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Finance Theories

Topic 7:

use different approach to find equilibrium

The Arbitrage Pricing Theory (APT)

Equilibrium → max SR_p

- Capm → same tangent portfolio = mkt portfolio
- APT → we will reach equilibrium if no arbitage opportunity → no incentive for price to change

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Agenda

1 Arbitrage and the Equilibrium Condition

2 The Arbitrage Pricing Theory (APT)

3 Implementation of the APT

... ...

1. ARBITRAGE AND THE EQUILIBRIUM CONDITION

1.1 Introduction

1.2 Definition

1.3 The Equilibrium Condition

1.1 Introduction

- The Arbitrage Pricing Theory (APT) was developed by Stephen Ross. The APT tries to overcome various weaknesses of the CAPM.
- Like the CAPM, the APT is an equilibrium theory explaining how E[r] of a risky asset is formed.
- However, the APT uses a different approach from the CAPM to determine asset prices in equilibrium.
- While the CAPM uses "risk-return dominance" argument in support of equilibrium price relation, the APT uses "arbitrage argument".

1.1 Introduction

- The APT starts with the assumption that asset returns are generated by a factor model and relies on approximate arbitrage argument.
- The theory does not require the existence of the market portfolio.

Arbitrage profit : is a trading strategy

• "risk-less" "abnormal" profit ; e.g. doesn't lose any money to invest but get return

1.2 Definition

abnormal profit

can get risk-less profit

- An arbitrage is an investment strategy that;
  - Requires no net initial investment
  - Results in risk-less profit *know payoff for sure = $1

you & mkt will short A & buy B

price still change

- EX: Consider two assets.

In future buy A and return it to owner

profit = $1

in future: sell B at $15

$P_{0,A}$: \$10 — 0.5 → Econ Boom: \$14; 0.5 → Econ Bust: \$8

① + $10

$P_{0,B}$: \$10 — 0.5 → Econ Boom: \$15; 0.5 → Econ Bust: \$9

− $10

to expensive

too cheap ; always better

Today, price of A & B are the same ⇒ short A and get $10 to invest in B (don't need to use your own money)

- An arbitrage portfolio can be formed by short selling A (says, 1 shares) and long B (1 shares) simultaneously.

1.2 Definition

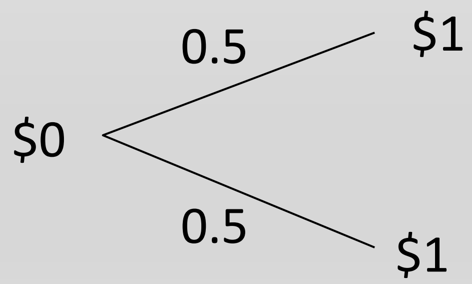
- This portfolio involves zero initial investment, i.e., zero-investment portfolio.
- Portfolio weight:

negative weight
positive weight

$$w_P = \Sigma_i w_i = w_A + w_B$$
$$= (-10/10) + (10/10) = 0$$

- Risk-less profit

Arbitrage portfolio:



1.3 The Equilibrium Condition

- Arbitrage opportunity creates excess demand and supply which causes security prices to change.
  - There will be excess demand for B and excess supply of A. Consequently, $P_{0,B}$ will rise and $P_{0,A}$ will drop.
- Actions by arbitrageurs will finally eliminate the arbitrage opportunity leaving no pressure for price to change further.
- Market equilibrium is reached when there is no arbitrage opportunity. $E[r_i]$
- The APT uses no arbitrage opportunity as the condition of capital market equilibrium.

start with

assume that asset return is from factor model

The factor model e.g. SIM

2. THE ARBITRAGE PRICING THEORY

less restrictive than CAPM

2.1 Assumptions

no assumption regarding mkt portfolio

can avoid Roll's

2.2 Derivation of the APT Relation

2.3 The Logic of the APT

2.4 The APT vs. CAPM

$r_i = \alpha_i + \beta_{i,1} F_1 + \ldots + \beta_{i,K} F_K + \varepsilon_i$

2.1 Assumptions

$r_i = \cancel{\alpha_i} + \beta_{i,1} F_1 + \varepsilon_i$ — (1)

$E[r_i] = \cancel{\alpha_i} + \beta_{i,1} E[F_1]$ — (2)

(1)-(2) : $r_i - E[r_i] = \beta_{i,1}(F_1 - E[F_1]) + \varepsilon_i$

$r_i = E[r_i] + \underbrace{\beta_{i,1}(F_1 - E[F_1])}_{\text{surprise from common factor}} + \varepsilon_i$

surprise from firm-specific

cause real return to deviate from expected return

1. Security returns are generated by a K-factor model.

theory never tell what is K

$$r_i = E[r_i] + \beta_{i,1} f_1 + \ldots + \beta_{i,K} f_K + \varepsilon_i$$

where r_i = rate of return on stock i

f_k = the deviation in value of a common factor k from its expected value, $f_k = F_k - E[F_k]$

$\beta_{i,k}$ = the sensitivity (or exposure) of return on i to the value of a common factor k ($\beta_{i,k}$ is called factor loading)

ε_i = return on i that is due to firm-specific surprise (unexplained by K factors)

$E[\varepsilon_i] = 0$, $Cov[\varepsilon_i, F_k] = 0$, $Cov[\varepsilon_i, \varepsilon_j] = 0$ and $Cov[F_k, F_{k+1}] = 0$

Explanation for a Factor Model

common factor

- EX: Assumed a single factor model with $\boxed{\text{GDP growth}}$ as the common factor (F). Asset i has $\beta_{i,Y}$ = 1.2, while $E[r_i]$ = 10% and r_F =2%. The consensus on GDP growth (Y) is 4%.

