Quiz 2

Imperfect Competition and Price-Setting

An economy has a competitive labor market, but an imperfectly competitive goods market. Each individual among a large population produces a heterogenous goods Q_i and sets the price P_i, and labor L_i is the only input in the production function, $Q_i = L_i$.
The demand function is $Q_i = Y(P_i/P)^{-\eta}, \eta > 1$, where P is the price index or aggregate price, and Y is the aggregate average real income. The utility function of the representative consumer is $U_i = C_i - L_i^{\gamma}/\gamma, \gamma > 1$ and C_i is the real consumption,ie.individual income divided by the price index. Individual income consists of sales profit of production,$(P_i - W)Q_i$, and labor income, WL_i, where W is the nominal wage. Hence, the utility can be expressed as;

$$U_i = \frac{(P_i - W)Q_i + WL_i}{P} - L_i^{\gamma}/\gamma$$

that the consumer chooses P_i and L_i to opitimize utility.
1) Insert the demand function into Equation(1) to find out the first order necessary condition (FONC) of P_i?
ANS:
For each individual i, the optimal problem is:

$$\max_{P_i, L_i} U_i = \frac{(P_i - W)Q_i + WL_i}{P} - L_i^{\gamma}/\gamma$$

$$Q_i = Y(P_i/P)^{-\eta}$$

Substitute Q_i into U_i :

$$U_i = \frac{(P_i - W)Y(P_i/P)^{-\eta} + WL_i}{P} - L_i^{\gamma}/\gamma$$

FONC for P_i is:

$$\frac{\partial U_i}{\partial P_i} : \quad \frac{Y(P_i/P)^{-\eta}}{P} + \frac{(-\eta)(P_i - W)Y(P_i/P)^{-\eta-1}}{P^2} = 0$$

$$\Rightarrow P_i = \frac{\eta}{\eta - 1} W$$

2) What's the relationship between relative price P_i/P and real wage W/P?
ANS:

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1} \frac{W}{P}$$

The relationship between relative price P_i/P and real wage W/P is depended by η the demand elastictiy of goods i.
3) What's the economic interpretation of the parameter η?Explain why the relationship in step2) represents the pricing under imperfect competition.
ANS:

since

$$\left|\frac{\partial Q_i/Q_i}{\partial P_i/P_i}\right| = \left|\frac{\partial Q_i}{\partial P_i}\frac{P_i}{Q_i}\right| = \eta$$

η is the demand elasticity for good i, which is also the elasticity between any two goods.
In step 2,

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1}\frac{W}{P}$$

$\frac{W}{P}$ is the real wage, representing the marginal cost for the producer; $\frac{\eta}{\eta-1}$ captures the markup chosen by producers for the special goods i. Since markup $\frac{\eta}{\eta-1} > 1$, it represents the pricing under imperfect competition.

4) Find out the FONC of labor input L_i, and express L_i as a function of real wage.

ANS:

FONC for L_i is:

$$\frac{\partial U_i}{\partial L_i}: \quad \frac{W}{P} - L_i^{\gamma-1} = 0$$

$$\Rightarrow L_i = \left(\frac{W}{P}\right)^{\frac{1}{\gamma-1}}$$

5) Derive the elasticity of labor input L_i to real wage.

ANS:

The elasticity of labor input L_i to real wage is.

$$\frac{\partial L_i/L_i}{\partial \frac{W}{P}/\frac{W}{P}} = \frac{\partial L_i}{\partial \frac{W}{P}}\frac{\frac{W}{P}}{L_i} = \frac{1}{\gamma - 1}$$

6) Due to the symmetry in this model, in equilibrium each individual inputs equal labor such that $L_i = L$, and results in equal production of each goods, $Q_i = Y$. In equilibrium, the real average output Y equals average labor input L.Combine with the result of step 4) to express Y as a function of real wage.

ANS:

$$Y = L = L_i = \left(\frac{W}{P}\right)^{\frac{1}{\gamma-1}}$$

7) Use results from 2) and 6) to express the relative price P_i/P as a function of Y.

ANS:

$$\begin{aligned}
\frac{P_i}{P} &= \frac{\eta}{\eta - 1}\frac{W}{P} \\
Y &= \left(\frac{W}{P}\right)^{\frac{1}{\gamma-1}} \\
\frac{P_i}{P} &= \frac{\eta}{\eta - 1}Y^{\gamma-1}
\end{aligned}$$

8) In equilibrium every producer sets the same price,such that the relative price is one. Use this condition in the equation obtained in 7) to solve for the equilibrium output Y.

ANS:

$$\begin{aligned} 1 &= \frac{P_i}{P} = \frac{\eta}{\eta - 1} Y^{\gamma-1} \\ Y &= \left(\frac{\eta - 1}{\eta}\right)^{\frac{1}{\gamma-1}} \end{aligned}$$

9) Assume the aggregate real demand function is $Y = M/P$,where M is the money supply,solve for the equilibrium price.
ANS:

$$\begin{aligned} Y &= \left(\frac{\eta - 1}{\eta}\right)^{\frac{1}{\gamma-1}} = \frac{M}{P} \\ P &= \left(\frac{\eta - 1}{\eta}\right)^{-\frac{1}{\gamma-1}} M \end{aligned}$$

10) For what value of η will the economy have perfect competition? Compare the output level Y under this condition and the output under imperfect competition.
ANS:
When the elasticity η approaches to infinity, then the markup $\frac{\eta}{\eta-1}$ limits to 1, the economy have perfect competition.
Under perfect competition, $Y = 1$
Under imperfect competition, $Y = \left(\frac{\eta-1}{\eta}\right)^{\frac{1}{\gamma-1}} < 1$.
Therefore, the output level Y is smaller than the output under imperfect competition.
11) Use this model to explain whether imperfect competition implies non-neutrality of money.
ANS: No. Note that

$$\begin{aligned} Y &= \left(\frac{\eta - 1}{\eta}\right)^{\frac{1}{\gamma-1}} \\ P &= \left(\frac{\eta - 1}{\eta}\right)^{-\frac{1}{\gamma-1}} M \end{aligned}$$

In equilibrium, the money supply only affects the price level and has no influence on output, so it is incorrect to say that imperfect competition implies non-neutrality.