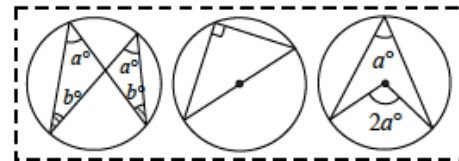# SOLUTION OF TRIANGLES

- ONE PAGE NOTE (OPN)
- WORKSHEET
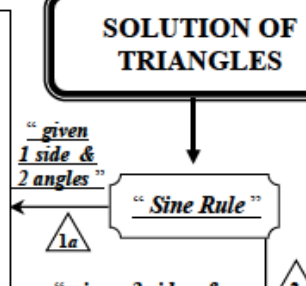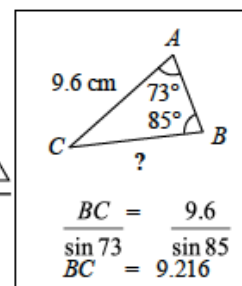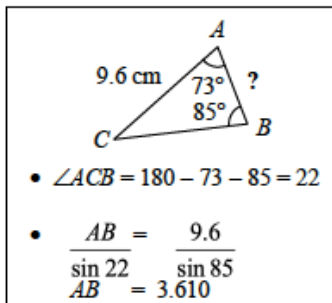
Encik Mohd Salleh Ambo

ONE PAGE NOTES (Coding Method)

CHAPTER 9 (FORM 4)
" SOLUTION OF TRIANGLES "

(circle theorem diagrams: $a°$, $b°$, $2a°$)

(cyclic quadrilateral: $x° + y° = 180$)

2

Area of triangle using the " *Heron's formula* "

$s = \dfrac{9.8 + 5.2 + 12.3}{2} = 13.65$ $\sim s =$ semi perimeter

$area \; \triangle ADC = \sqrt{13.65\,(13.65 - 9.8)\,(13.65 - 5.2)\,(13.65 - 12.3)}$

$= 24.485$

" given 3 sides "

9.8 cm, 5.2 cm, 12.3 cm, triangle A, D, C, ?

$\cos \angle ADC = \dfrac{9.8^2 + 5.2^2 - 12.3^2}{2(9.8)(5.2)}$

$\angle ADC = 106.07°$

" given 2 sides & 1 included angle "

A, 20 cm, 65°, B, 15 cm, C, ?

$AC^2 = 20^2 + 15^2 - 2(20)(15) \cos 65$

$AC = 19.27$

" Cosine Rule "   ②   ①

" Area of Triangle "   **1**

$area \; \triangle ABC = \dfrac{1}{2}(20)(15) \sin 65 = 135.95$ **1a**

∠ line / plane   ∠ 2 planes

SOLUTION OF TRIANGLES

A, h, B, b, C A, h, B, b, C

shortest distance A to BC
(perpendicular distance A to BC)

$area \; \triangle ABC = \dfrac{1}{2} \times b \times h$

A, 9.6 cm, 73°, 85°, C, B, ?

• $\angle ACB = 180 - 73 - 85 = 22$

• $\dfrac{AB}{\sin 22} = \dfrac{9.6}{\sin 85}$

$AB = 3.610$

1b

A, 9.6 cm, 73°, 85°, C, B, ?

" given 1 side & 2 angles "

$\dfrac{BC}{\sin 73} = \dfrac{9.6}{\sin 85}$

$BC = 9.216$

1a

" Sine Rule "

" given 2 sides & 1 non-included angle "   **2a**

" Ambiguous Case "

B, 30°, C', A **" only one $\triangle ABC$ "**

21.8 cm, M, N, ?, 68°, L, 23.2 cm, ∠ obtuse

• $\dfrac{\sin \angle LMN}{23.2} = \dfrac{\sin 68}{21.8}$

$\angle LMN = 80.65$ (acute)

$\angle LMN = 99.35$ (obtuse)

• $\angle LNM = 180 - 68 - 99.35 = 12.65$

2b

21.8 cm, M, N, ?, 68°, L, 23.2 cm, ∠ obtuse

$\dfrac{\sin \angle LMN}{23.2} = \dfrac{\sin 68}{21.8}$

$\angle LMN = 80.65$ (acute)

$\angle LMN = 99.35$ (obtuse)

[**HINT** : ∠acute + ∠obtuse = 180]

---

① sketch new $\triangle ABC'$ ②

9.4 cm, 5 cm, 30°, 70°, B, C', C, A

$\sim AB$ maintain
$\sim AC$ maintain
$\sim \angle ABC°$ maintain

9.4 cm, 110°, 5 cm, 30°, B, C, C', A

" HINT "

$\angle ACB + \angle AC'B = 180°$

5 cm, 9.4 cm, 30°, 110°, B, C', A

9.4 cm, 5 cm, 30°, 70°, B, C', A

WORKSHEET
TOPIC 9 : SOLUTION OF TRIANGLES
[Part C → 10 marks]

==

9.1 Sine Rule

9.1.1 Make and verify conjectures on the relationship between the ratio of length of sides of a triangle with the sine of the opposite angles, and hence define the sine rule.

[*the use of digital technology is encouraged*]

9.1.2 Solve triangles involving sine rule.

9.1.3 Determine the existence of ambiguous case of a triangle, and hence identify the conditions for such cases.

9.1.4 Solve triangles involving ambiguous cases.

9.1.5 Solve problems related to triangles using the sine rule.

9.2 Cosine rule

9.2.1 Verify the cosine rule.

9.2.2 Solve triangles involving the cosine rule.

9.2.3 Solve problems involving the cosine rule.

9.3 Area of a triangle

9.3.1 Derive the formula for area of triangles, and hence determine the area of a triangle.

9.3.2 Determine the area of a triangle using the Heron's formula.

9.3.3 Solve problems involving areas of triangles .

9.4 Application of sine rule, cosine rule and area of a triangle

9.4.1 Solve problems involving triangles.

==

==

9.1   Sine Rule

9.1.1   Make and verify conjectures on the relationship between the ratio of length of sides of a triangle with the sine of the opposite angles, and hence define the sine rule.

==

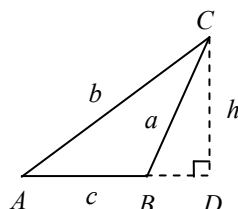**1**   The diagram shows a triangle ABC.
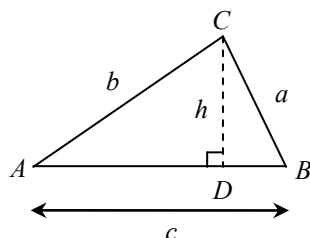
Show that $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$.

[2 marks]

Answer :

2   The diagram shows a triangle ABC.

Show that $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$.

