Advanced Macro II
The reference answer to homework 3

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Referring to the paper of Kwan and Chow (1996) and lecture slides, complete the following steps for the model:
For the FONCs on slides page 9.
1. Solve for the steady states on slides page 10. (4 points)
ANS:
The FOCs:
$E_t\left[\beta\frac{\overline{c}_t}{\overline{c}_{t+1}}R_{t+1}\right]=1$
$R_{t+1}=\left(1+(1-\alpha)\,\overline{k}_{t+1}^{-\alpha}\overline{z}_{t+1}^{\alpha}\right)/\overline{z}_{t+1}$
$\overline{c}_t+\overline{k}_{t+1}=\overline{k}_t^{1-\alpha}\overline{z}_t^{\alpha-1}+\overline{k}_t\overline{z}_t^{-1}$
$\ln\overline{z}_{t+1}=\mu+\varepsilon_{t+1}$
Steady state:
$E_t\left[\beta\frac{\ddot{c}}{\ddot{c}}\ddot{R}\right]=1$
$\ddot{R}=\left(1+(1-\alpha)\,\ddot{k}^{-\alpha}\ddot{z}^{\alpha}\right)/\ddot{z}$
$\ddot{c}+\ddot{k}=\ddot{k}^{1-\alpha}\ddot{z}^{\alpha-1}+\ddot{k}\ddot{z}^{-1}$
$\ln\ddot{z}=\mu$
$\Rightarrow$

$$\ddot{z}=e^{\mu}$$

$$\ddot{R}=1/\beta$$

$$\ddot{k}=\left(\frac{1-\alpha}{\ddot{R}\ddot{z}-1}\right)^{1/\alpha}\ddot{z}$$

$$\ddot{c}=\ddot{k}^{1-\alpha}\ddot{z}^{\alpha-1}+\ddot{k}\ddot{z}^{-1}-\ddot{k}$$

2. Loglinearize all the FONCs on slides page 9. (4 points)
ANS:
The FOCs:
$E_t\left[\beta\frac{\overline{c}_t}{\overline{c}_{t+1}}R_{t+1}\right]=1$
$R_{t+1}=\left(1+(1-\alpha)\,\overline{k}_{t+1}^{-\alpha}\overline{z}_{t+1}^{\alpha}\right)/\overline{z}_{t+1}$
$\overline{c}_t+\overline{k}_{t+1}=\overline{k}_t^{1-\alpha}\overline{z}_t^{\alpha-1}+\overline{k}_t\overline{z}_t^{-1}$
$\ln\overline{z}_{t+1}=\mu+\varepsilon_{t+1}$

$E_t\left[\beta\frac{\overline{c}_t}{\overline{c}_{t+1}}R_{t+1}\right]=1$
$E_t\left[\beta\frac{\ddot{c}e^{\widehat{c}_t}}{\ddot{c}e^{\widehat{c}_{t+1}}}\ddot{R}e^{\widehat{r}_{t+1}}\right]=1$

$E_t\left[\widehat{c}_t - \widehat{c}_{t+1} + \widehat{r}_{t+1}\right] = 0$

$R_{t+1} = \left(1 + (1-\alpha)\,\overline{k}_{t+1}^{-\alpha}\overline{z}_{t+1}^{\alpha}\right)/\overline{z}_{t+1}$
$\ddot{R}e^{\widehat{r}_{t+1}} = \left(1 + (1-\alpha)\,\ddot{k}^{-\alpha}e^{-\alpha\widehat{k}_{t+1}}\ddot{z}^{-\alpha}e^{\alpha\widehat{z}_{t+1}}\right)/\ddot{z}e^{\widehat{z}_{t+1}}$
$\widehat{r}_{t+1} + \widehat{z}_{t+1} = \frac{\ddot{R}\ddot{z}-1}{\ddot{R}\ddot{z}}(\alpha\widehat{z}_{t+1} - \alpha\widehat{k}_{t+1})$

