Monetary Policy, Inflation, and the Business Cycle

Chapter 1
Introduction

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The present monograph seeks to provide the reader with an overview of modern monetary theory. Over the past decade, monetary economics has been among the most fruitful research areas within macroeconomics. The efforts of many researchers to understand the relationship between monetary policy, inflation and the business cycle has led to the development of a framework–the so called New Keynesian model– that is widely used for monetary policy analysis. The following chapters offer an introduction to that basic framework and a discussion of its policy implications.

The need for a framework that can help us understand the links between monetary policy and the aggregate performance of an economy seems self-evident. On the one hand, and whether in their condition as consumers, workers or investors, citizens of modern societies have good reason to care about developments in inflation, employment, and other economy-wide variables, for those developments affect to an important degree people's opportunities to maintain or improve their standard of living. On the other hand, monetary policy, as conducted by central banks, has an important role in shaping those macroeconomic developments, both at the national and supranational levels. Changes in interest rates have a direct effect on the valuation of financial assets and their expected returns, as well as on the consumption and investment decisions of households and firms. Those decisions can in turn have consequences for GDP growth, employment and inflation. It is thus not surprising that the interest rate decisions made by the Fed, the ECB, or other prominent central banks around the world are given so much attention, not only by market analysts and the financial press, but also by the general public. It would thus seem important to understand how those interest rate decisions end up affecting the various measures of an economy's performance, both nominal and real. A key goal of monetary theory is to provide us with an account of the mechanisms through which those effects arise, i.e. the transmission mechanism of monetary policy.

Central banks do not change interest rates in an arbitrary or whimsical manner. Their decisions are meant to be purposeful, i.e. they seek to attain certain objectives, while taking as given the constraints posed by the workings of a market economy, in which the vast majority of economic decisions are made in a decentralized manner by a large number of individuals and firms. Understanding what should be the objectives of monetary policy and how the latter should be conducted in order to attain those objectives constitutes another important aim of modern monetary theory, in its normative dimension.

The following chapters present a framework that helps us understand both the transmission mechanism of monetary policy and the elements that come into play in the design of rules or guidelines for the conduct of monetary policy. The framework is, admittedly, highly stylized and should be viewed more as a pedagogical tool than a quantitative model that can be readily taken to the data. Nevertheless, and despite its simplicity, it contains the key elements (though not all the bells and whistles) found in the medium-scale monetary models that are currently being developed by the research teams of many central banks.[1]

The monetary framework that constitutes the focus of the present monograph has a core structure that corresponds to a Real Business Cycle (RBC) model, on which a number of elements characteristic of Keynesian models are superimposed. That confluence of elements has led some authors to label the new paradigm as the "new neoclassical synthesis."[2] In the following sections we describe briefly each of those two influences in turn, in order to provide some historical background to the framework developed in subsequent chapters.

1 Background: Real Business Cycle Theory and Classical Monetary Models

During the years following the seminal papers of Kydland and Prescott (1982) and Prescott (1986), Real Business Cycle (RBC) theory provided the main reference framework for the analysis of economic fluctuations, and became to a large extent the core of macroeconomic theory. The impact of the RBC revolution had both a methodological and a conceptual dimension.

From a *methodological* point of view, RBC theory established firmly the use of dynamic stochastic general equilibrium (DSGE) models as a central tool for macroeconomic analysis. Behavioral equations describing aggregate variables were thus replaced by first order conditions of intertemporal problems facing consumers and firms. Ad-hoc assumptions on the formation of expectations gave way to rational expectations. In addition, RBC economists

[1] See, e.g. , Bayoumi (2004), Coenen, McAdam and Straub (2006) and Erceg, Guerrieri and Gust (2006) for a description of the models under development at the International Monetary Fund, the European Central Bank and the Federal Reserve Board, respectively.

[2] See Goodfriend and King (1997).

stressed the importance of the quantitative aspects of modelling, as reflected in the central role given to the calibration, simulation and evaluation of their models.

The most striking dimension of the RBC revolution was, however, conceptual. It rested on three basic claims:

- *The efficiency of business cycles.* Thus, the bulk of economic fluctuations observed in industrialized countries could be interpreted as an equilibrium outcome resulting from the economy's response to exogenous variations in real forces (most importantly, technology), in an environment characterized by perfect competition and frictionless markets. According to that view, cyclical fluctuations did not necessarily signal an inefficient allocation of resources (in fact, the fluctuations generated by the standard RBC model were fully optimal). That view had an important corollary: stabilization policies may not be necessary or desirable, and they could even be counterproductive. This was in contrast with the conventional interpretation, tracing back to Keynes (1936), of recessions as periods with an inefficiently low utilization of resources, which could be brought to an end by means of economic policies aimed at expanding aggregate demand.

- *The importance of technology shocks as a source of economic fluctuations.* That claim derived from the ability of the basic RBC model to generate "realistic" fluctuations in output and other macroeconomic variables, even when variations in total factor productivity –calibrated to match the properties of the Solow residual– are assumed to be the only exogenous driving force. Such an interpretation of economic fluctuations was in stark contrast with the traditional view of technological change as a source of long-term growth, unrelated to business cycles.

- *The limited role of monetary factors.* Most importantly, given the subject of the present monograph, RBC theory sought to explain economic fluctuations with *no reference to monetary factors*, even abstracting from the existence of a monetary sector.

Its strong influence among academic researchers notwithstanding, the RBC approach had a very limited impact (if any) on central banks and other policy institutions. The latter continued to rely on large-scale macroeconometric models despite the challenges to their usefulness for policy evaluation (Lucas (1976)) or the largely arbitrary identifying restrictions underlying the estimates of those models (Sims (1980)).

The attempts by Cooley and Hansen (1989) and others to introduce a monetary sector in an otherwise conventional RBC model, while sticking to the assumptions of perfect competition and fully flexible prices and wages, were not perceived as yielding a framework that was relevant for policy analysis. As discussed in chapter 2, the resulting framework, which we refer to as the *classical* monetary model, generally predicts neutrality (or near neutrality) of monetary policy with respect to real variables. That finding is at odds with the widely held belief (certainly among central bankers) in the power of that policy to influence output and employment developments, at least in the short run. That belief is underpinned by a large body of empirical work, tracing back to the narrative evidence of Friedman and Schwartz (1963), up to the more recent work using time series techniques, as described in Christiano, Eichenbaum and Evans (1999).[3]

In additional to the empirical challenges mentioned above, the normative implications of classical monetary models have also led many economists to call into question their relevance as a framework for policy evaluation. Thus, those models generally yield as a normative implication the optimality of the Friedman rule –a policy that requires that central banks keep the short term nominal rate constant at a zero level–, even though that policy seems to bear no connection whatsoever with the monetary policies pursued (and viewed as desirable) by the vast majority of central banks. Instead, the latter are characterized by (often large) adjustments of interest rates in response to deviations of inflation and indicators of economic activity from their target levels.[4]

[3] An additional challenge to RBC models has been posed by the recent empirical evidence on the effects of technology shocks. Some of that evidence suggests that technology shocks generate a negative short-run comovement between output and labor input measures, thus rejecting a prediction of the RBC model that is key to its ability to generate fluctuations that resemble actual business cycles (see, e.g. Galí (1999) and Basu, Fernald, and Kimball (2006)). Other evidence suggests that the contribution of technology shocks to the business cycle has been quantitatively small (see, e.g. Christiano, Eichenbaum and Vigfusson (2003)), though investment-specific technology shocks may have played a more important role (Fisher (2006)). See Galí and Rabanal (2004) for a survey of the empirical evidence on the effects of technology shocks.

[4] An exception to that pattern is given by the Bank of Japan, which kept its policy rate

The conflict between theoretical predictions and evidence, and between normative implications and policy practice, can be viewed as a symptom that some elements that are important in actual economies may be missing in classical monetary models. As discussed below, those shortcomings are the main motivation behind the introduction of some Keynesian assumptions, while maintaining the RBC apparatus as an underlying structure.

2 The New Keynesian Model: Main Elements and Features[5]

Despite their different policy implications, there are important similarities between the RBC model and the new Keynesian monetary model. The latter, whether in the canonical form presented below or in its more complex extensions, has at its core some version of the RBC model. This is reflected in the assumption of (i) an infinitely-lived representative household, who seeks to maximize the utility from consumption and leisure, subject to an intertemporal budget constraint, and (ii) a large number firms with access to an identical technology, subject to exogenous random shifts. Though endogenous capital accumulation, a key element of RBC theory, is absent in canonical versions of the new Keynesian model, it is easy to incorporate and is a common feature of medium-scale versions.[6] Also, as in RBC theory, an equilibrium takes the form of a stochastic process for all the economy's endogenous variables, consistent with optimal intertemporal decisions by households and firms, given their objectives and constraints, and with the clearing of all markets.

The new Keynesian modelling approach, however, combines the DSGE structure characteristic of RBC models with assumptions that depart from those found in classical monetary models. Here is a list of some of the key elements and properties of the resulting models:

- *Monopolistic competition.* The prices of goods and inputs are set by pri-

at a zero level over the period 1999-2006. Few, however, would interpret that policy as the result of a deliberate attempt to implement the Friedman rule. Rather, it is generally viewed as a consequence of the zero lower bound on interest rates becoming binding, with the resulting inability of central bank to stimulate the economy out of a low growth and deflation trap.

[5] See Galí and Gertler (2007) for an extended introduction to the new Keynesian model and a discussion of its main features.

[6] See, e.g., Smets and Wouters (2003)

vate economic agents in order to maximize their objectives, as opposed to being determined by an anonymous Walrasian auctioneer seeking to clear all (competitive) markets at once.

- *Nominal rigidities.* Firms are subject to some constraints on the frequency with which they can adjust the prices of the goods and services they sell. Alternatively, firms may face some costs of adjusting those prices. The same kind of friction applies to workers in the presence of sticky wages.

- *Short run non-neutrality of monetary policy.* As a consequence of the presence of nominal rigidities, changes in short term nominal interest rates (whether chosen directly by the central bank or induced by changes in the money supply) are not matched by one-for-one changes in expected inflation, thus leading to variations in real interest rates. The latter bring about changes in consumption and investment and, as a result, on output and employment, since firms find it optimal to adjust the quantity of goods supplied to the new level of demand. In the long run, however, all prices and wages adjust, and the economy reverts back to its natural equilibrium.

