Advanced Macroeconomics II

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An Economic Applicaton of the Simple RBC Model

Estimating Economic Effects of Political Movements in China

Kwan and Chow, Journal of Comparative Economics 23, 192-208 (1996)

- **Investment** is determined by a central planner maximizing a multiperiod objective function.
- **Political events** are modeled by exogenous changes in the shocks to productivity and to investment.
- **Model parameters** are estimated with MLE.
- **Effects of the events** are measured by comparing the time paths generated by the model with and without the changes in the shocks.

Problem

What were the economic effects of the Great Leap Forward Movement in 1958-1962 and the Cultural Revolution in 1966-1969 in China?

Methodology Compare the historical time paths of the economy with the paths that would have prevailed absent the above events.

Tools

- An RBC model to explain the growth of Chinese economy.
- Solving the model with numerical method.
- Estimation: MLE
- Simulation.

Preference: from a social planner

$$E_t \sum_{i=1}^{\infty} \beta^{t+i} \log c_{t+i} \tag{1}$$

Technology:

Aggregate real output

$$Q_t = A_t K_t^{1-\alpha} L_t^{\alpha} \tag{2}$$

Denoting $q_t = Q/L$, $k = K/L$, net investment per laborer $i = I/L$,

$$\begin{aligned}
q_t &= A_t k_t^{1-\alpha} && (3)\\
q_t &= c_t + i_t && (4)\\
k_{t+1} &= k_t + i_t && (5)\\
\ln A_{t+1} &= \gamma + \ln A_t + \eta_{t+1} && (6)
\end{aligned}$$

Endowment and **information** as so defined in the previous baseline model.

Treatment on data

- Q: National income (Statistical Yearbook of China (SYC) 1994) devided by price deflator (national income in current price to national income in 1952 price).
- $Q_t = C_t + I_t$, in Chinese official statistics.
- Initial estimate of capital $K = 2213$(unit: 100 million yuan), from the estimate of Chow (1993b, p.821), In 1952, $k = K/L$. In later years, as defined by $k_{t+1} = k_t + i_t$. (An approximation)

$$\begin{aligned} K_{t+1} &= K_t + I_t \\ \frac{K_{t+1}}{L_t} &= \frac{K_t}{L_{t-1}}\frac{L_{t-1}}{L_t} + \frac{I_t}{L_t}, or \frac{K_{t+1}}{L_{t+1}}\frac{L_{t+1}}{L_t} = \frac{K_t}{L_t} + \frac{I_t}{L_t} \\ k_{t+1} &= k_t \frac{1}{1+n_t} + i_t, \quad or \;\; k_{t+1}(1+n_t) = k_t + i_t \end{aligned}$$

- Admitted shortcomings: treating technology, population, and labor force as exogenous. An important step however...

Detrending:

Convert the variables to stationary processes to avoid unit root problem in Eq. (6)

How? – To detrend all variables along their balanced growth paths.

$$z_t \equiv A_t^{1/\alpha}, \quad \bar{k}_{t+1} \equiv k_{t+1}/z_t, \quad \bar{c}_t \equiv c_t/z_t, \quad \bar{z}_t \equiv z_t/z_{t-1}. \tag{7}$$

Hence,

$$\begin{aligned}
\ln A_{t+1} &= \gamma + \ln A_t + \eta_{t+1} \\
\alpha \ln z_{t+1} &= \gamma + \alpha \ln z_t + \eta_{t+1} \\
\ln z_{t+1} &= \gamma/\alpha + \ln z_t + \eta_{t+1}/\alpha \text{ (unit root)}
\end{aligned}$$

Define

$$\mu \equiv \gamma/\alpha, \quad \varepsilon_t \equiv \eta_{t+1}/\alpha$$

$$\ln \bar{z}_{t+1} = \ln z_{t+1}/z_t = \mu + \varepsilon_{t+1}$$

Detrending:

Combining Eq. (3) to (5) to get the intertemporal budget constraint with only capital and consumption,

$$c_t + k_{t+1} = A_t k_t^{1-\alpha} + k_t, \tag{8}$$

transforming

$$\begin{aligned} c_t/z_t + k_{t+1}/z_t &= z_t^{\alpha-1} k_t^{1-\alpha} + (k_t/z_{t-1}) z_{t-1}/z_t \\ \bar{c}_t + \bar{k}_{t+1} &= \bar{k}_t^{1-\alpha} \bar{z}_t^{\alpha-1} + \bar{k}_t \bar{z}_t^{-1} \end{aligned} \tag{9}$$

The dynamic optimization problem under the transformed system

$$\max_{(c_t, k_{t+1})_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \ln \bar{c}_t \right]$$

[Why can we do this?]

$$\begin{aligned} s.t. \bar{c}_t + \bar{k}_{t+1} &= \bar{k}_t^{1-\alpha} \bar{z}_t^{\alpha-1} + \bar{k}_t \bar{z}_t^{-1} \\ \ln \bar{z}_{t+1} &= \mu + \varepsilon_{t+1} \end{aligned}$$

FONCs:

$$
\begin{aligned}
E_t\left[\beta\frac{\bar{c}_t}{\bar{c}_{t+1}}R_{t+1}\right] &= 1 && (10)\\
R_{t+1} &= \left(1+(1-\alpha)\bar{k}_{t+1}^{-\alpha}\bar{z}_{t+1}^{\alpha}\right)/\bar{z}_{t+1} && (11)\\
\bar{c}_t+\bar{k}_{t+1} &= \bar{k}_t^{1-\alpha}\bar{z}_t^{\alpha-1}+\bar{k}_t\bar{z}_t^{-1} && (12)\\
\ln\bar{z}_{t+1} &= \mu+\varepsilon_{t+1} && (13)
\end{aligned}
$$

Steady states (sorry for the abuse of notation):

$$
\begin{aligned}
\ddot{z} &= \\
\ddot{R} &= \\
\ddot{k} &= \\
\ddot{c} &=
\end{aligned}
$$

Log-linearization: (denote $\hat{x}_t \equiv \ln(\bar{x}_t / \ddot{x})$)

Economic Effects of Political Movements in China

Statistical Estimation I: solving the model

Postulate a linear recursive law of motion for $\hat{c}_t$ and $\hat{k}_t$

$$
\begin{aligned}
\hat{c}_t &= v_{ck}\hat{k}_t + v_{cz}\hat{z}_t \\
\hat{k}_{t+1} &= v_{kk}\hat{k}_t + v_{kz}\hat{z}_t
\end{aligned}
$$

Insert the return equation to Euler equation to delete $\hat{r}_t$, and insert the law of motion to the log-linearized budget constraint and Euler equation.

Undetermined coefficients:

$$v_{ck} \quad v_{cz} \quad v_{kk} \quad v_{kz} \ ??$$

After getting these coefficients in terms of the parameters and steady state values, write the system as

$$\begin{bmatrix} \hat{k}_t \\ \hat{z}_t \end{bmatrix} = \begin{bmatrix} v_{kk} & v_{kz} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{k}_{t-1} \\ \hat{z}_{t-1} \end{bmatrix} + \begin{bmatrix} e_t \\ \varepsilon_t \end{bmatrix}.$$

To utilize the transformed data, write the system in log term,

$$\begin{bmatrix} \ln \bar{z}_t \\ \ln \bar{k}_t \end{bmatrix} = \begin{bmatrix} \mu \\ g \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ G_1 & G_2 \end{bmatrix} \begin{bmatrix} \ln \bar{z}_{t-1} \\ \ln \bar{k}_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ e_t \end{bmatrix}$$

[What are g, G_1 and G_2?]