[2 marks]

Answer :

MIND think :
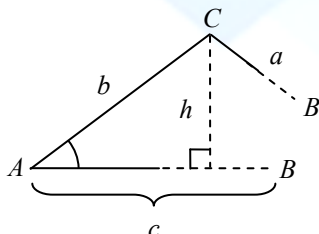
> **SINE RULE**
>
> ~ For any triangle ABC,
>
> $\dfrac{a}{\sin A} =$ **OR** $\dfrac{\sin A}{a} =$

==

9.1.3   Determine the existence of ambiguous case of a triangle, and hence identify the conditions for such cases.

==

3   The diagram shows an incomplete triangle *ABC*.

If ∠*A*, length of *a* and length of *b* are fixed.
For each of the following cases, state the number of possible triangle/s that can be formed.

Answer :

$a < h$	$a = h$	$a \geq b$	$h < a < b$

4   In the answer space, mark (✓) for the triangle which exist ambiguous case. If (✓), sketch the another difference triangle on the same diagram.
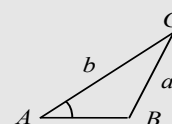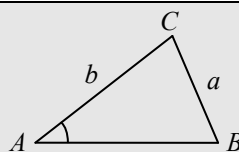
Answer :

(a)	(b)	(c)
8 cm 5 cm 65°	8 cm 5 cm 35°	8 cm 115° 5 cm
☐	☐	☐

(d)	(e)	(f)
5 cm 35° 8 cm	5 cm 35° 8 cm	8 cm 5 cm 35°
☐	☐	☐

MIND think :

AMBIGUOUS exists if :

~ given ☐ sides and ☐ non-included angle.

~ ∠*A* is an ☐ angle, **and** *a* ☐ *b*.

5   *PQR* is a triangle where *PQ* = 12 cm, *QR* = 6.8 cm, and $\angle RPQ = 33°$. Sketch the two possible triangles *PQR*. Hence, find the two possible values of $\angle PQR$.   (*Ans* : 40.97°, 73.03°)
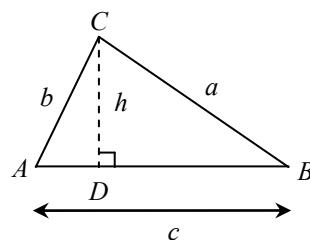
[5 marks]

Answer :

6   In a triangle *DEF*, *DE* = 8 cm, *DF* = 10 cm, and $\angle DFE = 50°$. Sketch the two possible triangles *DEF*.   Hence,   find the probable values of the length of   *EF*. (*Ans* : 4.122, 8.734)

[5 marks]

Answer :

==

9.2   ***Cosine rule***

    9.2.1   **Verify the cosine rule.**

==

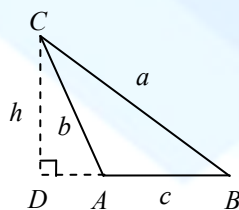**7**   The diagram show a triangle *ABC*.

Show that $a^2 = b^2 + c^2 - 2bc \cos A$.

[3 marks]

Answer :

8   The diagram shows a triangle ABC.

Show that $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

[3 marks]

Answer :

MIND think :

COSINE RULE
~ For any triangle ABC,
$b^2 =$ **OR** $\cos B =$
$c^2 =$ **OR** $\cos C =$

==
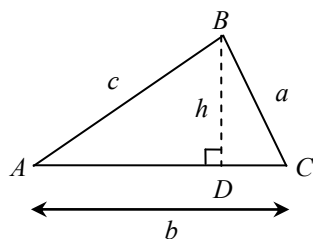
9.3   Area of a triangle

   9.3.1   Derive the formula for area of triangles, and hence determine the area of a triangle.

   9.3.2   Determine the area of a triangle using the Heron's formula.

==

9   The diagram shows a triangle ABC.
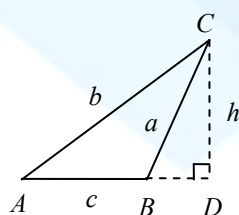

Show that the area of triangle $ABC = \dfrac{1}{2}ab\sin C = \dfrac{1}{2}bc\sin A$.

[2 marks]

Answer :

10   The diagram shows a triangle ABC.

Show that the area of triangle $ABC = \dfrac{1}{2}ab\sin C$.
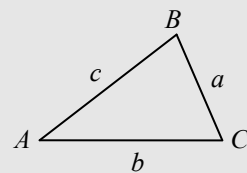
[2 marks]

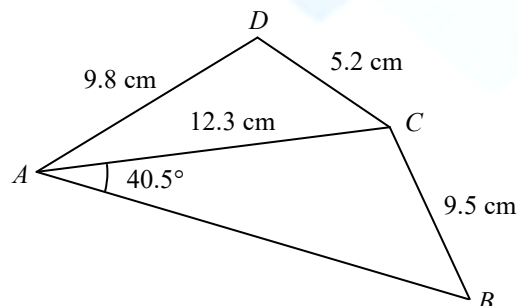*Answer :*

MIND think :

HERON'S FORMULA

~ For any triangle ABC,

- semi perimeter, s =

- area of triangle ABC =

PAPER 2

⟹ *2 dimensional surfaces*

11   The diagram shows a quadrilateral *ABCD* such that ∠*ABC* is acute.

(a)   Calculate

    (i)   ∠*ABC*,   (*Ans* : 57.23)

    (ii)   ∠*ADC*,   (*Ans* : 106.07)
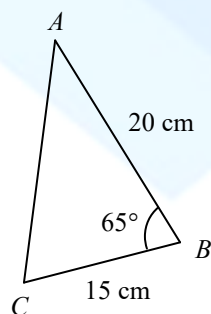
    (iii)   the area, in cm², of quadrilateral *ABCD*.   (*Ans* : 82.37)
                                                                                            [8 marks]

(b)   A triangle *A′B′C′* has the same measurements as those given for triangle *ABC*, that is, *A′C′* = 12.3 cm, *C′B′* = 9.5 cm, and ∠*B′A′C′* = 40.5°, but which is different in shape to triangle *ABC*.

    (i)   Sketch the triangle *A′B′C′*,

    (ii)   State the size of ∠*A′B′C′*.   (*Ans* : 122.77)
                                                                                            [2 marks]
   [*2004, No.13*]

Answer :

12   The diagram shows a triangle ABC.

(a)   Calculate the length, in cm, of AC.   (*Ans* : 19.27) [2 marks]

(b)   A quadrilateral $ABCD$ is now formed so that AC is a diagonal, $\angle ACD = 40°$ and $AD = 16$ cm. Calculate the two possible values of $\angle ADC$.

