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Deep Learning for Symbolic Math

Data Science Retreat Batch 21
Wenjuan Yang

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What is Symbolic Math?

- Simple example: $$\int x dx = \frac{x^2}{2}$$ $$\int_a^b$$
- Complex example: $$\int (x^2(\tan(x)^2 + 1) + 2x\tan(x) + 1)dx = x^2\tan(x) + x$$
- More complex example:

$$\int \frac{16x^3 - 42x^2 + 2x}{(-16x^8 + 112x^7 - 204x^6 + 28x^5 - x^4 + 1)^{1/2}} dx = \sin^{-1}(4x^4 - 14x^3 + x^2)$$

Ref: Lample, Guillaume, and François Charton. "Deep learning for symbolic mathematics."

Can Deep Learning Learn Math?

- Difference

  Language: Statistic way of learning

  Math: Rule-based inference

- Similarity

  Pattern recogination

Background

Symbolic Math

Deep learning for symbolic mathematics

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Abstract

Neural networks have a reputation for being better at solving statistical or approximate problems than at performing calculations or working with symbolic data. In this paper, we show that they can be surprisingly good at more elaborated tasks in mathematics, such as symbolic integration and solving differential equations. We propose a syntax for representing mathematical problems, and methods for generating large datasets that can be used to train sequence-to-sequence models. We achieve results that outperform commercial Computer Algebra Systems such as Matlab or Mathematica.

Facebook GitHub: https://github.com/facebookresearch/SymbolicMathematics

Code released on Mar 15

Outline

1. *Data Generation*
2. *Transformer Model*
3. *Results*
4. *Outlook*

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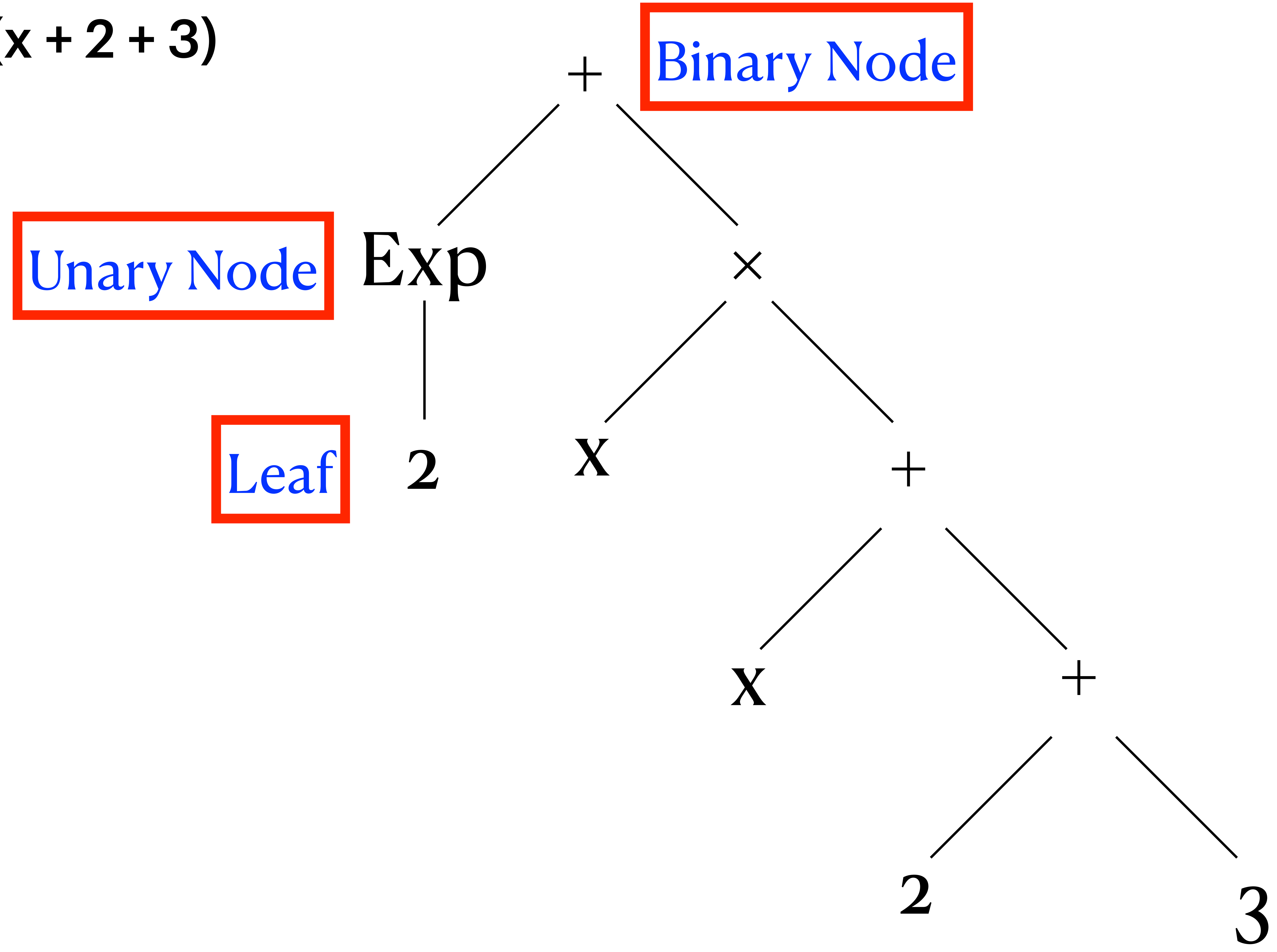
How to Generate Data?