$\overline{c}_t + \overline{k}_{t+1} = \overline{k}_t^{1-\alpha}\overline{z}_t^{\alpha-1} + \overline{k}_t\overline{z}_t^{-1}$
$\ddot{c}e^{\widehat{c}_t} + \ddot{k}e^{\widehat{k}_{t+1}} = \ddot{k}^{1-\alpha}e^{(1-\alpha)\widehat{k}_t}\ddot{z}^{\alpha-1}e^{(\alpha-1)\widehat{z}_t} + \ddot{k}e^{\widehat{k}_t}\ddot{z}e^{-\widehat{z}_t}$
$\ddot{c}\widehat{c}_t + \ddot{k}\widehat{k}_{t+1} = \ddot{R}\ddot{k}(\widehat{k}_t - \widehat{z}_t)$

$\ln\overline{z}_{t+1} = \mu + \varepsilon_{t+1}$
$\ln\ddot{z}e^{\widehat{z}_{t+1}} = \mu + \varepsilon_{t+1}$
$\widehat{z}_{t+1} = \varepsilon_{t+1}$

Summarize the log-linearized equations as follows:

$$E_t\left[\widehat{c}_t - \widehat{c}_{t+1} + \widehat{r}_{t+1}\right] = 0$$

$$\widehat{r}_{t+1} = -\widehat{z}_{t+1} + \alpha\frac{\ddot{R}\ddot{z}-1}{\ddot{R}\ddot{z}}(\widehat{z}_{t+1} - \widehat{k}_{t+1})$$

$$\ddot{c}\widehat{c}_t + \ddot{k}\widehat{k}_{t+1} = \ddot{R}\ddot{k}(\widehat{k}_t - \widehat{z}_t)$$

$$\widehat{z}_{t+1} = \varepsilon_{t+1}$$

Then you proceed to solve the model into the recursive law of motion.
3. Solve for the undetermined coeffcients v_{ck} v_{cz} v_{kk} v_{kz}.(6 points)
ANS:
Postulate a linear recursive law of motion for $\widehat{c}_t$ and $\widehat{k}_{t+1}$

$$\widehat{c}_t = v_{ck}\widehat{k}_t + v_{cz}\widehat{z}_t \tag{1}$$

$$\widehat{k}_{t+1} = v_{kk}\widehat{k}_t + v_{kz}\widehat{z}_t \tag{2}$$

Insert the return equation to Euler equation to delete $\widehat{r}_{t+1}$, and insert the law of motion to the log-linearized budget constraint and Euler equation.
we can get
$\ddot{c}\left(v_{ck}\widehat{k}_t + v_{cz}\widehat{z}_t\right) + \ddot{k}\left(v_{kk}\widehat{k}_t + v_{kz}\widehat{z}_t\right) = \ddot{R}\ddot{k}(\widehat{k}_t - \widehat{z}_t)$
$\left(\ddot{c}v_{ck} + \ddot{k}v_{kk} - \ddot{R}\ddot{k}\right)\widehat{k}_t + \left(\ddot{c}v_{cz} + \ddot{k}v_{kz} + \ddot{R}\ddot{k}\right)\widehat{z}_t = 0$
$\ddot{c}v_{ck} + \ddot{k}v_{kk} - \ddot{R}\ddot{k} = 0$
$\ddot{c}v_{cz} + \ddot{k}v_{kz} + \ddot{R}\ddot{k} = 0$
$v_{kk} = \ddot{R} - \frac{\ddot{c}}{\ddot{k}}v_{ck}$
$v_{kz} = -\ddot{R} - \frac{\ddot{c}}{\ddot{k}}v_{cz}$

