The framework of monetary policy

Classical Monetary Model: Money in a RBC

- Perfect competition
- Fully flexible prices
- Preference: household optimizing expected utility
- Production: firms optimizing profit
- Technology: exogenous process
- Information: rational expectation

Change the household preference :

- ➢ Money in the Utility Function
- ➢ Consumption and money Holding are separable

Separable Utility function

Consumption and money Holding are nonseparable

Nonseparable Utility function

- Imperfect competition in the goods market
- Price rigidity: some constraints are imposed on the price adjustment mechanism

The Basic New Keynesian Model

- The Households: Each household i consumes a basket of all goods C_t ,classical preference
- firms: a continuum of firms produces a differentiated food, acting as monopolistic competitors; Sticky price: firms adjust their price infrequently and the opportunity to adjust follows an exogenous Poisson process

The transmission of typical monetary policy

Main conclusion:

- Money's role is limited to the unit of account
- Money is neutral

Typical monetary policy:

- Interest rate instruments
  - ➢ **An exogenous stationary process for the Nominal Interest Rate**

    Nonfundamental fluctuation of interest rate, indeterminate equilibrium, indeterminate norminal variables
  - ➢ **A simple Inflation-Based Interest Rate Rule:**

    $$i_t = \rho + \phi_\pi \pi_t$$

    the price level to be uniquely determined.
- Money supply instruments
  - ➢ **An Exogenous Path for the Money Supply:**

    $$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$$

    the price level to be uniquely determined, but the contrast with empirical finding of response of its shocks

Main conclusion:

- Money's role is limited to the unit of account
- Money is netural
- The introduction of money, however, allows a money demand equation to be derived as

$$m_t - p_t = y_t - \eta i_t + const$$

Main conclusion:

- Money is not neutral
- Different monetary policy rules → Impacts on nominal interest rates→ Real balances →Labor supply and output.
- Long-run and short-run effects are the same, contradicting to empirical work

Main conclusion:

- Main variables: output gap, natural interest rate
- NKPC: $\pi_t = \beta E_t\left\{\pi_{t+1}\right\} + \kappa y_t$
- DISC: $y_t = -\frac{1}{\sigma}\left(\hat{i}_t - E_t\left\{\pi_{t+1}\right\} - \hat{r}_t^n\right) + E_t\left\{y_{t+1}\right\}$

Typical monetary policy:

- Interest rate rule: $\hat{i}_t = \phi_\pi \pi_t + \phi_y y_t + v_t$

  $v_t \uparrow, \pi_t \downarrow, y_t \downarrow, a_t \uparrow, y_t \downarrow, \pi_t \downarrow$
- Monely supply: $\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$

Objectives of monetary policy

No optimal monetary policy: Nominal variables are influenced by monetary policy.

- Objectives:

Welfare optimization

- Central banker:

a sequence of static optimization problems:

$$\max U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

$$s.t. C_t = A_t N_t^{1-\alpha}$$

- Implications:

The Friedman rule

Objectives:

To isolate the role of staggered price →To stabilize marginal costs at a level consistent with firm's desired markup, given the prices in place

- Optimal Interest Rate Rules(unrealistic)
- Simple rules in practice

  A Taylor-type interest rate rule; A constant money growth rule

Objectives:

short run tradeoffs between stabilizing inflation and real variables like output and employment

- an efficient steady state
  - ➢ Optimal Discretionary Policy
  - ➢ Optimal Policy under Commitment
- an distorted steady state
  - ➢ Optimal Discretionary Policy
  - ➢ Optimal Policy under Commitment

The Framework for chapter2-5

Key points for chapter2

Classical Monetary Model: Money in a RBC

Assumptions

- Perfect competition
- Fully flexible prices
- Preference: household optimizing expected utility
- Production: firms optimizing profit
- Technology: exogenous process
- Information: rational expectation

Household optimizing problem

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right)$$

$$s.t. P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t - T_t$$

$$\lim_{T \to \infty} E_t \left\{B_T\right\} \geq 0$$

$$U\left(C_t, N_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

Solution：

(1.1)
$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi}$$

(1.2)
$$Q_t = \beta E_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}}\right]$$

