6.1. (1). $U_c=[b\ \ Ab]=\begin{bmatrix}1&1\\0&3\end{bmatrix}$，秩$U_c=2$，系统完全能控，所以可以用状态反馈任意配置特征值.

(2) $U_c=[b\ Ab\ A^2b]=\begin{bmatrix}1&0&1&0&1&0\\0&1&0&-2&0&4\\0&0&0&0&0&0\end{bmatrix}$，秩$U_c=2$，系统不完全能控，所以不能通过状态反馈任意配置特征值.

6.2. $a^*(s)=(s+3)^3=s^3+9s^2+27s+27$

$$\dot{x}=\begin{bmatrix}0&1&0\\0&0&1\\-1&-1&-1\end{bmatrix}x+\begin{bmatrix}0\\0\\3\end{bmatrix}u.$$

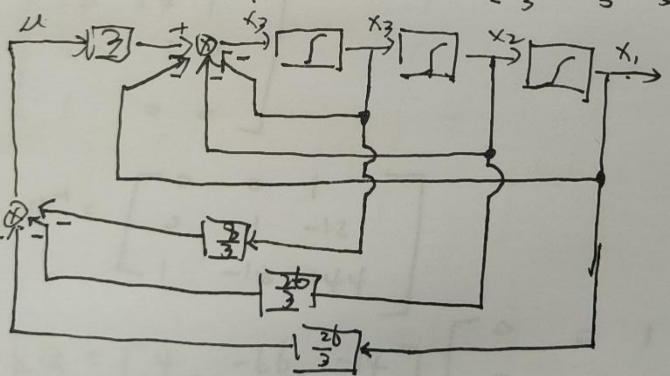
令 $u=-[k_1\ \ k_2\ \ k_3]x$，

$$\dot{x}=\begin{bmatrix}0&1&0\\0&0&1\\-1-3k_1&-1-3k_2&-1-3k_3\end{bmatrix}x$$

$a(s)=s^3+(1+3k_3)s^2+(1+3k_2)s+(1+3k_1)$.

$1+3k_3=9$，$1+3k_2=27$，$1+3k_3=27$.

即 $k_1=\frac{26}{3}$，$k_2=\frac{26}{3}$，$k_1=\frac{8}{3}$，$u=-\begin{bmatrix}\frac{26}{3}&\frac{26}{3}&\frac{8}{3}\end{bmatrix}x$.



6.3. $a^*(s)=(s+2)(s+4)(s+7)=s^3+13s^2+50s+56$.

$G(s)=\dfrac{1}{s(s+4)(s+8)}=\dfrac{1}{s^3+12s^2+32s}$

$$\dot{x}=\begin{bmatrix}0&1&0\\0&0&1\\0&-32&-12\end{bmatrix}x+\begin{bmatrix}0\\0\\1\end{bmatrix}u.$$

令 $u = -[k_1 \quad k_2 \quad k_3] x$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_1 & -32-k_2 & -12-k_3 \end{bmatrix} x$$

$a(s) = s^3 + (12+k_3)s^2 + (32+k_2)s + 3k_1$

$k_1 = 56$, $32+k_2 = 50$, $12+k_3 = 13$.

即 $k_1 = 56$, $k_2 = 18$, $k_3 = 1$.

6.4 $\det(sI-A) = \det\begin{bmatrix} s & 0 & 0 \\ -1 & s+6 & 0 \\ 0 & -1 & s+12 \end{bmatrix} = s^3 + 18s^2 + 72s$.

$a^*(s) = \prod_{i=1}^{3}(s-\lambda_i^*) = (s+2)(s+1-j)(s+1+j) = s^3+4s^2+6s+4$.

