5. 从求和点 ①.②.③. 则有

①: $\dot{x}_1 = x_2 + a_1 x_1$

②: $\dot{x}_2 = a_2 x_2 + x_3$

③: $\dot{x}_3 = a_3 x_3 + u$

$y = x_1 + du$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} a_1 & 1 & 0 \\ 0 & d_2 & 1 \\ 0 & 0 & a_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + du$$

7. $(x_1 + x_2) R_3 = -R_2 x_2 - L_2 \dot{x}_2$

$u = x_1 + [L_1 \dot{x}_1 + x_1 R_3 + x_2 R_3] / R_1$

$y = (x_1 + x_2) R_3$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{R_1+R_3}{L_1} & -\frac{R_3}{L_1} \\ -\frac{R_3}{L_2} & -\frac{R_2+R_1}{L_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{R_1}{L_1} \\ 0 \end{bmatrix} u$$

8.(1) $\dddot{y} + \ddot{y} + 4\dot{x} + 5y = 3u$

选择状态变量 $y = x_1$, $\dot{y} = x_2$, $\ddot{y} = x_3$, 有

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -5x_1 - 4x_2 - x_3 + 3u \\ y = x_1 \end{cases}$$

状态空间表达式则为

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

9.4) $g(s) = \dfrac{s^3 + s + 1}{s^3 + 6s^2 + 11s + 6}$

~~$g(s)$~~

化严格真有理式 $g(s) = \dfrac{Y(s)}{U(s)} = 1 + \dfrac{-6s^2 - 10s - 5}{s^3 + 6s^2 + 11s + 6} = 1 + g'(s).$

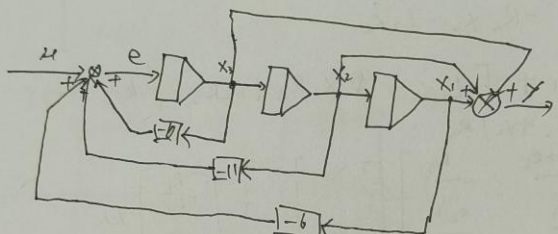
$g'(s) = \dfrac{Y'(s)}{U'(s)}$

$Y'(s) = U'(s)\dfrac{-6s^{-1} - 10s^{-2} - 5s^{-3}}{1 + 6s^{-1} + 11s^{-2} + 6s^{-3}}$

$E'(s) = U'(s)\dfrac{1}{1 + 6s^{-1} + 11s^{-2} + 6s^{-3}}$

即 $E'(s) = U(s) - 6s^{-1}E(s) - 11s^{-2}E(s) - 6s^{-3}E(s)$

$Y'(s) = -6s^{-1}E(s) - 10s^{-2}E(s) - 5s^{-3}E(s).$

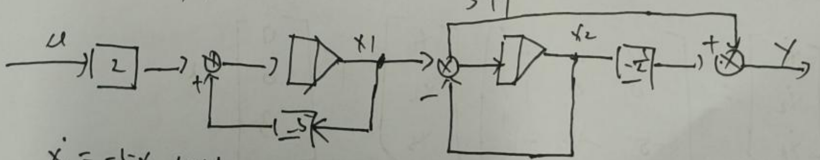


$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u$$

$$y = \begin{bmatrix}-6 & -11 & -6\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + u$$

10.12) $\dfrac{2s+1}{s^2+6s+5}$

$g(s) = \dfrac{2(s+\frac{1}{2})}{(s+1)(s+5)} = \dfrac{2}{s+5}\,\dfrac{s+\frac{1}{2}}{s+1}$



$\dot{x}_1 = -5x_1 + 2u$

$\dot{x}_2 = x_1 - x_2$

$y = x_1 - \frac{1}{2}x_2.$

即 $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-5 & 0\\ 1 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}2\\ 0\end{bmatrix}u$

$y = \begin{bmatrix}1 & -\frac{1}{2}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$