firms optimizing profit

(1.3)
$$\max_{(Y_t, N_t)} P_t Y_t - W_t N_t$$

(1.4) $$s.t. Y_t = A_t N_t^{1-\alpha}$$

(1.5) $$Y_t = A_t N_t^{1-\alpha}$$

Solution：

(1.6) $$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha}$$

Competitive equilibrium

Market clearing

(1.7) $$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi}$$

(1.8) $$Q_t = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right]$$

(1.9) $$P_t C_t + Q_t B_t \le B_{t-1} + W_t N_t - T_t$$

(1.10) $$Y_t = A_t N_t^{1-\alpha}$$

(1.11) $$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha}$$

(1.12) $$Y_t = C_t$$

Denote real wage as $W_t^r = \frac{W_t}{P_t}$,

(1.13) $$W_t^r = C_t^{\sigma} N_t^{\varphi}$$

(1.14) $$Y_t = A_t N_t^{1-\alpha}$$

(1.15) $$W_t^r = (1-\alpha) A_t N_t^{-\alpha}$$

(1.16) $$Y_t = C_t$$

Taking logs and denote $x_t = \log X_t$

(1.17) $$w_t^r = \sigma c_t + \varphi n_t$$

(1.18) $$y_t = a_t + (1-\alpha) n_t$$

(1.19) $$w_t^r = a_t - \alpha n_t + \log(1-\alpha)$$

(1.20) $$y_t = c_t$$

We can solve this equation as

$$y_t = \psi_{ya} a_t + \vartheta_y \qquad n_t = \psi_{na} a_t + \vartheta_n \qquad w_t^r = \psi_{wa} a_t + \vartheta_w$$

Take log the Euler equation(1.8)

(1.21) $$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\left[i_t - E_t\{\pi_{t+1}\} - \rho\right]$$

Define real interest rate as $r_t \equiv i_t - E_t\{\pi_{t+1}\}$

$$r_t = \rho + \sigma E_t\{\Delta y_{t+1}\} = \rho + \sigma \psi_{ya} E_t\{\Delta a_{t+1}\}$$

Monetary policy instruments

- Interest rate

An Exogenous Path for the Nominal Interest Rate:

- The nominal interest rate as an exogenous stationary process $\{i_t\}$
- Rewrite the Fisherian equation as

$$p_{t+1} = p_t + i_t - r_t + \xi_{t+1}$$

- $E_t\left\{\overline{\zeta_{t+1}}\right\} = 0$ for all t, which may not be related to economic fundamentals, the inflation or price level determination is consistent with the equilibrium.
- Equilibrium fluctuation caused: indeterminate equilibrium. The price level indeterminancy also causes indeterminancy in other norminal variables such as money supply or wage.

A simple Inflation-Based Interest Rate Rule

- $i_t = \rho + \phi_\pi \pi_t$

- $\phi_\pi > 1$: $\pi_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t\left\{\hat{r}_{t+k}\right\}$, $a_t = \rho_a a_{t-1} + \varepsilon_t^a$,then

$$\hat{r}_t = -\sigma\psi_{ya}(1-\rho_a)a_t$$

$$\pi_t = -\frac{\sigma\psi_{ya}(1-\rho_a)}{\phi_\pi - \rho_a} a_t$$

$\phi_\pi \leq 1$, $\pi_{t+1} = \phi_\pi \pi_t - \hat{r}_t + \xi_{t+1}$

Central bank needs to adjust nominal interest rates more than one for one in response to changes in inflation, for the price level to be uniquely determined.

- Money supply

An Exogenous Path for the Money Supply

- money supply has an exogenous path, i.e. it does not respond to other economic variables.
- Let $u_t \equiv (1+\eta)^{-1}(\eta r_t - y_t)$,then we have

(1.22) $$p_t = m_t + \sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k E_t\left\{\Delta m_{t+k}\right\} + u_t', u_t' = \sum_{k=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k E_t\left\{u_{t+k}\right\}$$

An arbitrary exogenous path for the money supply always determines the price level uniquely.

- Let $u_t'' = \eta^{-1}(u_t' + y_t)$,then we have

(1.23) $$i_t = \eta^{-1}[y_t - (m_t - p_t)] = \eta^{-1}\sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k E_t\left\{\Delta m_{t+k}\right\} + u_t''$$

i_t can be uniquely determined under exogenous money supply.

- if $\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$,then

$$p_t = m_t + \frac{\eta\rho_m}{1+\eta(1-\rho_m)}\Delta m_t$$

$$i_t = \frac{\rho_m}{1+\eta(1-\rho_m)} \Delta m_t$$

Money in the Utility Function

Separable Utility

- **household's problem**

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

$$s.t. P_t C_t + Q_t A_{t+1} + (1-Q_t) M_t \le A_t + W_t N_t - T_t$$

$$\lim_{T\to\infty} E_t \{A_T\} \ge 0 \, \forall t$$

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{(M_t/P_t)^{1-\nu}}{1-\nu} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

- Solution:

- Real equilibrium as captured before is not affected. it is as in the cashless economy, that monetary policy does not affect real variables.
- The introduction of money, however, allows a money demand equation to be derived form

$$\frac{M_t}{P_t} = C_t^{\frac{\sigma}{\nu}} [1-\exp\{-i_t\}]^{-\frac{1}{\nu}}$$

We get $$m_t - p_t = y_t - \eta i_t + const$$

Nonseparable Utility

- **household's problem**

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

$$s.t. P_t C_t + Q_t A_{t+1} + \left(1-Q_t\right) M_t \le A_t + W_t N_t - T_t$$

$$\lim_{T\to\infty} E_t\left\{A_T\right\} \ge 0 \,\forall t$$

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{X_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

$$X_t \equiv \left[\left(1-\theta\right) C_t^{1-v} + \theta\left(\frac{M_t}{P_t}\right)^{1-v}\right]^{\frac{1}{1-v}}, v \ne 1$$

$$\equiv C_t^{1-\theta}\left(\frac{M_t}{P_t}\right)^{\theta}, v = 1$$

- Solution:

