2604639 Finance Theories

Exercises and Solutions for Topics 4-7

Topic 2

CHAPTER 6

Problem Sets

1. (d) While a higher or lower trading costs are not necessarily an indication of an investor's tolerance for risk, risk-averse investors may prefer the portfolios associated with lower trading costs (less actively managed, and therefore less risk). Investors with a higher degree of risk aversion will not want (a) higher risk premiums (as they are associated with higher risks), (b) riskier portfolios (higher standard deviation), or (c) lower Sharpe Ratios (any investor will always prefer investment portfolios with higher Sharpe ratios)

4. a. The expected cash flow is: $(0.5 \times \$70{,}000) + (0.5 \times 200{,}000) = \$135{,}000$.

   With a risk premium of 8% over the risk-free rate of 2%, the required rate of return is 10%. Therefore, the present value of the portfolio is:

   $\$135{,}000/1.10 = \mathbf{\$122{,}727}$

   b. If the portfolio is purchased for \$122,727 and provides an expected cash inflow of \$135,000, then the expected rate of return ($E([r])$ is as follows:

   $\$122{,}727 \times (1 + E[r]) = \$135{,}000$

   Therefore, $E(r)$ = **10%**. The portfolio price is set to equate the expected rate of return with the required rate of return.

   c. If the risk premium over T-bills is now 12%, then the required return is:

   $2\% + 12\% = 14\%$

   The present value of the portfolio is now: $\$135{,}000/1.14 = \mathbf{\$118{,}421}$

   d. For a given expected cash flow, portfolios that command greater risk premiums must sell at lower prices. The extra discount from expected value is a penalty for risk.

5. When we specify utility by $U = E(r) - 0.5A\sigma^2$, the utility level for T-bills is: 0.02

   The utility level for the risky portfolio is:

   $U = 0.07 - 0.5 \times A \times (0.18)^2 = 0.07 - 0.0162 \times A$

   In order for the risky portfolio to be preferred to T-bills, the following must hold:

   $0.07 - 0.0162 \times A > 0.02 \rightarrow A < 0.05/0.0162 = \mathbf{3.09}$

A must be less than 3.09 for the risky portfolio to be preferred to T-bills.

Appendix

1. By year-end, the $50,000 investment will grow to: $50,000 × 1.06 = $53,000
   Without insurance, the probability distribution of end-of-year wealth is:

	Probability	Wealth
No fire	0.999	$253,000
Fire	0.001	53,000

For this distribution, expected utility is computed as follows:

$$E(U[W]) = (0.999 \times ln[253{,}000]) + (0.001 \times ln[53{,}000]) = 12.439582$$

The certainty equivalent is:

$$W_{CE} = e^{12.439582} = \$252{,}604.85$$

With fire insurance, at a cost of $*P*, the investment in the risk-free asset is:

$$\$(50{,}000 - P)$$

Year-end wealth will be certain (since you are fully insured) and equal to:

$$(\$[50{,}000 - P] \times 1.06) + \$200{,}000$$

Solve for P in the following equation:

$$(\$[50{,}000 - P] \times 1.06) + \$200{,}000 = \$252{,}604.85 \Rightarrow P = \$372.78$$

This is the most you are willing to pay for insurance. Note that the expected loss is "only" $200, so you are willing to pay a substantial risk premium over the expected value of losses. The primary reason is that the value of the house is a large proportion of your wealth.