stock i against GDP growth

$E[F_Y]$ ← expected value of GDP growth

factor

  - r_i can be written in a single factor model as;

actual return on i

firm-specific event

GDP

$$r_i = 0.10 + 1.2 \cdot f_Y + \varepsilon_i \quad \text{or} \quad (r_i - r_F) = 0.08 + 1.2 \cdot f_Y + \varepsilon_i$$

no surprise

  - If Y turns out to be 4%, then f_Y=0% and r_i = 10.0%

1% more than expectation

1.2 times of surprise

  - If Y turns out to be 5%, then f_Y = 1% and r_i = 11.2%
  - If Y turns out to be 3%, then f_Y = –1% and r_i = 8.8%

Explanation for a Factor Model

- EX: Asset returns are generated by a two-factor model (Y = GDP growth and IR = change in interest rate)

Stock i

performance doesn't go with mkt much → $\beta_{i,Y} < 1$

$$r_i = E[r_i] + \beta_{i,Y} \cdot f_Y + \beta_{i,IR} \cdot f_{IR} + \varepsilon_i$$

int rate in mkt ↑ by 1% more than E[int rate]

have contract with govt

should be negative

  - For stock of a regulated electric-power utility firm, its operating cash flows are insensitive to changes in business cycle. With rather stable operating cash flows, an increase in interest rate with hurt the value of its business.

stable cash flow

  - In this case the model for stock i may look like this;

$$r_i = E[r_i] + 0.2 \cdot f_Y - 0.4 \cdot f_{IR} + \varepsilon_i$$

2.1 Assumptions

2. There are large number of securities such that well-diversified portfolios could be constructed with negligible idiosyncratic risks. firm-specific risk
"unsystematic risk"

3. A diversified portfolio, P, that is not exposed to any factor risk ($\beta_{P,1} = \ldots = \beta_{P,K} = 0$) must offer the risk-free rate

zero sensitivity

zero exposure to any pressure

$$E[r_P] = r_F$$

form port is such a way that
no exposure to any risk

4. There always exist portfolios that are exposed to the risk of only a single factor, k.

return generating process has k factor

$$r_P = E[r_P] + \beta_{P,k} \cdot f_k + \varepsilon_P$$

always can find β against 1 factor

1

can be used to represent return of factor k

portfolio track movement of GDP perfectly

$$E[r_i] = r_F + \beta_i (E[r_M] - r_F)$$

$F = GDP$

r_i

$= \% \Delta GDP$

2.1 Assumptions

$\underbrace{E[r_{Pk}] - r_F}_{\text{risk premium against factor } k}$

5. A portfolio, P_k, that has unitary risk of factor k, $\beta_{P,k} = 1$, offers a risk premium associated with the factor risk

$$E[r_{Pk}] = E[r_k]$$

Portfolio P_k is called a factor portfolio and $E[r_{Pk}]-r_F$ reflects the risk premium on factor k.

- EX: Assume the common factor is GDP growth. If we construct a portfolio of financial assets such that the portfolio's return tracks the movement of GDP growth 1:1 ($\beta_{P,Y} = 1$), then the expected excess return on this portfolio reflects the risk premium associated with GDP growth uncertainty.

2.2 Derivation of the APT

to derive E[r_i]

- Factor models do not explain how E[r_i] is reached in equilibrium.

How is E[r_i] formed in equilibrium

- To derive the APT, we assume that asset returns are generated by a two-factor model.

$$r_i = \underbrace{E[r_i]} + \beta_{i,1}f_1 + \beta_{i,2}f_2 + \varepsilon_i$$

- Investors can construct a zero-beta (0 systematic risk) zero-investment portfolio, such that;

have to short sell & long some stocks

$$\Sigma_i w_i = 0$$

$$\beta_{p,1} = \Sigma_i w_i \beta_{i,1} = 0$$

$$\beta_{p,2} = \Sigma_i w_i \beta_{i,2} = 0$$

weight รวมกันได้ 0 เพราะมีทั้ง short-sell & buy

don't use money & create port with no unsystematic risk
⇓
arbitrage port
⇓
E[r_i] should be zero

where i = 1 to N

↳ as long as it is large enough to eliminate unsystematic risk

2.2 Derivation of the APT

- The return on this portfolio can be written as;

driving by two factor model

$$r_P = \Sigma_i w_i r_i$$
$$= \Sigma_i w_i (E[r_i] + \beta_{i,1} f_1 + \beta_{i,2} f_2 + \varepsilon_i)$$
$$= \Sigma_i w_i E[r_i] + f_1 \Sigma_i w_i \underbrace{\beta_{i,1}}_{\beta_{P,1}=0} + f_2 \Sigma_i w_i \underbrace{\beta_{i,2}}_{\beta_{P,2}=0} + \Sigma_i w_i \varepsilon_i$$

- We assume that N is large enough, so that $\Sigma_i w_i \varepsilon_i \approx 0$. diversify enough

- $\Sigma_i w_i \beta_{i,1}$ and $\Sigma_i w_i \beta_{i,2} = 0$ by construction. Hence,

what left

$$r_P = \Sigma_i w_i E[r_i] \quad \xrightarrow{\text{take } E[\,]} \quad \text{and} \quad E[r_P] = \Sigma_i w_i E[r_i]$$

- In equilibrium, $E[r_P] = 0$ for an arbitrage portfolio.

2.2 Derivation of the APT

- To sum up, by construction of an arbitrage portfolio, we have

$$\Sigma_i w_i = 0, \quad \overbrace{\Sigma_i w_i \beta_{i,1}}^{\beta_{p,1}} = 0 \quad \text{and} \quad \overbrace{\Sigma_i w_i \beta_{i,2}}^{\beta_{p,2}} = 0$$

which imply that

$$\overbrace{\Sigma_i w_i E[r_i]}^{E[r_p]} = 0$$

summation of w_i

- In matrix form; $\mathbf{w}^T\mathbf{1} = 0$, $\mathbf{w}^T\boldsymbol{\beta}_1 = 0$ and $\mathbf{w}^T\boldsymbol{\beta}_2 = 0$ imply that $\mathbf{w}^T\mathbf{E[r]} = 0$.