    (*Ans* : 50.73, 129.27) [2 marks]

(c)   By using the acute angle $\angle ADC$ from (b), calculate

    (i)   the length, in cm, of CD,   (*Ans* : 24.89)

    (ii)   the area, in cm², of the quadrilateral $ABCD$.   (*Ans* : 290.1)

                                                            [6 marks]
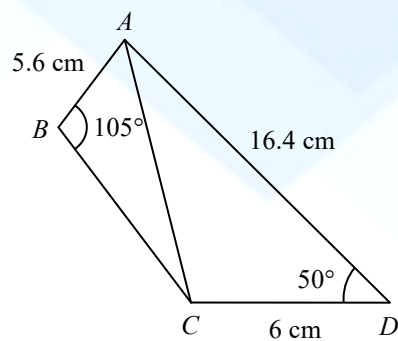
                                                          [2005, No.12]

Answer :

13   The diagram shows a quadrilateral *ABCD*.
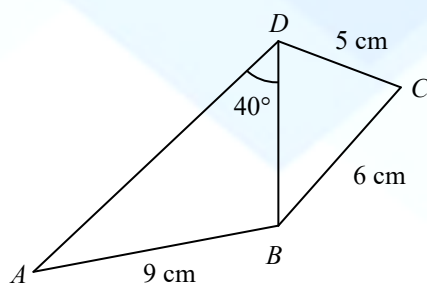
The area of triangle *BCD* is 13 cm^2 and $\angle BCD$ is acute. Calculate

(a) $\angle BCD$,   (*Ans* : 60.074) [2 marks]

(b)   the length, in cm, of *BD*,   (*Ans* : 5.5738) [2 marks]

(c) $\angle ABD$,   (*Ans* : 116.54) [3 marks]

(d)   the area, in cm^2, of quadrilateral *ABCD*.   (*Ans* : 35.439) [3 marks]

[***2006, No.13***]

Answer :

14   The diagram shows a quadrilateral *ABCD*.

(a)   Calculate

   (i)   the length, in cm, of *AC*,   (*Ans* : 13.359)

   (ii)   ∠*ACB*.   (*Ans* : 23.89)
                                                                                    [4 marks]

(b)   Point *A′* lies on *AC* such that *A′B* = *AB*.

   (i)   Sketch △*A′BC*.

   (ii)   Calculate the area, in cm², of △*A′BC*.   (*Ans* : 13.785)
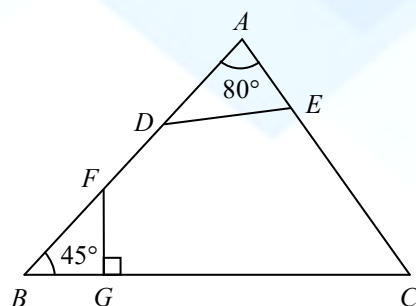   [6 marks]
                                                                              [***2007, No.15***]

Answer :

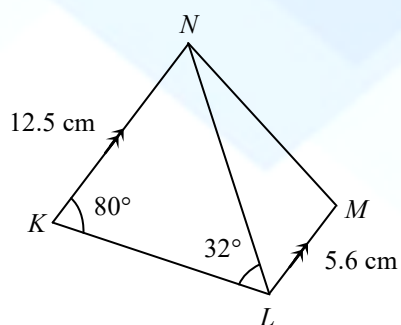**15** In the diagram, *ABC* is a triangle. *ADFB*, *AEC* and *BGC* are straight lines. The straight line *FG* is perpendicular to *BC*.

It is given that *BD* = 19 cm, *DA* = 16 cm, *AE* = 14 cm, ∠*DAE* = 80° and ∠*FBG* = 45°

(a) Calculate the length, in cm, of

   (i)   *DE*,   (*Ans* : 19.34)

   (ii) *EC*.   (*Ans* : 16.21)
                                                                                                  [5 marks]

(b) The area of triangle *DAE* is twice the area of triangle *FBG*.
      Calculate the length, in cm, of *BG*.   (*Ans* : 10.502) [4 marks]

(c) Sketch triangle *A'B'C'* which has a different shape from triangle *ABC* such that *A'B'* = *AB*,
      A'C' = *AC* and ∠*A'B'C'* = ∠*ABC*.   [1 mark]

                                                                                    [***2008, No.14***]

Answer :

16   The diagram shows a trapezium *KLMN*. *KN* is parallel to *LM* and ∠*LMN* is obtuse.

Find

(a)   the length, in cm, of *LN*,   (*Ans* : 23.23) [2 marks]

(b)   the length, in cm, of *MN*,   (*Ans* : 21.76) [3 marks]

(c)   ∠*LMN*,   (*Ans* : 98.20) [3 marks]

(d)   the area, in cm², of triangle *LMN*.   (*Ans* : 60.31) [2 marks]
   [*2009, No.12*]

Answer :

17 *Solutions by scale drawing will **not** be accepted.*
The diagram shows triangle *ABC* and triangle *CDE* where *BCE* and *ACD* are straight lines.
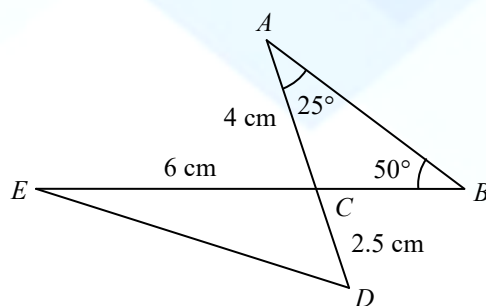
(a)   Calculate the length, in cm, of

    (i)   *BC*,   (*Ans* : 2.207)

    (ii)   *DE*,   (*Ans* : 7.072)
                                                                                          [5 marks]

(b)   Point *C′* lies on *BE* such that *AC′* = *AC*.

    (i)   Sketch triangle *AC′B*.

    (ii)   Find $\angle AC'B$.   (*Ans* : 75°)

    (iii)   Calculate the area, in cm², of triangle *AC′B*.   (*Ans* : 8.264)
                                                                                          [5 marks]
                                                                                  [***2010, No.13***]

Answer :

18 *Solutions by scale drawing will **not** be accepted.*
The diagram shows ΔPQR and ΔTQR.
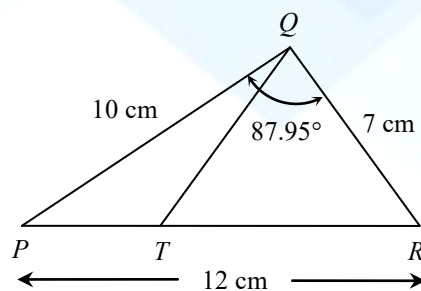
It is given that $\angle PQR = 87.95°$, $PQ = 10$ cm, $PR = 12$ cm and $TQ = QR = 7$ cm.

(a)   Find
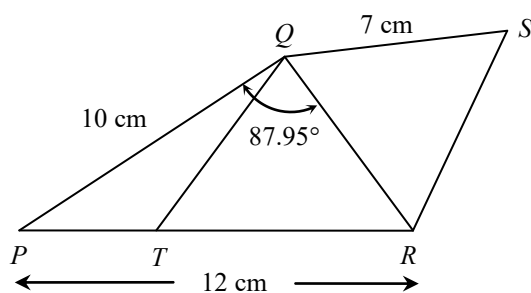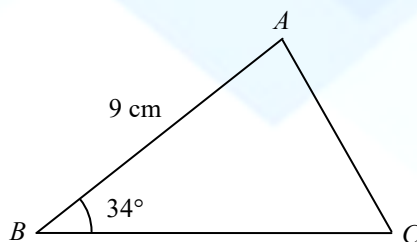
    (i) $\angle PRQ$,   (*Ans* : 56.39°)

    (ii)   the length, in cm, of *TR*,   (*Ans* : 7.750)

    (iii)   the area, in cm², of ΔPQT.   (*Ans* : 12.39)
[7 marks]

(b)   In the diagram, ΔSQR is the image of ΔTQR under the reflection in the *QR*.

