IT 607
ASSIGNMENT 1
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GENERAL DESCRIPTION

The aim of this simulation exercise is to study the capacity of wireless channels under different scenarios in terms of the knowledge of the channel available at both the transmitting and receiving end against the signal to noise ratio. The type of fading is Rayleigh flat fading and the channel model is shown below:

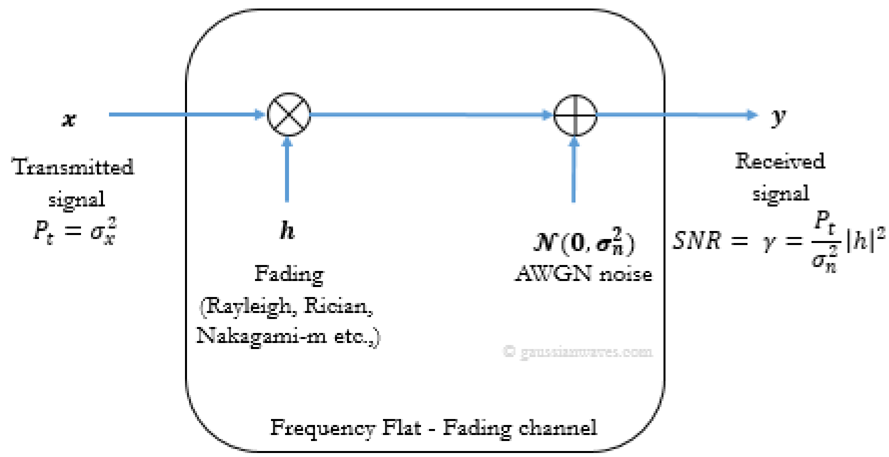


Fig. 1: Rayleigh Flat Fading Channel Model

Where, h is the flat fading complex channel impulse response that is held constant for each block of N transmitted symbols, P_t is the average input power at the transmit antenna, $\gamma = \frac{P_t}{\sigma_n^2}|h|^2$ is the signal-to-noise ratio (SNR) at the receiver input and σ_n^2 is the noise power of the channel which is assumed to be equal to one.

A. CSI is known at the transmitter and receiver

When both the transmitter and the receiver have the CSI, the transmitter can adapt its transmission strategy relative to this CSI and there is no notion of capacity versus outage where the transmitter sends bits that cannot be decoded.

This is because the transmitter knows the channel and thus will not send bits unless they can be decoded correctly.

The mechanism that allows for the transmitter to know the received CSI at the receiver is the feedback control that goes back to the transmitter from the receiver.

Capacity of such a channel can be given by

$$C = \int Blog_2(1+\gamma)p(\gamma)d\gamma \tag{1}$$

But the transmitter knows the instantaneous received CSI and can adapt power accordingly following the condition

$$\int P(\gamma)p(\gamma)d\gamma \leq \bar{P} \tag{2}$$

Implementing this condition will lead to the Capacity formula being given by:

$$C = \int_{\gamma_0}^{\infty} Blog_2(\frac{\gamma}{\gamma_0})p(\gamma)d\gamma \tag{3}$$

Since γ is time-varying, the maximizing power adaptation policy is a water-pouring formula in time that depends on the fading statistics $p(\gamma)$ only through the cutoff value γ_0.

The cutoff SNR γ_0 chosen for optimal power allocation is given by:

$$\int_{\gamma_0}^{\infty} (\frac{1}{\gamma_0} - \frac{1}{\gamma})p(\gamma)d\gamma = 1 \tag{4}$$

In the case of our model where cut-off SNR is normalized $\gamma = 1$ the above formula simplifies to:

$$C = \mathbb{E}\left\{ Blog_2\left(\frac{P_t}{\sigma_n^2}\left|h\right|^2\right)\right\} \tag{5}$$

