CHULALONGKORN BUSINESS SCHOOL

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Stochastic dominance]– normative theory
mean-variance criterian

2604-639

Finance Theories

can do empirical test

positive theory

Topic 6:

The Capital Asset Pricing Model (CAPM)

E[r_i] is form in equilibrium in relation to risk_i

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Agenda

1. INTRODUCTION AND ASSUMPTIONS

1.1 Introduction

- Finance theories covered so far are normative economic theories.
- Lectures 4-5 examine the modern portfolio theory which analyzes investors' asset demand given asset return distribution. The theory suggests how investors should form their portfolios of risky assets.
- In the next two lectures, we will study how investors' asset demand determines the relation between assets' risk and return in a market equilibrium. Two theories will be studied, The Capital Asset Pricing Model (CAPM) and The Arbitrage Pricing Theory (APT). Both theories are positive economic theories.

have same objective

try to find expected return in equilibrium

1.1 Introduction

- The Capital Asset Pricing Model (CAPM) is a model describing how expected returns on risky assets are formed in equilibrium.
- The CAPM is based on the portfolio theory pioneered by Harry Markowitz (mean-variance analysis) and James Tobin (CML).
- William Sharpe, John Lintner and Jan Mossin are credited for the development of the CAPM.

capital mkt line

MVC → CML → CAPM

market portfolio

1.2 Assumptions

- The CAPM relies on a set of rather restrictive assumptions.

1. Assumptions regarding individuals’ behavior
In term of axiom of choice (a) All investors are rational mean-variance optimizers (i.e., assets returns are normally distributed). ↳ can assume that utility function is quadratic function
(b) All investors have identical single period investment horizon (i.e., investment decision is made once today). invest in ST as E[i] will ↑ LT or ST bond → have multi-investment horizon ↳ have reinvestment risk E[i] ↑
same efficient frontier (c) All investors have homogeneous expectations regarding $E[r_i]$’s and σ_i’s of all assets (i.e., identical input list is used in optimization). This assumption is consistent with the assumption that all relevant information is publicly available.

lot size regulative → In reality, asset is not divisible

Human capital cannot be trade → it is privately held

1.2 Assumptions

2. Assumptions regarding capital market structure

(a) All assets are divisible and publicly held. They are tradable on public exchanges.

where investors can borrow/lend

(b) There exists a risk-free asset such that investors can borrow or lend unlimited amount at r_F.

cannot borrow at risk-free rate!

only govt can do this

(c) The capital market is perfect (i.e., frictionless).

- No transaction costs and taxes
- Equal and costless access to information
- No restrictions on short sale
- Investors are price takers

mkt price is the fair price

each of investor is small action doesn't affect mkt

but aggregated action of a lot investors can move price

2. THE CAPITAL ASSET PRICING MODEL

2.1 The Capital Market Line (CML)

2.2 Derivation of the CAPM

2.3 The CAPM and Equilibrium Expected Returns

2.4 Properties of the CAPM

2.5 Applications of the CAPM

2.6 Extensions of the CAPM

2.1 The Capital Market Line (CML)

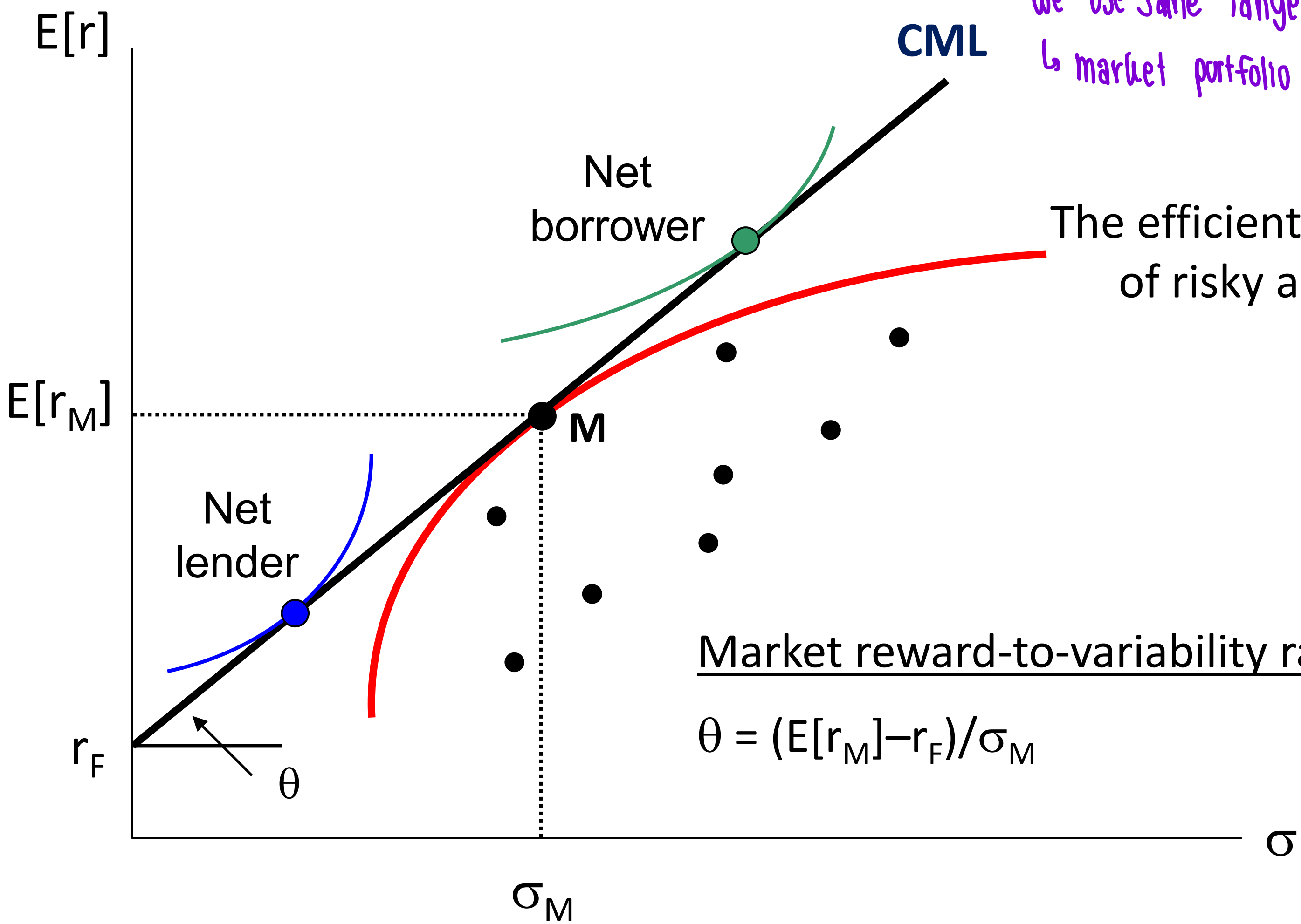
- The above assumptions imply that all investors must face the same CAL and hold the same optimal risky portfolio (i.e., same component stocks and weights).
- In equilibrium;
  - Asset prices must be adjusted until there is no excess demand and supply on financial assets.
  - The total wealth invested on risk-free asset must be zero (total borrowing = total lending).
  - All existing risky assets must be held in the optimal risky (or the tangent) portfolio, whereby assets are held according to market value weights.

2.1 The Capital Market Line (CML)

- The optimal risky portfolio is called “the market portfolio (M)” because it consists of all risky assets traded in the market.
- The CAL that connects the risk-free rate and the market portfolio is called the “Capital Market Line” or CML.

The Capital Market Line (CML) and the Two-Fund Separation Property

we use same tangent point
↳ market portfolio



Market reward-to-variability ratio

$\theta = (E[r_M] - r_F)/\sigma_M$

2.1 The Capital Market Line

- EX: There are 3 risky assets, A, B and C, and 3 investors, X, Y and Z. The tangent portfolio is the same for all 3 investors, that is, $w_T = (w_A, w_B, w_C) = (0.25, 0.50, 0.25)$.

  weight that Mr. X, Y, Z use to form tangent port

  amount of money lend & borrow must be the same ⇒ equilibrium

- The total wealth for each investor, X=\$500, Y=\$1,000 and Z=\$1,500. Their capital allocations "RF + the Risky Portfolio" are X= \$100+\$400, Y=\$200+\$800 and Z=–\$300+1,800.

  – net borrower

  borrow to short-sell

- The table shows the value of investment in each assets.

Investor	RF	A	B	C	Total
X	100	100	200	100	**500**
Y	200	200	400	200	**1,000**
Z	–300	450	900	450	**1,500**
Total	**0**	**750**	**1,500**	**750**	**3,000**

market cap of A

weight of mkt portfolio is the same as weight of tangent point

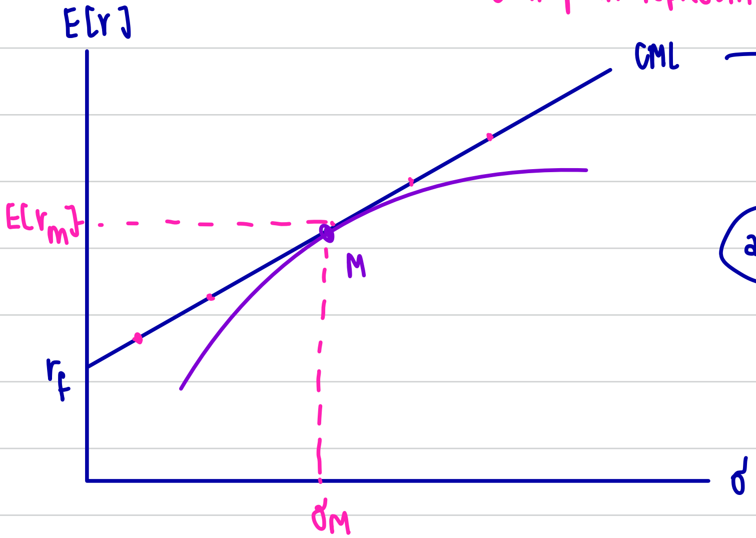
2.1 The Capital Market Line

- In equilibrium, the total dollars holding in each risky asset must be equal to the asset's market capitalization.
- The weight of each asset in the market portfolio is;
  - $w_A = V_A/V_M = 750/3{,}000 = 0.25$
  - $w_B = V_B/V_M = 1{,}500/3{,}000 = 0.50$
  - $w_C = V_C/V_M = 750/3{,}000 = 0.25$
- In equilibrium, the tangent portfolio is the market portfolio.
- The total investment in the RF asset is \$0.