It is important to note that the three ingredients above were already central to the new Keynesian literature that emerged in the late 1970s and 1980s, and which developed in parallel with RBC theory. The models used in that literature, however, were often static or used reduced form equilibrium conditions that were not derived from explicit dynamic optimization problems facing firms and households. The emphasis of much of that work was instead on providing microfoundations, based on the presence of small menu costs, for the stickiness of prices and the resulting monetary non-neutralities.[7] Other papers emphasized the persistent effects of monetary policy on output, and the role that staggered contracts played in generating that persistence.[8] The novelty of the new generation of monetary models has been to embed those features in a fully-specified DSGE framework, thus adopting the formal modelling approach that has been the hallmark of RBC theory.

Not surprisingly, important differences with respect to RBC models emerge in the new framework. First, the economy's response to shocks is generally

[7] See, e.g., Akerlof and Yellen (1985), Mankiw (1985), Blanchard and Kiyotaki (1987), and Ball and Romer (1990).

[8] See, e.g., Fischer (1977) and Taylor (1980).

inefficient. Secondly, the non-neutrality of monetary policy resulting from the presence of nominal rigidities makes room for potentially welfare-enhancing interventions by the monetary authority, in order to minimize the existing distortions. Furthermore, those models are arguably suited for the analysis and comparison of alternative monetary regimes without being subject to the Lucas critique.[9]

2.1 Evidence of Nominal Rigidities and Monetary Policy Non-Neutrality

The presence of nominal rigidities and the implied real effects of monetary policy are two key ingredients of new Keynesian models. It would be hard to justify the use of a model with those distinctive features in the absence of evidence in support of their relevance. Next we briefly describe some of that evidence and provide the reader with some relevant references.

2.1.1 Evidence of Nominal Rigidities

Most attempts to uncover evidence on the existence and importance of price rigidities have generally relied on the analysis of micro data, i.e. data on the prices of individual goods and services.[10] In an early survey of that research, Taylor (1999) concludes that there is ample evidence of price rigidities, with the average frequency of price adjustment being about one year. In addition, he points to the very limited of evidence of synchronization of price adjustments, thus providing some justification for the assumption of staggered price setting commonly found in the new Keynesian model. The study of Bils and Klenow (2004), based on the analysis of the average frequencies of price changes for 350 product categories underlying the U.S. CPI, called into question that conventional wisdom by uncovering a median duration of prices between 4 and 6 months. Nevertheless, more recent evidence by Nakamura and Steinsson (2007), using data on the individual prices underlying the U.S. CPI and excluding price changes associated with sales, has led to a

[9] At least to the extent that the economy is sufficiently stable so that the log-linearized equilibrium conditions remain a good approximation, and that some of the parameters that are taken as "structural" (including the degree of nominal rigidities) can be viewed as approximately constant.

[10] See, e.g., Cecchetti (1986) and Kashyap (1995) for early works examining the patterns of prices of individual goods.

reconsideration of the Bils-Klenow evidence, with an upward adjustment of the estimated median duration to a range between 8 and 11 months. Evidence for the euro area, discussed in Dhyne et al. (2006), points to a similar distribution of price durations to that uncovered by Nakamura and Steinsson for the U.S..[11] It is worth mentioning that, in addition to evidence of substantial price rigidities, most studies find a large amount of heterogeneity in price durations across sectors/types of goods, with services being associated with the largest degree of price rigidities, and unprocessed food and energy with the lowest.

The literature also contains several studies based on micro data which provide analogous evidence of nominal rigidities for wages. Taylor (1999) surveys that literature and suggests an estimate of the average frequency of wage changes of about one year, the same as for prices. A significant branch of the literature on wage rigidities has focused on the possible existence of asymmetries that make wage cuts very rare or unlikely. Bewley's (1999) detailed study of firms' wage policies based on interviews with managers finds ample evidence of downward nominal wage rigidities. More recently, the multi-country study of Dickens et al. (2007) uncovers evidence of significant downward nominal and real wage rigidities in most of the countries in their sample.

2.1.2 Evidence of Monetary Policy Non-Neutralities

Monetary non-neutralities are, at least in theory, a natural consequence of the presence of nominal rigidities. As will be shown in chapter 3, if prices don't adjust in proportion to changes in the money supply (thus causing real balances to vary), or if expected inflation does not move one-for-one with the nominal interest rate when the latter is changed (thus leading to a change in the real interest rate), the central bank will generally be able to alter the level aggregate demand and, as a result, the equilibrium levels of output and employment. Is the evidence consistent with that prediction of models with nominal rigidities? And if so, are the effects of monetary policy interventions sufficiently important quantitatively to be relevant?

[11] In addition to studies based on the analysis of micro data, some researchers have conducted surveys of firms' pricing policies. See, e.g. Blinder et al. (1998) for the U.S. and Fabiani et al. (2005) for several countries in the euro area. The conclusions from the survey-based evidence tend to confirm the evidence of substantial price rigidities coming out of the micro-data analysis.

Unfortunately, identifying the effects of changes in monetary policy is not an easy task. The reason for this is well understood: an important part of the movements in whatever variable we take as the instrument of monetary policy (e.g. the short term nominal rate) are likely to be endogenous, i.e. the result of a deliberate response of the monetary authority to developments in the economy. Thus, simple correlations of interest rates (or the money supply) on output or other real variables cannot be used as evidence of non-neutralities, for the direction of causality could well go, fully or in part, from movements in the real variable (resulting from non-monetary forces) to the monetary variable. Over the years, a large literature has developed seeking to answer such questions while avoiding the pitfalls of a simple analysis of comovements. The main challenge facing that literature lies in identifying changes in policy that could be interpreted as autonomous, i.e. not the result of the central bank's response to movements in other variables. While alternative approaches have been pursued in order to meet that challenge, much of the recent literature has relied on time series econometrics techniques and, in particular, on structural (or identified) vector autoregressions.

The evidence displayed in Figure 1, taken from Christiano, Eichenbaum and Evans (1999), is representative of the findings in the recent literature seeking to estimate the effects of exogenous monetary policy shocks.[12] In the empirical model underlying Figure 1, monetary policy shocks are identified as the residual from an estimated policy rule followed by the Federal Reserve. That policy rule determines the level of the federal funds rate (taken to be the instrument of monetary policy), as a linear function of its own lagged values, current and lagged values of GDP, the GDP deflator, and an index of commodity prices, as well as the lagged values of some monetary aggregates. Under the assumption that neither GDP nor the two price indexes can respond contemporaneously to a monetary policy shock, the coefficients of the previous policy rule can be estimated consistently with OLS, and the fitted residual can be taken as an estimate of the exogenous monetary policy shock. The response over time of any variable of interest to that shock is then given by the estimated coefficients of a regression of the current value of that variable on the current and lagged values of the fitted residual from the first stage regression.

[12] Other references include Sims (1992), Galí (1992), Bernanke and Mihov (1995), and Uhlig (1995). Peersman and Smets (2003) provide similar evidence for the euro area. An alternative approach to identification, based on a narrative analysis of contractionary policy episodes can be found in Romer and Romer (1989).

Figure 1 shows the dynamic responses of the federal funds rate, (log) GDP, (log) GDP deflator, and the money supply (measured by M2), to an exogenous tightening of monetary policy. The solid line represents the estimated response, with the dashed lines capturing the corresponding 95 percent confidence interval. The scale on the horizontal axis measures the number of quarters after the initial shock. Note that the path of the funds rate itself, depicted in the top-left graph, shows an initial increase of about 75 basis points, followed by a gradual return to its original level. In response to that tightening of policy, GDP declines with a characteristic hump-shaped pattern. It reaches a trough after 5 quarters at a level about 50 basis point below its original level, and then it slowly reverts back to its original level. That estimated response of GDP can be viewed as evidence of sizable persistent real effects of monetary policy shocks. On the other hand, the (log) GDP deflator displays a flat response for over a year, after which it declines. That estimated sluggish response of prices to the policy tightening is generally interpreted as evidence of substantial price rigidities.[13] Finally, note that (log) M2 displays a persistent decline in the face of the rise in the federal funds rate, suggesting that the Fed needs to reduce the amount of money in circulation in order to bring about the increase in the nominal rate. The observed negative comovement between money supply and nominal interest rates is known as "liquidity effect". As discussed in chapter 2, it appears at odds with the predictions of a classical monetary model.

Having discussed the empirical evidence in support of the key assumptions underlying the new Keynesian framework, we end this introductory chapter with a brief description of the organization of the remaining chapters.

3 Organization of the Book

The book is organized in eight chapters, including this introduction. Chapters 2 through 7 progressively develop a unified framework, with new elements being incorporated in each chapter. Throughout the book we keep the references in the main text to a minimum, and add instead a section at the end of each chapter with notes on the literature, including references to some of

[13] Also, note that expected inflation hardly changes for several quarters and then declines. Combined with the path of the nominal rate, this implies a large and persistent increase in the real rate in response to the tightening of monetary policy, which provides another manifestation of the non-neutrality of monetary policy.

the key papers underlying the results presented in the chapter or containing extensions not covered in the book. In addition, each chapter contains a list of suggested exercises related to the material covered in the chapter. Next we describe briefly the content of the chapters.

In chapter 2 we introduce the assumptions on preferences and technology that will be used in most of the remaining chapters. We then determine and analyze the economy's equilibrium under the assumption of perfect competition in all markets, and fully flexible prices and wages. Those assumptions define what we label "the classical monetary economy," and which is characterized by neutrality of monetary policy and efficiency of the equilibrium allocation. In particular, the specification of monetary policy is shown to play a role only for the determination of nominal variables.

In the baseline model used in the first part of chapter 2, as in the rest of the book, money's role is limited to being the unit of account, i.e. the unit in terms of which prices of goods, labor services, and financial assets are quoted. Its potential role as a store of value (and hence as an asset in agents' portfolios) or as a medium of exchange is ignored. As a result, we generally do not need to specify a money demand function, unless monetary policy itself is specified in terms of a monetary aggregate, in which case we just postulate a simple log-linear money demand schedule. In the second part of chapter 2, however, we generate a motive to hold money by introducing real balances as an argument of the household's utility function, and examine its implications under the alternative assumptions of separability and non-separability of real balances. In the latter case, in particular, the result of monetary policy neutrality is shown to break down, even in the absence of nominal rigidities. The resulting non-neutralities, however, are shown to be of limited interest empirically.