2. a. With insurance coverage for one-half the value of the house, the premium is $100, and the investment in the safe asset is $49,900. By year-end, the investment of $49,900 will grow to: $49,900 × 1.06 = $52,894

   If there is a fire, your insurance proceeds will be $100,000, and the probability distribution of end-of-year wealth is:

	Probability	Wealth
No fire	0.999	$252,894
Fire	0.001	152,894

For this distribution, expected utility is computed as follows:

$$E(U[W]) = (0.999 \times ln[252{,}894]) + (0.001 \times ln[152{,}894]) = 12.4402225$$

The certainty equivalent is:

$$W_{CE} = e^{12.4402225} = \$252{,}766.77$$

b. With insurance coverage for the full value of the house, costing \$200, end-of-year wealth is certain, and equal to:

$$([\$50{,}000 - \$200] \times 1.06) + \$200{,}000 = \$252{,}788$$

Since wealth is certain, this is also the certainty equivalent wealth of the fully insured position.

c. With insurance coverage for 1½ times the value of the house, the premium is \$300, and the insurance pays off \$300,000 in the event of a fire. The investment in the safe asset is \$49,700. By year-end, the investment of \$49,700 will grow to: \$49,700 × 1.06 = \$52,682.

The probability distribution of end-of-year wealth is:

	Probability	Wealth
No fire	0.999	\$252,682
Fire	0.001	352,682

For this distribution, expected utility is computed as follows:

$$E(U[W]) = (0.999 \times ln[252{,}682]) + (0.001 \times ln[352{,}682]) = 12.4402205$$

The certainty equivalent is:

$$W_{CE} = e^{12.440222} = \$252{,}766.27$$

Therefore, full insurance dominates both over- and underinsurance. Overinsuring creates a gamble (you gain when the house burns down). Risk is minimized when you insure exactly the value of the house.

Topic 4

CHAPTER 6

Problem Sets

6. Answers will vary. Points on the curve are derived by solving for $E(r)$ in the following equation:
   $U = 0.02 = E(r) - 0.5A\sigma^2 = E(r) - 0.5 \times 3 \times \sigma^2$

   The values of $E(r)$, given the values of σ^2, are therefore:

σ	σ^2	$E(r)$
0.00	0.0000	0.0200
0.05	0.0025	0.0238
0.10	0.0100	0.0350
0.15	0.0225	0.0538
0.20	0.0400	0.0800
0.25	0.0625	0.1138

   The bold line in the graph on the next page (labeled Q6, for Question 6) depicts the indifference curve.

7. Repeating the analysis in Problem 6, utility is now:

   $$U = E(r) - 0.5A\sigma^2 = E(r) - 0.5 \times 4 \times \sigma^2 = 0.02$$

   The equal–utility combinations of expected return and standard deviation are presented in the table below. The indifference curve is the upward sloping line in the graph on the next page, labeled Q7 (for Question 7).

σ	σ^2	$E(r)$
0.00	0.0000	0.0200
0.05	0.0025	0.0250
0.10	0.0100	0.0400
0.15	0.0225	0.0650
0.20	0.0400	0.1000
0.25	0.0625	0.1450

   The indifference curve in Problem 7 differs from that in Problem 6 in slope. When A increases from 3 to 4, the increased risk aversion results in a greater slope for the indifference curve since more expected return is needed to compensate for additional σ.

13. Expected return = (0.7 × 12%) + (0.3 × 2%) = **9%**

    Standard deviation = 0.7 × 28% = **19.6%**

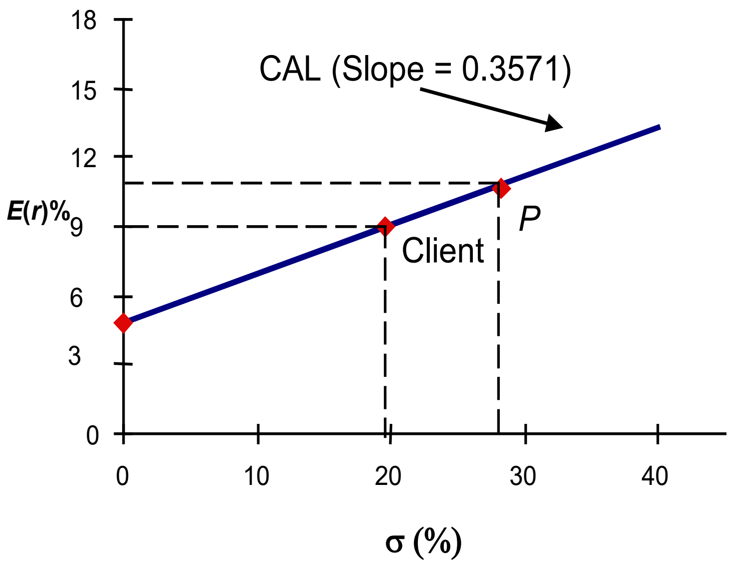
14. Investment proportions: **30.0%** in T-bills