Implication of the CML

- In equilibrium,
  - All investors hold the same risky portfolio and are different only in their capital allocations.
  - This leads to the Two-Fund Separation Property: Portfolio construction consists of 2 independent steps:
    - Security selection: Forming the optimal risky portfolio. This step is independent of investor's risk aversion and could be delegated to fund managers.
    - Capital allocation: choose the optimal proportion between the risk-free asset and the market portfolio.
  - The passive security selection strategy is MV efficient.

each point represent complete portfolio



equation for CML → expected return in equilibrium

$a + bx$

$$E[r_p] = r_f + \frac{E[r_m] - r_f}{\sigma_M} \times \sigma_p$$

2.1 The Capital Market Line

- The CML is the equilibrium condition representing the relation between $E[r_P]$ and σ_P of MV efficient portfolios.
- The CML equation is;

$$E[r_P] \;=\; r_F + \left(\frac{E[r_M] - r_F}{\sigma_M}\right) \times \sigma_P$$

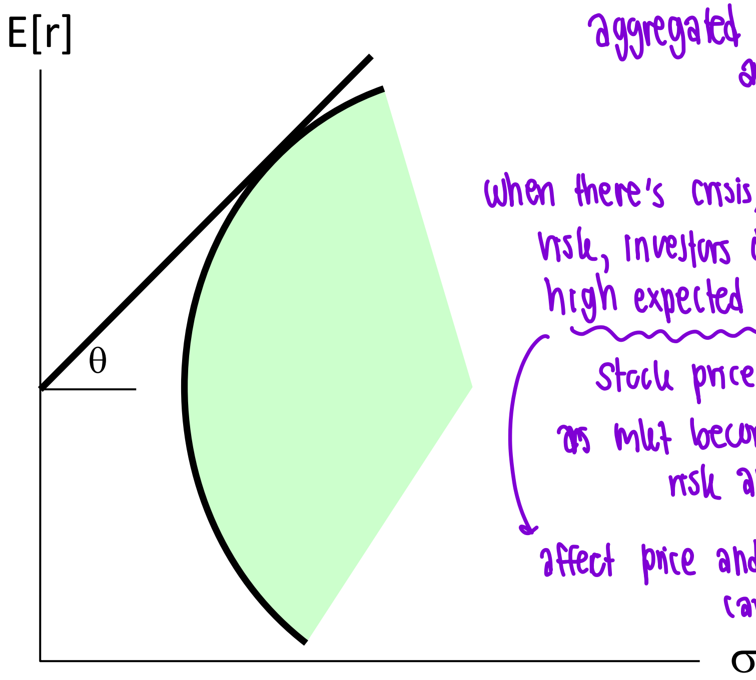
- The slope of the CML depends on the expected market risk premium, which in turn, depends on the aggregate risk aversion of the market.
- Drawback: The CML does not show how the expected return on any portfolio and any individual security (whether it is MV efficient or not) is formed in equilibrium.

The Market Risk Premium and Aggregate Risk Aversion

The slope of the CML depends on the RP_M, while the RP_M depends on the average risk aversion of all market participants.

sharpe ratio of mkt ; slope = $E(r_m - r_f) / \sigma_M$

aggregated risk aversion

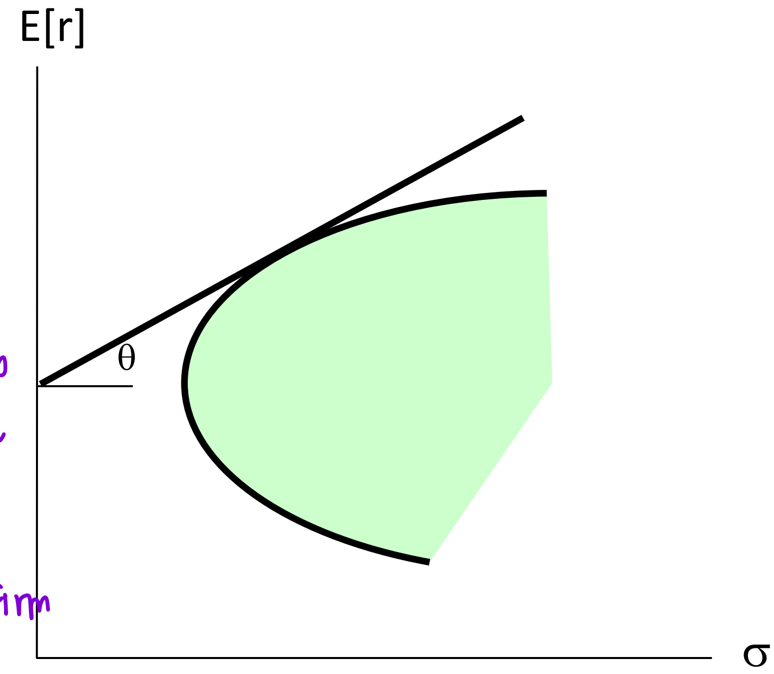


The CML with High Aggregate Risk Aversion

when there's crisis, to take risk, investors demand high expected return

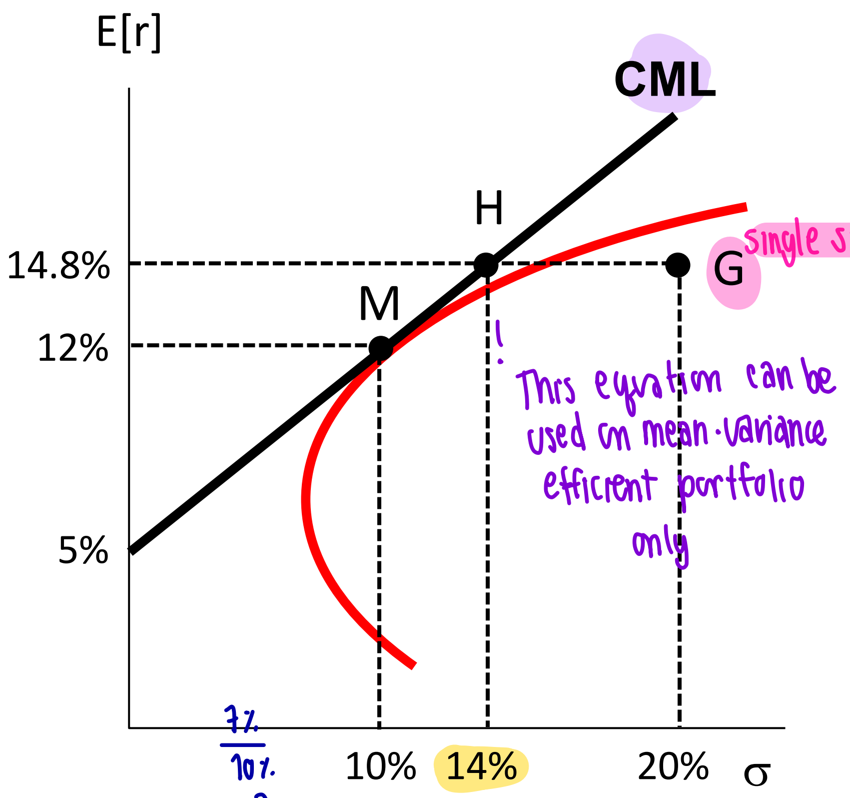
stock price will drop as mkt becomes more risk averse

affect price and cost of capital of firm



The CML with Low Aggregate Risk Aversion

The CML and Equilibrium Expected Return



$E[r_p] = 0.05 + 0.7\,\sigma_p$

Assume r_F = 5%, $E[RP_M]$ = 7% and σ_M = 10%. The CML equation is;

$$\mathbf{E[r_P] = 0.05 + (0.07/0.10)\,\sigma_P}$$

single stock → if we use this equation, it will give too high return

H is an efficient portfolio with $E[r_H]$ = 14.8% pa. and σ_H = 14%, while G is an inefficient asset with $E[r_G]$ = 14.8% pa. and σ_G = 20%.

mean-variance

Given the value of σ_H, $E[r_H]$ can be obtained from the CML equation.

$E[r_H] = 0.05 + (0.07/0.10)(0.14) = 0.148$

However, applying the CML on σ_G will overestimate $E[r_G]$ to 0.19.

2.2 Derivation of the CAPM (Simplified)

- Start with the Var-Cov Matrix and portfolio weights.

Portfolio Weights	w_1	w_2	...	w_{GE}	...	w_n
w_1	$Cov(R_1, R_1)$	$Cov(R_1, R_2)$	...	$Cov(R_1, R_{GE})$	...	$Cov(R_1, R_n)$
w_2	$Cov(R_2, R_1)$	$Cov(R_2, R_2)$	...	$Cov(R_2, R_{GE})$	...	$Cov(R_2, R_n)$
⋮	⋮	⋮		⋮		⋮
w_{GE}	$Cov(R_{GE}, R_1)$	$Cov(R_{GE}, R_2)$	...	$Cov(R_{GE}, R_{GE})$	...	$Cov(R_{GE}, R_n)$
⋮	⋮	⋮		⋮		⋮
w_n	$Cov(R_n, R_1)$	$Cov(R_n, R_2)$	...	$Cov(R_n, R_{GE})$	...	$Cov(R_n, R_n)$

if it is ⊕ sign

- Note: 1. $Cov[aX, bY] = ab \cdot Cov[X, Y]$

  2. $Cov[X, \Sigma_i a_i Y_i] = \Sigma_i Cov[X, a_i Y_i] = \Sigma_i a_i Cov[X, Y_i]$

$w_i + w_j$ || $i, j \to N$ stocks

2.2 Derivation of the CAPM (Simplified)

- The variance of the market portfolio is

$$\sigma_M^2 = \sum_i \sum_j w_i w_j Cov[r_i, r_j] = \sum_i w_i \{\sum_j w_j Cov[r_i, r_j]\}$$

rule 1

$$= \sum_i w_i \{w_1 Cov[r_i, r_1] + w_2 Cov[r_i, r_2] + \ldots + w_N Cov[r_i, r_N]\}$$

rule 2

$$= \sum_i w_i \{Cov[r_i, w_1 r_1] + Cov[r_i, w_2 r_2] + \ldots + Cov[r_i, w_N r_N]\}$$

$$= \sum_i w_i \, Cov[r_i, w_1 r_1 + w_2 r_2 + \ldots + w_N r_N]$$

$$= \sum_i w_i \, Cov[r_i, r_M]$$

$w_i \cdot \sigma_{i,m}^2 \to$ marginal contribution of stock i to the riskiness of the market

$$= w_1 Cov[r_1, r_M] + w_2 Cov[r_2, r_M] + \ldots + w_N Cov[r_N, r_M]$$

- **Conclusion:** Security *i*'s contribution to the risk of the market portfolio is measured by $w_i \times Cov[r_i, r_M]$.