✧

(1.24) $$\frac{W_t}{P_t} = N_t^{\varphi} X_t^{v-\sigma} C_t^{v}\left(1-\theta\right)^{-1}$$

(1.25) $$Q_t = \beta E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-v}\left(\frac{X_{t+1}}{X_t}\right)^{v-\sigma}\frac{P_t}{P_{t+1}}\right\}$$

(1.26) $$\frac{M_t}{P_t} = C_t\left(1-e^{-i_t}\right)^{-\frac{1}{v}}\left(\frac{\theta}{1-\theta}\right)^{\frac{1}{v}}$$

✧ Take log on the above equations, we have

$$w_t - p_t = \sigma c_t + \varphi n_t + \omega i_t - \log\left(1-\theta\right)$$

(1.27) $$where \quad k_m \equiv \frac{\overline{M}/\overline{P}}{\overline{C}} = \left[\frac{\theta}{\left(1-\beta\right)\left(1-\theta\right)}\right]^{\frac{1}{v}},$$

$$\chi = \frac{k_m\left(1-\beta\right)}{1+k_m\left(1-\beta\right)}, \omega = \frac{k_m \beta\left(1-\frac{\sigma}{v}\right)}{1+k_m\left(1-\beta\right)}$$

(1.28) $$c_t = E_t\left\{c_{t+1}\right\} - \frac{1}{\sigma}\left[i_t - E_t\left\{\pi_{t+1}\right\} - \omega E_t\left\{\Delta i_{t+1}\right\} - \rho\right]$$

(1.29) $$m_t - p_t = y_t - \eta i_t + const$$

- **Firms: as before**

- **Equilibrium:**

(1.30) $$w_t - p_t = \sigma c_t + \varphi n_t + \omega i_t - \log(1-\theta)$$

(1.31) $$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\left[i_t - E_t\{\pi_{t+1}\} - \omega E_t\{\Delta i_{t+1}\} - \rho\right]$$

(1.32) $$m_t - p_t = y_t - \eta i_t + const$$

(1.33) $$y_t = a_t + (1-\alpha) n_t$$

(1.34) $$y_t = c_t$$

(1.35) $$w_t - p_t = a_t - \alpha n_t$$

Solution：

$$y_t = \psi_{ya} a_t - \psi_{yi} i_t$$

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}\left[i_t - E_t\{\pi_{t+1}\} - \omega E_t\{\Delta i_{t+1}\} - \rho\right]$$

Combining $i_t = \rho + \phi_\pi \pi_t + v_t$,we can get π_t , i_t , y_t .

➢ Impact of monetary policy:

✧ inflation tends to increase

✧ the real rate tends to go down (as a result of the dominant effect of higher expected inflation)

Key points for chapter3

The Basic New Keynesian Model

- Workhorse for the analysis of monetary policy, fluctuations and welfare
- Two departures from the classical monetary economy:
  - ➢ Imperfect competition in the goods market: Each firm produces a differentiated good for which it sets the price

(instead of taking the price as given)

- Price rigidity: some constraints are imposed on the prie adjustment mechanism by assuming only a fraction of firms can reset their prices in any given period

3.1 the Households:

Assumptions:

- The economy is composed of a continuum of infinitely-lived individuals, whose total is normalized to unity.
- There exists a continuum of firms (goods) represented by the interval $i \in [0,1]$.
- Each household i consumes a basket of all goods C_t.
- The household supplies labor and saves in the form of nominal state contingent securities.
- Each firm i produces a differentiated food, which enters the consumption basket, and demands labor in a competitive labor market given wage.
- Household's preference:

$$E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right)$$

- The budget constraint

$$\int_0^1 P_t(i) C_t(t) di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t$$

The above representative households' problem can be solved in two steps:

- The agent allocates optimally his resources across each differentiated Goods in a purely static fashion.
- The agent solves a typical dynamic optimization problem, featuring intertemporal allocation of consumption and saving.

Step 1. Optimal (static) expenditure allocation

$$\max_{C_t(i)} C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$s.t. \int_0^1 P_t(i) C_t(i) di \equiv Z_t$$

Solution: let $P_t \equiv \left(\int_0^1 P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$,then

$$C_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} C_t$$

And we also have another relation: $P_t C_t = \int_0^1 P_t(i) C_t(i) di$

Step2: Intertemporal problem

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

$$s.t. P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + T_t$$

Solution:

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi}$$

$$Q_t = \beta E_t \left(\frac{C_t^{\sigma}}{C_{t+1}^{\sigma}} * \frac{P_t}{P_{t+1}} \right)$$

Loglinearization:

$$w_t - p_t = \sigma \hat{c}_t + \varphi n_t$$

$$0 = E_t\left[-\sigma\left(\hat{c}_{t+1} - \hat{c}_t\right) + \hat{i}_t - \pi_{t+1}\right]$$

Firms

- The market is populated by a continuum of firms acting as monopolistic competitors. Labor is the only input of production. The production function of firm i is given by

$$Y_t(i) = A_t F(N_t(i))$$

  - A_t : productivity (technology) level, common to all firms.
  - $F(.)$:a homogeneous function of degree one.
  - Labor is an economy-wide competitive factor. W_t is equal to all.
  - This also implies that all firms face a common nominal marginal cost MC_t.