6 STEPS

Random Tree

Example: Exp(2) + x (x + 2 + 3)



1

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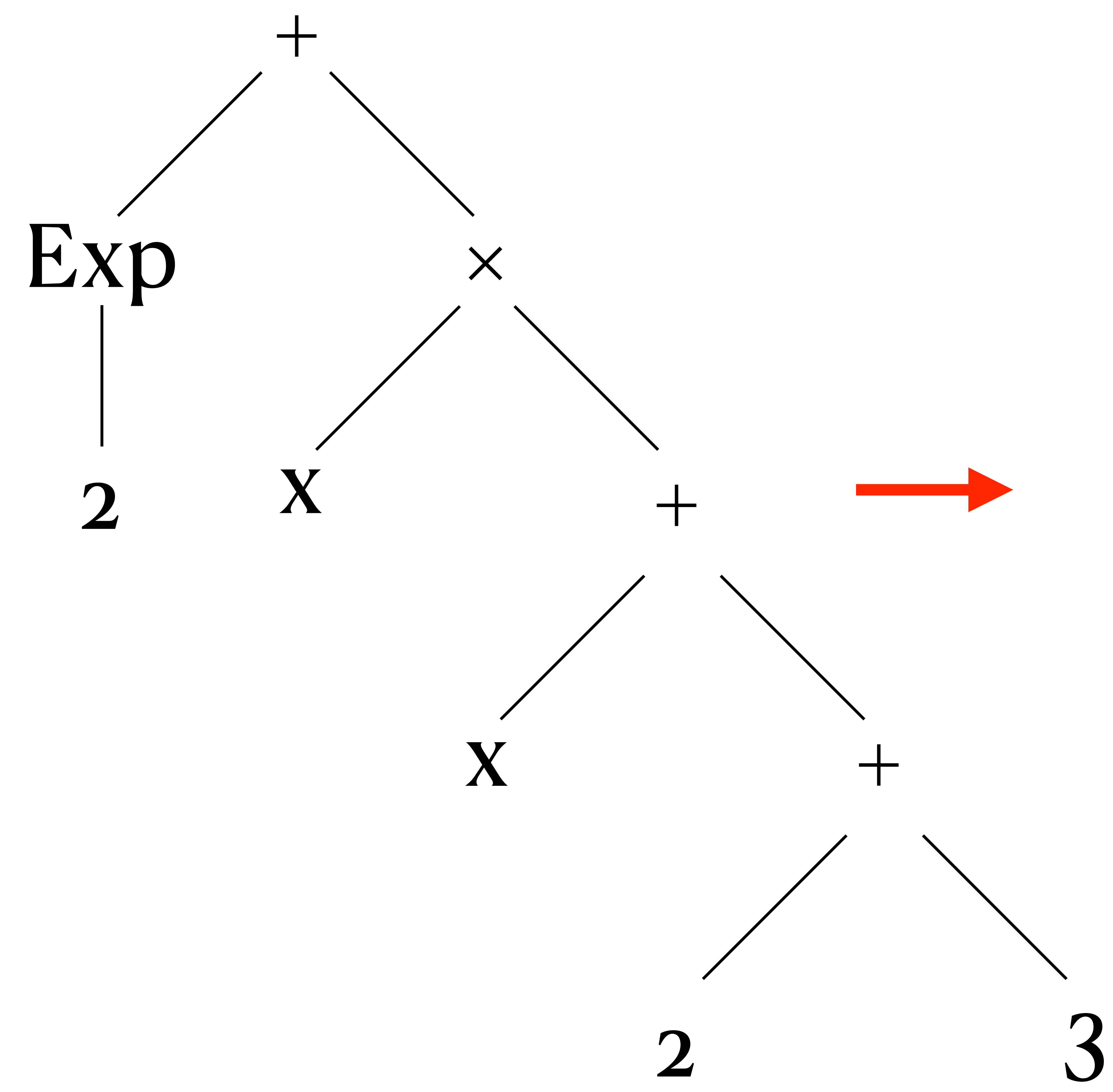
Random Tree → Prefix



Prefix

+ Exp 2 × x+ x + 2 3

Infix

Exp(2) + x (x + 2 + 3)

2

Prefix → Infix & Clean Data

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+ Exp 2 × x+ x +2 3 → Exp(2) + x × (x + 2 + 3)

∞ -∞ i ✗

3

Simplify

$\mathrm{Exp}(2) + x \times (x + 2 + 3) \longrightarrow \mathrm{Exp}(2) + x \times (x + 5)$ Infix

$\mathrm{Exp}(2) + x \times (x + 5) \longrightarrow + \ \mathrm{Exp}\ 2 \ \ \times\ x + x\ 5$ Prefix

Output

Differentiate

$$\text{Exp}(2) + x \times (x + 5) \longrightarrow 2 \times x + 5$$

Infix → Prefix

$2 \times x + 5 \longrightarrow + \times 2\ x\ 5$

Input

Generate Answer before Question

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Features(X)	Target(Y)
+ × 2 x 5	+ Exp 2 × x + x 5
Input	Output

Parallel Data Generation

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Two ways to generate data:

- Multiprocess thread pool

```
from multiprocessing.pool import ThreadPool
_FINISH = False
start = time.time()
with ThreadPool(processes=14) as p:
    out = []
    r = p.map_async(generate_bwd, 
[sequences_per_process]*process_runs, callback=out.append)
    r.wait()
    time.sleep(10)
    _FINISH = True    p.terminate()
```

- Ray

```
ray.init(num_cpus=cpu)
dataset = []
for _ in range(process_runs*cpu):
    try:
        out = ray_generate_bwd.remote(sequences_per_process)
        out = ray.get(out, timeout=sequences_per_process)
        dataset.extend(out)
```