marginal contribution of asset i to the return of market

$$r_M - r_f = w_1 r_1 + w_2 r_2 + \ldots + w_N r_N$$

marginal contribution of asset i to the risk premium of market

$w_i (r_i - r_f)$

2.2 Derivation of the CAPM (Simplified)

- The contribution of asset i to the risk premium of the market portfolio is $w_i(E[r_i]-r_F)$. Thus, the marginal reward-to-risk ratio for investment in asset i can be expressed as:

marginal contribution of asset i to the sharpe ratio of market

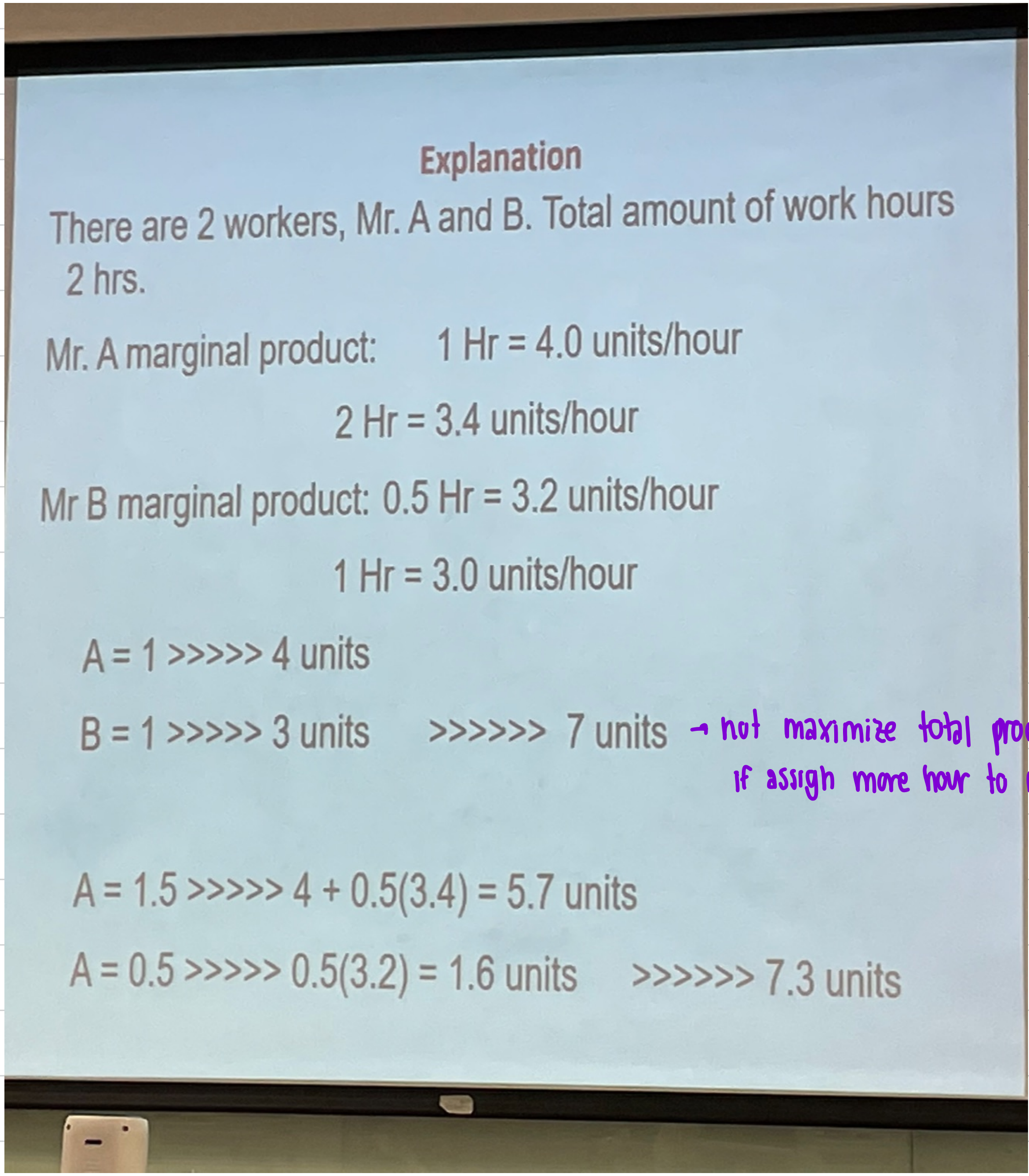
$$\frac{\cancel{w_i}(E[r_i]-r_F)}{\cancel{w_i}\sigma_{i,M}} = \frac{E[r_i]-r_F}{\sigma_{i,M}}$$

In equilibrium, these two should be equal

- The market portfolio is the tangent (MV efficient) portfolio. The reward-to-risk ratio for investment in the market portfolio can be expressed as

mkt sharpe ratio

$$\frac{E[r_M]-r_F}{\sigma_M^2}$$



→ not maximize total product yet
if assign more hour to A, need to decrease hour to B

$\underbrace{\text{marginal product of A = B then we maximize total product}}$

⇓

sharpe ratios of each stock are equal
then we can get optimized portfolio

2.2 Derivation of the CAPM (Simplified)

- In equilibrium, the return per unit of risk must be equal across securities and portfolios. In equilibrium, sharp ratios of each stock is equal

$$\frac{E[r_i] - r_F}{\sigma_{i,M}} = \frac{E[r_M] - r_F}{\sigma_M^2}$$

- To determine the risk premium of asset i, we rearrange the equation.

$$E[r_i] - r_F = \frac{\sigma_{i,M}}{\sigma_M^2}(E[r_M] - r_F)$$

- Define β_i as $\sigma_{i,M}/\sigma_M{}^2$. can be applied to any stock

do not assume that asset i is efficient

CAPM

$$E[r_i] = r_F + \beta_i(E[r_M] - r_F)$$

CML → only MV efficient portfolio

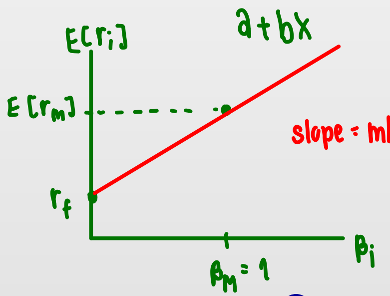
2.2 Derivation of the CAPM (Simplified)

$E[r] = r_f + \underline{\text{risk premium}}$ } compensation for time to defer consumption

- The CAPM equation;

e.g. PTT vs. AIS

β_i is the only data that explain cross-sectional expected return difference



slope = mkt risk premium

$$E[r_i] = r_F + (E[r_M]-r_F)\times\beta_i$$

Compensation for time value of money

The market price per 1 unit of risk

The risk level of security i

Compensation for systematic risk

not total risk

- Security's beta reflects the relation between returns on the asset and the market.

2.2 Derivation of the CAPM (Simplified)

- From

$$\sigma_M^2 = w_1\sigma_{1,M} + w_2\,\sigma_{2,M} + \ldots + w_N\sigma_{N,M}$$

- Divide by σ_M^2 both sides

$$1 = w_1(\sigma_{1,M}/\sigma_M^2) + w_2(\sigma_{2,M}/\sigma_M^2) + \ldots + w_N(\sigma_{N,M}/\sigma_M^2)$$

$$= w_1\beta_1 + w_2\beta_2 + \ldots + w_N\beta_N$$

- The market portfolio has the beta of 1.

$$\beta_i = \frac{cov(r_i, r_m)}{var(r_m)}$$

r_m

$\to var[r_m]$

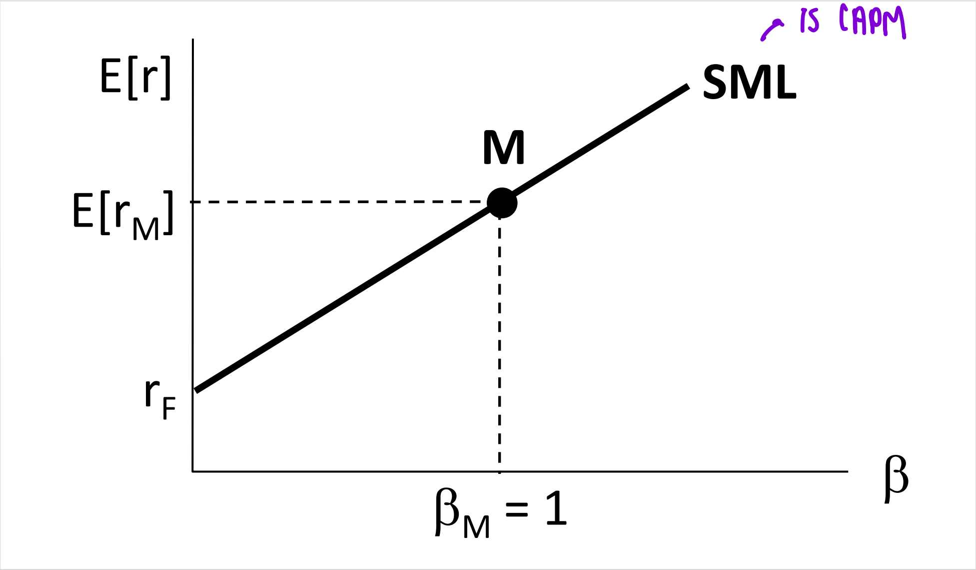
$= 1$

2.3 The CAPM and Equilibrium Expected Returns

- The graphical presentation of the CAPM is called the Security Market Line (SML).