$E_t\left[\widehat{c}_t - \widehat{c}_{t+1} + \widehat{r}_{t+1}\right] = 0$
$E_t\left[\widehat{c}_t - v_{ck}\widehat{k}_{t+1} + v_{cz}\widehat{z}_{t+1} + \widehat{r}_{t+1}\right] = 0$
$E_t\left[v_{ck}\widehat{k}_t + v_{cz}\widehat{z}_t - v_{ck}\left(v_{kk}\widehat{k}_t + v_{kz}\widehat{z}_t\right) + v_{cz}\widehat{z}_{t+1} + \widehat{r}_{t+1}\right] = 0$
$\widehat{r}_{t+1} = -\widehat{z}_{t+1} + \frac{\ddot{R}\ddot{z}-1}{\ddot{R}\ddot{z}}(\alpha\widehat{z}_{t+1} - \alpha\left(v_{kk}\widehat{k}_t + v_{kz}\widehat{z}_t\right))$
$v_{ck} - v_{ck}v_{kk} - \frac{\ddot{R}\ddot{z}-1}{\ddot{R}\ddot{z}}\alpha v_{kk} = 0$
$v_{cz} - v_{ck}v_{kz} - \frac{\ddot{R}\ddot{z}-1}{\ddot{R}\ddot{z}}\alpha v_{kz} = 0$
$v_{cz} - v_{ck}\left(-\ddot{R} - \frac{\ddot{c}}{\ddot{k}}v_{cz}\right) - \frac{\ddot{R}\ddot{z}-1}{\ddot{R}\ddot{z}}\alpha\left(-\ddot{R} - \frac{\ddot{c}}{\ddot{k}}v_{cz}\right) = 0$

$v_{ck} - v_{ck}\left(\ddot{R} - \frac{\ddot{c}}{\ddot{k}}v_{ck}\right) - \frac{\ddot{R}\ddot{z}-1}{\ddot{R}\ddot{z}}\alpha\left(\ddot{R} - \frac{\ddot{c}}{\ddot{k}}v_{ck}\right) = 0$
$\frac{\ddot{c}}{\ddot{k}}v_{ck}^2 + \left(1 - \ddot{R} + \alpha\frac{\ddot{R}\ddot{z}-1}{\ddot{R}\ddot{z}}\frac{\ddot{c}}{\ddot{k}}\right)v_{ck} - \alpha\frac{\ddot{R}\ddot{z}-1}{\ddot{z}} = 0$
denote $A = \alpha\frac{\ddot{R}\ddot{z}-1}{\ddot{R}\ddot{z}}$
$\ddot{c}v_{ck}^2 + \left(\ddot{k} - \ddot{R}\ddot{k} + A\ddot{c}\right)v_{ck} - A\ddot{k}\ddot{R} = 0$
$\Rightarrow$

$$v_{ck} = \frac{-\left(\ddot{k} - \ddot{R}\ddot{k} + A\ddot{c}\right) \underline{+} \sqrt{\left(\ddot{k} - \ddot{R}\ddot{k} + A\ddot{c}\right)^2 + 4A\ddot{k}\ddot{R}\ddot{c}}}{2\ddot{c}}$$

$$v_{kk} = \ddot{R} - \frac{\ddot{c}}{\ddot{k}}v_{ck}$$

$$v_{cz} = -\frac{(A + v_{ck})\ddot{R}\ddot{k}}{(A + v_{ck})\ddot{c} + \ddot{k}}$$

$$v_{kz} = -\ddot{R} - \frac{\ddot{c}}{\ddot{k}}v_{cz}$$

4. What are the parameters in the log variable VAR(1): g, $G_1$and G_2? (4 points)
ANS:
After getting these coefficients in terms of the parameters and steady state values, write the system as

$$\widehat{c}_t = v_{ck}\widehat{k}_t + v_{cz}\widehat{z}_t \tag{3}$$
$$\widehat{k}_{t+1} = v_{kk}\widehat{k}_t + v_{kk}\widehat{z}_t \tag{4}$$

$$\widehat{z}_{t+1} = \varepsilon_{t+1}$$

$\begin{bmatrix} \widehat{k}_t \\ \widehat{z}_t \end{bmatrix} = \begin{bmatrix} v_{kk} & v_{kz} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \widehat{k}_{t-1} \\ \widehat{z}_{t-1} \end{bmatrix} + \begin{bmatrix} \widehat{e}_t \\ \widehat{\varepsilon}_t \end{bmatrix}$
To utilize the transformed data,write the system in log term,
reminding that $\widehat{x}_t = \log\overline{x}_t - \log\ddot{x}$
$\begin{bmatrix} \log\overline{z}_t - \log\ddot{z} \\ \log\overline{k}_t - \log\ddot{k} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ v_{kz} & v_{kk} \end{bmatrix}\begin{bmatrix} \log\overline{z}_{t-1} - \log\ddot{z} \\ \log\overline{k}_{t-1} - \log\ddot{k} \end{bmatrix} + \begin{bmatrix} \widehat{e}_t \\ \widehat{\varepsilon}_t \end{bmatrix}$