$0.7 \times 25\% =$	**17.5%** in Stock A
$0.7 \times 32\% =$	**22.4%** in Stock B
$0.7 \times 43\% =$	**30.1%** in Stock C

15. Your reward-to-volatility (Sharpe) ratio: $S = \dfrac{0.12 - 0.02}{0.28} = \mathbf{0.3571}$

Client's reward-to-volatility (Sharpe) ratio: $S = \dfrac{0.09 - 0.02}{0.196} = \mathbf{0.3571}$

16.



17. a. $E(r_C) = r_f + y \times (E[r_P] - r_f) = 0.02 + y \times (0.12 - 0.02)$

If the expected return for the portfolio is 16%, then:

$$10\% = 2\% + 10\% \times y \Rightarrow y = \frac{0.10 - 0.02}{0.10} = \mathbf{0.8}$$

Therefore, in order to have a portfolio with expected rate of return equal to 10%, the client must invest 80% of total funds in the risky portfolio and 20% in T-bills.

b.

Client's investment proportions: **20.0%** in T-bills

$0.8 \times 25\% =$	**20.0%** in Stock A

$0.8 \times 32\% =$ **25.6%** in Stock B

$0.8 \times 43\% =$ **34.4%** in Stock C

c. $\sigma_C = 0.8 \times \sigma_P = 0.8 \times 28\% =$ **22.4%**

18. a. $\sigma_C = y \times 28\%$

If your client prefers a standard deviation of at most 12%, then:

$y = 0.12/0.28 = 0.4286 =$ **42.86%** invested in the risky portfolio.

b. $E(r_C) = 0.02 + 0.1 \times y = 0.02 + (0.1 \times 0.4286) = 0.0629 =$ **6.29%**

19. a. $$y^* = \frac{E(r_P) - r_f}{A\sigma_P^2} = \frac{0.18 - 0.08}{3.5 \times 0.28^2} = \frac{0.10}{0.2744} = 0.3644$$

Therefore, the client's optimal proportions are: **36.44%** invested in the risky portfolio and **63.56%** invested in T-bills.

b. $E(r_C) = 0.08 + 0.10 \times y^{\times} = 0.08 + (0.3644 \times 0.1) = 0.1164$ or **11.644%**

$\sigma_C = 0.3644 \times 28 =$ **10.203%**

CFA Questions

4. Indifference curve 2 because it is tangent to the CAL.

5. Point E

8. Expected return for equity fund = T-bill rate + Risk premium = 6% + 10% = 16%

Expected rate of return of the client's portfolio = (0.6 × 16%) + (0.4 × 6%) = 12%

Expected return of the client's portfolio = 0.12 × $100,000 = $12,000

(which implies expected total wealth at the end of the period = $112,000)

Standard deviation of client's overall portfolio = 0.6 × 14% = 8.4%

CHAPTER 7

Problem Sets

4. The parameters of the opportunity set are:

$E(r_S) = 20\%$, $E(r_B) = 12\%$, $\sigma_S = 30\%$, $\sigma_B = 15\%$, $\rho = 0.10$

From the standard deviations and the correlation coefficient, we generate the following covariance matrix (note that $\text{Cov}[r_S, r_B] = \rho \times \sigma_S \times \sigma_B$):

	Bonds	Stocks
Bonds	225	45
Stocks	45	900

The minimum variance portfolio is computed as follows:

$$w_{\text{Min}}(S) = \frac{\sigma_B^2 - \text{Cov}(r_S, r_B)}{\sigma_S^2 + \sigma_B^2 - 2\text{Cov}(r_S, r_B)} = \frac{225 - 45}{900 + 225 - (2 \times 45)} = 0.1739$$