- Sticky price: firms adjust their price infrequently and the opportunity to adjust follows an exogenous Poisson process. Each firm may reset its price with a constant probability $(1-\theta)$ independently of past history; with probability θ they cannot adjust price.
  - Thus, each period a measure $(1-\theta)$ of firms reset their prices
  - a fraction θ keeps their prices unchanged

Price setting and inflation

Aggregate price dynamics:

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(P_t^*\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Log-linear terms：

(1.1) $$p_t = \theta p_{t-1} + (1-\theta)\, p_t^*$$

$$\pi_t = (1-\theta)\left(P_t^* - P_{t-1}\right)$$

To understand inflation implies an understanding of why firms may want to choose to adjust their relative price periodically. We have to examine how p_t^* is set in order to understand the inflation dynamics.

Optimal price setting problem for firm i:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t\left[Q_{t,t+k}(P_t^* Y_{t+k|t} - \Psi_{t+k} Y_{t+k|t})\right]$$

$$s.t. Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} C_{t+k}$$

The FOC:

$$E_t\left\{\sum_{k=0}^{\infty} \theta^k Q_{t,t+k}\left[Y_{t+k|t} + P_t^* \frac{\partial Y_{t+k|t}}{\partial P_t^*} - \frac{\partial \Psi_{t+k}}{\partial Y_{t+k|t}} \frac{\partial Y_{t+k|t}}{\partial P_t^*}\right]\right\} = 0$$

with $\frac{\partial Y_{t+k|t}}{\partial P_t^*} = (-\varepsilon)\left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon-1}$, $\frac{C_{t+k}}{P_{t+k}} = (-\varepsilon)\frac{Y_{t+k|t}}{P_t^*}$ and define $\psi_{t+k|t} \equiv \frac{\partial \Psi_{t+k}}{\partial Y_{t+k|t}}$

Rearrange,we get

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k|t} \psi_{t+k|t}}{E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k|t}}$$

Log-linearization

(1.2) $$p_t^* = (1-\beta\theta) E_t \sum_{k=0}^{\infty} (\beta\theta)^k \left(mc_{t+k|t} + p_{t+k} \right) = (1-\beta\theta)\left(mc_{t|t} + p_{t+k} \right) + \beta\theta E_t \, p_{t+1}^*$$

Combine quation(1.1)and equation (1.1),we get the Inflation as

(1.3) $$\pi_t = \beta E_t \left\{ \pi_{t+1} \right\} + \frac{(1-\theta)(1-\beta\theta)}{\theta} mc_{t|t}$$

Equilibrium

Goods market clearing:

$$Y_t(i) = C_t(i)$$

$Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$, then $Y_t = C_t$

Combine it with the Euler equation of house holds

(1.4) $$y_t = E_t \left\{ y_{t+1} \right\} - \frac{1}{\sigma} \left(\hat{i}_t - E_t \left\{ \pi_{t+1} \right\} \right)$$

Labor market clearing:

$$N_t = \int_0^1 N_t(i) di$$

$$D_t = \left[\int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} di \right]^{1-\alpha}$$

$y_t = a_t + (1-\alpha) n_t - d_t$ and $d_t = 0$,then

(1.5) $$y_t = a_t + (1-\alpha) n_t$$

Average real marginal cost

(1.6) $$mc_t = w_t - p_t - (a_t - \alpha n_t)$$

Combine (1.5)and(1.6)

(1.7) $$mc_t = w_t - p_t - \frac{1}{1-\alpha}(a_t - \alpha y_t)$$

Combining

(1.8) $$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}$$

(1.9) $$y_{t+k} - y_{t+k|t} = \varepsilon(p_t^* - p_{t+k})$$

We get

(1.10) $$mc_{t+k|t} = mc_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}(p_t^* - p_{t+k})$$

substitute (1.10) into equation (1.2)

(1.11) $$p_t^* = (1-\beta\theta)E_t\left\{\sum_{k=0}^{\infty}(\beta\theta)^k\left[\Theta mc_{t+k} + p_{t+k}\right]\right\}, \Theta = \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$$

Rewrite this equation, we have

(1.12) $$p_t^* = (1-\beta\theta)\left(\Theta mc_t + p_t\right) + \beta\theta E_t p_{t+1}^*$$

Combine (1.1)and(1.12)

(1.13) $$\pi_t = \beta E_t\left\{\pi_{t+1}\right\} + \lambda mc_t, \lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$$

