Optical Flow

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1. Basic idea

2. Correlation-based optical flow

3. Differential-based and Dense optical flow

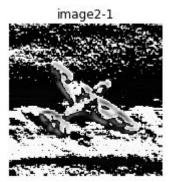
Optical Flow

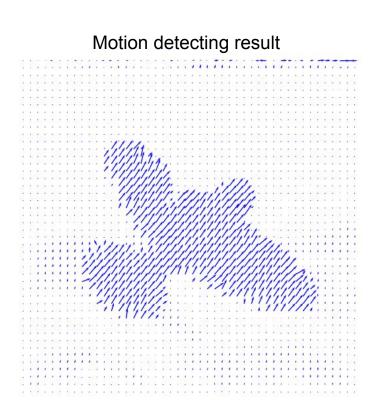
Basic idea



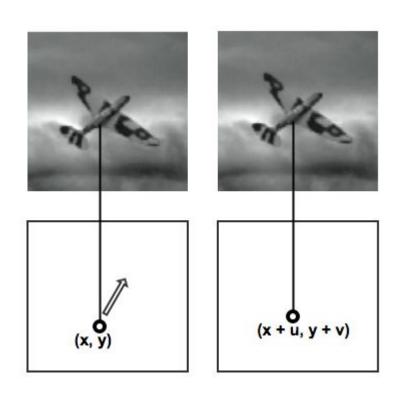


(original image1and image2 are captured from [1])





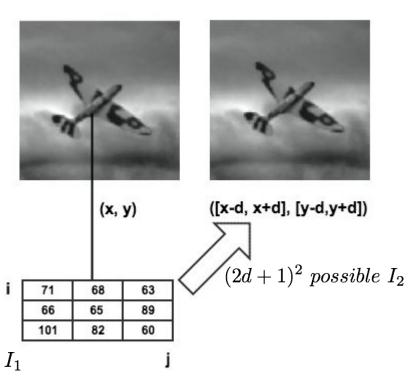
Basic idea



Important assumption

- 1. Constant brightness
- 2. Similar velocity for a block
- 3. Small displacement

Correlation-based



Differences measure methods:

 $SAD(Sum\ of\ absolute\ differences)$

$$\sum_{(i,j)\in W} |I_1(i,j) - I_2(dx + i, dy + j)|$$

 $ZSAD(Zero-mean\ Sum\ of\ absolute\ differences)$

$$\sum_{(i,j)\in W} |I_1(i,j) - \overline{I_1}(i,j) - I_2(dx+i,dy+j) + \overline{I_2}(dx+i,dy+j)|$$

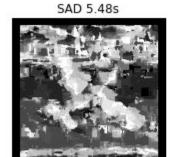
 $SSD(Sum\ of\ squared\ differences)$

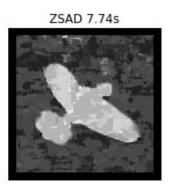
$$\sum_{(i,j)\in W} (I_1(i,j) - I_2(dx+i,dy+j))^2$$

 $ZSSD(Zero-mean\ Sum\ of\ squared\ differences)$

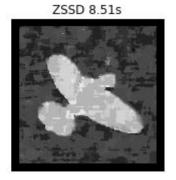
$$\sum_{(i,j)\in W} (I_1(i,j) - \overline{I_1}(i,j) - I_2(dx+i,dy+j) + \overline{I_2}(dx+i,dy+j))^2$$

No Gaussian filter:

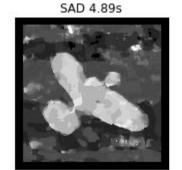




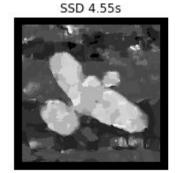


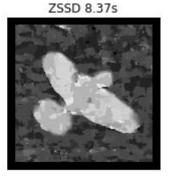


With Gaussian filter:









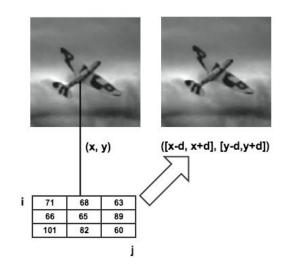
Similarity measure methods:

 $CC(Cross\ correlation)$

$$\sum_{(i,j)\in W} I_1(i,j)I_2(dx+i,dy+j)$$

 $NCC(Normalized\ Cross\ correlation)$

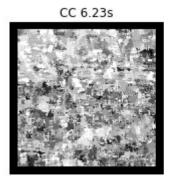
$$\frac{\sum_{(i,j)\in W} I_1(i,j)I_2(dx+i,dy+j)}{\sqrt{\sum_{(i,j)\in W} I_1(i,j)^2 \sum_{(i,j)\in W} I_2(dx+i,dy+j)^2}}$$

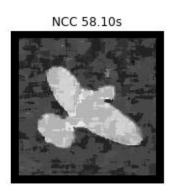


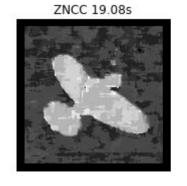
 $ZNCC(Zero-mean\ Normalized\ Cross\ correlation)$

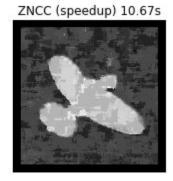
$$\frac{\sum_{(i,j)\in W} (I_1(i,j) - \overline{I_1}(i,j))(I_2(dx+i,dy+j) - \overline{I_2}(dx+i,dy+j))}{\sqrt{\sum_{(i,j)\in W} (I_1(i,j) - \overline{I_1}(i,j))^2 \sum_{(i,j)\in W} (I_2(dx+i,dy+j) - \overline{I_2}(dx+i,dy+j))^2}}$$

No Gaussian filter:

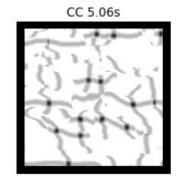


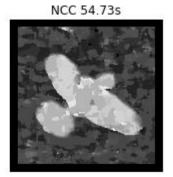


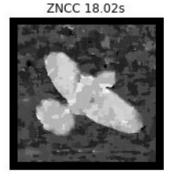


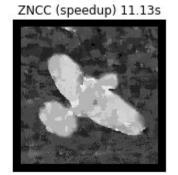


With Gaussian filter:

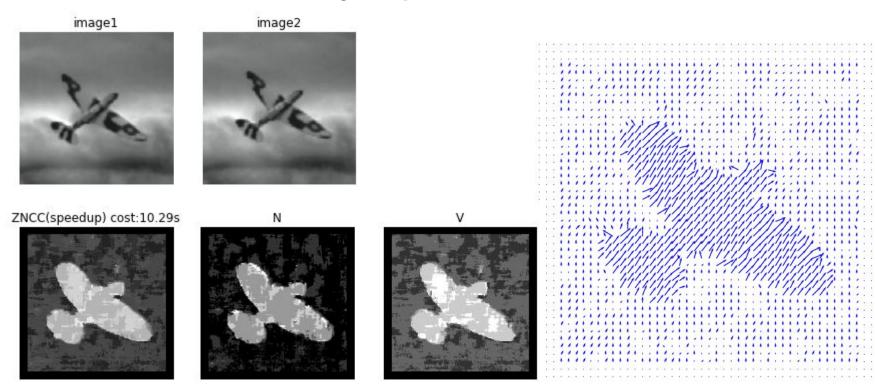




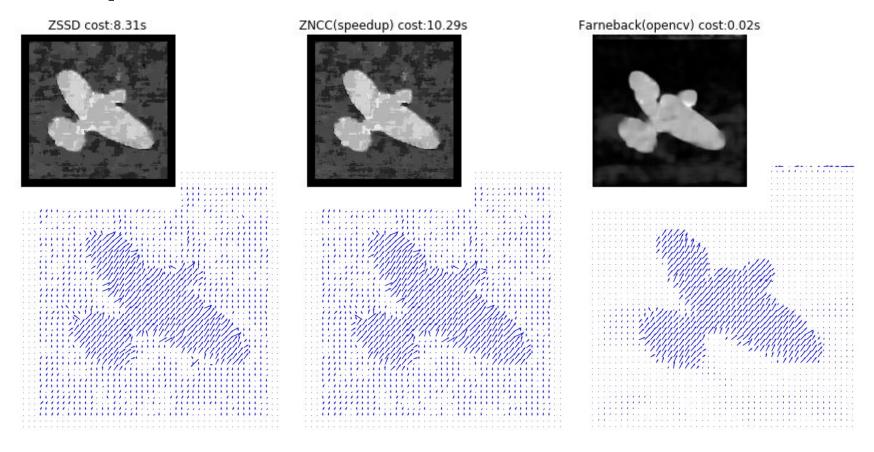




Belive me I am flying!



