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Problem 1 We are to clip the triangle T with vertices P, Q, R to the half-space H, where $\mathrm{P} = (2,\ 3,\ 0,\ 3)$, $\mathrm{Q} = (5,\ 2, -9,\ 3)$, $\mathrm{R} = (1,\ 11, -25,\ 7)$ in clip coordinates, and $\mathrm{H} = [1, 7, 9, 14]$ in normalized device coordinates

(a)Classify the points P, Q, R with respect to H. That is, which of these points lie inside, and which points lie outsize of H?

$$H \cdot P = \begin{bmatrix} 1 \\ 7 \\ 9 \\ 14 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ 0 \\ 3 \end{bmatrix} = 2 + 21 + 42 = 65 \rightarrow P \text{ is Outside}$$

$$H \cdot Q = \begin{bmatrix} 1 \\ 7 \\ 9 \\ 14 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 2 \\ -9 \\ 3 \end{bmatrix} = 5 + 14 + 81 + 42 = -20 \rightarrow Q \text{ is Inside}$$

$$H \cdot R = \begin{bmatrix} 1 \\ 7 \\ 9 \\ 14 \end{bmatrix} \cdot \begin{bmatrix} 11 \\ 11 \\ -25 \\ 7 \end{bmatrix} = 11 + 77 - 225 + 98 = -39 \rightarrow R \text{ is Inside}$$

(b)Find the intersection points (if any) between the edges of T and $\partial\mathrm{H}$.

$$\mathrm{t}_{PQ} = \frac{65}{65+20} = \frac{65}{85} = \frac{13}{17}$$

$$\mathrm{I}_{PQ} = \tfrac{4}{17}(2,3,0,3) + \tfrac{13}{17}(5,2,-9,3) = \left(\tfrac{8}{17}, \tfrac{12}{17}, 0, \tfrac{12}{17}\right) + \left(\tfrac{65}{17}, \tfrac{26}{17}, -\tfrac{117}{17}, \tfrac{39}{17}\right) = \left(\tfrac{73}{17}, \tfrac{38}{17}, -\tfrac{117}{17}, 3\right)$$

$$\mathrm{t}_{PR} = \frac{65}{65+39} = \frac{65}{104} = \frac{5}{8}$$

$$\mathrm{I}_{PR} = \tfrac{3}{8}(2,3,0,3) + \tfrac{5}{8}(1,11,-25,7) = \left(\tfrac{6}{8}, \tfrac{9}{8}, 0, \tfrac{9}{8}\right) + \left(\tfrac{5}{8}, \tfrac{55}{8}, -\tfrac{125}{8}, \tfrac{35}{8}\right) = \left(\tfrac{11}{8}, \tfrac{64}{8}, -\tfrac{125}{8}, \tfrac{44}{8}\right)$$

(c)Find the consecutive vertices of the resulting polygon.

$$\mathrm{I}_{PQ}\left(\tfrac{73}{17}, \tfrac{38}{17}, -\tfrac{117}{17}, 3\right),\ Q(5,2,-9,3), R(1,11,-25,7), \mathrm{I}_{PR}\left(\tfrac{11}{8}, 8, -\tfrac{125}{8}, \tfrac{11}{2}\right)$$

Problem 2 We are to clip the triangle T with vertices P, Q, R to the half-space H that corresponds to the near face of the camera view frustum, where $\mathrm{P} = (8, -12,\ 8,\ 4)$, $\mathrm{Q} = (8,\ 12, -20, -16)$, $\mathrm{R} = (20, -36, -4, -8)$ in clip coordinates.

(a)Find a homogeneous coordinate representation of H. Recall that with OpenGL conventions, the near plane is $\mathrm{z} = -1$ and the far plane is $\mathrm{z} = 1$ in NDC. Be careful of the inverted z-axis.

$\mathrm{H}: (0,0,-1,-1)$

(b)Classify the points P, Q, R with respect to H. That is, which of these points lie inside, and which points lie outside of H?

