


Deep Learning for Symbolic Math

Data Science Retreat Batch 21
Wenjuan Yang

What is Symbolic Math?

- Simple example: $\int x dx = \frac{x^2}{2}$ \int_a^b 
- Complex example: $\int (x^2(\tan(x)^2 + 1) + 2x \tan(x) + 1) dx = x^2 \tan(x) + x$
- More complex example:

$$\int \frac{16x^3 - 42x^2 + 2x}{(-16x^8 + 112x^7 - 204x^6 + 28x^5 - x^4 + 1)^{1/2}} dx = \sin^{-1}(4x^4 - 14x^3 + x^2)$$



Can Deep Learning Learn Math?

- Difference

Language: Statistic way of learning

Math: Rule-based inference

- Similarity

Pattern recognition

Background

Symbolic Math

DEEP LEARNING FOR SYMBOLIC MATHEMATICS

Guillaume Lample*
Facebook AI Research
glample@fb.com

François Charton*
Facebook AI Research
fcharton@fb.com

ABSTRACT

Neural networks have a reputation for being better at solving statistical or approximate problems than at performing calculations or working with symbolic data. In this paper, we show that they can be surprisingly good at more elaborated tasks in mathematics, such as symbolic integration and solving differential equations. We propose a syntax for representing mathematical problems, and methods for generating large datasets that can be used to train sequence-to-sequence models. We achieve results that outperform commercial Computer Algebra Systems such as Matlab or Mathematica.

2 Dec 2019

Facebook GitHub: <https://github.com/facebookresearch/SymbolicMathematics>

Code released on Mar 15

Outline

1. *Data Generation*

2. *Transformer Model*

3. *Results*

4. *Outlook*

Outline

1. *Data Generation*

2. *Transformer Model*

3. *Results*

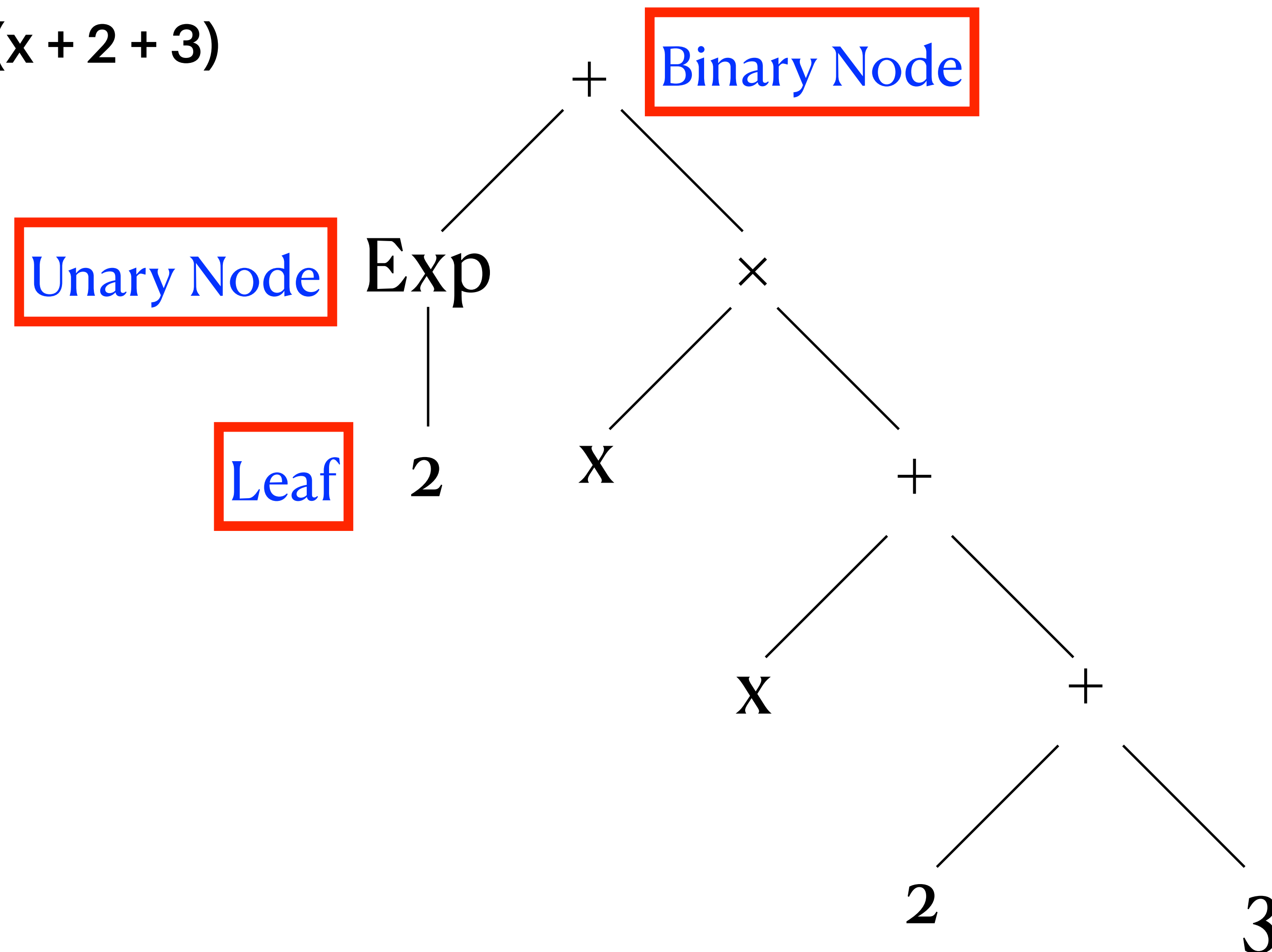
4. *Outlook*

How to Generate Data?