$\underbrace{[\quad\quad] \begin{bmatrix} \\ \end{bmatrix}}_{\text{these matrix are orthogonal if the product = 0}} = 0$ └ scalar

scalar

- A theory in linear algebra states that "if the fact that a vector (w) is orthogonal to N−1 vectors ($\mathbf{1}$, $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$) implies that it is orthogonal to the N^th vector ($\mathbf{E[r]}$), then the N^th vector can be expressed as a linear combination of the N−1 vectors.

$E[r] = 1 + a_1\beta_1 + a_2\beta_2$

↳ of 3 vectors

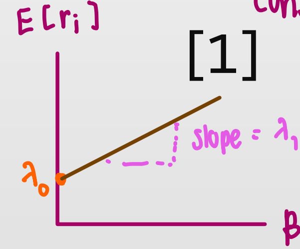
1, β_1 and β_2

2.2 Derivation of the APT

- Hence, $E[r_i]$ can be written as a linear combination of 1, $\beta_{i,1}$ and $\beta_{i,2}$ as;

coefficient in front of 1   coefficient   coefficient $\lambda_0, \lambda_1, \lambda_2$ are constant terms

$$E[r_i] = \lambda_0 + \beta_{i,1}\lambda_1 + \beta_{i,2}\lambda_2 \qquad [1]$$



⇒ should be 3-D plane

- [1] is the APT equation.
- The terms λ_k's are coefficients of a plane. (Note, if K = 1, λ_0 and λ_1 are coefficients of a straight line)
- How do we interpret λ_k's?

2.2 Derivation of the APT

normal portfolio

- First consider a portfolio with $\boxed{\Sigma_i w_i = 1}$, $\Sigma_i w_i \beta_{i,1} = 0$ and $\Sigma_i w_i \beta_{i,2} = 0$.

looking for weight

result = 0

result = 0

$$
\begin{aligned}
E[r_0] &= \Sigma_i w_i E[r_i] \\
&= \Sigma_i w_i (\lambda_0 + \beta_{i,1}\lambda_1 + \beta_{i,2}\lambda_2) \\
&= \lambda_0
\end{aligned}
$$

- Since this is a zero-beta portfolio, we interpret

represents

$$\lambda_0 = r_F$$

risk-free rate

2.2 Derivation of the APT

$\beta_{p,1} = 1$ $\beta_{p,2} = 0$

- First consider a portfolio with $\Sigma_i w_i = 1$, $\boxed{\Sigma_i w_i \beta_{i,1} = 1}$ and $\Sigma_i w_i \beta_{i,2} = 0$.

$$
\begin{aligned}
E[r_1] &= \Sigma_i w_i E[r_i] \\
&= \Sigma_i w_i \cdot (\lambda_0 + \overset{1}{\beta_{i,1}}\lambda_1 + \overset{0}{\beta_{i,2}}\lambda_2) \\
&= \overset{r_f}{\lambda_0} + \lambda_1 \\
\lambda_1 &= \underbrace{E[r_1] - r_F}
\end{aligned}
$$

mimic movement

- Thus, λ_1 is the risk premium associated with the 1st factor.
- Note: $E[r_1]$ is the expected return on the portfolio that has β=1 when measured against the 1st factor and β=0 when measured against the 2nd factor. This must be a factor portfolio for the 1st factor.

2.2 Derivation of the APT

- First consider a portfolio with $\Sigma_i w_i = 1$, $\Sigma_i w_i \beta_{i,1} = 0$ and $\Sigma_i w_i \beta_{i,2} = 1$.

$$
\begin{aligned}
E[r_1] &= \Sigma_i w_i E[r_i] \\
&= \Sigma_i w_i \cdot (\lambda_0 + \beta_{i,1}\lambda_1 + \beta_{i,2}\lambda_2) \\
&= \lambda_0 + \lambda_2 \\
\lambda_2 &= E[r_1] - r_F
\end{aligned}
$$

- Thus, λ_2 is the risk premium associated with the 2nd factor.
- Note: $E[r_1]$ is the expected return on the portfolio that has β=0 when measured against the 1st factor and β=1 when measured against the 2nd factor. This must be a factor portfolio for the 2nd factor.

2.2 Derivation of the APT

- So, the APT equation [1] can be rewritten as

$$E[r_i] \;=\; r_F + \beta_{i,1}\cdot(E[r_1]-r_F) + \beta_{i,2}\cdot(E[r_2]-r_F)$$

- The APT claims that for an arbitrary asset, its expected risk premium depends only on its exposure to common factors.

factor model: $r_i = \underbrace{E[r_i]}_{\text{in equilibrium}} + \beta_{i,1} f_1 + \beta_{i,2} f_2$

APT equation : $E[r_i] = \underbrace{\lambda_0}_{r_f} + \underbrace{\lambda_1}_{\text{risk premium associated with 1st factor}} \beta_{i,1} + \lambda_2 \beta_{i,2}$ ⇒ can be more than 2 factors

2.2 Derivation of the APT

- The above model can be generalized to the case where the return generating process is a K-factor model

$$r_i = E[r_i] + \beta_{i,1}f_1 + \ldots + \beta_{i,K}f_K + \varepsilon_i$$

- In this case, all securities and portfolios have expected returns described by;

$$E[r_i] = \lambda_0 + \beta_{i,1}\lambda_1 + \ldots + \beta_{i,K}\lambda_K$$

where $\lambda_0 = r_F$

$\lambda_k = E[r_k]-r_F$ is the RP on factor portfolio k.

$\beta_{i,k}$ = asset i's loading (or exposure) on factor k.