$\mathrm{P}: -12 < 0$ Inside $\mathrm{Q}: 36 > 0$ Outside $\mathrm{R}: 12 > 0$ Outside (c)Find the intersection points (if any) between the edges of T and $\partial\mathrm{H}$

$$t_{PQ} = \frac{-12}{-48} = \frac{1}{4}$$

$$I_{PQ} = \frac{3}{4}\begin{bmatrix}8\\-12\\8\\4\end{bmatrix} + \frac{1}{4}\begin{bmatrix}8\\12\\-20\\-16\end{bmatrix} = \begin{bmatrix}6\\-9\\6\\3\end{bmatrix} + \begin{bmatrix}2\\3\\-5\\-4\end{bmatrix} = \begin{bmatrix}8\\-6\\1\\-1\end{bmatrix}$$

$$t_{PR} = \frac{-12}{-24} = \frac{1}{2}$$

$$I_{PR} = \frac{1}{2}\begin{bmatrix}8\\-12\\8\\4\end{bmatrix} + \frac{1}{2}\begin{bmatrix}20\\-36\\-4\\-8\end{bmatrix} = \begin{bmatrix}4\\-6\\4\\2\end{bmatrix} + \begin{bmatrix}10\\-18\\-2\\-4\end{bmatrix} = \begin{bmatrix}14\\-24\\2\\-2\end{bmatrix}$$

(d)Find the consecutive vertices of the resulting polygon.

$P(8,-12,8,4),\ I_{PQ}(8,-6,1,-1),\ I_{PR}(14,-24,2,-2)$

<u>Problem 3</u> Considering triangle T with vertices in (device) clip space given by $\mathrm{P} = (10,\ 15,\ 20,\ 5)$, $\mathrm{Q} = (6,\ 9,\ 12,\ 3)$, $\mathrm{R} = (7,\ 14,\ 21,\ 14)$ and with scalar values $\nu_P = 60$, $\nu_Q = 162$, $\nu_R = 154$ associated to each vertex. We are to clip T to the half-space $\mathrm{H} = [1, 2, 3, -13]$ in device space. What are the vertices and associated scalar values of the clipped polygon?

$$H[PQR] = \begin{bmatrix}1\\2\\3\\-13\end{bmatrix} \cdot \begin{bmatrix}10 & 6 & 7\\15 & 9 & 14\\20 & 12 & 21\\5 & 3 & 14\end{bmatrix} = [35\ \ 21\ \ -84]$$ P&Q is Outside, R is Inside

$$\overline{RP} :\ S = \frac{h\cdot R}{h\cdot R - h\cdot P} = \frac{-84}{-84-35} = \frac{84}{119} = \frac{12}{17}$$

$$I_{\overline{RP}} = \frac{12}{17}(P-R)+R = \frac{12}{17}\begin{bmatrix}3\\1\\-1\\-9\end{bmatrix} + \begin{bmatrix}7\\14\\21\\14\end{bmatrix} = \begin{bmatrix}155/17\\250/17\\345/17\\130/17\end{bmatrix}$$

$$VI_{\overline{RP}} = \frac{12}{17}V_R + \frac{5}{17}V_P = \frac{12}{17}(154) + \frac{12}{17}(60) = \frac{2148}{17}$$

$$\overline{QR} :\ S = \frac{h\cdot Q}{h\cdot Q - h\cdot R} = \frac{21}{21+84} = \frac{21}{105} = \frac{1}{5}$$

$$I_{\overline{QR}} = \frac{1}{5}(P-Q)+Q = \frac{1}{5}\begin{bmatrix}1\\5\\9\\11\end{bmatrix} + \begin{bmatrix}6\\9\\12\\3\end{bmatrix} = \begin{bmatrix}31/5\\10\\69/5\\26/5\end{bmatrix}$$

$$VI_{\overline{QR}} = \frac{1}{5}V_Q + \frac{4}{5}V_R = \frac{1}{5}(162) + \frac{4}{5}(154) = \frac{778}{5}$$

$$[I_{\overline{QR}}\ \ R\ \ I_{\overline{RP}}] = \begin{bmatrix}31/5 & 7 & 155/17\\10 & 14 & 250/7\\69/5 & 21 & 345/17\\26/5 & 14 & 130/17\end{bmatrix}$$

$$VI_{\overline{QR}} = \frac{778}{5},\ V_R = 154,\ VI_{\overline{RP}} = \frac{2148}{17}$$