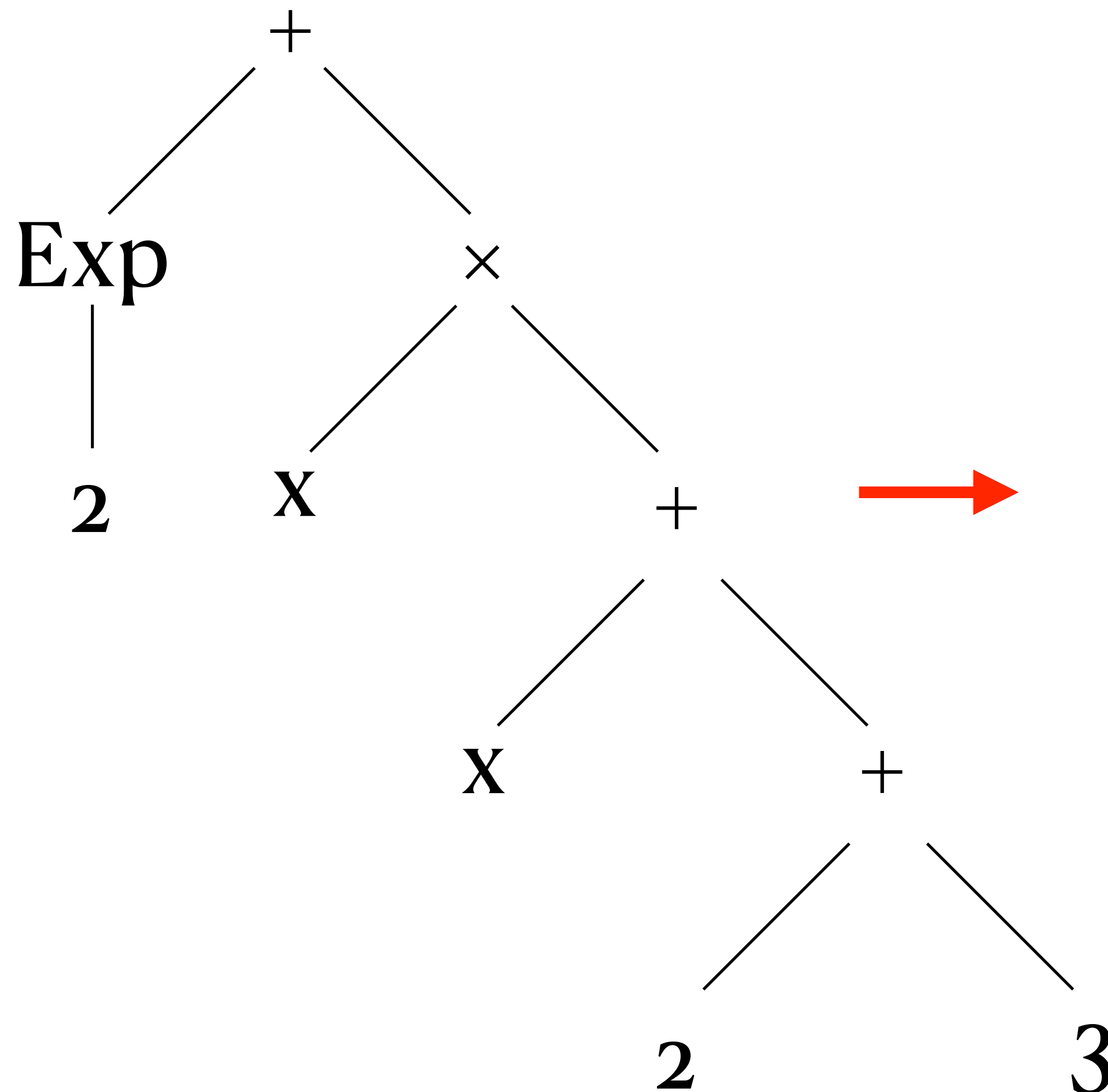
6 STEPS

Random Tree

Example: $\text{Exp}(2) + x(x + 2 + 3)$



Random Tree → Prefix



Prefix

+ Exp 2 × x + x + 2 3

Infix

Exp(2) + x (x + 2 + 3)



Prefix → Infix & Clean Data

+ Exp 2 × x + x + 2 3 → Exp(2) + x × (x + 2 + 3)

∞ - ∞ *i* ✗

Simplify

$\text{Exp}(2) + x \times (x + 2 + 3) \longrightarrow \text{Exp}(2) + x \times (x + 5)$ Infix

$\text{Exp}(2) + x \times (x + 5) \longrightarrow + \text{Exp } 2 \times x + x 5$ Prefix

Output

Differentiate

$$\text{Exp}(2) + x \times (x + 5) \longrightarrow 2 \times x + 5$$

Infix \rightarrow Prefix

$2 \times x + 5 \rightarrow + \times 2 x 5$

Input



Generate Answer before Question

Features(X)

+ × 2 x 5

Input

Target(Y)

+ Exp 2 × x + x 5

Output

Parallel Data Generation

Two ways to generate data:

- Multiprocess thread pool

```
from multiprocessing.pool import ThreadPool
_FINISH = False
start = time.time()
with ThreadPool(processes=14) as p:
    out = []
    r = p.map_async(generate_bwd,
[sequences_per_process]*process_runs, callback=out.append)
    r.wait()
    time.sleep(10)
    _FINISH = True    p.terminate()
```

- Ray

```
ray.init(num_cpus=cpu)
dataset = []
for _ in range(process_runs*cpu):
    try:
        out = ray_generate_bwd.remote(sequences_per_process)
        out = ray.get(out, timeout=sequences_per_process)
        dataset.extend(out)
```

