

Advanced Microeconomics II

Quiz 3

WISE, Xiamen University
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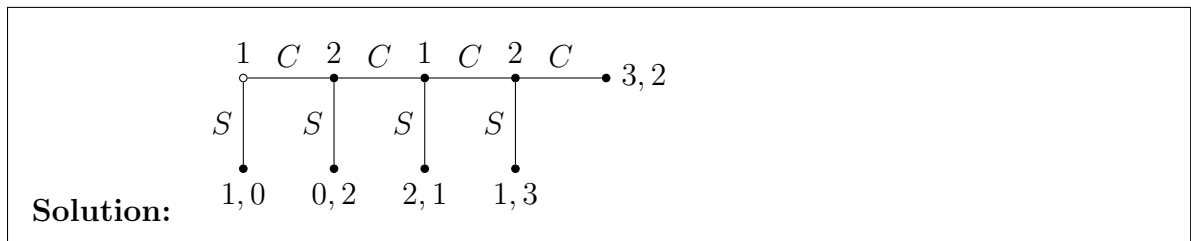
1. Consider the following extensive form game of perfect information, $\Gamma = \{\{1, 2\}, H, P, u_1, u_2\}$ where

- H consists of all sequences $C(t) = (C, \dots, C)$ of length t for $0 \leq t \leq 4$ and all sequences $S(t) = (C, \dots, C, S)$ consisting of $t - 1$ C s for $1 \leq t \leq 4$.
- $P(C(t)) = 1$ if t is even and $t < 4$, $P(C(t)) = 2$ if t is odd.

$$u_1(S(t)) = \begin{cases} (t+1)/2 & \text{if } t \text{ is odd} \\ t/2 - 1 & \text{if } t \text{ is even} \end{cases} \quad u_2(S(t)) = \begin{cases} (t-1)/2 & \text{if } t \text{ is odd} \\ t/2 + 1 & \text{if } t \text{ is even} \end{cases}$$

$$u_1(C(4)) = 3 \quad u_2(C(4)) = 2$$

(a) (2 points) Draw the game tree and associated payoffs.



(b) (2 points) Construct a sub-game perfect equilibrium of this game.

Solution: $s_1(\emptyset) = S, s_2(C) = S, s_1(CC) = S, s_2(CCC) = S$

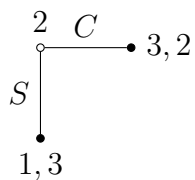
(c) (3 points) Construct a Nash equilibrium of this game that is not a sub-game perfect equilibrium.

Solution: There are three possible solutions.

- (i) $s_1(\emptyset) = S, s_2(C) = S, s_1(CC) = S, s_2(CCC) = C$
- (ii) $s_1(\emptyset) = S, s_2(C) = S, s_1(CC) = C, s_2(CCC) = S$
- (iii) $s_1(\emptyset) = S, s_2(C) = S, s_1(CC) = C, s_2(CCC) = C$

(d) (3 points) From the Nash equilibrium you constructed above define a sub-game and profitable one-shot deviation that deviates from the Nash equilibrium strategies only at the initial history. A game tree with arrows that reflect strategies is sufficient.

Solution: For the first and third Nash equilibrium identified above the following represents a subgame with a one-shot profitable deviation where the strategy is $s_2(\emptyset) = S$.



For the second Nash equilibrium identified above the following represents a subgame with a one-shot profitable deviation where the strategies are $s_1(\emptyset) = S, s_2(C) = S$.

