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Finance Theories

Topic 5:

The Single Index Model

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Drawbacks of Markowitz' approach to the MVC

Agenda

1 — The Single Index Model (SIM)

2 — Estimation of the Market Model

3 — Portfolio Construction under the SIM

... — ...

1. THE SINGLE INDEX MODEL

1.1 Motivations

1.2 The Single Factor Model

1.3 The Single Index (Market) Model

1.1 Motivations

- To construct the efficient frontier of risky assets, Markowitz model requires the full variance-covariance matrix as an important input (Markowitz, H., 1952, "Portfolio Selection", Journal of Finance, Vol. 7(1), p77-91.).
- There are some drawbacks of this approach.
  - A large numbers of estimation must be performed.
    - Time consuming
    - Tendency to obtain inconsistent estimates of inputs

analyst must know all industries

need to assign 1 team to do all estimations to avoid inconsistent

decentralize

SIM assumes $cov(\varepsilon_i, \varepsilon_j) = 0$

& กระจายงานให้คนอื่น

come from covariance stock

$cov(A,B) > 0$

$cov(B,C) > 0$

use full historical data will not cause this but using forecasted data might cause

$cov(A,C) < 0$

1.1 Motivations

- To obtain consistent estimates, security analysts must know all securities in the investable universe.
  - No division of labors

เกาะไปกับตลาด

  - ~~Tendency to obtain inconsistent estimates of inputs~~

diversify port

- Markowitz model has a strong implication for passive portfolio management. It implies that holding the market portfolio is optimal.

แต่ SIM → เน้น abnormal return

  - We also want a model for active portfolio management strategies.

1.1 Motivations

- Sharpe offers a simpler model for portfolio selection (Sharpe, W.F., 1963, “A Simplified Model for Portfolio Analysis”, Management Science, Vol. 9 (2), p277-293.).

still within MVC

- This model is called “The Single Index Model (SIM)”.
- The SIM relies on the assumption that security returns are driven by a single factor model.
- The SIM is based on the promise that covariances among securities’ returns arise from a common economic factor that affect the fortunes of all firms.

1.2 The Single Factor Model

- The model assumes that returns of all security is driven by a single factor, F. A single factor model is written as;

common to all stocks & drives returns of all stocks e.g. GDP inflation

$$r_i = \alpha_i + \beta_i \cdot F + \varepsilon_i$$

where r_i = return on a security i

F = a common factor driving returns of all securities

F change by 1% → r_i change by β %

β_i = the sensitivity of r_i to a change in value of F

α_i = the constant part of r_i

residual term

not driving by

ε_i = the random part of r_i which is unexplained by F

model becomes unbiased estimator

needed for run regression

$E[\varepsilon_i] = 0$, $Var[\varepsilon_i] = \sigma_{\varepsilon i}{}^2$ and $Cov[\varepsilon_i, F] = 0$ endogeneity

homoskedasticity

$Cov[\varepsilon_i, \varepsilon_j] = 0$

r_i, F and ε_i are random variables.

$r_{PTT} = \alpha_{PTT} + 0.8Y + \varepsilon_{PTT}$

constant

GDP

also cause r_{PPT} to change

$Y \uparrow \Delta y$ 1% → $r_{PPT}\uparrow$

How sensitive r_{PTT} to GDP

1.2 The Single Factor Model

- The single factor model can be written in another form.

[1] $\quad r_i = \cancel{\alpha_i} + \beta_i \cdot F + \varepsilon_i$

- Take expectation on both sides of [1], we obtain;

[2] $\quad E[r_i] = E[\alpha_i + \beta_i \cdot F + \varepsilon_i] = \cancel{\alpha_i} + \beta_i \cdot E[F]$; $E[\varepsilon_i] = 0$

- Subtracts [2] from [1];

[3] $\quad r_i - E[r_i] = \beta_i \cdot (F - E[F]) + \varepsilon_i$

- Therefore, the single factor model can be written as;

actual return on stock i

consists of ① expected return

② unexpected return

$$r_i = E[r_i] + \underbrace{\beta_i \cdot (F - E[F])}_{\text{Deviation from } E[r_i] \text{ due to a surprise in the value of the common factor, F.}} + \varepsilon_i$$

represent the same model

r_i = 13%

$E[r_i]$ = 10%

common factor

deviate from $E[r_i]$

ε_i: Deviation from $E[r_i]$ due to a surprise in firm- or industry-specific events.

Deviation from $E[r_i]$ due to a surprise in the value of the common factor, F.

1.2 The Single Factor Model

- Another way to think of the single factor model is that;

$$r_i \;=\; E[r_i] + \text{Unanticipated surprise}$$

- The unanticipated surprise could come from unexpected change in the broad economy or from unexpected event that is specific to the firm.

- We should also consider that each security will react to the unexpected change in the broad economy differently.

