$$\sum_{i \in N_0} x_{ij} + \sum_{i \in N_0} \sum_{k \in N_+} y_{ijk} = 1 \quad \forall j \in C$$

$$i \neq j \quad i \neq j \quad \langle i, j, k \rangle \in P$$

$$\sum_{j \in N_+} x_{0j} = 1$$

$$\sum_{i \in N_0} x_{i,c+1} = 1$$

2) 保证访问的次序, ps: 如果不进行这一步操作,有可能会造成成环(给出效果图)

$$u_i-u_j+1\leqslant (c+2)(1-x_{ij}) \quad orall i\in C, j\in \{N_+:j
eq i\}$$

3) 保证到达每个节点的卡车都能从那个节点出去

$$\sum_{egin{array}{c} i \in N_0 \ i
eq j \end{array}} x_{ij} = \sum_{egin{array}{c} k \in N_+ \ k
eq j \end{array}} x_{jk} \quad orall j \in C$$

4) 保证到每一个节点只有一个

$$egin{aligned} \sum_{egin{subarray}{c} j \in C \ j
eq i \ \langle i,j,k
angle \in P \end{aligned}} y_{ijk} \leqslant 1 \quad orall i \in N_0 \end{aligned}$$

$$egin{aligned} \sum_{i \, \in \, N_0} & \sum_{j \, \in \, C} & y_{ijk} \leqslant 1 & orall k \in N_+ \ i
eq k & \langle i, j, k
angle \in P \end{aligned}$$

5) 保证无人机的起点和终点卡车都已经到达

$$egin{aligned} 2y_{ijk} \leqslant \sum_{h \,\in\, N_0} x_{hi} + \sum_{l \,\in\, C} x_{lk} & orall i \in C, j \in \{C: j \,
eq i\}, k \ h
eq i & l
eq k \ \in \{N_+: \langle i, j, k
angle \in P\} \end{aligned}$$

$$y_{0\,jk}\leqslant \sum_{egin{array}{c} h\in N_0\ h
eq k \end{array}} x_{hk}\quad orall j\in C, k\in \{N_+:\langle 0,j,k
angle\in P\}$$

6) 保证放飞无人机的点在回收无人机的点之前

$$egin{aligned} u_k - u_i \geqslant 1 - (c+2) \left(1 - \sum_{egin{aligned} j \in C \ \langle i, j, k
angle \in P \end{aligned}} y_{ijk} \end{aligned}
ight) & orall i \in C, k \in \{N_+: k
eq i\}$$

7) 放飞和回收无人机时,无人机的飞到这一点的时间等于卡车到这一点的时间

$$egin{aligned} t_i' &\geqslant t_i - M \left(1 - \sum_{egin{aligned} j \in C & k \in N_+ \ j
eq i & \langle i, j, k
angle \in P \end{aligned}} egin{aligned} orall i \in C \ & t_i' \leqslant t_i + M \left(1 - \sum_{egin{aligned} j \in C & k \in N_+ \ j
eq i & \langle i, j, k
angle \in P \end{aligned}} egin{aligned} orall i \in C \end{aligned}
ight.$$

$$t_k' \geqslant t_k - M egin{pmatrix} 1 - \sum_{i \in N_0} & \sum_{j \in C} y_{ijk} \ i
eq k & \langle i, j, k
angle \in P \end{pmatrix} \quad orall k \in N_+$$

8) 保证无人机和时间约束

$$t_k \geqslant t_h + au_{hk} + s_L \left(\sum_{egin{array}{c} L \in C & m \in N_+ \ l
eq k & \langle k,l,m
angle \in P \end{array}} y_{klm}
ight) + s_R \left(\sum_{egin{array}{c} i \in N_0 & j \in C \ i
eq k & \langle i,j,k
angle \in P \end{array}} y_{ijk}
ight) - M(1-x_{hk})$$

 $\forall h \in N_0, k \in \{N_+ : k \neq h\}$

$$t_j'\geqslant t_i'+ au_{ij}'-Megin{pmatrix}1-\sum_{m{k}\,\in\,m{N}_+}&y_{ijk}\ \langle i,j,k
angle\in P\end{pmatrix} \quadorall j\in C',i\in\{N_0:i\,
eq j\}$$

$$t_k'\geqslant t_j'+ au_{jk}'+s_R-M egin{pmatrix} 1-\sum_{egin{array}{c}i\in N_0\ \langle i,j,k
angle \in P \end{pmatrix}} y_{ijk} & orall j\in C',k\in\{N_+:k
eq j\} \end{cases}$$

