CHAPTER 10 CONCLUSION

Key word: Microeconometrics, Financial econometrics, Nonparametric econometrics, Panel data econometrics, Time series econometrics.

Abstract: In this chapter, we first review what we have covered in the previous chapters, and then discuss other econometric courses needed for various fields of economics and finance.

10.1 Summary

Question: What have we learnt from this course?

In this chapter, we will first summarize what we have learnt in this book.

The modern econometric theory developed in this book is built upon the following fundamental axioms:

- Any economy can be viewed as a stochastic process governed by some probability law.
- Any economic phenomena can be viewed as a realization of the stochastic economic process.

The probability law of the data generating process can be called the "law of economic motions." The objective of econometrics is to infer the probability law of economic motions using observed data, and then use the obtained knowledge to explain what has happened, to predict what will happen, and to test economic theories and economic hypotheses.

Suppose the conditional pdf $f(y_t|\Psi_t)$ of Y_t given $\Psi_t = (X_t, Z^{t-1})$, is available. Then we can obtain various attributes of the conditional distribution of Y_t given Ψ_t, such as

- conditional mean;
- conditional variance;
- conditional skewness;
- conditional kurtosis;
- conditional quantile.

An important question in economic analysis is: what aspect of the conditional pdf will be important in economics and finance? Generally speaking, the answer is dictated by the nature of the economic problem one has at hand. For example, the efficient market hypothesis states that the conditional expected asset return given the past information is equal to the long-run market average return; rational expectations theory suggests that conditional expectational errors given the past information should be zero. In unemployment duration analysis, one should model the entire conditional distribution of the unemployment duration given the economic characteristics of the unemployed workers.

It should be emphasized that the conditional pdf or its various aspects only indicate a predictive relationship between economic variables, that is, when one can use some economic variables to predict other variables. The predictive relationship may or may not be the causal relationship between or among economic variables, which is often of central interest to economists. Economic theory often hypothesizes a causal relationship and such economic theory is used to interpret the predictive relationship as a causal relationship.

Economic theory or economic model is not a general framework that embeds an econometric model. In contrast, economic theory is often formulated as a restriction on the conditional pdf or its certain aspect. Such a restriction can be used to validate economic theory, and to improve forecasts if the restriction is valid or approximately valid.

Question: What is the role that economic theory plays in economic modeling?

- Indication of the nature (e.g., conditional mean, conditional variance, etc) of the relationship between Y_t and X_t : Which moments are important and of interest?
- Choice of economic variables X_t.
- Restriction on the functional form or parameters of the relationship.
- Helping judge causal relationships.

In summary, any economic theory can be formulated as a restriction on the conditional probability distribution of the economic stochastic process. Economic theory plays an important role in simplifying statistical relationships so that a parsimoneus econometric model can eventually capture essential economic relationships.

Motivated by the fact that economic theory often has implication on and only on the conditional mean of economic variables of interest, we first develop a comprehensive econometric theory for linear regression models where by linearity we mean the conditional mean is linear in parameters and not necessarily linear in explanatory variables. We start in Chapter 3 with the classical linear regression model, for which we develop a finite sample statistical theory when the regression disturbance is i.i.d. normally distributed, and is independent of the regressor. The normality assumption is crucial for the finite sample statistical theory. The essence of the classical theory for linear regression models is i.i.d., which implies conditional homoskedasticity and serial uncorrelatedness, which ensures the BLUE property for the OLS estimator. When conditional heteroskedasticity and autocorrelation exist, the GLS estimator illustrates how to restore the BLUE property by correcting conditional heteroskedasticity and differencing out serial correlation.

With the classical linear regression model as a benchmark, we have developed a modern econometric theory for linear regression models by relaxing the classical assumptions in subsequent chapters. First of all, we relax the normality assumption in Chapter 4. This calls for asymptotic analysis because finite sample theory is no longer possible. It is shown that when the sample size is large, the classical results are still approximately applicable for linear regression models with independent observations under conditional homoskedasticity. However, under conditional heteroskedasticity, the classical results, such as the popular t-test and F-test statistics, are no longer applicable, even if the sample size goes to infinity. This is due to the fact that the asymptotic variance of the OLS estimator has a different structure under conditional hetersokedasticity. We need to use White's (1980) heteroskedasticity-consistent variance estimator and use it to develop robust hypothesis tests. It is therefore important to test conditional homoskedasticity, and White (1980) develops a regression-based test procedure.

The asymptotic theory developed for linear regression models with independent observations in Chapter 4 is extended to linear regression models with time series observations. This covers two types of regression models: one is called a static regression model where the explanatory variables or regressors are exogenous variables. The other is called a dynamic regression model whose regressors include lagged dependent variables and exogenous variables. It is shown in Chapter 5 that when the asymptotic theory of Chapter 4 is applicable when the regression disturbance is a martingale difference sequence. Because of its importance, we introduce tests for martingale difference sequence of regression disturbances by checking serial correlation in the disturbance. The tests include the popular Lagrange multiplier test for serial correlation. We have also considered a Lagrange multiplier test for autoregressive conditional heteroskedasticity (ARCH) and discussed its implication on the inference of static and dynamic regression models respectively.