- Thus, we have the single factor model;

S&P 500 → not good as it includes only large stock

→ e.g. SET index

r_m

$$r_i \;=\; E[r_i] + \beta_i \cdot (F - E[F]) + \varepsilon_i$$

macro factor

need to find representative

return on stock mkt index ; reflect macro

1.3 The Single Index (Market) Model

- When the return on security market-wide index is used as the factor the model is called “the market model”.

to construct efficient frontier

$$r_i = \alpha_i + \beta_i \cdot r_M + \varepsilon_i$$

$E[r_i]$ σ_i^2 $\sigma_{i,j}$

where r_i = return on a security i

r_M = return on the market portfolio

β_i = the sensitivity of r_i to a change in value of r_M

α_i = the constant part of r_i

ε_i = the random part of r_i which is unexplained by F

$E[\varepsilon_i] = 0$, $Var[\varepsilon_i] = \sigma_{\varepsilon i}^2$ and $Cov[\varepsilon_i, r_M] = 0$

$Cov[\varepsilon_i, \varepsilon_j] = 0$ → แยกกันหาได้

r_i, r_M and ε_i are random variables.

1.3 The Single Index (Market) Model

- Based on the market model, the total risk of a security can be decomposed into 2 components have to be independent ~~cov(?)~~

total risk

constant

$$
\begin{aligned}
\mathrm{Var}[r_i] &= \mathrm{Var}[\alpha_i + \beta_i \cdot r_M + \varepsilon_i] \\
&= \cancel{\mathrm{Var}[\alpha_i]}^{\,0} + \mathrm{Var}[\beta_i \cdot r_M] + \mathrm{Var}[\varepsilon_i] \\
&= \beta_i^2 \cdot \sigma_M^2 + \sigma_{\varepsilon i}^2
\end{aligned}
$$

- ① Systematic risk = $\beta_i^2 \cdot \sigma_M^2$
- ② ~~Non~~ un-systematic risk = $\sigma_{\varepsilon i}^2$ firm specific risk
- This decomposition requires risk to be expressed as variance of return.

1.3 The Single Index (Market) Model

cov [r_i, r_i] = $\beta_i^2 \sigma_M^2$ ⇒ variance / systematic risk

- The SIM implies that it is the common factor, that generates correlation across securities.

$$\text{Cov}[r_i, r_j] \quad = \quad \text{Cov}[\alpha_i + \boxed{\beta_i \cdot r_M} + \varepsilon_i,\ \alpha_j + \boxed{\beta_j \cdot r_M} + \varepsilon_j]$$

- Since α's are constant and Cov[r_M, ε_i] = Cov[r_M, ε_j] = 0, we have $\boxed{\text{Cov}[r_i, r_j] = \text{Cov}[\underbrace{\beta_i \cdot r_M}_{\text{constant}}, \beta_j \cdot r_M].}$

variance of mkt return

$$\text{Cov}[r_i, r_j] \quad = \underbrace{\beta_i \cdot \beta_j}_{\text{คนละคนสามารถหาได้}} \cdot \sigma_M^2$$

mkt return to itself

Beta

$\sigma_i^2 = 10$

σ_M^2

10 + 1

$\sigma_{ij} = 11$

- Hence, the covariance between any pair of securities is determined by their β's.

* จำนวน estimated term ลดลง

- With 10 securities, only 10 β's and the σ_M^2 are needed to construct all covariance terms, compared to direct estimate of $(10^2/2)-10$ = 35 covariance terms under Markowitz.

$N = 10 \Rightarrow \sigma_i^2 = 10$ term, $\sigma_{ij} = \frac{10^2 - 10}{2} = 45$

don't need one group to know all stocks

↳ allow division of labor

1.3 The Single Index (Market) Model

$\beta_{PTT} = \dfrac{cov(r_{PTT}, r_M)}{Var[r_M]}$ SET index

$\overset{r_{i,t}}{y} = a + b\overset{r_{m,t}}{x} + e_{i,t}$ time series

$b = \dfrac{cov(x,y)}{var(x)}$

- Furthermore,

$$
\begin{aligned}
Cov[r_i, r_M] &= Cov[\alpha_i+\beta_i \cdot r_M+\varepsilon_i, r_M] \\
&= Cov[\beta_i \cdot r_M, r_M] \\
&= \beta_i \cdot Cov[r_M, r_M] \\
&= \beta_i \cdot Var[r_M] \\
\beta_i &= Cov[r_i, r_M] / Var[r_M]
\end{aligned}
$$

- We also have, don't use

$$
\begin{aligned}
Corr[r_i, r_j] &= \beta_i \cdot \beta_j \cdot \sigma_M^2 / \sigma_i \cdot \sigma_j \\
&= Corr[r_i, r_M] \times Corr[r_j, r_M]
\end{aligned}
$$

2. ESTIMATION OF THE MARKET MODEL

2.1 The Form of the Regression Model

2.2 The Estimation Inputs and Outputs

2.3 The SIM and Diversification

2.1 The Form of the Regression Model

- Because the SIM is linear, it could be estimated using the univariate linear regression.

