Changxv's Contest Template Library

Mofish Automaton

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Contest (1)

template.cpp

```cpp
#include "bits/stdc++.h"
#define sz(x) (int)(x).size()
#define all(x) begin(x), end(x)
#define vc vector

using namespace std;
using ll = int64_t;

int main() {
  cin.tie(0)->sync_with_stdio(0);

}
```

debug.cpp

```cpp
template<class U, class T>
enable_if_t<is_same_v<U, ostream>, U&>
operator << (U &os, T v) {
  os << "{ ";
  for (auto e: v) os << e << ' ';
  return os << '}';
}
ostream &operator << (ostream &os, pair<int, int> p) {
  return os << '(' << p.first << ", " << p.second << ')';
}
void dbg() { cerr << "\n"; }
template<class T, class ...U>
void dbg(T e, U ...a) { cerr << e << ' '; dbg(a...); }
#define err(A...) cerr << #A ": ", dbg(A)
```

check.sh

```sh
#!/bin/bash
while true; do
  ./gen
  ./a > myout.txt
  ./b > brute.txt
  diff myout.txt brute.txt
  if [ $? -ne 0 ]; then break; fi
  echo AC
done
```

Makefile

```
CXX = g++
CXXFLAGS = -O2 -std=c++17 -Wall -Wextra -Wconversion -Wno-
    unused-result -pedantic -Wshadow -Wformat=2 -Wfloat-equal
    -Wlogical-op -Wshift-overflow=2 -Wduplicated-cond -Wcast-
    qual -Wcast-align
DEBUGFLAGS = -fsanitize=undefined -fsanitize=address -fno-
    sanitize-recover=all -fstack-protector -D_GLIBCXX_DEBUG -
    D_GLIBCXX_DEBUG_PEDANTIC -D_FORTIFY_SOURCE=2
CXXFLAGS += $(DEBUGFLAGS)
```

.vimrc

```
set hls ic is scs ru nu ls=2 so=7 bs=2 ai cin noswf aw ar sr ts
    =2 sw=2 mouse=a
syn on
no ; :
nn U <c-r>
no <bs> :noh<cr>
no <space>a ggVG
no <f2> :vs in.txt<cr>
no <f9> :!make %:r<cr>
no <f5> :!time ./%:r < in.txt<cr>
ca Hash w !cpp -dD -P -fpreprocessed \| tr -d '[:space:]' \|
    md5sum \| cut -c-8
```

troubleshoot.txt

```
Pre-submit:
Write a few simple test cases if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all data structures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a teammate.
Ask the teammate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a teammate do it.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your teammates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need?
Are you clearing all data structures between test cases?
```

Mathematics (2)

2.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned} \Rightarrow \begin{aligned} x &= \frac{ed - bf}{ad - bc} \\ y &= \frac{af - ec}{ad - bc} \end{aligned}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A_i'}{\det A}$$

where A_i' is A with the i'th column replaced by b.

2.2 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k - c_1 x^{k-1} - \cdots - c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.3 Trigonometry

$$\begin{aligned} \sin(v + w) &= \sin v \cos w + \cos v \sin w \\ \cos(v + w) &= \cos v \cos w - \sin v \sin w \\ \tan(v + w) &= \frac{\tan v + \tan w}{1 - \tan v \tan w} \end{aligned}$$

$$\sin v + \sin w = 2\sin\frac{v+w}{2}\cos\frac{v-w}{2}$$
$$\sin v - \sin w = 2\cos\frac{v+w}{2}\sin\frac{v-w}{2}$$
$$\cos v + \cos w = 2\cos\frac{v+w}{2}\cos\frac{v-w}{2}$$
$$\cos v - \cos w = -2\sin\frac{v+w}{2}\sin\frac{v-w}{2}$$

$$(V+W)\tan(v-w)/2 = (V-W)\tan(v+w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$a\cos x + b\sin x = r\cos(x-\phi)$$
$$a\sin x + b\cos x = r\sin(x+\phi)$$

where $r = \sqrt{a^2+b^2}, \phi = \text{atan2}(b, a)$.

2.4 Geometry

2.4.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \dfrac{a+b+c}{2}$

Area: $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R = \dfrac{abc}{4A}$

Inradius: $r = \dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2+2c^2-a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc\left[1-\left(\frac{a}{b+c}\right)^2\right]}$$

Law of sines: $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c} = \dfrac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Law of tangents: $\dfrac{a+b}{a-b} = \dfrac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$

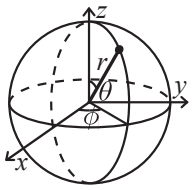
2.4.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef\cdot\sin\theta = F\tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

2.4.3 Spherical coordinates



$$x = r\sin\theta\cos\phi \qquad r = \sqrt{x^2+y^2+z^2}$$
$$y = r\sin\theta\sin\phi \qquad \theta = \text{acos}(z/\sqrt{x^2+y^2+z^2})$$
$$z = r\cos\theta \qquad \phi = \text{atan2}(y, x)$$

2.4.4 Pick's theorem

B = number of lattice points on the boundary of the polygon.

I = number of lattice points in the interior of the polygon.

$$Area = \frac{B}{2} + I - 1$$

2.5 Derivatives/Integrals

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\tan x = 1+\tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$
$$\int\tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x\sin ax = \frac{\sin ax - ax\cos ax}{a^2}$$
$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\text{erf}(x) \qquad \int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

2.6 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1}-c^a}{c-1}, c\neq 1$$

$$1^2+2^2+3^2+\cdots+n^2 = \frac{n(2n+1)(n+1)}{6}$$
$$1^3+2^3+3^3+\cdots+n^3 = \frac{n^2(n+1)^2}{4}$$
$$1^4+2^4+3^4+\cdots+n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

2.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, (-\infty < x < \infty)$$

2.8 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x xp_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x-\mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX+bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX+bY) = a^2V(X) + b^2V(Y).$$

2.8.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\text{Bin}(n, p)$, $n = 1, 2, \ldots, 0 \leq p \leq 1$.

$$p(k) = \binom{n}{k}p^k(1-p)^{n-k}$$

$$\mu = np,\ \sigma^2 = np(1-p)$$

$\text{Bin}(n, p)$ is approximately $\text{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each wich yields success with probability p is $\text{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1},\ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p},\ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\text{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda,\ \sigma^2 = \lambda$$

2.8.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\text{U}(a, b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2},\ \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\text{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda},\ \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.9 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let $X_1, X_2, \ldots$ be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P} = (p_{ij})$, with $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$, and $\mathbf{p}^{(n)} = \mathbf{P}^n\mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)} = \Pr(X_n = i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

π is a stationary distribution if $\pi = \pi\mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i = \frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i. π_j/π_i is the expected number of visits in state j between two visits in state i.

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k\to\infty}\mathbf{P}^k = \mathbf{1}\pi$.

A Markov chain is an A-chain if the states can be partitioned into two sets $\mathbf{A}$ and $\mathbf{G}$, such that all states in $\mathbf{A}$ are absorbing ($p_{ii} = 1$), and all states in $\mathbf{G}$ leads to an absorbing state in $\mathbf{A}$. The probability for absorption in state $i \in \mathbf{A}$, when the initial state is j, is $a_{ij} = p_{ij} + \sum_{k\in\mathbf{G}} a_{ik}p_{kj}$. The expected time until absorption, when the initial state is i, is $t_i = 1 + \sum_{k\in\mathbf{G}} p_{ki}t_k$.

Data structures (3)

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change `null_type`.
Time: $\mathcal{O}(\log N)$

1ee20c74, 24 lines

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

/* basic */
begin(), end(), size(), empty(), clear()
insert(pair<key, T>), erase(iter), erase(key), operator[]
find(key), lower_bound(key), upper_bound(key)

/* advanced */
order_of_key(key); // number of element less than key

// return the iter of order+1, end() if too large
find_by_order(size_type order);

// join the other if their ranges don't intersect
join(tree &other);

// split tr and overide other with element larger than key
tr.split(key, tree &other);
```

HashMap.h

Description: Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

08664751, 14 lines

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/hash_policy.hpp>
struct chash {
  const uint64_t C = ll(4e18 * acos(0)) | 71;
  const int RANDOM = chrono::high_resolution_clock::now().
      time_since_epoch().count();
  ll operator()(ll x) const {
    return __builtin_bswap64((x^RANDOM)*C);
  }
};
template<class K, class V> using ht = __gnu_pbds::gp_hash_table
    <K,V,chash>;
template<class K, class V> V get(ht<K, V> &u, K x) {
  auto it = u.find(x);
  return it == end(u) ? 0 : it->s;
}
```

Rope.h

Description: STL BST
Time: $\mathcal{O}(\log N)$

3bbc1517, 16 lines

```
#include <ext/rope>
using namespace __gnu_cxx;
rope<T> rp;
crope rp; // same for rope<char>

/* basic */
size(), operator[]
push_back(key);
insert(pos, x);
erase(pos, x);
replace(pos, x);
substr(pos, x);

/* advanced */
rope<int> *his[N];
his[i] = new rope<int> (*his[i - 1]);
```

SegmentTree.h

Description: Zero-indexed segment tree. Bounds are inclusive to the left and exclusive to the right.
Time: $\mathcal{O}(\log N)$

dea81b51, 49 lines

```
struct Seg {
  int n, h;
  vc<int> s, add;
  Seg(int n): n(n), h(32 - __builtin_clz(n)), s(n * 2),
              add(n) {}
  void update(int x,int v, int l) {
    s[x] += v * l;
    if (x < n) add[x] += v;
  }
  void push(int l, int r) {
    int b = h, k = 1 << (h - 1);
    for (l += n, r += n - 1; b > 0; --b, k /= 2)
      for (int x = l >> b; x <= r >> b; ++x) if (add[x]) {
        update(x * 2, add[x], k);
        update(x * 2 + 1, add[x], k);
        add[x] = 0;
      }
  }
  void pull(int l, int r) {
    int k = 2;
    for(l += n, r += n - 1; l > 1; k *= 2) {
      l /= 2, r /= 2;
      for (int x = r; x >= l; --x) {
        s[x] = s[x * 2] + s[x * 2 + 1] + add[x] * k;
      }
    }
  }
  void modify(int l, int r, int v) { // hash-1
    push(l, l + 1);
    push(r - 1, r);
    int l0 = l, r0 = r, k = 1;
    for (l += n, r += n; l < r; l /= 2, r /= 2, k *= 2) {
      if (l & 1) update(l++, v, k);
      if (r & 1) update(--r, v, k);
    }
    pull(l0, l0 + 1);
```

```
    pull(r0 - 1, r0);
  } // hash—1 = 3fc22174
  int query(int l, int r) {
    push(l, l + 1);
    push(r - 1, r);
    int res = 0;
    for (l += n, r += n; l < r; l /= 2, r /= 2) {
      if (l & 1) res += s[l++];
      if (r & 1) res += s[--r];
    }
    return res;
  }
};
```

LazySegmentTree.h

Description: Segment tree with ability to add or set values of large intervals, and compute max of intervals. Can be changed to other things. Use with a bump allocator for better performance, and SmallPtr or implicit indices to save memory.
Usage: `Node* tr = new Node(v, 0, sz(v));`
Time: $\mathcal{O}(\log N)$.

"../various/BumpAllocator.h" 86334bd5, 52 lines

```
constexpr int INF = numeric_limits<int>::max() / 2;
struct Seg {
  Seg *ls = 0, *rs = 0;
  int l, r, mset = INF, madd = 0, val = -INF;
  Seg(int l,int r): l(l), r(r){}
  Seg(vc<int> &v, int l, int r): l(l), r(r) {
    if (l + 1 < r) {
      int mi = l + (r - l)/2;
      ls = new Seg(v, l, mi), rs = new Seg(v, mi, r);
      pull();
    } else val = v[l];
  } // 972dfe1b
  void pull() { val = max(ls->val, rs->val); }
  void push() {
    if (!ls) {
      int mi = l + (r - l)/2;
      ls = new Seg(l, mi), rs = new Seg(mi, r);
    }
    if (mset != INF)
      ls->set(l,r,mset), rs->set(l,r,mset), mset = INF;
    else if (madd)
      ls->add(l,r,madd), rs->add(l,r,madd), madd = 0;
  }
  int query(int L, int R) {
    if (R <= l || r <= L) return -INF;
    if (L <= l && r <= R) return val;
    push();
    return max(ls->query(L, R), rs->query(L, R));
  }
  void set(int L, int R, int x) {
    if (R <= l || r <= L) return;
    if (L <= l && r <= R) mset = val = x, madd = 0;
    else {
      push();
      ls->set(L, R, x), rs->set(L, R, x);
      pull();
    }
  }
  void add(int L, int R, int x) {
    if (R <= l || r <= L) return;
    if (L <= l && r <= R) {
      if (mset != INF) mset += x;
      else madd += x;
      val += x;
    }
    else {
      push();
      ls->add(L, R, x), rs->add(L, R, x);
      pull();
    }
  }
};
```

PersistentSegmentTree.h

Description: Segment tree with persistent ability. Can be changed to other things. Use with a bump allocator for better performance, and SmallPtr or implicit indices to save memory.
Usage: `Node* tr = null; tr->insert(-INF, INF, pos, val);`
Time: $\mathcal{O}(\log N)$.

"../various/BumpAllocator.h" 7569d05b, 25 lines

```
struct Seg {
  static Seg *null;
  Seg *ls = this, *rs = this;
  int val = 0;
  Seg() {}
  Seg(int val): ls(null), rs(null), val(val) {}
  Seg(Seg *ls, Seg *rs): ls(ls), rs(rs) {
    val += ls->val;
    val += rs->val;
  }
  Seg *insert(int l, int r, int pos, int v) {
    if (pos < l || pos >= r) return this;
    if (l + 1 == r) return new Seg(val + v);
    int mi = l + (r - l) / 2;
    return new Seg(ls->insert(l, mi, pos, v),
                   rs->insert(mi, r, pos, v));
  }
  int query(Seg *p, int l, int r, int k) {
    if (l + 1 == r) return l;
    int mi = l + (r - l) / 2, cnt = ls->val - p->ls->val;
    return cnt >= k ? ls->query(p->ls, l, mi, k) :
                      rs->query(p->rs, mi, r, k - cnt);
  }
};
Seg *Seg::null = new Seg;
```

UnionFindRollback.h

Description: Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().
Usage: `int t = uf.time(); ...; uf.rollback(t);`
Time: $\mathcal{O}(\log N)$

005e7e70, 22 lines

```
struct RollbackUF {
  vc<int> e;
  vc<pair<int, int>> st;
  RollbackUF(int n) : e(n, -1) {}
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : find(e[x]); }
  int time() { return sz(st); }
  void rollback(int t) {
    for (int i = sz(st); i-- > t;)
      e[st[i].first] = st[i].second;
    st.resize(t);
  }
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return 0;
    if (e[a] > e[b]) swap(a, b);
    st.emplace_back(a, e[a]);
    st.emplace_back(b, e[b]);
    e[a] += e[b]; e[b] = a;
    return 1;
  }
};
```

Matrix.h

Description: Basic operations on square matrices.
Usage: `Matrix<int, 3> A;`
`A.d = {{{1,2,3}}, {{4,5,6}}, {{7,8,9}}};`
`vector<int> vec = {1,2,3};`
`vec = (A^N) * vec;`

ed7aad73, 31 lines

```
template<class T, int N>
struct Matrix {
  using M = Matrix;
  array<array<T, N>, N> d{};
  M operator * (const M &m) const {
    M a;
    for (int i = 0; i < N; ++i)
      for (int k = 0; k < N; ++k)
        for (int j = 0; j < N; ++j)
          a.d[i][j] += d[i][k] * m.d[k][j];
    return a;
  }
  vc<T> operator * (const vc<T> &vec) const {
    vc<T> ret(N);
    for (int i = 0; i < N; ++i)
      for (int j = 0; j < N; ++j)
        ret[i] += d[i][j] * vec[j];
    return ret;
  }
  M operator ^ (int p) const {
    assert(p >= 0);
    M a, b(*this);
    for (int i = 0; i < N; ++i) a.d[i][i] = 1;
    while (p) {
      if (p&1) a = a * b;
      b = b * b;
      p /= 2;
    }
    return a;
  }
};
```

LineContainer.h

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming ("convex hull trick").
Time: $\mathcal{O}(\log N)$

7f54e44f, 30 lines

```
struct Line {
  mutable ll k, m, p;
  bool operator < (const Line &o) const { return k < o.k; }
  bool operator < (ll x) const { return p < x; }
};

struct LineContainer: multiset<Line, less<>> {
  // for doubles, use INF = 1 / .0, div(a, b) = a / b
  static const ll INF = numeric_limits<ll>::max();
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b);
  }
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = INF, 0;
    if (x->k == y->k) x->p = x->m > y->m ? INF : -INF;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y=x)!=begin()&&(--x)->p>=y->p) isect(x, erase(y));
  }
  ll query(ll x) {
```

```
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

LiChaoTree.h

Description: LiChao tree for dynamic convex hull trick. can query at any point and insert lines/segments on ranges.
Time: $\mathcal{O}(n \log n)$

64418e87, 65 lines

```
template <typename T>
struct LichaoTree {
  const T INF = numeric_limits<T>::max();
  struct line {
    T a, b;
    line(T a, T b) : a(a), b(b) {}
    T operator()(T x) const { return a * x + b; }
  };
  int n;
  vector<line> fs;
  vector<T> xs;

  int index(T x) const { return lower_bound(xs.begin(), xs.end
      (), x) - xs.begin(); }

  void update(T a, T b, int l, int r) {
    line g(a, b);
    for(l += n, r += n; l < r; l >>= 1, r >>= 1) {
      if(l & 1) descend(g, l++);
      if(r & 1) descend(g, --r);
    }
  }

  void descend(line g, int i) {
    int l = i, r = i + 1;
    while(l < n) l <<= 1, r <<= 1;
    while(l < r) {
      int c = (l + r) >> 1;
      T xl = xs[l - n], xr = xs[r - 1 - n], xc = xs[c - n];
      line &f = fs[i];
      if(f(xl) <= g(xl) && f(xr) <= g(xr)) return;
      if(f(xl) >= g(xl) && f(xr) >= g(xr)) {
        f = g;
        return;
      }
      if(f(xc) > g(xc)) swap(f, g);
      if(f(xl) > g(xl))
        i = i << 1 | 0, r = c;
      else
        i = i << 1 | 1, l = c;
    }
  }

  Tree(const vector<T> &xs_) : xs(xs_) {
    sort(xs.begin(), xs.end());
    xs.erase(unique(xs.begin(), xs.end()), xs.end());
    n = xs.size();
    fs.assign(n << 1, line(T(0), INF));
  }

  // add f(x) = ax + b
  void add_line(T a, T b) { update(a, b, 0, n); }

  // add f(x) = ax + b (x in [xl, xr))
  void add_segment(T a, T b, T xl, T xr) {
    int l = index(xl), r = index(xr);
    update(a, b, l, r);
  }

  T get_min(T x) const {
    int i = index(x);
    T res = INF;
    for(i += n; i; i >>= 1) res = min(res, fs[i](x));
    return res;
  }
};
```

Treap.h

Description: A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.
Time: $\mathcal{O}(\log N)$

0dd1013a, 54 lines

```
struct Node {
  Node *l = 0, *r = 0;
  int val, y, c = 1;
  Node(int val): val(val), y(rand()) {}
  void pull() {
    c = 1;
    if (l) c += l->c;
    if (r) c += r->c;
  }
};

struct Treap {
  Node *tr = 0;
  int cnt(Node *n) { return n ? n->c : 0; }
  // split is [, )
  pair<Node*, Node*> split(Node* n, int k) { // hash-1
    if (!n) return {};
    if (cnt(n->l) >= k) { // "n->val >= k" for lower_bound(k)
      auto pa = split(n->l, k);
      n->l = pa.second;
      n->pull();
      return {pa.first, n};
    } else {
      auto pa = split(n->r, k - cnt(n->l)); // or just "k"
      n->r = pa.first;
      n->pull();
      return {n, pa.second};
    }
  } // hash-1 = 2753f4a9
  Node* merge(Node* l, Node* r) { // hash-2
    if (!l) return r;
    if (!r) return l;
    if (l->y > r->y) {
      l->r = merge(l->r, r);
      l->pull();
      return l;
    } else {
      r->l = merge(l, r->l);
      r->pull();
      return r;
    }
  } // hash-2 = 5e0342f4
  Node *ins(Node *t, Node *n, int pos) {
    auto pa = split(t, pos);
    return merge(merge(pa.first, n), pa.second);
  }
  // Example application: move the range [l, r) to index k
  void move(Node *&t, int l, int r, int k) {
    Node *a, *b, *c;
    tie(a,b) = split(t, l); tie(b,c) = split(b, r - l);
    if (k <= l) t = merge(ins(a, b, k), c);
    else t = merge(a, ins(c, b, k - r));
  }
};
```

FenwickTree.h

Description: Computes partial sums a[0] + a[1] + ... + a[pos - 1], and updates single elements a[i], taking the difference between the old and new value.
Time: Both operations are $\mathcal{O}(\log N)$.

e77418e0, 23 lines

```
template<class T> struct FT {
  vc<T> s;
  FT(int n): s(n) {}
  void update(int p, T v) {
    for (; p < sz(s); p |= p + 1) s[p] += v;
  }
  T query(int p) {
    T res = 0;
    for (; p > 0; p &= p - 1) res += s[p - 1];
    return res;
  }
// hash-1
  int lower_bound(T sum) {// min pos st sum of [0, pos] >= sum
  // Returns n if no sum is >= sum, or -1 if empty sum is.
    if (sum <= 0) return -1;
    int p = 0;
    for (int pw = 1 << 25; pw; pw >>= 1) {
      if (p + pw <= sz(s) && s[p + pw - 1] < sum)
        p += pw, sum -= s[p - 1];
    }
    return p;
  } // hash-1 = 18c85520
};
```

FenwickTree2d.h

Description: Computes sums a[i,j] for all i<I, j<J, and increases single elements a[i,j]. Requires that the elements to be updated are known in advance (call fakeUpdate() before init()).
Time: $\mathcal{O}\left(\log^2 N\right)$. (Use persistent segment trees for $\mathcal{O}(\log N)$.)

"FenwickTree.h" f4066bfa, 23 lines

```
struct FT2 {
  vector<vc<int>> ys; vector<FT> ft;
  FT2(int limx): ys(limx) {}
  void fakeUpdate(int x, int y) {
    for (; x < sz(ys); x |= x + 1) ys[x].push_back(y);
  }
  void init() {
    for (vc<int> &v : ys) sort(all(v)), ft.emplace_back(sz(v));
  }
  int ind(int x, int y) {
    return (int)(lower_bound(all(ys[x]), y) - ys[x].begin());
  }
  void update(int x, int y, ll dif) {
    for (; x < sz(ys); x |= x + 1)
      ft[x].update(ind(x, y), dif);
  }
  ll query(int x, int y) {
    ll sum = 0;
    for (; x; x &= x - 1)
      sum += ft[x - 1].query(ind(x - 1, y));
    return sum;
  }
};
```

RMQ.h

Description: Range Minimum Queries on an array. Returns min(V[a], V[a + 1], ... V[b - 1]) in constant time.
Usage: `RMQ rmq(values); rmq.query(inclusive, exclusive);`
Time: $\mathcal{O}(|V| \log |V| + Q)$

17539037, 16 lines

```
template<class T> struct RMQ {
  vc<vc<T>> jmp;
  RMQ(vc<T> &a): jmp(1, a) {
```

```
    for (int pw = 1, k = 1; pw * 2 <= sz(a); pw *= 2, ++k) {
      jmp.emplace_back(sz(a) - pw * 2 + 1);
      for (int j = 0; j < sz(jmp[k]); ++j) {
        jmp[k][j] = min(jmp[k - 1][j], jmp[k - 1][j + pw]);
      }
    }
  }
  T query(int a, int b) {
    assert(a < b); // or return inf if a == b
    int dep = 31 - __builtin_clz(b - a);
    return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
  }
};
```

MoQueries.h

Description: Answer interval or tree path queries by finding an approximate TSP through the queries, and moving from one query to the next by adding/removing points at the ends. If values are on tree edges, change `step` to add/remove the edge (a, c) and remove the initial `add` call (but keep `in`).
Time: $\mathcal{O}\left(N\sqrt{Q}\right)$

3d59ce61, 49 lines