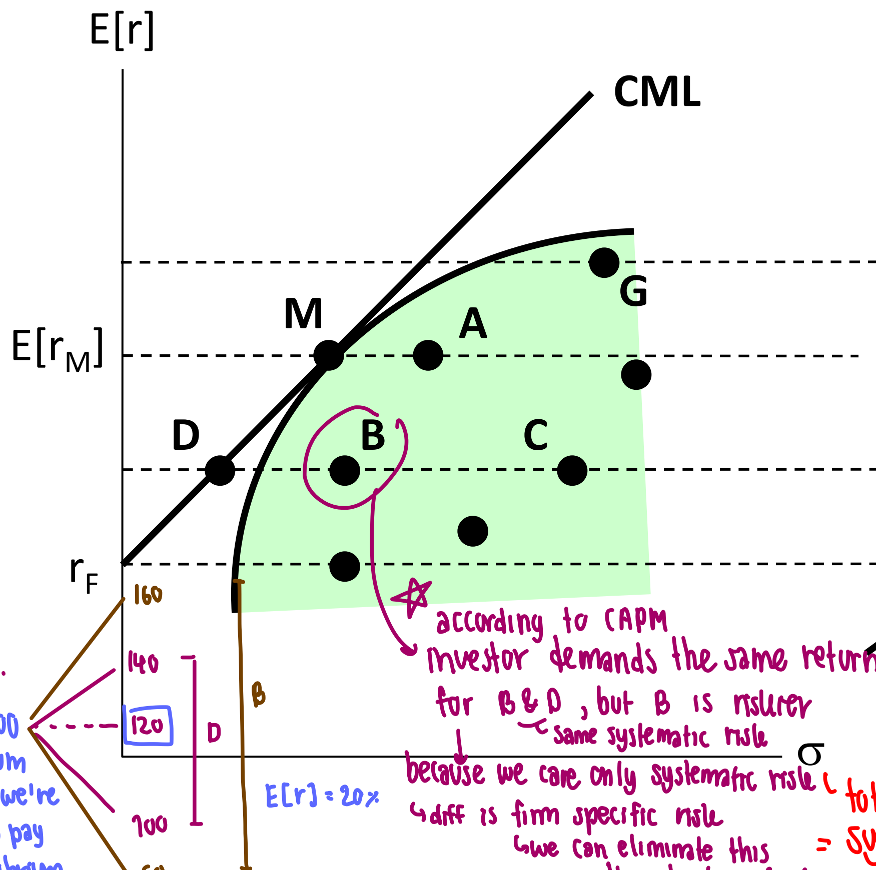
  The interception of the SLM = r_F

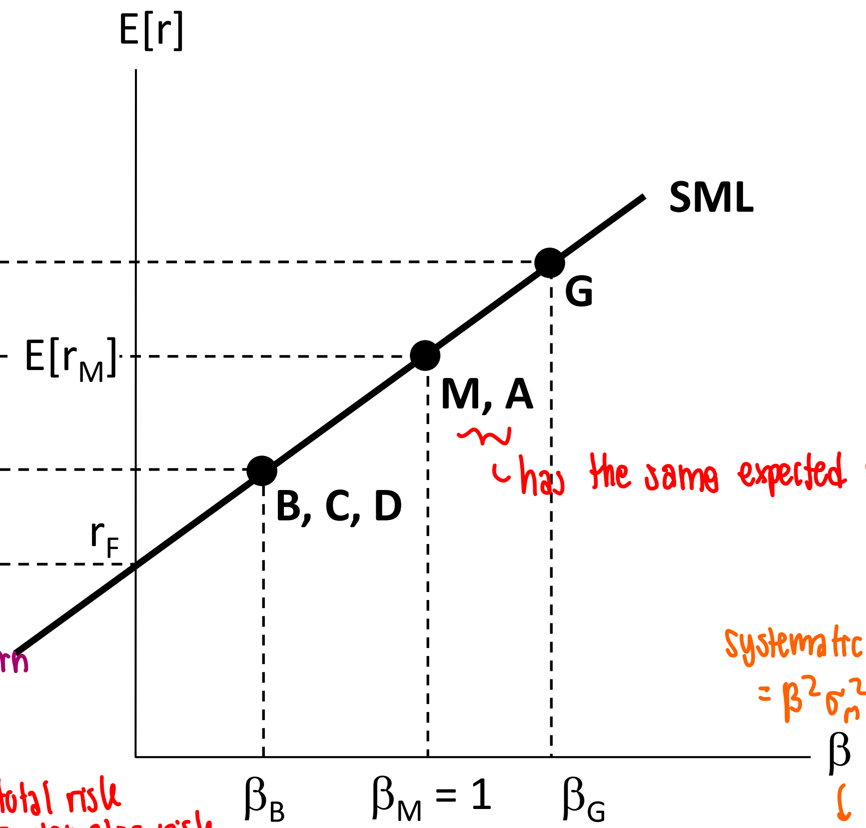
  The slope of the SML = $E[r_M]-r_F$ = $E[R_M]$



- In equilibrium, all assets and portfolios must lie on the SML.

CML vs. SML





e.g.
$100 maximum price we're will to pay = equilibrium price

160
140
$\boxed{120}$
100
80
D
B
E[r] = 20%

according to CAPM investor demands the same return for B & D, but B is riskier
↳ same systematic risk
because we care only systematic risk ↳ total risk = systematic risk
↳ diff is firm specific risk
↳ we can eliminate this through diversification
no cost
no transaction
↳ don't charge premium in model
must give fair price

↳ has the same expected return

systematic risk $= \beta^2 \sigma_M^2$

exposure to σ_M^2 market risk

The Capital Market Line

Only efficient assets and portfolios lie on the CML

The Security Market Line

All assets and portfolios lie on the SML

$E[P_{B,1}] = 109$, $P_{B,0} = 100$ } $E[r_B] = 9\%$

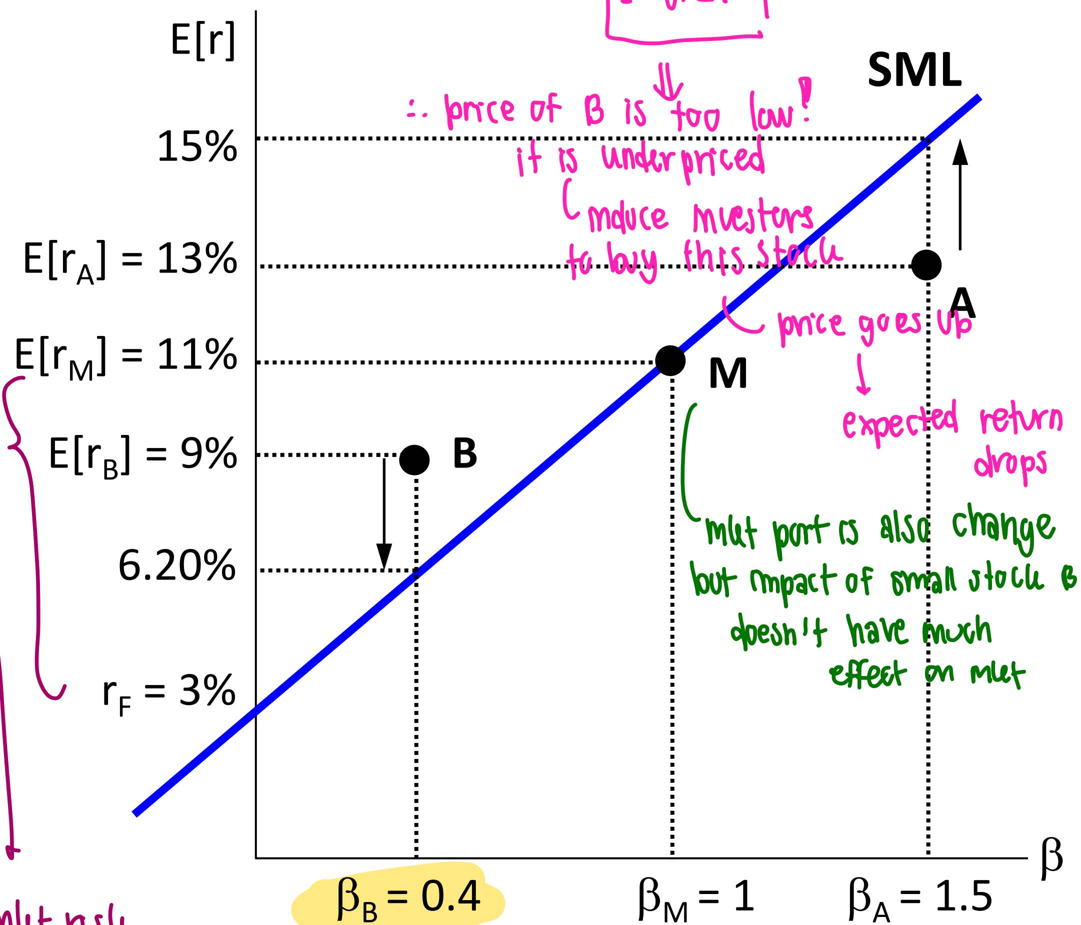
$E[P_{A,1}] = 113$, $P_{A,0} = 100$ } $E[r_A] = 13\%$ return based on current price → the price is too high → investors sell this stock → price ↓, $E[r_A]$ ↑ → equilibrium

Adjustments of E[r] to Equilibrium E[r]

from CAPM $= 3\% + 0.4 \cdot 8\%$ $\boxed{= 6.2\%}$

∴ price of B is too low! if is underpriced (induce investors to buy this stock

Recall; $E[r] = (E[P_1+D_1]/P_0) - 1$



mkt risk premium = 8%

Security A is overpriced. This will induce investors to sell A causing P_0 of A to drop. As P_0 drops, $E[r_A]$ will increase. Once $E[r_A]$ is on the SML, there will be no excess supply, and hence, no pressure on price.

Security B is under-priced. This will induce investors to buy B causing $E[r_B]$ to drop. Once $E[r_B]$ is on the SML, there will be no excess demand.

2.3 The CAPM and Equilibrium Expected Returns

total risk

- EX: Two stocks, i and j, have $\sigma_i = 20\%$ and $\sigma_j = 15\%$ pa, while $\rho_{i,M} = 0.6$ and $\rho_{j,M} = 0.8$. Furthermore, $E[R_M] = 8\%$, $\sigma_M = 10\%$ and $r_F = 3\%$ pa. Calculate total risk, systematic risk and E[r] on i and j.
  - $\sigma_{i,M} = 120$ and $\sigma_{j,M} = 120$. Hence, $\beta_i = \beta_j = 1.2$
  - $E[r_i] = E[r_j] = 3\% + 1.2(8\%) = 12.6\%$
  - Note: Higher E[r] implied lower offered price.
  - Why should investors demand the same level of E[r] on i and j when σ_i is higher than σ_j?
  - What would happen to the portfolio of an investor who forms E[r] on the total risk rather than the systematic risk?

$$\beta_i = \frac{\sigma_{i,M}}{\sigma_M^2}$$

$$= \frac{\rho_{i,M} \cdot \sigma_i \cdot \sigma_M}{\sigma_M^2}$$

$$= \frac{(0.6 \times 0.2 \times (0.10))}{(0.10)^2}$$

$$= 1.20$$

⇓

Investors demand same return for the same β_i

↳ we ignore unsystematic risk, concern only β_i (systemetric risk)

If we take total risk ↳ can be diversified

↳ price we willing to buy is too low

⇓

port is not diversified anymore

2.4 Properties of the CAPM

- The risk premium on an individual asset is the product of the market risk premium and the beta coefficient of the asset.

exposure to mkt risk

$$E[r_i]-r_F = (E[r_M]-r_F)\times\beta_i$$

- Beta measures the extent to which returns on security and market move together, i.e., $\beta_i = (\sigma_{i,M}/\sigma_M^2)$.
- Beta can be viewed as a measure of systematic risk of an asset. The CAPM relation shows that investors only demand compensation for systematic risk of an asset.

2.4 Properties of the CAPM

- Portfolio beta (β_P) is a value-weighted average of betas of all securities contained in the portfolio.

$$\beta_P = \Sigma_i w_i \cdot \beta_i$$

where $w_i = V_i / \Sigma_i V_i$

V_i = the value of security i in the portfolio

$\Sigma_i V_i$ = the total value of the portfolio

2.5 Applications of the CAPM

- Two main applications of the CAPM
  - Corporate finance: Project or asset valuation
  - Investment management industry: Measuring fund performance

CFO has to make financial decision : CFO → Investment decision / financial decision

LIT decision making process

calculate NPV → need discount rate (cost of capital) from CAPM

tells whether to invest or not

$PV = \frac{FV}{(1+r)^t}$

use CAPM to find r

Fisher of separation → assume no risk → use r_f as cost of capital

↳ risk → adjusted risk premium to r_f

affect on innovation, GDP

Project Evaluation

- $E[r_i]$ from the CAPM represents the required rate of return by shareholders. Firm could use this $E[r_i]$ as the cost of equity.
- The NPV of any project i can be estimated from the discount cash flow model as;

$$NPV_0 = \sum_{t=0}^{T} \frac{E[FCF_t]}{(1 + k_E)^t}$$

or risk adjusted return / cost of capital

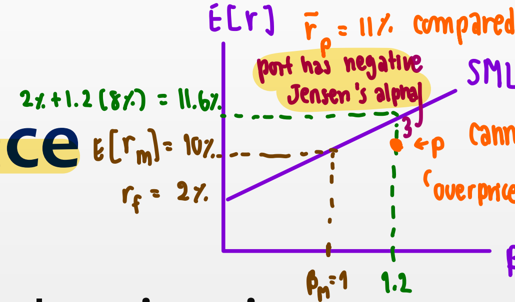
(cost of capital

where $E[FCF_t]$ = the expected free cash flows from the project to shareholders at time t.