Combining （1.5）and(1.6)and household FOCs, we have：

(1.14) $$mc_t = \frac{\varphi+\alpha+\sigma-\alpha\sigma}{1-\alpha}y_t - \frac{\varphi+1}{1-\alpha}a_t$$

Let $mc_t = 0$,we get the natural output y_t^n ,

(1.15) $$y_t^n = \frac{\varphi+1}{\varphi+\alpha+\sigma-\alpha\sigma}a_t$$

Definition: Output gap $y_t = y_t - y_t^n$

Rewrite (1.14),we have

(1.16) $$mc_t = \left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) y_t$$

Rewrite(1.13),we have NKPC

(1.17) $$\pi_t = \beta E_t \left\{ \pi_{t+1} \right\} + \kappa y_t, \kappa = \lambda \left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right)$$

Definition: natural rate of interest

$$\hat{r}_t^n = \sigma E_t \left\{ y_{t+1}^n - y_t^n \right\}$$

Rewrite the household FOCs, we get DIS equation:

(1.18) $$y_t = -\frac{1}{\sigma} \left(\hat{i}_t - E_t \left\{ \pi_{t+1} \right\} - \hat{r}_t^n \right) + E_t \left\{ y_{t+1} \right\}$$

Let $\hat{r}_t \equiv \hat{i}_t - E_t \left\{ \pi_{t+1} \right\}$,we get $y_t = -\frac{1}{\sigma} \left(\hat{r}_t - \hat{r}_t^n \right) + E_t \left\{ y_{t+1} \right\}$

Thus we get NKPC

(1.19) $$\pi_t = \beta E_t \left\{ \pi_{t+1} \right\} + \kappa y_t, \kappa = \lambda \left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right)$$

DIS

(1.20) $$y_t = -\frac{1}{\sigma} \left(\hat{i}_t - E_t \left\{ \pi_{t+1} \right\} - \hat{r}_t^n \right) + E_t \left\{ y_{t+1} \right\}$$

Monetary policy Rules

Equilibrium under an interest rate rule

(1.21) $$\hat{i}_t = \phi_\pi \pi_t + \phi_y y_t + v_t$$

Then we get the two equations

(1.22) $$y_t = -\frac{1}{\sigma} \left(\phi_\pi \pi_t + \phi_y y_t - E_t \left\{ \pi_{t+1} \right\} - \left(\hat{r}_t^n - v_t \right) \right) + E_t \left\{ y_{t+1} \right\}$$

(1.23) $$\pi_t = \beta E_t\left\{\pi_{t+1}\right\} + \kappa y_t, \kappa = \lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$$

Given that both y_t and π_t are nonpredetermined, the solution to the above equations is unique if and only in the following constraint

$$\kappa\left(\phi_\pi - 1\right) + \left(1-\beta\right)\phi_y > 0$$

The effects of a monetary policy shock

(1.24) $$\pi_t = \beta E_t\left\{\pi_{t+1}\right\} + \kappa y_t$$

(1.25) $$y_t = -\frac{1}{\sigma}\left(\hat{i}_t - E_t\left\{\pi_{t+1}\right\} - \hat{r}_t^n\right) + E_t\left\{y_{t+1}\right\}$$

(1.26) $$\hat{i}_t = \phi_\pi \pi_t + \phi_y y_t + v_t$$

(1.27) $$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

We get

(1.28) $$\pi_t = -\kappa\Lambda_v v_t$$

(1.29) $$y_t = -\left(1-\beta\rho_v\right)\Lambda_v v_t$$

(1.30) $$\hat{r}_t = \sigma(1-\rho_v)\left(1-\beta\rho_v\right)\Lambda_v v_t$$

(1.31) $$\hat{i}_t = \left[\sigma(1-\rho_v)\left(1-\beta\rho_v\right) - \kappa\rho_v\right]\Lambda_v v_t$$

The effects of a technology shock

(1.32) $$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

(1.33) $$\hat{r}_t^n \equiv \sigma E_t\left\{\Delta y_{t+1}^n\right\} = -\sigma\psi_{ya}^n(1-\rho_a)a_t$$

(1.34) $$y_t = -\sigma\psi_{ya}^n(1-\rho_a)\left(1-\beta\rho_a\right)\Lambda_a a_t$$

(1.35) $$\pi_t = -\sigma\psi_{ya}^n(1-\rho_a)\kappa\Lambda_a a_t$$

(1.36) $$y_t = \psi_{ya}^n\left[1-\sigma(1-\rho_a)\left(1-\beta\rho_a\right)\Lambda_a\right]a_t$$

an exogenous money supply Δm_t

define log real money balance as $\hat{l}_t$

Rewrite the DIS equation as

(1.37) $$(1+\sigma\eta)\, y_t = \sigma\eta E_t\{y_{t+1}\} + \hat{l}_t + \eta E_t\{\pi_{t+1}\} + \eta \hat{r}_t^n - y_t^n$$