```
void add(int ind, int end) { ... } // add a[ind] (end = 0 or 1)
void del(int ind, int end) { ... } // remove a[ind]
int calc() { ... } // compute current answer

vc<int> mo(vc<array<int, 2>> Q) { // hash-1
  int L = 0, R = 0, blk = 350; // ~N/sqrt(Q)
  vc<int> s(sz(Q)), res = s;
#define K(x) make_pair(x[0] / blk, x[1] ^ -(x[0] / blk & 1))
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
  for (int qi: s) {
    array<int, 2> q = Q[qi];
    while (L > q.first) add(--L, 0);
    while (R < q.second) add(R++, 1);
    while (L < q.first) del(L++, 0);
    while (R > q.second) del(--R, 1);
    res[qi] = calc();
  }
  return res;
} // hash-1 = 844328d6

vc<int> moTree(vc<array<int, 2>> Q,vc<vc<int>> &ed,int root=0){
      // hash-2
  int N = sz(ed), pos[2] = {}, blk = 350; // ~N/sqrt(Q)
  vc<int> s(sz(Q)), res = s, I(N), L(N), R(N), in(N), par(N);
  add(0, 0), in[0] = 1;
  auto dfs = [&](int x, int p, int dep, auto &f) -> void {
    par[x] = p;
    L[x] = N;
    if (dep) I[x] = N++;
    for (int y : ed[x]) if (y != p) f(y, x, !dep, f);
    if (!dep) I[x] = N++;
    R[x] = N;
  };
  dfs(root, -1, 0, dfs);
#define K(x) make_pair(I[x[0]]/blk, I[x[1]] ^ -(I[x[0]]/blk&1))
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
  for (int qi : s) for(int end = 0; end < 2; ++end) {
    int &a = pos[end], b = Q[qi][end], i = 0;
#define step(c) { if (in[c]) { del(a, end); in[a] = 0; } \
                  else { add(c, end); in[c] = 1; } a = c; }
    while (!(L[b] <= L[a] && R[a] <= R[b]))
      I[i++] = b, b = par[b];
    while (a != b) step(par[a]);
    while (i--) step(I[i]);
    if (end) res[qi] = calc();
  }
  return res;
} // hash-2 = 01876993
```

Numerical (4)

4.1 Polynomials and recurrences

Polynomial.h

c9b7b07a, 17 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for (int i = sz(a); i--;) (val *= x) += a[i];
    return val;
  }
  void diff() {
    rep(i,1,sz(a)) a[i-1] = i*a[i];
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
    a.pop_back();
  }
};
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: `polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

"Polynomial.h" 088a8d2d, 23 lines

```
vc<double> polyRoots(Poly p, double xmin, double xmax) {
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vc<double> ret;
  Poly der = p;
  der.diff();
  auto dr = polyRoots(der, xmin, xmax);
  dr.emplace_back(xmin-1);
  dr.emplace_back(xmax+1);
  sort(all(dr));
  for (int i = 0; i < sz(dr) - 1; ++i) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      for (int it = 0; it < 60; ++it) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.emplace_back((l + h) / 2);
    }
  }
  return ret;
}
```

PolyInterpolate.h

Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: $p(x) = a[0] * x^0 + ... + a[n-1] * x^{n-1}$. For numerical precision, pick $x[k] = c * \cos(k/(n-1) * \pi), k = 0 \dots n-1$.
Time: $\mathcal{O}\left(n^2\right)$

bf0054d4, 14 lines

```
vc<double> interpolate(vc<double> x, vc<double> y, int n) {
  vc<double> res(n), temp(n);
  for (int k = 0; k < n - 1; ++k)
    for (int i = k + 1; i < n; ++i)
      y[i] = (y[i] - y[k]) / (x[i] - x[k]);
  double last = 0; temp[0] = 1;
  for (int k = 0; k < n; ++k)
    for (int i = 0; i < n; ++i) {
      res[i] += y[k] * temp[i];
      swap(last, temp[i]);
      temp[i] -= last * x[k];
    }
  return res;
}
```

BerlekampMassey.h

Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
Usage: `berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`
Time: $\mathcal{O}\left(N^2\right)$

"../number-theory/ModPow.h" 9ce628c7, 22 lines

```
vc<ll> berlekampMassey(vc<ll> s) {
  int n = sz(s), L = 0, m = 0;
  vc<ll> C(n), B(n), T;
  C[0] = B[0] = 1;

  ll b = 1;
  for (int i = 0; ++m, i < n; ++i) {
    ll d = s[i] % mod;
    for (int j = 1; j <= L; ++j)
      d = (d + C[j] * s[i - j]) % mod;
    if (!d) continue;
    T = C; ll coef = d * modpow(b, mod-2) % mod;
    for (int j = m; j < n; ++j)
      C[j] = (C[j] - coef * B[j - m]) % mod;
    if (2 * L > i) continue;
    L = i + 1 - L; B = T; b = d; m = 0;
  }

  C.resize(L + 1); C.erase(C.begin());
  for (ll &x : C) x = (mod - x) % mod;
  return C;
}
```

LinearRecurrence.h

Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \dots \geq n-1]$ and $tr[0 \dots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
Usage: `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`
Time: $\mathcal{O}\left(n^2 \log k\right)$

7ec87418, 28 lines

```
using Poly =  vc<ll>;
ll linearRec(Poly S, Poly tr, ll k) {
  int n = sz(tr);

  auto combine = [&](Poly a, Poly b) {
    Poly res(n * 2 + 1);
    for (int i = 0; i <= n; ++i)
      for (int j = 0; j <= n; ++j)
        res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
    for (int i = 2 * n; i > n; --i) for (int j = 0; j < n; ++j)
      res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) % mod;
    res.resize(n + 1);
    return res;
  };

  Poly pol(n + 1), e(pol);
  pol[0] = e[1] = 1;

  for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
```

```
  }

  ll res = 0;
  for (int i = 0; i < n; ++i)
    res = (res + pol[i + 1] * S[i]) % mod;
  return res;
}
```

4.2 Optimization

GoldenSectionSearch.h

Description: Finds the argument minimizing the function f in the interval $[a, b]$ assuming f is unimodal on the interval, i.e. has only one local minimum. The maximum error in the result is eps. Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.
Usage: `double func(double x) { return 4+x+.3*x*x; }`
`double xmin = gss(-1000,1000,func);`
Time: $\mathcal{O}(\log((b-a)/\epsilon))$

31d45b51, 14 lines

```
double gss(double a, double b, double (*f)(double)) {
  double r = (sqrt(5)-1)/2, eps = 1e-7;
  double x1 = b - r*(b-a), x2 = a + r*(b-a);
  double f1 = f(x1), f2 = f(x2);
  while (b-a > eps)
    if (f1 < f2) { //change to > to find maximum
      b = x2; x2 = x1; f2 = f1;
      x1 = b - r*(b-a); f1 = f(x1);
    } else {
      a = x1; x1 = x2; f1 = f2;
      x2 = a + r*(b-a); f2 = f(x2);
    }
  return a;
}
```

SimulatedAnnealing.h

Description: Simple SA with exponential annealing

e28ac420, 52 lines

```
typedef int numt;
numt solve() {
    // ADD: return the value of the current state
    return 0;
}

int main() {
    clock_t timer = clock();

    const double Tt = 1.9;
    double et = 0.0;
    double uphill = 1.;
    const double up_inc = 0.01;
    double f = 0.9999;
    double t0 = 100; // can initialize with delta / ln(0.8)
    double temp = t0;

    // ADD: initialize initial state
    numt curr = solve();
    numt res = curr;

    while (et < Tt) {
        // ADD: random move

        uphill *= (1. - up_inc);
        numt s = solve();
        // reverse if maximizing
        if (s < curr) {
            curr = s;
        } else {
            ll x = rand() + 1ll;
            ll y = rand() + 1ll;
            x %= y;
            // (s - curr) if maximizing
            if (x / (double) y <= exp((curr - s) / temp)) {
                // reverse if maximizing
                if (s > curr) uphill += up_inc;
                curr = s;
            } else {
                // ADD: move back
            }
        }

        // max if maximizing
        res = min(res, curr);

        if (uphill > 0.02) temp *= f;
        if (uphill < 0.001) temp /= f;

        et = (clock() - timer) / double(CLOCKS_PER_SEC);
    }
}
```

Integrate.h

Description: Simple integration of a function over an interval using Simpson's rule. The error should be proportional to h^4, although in practice you will want to verify that the result is stable to desired precision when epsilon changes.

a89f8870, 7 lines

```
template<class F>
double quad(double a, double b, F f, const int n = 1000) {
  double h = (b - a) / 2 / n, v = f(a) + f(b);
  for (int i = 1; i < n * 2; ++i)
    v += f(a + i*h) * (i&1 ? 4 : 2);
  return v * h / 3;
}
```

IntegrateAdaptive.h

Description: Fast integration using an adaptive Simpson's rule.
Usage: `double sphereVolume = quad(-1, 1, [](double x) {`
`return quad(-1, 1, [&](double y) {`
`return quad(-1, 1, [&](double z) {`
`return x*x + y*y + z*z < 1; });});});`

898d1125, 14 lines

```
#define S(a,b) (f(a) + 4*f((a+b) / 2) + f(b)) * (b-a) / 6

template <class F>
double rec(F& f, double a, double b, double eps, double S) {
  double c = (a + b) / 2;
  double S1 = S(a, c), S2 = S(c, b), T = S1 + S2;
  if (abs(T - S) <= 15 * eps || b - a < 1e-10)
    return T + (T - S) / 15;
  return rec(f, a, c, eps / 2, S1) + rec(f, c, b, eps / 2, S2);
}
template<class F>
double quad(double a, double b, F f, double eps = 1e-8) {
  return rec(f, a, b, eps, S(a, b));
}
```

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \leq b$, $x \geq 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.
Usage: `vvd A = {{1,-1}, {-1,1}, {-1,-2}};`
`vd b = {1,1,-4}, c = {-1,-1}, x;`
`T val = LPSolver(A, b, c).solve(x);`
Time: $\mathcal{O}(NM * \#pivots)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}(2^n)$ in the general case.

0d3ca651, 78 lines

```
using T = double; // long double, Rational, double + mod<P>...
using vd = vc<T>;
using vvd = vc<vd>;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j

struct LPSolver {
  int m, n;
  vi N, B;
  vvd D;

  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
      for (int i = 0; i < m; ++i)
        for (int j = 0; j < n; ++j) D[i][j] = A[i][j];
      for (int i = 0; i < m; ++i)
        B[i] = n+i, D[i][n] = -1, D[i][n+1] = b[i];
      for (int j = 0; j < n; ++j)
        N[j] = j, D[m][j] = -c[j];
      N[n] = -1, D[m+1][n] = 1;
    }

  void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    for (int i = 0; i < m + 2; ++i)
      if (i != r && abs(D[i][s]) > eps) {
        T *b = D[i].data(), inv2 = b[s] * inv;
        for (int j = 0; j < n + 2; ++j) b[j] -= a[j] * inv2;
        b[s] = a[s] * inv2;
      }

    for (int j = 0; j < n + 2; ++j)
      if (j != s) D[r][j] *= inv;
    for (int i = 0; i < m + 2; ++i)
      if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  }

  bool simplex(int phase) {
    int x = m + phase - 1;
    for (;;) {
      int s = -1;
      for (int j = 0; j <= n; ++j)
        if (N[j] != -phase) ltj(D[x]);
      if (D[x][s] >= -eps) return true;
      int r = -1;
      for (int i = 0; i < m; ++i) {
        if (D[i][s] <= eps) continue;
        if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                     < MP(D[r][n+1] / D[r][s], B[r])) r = i;
      }
      if (r == -1) return false;
      pivot(r, s);
    }
  }

  T solve(vd &x) {
    int r = 0;
    for (int i = 1; i < m; ++i)
      if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) {
      pivot(r, n);
      if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
```

```
      for (int i = 0; i < m; ++i) if (B[i] == -1) {
        int s = 0;
        for (int j = 1; j <= n; ++j) ltj(D[i]);
        pivot(i, s);
      }
    }
    bool ok = simplex(1); x = vd(n);
    for (int i = 0; i < m; ++i)
      if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
  }
};
```

4.3 Matrices

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}\left(N^3\right)$

c5cc96b2, 17 lines

```
auto det = [](vc<vc<double>> &a) {
  int n = sz(a); double res = 1;
  for (int i = 0; i < n; ++i) {
    int b = i;
    for (int j = i + 1; j < n; ++j)
      if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
    if (i != b) swap(a[i], a[b]), res *= -1;
    res *= a[i][i];
    if (res == 0) return 0;
    for (int j = i + 1; j < n; ++j) {
      double v = a[j][i] / a[i][i];
      if (v != 0)
        for (int k = i + 1; k < n; ++k) a[j][k] -= v * a[i][k];
    }
  }
  return res;
};
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}\left(N^3\right)$

88ad2892, 18 lines

```
const ll md = 12345;
auto det = [md](vc<vc<ll>> &a) { //2612f2d6
  int n = sz(a); ll ans = 1;
  for (int i = 0; i < n; ++i) {
    for (int j = i + 1; j < n; ++j) {
      while (a[j][i] != 0) { // gcd step
        ll t = a[i][i] / a[j][i];
        if (t) for (int k = i; k < n; ++k)
          a[i][k] = (a[i][k] - a[j][k] * t) % md;
        swap(a[i], a[j]);
        ans *= -1;
      }
    }
    ans = ans * a[i][i] % md;
    if (!ans) return 0;
  }
  return (ans + md) % md;
};
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}\left(n^2m\right)$

9681fb61, 37 lines

```
using vd = vc<double>;
constexpr double eps = 1e-12;

auto solveLinear = [](vc<vd> &A, vd &b, vd &x) { //790aae09
  int n = sz(A), m = sz(x), rank = 0, br, bc;
  if (n) assert(sz(A[0]) == m);
  vc<int> col(m); iota(all(col), 0);
  for (int i = 0; i < n; ++i) {
    double v, bv = 0;
    for (int r = i; r < n; ++r)
      for (int c = i; c < m; ++c)
        if ((v = fabs(A[r][c])) > bv) br = r, bc = c, bv = v;
    if (bv <= eps) {
      for (int j = i; j < n; ++j)
        if (fabs(b[j]) > eps) return -1;
      break;
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    for (int j = 0; j < n; ++j) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    for (int j = i + 1; j < n; ++j) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      for (int k = i + 1; k < m; ++k) A[j][k] -= fac * A[i][k];
    }
    ++rank;
  }
  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    for (int j = 0; j < i; ++j) b[j] -= A[j][i] * b[i];
  }
  return rank; // (multiple solutions if rank < m)
};
```

SolveLinear2.h

Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:

"SolveLinear.h" c8e85a5f, 11 lines

```
for (int j = 0; j < n; ++j)
  if (j != i) // instead of rep(j,i+1,n)

// ... then at the end:
x.assign(m, undefined);
for (int i = 0; i < rank; ++i) {
  for (int j = rank; j < m; ++j)
    if (fabs(A[i][j]) > eps) goto fail;
  x[col[i]] = b[i] / A[i][i];
fail:;
}
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.
Time: $\mathcal{O}\left(n^2m\right)$

37a74b48, 33 lines

```
using bs = bitset<1000>;

auto solveLinear = [](vc<bs> &A, vc<int> &b, bs &x, int m) {
  int n = sz(A), rank = 0, br;
  assert(m <= sz(x));
  vc<int> col(m); iota(all(col), 0);
  for (int i = 0; i < n; ++i) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      for (int j = i; j < n; ++j) if(b[j]) return -1;
      break;
    }
    int bc = int(A[br]._Find_next(i - 1));
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    for (int j = i + 1; j < n; ++j) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
    for (int j = i + 1; j < n; ++j) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    }
    ++rank;
  }
  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    for (int j = 0; j < i; ++j) b[j] ^= A[j][i];
  }
  return rank; // (multiple solutions if rank < m)
}; //f6c54262
```

LinearBase.h

Description: LinearBase.
Time: $\mathcal{O}\left(\log N\right)$

022c9f27, 50 lines

```
int n,m;
ll lb[63],bin[63];
vector<ll> ve;
vector<int> bit;

void Ins(ll x){
  for (int i = 62; ~i; --i) {
    if(!x)return ;
    if(~x&bin[i])continue;
    if(lb[i])x^=lb[i];
    else{
      for (int j = 0; j < i; ++j) if(x&bin[j])x^=lb[j];
      for (int j = i + 1; j < 63; ++j) if(lb[j]&bin[i])lb[j]^=x
          ;
      lb[i]=x;
      return ;
    }
  }
}

ll Max(){
  ll ans=0;
  for (int i = 0; i < 63; ++i) if(lb[i])ans^=lb[i];
  return ans;
}

ll Kth(ll K){
  if((int)ve.size()!=n)--K;
  if(K>bin[ve.size()]-1)return -1;
  ll ans=0;
  for (int i = 0; i < sz(ve); ++i) if(K&bin[i])ans^=ve[i];
  return ans;
}

ll Rank(ll x){
  ll ans=0;
  for (int i = 0; i < sz(bit); ++i) if(x&bin[bit[i]])ans|=bin[i
      ];
  return ans;
}

void Merge(ll b[]){
  for (int i = 0; i < 63; ++i) Ins(b[i]);
}
```

```
void solve() {
  bin[0]=1;
  for (int i = 0; i < 63; ++i) bin[i]=bin[i-1]<<1ll;
  // insert(xi)
  for (int i = 0; i < 63; ++i)
    if(lb[i])ve.push_back(lb[i]),bit.push_back(i);
}
```

MatrixInverse.h

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1})$ (mod p^k) where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.
Time: $\mathcal{O}(n^3)$

453d5d7a, 35 lines

```
int matInv(vc<vc<double>>& A) {
  int n = sz(A); vi col(n);
  vc<vc<double>> tmp(n, vc<double>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;

  rep(i,0,n) {
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
    if (fabs(A[r][c]) < 1e-12) return i;
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    double v = A[i][i];
    rep(j,i+1,n) {
      double f = A[j][i] / v;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] -= f*A[i][k];
      rep(k,0,n) tmp[j][k] -= f*tmp[i][k];
    }
    rep(j,i+1,n) A[i][j] /= v;
    rep(j,0,n) tmp[i][j] /= v;
    A[i][i] = 1;
  }

  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    double v = A[j][i];
    rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
  }

  rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
  return n;
}
```

MatrixInverse-mod.h

Description: Invert matrix A modulo a prime. Returns rank; result is stored in A unless singular (rank < n). For prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1})$ (mod p^k) where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.
Time: $\mathcal{O}(n^3)$

"../number-theory/ModPow.h" a6f68f90, 36 lines

```
int matInv(vector<vector<ll>>& A) {
  int n = sz(A); vi col(n);
  vector<vector<ll>> tmp(n, vector<ll>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;

  rep(i,0,n) {
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n) if (A[j][k]) {
      r = j; c = k; goto found;
    }
    return i;
found:
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n) swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    ll v = modpow(A[i][i], mod - 2);
    rep(j,i+1,n) {
      ll f = A[j][i] * v % mod;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] = (A[j][k] - f*A[i][k]) % mod;
      rep(k,0,n) tmp[j][k] = (tmp[j][k] - f*tmp[i][k]) % mod;
    }
    rep(j,i+1,n) A[i][j] = A[i][j] * v % mod;
    rep(j,0,n) tmp[i][j] = tmp[i][j] * v % mod;
    A[i][i] = 1;
  }

  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    ll v = A[j][i];
    rep(k,0,n) tmp[j][k] = (tmp[j][k] - v*tmp[i][k]) % mod;
  }

  rep(i,0,n) rep(j,0,n)
    A[col[i]][col[j]] = tmp[i][j] % mod + (tmp[i][j] < 0 ? mod : 0);
  return n;
}
```

Tridiagonal.h

Description: $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i,\ 1 \le i \le n,$$

where a_0, a_{n+1}, b_i, c_i and d_i are known. a can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, ..., -1, 1\}, \{0, c_1, c_2, \ldots, c_n\}, \{b_1, b_2, \ldots, b_n, 0\}, \{a_0, d_1, d_2, \ldots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.
If $|d_i| > |p_i| + |q_{i-1}|$ for all i, or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.
Time: $\mathcal{O}(N)$

7a2f067a, 26 lines

```
using T = double;
vc<T> tridiagonal(vc<T> diag, const vc<T>& super,
    const vc<T>& sub, vc<T> b) {
  int n = sz(b); vi tr(n);
  rep(i,0,n-1) {
    if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i] == 0
      b[i+1] -= b[i] * diag[i+1] / super[i];
      if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
      diag[i+1] = sub[i]; tr[++i] = 1;
    } else {
      diag[i+1] -= super[i]*sub[i]/diag[i];
      b[i+1] -= b[i]*sub[i]/diag[i];
    }
  }
  for (int i = n; i--;) {
    if (tr[i]) {
      swap(b[i], b[i-1]);
      diag[i-1] = diag[i];
      b[i] /= super[i-1];
    } else {
      b[i] /= diag[i];
      if (i) b[i-1] -= b[i]*super[i-1];
    }
  }
  return b;
}
```

4.4 Fourier transforms

FastFourierTransform.h

Description: fft(a) computes $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$ for all k. N must be a power of 2. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2) \log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16}; higher for random inputs). Otherwise, use NTT/FFTMod.
Time: $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ ($\sim 1s$ for $N = 2^{22}$)

adeeddae, 44 lines

```
using C = complex<double>;
using vd = vc<double>;

auto fft = [&](vc<C> &a) {
  int n = sz(a), L = 31 - __builtin_clz(n);
  static vc<C> R(2, 1), rt(2, 1); // R can be long double
  for (static int k = 2; k < n; k *= 2) {
    R.resize(n), rt.resize(n);
    auto x = polar(1., acos(-1) / k);
    for (int i = k; i < k * 2; ++i)
      rt[i] = R[i] = i & 1 ? R[i / 2] * x : R[i / 2];
  }
  vc<int> rev(n);
  for (int i = 0; i < n; ++i)
    rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  for (int i = 0; i < n; ++i)
    if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += k * 2) {
      auto it1 = &a[i], it2 = it1 + k;
      for (int j = 0; j < k; ++j, ++it1, ++it2) {
        auto x = (double *)&rt[j + k], y = (double *)it2;
        C z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1] * y[0]);
        *it2 = *it1 - z;
        *it1 += z;
      }
    }
}; //bfe3257c
auto conv = [&](vd &a,vd &b) {
  if (a.empty() || b.empty()) return vd();
  vd res(sz(a) + sz(b) - 1);
  int L = 32 - __builtin_clz(sz(res) - 1), n = 1 << L;
  vc<C> in(n), out(n);
  copy(all(a), begin(in));
  for (int i = 0; i < sz(b); ++i) in[i].imag(b[i]);
  fft(in);
  for (C &x: in) x *= x;
  for (int i = 0; i < n; ++i)
    out[i] = in[-i & (n - 1)] - conj(in[i]);
  fft(out);
  for (int i = 0; i < sz(res); ++i)
    res[i] = imag(out[i]) / (n * 4);
  return res;
}; //13bd14b6
```

FastFourierTransformMod.h

Description: Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice 10^{16} or higher). Inputs must be in $[0, \text{mod})$.
Time: $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

"FastFourierTransform.h" 8b4a5cbd, 21 lines

```
auto convMod = [&](vc<int> &a, vc<int> &b, int M) {
  if (a.empty() || b.empty()) return vc<ll>();
  vc<ll> res(sz(a) + sz(b) - 1);
  int B=32-__builtin_clz(sz(res)),n=1<<B, cut = int(sqrt(M));
  vc<C> L(n), R(n), outs(n), outl(n);
  for (int i = 0; i < sz(a); ++i) L[i] = C(a[i]/cut, a[i]%cut);
  for (int i = 0; i < sz(b); ++i) R[i] = C(b[i]/cut, b[i]%cut);
  fft(L), fft(R);
  for (int i = 0; i < n; ++i) {
    int j = -i & (n - 1);
    outl[j] = (L[i] + conj(L[j])) * R[i] / (n * 2.);
    outs[j] = (L[i] - conj(L[j])) * R[i] / (n * 2.) / 1i;
  }
  fft(outl), fft(outs);
  for (int i = 0; i < sz(res); ++i) {
    ll av = ll(real(outl[i]) + .5),cv = ll(imag(outs[i]) + .5),
       bv = ll(imag(outl[i]) + .5) + ll(real(outs[i]) + .5);
    res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
  }
  return res;
};
```