$\bar{k} = [a_0^*-a_0,\ a_1^*-a_1,\ a_2^*-a_2] = [4, -66, -14]$.

$$P = [b \quad Ab \quad A^2b]\begin{bmatrix} a_1 & a_2 & 1 \\ a_2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 72 & 18 & 1 \\ 18 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 72 & 18 & 1 \\ 12 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$Q = P^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -12 \\ 1 & -18 & 144 \end{bmatrix}$$

$$k = \bar{k}Q = [4, -66, -14]\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -12 \\ 1 & -18 & 144 \end{bmatrix} = [-14, 186, -1220]$$

6-9 (1). $\det(sI - A^T) = \begin{vmatrix} s+1 & 0 & -1 \\ 2 & s+1 & 0 \\ 3 & -1 & s+1 \end{vmatrix} = s^3 + 3s^2 + 6s + 6.$

$a_1 = 3,\ a_2 = 6,\ a_3 = 6.$

$a^*(s) = (s+3)(s+4)(s+5) = s^3 + 12s^2 + 47s + 60.$

$a_1^* = 12,\ a_2^* = 47,\ a_3^* = 60.$

$\tilde{E}^T = [a_3^* - a_3,\ a_2^* - a_2,\ a_1^* - a_1] = [54\ \ 41\ \ 9]$

$Q = [C^T\ A^TC^T\ (A^T)^2C^T]\begin{bmatrix} a_2 & a_1 & 1 \\ a_1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 & -1 \\ 1 & -3 & 5 \\ 0 & -2 & 2 \end{bmatrix}\begin{bmatrix} 6 & 3 & 1 \\ 3 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 0 & 1 \\ -4 & -2 & 0 \end{bmatrix}$

$P = Q^{-1} = -\frac{1}{8}\begin{bmatrix} 2 & -2 & 2 \\ -4 & 4 & 0 \\ -4 & -4 & -4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$E^T = \tilde{E}^T P = \left[\frac{23}{2}\ \ -\frac{5}{2}\ \ -9\right]$

$\dot{\hat{x}} = (A - EC)\hat{x} + Bu + Ey$

$= \left\{\begin{bmatrix} -1 & -2 & -3 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} - \begin{bmatrix} \frac{23}{2} \\ -\frac{5}{2} \\ -9 \end{bmatrix}[1\ \ 1\ \ 0]\right\}\hat{x} + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}u + \begin{bmatrix} \frac{23}{2} \\ -\frac{5}{2} \\ -9 \end{bmatrix}y$

$= \begin{bmatrix} -\frac{25}{2} & -\frac{27}{2} & -3 \\ \frac{5}{2} & \frac{3}{2} & 1 \\ 10 & 9 & -1 \end{bmatrix}\hat{x} + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}u + \begin{bmatrix} \frac{23}{2} \\ -\frac{5}{2} \\ -9 \end{bmatrix}y$

(2) 令 $\bar{x} = Px = \begin{bmatrix} D \\ C \end{bmatrix}x = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}x.$

$P^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \end{bmatrix} = PAP^{-1}x + Pbu = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} =$$

$$\begin{bmatrix} -1 & -1 & 1 \\ 1 & -1 & 0 \\ -2 & -2 & -1 \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} u \quad y = CP^{-1}\bar{x} = [0 \ \ 0 \ \ 1] \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}$$

$$a^*(s) = (s+3)(s+4) = s^2+7s+12$$

设 $\bar{E} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$

$$(\bar{A}_{11} - \bar{E}\bar{A}_{21}) = \left\{ \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} \bar{e}_1 \\ \bar{e}_2 \end{bmatrix} [-2 \ \ -2] \right\}$$

$$= \begin{bmatrix} 2\bar{e}_1 - 1 & 2\bar{e}_1 - 1 \\ 2\bar{e}_2 + 1 & 2\bar{e}_2 - 1 \end{bmatrix}$$

$$\det[sI - (\bar{A}_{11} - \bar{E}\bar{A}_{21})] = \det \begin{bmatrix} s - 2\bar{e}_1 + 1 & 1 - 2\bar{e}_1 \\ -2\bar{e}_2 - 1 & s - 2\bar{e}_2 + 1 \end{bmatrix}$$