11. 1).

$$g(s) = \frac{c_1}{s} + \frac{c_2}{s+2} + \frac{c_3}{s+3}$$

$$c_1 = \lim_{s\to 0} \frac{6(s+1)}{(s+2)(s+3)} = 1$$

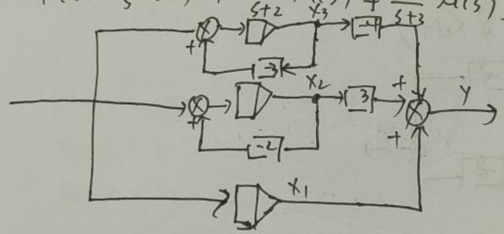
$$c_2 = \lim_{s\to -2} \frac{6(s+1)}{s(s+3)} = 3.$$

$$c_3 = \lim_{s\to -3} \frac{6(s+1)}{s(s+2)} = -4$$

$$g(s) = \frac{1}{s} + \frac{3}{s+2} + \frac{-4}{s+3}$$

$$\begin{cases} X_1(s) = \frac{1}{s} u(s) \\ X_2(s) = \frac{1}{s+2} u(s) \\ X_3(s) = \frac{1}{s+3} u(s) \end{cases}$$

$$Y(s) = \frac{1}{s} u(s) + \frac{3}{s+2} u(s) + \frac{-4}{s+3} u(s).$$



$$\dot{x}_1 = u$$

$$\dot{x}_2 = -2x_2 + u$$

$$\dot{x}_3 = -3x_3 + u$$

$$y(t) = x_1(t) + 3x_2(t) - 4x_3(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(2) $g(s) = \frac{c_1}{s+1} + \frac{c_2}{s+5}$

$c_1 = \lim_{s \to -1} \frac{2s+1}{s+5} = -\frac{1}{4}$

$c_2 = \lim_{s \to -5} \frac{2s+1}{s+1} = \frac{9}{4}$

$g(s) = \frac{Y(s)}{U(s)} = \frac{-\frac{1}{4}}{s+1} + \frac{\frac{9}{4}}{s+5}$

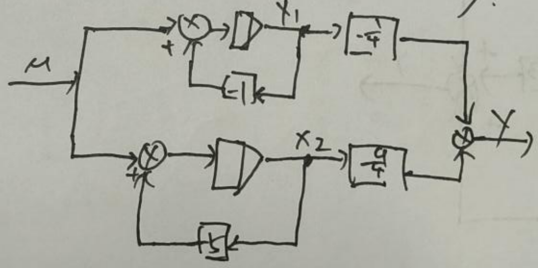
$Y(s) = \frac{-\frac{1}{4}}{s+1} U(s) + \frac{\frac{9}{4}}{s+5} U(s).$

$$\begin{cases} X_1(s) = \frac{1}{s+1} U(s) \\ X_2(s) = \frac{1}{s+5} U(s). \end{cases}$$

$\dot{x}_1(t) = -x_1 + u$

$\dot{x}_2(t) = -5x_2 + u$

$y(t) = -\frac{1}{4} x_1(t) + \frac{9}{4} x_2(t).$



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} -\frac{1}{4} & \frac{9}{4} \end{bmatrix} \begin{bmatrix} -\frac{1}{4} & \frac{9}{4} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

12. $X_1(s) = \frac{1}{s} X_2(s)$

$X_2(s) = \frac{1}{T_m s} X_3(s)$.

$X_3(s) = \left\{ [U(s) - X_1(s)]k - X_2(s) \right\} \frac{1}{T_m s + 1}$

$Y(s) = X_1(s)$.

$\dot{x}_3(t) = -\frac{k}{T_m} x_1(t) - \frac{1}{T_m} x_2(t) - \frac{1}{T_m} x_3(t) + \frac{k}{T_m} u(t)$

$\dot{x}_2(t) = \frac{1}{T_m} x_3(t)$

$\dot{x}_1(t) = x_2(t)$

$y(t) = x_1(t)$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{1}{T_m} \\ -\frac{k}{T_m} & -\frac{1}{T_m} & -\frac{1}{T_m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{k}{T_m} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

13. $X_1(s) = Y_1(s)$

$X_2(s) = Y_2(s)$.