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Statistical Estimation II: Estimating the model

With $y_t = (\ln \bar{z}_t, \ln \bar{k}_t)'$ and $x_t = (1, \ln \bar{z}_{t-1}, \ln \bar{k}_{t-1})'$, and

$$\Gamma = \begin{bmatrix} \mu & 0 & 0 \\ g & G_1 & G_2 \end{bmatrix}, \ \xi_t = \begin{bmatrix} \varepsilon_t \\ e_t \end{bmatrix}$$

$$y_t = \Gamma x_t + \xi_t$$

With T observations, $Y = (y_1, ..., y_T)'$, $X = (x_1, ...x_T)'$ and $\Xi = (\xi_1, ...\xi_T)'$, the stacked form is

$$Y = X\Gamma' + \Xi$$

Estimation procedure

Assumption: $\xi_t \sim i.i.d.N(0,\Sigma)$, $\Sigma = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$.

Likelihood function:

$$L = \frac{1}{(2\pi)^T} |\Sigma|^{-T/2} \exp[-\frac{1}{2}\Sigma_{t=1}^T (y_t - \Gamma x_t)'(\Sigma)^{-1}(y_t - \Gamma x_t)]$$

Using the concentrated log-likelihood function

$$\ln L = const - (T/2) \ln |T^{-1}(Y - X\Gamma')'(Y - X\Gamma')|$$

What's crucial in the estimation?

Data construction for y_t and x_t!

We have in the beginning: k_t, q_t, i_t, c_t.

For y_t, we need $\ln \bar{k}_t = \ln(k_t / z_{t-1})$

$$\ln z_t = \ln(q_t - (1-\alpha) \ln k_t)/\alpha$$

So the data itself is a function of the underlying parameter sets (α, γ, β). Each time of likelihood calculation given parameter set needs a lot of computation.

$$\ln A_t = \alpha \ln z_t = \ln q_t - (1-\alpha) \ln k_t .$$

Then the productivity process

$$\begin{aligned} \ln A_t &= \gamma + \ln A_{t-1} + \eta_t \\ \ln q_t - (1-\alpha) \ln k_t &= \gamma + \ln q_{t-1} - (1-\alpha) \ln k_{t-1} + \eta_t \end{aligned}$$

What's crucial in the estimation?

Writing out the second equation using $\ln \bar{k}_t = \ln(k_t / z_{t-1})$

$$
\begin{aligned}
\ln \bar{k}_t &= g + G_1 \ln \bar{z}_{t-1} + G_2 \ln \bar{k}_{t-1} + e_t \\
\ln k_t &= g + [\ln q_t - (1-\alpha)\ln k_t] / \alpha \\
&\quad + G_1 [\Delta \ln q_{t-1} - (1-\alpha)\Delta \ln k_{t-1}] / \alpha \\
&\quad + G_2 [\ln k_{t-1} - (\ln q_{t-1} - (1-\alpha)\ln k_{t-1}) / \alpha] + e_t
\end{aligned}
$$

So the data itself is a function of the underlying parameter set (α, γ, β). Each time of likelihood calculation given parameter set needs the computation of productivity.

Implementing the MLE, you will obtain the parameter estimate

$$\tilde{\theta}_{MLE} = \left(\tilde{\alpha}, \tilde{\gamma}, \tilde{\beta}\right).$$

Now each period, your estimated model gives predictions on productivity and capital.

$$\begin{aligned}\ln \tilde{A}_t &= \tilde{\gamma} + \ln A_{t-1} \\ &= \tilde{\gamma} + \ln q_{t-1} - (1-\alpha)\ln k_{t-1}\end{aligned}$$

The residual term is

$$\begin{aligned}\tilde{\eta}_t &= \ln A_t - \ln \tilde{A}_t \\ &= \ln q_t - (1-\alpha)\ln k_t - \left[\tilde{\gamma} + \ln q_{t-1} - (1-\alpha)\ln k_{t-1}\right].\end{aligned}$$

Likewise, you obtain $\ln \tilde{k}_t$ and $\tilde{e}_t$.

