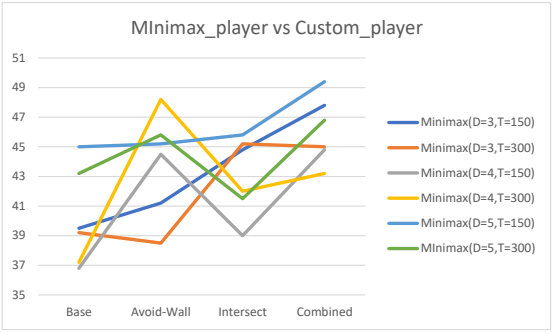
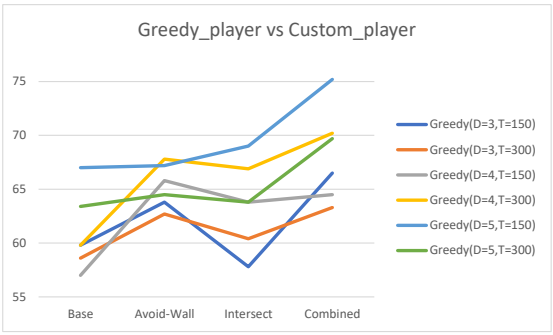
Heuristic	Minimax	Greedy	Depth limit	Time limit
Base	win rate(%)	win rate(%)		(ms)
	39.5	59.8	3	150
	36.8	57	4	150
	45	67	5	150
	39.2	58.6	3	300
	37.2	59.8	4	300
	43.2	63.4	5	300
Avoid-Wall	win rate(%)	win rate(%)		(ms)
	41.2	63.8	3	150
	44.5	65.8	4	150
	45.2	67.2	5	150
	38.5	62.7	3	300
	48.2	67.8	4	300
	45.8	64.5	5	300
Intersect	win rate(%)	win rate(%)		(ms)
	44.8	57.8	3	150
	39	63.8	4	150
	45.8	69	5	150
	45.2	60.4	3	300
	42	66.9	4	300
	41.5	63.8	5	300
Combined	win rate(%)	win rate(%)		(ms)
	47.8	66.5	3	150
	44.8	64.5	4	150
	49.4	75.2	5	150
	45	63.3	3	300
	43.2	70.2	4	300
	46.8	69.7	5	300





Advanced Heuristic
- What features of the game does your heuristic incorporate, and why do you think those features matter in evaluating states during search?
- Analyze the search depth your agent achieves using your custom heuristic. Does search speed matter more or less than accuracy to the performance of your heuristic?

For this project, I decided to implement the advanced heuristics. I ran 100 rounds of fair games with depth of 3, 4 and 5, and with time limit of 150 ms and 300 ms on each of base, avoid-wall, intersect and combined heuristics. I chose to run fair games, meaning both player and computer get to play set of rounds where they are they have the first move becuase, I believe that there is advantage to either first player or the second player.
First approach I took was to stay away from the wall. Through some research, I found out that going near the wall limits player's move.
For example, if a player is 1 unit away from a wall, the wall limits two moves of the player. If the player is right by the wall, the wall limits four moves of the player. If the player is 1 unit away from walls on both x and y coordinates, the walls limit four moves of the player. Finally if the player is at the corner of the board, the player only has maximum of three moves.
These pattern implies that the nearer the player is to walls, the more likely that the player is going to lose. The result, as shown above, aligns with my theory. Although there is one case where avoid-wall heuristic played worse than the base heuristic, in general avoid-wall heuristic shows better performance than base heuristic.
Secon approach I took was to limit the opponent's moves. I thought if the custom player chases after the opponent blocks the opponent, eventually the player will be able to trap the opponent. Therefore, I designed a heuristic that puts more weight on a move where opponent will have lesser moves, and at the same time put even more weight on a move that intersect's with the opponent's available moves.
According to the graph, this heuristic does seem to show better performance than the base heuristic in general, but not as good performance as avoid-wall heuristic.
This led to my third approach to combine both heuristics. By combining the two heuristics, custom player tries to avoid goind near the walls and limit the opponent's moves at that same time. Based on the result, this approach shows the best performance in general.

The result of this experiment was quite different than my expectations. I thought the alpha-beta pruning with iterative deepening method would perform much better than the minimax player even with the base heuristic, because alpha-beta method is more efficient meaning it can go deeper into the game. However, the win rate was much lower than 50% even with advanced heuristics. While the use of advanced heuristics showed better performance, it still had win rate that was lower than 50%. On the other hand, against the greedy player, the custom player showed much better performance.
According to the graph, the deeper the search depth, the better the result is in general. However, because this is small sized sample, there times that the result if greatly off the pattern. Other thing to note is that the increased time limit does not seem to generate much different result. The win rate did not change much at all even with longer time limit. However, because when depth limit was bigger the win rate was higher, as time limit gets longer allowing the depth limit to be deeper, the win rate of custom player will get higher.