NumberTheoreticTransform.h

Description: ntt(a) computes $\hat{f}(k) = \sum_x a[x] g^{xk}$ for all k, where $g = \text{root}^{(mod-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a. For arbitrary modulo, see FFTMod. conv(a, b) = c, where $c[x] = \sum a[i]b[x-i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in [0, mod).
Time: $\mathcal{O}(N \log N)$

"../number-theory/ModPow.h" 0bbe7f0e, 39 lines

```
constexpr ll md = 998244353, root = 62;
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 << 21
// and 483 << 21 (same root). The last two are > 10^9.
auto ntt = [&](vc<Mod> &a) {
  int n = sz(a), L = 31 - __builtin_clz(n);
  static vc<Mod> rt(2, 1);
  for (static int k = 2, s = 2; k < n; k *= 2, ++s) {
    rt.resize(n);
    array<Mod, 2> z{1, mdPow(root, md >> s)};
    for (int i = k; i < k * 2; ++i)
      rt[i] = rt[i / 2] * z[i & 1];
  }
  vc<int> rev(n);
  for (int i = 0; i < n; ++i)
    rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  for (int i = 0; i < n; ++i)
    if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0 ; i < n; i += k * 2) {
      auto it1 = &a[i], it2 = it1 + k;
      for (int j = 0; j < k; ++j, ++it1, ++it2) {
        Mod z = rt[j + k] * *it2;
        *it2 = *it1 - z, *it1 += z;
      }
    }
}; //7c5a0cf5
auto conv = [&](vc<Mod> a, vc<Mod> b) -> vc<Mod> {
  if (a.empty() || b.empty()) return {};
  int s = sz(a) + sz(b) - 1,
      n = 1 << (32 - __builtin_clz(s - 1));
  Mod inv = md - (md - 1) / n;
  vc<Mod> L(a), R(b), out(n);
  L.resize(n), R.resize(n);
  ntt(L), ntt(R);
  for (int i = 0; i < n; ++i)
    out[-i & (n - 1)] = L[i] * R[i] * inv;
  ntt(out);
  return {out.begin(), out.begin() + s};
}; //75d60f42
```

PolyInverse.h

Description: Compute the inversion of the Polynomial.
Time: $\mathcal{O}(n \log n)$

"NumberTheoreticTransform.h" ad42700a, 26 lines

```
auto invIter = [&](vc<Mod> &a, vc<Mod> &in, vc<Mod> &b) {
  int n = sz(in);
  vc<Mod> out(n);
  copy(a.begin(), a.begin() + min(sz(a), n), out.begin());
  auto conv = [&] {
    ntt(out);
    for (int i = 0; i < n; ++i) out[i] *= in[i];
    ntt(out), reverse(out.begin() + 1, out.end());
  };
  conv(), fill(out.begin(), out.begin() + sz(b), 0), conv();
  b.resize(n);
  Mod inv = md - (md - 1) / n; inv *= inv;
  for (int i = n / 2; i < n; ++i)
    b[i] = out[i].x ? inv * (md - out[i].x) : 0;
}; //787b29bc
auto polyInv = [&](vc<Mod> &a) -> vc<Mod> {
  if (a.empty()) return {};
  vc<Mod> b{mdPow(a[0], md - 2)};
  b.reserve(sz(a));
  while (sz(b) < sz(a)) {
    vc<Mod> in(sz(b) * 2);
    copy(all(b), in.begin()), ntt(in);
    invIter(a, in, b);
  }
  return {b.begin(), b.begin() + sz(a)};
};
```

PolySqrt.h

Description: Compute the sqrt of a, where a is a polynomial and a[0] = 1(if not, ModSqrt is required).
Time: $\mathcal{O}(n \log n)$

"PolyInverse.h" a0bcdc24, 24 lines

```
auto polySqrt = [&](vc<Mod> &a) -> vc<Mod> {
  if (a.empty()) return {};
  vc<Mod> b{1}, ib{1};
  b.reserve(sz(a)), ib.reserve(sz(a));
  auto conv = [&](vc<Mod> &a, vc<Mod> &b) {
    ntt(a);
    for (int i = 0; i < sz(a); ++i) a[i] *= b[i];
    ntt(a), reverse(a.begin() + 1, a.end());
  };
  while (sz(b) < sz(a)) {
    int h = sz(b), n = h * 2;
    vc<Mod> in(n), out(n);
    copy(all(ib), in.begin()), ntt(in);
    copy(all(b), out.begin());
    conv(out, out), fill(out.begin(), out.begin() + h, 0);
    Mod inv = md - (md - 1) / n;
    for (int i = h; i < min(n, sz(a)); ++i)
      out[i] = out[i] * inv - a[i];
    conv(out, in), b.resize(n), inv *= (md / 2);
    for (int i = h; i < n; ++i) b[i] = out[i] * inv;
    if (sz(b) < sz(a)) invIter(b, in, ib);
  }
  return {b.begin(), b.begin() + sz(a)};
};
```

PolyExp.h

Description: Compute the Exp of a, where a is a polynomial and a[0] = 0.
Time: $\mathcal{O}(n \log n)$

"PolyInverse.h", "../number-theory/ModInverse.h" b11b7ae3, 36 lines

```
auto deri = [&](vc<Mod> a) {
  for (int i = 1; i < sz(a); ++i) a[i - 1] = a[i] * i;
  a.pop_back();
  return a;
};
auto polyExp = [&](vc<Mod> &a) -> vc<Mod> {
  if (a.empty()) return {};
  vc<Mod> b{1}, ib{1};
  b.reserve(sz(a)), ib.reserve(sz(a));
  auto conv = [&](vc<Mod> &a, vc<Mod> &b) {
    ntt(a);
    for (int i = 0; i < sz(a); ++i) a[i] *= b[i];
    ntt(a), reverse(a.begin() + 1, a.end());
  };
  while (sz(b) < sz(a)) {
    int h = sz(b), n = h * 2;
    Mod inv = md - (md - 1) / n;
    vc<Mod> db(n), dib(n), A(deri(b)), B(n);
    copy(all(ib), dib.begin()), ntt(dib);
    copy(all(b), db.begin()), ntt(db);
    A.resize(n), conv(A, dib);
    for (int i = 0; i < n; ++i) B[i] = db[i] * dib[i];
    ntt(B), reverse(B.begin() + 1, B.end());
    fill(B.begin(), B.begin() + h, 0);
    vc<Mod> da(deri(vc<Mod>(a.begin(), a.begin() + h)));
    da.resize(n), ntt(da), conv(B, da);
    for (int i = min(n, sz(a)) - 1; i >= h; --i)
      A[i] = (A[i - 1] - B[i - 1] * inv) * iv[i] * inv - a[i];
    fill(A.begin(), A.begin() + h, 0), conv(A, db);
    b.resize(n);
    for (int i = h; i < n; ++i)
      b[i] = A[i].x ? inv * (md - A[i].x) : 0;
    if (sz(b) < sz(a)) invIter(b, dib, ib);
  }
  return {b.begin(), b.begin() + sz(a)};
};
```

PolyLn.h

Description: Compute the Ln of a, where a is a polynomial and a[0] = 1.
Time: $\mathcal{O}(n \log n)$

"PolyInverse.h", "../number-theory/ModInverse.h" 9d4be733, 23 lines

```
auto deri = [&](vc<Mod> a) {
  for (int i = 1; i < sz(a); ++i) a[i - 1] = a[i] * i;
  a.pop_back();
  return a;
};
auto inte = [&](vc<Mod> a) {
  for (int i = sz(a) - 1; i >= 1; --i)
    a[i] = a[i - 1] * iv[i];
  a[0] = 0;
  return a;
};
auto polyLn = [&](vc<Mod> &a) -> vc<Mod> {
  if (a.empty()) return {};
  int n = 1 << (32 - __builtin_clz(2 * sz(a) - 2));
  Mod inv = md - (md - 1) / n;
  vc<Mod> b = polyInv(a), c = deri(a);
  b.resize(n), c.resize(n);
  ntt(b), ntt(c);
  for (int i = 0; i < n; ++i) b[i] = b[i] * c[i] * inv;
  ntt(b), reverse(b.begin() + 1, b.end());
  b = inte(b);
```

```
  return {b.begin(), b.begin() + sz(a)};
};
```

PolyPow.h

Description: Compute the $A^m \pmod{x^n}$, where the A is a polynomial and n is the size of A. Notice that you should change the constant term of the PolyExp.
Time: $\mathcal{O}(n \log n)$

"PolyLn.h", "PolyExp.h" 0528559c, 14 lines

```
auto polyPow = [&](vc<Mod> &a, ll k) {
  int n = sz(a), t = n;
  for (int i = 0; i < n; ++i) if (a[i].x) { t = i; break; }
  if (t * k >= n) return vc<Mod>(n, 0);
  vc<Mod> b(a.begin() + t, a.end());
  b.resize(n - t * k);
  Mod a0 = b[0];
  b = polyLn(b);
  ll k1 = k % md;
  for (Mod &e: b) e *= k1;
  b = polyExp(b, mdPow(a0, k % (md - 1)));
  b.insert(b.begin(), t * k, 0);
  return b;
};
```

PolyDivision.h

Description: Polynomial floor division. No leading 0's.
Time: $\mathcal{O}(n \log n)$

"NumberTheoreticTransform.h" 0c82d61c, 13 lines

```
auto polyDiv = [&](vc<Mod> a, vc<Mod> b)
               -> pair<vc<Mod>, vc<Mod>> {
  if (sz(a) < sz(b)) return {vc<Mod>(), a};
  int n = sz(a) - sz(b) + 1;
  vc<Mod> da(a.rbegin(), a.rend()), db(b.rbegin(), b.rend());
  da.resize(n), db.resize(n);
  da = conv(da, polyInv(db));
  da.resize(n), reverse(all(da));
  auto c = conv(da, b);
  a.resize(sz(b) - 1);
  for (int i = 0; i < sz(a); ++i) a[i] -= c[i];
  return {da, a};
};
```

FastSubsetTransform.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x \oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of a must be a power of two.
Time: $\mathcal{O}(N \log N)$

464cf360, 16 lines

```
void FST(vi& a, bool inv) {
  for (int n = sz(a), step = 1; step < n; step *= 2) {
    for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) {
      int &u = a[j], &v = a[j + step]; tie(u, v) =
        inv ? pii(v - u, u) : pii(v, u + v); // AND
        inv ? pii(v, u - v) : pii(u + v, u); // OR
        pii(u + v, u - v);                   // XOR
    }
  }
  if (inv) for (int& x : a) x /= sz(a); // XOR only
}
vi conv(vi a, vi b) {
  FST(a, 0); FST(b, 0);
  rep(i,0,sz(a)) a[i] *= b[i];
  FST(a, 1); return a;
}
```

Number theory (5)

5.1 Modular arithmetic

ModularArithmetic.h

Description: Operators for modular arithmetic. You need to set mod to some number first and then you can use the structure.

"Euclid.h" b0b6316b, 26 lines

```
constexpr ll md = 998244353; // change to something else
struct Mod {
  ll x;
  Mod(ll x = 0): x(x) {}
  Mod operator+(Mod b) {ll y=x+b.x;return y<md ? y : y - md; }
  Mod operator-(Mod b) { return x - b.x + (x < b.x ? md : 0); }
  Mod operator * (Mod b) { return x * b.x % md; }
  Mod operator / (Mod b) { return *this * inv(b); }
  void operator += (Mod b) { x += b.x; x < md ?: x -= md; }
  void operator *= (Mod b) { (x *= b.x) %= md; }
  void operator -= (Mod b) { x -= b.x; -x < 0 ?: x += md; }
  void operator /= (Mod b) { *this *= inv(b); }
  Mod inv(Mod a) {
    ll x, y, g = euclid(a.x, md, x, y);
    assert(g == 1);
    return (x + md) % md;
  }
  Mod operator ^ (ll e) {
    if (!e) return 1;
    Mod r = *this ^ (e / 2); r = r * r;
    return e & 1 ? *this * r : r;
  }
  friend ostream &operator << (ostream &os, Mod m) {
    return os << m.x;
  }
};
```

ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime.
Time: $\mathcal{O}(n)$

6cc69c62, 7 lines

```
constexpr ll md = 1e9 + 7, LIM = 2e4 + 1; // change this
auto mdInv = [&] {
  vc<ll> inv(LIM); inv[1] = 1;
  for (int i = 2; i < LIM; ++i)
    inv[i] = md - (md / i) * inv[md % i] % md;
  return inv;
};
```

ModPow.h

Time: $\mathcal{O}(\log n)$

"ModularArithmetic.h" b807a56c, 7 lines

```
constexpr ll md = 1000000007; // change this
auto mdPow = [&](Mod b, ll e) {
  Mod res = 1;
  for (; e; b *= b, e /= 2)
    if (e & 1) res *= b;
  return res;
};
```

ModLog.h

Description: Returns the smallest $x > 0$ s.t. $a^x = b \pmod{m}$, or -1 if no such x exists. modLog(a,1,m) can be used to calculate the order of a.
Time: $\mathcal{O}\left(\sqrt{m}\right)$

c040b88d, 11 lines

```
ll modLog(ll a, ll b, ll m) {
  ll n = (ll) sqrt(m) + 1, e = 1, f = 1, j = 1;
  unordered_map<ll, ll> A;
  while (j <= n && (e = f = e * a % m) != b % m)
    A[e * b % m] = j++;
  if (e == b % m) return j;
  if (__gcd(m, e) == __gcd(m, b))
    rep(i,2,n+2) if (A.count(e = e * f % m))
      return n * i - A[e];
  return -1;
}
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \leq a, b \leq c \leq 7.2 \cdot 10^{18}$.
Time: $\mathcal{O}(1)$ for modmul, $\mathcal{O}(\log b)$ for modpow

c36634fb, 6 lines

```
using ull = uint64_t;
constexpr ll md = 1e9 + 7; // change this
auto mdMul = [&] (ull a, ull b) {
  ll ret = a * b - md * ull(1.L / md * a * b);
  return ret + md * (ret < 0) - md * (ret >= (ll)md);
};
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod{p}$ ($-x$ gives the other solution). p should be an odd prime and $0 \leq a < p$.
Time: $\mathcal{O}\left(\log^2 p\right)$ worst case, $\mathcal{O}(\log p)$ for most p

"ModPow.h" 3e061b4f, 18 lines

```
auto mdSqrt = [&](ll a) -> ll {
  if (!a) return 0;
  if (mdPow(a, md / 2) != 1) return -1;
  if (md % 4 == 3) return mdPow(a, (md + 1) / 4);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
  ll s = md - 1, n = 2;
  int r = 0, m;
  while (s % 2 == 0) ++r, s /= 2;
  while (mdPow(n, md / 2) != md - 1) ++n;
  ll x = mdPow(a, (s + 1) / 2), b = mdPow(a, s), g = mdPow(n, s
       );
  for (; ; r = m) {
    ll t = b;
    for (m = 0; m < r && t != 1; ++m) (t *= t) %= md;
    if (!m) return x;
    ll gs = mdPow(g, 1ll << (r - m - 1));
    g = gs * gs % md, (x *= gs) %= md, (b *= g) %= md;
  }
};
```

ModSum.h

Description: Sums of mod'ed arithmetic progressions.
modsum(to, c, k, m) $= \sum_{i=0}^{\text{to}-1} (ki + c) \% m$. divsum is similar but for floored division.
Time: $\log(m)$, with a large constant.

5c5bc571, 16 lines

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (!k) return res;
  ull to2 = (to * k + c) / m;
  return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k);
}

ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
}
```

ModLattice.h

Description: Counts # of lattice points (x, y) in the triangle $1 \leq x, 1 \leq y, ax + by \leq s \pmod{2^{64}}$ and related quantities.

Time: $\mathcal{O}(\log ab)$

23cbf689, 20 lines

```
using ul = uint64_t;
ul sum2(ul n) { return n/2*((n-1)|1); } // sum(0..n-1)
// \return |{(x,y) | 1 <= x, 1 <= y, a*x+b*y <= S}|
//         = sum_{i=1}^{qs} (S-a*i)/b
ul triSum(ul a, ul b, ul s) { assert(a > 0 && b > 0);
  ul qs = s/a, rs = s%a; // ans = sum_{i=0}^{qs-1}(i*a+rs)/b
  ul ad = a/b*sum2(qs)+rs/b*qs; a %= b, rs %= b;
  return ad+(a?triSum(b,a,a*qs+rs):0); // reduce if a >= b
} // then swap x and y axes and recurse

// \return sum_{x=0}^{n-1} (a*x+b)/m
//         = |{(x,y) | 0 < m*y <= a*x+b < a*n+b}|
// assuming a*n+b does not overflow
ul divSum(ul n, ul a, ul b, ul m) { assert(m > 0);
  ul extra = b/m*n; b %= m;
  return extra+(a?triSum(m,a,a*n+b):0); }
// \return sum_{x=0}^{n-1} (a*x+b)%n
ul modSum(ul n, ll a, ll b, ul m) { assert(m > 0);
  a = (a%m+m)%m, b = (b%m+m)%m;
  return a*sum2(n)+b*n-m*divSum(n,a,b,m); }
```

ModArith.h

Description: Statistics on mod'ed arithmetic series. `minBetween` and `minRemainder` both assume that $0 \le L \le R < B$, $AB < 2^{62}$

"Euclid.h" f68a6d97, 40 lines

```
ll minBetween(ll A, ll B, ll L, ll R) {
  // min x s.t. exists y s.t. L <= A*x-B*y <= R
  A %= B;
  if (L == 0) return 0;
  if (A == 0) return -1;
  ll k = cdiv(L,A); if (A*k <= R) return k;
  ll x = minBetween(B,A,A-R%A,A-L%A); // min x s.t. exists y
  // s.t. -R <= Bx-Ay <= -L
  return x == -1 ? x : cdiv(B*x+L,A); // solve for y
}

// find min((Ax+C)%B) for 0 <= x <= M
// aka find minimum non-negative value of A*x-B*y+C
// where 0 <= x <= M, 0 <= y
ll minRemainder(ll A, ll B, ll C, ll M) {
  assert(A >= 0 && B > 0 && C >= 0 && M >= 0);
  A %= B, C %= B; ckmin(M,B-1);
  if (A == 0) return C;
  if (C >= A) { // make sure C<A
    ll ad = cdiv(B-C,A);
    M -= ad; if (M < 0) return C;
    C += ad*A-B;
  }
  ll q = B/A, new_B = B%A; // new_B < A
  if (new_B == 0) return C; // B-q*A

  // now minimize A*x-new_B*y+C
  // where 0 <= x,y and x+q*y <= M, 0 <= C < new_B < A
  // q*y -> C-new_B*y
  if (C/new_B > M/q) return C-M/q*new_B;
  M -= C/new_B*q; C %= new_B; // now C < new_B

  // given y, we can compute x = ceil[((B-q*A)*y-C)/A]
  // so x+q*y = ceil((B*y-C)/A) <= M
  ll max_Y = (M*A+C)/B; // must have y <= max_Y
  ll max_X = cdiv(new_B*max_Y-C,A); // must have x <= max_X
  if (max_X*A-new_B*max_Y+C >= new_B) --max_X;
  // now we can remove upper bound on y
  return minRemainder(A,new_B,C,max_X);
}
```

5.2 Primality

FastEratosthenes.h

Description: Prime sieve for generating all primes smaller than LIM.

Time: LIM=1e9 $\approx$ 1.5s

22899edc, 24 lines

```
constexpr int LIM = 1e7;
bitset<LIM> is_pr;
auto eratosthenes = [&] {
  const int S = round(sqrt(LIM)), R = LIM / 2;
  array<bool, S + 1> sieve{};
  vc<pair<int, int>> cp;
  for (int i = 3; i <= S; i += 2) if (!sieve[i]) {
    cp.emplace_back(i,  i * i / 2);
    for (int j = i * i; j <= S; j += i * 2) sieve[j] = 1;
  }
  vc<int> pr = {2};
  pr.reserve(int(LIM / log(LIM) * 1.1));
  for (int L = 1; L <= R; L += S) {
    array<bool, S> block{};
    for (auto &[p, idx]: cp)
      for (int i = idx; i < S + L; idx = (i += p))
        block[i - L] = 1;
    for (int i = 0; i < min(S, R - L); ++i) {
      if (!block[i]) pr.emplace_back((L + i) * 2 + 1);
    }
  }
  for (int i: pr) is_pr[i] = 1;
  return pr;
};
```

EulerSieve.h

Description: Calculate some arithmetic function in linear time.

Time: $\mathcal{O}(n)$

2d119e47, 27 lines

```
constexpr int LIM = 1e6 + 1; // change this
vc<int> pr, d(LIM), w(d), mu(d), phi(d);
bitset<LIM> n_pr;
auto eulerSieve = [&] {
  pr.reserve(int(LIM / log(LIM) * 1.1));
  phi[1] = d[1] = w[1] = mu[1] = n_pr[1] = 1;
  for (int i = 2; i < LIM; ++i) {
    if (!n_pr[i]) {
      pr.emplace_back(i);
      phi[i] = i - 1, mu[i] = -1, d[i] = 2, w[i] = i + 1;
    }
    for (auto p: pr) {
      int t = p * i;
      if (t >= LIM) break;
      n_pr[t] = 1;
      if (i % p == 0) {
        phi[t] = phi[i] * p, mu[t] = 0;
        d[t] = d[i] * 2 - d[i / p];
        w[t] = w[i] + p * (w[i] - w[i / p]);
      } else {
        phi[t] = phi[i] * (p - 1), mu[t] = -mu[i];
        d[t] = d[i] * 2;
        w[t] = w[i] * (p + 1);
      }
    }
  }
};
```

MultiplicativePrefix.h

Description: $\sum_{i=1}^{N} f(i)$ where $f(i) = \prod \texttt{val}[e]$ for each p^e in the factorization of i. Must satisfy $\texttt{val}[1] = 1$. Generalizes to any multiplicative function with $f(p) = p^{\text{fixed power}}$.

Time: $\mathcal{O}\left(\sqrt{N}\right)$

"Eratosthenes.h" 3151ead4, 11 lines

```
vmi val;
mi get_prefix(ll N, int p = 0) {
  mi ans = N;
  for (; S.primes.at(p) <= N / S.primes.at(p); ++p) {
    ll new_N = N / S.primes.at(p) / S.primes.at(p);
    for (int idx = 2; new_N; ++idx, new_N /= S.primes.at(p)) {
      ans += (val.at(idx) - val.at(idx - 1)) * get_prefix(new_N
          , p + 1);
    }
  }
  return ans;
}
```

MillerRabin.h

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.

Time: 7 times the complexity of $a^b \mod c$.

"ModMulLL.h" 60dcd132, 12 lines

```
bool isPrime(ull n) {
  if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
  ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
      s = __builtin_ctzll(n-1), d = n >> s;
  for (ull a : A) {   // ^ count trailing zeroes
    ull p = modpow(a%n, d, n), i = s;
    while (p != 1 && p != n - 1 && a % n && i--)
      p = modmul(p, p, n);
    if (p != n-1 && i != s) return 0;
  }
  return 1;
}
```

Factor.h

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).

Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

"MillerRabin.h" a33cf6ef, 18 lines

```
ull pollard(ull n) {
  auto f = [n](ull x) { return modmul(x, x, n) + 1; };
  ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
  while (t++ % 40 || __gcd(prd, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
    x = f(x), y = f(f(y));
  }
  return __gcd(prd, n);
}
vector<ull> factor(ull n) {
  if (n == 1) return {};
  if (isPrime(n)) return {n};
  ull x = pollard(n);
  auto l = factor(x), r = factor(n / x);
  l.insert(l.end(), all(r));
  return l;
}
```

5.3 Divisibility

Euclid.h

Description: Finds two integers x and y, such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in __gcd instead. If a and b are coprime, then x is the inverse of $a \pmod b$.

33ba8f91, 5 lines

```
ll euclid(ll a, ll b, ll &x, ll &y) {
  if (!b) return x = 1, y = 0, a;
  ll d = euclid(b, a % b, y, x);
  return y -= a / b * x, d;
```

```
}
```

Euclid2.h

Description: finds smallest $x \geq 0$ such that $L \leq Ax \pmod P \leq R$.

0b3047f4, 9 lines