$k_E = r_F + (E[r_M] - r_F) \times \beta_i$ (This equation is called risk-adjusted cost of capital)

β_i = equity beta of project i

Fund Performance



compared with SET index (benchmark)
⇓
cannot compare directly
↓
because port is riskier than the market
↓
port actually underperform

- The SML provides a benchmark for evaluating investment performance of a fund manager.
- Without any stock selection skill, an average investor would expect security returns to fall on the SML. Such expected returns are considered fairly priced.
- Active portfolio managers with preferable skills must be able to identify securities that realize non-zero alphas and invest accordingly.
- We should expect the portfolio of a skillful fund manager to have positive alpha ($\alpha_P > 0$).

contains r_f → should not be counted

$$r_{p,t} = \alpha_p + \beta_p r_{Mt} + \varepsilon_{p,t}$$

Fund Performance

- To estimate realized alpha, we use the following time series regression.

$$r_{P,t} - r_{F,t} = \alpha_P + \beta_P \cdot (r_{M,t} - r_{F,t}) + \varepsilon_{P,t}$$

where $r_{P,t}$, $r_{F,t}$ and $r_{M,t}$ = return on the portfolio, risk-free asset and the market index at time t

β_P = portfolio beta

- Note $\alpha_P = \Sigma_i w_i \cdot \alpha_i$ and $\beta_P = \Sigma_i w_i \cdot \beta_i$
- The alpha of a passive portfolio is expected to be zero.

risk premium of market

risk premium of portfolio

→ run regression in excess return form

T	NAV	SET	$r_{p,t}$	$r_{M,t}$	$r_{f,t}$
0	x	x			
.	x	x			
.	x	x			
.					
.					
.					
.					
.					

today

Estimates of Individual Mutual Fund Alphas, 1972-1991

If includes expenses → most of active ports have negative α

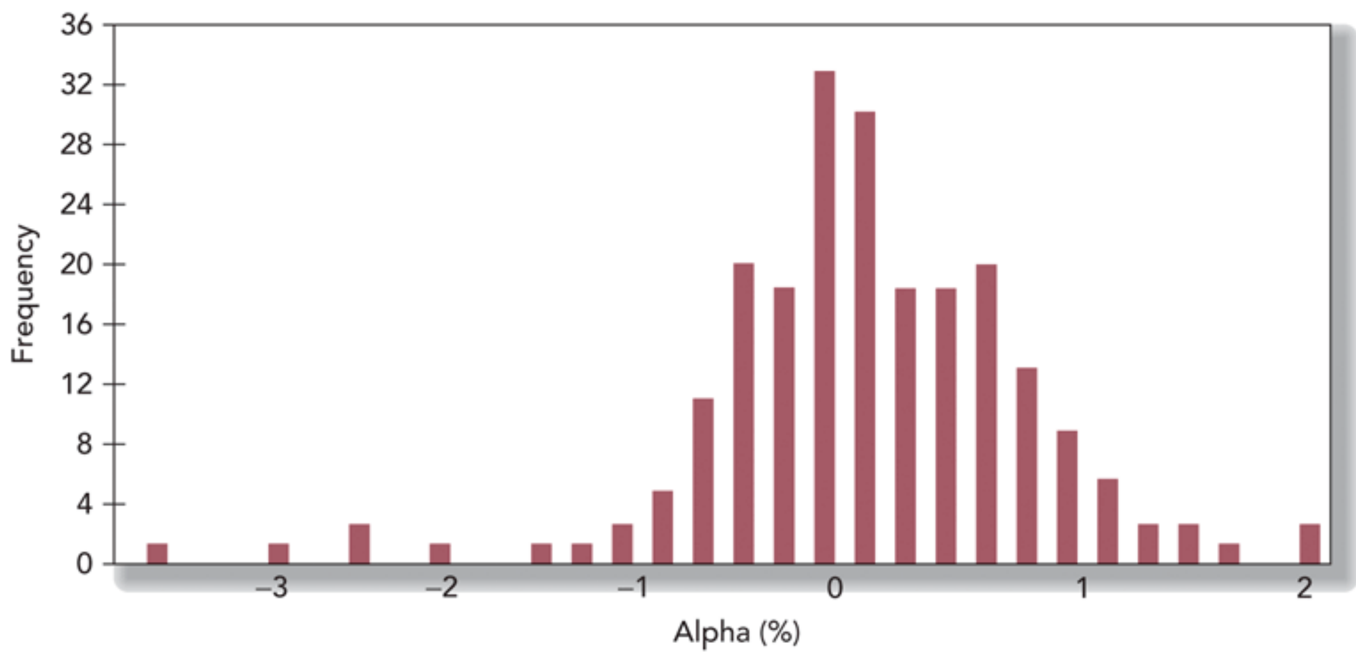


FIGURE 9.4 Estimates of individual mutual fund alphas, 1972–1991

This is a plot of the frequency distribution of estimated alphas for all-equity mutual funds with 10-year continuous records.

Source: Burton G. Malkiel, "Returns from Investing in Equity Mutual Funds 1971–1991," *Journal of Finance* 50 (June 1995), pp. 549–72. Reprinted by permission of the publisher, Blackwell Publishing, Inc.

next week

2.6 Extensions of the CAPM

- The CAPM relies on a set of restrictive assumptions.
- There have been attempts to generalize the model to accommodate more realistic assumptions, resulting in various extensions of the model.
- Although parts of the basic model may change in important ways, the fundamental distinction between systematic risk and diversifiable risk remains.

2.6 Extensions of the CAPM

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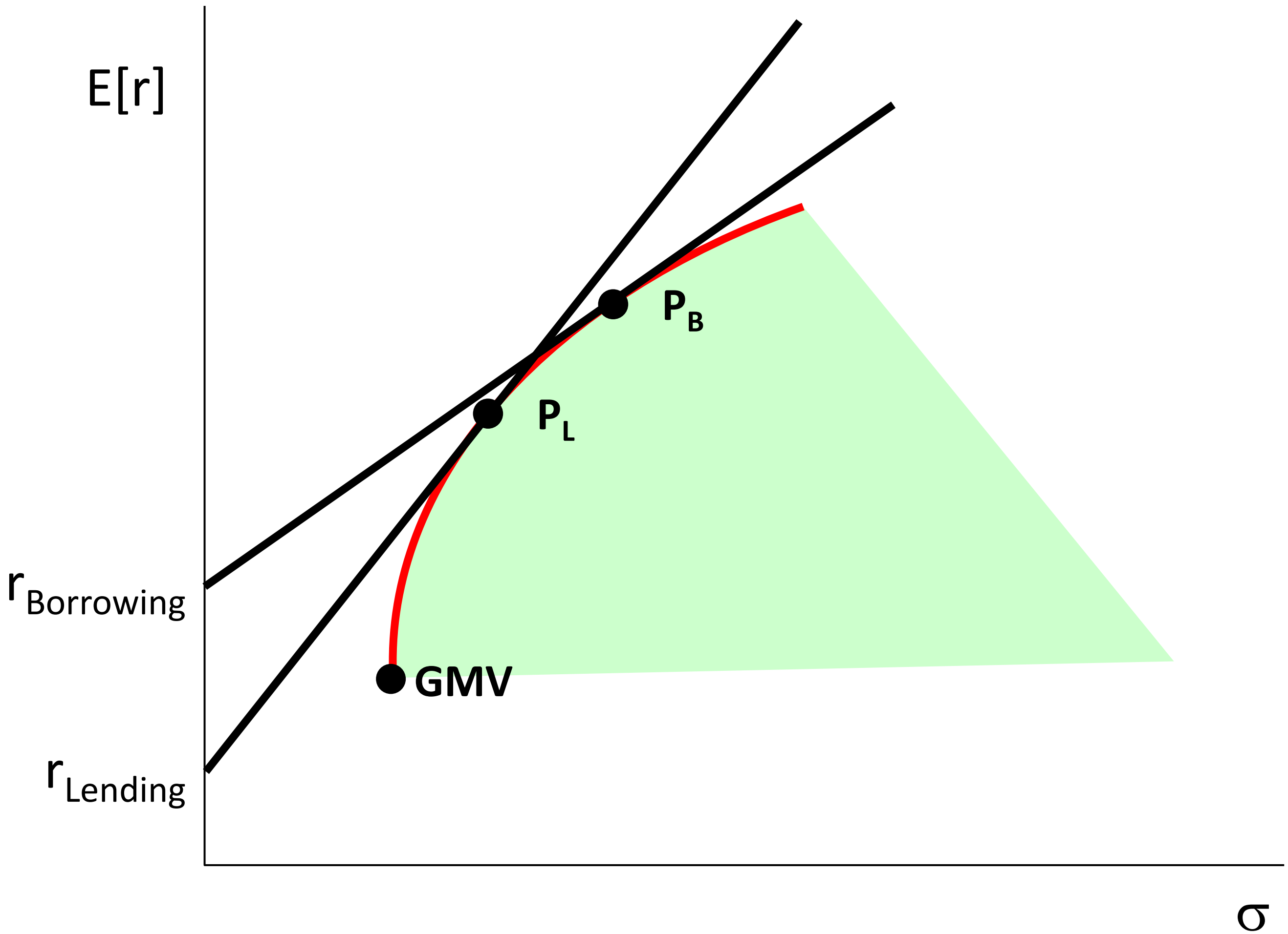
Black's Zero Beta Model

- If lending and borrowing interest rates are different, investors who are net lenders and net borrowers will have different tangent portfolios.
  - Although this will not change the efficient frontier of risky assets, the market portfolio will no longer be the optimal risky portfolio for all investors.
- Black use the following properties of the efficient frontier to derive the Zero Beta CAPM.
  - A combination of portfolios on the efficient frontier are MV efficient. Since the market portfolio is a combination of tangent portfolios of all investors, the market portfolio must be MV efficient.

