IT 607 ASSIGNMENT 2

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I. Assignment 2.1

Use the outage probability expressions derived in class to plot the outage probability versus SNR for Selective Combining, and Maximum Ratio Combining, diversity techniques. Pick a fixed value for the threshold SNR, These expressions are also available in the book by Goldsmith. Plot for the number of diversity branches M = 1; 2; 4; 8; and 12, and SNR in the range 0 to 30dB in steps of 5dB. Comment on the diversity and array gains as M increases.

II. Selection combining Technique

For Selection Combining technique, the formula for outage probability is given in figure 1

Matlab Code

snrdB=0:0.5:30;
$h = (randn(1, 10000) + 1i * randn(1, 10000))/\sqrt(2);$
sigmaz=1;
$snr = 10.^(snrdB/10);$
$P = (sigmaz^2) * snr./(mean(abs(h).^2));$
$snrm = mean(abs(h).^2). * P/(sigmaz^2);$
x = 10;
M1 = 1;
M2 = 2;
M3 = 4;
M4 = 8;
M5 = 12;
$Pout1 = (1 - exp(-x./snrm)).^M1;$
$Pout2 = (1 - exp(-x./snrm)).^M2;$
$Pout3 = (1 - exp(-x./snrm)).^M3;$
$Pout4 = (1 - exp(-x./snrm)).^M4;$
$Pout5 = (1 - exp(-x./sn_m)).^M5;$
plot(snrdB, Pout1,'blue'); hold on;
plot(snrdB, Pout2,'black'); hold on;
plot(snrdB, Pout3,'green'); hold on;
plot(snrdB, Pout4,'red'); hold on;
plot(snrdB, Pout5,'yellow');

$$P_o(\gamma_o) = \prod_{i}^{M}[p(\gamma_i < \gamma_o)] = \prod_{i}^{M}[1 - e^{\frac{\gamma_o}{\bar{\gamma}_i}})]$$

Figure 1: Equation

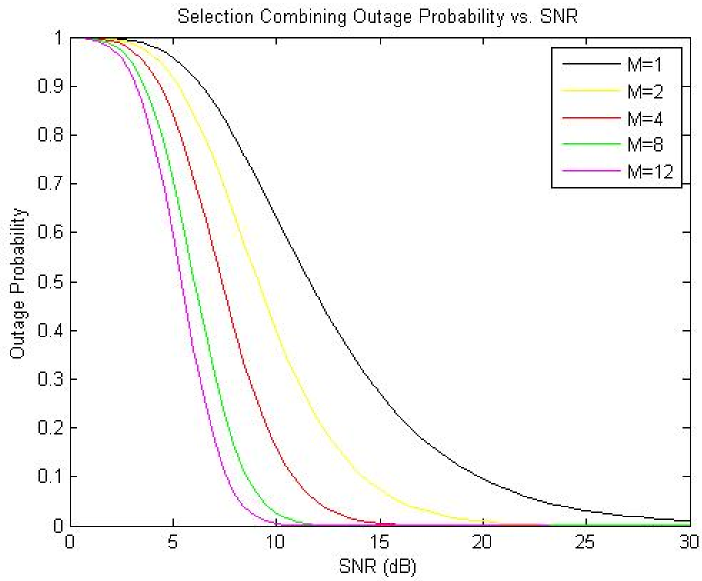

Figure 2: Outage Probability in Selection Combining

legend('M=1','M=2','M=4','M=8','M=12'); title('Outage Probability vs. SNR'); xlabel('SNR (dB)'); ylabel('Outage Probability');

III. Maximal Ratio Combining

formular for outage probability for Maximal ratio combining is shown in Figure 3 Matlab Code

snrdB=0:0.5:30;
$h = (randn(1, 10000) + 1i * randn(1, 10000))/\sqrt(2);$
$sigmaz = 1;$
$snr = 10.^{(}snrdB/10);$
$P = (sigma_z^2) * snr./(mean(abs(h).^2));$
$snr_m = mean(abs(h).^2). * P/(sigma_z^2);$
$x = 10;$
$a = -x./snr_m;$
$b = x./snr_m;$