Below is the curve of Capacity against SNR

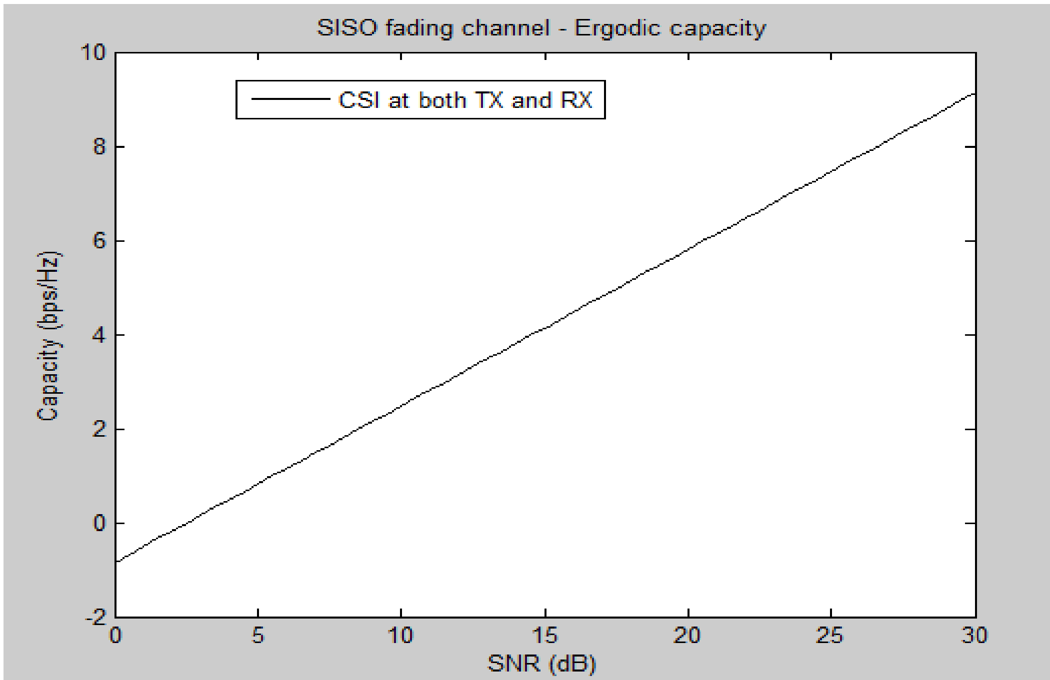


Fig. 2: Channel Capacity with CSI at Transmitter and Receiver

B. CSI is known at the receiver only

The capacity of a Single Input Single Output (SISO) link with perfect knowledge of the channel at the receiver (receiver CSI) is given as:

$$C = \int Blog_2(1+\gamma)p(\gamma)d\gamma \qquad (6)$$

Where the SNR γ is:

$$\gamma = \frac{P_t}{\sigma_n^2}\left|h\right|^2 \qquad (7)$$

From the fact that the Ergodic channel capacity is defined as the statistical average of the mutual information, where the expectation is taken over $\left|h\right|^2$

$$C_{erg} = \mathbb{E}\left\{Blog_2\left(1+\frac{P_t}{\sigma_n^2}\left|h\right|^2\right)\right\} \qquad (8)$$

We get the expression for the capacity of the channel when the CSI is known at the receiver only.

Plotting against SNR in MATLAB we get the following curve:

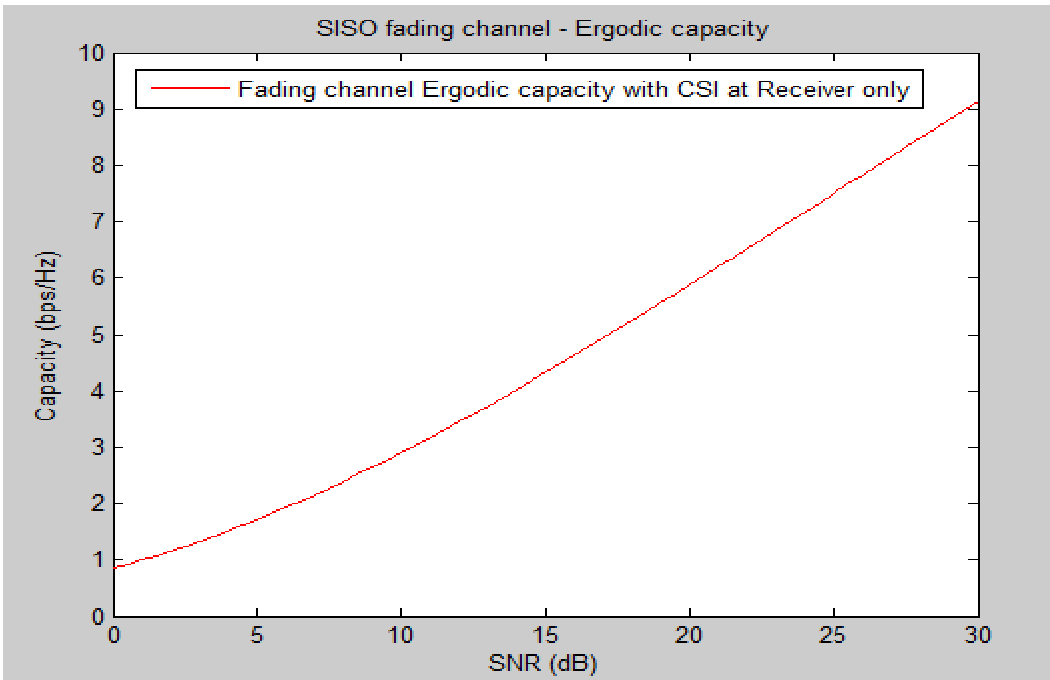


Fig. 3: Channel Capacity with CSI at Receiver only

C. Channel inversion power control is used

In this case the transmitter uses the channel side information (CSI) to maintain a constant received power, i.e., it inverts the channel fading.

The channel then appears to the encoder and decoder as a time-invariant AWGN channel.

The fading channel capacity with channel inversion is just the capacity of an AWGN channel with SNR

$$\sigma = \frac{1}{\mathbb{E}(1/\gamma)} \tag{9}$$

$$C = Blog_2(1+\sigma) = Blog_2(1+\frac{1}{\mathbb{E}(1/\gamma)}) \qquad (10)$$

$$C = Blog_2(1+\frac{\sigma_n^2}{P_t\mathbb{E}[|h|^2]}) \qquad (11)$$

Plotting against SNR in MATLAB we get the following curve:

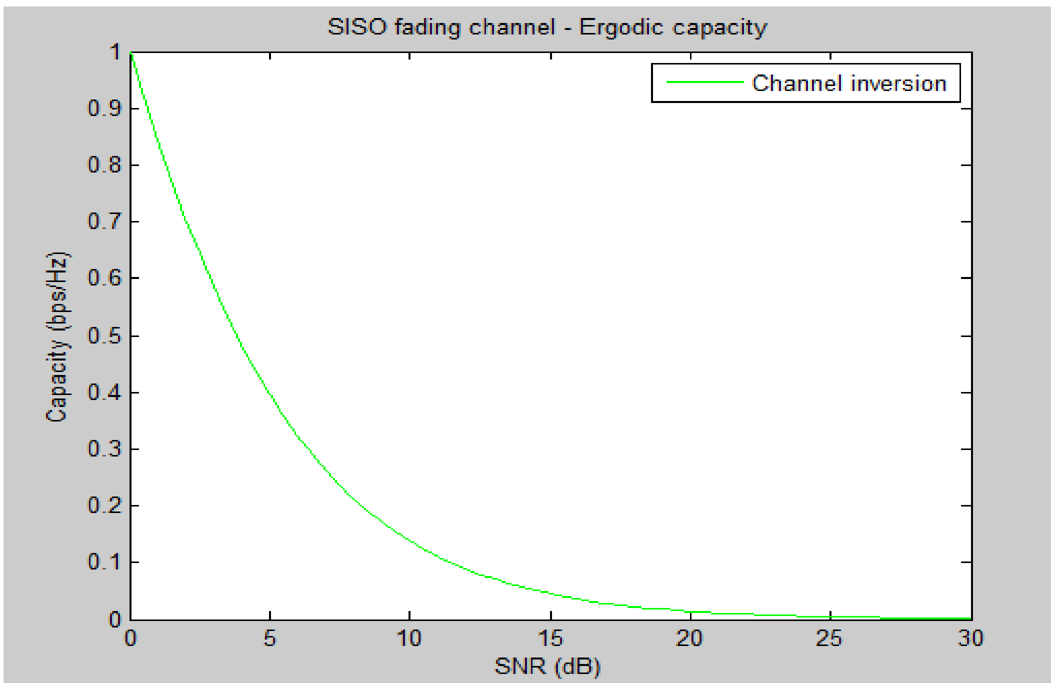


Fig. 4: Channel Inversion Capacity

Channel inversion capacity exhibit a large data rate reduction relative to Shannon capacity in extreme fading environments. As can be deduced above, in Rayleigh fading $\mathbb{E}(1/\gamma)$ is infinite, and thus the inverted capacity given bounds to zero.

D. Maximum outage capacity

Capacity with outage is defined as the maximum data rate that can be transmitted over a channel with some outage probability corresponding to the probability that the transmission cannot be decoded with negligible error probability. In other words, capacity with outage allows bits sent over a given transmission burst to be decoded at the end of the burst with some probability that these bits will be decoded incorrectly.

