Advanced Microeconomics II
Problem Set 2

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1. Each of n people chooses whether or not to contribute a fixed amount toward the provision of a public good. The good is provided if and only if at least k people contribute, where $2 \leq k \leq n$; if it is not provided, contributions are not refunded. Each person ranks outcomes from best to worst as follows:

   (i) any outcome in which the good is provided and she does not contribute,

   (ii) any outcome in which the good is provided and she contributes,

   (iii) any outcome in which the good is not provided and she does not contribute,

   (iv) any outcome in which the good is not provided and she contributes.

   (a) Formulate this situation as a strategic game.

   (b) Find its Nash equilibria.

2. (Gibbons 1.2) Players 1 and 2 are bargaining over how to split five dollars. Both players simultaneously name shares they would like to have, s_1 and s_2, where $0 \leq s_i \leq 5, i = 1, 2$. If $s_1 + s_2 \leq 5$, then the players receive the shares they named; if $s_1 + s_2 > 5$, then both players receive zero.

   (a) Consider a symmetric (non-pure) mixed strategy equilibrium, where each player chooses between two numbers a and b, $a < b$.

      i. Prove or disprove that to be a mixed strategy equilibrium $a + b \leq 5$.

      ii. Prove or disprove that to be a mixed strategy equilibrium $a + b \geq 5$.

      iii. Derive such a mixed strategy equilibrium where $a = 1$ and $b = 4$.

3. Consider the finite strategic game $G = \{N, (A_i), (u_i)\}$. Let $B_i(\alpha_{-i})$ be the best-response of player i in G. Let $B(\alpha) = \times_{i \in N} B_i(\alpha_{-i})$. Prove or disprove that for all $b \in B(\alpha)$, for all $i \in N$, $U_i(b) \geq U_i(\alpha'_i, b_{-i})$ for all $\alpha'_i \in \Delta(A_i)$.

4. Consider the strategic game described in the following table.

	L	R
U	$3, 3$	$1, 4$
D	$4, 1$	$0, 0$

   (a) What are the set of mixed strategy Nash equilibria for this game.

   (b) Construct a correlated equilibrium for this game with payoffs that are equal to the payoffs in one of the Nash equilibria you constructed in (a).

   (c) Construct a correlated equilibrium where payoffs for each player are equal to 2.5.