$sum_all1 = 0;$
$fork = 1 : 1$
$sum_all1 = sum_all1 + ((b.^(k-1))/prod(1 : (k-1)));$
end

$P_out1 = 1 - (exp(a). * sum_all1);$
$sum_all2 = 0;$
$fork = 1 : 2$
$sum_all2 = sum_all2 + ((b.^(k-1))/prod(1 : (k-1)));$
end

$P_out2 = 1 - (exp(a). * sum_all2);$
$sum_all3 = 0;$
$fork = 1 : 4$
$sum_all3 = sum_all3 + ((b.^(k-1))/prod(1 : (k-1)));$ end

$P_out3 = 1 - (exp(a). * sum_all3);$
$sum_all4 = 0;$
$fork = 1 : 8$
$sum_all4 = sum_all4 + ((b.^(k-1))/prod(1 : (k-1)));$
end

$P_out4 = 1 - (exp(a). * sum_all4);$
$sum_all5 = 0;$
$fork = 1 : 12$
$sum_all5 = sum_all5 + ((b.^(k-1))/prod(1 : (k-1)));$
end

$P_out5 = 1 - (exp(a). * sum_all5);$
$plot(snrdB, P_out1,' blue'); holdon;$
$plot(snrdB, P_out2,' black'); holdon;$
$plot(snrdB, P_out3,' green'); holdon;$
$plot(snrdB, P_out4,' red'); holdon;$
$plot(snrdB, P_out5,' yellow');$
$legend('M = 1',' M = 2',' M = 4',' M = 8',' M = 12');$
$title('OutageProbabilityvs.SNR');$
$xlabel('SNR(dB)');$
$ylabel('OutageProbability');$

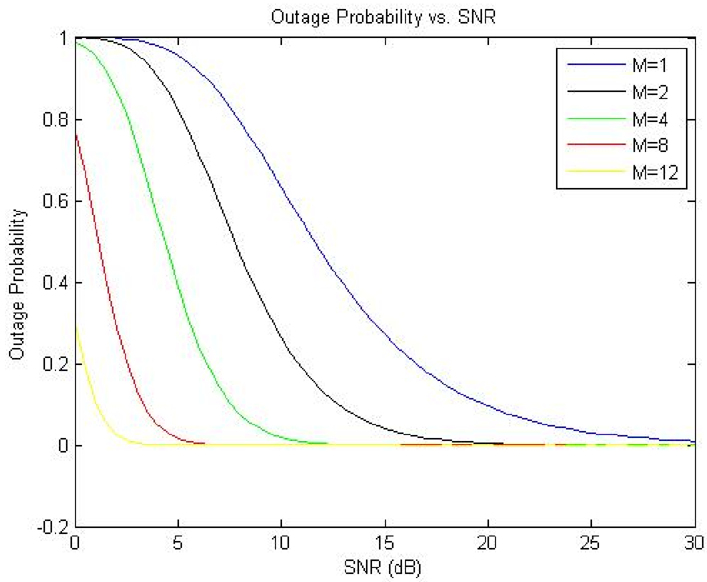

Figure 3:

$$P_{out} = p(\gamma_\Sigma < \gamma_0) = \int_0^{\gamma_0} p_{\gamma_\Sigma}(\gamma) d\gamma = 1 - e^{-\gamma_0/\overline{\gamma}} \sum_{k=1}^{M} \frac{(\gamma_0/\overline{\gamma})^{k-1}}{(k-1)!}.$$

Figure 4:

IV. Assignment 2.2

Use simulation to plot the bit error probability versus SNR for Selective Combining, Maximum Ratio Combining, and Equal Gain Combining diversity techniques using QPSK modulation. The simulation system is shown in the

figure below, assuming a block fading channel model with block length N as a parameter of choice. You should generate enough random input bits bk and the corresponding channel realizations to obtain a smooth error probability plot. Perform the simulation for the number of diversity branches M = 1; 2; 4 and 8, and for SNR in the range 0 to 20dB in steps of 5dB.
Simlation Diagram