ᏒᏗᏁᏒᏫᏗ

→ return of portfolio that mimic / tract GDP perfectly 1-to-1 with GDP

$(E[r_Y] - r_f)$

risk premium associated with GDP

2.3 The Logic of the APT

r_f → $(E[r_\pi] - r_f)$

$$E[r_i] = \lambda_0 + \lambda_1 \cdot \beta_{i,Y} + \lambda_2 \cdot \beta_{i,\pi}$$

- **EX:** Assume returns on financial securities is generated by the following two-factor model.

premium associated with GDP

can hedge inflation

procyclical

$$r_i = E[r_i] + 1.8 \cdot f_Y - 0.2 \cdot f_\pi + u_i$$

λ_1

- Returns on individual assets have two common factors, GDP growth (Y) and Inflation (π). The term u_i represents firm-specific factors
- Note, f_Y, f_π and u_i are r.v.'s while $E[r_i]$ is a constant
- $\beta_{i,Y} = 1.8$ means a deviation of the return on the GDP factor portfolio by 1% from its expected value will cause r_i to move by 1.8% from $E[r_i]$.

sensitivity

2.3 The Logic of the APT

- $\beta_{i,\pi} = -0.2$ means a deviation of the return on the Inflation factor portfolio by 1% from its expected value will cause r_i to move by –0.2% away from $E[r_i]$.
- Returns on asset i have positive correlation with returns on the GDP factor portfolio and negative correlation with returns on the Inflation factor portfolio. .

2.3 The Logic of the APT

$i = PTT$

Find expected return in equilibrium

- According to the APT, asset i's expected return is

risk premium risk premium

$$E[r_i] = r_F + 1.8 \cdot E[R_Y] - 0.2 \cdot E[R_\pi]$$

- $E[R_Y] = E[r_Y] - r_F$ is the expected risk premium on the GDP factor portfolio (a portfolio whose β is 1 relative to GDP and 0 relative to inflation).
- $E[R_\pi] = E[r_\pi] - r_F$ is the expected risk premium on the Inflation factor portfolio.
- $\beta_{i,Y} \cdot E[R_Y]$ and $\beta_{i,\pi} \cdot E[R_\pi]$ represent the premium required on asset i for bearing GDP and inflation risks.

2.3 The Logic of the APT

start with \$100

short sell 20% on inflation port
60% on risk-free

- If $r_F = 4\%$, $E[R_Y] = 6\%$, $E[R_\pi] = -1\%$

$$E[r_i] = 4 + 1.8(6) - 0.2(-1) = 15.0\%$$

if they're not the same, we can do arbritage

- Note:

$\Sigma = 1$ portfolio weight

$$E[r_i] = r_F + 1.8 \cdot (E[r_Y] - r_F) - 0.2 \cdot (E[r_\pi] - r_F)$$

$$= -0.6 \cdot \underset{4\%}{r_F} + 1.8 \cdot \underset{10\%}{E[r_Y]} - 0.2 \cdot \underset{3\%}{E[r_\pi]}$$

= 15% as well

↳ has same systematic risk as asset i

- Portfolio P can be formed to replicate asset i using the following weights
  - 1.8 in the GDP factor portfolio
  - −0.2 in the Inflation factor portfolio
  - 1−1.8−(−0.2) = −0.6 in RF asset

want to short-sell this port
short GDP 180%
long inflation 20%
risk free 60%
left 100% to buy stock i

2.3 The Logic of the APT

$$E[r_P] = -0.6(4) + 1.8(10) - 0.2\cdot(3)$$

$$= -2.4 + 18.0 - 0.6$$

$$= 15.0\%$$

- The coefficients $\beta_{i,k}$'s can be viewed as portfolio weights.
- If the current market price of i implies that $E[r_i]$ = 15.7% rather than 15.0%, what should happen?

  higher than rate imply by APT

  • short sell asset with same risk as asset i
  mimic portfolio
- We could form a replicating portfolio, P, that have the same factor risk as asset i but whose expected return is 15.0%. Then short-sell this portfolio and use the proceed to buy asset i.

  APT

  ถ้า lower [$E[r_i] < E[r_p]$]
  • long asset i

2.3 The Logic of the APT

$E[r_p] = 15\%$

cost of short sell p is 15%

- Consider the following portfolio
  - Short sell \$180 of GDP factor portfolio
  - Buy \$20 of Inflation factor portfolio
  - Buy (lend) \$60 of RF asset
  - Buy \$100 of asset i. → return: $E[r_i] = 15.7\%$ ⇒ not equilibrium

Short sell P

↓ will start to go down until $E[r_i] = E[r_p]$ equilibrium

- This portfolio has the following characteristic
  - Requires zero initial investment
  - Bears no factor risk (and no idiosyncratic risk)
  - Pays 0.7% for sure