$$= s^2 + (2 - 2\bar{e}_1 - 2\bar{e}_2)s - 4\bar{e}_1 + 2.$$

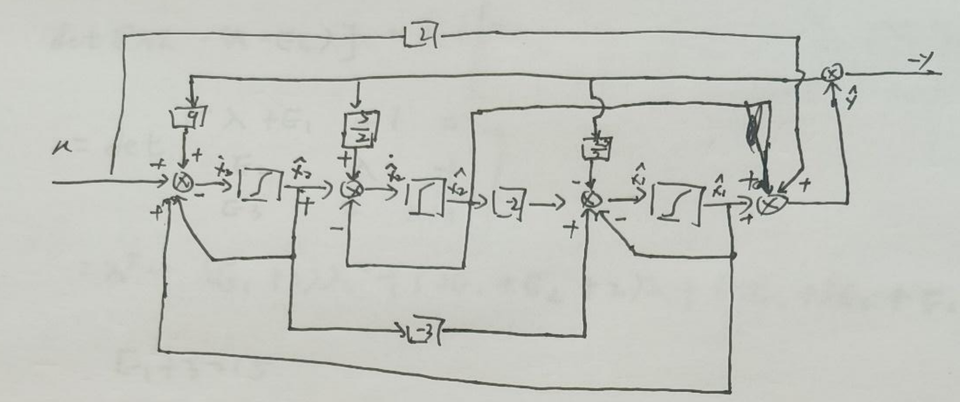
$$\begin{cases} 2 - 2\bar{e}_1 - 2\bar{e}_2 = 7 \\ -4\bar{e}_1 + 2 = 12. \end{cases}$$

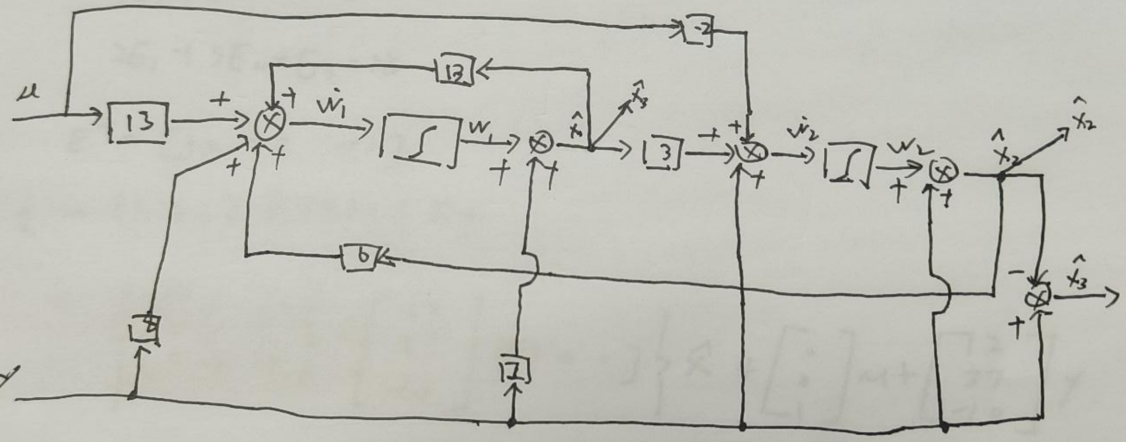
$$\bar{e}_1 = -\frac{5}{2} \quad \bar{e}_2 = 0$$

所以 $\dot{w} = (\bar{A}_{11} - \bar{E}\bar{A}_{21})w + (\bar{B}_1 - \bar{E}\bar{B}_2)u + [\bar{A}_{12} - \bar{E}\bar{A}_{22} + (\bar{A}_{11} - \bar{E}\bar{A}_{21})\bar{E}]y$

$$= \begin{bmatrix} -6 & -6 \\ 1 & -1 \end{bmatrix} w + \begin{bmatrix} 6 \\ 0 \end{bmatrix} u + \begin{bmatrix} \frac{27}{2} \\ -\frac{5}{2} \end{bmatrix} y$$

(2).