- To estimate the market model, the excess return of a security, $r_i - r_F$, is regressed on the excess return on the market index, $r_M - r_F$. That is,

academic

need more to compensate risk

residual error
unsystematic risk
firm-specific risk

$$\underbrace{\left(r_{i,t} - \underbrace{r_{F,t}}_{\text{risk-free}}\right)}_{\text{excess return}} = \alpha_i + \beta_i\left(r_{M,t} - r_{F,t}\right) + \varepsilon_{i,t} \qquad [1]$$

has special meaning

บ่งบอกความสามารถของหุ้น
นำหุ้นที่ α สูงเข้า portfolio

α = 0 ไม่ดี ทำไรไม่ได้
α < 0 short sell
α > 0 เพิ่ม weight

- Equation [1] is also known as the Security Characteristic Line (SCL)

- Excess returns are used instead of raw returns, because we should expect a risky asset to earn return in excess of the risk-free return. This way, a_i can be interpretated as an abnormal return.

2.1 The Form of the Regression Model

use raw return of stock i and market

- Note that the industry version of the SCL uses raw return.

raw return

$$r_{i,t} = a_i + b_i \cdot r_{M,t} + u_{i,t} \qquad [2]$$

- To see whether equations [1] and [2] will obtain consistent estimates of the slope term, rewrite [1]

β_i

should use 1st equation

if this represents constant (only when $r_{F,t}$ is constant)

$$r_{i,t} = \alpha_i + (1 - \beta_i) \cdot r_{F,t} + b_i \cdot r_{M,t} + \varepsilon_{i,t} \qquad [3]$$

- If $r_{F,t}$ is constant over the estimation period, [1] and [2] will yield consistent slope term.

- However, the intercept term in [2] is really an estimate of $\alpha_i+(1-\beta_i)\cdot r_{F,T}$, which is not the abnormal return.

2.1 The Form of the Regression Model

- Taking expectation on both sides of Equation [1].

expected risk premium on stock i comes from 2 parts

unsystematic risk หายไป $\varepsilon_{i,t}$

$$\underbrace{\mathbf{E}[\mathbf{r_i}] - \mathbf{r_F}}_{\text{expected risk premium}} = \underbrace{\boldsymbol{\alpha}_\mathbf{i}}_{\text{Non-market return}} + \underbrace{\boldsymbol{\beta}_\mathbf{i}(\mathbf{E}[\mathbf{r_M}] - \mathbf{r_F})}_{\text{Expected risk premium compensated for systematic risk of the security}}$$

Non-market return

Expected risk premium compensated for systematic risk of the security

α_i is the abnormal return known as Jensen's Alpha. It is the part of excess return that bares no risk, Var[α_i]=0. When α_i > 0, the expected return on the security is more than compensates for its systematic risk. Such securities would be in demand. Competition in the capital market will finally drive α_i down to zero.

A major task of security analysts is to identify securities whose $\alpha_i \neq 0$ before others could.

เป็น 0 ไม่ดี → ใช้ทำไรไม่ได้

Run SCL equation for PTT → run time-series regression

1) Collect price and compute price return
mkt mkt return

t	r_{PTT}	r_{SET}	r_f	$r_{PTT} - r_f$
0	x	x	x	
1	x	x	x	
2	x	x	x	
⋮				
T				

2.2 The Estimation Inputs and Outputs

- To estimate the market model or SCL on a security, the following inputs must be prepared.

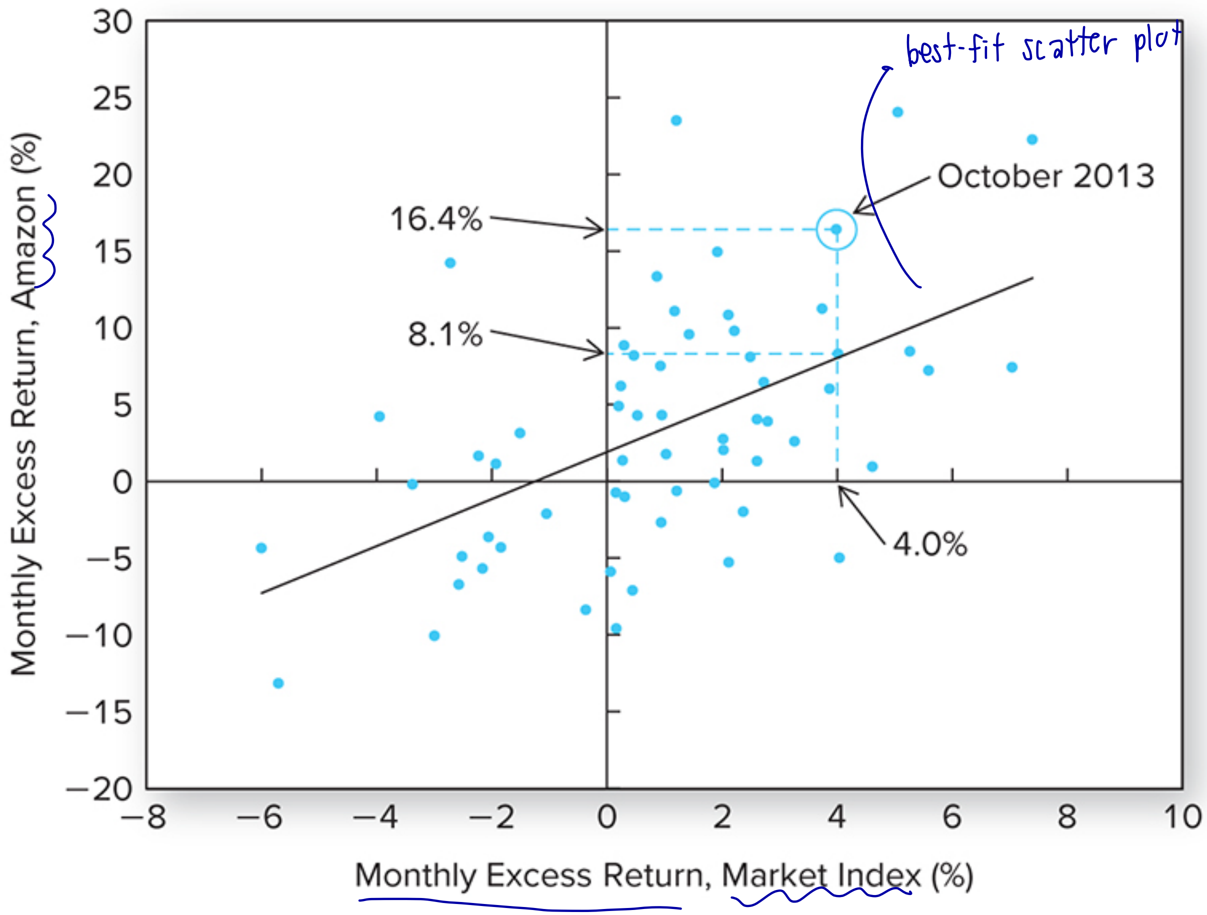
  [1] Historical returns on the stock and the market index. Historical risk-free rates.

  [2] Calculate historical excess returns on the stock and the market index.

- The following graph show the SCL regression for Amazon stock using 5 years of monthly returns.

SCL for Amazon (5 years of monthly return ending Jun 2018)

$r_{i,t} - r_{f,t}$



$r_{M,t} - r_{f,t}$

SCL for Amazon (5 years of monthly return ending Jun 2018)

$Var[r_{AMAZON}]$

Regression Statistics	
Multiple *R*	0.5351
R-Square	0.2863
Adjusted *R*-Square	0.2742
Standard Error	0.0686
Observations	60

line capture scatter plot ไม่ดี

market return can explain stock return only 28.63%

ที่เหลือ belong to firm specific risk

unsystematic risk $\varepsilon_{i,t}$ สูง

residual เยอะ

	Coefficients	Standard Error	*t*-statistic	*p*-value
Intercept	0.0192	0.0093	2.0645	0.0434
Market index	1.5326	0.3150	4.8648	0.0000

$\beta_{AMAZON} = +1.5326$

$\alpha_{AMAZON} = +0.0192$

ก่อนหน้านี้อธิบายหุ้นตัวเดียวโดยใช้ market return

2.3 The SIM and Portfolio Diversification

- We may view a portfolio of securities as an asset. Apply the SIM to a portfolio and use excess return form, $R = r - r_F$.

$$R_P = \alpha_P + \beta_P \cdot R_M + \varepsilon_P$$

- Portfolio return can also be written as

$$R_P = \Sigma w_i \cdot R_i$$

$$= \Sigma w_i \cdot (\alpha_i + \beta_i \cdot R_M + \varepsilon_i)$$

$$= \Sigma w_i \cdot \alpha_i + R_M \cdot \Sigma w_i \cdot \beta_i + \Sigma w_i \cdot \varepsilon_i$$

... of portfolio is the weighted average of each stock

$\alpha_i > 0$

$\alpha_j < 0$

$$E[R_P] = \Sigma w_i \cdot \alpha_i + E[R_M] \cdot \Sigma w_i \cdot \beta_i$$

- Therefore,

เอาแต่ละตัวมา weight

$$\alpha_P = \Sigma w_i \cdot \alpha_i \;;\; \beta_P = \Sigma w_i \cdot \beta_i \;;\; \varepsilon_P = \Sigma w_i \cdot \varepsilon_i$$