- Assume the model is true, the actual data are generated exactly by the dynamic equation with exogenous stochastic shocks

$$y_t = f(y_{t-1}, \theta) + \varepsilon_t$$

- With the estimated parameters

$$y_t = \tilde{f}(y_{t-1}, \tilde{\theta}) + \tilde{\varepsilon}_t$$

- In a counter-factual study, we ask what if $\tilde{\varepsilon}_t$ is different, i.e. $\check{\varepsilon}_t$? –> y_t changes to $\check{y}_t$.
- The difference, $y_t - \check{y}_t$, is the effect of $(\tilde{\varepsilon}_t - \check{\varepsilon}_t)$.
- The "counter-factual" $\check{y}_t$ tells what y_t would have been, had $\check{\varepsilon}_t$ happened.

Economic Effects of Political Movements in China

Simulation: A counter-factual study on the Great Leap Forward in 1958-1962.

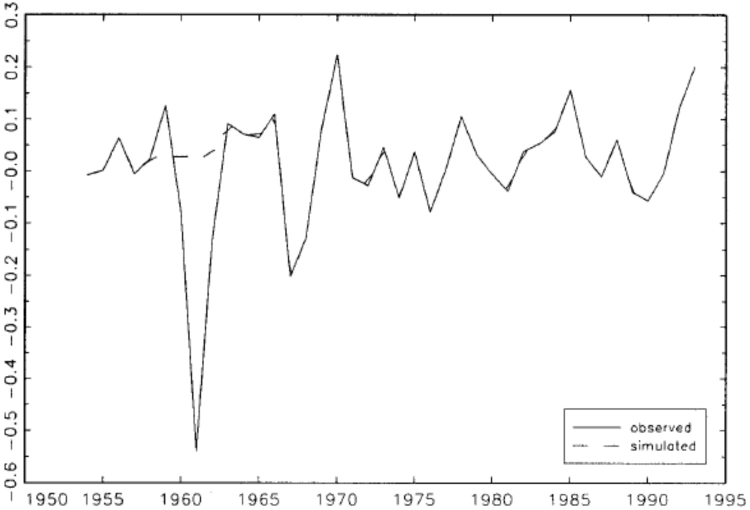

FIG. 1. Observed and simulated residual 1.

Economic Effects of Political Movements in China

Simulation: A counter-factual study on the Great Leap Forward in 1958-1962.

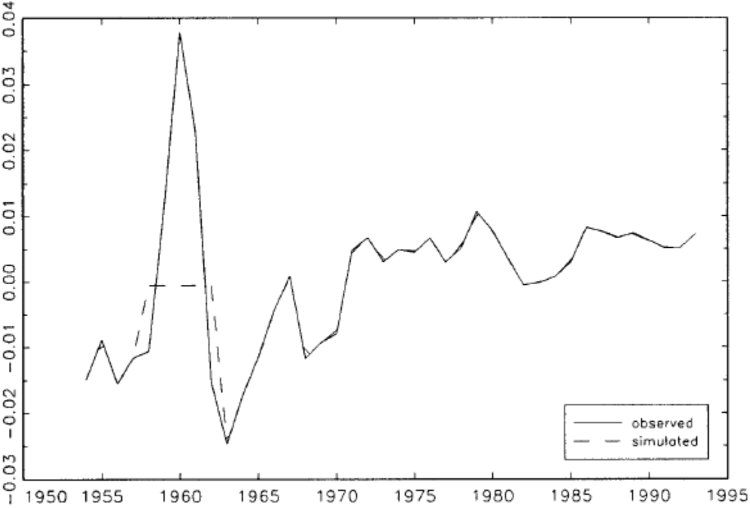

FIG. 2. Observed and simulated residual 2.

Economic Effects of Political Movements in China

Simulation: A counter-factual study on the Great Leap Forward in 1958-1962.

Question: What if the Great Leap Forward had not happened?