$w_{\text{Min}}(B) = 1 - 0.1739 = 0.8261$

The minimum variance portfolio mean and standard deviation are:

$E(r_{\text{Min}}) = (0.1739 \times 0.20) + (0.8261 \times 0.12) = 0.1339 = 13.39\%$

$$\sigma_{\text{Min}} = (w_S^2\sigma_S^2 + w_B^2\sigma_B^2 + 2w_S w_B \text{Cov}[r_S, r_B])^{1/2}$$

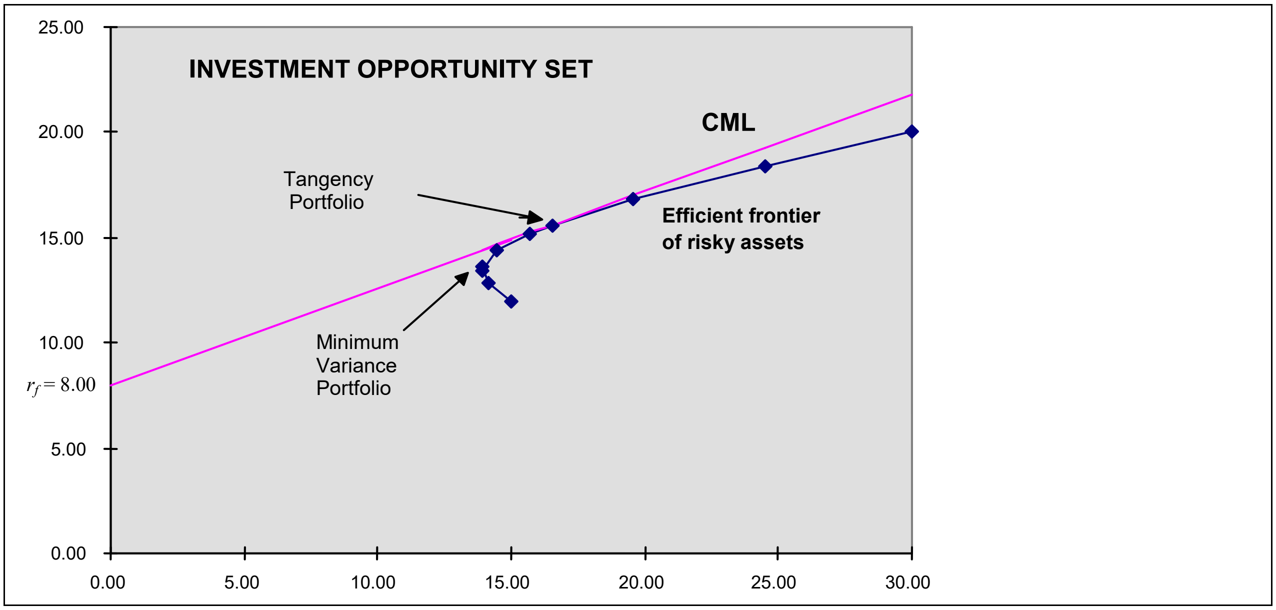
$$= ([0.1739^2 \times 900] + [0.8261^2 \times 225] + [2 \times 0.1739 \times 0.8261 \times 45])^{1/2}$$

= **13.92%**

5.

Proportion in Stock Fund	Proportion in Bond Fund	Expected Return	Standard Deviation	
0.00%	100.00%	12.00%	15.00%	
17.39	82.61	13.39	13.92	minimum variance
20.00	80.00	13.60	13.94	
40.00	60.00	15.20	15.70	
45.16	54.84	15.61	16.54	tangency portfolio
60.00	40.00	16.80	19.53	
80.00	20.00	18.40	24.48	
100.00	0.00	20.00	30.00	

Graph shown below.



6. See the graph (Problem 5). It indicates that the optimal portfolio is the tangency portfolio with expected return approximately **15.6%** and standard deviation approximately **16.5%**.