For many static regression models, it is evident that the regression disturbance displays serial correlation. This affects the asymptotic variance of the OLS estimator. When serial correlation is of a known structure up to a few unknown parameter, we can use the Ornut-Cochrance procedure to obtain asymptotically efficient estimator for regression parameters. When serial correlation is of unknown form, we have to use a long-run variance estimator to estimate the asymptotic variance of the OLS estimator. A leading example is the kernel-based estimator such as the Newey-West variance estimator. With such a variance estimator, robust test procedures for hypotheses of interest can be constructed. These are discussed in Chapter 6.

The estimation and inference of linear regression models are complicated when the orthoganality condition of $E(\varepsilon_t|X_t) = 0$ does not hold, which can arise due to measurement errors, simultaneous equations bias, omitted variables, and so on. In Chapter 7 we discuss a popular method—the two stage least squares—to estimate model parameters in such scenarios.

Chapter 8 introduces the GMM method, which is particularly suitable for estimating both linear and nonlinear econometric models that can be characterized by a set of moment conditions. A prime economic example is the rational expectations theory, which is often characterized by an Euler equation. In fact, the GMM method provides a convenient framework to view most econometric estimators, including the least squares, and instrumental variables regression.

Chapter 9 discusses conditional probability distribution models and other econometric models that can be estimated by using pseudo probability likelihood methods. Conditional distribution models have found wide applications in economics and finance, and MLE is the most popular and most efficient method to estimate parameters in conditional distribution models. On the other hand, many econometric models can be conveniently estimated by using a pseudo likelihood function. These include nonlinear least squares, ARMA, GARCH models, as well as limited dependent variables and discrete choice models. Such an estimation method is called the Quasi-MLE. There is an important difference between MLE and QMLE. The forms of their asymptotic variances are different. In certain sense, the asymptotic variance of MLE is similar in structure to the asymptotic variance of the OLS estimator under conditional homoskedasticity and serial uncorrelatedness, while the asymptotic variance of the QMLE is similar in structure to the asymptotic variance of the OLS estimator under conditional heteroskedasticity and autocorrelation.

Chapters 2 to 9 are treated in a unified and coherent manner. The theory is constructed progressively from the simplest classical linear regression models to nonlinear expectations models and then to conditional distributional models. The book has emphasized the important implication of conditional heteroskedasticity and autocorrelation as well as misspecification of conditional distributional models on the asymptotic variance of the related econometric estimators. With a good command of the econometric theory developed in Chapters 2 to 9, we can conduct a variety of empirical analysis in economics and finance, including all motivating examples introduced in Chapter 1. In addition to asymptotic theory, the book has also shown students how to do asymptotic analysis via the progressive development of the asymptotic theory in Chapters 2 to 9. Moreover, we have also introduced a variety of basic asymptotic analytic tools concepts, including various convergence concepts, limit theorems, and basic time series concepts and models.

10.2 Directions for Further Study in Econometrics

The econometric theory presented in this book has laid down a solid foundation in econometric study. However, it does not cover all econometric theory. For example, we only cover stationary time series models, nonstationary time series models, such as unit root models and cointegrated models, have not been covered, which call for a different asymptotic theory (see, e.g., Hamilton 1994). Panel data models also require a separate and independent treatment (see, e.g., Hsiao 2002). Due to the unique features of financial time series, particularly high-frequency financial time series, financial econometrics has emerged as a new field in econometrics that is not covered by standard time series econometrics. On the other hand, although our theory can be applied to models for limited dependent variables and discrete choice variables, more detailed treatment and comprehensive coverage are needed. Moreover, topics on asymptotic analytic tools may be covered to train students' asymptotic analysis ability in a more comprehensive manner.

About the Author: Yongmiao Hong received his Bachelor Degree in Physics in 1985, and his MA degree in Economics in 1988, both from Xiamen University. He received his PHD in Economics from University of California, San Diego, in 1993. In the same year, he became a tenure track assistant professor in Department of Economics, Cornell University, where he became a tenured faculty in 1998, and a full professor in 2001. He has also been a special-term visiting professor in the School of Economics and Management, Tsinghua University since 2002, and a Cheung Kong Visiting Professor in the Wang Yanan Institute for Studies in Economics (WISE), Xiamen University, since 2005. He is the President of the Chinese Economists Society in North America, 2009-2010. Yongmiao Hong's research interests have been econometric theory, time series analysis, financial econometrics, and empirical study on the Chinese economy and financial markets. He has published dozens of academic papers in a number of top academic journals in economics, finance and statistics, such as *Econometrica*, *Journal of Political Economy*, *Journal of Quarterly Economics*, *Review of Economic Studies*, *Review of Economics and Statistics*, *Review of Financial Studies*, *Journal of Econometrics*, *Econometric Theory*, *Biometrika*, *Journal of Royal Statistical Society Series B*, and *Journal of American Statistical Association.*