Chapter 3 introduces the basic new Keynesian model, by adding product differentiation, monopolistic competition and staggered price setting to the framework developed in chapter 2. Labor markets are still assumed to be competitive. We derive the solution to the optimal price setting problem of a firm in that environment, and the resulting inflation dynamics. The log-linearization of the optimality conditions of households and firms, combined with some market clearing conditions, leads to the canonical representation of the model's equilibrium, which includes the new Keynesian Phillips curve, a dynamic IS equation and a description of monetary policy. Two variables play a central role in the equilibrium dynamics: the output gap and the natural rate of interest. The presence of sticky prices is shown to make monetary policy non-neutral. This is illustrated by analyzing the economy's response to two types of shocks: an exogenous monetary policy shock and a technology shock.

In chapter 4, we discuss the role of monetary policy in the basic new Keynesian model from a normative perspective. In particular, we show that, under some assumptions, it is optimal to pursue a policy that fully stabilizes the price level ("strict inflation targeting") and discuss alternative ways in which that policy can be implemented (optimal interest rate rules). We then discuss the likely practical difficulties in the implementation of the optimal policy, which motivates the introduction and analysis of simple monetary policy rules, i.e. rules that can be implemented with little or no knowledge of the economy's structure and/or realization of shocks. A welfare-based loss function that can be used for the evaluation and comparison of those rules is then derived and applied to two simple rules: a Taylor rule and a constant money growth rule.

A common criticism of the analysis of optimal monetary policy contained in chapter 4 is the absence of a conflict between inflation stabilization and output gap stabilization in the basic new Keynesian model. In chapter 5 we address that criticism by appending an exogenous additive shock to the new Keynesian Phillips curve, thus generating a meaningful policy tradeoff. In that context, and following the analysis in Clarida, Galí and Gertler (1999), we discuss the optimal monetary policy under the alternative assumptions of discretion and commitment, emphasizing the key role played by the forward-looking nature of inflation as a source of the gains from commitment.

Chapter 6 extends the basic new Keynesian framework by introducing imperfect competition and staggered nominal wage setting in labor markets, in coexistence with staggered price setting and modelled in an analogous way, following the work of Erceg, Henderson and Levin (2000). The presence of sticky nominal wages, and the consequent variations in wage markups, render a policy aimed at fully stabilizing price inflation suboptimal. The reason is that fluctuations in wage inflation, in addition to variations in price inflation and the output gap, generate a resource misallocation and a consequent welfare loss. Thus, the optimal policy is one that seeks to strike the right balance between stabilization of those three variables. For a broad range of parameters, however, the optimal policy can be approximated well by a policy that stabilizes a weighted average of price and wage inflation, where the proper weights are function of the relative stickiness of prices and wages.

In chapter 7 we develop a small open economy version of the basic new Keynesian model. The analysis of the resulting model yields several results. First, we show that the equilibrium conditions have a canonical representation analogous to that of the closed economy, including a new Keynesian Phillips curve, a dynamic IS equation and an interest rate rule. In general, though, both the natural level of output and the natural real rate are a function of foreign, as well as domestic, shocks. Secondly, and under certain assumptions, the optimal policy consists in fully stabilizing domestic inflation, while accommodating the changes in the exchange rate (and, as a result, in CPI inflation) necessary to bring about desirable changes in the relative price of domestic goods. Thus, in general, policies that seek to stabilize the nominal exchange rate, including the limiting case of an exchange rate peg, are likely to be suboptimal.

Finally, in chapter 8 we review some of the general lessons that can be drawn from the previous chapters. In doing so we focus on two key insights generated by the new framework, namely, the key role of expectations in shaping the effects of monetary policy, and the importance of the natural levels of output and the interest rate for the design of monetary policy. We end the chapter by describing briefly some of the extensions of the basic new Keynesian model that have not been covered in the book, and by discussing some of the recent developments in the literature.

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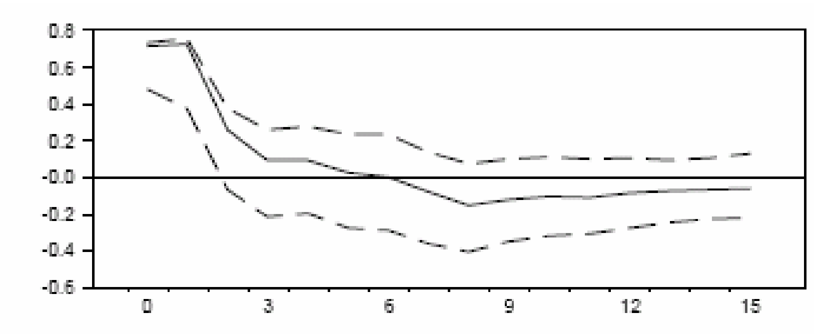
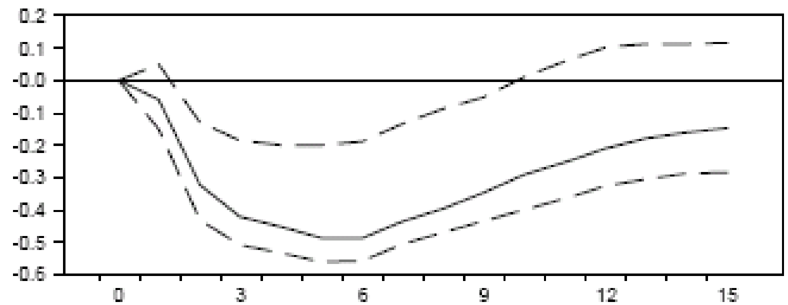
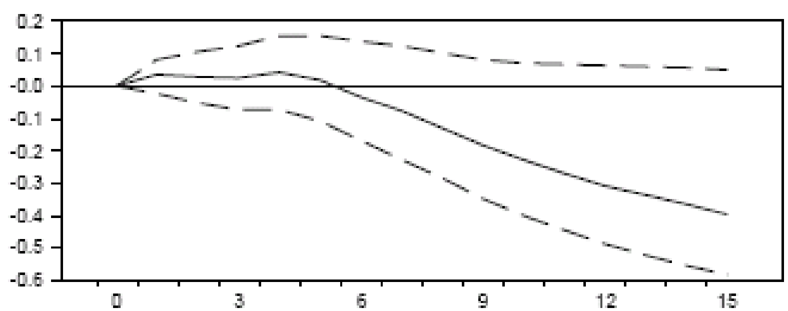
Figure 1. Estimated Dynamic Response to a Monetary Policy Shock



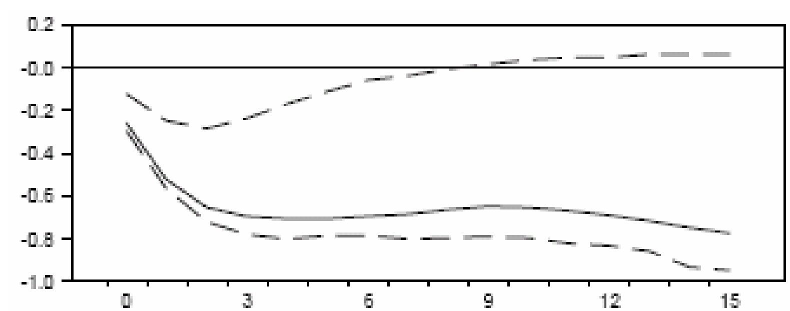
Federal funds rate



GDP



GDP deflator



M2

Source: Christiano, Eichenbaum and Evans (1999)

Monetary Policy, Inflation, and the Business Cycle

Chapter 2
A Classical Monetary Model

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In the present chapter we lay out a simple model of a classical monetary economy, featuring perfect competition and fully flexible prices in all markets. As stressed below, many of the predictions of that classical economy are strongly at odds with the evidence reviewed in chapter 1. That notwithstanding, we view the analysis of the classical economy as providing a benchmark that will be useful in subsequent chapters when some of its strong assumptions are relaxed. It also allows us to introduce some notation, as well as the assumptions on preferences and technology that are used in the remainder of the book.

Following much of the recent literature, our baseline classical model attaches a very limited role for money. Thus, in the first four sections the only explicit role played by money is to serve as a unit of account. In that case, and as shown below, whenever monetary policy is specified in terms of an interest rate rule, we do not need to make any reference whatsoever to the quantity of money in circulation in order to determine the economy's equilibrium. When the specification of monetary policy involves the money supply, we postulate a "conventional" money demand equation in order to close the model, without taking a stand on its microfoundations. In section 5, we introduce an explicit role for monay, beyond that of serving as a unit of account. In particular we analyze a model in which real balances are assumed to generate utility to households, and explore the implications for monetary policy of alternative assumptions on the properties of that utility function.

Independently of how money is introduced, the proposed framework assumes a representative household solving a dynamic optimization problem. That problem and the associated optimality conditions are described in section 1. Section 2 introduces the representative firm's technology and determines its optimal behavior, under the assumption of price and wage-taking. Section 3 characterizes the equilibrium, and shows how real variables are uniquely determined, independently of monetary policy. Section 4 discusses the determination of the price level and other nominal variables under alternative monetary policy rules. Finally, section 5 analyzes a version of the model with money in the utility function, and discusses the extent to which the conclusions drawn from our earlier analysis need to be modified under that assumption.

1 Households

The representative household seeks to maximize the objective function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \; U(C_t, N_t) \tag{1}$$

where C_t is the quantity consumed of the single good, and N_t denotes hours of work or employment.[1] The period utility $U(C_t, N_t)$ is assumed to be continuous and twice differentiable, with $U_{c,t} \equiv \frac{\partial U(C_t,N_t)}{\partial C_t} > 0$, $U_{cc,t} \equiv \frac{\partial^2 U(C_t,N_t)}{\partial C_t^2} \leq 0$, $U_{n,t} \equiv \frac{\partial U(C_t,N_t)}{\partial N_t} \leq 0$, and $U_{nn,t} \equiv \frac{\partial^2 U(C_t,N_t)}{\partial N_t^2} \leq 0$. In words, the marginal utility of consumption $U_{c,t}$ is assumed to be positive and non-increasing, while the marginal disutility of labor, $-U_{n,t}$, is positive and non-decreasing.

Maximization of (1) is subject to a sequence of flow budget constraints given by

$$P_t \; C_t + Q_t \; B_t \leq B_{t-1} + W_t \; N_t - T_t \tag{2}$$

for $t = 0, 1, 2, ...$ P_t is the price of the consumption good,. W_t denotes the nominal wage, B_t represents the quantity of one-period nominally riskless discount bonds purchased in period t, and maturing in period $t+1$. Each bond pays one unit of money at maturity, and its price is Q_t. T_t represents lump-sum additions or subtractions to period income (e.g. lump-sum taxes, dividends, etc.), expresed in nominal terms. When solving the problem above, the household is assumed to take as given the price of the good, the wage and the price of bonds.