- Assume $\varepsilon_t = \begin{bmatrix} \eta_t & e_t \end{bmatrix}' = 0_{2\times 1}$, for $t = 1958, ..., 1962$.
- Keep $\tilde{\varepsilon}_t = \begin{bmatrix} \tilde{\eta}_t & \tilde{e}_t \end{bmatrix}'$ for all other t.
- Up to 1957: $\tilde{f}(y_{t-1}, \tilde{\theta}) + \tilde{\varepsilon}_t = y_t$, realized data.
- In 1958: $\check{y}_t \equiv \tilde{f}(y_{t-1}, \tilde{\theta})$, the simulated data begin to diverge from the realized ones.
  - From 1959 to 1962: $\check{y}_t \equiv \check{f}(\check{y}_{t-1}, \tilde{\theta})$.
- From 1963 onwards: $\check{y}_t \equiv \check{f}(\check{y}_{t-1}, \tilde{\theta}) + \tilde{\varepsilon}_t$.

Economic Effects of Political Movements in China

Simulation: A counter-factual study on the Great Leap Forward in 1958-1962.

Question: What if the Great Leap Forward had not happened?

Answer

- Compare $\check{y}_t$ and y_t, i.e., $\begin{bmatrix} \ln \check{A}_t & \ln \check{k}_t \end{bmatrix}'$ and $\begin{bmatrix} \ln A_t & \ln k_t \end{bmatrix}'$.
- $\check{A}_t$ actually helps to pin down $\check{q}_t$, as

$$\ln q_t = \ln A_t + (1 - \alpha) \ln k_t .$$

- Convert the variables to their levels from log terms.
- What about consumption? – Simply use the budget constraint in level.

Economic Effects of Political Movements in China

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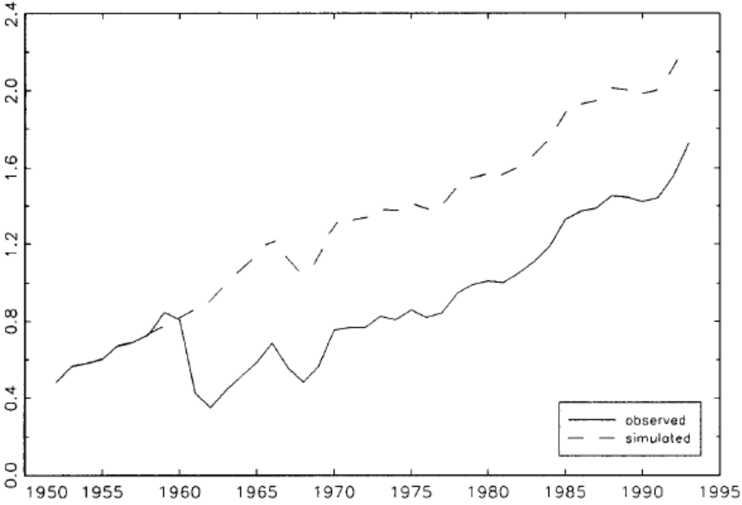

FIG. 6. Observed and simulated Solow residual (in log).

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Simulation: A counter-factual study on the Great Leap Forward in 1958-1962.

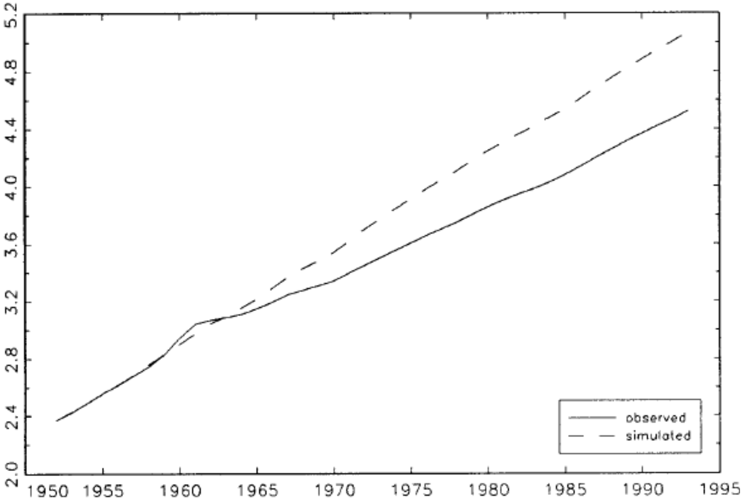

FIG. 5. Observed and simulated capital (in log).