(1.38) $$\hat{l}_{t-1} = \hat{l}_t + \pi_t - \Delta m_t$$

(1.39) $$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa y_t$$

The effects of a monetary policy shock

(1.40) $$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$$

$$\Delta m_t \uparrow, y_t \uparrow, \pi_t \uparrow, \hat{l}_t \uparrow, \hat{i}_t \uparrow, \hat{r}_t \downarrow$$

The effects of a technology shock

$$a_t \uparrow, \hat{r}_t^n \downarrow, y_t \uparrow, \pi_t \uparrow, n_t \downarrow, \hat{r}_t \uparrow, \hat{i}_t \approx 0, y_t^n \uparrow, y_t \downarrow$$

Key points for chapter4

4.1 The efficient allocation

The efficient allocation can be determined by a social planner's problem

$\max U(C_t, N_t)$

s.t.

$C_t(i) = A_t F(N_t(i))$

For all $\mathrm{i} \in [0,1]$ with

$$C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$N_t = \int_0^1 N_t(i) \, di$$

The basic New Keynesian model developed in Chapter 3 is characterized by two distortions

a. The presence of market power in goods markets, exercised by monopolistically competitive firms.
b. Infrequent adjustment of prices by firms.

4.2 Sources of sub optimality in the basic New Keynesian model

Assumptions for simplicity:

a. An optimal subsidy in place to offset the market power distortion
b. The economy starts without relative price distortion

Requirement of the optimal policy

a. To stabilize marginal costs at a level consistent with firm's desired markup, given the prices in place.

Using the notation for the log-linearized model in Chapter 3, the optimal policy requires that for all t

$$\tilde{y}_t = 0$$
$$\tilde{\pi}_t = 0$$

The dynamic IS equation then implies

$$\hat{i}_t = \hat{r}_t^n$$

Stabilizing output is not desirable in and of itself. As usually

$\hat{y}_t \neq \tilde{y}_t$

Price stability emerges as a feature of the optimal policy even though, a priori, the policy maker does not attach any weight to such an objective.

The non-policy block:

$$\tilde{y}_t = -\frac{1}{\sigma}\left(\hat{i}_t - E_t\left\{\hat{\pi}_{t+1}\right\} - \hat{r}_t^n\right) + E_t\left\{\tilde{y}_{t+1}\right\}$$

$$\hat{\pi}_t = \beta E_t\left\{\hat{\pi}_{t+1}\right\} + \kappa\tilde{y}_t$$

The policy block:

a. An exogenous interest rate rule

b. An interest rate rule with an endogenous component

c. A forward-looking interest rate rule

An exogenous interest rate rule:

So we have:

$$\begin{bmatrix} \tilde{y}_t \\ \hat{\pi}_t \end{bmatrix} = A_0 \begin{bmatrix} E_t\left\{\tilde{y}_{t+1}\right\} \\ E_t\left\{\hat{\pi}_{t+1}\right\} \end{bmatrix}$$

Where

$$A_0 = \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix}$$

An interest rate rule with an endogenous component:

$$\hat{i}_t = \hat{r}_t + \phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t$$

$$A_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \kappa\sigma & \kappa + \beta\left(\sigma + \phi_y\right) \end{bmatrix}$$

$$\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$$

For uniqueness of solution, both eigenvalues of matrix A_T should lie within the unit circle, which implies:

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$$

A forward –looking interest rate rule:

$$\hat{i}_t = \hat{r}_t^n + \phi_\pi E_t\{\hat{\pi}_{t+1}\} + \phi_y E_t\{\tilde{y}_{t+1}\}$$

It the implied dynamics are described by the system

$$A_F \equiv \begin{bmatrix} 1-\sigma^{-1}\phi_y & -\sigma^{-1}\phi_\pi \\ \kappa\left(1-\sigma^{-1}\phi_y\right) & \beta - \kappa\sigma^{-1}\phi_\pi \end{bmatrix}$$

The conditions for a unique equilibrium are twofold and given by:

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$$
$$\kappa(\phi_\pi - 1) + (1+\beta)\phi_y < 2\sigma(1+\beta)$$

Practical shortcomings of optimal policy rules:

Unrealistic: Both require that the policy rate be adjusted one-for-one with the natural rate of interest, thus implicitly assuming observation of the latter variable. Implication on knowledge of

a. the economy’s "true model"
b. the values taken by all its parameters
c. the realized value (observed in real time) of all the shocks.

Unobservability of output gap is not binding. Simple rules are needed in practice.

A Taylor-type interest rate rule:

$$i_t = p + \phi_\pi \pi_t + \phi_t \hat{y}_t$$

as a driving force proportional to the deviations of natural output from steady state, instead of an exogenous monetary policy shock.

The equilibrium dynamics:

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = A_T \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + B_T\left(\hat{r}_t^n - v_t\right)$$

The following equality holds:

$$\hat{r}_t^n - v_t = -\sigma\psi_{y\alpha}^n\left(1-\rho_\alpha\right)\alpha_t - \phi_y\psi_{y\alpha}^n\alpha_t$$

A simple Taylor-type rule that responds aggressively to movements in inflation can approximate arbitrarily well the optimal policy.