unsystematic risk cancel each other in p & stock i

cannot eliminate unsystematic risk

⇓

APT is approximation

can do on small group not whole mkt like CAPM

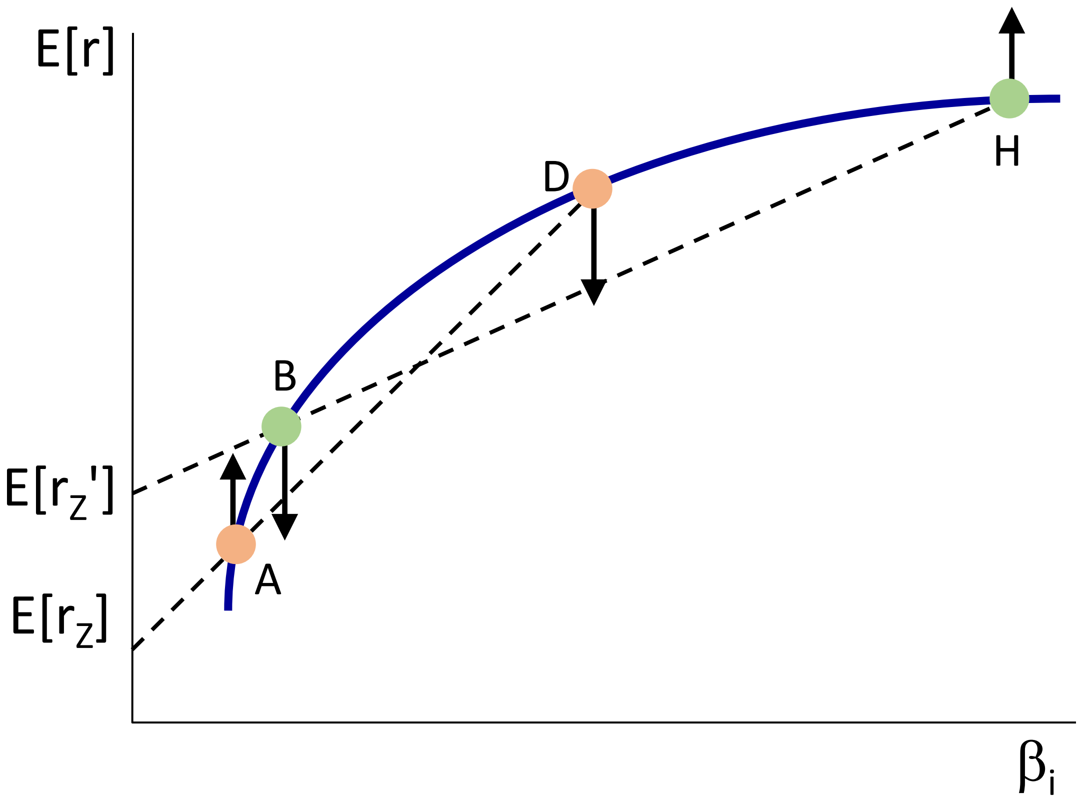
arbitrage

2.3 The Logic of the APT is an approximation

- This would be an arbitrage.
- Hence, in absence of arbitrage, equation [2] must hold.

- What if an asset also bears idiosyncratic risk?
- Since it cannot be replicated by other assets, in particular the factor portfolios, [2] need not hold.
- However, in the presence of idiosyncratic risk, deviations from [2] cannot be pervasive. In other words, for most assets, [2] must be approximately correct.

Infeasible Relationship b/w $E[r_i]$ and β_i a One-Factor Model



The dashed line connecting securities B and H represents various combinations of B and H. $E(R_Z')$ can be obtained from short selling ~~M~~ H and long B such that its systematic risk is zero. Similarly, $E(R_Z)$ can be obtained from short selling D and long A.

An arbitrage portfolio can be obtained by short selling portfolio Z (short A and long D) and long portfolio Z' (short H and long B). This will drive the prices of D and B to rise and A and H to drop.

An Example

↗ short B

overprice

เหมือนถูก discount น้อยไป

- **EX:** Portfolio B: $\beta_B = 0.50$, $E[r_B] = 6\%$, $E[r_M] = 10\%$, $r_F = 4\%$ $(E[R_M] = 6\%)$
  - Replicating portfolio B by forming portfolio D consisting of 50% risk-free asset and 50% market portfolio.

$\beta_p = \sum_i w_i \beta_i$

risk-free 50% คือ mkt port 50% → beta ของ mkt = 1 β_M

$$\beta_D = (0.5\times0)+(0.5\times1.00) = 0.5$$

r_F $E[r_M]$

$$E[r_D] = (0.5\times4)+(0.5\times10) = 7\%$$

→ mimicked port expected return is higher than that of port B

  - Now we can form an arbitrage strategy by short selling portfolio B and take a long position on portfolio D.

$$E[r_{ARB}] = 0.07 - 0.06 = 0.01 \text{ or } 1.00\%$$

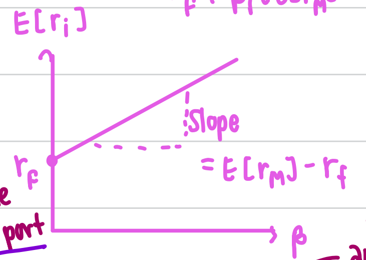
risk-less $\beta_{ARB} = 0.50 - 0.50 = 0$

market return as common factor

when there's only mkt return as common factor → like CAPM $r_i = \underline{E[r_i]} + \beta_i r_M + \varepsilon_i$

$E[r_i] = \lambda_0 + \beta_i \times \lambda_1$

$= r_f + \beta_i (E[r_M] - r_f)$



homogenous

assume everyone & hold mkt port

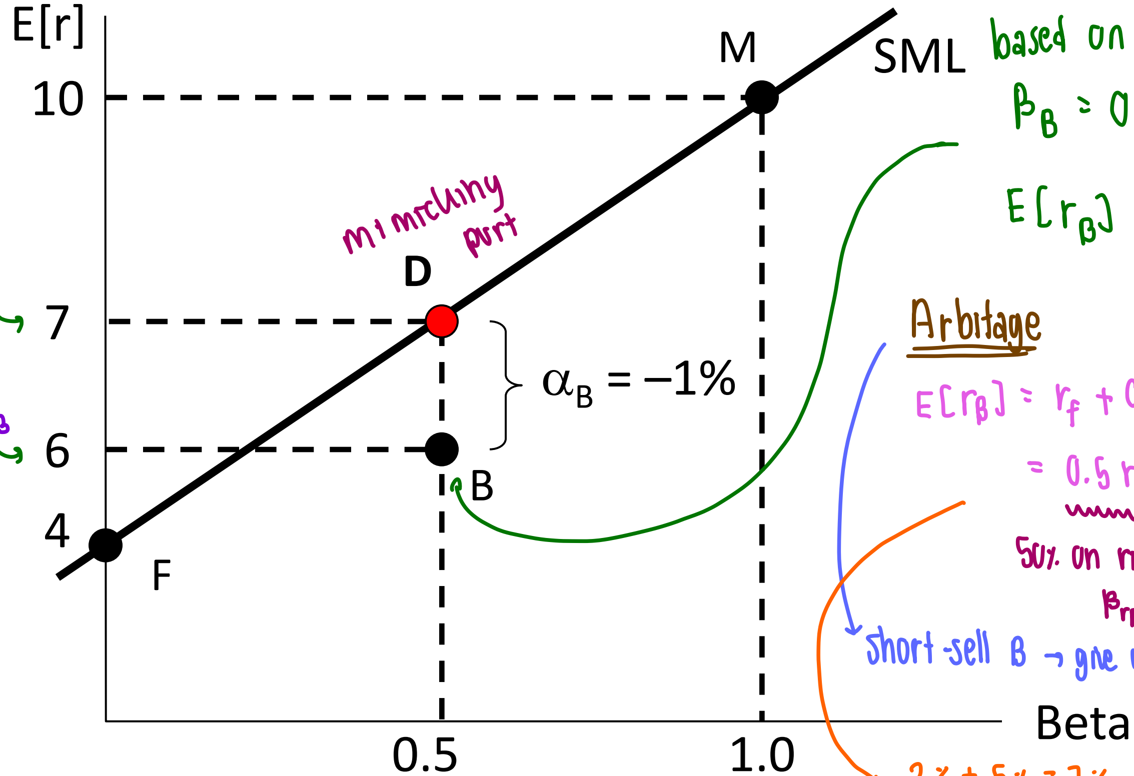
CAPM is a special case of APT ??