Outline

1. *Data Generation*

2. *Transformer Model*

3. *Results*

4. *Outlook*

Transformer Model

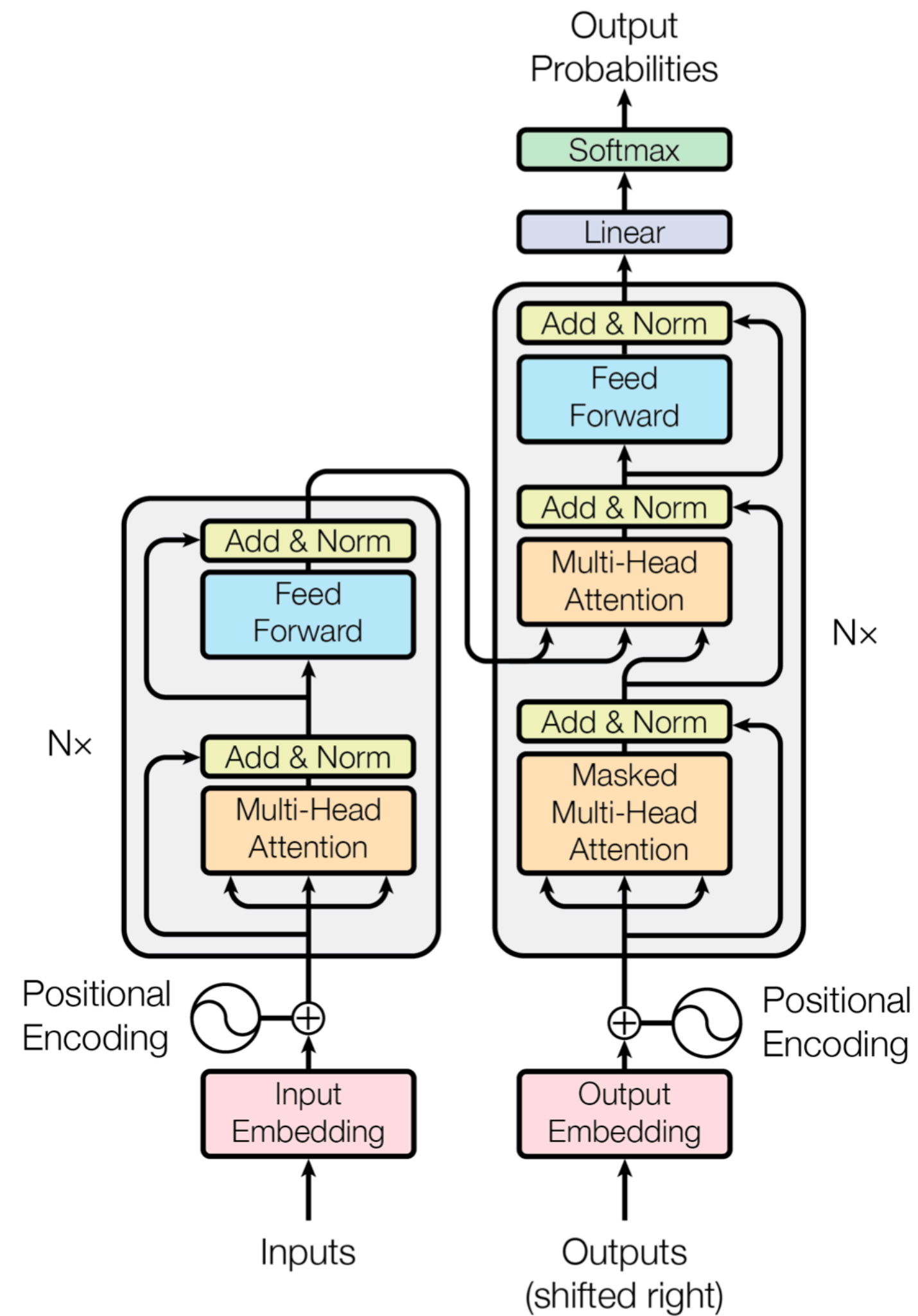


Figure from Vaswani, Ashish, et al. "Attention is all you need." (2017)