$$\begin{bmatrix} \log \overline{z}_t \\ \log \overline{k}_t \end{bmatrix} = \begin{bmatrix} \mu \\ (1 - v_{kk}) \log \ddot{k} - v_{kz}\mu \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ v_{kz} & v_{kk} \end{bmatrix} \begin{bmatrix} \log \overline{z}_{t-1} \\ \log \overline{k}_{t-1} \end{bmatrix} + \begin{bmatrix} \widehat{\varepsilon}_t \\ \widehat{e}_t \end{bmatrix}$$

Therefore,

$$g = (1 - v_{kk}) \log \ddot{k} - v_{kz}\mu$$

$$G_1 = v_{kz}$$

$$G_2 = v_{kk}$$

Simulation study using the data set in Data_KwanChow1996.xls in the folder of "DataCodes" of the teaching page.

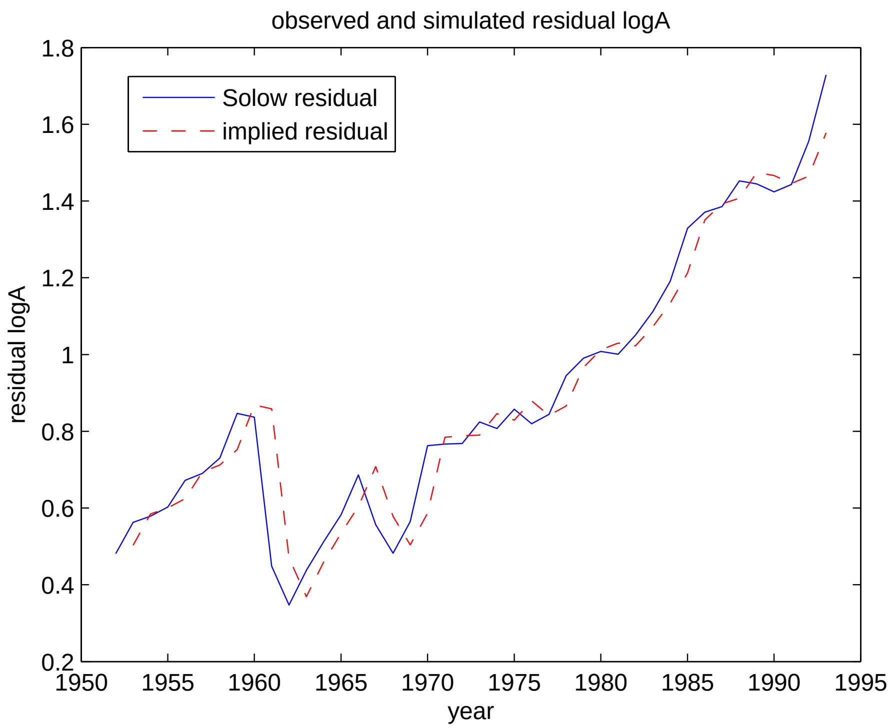
5. Take the MLE estimate of authors $\left(\widetilde{\alpha}, \widetilde{\beta}, \widetilde{\gamma}\right) = (0.7495; 0.9999; 0.0218)$to construct the Solow residual series and the model implied series, i.e.

$\ln A_t = \ln q_t - (1 - \alpha) \ln k_t$

$\ln \widetilde{A}_t = \widetilde{\gamma} + \ln q_{t-1} - (1 - \alpha) \ln k_{t-1}$

plot them on one graph. Please label the series properly. (5 points)

ANS:



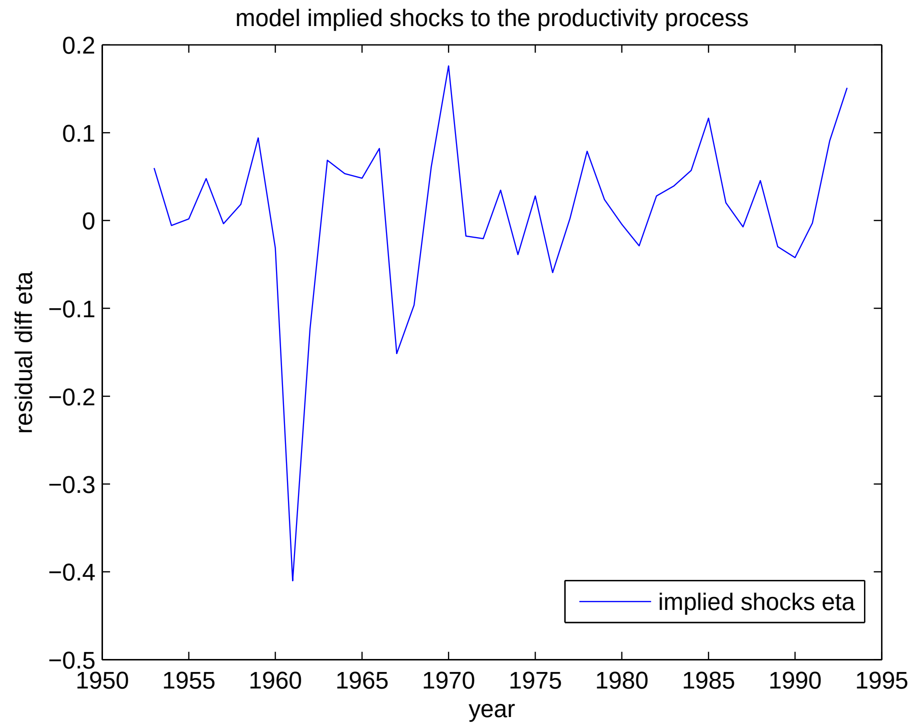
6. Compute the model implied shocks to the productivity process,

$\widetilde{\eta}_t = \ln A_t - \ln \widetilde{A}_t$

and plot it. What is the variance of $\widetilde{\eta}_t$.(2 points)

ANS:

$var(\widetilde{\eta}_t) = 0.0086$

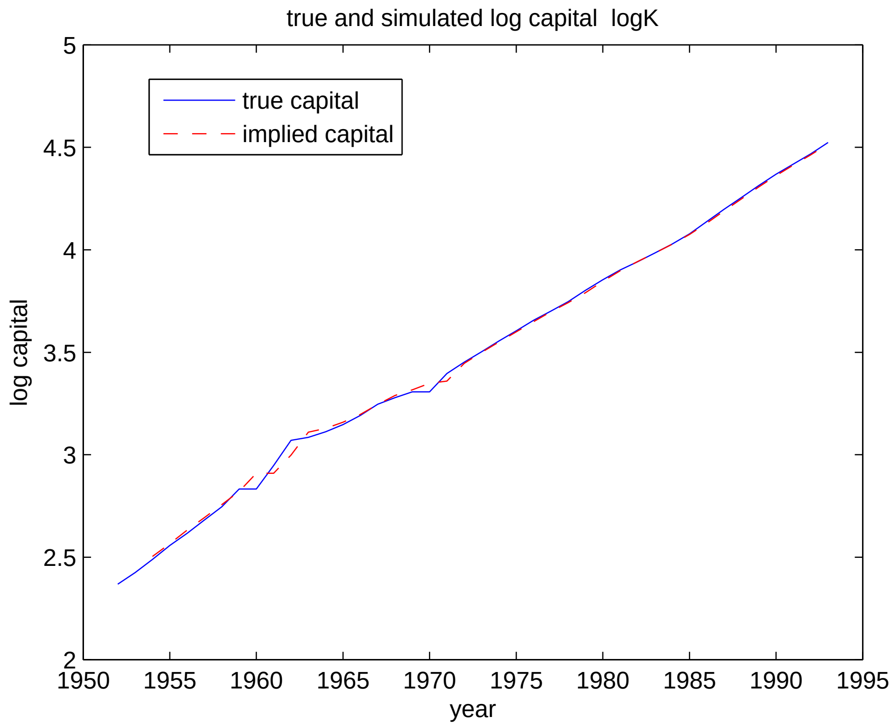


7. Compute the model implied log capital, $\ln \widetilde{k}_t$ using information up t, i.e.

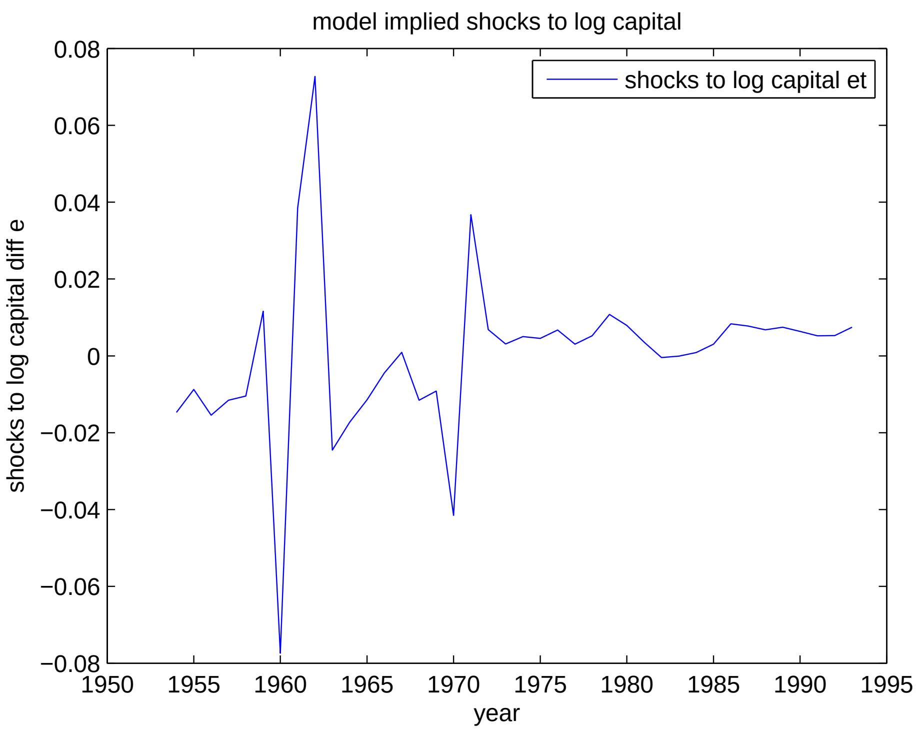
$$\begin{aligned}\ln \widetilde{k}_t &= g + \left[\ln q_{t-1} - (1-\alpha)\ln k_{t-1}\right]/\alpha \\ &\quad + G_1 \left[\triangle \ln q_{t-1} - (1-\alpha)\triangle \ln k_{t-1}\right]/\alpha \\ &\quad + G_2 \left[\ln k_{t-1} - (\ln q_{t-2} - (1-\alpha)\ln k_{t-2})\right]/\alpha\end{aligned}$$

and plot the implied capital, $\ln \widetilde{k}_t$; and true capital, $\ln k_t$; processes on one graph.Please label the series properly. (5 points)

ANS:

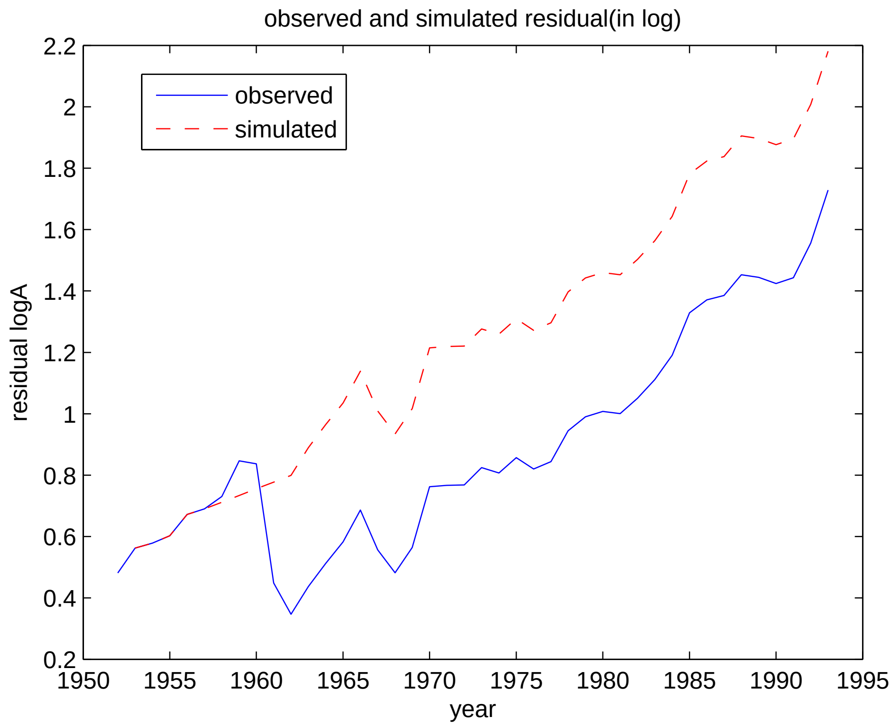


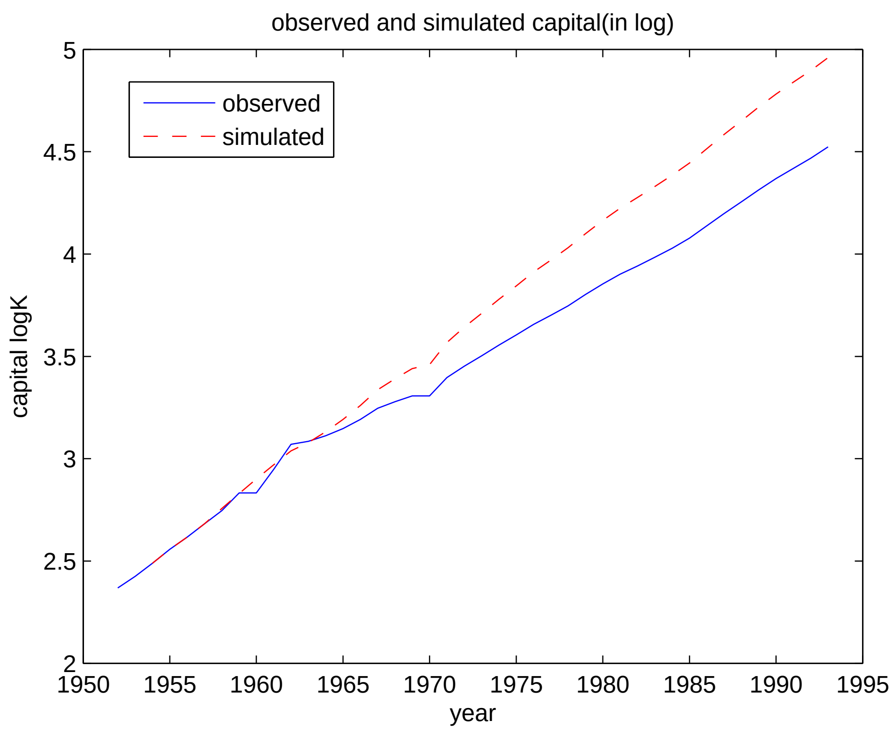
8. Compute the model implied shocks to log capital,
$\widetilde{e}_t = \ln k_t - \ln \widetilde{k}_t$
and plot it. What is the variance of $\widetilde{e}_t$.(2 points)
ANS:
$var(\widetilde{e}_t)$ =4.8123e-004

