Advanced Macroeconomics
Final Exam

Name:____________________________ Enrollment number:_____________________________

This is a **closed-book** exam. Total time is *90 minutes* . Please make sure to follow instructions and label all answers carefully. Total points available to the exam: 35.5 points.

Question 1 (4 points)

There are 4 data series with sample size $T = 1000$ that are generated according to the following data generating processes:

(i) AR(1): $y_t = 0.96y_{t-1} + \varepsilon_t,\ \ \varepsilon_t \overset{i.i.d.}{\sim} N(0,1)$

(ii). AR(2): $y_t = 1.3y_{t-1} - 0.5y_{t-2} + \varepsilon_t,\ \ \varepsilon_t \overset{i.i.d.}{\sim} N(0,1)$

(iii). MA(1): $y_t = \varepsilon_t + 0.8\varepsilon_{t-1},\ \ \varepsilon_t \overset{i.i.d.}{\sim} N(0,1)$

(iv). ARMA(1,1): $y_t = 0.9y_{t-1} + \varepsilon_t - 0.6\varepsilon_{t-1},\ \ \varepsilon_t \overset{i.i.d.}{\sim} N(0,1)$.

Please select from Figure 1 (panels (a) to (e), on page 5) their corresponding Autocorrelation functions (ACF).

Process	(i)	(ii)	(iii)	(iv)
Correponding ACF from Figure 1				

Question 2. (7 points)

Suppose after solving a DSGE model, you get the following recursive solution to its log-linearized approximation in terms of the endogenous state variable: capital $\tilde{k}_t$, and exogenous state variable: productivity $\tilde{z}_t$:

$$\begin{aligned} \tilde{k}_t &= \eta_{kk}\tilde{k}_{t-1} + \eta_{kz}\tilde{z}_t && (1) \\ \tilde{z}_t &= \phi\tilde{z}_{t-1} + \varepsilon_t && (2) \end{aligned}$$

1) Insert equation (2) into (1) to express $\tilde{k}_t$ as a function of lagged state variables $\left(\tilde{k}_{t-1}, \tilde{z}_{t-1}\right)$and shocks ε_t, label this expression as equation (3); (1 point)

2) Define $x_t \equiv \begin{bmatrix} \tilde{k}_t & \tilde{z}_t \end{bmatrix}'$, $v_t \equiv \begin{bmatrix} 0 & \varepsilon_t \end{bmatrix}'$,.re-write equation (2) and (3) in a VAR form of $x_t = Ax_{t-1} + Bv_t$. Express A and B in terms of original coefficients η_{kk}, η_{kz} and ϕ? (2 points)

3) One can solve the contemporaneous variance-covariance matrix $\Gamma(0) = E\left(x_t x_t'\right)$ of x_t as a solution to $\Gamma(0) = A\Gamma(0)A' + \Sigma$, where $\Sigma = BE(v_t v_t')B'$. Assume $\eta_{kk} = 0.965$, $\eta_{kz} = 0.075$, $\phi = 0.95$, $\sigma_\varepsilon^2 = E(\varepsilon_t^2) = 0.712$, then you can get

$$\Gamma(0) = \begin{bmatrix} 14.032 & 6.625 \\ 6.625 & 7.303 \end{bmatrix},$$

what are the following second moments implied by your model parameterization? (2 points)

σ_k^2 (variance of $\tilde{k}_t$)		σ_{kz} (covariance between $\tilde{k}_t$ and $\tilde{z}_t$)	
σ_z^2 (variance of $\tilde{z}_t$)		ρ_{kz} (correlation between $\tilde{k}_t$ and $\tilde{z}_t$)	

4) The 1st-order cross-covariance $\Gamma(1) \equiv E\left(x_t x_{t-1}'\right) = A\Gamma(0)$. Based on the above information, what are the following 1st-order cross-covariance implied by your model parameterization? (2 points)

$cov\left(\tilde{k}_t, \tilde{k}_{t-1}\right)$		$cov\left(\tilde{z}_t, \tilde{k}_{t-1}\right)$		$cov\left(\tilde{z}_t, \tilde{z}_{t-1}\right)$		$cov\left(\tilde{k}_t, \tilde{z}_{t-1}\right)$	

Question 3. (6 points)

If one aims to remove a long run trend τ_t from a time series y_t, the Hodrick-Prescott (HP) filter is a frequently used tool.

$$\min_{\tau_t} \left\{ \sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} \left[(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})\right]^2 \right\} \tag{4}$$

1). Describe briefly the implied "trade off" of the HP filter. (1 point)