9) 保证续航 e 为续航时间

$$egin{aligned} t_k'-(t_j'- au_{ij}')&\leqslant e+M(1-y_{ijk}) &orall k\in N_+, j\in\{C:j
eq k\}, i\ &\in\{N_0:\langle i,j,k
angle\in P\} \end{aligned}$$

10) 得到一个次序如果两个客户都由卡车访问,并且卡车先访问客户 i 再访问客户 j,那么 p_ij 就为1,这有助于在后续的优化模型中确保顺序的一致性

$$u_i-u_j\geqslant 1-(c+2)p_{ij}\quad orall i\in C, j\in\{C:j
eq i\}$$

$$u_i-u_j\leqslant -1+(c+2)(1-p_{ij})\quad orall i\in C, j\in \{C:j
eq i\}$$

$$p_{ij}+p_{ji}=1 \quad orall i \in C, j \in \{C: j
eq i\}$$

$$egin{aligned} t_l' \geqslant t_k' - M \ & 3 - \sum_{egin{aligned} j \in C \ j
eq l \end{aligned}} y_{ijk} - \sum_{egin{aligned} m \in C \ m \in N_+ \ (l,m,n) \in P \ j
eq l \end{aligned}} y_{lmn} - p_{il} \ & \sum_{egin{aligned} \langle i,j,k \rangle \in P \ m
eq i \ \langle l,m,n \rangle \in P \ m
eq l \ n
eq k \end{aligned}} y_{lmn} - p_{il} \ & \sum_{egin{aligned} \langle i,j,k \rangle \in P \ j
eq l \end{aligned}} y_{lmn} - p_{il} \ & \sum_{egin{aligned} \langle i,j,k \rangle \in P \ j
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eq l \ & \sum_{egin{aligned} \langle i,j,k \rangle \in P \ j \ eq l \ & \sum_{egin{aligned} \langle i,j,k \rangle \in P \ j \ eq l \ & \sum_{egin{aligned} \langle i,j,k \rangle \in P \ ij \ eq l \ & \sum_{egin{aligned} \langle i,j,k \rangle \in P \ ij \ eq l \ & \sum_{egin{aligned} \langle i,j,k \rangle \in P \ ij \ eq l \ & \sum_{egin{aligned} \langle i,j,k \rangle \in P \ ij \ & \sum_{egin{aligned} \langle i,j,k \rangle \in P \ ij \ & \sum_{egin{aligned} \langle i,j,k \rangle \in P \ ij \ & \sum_{egin{aligned} \langle i,j,k \rangle \in P \ ij \ & \sum_{egin{aligned} \langle i,j,k \rangle \in P \ ij \ & \sum_{egin{aligned} \langle i,j,k \rangle \in P$$

11) 其他约束条件

 $t_0 = 0$

 $t_0' = 0$

 $p_{0j}=1 \quad orall j \in C$

 $x_{ij} \in \{0,1\} \quad orall i \in N_0, j \in \{N_+: j
eq i\}$

 $y_{ijk} \in \{0,1\} \quad orall i \in N_0, j \in \{C: j
eq i\}, k \in \{N_+: \langle i,j,k
angle \in P\}$

 $1\leqslant u_i\leqslant c+2 \quad orall i\in N_+$

 $t_i\geqslant 0 \quad orall i\in N$

 $t_i'\geqslant 0 \quad orall i\in N$

 $p_{ij} \in \{0,1\} \quad \forall i \in N_0, j \in \{C: j \neq i\}.$

(1) 由于上述问题变量较多,使用 Pvthon 求解较为方便。求解的代码如下:

求解结果为:

Time cost: 1.7875301

Start:0, end:6

Truck: $0 (0.00) \rightarrow 2 (0.89)$ Truck: 1 (1.10) ->3 (1.62) Truck: 2 (0.89) ->1 (1.10) Truck: 3 (1.62) ->6 (1.79) $UAV: 0 (0.00) \rightarrow 5 (0.42) \rightarrow 2 (0.89)$ $UAV: 2(0.89) \rightarrow 4(1.37) \rightarrow 6(1.79)$

对应的方案为:

卡车路线为:

0 (发点) (时间 0.00) ->2 (时间 0.89) ->1 (时间 1.10) ->3 (时间 1.62) ->6 (终 点) (时间1.79)

无人机线路为:

0(放出, 时间 0.00) ->5(送货, 时间 0.42) ->2(收回, 时间 0.89) 2(放出,时间 0.89)->4(送货,时间 1.37)->6(收回,时间 1.79)

(2)结合(1)的求解结果 2.66, (2)提出的模型所需时间 1.78 更短, 因此(2)