Transformer Model

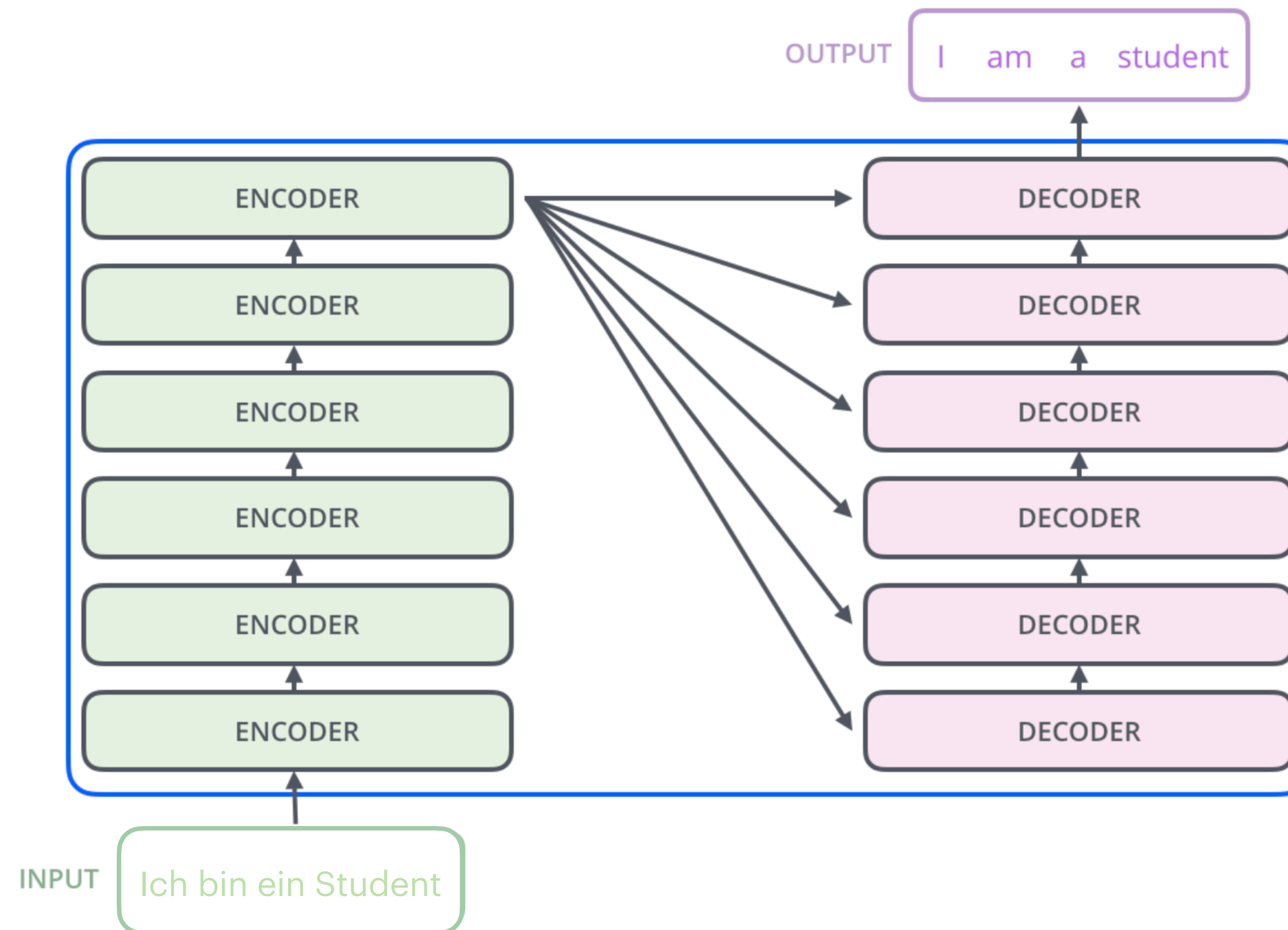


Figure from Jay Alammar <http://jalammar.github.io/illustrated-transformer/>

Transformer Model

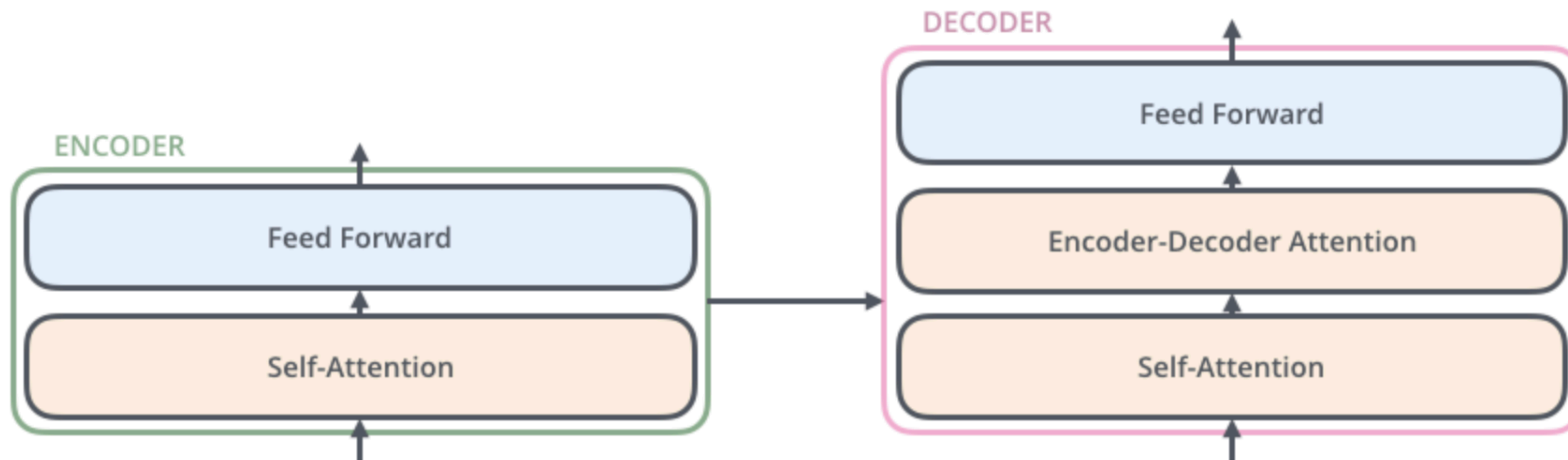


Figure from Jay Alammar <http://jalammar.github.io/illustrated-transformer/>

Transformer Model

Hyperparameters

batch_size = 32

d_model = 512

sequence_length = 512

num_layers = 6