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Transformer Model

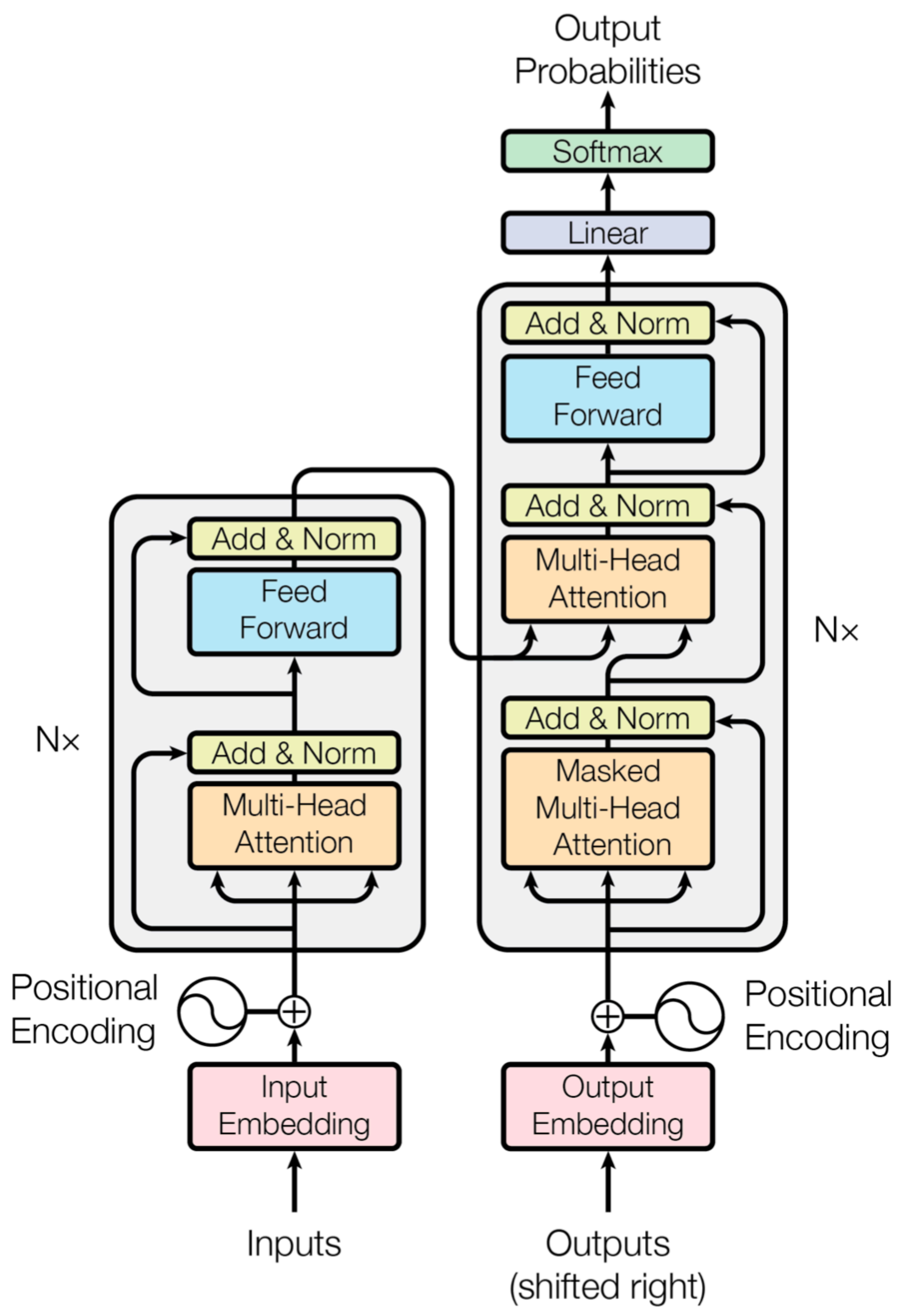

Figure from Vaswani, Ashish, et al. "Attention is all you need."(2017)