It is given by:

$$C = (1 - P_o) B log_2(1 + \gamma) \qquad (12)$$

$$C = (1 - P_o) B log_2(1 + \frac{P_t}{\sigma_n^2} \mathbb{E}[|h|^2]) \qquad (13)$$

Where P_o is the outage probability, that is, the probability that the actual SNR at the receiver is less than the SNR assumed by the transmitter and upon which the data rate is based.

Picking an arbitrary outage probability of 0.25 we get the following curve:

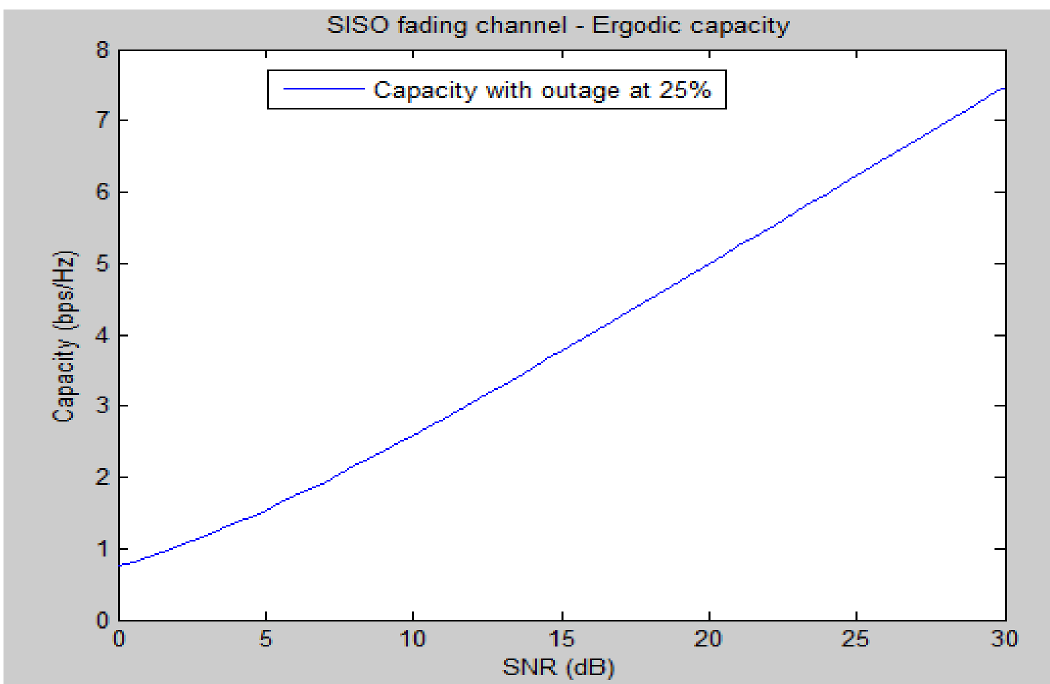


Fig. 5: Channel Capacity with outage $P_0 = 0.25$

E. AWGN channel capacity with the same average SNR as the Rayleigh channel.

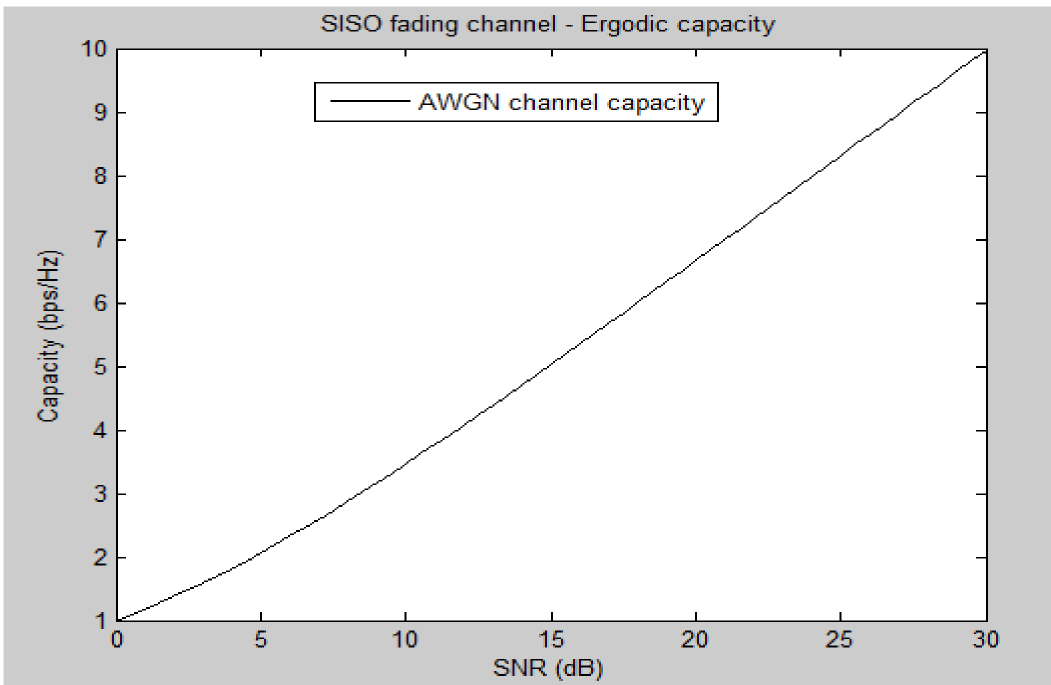
Shannon Capacity of an AWGN channel is given by:

$$C = Blog_2(1 + \gamma) \qquad (14)$$

If we consider the case where we have the same average SNR as the Rayleigh channel we can express the above as

$$C = Blog_2 \left(1 + \frac{P_t}{\sigma_n^2}\mathbb{E}[|h|^2]\right) \qquad (15)$$

Plotting against SNR in MATLAB we get the following curve:



The above curve gives the absolute bound for the maximum theoretical data rate that can be obtained, it neglects considerations of the magnitude of delay of information transit between the source and destination and does not consider the complexity of the system required to achieve this rate.

COMPARISON BETWEEN TIME INVARIANT AWGN AND RECEIVER CSI

Consider the case where we compare the AWGN Capacity obtained above and the Capacity of the channel under going flat-fading with Receiver CSI.

Jensens inequality states that for any concave function $f(x)$, where x is a random variable

$$E[f(X)] \leq f(E[X]) \tag{16}$$

Applying Jensens inequality to Ergodic capacity in equation (15)

$$\mathbb{E}\left[log_2\left(1 + \frac{P_t}{\sigma_n^2}|h|^2\right)\right] \leq log_2\left(1 + \frac{P_t}{\sigma_n^2}\mathbb{E}[|h|^2]\right) \tag{17}$$

This implies that the Ergodic capacity of a fading channel cannot exceed that of an AWGN channel with constant gain.

This is corroborated in the simulation output displayed below.

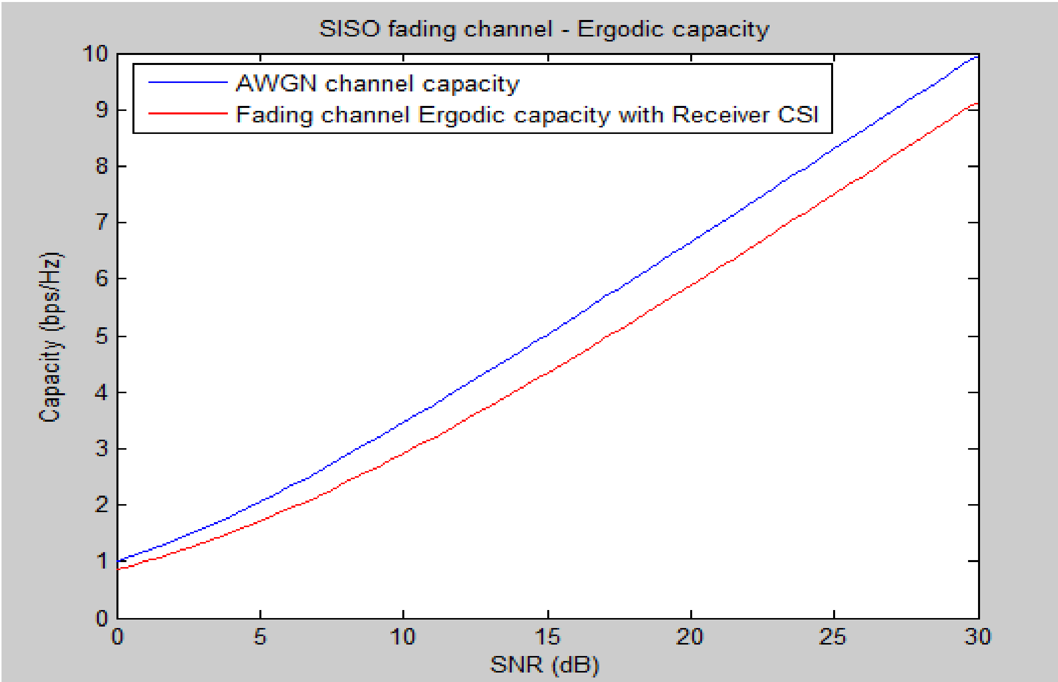


Fig. 6: Comparing AWGN capacity and Receiver CSI Capacity

We see that the Shannon capacity of a fading channel with receiver CSI only is less than the Shannon capacity of an AWGN channel with the same average SNR. In other words, fading reduces Shannon capacity when only the receiver has CSI.