```
ll cdiv(ll x, ll y) { return (x+y-1)/y; }
ll bet(ll P, ll A, ll L, ll R) {
  if (A == 0) return L == 0 ? 0 : -1;
  ll c = cdiv(L,A); if (A*c <= R) return c;
  ll B = P%A; // P = k*A+B,  L <= A(x-Ky)-By <= R
  // => -R <= By % A <= -L
  auto y = bet(A,B,A-R%A,A-L%A);
  return y == -1 ? y : cdiv(L+B*y,A)+P/A*y;
}
```

CRT.h

Description: Chinese Remainder Theorem.

`crt(a, m, b, n)` computes x such that $x \equiv a \pmod m$, $x \equiv b \pmod n$. If $|a| < m$ and $|b| < n$, x will obey $0 \leq x < \text{lcm}(m, n)$. Assumes $mn < 2^{62}$.

Time: $\log(n)$

"euclid.h" 04d93a45, 7 lines

```
ll crt(ll a, ll m, ll b, ll n) {
  if (n > m) swap(a, b), swap(m, n);
  ll x, y, g = euclid(m, n, x, y);
  assert((a - b) % g == 0); // else no solution
  x = (b - a) % n * x % n / g * m + a;
  return x < 0 ? x + m*n/g : x;
}
```

5.3.1 Bézout's identity

For $a \neq, b \neq 0$, then $d = gcd(a, b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

5.4 Fractions

FracInterval.h

Description: Given fractions $a < b$ with non-negative numerators and denominators, finds fraction f with lowest denominator such that $a < f < b$. Should work with all numbers less than 2^{62}.

1860f3b3, 6 lines

```
pl bet(pl a, pl b) {
  ll num = a.f/a.s; a.f -= num*a.s, b.f -= num*b.s;
  if (b.f > b.s) return {1+num,1};
  auto x = bet({b.s,b.f},{a.s,a.f});
  return {x.s+num*x.f,x.f};
}
```

ContinuedFractions.h

Description: Given N and a real number $x \geq 0$, finds the closest rational approximation p/q with $p, q \leq N$. It will obey $|p/q - x| \leq 1/qN$.

For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. ($p_k/q_k$ alternates between $> x$ and $< x$.) If x is rational, y eventually becomes ∞; if x is the root of a degree 2 polynomial the a's eventually become cyclic.

Time: $\mathcal{O}(\log N)$

dd6c5e10, 21 lines

```
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
pair<ll, ll> approximate(d x, ll N) {
  ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x;
  for (;;) {
    ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf),
       a = (ll)floor(y), b = min(a, lim),
       NP = b*P + LP, NQ = b*Q + LQ;
    if (a > b) {
      // If b > a/2, we have a semi-convergent that gives us a
      // better approximation; if b = a/2, we *may* have one.
      // Return {P, Q} here for a more canonical approximation.
      return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)) ?
        make_pair(NP, NQ) : make_pair(P, Q);
    }
    if (abs(y = 1/(y - (d)a)) > 3*N) {
      return {NP, NQ};
    }
    LP = P; P = NP;
    LQ = Q; Q = NQ;
  }
}
```

FracBinarySearch.h

Description: Given f and N, finds the smallest fraction $p/q \in [0, 1]$ such that $f(p/q)$ is true, and $p, q \leq N$. You may want to throw an exception from f if it finds an exact solution, in which case N can be removed.

Usage: `fracBS([](Frac f) { return f.p>=3*f.q; }, 10); // {1,3}`

Time: $\mathcal{O}(\log(N))$

27ab3ee7, 25 lines

```
struct Frac { ll p, q; };

template<class F>
Frac fracBS(F f, ll N) {
  bool dir = 1, A = 1, B = 1;
  Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N]
  if (f(lo)) return lo;
  assert(f(hi));
  while (A || B) {
    ll adv = 0, step = 1; // move hi if dir, else lo
    for (int si = 0; step; (step *= 2) >>= si) {
      adv += step;
      Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
      if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
        adv -= step; si = 2;
      }
    }
    hi.p += lo.p * adv;
    hi.q += lo.q * adv;
    dir = !dir;
    swap(lo, hi);
    A = B; B = !!adv;
  }
  return dir ? hi : lo;
}
```

5.5 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \;\; b = k \cdot (2mn), \;\; c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

5.6 Lifting the Exponent

For $n > 0$, p prime, and ints x, y s.t. $p \nmid x, y$ and $p|x - y$:

- $p \neq 2$ or $p = 2, 4|x - y \implies v_p(x^n - y^n) = v_p(x - y) + v_p(n)$.
- $p = 2, 2|n \implies v_2(x^n - y^n) = v_2((x^2)^{n/2} - (y^2)^{n/2})$.

5.7 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

5.8 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

$n \leq$	10^1	10^2	10^3	10^4	10^5	10^6	10^7	10^8	10^9	10^{10}
$\max\{\omega(n)\}$	2	3	4	5	6	7	8	8	9	10
$\max\{d(n)\}$	4	12	32	64	128	240	448	768	1344	2304
$n \leq$	10^{11}	10^{12}	10^{13}	10^{14}	10^{15}	10^{16}	10^{17}	10^{18}		
$\max\{\omega(n)\}$	10	11	12	12	13	13	14	15		
$\max\{d(n)\}$	4032	6720	10752	17280	26880	41472	64512	103680		

5.9 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \leq m \leq n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

Combinatorial (6)

6.1 The Twelvefold Way

Counts the # of functions $f : N \to K, |N| = n, |K| = k$. The elements in N and K can be distinguishable or indistinguishable, while f can be injective (one-to-one) of surjective (onto).

N	K	none	injective	surjective
dist	dist	k^n	$\frac{k!}{(k-n)!}$	$k!S(n, k)$
indist	dist	$\binom{n+k+1}{n}$	$\binom{k}{n}$	$\binom{n-1}{n-k}$
dist	indist	$\sum_{t=0}^{k} S(n, t)$	$[n \leq k]$	$S(n, k)$
indist	indist	$\sum_{t=1}^{k} p(n, t)$	$[n \leq k]$	$p(n, k)$

Here, $S(n,k)$ is the Stirling number of the second kind, and $p(n,k)$ is the partition number.

6.2 Permutations

6.2.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
n	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

IntPerm.h
Description: Permutation -> integer conversion. (Not order preserving.) Integer -> permutation can use a lookup table.
Time: $\mathcal{O}(n)$
044568e0, 6 lines

```
int permToInt(vi& v) {
  int use = 0, i = 0, r = 0;
  for(int x:v) r = r * ++i + __builtin_popcount(use & -(1<<x)),
    use |= 1 << x;                    // (note: minus, not ~!)
  return r;
}
```

6.2.2 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n)\frac{x^n}{n!} = \exp\left(\sum_{n\in S}\frac{x^n}{n}\right)$$

6.2.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

Derangements.h
Description: Generates the i:th derangement of S_n (in lexicographical order).
155b14b9, 38 lines

```
template <class T, int N>
struct derangements {
  T dgen[N][N], choose[N][N], fac[N];
  derangements() {
    fac[0] = choose[0][0] = 1;
    memset(dgen, 0, sizeof(dgen));
    rep(m,1,N) {
      fac[m] = fac[m-1] * m;
      choose[m][0] = choose[m][m] = 1;
      rep(k,1,m)
        choose[m][k] = choose[m-1][k-1] + choose[m-1][k];
    }
  }
  T DGen(int n, int k) {
    T ans = 0;
    if (dgen[n][k]) return dgen[n][k];
    rep(i,0,k+1)
      ans += (i&1?-1:1) * choose[k][i] * fac[n-i];
    return dgen[n][k] = ans;
  }
  void generate(int n, T idx, int *res) {
    int vals[N];
    rep(i,0,n) vals[i] = i;
    rep(i,0,n) {
      int j, k = 0, m = n - i;
      rep(j,0,m) if (vals[j] > i) ++k;
      rep(j,0,m) {
        T p = 0;
        if (vals[j] > i) p = DGen(m-1, k-1);
        else if (vals[j] < i) p = DGen(m-1, k);
        if (idx <= p) break;
        idx -= p;
      }
      res[i] = vals[j];
      memmove(vals + j, vals + j + 1, sizeof(int)*(m-j-1));
    }
  }
};
```

6.2.4 Involutions

An involution is a permutation with maximum cycle length 2, and it is its own inverse.

$$a(n) = a(n-1) + (n-1)a(n-2)$$

$$a(0) = a(1) = 1$$

1,1,2,4,10,26,76,232,764,2620,9496,35696,140152

6.2.5 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|}\sum_{g\in G}|X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n}\sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n}\sum_{k|n} f(k)\phi(n/k).$$

6.3 Partitions and subsets

6.3.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(n,k) = p(n-1,k-1) + p(n-k,k)$$

$$p(0,0) = p(1,n) = p(n,n) = p(n,n-1) = 1$$

For partitions with any number of parts, $p(n)$ obeys

$$p(0) = 1,\ p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}} (-1)^{k+1} p(n-k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

6.3.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod p$.

6.3.3 Binomials

multinomial.h
Description: Computes $\binom{k_1+\cdots+k_n}{k_1,k_2,\ldots,k_n} = \frac{(\sum k_i)!}{k_1!k_2!\ldots k_n!}$.
a0a3128f, 6 lines

```
ll multinomial(vi& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  rep(i,1,sz(v)) rep(j,0,v[i])
    c = c * ++m / (j+1);
  return c;
}
```

6.4 General purpose numbers

6.4.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t-1}$ (FFT-able).
$B[0,\ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!}f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

6.4.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k)$$

$$c(0,0) = 1, c(0,n) = c(n,0) = 0$$

$$\textstyle\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

6.4.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

6.4.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

6.4.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

6.4.6 Labeled unrooted trees

\# on n vertices: n^{n-2}
\# on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
\# with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

6.4.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

6.5 Young Tableaux

Let a **Young diagram** have shape $\lambda = (\lambda_1 \geq \cdots \geq \lambda_k)$, where λ_i equals the number of cells in the i-th (left-justified) row from the top. A **Young tableau** of shape λ is a filling of the $n = \sum \lambda_i$ cells with a permutation of $1 \ldots n$ such that each row and column is increasing.

Hook-Length Formula: For the cell in position (i,j), let $h_\lambda(i,j) = |\{(I,J)|i \leq I, j \leq J, (I = i \text{ or } J = j)\}|$. The number of Young tableaux of shape λ is equal to $f^\lambda = \frac{n!}{\prod h_\lambda(i,j)}$.

Schensted's Algorithm: converts a permutation σ of length n into a pair of Young Tableaux $(S(\sigma), T(\sigma))$ of the same shape. When inserting $x = \sigma_i$,

1. Add x to the first row of S by inserting x in place of the largest y with $x < y$. If y doesn't exist, push x to the end of the row, set the value of T at that position to be i, and stop.
2. Add y to the second row using the same rule, keep repeating as necessary.

All pairs $(S(\sigma), T(\sigma))$ of the same shape correspond to a unique σ, so $n! = \sum (f^\lambda)^2$. Also, $S(\sigma^R) = S(\sigma)^T$.

Let $d_k(\sigma), a_k(\sigma)$ be the lengths of the longest subseqs which are a union of k decreasing/ascending subseqs, respectively. Then $a_k(\sigma) = \sum_{i=1}^{k} \lambda_i, d_k(\sigma) = \sum_{i=1}^{k} \lambda_i^*$, where λ_i^* is size of the i-th column.

6.6 General prupose theorems

6.6.1 Identities

Vandermonde Convolution: $\binom{m+n}{r} = \sum_{k=0}^{r}\binom{m}{k}\cdot\binom{r}{n-k}$.

Hockey Stick: $\binom{n+1}{r+1} = \sum_{i=r}^{n}\binom{i}{r}$.

6.6.2 Cycle Lemma

Any sequence of mX's and nY's, where $m > n$ has exactly $m-n$ cyclic permutations which are dominating, and $m - kn$ which are k-dominating. To find them, arrange sequence in a circle and repeatedly remove adjacent pairs XY. The remaining X's were each the start of a dominating permutation.

6.6.3 Sprague Grundy theorem

Every impartial game is equivalent to a nimber. Nimbers are de-fined inductively as
$*0 = \{\}, *1 = *0, *2 = *0, *1, *(n+1) = *n \cup n$, and corresponds to a heap of size n. The formula for adding positions is

$$S + S' = S + s' | s' \in S' \cup s + S' | s \in S$$

$$a + b = a \oplus b + 2(a \& b)$$

Define minimum exclusion $M : \phi(N) \to N$ by $M(S) =$ the least non-negative integer not in S. Let
$C = (M(A) \oplus B) \cup (M(B) \oplus A)$. Then $M(C) = M(A) \oplus M(B)$. Define $SG(S) = M(\{SG(s)|s \in S\})$. $SG(Nim_k) = k$ by strong induction. Game is losing iff $SG(S) = 0$. Theorem:
$SG(A + B) = SG(A) \oplus SG(B)$.

6.6.4 Partisan Game

Can define the negative of a game by interchanging L and R's possible moves. Define $G = 0$ if first player loses. $G = H$ if $G + (-H) = 0$. A cold game is one which moving only hurts players. In this case we never have G fuzzy 0, so G is representable as an integer, thus calculable by DP.

6.6.5 Matrices for operators

Matrices for xor, and, and or are: $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}, \begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}$ with inverses: $\begin{bmatrix}-1 & 1\\ 1 & 0\end{bmatrix}, \begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$.

6.6.6 Prufer sequences

The set of labeled trees on n vertices corresponds bijectively to the set of Prufer sequences of length $n-2$. To convert a tree into a Prufer sequence, repeatedly remove the leaf with the smallest label, and write down its neighbor. To convert sequence to tree, first set the degree of each vertex to $n_v + 1$, where n_v is the number of times the vertex appears in the sequence. Then for each i, find lowest j with degree 1, add edge a_i, j, and decrease the degrees of a_i and j by 1. After this, two nodes of degree 1 remain - connect them.

This can be used to calculate number of labeled trees in a complete bipartite graph - $l^{r-1} \cdot r^{l-1}$.

6.6.7 Tournament Graphs

There exists a Hamiltonian path on any tournament graphs - use induction to find. Cycle if strongly connected. TFAE: T is 1. transitive. 2. is strict total ordering. 3. is acyclic. 4. has no cycle of length 3. 5. The outdegrees are $\{0, 1, \cdots, n-1\}$. 6. has exactly one Hamiltonian path.

6.6.8 Landau's theorem

A sequence of numbers is called a score sequence if for each subset S, sum of numbers in S is at least $\binom{|S|}{2}$ and sum of all numbers is $\binom{n}{2}$.

This score sequence represents the outdegrees of a vertex in a tournament graph.

6.6.9 Dilworth's / Hall's / Mirsky's theorem

Maximum antichain has same size as minimum chain decomposition.

Maximum chain size has same size as minimum antichain decomposition.

To compute size, model as bipartite graph with two copies of vertices - $v_i n$ and $v_o ut$. Distinct representatives can be chosen for a family of sets S iff every subfamily W of S has at least $|W|$ elements in their union. E.g. Left side of bipartite graph can be fully matched iff each subset has sufficient "degree".

6.6.10 Laplacian Matrix and Kirchoff's Theorem

Laplacian matrix is defined as $L = D - A$, where D is the degree matrix (diagonal), and A the adjacency matrix.

Kirchoff's Theorem states that the number of spanning trees in a graph is any cofactor of the Laplacian.

To calculate that, remove the first row and column and calculate the determinant of the remaining matrix.

6.7 Other

NimProduct.h

Description: Product of nimbers is associative, commutative, and distributive over addition (xor). Forms finite field of size 2^{2^k}. Defined by $ab = \text{mex}(\{a'b + ab' + a'b' : a' < a, b' < b\})$. Application: Given 1D coin turning games G_1, G_2 $G_1 \times G_2$ is the 2D coin turning game defined as follows. If turning coins at $x_1, x_2, \ldots, x_m$ is legal in G_1 and $y_1, y_2, \ldots, y_n$ is legal in G_2, then turning coins at all positions (x_i, y_j) is legal assuming that the coin at (x_m, y_n) goes from heads to tails. Then the grundy function $g(x, y)$ of $G_1 \times G_2$ is $g_1(x) \times g_2(y)$.

9bba25d6, 17 lines

```
using ull = uint64_t;
ull _nimProd2[64][64];
ull nimProd2(int i, int j) {
  if (_nimProd2[i][j]) return _nimProd2[i][j];
  if ((i & j) == 0) return _nimProd2[i][j] = 1ull << (i|j);
  int a = (i&j) & -(i&j);
  return _nimProd2[i][j] = nimProd2(i ^ a, j) ^ nimProd2((i ^ a
      ) | (a-1), (j ^ a) | (i & (a-1)));
}
ull nimProd(ull x, ull y) {
  ull res = 0;
  for (int i = 0; (x >> i) && i < 64; i++)
    if ((x >> i) & 1)
      for (int j = 0; (y >> j) && j < 64; j++)
        if ((y >> j) & 1)
          res ^= nimProd2(i, j);
  return res;
}
```

DeBruijnSeq.h

Description: Recursive FKM, given alphabet $[0, k)$ constructs cyclic string of length k^n that contains every length n string as substr.

a7faa508, 13 lines

```
vi dseq(int k, int n) {
  if (k == 1) return {0};
  vi res, aux(n+1);
  function<void(int,int)> gen = [&](int t, int p) {
    if (t > n) { // consider lyndon word of len p
      if (n%p == 0) FOR(i,1,p+1) res.pb(aux[i]);
    } else {
      aux[t] = aux[t-p]; gen(t+1,p);
      FOR(i,aux[t-p]+1,k) aux[t] = i, gen(t+1,t);
    }
  };
  gen(1,1); return res;
}
```

MatroidIntersect.h

Description: Computes a set of maximum size which is independent in both graphic and colorful matroids, aka a spanning forest where no two edges are of the same color. In general, construct the exchange graph and find a shortest path. Can apply similar concept to partition matroid.
Usage: `MatroidIsect<Gmat,Cmat> M(sz(ed),Gmat(ed),Cmat(col))`
Time: $\mathcal{O}\left(GI^{1.5}\right)$ calls to oracles, where G is size of ground set and I is size of independent set.

"../graphs (12)/DSU/DSU (7.6).h" d0051cd5, 51 lines

```
struct Gmat { // graphic matroid
  int V = 0; vpi ed; DSU D;
  Gmat(vpi _ed):ed(_ed) {
    map<int,int> m; each(t,ed) m[t.f] = m[t.s] = 0;
    each(t,m) t.s = V++;
    each(t,ed) t.f = m[t.f], t.s = m[t.s];
  }
  void clear() { D.init(V); }
  void ins(int i) { assert(D.unite(ed[i].f,ed[i].s)); }
  bool indep(int i) { return !D.sameSet(ed[i].f,ed[i].s); }
};
struct Cmat { // colorful matroid
  int C = 0; vi col; V<bool> used;
  Cmat(vi col):col(col) {each(t,col) ckmax(C,t+1); }
  void clear() { used.assign(C,0); }
  void ins(int i) { used[col[i]] = 1; }
  bool indep(int i) { return !used[col[i]]; }
};
template<class M1, class M2> struct MatroidIsect {
  int n; V<bool> iset; M1 m1; M2 m2;
  bool augment() {
    vi pre(n+1,-1); queue<int> q({n});
    while (sz(q)) {
      int x = q.ft; q.pop();
      if (iset[x]) {
        m1.clear(); F0R(i,n) if (iset[i] && i != x) m1.ins(i);
        F0R(i,n) if (!iset[i] && pre[i] == -1 && m1.indep(i))
          pre[i] = x, q.push(i);
      } else {
        auto backE = [&]() { // back edge
          m2.clear();
          F0R(c,2)F0R(i,n)if((x==i||iset[i])&&(pre[i]==-1)==c){
            if (!m2.indep(i))return c?pre[i]=x,q.push(i),i:-1;
            m2.ins(i); }
          return n;
        };
        for (int y; (y = backE()) != -1;) if (y == n) {
          for(; x != n; x = pre[x]) iset[x] = !iset[x];
          return 1; }
      }
    }
    return 0;
  }
  MatroidIsect(int n, M1 m1, M2 m2):n(n), m1(m1), m2(m2) {
    iset.assign(n+1,0); iset[n] = 1;
    m1.clear(); m2.clear(); // greedily add to basis
    R0F(i,n) if (m1.indep(i) && m2.indep(i))
      iset[i] = 1, m1.ins(i), m2.ins(i);
    while (augment());
  }
};
```

Graph (7)

7.1 Fundamentals

BellmanFord.h

Description: Calculates shortest paths from s in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes $V^2 \max|w_i| < \sim 2^{63}$.
Time: $\mathcal{O}(VE)$

7a4810de, 27 lines

```
constexpr ll INF = numeric_limits<ll>::max();

struct Ed {
  int a, b, w, s() { return a < b ? a : -a; }
  Ed(int a, int b, int w): a(a), b(b), w(w) {}
};
struct Node { ll dist = INF; int prev = -1; };

auto bellmanFord = [](vc<Node> &nodes, vc<Ed> &eds, int s) {
  nodes[s].dist = 0;
  sort(all(eds), [](Ed a, Ed b) { return a.s() < b.s(); });

  int lim = sz(nodes) / 2 + 2; // /3+100 with shuffled vertices
  for (int i = 0; i < lim; ++i) for (Ed ed: eds) {
    Node cur = nodes[ed.a], &dest = nodes[ed.b];
    if (abs(cur.dist) == INF) continue;
    ll d = cur.dist + ed.w;
    if (d < dest.dist) {
      dest.prev = ed.a;
      dest.dist = (i < lim - 1 ? d : -INF);
    }
  }
  for (int i = 0; i < lim; ++i) for (Ed e : eds) {
    if (nodes[e.a].dist == -INF)
      nodes[e.b].dist = -INF;
  }
}; // 96199983
```

FloydWarshall.h

Description: Calculates all-pairs shortest path in a directed graph that might have negative edge weights. Input is an distance matrix m, where $m[i][j] = \texttt{inf}$ if i and j are not adjacent. As output, $m[i][j]$ is set to the shortest distance between i and j, inf if no path, or -inf if the path goes through a negative-weight cycle.
Time: $\mathcal{O}\left(N^3\right)$

379909b1, 14 lines

```
constexpr ll INF = numeric_limits<ll>::max();

auto floydWarshall = [](vc<vc<ll>> &m) {
  int n = sz(m);
  for (int i = 0; i < n; ++i) m[i][i] = min(m[i][i], 0ll);
  for (int k = 0; k < n; ++k)
    for (int i = 0; i < n; ++i)
      for (int j = 0; j < n; ++j)
        if (m[i][k] != INF && m[k][j] != INF)
          m[i][j] = min(m[i][j], max(m[i][k] + m[k][j], -INF));
  for (int k = 0; k < n; ++k) if (m[k][k] < 0)
    for (int i = 0; i < n; ++i) for (int j = 0; j < n; ++j)
      if (m[i][k] != INF && m[k][j] != INF) m[i][j] = -INF;
}; // ca5992f4
```

7.2 Network flow

PushRelabel.h

Description: Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only.
Time: $\mathcal{O}\left(V^2\sqrt{E}\right)$

18d53978, 62 lines

```
struct PushRelabel {
  struct Edge {
    int v, rev;
    ll f, c;
  };
  vc<vc<Edge>> g;
  vc<int> h, gap;
  vc<ll> ec;
  vc<vc<int>> hv;
  vc<Edge*> cur;
  PushRelabel(int n):
    g(n), ec(n), h(n), cur(n), hv(n * 2), gap(n * 2) {}
  void addEdge(int x, int y, ll c, ll rc = 0) {
    if (x == y) return;
    Edge a{y, sz(g[y]), 0, c}, b{x, sz(g[x]), 0, rc};
    g[x].emplace_back(a);
    g[y].emplace_back(b);
  }
  void addFlow(Edge &e, ll f) {
    Edge &re = g[e.v][e.rev];
    if (!ec[e.v] && f) hv[h[e.v]].emplace_back(e.v);
    e.f += f, e.c -= f, ec[e.v] += f;
    re.f -= f, re.c += f, ec[re.v] -= f;
  }
```