A constant money growth rule:

A constant growth rate for the money supply associated with Friedman (1960) consistent with zero inflation in the steady state

$$\triangle m_t = 0$$

for all t.

A money market clearing condition to augment with DIS and NKPC

$$l_t = y_t - \eta i_t - \zeta_t$$

Rewrite the money market equilibrium condition in terms of deviations

$$l_t^+ \equiv I_t + \zeta_t$$

In addition, using the definition and together with the assumed rule:

$$\hat{l}_{t-1}^+ \equiv \hat{I}_t^+ + \pi_t - \triangle\zeta_t$$

The equilibrium dynamics

$$A_{M,0}\begin{bmatrix}\tilde{y}_t \\ \pi_t \\ \hat{I}^{+}_{t-1}\end{bmatrix} = A_{M,1}\begin{bmatrix}E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ \hat{I}^{+}_{t}\end{bmatrix} + B_M\begin{bmatrix}\hat{r}^n_t \\ \hat{y}^n_t \\ \Delta\zeta_t\end{bmatrix}$$

The natural output and the natural rate of interest vary in response to technology shocks.

The degree of stability of money demand is a key element in determining the desirability of a money growth rule.

Key points for chapter5

In Chapter 4, we discussed optimal monetary policy under the condition that:

a. The staggered price setting was the only relevant distortion;
b. Distortion from monopolistic competition was corrected by a wage subsidy;
c. Policy that seeks to replicate the flexible price equilibrium allocation is both feasible and optimal.

In reality, there could be real imperfections other than staggered price.

a. Central banks may face short run tradeoffs between stabilizing inflation and real variables like output and employment.
b. The existence of such a tradeoff makes it desirable for the central bank to be able to commit to a state-contingent policy plan

The Monetary Policy Problem: The Case of an Efficient Steady State

the welfare losses are, up to a second-order approximation, proportional to

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t\left(\pi_t^2+\alpha_x x_t^2\right)\right\}$$

The disturbance is exogenous with respect to monetary policy.

a. The central bank will take the current and anticipated values of u_t as given when solving its policy problem.

Time variations in the gap between the efficient and natural levels of output reflected in fluctuations in u_t a tradeoff for the monetary authority.

a. u_t makes it impossible to attain simultaneously zero inflation and an efficient level of activity.
b. Sources of the cost-push shock u_t-exogenous changes in desired price or wage markups, or fluctuations in labor income taxes.
c. Assume an exogenous AR(1) process of u_t as

$$u_t=\rho_u u_{t-1}+\varepsilon_t^u$$

d. The DIS equation is needed to implement the policy

$$x_t=-\frac{1}{\sigma}(\hat{i}_t-E_t\left\{\hat{\pi}_{t+1}\right\}-r_t^e)+E_t\left\{x_{t+1}\right\}$$

e. The forward-looking nature of constraint in the policy problem requires the specification of the extent to which the central bank can credibly commit in advance to future policy actions.

$$\pi_t=\beta E_t\left\{\pi_{t+1}\right\}+\kappa x_t+u_t$$

Optimal policy under discretion

The central bank makes whatever decision is optimal each period without committing itself to any future actions. In each period, the central bank's problem:

$$\min_{(x_t,\pi_t)} \pi_t^2 + \alpha_x x_t^2$$

s.t.

$$\pi_t = \kappa x_t + v_t$$

The optimality condition:

$$x_t = -\frac{\kappa}{\alpha_x}\pi_t$$

The central bank lets the output gap and inflation deviate from their targets in proportion to the current value of the cost-push shock.

$$\pi_t = \alpha_x \psi_{u_t}$$
$$x_t = -\kappa \psi_{u_t}$$

The central bank finds it optimal to accommodate partly the inflationary pressures resulting from the cost-push shock by letting inflation rise.

However, that the increase in inflation is smaller than the increase that would be obtained if the output gap remained unchanged, in which case

$$\pi_t = \frac{1}{1-\beta\rho_u}u_t$$

The impact on inflation is dampened by the negative response of the output gap.

The implied response of inflation leads naturally to a permanent change in the price level.

Optimal Discretionary Policy: Implementation

Analysis shows that for optimal discretionary policy, the following condition should hold:

$$x_t = -\frac{\kappa}{\alpha_x}\pi_t$$

let's first determine the equilibrium interest rate under the optimal policy as a function of u_t.

