0 : \frac{\beta_1}{\beta_2} = r$ where r is a known constant.

Define $\theta = \frac{\beta_1}{\beta_2}$. Let $\hat{\beta} = (\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2)$ be the OLS estimators of the above model, and let $\hat{V}$ be an estimates of the asymptotic variance matrix of $\hat{\beta}$. Set $\hat{\theta} = \hat{\beta}_1 / \hat{\beta}_2$. Define

$$\hat{H}_1 = \begin{pmatrix} 0 \\ \frac{1}{\hat{\beta}_2} \\ -\frac{\hat{\beta}_1}{\hat{\beta}_2^2} \end{pmatrix}$$

so that the standard error of $\hat{\theta}$ is $s(\hat{\theta}) = (n^{-1}\hat{H}_1' \hat{V} \hat{H}_1)^{1/2}$

A t-statistic for H_0 is

$$t_{1n} = \frac{(\hat{\beta}_1 / \hat{\beta}_2 - r)}{s(\hat{\theta})}$$

An alternative statistic can be constructed through reformulating H_0 as

$$H_0 : \beta_1 - r\beta_2 = 0$$

and a t-statistic is

$$t_{2n} = \frac{(\hat{\beta}_1 - r\hat{\beta}_2)}{(n^{-1} H_2' \hat{V} H_2)^{1/2}}$$

where

$$H_2 = \begin{pmatrix} 0 \\ 1 \\ -r \end{pmatrix}$$

Let $x_{1,i}$ and $x_{2,i}$ be mutually independent $\sim(0,1)$, ε_i be an independent $\sim(0,\sigma^2)$ draw with $\sigma=3$, and let $\beta_0 = 0$ and $\beta_1 = 1$. Perform a Monte Carlo simulation to compare the actual rejection probabilities ($P(t_n < -1.645)$ and $P(t_n > 1.645)$) of t_{1n} and t_{2n} for $\beta_2 = 0.1$, 0.25, 0.50, 0.75 and 1.0, and n among 100 and 500.

Please interpret your simulation results.