```
  ll maxFlow(int S, int T) {
    int n = sz(g);
    h[S] = n, ec[T] = 1, gap[0] = n - 1;
    for (int i = 0; i < n; ++i) cur[i] = g[i].data();
    for (auto &e: g[S]) addFlow(e, e.c);
    if (hv[0].empty()) return 0;
    for (int h_now = 0; h_now >= 0; ) {
      int u = hv[h_now].back();
      hv[h_now].pop_back();
      while (ec[u] > 0) {
        if (cur[u] == g[u].data() + sz(g[u])) {
          h[u] = numeric_limits<int>::max();
          for (auto &e : g[u]) {
            if (e.c && h[u] > h[e.v] + 1) {
              h[u] = h[e.v] + 1;
              cur[u] = &e;
            }
          }
          ++gap[h[u]];
          if(!--gap[h_now] && h_now < n) {
            for (int i = 0; i < n; ++i) {
              if(h_now < h[i] && h[i] < n) {
                --gap[h[i]];
                h[i] = n + 1;
              }
            }
          }
          h_now = h[u];
        } else if (cur[u]->c && h[u] == h[cur[u]->v] + 1) {
          addFlow(*cur[u], min(ec[u], cur[u]->c));
        } else ++cur[u];
      }
      while (h_now >= 0 && hv[h_now].empty()) --h_now;
    }
    return -ec[S];
  }
  bool leftOfMinCut (int a) { return h[a] >= sz(g); }
};
```

MinCostMaxFlow.h

Description: Min-cost max-flow. Note that negative cost cycles are not supported.
Time: Approximately $\mathcal{O}\left(E^2\right)$

c7cdbcb7, 91 lines

```
#include <bits/extc++.h>

constexpr ll INF = numeric_limits<ll>::max() / 2;

struct MCMF {
  struct Edge {
    int v, back;
    ll f, c;
  };
  int s, t;
  vc<vc<Edge>> g;
  vc<ll> dis, p;
  vc<int> vis;
  MCMF(int n): g(n), dis(n), p(n), vis(n) {}
  void add_edge(int x, int y, ll f, ll c) {
    Edge a{y, sz(g[y]), f, c}, b{x, sz(g[x]), 0, -c};
    g[x].emplace_back(a);
    g[y].emplace_back(b);
  };
  bool setpi() {
    queue<int> Q;
    fill(all(dis), INF);
    dis[t] = 0;
    Q.emplace(t);
    while (!Q.empty()) {
      int u = Q.front();
      Q.pop();
      vis[u] = 0;
      for (auto e: g[u]) {
        auto re = g[e.v][e.back];
        if (re.f && dis[e.v] > dis[u] + re.c) {
          dis[e.v] = dis[u] + re.c;
          if (!vis[e.v]) {
            vis[e.v] = 1;
            Q.emplace(e.v);
          }
        }
      }
    }
    return dis[s] != INF;
  }
  bool path() {
    fill(all(dis), INF);
    __gnu_pbds::priority_queue<pair<ll, int>> Q;
    vc<decltype(Q)::point_iterator> it(sz(p));
    dis[t] = 0;
    it[t] = Q.push({-dis[t], t});
    while (!Q.empty()) {
      int u = Q.top().second;
      Q.pop();
      for (auto e: g[u]) {
        auto re = g[e.v][e.back];
        if (re.f && dis[e.v] > dis[u] + re.c) {
          dis[e.v] = dis[u] + re.c;
          if (it[e.v] == Q.end())
            it[e.v] = Q.push({-dis[e.v], e.v});
          else Q.modify(it[e.v], {-dis[e.v], e.v});
        }
      }
    }
    return dis[s] != INF;
  }
  ll dfs(int u, ll flow) {
    if (u == t || !flow) return flow;
    vis[u] = 1;
    ll w = flow;
    for (auto &e: g[u])
      if (e.f && !vis[e.v] && !e.c) {
        ll tmp = dfs(e.v, min(e.f, w));
        w -= tmp, e.f -= tmp, g[e.v][e.back].f += tmp;
        if (!w) return flow;
      }
    return flow - w;
  }
  pair<ll, ll> mincost_flow(int S, int T) {
    s = S, t = T;
    if (!setpi()) return {};
    ll flow = 0, cost = 0, delta = 0, tmp;
    do {
      for (int u = 0; u < sz(g); ++u)
        for (auto &e: g[u]) e.c += dis[e.v] - dis[u];
      delta += dis[s];
      do {
        fill(all(vis), 0);
        tmp = dfs(s, INF);
        flow += tmp, cost += tmp * delta;
      }while (tmp);
    }while (path());
    return {flow, cost};
  }
};
```

MinCut.h

Description: After running max-flow, the left side of a min-cut from s to t is given by all vertices reachable from s, only traversing edges with positive residual capacity.

GlobalMinCut.h

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.
Time: $\mathcal{O}\left(V^3\right)$

8b0e19d6, 21 lines

```
pair<int, vi> globalMinCut(vector<vi> mat) {
  pair<int, vi> best = {INT_MAX, {}};
  int n = sz(mat);
  vector<vi> co(n);
  rep(i,0,n) co[i] = {i};
  rep(ph,1,n) {
    vi w = mat[0];
    size_t s = 0, t = 0;
    rep(it,0,n-ph) { // O(V^2) -> O(E log V) with prio. queue
      w[t] = INT_MIN;
      s = t, t = max_element(all(w)) - w.begin();
      rep(i,0,n) w[i] += mat[t][i];
    }
    best = min(best, {w[t] - mat[t][t], co[t]});
    co[s].insert(co[s].end(), all(co[t]));
    rep(i,0,n) mat[s][i] += mat[t][i];
    rep(i,0,n) mat[i][s] = mat[s][i];
    mat[0][t] = INT_MIN;
  }
  return best;
}
```

GomoryHu.h

Description: Given a list of edges representing an undirected flow graph, returns edges of the Gomory-Hu tree. The max flow between any pair of vertices is given by minimum edge weight along the Gomory-Hu tree path.
Time: $\mathcal{O}\left(V\right)$ Flow Computations

"PushRelabel.h" 0418b3cd, 13 lines

```
typedef array<ll, 3> Edge;
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
  vector<Edge> tree;
  vi par(N);
  rep(i,1,N) {
    PushRelabel D(N); // Dinic also works
    for (Edge t : ed) D.addEdge(t[0], t[1], t[2], t[2]);
    tree.push_back({i, par[i], D.calc(i, par[i])});
    rep(j,i+1,N)
      if (par[j] == par[i] && D.leftOfMinCut(j)) par[j] = i;
  }
  return tree;
}
```

7.3 Matching

HopcroftKarp.h

Description: Fast bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.
Usage: vi btoa(m, -1); hopcroftKarp(g, btoa);
Time: $\mathcal{O}\left(\sqrt{V}E\right)$

f612e443, 42 lines

```
bool dfs(int a, int L, vector<vi>& g, vi& btoa, vi& A, vi& B) {
  if (A[a] != L) return 0;
  A[a] = -1;
  for (int b : g[a]) if (B[b] == L + 1) {
    B[b] = 0;
    if (btoa[b] == -1 || dfs(btoa[b], L + 1, g, btoa, A, B))
```

```
      return btoa[b] = a, 1;
  }
  return 0;
}

int hopcroftKarp(vector<vi>& g, vi& btoa) {
  int res = 0;
  vi A(g.size()), B(btoa.size()), cur, next;
  for (;;) {
    fill(all(A), 0);
    fill(all(B), 0);
    cur.clear();
    for (int a : btoa) if(a != -1) A[a] = -1;
    rep(a,0,sz(g)) if(A[a] == 0) cur.push_back(a);
    for (int lay = 1;; lay++) {
      bool islast = 0;
      next.clear();
      for (int a : cur) for (int b : g[a]) {
        if (btoa[b] == -1) {
          B[b] = lay;
          islast = 1;
        }
        else if (btoa[b] != a && !B[b]) {
          B[b] = lay;
          next.push_back(btoa[b]);
        }
      }
      if (islast) break;
      if (next.empty()) return res;
      for (int a : next) A[a] = lay;
      cur.swap(next);
    }
    rep(a,0,sz(g))
      res += dfs(a, 0, g, btoa, A, B);
  }
}
```

DFSMatching.h

Description: Simple bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.
Usage: `vi btoa(m, -1); dfsMatching(g, btoa);`
Time: $\mathcal{O}(VE)$

522b9861, 22 lines

```
bool find(int j, vector<vi>& g, vi& btoa, vi& vis) {
  if (btoa[j] == -1) return 1;
  vis[j] = 1; int di = btoa[j];
  for (int e : g[di])
    if (!vis[e] && find(e, g, btoa, vis)) {
      btoa[e] = di;
      return 1;
    }
  return 0;
}
int dfsMatching(vector<vi>& g, vi& btoa) {
  vi vis;
  rep(i,0,sz(g)) {
    vis.assign(sz(btoa), 0);
    for (int j : g[i])
      if (find(j, g, btoa, vis)) {
        btoa[j] = i;
        break;
      }
  }
  return sz(btoa) - (int)count(all(btoa), -1);
}
```

MinimumVertexCover.h

Description: Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

"DFSMatching.h" da419632, 20 lines

```
vi cover(vector<vi>& g, int n, int m) {
  vi match(m, -1);
  int res = dfsMatching(g, match);
  vector<bool> lfound(n, true), seen(m);
  for (int it : match) if (it != -1) lfound[it] = false;
  vi q, cover;
  rep(i,0,n) if (lfound[i]) q.push_back(i);
  while (!q.empty()) {
    int i = q.back(); q.pop_back();
    lfound[i] = 1;
    for (int e : g[i]) if (!seen[e] && match[e] != -1) {
      seen[e] = true;
      q.push_back(match[e]);
    }
  }
  rep(i,0,n) if (!lfound[i]) cover.push_back(i);
  rep(i,0,m) if (seen[i]) cover.push_back(n+i);
  assert(sz(cover) == res);
  return cover;
}
```

WeightedMatching.h

Description: Min cost perfect bipartite matching. Negate costs for max cost.
Time: $\mathcal{O}(N^3)$

64268b4e, 57 lines

```
template<typename T>
int MinAssignment(const vector<vector<T>> &c) {
  int n = c.size(), m = c[0].size();       // assert(n<=m);
  vector<T> v(m), dist(m);                 // v: potential
  vector<int> L(n, -1), R(m, -1);          // matching pairs
  vector<int> index(m), prev(m);
  iota(index.begin(), index.end(), 0);

  auto residue = [&](int i, int j) { return c[i][j] - v[j]; };
  for (int f = 0; f < n; ++f) {
    for (int j = 0; j < m; ++j) {
      dist[j] = residue(f, j); prev[j] = f;
    }
    T w; int j, l;
    for (int s = 0, t = 0;;) {
      if (s == t) {
        l = s; w = dist[index[t++]];
        for (int k = t; k < m; ++k) {
          j = index[k]; T h = dist[j];
          if (h <= w) {
            if (h < w) { t = s; w = h; }
            index[k] = index[t]; index[t++] = j;
          }
        }
        for (int k = s; k < t; ++k) {
          j = index[k];
          if (R[j] < 0) goto aug;
        }
      }
      int q = index[s++], i = R[q];
      for (int k = t; k < m; ++k) {
        j = index[k];
        T h = residue(i,j) - residue(i,q) + w;
        if (h < dist[j]) {
          dist[j] = h; prev[j] = i;
          if (h == w) {
            if (R[j] < 0) goto aug;
            index[k] = index[t]; index[t++] = j;
          }
        }
      }
    }
  aug:
    for(int k = 0; k < l; ++k)
      v[index[k]] += dist[index[k]] - w;
    int i;
    do {
      R[j] = i = prev[j];
      swap(j, L[i]);
    } while (i != f);
  }
  T ret = 0;
  for (int i = 0; i < n; ++i) {
    ret += c[i][L[i]]; // (i, L[i]) is a solution
  }
  return ret;
}
```

GaleShapley.h

Description: Gale-Shapley algorithm for the stable marriage problem. madj[i][j] is the jth highest ranked woman for man i. fpref[i][j] is the rank woman i assigns to man j. Returns a pair of vectors(mpart, fpart), where mpart[i] gives the partner of man i, and fpart is analogous.
Time: $\mathcal{O}(n^2)$

ac5ecc71, 26 lines

```
pair<vector<int>, vector<int>> stable_marriage(vector<vector<
    int>> & madj, vector<vector<int>>& fpref) {
  int n = madj.size();
  vector<int> mpart(n, -1), fpart(n, -1);
  vector<int> midx(n);
  queue<int> mfree;
  for (int i = 0; i < n; i++) {
    mfree.push(i);
  }
  while (!mfree.empty()) {
    int m = mfree.front();
    mfree.pop();
    int f = madj[m][midx[m]++];
    if (fpart[f] == -1) {
      mpart[m] = f;
      fpart[f] = m;
    } else if (fpref[f][m] < fpref[f][fpart[f]]) {
      mpart[fpart[f]] = -1;
      mfree.push(fpart[f]);
      mpart[m] = f;
      fpart[f] = m;
    } else {
      mfree.push(m);
    }
  }
  return {mpart, fpart};
}
```

GeneralMatching.h

Description: Matching for general graphs. Fails with probability N/mod.
Time: $\mathcal{O}(N^3)$

"../numerical/MatrixInverse-mod.h" cb191272, 40 lines

```
vector<pii> generalMatching(int N, vector<pii>& ed) {
  vector<vector<ll>> mat(N, vector<ll>(N)), A;
  for (pii pa : ed) {
    int a = pa.first, b = pa.second, r = rand() % mod;
    mat[a][b] = r, mat[b][a] = (mod - r) % mod;
  }

  int r = matInv(A = mat), M = 2*N - r, fi, fj;
  assert(r % 2 == 0);
```

```
  if (M != N) do {
    mat.resize(M, vector<ll>(M));
    rep(i,0,N) {
      mat[i].resize(M);
      rep(j,N,M) {
        int r = rand() % mod;
        mat[i][j] = r, mat[j][i] = (mod - r) % mod;
      }
    }
  } while (matInv(A = mat) != M);

  vi has(M, 1); vector<pii> ret;
  rep(it,0,M/2) {
    rep(i,0,M) if (has[i])
      rep(j,i+1,M) if (A[i][j] && mat[i][j]) {
        fi = i; fj = j; goto done;
    } assert(0); done:
    if (fj < N) ret.emplace_back(fi, fj);
    has[fi] = has[fj] = 0;
    rep(sw,0,2) {
      ll a = modpow(A[fi][fj], mod-2);
      rep(i,0,M) if (has[i] && A[i][fj]) {
        ll b = A[i][fj] * a % mod;
        rep(j,0,M) A[i][j] = (A[i][j] - A[fi][j] * b) % mod;
      }
      swap(fi,fj);
    }
  }
  return ret;
}
```

7.4 DFS algorithms

SCC.h

Description: Finds strongly connected components in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.

Usage: `scc(graph, [&](vi& v) { ... })` visits all components in reverse topological order. `comp[i]` holds the component index of a node (a component only has edges to components with lower index). `ncomps` will contain the number of components.

Time: $\mathcal{O}(E+V)$

a356aae9, 21 lines

```
int tim = 0;
vc<int> low(n), comp(n, -1), st, topo;
function<int(int)> dfs = [&](int u) {
  int lw = low[u] = ++tim, x;
  st.emplace_back(u);
  for (int v: g[u]) if (comp[v] < 0)
    lw = min(lw, low[v] ?: dfs(v));

  if (lw == low[u]) {
    topo.emplace_back(u);
    do {
      comp[x = st.back()] = u;
      st.pop_back();
    } while (x != u);
  }
  return low[u] = lw;
};
auto tarjan = [&] {
  for (int i = 0; i < n; ++i) if (comp[i] < 0) dfs(i);
  reverse(all(topo));
};
```

BiconnectedComponents.h

Description: Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.

Usage: `int eid = 0; ed.resize(N);`
`for each edge (a,b) {`
`ed[a].emplace_back(b, eid);`
`ed[b].emplace_back(a, eid++); }`
`bicomps([&](const vi& edgelist) {...});`

Time: $\mathcal{O}(E+V)$

2965e537, 33 lines

```
vi num, st;
vector<vector<pii>> ed;
int Time;
template<class F>
int dfs(int at, int par, F& f) {
  int me = num[at] = ++Time, e, y, top = me;
  for (auto pa : ed[at]) if (pa.second != par) {
    tie(y, e) = pa;
    if (num[y]) {
      top = min(top, num[y]);
      if (num[y] < me)
        st.push_back(e);
    } else {
      int si = sz(st);
      int up = dfs(y, e, f);
      top = min(top, up);
      if (up == me) {
        st.push_back(e);
        f(vi(st.begin() + si, st.end()));
        st.resize(si);
      }
      else if (up < me) st.push_back(e);
      else { /* e is a bridge */ }
    }
  }
  return top;
}

template<class F>
void bicomps(F f) {
  num.assign(sz(ed), 0);
  rep(i,0,sz(ed)) if (!num[i]) dfs(i, -1, f);
}
```

2SAT.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a||b)\&\&(!a||c)\&\&(d||!b)\&\&\ldots$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).

Usage: `TwoSat ts(number of boolean variables);`
`ts.either(0, ~3); // Var 0 is true or var 3 is false`
`ts.setValue(2); // Var 2 is true`
`ts.atMostOne({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true`
`ts.solve(); // Returns true iff it is solvable`
`ts.values[0..N-1] holds the assigned values to the vars`

Time: $\mathcal{O}(N+E)$, where N is the number of boolean variables, and E is the number of clauses.

5f970691, 56 lines

```
struct TwoSat {
  int N;
  vector<vi> gr;
  vi values; // 0 = false, 1 = true

  TwoSat(int n = 0) : N(n), gr(2*n) {}

  int addVar() { // (optional)
    gr.emplace_back();
    gr.emplace_back();
    return N++;
  }

  void either(int f, int j) {
    f = max(2*f, -1-2*f);
    j = max(2*j, -1-2*j);
    gr[f].push_back(j^1);
    gr[j].push_back(f^1);
  }
  void setValue(int x) { either(x, x); }

  void atMostOne(const vi& li) { // (optional)
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    rep(i,2,sz(li)) {
      int next = addVar();
      either(cur, ~li[i]);
      either(cur, next);
      either(~li[i], next);
      cur = ~next;
    }
    either(cur, ~li[1]);
  }

  vi val, comp, z; int time = 0;
  int dfs(int i) {
    int low = val[i] = ++time, x; z.push_back(i);
    for(int e : gr[i]) if (!comp[e])
      low = min(low, val[e] ?: dfs(e));
    if (low == val[i]) do {
      x = z.back(); z.pop_back();
      comp[x] = low;
      if (values[x>>1] == -1)
        values[x>>1] = x&1;
    } while (x != i);
    return val[i] = low;
  }

  bool solve() {
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;
    rep(i,0,2*N) if (!comp[i]) dfs(i);
    rep(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;
    return 1;
  }
};
```

EulerWalk.h

Description: Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.

Time: $\mathcal{O}(V+E)$

780b64a9, 15 lines

```
vi eulerWalk(vector<vector<pii>>& gr, int nedges, int src=0) {
  int n = sz(gr);
  vi D(n), its(n), eu(nedges), ret, s = {src};
  D[src]++; // to allow Euler paths, not just cycles
  while (!s.empty()) {
    int x = s.back(), y, e, &it = its[x], end = sz(gr[x]);
    if (it == end){ ret.push_back(x); s.pop_back(); continue; }
    tie(y, e) = gr[x][it++];
    if (!eu[e]) {
      D[x]--, D[y]++;
      eu[e] = 1; s.push_back(y);
    }}
  for (int x : D) if (x < 0 || sz(ret) != nedges+1) return {};
```

```
  return {ret.rbegin(), ret.rend()};
}
```

7.5 Coloring

EdgeColoring.h

Description: Given a simple, undirected graph with max degree D, computes a $(D+1)$-coloring of the edges such that no neighboring edges share a color. (D-coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)

Time: $\mathcal{O}(NM)$

e210e25f, 31 lines

```
vi edgeColoring(int N, vector<pii> eds) {
  vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
  for (pii e : eds) ++cc[e.first], ++cc[e.second];
  int u, v, ncols = *max_element(all(cc)) + 1;
  vector<vi> adj(N, vi(ncols, -1));
  for (pii e : eds) {
    tie(u, v) = e;
    fan[0] = v;
    loc.assign(ncols, 0);
    int at = u, end = u, d, c = free[u], ind = 0, i = 0;
    while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)
      loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
    cc[loc[d]] = c;
    for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd])
      swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
    while (adj[fan[i]][d] != -1) {
      int left = fan[i], right = fan[++i], e = cc[i];
      adj[u][e] = left;
      adj[left][e] = u;
      adj[right][e] = -1;
      free[right] = e;
    }
    adj[u][d] = fan[i];
    adj[fan[i]][d] = u;
    for (int y : {fan[0], u, end})
      for (int& z = free[y] = 0; adj[y][z] != -1; z++);
  }
  rep(i,0,sz(eds))
    for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i];
  return ret;
}
```

7.6 Heuristics

MaximalCliques.h

Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.

Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs

b0d5b15b, 12 lines

```
typedef bitset<128> B;
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R={}) {
  if (!P.any()) { if (!X.any()) f(R); return; }
  auto q = (P | X)._Find_first();
  auto cands = P & ~eds[q];
  rep(i,0,sz(eds)) if (cands[i]) {
    R[i] = 1;
    cliques(eds, f, P & eds[i], X & eds[i], R);
    R[i] = P[i] = 0; X[i] = 1;
  }
}
```

MaximumClique.h

Description: Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.

Time: Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs.

f7c0bc8c, 49 lines

```
typedef vector<bitset<200>> vb;
struct Maxclique {
  double limit=0.025, pk=0;
  struct Vertex { int i, d=0; };
  typedef vector<Vertex> vv;
  vb e;
  vv V;
  vector<vi> C;
  vi qmax, q, S, old;
  void init(vv& r) {
    for (auto& v : r) v.d = 0;
    for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
    sort(all(r), [](auto a, auto b) { return a.d > b.d; });
    int mxD = r[0].d;
    rep(i,0,sz(r)) r[i].d = min(i, mxD) + 1;
  }
  void expand(vv& R, int lev = 1) {
    S[lev] += S[lev - 1] - old[lev];
    old[lev] = S[lev - 1];
    while (sz(R)) {
      if (sz(q) + R.back().d <= sz(qmax)) return;
      q.push_back(R.back().i);
      vv T;
      for(auto v:R) if (e[R.back().i][v.i]) T.push_back({v.i});
      if (sz(T)) {
        if (S[lev]++ / ++pk < limit) init(T);
        int j = 0, mxk = 1, mnk = max(sz(qmax) - sz(q) + 1, 1);
        C[1].clear(), C[2].clear();
        for (auto v : T) {
          int k = 1;
          auto f = [&](int i) { return e[v.i][i]; };
          while (any_of(all(C[k]), f)) k++;
          if (k > mxk) mxk = k, C[mxk + 1].clear();
          if (k < mnk) T[j++].i = v.i;
          C[k].push_back(v.i);
        }
        if (j > 0) T[j - 1].d = 0;
        rep(k,mnk,mxk + 1) for (int i : C[k])
          T[j].i = i, T[j++].d = k;
        expand(T, lev + 1);
      } else if (sz(q) > sz(qmax)) qmax = q;
      q.pop_back(), R.pop_back();
    }
  }
  vi maxClique() { init(V), expand(V); return qmax; }
  Maxclique(vb conn) : e(conn), C(sz(e)+1), S(sz(C)), old(S) {
    rep(i,0,sz(e)) V.push_back({i});
  }
};
```

MaximumIndependentSet.h

Description: To obtain a maximum independent set of a graph, find a max clique of the complement. If the graph is bipartite, see MinimumVertexCover.

GraphClique.h

Description: Max clique N<64. Bit trick for speed. clique solver calculates both size and consitution of maximum clique uses bit operation to accelerate searching graph size limit is 63, the graph should be undirected can optimize to calculate on each component, and sort on vertex degrees can be used to solve maximum independent set

15b35db5, 82 lines

```
class clique {
  public:
  static const long long ONE = 1;
  static const long long MASK = (1 << 21) - 1;
  char* bits;
  int n, size, cmax[63];
  long long mask[63], cons;
  // initiate lookup table
  clique() {
    bits = new char[1 << 21];
    bits[0] = 0;
    for (int i = 1; i < (1<<21); ++i)
      bits[i] = bits[i >> 1] + (i & 1);
  }
  ~clique() {
    delete bits;
  }
  // search routine
  bool search(int step,int siz,LL mor,LL con);
  // solve maximum clique and return size
  int sizeClique(vector<vector<int> >& mat);
  // solve maximum clique and return set
  vector<int>getClq(vector<vector<int> >&mat);
};
// step is node id, size is current sol., more is available
     mask, cons is constitution mask
bool clique::search(int step, int size,
                    LL more, LL cons) {
  if (step >= n) {
    if (size > this->size) {
      // a new solution reached
      this->size = size;
      this->cons = cons;
    }
    return true;
  }
  long long now = ONE << step;
  if ((now & more) > 0) {
    long long next = more & mask[step];
    if (size + bits[next & MASK] +
        bits[(next >> 21) & MASK] +
        bits[next >> 42] >= this->size
     && size + cmax[step] > this->size) {
      // the current node is in the clique
      if (search(step+1,size+1,next,cons|now))
        return true;
    }
  }
  long long next = more & ~now;
  if (size + bits[next & MASK] +
      bits[(next >> 21) & MASK] +
      bits[next >> 42] > this->size) {
    // the current node is not in the clique
    if (search(step + 1, size, next, cons))
      return true;
  }
  return false;
}
// solve maximum clique and return size
int clique::sizeClique(vector<vector<int> >& mat) {
  n = mat.size();
  // generate mask vectors
  for (int i = 0; i < n; ++i) {
    mask[i] = 0;
    for (int j = 0; j < n; ++j)
      if (mat[i][j] > 0) mask[i] |= ONE << j;
  }
  size = 0;
  for (int i = n - 1; i >= 0; --i) {
    search(i + 1, 1, mask[i], ONE << i);
    cmax[i] = size;
  }
```