2.3 The SIM and Portfolio Diversification

- The variance of the portfolio

$$
\begin{aligned}
Var[R_P] &= Var[\Sigma w_i\alpha_i + R_M \cdot \Sigma w_i\beta_i + \Sigma w_i\varepsilon_i] \\
&= \cancel{Var[\Sigma w_i \underset{\text{constant}}{\alpha_i}]}^{\,0} + Var[R_M \cdot \overset{\text{constant}}{\Sigma w_i\beta_i}] + Var[\Sigma w_i\varepsilon_i] \\
&= (\Sigma w_i\beta_i)^2 \cdot Var[R_M] + \Sigma w_i^2 Var[\varepsilon_i] \\
&= (\Sigma w_i\beta_i)^2 \cdot \sigma_M^2 + \Sigma w_i^2 \cdot \sigma_{\varepsilon i}^2 \\
&= \underbrace{\beta_p^2 \cdot \sigma_M^2}_{\text{systematic risk}} + \underbrace{\sigma_{\varepsilon p}^2}_{\text{unsystematic risk}}
\end{aligned}
$$

ε_i as return on stock i

$var[w_1\varepsilon_1 + w_2\varepsilon_2 + \ldots]$

$var[\varepsilon_i] + cov(\varepsilon_i, \varepsilon_j)$

assume this is zero (unlike Markowitz)

- As the risk-free rate is a constant term, $Var[R_M] = Var[r_M - r_F] = Var[r_M]$

- The SIM assumes $Cov[\varepsilon_i, \varepsilon_j] = 0$, thus $\sigma_{\varepsilon p}^2 = \Sigma w_i^2 \cdot \sigma_{\varepsilon i}^2$.

ต่างจาก Markowitz

ข้อมูลเยอะไม่ใช่ว่าจะดี | SIM → partial info

2.3 The SIM and Portfolio Diversification

- As more and more securities are included in the portfolio;
  - $\sigma_{\varepsilon p}^{2}$ approaches 0
    - Assume an equally-weighted portfolio ($w_i = 1/N$)

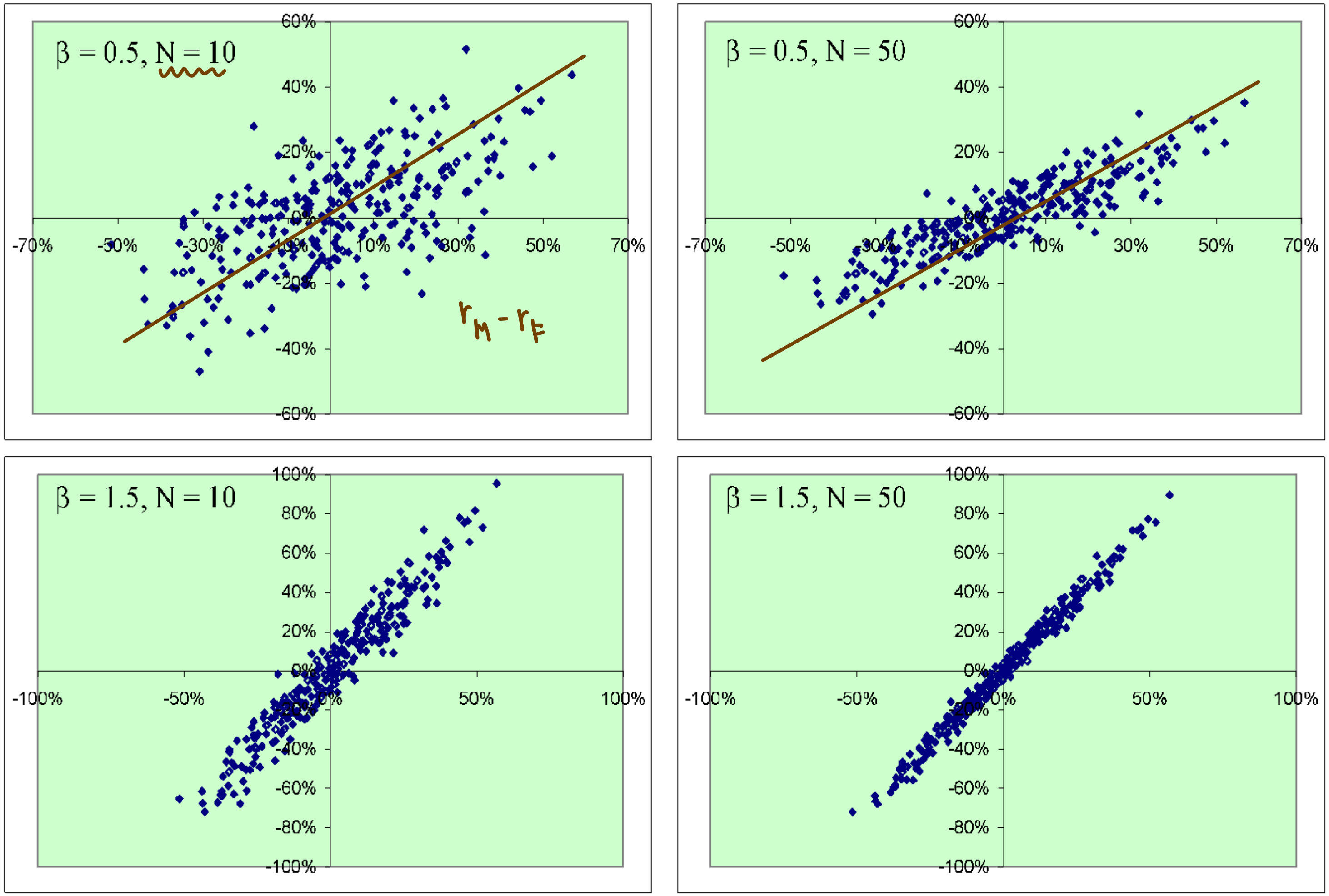
$$
\begin{aligned}
Var[\varepsilon_p] &= \Sigma\, (1/N)^2\, Var[\varepsilon_i] \\
&= (1/N)^2\, \Sigma\, Var[\varepsilon_i] \\
&= (1/N)\, ave[\sigma_{\varepsilon i}^{2}] \rightarrow 0 \text{ (as } N \rightarrow \infty)
\end{aligned}
$$