    Find the length, in cm, of *PS*.   (*Ans* : 16.62) [3 marks]
[*2011, No.14*]

Answer :

19   *Solutions by scale drawing will* ***not*** *be accepted.*

The diagram shows triangle ABC such that $\angle ABC = 34°$ and $AB = 9$ cm.

It is given that the area of triangle ABC is 28 cm^2.

(a)   Calculate

   (i)   the length, in cm, of BC,   (*Ans* : 11.13) [2 marks]

   (ii)   the length, in cm, of AC,   (*Ans* : 6.228) [2 marks]

   (iii) $\angle ACB$.   (*Ans* : 53.91°) [2 marks]

(b)   Point C' lies on BC such that $AC' = AC$.

   (i)   Sketch the triangle ABC'.

   (ii)   Calculate the area, in cm^2, of the triangle ABC'.   (*Ans* : 9.544)
                                                          [4 marks]
                                                   [***2012, No.14***]

Answer :

20   The diagram shows trapezium *PQRS*.

(a)   Calculate
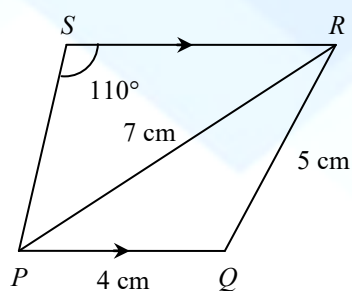
    (i) $\angle QPR$,   (*Ans* : 44.42°)

    (ii)   the length, in cm, of *PS*.   (*Ans* : 5.214)

                                                    [5 marks]

(b)   The straight line *PQ* is extended to *Q'* such that $QR = Q'R$.

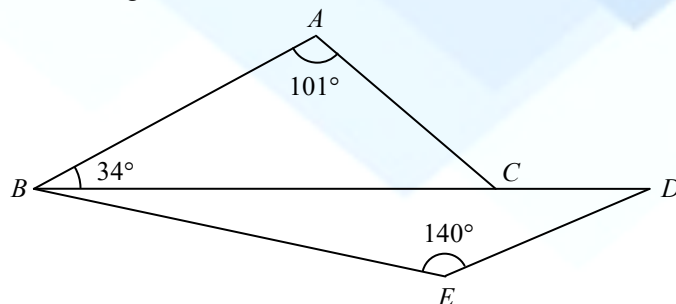   (i)   Sketch the trapezium *PQ'RS*,

    (ii)   Calculate the area, in cm², of $\triangle QQ'R$.   (*Ans* : 4.900)

                                                    [5 marks]

                                                  [2013, No.13]

Answer :

21   The diagram shows two triangles *ABC* and *BDE*.

It is given that *BE* = 8.5 cm, *DE* = 4.6 cm and *AC* = 5.8 cm.

(a)   Calculate

    (i)   the length, in cm, of *BC*,   (*Ans* : 10.18)

    (ii)   the length, incm, of *CD*,   (*Ans* : 2.2)

    (iii)   the area, in cm², of $\triangle ABC$.   (*Ans* : 20.88)

                                                   [8 marks]

(b)   (i)   Sketch a $\triangle A'B'C'$ which has a different shape from $\triangle ABC$ such that $A'B' = AB$, $A'C' = AC$ and $\angle A'B'C' = \angle ABC$.

    (ii)   Hence, state the size of $\angle B'A'C'$.   (*Ans* : 11)

                                                   [2 marks]

                                                          [**2014, No.13**]

Answer :

22   The diagram shows a quadrilateral *PQRS*.
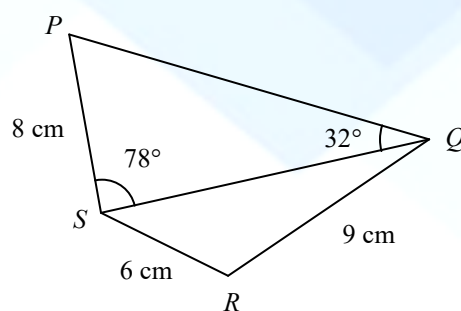
(a)   Find

　　　(i)   the length, in cm, of *QS*,   (*Ans* :   14.19)

　　　(ii)   ∠*QRS*,   (*Ans* :   141.36)

　　　(iii)   the area, in cm², of the quadrilateral   *PQRS*.   (*Ans* :   72.38)
                                                                              [8 marks]

(b)   (i)   Sketch a triangle *S′Q′R′* which has a different shape from triangle *SQR* such that *S′R′* = *SR*,   *S′Q′* = *SQ*   and ∠*S′Q′R′* = ∠*SQR*.

　　　(ii)   Hence, state ∠*S′R′Q′*.   (*Ans* :   38.64)
                                                                              [2 marks]
                                                                              [***2015, No.14***]

Answer :

23 The diagram shows a cyclic quadrilateral *ABCD*.
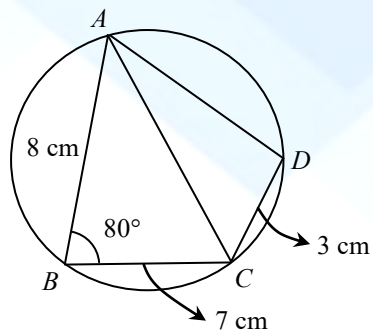
(a) Calculate

    (i) the length, in cm, of *AC*,   (*Ans* : 9.672)

    (ii) ∠*ACD*,   (*Ans* : 62.21)

                                                  [6 marks]

(b) Find

    (i) the area, in cm^2, of Δ*ABC*.   (*Ans* : 27.57)

    (ii) the shortest distance, in cm, from point *B* to *AC*.   (*Ans* : 5.701)

                                                    [4 marks]

                                                 [**2016, No.15**]

Answer :

24   *Solution by scale drawing is* **not** *accepted.*

The diagram shows a quadrilateral $ABCD$ such that AC and BD are straight lines.
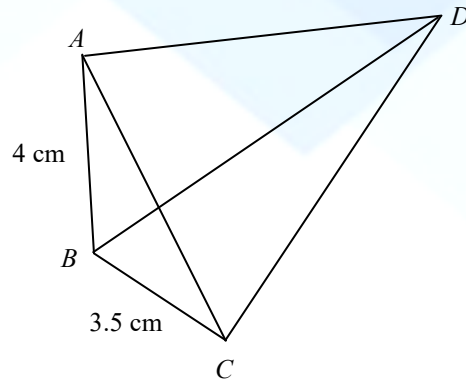
It is given that the area of $\triangle ABC = 6$ cm^2 and $\angle ABC$ is obtuse.