Transformer Model

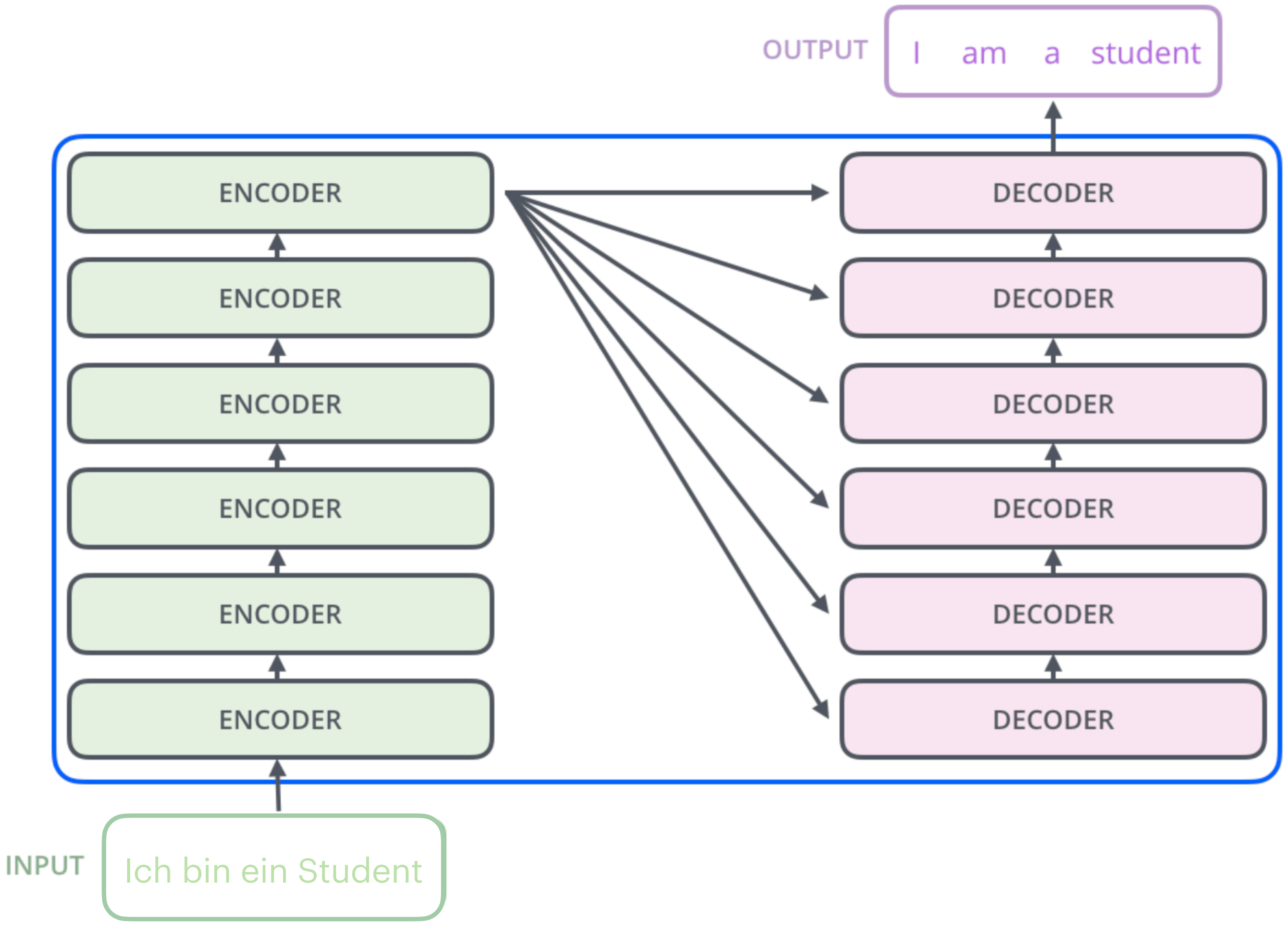

Figure from Jay Alammar http://jalammar.github.io/illustrated-transformer/

Transformer Model

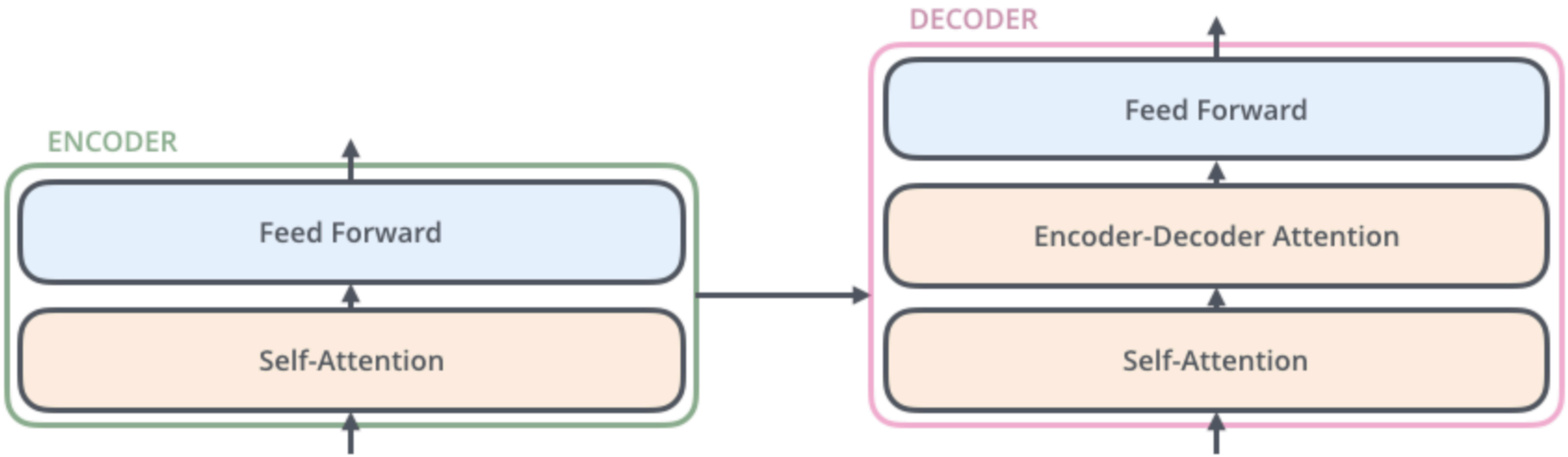

Figure from Jay Alammar http://jalammar.github.io/illustrated-transformer/

Transformer Model

Hyperparameters

batch_size = 32 d_model = 512

sequence_length = 512 num_layers = 6

num_heads = 8 dropout_rate = 0.1