Tangent Portfolios between Net Lenders and Borrowers



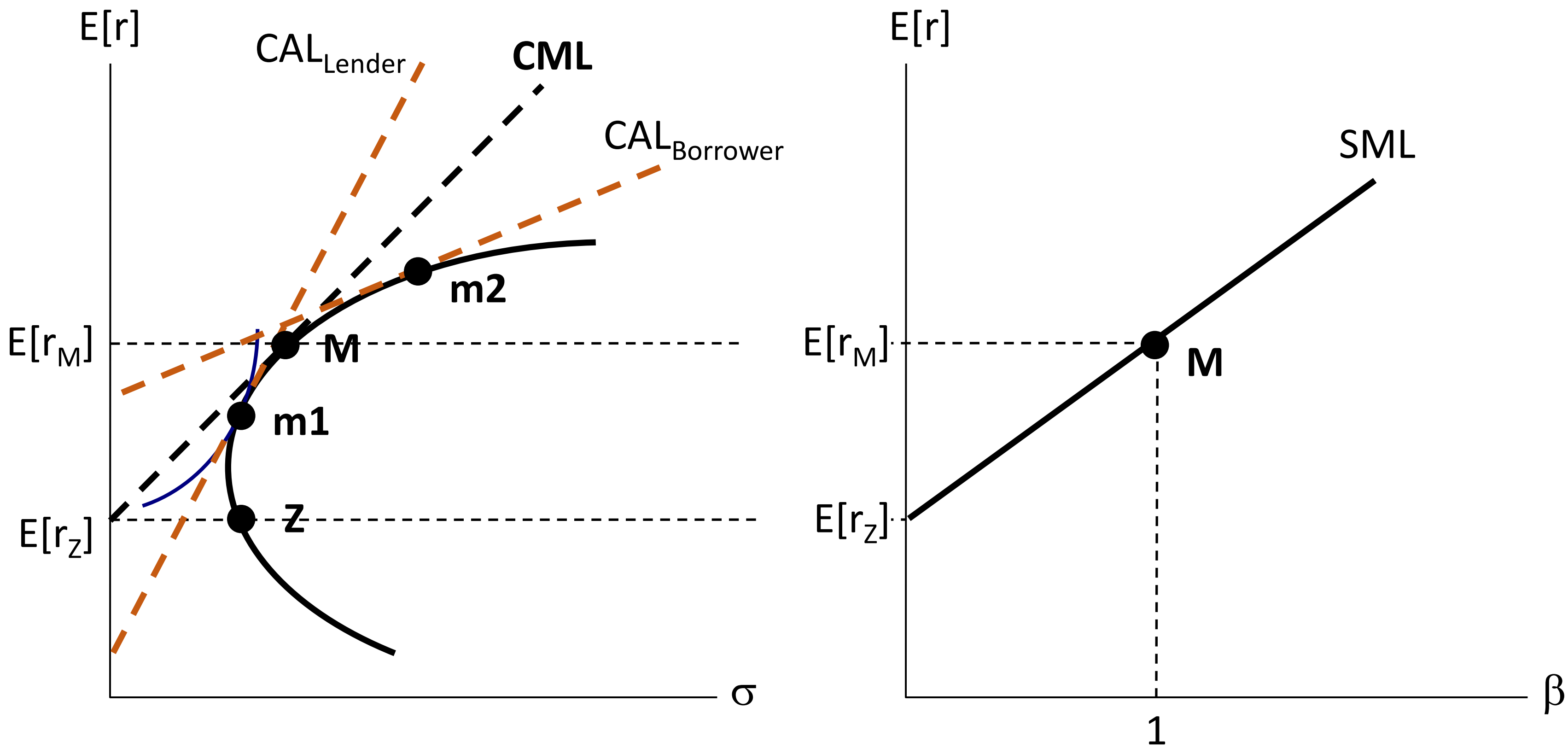
Black's Zero Beta Model

- All MV efficient portfolios have companion portfolios on the bottom half of the frontier that are uncorrelated. The companion portfolio is called the "zero beta portfolio"
- Returns on individual assets can be expressed as linear combinations of an MV efficient portfolio and its zero-beta companion.

$$E[r_i] = E\left[r_{Z(P)}\right] + \left(E[r_P] - E\left[r_{Z(P)}\right]\right) \times \frac{\sigma_{i,P}}{\sigma_P^2}$$

- The above equation must be true for the market portfolio as well. That is, $E[r_i] = E\left[r_{Z(M)}\right] + \left(E[r_M] - E\left[r_{Z(M)}\right]\right) \times \frac{\sigma_{i,M}}{\sigma_M^2}$
- Hence, the CAPM relation could be derived without risk-free borrowing and lending.

Black's Zero Beta Model: No Risk-Free Asset



$$\mathbf{E[r_i] = E[r_Z] + (E[r_M] - E[r_Z]) \times \beta_i}$$

where $\beta_i = \sigma_{i,M} / \sigma_M^2$

Black's Zero Beta Model

- Implications
  - The higher cost of borrowing may impede the ability of risk-tolerant investors to leverage up their portfolios.
  - These investors will tilt toward high-β assets instead causing prices of high-β stocks to increase and their E[r] to drop below E[r] from the CAPM.
  - Further, as $E[r_Z]$ is likely to be higher than r_F. The SML will be flatter in Zero Beta Model than the CAPM.

Effects of Non-tradable Assets

- An individual wealth may consist of ownership of a private business, human capital and portfolio of financial assets.
- A significant part of an individual wealth is not tradable.
- These considerations imply that investors may derive different optimal portfolios of tradable assets.
- For example, an owner of a family business may want to avoid investments in assets that are highly correlated with performance of his private business.
- In general, investors should avoid stock returns that are highly correlated with their personal income.

Effects of Non-tradable Assets

- Portfolio of traded assets that best hedge the risk of typical individual would enjoy elevated demand resulting in lower E[r] than what implied by the CAPM.
- When analyzing the value of a stock, investors will be concerned with
  - The risk of the stock relative to the market portfolio of traded assets, and
  - The hedging property of the stock against bad prospects of their future incomes (or total wealth).

Intertemporal CAPM (I-CAPM)

- By allowing for multiperiod investment horizon, additional investment risks start coming into consideration.
  - Investment opportunities could change when investors have to rollover their investment in the future.
  - Real return could change in the future (due to lower nominal return and/or higher inflation) causing actual purchasing power to be lower over time.
- Investors should care not only the risk and return of their wealth measured in $ term, but also the stream of consumption that wealth can buy for them.

Intertemporal CAPM (I-CAPM)

- To the extent that returns on some securities are correlated with changes in these “extra market risk”, such as inflation and investment opportunities, a portfolio can be formed to hedge these risks.
- The extra demand for assets that can be used to hedge extra market risks would drive up their prices and reduce their risk premium relative to the prediction of the CAPM.
- Multi-factor CAPM or Intertemporal CAPM (ICAPM) developed by Robert Merton tries to model multiple systematic risks into E[r]. The general form of ICAPM is;

$$E[r_i]-r_F = \beta_{i,M}(E[r_M]-r_F) + \beta_{i,1}(E[r_1]-r_F) + \ldots + \beta_{i,K}(E[r_K]-r_F)$$

Consumption CAPM (C-CAPM)

- To apply ICAPM, one needs to identify sources of intertemporal risks in asset returns and specify their relative importance to investors. The theory itself gives little information on these factors.
- One approach is to collapse many risk factors into one, that is consumption risk.

$$E[r_i]-r_F = \beta_{i,C}(E[r_C]-r_F)$$

- The RP of asset i is a function of its consumption risk ($\beta_{i,C}$), where C is the consumption-tracking portfolio (i.e., the portfolio with the highest correlation to consumption growth).

Consumption CAPM (C-CAPM)

- CCAPM views risk from the perspective of consumption risk.
- Investors will value additional income more highly during difficult economic time (when resources are scarce) than when during affluent time (then consumption is abundant)
- Assets will be viewed as riskier in terms of consumption if it has positive correlation with consumption growth.
- The CCAPM is similar to the original CAPM. It is a single factor linear model.

3. EMPIRICAL TESTS OF THE CAPM

3.1 Introduction

should test hypothesis of theory by using actual data

- The usefulness of a positive economic theory, like the CAPM, should be judged on the ground that the theory can explain and predict actual economic phenomenon (positive test).
- To evaluate finance theories, we perform empirical studies. That is, we test whether the prediction of the theory is supported by the actual (or observed) data.

CAPM predicts $E[r_i]$

in equilibrium, β_i drives $E[r_i]$

investors are risk aversed

risk premium should > 0

need compensation

$E[r_i] = f(\beta_i)$

3.2 The Ex-Post Form of the CAPM

expected return

- The CAPM is an expectation model.

we cannot observe expected return

- To justify the test of the ex-ante (i.e., before the event) model using historical or observed data, we must make one of these 2 assumptions.

— we can use actual return to test

in average, what investors expects is true (unbiased) (should be long enough

[1] Investors have rational expectation which implies that realized returns on securities are on average reflect what investors expected to happen (i.e., unbiased expectation).

or

[2] Security returns follow the market model.

we can transform from
ex-ante to ex-post return

3.2 The Ex-Post Form of the CAPM

the market model

- Based on [2]

replace CAPM $r_f + \beta_i (E[r_m] - r_f)$

$$r_{i,t} = \boxed{E[r_{i,t}]} + \beta_i \cdot \delta_{M,t} + \varepsilon_{i,t}$$

← market model
empirical model based on actual data

surprise of return from mkt portfolio

where $\delta_{M,t} = r_{M,t} - E[r_{M,t}]$

$E[\delta_{M,t}] = 0$

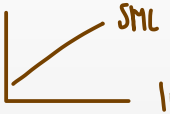
- Substitute the CAPM for $E[r_{i,t}]$ on the RHS and rearrange

ex-post version of CAPM →

$$(r_{i,t} - r_{F,t}) = (r_{M,t} - r_{F,t}) \times \beta_i + \varepsilon_{i,t}$$

→ can test this because it is in actual

- This is the ex-post form of the CAPM.
- It represents the testable form of the CAPM.



CAPM : $E[r_i] = f(\beta_i)$ +

$\bar{r}_i = a + b\beta_i + e$

3.2 The Ex-Post Form of the CAPM

should be $\overline{r_i - r_f}$ to match

$H_0 : b = \overline{r_M - r_f}$ risk premium

running regression across i
Cross-sectional regression
only 1 period

test CAPM from return from single stock
contain alot of error
⇓
test CAPM from portfolio to minimize error

- Tests of the CAPM commonly follow two steps
- Rank securities in the sample according to their betas (β_i's) and form beta-sorted portfolios.

N = 500 stocks , n_i = 10 in each port ⇒ need to run 50 portfolios

one for each port

50 $\overline{r_p - r_f}$ 50 b_p's

- First Pass (Time Series Regression): SCL

security characteristic line

$$(r_{P,t} - r_{F,t}) = a_P + b_P \cdot (r_{M,t} - r_{F,t}) + e_{P,t} \qquad [1]$$

- Second Pass (Cross-sectional Regression): SML

SML

$$\overline{(r_P - r_F)} = \gamma_0 + \gamma_1 \cdot b_P + u_P \qquad [2]$$

risk premium

compare to CAPM: $E[r_p] - r_f = \beta_p(E[r_m] - r_f)$

should be equal to average mkt risk premium

$H_0 : \gamma_1 = \text{average}(r_{M,t} - r_{f,t})$

mkt model is time-series model

50 portfolios
each port contain 10 stock
N = 500

rank first
then form
portfolio from beta

$r_A \Rightarrow \beta_A$

$r_B \Rightarrow \beta_B$

β_{lowest} } β_{p_1}

β

⋮ } β_{p_2}

$\beta_{highest}$

3.2 The Ex-Post Form of the CAPM

- Predictions made by the CAPM to be tested in Eq [2]
  - to be strict — compare two factors → use F-test
  - $\gamma_0 = 0$ and $\gamma_1 = \text{average}(r_{M,t} - r_{F,t})$
  - Beta should be the only factor explaining variation in the rate of return across assets. Other terms such as σ_e^2, dividend yield, P/E ratios or firm size should have no explanatory power. if CAPM is correct, only systematic risk matters

$$\overline{(r_P - r_F)} = \gamma_0 + \gamma_1 b_P + \gamma_2 \sigma_{eP}^2 + \gamma_3 DIV_P + \gamma_4 SIZE_P + u_P$$

= 0 then CAPM is correct

  - The relationship should be linear in beta.

but small firm is more riskier than big firm

$$\overline{(r_P - r_F)} = \gamma_0 + \gamma_1 b_P + \gamma_2 b_P^2 + u_P$$

* According to CAPM, size of firm doesn't matter

3.2 The Ex-Post Form of the CAPM

- To run equation [1], proxies for the risk-free rate and the market return must be obtained.
  - Generally, return or yield on government security such as T-bill or T-bond are used as a proxy for r_F.
    ↳ use YTM to represent r_F
  - A market-wide security index is used to proxy the market portfolio.
    ↳ represent $E[r_m]$
    Dow Jones is price-weighted
    - Both value-weighted and equally-weighted indexes have been used in the literature.

because based of CAPM

mut port : each of them has valued-weight

3.3 Early Results

- The empirical studies prior to Fama and French (1992) tend to agree on the following conclusions.

doesn't support CAPM

*
- γ_0 is significant different from zero and γ_1 is less than RP_M

support CAPM

- The simple linear model fits the data best (the coefficient on is insignificant $\beta_P{}^2$).

support CAPM

- β_P dominates σ^2_{eP} as a measure of risk (E[r]'s are not affected by nonsystematic risk).

