ORDERED OUTCOME AND BRACKET-VALUED REGRESSOR

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- Political science has long been interested in individual's preferences and opinions
  - Approval ratings (Canes-Wrone and De Marchi, 2002; Kriner and Schwartz, 2009)
  - Redistribution (Alt and Iversen, 2017; Magni, 2021)
  - Immigration policy (Hainmueller and Hiscox, 2007)
  - Trade policy (Scheve and Slaughter, 2001; Mayda and Rodrik, 2005; Wu, 2022)
- Usually measured in ordinal scale

  Strongly Disagree to *Strongly Agree*

- Often regressed on covariates measured in brackets, such as income, asset, or education

  Less than $5,000, *$5,000 – $10,000*, ..., *More than $100,000*

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COMMON APPROACH

- Ordered Probit / Logit
  - Assume that there is no interpretation problem
  - Assume that the distribution follows normal / logistic
  - Assume that all measures are precise
- However, these assumptions are ***strong***, and if not met, may lead to biased results, sometimes even ***flip*** the sign (Manski, 1988; Greene and Hensher, 2010; Bond and Lang, 2018)

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EXAMPLE: ORDERED PROBIT

- Assume $\epsilon \sim F(\cdot)$
- MLE for **Ordered Probit** model:

$$\arg\max_{\beta_1,\beta_2,\alpha} \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} \log\left(\Phi(\alpha_j,\cdot) - \Phi(\alpha_{j-1},\cdot)\right)\right]$$

- MLE for the **true** model

$$\arg\max_{\beta_1,\beta_2,\alpha} \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} \log\left(F(\alpha_j,\cdot) - F(\alpha_{j-1},\cdot)\right)\right]$$

EXAMPLE: ORDERED PROBIT

- Increase in sample size does not make the two distributions closer
- The Ordered Probit estimator is biased, but the size and direction of the bias depend on the shape of F
- Imprecise measure of v may lead to another bias, by affecting the log likelihood function. The size and direction of the bias also depend on the shape of F

PREVIOUS WORKS

	Interpretation	Distribution	Imprecise
Anchored Vignettes (King et al., 2004; King and Wand, 2007)	✓		
Semiparametric (Lewbel, 2000; Lee, 1992; Liu and Yu, 2024) Sensitivity Analysis (Bloem, 2022)	✓	✓	

THIS PAPER

	Interpretation	Distribution	Imprecise
Anchored Vignettes	✓		
Semiparametric Sensitivity Analysis	✓	✓	
Semiparametric Partial Identification Manski and Tamer (2002); Wang and Chen (2022)	✓	✓	✓

PROBLEM SETTING

True DGP:

$$Y^* = X^T\beta_1 + v^T\beta_2 + \epsilon \tag{1}$$

Observation:

$$Y = \begin{cases} 0 & Y^* \leq 0 \\ 1 & 0 \leq Y^* \leq \alpha_1 \\ \vdots & \\ k & \alpha_{k-1} \leq Y^* \leq \alpha_k \end{cases} \tag{2}$$

Also, we do not observe v directly, the lower bound of v_0 and the upper bound of v_1

ASSUMPTIONS

Assumption (1)

$Q_\tau(\epsilon \mid X, v) = 0$

Assumption (2)

$\mathbb{P}(\epsilon \mid X, v, v_0, v_1) = \mathbb{P}(\epsilon \mid X, v)$

Assumption (3)

$\beta_2 > 0$

GMMS ESTIMATOR

Let $\lambda_{mn}(\cdot) = \mathbb{1}_{\{\mathbb{P}(Y_i > m | X_i, V_{1i}, V_{0i}) > (1-\tau)\}}$

$$\Theta_n = \{b : S_n(b) \geq \max_{C \in \mathcal{B}} S_n(c) - \varepsilon_n\}$$

, where

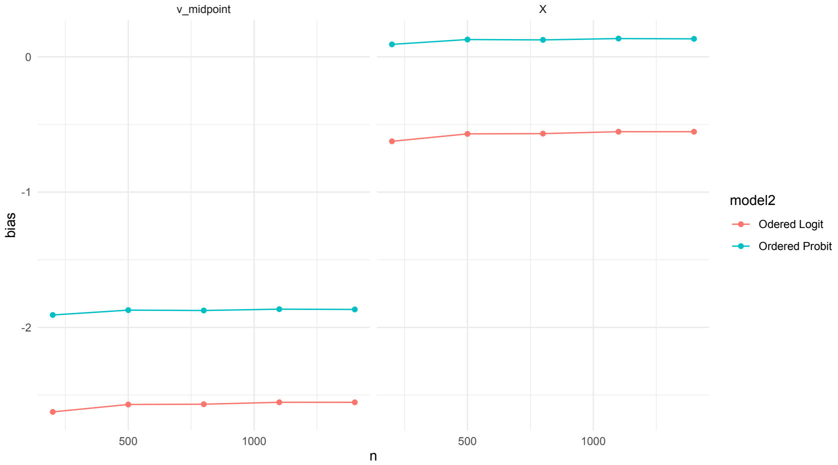
$$S_n(b^s) = \frac{1}{n} \sum_{i=1}^{n} \sum_{m=1}^{M-1} \Big(\mathbb{P}(Y_i > m \mid X_i, V_{0i}, V_{1i}) - (1 - tau) \Big)$$
$$\Big[\lambda_{mn}(X_i, V_{0i}, V_{1i}) \cdot \mathrm{sgn}(\tilde{X}_i' b + V_i^1 + b_{1m}) +$$
$$(1 - \lambda_{mn}(X_i, V_{0i}, V_{1i})) \cdot \mathrm{sgn}(\tilde{X}_i' b + V_{0i} + b_{1m}) \Big]$$

, for some $\varepsilon_n = \frac{\ln(n)}{n} > 0$.

SIMULATION

- $X \sim \mathbb{N}(1, 4)$
- $V \sim \mathbb{N}(0, 2)$
- $\epsilon \sim Weibull(10, 10)$

ORDERED PROBIT AND LOGIT



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