V. Selection Combining

Bit error rate is calculated by the formular

Matlab Code
clear $N = 10^6$;

$ip = rand(1, N) > 0.5; s = 2 * ip - 1;$

$nRx = [12345678]; Eb_N0_dB = [0:20];$

$forjj = 1 : length(nRx)$

$forii = 1 : length(Eb_N0_dB)$

$n = 1/sqrt(2) * [randn(nRx(jj), N) + j * randn(nRx(jj), N)]; h = 1/sqrt(2) * [randn(nRx(jj), N) + j * randn(nRx(jj), N)];$

$sD = kron(ones(nRx(jj), 1), s); y = h. * sD + 10^(- Eb_N0_dB(ii)/20) * n;$

$hPower = h. * conj(h);$

$[hMaxValind] = max(hPower, [], 1); hMaxValMat = kron(ones(nRx(jj), 1), hMaxVal);$

$ySel = y(hPower == hMaxValMat); hSel = h(hPower == hMaxValMat);$

$yHat = ySel./hSel; yHat = reshape(yHat, 1, N);$

$ipHat = real(yHat) > 0;$

$nErr(jj, ii) = size(find([ip - ipHat]), 2);$

end

end

$simBer = nErr/N;$

$EbN0Lin = 10.^(Eb_N0_dB/10); theoryBer_nRx1 = 0.5. * (1 - 1 * (1 + 1./EbN0Lin).^(- 0.5)); theoryBer_nRx2 = 0.5. * (1 - 2 * (1 + 1./EbN0Lin).^(- 0.5) + (1 + 2./EbN0Lin).^(- 0.5)); theoryBer_nRx4 = 0.5. * (1 - 4 * (1 + 1./EbN0Lin).^(- 0.5) + 6 * (1 + 2./EbN0Lin).^(- 0.5) + (-4) * (1 + 3./EbN0Lin).^(- 0.5) + (1 + 4./EbN0Lin).^(- 0.5));$

close all
figure

$$P_e = \frac{1}{2}\sum_{k=0}^{N}(-1)^k\binom{N}{k}\left(1+\frac{k}{(E_b/N_0)}\right)^{-1/2}.$$

Figure 5:

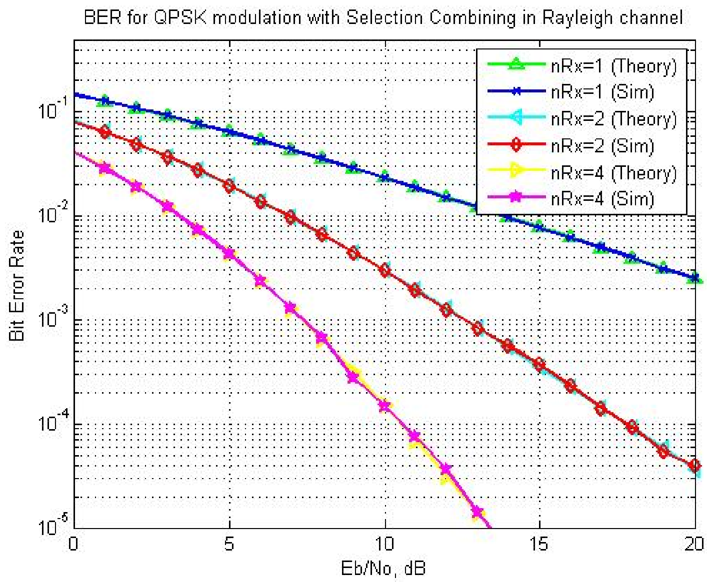

Figure 6:

hold on $semilogy(Eb_N0_dB, simBer(1,:),' bx-',' LineWidth', 2); semilogy(Eb_N0_dB, theoryBer_nRx2,' c- <' ,' LineWidth', 2); semilogy(Eb_N0_dB, simBer(2,:),' rd-',' LineWidth', 2); semilogy(Eb_N0_dB, theoryBer_nRx4,' y- >' ,' LineWidth', 2); semilogy(Eb_N0_dB, simBer(4,:),' mp-',' LineWidth', 2);$