    - A well-diversified portfolio is only exposed to systematic risk.

Hypothetical Portfolios

$\sigma_{\varepsilon p}^2 \downarrow \quad R^2 \uparrow$

As we increase more and more stocks



The number of securities in the portfolio has no bearing on its systematic risk (β). In contrast, firm-specific risks (σ_ε) become increasingly irrelevant as the portfolio becomes more diversified.

3. PORTFOLIO CONSTRUCTION UNDER SIM

3.1 The Objective Function

3.2 The Treynor-Black Approach

3.3 The SIM vs Markowitz Model

3.1 The Objective Function

- To identify the optimal risky portfolio, P, we start as follows.
- Assume there are N individual assets, and their returns are written in the excess return form, $R_i = r_i - r_F$.
  - Since r_F is assumed to be constant, $Var[R_i] = Var[r_i - r_F] = Var[r_i]$.
- Choose an appropriate market index and run SCL regression on each individual asset.
  - Obtain model inputs, namely, $E[R_M]$, σ_M^2, α_i's, β_i's, and $\sigma_{\varepsilon i}$'s.
- The next step is to set up the objective function.

From topic 4 slide

SIM : $E[r_p] = E[\alpha_p] + \beta_p \cdot E[R_M] + E[\varepsilon_p]$ → 0 by defination

$\sum w_i \alpha_i + (\sum w_i \beta_i) E[R_M]$

$\sigma_p^2 = \beta_p^2 \sigma_M^2 + \sigma_{\varepsilon p}^2$

systematic, unsystematic

$(\sum w_i \beta_i)^2$, $\sum w_i^2 \sigma_{\varepsilon i}^2 \to 0$

4.2 The Efficient Portfolio with RF Asset

Look for weight

- The objective function for finding the optimal risky portfolio;

sharpe ratio

$$\text{Max.} \quad SR_P = \frac{E[r_P] - r_F}{\sigma_P}$$

$$\text{s.t.} \quad \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{i,j} = \sigma_P^2$$

$$\sum_{i=1}^{N} w_i E[r_i] = E[r_P]$$

$$\sum_{i=1}^{N} w_i = 1$$

$$w_i \geq 0 \quad \forall\, i$$

$$\text{Max.} \quad SR_P = \frac{E[r_P] - r_F}{\sigma_P}$$

$$\text{s.t.} \quad \mathbf{w}^{\mathbf{T}}\mathbf{V}\mathbf{w} = \sigma_P^2$$

$$\mathbf{w}^{\mathbf{T}}\mathbf{r} = E[r_P]$$

$$\mathbf{w}^{\mathbf{T}}\mathbf{1} = 1$$

$$w_i \geq 0 \quad \forall\, i$$

3.1 The Objective Function

can be not well diversified ← ▸ allow fund manager can express active skill ↳ α selection skill

as long as everybody agree with market

▸ β_i & β_j can be calculated independently

use partial information

N stocks → estimate

α_i	N terms	3N come from 1 regression
β_i	N	
$\sigma^2_{\varepsilon i}$	N	
$E[R_M]$	1	
σ^2_M	1	

- The objective function is;

$$\text{Max.} \quad SR_P = \frac{E[R_P]}{\sigma_P}$$

where fund manager can show his ability to choose stock

$$\text{s.t.} \quad \sum_{i=1}^{N} w_i \alpha_i + E[R_M] \sum_{i=1}^{N} w_i \beta_i = E[R_P]$$

→ ใส่หุ้นที่ α สูงใส่ port

$\alpha_i > 0$

$w_i > 0$

then sharpe ratio ↑

but we might incur higher unsystematic risk

$$\sigma^2_M \left(\sum_{i=1}^{N} w_i \beta_i \right)^2 + \sum_{i=1}^{N} w_i^2 \sigma^2_{\epsilon i} = \sigma^2_P$$