Consider an inflation-targeting rule

$$i_t = r_t^e + \phi_\pi \pi_t$$

a determinate equilibrium exists if and only if

$$\phi_\pi > 1$$

Equivalently

$$\kappa\sigma > \alpha_x$$

$$i_t = r_t^e + \Theta_i u_t + \phi_\pi \pi_t$$

Optimal Policy under Commitment

Assume the central bank is able to commit, with full credibility, to a policy plan.

$$\min_{\{x_t,\pi_t\}_{t=0}^{\infty}} \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \alpha_x x_t^2\right)$$

subject to the sequence of constraints

$$\pi_t = \beta E_t\left\{\pi_{t+1}\right\} + \kappa x_t + u_t$$

From the FOC of

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}\left(\pi_t^2 + \alpha_x x_t^2\right) + \gamma_t \left(\pi_t - \kappa x_t - \beta \pi_{t+1}\right) \right]$$

$$x_0 = -\frac{\kappa}{\alpha_x} \pi_0$$

$$x_t = x_{t-1} - \frac{\kappa}{\alpha_x} \pi_t$$

$$\hat{p}_t = \delta \hat{p}_{t-1} + \frac{\delta}{1 - \delta \beta \rho_u} u_t$$

$$\delta \equiv \frac{1 - \sqrt{1 - 4\beta\alpha^2}}{2\alpha\beta} \in (0,1)$$

$$x_t = \delta x_{t-1} - \frac{\kappa\delta}{\alpha_x (1 - \delta\beta\rho_u)} u_t$$

for t =1,2,3, ...with the response at the time of the shock (t =0) being given by

$$x_0 = -\frac{\kappa\delta}{\alpha_x (1 - \delta\beta\rho_u)} u_0$$

Discretionary policy: both the output gap and inflation return to their zero initial value once the shock has vanished.

Commitment: deviations in the output gap and inflation from target persist well beyond the life of the shock.

a. Improvement in the output gap/inflation tradeoff initially.
b. Forward-looking nature of ináation. Iterating the NKPC forward

$$\pi_t = \kappa x_t + \kappa \sum_{k=1}^{\infty} \beta^k E_t \{x_{t+k}\} + u_t$$

c. Due to convexity of loss function, the dampening of deviations in the period of shock improves welfare.

$$i_t = r_t^e - \left[\phi_p + (1-\delta)\left(1 - \frac{\sigma\kappa}{\alpha_x}\right)\right]\sum_{k=0}^{t}\delta^{k+1}u_{t-k} + \phi_p \hat{p}_t$$

The presence of uncorrected real imperfections generate a permanent gap between the natural and the efficient levels of output

$$-\frac{U_N}{U_C} = MPN(1-\Phi)$$

$$\Phi = 1 - \frac{1}{\mu}$$

Optimal Discretionary Policy

$$\min_{(x_t, \pi_t)} \frac{1}{2}\left(\pi_t^2 + \alpha_x \hat{x}_t^2\right) - \Lambda \hat{x}_t$$

s.t.

$$\pi_t = \kappa \hat{x}_t + v_t$$

a more expansionary policy than that given in the absence of a steady state distortion.

$$\pi_t = \frac{\Lambda\kappa}{\kappa^2 + \alpha_x(1-\beta)} + \alpha_x \Psi u_t$$

$$\hat{x}_t = \frac{\Lambda(1-\beta)}{\kappa^2 + \alpha_x(1-\beta)} - \kappa\Psi u_t$$

The distorted steady state

a. Does not affect the responses of the output gap and inflation to shocks under the optimal policy.
b. Affects the average levels of inflation and the output gap around which the economy fluctuates.

The inefficiently low natural level of output and employment

a. Leads to positive average inflation as a consequence of the central bank's incentive to push output above its natural steady state level.

b. Increases the incentive of keeping inflation higher than zero.

Optimal Policy under Commitment

Under credible commitment, the central bank solves a dynamic problem

$$L = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}\left(\pi_t^2 + \alpha_x \hat{x}_t^2\right) - \Lambda \hat{x}_t + \gamma_t \left(\pi_t - \kappa \hat{x}_t - \beta \pi_{t+1}\right) \right]$$

FONCs

$$\alpha_x \hat{x}_t - \kappa \gamma_t - \Lambda = 0$$

$$\pi_t + \gamma_t \text{-} \gamma_{t-1} = 0$$

The FONCs can be combined to yield the difference equation for the log price level

$$\hat{p}_t = a\hat{p}_{t-1} + \alpha\beta E_t\left\{\hat{p}_{t+1}\right\} + \alpha_x \kappa \Lambda + \alpha u_t$$

The corresponding path for the output gap is given by

$$\hat{x}_t = \delta \hat{x}_{t-1} - \frac{\kappa\delta}{\alpha_x\left(1\text{-}\delta\beta\rho_u\right)} u_t + \Lambda\left[1 - \delta\left(1 + \frac{\kappa^2}{\alpha_x\left(1-\delta\beta\right)}\right)\right]$$

a. As with the discretionary policy, an identical stabilization bias exists.

b. Also, the response to a cost-push shock is not affected by the presence of a distorted steady state.

c. In the presence of a distorted steady state, an additional difference arises between the discretionary and commitment policies.

The existence of zero average inflation is observationally equivalent to that of an economy with an efficient steady state.

a. Anticipation of public can improve the short run tradeoff facing the central bank.
b. Commitment avoids the inflation bias under discretionary policy.