6.10. (1) $a(s)=s(s+1)(s+2)=s^3+3s^2+2s$.

$$\dot{x}=\begin{bmatrix}0&1&0\\0&0&1\\0&-2&-3\end{bmatrix}x+\begin{bmatrix}0\\0\\1\end{bmatrix}u\ ,\quad y=\begin{bmatrix}1&0&0\end{bmatrix}x$$

$a^*(s)=(s+3)(s+\frac{1}{2}-j\frac{\sqrt{3}}{2})(s+\frac{1}{2}+j\frac{\sqrt{3}}{2})=s^3+4s^2+4s+3$、

引入状态反馈 $u=-[k_1\ k_2\ k_3]x$，有 $k_1=3$，$k_2+2=4$，$k_3+3=4$

于是状态反馈 $u=-[k_1\ k_2\ k_3]x=-[3\ 2\ 1]x$

(2). $a^*(s) = (s+5)^3 = s^3 + 15s^2 + 75s + 125$.

$$\det[\lambda I - (A - EC)] = \det\left\{\begin{bmatrix} \lambda & & \\ & \lambda & \\ & & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} + \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\right\}$$

$$= \det\begin{bmatrix} \lambda + E_1 & -1 & 0 \\ E_2 & \lambda & -1 \\ E_3 & 2 & \lambda+3 \end{bmatrix}$$

$$= \lambda^3 + (E_1 + 3)\lambda^2 + (3E_1 + E_2 + 2)\lambda + (2E_1 + 3E_2 + E_3).$$

$E_1 + 3 = 15$

$3E_1 + E_2 + 2 = 75.$

$2E_1 + 3E_2 + E_3 = 125$

$E^T = [12 \quad 37 \quad -10].$

$$\dot{\hat{x}} = (A - EC)\hat{x} + Bu + Ey$$

$$= \left\{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} - \begin{bmatrix} 12 \\ 37 \\ -10 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\right\}\hat{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u + \begin{bmatrix} 12 \\ 37 \\ -10 \end{bmatrix}y.$$

$$= \begin{bmatrix} -12 & 1 & 0 \\ -37 & 0 & 1 \\ 10 & -2 & -3 \end{bmatrix}\hat{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u + \begin{bmatrix} 12 \\ 37 \\ -10 \end{bmatrix}y.$$

(3). $\bar{x} = Px = \begin{bmatrix} D \\ c \end{bmatrix}x = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}x.$

有 $P^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \\ \dot{\bar{x}}_3 \end{bmatrix} = PAP^{-1}x + Pbu = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -2 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}u, \quad y = CP^{-1}x = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{bmatrix}$$

$a^*(s) = (s+5)^2 = s^2 + 10s + 25.$

设 $\bar{E} = \begin{bmatrix} \bar{e}_1 \\ \bar{e}_2 \end{bmatrix}$，有 $(\bar{A}_{11} - \bar{E}\bar{A}_{21}) = \left\{ \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} \bar{e}_1 \\ \bar{e}_2 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} \right\} =$

$\begin{bmatrix} -3 & -2-\bar{e}_1 \\ 1 & -\bar{e}_2 \end{bmatrix}$

于是 $\det[sI - (\bar{A}_{11} - \bar{E}\bar{A}_{21})] = \det \begin{bmatrix} s+3 & 2+\bar{e}_1 \\ -1 & s+\bar{e}_2 \end{bmatrix} =$