```
  return size;
}
// calls sizeClique and restore cons
vector<int> clique::getClq(
    vector<vector<int> >& mat) {
  sizeClique(mat);
  vector<int> ret;
  for (int i = 0; i < n; ++i)
    if ((cons&(ONE<<i)) > 0) ret.push_back(i);
  return ret;
}
```

CycleCounting.h

Description: Counts 3 and 4 cycles

99799f29, 62 lines

```
#define P 1000000007
#define N 110000

int n, m;
vector <int> go[N], lk[N];

int w[N];
int circle3(){ // hash=1
  int ans=0;
  for (int i = 1; i <= n; i++)
    w[i]=0;

  for (int x = 1; x <= n; x++) {
    for(int y:lk[x])w[y]=1;

    for(int y:lk[x])for(int z:lk[y])if(w[z]){
      ans=(ans+go[x].size()+go[y].size()+go[z].size()-6)%P;
    }

    for(int y:lk[x])w[y]=0;
  }
  return ans;
} // hash=1 = 719dcec9

int deg[N], pos[N], id[N];

int circle4(){ // hash=2
  for (int i = 1; i <= n; i++)
    w[i]=0;
  int ans=0;
  for (int x = 1; x <= n; x++) {
    for(int y:go[x])for(int z:lk[y])if(pos[z]>pos[x]){
      ans=(ans+w[z])%P;
      w[z]++;
    }
    for(int y:go[x])for(int z:lk[y])w[z]=0;
  }
  return ans;
} // hash=2 = 39b3aaf4

inline bool cmp(const int &x,const int &y){
  return deg[x]<deg[y];
}

void init() {
  scanf("%d%d", &n, &m);
  for (int i = 1; i <= n; i++)
    deg[i] = 0, go[i].clear(), lk[i].clear();;
  while (m--) {
    int a,b;
    scanf("%d%d",&a,&b);
    deg[a]++;deg[b]++;
    go[a].push_back(b);go[b].push_back(a);
  }
  for (int i = 1; i <= n; i++)
    id[i] = i;
  sort(id+1,id+1+n,cmp);
  for (int i = 1; i <= n; i++) pos[id[i]]=i;
  for (int x = 1; x <= n; x++)
    for(int y:go[x])
      if(pos[y]>pos[x])lk[x].push_back(y);
}
```

7.7 Trees

BinaryLifting.h

Description: Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.
Usage: `auto jmp = treeJump(par); lca(jmp, dep, a, b);`
Time: construction $\mathcal{O}(N \log N)$, queries $\mathcal{O}(\log N)$

9fb3804b, 22 lines

```
auto treeJump = [](vc<int> &a) {
  int d = 32 - __builtin_clz(sz(a));
  vc<vc<int>> jmp(d, a);
  for (int i = 1; i < d; ++i) for (int j = 0; j < sz(a); ++j)
    jmp[i][j] = jmp[i - 1][jmp[i - 1][j]];
  return jmp;
};
auto lca = [](vc<vc<int>> &tbl, vc<int> &dep, int a, int b) {
  if (dep[a] < dep[b]) swap(a, b);
  auto jmp = [&](int nod, int h) {
    for (int i = 0; i < sz(tbl); ++i) if (h & (1 << i))
      nod = tbl[i][nod];
    return nod;
  };
  a = jmp(a, dep[a] - dep[b]);
  if (a == b) return a;
  for (int i = sz(tbl); i--; ) {
    int c = tbl[i][a], d = tbl[i][b];
    if (c != d) a = c, b = d;
  }
  return tbl[0][a];
};
```

LCA.h

Description: Data structure for computing lowest common ancestors in a tree (with 0 as root). g should be an adjacency list of the tree, either directed or undirected.
Time: $\mathcal{O}(N \log N + Q)$

"../data-structures/RMQ.h" 653987bf, 19 lines

```
struct LCA {
  int T = 0;
  vc<int> tim, pth, ret;
  RMQ<int> rmq;
  LCA(vc<vc<int>> &g): tim(sz(g)), rmq((dfs(g, 0, -1), ret)) {}
  void dfs(vc<vc<int>> &g, int u, int p) {
    tim[u] = T++;
    for (int v: g[u]) if (v != p) {
      pth.emplace_back(u), ret.emplace_back(tim[u]);
      dfs(g, v, u);
    }
  }
  int lca(int a, int b) {
    if (a == b) return a;
    tie(a, b) = minmax(tim[a], tim[b]);
    return pth[rmq.query(a, b)];
  }
  //dist(a,b){return depth[a] + depth[b] - 2*depth[lca(a,b)];}
};
```

CompressTree.h

Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S| - 1$) pairwise LCA's and compressing edges. Returns a list of (par, orig_index) representing a tree rooted at 0. The root points to itself.
Time: $\mathcal{O}(|S| \log |S|)$

"LCA.h" 9775a02f, 21 lines

```
typedef vector<pair<int, int>> vpi;
vpi compressTree(LCA& lca, const vi& subset) {
  static vi rev; rev.resize(sz(lca.time));
  vi li = subset, &T = lca.time;
  auto cmp = [&](int a, int b) { return T[a] < T[b]; };
  sort(all(li), cmp);
  int m = sz(li)-1;
  rep(i,0,m) {
    int a = li[i], b = li[i+1];
    li.push_back(lca.lca(a, b));
  }
  sort(all(li), cmp);
  li.erase(unique(all(li)), li.end());
  rep(i,0,sz(li)) rev[li[i]] = i;
  vpi ret = {pii(0, li[0])};
  rep(i,0,sz(li)-1) {
    int a = li[i], b = li[i+1];
    ret.emplace_back(rev[lca.lca(a, b)], b);
  }
  return ret;
}
```

HLD.h

Description: Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most log(n) light edges. Code does additive modifications and max queries, but can support commutative segtree modifications/queries on paths and subtrees. Takes as input the full adjacency list. VALS_EDGES being true means that values are stored in the edges, as opposed to the nodes. All values initialized to the segtree default. Root must be 0.
Time: $\mathcal{O}\left((\log N)^2\right)$

"../data-structures/SegmentTree.h" 4f3827a7, 46 lines

```
template<bool EDGE> struct HLD {
  int n, tim = 0;
  vc<vc<int>> g;
  vc<int> par, siz, dep, rt, pos;
  Seg tr;
  HLD(vc<vc<int>> g): n(sz(g)), g(g), par(n, -1), siz(n, 1),
                      dep(n), rt(n), pos(n), tr(n) {
                      dfsSiz(0); dfsHld(0); }
  void dfsSiz(int u) {
    if (~par[u]) g[u].erase(find(all(g[u]), par[u]));
    for (auto &v: g[u]) {
      par[v] = u, dep[v] = dep[u] + 1;
      dfsSiz(v);
      siz[u] += siz[v];
      if (siz[v] > siz[g[u][0]]) swap(v, g[u][0]);
    }
  } //f3bc4162
  void dfsHld(int u) {
    pos[u] = tim++;
    for (auto v: g[u]) {
      rt[v] = (v == g[u][0] ? rt[u] : v);
      dfsHld(v);
    }
  } //bac86e27
  template<class F> void process(int u, int v, F op) {
    for (; rt[u] != rt[v]; v = par[rt[v]]) {
      if (dep[rt[u]] > dep[rt[v]]) swap(u, v);
      op(pos[rt[v]], pos[v] + 1);
    }
    if (dep[u] > dep[v]) swap(u, v);
```

```
    op(pos[u] + EDGE, pos[v] + 1);
  } //974632cb
  void modifyPath(int u, int v, int val) {
    process(u, v, [&](int l, int r) { tr.modify(l, r, val); });
  }
  int queryPath(int u, int v) {
    int res = 0;
    process(u, v, [&](int l, int r) {
      (res += tr.query(l, r)) %= P;
    });
    return res;
  }
  int querySubtree(int u) { // modifySubtree is similar
    return tr.query(pos[u] + EDGE, pos[u] + siz[u]);
  }
};
```

LinkCutTree.h

Description: link-cut Tree. Supports BST-like augmentations. (Can be used in place of HLD). Current implementation supports update value at a node, and query max on a path.
Time: All operations take amortized $\mathcal{O}(\log N)$.

2534771f, 77 lines

```
struct Node {
  Node *p, *pp, *c[2];
  bool flip;
  int val, xval;
  void push() {
    if (flip) {
      if (c[0]) c[0]->flip ^= 1;
      if (c[1]) c[1]->flip ^= 1;
      swap(c[0], c[1]), flip = 0;
    }
  }
  void pull() {
    xval = val;
    if (c[0]) xval ^= c[0]->xval;
    if (c[1]) xval ^= c[1]->xval;
  }
  bool dir() { return this == p->c[1]; }
  void rot() {
    bool t = dir();
    Node *y = p, *&z = c[!t];
    p = y->p;
    if (p) p->c[y->dir()] = this;
    if (z) z->p = y;
    y->c[t] = z;
    y->p = this;
    (z = y)->pull();
  }
  void g() { if (p) p->g(), pp = p->pp; push(); }
  void splay() {
    for (g(); p; rot()) if (p->p)
      (p->dir() == dir() ? p : this)->rot();
    pull();
  }
  Node *access() {
    for (Node *y = 0, *z = this; z; y = z, z = z->pp) {
      z->splay();
      if (z->c[1]) z->c[1]->pp = z, z->c[1]->p = 0;
      if (y) y->p = z;
      z->c[1] = y;
      z->pull();
    }
    splay();
    flip ^= 1;
    return this;
  } // 2060f82f
};
```

```
struct LinkCut {
  vc<Node> nodes;
  LinkCut(int n): nodes(n) {}
  bool cut(int u, int v) {
    Node *y = nodes[v].access();
    Node *x = nodes[u].access();
    if (x->c[0] != y || y->c[1]) return 0;
    x->c[0] = y->p = y->pp = 0;
    x->pull();
    return 1;
  }
  bool is_connected(int u, int v) {
    if (u == v) return 1;
    Node *x = nodes[u].access();
    Node *y = nodes[v].access();
    return x->p;
  }
  bool link(int u, int v) {
    if (is_connected(u, v)) return 0;
    nodes[u].access()->pp = &nodes[v];
    return 1;
  } // 892ea56a
  void update(int u, int c) {
    nodes[u].access()->val = c;
  }
  int query(int u, int v) {
    nodes[v].access();
    return nodes[u].access()->xval;
  }
};
```

Centroid.h

Description: The centroid of a tree of size N is a vertex such that after removing it, all resulting subtrees have size at most $\frac{N}{2}$. Supports updates in the form "add 1 to all verts v such that $dist(x,v) \leq y$."
Memory: $\mathcal{O}(N \log N)$
Time: $\mathcal{O}(N \log N)$ build, $\mathcal{O}(\log N)$ update and query

907e21e3, 54 lines

```
void ad(vi& a, int b) { ckmin(b,sz(a)-1); if (b>=0) a[b]++; }
void prop(vi& a) { R0F(i,sz(a)-1) a[i] += a[i+1]; }
template<int SZ> struct Centroid {
  vi adj[SZ]; void ae(int a,int b){adj[a].pb(b),adj[b].pb(a);}
  bool done[SZ]; // processed as centroid yet
  int N,sub[SZ],cen[SZ],lev[SZ]; // subtree size, centroid anc
  int dist[32-__builtin_clz(SZ)][SZ]; // dists to all ancs
  vi stor[SZ], STOR[SZ];
  void dfs(int x, int p) { sub[x] = 1;
    each(y,adj[x]) if (!done[y] && y != p)
      dfs(y,x), sub[x] += sub[y];
  }
  int centroid(int x) {
    dfs(x,-1);
    for (int sz = sub[x];;) {
      pi mx = {0,0};
      each(y,adj[x]) if (!done[y] && sub[y] < sub[x])
        ckmax(mx,{sub[y],y});
      if (mx.f*2 <= sz) return x;
      x = mx.s;
    }
  }
  void genDist(int x, int p, int lev) {
    dist[lev][x] = dist[lev][p]+1;
    each(y,adj[x]) if (!done[y] && y != p) genDist(y,x,lev); }
  void gen(int CEN, int _x) { // CEN = centroid above x
    int x = centroid(_x); done[x] = 1; cen[x] = CEN;
    sub[x] = sub[_x]; lev[x] = (CEN == -1 ? 0 : lev[CEN]+1);
    dist[lev[x]][x] = 0;
    stor[x].rsz(sub[x]),STOR[x].rsz(sub[x]+1);
    each(y,adj[x]) if (!done[y]) genDist(y,x,lev[x]);
    each(y,adj[x]) if (!done[y]) gen(x,y);
  }
  void init(int _N) { N = _N; FOR(i,1,N+1) done[i] = 0;
    gen(-1,1); } // start at vert 1
  void upd(int x, int y) {
    int cur = x, pre = -1;
    R0F(i,lev[x]+1) {
      ad(stor[cur],y-dist[i][x]);
      if (pre != -1) ad(STOR[pre],y-dist[i][x]);
      if (i > 0) pre = cur, cur = cen[cur];
    }
  } // call propAll() after all updates
  void propAll() { FOR(i,1,N+1) prop(stor[i]), prop(STOR[i]); }
  int query(int x) { // get value at vertex x
    int cur = x, pre = -1, ans = 0;
    R0F(i,lev[x]+1) { // if pre != -1, subtract those from
      ans += stor[cur][dist[i][x]]; // same subtree
      if (pre != -1) ans -= STOR[pre][dist[i][x]];
      if (i > 0) pre = cur, cur = cen[cur];
    }
    return ans;
  }
};
```

DirectedMST.h

Description: Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.
Time: $\mathcal{O}(E \log V)$

"../data-structures/UnionFindRollback.h" 39e620f1, 60 lines

```
struct Edge { int a, b; ll w; };
struct Node {
  Edge key;
  Node *l, *r;
  ll delta;
  void prop() {
    key.w += delta;
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  }
  Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r)));
  return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
  RollbackUF uf(n);
  vector<Node*> heap(n);
  for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e});
  ll res = 0;
  vi seen(n, -1), path(n), par(n);
  seen[r] = r;
  vector<Edge> Q(n), in(n, {-1,-1}), comp;
  deque<tuple<int, int, vector<Edge>>> cycs;
  rep(s,0,n) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {
      if (!heap[u]) return {-1,{}};
      Edge e = heap[u]->top();
      heap[u]->delta -= e.w, pop(heap[u]);
      Q[qi] = e, path[qi++] = u, seen[u] = s;
      res += e.w, u = uf.find(e.a);
```

```
      if (seen[u] == s) {
        Node* cyc = 0;
        int end = qi, time = uf.time();
        do cyc = merge(cyc, heap[w = path[--qi]]);
        while (uf.join(u, w));
        u = uf.find(u), heap[u] = cyc, seen[u] = -1;
        cycs.push_front({u, time, {&Q[qi], &Q[end]}});
      }
    }
    rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
  }

  for (auto& [u,t,comp] : cycs) { // restore sol (optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto& e : comp) in[uf.find(e.b)] = e;
    in[uf.find(inEdge.b)] = inEdge;
  }
  rep(i,0,n) par[i] = in[i].a;
  return {res, par};
}
```

DominatorTree.h

Description: Dominator Tree.

05f441f7, 107 lines

```
#define N 110000 //max number of vertices

vector<int> succ[N], prod[N], bucket[N], dom_t[N];
int semi[N], anc[N], idom[N], best[N], fa[N], tmp_idom[N];
int dfn[N], redfn[N];
int child[N], size[N];
int timestamp;

void dfs(int now) { // hash-1
  dfn[now] = ++timestamp;
  redfn[timestamp] = now;
  anc[timestamp] = idom[timestamp] = child[timestamp] = size[
      timestamp] = 0;
  semi[timestamp] = best[timestamp] = timestamp;
  int sz = succ[now].size();
  for(int i = 0; i < sz; ++i) {
    if(dfn[succ[now][i]] == -1) {
      dfs(succ[now][i]);
      fa[dfn[succ[now][i]]] = dfn[now];
    }
    prod[dfn[succ[now][i]]].push_back(dfn[now]);
  }
} // hash-1 = 6412bfd6

void compress(int now) { // hash-2
  if(anc[anc[now]] != 0) {
    compress(anc[now]);
    if(semi[best[now]] > semi[best[anc[now]]])
      best[now] = best[anc[now]];
    anc[now] = anc[anc[now]];
  }
} // hash-2 = 1c9444eb

inline int eval(int now) { // hash-3
  if(anc[now] == 0)
    return now;
  else {
    compress(now);
    return semi[best[anc[now]]] >= semi[best[now]] ? best[now]
      : best[anc[now]];
  }
} // hash-3 = 4e235f39

inline void link(int v, int w) { // hash-4
  int s = w;
  while(semi[best[w]] < semi[best[child[w]]]) {
    if(size[s] + size[child[child[s]]] >= 2*size[child[s]]) {
      anc[child[s]] = s;
      child[s] = child[child[s]];
    } else {
      size[child[s]] = size[s];
      s = anc[s] = child[s];
    }
  }
  best[s] = best[w];
  size[v] += size[w];
  if(size[v] < 2*size[w])
    swap(s, child[v]);
  while(s != 0) {
    anc[s] = v;
    s = child[s];
  }
} // hash-4 = 270548fd

// idom[n] and other vertices that cannot be reached from n
    will be 0
void lengauer_tarjan(int n) { // n is the root's number // hash
    -5
  memset(dfn, -1, sizeof dfn);
  memset(fa, -1, sizeof fa);
  timestamp = 0;
  dfs(n);
  fa[1] = 0;
  for(int w = timestamp; w > 1; --w) {
    int sz = prod[w].size();
    for(int i = 0; i < sz; ++i) {
      int u = eval(prod[w][i]);
      if(semi[w] > semi[u])
        semi[w] = semi[u];
    }
    bucket[semi[w]].push_back(w);
    //anc[w] = fa[w]; link operation for o(mlogm) version
              link(fa[w], w);
    if(fa[w] == 0)
      continue;
    sz = bucket[fa[w]].size();
    for(int i = 0; i < sz; ++i) {
      int u = eval(bucket[fa[w]][i]);
      if(semi[u] < fa[w])
        idom[bucket[fa[w]][i]] = u;
      else
        idom[bucket[fa[w]][i]] = fa[w];
    }
    bucket[fa[w]].clear();
  }
  for(int w = 2; w <= timestamp; ++w) {
    if(idom[w] != semi[w])
      idom[w] = idom[idom[w]];
  }
  idom[1] = 0;
  for(int i = timestamp; i > 1; --i) {
    if(fa[i] == -1)
      continue;
    dom_t[idom[i]].push_back(i);
  }
  memset(tmp_idom, 0, sizeof tmp_idom);
  for (int i = 1; i <= timestamp; i++)
    tmp_idom[redfn[i]] = redfn[idom[i]];
  memcpy(idom, tmp_idom, sizeof idom);
} // hash-5 = f49c4046
```

7.8 Math

7.8.1 Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \cdots \geq d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

7.8.2 Number of Spanning Trees

Create an $N \times N$ matrix `mat`, and for each edge $a \to b \in G$, do `mat[a][b]--, mat[b][b]++` (and `mat[b][a]--, mat[a][a]++` if G is undirected). Remove the ith row and column and take the determinant; this yields the number of directed spanning trees rooted at i (if G is undirected, remove any row/column).

Geometry (8)

8.1 Geometric primitives

Point.h

Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

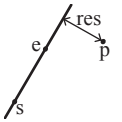
6d296e7d, 29 lines

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T> struct Point {
  using P = Point;
  T x, y;
  explicit Point(T x = 0, T y = 0): x(x), y(y) {}
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
  P operator + (P p) { return P(x + p.x, y + p.y); }
  P operator - (P p) { return P(x - p.x, y - p.y); }
  P operator * (T d) { return P(x * d, y * d); }
  P operator / (T d) { return P(x / d, y / d); }
  T dot(P p) { return x * p.x + y * p.y; }
  T cross(P p) { return x * p.y - y * p.x; }
  T cross(P a, P b) { return (a - *this).cross(b - *this); }
  T dist2() { return x * x + y * y; }
  double dist() { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() { return atan2(y, x); }
  P unit() { return * this / dist(); } // make dist() = 1
  P perp() { return P(-y, x); } // rotates +90 degrees
  P norm() { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the origin
  P rotate(double a) {
    return P(x * cos(a) - y * sin(a),x * sin(a) + y * cos(a));
  }
  friend ostream &operator<<(ostream &os, P p) {
    return os << "(" << p.x << "," << p.y << ")";
  }
};
```

lineDistance.h

Description:

Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

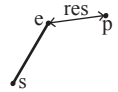


"Point.h"

e73148e8, 3 lines

```
template<class P> double lineDist(P s, P e, P p) {
  return (double)s.cross(e, p) / (e - s).dist();
}
```

SegmentDistance.h



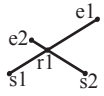
Description:
Returns the shortest distance between point p and the line segment from point s to e. P should be double.

"Point.h" 0a3fc1b3, 5 lines

```
template<class P> double segDist(P s, P e, P p) {
  if (s == e) return (p - s).dist();
  auto d = (e - s).dist2(), t = clamp((p - s).dot(e - s),0.,d);
  return ((p - s) * d - (e - s) * t).dist() / d;
}
```

SegmentIntersection.h

Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



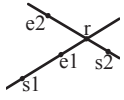
Usage:
```
vector<P> inter = segInter(s1,e1,s2,e2);
if (sz(inter)==1)
cout << "segments intersect at " << inter[0] << endl;
```

"Point.h", "OnSegment.h" e2e771e5, 12 lines

```
template<class P> vc<P> segInter(P a, P b, P c, P d) {
  auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
    return {(a * ob - b * oa) / (ob - oa)};
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d);
  return {all(s)};
}
```

lineIntersection.h

Description:
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.



Usage:
```
auto res = lineInter(s1,e1,s2,e2);
if (res.first == 1)
cout << "intersection point at " << res.second << endl;
```

"Point.h" a01f815e, 8 lines

```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d};
}
```

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 $\Leftrightarrow$ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

Usage:
```
bool left = sideOf(p1,p2,q)==1;
```

"Point.h" ca466558, 8 lines

```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P> int sideOf(P s, P e, P p, double eps) {
  auto a = (e - s).cross(p - s);
  double l = (e - s).dist() * eps;
  return (a > l) - (a < -l);
}
```

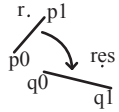
OnSegment.h

Description: Returns true iff p lies on the line segment from s to e. Use (segDist(s,e,p)<=epsilon) instead when using Point<double>.

"Point.h" c597e874, 3 lines

```
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

linearTransformation.h



Description:
Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r. P should be double.

"Point.h" 1894dbcb, 5 lines

```
template<class P>
P linearTransformation(P p0, P p1, P q0, P q1, P r) {
  P dp = p1 - p0, num(dp.cross(q1 - q0), dp.dot(q1 - q0));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

LineProjectionReflection.h

Description: Projects point p onto line ab. Set refl=true to get reflection of point p across line ab insted. The wrong point will be returned if P is an integer point and the desired point doesn't have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow.