optimizer = Adam learning_rate =CustomSchedule

loss, accuracy = CustomSchedule

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Paper Results

Training set size = 40M, Test set size = 5000

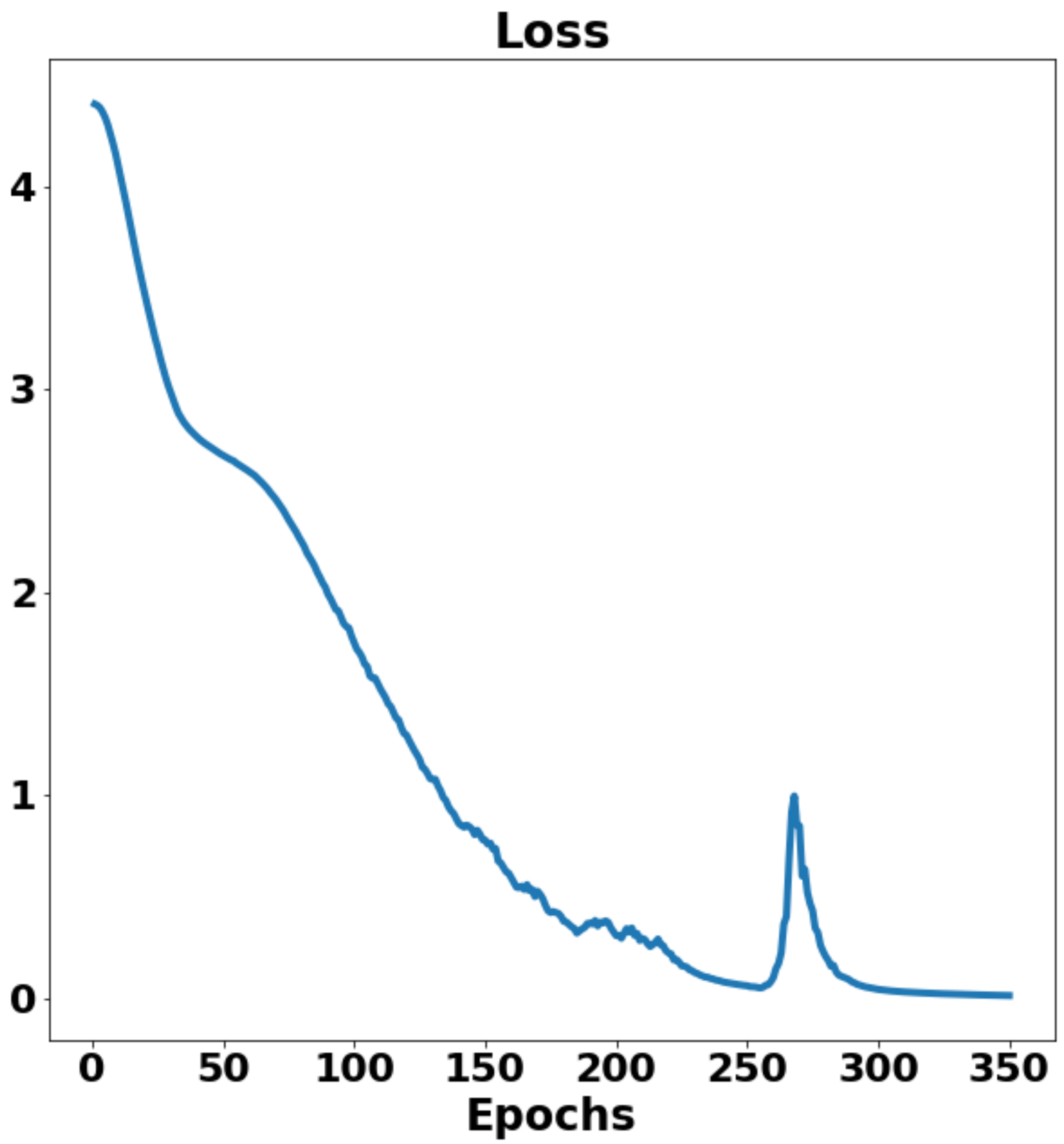
Compare with commercial software

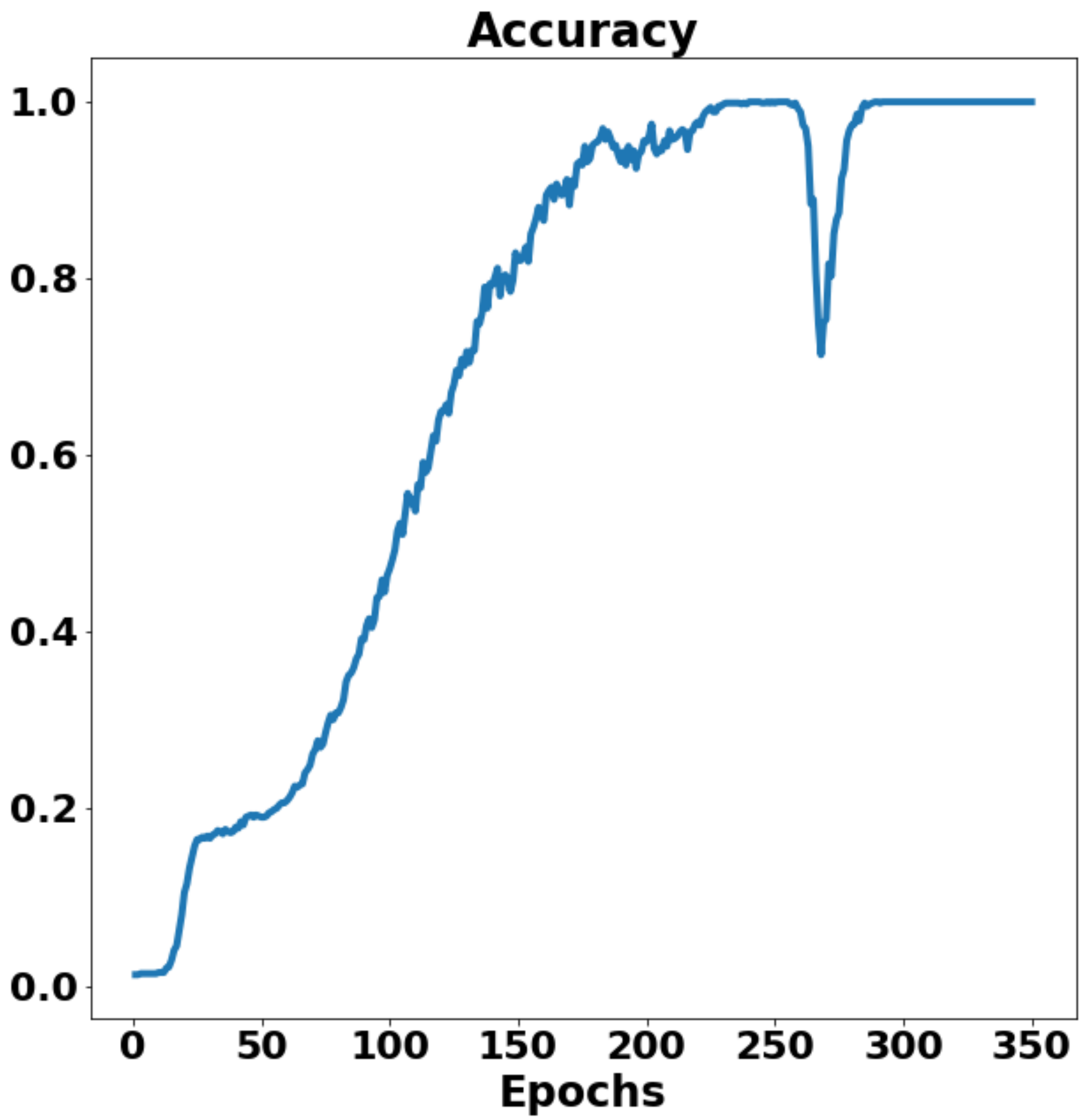
	Integration (BWD)
Mathematica (30s)	84.0
Matlab	65.2
Maple	67.4
Beam size 1	98.4
Beam size 10	99.6
Beam size 50	99.6