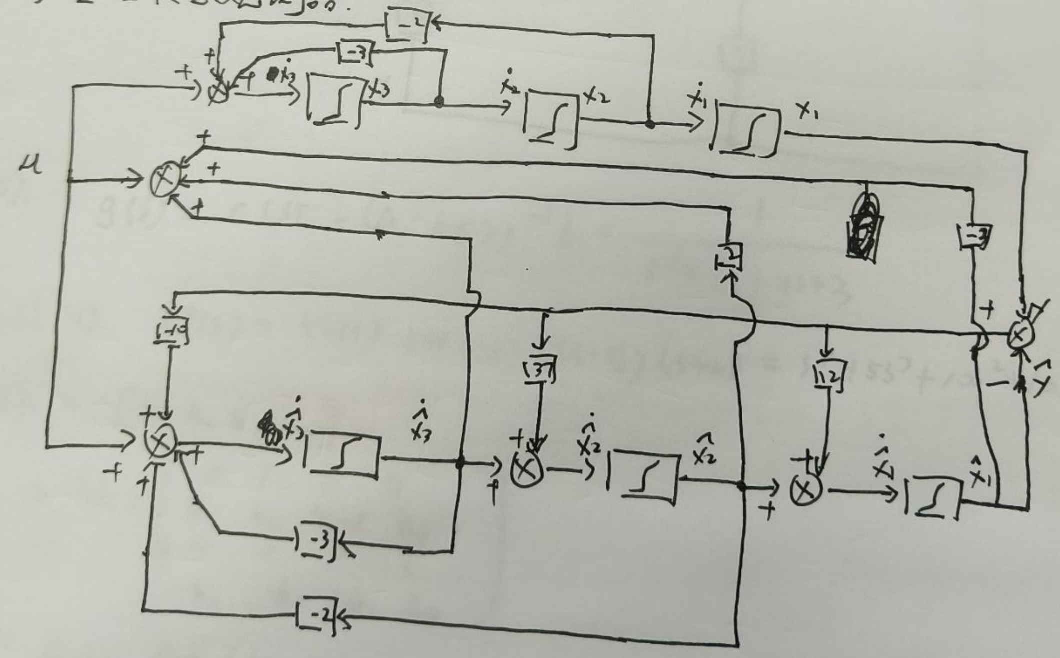
$s^2 + (\bar{e}_2+3)s + 3\bar{e}_2 + \bar{e}_1 + 2.$

比较系数 $\bar{e}_1 = 2$，$\bar{e}_2 = 7$.

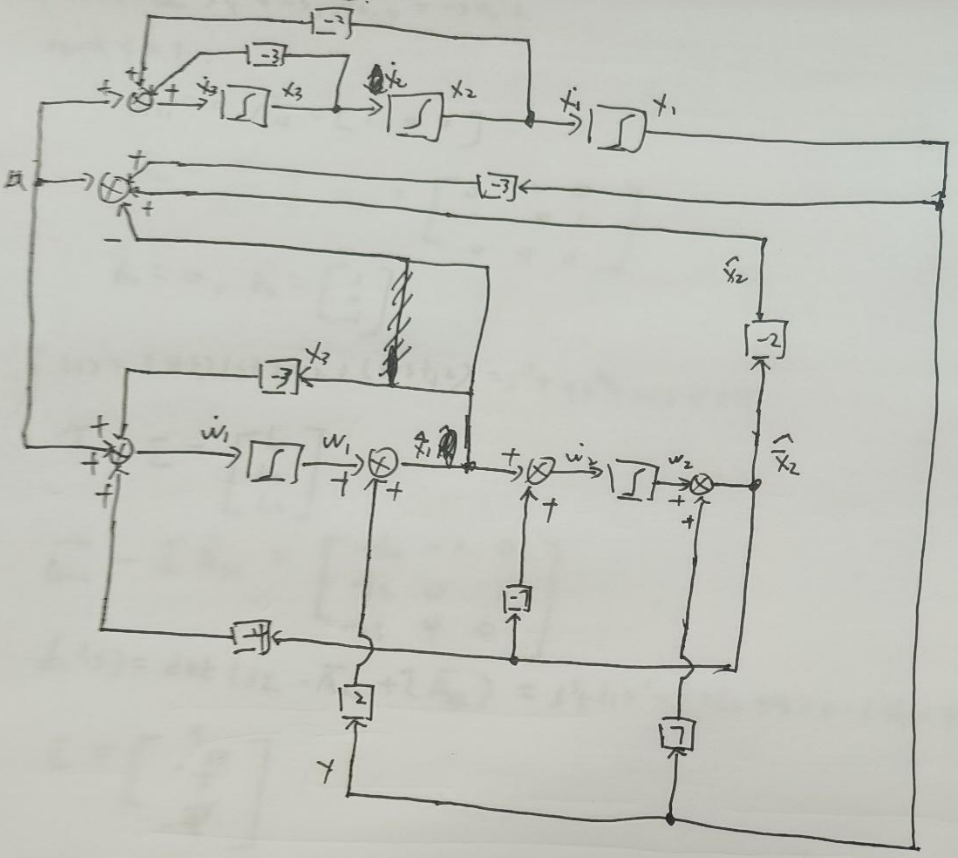
$\dot{w} = (\bar{A}_{11} - \bar{E}\bar{A}_{21})w + (\bar{B}_1 - \bar{E}\bar{B}_2)u + [\bar{A}_{12} - \bar{E}\bar{A}_{22} + (\bar{A}_{11} - \bar{E}\bar{A}_{21})\bar{E}]y$

$= \begin{bmatrix} -3 & -4 \\ 1 & -7 \end{bmatrix} w + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} -34 \\ -47 \end{bmatrix} y$

(4) 全维状态观测器.



