Advanced Microeconomics II

WISE, Xiamen University

Spring 2012 Final

1. (5 points) Define precisely a subgame perfect equilibrium of an extensive game with perfect information.

2. (5 points) Define precisely a T-period finitely repeated game of the strategic game $G = \{N, (A_i), (\succeq_i)\}$.

3. (5 points) Define precisely an assessment in an extensive game.

4. (5 points) Define precisely a perfect Bayesian equilibrium of a Bayesian extensive game with observable actions.

5. (10 points) In the following two-person infinitely repeated bargaining game only decimal divisions are possible, i.e. the set of possible divisions are $\{(1, 0), (.9, .1), \ldots, (.1, .9), (0, 1)\}$. Consider the following stationary strategies: For each player i, player i always proposes $x = (x_1, x_2)$ and accepts a proposal y if and only if $y_i \geq x_i$. (Note that the offer x is the same for every player.) For what divisions of the pie are such stationary strategies sub-game perfect equilibria if $\delta_1 = \delta_2 = .85$?

6. Consider the following stage game of a discounting-infinitely repeated game where both players have the same discount factor.

		Player 2	
		A	B
Player 1	A	$4, 4$	$0, 9$
	B	$9, 0$	$1, 1$

   (a) (5 points) What is the minimum discount factor required to sustain the infinitely repeated outcome (A, A)?

   (b) (5 points) What is the minimum discount factor required to sustain the infintely repeated cycle $((A, B), (B, A), (B, B))$?

7. Consider a 2-consumer economy in which consumer i's preferences are given by $u_i(x_i, g) = x_i + \theta_i g - g^2/2$, where x_i is consumer i's consumption of the private good, g is the quantity of the public good, and θ_i is i's marginal valuation of the public good, which is private information for each consumer. What is publicly known, however, is that for each player, $\theta_i \sim U[3, 4]$ independently across the two players. The total amount of the public good g is determined by the sum of the individual contributions, i.e., $g = g_1 + g_2$. Each consumer is endowed with 5 units of the private good that can be converted into the public good in a 1-for-1 fashion. That is, each consumer faces a budget constraint of the form $x_i + g_i \leq 5$.

   (a) (3 points) Given the realizations of θ_1 and θ_2, what is the amount of the public good that maximizes the sum of the two agents utilities?

   (b) Now suppose the public good is provided by private contributions and consider a Bayesian Nash equilibrium where each consumer only knows his own θ_i.

i. (3 points) What is the ex-ante expected level of the public good provided in such an equilibrium?

ii. (3 points) What is the actual equilibrium amount of the public good given the realizations of θ_1 and θ_2?

iii. (1 point) How does the equilibrium amount of the public good compare to the utility maximizing amount?

iv. (5 points) What is the set of pure-strategy Bayesian Nash equilibria?

8. The value of a used car is v_b for the buyer and v_s for the seller; v_b and v_s are common knowledge. Assume $v_b > v_s$. This game has two stages. In the first stage, the buyer offers a price p to the seller. In the second stage, the seller observes the offer and decides whether to accept the price in exchange for the car.

   (a) (3 points) Model this is an extensive game of perfect information.

   (b) (3 points) Find the sub-game perfect Nash equilibria of this game.

   (c) Now assume that there are three possible qualities of cars: high, medium and low. The seller knows the quality of her car precisely, but the buyer does not. In the first stage, nature decides the car type. In the second stage the buyer makes a price offer. In the third stage the seller observes the offer (and the choice of nature) and decides whether to accept or reject the offer. The ex-ante probability distribution over the possible states of car quality and the value of the car to the buyer and seller in each state, all of which are common knowledge, are shown in the following table.

Car Quality	High	Medium	Low
Probability	1/3	1/3	1/3
v_b	18	13	9
v_s	15	10	8

   i. (3 points) Model this as a Bayesian extensive game with observable actions.

   ii. (1 point) Can this be modelled as a signalling game? Why or why not?

   iii. (5 points) Find the Perfect Bayesian Equilibria of this game.

9. Two partners must dissolve their partnership. Partner 1 currently owns share s of the partnership, partner 2 owns share $1 - s$. The partners agree to play the following game: partner 1 names a price p for the whole partnership, and partner 2 then chooses either to buy partner 1's share for ps or to sell her share to partner 1 for $p(1-s)$. Suppose it is common knowledge that the partners' valuations for owning the whole partnership are independently and uniformly distributed on $[0, 1]$, but that each partner's valuation is private information.

   (a) (3 points) Model this as a Bayesian extensive game with observable actions.

   (b) (1 point) Can this be modelled as a signalling game? Why or why not?

   (c) (3 points) Describe the strategy of player 2 in a perfect Bayesian equilibrium.

   (d) (4 points) Describe the strategy of player 1 in a perfect Bayesian equilibrium.

   (e) (4 points) Describe a system of beliefs that completes the description of a perfect Bayesian equilibrium .

10. Let $\Gamma = \{N, H, P, f_c, (\mathcal{I}_i), (\succeq_i)\}$ be a finite extensive game with perfect recall.

(a) (5 points) Construct a Γ that has a weak perfect Bayesian equilibrium which is not a subgame perfect equilibrium outcome. Clearly identify the equilibrium.

(b) (5 points) Construct a Γ that has a subgame perfect equilibrium outcome which is not a weak perfect Bayesian equilibrium outcome. Clearly identify the equilibrium.

(c) (5 points) Construct a Γ that has a perfect Bayesian equilibrium outcome which is not a sequential equilibrium outcome. Clearly identify the equilibrium.