Ref: Lample, Guillaume, and François Charton. "Deep learning for symbolic mathematics."

Results - Stage 1

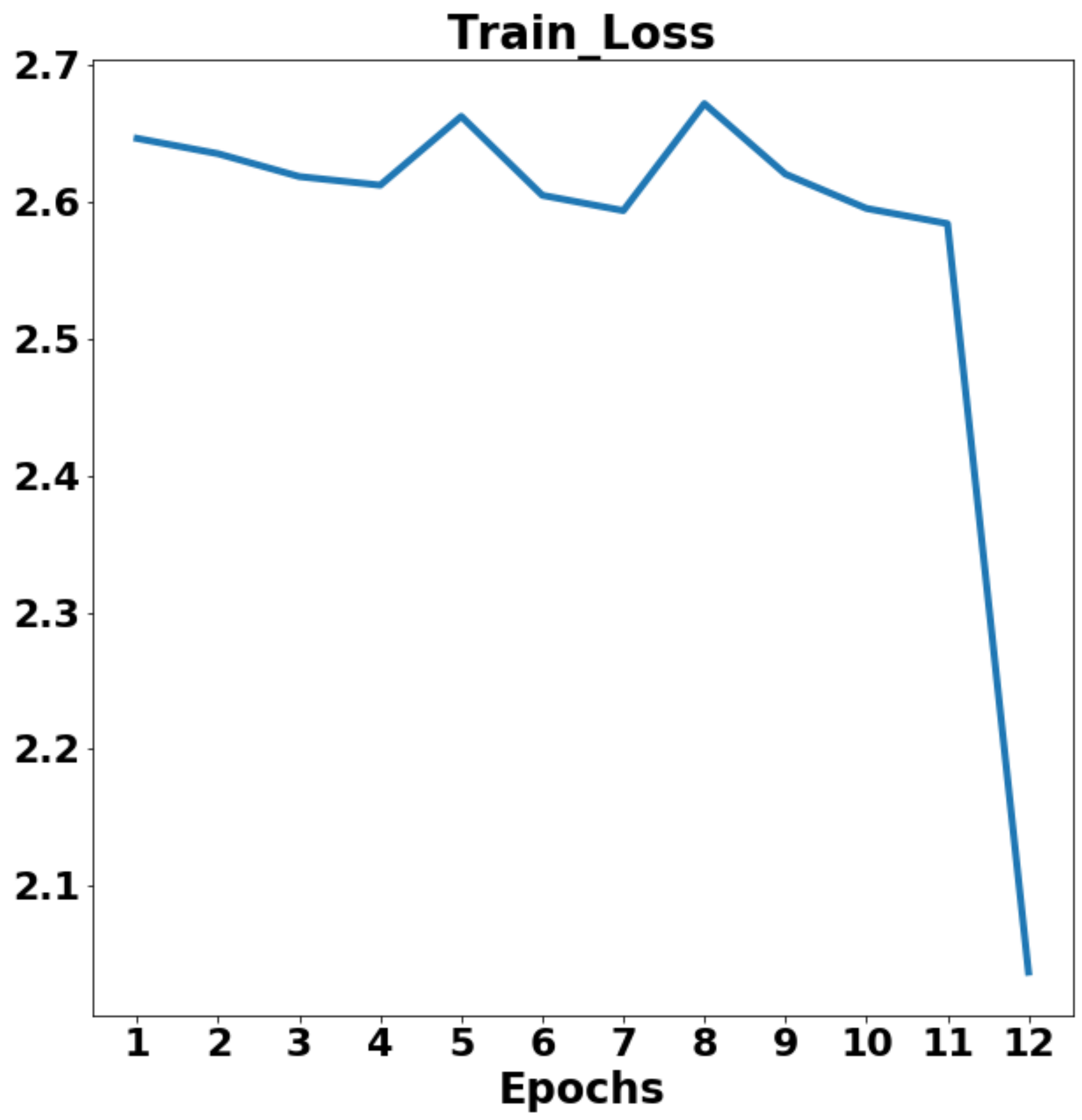
Training set size = 64