num_heads = 8

dropout_rate = 0.1

optimizer = Adam

learning_rate = CustomSchedule

loss, accuracy = CustomSchedule

Outline

1. *Data Generation*

2. *Transformer Model*

3. *Results*

4. Outlook

Paper Results

Training set size = 40M, Test set size = 5000

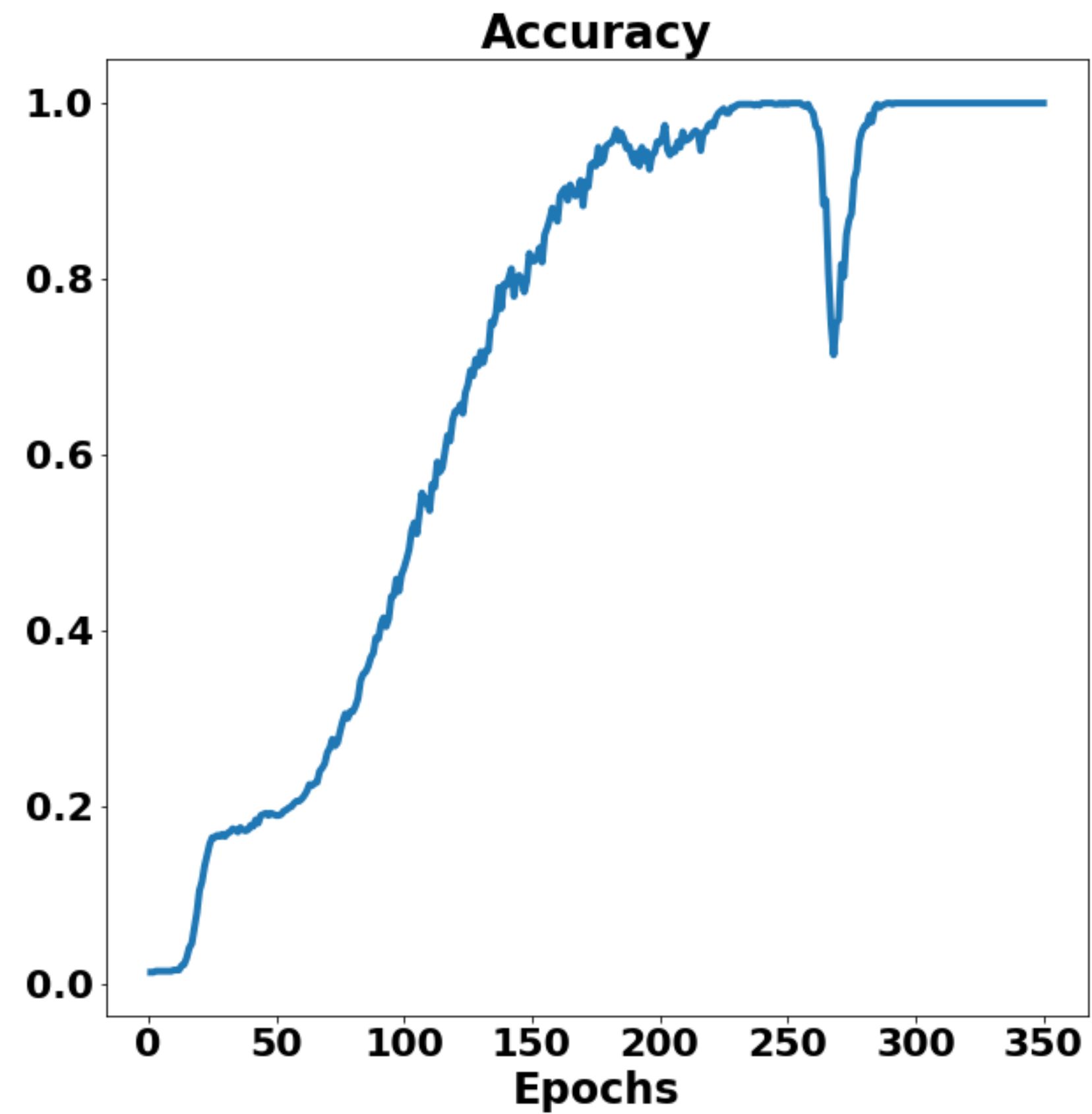
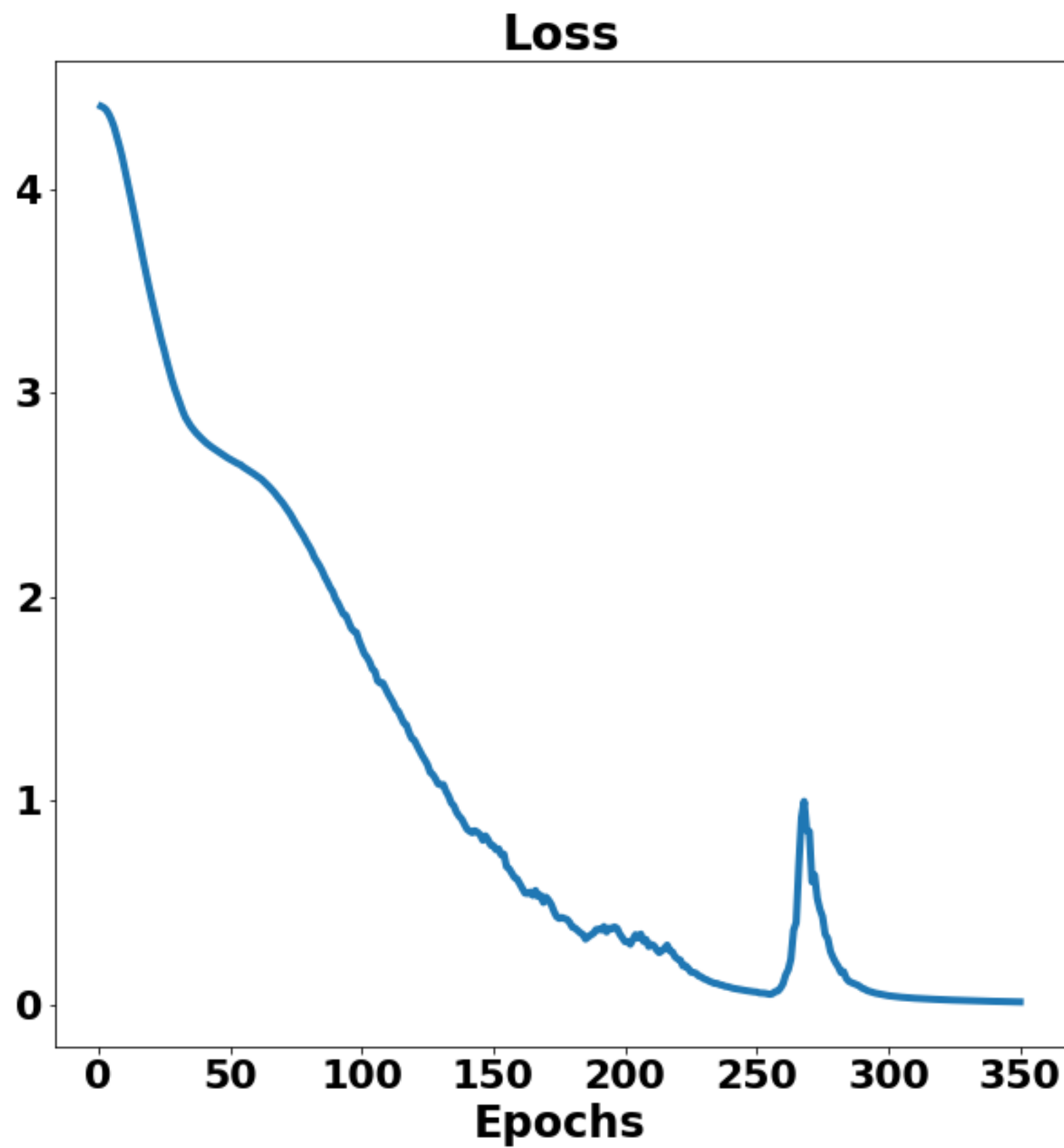
Compare with commercial software