"Point.h" b5562d9e, 5 lines

```
template<class P>
P lineProj(P a, P b, P p, bool refl=false) {
  P v = b - a;
  return p - v.perp()*(1+refl)*v.cross(p-a)/v.dist2();
}
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

Usage:
```
vector<Angle> v = {w[0], w[0].t360() ...}; // sorted
int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }
// sweeps j such that (j-i) represents the number of positively
oriented triangles with vertices at 0 and i
```

a4aefb74, 34 lines

```
struct Angle {
  ll x, y;
  int t;
  Angle(ll x, ll y, int t = 0): x(x), y(y), t(t) {}
  int half() const {
    assert(x || y);
    return y < 0 || (y == 0 && x < 0);
  }
  Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
  Angle t180() const { return {-x, -y, t + half()}; }
  Angle t360() const { return {x, y, t + 1}; }
  bool operator < (Angle r) const {
  // add a.dist2() and b.dist2() to also compare distances
    return make_tuple(t, half(), y * r.x) <
           make_tuple(r.t, r.half(), x * r.y);
  }
  Angle operator + (Angle r) { // + vector r
    Angle c(x + r.x, y + r.y, t);
    if (t180() < c) --c.t;
    return c.t180() < *this ? c.t360() : c;
  }
  Angle operator - (Angle r) { // - angle r
    int tt = t - r.t; r.t = t;
    return {x * r.x + y * r.y, y * r.x - x * r.y, tt - (*this <
          r)};
  }
};

// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
array<Angle, 2> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  if (b < a.t180()) return {a, b};
  return {b, a.t360()};
}
```

angleCmp.h

Description: Useful utilities for dealing with angles of rays from origin. OK for integers, only uses cross product. Doesn't support (0,0). 6edd25f3, 22 lines

```
template <class P>
bool sameDir(P s, P t) {
  return s.cross(t) == 0 && s.dot(t) > 0;
}
// checks 180 <= s..t < 360?
template <class P>
bool isReflex(P s, P t) {
  auto c = s.cross(t);
  return c ? (c < 0) : (s.dot(t) < 0);
}
// operator< (s,t) for angles in [base,base+2pi)
template <class P>
bool angleCmp(P base, P s, P t) {
  int r = isReflex(base, s) - isReflex(base, t);
  return r ? (r < 0) : (0 < s.cross(t));
}
// is x in [s,t] taken ccw? 1/0/-1 for in/border/out
template <class P>
int angleBetween(P s, P t, P x) {
  if (sameDir(x, s) || sameDir(x, t)) return 0;
  return angleCmp(s, x, t) ? 1 : -1;
}
```

8.2 Circles

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection. Assert if inf intersections.

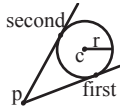
"Point.h" ee89edf1, 11 lines

```
template<class P>
bool circleInter(P a,P b,double r1,double r2,array<P, 2>&out) {
  if (a == b) { assert(r1 != r2); return 0; }
  P vec = b - a;
  double d2 = vec.dist2(), sum = r1 + r2, dif = r1 - r2,
         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;
  if (sum * sum < d2 || dif * dif > d2) return 0;
  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
  out = {mid + per, mid - per};
  return 1;
```

```
}
```

CircleTangents.h

Description:
Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0. Assert if circles are indentified.



"Point.h" 58caf090, 13 lines

```
template<class P>
vc<array<P, 2>> tangents(P c1, double r1, P c2, double r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0)  { assert(h2 != 0); return {}; }
  vc<array<P, 2>> out;
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  }
  if (h2 == 0) out.pop_back();
  return out;
}
```

CircleLine.h

Description: Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

"Point.h" e0cfba4a, 9 lines

```
template<class P>
vector<P> circleLine(P c, double r, P a, P b) {
  P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2();
  double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2();
  if (h2 < 0) return {};
  if (h2 == 0) return {p};
  P h = ab.unit() * sqrt(h2);
  return {p - h, p + h};
}
```

CirclePolygonIntersection.h

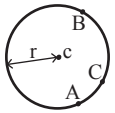
Description: Returns the area of the intersection of a circle with a ccw polygon.
Time: $\mathcal{O}(n)$

"../../content/geometry/Point.h" a1ee63d6, 19 lines

```
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
  auto tri = [&](P p, P q) {
    auto r2 = r * r / 2;
    P d = q - p;
    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
    if (t < 0 || 1 <= s) return arg(p, q) * r2;
    P u = p + d * s, v = p + d * t;
    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
  };
  auto sum = 0.0;
  rep(i,0,sz(ps))
    sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
  return sum;
}
```

Circumcircle.h

Description:
The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h" 29744944, 10 lines

```
template<class P>
double ccRadius(P A, P B, P C) {
  return (B - A).dist() * (C - B).dist() * (A - C).dist() /
         abs((B - A).cross(C - A)) / 2;
}
template<class P>
P ccCenter(P A, P B, P C) {
  P b = C - A, c = B - A;
  return A + (b*c.dist2()-c*b.dist2()).perp() / b.cross(c) / 2;
}
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.
Time: expected $\mathcal{O}(n)$

"circumcircle.h" 09dd0aa4, 17 lines

```
pair<P, double> mec(vector<P> ps) {
  shuffle(all(ps), mt19937(time(0)));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0;
    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
      o = (ps[i] + ps[j]) / 2;
      r = (o - ps[i]).dist();
      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
        o = ccCenter(ps[i], ps[j], ps[k]);
        r = (o - ps[i]).dist();
      }
    }
  }
  return {o, r};
}
```

kIntersection.h

Description: Given n cirlce, for all $k \leq n$, computes the area of regions part of at least k circles.
Time: $\mathcal{O}(n^2)$

0e6b2d79, 65 lines

```
const int N = 22222;
const double EPS = 1e-8;
const double PI = acos(-1.0);
typedef complex<double> Point;
int n, m;
double r[N], result[N];
Point c[N];
pair<double, int> event[N];
int sgn (double x) {return x < -EPS? -1: x < EPS? 0: 1;}
double det (const Point &a, const Point &b) { return a.real() *
      b.imag() - a.imag()
  * b.real();}
  void addEvent (double a, int v) {
    event[m ++] = make_pair(a, v);
  }
void addPair (double a, double b) {
  if (sgn(a - b) <= 0) {
    addEvent(a, +1);
    addEvent(b, -1);
  } else {
    addPair(a, +PI);
    addPair(-PI, b);
  }
}
Point polar (double t) { return Point(cos(t), sin(t)); }
Point radius (int i , double t) {
  return c[i] + polar(t) * r[i];
}
void solve () {
  // result [k] : the total area covered no less than k times
  memset(result, 0, sizeof(result));
  for (int i = 0; i < n; ++ i) {
    m = 0;
    addEvent(-PI, 0);
    addEvent(+PI, 0);
    for (int j = 0; j < n; ++ j) {
      if (i != j) {
        if (sgn(abs(c[i] - c[j]) - abs(r[i] - r[j])) <= 0) {
          if (sgn(r[i] - r[j]) <= 0) {
            addPair(-PI, +PI);
          }
        } else {
          if (sgn(abs(c[i] - c[j]) - (r[i] + r[j])) >= 0) {
            continue;
          }
          double d = abs(c[j] - c[i]);
          Point b = (c[j] - c[i]) / d * r[i];
          double t = acos((r[i] * r[i] + d * d - r[j] * r[j]) /
                (2 *
                r[i] * d));
          Point a = b * polar(-t);
          Point c = b * polar(+t);
          addPair(arg(a), arg(c));
        }
      }
    }
    sort(event, event + m);
    int count = event[0].second;
    for (int j = 1; j < m; ++ j) {
      double delta = event[j].first - event[j - 1].first;
      result[count] += r[i] * r[i] * (delta - sin(delta));
      result[count] += det(radius(i , event[j - 1].first),
           radius(i ,
            event[j].first));
      count += event[j].second;
    }
  }
}
```

8.3 Polygons

InsidePolygon.h

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.
Usage: `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`
`bool in = inPolygon(v, P{3, 3}, false);`
Time: $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" 4603b981, 11 lines

```
template<class P>
bool inPolygon(vc<P> &p, P a, bool strict = 1) {
  int cnt = 0, n = sz(p);
  for (int i = 0; i < n; ++i) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict;
    //or: if (segDist(p[i], q, a) <= eps) return !strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) > 0;
  }
  return cnt;
}
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" f1230037, 6 lines

```
template<class T>
T polygonArea2(vector<Point<T>>& v) {
  T a = v.back().cross(v[0]);
  rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
  return a;
}
```

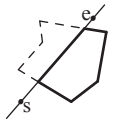
PolygonCenter.h

Description: Returns the center of mass for a polygon.
Time: $\mathcal{O}(n)$

"Point.h" 9706dcc8, 9 lines

```
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
  P res(0, 0); double A = 0;
  for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
    A += v[j].cross(v[i]);
  }
  return res / A / 3;
}
```

PolygonCut.h



Description:
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.
Usage: `vector<P> p = ...;`
`p = polygonCut(p, P(0,0), P(1,0));`

"Point.h", "lineIntersection.h" f2b7d494, 13 lines

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0;
    if (side != (s.cross(e, prev) < 0))
      res.push_back(lineInter(s, e, cur, prev).second);
    if (side)
      res.push_back(cur);
  }
  return res;
}
```

ConvexHull.h

Description:
Returns a vector of the points of the convex hull in counterclockwise order. Points on the edge of the hull between two other points are not considered part of the hull.
Time: $\mathcal{O}(n \log n)$

"Point.h" 31095458, 13 lines

```
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
  if (sz(pts) <= 1) return pts;
  sort(all(pts));
  vector<P> h(sz(pts)+1);
  int s = 0, t = 0;
  for (int it = 2; it--; s = --t, reverse(all(pts)))
    for (P p : pts) {
      while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
      h[t++] = p;
    }
  return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}
```

PolygonDiameter.h

Description: Calculates the max squared distance of a set of points.

"ConvexHull.h" 5596d386, 19 lines

```
vector<pii> antipodal(const vector<P>& S, vi& U, vi& L) {
  vector<pii> ret;
  int i = 0, j = sz(L) - 1;
  while (i < sz(U) - 1 || j > 0) {
    ret.emplace_back(U[i], L[j]);
    if (j == 0 || (i != sz(U)-1 && (S[L[j]] - S[L[j-1]])
          .cross(S[U[i+1]] - S[U[i]]) > 0)) ++i;
    else --j;
  }
  return ret;
}

pii polygonDiameter(const vector<P>& S) {
  vi U, L; tie(U, L) = ulHull(S);
  pair<ll, pii> ans;
  trav(x, antipodal(S, U, L))
    ans = max(ans, {(S[x.first] - S[x.second]).dist2(), x});
  return ans.second;
}
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).
Time: $\mathcal{O}(N)$

"Point.h" c571b8ed, 12 lines

```
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
  int n = sz(S), j = n < 2 ? 0 : 1;
  pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
  rep(i,0,j)
    for (;; j = (j + 1) % n) {
      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
        break;
    }
  return res.second;
}
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 71446bda, 14 lines

```
typedef Point<ll> P;

bool inHull(const vector<P>& l, P p, bool strict = true) {
  int a = 1, b = sz(l) - 1, r = !strict;
  if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<= -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
  }
  return sgn(l[a].cross(l[b], p)) < r;
}
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: $\bullet$ $(-1,-1)$ if no collision, $\bullet$ $(i,-1)$ if touching the corner i, $\bullet$ (i,i) if along side $(i,i+1)$, $\bullet$ (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(\log n)$

"Point.h" 7cf45b1b, 39 lines

```
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo + 1 < hi) {
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
  }
  return lo;
}

#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
  int endA = extrVertex(poly, (a - b).perp());
  int endB = extrVertex(poly, (b - a).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};
  array<int, 2> res;
  rep(i,0,2) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    }
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  }
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    }
  return res;
}
```

HalfPlaneIntersection.h

Description: Data structure that dynamically keeps track of the intersection of halfplanes. Use is straigntforward. Area should be able to be kept dynamically with some modifications. NOTE. REMOVE t LOGIC FROM ANGLE WHEN IMPLEMENTING
Usage: `HalfplaneSet hs;`
`hs.Cut({0, 0}, {1, 1});`
`double best = hs.Maximize({1, 2});`
Time: $\mathcal{O}(\log n)$

"Point.h", "LineIntersection.h", "Angle.h" 776b5343, 62 lines

```
struct HalfplaneSet : multimap<Angle, Point> {
  using Iter = multimap<Angle, Point>::iterator;

  HalfplaneSet() {
    insert({{+1, 0}, {-kInf, -kInf}});
    insert({{0, +1}, {+kInf, -kInf}});
    insert({{-1, 0}, {+kInf, +kInf}});
    insert({{0, -1}, {-kInf, +kInf}});
```

```
  }

  Iter get_next(Iter it) {
    return (next(it) == end() ? begin() : next(it)); }
  Iter get_prev(Iter it) {
    return (it == begin() ? prev(end()) : prev(it)); }
  Iter fix(Iter it) { return it == end() ? begin() : it; }

  // Cuts everything to the RIGHT of a, b
  // For LEFT, just swap a with b
  void Cut(Angle a, Angle b) {
    if (empty()) return;
    int old_size = size();

    auto eval = [&](Iter it) {
      return sgn(det(a.p(), b.p(), it->second)); };
    auto intersect = [&](Iter it)  {
      return LineIntersection(a.p(), b.p(),
          it->second, it->first.p() + it->second);
    };

    auto it = fix(lower_bound(b - a));
    if (eval(it) >= 0) return;

    while (size() && eval(get_prev(it)) < 0)
      fix(erase(get_prev(it)));
    while (size() && eval(get_next(it)) < 0)
      it = fix(erase(it));

    if (empty()) return;

    if (eval(get_next(it)) > 0) it->second = intersect(it);
    else it = fix(erase(it));
    if (old_size <= 2) return;
    it = get_prev(it);
    insert(it, {b - a, intersect(it)});
    if (eval(it) == 0) erase(it);
  }

  // Maximizes dot product
  double Maximize(Angle c) {
    assert(!empty());
    auto it = fix(lower_bound(c.t90()));
    return dot(it->second, c.p());
  }

  double Area() {
    if (size() <= 2) return 0;
    double ret = 0;
    for (auto it = begin(); it != end(); ++it)
      ret += cross(it->second, get_next(it)->second);
    return ret;
  }
};
```

PointInPolygonOrTangentToPolygon.h

Description: C : counter_clockwise(C[0] == C[N]), $N \geq 3$. return highest point in $C \leftarrow P$(clockwise) or -1 if strictly in P. Polygon is strongly convex, $C[i] \neq P$

c561a7e5, 24 lines

```
int convex_tangent (vector<pi>&C,pi P,int up = 1){
  auto sign = [&](lint c){return c> 0 ?  up : c ==0 ? 0 : -up;
      };
  auto local =[&](pi P,pi a,pi b,pi c){
    return sign (ccw(P,a,b))<=0 && sign(ccw(P,b,c))>= 0;
  };
  int N = C.size()-1,s =0,e = N,m;
  if(local(P,C[1],C[0],C[N-1]))return 0;
  while (s + 1 < e) {
    m = (s + e) / 2;
    if (local(P, C[m - 1], C[m], C[m + 1])) return m;
    if (sign(ccw(P, C[s], C[s + 1])) < 0) { // up
      if (sign(ccw(P, C[m], C[m + 1])) > 0) e = m;
      else if (sign(ccw(P, C[m], C[s])) > 0) s = m;
      else e = m;
    } else { // down
      if (sign(ccw(P, C[m], C[m + 1])) < 0) s = m;
      else if (sign(ccw(P, C[m], C[s])) < 0) s = m;
      else e = m;
    }
  }
  if (s && local(P, C[s - 1], C[s], C[s + 1])) return s;
  if (e != N && local(P, C[e - 1], C[e], C[e + 1])) return e;
  return -1;
}
```

8.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n \log n)$

"Point.h" d58e8aa8, 17 lines

```
using P = Point<double>;
auto closest = [](vc<P> v) {
  assert(sz(v) > 1);
  set<P> S;
  sort(all(v), [](P a, P b) { return a.y < b.y; });
  pair<ll,pair<P, P>> ret{numeric_limits<ll>::max(),{P(),P()}};
  int j = 0;
  for (P p: v) {
    P d(1 + sqrt(ret.first), 0);
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
    for (; lo != hi; ++lo)
      ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
    S.insert(p);
  }
  return ret.second;
}; // 4e29a1c8
```

ManhattanMST.h

Description: Given N points, returns up to 4*N edges, which are guaranteed to contain a minimum spanning tree for the graph with edge weights w(p, q) = —p.x - q.x— + —p.y - q.y—. Edges are in the form (distance, src, dst). Use a standard MST algorithm on the result to find the final MST.
Time: $\mathcal{O}(N \log N)$

"Point.h" df6f59d0, 23 lines

```
typedef Point<int> P;
vector<array<int, 3>> manhattanMST(vector<P> ps) {
  vi id(sz(ps));
  iota(all(id), 0);
  vector<array<int, 3>> edges;
  rep(k,0,4) {
    sort(all(id), [&](int i, int j) {
          return (ps[i]-ps[j]).x < (ps[j]-ps[i]).y;});
    map<int, int> sweep;
    for (int i : id) {
      for (auto it = sweep.lower_bound(-ps[i].y);
                it != sweep.end(); sweep.erase(it++)) {
        int j = it->second;
        P d = ps[i] - ps[j];
        if (d.y > d.x) break;
        edges.push_back({d.y + d.x, i, j});
      }
      sweep[-ps[i].y] = i;
    }
    for (P& p : ps) if (k & 1) p.x = -p.x; else swap(p.x, p.y);
  }
  return edges;
}
```

kdTree.h

Description: KD-tree (2d, can be extended to 3d)

"Point.h" bac5b040, 63 lines

```
typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node {
  P pt; // if this is a leaf, the single point in it
  T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
  Node *first = 0, *second = 0;

  T distance(const P& p) { // min squared distance to a point
    T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
    T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
    return (P(x,y) - p).dist2();
  }

  Node(vector<P>&& vp) : pt(vp[0]) {
    for (P p : vp) {
      x0 = min(x0, p.x); x1 = max(x1, p.x);
      y0 = min(y0, p.y); y1 = max(y1, p.y);
    }
    if (vp.size() > 1) {
      // split on x if width >= height (not ideal...)
      sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
      // divide by taking half the array for each child (not
      // best performance with many duplicates in the middle)
      int half = sz(vp)/2;
      first = new Node({vp.begin(), vp.begin() + half});
      second = new Node({vp.begin() + half, vp.end()});
    }
  }
};

struct KDTree {
  Node* root;
  KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

  pair<T, P> search(Node *node, const P& p) {
    if (!node->first) {
      // uncomment if we should not find the point itself:
      // if (p == node->pt) return {INF, P()};
      return make_pair((p - node->pt).dist2(), node->pt);
    }

    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

    // search closest side first, other side if needed
    auto best = search(f, p);
    if (bsec < best.first)
      best = min(best, search(s, p));
    return best;
  }

  // find nearest point to a point, and its squared distance
  // (requires an arbitrary operator< for Point)
  pair<T, P> nearest(const P& p) {
    return search(root, p);
  }
};
```

DelaunayTriangulation.h

Description: Computes the Delaunay triangulation of a set of points. Each circumcircle contains none of the input points. If any three points are collinear or any four are on the same circle, behavior is undefined.
Time: $\mathcal{O}(n^2)$

"Point.h", "3dHull.h" c0e7bcf0, 10 lines

```
template<class P, class F>
void delaunay(vector<P>& ps, F trifun) {
  if (sz(ps) == 3) { int d = (ps[0].cross(ps[1], ps[2]) < 0);
    trifun(0,1+d,2-d); }
  vector<P3> p3;
  for (P p : ps) p3.emplace_back(p.x, p.y, p.dist2());
  if (sz(ps) > 3) for(auto t:hull3d(p3)) if ((p3[t.b]-p3[t.a]).
      cross(p3[t.c]-p3[t.a]).dot(P3(0,0,1)) < 0)
    trifun(t.a, t.c, t.b);
}
```

FastDelaunay.h

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ... }, all counter-clockwise.
Time: $\mathcal{O}(n \log n)$

"Point.h" eefdf506, 88 lines

```
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

struct Quad {
  Q rot, o; P p = arb; bool mark;
  P& F() { return r()->p; }
  Q& r() { return rot->rot; }
  Q prev() { return rot->o->rot; }
  Q next() { return r()->prev(); }
} *H;

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
  lll p2 = p.dist2(), A = a.dist2()-p2,
      B = b.dist2()-p2, C = c.dist2()-p2;
  return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
}
Q makeEdge(P orig, P dest) {
  Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
  H = r->o; r->r()->r() = r;
  rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
  r->p = orig; r->F() = dest;
  return r;
}
void splice(Q a, Q b) {
  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next());
  splice(q->r(), b);
  return q;
}

pair<Q,Q> rec(const vector<P>& s) {
  if (sz(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
    if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = s[0].cross(s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
  }

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
  Q A, B, ra, rb;
  int half = sz(s) / 2;
  tie(ra, A) = rec({all(s) - half});
  tie(B, rb) = rec({sz(s) - half + all(s)});
  while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
         (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
      Q t = e->dir; \
      splice(e, e->prev()); \
      splice(e->r(), e->r()->prev()); \
      e->o = H; H = e; e = t; \
    }
  for (;;) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
      base = connect(RC, base->r());
    else
      base = connect(base->r(), LC->r());
  }
  return { ra, rb };
}

vector<P> triangulate(vector<P> pts) {
  sort(all(pts));  assert(unique(all(pts)) == pts.end());
  if (sz(pts) < 2) return {};
  Q e = rec(pts).first;
  vector<Q> q = {e};
  int qi = 0;
  while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
  q.push_back(c->r()); c = c->next(); } while (c != e); }
  ADD; pts.clear();
  while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
  return pts;
}
```

8.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c355, 6 lines

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilist) {
  double v = 0;
  for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
  return v / 6;
}
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

8058aeda, 32 lines

```
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }
};
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.
Time: $\mathcal{O}(n^2)$

"Point3D.h" 5b45fc3f, 49 lines

```
typedef Point3D<double> P3;

struct PR {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
  assert(sz(A) >= 4);
  vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
  vector<F> FS;
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
      q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
  };
  rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);

  rep(i,4,sz(A)) {
    rep(j,0,sz(FS)) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
```

```
    }
    int nw = sz(FS);
    rep(j,0,nw) {
      F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
  return FS;
};
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

611f0797, 8 lines

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);
  return radius*2*asin(d/2);
}
```

Strings (9)

KMP.h

Description: p[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123). Can be used to find all occurrences of a string.

Time: $\mathcal{O}(n)$

f84022ea, 17 lines

```
auto getBorder = [](string s) {
  vc<int> p(sz(s));
  for (int i = 1; i < sz(s); ++i) {
    int g = p[i - 1];
    while (g && s[g] != s[i]) g = p[g - 1];
    p[i] = g + (s[g] == s[i]);
  }
  return p;
}; // 43f874fa

auto KMP = [&getBorder](string s, string pat) {
  vc<int> p = getBorder(pat + '\0' + s), ans;
  for (int i = sz(p) - sz(s); i < sz(p); ++i) {
    if (p[i] == sz(pat)) ans.emplace_back(i - 2 * sz(pat));
  }
  return ans;
}; // 8ffe484c
```

Zfunc.h

Description: z[x] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)

Time: $\mathcal{O}(n)$

3ae5260c, 12 lines

```
vi Z(string S) {
  vi z(sz(S));
  int l = -1, r = -1;
  rep(i,1,sz(S)) {
    z[i] = i >= r ? 0 : min(r - i, z[i - l]);
    while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]])
      z[i]++;
    if (i + z[i] > r)
      l = i, r = i + z[i];
  }
  return z;
}
```

Manacher.h

Description: For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).