$X_1(s) = [U_1(s) - X_2(s)] \frac{c}{s+a}$

$X_2(s) = [U_2(s) - X_1(s)] \frac{d}{s+b}$

$\dot{x}_1(t) = -a x_1(t) - c x_2(t) + c u_1(t)$

$\dot{x}_2(t) = -d x_1(t) - b x_2(t) + d u_2(t)$

$y_1(t) = x_1(t)$

$y_2(t) = x_2(t)$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -a & -c \\ -d & -b \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

14.12. $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 2 \\ -12 & -7 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 5 \\ 7 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

$\cancel{\lambda A - I}$ $|\lambda I - A| = \begin{vmatrix} \lambda & -1 & 0 \\ -3 & \lambda & -2 \\ 12 & 7 & \lambda+6 \end{vmatrix} = (\lambda+1)(\lambda+2)(\lambda+3) = 0$

解得 $\lambda_1 = -1$, $\lambda_2 = -2$, $\lambda_3 = -3$

特征向量

a. 对于 $\lambda_1 = -1$，有 $\begin{bmatrix} -1 & -1 & 0 \\ -3 & -1 & -2 \\ 12 & 7 & 5 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \\ v_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_{11} \\ v_{12} \\ v_{13} \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

b. 对于 $\lambda_2 = -2$，有 $\begin{bmatrix} -2 & -1 & 0 \\ -3 & -2 & -2 \\ 12 & 7 & 4 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \\ v_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_{11} \\ v_{12} \\ v_{13} \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}$

c. 对于 $\lambda_2 = -3$，有 $\begin{bmatrix} -3 & -1 & 0 \\ -3 & -3 & -2 \\ 12 & 7 & 3 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \\ v_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_{11} \\ v_{12} \\ v_{13} \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix}$

用构造P，$P = \begin{bmatrix} 1 & 2 & 1 \\ -1 & -4 & -3 \\ -1 & 1 & 3 \end{bmatrix}$ $\quad P^{-1} = \begin{bmatrix} \frac{9}{2} & \frac{5}{2} & 1 \\ -3 & -2 & -1 \\ \frac{5}{2} & \frac{3}{2} & 1 \end{bmatrix}$

求 $\bar{A}$、$\bar{B}$. $\bar{A} = P^{-1}AP = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

$\bar{B} = P^{-1}B = \begin{bmatrix} \frac{37}{2} & -27 \\ -15 & -20 \\ \frac{27}{2} & 16 \end{bmatrix}$

$\dot{\bar{x}} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \bar{x} + \begin{bmatrix} \frac{37}{2} & -27 \\ -15 & -20 \\ \frac{27}{2} & 16 \end{bmatrix} u.$

17. $A=\begin{bmatrix}-2 & 1 & 0\\ 0 & -3 & 0\\ 0 & 1 & -4\end{bmatrix}$ $B=\begin{bmatrix}-1 & -1\\ 1 & 4\\ 2 & -3\end{bmatrix}$ $C=\begin{bmatrix}1 & 1 & 1\\ -2 & -1 & 0\end{bmatrix}$ $D=0$.

