Solution to P.S.2

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1. Solution

   - If the individual does not take the risk, his expected utility would be $U(W)$.
   - If the individual takes the risk, his expected utility is

   $$E[U(\tilde{\varepsilon})] = pU(W+\epsilon) + (1-p)U(W-\epsilon) \tag{1}$$

   , taking second-order Taylor expansion on the term $U(W+\epsilon)$ and $U(W-\epsilon)$ around the point W:

   $$U(W+\epsilon) = U(W) + \epsilon U'(W) + \frac{1}{2}\epsilon^2 U''(W) \tag{2}$$
   $$U(W-\epsilon) = U(W) - \epsilon U'(W) + \frac{1}{2}\epsilon^2 U''(W) \tag{3}$$

   substituting (2) and (3) into (1), we could get

   $$E[U(\tilde{\varepsilon})] = U(W) + (2p-1)\epsilon U'(W) + \frac{1}{2}\epsilon^2 U''(W) \tag{4}$$

   - Since the individual is indifferent between taking or not taking the risk, i.e.

   $$U(W) = U(W) + (2p-1)\epsilon U'(W) + \frac{1}{2}\epsilon^2 U''(W)$$

   solve out the equation, we could get $p = \frac{\epsilon R(W)+2}{4}$.

- Thus, when the probability of winning equals $\frac{\epsilon R(W)+2}{4}$, the individual is indifferent between taking the risk or not.

2. Solution

- The expected excess return of taking the risk is

$$p(W+\epsilon)+(1-p)(W-\epsilon)-W=(2p-1)\epsilon$$

- As we can see, the Arrow's risk premium $\theta = 2p-1$ determines the excess return when the payoff $\epsilon$is given.