Economic Effects of Political Movements in China

Simulation: A counter-factual study on the Great Leap Forward in 1958-1962.

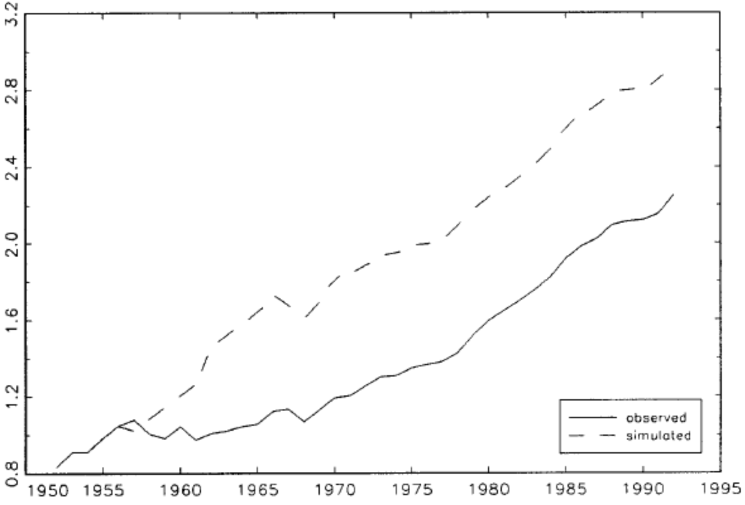

FIG. 4. Observed and simulated consumption (in log).

Economic Effects of Political Movements in China

Simulation: A counter-factual study on the Great Leap Forward in 1958-1962.

TABLE 3

SIMULATION/OBSERVED LEVEL IN 1992

	Great leap	Cultural revolution	Both
Output	2.0031	1.2033	2.7130
Consumption	2.0047	1.2022	2.7261
Capital	1.7208	1.1537	2.1687
Steady state	2.1074	1.2204	2.9238

Note. $(\hat{\alpha}, \hat{\beta}, \hat{\gamma}) = (0.7495, 0.9999, 0.0218)$.

Economic Effects of Political Movements in China

Simulation: A counter-factual study on the Great Leap Forward in 1958-1962.

Question

How a new steady state is computed with the simulated data?

Economic Effects of Political Movements in China

Simulation: a counter-factual study on the Great Leap Forward in 1958-1962.

Robustness check

How sensitive are your results to the parameter estimates, and modeling assumptions?

Economic Effects of Political Movements in China

Simulation: a counter-factual study on the Great Leap Forward in 1958-1962.

TABLE 4

SIMULATION/OBSERVED LEVEL IN 1992

	Great leap	Cultural revolution	Both
Output	2.5446	1.2355	3.6549
Consumption	2.5680	1.2349	3.7277
Capital	1.9708	1.1643	2.5461
Steady state	3.2856	1.3111	5.2465

Note. α fixed at 0.5, $\hat{\beta} = 0.9715$, $\hat{\gamma} = 0.0083$.

Economic Effects of Political Movements in China

Simulation: a counter-factual study on the Great Leap Forward in 1958-1962.

TABLE 5

SIMULATION/OBSERVED LEVEL IN 1992

	Great leap	Cultural revolution	Both
Output	2.2907	1.2217	3.2082
Consumption	2.3008	1.2207	3.2459
Capital	1.8614	1.1597	2.3796
Steady state	2.6306	1.2648	3.9152

Note. α fixed at 0.6, $\hat{\beta} = 0.9817$, $\hat{\gamma} = 0.0132$.