IEOR E4601: Dynamic Pricing and Revenue Management
Lecture 7: Assortment optimization under MNL

1 Study Guide

By the end of this lecture, you should be able to

1. State the assortment optimization problem and explain its connection to price optimization.
2. Derive the alternative formulation that leads to the binary search algorithm.
3. Perform the binary search algorithm.
4. Derive the alternative formulation that leads to the direct search algorithm.
5. Perform the direct search algorithm.

2 Recall of MNL and problem statement

Revenue is

$$R^* = \max_{S \subset \{1,\dots,n\}} \sum_{i \in S} \frac{p_i e^{u_i}}{1 + \sum_{j \in S} e^{u_j}}$$

Let $V_i = e^{u_i}$, ~~them~~ called attraction of i

$$R^* = \max_{S \subset \{1,2,\dots,n\}} \sum_{i \in S} \frac{p_i V_i}{1 + \sum_{j \in S} V_j}$$

Discussion: How can assortment optimization be considered as a special case of price optimization?

3 Binary search algorithm

$$R^* = \min z$$
$$\text{s.t.} \quad z \geq \max_{S \subseteq \{1,\dots,n\}} \frac{\sum_{i \in S} p_i v_i}{1 + \sum_{j \in S} v_j}$$

$$= \min z$$
$$\text{s.t.} \quad z \geq \frac{\sum p_i v_i}{1 + \sum_{j \in S} v_j} \quad \forall S \subset \{1, n\} \Leftarrow \text{for any subset}$$

$$= \min z$$
$$\text{s.t.} \quad z(1 + \sum_{j \in S} v_j) \geq \sum_{j \in S} p_i v_i$$

$$= \min z$$
$$\text{s.t.} \quad z \geq \sum_{i \in S} v_i (p_i - z) \quad \forall S \subset \{1, \dots, n\}$$

$$= \min z \quad \text{s.t.} \quad z \geq \max_S v_i (p_i - z)$$

if we know z, the maximiou will be achieve by picking ~~Sop~~ all x_i, as long as $p_i \geq z$

- Therefore, the optimal solution S^*, must be that $\{ i \mid p_i \geq z \}$ for some z

let's order products by price, s.t. $p_1 \geq p_2 \geq \dots \geq p_n$.

- Then S^* has the form $\{p_1, \dots p_k\}$ for some k ($[K]$).
  This is called a revenue ordered assortment
- Implication : we only have to check $S : [1], [2], \dots [n]$, $O(n)$

Init: $z^u = p_1$, $z^L = 0$

while $|z^u - z^L| > \varepsilon$, do:

Let $\hat{z} = \dfrac{z^u + z^L}{2}$

check whether $\hat{z}$ satisfy

$$\hat{z} \geq \max_S \sum v_i (p_i - \hat{z}) , \ \forall \ S \in \{[1], [2] \ldots [n]\}$$

if yes, then make $z^u = \hat{z}$

else make $z^L = \hat{z}$