(a)   Find

(i) $\angle ABC$,   (*Ans* : 121)

(ii)   the length, in cm, of AC,   (*Ans* : 6.532)

(iii) $\angle BAC$,   (*Ans* : 27.34)

[7 marks]

(b)   Given $BD = 7.3$ cm and $\angle BCD = 90°$, calculate the area, in cm^2, of $\triangle ACD$.   (*Ans* : 17.808) [3 marks]

[*2019, No.13*]

Answer   :

⇒ *Part C ~ 2 dimensional and 3 dimensional surfaces*

25   The diagram shows a tent $VABC$ in the shape of a pyramid with triangle ABC as the horizontal base. V is the vertex of the tent and the angle between the inclined plane VBC and the base is 50°.
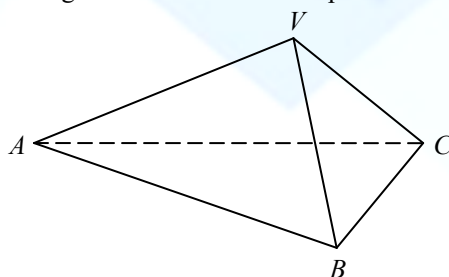
Given that *VB = VC* = 2.2 m and *AB = AC* = 2.6 m. Calculate

(a)   the length of *BC* if the area of the base is 3 m²,
      (*Ans* : 2.700) [3 marks]

(b)   the length of *AV* if the angle between *AV* and the base is 25°,
      (*Ans* : 3.149) [3 marks]

(c)   the area of triangle *VAB*.   (*Ans* : 2.829) [4 marks]
                                                                              [*2003, No.15*]

Answer :

26   *Solutions by scale drawing is not accepted.*

The diagram shows a quadrilateral *ABCD* on a horizontal plane.
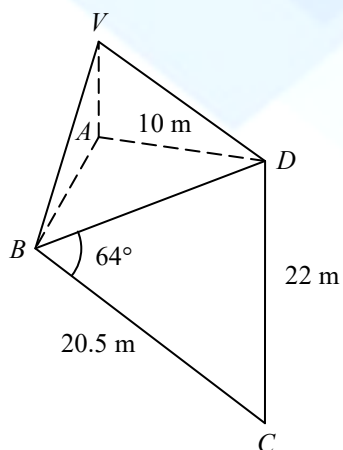
VBDA is a pyramid such that *AB* = 12 m and *V* is 5 m vertically above *A*. Find

(a)   ∠ *BDC*,   (*Ans* : 56.88)   [2 marks]

(b)   the length, in cm, of *BD*.   (*Ans* : 21.01)   [3 marks]

(c)   the area, in m², of inclined plane *BVD*.   (*Ans* : 62.64)   [5 marks]

[*2017, No.15*]

Answer   :

27 *Solution by scale drawing is not accepted.*

The diagram shows a transparent prism with a rectangular base *ABCD*. The inclined surface *ABFE* is a square with sides 12 cm and the inclined surface *CDEF* is a rectangle. *AED* is a uniform cross section of the prism. *BDE* is a shaded plane in the prism.
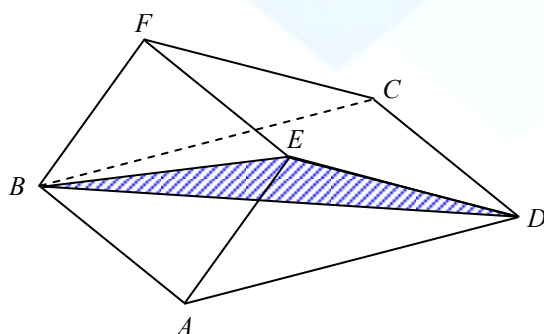
It is given that $\angle ADE = 37°$ and $\angle EAD = 45°$. Find

(a)   the length, in cm, of *DE*,   (*Ans* : 14.099) [2 marks]

(b)   the area, in cm², of the shaded plane,   (*Ans* : 119.06) [6 marks]

(c)   the shortest length, in cm, from point *E* to the straight line *BD*.   (*Ans* : 10.204) [2 marks]

[2018, No.14]

Answer   :

FORECAST

28   The diagram shows a triangle ABC.
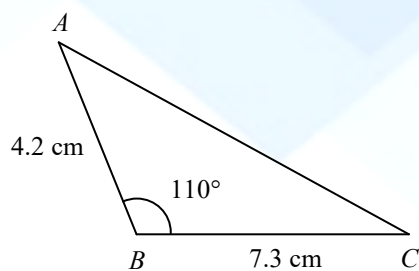
(a)   Calculate the length, in cm, of AC.   (*Ans* : 9.587)   [2 marks]

(b)   If the length of AB is extanded to D such that angle $ADC = 48°$, find the length, in cm, of BD.   (*Ans* : 8.673)   [3 marks]

(c)   Find the area, in cm², of triangle ADC.   (*Ans* : 44.16)   [3 marks]

(d)   Find the shortest distance, in, from C to AD.   (*Ans* : 6.860)   [2 marks]

Answer   :

29   The diagram shows a triangle ABC.
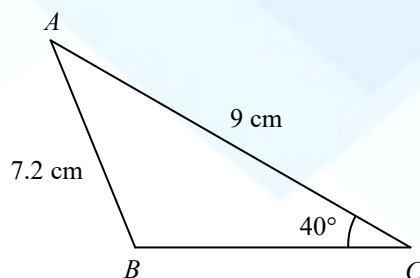
(a)   Calculate obtuse angle ABC.   (*Ans* : 126.54°) [3 marks]

(b)   Sketch and label another triangle which is different from triangle ABC, such that the lengths of AB and AC, and the $\angle ACB$ are maintained.
Hence, find the area, in cm², of $\triangle AB'C$.   (*Ans* : 32.34) [4 marks]

(c)   If the length of AB is reduced while the length of AC and $\angle ACB$ are maintained, so that only one triangle $AB'C$ can be formed.