In addition to (2), we assume that the household is subject to a solvency constraint that prevents it from engaging in Ponzi-type schemes. For our purposes the following constraint is sufficient:

$$\lim_{T \to \infty} E_t\{B_T\} \geq 0 \tag{3}$$

foa all t.

[1] Alternatively, N_t can be interpreted as the number of household members employed, assuming a large household and ignoring integer constraints.

1.0.1 Optimal Consumption and Labor Supply

The optimality conditions implied by the maximization of (1) subject to (2) are given by:

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \tag{4}$$

$$Q_t = \beta \; E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\} \tag{5}$$

for $t = 0, 1, 2, ...$

The previous optimality conditions can be derived using a simple variational argument. Let us first consider the impact on utility of a small departure, in period t, from the household's optimal plan. That departure consists of an increase in consumption dC_t and and increase in hours dN_t, while keeping the remaining variables unchanged (including consumption and hours in other periods). If the household was following an optimal plan to begin with, it must be the case that

$$U_{c,t} \; dC_t + U_{n,t}.dN_t = 0$$

for any pair (dC_t, dN_t) satisfying the budget constraint, i.e.

$$P_t \; dC_t = W_t.dN_t$$

for otherwise it would be possible to raise utility by increasing (or decreasing) consumption and hours, thus contradicting the assumption that the household is on an optimal plan. Note that by combining both equations we obtain the optimality condition (4).

Similarly, we can consider the impact on expected utility as of time t of a reallocation of consumption between periods t and $t+1$, while keeping consumption in any period other than t and $t+1$, and hours worked (in all periods) unchanged. If the household is optimizing it must be the case that

$$U_{c,t} \; dC_t + \beta \; E_t\{U_{c,t+1}.dC_{t+1}\} = 0$$

for any pair (dC_t, dC_{t+1}) satisfying

$$P_{t+1}dC_{t+1} = -\frac{P_t}{Q_t} \; dC_t$$

where the latter equation determines the increase in consumption expenditures in period $t+1$ made possible by the additional savings $-P_t dC_t$ allocated into one-period bonds. Combining the two previous equations we obtain the intertemporal optimality condition (5).

In much of what follows we assume that the period utility takes the form:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

The consumer's optimality conditions (4) and (5) thus become:

$$\frac{W_t}{P_t} = C_t^{\sigma}\ N_t^{\varphi} \tag{6}$$

$$Q_t = \beta\ E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \tag{7}$$

Note, for future reference, that equation (6) can be re-written in log-linear form as follows:

$$w_t - p_t = \sigma\ c_t + \varphi\ n_t \tag{8}$$

where lower case letters denote the natural logs of the corresponding variable (i.e. $x_t \equiv \log X_t$). The previous condition can be interpreted as a competitive labor supply schedule, determining the quantity of labor supplied as a function of the real wage, given the marginal utility of consumption (which under our assumptions is a function of consumption only).

As shown in Appendix 1, a log-linear approximation of (7) around a steady state with constant rates of inflation and consumption growth is given by

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\ (i_t - E_t\{\pi_{t+1}\} - \rho) \tag{9}$$

where we have defined $i_t \equiv -\log Q_t$ and $\rho \equiv -\log \beta$. Notice that i_t corresponds to the log of the gross yield on the one-period bond; we henceforth refer to it as the *nominal interest rate*.[2] Similarly, ρ can be interpreted as the household's discount rate.

[2] The yield on the one period bond is defined by $Q_t \equiv (1 + yield)^{-1}$. Note that $i_t \equiv -\log Q_t = \log(1 + yield_t) \simeq yield_t$ where the latter approximation will be accurate as long as the nominal yield is "small."

While the previous framework does not explicitly introduce a motive for holding money balances, in some cases it will be convenient to postulate a demand for real balances with a log-linear form given by (up to an additive constant):

$$m_t - p_t = y_t - \eta \ i_t \tag{10}$$

where $\eta \geq 0$ denotes the interest semi-elasticity of money demand.

A money demand equation similar to (10) can be derived under a variety of assumptions. For instance, in section 5 below we derive it as an optimality condition for the household when money balances yield utility.

2 Firms

We assume a representative firm whose technology is described by a production function given by

$$Y_t = A_t \ N_t^{1-\alpha} \tag{11}$$

where A_t represents the level of technology. We assume $a_t \equiv \log A_t$ evolves exogenously according to some stochastic process.

Each period the firm maximizes profits

$$P_t \ Y_t - W_t \ N_t \tag{12}$$

subject to (11), and taking the price and wage as given.

Maximization of (12) subject to (11) yields the optimality condition

$$\frac{W_t}{P_t} = (1-\alpha) \ A_t \ N_t^{-\alpha} \tag{13}$$

i.e. the firm hires labor up to the point where its marginal product equals the real wage. Equivalently, the marginal cost, $\frac{W_t}{(1-\alpha)A_t \ N_t^{-\alpha}}$, must be equated to the price, P_t.

In log-linear terms, we have

$$w_t - p_t = a_t - \alpha \ n_t + \log(1-\alpha) \tag{14}$$

which can be interpreted as labor demand schedule, mapping the real wage into the quantity of labor demanded, given the level of technology.

3 Equilibrium

Our baseline model abstracts from aggregate demand components like investment, government purchases, or net exports. Accordingly, the goods market clearing condition is given by

$$y_t = c_t \tag{15}$$

i.e. all output must be consumed.

By combining the optimality conditions of households and firms with (15) and the log-linear aggregate production relationship

$$y_t = a_t + (1 - \alpha)\ n_t \tag{16}$$

we can determine the equilibrium levels of employment and output, as a function of the level of technology:

$$n_t = \psi_{na}\ a_t + \vartheta_n \tag{17}$$

$$y_t = \psi_{ya}\ a_t + \vartheta_y \tag{18}$$

where $\psi_{na} \equiv \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha}$, $\vartheta_n \equiv \frac{\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$, $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$, and $\vartheta_y \equiv (1-\alpha)\vartheta_n$.

Furthermore, given the equilibrium process for output, we can use (9) to determine the implied real interest rate, $r_t \equiv i_t - E_t\{\pi_{t+1}\}$

$$\begin{aligned} r_t &= \rho + \sigma\ E_t\{\Delta y_{t+1}\} \\ &= \rho + \sigma\psi_{ya}\ E_t\{\Delta a_{t+1}\} \end{aligned} \tag{19}$$

Finally, the equilibrium real wage, $\omega_t \equiv w_t - p_t$, is given by

$$\begin{aligned} \omega_t &= a_t - \alpha\ n_t + \log(1-\alpha) \\ &= \psi_{\omega a}\ a_t + \vartheta_\omega \end{aligned} \tag{20}$$

where $\psi_{\omega a} \equiv \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$ and $\vartheta_\omega \equiv \frac{(\sigma(1-\alpha)+\varphi)\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$.

Notice that the equilibrium dynamics of employment, output, and the real interest rate are determined *independently of monetary policy*. In other words, monetary policy is *neutral* with respect to those real variables. In our simple model output and employment fluctuate in response to variations in technology, which is assumed to be the only real driving force.[3] In particular, output always rises in the face of a productivity increase, with the size of the increase being given by $\psi_{ya} > 0$. The same is true for the real wage. On the other hand, the sign of the employment is ambiguous, depending on whether σ (which measured the strength of the wealth effect of labor supply) is larger or smaller than one. When $\sigma < 1$, the substitution effect on labor supply resulting from a higher real wage dominates the negative effect caused by a smaller marginal utility of consumption, leading to an increase in employment. The converse is true whenever $\sigma > 1$. When the utility of consumption is logarithmic ($\sigma = 1$) employment remains unchanged in the face of technology variations, for substitution and wealth effects exactly cancel one another. Finally, the response of the real interest rate depends critically on the time series properties of technology. If the current improvement in technology is transitory, so that $E_t\{a_{t+1}\} < a_t$, then the real rate will go down. Otherwise, if technology is expected to keep improving, then $E_t\{a_{t+1}\} > a_t$ and the real rate will increase with a rise in a_t.

What about nominal variables, like inflation or the nominal interest rate? Not surprisingly, and in contrast with real variables, their equilibrium behavior cannot be determined uniquely by real forces. Instead, it requires that we specify how monetary policy is conducted. Below we consider several monetary policy rules and their implied outcomes.

4 Monetary Policy and Price Level Determination

We start by examining the implications of some interest rate rules. Later we introduce rules that involve monetary aggregates. In all cases we make use of the Fisherian equation:

$$i_t = E_t\{\pi_{t+1}\} + r_t \tag{21}$$

which implies that the nominal rate adjusts one-for-one with expected inflation, given a real interest rate that is determined exclusively by real factors, as in (19).

[3] It would be straightforward to introduce other real driving forces like variations in government purchases or exogenous shifts in preferences. In general, real variables will affected by all those real shocks in equilibrium.

4.1 An Exogenous Path for the Nominal Interest Rate

Let us firs consider the case of the nominal interest rate following an *exogenous* stationary process $\{i_t\}$. Without loss of generality we assume that i_t has mean ρ, which is consistent with a steady state with zero inflation and no secular growth. Notice that a particular case of this rule corresponds to a constant interest rate $i_t = i = \rho$, for all t.

Using (21) we can write,

$$E_t\{\pi_{t+1}\} = i_t - r_t$$

where, as discussed above, r_t is determined independently of the monetary policy rule.

Note that expected inflation is pinned down by the previous equation. But actual inflation is not. Since there is no other condition that can be used to determine inflation, it follows that any path for the price level that satisfies

$$p_{t+1} = p_t + i_t - r_t + \xi_{t+1}$$

is consistent with equilibrium, where ξ_{t+1} is a shock, possibly unrelated to economic fundamentals, satisfying $E_t\{\xi_{t+1}\} = 0$ for all t. Such shocks are often referred to in the literature as *sunspot* shocks. We refer to an equilibrium in which such non-fundamental factors may cause fluctuations in one or more variables as an *indeterminate* equilibrium. In the example above, we have thus shown how an exogenous nominal interest rate leads to *price level indeterminacy.*

Notice that when (10) is operative the equilibrium path for the money supply (which is endogenous under the present policy regime) is given by

$$m_t = p_t + y_t - \eta\, i_t$$

Hence, the money supply will inherit the indeterminacy of p_t. The same will be true of the nominal wage (which, in logs, equals the real wage, which determined by (20), plus the price leve, which is indeterminate).