	Integration (BWD)
Mathematica (30s)	84.0
Matlab	65.2
Maple	67.4
Beam size 1	98.4
Beam size 10	99.6
Beam size 50	99.6

Ref: Lample, Guillaume, and François Charton. "Deep learning for symbolic mathematics."

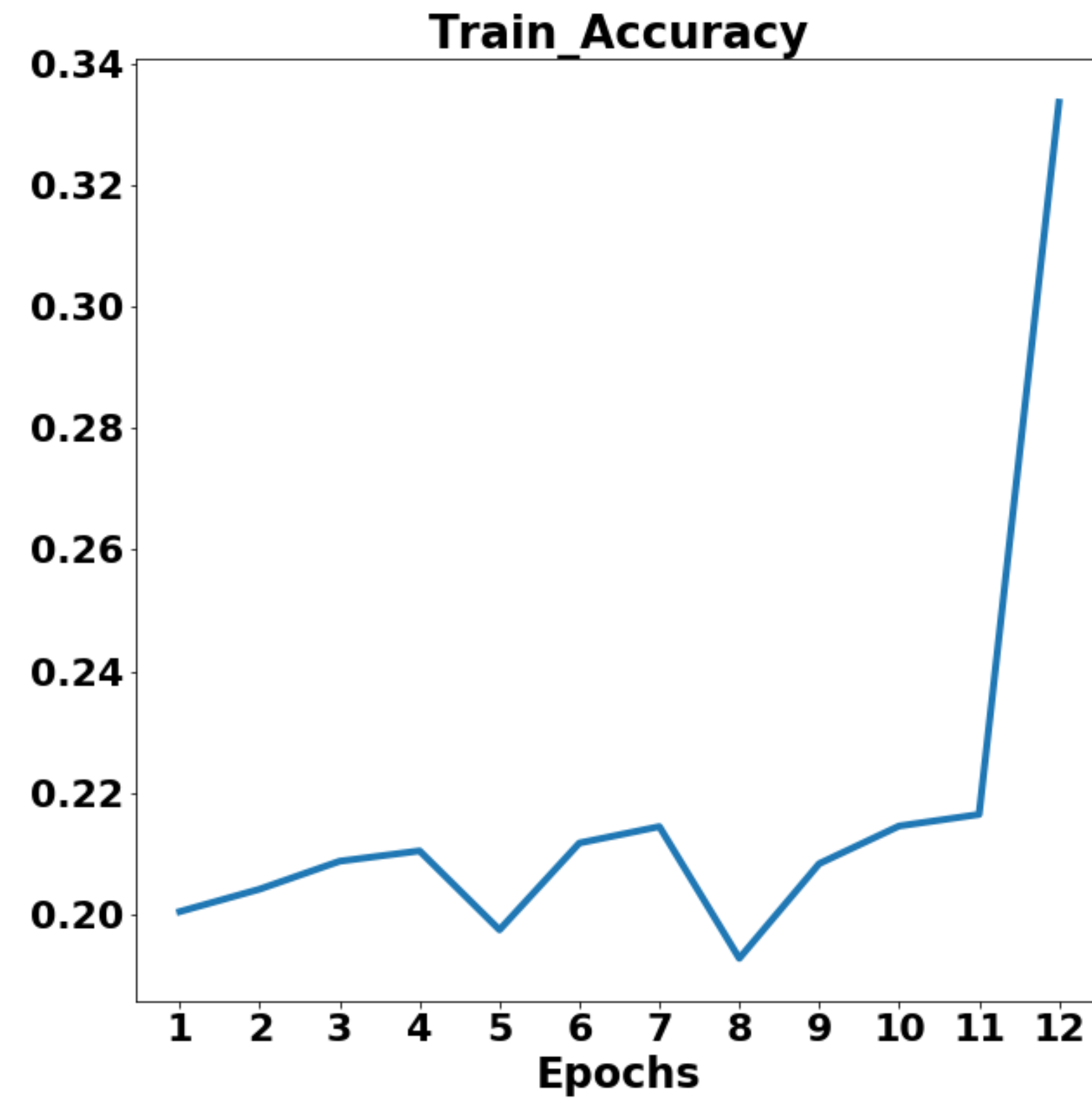
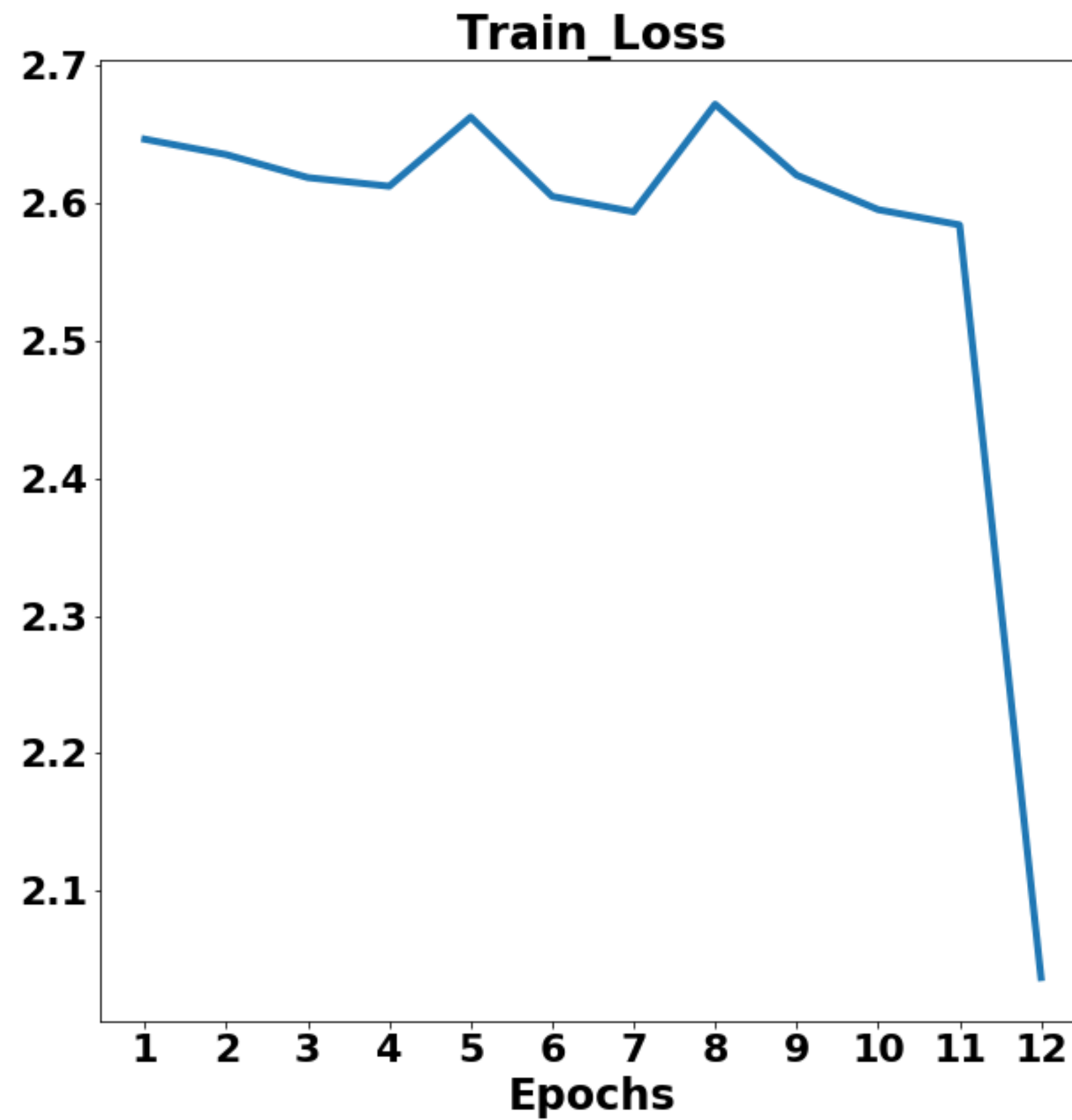
Results - Stage 1

