# Assignment 3

STAT314/STAT461

Set: Tue, Aug-24. Due: Fri Sept-17

**Please type everything in either Word or LaTex, and submit it as a PDF file via Learn. No handwritten submissions! (And no scans of handwritten submissions, please).**

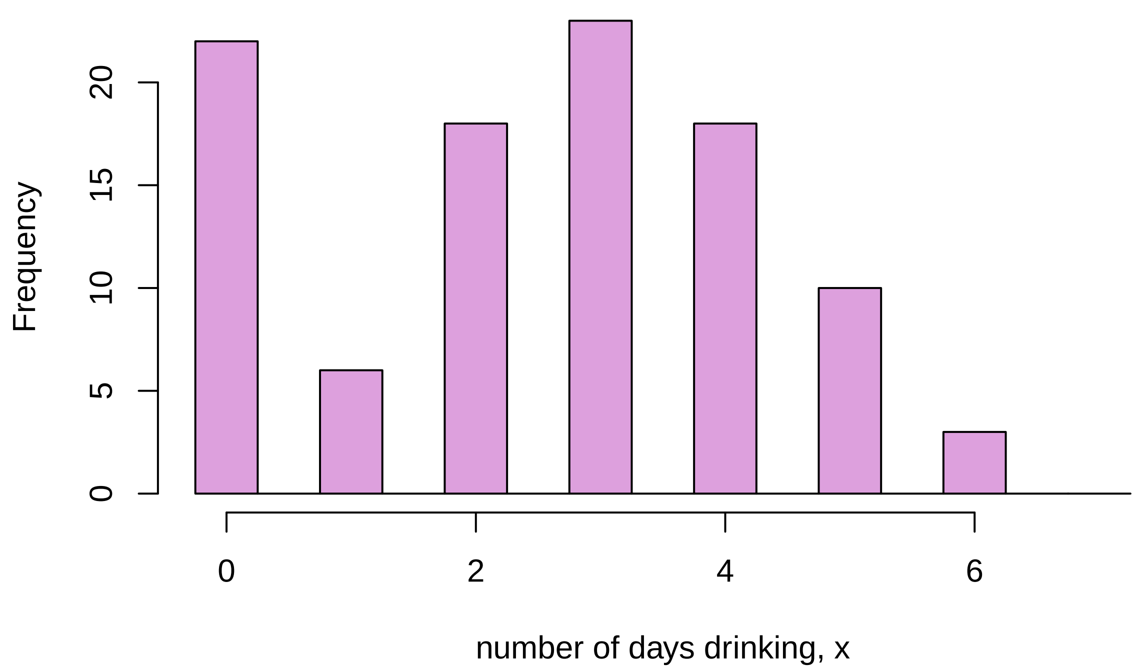
**Show your workings: equations for theoretical problems; code and (relevant) output for the computational problems. It is not sufficient to report an answer. Don't forget to include intermediate steps and explain your way of thinking. You may get points for thinking in the right direction even if you don't get the answer exactly right.**

**If you need an extention, please ask for it in advance. Late submissions will not be accepted.**

## Problem 1.

Let's get back to the drinkers and teetotallers data.

```
x <- rep(0:6,c(22,6,18,23,18,10,3))
hist(x,seq(-.25,7.25,.5),col='plum',xlab='number of days drinking, x',main='')
```



Consider the zero-inflated binomial model. Let $z_i = 1$ if the person $i$ drinks alcohol at least sometimes, and $z_i = 0$ otherwise. And let $x_i$ be the number of days in the past week the person $i$ reported as alcohol drinking days.

$$Pr(x_i = 1|z_i = 0) = 1 - Pr(x_i = 0|z_i = 0) = 0$$

$$x_i|z_i = 1 \sim \text{BIN}(7, p)$$

and the prior for $p$:

$$p \sim Beta(a, b)$$

for some prior parameters $a$ and $b$.

Furthermore, assume that $z_i$ has a further Bernoulli distribution with probability $\omega$:

$$z_i \sim \text{Bern}(\omega),$$

and that the probability parameter $\omega$ also has a beta prior (with parameters different from those for the prior of $p$):

$$\omega \sim Beta(a_\omega, b_\omega)$$

(a) Derive the posterior conditional distribution of $z_i$ given $\omega$ and $x_i$.

Hint: $z_i$ can only take two values, so it is enough to evaluate the $Pr(z_i = 1|x_i, \omega, p)$. The solution for $x_i > 0$ should be obvious. So, the only remaining challenge is to find $Pr(z_i = 1|x_i = 0, \omega, p)$. Use Bayes' formula. **4pt**

(b) Run the zero-inflated and the standard binomial model using WinBUGS. Calculate the DIC for both models. Which model is a better fit statistically? **3pt**

## Problem 2.

You have two coins in your pocket. One is a standard fair coin, so that P(heads)=0.50. The other one is a two-sided coin, so that P(heads)=1. You draw one of the two coins at random and toss it twice. The result is "heads" and "heads".

(a) Use Bayes' formula to obtain the probability that you have selected the two-sided coin, given your observations. **1pt**

(b) How many consecutive "heads" do you need to observe to be at least 90% certain that the coin is two-sided? **1pt**

(c) Consider a classical approach to testing the null hypothesis: the coin is fair. Let the number of "heads" observed after two tosses be the test statistic. What is the associated p-value. (Hint: look up the definition of the classical p-value.) **1pt**