Comparison



Differencial-based

& Dense optical flow

Differencial-based Derive

Image brightness constancy[5]:

from a short interval t1 to t2, while an object may change position, the reflectivity and illumination will remain constant.

$$f(x + \triangle x, y + \triangle y, t + \triangle t) \approx f(x, y, t)$$

According to Taylor series expansion:

$$f(x + \triangle x, y + \triangle y, t + \triangle t) = f(x, y, t) \frac{\partial}{\partial x} \triangle x + \frac{\partial}{\partial y} \triangle y + \frac{\partial}{\partial z} \triangle z + h.o.t$$

$$\nabla I \bullet + I_t = 0$$

HORN & SCHUNCK METHOD[6]

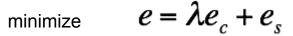
use masks to calculate fx, fy,ft:

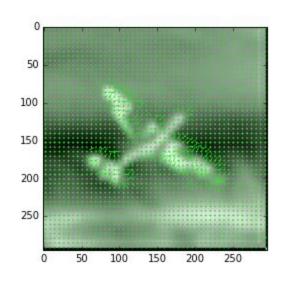
$$\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$$

$$mask1 \qquad mask2 \qquad mask3 \qquad mask4$$

$$e_s = \iint ((u_x^2 + u_y^2) + (v_x^2 + v_y^2)) dxdy$$

$$e_c = \iint (I_x u + I_y v + I_t)^2 dx dy$$





LUCAS & KANADE METHOD[6]

$$\min \sum_{i} (f_{xi}u + f_{yi}v + f_t)^2$$

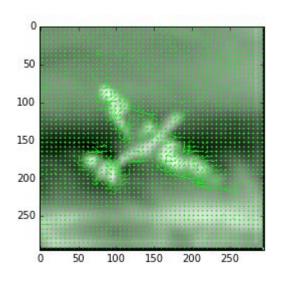
$$\sum (f_{xi}u + f_{yi}v + f_{ti})f_{xi} = 0$$

$$\sum (f_{xi}u + f_{yi}v + f_{ti})f_{yi} = 0$$

$$\sum f_{xi}^2 u + \sum f_{xi} f_{yi} v = -\sum f_{xi} f_{ti}$$

$$\sum f_{xi} f_{yi} u + \sum f_{yi}^2 v = -\sum f_{yi} f_{ti}$$

$$\begin{bmatrix} \sum f_{xi}^2 & \sum f_{xi}f_{yi} \\ \sum f_{xi}f_{yi} & \sum f_{xi}^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -\sum f_{xi}f_{ti} \\ -\sum f_{yi}f_{ti} \end{bmatrix}$$



Gunnar Farneback method

Dense optical flow

_____ A optimal method to track all the pixels in an image

Farneback method uses Polynomial Expansion to approximate the neighbors of a pixel[7].

Demo with Dense techniques

- Extract the moving object from the static background[8]

 - ⇒ Get background with threshold

- Track the moving object
 - ➡ findContours() in opencv
 - ⇒ Draw the rectangle edge of the moving object

Speed up with numba!

Thanks

Reference

- [1] Nixon M. Feature extraction & image processing[M]. Academic Press, 2008.
- [2] Giachetti, A., Matching Techniques to Compute Image Motion, Image Vision Comput., 18(3), pp. 247–260, 2000
- [3] Correlation based similarity measures-Summary, https://siddhantahuja.wordpress.com/tag/normalized-cross-correlation/
- [4] Zero Mean Normalized Cross-Correlatio, https://martin-thoma.com/zero-mean-normalized-cross-correlation/#tocAnchor-1-2
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- [6] Coarse-to-fine Optical Flow, http://eric-yuan.me/coarse-to-fine-optical-flow/
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- [8] SuganyaDevi, K., N. Malmurugan, and R. Sivakumar. "Efficient foreground extraction based on optical flow and SMED for road traffic analysis." *International Journal of Cyber-Security and Digital Forensics (IJCSDF)* 1.3 (2012): 177-182.