(i)   Sketch the triangle $AB'C$.   [1 mark]

(ii)   Calculate the length, in cm, of AB'.   (*Ans* : 5.785) [2 marks]

Answer   :

30   The diagram shows a triangle *PQR*.
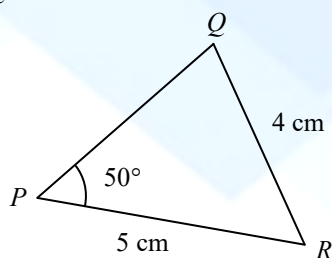
(a)   Calculate $\angle PRQ$.   (*Ans* : 56.75°)   [3 marks]

(b)   Sketch and label another triangle that is different from triangle *PQR* in the
        diagram, such that the lengths of *PR* and *RQ*, and the angle *RPQ* remain the
        same.
        Hence, using the cosine rule, find the length of *PQ'*.   (*Ans* : 2.061)   [4 marks]

(c)   If the length of *RQ* is shortened while the length of *PR* and $\angle RPQ$ are
        maintained, such that only one triangle *PQ'R* can be formed

        (i)   Sketch the triangle *PQ'R*.   [1 mark]

        (ii)   Find the area, in cm², of the new triangle formed.   (*Ans* : 6.155)   [2 marks]

Answer   :

31   The diagram shows a quadrilateral *PQRS*.
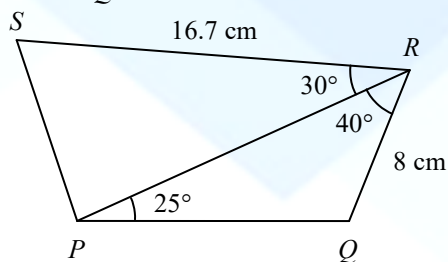
(a)   Calculate the length, in cm, of

(i)   *PR*,   (*Ans* : 17.16)

(ii)   *PS*,   (*Ans* : 8.775)

(iii)   ∠*RPS*,   (*Ans* : 72.09)
                                                                                                [7 marks]

(b)   Point *P′* lies on *PR* such that *PS* = *P′S*.
Calculate the area, in cm², of triangle *PP′S*.   (*Ans* : 22.53)   [3 marks]

Answer :

32   The diagram shows two triangles ABC and BCD, where $\angle ACB$ and $\angle BCD$ are obtuse.
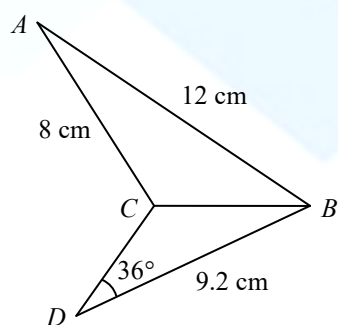
If the area of triangle ABC is 20 cm^2, calculate

(a) $\angle BAC$,   (Ans : 24.62) [2marks]

(b)   the length, in cm, of BC,   (Ans : 5.784) [2 marks]

(c)   the length, in cm, of AD.   (Ans : 12.14) [6 marks]

Answer :

33 (a) In the diagram, CFE is a straight line.
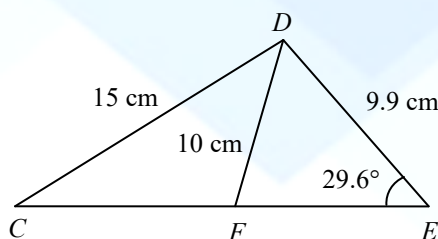
Calculate the length, in cm, of CFE. (*Ans* : 22.79)

[4 marks]

(b) In the diagram, P, Q, R, and S are four points on a horizontal groud. A surveyor want to measure the distance PR and PS. The surveyor knows that PQ = 3.2 km.
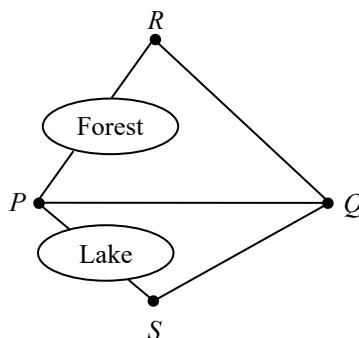
(i) The distance PS cannot be measure directly because there is a lake between P and S. By measure, the surveyor found that $\angle QPS$ = 73°30′ and $\angle PQS$ = 44°30′.
Find the distance, in km, PS. (*Ans* : 2.540) [3 marks]

(ii) The distance PR also cannot be measure directly because there is a forest between R and P. By measure, the surveyor found that RQ = 4.4 km and $\angle PQR$ = 62°20′.
Find the distance, in km, PR. (*Ans* : 4.065) [3 marks]

Answer :

34 (a) The diagram shows a triangle *PQR*. *PR* is horizontal.
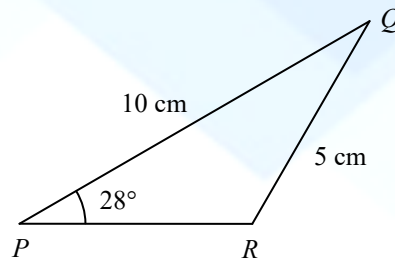
Calculate

(i) the bearing of *Q* from *R*, (*Ans* : 020.13°)

(ii) the area of the new triangle if *PR* is extended, while the lengths of *PQ* and *QR*, and ∠QPR are maintained. (*Ans* : 24.76)

[5 markah]

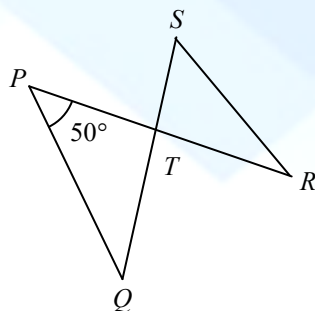(b) In the diagram, *KLMN* is a cyclic quadrilateral of a circle with centre *O*.
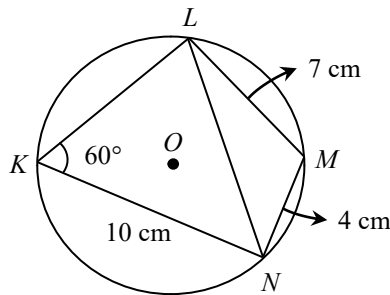
Calculate

(i) the length, in cm, of *LN*, correct to two decimal places, (*Ans* : 9.64)

(ii) ∠KNL. (*Ans* : 56.06)

[5 marks]

Answer :

35   (a)   In the diagram, *PTR* and *QTS* are straight lines.

Given that *RS* = 6 cm, *RT* = 4 cm, *TS* = 3 cm, and *PQ* = 14 cm, calculate

(i)   ∠*RTS*,   (*Ans* : 117.28)

(ii)   the length, in cm, of *QT*.   (*Ans* : 12.07)

[4 marks]

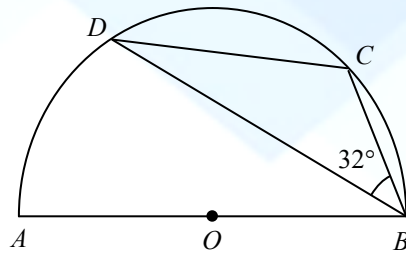(b)   The diagram shows a circle with centre *O* and a radius of 6 cm.
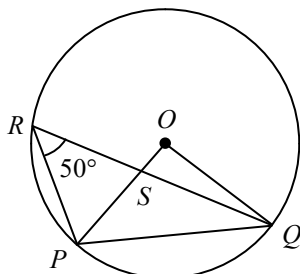
Given that ∠*PRQ* = 50° and *PR* = 5 cm, calculate

(i)   the length, in cm, of *PQ*,   (*Ans* : 9.193)

(ii)   the area, in cm², of triangle *PSQ*.   (*Ans* : 12.52)

[6 marks]

Answer :

36 (a) In the diagram, $ABCD$ is a semi circle with centre O and radius 10 cm.