$axis([02010^-50.5])$ $set(gca,' xtick', [0, 5, 10, 15, 20]);$ grid on
$legend('nRx = 1(Theory)',' nRx = 1(Sim)',' nRx = 2(Theory)',' nRx = 2(Sim)',' nRx = 4\ (Theory)',' nRx = 4(Sim)'); xlabel('Eb/No, dB'); ylabel('BitErrorRate'); title('BERforQPSKmodulationwithSelectionCombininginRayleigh$
%Ploting

VI. Maximal Ratio Combining

Matlab Code
clear $N = 10^6;$

$ip = rand(1, N) > 0.5; s = 2 * ip - 1;$

$nRx = [12345678]; Eb_N0_dB = [0 : 20];$

$forjj = 1 : length(nRx)$

$forii = 1 : length(Eb_N0_dB)$

$n = 1/sqrt(2) * [randn(nRx(jj), N) + j * randn(nRx(jj), N)]$; $h = 1/sqrt(2) * [randn(nRx(jj), N) + j * randn(nRx(jj), N)]$;

$sD = kron(ones(nRx(jj), 1), s)$; $y = h. * sD + 10^(- Eb_N0_dB(ii)/20) * n$;

$yHat = sum(conj(h). * y, 1)./sum(h. * conj(h), 1)$;

$ipHat = real(yHat) > 0$;

$nErr(jj, ii) = size(find([ip - ipHat]), 2)$;

end

end

$simBer = nErr/N$;

$EbN0Lin = 10.^(Eb_N0_dB/10)$; $theoryBer_nRx1 = 0.5. * (1 - 1 * (1 + 1./EbN0Lin).^(- 0.5))$; $p = 1/2 - 1/2 * (1 + 1./EbN0Lin).^(- 1/2)$; $theoryBer_nRx2 = p.^2. * (1 + 2 * (1 - p))$; $theoryBer_nRx4 = p.^4. * (1 + 4 * (1 - p) + 10 * ((1 - p).^2) + 20 * ((1 - p).^3))$;

close all

figure

hold on $semilogy(Eb_N0_dB, simBer(1, :),' bx-',' LineWidth', 2)$; $semilogy(Eb_N0_dB, theoryBer_nRx2,' c- <' ,' LineWidth', 2)$; $semilogy(Eb_N0_dB, simBer(2, :),' rd-',' LineWidth', 2)$; $semilogy(Eb_N0_dB, theoryBer_nRx4,' y- >' ,' LineWidth', 2)$; $semilogy(Eb_N0_dB, simBer(4, :),' mp-',' LineWidth', 2)$;

$axis([02010^-50.5])$ set(gca,'xtick',[0, 5, 10, 15, 20]); grid on legend('nRx=1 (Theory)', 'nRx=1 (Sim)', 'nRx=2 (Theory)', 'nRx=2 (Sim)', 'nRx=4 (Theory)', 'nRx=4 (Sim)'); xlabel('Eb/No, dB'); ylabel('Bit Error Rate'); title('BER in Maximal Ratio Combining in Rayleigh channel');

Simulation Results

VII. Equal Gain Combining

clear $N = 10^6$;

$ip = rand(1, N) > 0.5$; $s = 2 * ip - 1$;

$nRx = [12345678]$; $Eb_N0_dB = [0 : 20]$;

$forjj = 1 : length(nRx)$

$forii = 1 : length(Eb_N0_dB)$

$n = 1/sqrt(2) * [randn(nRx(jj), N) + j * randn(nRx(jj), N)]$; $h = 1/sqrt(2) * [randn(nRx(jj), N) + j * randn(nRx(jj), N)]$;

$sD = kron(ones(nRx(jj), 1), s)$; $y = h. * sD + 10^(- Eb_N0_dB(ii)/20) * n$;

$yHat = y. * exp(-j * angle(h))$; $yHat = sum(yHat, 1)$;

$ipHat = real(yHat) > 0$;

$nErr(jj, ii) = size(find([ip - ipHat]), 2)$;

end

end

$simBer = nErr/N$;

