Recitation n° 3 : Binary Search

02/17/19

Recap

⊕ $p_1 \geq \dots \geq p_n$

$\Rightarrow S^* = \{1, \dots, k\}$ for some $k \in \{1 \dots n\}$

where $S^* \in \arg\max \sum_{j \in S} p_j \frac{v_j}{1 + \sum_{i \in S} v_i}$,

$v_j = e^{u_j}$, $u_j = \beta^T x_j$.

⊕ We might just enumerate all sets

$\{1\}, \{1, 2\}, \dots, \{1, 2, \dots, n\}$.

This is n comparisons. We can do $O(\log(n))$ with binary search. | quick example: "Guess a number between 0 and 100"

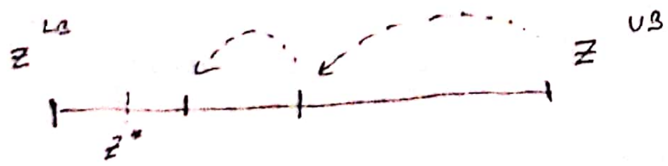
Binary search

⊕ We want $z^* = \min z$

(1) $z \geq \sum_{j \in S} (p_j - z) \cdot v_j \quad \forall S \subseteq \{1 \dots n\}$.

We know: $z^* \leq p_1 = z^{UB}$ because z_1 satisfies (1)

$z^{LB} = 0 < z^*$ because 0 does not satisfy (1).



TA

Candidate : $z^* = \frac{z^{UB} + z^{LB}}{2}$.

So if we want precision ε :

- $z^{UB} = p_1, \quad z^{LB} = 0$
- while $(|z^{UB} - z^{LB}| > \varepsilon)$

$$\left\{ \begin{array}{l} \oplus \text{ TEST: } \hat{z} = \frac{z^{UB} + z^{LB}}{2} \text{ is s.t. } \hat{z} \geq \sum_{j \in S} (p_j - \hat{z}) v_j \quad \forall S \subseteq \{1 \dots n\} \\ \oplus \text{ if TEST = TRUE, } \left\{ \begin{array}{l} z^{UB} \leftarrow \hat{z} \\ z^{LB} \leftarrow z^{LB} \end{array} \right\} \\ \oplus \text{ if TEST = FALSE, } \left\{ \begin{array}{l} z^{LB} \leftarrow \hat{z} \\ z^{UB} \leftarrow z^{UB} \end{array} \right\} \end{array} \right.$$

→ We divide the length of $|z^{UB} - z^{LB}|$ by 2 at every iteration → « binary search. »

→ « TEST » can be reduced to « $\hat{z} \geq \sum_{j \in S} (p_j - \hat{z}) \cdot v_j$;
$\forall S = \{1\}, \{1,2\}, \dots, \{1, \dots, n\}$ »

$\boxed{EX}$ $n = 6$, $(p_j)_{j \in \{1 \dots n\}} = (32, 30, 27, 15, 14, 6)$

$(v_j)_{j \in \{1 \dots n\}} = (4, 2, 5, 25, 30, 28)$

ⓐ Compute $\sum_{j=1}^{k} p_j v_j$, $\sum_{j=1}^{k} v_j$, $\forall k = 1 \dots n$.

$$\left(\sum_{j=1}^{k} p_j v_j\right)_{k=1\dots n} = (128,\ 188,\ 323,\ 698,\ 1128,\ 1288)$$

$$\left(\sum_{j=1}^{k} v_j\right)_{k=1\dots n} = (4,\ 6,\ 11,\ 36,\ 66,\ 94)$$

(b) Let us run the algorithm: $\boxed{\varepsilon = 4.}$

iteration	Z^{UB}	Z^{LB}	$\hat{Z} = \frac{Z^{UB}+Z^{LB}}{2}$	$\sum_{j=1}^{k} p_j v_j - \hat{Z}\cdot\sum_{j=1}^{k} v_j,\ k=1\dots 6$
0	32	0	16	(64, 92, 147, 122, -62, -218)
1	32	16	24	(32, 44, 59, -166, -466, -970
2	32	24	28 $= \hat{Z}_2$	(16, 20, 15, -310, -730, -1346)

We stop because $\hat{Z}_2 \geq \sum_{j=1}^{k} p_j v_j - \hat{Z}_2 \cdot \sum_{j=1}^{k} v_j,\ k=1\dots 6$

and $|\hat{Z}_2 - Z_{LB}| = |28 - 24| = 4 = \varepsilon.$