降维状态观测器结构图.



(5). $g(s) = c(sI-(A-bk))^{-1}b = \dfrac{1}{s^3+4s^2+4s+3}$

6.11(4). $f^*(s) = (s+1)(s+1-j)(s+1+j)(s+2) = s^4+5s^3+10s^2+10s+4$.

令 $k = -[k_1\ k_2\ k_3\ k_4]$.

$$A-bk = \begin{bmatrix} 0 & 1 & 0 & 0 \\ k_1 & k_2 & k_3-2 & k_4 \\ 0 & 0 & 0 & 1 \\ -k_1 & -k_2 & 4-k_3 & -k_4 \end{bmatrix}$$

知 $f(s) = \det(sI-A+bk) = s^4+(k_4-k_2)s^3+(k_3-k_1-4)s^2+2k_2 s+2k_1$

$k = -[2\ 5\ 16\ 10]$.

(2). 特征值 $\lambda_1 = -3$, $\lambda_{2,3} = -3 \pm j2$.

rank $C = 1$.

$$\bar{A}_{11} = 0,\ \bar{A}_{12} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

$$\bar{A}_{21} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\ \bar{A}_{22} = \begin{bmatrix} 0 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 4 & 0 \end{bmatrix}$$

$$\bar{B}_1 = 0,\ \bar{B}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$f(s) = (s+3)(s+3-j2)(s+3+j2) = s^3 + 9s^2 + 31s + 39.$$

令 $\bar{L} = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}$

$$\bar{A}_{22} - \bar{L}\bar{A}_{12} = \begin{bmatrix} -l_1 & -2 & 0 \\ -l_2 & 0 & 1 \\ -l_3 & 4 & 0 \end{bmatrix}$$

$$f_L(s) = \det(sI - \bar{A}_{22} + \bar{L}\bar{A}_{12}) = s^3 + l_1 s^2 - (2l_2 + 4)s - (2l_3 + 4l_1).$$

$$\bar{L} = \begin{bmatrix} 9 \\ -\frac{35}{2} \\ -\frac{75}{2} \end{bmatrix}$$

$$\bar{A}_{22} - \bar{L}\bar{A}_{12} = \begin{bmatrix} -9 & -2 & 0 \\ \frac{35}{2} & 0 & 1 \\ \frac{75}{2} & 4 & 0 \end{bmatrix}$$

$$(\bar{A}_{22} - \bar{L}\bar{A}_{12})\bar{L} + (\bar{A}_{21} - \bar{L}\bar{A}_{11}) = (\bar{A}_{22} - \bar{L}\bar{A}_{12})\bar{L} = \begin{bmatrix} -46 \\ 120 \\ \frac{535}{2} \end{bmatrix}$$

$$\bar{B}_2 - \bar{L}\bar{B}_1 = \bar{B}_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\dot{z} = \begin{bmatrix} -9 & -2 & 0 \\ \frac{35}{2} & 0 & 1 \\ \frac{75}{2} & 4 & 0 \end{bmatrix} z + \begin{bmatrix} -46 \\ 120 \\ \frac{535}{2} \end{bmatrix} y + \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} u.$$

(3). $$\hat{x} = \begin{bmatrix} y \\ z + \bar{L}y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \bar{L} & I_3 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 9 & 1 & 0 & 0 \\ -17.5 & 0 & 1 & 0 \\ -37.5 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$u = v + \begin{bmatrix} 28 & 16 & 10 \end{bmatrix} \hat{x}$$

这里 v 为标量参考输入.