Empirical SML

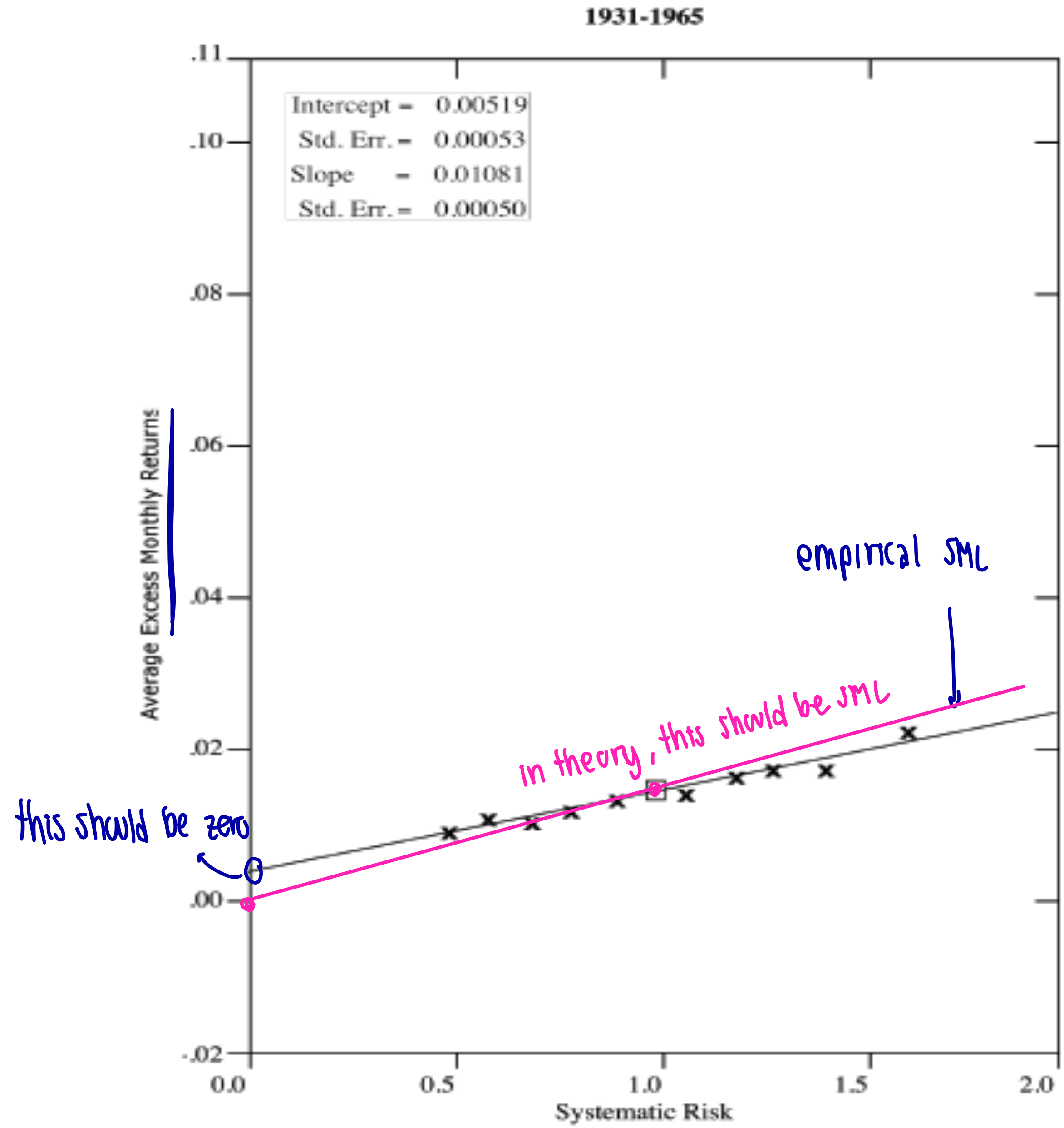
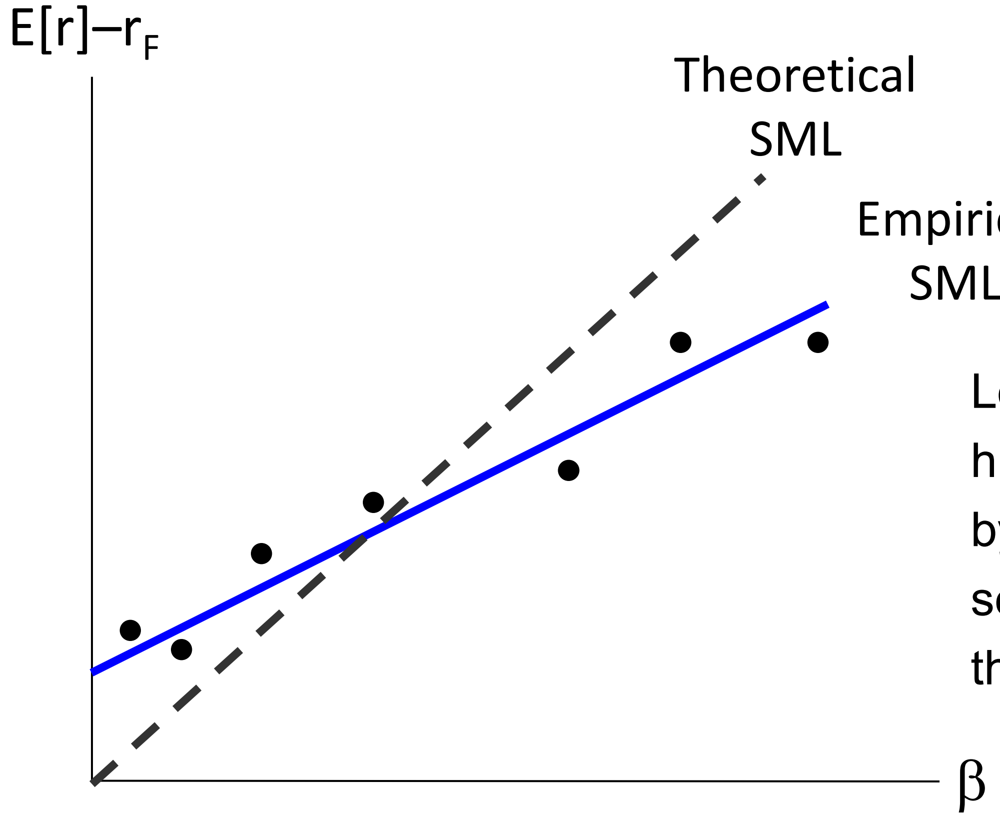


Figure 7 Average monthly returns versus systematic risk for the 35-year period 1931-65 for the ten portfolios and the market portfolio.

Source: Black, Jensen and Scholes (1972)

Empirical vs. Theoretical SML



Low-beta securities earn higher returns than predicted by the CAPM. High-beta securities earn lower returns than predicted by the CAPM.

3.3 Early Results

- Toward the end of 1970s, there were more evidence that factors other than β_P are successful in explaining portfolio returns not captured by beta.
  - Basu (1977): Low P/E portfolios earn higher returns than predicted by the CAPM.
  - Banz (1981): Smaller firms tend to have higher returns than bigger firms.

size

outperform

low mkt cap

high mkt cap

low $\frac{P}{E}$ ⇒ value stock ⇒ tend to outperform growth stock in average

⇓

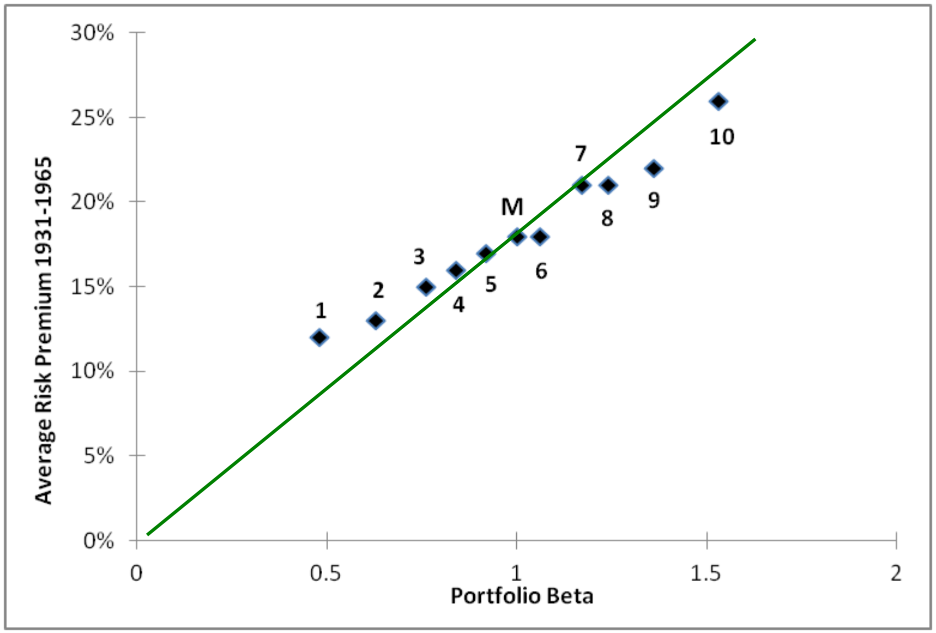
even hold same beta mkt constant

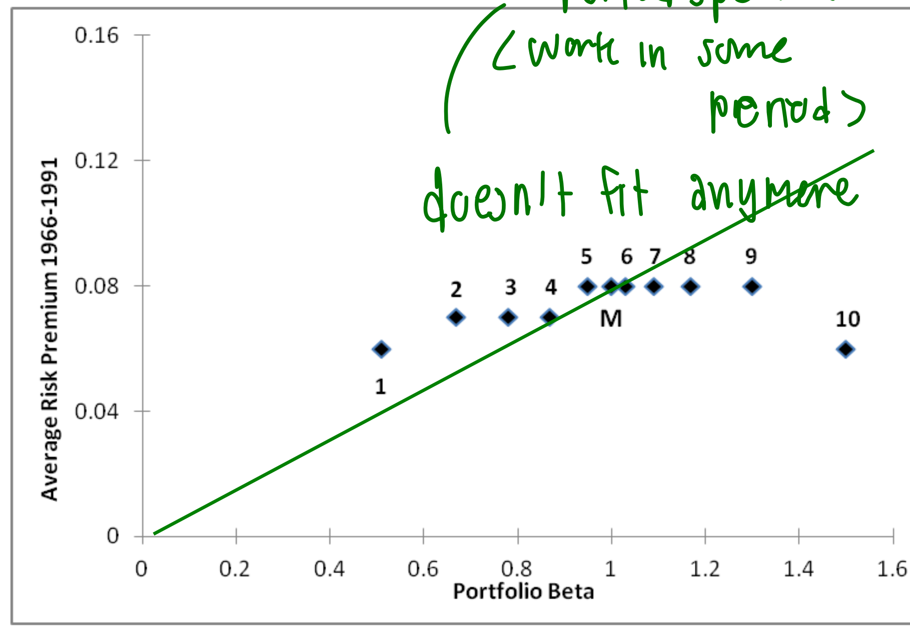
high $\frac{P}{E}$ ⇒ growth stock

expected earning will grow

The CAPM doe not seem to work well during 1960-90.