Time: $\mathcal{O}(N)$

e7ad794a, 13 lines

```
array<vi, 2> manacher(const string& s) {
  int n = sz(s);
  array<vi,2> p = {vi(n+1), vi(n)};
  rep(z,0,2) for (int i=0,l=0,r=0; i < n; i++) {
    int t = r-i+!z;
    if (i<r) p[z][i] = min(t, p[z][l+t]);
    int L = i-p[z][i], R = i+p[z][i]-!z;
    while (L>=1 && R+1<n && s[L-1] == s[R+1])
      p[z][i]++, L--, R++;
    if (R>r) l=L, r=R;
  }
  return p;
}
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.

Usage: rotate(v.begin(), v.begin()+minRotation(v), v.end());

Time: $\mathcal{O}(N)$

d07a42d1, 8 lines

```
int minRotation(string s) {
  int a=0, N=sz(s); s += s;
  rep(b,0,N) rep(k,0,N) {
    if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1); break;}
    if (s[a+k] > s[b+k]) { a = b; break; }
  }
  return a;
}
```

LyndonFactor.h

Description: A string is "simple" if it is strictly smaller than any of its own nontrivial suffixes. The Lyndon factorization of the string s is a factorization $s = w_1 w_2 \dots w_k$ where all strings w_i are simple and $w_1 \geq w_2 \geq \cdots \geq w_k$. Min rotation gets min index i such that cyclic shift of s starting at i is minimum.

Time: $\mathcal{O}(N)$

af38ba1b, 19 lines

```
vs duval(str s) {
  int N = sz(s); vs factors;
  for (int i = 0; i < N; ) {
    int j = i+1, k = i;
    for (; j < N && s[k] <= s[j]; ++j) {
      if (s[k] < s[j]) k = i;
      else ++k;
    }
    for (; i <= k; i += j-k) factors.pb(s.substr(i,j-k));
  }
  return factors;
}
int minRotation(str s) {
  int N = sz(s); s += s;
  vs d = duval(s); int ind = 0, ans = 0;
  while (ans+sz(d[ind]) < N) ans += sz(d[ind++]);
  while (ind && d[ind] == d[ind-1]) ans -= sz(d[ind--]);
  return ans;
}
```

SuffixArray.h

Description: Builds suffix array for a string. sa[i] is the starting index of the suffix which is i'th in the sorted suffix array. The returned vector is of size $n+1$, and sa[0] = n. The ht array contains longest common prefixes for neighbouring strings in the suffix array: ht[i] = ht(sa[i], sa[i-1]), ht[0] = 0. The input string must not contain any zero bytes.

Time: $\mathcal{O}(n \log n)$

195132ce, 24 lines

```
struct SA {
  vc<int> sa, ht, rk;
  SA(string s, int lim = 256) {
    int n = sz(s) + 1, k = 0, a, b, i, j, p;
    vc<int> x(all(s) + 1), y(n), ws(max(n, lim));
    rk = sa = ht = y, iota(all(sa), 0);
    for (j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
      p = j, iota(all(y), n - j);
      for (i = 0; i < n; ++i) if (sa[i] >= j) y[p++] = sa[i]-j;
      fill(all(ws), 0);
      for (i = 0; i < n; ++i) ++ws[x[i]];
      for (i = 1; i < lim; ++i) ws[i] += ws[i - 1];
      for (i = n; i--; ) sa[--ws[x[y[i]]]] = y[i];
      swap(x, y), p = 1, x[sa[0]] = 0;
      for (i = 1; i < n; ++i) {
        a = sa[i - 1], b = sa[i];
        x[b] = (y[a]==y[b] && y[a+j]==y[b+j]) ? p - 1 : p++;
      }
    }
    for (i = 1; i < n; ++i) rk[sa[i]] = i;
    for (i = 0; i < n - 1; ht[rk[i++]] = k)
      for (k && --k, j = sa[rk[i] - 1]; s[i+k] == s[j+k]; ++k);
  }
};
```

SuffixAutomaton.h

Description: Suffix automaton. Constructs a DAG efficiently maintaining equivalence classes of string occurrences. LOOK AT THE PICTURE!!! Each distinct string is some path through the automaton. Each occurrence of string w is a path from its node to some terminal node. At most 2N states and 3N edges in the whole automaton. Many things done by DP, add calculations in init()

Time: $\mathcal{O}(n\alpha)$ or $\mathcal{O}(nlog\alpha)$ If you need suffix tree, use suffix links in SA for reversed string.

<bits/stdc++.h> 70b19e8e, 92 lines

```
using namespace std;
struct state {
    int len, link;
    map<char, int> next;
    state() : len(0), link(-1) {}
};
struct suffix_automaton {
  string input;
  vector <state> st;
  int last, size;
  vi top; vector<ll> cnt; vector<bool> odw;
  suffix_automaton(const string &s) : input(s), last(0), size
      (1) {
    st.push_back(state());
    trav(c, s) add_letter(c);
    init();
  }
  void dfs(int x) {
    odw[x] = 1;
    for (auto [lett, node] : st[x].next)
      if (!odw[node]) dfs(node);
    top.push_back(x);
  }
  void init() {
    int p = last;
    cnt.resize(size, 0); odw.resize(size, 0);
```

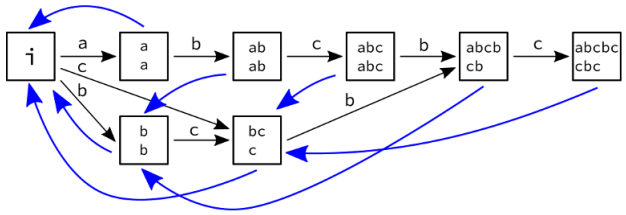
```
    while (p > 0) cnt[p]++, p = st[p].link;
    dfs(0);
    reverse(all(top)); assert(top[0] == 0);
    for (int i = sz(top)-1; i>0; --i) {
      for (auto [lett, node] : st[top[i]].next) {
        cnt[top[i]] += cnt[node]; //dp calculations here
      }
    }
  }
  void add_letter(char c) {
    st.push_back(state());
    int cur = size++;
    st[cur].len = st[last].len + 1;
    int p = last;
    while (p != -1 && !st[p].next.count(c)) {
      st[p].next[c] = cur;
      p = st[p].link;
    }
    if (p == -1) {
      st[cur].link = 0;
    } else {
      int q = st[p].next[c];
      if (st[p].len + 1 == st[q].len) {
        st[cur].link = q;
      } else {
        st.push_back(state());
        int clone = size++;
        st[clone].len = st[p].len + 1;
        st[clone].next = st[q].next;
        st[clone].link = st[q].link;
        while (p != -1 && st[p].next[c] == q) {
          st[p].next[c] = clone;
          p = st[p].link;
        }
        st[q].link = st[cur].link = clone;
      }
    }
    last = cur;
  }
  int search(const string &s) {
    int q = 0;
    trav(c, s) {
      if (st[q].next.find(c) == st[q].next.end()) return 0;
      q = st[q].next[c];
    }
    return q;
  }
  ll count_occs(string &s) { return cnt[search(s)]; }
  string lcs(const string &T) {
    int v = 0, l = 0, best = 0, bestpos = 0;
    rep(i, 0, sz(T)) {
      while (v && !st[v].next.count(T[i])) {
        v = st[v].link;
        l = st[v].len;
      }
      if (st[v].next.count(T[i])) {
        v = st [v].next[T[i]];
        l++;
      }
      if (l > best) {
        best = l;
        bestpos = i;
      }
    }
    return T.substr(bestpos - best + 1, best);
  }
};
```

RunEnumerate.h

Description: Find all (i, p) such that s.substr(i,p) == s.substr(i+p,p). No two intervals with the same period intersect or touch. Also look at comments below.
Time: $\mathcal{O}(N \log N)$

"SuffixArray.h" 34814a13, 14 lines

```
vector<array<int,3>> solve(string s) {
  int N = sz(s); SuffixArray A(s);
  reverse(all(s));
  SuffixArray B(s);
  vector<array<int,3>> runs;
  for (int p = 1; 2*p <= N; ++p) { // do in O(N/p) for period p
    for (int i = 0, lst = -1; i+p <= N; i += p) {
      int l = i-B.get_lcp(N-i-p,N-i), r = i-p+A.get_lcp(i,i+p);
      if (l > r || l == lst) continue;
      runs.pb({lst = l,r,p}); // for each i in [l,r],
    } // s.substr(i,p) == s.substr(i+p,p)
  }
  return runs;
}
```



Hashing.h

Description: Self-explanatory methods for string hashing. 706b23b8, 44 lines

```
// Arithmetic mod 2^64-1. 2x slower than mod 2^64 and more
// code, but works on evil test data (e.g. Thue-Morse, where
// ABBA... and BAAB... of length 2^10 hash the same mod 2^64).
// "typedef ull H;" instead if you think test data is random,
// or work mod 10^9+7 if the Birthday paradox is not a problem.
struct H {
  typedef uint64_t ull;
  ull x; H(ull x=0) : x(x) {}
  H operator+(H o) { return x + o.x + (x + o.x < x); }
  H operator-(H o) { return *this + ~o.x; }
  H operator*(H o) { auto m = (__uint128_t)x * o.x;
    return H((ull)m) + (ull)(m >> 64); }
  ull get() const { return x + !~x; }
  bool operator==(H o) const { return get() == o.get(); }
  bool operator<(H o) const { return get() < o.get(); }
};
static const H C = (ll)1e11+3; // (order ~ 3e9; random also ok)

struct HashInterval {
  vector<H> ha, pw;
  HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
    pw[0] = 1;
    rep(i,0,sz(str))
      ha[i+1] = ha[i] * C + str[i],
      pw[i+1] = pw[i] * C;
  }
  H hashInterval(int a, int b) { // hash [a, b)
    return ha[b] - ha[a] * pw[b - a];
  }
};

vector<H> getHashes(string& str, int length) {
  if (sz(str) < length) return {};
  H h = 0, pw = 1;
  rep(i,0,length)
    h = h * C + str[i], pw = pw * C;
  vector<H> ret = {h};
  rep(i,length,sz(str)) {
    ret.push_back(h = h * C + str[i] - pw * str[i-length]);
  }
  return ret;
}

H hashString(string& s){H h{}; for(char c:s) h=h*C+c;return h;}
```

AhoCorasick.h

Description: Aho-Corasick automaton, used for multiple pattern matching. Initialize with AhoCorasick ac(patterns); the automaton start node will be at index 0. find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(−, word) finds all words (up to $N\sqrt{N}$ many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input. For large alphabets, split each symbol into chunks, with sentinel bits for symbol boundaries.
Time: construction takes $\mathcal{O}(26N)$, where N = sum of length of patterns. find(x) is $\mathcal{O}(N)$, where N = length of x. findAll is $\mathcal{O}(NM)$. f35677c4, 66 lines

```
struct AhoCorasick {
  enum {alpha = 26, first = 'A'}; // change this!
  struct Node {
    // (nmatches is optional)
    int back, next[alpha], start = -1, end = -1, nmatches = 0;
    Node(int v) { memset(next, v, sizeof(next)); }
  };
  vector<Node> N;
  vi backp;
  void insert(string& s, int j) {
    assert(!s.empty());
    int n = 0;
    for (char c : s) {
      int& m = N[n].next[c - first];
      if (m == -1) { n = m = sz(N); N.emplace_back(-1); }
      else n = m;
    }
    if (N[n].end == -1) N[n].start = j;
    backp.push_back(N[n].end);
    N[n].end = j;
    N[n].nmatches++;
  }
  AhoCorasick(vector<string>& pat) : N(1, -1) {
    rep(i,0,sz(pat)) insert(pat[i], i);
    N[0].back = sz(N);
    N.emplace_back(0);

    queue<int> q;
    for (q.push(0); !q.empty(); q.pop()) {
      int n = q.front(), prev = N[n].back;
      rep(i,0,alpha) {
        int &ed = N[n].next[i], y = N[prev].next[i];
        if (ed == -1) ed = y;
        else {
          N[ed].back = y;
          (N[ed].end == -1 ? N[ed].end : backp[N[ed].start])
            = N[y].end;
          N[ed].nmatches += N[y].nmatches;
          q.push(ed);
        }
      }
    }
  }
  vi find(string word) {
```

```
    int n = 0;
    vi res; // ll count = 0;
    for (char c : word) {
      n = N[n].next[c - first];
      res.push_back(N[n].end);
      // count += N[n].nmatches;
    }
    return res;
  }
  vector<vi> findAll(vector<string>& pat, string word) {
    vi r = find(word);
    vector<vi> res(sz(word));
    rep(i,0,sz(word)) {
      int ind = r[i];
      while (ind != -1) {
        res[i - sz(pat[ind]) + 1].push_back(ind);
        ind = backp[ind];
      }
    }
    return res;
  }
};
```

Various (10)

10.1 Intervals

IntervalContainer.h

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).
Time: $\mathcal{O}(\log N)$

edce4766, 23 lines

```
set<pii>::iterator addInterval(set<pii>& is, int L, int R) {
  if (L == R) return is.end();
  auto it = is.lower_bound({L, R}), before = it;
  while (it != is.end() && it->first <= R) {
    R = max(R, it->second);
    before = it = is.erase(it);
  }
  if (it != is.begin() && (--it)->second >= L) {
    L = min(L, it->first);
    R = max(R, it->second);
    is.erase(it);
  }
  return is.insert(before, {L,R});
}

void removeInterval(set<pii>& is, int L, int R) {
  if (L == R) return;
  auto it = addInterval(is, L, R);
  auto r2 = it->second;
  if (it->first == L) is.erase(it);
  else (int&)it->second = L;
  if (R != r2) is.emplace(R, r2);
}
```

IntervalCover.h

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add `|| R.empty()`. Returns empty set on failure (or if G is empty).
Time: $\mathcal{O}(N \log N)$

9e9d8de7, 19 lines

```
template<class T>
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
  vi S(sz(I)), R;
  iota(all(S), 0);
  sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
  T cur = G.first;
  int at = 0;
  while (cur < G.second) { // (A)
    pair<T, int> mx = make_pair(cur, -1);
    while (at < sz(I) && I[S[at]].first <= cur) {
      mx = max(mx, make_pair(I[S[at]].second, S[at]));
      at++;
    }
    if (mx.second == -1) return {};
    cur = mx.first;
    R.push_back(mx.second);
  }
  return R;
}
```

ConstantIntervals.h

Description: Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.
Usage: `constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});`
Time: $\mathcal{O}\left(k \log \frac{n}{k}\right)$

753a4cd9, 19 lines

```
template<class F, class G, class T>
void rec(int from, int to, F& f, G& g, int& i, T& p, T q) {
  if (p == q) return;
  if (from == to) {
    g(i, to, p);
    i = to; p = q;
  } else {
    int mid = (from + to) >> 1;
    rec(from, mid, f, g, i, p, f(mid));
    rec(mid+1, to, f, g, i, p, q);
  }
}
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
  if (to <= from) return;
  int i = from; auto p = f(i), q = f(to-1);
  rec(from, to-1, f, g, i, p, q);
  g(i, to, q);
}
```

10.2 Misc. algorithms

TernarySearch.h

Description: Find the smallest i in $[a, b]$ that maximizes $f(i)$, assuming that $f(a) < \ldots < f(i) \geq \cdots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the $<$ marked with (A) to $<=$, and reverse the loop at (B). To minimize f, change it to $>$, also at (B).
Usage: `int ind = ternSearch(0,n-1,[&](int i){return a[i];});`
Time: $\mathcal{O}(\log(b-a))$

9155b4fb, 11 lines

```
template<class F>
int ternSearch(int a, int b, F f) {
  assert(a <= b);
  while (b - a >= 5) {
    int mid = (a + b) / 2;
    if (f(mid) < f(mid+1)) a = mid; // (A)
    else b = mid+1;
  }
  rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
  return a;
}
```

LIS.h

Description: Compute indices for the longest increasing subsequence.
Time: $\mathcal{O}(N \log N)$

2932a052, 17 lines

```
template<class I> vi lis(const vector<I>& S) {
  if (S.empty()) return {};
  vi prev(sz(S));
  typedef pair<I, int> p;
  vector<p> res;
  rep(i,0,sz(S)) {
    // change 0 -> i for longest non-decreasing subsequence
    auto it = lower_bound(all(res), p{S[i], 0});
    if (it == res.end()) res.emplace_back(), it = res.end()-1;
    *it = {S[i], i};
    prev[i] = it == res.begin() ? 0 : (it-1)->second;
  }
  int L = sz(res), cur = res.back().second;
  vi ans(L);
  while (L--) ans[L] = cur, cur = prev[cur];
  return ans;
}
```

FastKnapsack.h

Description: Given N non-negative integer weights w and a non-negative target t, computes the maximum S <= t such that S is the sum of some subset of the weights.
Time: $\mathcal{O}(N \max(w_i))$

14f7b939, 18 lines

```
int knapsack(vc<int> w, int t) {
  int a = 0, b = 0, x;
  while (b < sz(w) && a + w[b] <= t) a += w[b++];
  if (b == sz(w)) return a;
  int m = *max_element(all(w));
  vc<int> u, v(2*m, -1);
  v[a+m-t] = b;
  for (int i = b; i < sz(w); ++i) {
    u = v;
    for (int x = 0; x < m; ++x)
      v[x+w[i]] = max(v[x+w[i]], u[x]);
    for (x = 2*m; --x > m;)
      for (int j = max(0, u[x]); j < v[x]; ++j)
      v[x-w[j]] = max(v[x-w[j]], j);
  }
  for (a = t; v[a+m-t] < 0; a--) ;
  return a;
}
```

10.3 Dynamic programming

KnuthDP.h

Description: When doing DP on intervals: $a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \leq f(a,d)$ and $f(a,c) + f(b,d) \leq f(a,d) + f(b,c)$ for all $a \leq b \leq c \leq d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.
Time: $\mathcal{O}\left(N^2\right)$

DivideAndConquerDP.h

Description: Given $a[i] = \min_{lo(i) \leq k < hi(i)}(f(i,k))$ where the (minimal) optimal k increases with i, computes $a[i]$ for $i = L..R-1$.
Time: $\mathcal{O}\left((N + (hi - lo)) \log N\right)$

d38d2b27, 18 lines

```
struct DP { // Modify at will:
  int lo(int ind) { return 0; }
  int hi(int ind) { return ind; }
  ll f(int ind, int k) { return dp[ind][k]; }
  void store(int ind, int k, ll v) { res[ind] = pii(k, v); }

  void rec(int L, int R, int LO, int HI) {
    if (L >= R) return;
    int mid = (L + R) >> 1;
```

```
    pair<ll, int> best(LLONG_MAX, LO);
    rep(k, max(LO,lo(mid)), min(HI,hi(mid)))
      best = min(best, make_pair(f(mid, k), k));
    store(mid, best.second, best.first);
    rec(L, mid, LO, best.second+1);
    rec(mid+1, R, best.second, HI);
  }
  void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
};
```

10.4 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` failures generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (1), 0-divs (4), infinities (8) and denormals (16).

10.5 Optimization tricks

`__builtin_ia32_ldmxcsr(40896);` disables denormals (which make floats 20x slower near their minimum value).

10.5.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K)) if (i & 1 << b) D[i] += D[i^(1 << b)];` computes all sums of subsets.

10.5.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize for loops and optimizes floating points better (assumes associativity and turns off denormals).
- `#pragma GCC target ("avx,avx2")` can double performance of vectorized code, but causes crashes on old machines. Also consider older `#pragma GCC target ("sse4")`.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

FastMod.h

Description: Compute $a\%b$ about 5 times faster than usual, where b is constant but not known at compile time. Returns a value congruent to a (mod b) in the range $[0, 2b)$.

751a02f9, 8 lines

```
typedef unsigned long long ull;
struct FastMod {
  ull b, m;
  FastMod(ull b) : b(b), m(-1ULL / b) {}
  ull reduce(ull a) { // a % b + (0 or b)
    return a - (ull)((__uint128_t(m) * a) >> 64) * b;
  }
};
```

FastInput.h

Description: Read an integer from stdin. Usage requires your program to pipe in input from file.
Usage: `./a.out < input.txt`
Time: About 5x as fast as cin/scanf.

7b3c7051, 17 lines

```
inline char gc() { // like getchar()
  static char buf[1 << 16];
  static size_t bc, be;
  if (bc >= be) {
    buf[0] = 0, bc = 0;
    be = fread(buf, 1, sizeof(buf), stdin);
  }
  return buf[bc++]; // returns 0 on EOF
}

int readInt() {
  int a, c;
  while ((a = gc()) < 40);
  if (a == '-') return -readInt();
  while ((c = gc()) >= 48) a = a * 10 + c - 480;
  return a - 48;
}
```

BumpAllocator.h

Description: When you need to dynamically allocate many objects and don't care about freeing them. "new X" otherwise has an overhead of something like 0.05us + 16 bytes per allocation.

745db225, 8 lines

```
// Either globally or in a single class:
static char buf[450 << 20];
void* operator new(size_t s) {
  static size_t i = sizeof buf;
  assert(s < i);
  return (void*)&buf[i -= s];
}
void operator delete(void*) {}
```

SmallPtr.h

Description: A 32-bit pointer that points into BumpAllocator memory.

"BumpAllocator.h" 2dd6c977, 10 lines

```
template<class T> struct ptr {
  unsigned ind;
  ptr(T* p = 0) : ind(p ? unsigned((char*)p - buf) : 0) {
    assert(ind < sizeof buf);
  }
  T& operator*() const { return *(T*)(buf + ind); }
  T* operator->() const { return &**this; }
  T& operator[](int a) const { return (&**this)[a]; }
  explicit operator bool() const { return ind; }
};
```

BumpAllocatorSTL.h

Description: BumpAllocator for STL containers.
Usage: `vector<vector<int, small<int>>> ed(N);`

bb66d422, 14 lines

```
char buf[450 << 20] alignas(16);
size_t buf_ind = sizeof buf;

template<class T> struct small {
  typedef T value_type;
  small() {}
  template<class U> small(const U&) {}
  T* allocate(size_t n) {
    buf_ind -= n * sizeof(T);
    buf_ind &= 0 - alignof(T);
    return (T*)(buf + buf_ind);
  }
  void deallocate(T*, size_t) {}
};
```

Unrolling.h

520e76d6, 5 lines

```
#define F {...; ++i;}
int i = from;
while (i&3 && i < to) F // for alignment, if needed
while (i + 4 <= to) { F F F F }
while (i < to) F
```

SIMD.h

Description: Cheat sheet of SSE/AVX intrinsics, for doing arithmetic on several numbers at once. Can provide a constant factor improvement of about 4, orthogonal to loop unrolling. Operations follow the pattern `"_mm(256)?_name_(si(128|256)|epi(8|16|32|64)|pd|ps)"`. Not all are described here; grep for `_mm_` in `/usr/lib/gcc/*/4.9/include/` for more. If AVX is unsupported, try 128-bit operations, "emmintrin.h" and `#define __SSE__` and `__MMX__` before including it. For aligned memory use `_mm_malloc(size, 32)` or `int buf[N] alignas(32)`, but prefer loadu/storeu.

551b8204, 43 lines

```
#pragma GCC target ("avx2") // or sse4.1
#include "immintrin.h"

typedef __m256i mi;
#define L(x) _mm256_loadu_si256((mi*)&(x))

// High-level/specific methods:
// load(u)?_si256, store(u)?_si256, setzero_si256, _mm_malloc
// blendv_(epi8|ps|pd) (z?y:x), movemask_epi8 (hibits of bytes)
// i32gather_epi32(addr, x, 4): map addr[] over 32-b parts of x
// sad_epu8: sum of absolute differences of u8, outputs 4xi64
// maddubs_epi16: dot product of unsigned i7's, outputs 16xi15
// madd_epi16: dot product of signed i16's, outputs 8xi32
// extractf128_si256(, i) (256->128), cvtsi128_si32 (128->lo32)
// permute2f128_si256(x,x,1) swaps 128-bit lanes
// shuffle_epi32(x, 3*64+2*16+1*4+0) == x for each lane
// shuffle_epi8(x, y) takes a vector instead of an imm

// Methods that work with most data types (append e.g. _epi32):
// set1, blend (i8?x:y), add, adds (sat.), mullo, sub, and/or,
// andnot, abs, min, max, sign(1,x), cmp(gt|eq), unpack(lo|hi)

int sumi32(mi m) { union {int v[8]; mi m;} u; u.m = m;
  int ret = 0; rep(i,0,8) ret += u.v[i]; return ret; }
mi zero() { return _mm256_setzero_si256(); }
mi one() { return _mm256_set1_epi32(-1); }
bool all_zero(mi m) { return _mm256_testz_si256(m, m); }
bool all_one(mi m) { return _mm256_testc_si256(m, one()); }

ll example_filteredDotProduct(int n, short* a, short* b) {
  int i = 0; ll r = 0;
  mi zero = _mm256_setzero_si256(), acc = zero;
  while (i + 16 <= n) {
    mi va = L(a[i]), vb = L(b[i]); i += 16;
    va = _mm256_and_si256(_mm256_cmpgt_epi16(vb, va), va);
    mi vp = _mm256_madd_epi16(va, vb);
    acc = _mm256_add_epi64(_mm256_unpacklo_epi32(vp, zero),
      _mm256_add_epi64(acc, _mm256_unpackhi_epi32(vp, zero)));
  }
  union {ll v[4]; mi m;} u; u.m = acc; rep(i,0,4) r += u.v[i];
  for (;i<n;++i) if (a[i] < b[i]) r += a[i]*b[i]; // <- equiv
  return r;
}
```