4.2 A Simple Inflation-Based Interest Rate Rule

Suppose that the central bank adjusts the nominal interest rate according to the rule

$$i_t = \rho + \phi_\pi\, \pi_t$$

where $\phi_\pi \geq 0$.

Combining the previous rule with the Fisherian equation (21) we obtain

$$\phi_\pi \; \pi_t = E_t\{\pi_{t+1}\} + \widehat{r}_t \tag{22}$$

where $\widehat{r}_t \equiv r_t - \rho$. We distinguish between two cases, depending on whether the coefficient on inflation in the above rule, ϕ_π, is larger or smaller than one.

If $\phi_\pi > 1$, the previous difference equation has only one stationary solution, i.e. a solution that remains in a neighborhood of the steady state. That solution can be obtained by solving (22) forward, which yields

$$\pi_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} \; E_t\{\widehat{r}_{t+k}\} \tag{23}$$

The previous equation fully determines inflation (and, hence, the price level) as a function of the path of the real interest rate, which in turn is a function of fundamentals, as shown in (19). Consider, for the sake of illustration, the case in which technology follows the stationary AR(1) process

$$a_t = \rho_a \; a_{t-1} + \varepsilon_t^a$$

where $\rho_a \in [0, 1)$. Then (19) implies $\widehat{r}_t = -\,\sigma\psi_{ya}(1-\rho_a)\; a_t$, which combined with (23) yields the following expression for equilibrium inflation:

$$\pi_t = -\frac{\sigma\psi_{ya}(1-\rho_a)}{\phi_\pi - \rho_a} \; a_t$$

Note that a central bank following a rule of the form considered here can influence the degree of inflation volatility by choosing the size of ϕ_π. The larger is the latter parameter the smaller will be the impact of the real shock on inflation.

On the other hand, if $\phi_\pi < 1$ the stationary solutions to (22) take the form

$$\pi_{t+1} = \phi_\pi \; \pi_t - \widehat{r}_t + \xi_{t+1} \tag{24}$$

where $\{\xi_t\}$ is, again, an arbitrary sequence of shock, possibly unrelated to fundamentals, satisfying $E_t\{\xi_{t+1}\} = 0$ all t.

Accordingly, any process $\{\pi_t\}$ satisfying (24) is consistent with equilibrium, while remaining in a neighborhood of the steady state. So, as in the case of an exogenous nominal rate, the price level (and, hence, inflation and the nominal rate) are not determined uniquely when the interest rate rule implies a weak response of the nominal rate to changes in inflation. More specifically, the condition for a determinate price level, $\phi_\pi > 1$, requires that the central bank adjust nominal interest rates more than one-for-one in response to any change in inflation, a property known as the *Taylor principle*. The previous result can be viewed as a particular instance of the need to satisfy the *Taylor principle* in order for an interest rate rule to bring about a determinate equilibrium.

4.3 An Exogenous Path for the Money Supply

Suppose that the central bank sets an exogenous path for the money supply $\{m_t\}$. Using (10) to eliminate the nominal interest rate in (21), we can derive the following difference equation for the price level:

$$p_t = \left(\frac{\eta}{1+\eta}\right) E_t\{p_{t+1}\} + \left(\frac{1}{1+\eta}\right) m_t + u_t$$

where $u_t \equiv (1+\eta)^{-1}(\eta\, r_t - y_t)$ evolves independently of $\{m_t\}$.

Assuming $\eta > 0$ and solving forward we obtain:

$$p_t = \frac{1}{1+\eta} \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k E_t\{m_{t+k}\} + u'_t$$

where $u'_t \equiv \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k E_t\{u_{t+k}\}$ is, again, independent of monetary policy.

Equivalently, we can rewrite the previous expression in terms of expected future growth rate of money:

$$p_t = m_t + \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k E_t\{\Delta m_{t+k}\} + u'_t \tag{25}$$

Hence, we see how an arbitrary exogenous path for the money supply always determines the price level uniquely. Given the price level, as determined above, we can then use (10) to solve for the nominal interest rate:

$$
\begin{aligned}
i_t &= \eta^{-1} \left[y_t - (m_t - p_t) \right] \\
&= \eta^{-1} \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t \{ \Delta m_{t+k} \} + u_t''
\end{aligned}
$$

where $u_t'' \equiv \eta^{-1}(u_t' + y_t)$ is independent of monetary policy.

As an example, consider the case in which money growth follows an AR(1) process.

$$\Delta m_t = \rho_m \, \Delta m_{t-1} + \varepsilon_t^m$$

For simplicity let us assume the absence of real shocks, thus implying a constant output and a constant real rate. Without loss of generality, we set $r_t = y_t = 0$ for all t. Then it follows from (25) that

$$p_t = m_t + \frac{\eta \rho_m}{1 + \eta(1 - \rho_m)} \, \Delta m_t$$

Hence, in response to an exogenous monetary policy shock, and as long as $\rho_m > 0$ (the empirically relevant case, given the observed positive autocorrelation of money growth), the price level should respond more than one-for-one with the increase in the money supply, a prediction which contrasts starkly with the sluggish response of the price level observed in empirical estimates of the effects of monetary policy shocks, as discussed in chapter 1.

The nominal interest rate is in turn given by

$$i_t = \frac{\rho_m}{1 + \eta(1 - \rho_m)} \, \Delta m_t$$

i.e. in response to an expansion of the money supply, an as long as $\rho_m > 0$, the nominal interest rate is predicted to go up. In other words, the model implies the absence of a *liquidity effect*, in contrast with the evidence discussed in chapter 1.

4.4 Optimal Monetary Policy

The analysis of the baseline classical economy above has shown that while real variables are independent of monetary policy, the latter can have important implications for the behavior of nominal variables and, in particular, of prices. Yet, and given that the household's utility is a function of consumption and hours only–two real variables that are invariant to the way monetary policy is conducted– it follows that there is no policy rule that is better than any other. Thus, in the classical model above, a policy that generates large fluctuations in inflation and other nominal variables (perhaps as a consequence of following a policy rule that does not guarantee a unique equilibrium for those variables) is no less desirable that one that succeeds in stabilizing prices in the face of the same shocks.

The previous result, which is clearly extreme and empirically unappealing, can be overcome once we consider versions of the classical monetary model in which a motive to keep part of household's wealth in the form of monetary assets is introduced explicitly. Section 5 discusses one such model, in which real balances are assumed to yield utility.

Our overall assessment of the classical monetary model as a framework to understand the joint behavior of nominal and real variables and their connection to monetary policy cannot be positive. The model cannot explain the observed real effects of monetary policy on real variables. Its predictions regarding the response of the price level, the nominal rate and the money supply to exogenous monetary olicy shocks are also in conflict with the empirical evidence. Those empirical failures are the main motivation behind the introduction of nominal frictions in othersise similar model, a task that we undertake in chapter 3.

5 Money in the Utility Function[4]

In the model developed in the previous sections, and in much of the recent monetary literature, the only role played by money is to serve as a numéraire, i.e. unit of account in which prices, wages and securities' payoffs are stated. Economies with that characteristic are often referred to as *cashless economies*. Whenever we have postulated a simple log linear money demand function, we have done so in an ad-hoc manner, without an explicit justification for why agents would want to hold an asset that is dominated in return by bonds, while having identical risk properties. Even though in the analysis of subsequent chapters we will stick to the assumption of a cashless economy,

[4] The reader may skip this section and proceed to section 6 without any loss of continuity.

it is useful to understand how the basic framework can incorporate a role for money other than that of a unit of account and, in particular, how it can generate a demand for money. The discussion in the present section focuses on models that achieve the previous objective by assuming that real balances are an argument of the utility function.

The introduction of money in the utility function requires that we modify the household 's problem in two ways. First, preferences are now given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \, U\left(C_t, \frac{M_t}{P_t}, N_t\right) \tag{26}$$

where M_t denotes holdings of money in period t. We assume that period utility is increasing and concave in real balances M_t/P_t. Secondly, the flow budget constraint incorporates monetary holdings explicitly, taking the following form:

$$P_t \; C_t + Q_t \; B_t + M_t \leq B_{t-1} + M_{t-1} + W_t \; N_t - T_t$$

Letting $\mathcal{A}_t \equiv B_{t-1} + M_{t-1}$ denote total financial wealth at the beginning of the period t (i.e. before consumption and portfolio decisions are made), we can rewrite the previous flow budget constraint as:

$$P_t \; C_t + Q_t \mathcal{A}_{t+1} + (1 - Q_t) \; M_t \leq \mathcal{A}_t + W_t \; N_t - T_t \tag{27}$$

with the solvency constraint taking now the form $\lim_{T \to \infty} E_t\{\mathcal{A}_T\} \geq 0$.

The previous representation of the budget constraint can be thought of as equivalent to that of an economy in which all financial assets (represented by $\mathcal{A}_t$) yield a gross nominal return Q_t^{-1} $(= \exp\{i_t\})$, and where agents can purchase the utility-yielding "services" of money balances at a unit price $(1 - Q_t) = 1 - \exp\{-i_t\} \simeq i_t$. Thus, we see that the implicit price for money services roughly corresponds to the nominal interest rate, which in turn is the opportunity cost of holding one's financial wealth in terms of monetary assets, instead of interest-bearing bonds.

Consider next the household's problem, which consists of maximizing (26) subject to (27). Two of the implied optimality conditions are the same as those obtained for the cashless model, i.e. (6) and (7), with the marginal utility terms being now defined over (and evaluated at) the triplet $(C_t, \frac{M_t}{P_t}, N_t)$. In addition to (6) and (7), there is an additional optimality condition given by

$$\frac{U_{m,t}}{U_{c,t}} = 1 - \exp\{-i_t\} \tag{28}$$

where $U_{m,t} \equiv \frac{\partial U\left(C_t, \frac{M_t}{P_t}, N_t\right)}{\partial (M_t/P_t)} > 0$.

Again, in order to derive that optimality condition we can use a simple variational argument. Suppose that the household is considering a deviating from the optimal plan by adjusting consumption and money holdings in period t by amounts dC_t and dM_t respectively, while keeping all other variables unchanged at their optimal values. Optimality of the initial plan requires that utility cannot be raised as a result of the deviation, i.e.

$$U_{c,t} \ dC_t + U_{m,t} \ \frac{1}{P_t} \ dM_t = 0$$

for any pair (dC_t, dM_t) satisfying

$$P_t \ dC_t + (1 - Q_t) \ dM_t = 0$$

which guarantees that the budget constraint is met without the need to adjust any other variable. Combining the previous two equations and using the definition of the nominal rate $i_t = -\log Q_t$ yields the optimality condition (28).