$EbN0Lin = 10.^(Eb_N0_dB/10)$;

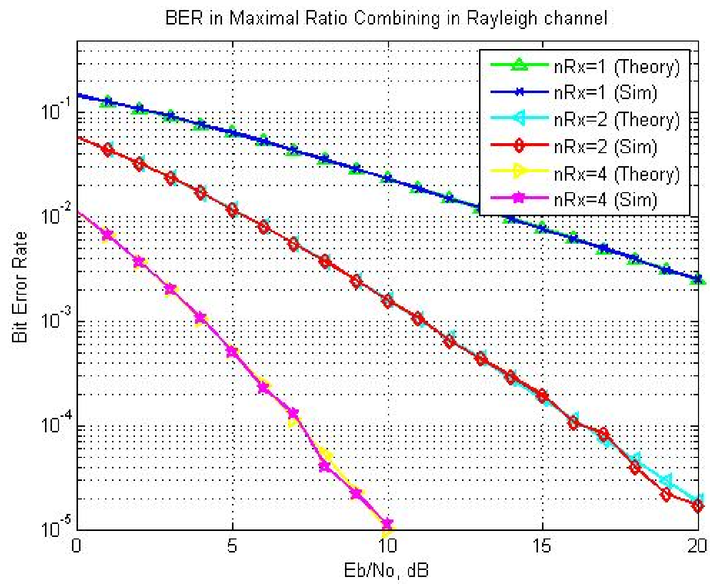

Figure 7:

close all figure

$semilogy(Eb_N0_dB, theoryBer_nRx1,' bp-',' LineWidth', 2); semilogy(Eb_N0_dB, simBer(1,:),' bx-',' LineWidth', 2);$
hold on $semilogy(Eb_N0_dB, theoryBer_nRx2,' rd-',' LineWidth', 2); semilogy(Eb_N0_dB, simBer(2,:),' rd-',' LineWidth', 2);$

$axis([02010^-50.5])$ set(gca,'xtick',[0, 5, 10, 15, 20]);
grid on
legend('nRx=1', 'nRx=2');
xlabel('Eb/No, dB');
ylabel('Bit Error Rate');
title('BER in Equal Gain Combining in Rayleigh channel');

Simulation results

VIII. Conclusion

Simulation shows that the outage probability for Selection and Maximal ratio combining generarry decreases as number of branches increase
For Selection combining, the outage probability drop rapidly as SNR increases

The bit error probabilities for Selection Combining, Maximal ration and equal gain are more less as compared to the outage probabilities for same cases

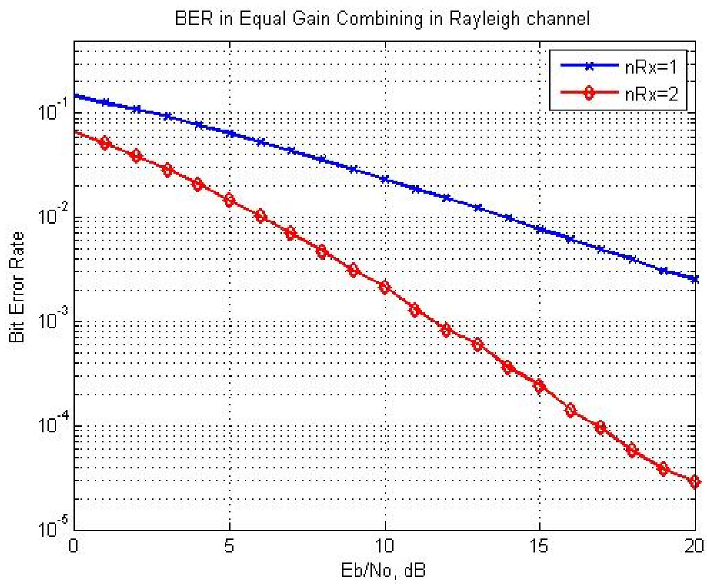
BER in Equal Gain Combining in Rayleigh channel
nRx=1
nRx=2
Bit Error Rate
Eb/No, dB

Figure 8: