Advanced Microeconomics II
Quiz 2

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1. Each of 3 people chooses whether or not to contribute \$10 toward the provision of a public good. The good is provided if and only if at least 2 people contribute, if it is not provided, contributions are not refunded. Denote the contribution of player i by a_i. Each player values the public good at \$30.

   (a) (3 points) Formulate this situation as a strategic game.

   Solution: This strategic game consists of:

   - $N = \{1, 2, 3\}$
   - For each player $i \in N$, A_i=$\{0,10\}$
   - For each player $i \in N$, for each $a \in \times_{j \in N} A_j$

   $$U_i(a) = 30I(\sum_{j=1}^{3} a_j \geq 20) - a_i,$$

   where $I(\sum_{j=1}^{3} a_j \geq 20)$ is an indicator function equal to one if $\sum_{j=1}^{3} a_j \geq 20$ and zero otherwise.

   (b) (2 points) Find a pure strategy Nash equilibrium.

   Solution: There exist two kinds of pure strategy Nash equilibria:

   - $\{a : \sum_1^3 a_j = 20\}$
   - $\{a : \sum_1^3 a_j = 0\}$

   (c) (5 points) Find a non-degenerate mixed strategy Nash equilibrium.

   Solution: For there to be a non-degenerate mixed-strategy Nash Equilibria, a player needs to be indifferent between his two strategies. Consider a symmetric mixed strategy Nash equilibrium where each player contributes with probability α.

   $$U_i(0, \alpha_{-i}) = \alpha^2 30 + (1 - \alpha^2)0,$$
   $$U_i(10, \alpha_{-i}) = (1 - (1 - \alpha)^2)30 + (1 - \alpha)^2 0 - 10$$

   These are equal if $\alpha \in \{(3-\sqrt{3})/6, (3+\sqrt{3})/6\}$. Hence, $\alpha \in \{(3-\sqrt{3})/6, (3+\sqrt{3})/6\}$ is a non-degenerate mixed strategy equilibrium.