In order to be able to make any statements about the consequences of having money in the utility function we need to be more precise about the way money balances interact with other variables in yielding utility. In particular, whether the utility function is separable or not in real balances determines the extent to which the neutrality properties derived above for the cashless economy carry over to the economy with money in the utility function. We illustrate that point by considering, in turn, two example economies with separable and non-separable utility.

5.1 An Example with Separable Utility

We specify the household's utility function to have the functional form

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{(M_t/P_t)^{1-\nu}}{1-\nu} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

Note that, given the separability of real balances, neither $U_{c,t}$ nor $U_{n,t}$ depend on the level of real balances. As a result, (6) and (7) (as well as their log-linear counterparts, (8) and (9)) continue to hold unchanged. It follows that we can determine the equilibrium values for output, employment, the real rate and the real wage following the same steps as above, and without any reference to monetary policy.

The introduction of money in the utility function, allows us to derive a money demand equation from the household's optimal behavior. Using the above specification of utility we can rewrite the optimality condition (28) as:

$$\frac{M_t}{P_t} = C_t^{\sigma/\nu} \ (1 - \exp\{-i_t\})^{-1/\nu} \tag{29}$$

which can be naturally interpreted as a demand for real balances. The latter is increasing in consumption and inversely related to the nominal interest rate, as in conventional specifications.

Using the first-order Taylor approximation $\log(1 - \exp\{-i_t\}) \simeq const. + \frac{1}{\exp\{i\}-1} \ i_t$, we can rewrite (29) in approximate log-linear form (and up to an uninteresting constant) as:

$$m_t - p_t = \frac{\sigma}{\nu} \ c_t - \eta \ i_t \tag{30}$$

where $\eta \equiv \frac{1}{\nu(\exp\{i\}-1)} \simeq \frac{1}{\nu i}$ is the implied interest semi-elasticity of money demand.

The particular case of $\nu = \sigma$ is an appealing one, since it implies a unit elasticity with respect to consumption. Under that assumption, we obtain a conventional linear demand for real balances

$$\begin{aligned} m_t - p_t &= c_t - \eta \ i_t \\ &= y_t - \eta \ i_t \end{aligned} \tag{31}$$

where the second equality holds in our baseline model economy, in which all output is consumed. The previous specification is often assumed in subsequent chapters, without the need to invoke its source explicitly.

As in the analysis of the cashless economy, the usefulness of (30) (or (31)) is confined to the determination of the equilibrium values for inflation and other nominal variables whenever the description of monetary policy involves the quantity of money in circulation. Otherwise, the only use of the money demand equation is to determine the quantity of money that the central bank will need to supply in order to support, in equilibrium, the nominal interest rate implied by the policy rule.

5.2 An Example with Non-Separable Utility

Let us consider next an economy in which period utility is given by

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{X_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

where X_t is a composite index of consumption and real balances defined as follows

$$\begin{aligned} X_t &\equiv \left[(1-\vartheta)\ C_t^{1-\nu} + \vartheta\ \left(\frac{M_t}{P_t}\right)^{1-\nu}\right]^{\frac{1}{1-v}} \qquad for\ \nu \neq 1 \\ &\equiv C_t^{1-\vartheta}\ \left(\frac{M_t}{P_t}\right)^{\vartheta} \qquad for\ \nu = 1 \end{aligned}$$

with ν represents the (inverse) elasticity of substitution between consumption and real balances, and ϑ the relative weight of real balances in utility.

Notice that the marginal utilities of consumption and real balances are now given, respectively, by

$$U_{c,t} = (1-\vartheta)\ X_t^{\nu-\sigma}\ C_t^{-\nu}$$

$$U_{m,t} = \vartheta\ X_t^{\nu-\sigma}\ \left(\frac{M_t}{P_t}\right)^{-\nu}$$

whereas the marginal (dis)utility of labor is, as before, given by $U_{n,t} = -N_t^{\varphi}$. The optimality conditions of the household's problem, (4), (5) and (28), can now be written as:

$$\frac{W_t}{P_t} = N_t^{\varphi}\ X_t^{\sigma-\nu}\ C_t^{\nu}\ (1-\vartheta)^{-1} \tag{32}$$

$$Q_t = \beta\ E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\nu}\left(\frac{X_{t+1}}{X_t}\right)^{\nu-\sigma}\ \frac{P_t}{P_{t+1}}\right\} \tag{33}$$

$$\frac{M_t}{P_t} = C_t\ \left(1-\exp\{-i_t\}\right)^{-\frac{1}{\nu}}\ \left(\frac{\vartheta}{1-\vartheta}\right)^{\frac{1}{\nu}} \tag{34}$$

Notice that in the particular case in which the intertemporal and intratemporal elasticities of substitution coincide (i.e. $\nu = \sigma$), optimality conditions (32) and (33) match exactly those obtained in the case of separable utility, and thus lead to the same equilibrium implications derived for that case and discussed in the previous subsection..

In the general case, however, both the labor supply equation (32) and the Euler equation (33) are influenced by the level of real balances, through the dependence of the index X_t on the latter. The level of real balances depends, in turn, on the nominal interest rate (as implied by (34)). Those features imply that monetary policy is no longer neutral in the case of non-separable utiillity considered here. In particular, to the extent that different monetary policy rules have different implications for the path of the nominal rate (as will generally be the case), they will also have different effects on real balances and–through the latter's influence on the marginal utility of consumption–on the position of the labor supply schedule and, hence, on employment and output. This mechanism is analyzed formally below.

Notice that the implied money demand equation (34) can be rewritten in log-linear form (and up to an additive constant) as in (31) above, i.e.

$$m_t - p_t = c_t - \eta \ i_t \tag{35}$$

where, again, $\eta = \frac{1}{\nu(\exp\{i\}-1)}$. Thus, the implied interest semi-elasticity of demand η is now proportional to the elasticity of substitution between real balances and consumption, ν^{-1}.

On the other hand, log-linearization of (32) around the zero inflation steady state yields

$$w_t - p_t = \sigma c_t + \varphi n_t + (\nu - \sigma)(c_t - x_t)$$

Log-linearizing the expression defining X_t around a zero inflation steady state, and combining the resulting expression with (34) we obtain

$$\begin{aligned} w_t - p_t &= \sigma c_t + \varphi n_t + \chi(\nu - \sigma) \ [c_t - (m_t - p_t)] \\ &= \sigma c_t + \varphi n_t + \chi\eta(\nu - \sigma) \ i_t \end{aligned}$$

where $\chi \equiv \frac{\vartheta^{\frac{1}{\nu}}(1-\beta)^{1-\frac{1}{\nu}}}{(1-\vartheta)^{\frac{1}{\nu}}+\vartheta^{\frac{1}{\nu}}(1-\beta)^{1-\frac{1}{\nu}}} \in [0,1)$, and where the second equality makes use of (35).

For future reference it is convenient to rewrite the previous optimality conditions in terms of the steady state ratio $k_m \equiv \frac{M/P}{C}$, i.e. the inverse consumption velocity. Using the money demand equation, we have $k_m = \left(\frac{\vartheta}{(1-\beta)(1-\vartheta)}\right)^{\frac{1}{\nu}}$. Noting that $\chi = \frac{k_m(1-\beta)}{1+k_m(1-\beta)}$, and using the definition of η

evaluated at the zero inflation steady state we can rewrite the optimality condition above as

$$w_t - p_t = \sigma c_t + \varphi n_t + \omega \ i_t \tag{36}$$

where $\omega \equiv \frac{k_m \beta (1-\frac{\sigma}{\nu})}{1+k_m(1-\beta)}$. Thus, we see that the sign of the effect of the nominal interest rate on labor supply is determined by the sign of $\nu - \sigma$. When $\nu > \sigma$ (implying $\omega > 0$) the reduction in real balances induced by an increase in the nominal rate brings down the marginal utility of consumption (for any given c_t), lowering the quantity of labor supplied at any given real wage. The opposite effect obtains when $\nu < \sigma$. Note, however, that $\nu \simeq \frac{1}{i\eta}$ is likely to be larger than σ for any plausible values of η and σ. Thus, the case of $U_{cm} > 0$ (and hence $\omega > 0$) appears as the most plausible one, conditional on the specification of preferences analyzed here.

The corresponding log-linear approximation to (33) is given by

$$\begin{aligned}
c_t &= E_t\{c_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - (\nu - \sigma)\ E_t\{(c_{t+1} - x_{t+1}) - (c_t - x_t)\} - \rho\right) \\
&= E_t\{c_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \chi(\nu - \sigma)\ E_t\{\Delta c_{t+1} - \Delta(m_{t+1} - p_{t+1})\} - \rho\right) \\
&= E_t\{c_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \omega\ E_t\{\Delta i_{t+1}\} - \rho\right) \qquad (37)
\end{aligned}$$

where, again, the last equality makes use of (35). Thus, when $\nu > \sigma$ (and, hence, $\omega > 0$) the anticipation of a nominal rate increase (and, hence, of a decline in real balances), lowers the expected one period ahead marginal utility of consumption (for any expected c_{t+1}), which induces an increase in current consumption (in order to smooth marginal utility over time).

In order to reflect the changes implied by non-separable utility, we need to modify the economy's log-linearized equilibrium conditions. Thus, combining (36) with the labor demand schedule (14) we obtain the following labor market clearing condition:

$$\sigma c_t + \varphi n_t + \omega \ i_t = y_t - n_t + \log(1-\alpha) \tag{38}$$

which we can rewrite, using the goods market clearing condition (15) and the log-linear production relationship (16) as (ignoring an uninteresting additive constant):

$$y_t = \psi_{ya}\ a_t - \psi_{yi}\ i_t \tag{39}$$

where $\psi_{yi} \equiv \frac{\omega(1-\alpha)}{\sigma+\varphi+\alpha(1-\sigma)}$.

Condition (39) points to a key implication of the property of non-separability ($\omega \neq 0$): equilibrium output is no longer invariant to monetary policy, at least to the extent that the latter implies variations in the nominal interest rate. In other words, monetary policy is not neutral. As a result, equilibrium condition (39) does not suffice to determine the equilibrium level of output, in contrast with the economy with separable utility analyzed above. In order to pin down the equilibrium path of output and other endogenous variables we need to combine (39) with the remaining equilibrium conditions, including a description of monetary policy.