Training set size = 64



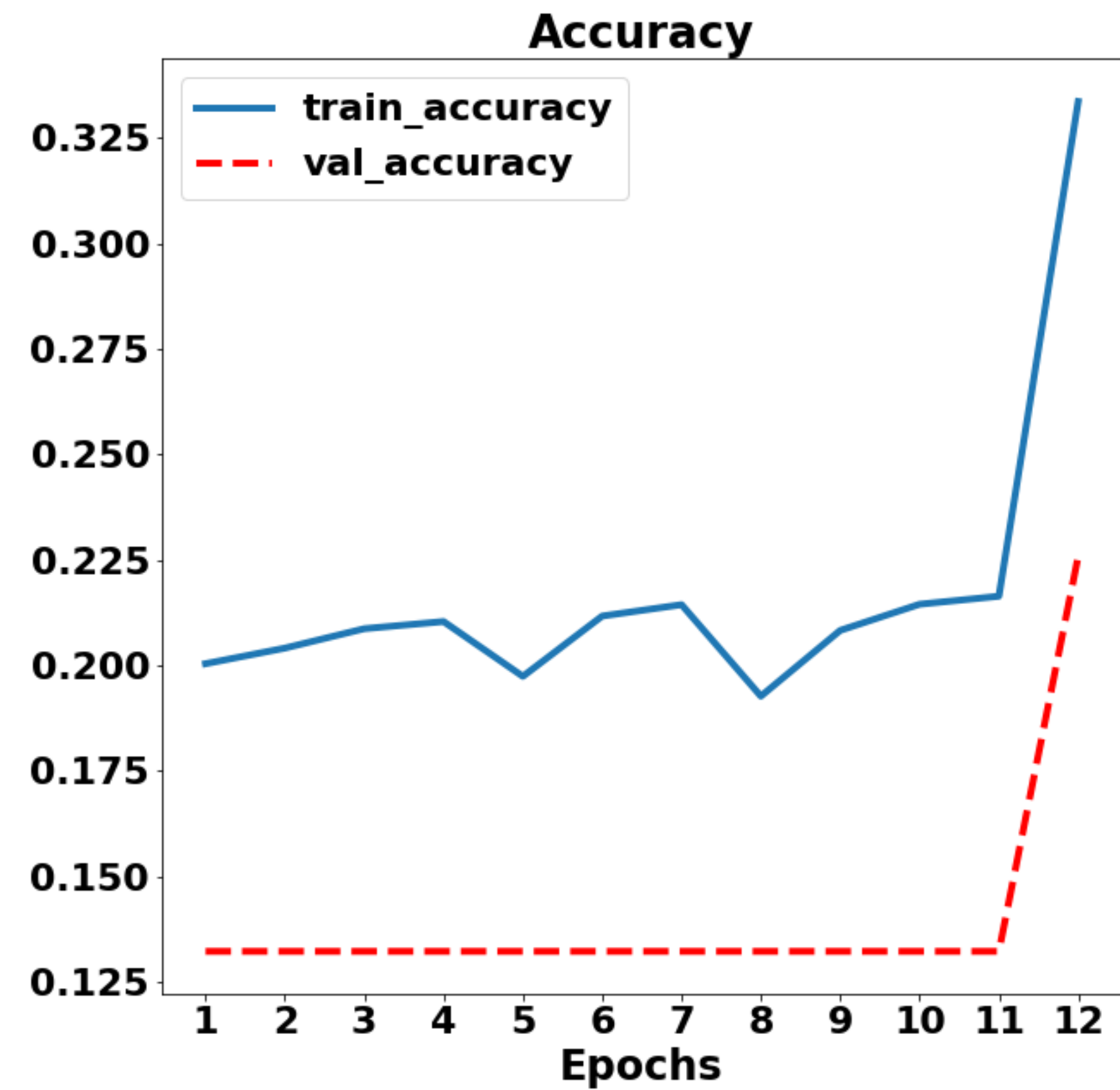
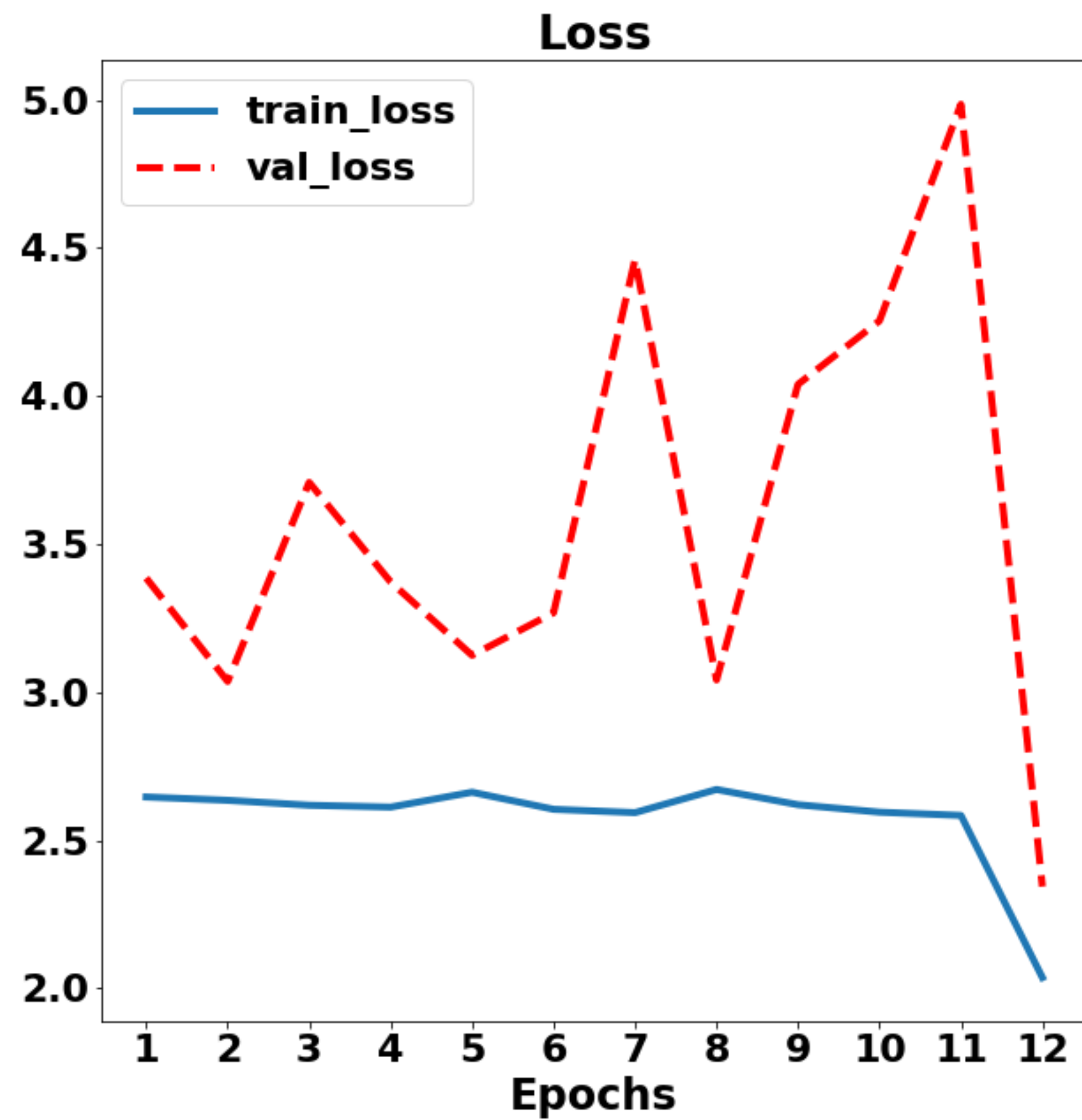
Results - Stage 2

Training set size=100,000, Validation set size=9263



Results - Stage 2

Training set size=100,000, Validation set size=9263



Outline

1. *Data Generation*

2. *Transformer Model*

3. *Results*

4. *Outlook*

Outlook

- Generate more data and use more GPUs to train the model
- Compare performance of different dataset size
- Give a lower limit of dataset size
- Generalize to complex number

Reference

1. Lample, Guillaume, and François Charton. "Deep learning for symbolic mathematics." *arXiv preprint arXiv:1912.01412* (2019).
2. Vaswani, Ashish, et al. "Attention is all you need." *Advances in neural information processing systems*. 2017.
3. Lample, Guillaume, and François Charton. "Deep learning for symbolic mathematics." Spotlight, Lunch&Learn, Author Speaking. <https://aisc.ai.science/events/2020-02-18/>
4. Jay Alammam "The Illustrated Transformer." <http://jalammar.github.io/illustrated-transformer/>

Acknowledgement



Thank you

Email: yangwj2011@gmail.com

Github: <https://github.com/janeyoung2018/symbolic-math>

LinkedIn: <https://www.linkedin.com/in/wenjuan-yang-3664405a/>