; no α

เกิดขึ้นเพราะเราไป concentrate หุ้นใดหุ้นหนึ่ง

$$\sum_{i=1}^{N} w_i = 1$$

$$w_i \geq 0 \quad \forall i$$

3.1 The Objective Function

- In active portfolio management, security analysts try to uncover securities whose $\alpha_i \neq 0$.
- To increase SR_P, the optimization procedure will tend to assign more weights on securities with $\alpha_i > 0$ and less or even negative weights on securities with $\alpha_i < 0$, so that;

$$\alpha_P = \sum_{i=1}^{N} w_i \alpha_i > 0$$

- This also means that the optimal risky portfolio will not be fully diversified as it moves away from the market index.
- In this case, the optimal risky portfolio will carry certain level of unsystematic (idiosyncratic) risk, $\sigma^2_{\varepsilon P}$.

3.1 The Objective Function

- The tradeoff between abnormal return and idiosyncratic risk is called the information ratio.

measure how good active manager is

↳ good manager should have higher ratio

high alpha,
low unsystematic risk

- For individual asset, the information ratio is;

$$\text{Information Ratio of stock i} = \frac{\alpha_i}{\sigma_{\varepsilon i}}$$

abnormal return row stock

unsystematic risk

3.2 Treynor-Black Approach

- Treynor and Black developed techniques to identify the optimal risky portfolio, which happens to consist of 2 sub-portfolios, namely, the passive portfolio and the active portfolio.
- The passive portfolio is an investment in the market index.
- The active portfolio represents the attempt by portfolio managers to capture mispriced assets.
- They add one more asset, the market index, into the objective function.
    - Note: $\alpha_{N+1} = \alpha_M = 0$, $\beta_{N+1} = \beta_M = 1$, and $\sigma^2_{\varepsilon,N+1} = \sigma^2_{\varepsilon M} = 0$

3.2 Treynor-Black Approach

- The objective function is;

$$\text{Max.} \quad SR_P = \frac{E[R_P]}{\sigma_P}$$

$$\text{s.t.} \quad \sum_{i=1}^{N+1} w_i \alpha_i + E[R_M] \sum_{i=1}^{N+1} w_i \beta_i = E[R_P]$$

$$\sigma_M^2 \left(\sum_{i=1}^{N+1} w_i \beta_i \right)^2 + \sum_{i=1}^{N+1} w_i^2 \sigma_{\epsilon i}^2 = \sigma_P^2$$

$$\sum_{i=1}^{N+1} w_i = 1$$

$$w_i \geq 0 \quad \forall\, i$$

3.2 Treynor-Black Approach

- The contribution of each component to the Sharpe's Ratio of the optimal risky portfolio is shown in the following equation.

$$SR_P^2 = SR_M^2 + \left(\frac{\alpha_A}{\sigma_{\varepsilon A}}\right)^2$$

where SR_P and SP_M are Sharpe's Ratios of the optimal risky portfolio and the market index

α_A is the Jensen's alpha of the active portfolio

$\sigma_{\varepsilon A}$ is the idiosyncratic risk of the active portfolio.

3.2 Treynor-Black Approach

- The optimal weight in each individual asset (w_i*) is found to be;

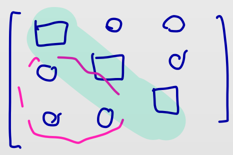
$$w_i^* = w_A^* \frac{\alpha_i/\sigma_{\varepsilon i}^2}{\sum_{i=1}^{N}(\alpha_i/\sigma_{\varepsilon i}^2)}$$

where w_A* is the portfolio weight on the active portfolio

- The information ratio play a central role in active portfolio management.

3.3 The SIM vs Markowitz Model

- Markowitz Portfolio Selection Model requires the following estimates as the input to the model.
  - $E[r_i]$ = N terms
  - $Var[r_i]$ = N terms
  - $Cov[r_i, r_M]$ = $(N^2 - N)/2$ terms



- Total number of estimated variables = $2N + (N^2 - N)/2$
- With 100 securities, the number of estimates = 5,150
- With 500 securities, the number of estimates = 125,750

3.3 The SIM vs Markowitz Model

- The SIM requires the following estimates as the input to the model.
  - α_i = N terms
  - β_i = N terms
  - $Var[\varepsilon_i]$ = N terms
  - $E[r_M]$ = 1 term
  - $Var[r_M]$ = 1 term

For each security i, one regression of the market model yields these 3 coefficients.