One such additional condition can be obtained by imposing the goods market clearing condition $y_t = c_t$ on Euler equation (37), which yields an equation relating the nominal interest rate to the expected path of output and expected inflation:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \omega\ E_t\{\Delta i_{t+1}\} - \rho\right) \tag{40}$$

Finally, we need an equation which describes how monetary policy is conducted. For the purposes of illustration we assume that the central bank follows the simple inflation-based interest rate rule

$$i_t = \rho + \phi_\pi\ \pi_t + v_t \tag{41}$$

where v_t now represents an exogenous policy disturbance, assumed to follow the stationary AR(1) process

$$v_t = \rho_v\ v_{t-1} + \varepsilon_t^v$$

Similarly, and for concreteness, we assume that the technology parameter follows the AR(1) process

$$a_t = \rho_a\ a_{t-1} + \varepsilon_t^a$$

Using (41) to eliminate the nominal rate in (39) and (40), and combining the resulting two equations we can obtain (after some algebraic manipulation) the following closed form expressions for the equilibrium level of inflation, the nominal rate, and output:

$$\begin{aligned}
\pi_t &= -\frac{\sigma(1-\rho_a)\psi_{ya}}{\phi_\pi(1+\omega\psi)(1-\Theta\rho_a)}\ a_t - \frac{1+(1-\rho_v)\omega\psi}{\phi_\pi(1+\omega\psi)(1-\Theta\rho_v)}\ v_t \\
i_t &= -\frac{\sigma(1-\rho_a)\psi_{ya}}{(1+\omega\psi)(1-\Theta\rho_a)}\ a_t - \frac{\rho_v}{\phi_\pi(1+\omega\psi)(1-\Theta\rho_v)}\ v_t \\
y_t &= \psi_{ya}\left(1+\frac{\sigma(1-\rho_a)\psi_{yi}}{(1+\omega\psi)(1-\Theta\rho_a)}\right)\ a_t + \frac{\rho_v\psi_{yi}}{\phi_\pi(1+\omega\psi)(1-\Theta\rho_v)}\ v_t
\end{aligned}$$

where $\Theta \equiv \frac{1+\omega\psi\phi_\pi}{(1+\omega\psi)\phi_\pi}$ and $\psi \equiv \frac{\alpha+\varphi}{\sigma(1-\alpha)+\alpha+\varphi}$.

A few remarks regarding the impact of monetary policy on the economy's equilibrium are in order. First, note that the interest rate multiplier of output, conditional on an exogenous monetary policy shock is given by $\frac{dy_t}{di_t} = \frac{dy_t/dv_t}{di_t/dv_t} = -\psi_{yi}$. In order to get a sense for the magnitude of that multiplier, recall that $\psi_{yi} \equiv \frac{\omega(1-\alpha)}{\sigma+\varphi+\alpha(1-\sigma)}$. Let us assume parameter values $\sigma = \varphi = 1$ and $\alpha = 1/3$, as in the baseline calibration that will be introduced in chapter 3. Using the definition of ω, and the fact that $\nu = \frac{1}{\eta\rho}$ is "large" for any reasonable values of η, we have $\psi_{yi} \simeq \frac{k_m}{3}$, and so the size of the inverse velocity k_m is a key determinant of the quantitative importance of monetary non-neutralities in the model. Unfortunately, the magnitude of k_m depends crucially on the definition of money used. Thus, and focusing on postwar U.S. data, $k_m \simeq 0.3$ if we take the monetary base as the relevant measure of money.[5] In that case we have $\psi_{yi} \simeq 0.1$, which implies a relative small multiplier: a monetary policy shock that raised the nominal rate by one percentage point (expressed at an annual rates) would generate a decrease in output of about 0.025 percent. By way of contrast, if we use $M2$ as the definition of money, we have $k_m \simeq 3$ and so the impact on output of an analogous monetary policy shock is a 0.25 percent decline. The latter value, while small, appears to be closer to the estimated output effects of a monetary policy shock found in the literature. Yet, even in the latter case, there are other aspects of the transmission of monetary policy shocks implied by the model that are clearly at odds with the evidence, e.g. the response of inflation and the real interest rate. Thus, note that

$$\frac{d\pi_t}{di_t} = \frac{d\pi_t/dv_t}{di_t/dv_t} = (1+(1-\rho_v)\omega\psi)\ \rho_v^{-1} > 0$$

[5] This is the approach followed in Woodford (2003, chapter 2).

$$\frac{dr_t}{di_t} = 1 - \frac{dE_t\{\pi_{t+1}\}/dv_t}{di_t/dv_t} = -(1-\rho_v)\omega\psi < 0$$

i.e. in response to a monetary policy shock that raises the nominal interest rate and lowers output, inflation tends to increase, and the real rate to go down (as a result of the dominant effect of higher expected inflation). This contrasts with the downward adjustment of inflation and the rise in the real rate observed as part of the economy's response of the economy following a contractionary monetary policy shock.

Finally, there is an additional argument that can be brought up and which calls into question the relevance of the transmission mechanism underlying the classical model with non-separable preferences and which has to do with its implications regarding the long-run effects of monetary policy. To see this, consider an exogenous monetary policy intervention that raises the nominal rate permanently. The implied permanent change in output is determined by (39), and given by $-\psi_{yi}$. Thus, the long-run trade-off between output and the nominal rate is identical to the short-run trade-off. How about the inflation-output trade-off? Equation (40), evaluated at the steady state, requires a long-run increase in inflation of the same size as the increase in the nominal rate. Hence the long-run trade-off between inflation and output is also given by $-\psi_{yi}$. But note that the same coefficient describes the short-run output-inflation trade-off since, in the relevant case of a permanent policy change ($\rho_v = 1$), we have $\frac{dy_t/dv_t}{d\pi_t/dv_t} = -\psi_{yi}$.

As argued above, for a most plausible range of parameter values we have $\psi_{yi} > 0$. Thus, in the present model a permanent increase in inflation will be associated with a permanent decline in output. Given the determinants of ψ_{yi}, whether that long-run trade-off is large or small will largely depend on the size of inverse velocity k_m and, hence, on the relevant measure of money. Thus, the lack of a significant empirical relationship between long-run inflation and economic activity (at least at low levels of inflation), suggests a low value for k_m and ψ_{yi}, as implied by a narrow definition of money. Unfortunately, in the present model, and as argued above, any calibration with the desirable feature of a negligible long-run trade-off will also be associated with negligible (and hence counterfactual) short run effects of monetary policy.

5.3 Optimal Monetary Policy in a Classical Economy with Money in the Utility Function

In this section we derive the form of the optimal monetary policy in the presence of money in the utility function. We start by laying out and solving the problem facing a hypothetical social planner seeking to maximize the utility of the representative household.

Note that, under our assumptions, there are no aggregate intertemporal links in our simple model: even though each individual household can reallocate its own consumption over time through financial markets, there are no mechanisms that make this possible for the economy as a whole. Thus, the social planner would solve a sequence of static problems of the form

$$\max U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

subject to the resource constraint

$$C_t = A_t \; N_t^{1-\alpha}$$

The optimality conditions for that problem are given by

$$-\frac{U_{n,t}}{U_{c,t}} = (1-\alpha) \; A_t N_t^{-\alpha} \tag{42}$$

$$U_{m,t} = 0 \tag{43}$$

Condition (42) requires that the marginal rate of substitution between hours of work and consumption be equated to the marginal product of labor. Condition (43) equates the marginal utility of real balances to the "social" marginal cost of producing real balances, which is implicitly assumed to be zero in our setting.

Under what conditions the equilibrium of the decentralized economy satisfies efficiency conditions (42) and (43)? We first note that condition (42) is implied by the combined effect of profit maximization by firms (which equates the real wage to the marginal product of labor; see equation (13)) and the optimal labor supply choice by the household (which equates the real wage to the marginal rate of substitution between hours of work and consumption; see equation (4)). Hence, (42) will be satisfied independently of monetary policy. On the other hand, and as shown above, the household's optimal choice of money balances requires

$$\frac{U_{m,t}}{U_{c,t}} = 1 - \exp\{-i_t\}$$

Accordingly, efficiency condition (43) will be satisfied.if and only if $i_t = 0$ for all t, a policy known as the *Friedman rule*. The rationale for that policy is quite intuitive: while the social cost of producing real balances is zero, the private (opportunity) cost is given by the nominal interest rate. As a result, only when the nominal interest rate is zero are the private and social costs of holding money equated. Note that such a policy implies an average (steady state) rate of inflation

$$\pi = -\rho < 0$$

i.e. prices will decline on average at the rate of time preference. In other words: under the Friedman rule the economy experiences a (moderate) deflation in the long-run.

Implementation of the Friedman rule requires some discussion. As shown earlier a policy rule of the form $i_t = 0$ for all t leaves the price level indeterminate in our model. Even though that indeterminacy should not have any welfare consequences (since (42) and (43) pin down consumption, employment and real balances uniquely), a central bank could avoid that indeterminacy by following a rule of the form

$$i_t = \phi\ (r_{t-1} + \pi_t)$$

for some $\phi > 1$. Combined with (21) that rule implies the difference equation

$$E_t\{i_{t+1}\} = \phi\ i_t$$

whose only stationary solution is $i_t = 0$ for all t. Under that rule, equilibrium inflation is fully predictable and given by

$$\pi_t = -r_{t-1}$$

More generally, any rule that makes the central bank adjust its policy settings (e.g. the money supply) to guarantee that current inflation moves inversely, and one-for-one with the lagged real interest rate will imply a zero nominal interest rate and, thus, an efficient amount of real balances.

6 Notes on the Literature

The modelling approach favored in much of the recent monetary literature, and the one adopted in the present book (with the exception of section 5 of this chapter), does not incorporate monetary assets ("money") explicitly in the analysis. Under that approach the main role played by money is that of a unit of account. Such model economies can be viewed as a limiting case (the cashless limit) of an economy in which money is valued and held by households. Woodford (2003) provides a detailed discussion and a forceful defense of that approach.

Models that introduce monetary assets explicitly rely on one of two alternative formalisms in order to generate a demand for an asset that–as is the case with money–is dominated in its rate of return by alternative assets that have identical risk characteristics: they either assume (i) that real balances generate utility to households or, alternatively, (ii) that the presence of some transaction costs in the purchases of goods can be reduced by household's holding of monetary assets.