Given that BC = 4 cm and $\angle CBD$ = 32°. Calculate

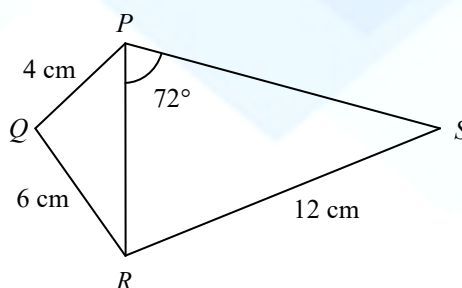(i) the length, in cm, of CD,   (*Ans* : 10.596)

(ii) the length, in cm, of AD.   (*Ans* : 14.501)

[6 marks]

(b) The diagram shows a pyramid with a horizontal triangular base ABC.
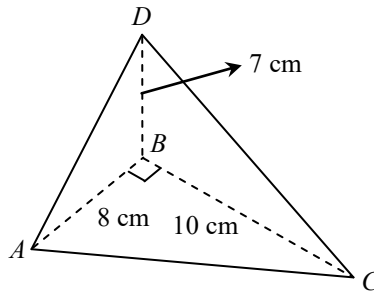
Given AB = 8 cm, BC = 10 cm and $\angle ABC$ = 90°. Vertes D is 7 cm vertically above B. Calculate the area, in cm², of the slanting surface ADC. (*Ans* : 60.07)

[4 marks]

Answer :

37 (a) In the diagram, $\sin \angle PQR = \dfrac{12}{13}$, where $\angle PQR$ is obtuse.
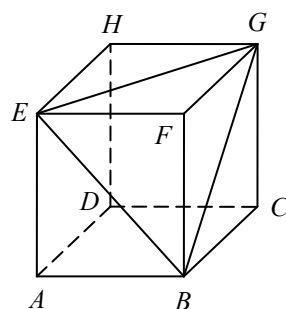
P

4 cm

72°

Q

S

6 cm

12 cm

R

Calculate

(i) the length, in cm, of *PR*, (*Ans* : 8.39)

(ii) $\angle PSR$. (*Ans* : 41.70)

[5 marks]

(b) The diagram shows a cuboid with a square base of 4 cm, and a height of 6 cm.

H   G

E

F

D   C

A   B

Calculate

(i) $\angle EGB$, (*Ans* : 66.91)

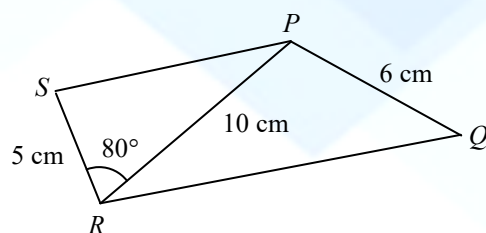(ii) the area, in cm², of triangle *EGB*. (*Ans* : 18.76)
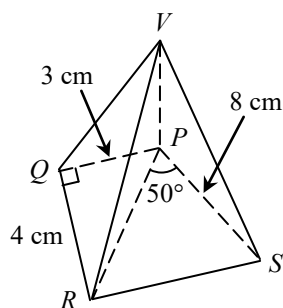
[5 marks]

Answer :

38 (a) In the diagram, $\cos \angle QPR = -\dfrac{3}{5}$.

Calculate

(i)   the length, in cm, of *RQ*,   (*Ans* : 14.422)

(ii) $\angle PSR$.   (*Ans* : 71.66)

[5 marks]

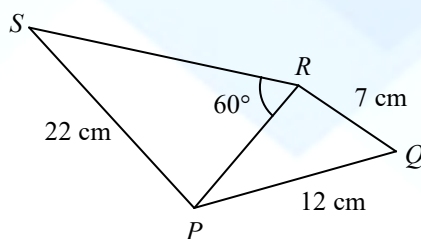(b) The diagram shows a pyramid with a horizontal quadrilateral base.

The apex *V* is 12 cm vertically above *P* and $\angle RPS$ is 50°. Calculate

(i)   the length, in cm, of *RS*,   (*Ans* : 6.130)

(ii)   the area, in cm², of triangle *VRS*.   (*Ans* : 39.84)

[5 marks]

Answer :

39 (a) In the diagram, $\sin \angle PQR = \dfrac{3}{5}$, where $\angle SPR$ is obtuse.
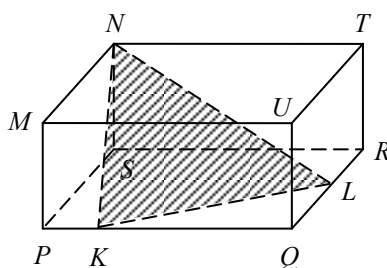

.

Calculate

(i) the length of *PR*, correct to three significant figures, (*Ans* : 7.66)

(ii) $\angle SPR$. (*Ans* : 102.45)

[5 marks]

(b) The diagram shows a cuboid, where *PQ* = 8 cm, *QR* = 6 cm and *MP* = 4 cm. *L* is the midpoint of *QR* and *PK* : *KQ* = 1 : 3.
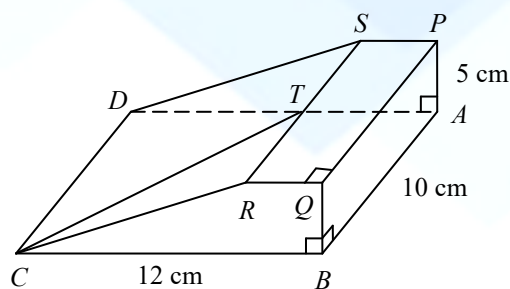

Calculate

(i) the length, in cm, of *NL*, (*Ans* : 9.434)

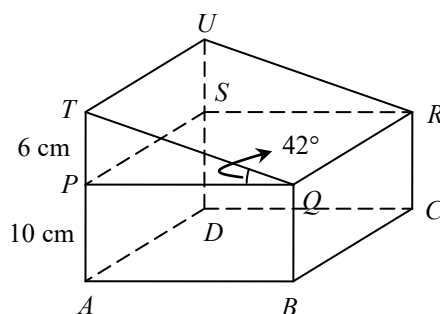(ii) the area, in cm², of triangle *KLN*. (*Ans* : 24.92)

[5 marks]

Answer :

40 (a) The diagram shows a right prism with a horizontal rectangular base *ABCD*. Trapezium *BCRQ* is the uniform cross section. *T* is the midpoint of *RS*.

