IEOR E4601: Dynamic Pricing and Revenue Management
Assignment 2

Notes

- All questions in this homework must be individual work. You are allowed to discuss the assignment with others but please mention in your write-up if you have discussed the solution with someone.
- For the questions involving data, you should also submit your programming code (Python, R, MATLAB or Excel) along with the answers.

Homework Exercises

1. **(Single Fare Class Overbooking)** In class, we have seen a problem of finding optimal overbooking amount for single fare class. Suppose the capacity is C seats, the price per seat is fixed at p, the number of no-show customers is a random variable $Y \sim$ cdf $F(\cdot)$ and the number of overbooked tickets is q. Assume we are able to sell all $(C+q)$ seats for any q. The profit or the revenue function is as follows.

$$\Pi(q) = p * q - \gamma * E_Y[(q-Y)_+],$$

where $\gamma > p$, is the penalty per customer who can't get a seat. Suppose Y is a Poisson random variable with mean 5, $p = 100, \gamma = 300, C = 100$.

   (a) Compute the optimal overbooking limit, q^* that maximizes the expected profit.

   (b) For q^* computed in part a), find the expected penalty paid due to overbooking.

2. **(Callable Tickets)**. US Tennis Association (USTA) is trying to devise a strategy to sell tickets for the Men's Quarterfinals for US Open 2017 in New York. The stadium has a seating capacity of 20000. Since the quarterfinals are held on a weekday, the demand is quite uncertain (assume all quarter finals are held on a single day and there is a single ticket that grants admission for all four quarter finals). USTA plans to offer a low-price ticket of \$50 with a restriction that USTA can buy it back from the customer anytime at the price of \$75. USTA estimates that there is enough demand for the low-price tickets and any number will be sold. USTA also plans to offer a high-price ticket of \$100 that does not have any restrictions. However, demand for the high-price tickets is uncertain and USTA estimates that is uniformly distributed between 8000 and 17000. Assume that all the low-price demand arrives before the first high-price demand arrives and there is enough demand for low price tickets in the initial period that any number of low-price tickets can be sold. However, no low-price tickets can be sold after high price demand arrives.

   (a) What is the optimal number of tickets that USTA should offer at high-price initially to maximize the expected revenue from sales?

   (b) What is the expected revenue if USTA offers the high-price tickets as computed in part a)?

3. **(Multiple Fare Class Protection Levels)**. Suppose there are 3 fare classes on a particular airline route. Let the random demand for the three fare classes be given by D_1, D_2 and D_3, where D_3 is the lowest fare class demand that arrives first, then D_2, and D_1, the highest fare class demand arrives last. The fares for the three classes are $p_3 = 100, p_2 = 200, p_1 = 300$. Suppose we have $C = 10$ seats in total and the demand distributions are given as follows.

$$D_3 = \begin{cases} 9 & \text{with prob. } \frac{1}{2} \\ 6 & \text{with prob. } \frac{1}{2} \end{cases}$$

$$D_2 = \begin{cases} 9 & \text{with prob. } \frac{1}{4} \\ 6 & \text{with prob. } \frac{1}{2} \\ 2 & \text{with prob. } \frac{1}{4} \end{cases}$$

$$D_1 = \begin{cases} 9 & \text{with prob. } \frac{1}{4} \\ 2 & \text{with prob. } \frac{3}{4} \end{cases}$$

Find the optimal protection levels y_1^* and y_2^* for D_1 and $D_1 + D_2$ to maximize the expected revenue.

4. **(Multinomial Logit Model)** Consider the multinomial logit (MNL) model we have discussed in class. Suppose the utility of product j is a linear function of the price p_j of item j. Then, the MNL model is given as follows. For any product $j = 1, \ldots, n$,

$$u_j = \beta_0 - \beta_1 p_j$$
$$v_j = e^{u_j}$$
$$P(j \mid S) = \frac{v_j}{1 + \sum_{i \in S} v_i}$$

Suppose there are $N = 10$ different substitutable products with prices $p_1 = 1, p_2 = 2, p_3 = 3, p_4 = 4, p_5 = 5, p_6 = 6, p_7 = 7, p_8 = 8, p_9 = 9, p_{10} = 10$. Suppose we are given historical data in a separate file *AssortmentSample.xlsx* about what set was offered and what product was selected (possibly no-purchase, i.e., product 0). Each row denotes the assortment offered and the corresponding choice of the customer. The following table shows a sample line of the dataset.

No-purchase0	item1	item2	item3	item4	item5	item6	item7	item8	item9	item10	Choice
1	0	1	1	0	0	0	0	0	1	1	0

Here a 1 in a column corresponding to a particular item denotes that the item is in the assortment and 0 denotes that the item is not in the assortment. The last column denotes the item chosen by the customer. In the instance we gave, the offer set is $\{2, 3, 9, 10\}$ and the customer chooses to not purchase.

(a) Give the first order optimality condition for MLE for β_0, β_1.

(b) Use the numbers for β_1 and β_0 you get above to find the optimal assortment we should offer to customers.

(c) Now, we consider to reset the prices for all products. Use the numbers for β_1 and β_0 you get above and suppose all the products will be offered to the customers, find the optimal prices for these N products.