The first of those approaches – money in the utility function – traces back to Sidrauski (1967), who introduced that assumption in an otherwise standard neoclassical growth model (with inelastic labor supply). Woodford (2003) offers a detailed analysis of the implications of alternative assumptions on the specification of utility and, in particular, of the likely degree of monetary non-neutralities arising from the non-separability of real balances. Walsh (2003, chapter 2) develops a real business cycle model with money in the utility function, and analyzes the equilibrium properties of a calibrated version of that model. Both analyses conclude, in a way consistent with the discussion above, that even under a utility that is non-separable in real balances, the real effects of monetary policy are quantitatively very small for plausible calibrations of the models.

A common approach to the modelling of a transactions motive for holding money builds on the assumption, originally due to Clower (1967), that cash mush be held in advance in order to purchase certain goods. Early examples of classical monetary models in which a demand for money is generated by postulating a cash-in-advance constraint can be found in the work of Lucas (1982) and Svensson (1985). Cooley and Hansen (1989) analyze an otherwise standard real business cycle models augmented with a cash-in-advance constraint for consumption goods, showing that monetary policy is near-neutral for plausible calibrations of that model. Walsh (2003, chapter

3) provides a detailed description of classical monetary models with cash-in-advance constraints and their implications for the role of monetary policy.

The practice, followed in the present monograph, of appending a money demand equation to a set of equilibrium conditions that have been derived in the context of cashless economy is often found in the literature. King and Watson (1995) is an example of that practice.

The analysis of the form of the optimal monetary policy in a classical economy goes back to Friedman (1969), where a case is made for a policy that keeps the nominal interest rate constant at a zero level. More recent treatments of the conditions under which is optimal include Woodford (1990) and Correia and Teles (1999).

Finally, the reader can find two useful discussions of the notion of monetary neutrality and its evolution in macroeconomic thinking in Patinkin (1987) and Lucas (1996).

Appendix 1: Some Useful Log-Linear Approximations

Euler equation

We can rewrite the consumer's Euler equation as

$$1 = E_t\{\exp(i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho)\} \tag{44}$$

In a perfect foresight steady state with constant inflation π and constant growth γ we must have:

$$i = \rho + \pi + \sigma\gamma$$

with the steady state real rate being given by

$$\begin{aligned} r &\equiv i - \pi \\ &= \rho + \sigma\gamma \end{aligned}$$

A first-order Taylor expansion of $\exp(i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho)$ around that steady state yields:

$$\begin{aligned} \exp(i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho) &\simeq 1 + (i_t - i) - \sigma(\Delta c_{t+1} - \gamma) - (\pi_{t+1} - \pi) \\ &= 1 + i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho \end{aligned}$$

which can be used in (44) to obtain, after some rearrangement of terms, the log-linearized Euler equation

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\,(i_t - E_t\{\pi_{t+1}\} - \rho)$$

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Exercises

1. Optimality Conditions under Non-Separable Leisure

Derive the log-linearized optimality conditions of the household problem under the following specification of the period utility function with non-separable leisure.

$$U(C_t, N_t) = \frac{1}{1-\sigma} \left[C_t \, (1-N_t)^{\nu} \right]^{1-\sigma}$$

2. Alternative Interest Rules for the Classical Economy

Consider the simple classical economy described in the text, in which the following approximate equilibrium relationships must be satisfied

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} \left(i_t - E_t\{\pi_{t+1}\} - \rho \right)$$

and

$$\begin{aligned} r_t &\equiv i_t - E_t\{\pi_{t+1}\} \\ &= \rho + \sigma \; E_t\{\Delta y_{t+1}\} \end{aligned}$$

and where y_t and, hence, r_t, are determined independently of monetary policy. Next you are asked to analyze, in turn, two alternative monetary policy rules and their implications. When relevant, we assume that the money market clearing condition takes the form

$$m_t - p_t = y_t - \eta \; i_t + \varepsilon_t^m$$

where ε_t^m is a stochastic money demand disturbance.

a) *Strict Inflation Targeting.*

(i) Derive an interest rate rule that guarantees full stabilization of inflation, i.e. $\pi_t = \pi^*$ for all t where π^* is an inflation target assumed to be "close to" zero (so that the log-linearized equilibrium conditions remain valid).

(ii) Determine the behavior of money growth that is consistent with the strict inflation targeting policy analyzed in (i).

(iii) Explain why a policy characterized by a constant rate of money growth $\Delta m_t = \pi$ will generally not succeed in stabilizing inflation in that economy.

b) *Price Level Targeting.*

(i) Consider the interest rate rule

$$i_t = \rho + \phi_p \, (p_t - p^*)$$

where $\phi_p > 0$, and p^* is a (constant) target for the (log) price level. Determine the equilibrium behavior of the price level under this rule. (hint: you may find it useful to introduce a new variable $\widehat{p}_t \equiv p_t - p^*$ –the deviation of the price level from target–to ease some of the algebraic manipulations).

(ii) Consider instead the money targeting rule

$$m_t = p^*$$

Determine the equilibrium behavior of the price level under this rule.

(iii) Show that the money targeting rule considered in (ii) can be combined with the money market clearing condition and rewritten as a price-level targeting rule of the form

$$i_t = \rho + \psi \, (p_t - p^*) + u_t$$

where ψ is a coefficient and u_t is a stochastic process to be determined.

(iv) Suppose that the central bank wants to minimize the volatility of the price level. Discuss the advantages and disadvantages of the interest rate rule in (i) versus the money targeting rule in (ii) in light of your findings above.

3. Nonseparable Preferences and Money Superneutrality

Assume that the representative consumer's period utility is given by:

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{1}{1-\sigma}\left[(1-\vartheta)\, C_t^{1-\nu} + \vartheta \, \left(\frac{M_t}{P_t}\right)^{1-\nu}\right]^{\frac{1-\sigma}{1-\nu}} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

a) Derive the optimality conditions of the associated consumer's problem.

b) Assume that the representative firm has access to a simple technology $Y_t = N_t$ and that the monetary authority keeps a constant money growth γ_m. Derive the economy's steady state equilibrium under the assumption of perfect competition.

c) Discuss the effects on inflation and output of a permanent change in the rate of money growth γ_m, and relate it to the existing evidence.

4. Optimal Monetary Policy in a Classical Economy with an Exact Equilibrium Representation

Consider a version of the classical economy with money in the utility function, where the representative consumer maximizes $E_0 \sum_{t=0}^{\infty} \beta^t \, U\left(C_t, \frac{M_t}{P_t}, N_t\right)$ subject to the sequence of dynamic budget constraints

$$P_t C_t + M_t + Q_t B_t \leq M_{t-1} + B_{t-1} + W_t \; N_t - T_t$$

Assume a period utility given by:

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \log C_t + \log \frac{M_t}{P_t} - \frac{{N_t}^{1+\varphi}}{1+\varphi} \tag{45}$$

Suppose there is a representative perfectly competitive firm, producing the single consumption good. The firm has access to the linear production function $Y_t(i) = A_t \; N_t(i)$, where productivity evolves according to:

$$\frac{A_t}{A_{t-1}} = (1+\gamma_a) \; \exp\{\varepsilon_t^a\}$$

with $\{\varepsilon_t^a\}$ is an i.i.d. random process, normally distributed, with mean 0 and variance σ_a^2.

The money supply varies exogenously according to the process

$$\frac{M_t}{M_{t-1}} = (1+\gamma_m) \; \exp\{\varepsilon_t^m\} \tag{46}$$

where $\{\varepsilon_t^m\}$ is an i.i.d., normally distributed process with mean 0 and variance σ_m^2 . We assume that $\{\varepsilon_t^m\}$ evolves exogenously, outside the control of the monetary authority (e.g., could reflect shocks in the monetary multiplier that prevent the monetary authority from fully controlling the money supply.). Finally, we assume that all output is consumed, so that in equilibrium $Y_t = C_t$ for all t.

a) Derive the optimality conditions for the problem of households and firms.

b) Determine the equilibrium levels of aggregate employment, output, and inflation (Hint: show that a constant velocity $\frac{P_t Y_t}{M_t} = V$ for all t is a solution)

c) Discuss how utility depends on the two parameters describing monetary policy, γ_m and σ_m^2 (recall that the nominal interest rate is constrained to be non-negative, i.e., $Q_t \leq 1$ for all t). Show that the optimal policy must satisfy the Friedman rule ($i_t = 0$ all t) and discuss alternative ways of supporting that rule in equilibrium.

5. A Shopping Time Model (based on Walsh (2003)).

Assume that the transactions technology is such that consuming C_t requires a quantity of shopping time $N_t^s = s\left(C_t, \frac{M_t}{P_t}\right)$, where $s_c > 0$ and $s_m \leq 0$. Hence the amount of time diverted from leisure is given by $N_t + N_t^s$, where N_t denotes hours of work. Let the original period utility be given by $V(C_t, L_t)$ where $L_t = 1 - N_t - N_t^s$ denotes leisure.

a) Derive the condition determining the optimal allocation of time.

b) Derive the implied utility function in terms of consumption, hours and real balances, and discuss its properties.

6. A Model with Cash and Credit Goods

Assume that the utility of the representative household is given by:

$$V(C_{1t}, C_{2t}, N_t) \tag{47}$$

where C_{1t} denotes consumption of a "cash-good" (i.e., a good that requires cash in order to be purchased), C_{2t} is consumption of a "credit-good" (which does not require cash), and N_t is labor supply. For simplicity, let us assume that the price of the two goods is identical and equal to P_t (this will be the case if the production function of the representative firm is given by $Y_{1t} + Y_{2t} = N_t$ and there is perfect competition). Purchases of cash-goods have to be settled in cash, whereas credit goods can be financed by issuing one-period riskless nominal bonds.

The budget constraint is given by

$$P_t\ (C_{1t} + C_{2t}) + Q_t\ B_t + M_t = B_{t-1} + M_{t-1} + W_t N_t + T_t$$

Finally, the CIA constraint is given by

$$P_t\ C_{1t} \leq M_{t-1} + T_t$$

where, in equilibrium, $T_t = \Delta M_t$, i.e. transfers to households correspond to money tranfers made by the central bank, and which consumers take as given. For simplicity we assume no uncertainty.

a) Derive the first order conditions associated with the household's problem

b) Note that whenever the CIA contraint is binding we can define a reduced form period utility:

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) \equiv V\left(\frac{M_t}{P_t}, C_t - \frac{M_t}{P_t}, N_t\right)$$

where $C_t = C_{1t} + C_{2t}$. Show that $U_m \geq 0$, given the optimality conditions de rived in a).