factor model & no arbitrage opportunity

No, they're diff in argument

Example: Disequilibrium and Adjustment

r_M : the factor



based on β_B → 7

based on $P_{0,B}$ → 6

based on theory :

$\beta_B = 0.5$

$E[r_B] = 4\% + 0.5(6\%) = 7\%$

Arbitage

$E[r_B] = r_f + 0.5(E[r_M] - r_F)$

$= 0.5 r_f + 0.5 E[r_M]$

50% on risk-free $\beta_{rf} = 0$ — 50% on mkt port $\beta_M = 1$

Short-sell B → give up $E[r_B] = 6\%$

1%

⇓

price of B will start to drop & $E[r_B]$ ↑ as everyone do the same

$2\% + 5\% = 7\%$

The SML is expressed as; $E[r_i]-r_F = \beta_i (E[r_M]-r_F)$

$E[r_B]-r_F = 6-4 = 2 = -1 + 0.5(E[r_M]-r_F)$

Portfolio B's alpha

2.4 The APT vs. CAPM

- When assuming the market model (single factor model), the APT and the CAPM relations are consistent. However, both are two different theories. argument to explain equilibrium
- everyone try to maximize sharpe's ratio
  The CAPM uses "risk-return dominance" argument in support of equilibrium price relation. Aggregate actions by large number of investors are needed to bring back equilibrium.
- The APT uses "no arbitrage" argument. Only actions by a small number of arbitrageur with large transactions are needed to bring back equilibrium.

2.4 The APT vs. CAPM

- The principal strength of the APT approach is that it is based on the no arbitrage argument.
- Because no arbitrage conditions should hold for any subset of securities, it is not necessary to identify the market portfolio to test the APT.
- APT is normally tested over a class of assets such as common stocks.
- However, APT applies to well diversified portfolios and not necessarily to individual stocks. A small number of stocks could have their expected return-beta relationship in violation of the APT.

2.4 The APT vs. CAPM

- The APT is very general. It allows equilibrium to be described in terms of any multi-index model, as opposed to the single-index model in the CAPM.
- However, the APT gives no guidance concerning the determination of the relevant risk factors (F_k) and their risk premium (λ_k).
- In contrast, the CAPM indicates the market portfolio whose risk premium is positive.

2.4 The APT vs. CAPM

- Despite its apparent advantages, the APT does not fully dominate the CAPM.
- Most academia seem to agree that factors other than the market beta are needed to fully describe variation in security returns. But no agreement has been reached on what these extra factors should be.

3. IMPLEMENTATION OF THE APT

3.1 Implementation Process

3.2 Where to Look for Common Factors?

3.3 Smart Betas Mutual Funds

Drawbacks on ATP

① k = ? never tell how many factors should be put in model

② What are common factor? (CAPM use expected return on mkt as common factor)

3.1 The Implementation Process

- Like the CAPM, the APT can be applied to measure fund performance and equity cost of capital.
- The implementation of the APT involves three steps.
  - Identify the factors
    should be macro in nature as it affects all firms
  - Estimate factor risk premia
    risk premium on factor → $(E[r_k] - r_F)$
  - Estimate factor loadings (i.e., betas) of assets. $\beta_{i,k}$

3.2 Where to Look for Common Factors?

- The APT does not spell out what systematic risks are.
- To implement the APT, common factors must be defined. Then, proxies for the factors must be identified.
- It is logical to believe that these factors represent macro risks that affect all securities.
- It is preferable if we could 1) construct common factors that have economic meanings and 2) represent the common factors as risk premium on the risk factor.

3.2 Where to Look for Common Factors?

- Three approaches have been employed in the literature to construct APT factors.
  - Statistic-based factors
    we what is in mkt
    factors analysis, principal component analysis
    derive factor from return of stock
    difficult to draw acedemic meaning from this
  - Macroeconomic-based factors
    GDP curve, credit spread of yield curve
    difficult to interprete in return
    not in form of return on investment
  - Characteristic-based factors.
    - size,
    - MTB
    characteristic of firms
    - Momentum (characteristic of stock return)
    - etc.

Statistical-Based Factors

- Procedure
  - Collect time series of stock returns
  - include all stock in mkt
    Compute the Var-Cov matrix of stock returns
  - to combine stock into port
    Use principal components analysis or factor analysis to extract “factors” from the Var-Cov matrix. A factor is represented by a group (portfolio) of stocks
  - Estimate risk premium associated with each factor
  - Estimate factor loadings for individual assets by a time series regression
- The drawback is that it is very hard to provide economic meanings to the estimated factors.