But it can also be observed that there is a much lower performance difference between the capacities of AWGN and Rayleigh channels then one would expect. This is highly indicative that the coding of fading channels will yield considerable coding gain for large SNR.

CONCLUSION

A plot with all the above curves together is provided below

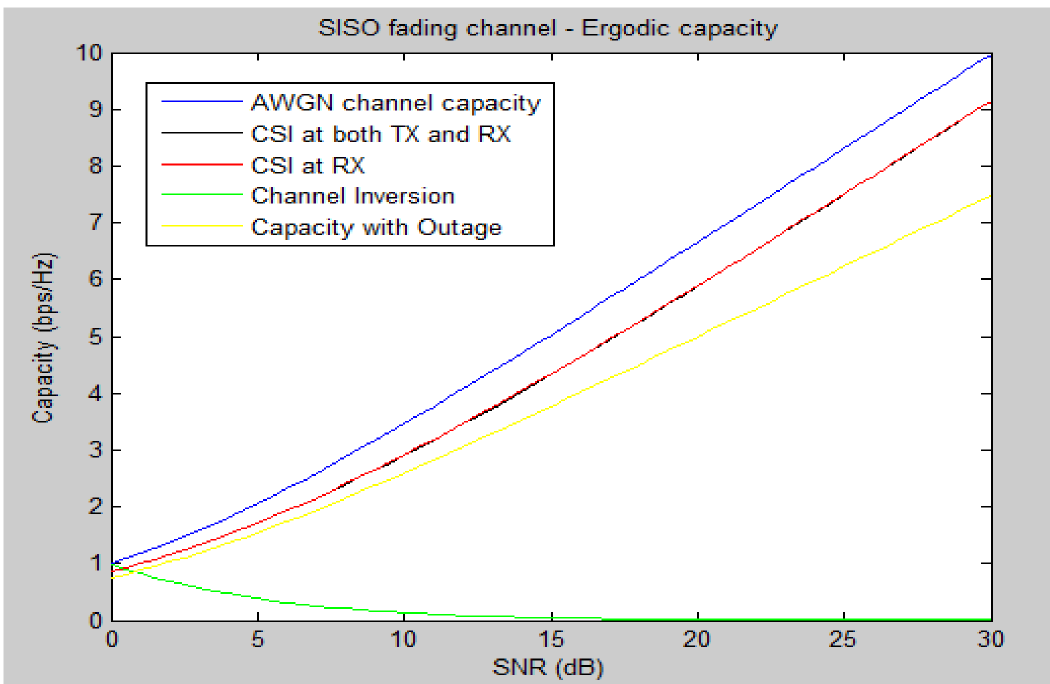


Fig. 7: Capacities under different case scenarios

The AWGN channel has a larger capacity than that of all fading channels. However, at low SNR, fading channels with transmitter and receiver CSI have the same capacity as the AWGN channel.

The simulation also shows that the difference between the capacity curves under transmitter and receiver CSI and receiver CSI only are negligible in all cases.

This means that transmitter adaptation yields a negligible capacity gain compared to using only receiver CSI.

Nevertheless, the power adaptation scheme with transmitter and receiver CSI requires a more complex transmitter and a feedback path for sending the CSI from the receiver to transmitter, but the decoder in the receiver is simpler.

On the other hand, the non-adaptive policy leads to a simple transmission scheme, but its code design must make use of the channel correlation statistics and the decoder complexity is increased.

Channel inversion uses codes designed for the AWGN channel and are easy to implement, but there is a large capacity loss in severe fading conditions.

It is much less power-efficient in a fading channel (Capacity tending to zero under Rayleigh fading), compared to water-filling, since the majority of the power is used to invert the bad channel condition.

However, channel inversion achieves a fixed rate in all fading states. Thus, channel inversion eliminates the delay associated with the time-scale of channel variations.