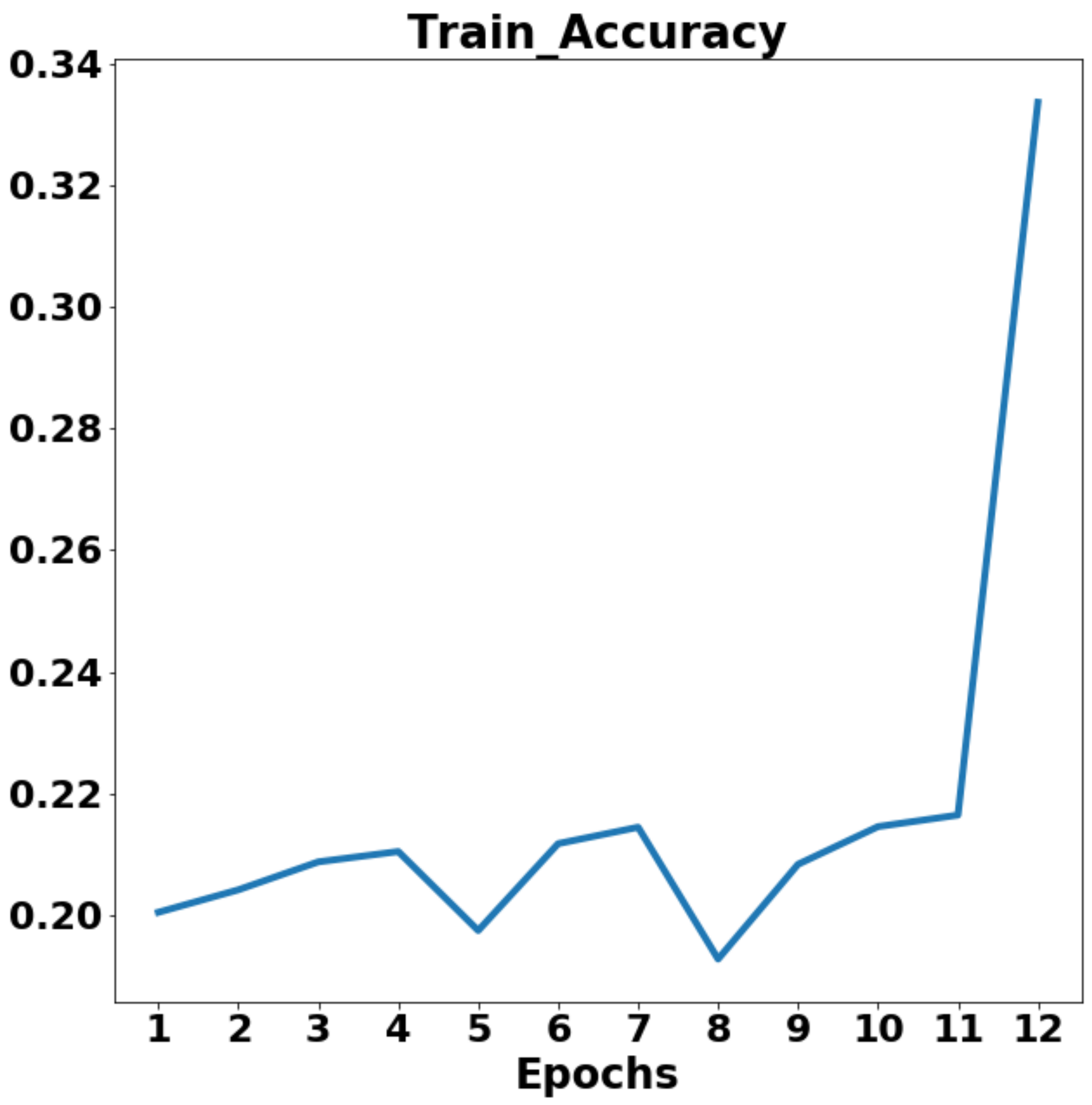




Results - Stage 2

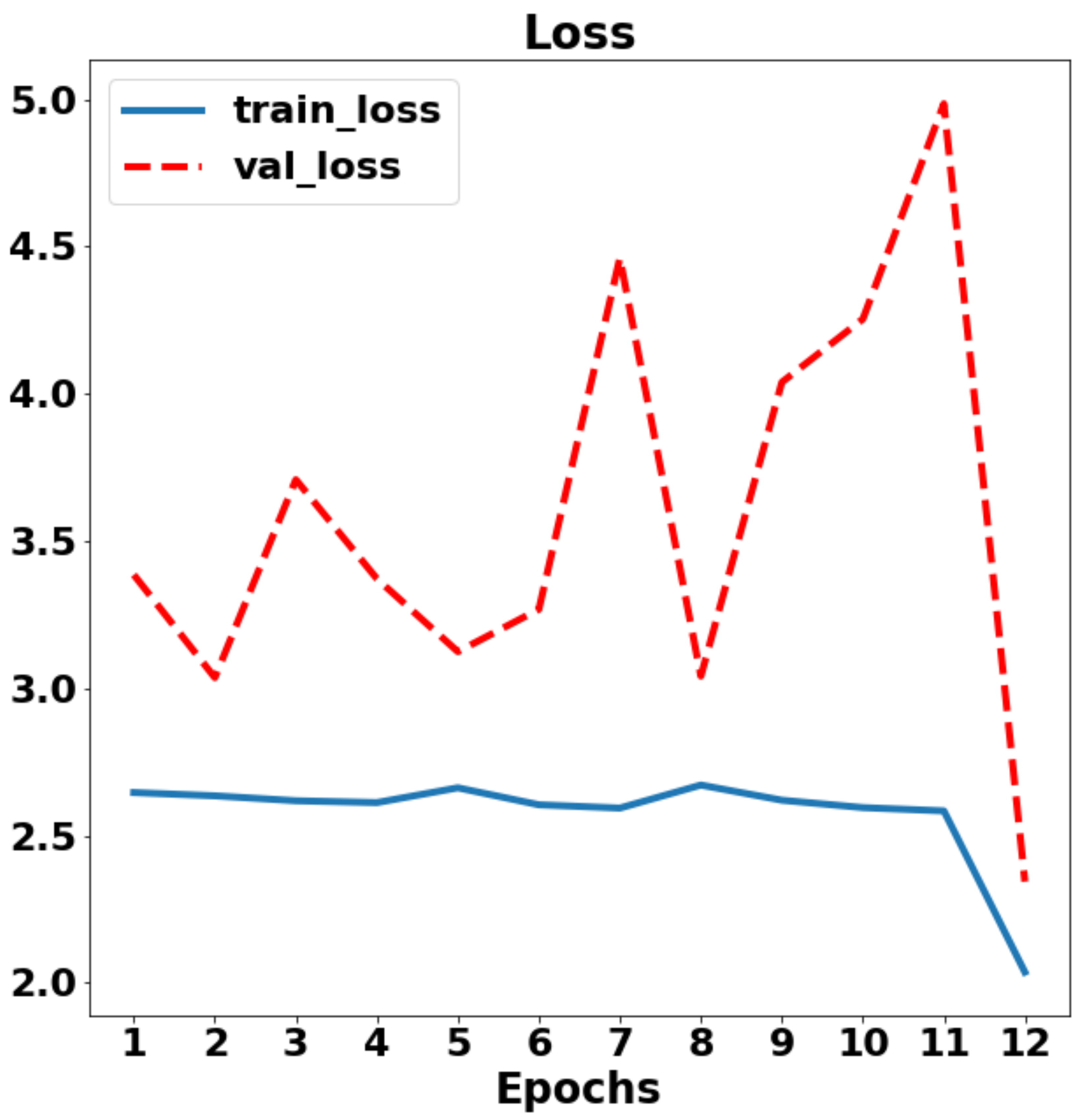
Training set size=100,000, Validation set size=9263