2). Describe briefly the importance of the parameter λ. (1 point)

3). What happens to τ_t, the variance of the cyclical component and the variance of the change of trend growth if

(a) $\lambda \to 0$. (1 point)

(b) $\lambda \to \infty$. (1 point)

4) Show that the HP filter can be written as a linear filter. (2 point) *(Hint: given λ, the solution of τ_t amounts to solve the optimization problem in equation (4).)*

Question 4. (6 points)

The population spectrum of Y can be defined as

$$S_Y(\omega) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j e^{-i\omega j} \tag{5}$$

1) Please demonstrate that the spectrum can be derived as

$$S_Y(\omega) = \frac{1}{2\pi} \left\{ \gamma_0 + 2 \sum_{j=-\infty}^{\infty} \gamma_j \cos(\omega j) \right\}. \text{ (3 points)} \tag{6}$$

2) For an MA(1) process $Y_t = \varepsilon_t + 0.8\varepsilon_{t-1}$, derive its spectrum. (2 points) And plot the spectrum between the interval of $[0, \pi]$. (1 point)

Question 5. Welfare cost of business cycle fluctuation in different regimes (6 points)

In an economy where the representative agent has CRRA utility, the welfare cost λ is determined both by the variance in consumption growth σ^2 and the risk aversion parameter γ :

$$\lambda = \frac{1}{2}\sigma^2\gamma \tag{7}$$

To calculate the welfare cost, we can calibrate σ^2 from per capita consumption data, and for γ, we exploit an intertemporal optimality condition for consumption growth μ with respect to γ, interest rate r, and the consumer's subjective discount rate ρ:

$$\mu = \frac{1}{\gamma}(r - \rho) \tag{8}$$

In the Lucas' calibration study, he has calibrate the welfare cost for the whole time span of 1947-2001. But we know that this could blur the picture, as there are very different periods in this sample, where pre-1984, the variance of economic activity is much higher than in the post-1984 period. Further, when variance is higher, the interest rate not only fluctuates widely but also stays at higher level. Would this make difference on the calibration of welfare cost? The following table gives you some basic information on the parameter estimates in these two periods.

	1957-1984	post-1984
σ^2	0.036	0.02
μ	0.037	0.033
r	0.08	0.035

1. According to the above information, what are the upper bound of $\bar{\gamma}$ for each of these two periods? (Assume $\rho = 0$ to get the upper bound.) (2 points)

	1957-1984	post-1984
$\bar{\gamma}$		

2. Calculate the welfare costs for both periods in the following table, given the upper bound of γ you obtained from last step.(2 points)

	1957-1984	post-1984
$\lambda(\bar{\gamma})$		

3. What drives the difference in the welfare cost? (1 point)

4. Is the risk aversion parameter independent from variance σ^2 from your exercise? (1 point)

Question 6. The equity premium puzzle (6.5 points)

In a simple asset pricing model with multi-asset environment, the equilibrium conditions derived from the representative agent's maximization problem are as follows:

$$\beta E_t \left\{ \frac{u'(c_{t+1})}{u'(c_t)} R^f_{t+1} \right\} - 1 = 0 \tag{9}$$

$$E_t \left\{ \frac{u'(c_{t+1})}{u'(c_t)} \left[R^e_{t+1} - R^f_{t+1} \right] \right\} = 0 \tag{10}$$

where R^f_{t+1} and R^e_{t+1} denote the gross returns associated with a risk-free and risky asset. The difference $R^e_{t+1} - R^f_{t+1}$ is referred to as the equity premium.

According to historical data from 1889 to 2004, sample averages (standard deviations) of $r^f_{t+1} (= R^f_{t+1} - 1)$ and $r^e_{t+1} (= R^e_{t+1} - 1)$ are 1.11%(5.61%) and 7.48% (16.04%) respectively. You wonder whether a CRRA preference specification can account for this pattern, i.e. with

$$\frac{u'(c_{t+1})}{u'(c_t)} = \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma}.$$

Consumption growth rate in the same sample is 1.79%. Assume $\beta = 0.99$.

1. Compute the left hand sides of equations (9) and (10) in the following table with respect to different values of risk aversion γ.(3 points)

γ	$\beta E \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} R^f_{t+1} \right\} - 1$	$E \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \left[R^e_{t+1} - R^f_{t+1} \right] \right\}$
0		
1		
4		
10		
20		

2. According to the above results, what value of γ do you need for fitting the equations (9) and (10) respectively? Are they consistent to the equilibrium? (1.5 points)

3. What says the "equity premium puzzle"? (2 points)