Macroeconomic-Based Factors

- Elton, Gruber and Mei (1994) use 6 macro factors to represent systematic risks.
  - Unexpected change in yield curve (YC)
  - The level of interest rate (IR)
  - Foreign exchange rate (EX)
  - The business cycle (Y)
  - Inflation rate (INF)
  - Index that summarize other macro factors (OTH)

$$r_i = r_F + \beta_{i,YC} \cdot RP_{YC} + \beta_{i,IR} \cdot RP_{IR} + \beta_{i,EX} \cdot RP_{EX} + \beta_{i,Y} \cdot RP_Y + \beta_{i,INF} \cdot RP_{INF} + \beta_{i,OTH} \cdot RP_{OTH} + \varepsilon_i$$

Macroeconomic-Based Factors

- To estimate the risk premium associated with each risk source;
  - Estimate factor loadings (betas) of a large sample of stocks using time series regression of monthly stock returns against 6 factors.
  - To estimate risk premium of each risk factor, in each month, regress return on each stock against the 6 betas estimated. The estimated coefficients are factor risk premia. To reduce sampling errors, run the regression in each month for 12 months and take the average values of the coefficients.

Estimated Factor Risk Premia and Factor Loadings for Niagara Mohawk (% per month)

measurement is not in asset return ↓ difficult to interprete

Factor	Factor Risk Premium	Factor Betas for Niagara Mohawk
Term structure	0.425	1.0615
Interest rates	−0.051	−2.4167
Exchange rates	−0.049	1.3235
Business cycle	0.041	0.1292
Inflation	−0.069	−0.5220
Other macro factors	0.530	0.3046

$$r_i = r_F + 0.425\beta_{i,TS} - 0.051\beta_{i,IR} - 0.049\beta_{i,EX} + 0.041\beta_{i,Y} - 0.069\beta_{i,INF} + 0.530\beta_{i,OTH}$$

$$= r_F + 0.72\%$$

Characteristic-Based Factors

- Fama and French (1996) propose a three-factor model, where the extra-market factors are represented by firm's characteristics, namely, Size (market capitalization of equity) and BTM (book value over market value of equity).
- The model was motivated by empirical findings that firm's size and BTM have explanatory power on cross-sectional differences in stock returns.
- It is difficult to provide economic meanings to these extra-market factors. For example, how does Size represent significant source of risk common to most investors.

Characteristic-Based Factors

- Fama and French (1996) observe that average returns of small stocks and high BTM stocks are higher than predicted by the SML. Hence, Size and BTM may proxy systematic risk not captured by market beta.

to make it in form of return → pg. 49

- FF (1996) proposed the three-factor model.

$$E[r_i] - r_F = \beta_i \cdot (E[r_M] - r_F) + s_i \cdot E[SMB] + h_i \cdot E[HML]$$

where E[SMB] is E[r] on a portfolio with long positions on small stocks and short positions on big stocks

E[HML] is E[r] on a portfolio with long positions on high BTM stocks and short positions on low BTM stocks

Characteristic-Based Factors

- To create portfolios that track the Size and BTM factors, FF sort firms by Size and BTM. The next table shows how factor risk premium (SMB and HML) are calculated. SMB and HML are constructed as returns on zero-investment portfolios.
- FF then run two-pass regressions to test the three-factor model. The results are shown in the following table.
- They claimed that the three-factor model does not suffer from model miss specification ($a = 0$). All three factors are statistically significant and the R2's are higher than SML.
- FF claimed that the three-factor model perform better than the SML.

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Two-Way Sorting Procedure

small stocks portfolio

big stock portfolio

want only extreme

Book-to-Market Ratio	Size: Small	Size: Medium	Size: Big
High	S/H	M/H	B/H
Medium	S/M	M/M	B/M
Low	S/L	M/L	B/L

① based on BTM first ⇓ further sort it based on size

To construct SMB

- Sort stocks into 3 groups based on BTM from High to Low.
- In each BTM-sorted group, sort stocks into 3 group based on market cap from Small to Big
- SMB_t = difference in returns on portfolios (S/H+S/M+S/L) and (B/H+B/M+B/L)

To construct HML not be related to size

- Similar procedure, but start by sorting stock based on Size from Small to Big
- HML_t = difference in returns on portfolios (S/H+M/H+B/H) and (S/L+M/L+B/L)

β_i $\quad r_{M,t} - r_{F,t}$

s_i $\quad SMB_t \rightarrow \bar{r}_{small} - \bar{r}_{big}$

h_i $\quad HML_t$

from time-series regression

In CAPM → calculate β by → $(r_{i,t} - r_{F,t}) = \alpha_i + \beta_i (r_{M,t} - r_{F,t})$

CAPM alone might undervalue cost of equity as there's no SMB term

$+ s_i SMB_t$

$+ h_i HML_t$

If you ignore this (CAPM)

and for small firm overestimate

e.g. PIT (large stock)

↳ return correlates with β more → SMB ⊖

Results from the 1st Pass Regression

$$(r_{i,t}-r_{F,t}) = a_i + b_i \cdot (r_{M,t}-r_{F,t}) + s_i \cdot (r_{S,t}-r_{B,t}) + h_i \cdot (r_{H,t}-r_{L,t}) + e_{i,t}$$

	B/M	Size	Excess Return	*a*	*b*	*s*	*h*	*t(a)*	*t(b)*	*t(s)*	*t(h)*	R^2
S/L	0.55	22.39	0.61	−0.42	1.06	1.39	0.09	−4.34	30.78	19.23	1.73	0.91
S/M	1.11	22.15	1.05	−0.01	0.97	1.16	0.37	−0.18	53.55	19.49	9.96	0.96
S/H	2.83	19.05	1.24	−0.03	1.03	1.12	0.77	−0.73	67.32	39.21	26.97	0.98
M/L	0.53	55.85	0.70	−0.06	1.04	0.59	−0.12	−1.29	55.83	18.01	−4.30	0.96
M/M	1.07	55.06	0.95	−0.01	1.05	0.47	0.34	−0.15	32.98	17.50	9.50	0.96
M/H	2.18	53.21	1.13	−0.04	1.08	0.53	0.73	−0.90	47.85	8.99	11.12	0.97
B/L	0.43	94.65	0.58	0.02	1.02	−0.10	−0.23	0.88	148.09	−6.88	−13.52	0.98
B/M	1.04	92.06	0.72	−0.09	1.01	−0.14	0.34	−1.76	61.61	−4.96	13.66	0.95
B/H	1.87	89.53	1.00	−0.09	1.06	−0.07	0.84	−1.40	52.12	−0.86	21.02	0.93

