Advanced Microeconomics II
Quiz 1

Spring 2013

1. Consider a homogeneous product produced by two different firms, firm 1 and firm 2. Individual firm demand for the product is given by

$$q_i(p_1, p_2) = \begin{cases} 1 - p_i & \text{if } p_i < p_{-i} \\ (1 - p_i)/2 & \text{if } p_i = p_{-i} \\ 0 & \text{otherwise.} \end{cases}$$

Each firm offers to match the competing firm's price so that, for example, if firm 1 charges a price p_1 and firm 2 charges a higher price p_2, then the price faced by consumers would be p_1. Firms simultaneously choose prices and both firms produce their products at constant marginal cost equal to zero.

(a) (3 points) Write down the normal form of this game.

> **Solution:** $N = \{1, 2\}$, $A_i = [0, \infty)$;
>
> $$u_i(a_1, a_2) = \begin{cases} p_i(1 - p_i)/2 & \text{if } p_i < p_j \\ p_j(1 - p_j)/2 & \text{otherwise.} \end{cases}$$

(b) (4 points) Find the set of symmetric Nash equilibria for this game, i.e., the equilibria where both firms use the same strategy, or prove that the set is empty.

> **Solution:** Any pair (p, p) such that $0 \leq p \leq 0.5$ is a Nash equilibrium of this game.
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> $$u_1(a_1, a_2) = p_i(1 - p_i)/2;$$
> $$u_2(a_1, a_2) = p_i(1 - p_i)/2$$

(c) (3 points) Construct an asymmetric Nash equilibrium for this game or prove that one does not exist.

> **Solution:** Any pair (p_1, p_2) such that either $p_1 = 0.5 < p_2$ or $p_2 = 0.5 < p_1$ is an asymmetric Nash equilibrium of this game.