

Quiz 1 solution
Econometrics II Spring, 2013

When the estimator $\bar{\theta}_{2n}$ is the constrained MLE, show that the Neyman $C(\alpha)$ test coincides with the score test.

Solution:

$$C(\alpha) = \frac{1}{n} \left[\frac{\partial L n(\bar{\theta}_n)}{\partial \theta_1} - I_{12}(\bar{\theta}_n) I_{22}^{-1}(\bar{\theta}_n) \frac{\partial L n(\bar{\theta}_n)}{\partial \theta_2} \right]' I^{11}(\bar{\theta}_n) \left[\frac{\partial L n(\bar{\theta}_n)}{\partial \theta_1} - I_{12}(\bar{\theta}_n) I_{22}^{-1}(\bar{\theta}_n) \frac{\partial L n(\bar{\theta}_n)}{\partial \theta_2} \right]$$

Note that, when $\bar{\theta}_{2n}$ is the constrained MLE, $\frac{\partial L n(\bar{\theta}_n)}{\partial \theta_2} = 0$. Thus,

$$C(\alpha) = \frac{1}{n} \left[\frac{\partial L n(\bar{\theta}_n)}{\partial \theta_1} \right]' I^{11}(\bar{\theta}_n) \left[\frac{\partial L n(\bar{\theta}_n)}{\partial \theta_1} \right]$$

which is of the form of score test.