Assignment - 5

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CSE - 41

(1). Given that, $n = 2000$

$\mu = 2040$ hrs

$\sigma = 60$

$z = \frac{x-\mu}{\sigma} = 1.833$

(a). For $x = 2150$

$P(x > 2150) = P(z > 1.833)$

$= 0.5 - (P(0 < z < 1.833))$

$= 0.5 - A(1.833)$

$= 0.5 - 0.4664$

$= 0.0336$

∴ No. of bulbs expected to burn for more than 2150 hrs.
$\Rightarrow 0.0336 \times 2000 = 67$

(b). $x = 1950$, $z = \frac{x-\mu}{\sigma} = -1.5$

$P(x < 1950) = P(z < -1.5) = 0.5 - 0.4332$
$= 0.668$

No. of bulbs expected to burn for less than

1950 hrs $= 0.668 \times 2000$
$= 133.58$
$= 134$ (approx)

(c). $x = 1920$, $z = \frac{x-\mu}{\sigma} = -2$

$P(1920 < x < 2160) = P(-2 < z < 2)$
$= 2 \times P(0 < z < 2)$
$= 2 \times 0.4772 = 0.9544$

No. of bulbs required $= 0.9544 \times 2000$

$= 1909$

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(2) Given, $\mu = 20$

$\sigma = 3$

$n = 1000$

(a). $z = \dfrac{25-20}{3} = 5/3 = 1.67$

$P(x > 25) = P(z > 1.67)$

$= 0.0475$

Packages $= 0.0475 \times 1000$

$= 47.5$

(b). $z = \dfrac{18-20}{3} = \dfrac{-2}{3} = -0.67$

$P(z < -0.67) = 0.2514$

Packages $= 0.2514 \times 1000$

$= 251$ packages

(c) Packages b/w 19 and 22

For 19 kgs:

$z = \left(\dfrac{19-20}{3}\right) = -\dfrac{1}{3} = -0.33$

$P(z < -0.33) = 0.3707$

For 22 kgs:

$z = \dfrac{22-20}{3} = \dfrac{2}{3} = 0.67$

$P(z < 0.67) \simeq 0.7486$

Probability between then:

$0.7486 - 0.3707$

$= 6.339$

(3) $\Rightarrow$ $\mu = 78$ Yrs , $\sigma = 5$ yrs $n = 64$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{79 - 78}{5/\sqrt{64}} = 1.6$$

$$P(\bar{x} > 79) = P(z > 1.6) = 0.0548$$

$\therefore$ The Probability that sample mean 6 mm,

$= 0.\cancel{00135}0548$

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(4). $\mu = 5$ mm

$\sigma = 2$ mm

$n = 36$

$$z = \frac{6-5}{2/\sqrt{36}} = 3$$

$$P(\bar{x} > 6) = P(z > 3) = 0.5 - A(z)$$

$= 0.5 - 0.4731$

$= 0.00135$

$\therefore$ The Probability that sample mean of 6 samples $= 0.00135$

(5). Estimate:

An estimate is a Value Calculated from Sample data that is used to oppurtunities an unknown the Population parameter.

Types:

(1). Point Estimate:

A point estimate is a single numerical Value used to estimate population.

Ex: Sample mean is used to estimate population mean (m).

(2). Interval estimate:

An interval estimate provides a range of Values neither the population parameter is likely to lie

estimate ± merge of error.

Ex: A 95% Confidence interval for mean height might be: $165 \pm 3 \Rightarrow (162, 168)$

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(6). (1). Unbiasedness:

An estimator is unbiased if it's expected Values equals the true population parameter.

$$E(\hat{\theta}) = \theta$$

② Consistency:

- An estimator is Convinient if it becomes the closer to the true parameter as the Sample Size increases

$$\hat{\theta}_n \to \theta \quad \text{as} \quad n \to \infty$$

③. Efficiency:

- Among all unbiased estimators the one with the smallest Variance is the most efficient.

④. Sufficiency:

- An estimator is Sufficient if it uses all the estimation in the Sample about Parameter.

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④. Let $x_1, x_2, x_3 \ldots\ldots x_n$ be random Sample drawn

$$E(\bar{x}_1) = E\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) = \frac{1}{n} E(x_1 + x_2 + \cdots + x_n)$$

$$= \frac{1}{n}\left[E(x_1) + E(x_2) + E(x_3) + \cdots + E(x_n)\right]$$

$$= \frac{1}{n}\left[\mu + \mu + \cdots\cdots + \mu\right]$$

$$= \frac{1}{n}(n\mu)$$

$$= \mu$$

$$\therefore E[\bar{x}] = \mu.$$

(8). $n = 125$

$\sigma = 5$

$E = z_{\alpha/2} (\sigma/\sqrt{n})$

$= 1.96 (5/\sqrt{125})$

$= 1.96 (5/11.18)$

$= 1.96 \times 0.447$

$= 0.8765.$

(9) $x = 18$

$n = 90$

(a) $\hat{p} = \frac{x}{n}$ $\therefore \hat{p} = \frac{18}{90} = 0.2$

(b). Expected $= 600 \times 0.2 = 120$ individuals.

(10). $n = 120, \quad x = 24$

(a). $\hat{p} = \frac{x}{n} = \frac{24}{120} = 0.2$

(b). Expected $= 800 \times 0.2$

$= 160$ patients.

(11) (i) $\bar{x} = \dfrac{4+3+3+2+5+4+3}{7}$

$= \dfrac{27}{7} \simeq 3.4286$

$\therefore \bar{\lambda} = 3.43$

(ii). Expected $= 30 \times 3.43 = 102.86$

103 Admissions.

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(12) (i) $\hat{\lambda} = \dfrac{4+7+6+5+8+6+7}{7} = 6.14$

$\therefore$ MLE for $\lambda = 6.14$ Customers per 30 min.

(ii). Expected, $= 4 \times 6.14 = 24.56$

$= 25$ Customers.

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(13). $x = \{70, 75, 80, 85, 90\}$

$\hat{\mu} = \dfrac{70+75+80+85+90}{5} = 80$

$\hat{\sigma}^2 = \dfrac{1}{n} \sum (x_i - \hat{\mu})^2$

$= \dfrac{250}{5} = 50$

$\therefore \hat{\sigma} = 7.07$

Variance $= 50$

Mean $= 80$

Std. deviation $= 7.07$

(14)

$$\hat{\mu} = \frac{32+35+30+31+34}{5}$$

$$= 32.4°C$$

$$\sigma^2 = \frac{1}{n} \sum (x_i - \hat{\mu})^2$$

$$\hat{\sigma}^2 = 3.44$$

$$\sigma = 1.85°C$$

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(15)

$$\bar{x} = \frac{1+2+3+4+5}{5} = 3$$

$$\bar{y} = \frac{20000+12000+15000+12000+10000}{5}$$

$$= 15000$$

$$b = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$b = \frac{-26000}{10} = -2600$$

$$a = 15000 - (-2600)(3) = 15000 + 7800 = 22800$$

The linear regression is in form $\hat{y} = a + bx$

$$\hat{y} = 22800 - 2600x$$

Substitute $x = 3.5$ years into equation

$$\hat{y} = 22800 - 2600(3.5) = 22800 - 9100$$

$$= 13700.$$