Table 13.5

Three-factor regressions for portfolios formed from sorts on size and book-to-market ratio (B/M)

Source: James L. Davis, Eugene F. Fama, and Kenneth R. French, “Characteristics, Covariances, and Average Returns, 1929 to 1997,” *Journal of Finance* 55, no. 1 (2000), pp. 396. Reprinted by the permission of the publisher, Blackwell Publishing, Inc.

FF Three-Factor Model on Amazon

correlated to $r_{B,t}$ rather than $r_{S,t}$

growth

$$(r_{i,t}-r_{F,t}) = a_i + b_i \cdot (r_{M,t}-r_{F,t}) + s_i \cdot (r_{S,t}-r_{B,t}) + h_i \cdot (r_{H,t}-r_{L,t}) + e_{i,t}$$

	Single-Factor Model		Three-Factor Model	
	Regression Coefficient	*t*-Statistic	Regression Coefficient	*t*-Statistic
Intercept (alpha)	1.916%	2.065	1.494%	1.790
$r_M - r_f$	1.533	4.865	1.612	5.866
SMB			−0.689	−2.126
HML			−1.133	−3.304
R-square	.286		.455	
Residual std. dev.	6.864%		6.101%	

coefficient is negative too so that it go together

Table 10.1

Estimates of single-index and three-factor Fama-French regressions for Amazon, monthly data, 5 years ending June 2018.

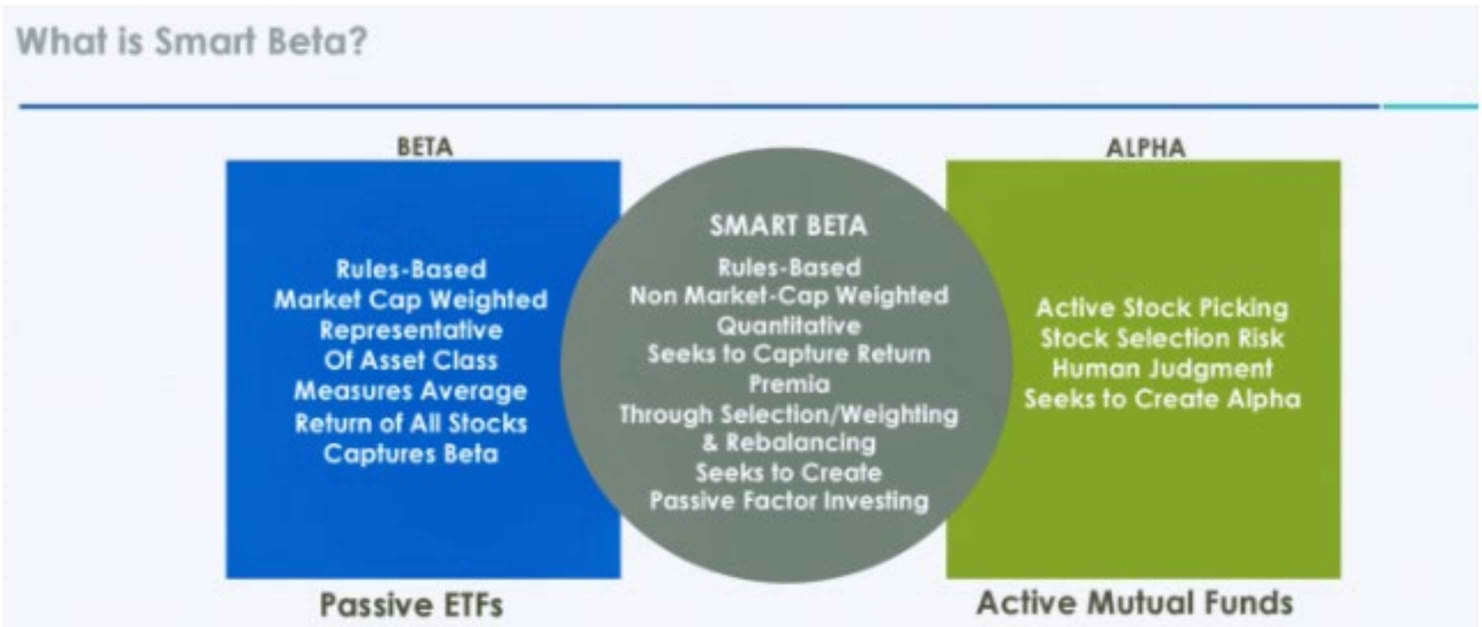
Low BTM = growth

3.3 Smart Betas Mutual Funds

- Implication from multifactor asset pricing models (APT and CAPM extensions)
  - As there are more than one systematic source of risk, investors need to think about how much exposure they wish to establish to each systematic factor in their portfolios.
  - Risk premium comes from exposure to various risk factors. To evaluate investment performance, portfolio alpha need to be calculated controlling for each of them.

3.3 Smart Betas Mutual Funds

- A smart-beta investment strategy is an investment strategy analogous to market index funds.
- While a market index fund tracks performance of a broad market index, a smart-beta fund is designed to provide exposure to specific characteristics, such as size, value, growth, momentum or volatility.
- Smart-beta funds allow investors to tailor portfolio exposure either toward or away from a range of extra-market risk factors using easy-to-trade index-like products.



https://www.riachannel.com/smart-beta-will-lead-great-migration-mutual-funds/