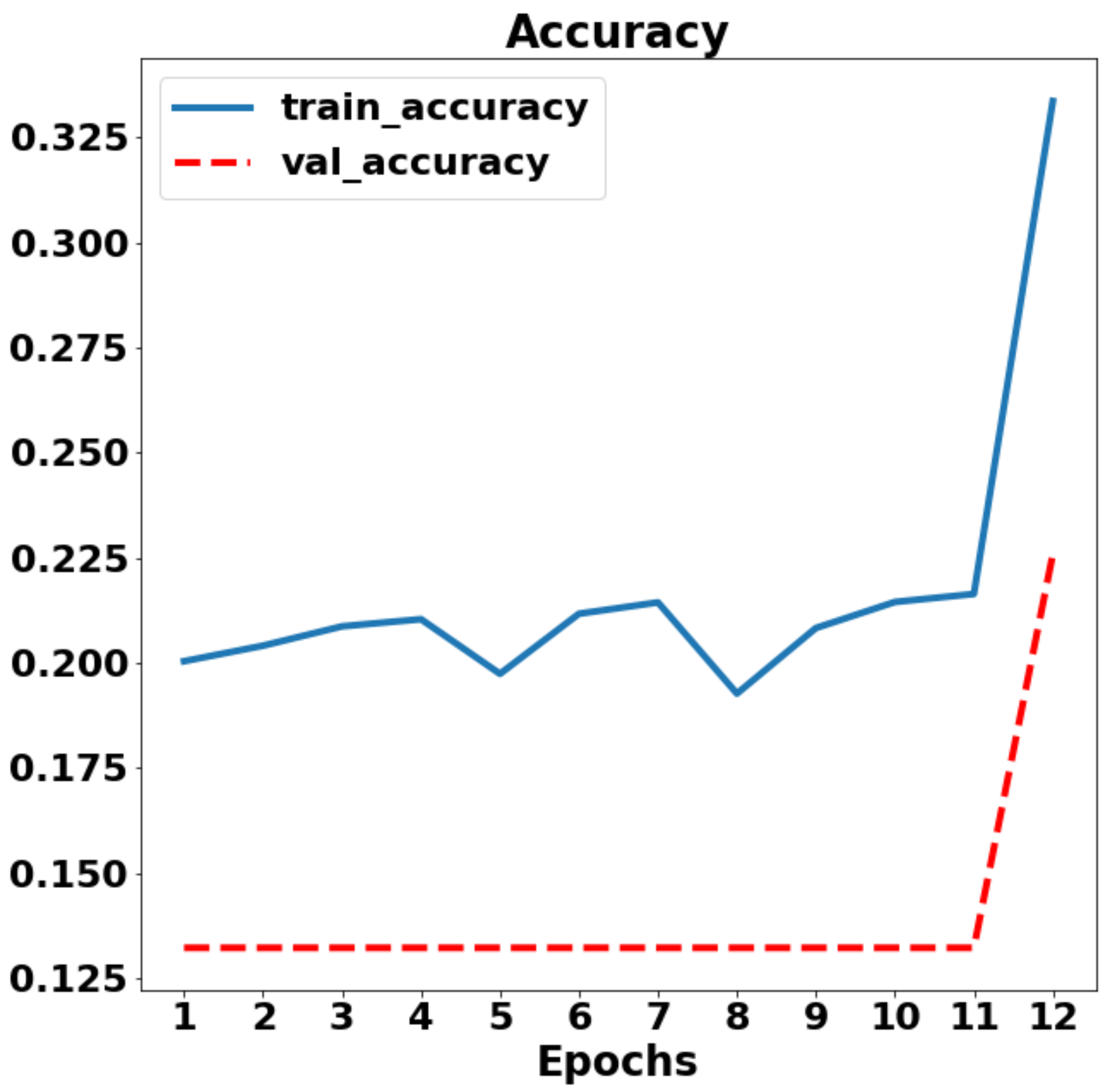




Results - Stage 2

Training set size=100,000, Validation set size=9263





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Outlook

- Generate more data and use more GPUs to train the model
- Compare performance of different dataset size
- Give a lower limit of dataset size
- Generalize to complex number

Reference

1. Lample, Guillaume, and François Charton. "Deep learning for symbolic mathematics." *arXiv preprint arXiv:1912.01412* (2019).
2. Vaswani, Ashish, et al. "Attention is all you need." *Advances in neural information processing systems*. 2017.
3. Lample, Guillaume, and François Charton. "Deep learning for symbolic mathematics." Spotlight, Lunch&Learn, Author Speaking. https://aisc.ai.science/events/2020-02-18/
4. Jay Alammar "The Illustrated Transformer." http://jalammar.github.io/illustrated-transformer/

Acknowledgement

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Thank you

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