$$G(s)=C(sI-A)^{-1}B+D.$$

$$sI-A=\begin{bmatrix}s+2 & -1 & 0\\ 0 & s+3 & 0\\ 0 & -1 & s+4\end{bmatrix}$$

$$(sI-A)^{-1}=\begin{bmatrix}\frac{1}{s+2} & \frac{1}{(s+2)(s+3)} & 0\\ 0 & \frac{1}{s+3} & 0\\ 0 & \frac{1}{(s+3)(s+4)} & \frac{1}{s+4}\end{bmatrix}$$

$$C(sI-A)^{-1}=\begin{bmatrix}1 & 1 & 1\\ -2 & -1 & 0\end{bmatrix}\begin{bmatrix}\frac{1}{s+2} & \frac{1}{(s+2)(s+3)} & 0\\ 0 & \frac{1}{s+3} & 0\\ 0 & \frac{1}{(s+3)(s+4)} & \frac{1}{s+4}\end{bmatrix}$$

$$=\begin{bmatrix}\frac{1}{s+2} & \frac{s^2+8s+14}{(s+2)(s+3)(s+4)} & \frac{1}{s+4}\\ \frac{-2}{s+2} & \frac{-s-4}{(s+2)(s+3)} & 0\end{bmatrix}$$

$$G(s)=C(sI-A)^{-1}B=\begin{bmatrix}\frac{2s+7}{(s+3)(s+4)} & \frac{10s+26}{(s+2)(s+3)(s+4)}\\ \frac{1}{s+3} & \frac{-2s-10}{(s+2)(s+3)}\end{bmatrix}$$

18. $G(s)=[I+G_0(s)H]^{-1}G_0(s)$.

$$G_0(s)H=\begin{bmatrix}\frac{1}{s+1} & -\frac{1}{s}\\ 2 & \frac{1}{s+2}\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{s+1} & -\frac{1}{s}\\ 2 & \frac{1}{s+2}\end{bmatrix}$$

$$I+G_0(s)H=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}\frac{1}{s+1} & -\frac{1}{s}\\ 2 & \frac{1}{s+2}\end{bmatrix}=\begin{bmatrix}\frac{s+2}{s+1} & -\frac{1}{s}\\ 2 & \frac{s+3}{s+2}\end{bmatrix}$$

$$[I+G_0(s)H]^{-1}=\begin{bmatrix}\frac{s(s+1)(s+3)}{(s+2)(s^2+5s+2)} & \frac{s+1}{s^2+5s+2}\\ \frac{-2s(s+1)}{s^2+5s+2} & \frac{s(s+2)}{s^2+5s+2}\end{bmatrix}$$

~~故 $G(s)=$~~

故 $G(s) = [I + G_0(s)H]^{-1} G_0(s)$

$$= \begin{bmatrix} \dfrac{3s^2+9s+4}{(s+2)(s^2+5s+2)} & \dfrac{-(s+1)}{s^2+5s+2} \\ \dfrac{2s(s+1)}{s^2+5s+2} & \dfrac{3s+2}{s^2+5s+2} \end{bmatrix}$$

19. 对差分方程取 z 变换.

$z^2 Y(z) + 5zY(z) + 3Y(z) = zU(z) + 2U(z)$.

$g(z) = \dfrac{Y(z)}{U(z)} = \dfrac{z+2}{z^2+5z+3}$.

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k).$$

$$y = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

20. $x_1(k+1) = x_2(k)$

$x_2(k+1) = x_1(k) + 3x_2(k) + 2u(k)$

$y(k) = x_1(k) + x_2(k)$.

$zX_1(z) = X_2(z)$.

$zX_2(z) = X_1(z) + 3X_2(z) + 2U(z)$.

$Y(z) = X_1(z) + X_2(z)$.

$Y(z) = X_1(z) + zX_1(z) = (z+1)X_1(z)$.

$X_1(z) = \dfrac{Y(z)}{z+1}$.

$\cancel{z^2 X_1(z) = X_1(z) + 3zX_1(z)}$

$z^2 X_1(z) = X_1(z) + 3zX_1(z) + 2U(z)$.

$(z^2-3z-1)X_1(z) = 2U(z)$.

$(z^2-3z-1)\dfrac{Y(z)}{z+1} = 2U(z)$. $\quad \dfrac{Y(z)}{U(z)} = \dfrac{2(z+1)}{(z^2-3z-1)}$.