If $RQ = \dfrac{1}{3} BC$, calculate the angle between the line *CT* and the base *ABCD*. (*Ans* : 27.92)

[2 marks]

(b) The diagram shows a solid formed by a right prism and a cuboid. The rectangular surface, *QRUT*, is inclined. The angle between the line *TR* and the plane *PQRS* is 30°.
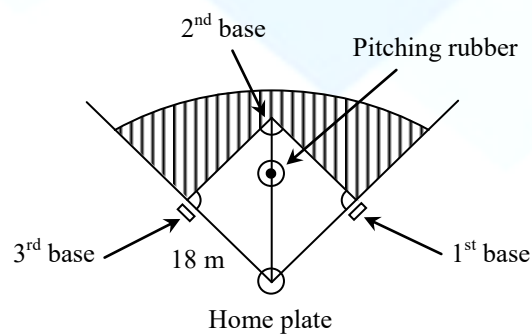

Calculate

(i) the lengths, in cm, of *PQ*, *TR*, and *AR*, (*Ans* : 6.664, 12, 14.42) [4 marks]

(ii) the area, in cm², of triangle *TAR*. (*Ans* : 83.13) [4 marks]

Answer :

41   (a)   The diagram shows a baseball diamond playing field in the shape of a square of side 18 m. The pitching rubber is located 13.8 m from the home plate on a line joining home plate and second base.

Calculate the distance, in m, from the pitching rubber to

(i)   second base,   (*Ans* : 11.66)   [2 marks]

(ii)   third base.   (*Ans* : 12.77)   [2 markah]

(b)   In a triangle *DEF*, *DE* = 8 cm, *DF* = 10 cm, and ∠*DFE* = 50°. Sketch the two possible triangles *DEF*. Hence, find the probable values of the length of *EF*.   (*Ans* : 4.122, 8.734)
                                                                                              [6 markah]

Answer :

42   Coast guard station A is located 184 km due west of station B. A cruiser at sea sends an emergency call that is received by each station. The call to station A indicates that the location of the cruiser is 40° east of north and the call to the station B indicates that the location of the cruiser is 30° west of north.
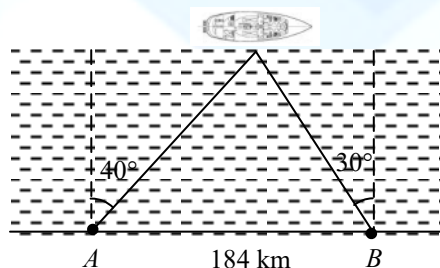

(a)   How far is each station from the cruiser?   (*Ans* : 150, 169.58) [3 marks]

(b)   A speedboat capable of speeding 100 km per hour is despatched from the station B to the cruiser. After travelling for $\dfrac{1}{2}$ an hour, the speedboat encounters heavy crosswinds and strong currents from the west. The crew finds that the speedboat of off course by 20°.
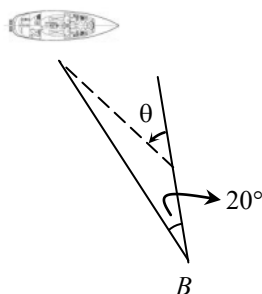
(i)   How far is the speedboat from the cruiser ?   (*Ans* : 104.43) [2 marks]

(ii)   Through what angle should the speedboat turn to correct its course?   (*Ans* : 29.44) [2 marks]

(iii) How much time has been added to the trip because of this problem?   (*Ans* : 2.658) [1 mark]

(c)   Since the cruiser needs assitance badly, a helicopter which is capable flying 200 km per hour is deispatched from station A. After 15 minutes journey, the pilot discovered that he was 10° off course.
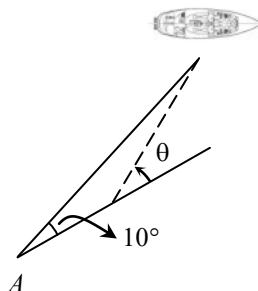
What is the average speed, in kmh⁻¹, should the pilot maintain, so that the total time to reach the cruiser is not more than 30 minutes.   (*Ans* : 241.3) [2 marks]

Answer :

CONTINUOUS EXERCISES

43   Given that the lengths of the two sides of a parallelogram are 68.2 cm and 83.3 cm. One of its diagonal
has a length of 42.5 cm. Calculate

    (a)   the angles of the parallelogram,   (*Ans* : 30.56, 149.44)

    (b)   the length, in cm, of another diagonal,   (*Ans* : 146.20)

    (c)   the area, in cm², of the parallelogram.   (*Ans* : 2888.48)

[7 marks]

    Answer :

44   In a parallelogram, the adjacent angles of a diagonal of length 76.33 cm are 52.2° and 41.45°
respectively. Find

    (a)   the length, in cm, of the sides of the parallelogram,   (*Ans* : 50.63, 60.44)

    (b)   the area, in cm², of the parallelogram.   (*Ans* : 3053.62)

[7 marks]

    Answer :

45   The angles of a triangle are in the ratio 5 : 10 : 21, and the shortest side is 35.64 cm. Find

(a)   the length, in cm, of the longest side,   (*Ans* : 81.46)

(b)   the area, in cm², of the triangle.   (*Ans* : 1112.00)

                                                                                                          [5 marks]

Answer :

46   The perimeter of △*PQR* is 40 cm and ∠*P* : ∠*Q* : ∠*R* = 1 : 2 : 6. Find

(a)   the length, in cm, of the sides *p*, *q* and *r*.   (*Ans* : 7.392, 13.890, 18.717)

(b)   the area, in cm², of the △*PQR*.   (*Ans* : 44.459)

                                                                                                          [10 marks]

Answer :

47   The length of the sides of a triangle are *x* −2, 2*x* + 7 and 2*x* + 8. Given the perimeter of the triangle is 63 cm.

(a)   the value of *x*,   (*Ans* : 10)

(b)   the area, in cm², of the triangle.   (*Ans* : 107.99)

                                                                                                          [5 marks]

Answer :

48 Darren want to prepare a greeting card in the shape of a triangle. The lengths of two sides of the triangle are 8 cm and 11 cm respectively, and the area of the is 30 cm^2.

(a) Sketch the two possible triangles.

(b) Hence, find the possible lengths, in cm, of the third side. (*Ans* : 7.501, 17.713)

[7 marks]

Answer :

49 A regular pentagon has sides of 5 cm. Find

(a) the length, in cm, of its diagonal. (*Ans* : 8.090)

(b) the area, in cm^2, of the regular pentagon. (*Ans* : 43.009)

[7 marks]

Answer :

50 If the three sides of triangle are a, b and $\sqrt{a^2 + b^2 + ab}$.

Find the greater angle of this triangle. (*Ans* : 120)

[3 marks]

Answer :

51 Given that the ratio of the sides of a triangle is $7 : 4\sqrt{3} : 13$, find its smallest angle.   (*Ans* : 20.92)

[3 marks]

Answer :

52 Given $\triangle PQR$ in which $\sin P : \sin Q : \sin R = 3 : 5 : 7$. Find its largest angle.   (*Ans* : 120)

[3 marks]

Answer :

53 Given $\triangle ABC$ in which $(b + c) : (c + a) : (a + b) = 7 : 8 : 9$. Find $\sin A : \sin B : \sin C$.
(*Ans* : $5 : 4 : 3$)

[3 marks]

Answer :

54 Given $\triangle ABC$ in which $(b + c) : (c + a) : (a + b) = 4 : 5 : 6$. Find $\angle A$.   (*Ans* : 120)

[3 marks]

Answer :