- Total number of estimated variables = 3N+2

SIM use partial information

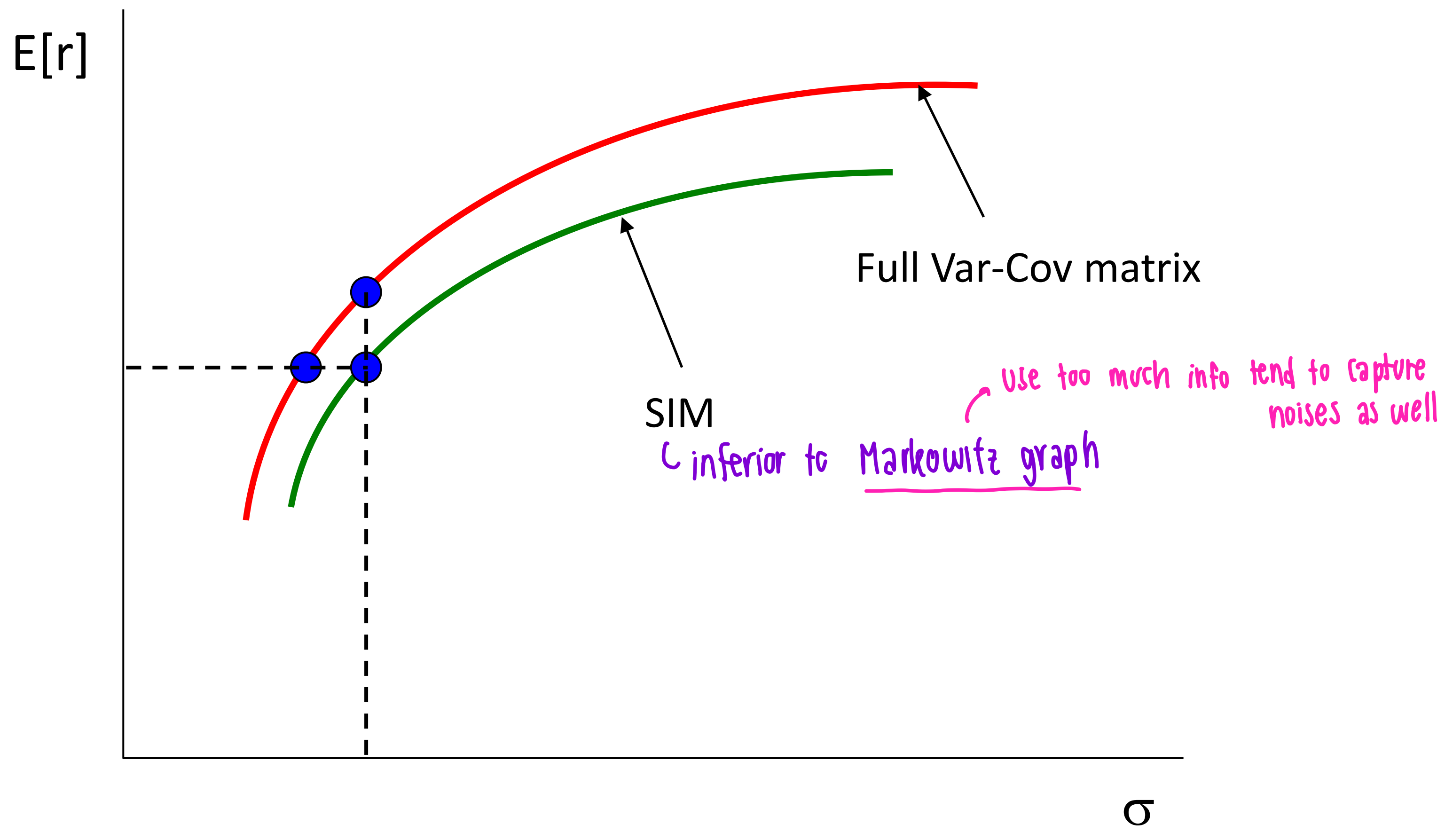
alot smaller

- With 100 securities, the number of estimates = 302
- With 500 securities, the number of estimates = 1,502

3.3 The SIM vs Markowitz Model

- When applied in-sample, the efficient frontier from SIM is inferior to Markowitz's model.
- However, when applied out-of-sample the performance of both models depends on how reliable the estimated variables are. There is nothing to guarantee that Markowitz's model will outperform SIM in an out-of-sample setting.

The Efficient Frontiers: Markowitz vs. SIM (in-sample test)



" Parsimonious "

" Parsimony"

↳ a more comprecate model need not be better than simplier model

try to ignore noises

3.3 The SIM vs Markowitz Model

- The SIM has clear practical advantages over the full variance-covariance model. The SIM allows division of labor and decentralization of macro and security analysis.
- The SIM can be extended to optimize the portfolio with active management.
  - In an active portfolio, an investor overweighs securities whose α's > 0 and under-weighs securities whose α's < 0 expecting that $\alpha_P = \Sigma w_i \alpha_i > 0$.
  - The trade-off for active management is that the portfolio will not be fully diversified.

Exercises

1. Use the data from the accompany data file. Choose 3 stock and run the SCL equation for each stock.

- Which price/index series do you use?
- Which sampling period and sampling interval do you use?
- Interpret the regression results?