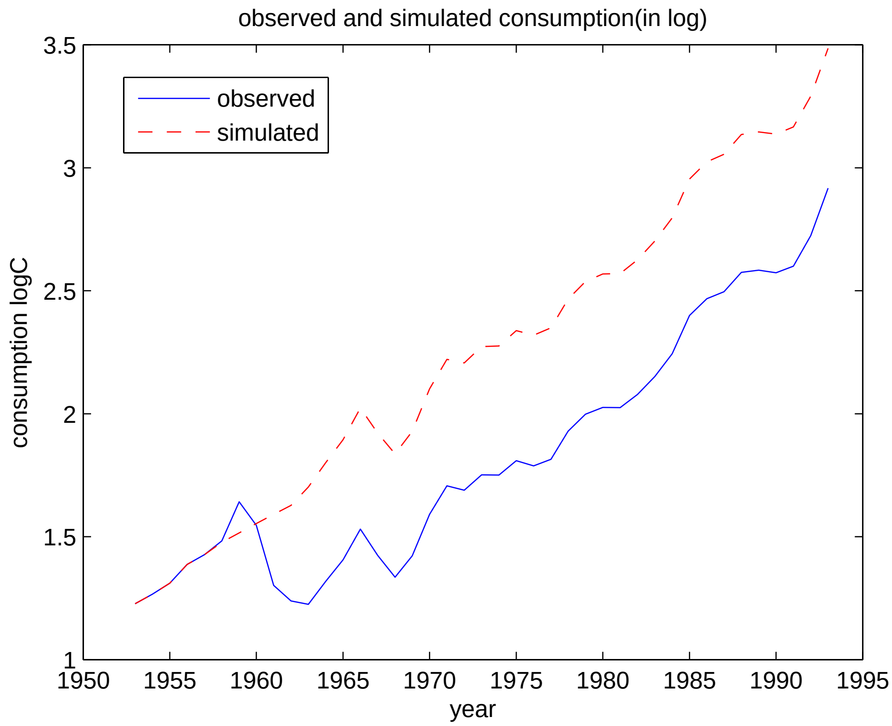


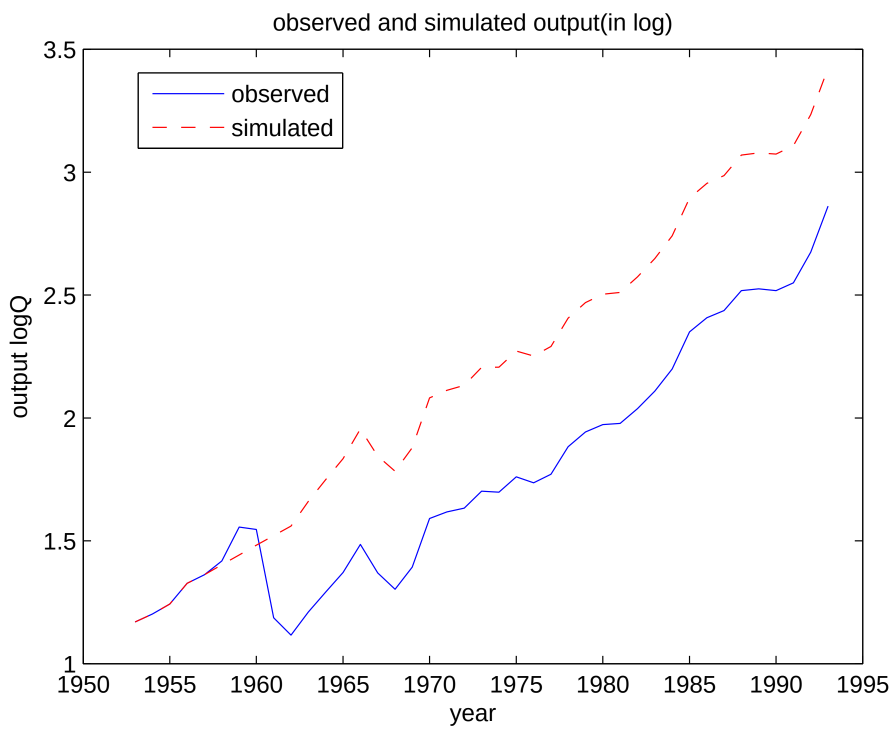
9. Assume $\widetilde{\eta}_t = 0$, $\widetilde{e}_t = 0$ for t= 1958, ...,1962,simulate logged Solow residual, capital and output. Replicate Fig.6, Fig.5 and Fig.3 in the paper respectively. (Note, your figure can be slightly different as we set the disturbance terms during the Great Leap Forward completely to zero.) (7 points)

ANS:









10. Compute the ratios of Simulation/Observed levels for these three series in 1992, using your results. And compare them with the first column of Table 3. (1 point)

ANS:

Table 1: Simulation/Observed level in 1992

	Great leap	Cultural revolution	Both
Output	1.7499	1.1388	1.9928
Consumption	1.7634	1.1408	2.0117
Capital	1.5350	1.1051	1.6963

Note: Cultural revolution refers year 1966-1969 and Great leap refers year 1958-1962.