CAPM result can be period specific
(work in some period)





doesn't fit anymore

Source: Fisher Black (1993), “Beta and Return”, Journal of Portfolio Management.

not only CAPM can reflect return

3.4 Roll's Critique

- Roll (1977) argues that the CAPM is not testable because

  correct way to test

  - The only legitimate test of the CAPM is whether the market portfolio is MV efficient.

    reject → just proxy isn't MV efficient

    ↳ should contain all risky assets, but we only use proxy to test CAPM.

  - Since the market portfolio consists of all assets (tradable and non-tradable), the market portfolio is not observable.

    most assets cannot be trade

    cannot do this

    ∴ Thus, CAPM cannot be tested / verified

- To understand Roll's argument, recall the Zero Beta Model. The expected return on any asset can be written as.

$$E[r_i] = E[r_{Z(M)}] + \beta_i \cdot (E[r_M] - E[r_{Z(M)}])$$

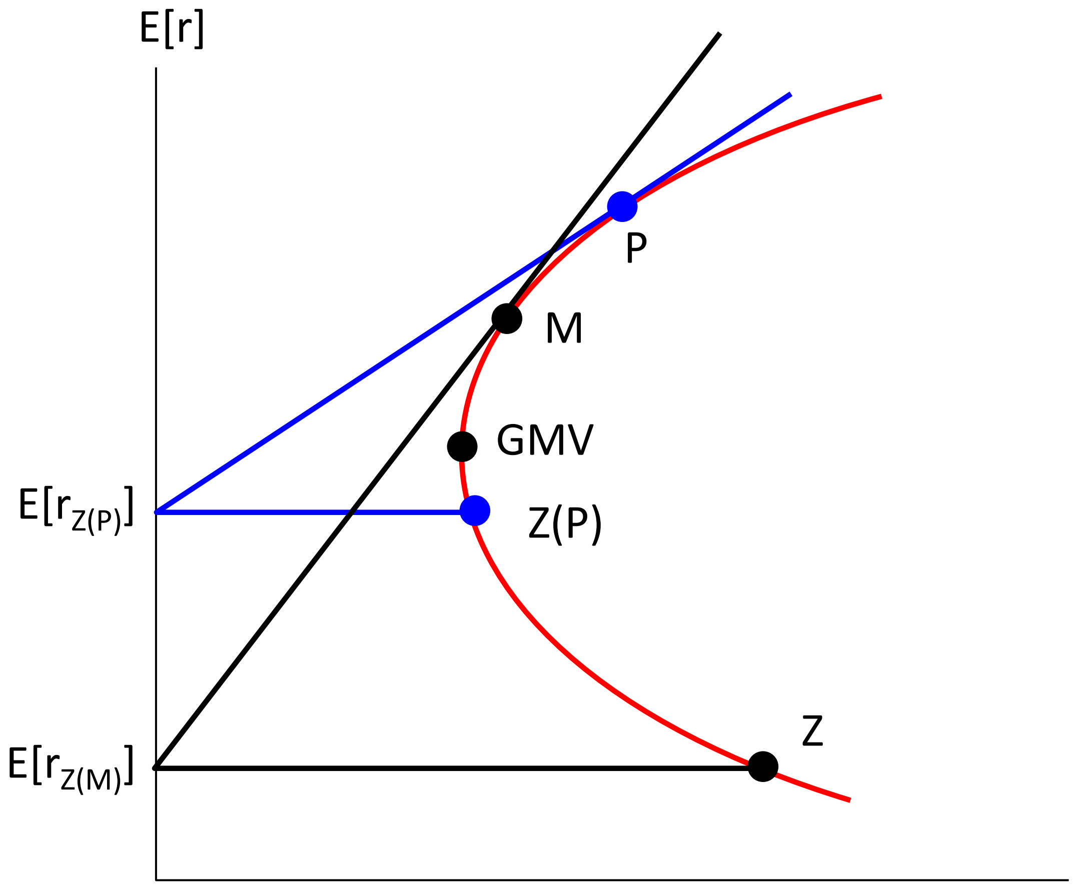
- In fact, any portfolio on the efficient frontier can be chosen such that

$$E[r_i] = E[r_{Z(P)}] + \beta_i \cdot (E[r_M] - E[r_{Z(P)}])$$

3.4 Roll's Critique

- Because the E[r] on any asset can be written as a linear function of its beta measured relative to any efficient index, it is not necessary to know the market portfolio.
- Empirical findings that SML holds only imply that the market proxy used is MV efficient. Empirical findings that SML does not hold only imply that the market proxy used is MV inefficient. They neither accept nor reject the CAPM.

Properties of the Efficient Frontier



Capm : $E[r_i] = r_f$ + risk premium

exposure to mkt risk

$\beta_i \times \underbrace{(E[r_M] - r_f)}$ → mkt risk premium

"

1.2 times of mkt

Roll's → ① CAPM is not testible

↳ mkt portfolio is mean-variance efficient ?

cannot be observed

② size and BVE / MVE can explain cross-sectional variance better than β

book - to - mkt ratio

BTM

↳ high → value stock

↳ low → growth

expect HML to be positive

because value stock is riskier than growth stock

↓

in long run, value stock will give higher return

* Most value firm will experiance drop in BV

↓

higher risk

3.5 Fama and French Model

- Fama and French (1992) published a landmark study on the cross-sectional relation between return and risk.
  - They run a two-pass sort to separate the effect of β from size. The result is shown in the following table.
  - Reading across a row (holding size constant), there seems to be very weak relation between β and return.
  - They then run multiple regression with security returns as the dependent variable. They conclude that size can explain variation in security returns better than beta.

Two-Pass Sort by Size and Beta Average Monthly Return over 1963–1990

sort them further by mkt cap

rank all stocks from low to high beta ; suppose 10 ports

	Low Beta	3rd Decile	Ave of 5th & 6th	8th Decile	High Beta
1 Small Equity	1.71	1.79	1.50	1.63	1.42
3rd Decile	1.12	1.17	1.19	1.36	0.76
Ave. of 5th & 6th	1.21	1.33	1.25	1.16	1.05
8th Decile	1.09	1.37	1.13	1.02	0.94
10 Large Equity	1.01	1.10	0.91	0.71	0.56

1

10

50 stocks

5 stocks

Beta increases

return should keep increasing if CAPM corrects

highest in beta with smallest size

size can reflect return more

has lowest beta and smallest in size

▶ small firms make investors exposed to higher risk than big firms

higher capital

- better support capital & govt
- can endure crisis

3.5 Fama and French Model

size related risk
BTM related risk

- Fama and French (1996) observe that average returns of small stocks and high B/M stocks are higher than predicted by the SML. Thus, size and B/M may be proxies for systematic risk not captured by market beta.

$\bar{r}_{small} - \bar{r}_{big\ firm}$

long position on   short position on

- FF (1996) proposed the three-factor model.

higher return than big firm
small minus big   regardless their beta
high minus low

$$E[r_i]-r_F = b_i\cdot(E[r_M]-r_F) + s_i\cdot E[SMB] + h_i\cdot E[HML]$$

$\bar{r}_{High\ BTM} - \bar{r}_{Low\ BTM}$

where E(SMB) is the expected return on a portfolio of long small stocks and short large stocks

E(HML) is the expected return on a portfolio of long high B/M stocks and short low B/M stocks

3.6 Conclusion

- The original form of the CAPM has failed in various empirical tests. Where do we go from here?
- Two main possible reasons for the failure of the CAPM.
  - , nothing wrong with theory
  - Bad Research Design: Validity of the market proxy, Assumptions implied in the market model (i.e., noises in observed returns, properties of the error term, time-varying beta, etc.) CAPM (Long-run return) ← but you use daily return to use CAPM → use mkt return as proxy but use only fraction of mkt index S&P500
    - These are research design issues. Not the problem with the theory itself.
    - The problems could be partially solved by improving market proxies and/or improving statistical methods.

3.6 Conclusion

rely on a set of very strong assumption

- The CAPM is Invalid: The CAPM relies on some very restrictive assumptions.
  - These are the problems that directly challenge the validity of the concept. These problems could be solved by either;
  - relax some assumptions of the CAPM and derive extension versions of the CAMP, or
  - derive a new asset pricing theory. → APT

3.6 Conclusion

- The CAPM suffers from many empirical shortcomings and the single-index CAPM is considered out-of-date.
- However, there is no broad consensus on a better replacement for the basic model. There is no consensus on which extension models work best or whether they work better than the basic model.

SMB & HML factors

CAPM model is not enough to capture systematic risk

- In academic, there is a trend toward adding extra-market risk factors into modelling expected return, i.e., multi-factor models.
- One popular model is Fama-French three factor model. As their risk factors are empirically driven, the economic logic behind these factors is subject to much debate.

3.6 Conclusion

- In the industry, the basic CAPM model is still an accepted norm.

  ↳ because it is easy to intepret

- Practitioners and academic found the decomposition of total risk into systematic and unsystematic risks and use of security beta to measure its systematic risk are attractive.
- The basic form of the CAPM is still used extensively in corporate finance to estimate the cost of equity capital.
- The concept of alpha is also extensively used in the investment management community to measure fund performance.

Exercises

1. Explain how you could evaluate and compare investment performance of two equity mutual funds.

2. According to the CAPM framework, which of the following characteristics of a portfolio reflects the degree of risk aversion of the investor? Explain.

   a) the percent of the investor's portfolio held in the market portfolio

   b) the of the investor's portfolio

   c) the expected return on the investor's portfolio, you can tell how risk averse he is.