7. The proportion of the optimal risky portfolio invested in the stock fund is given by:

$$w_S = \frac{(E[r_S]-r_f)\times\sigma_B^2-(E[r_B]-r_f)\times\mathrm{Cov}(r_S,r_B)}{(E[r_S]-r_f)\times\sigma_B^2+(E[r_B]-r_f)\times\sigma_S^2-(E[r_S]-r_f+E[r_B]-r_f)\times\mathrm{Cov}(r_S,r_B)}$$

$$= \frac{([.20-.08]\times 225)-([.12-.08]\times 45)}{([.20-.08]\times 225)+([.12-.08]\times 900)-([.20-.08+.12-.08]\times 45)} = \mathbf{0.4516}$$

$$w_B = 1-0.4516 = \mathbf{0.5484}$$

The mean and standard deviation of the optimal risky portfolio are:

$E(r_P) = (0.4516 \times .20) + (0.5484 \times .12) = .1561$ or **15.61%**

$\sigma_P = ([0.4516^2 \times 900] + [0.5484^2 \times 225] + [2 \times 0.4516 \times 0.5484 \times 45])^{1/2} =$ **16.54%**

Note: This is consistent with the graphical approximation in Problem 6.

8. Using the calculations from Problems 5 and 7, the reward-to-volatility ratio of the optimal CAL is:

$$\frac{E(r_p)-r_f}{\sigma_p} = \frac{.1561-.08}{.1654} = 0.4603$$

Note: Slight differences may be due to rounding intermediate steps.

9. a. If you require that your portfolio yield an expected return of 14%, then you can find the corresponding standard deviation from the optimal CAL:

$$.14 = E(r_C) = r_f + \frac{E(r_p) - r_f}{\sigma_P}\sigma_C = .08 + \frac{.1561 - .08}{.1654} \times \sigma_c = .08 + 0.4603 \times \sigma_C \rightarrow$$

$$\sigma_C = .1303$$

If $E(r_C)$ = 14%, then the standard deviation of the portfolio is **13.03%**.

b. To find the proportion invested in the money market fund, remember that the mean of the complete portfolio (here, 14%) is an average of the T-bill rate and the optimal combination of stocks and bonds (P). Let y be the proportion invested in the portfolio P. The mean of any portfolio along the optimal CAL is:

$$E(r_C) = (1 - y) \times r_f + y \times E(r_P) = r_f + y \times (E[r_P] - r_f) = .08 + y \times (.1561 - .08)$$

Setting $E(r_c)$ = 14%, we find: y = 0.7881 and $(1 - y)$ = **0.2119** (the proportion invested in the money market fund).

To find the proportions invested in each of the funds, multiply 0.7884 times the respective proportions of stocks and bonds in the optimal risky portfolio:

Proportion of stocks in complete portfolio = 0.7884 × 0.4516 = **0.3559**

Proportion of bonds in complete portfolio = 0.7884 × 0.5484 = **0.4322**

Note: Slight differences may be due to rounding intermediate steps.

10. Using only the stock and bond funds to achieve a portfolio expected return of 14%, we must find the appropriate proportion in the stock fund (w_S) and the appropriate proportion in the bond fund ($w_B = 1 - w_S$) as follows:

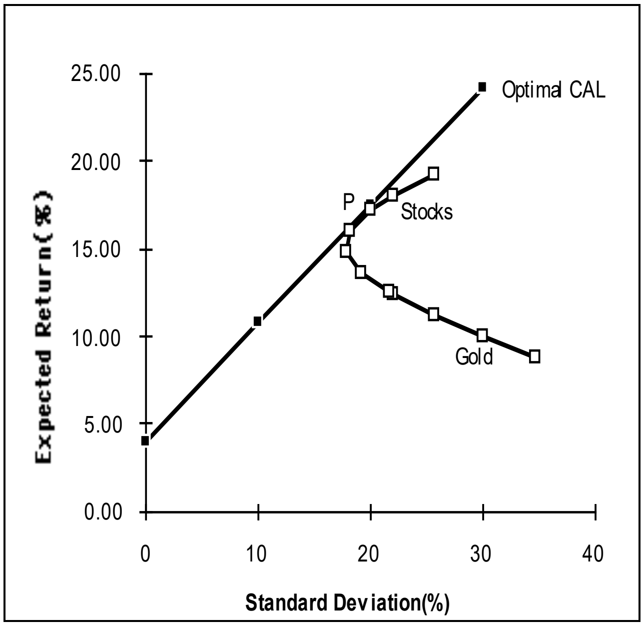
$$0.14 = 0.20 \times w_s + 0.12 \times (1 - w_s) = 0.12 + 0.08 \times w_s \Rightarrow w_s = 0.25$$

The proportions are 25% invested in the stock fund and 75% in the bond fund. The standard deviation of this portfolio will be:

$$\sigma_P = ([0.25^2 \times 900] + [0.75^2 \times 225] + [2 \times 0.25 \times 0.75 \times 45])^{1/2} = \mathbf{14.13\%}$$

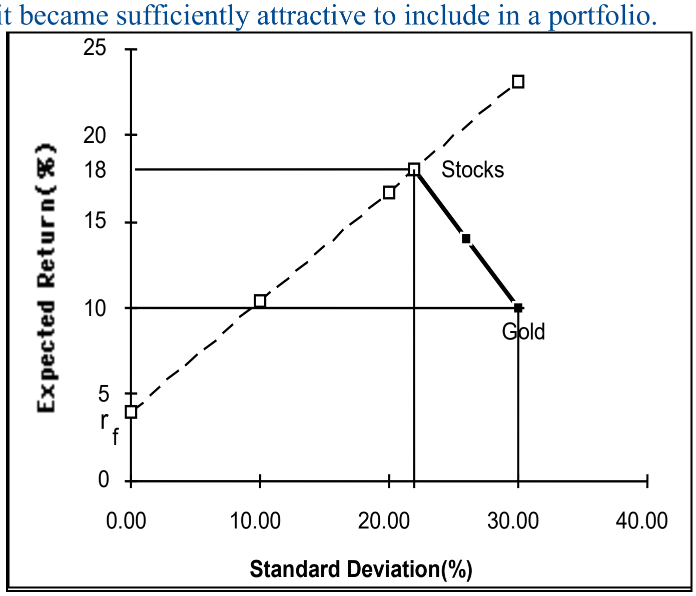
This is considerably greater than the standard deviation of 13.03% achieved using T-bills and the optimal portfolio.

11.



a. Even though it seems that gold is dominated by stocks, gold might still be an attractive asset to hold as a *part* of a portfolio. If the correlation between gold and stocks is sufficiently low, gold will be held as a component in a portfolio, specifically, the optimal tangency portfolio.

b. If the correlation between gold and stocks equals +1, then no one would hold gold. The optimal CAL would be composed of bills and stocks only. Since the set of risk/return combinations of stocks and gold would plot as a straight line with a negative slope (see the following graph), these combinations would be dominated by the stock portfolio. Of course, this situation could not persist. If no one desired gold, its price would fall, and its expected rate of return would increase until it became sufficiently attractive to include in a portfolio.



13. False. If the borrowing and lending rates are not identical, then, depending on the tastes of the individuals (that is, the shape of their indifference curves), borrowers and lenders could have

different optimal risky portfolios.

17. The correct choice is (c). Intuitively, we note that since all stocks have the same expected rate of return and standard deviation, we choose the stock that will result in lowest risk. This is the stock that has the lowest correlation with Stock A.

    More formally, we note that when all stocks have the same expected rate of return, the optimal portfolio for any risk-averse investor is the global minimum variance portfolio (*G*). When the portfolio is restricted to Stock A and one additional stock, the objective is to find *G* for any pair that includes Stock A and then select the combination with the lowest variance. With two stocks, I and J, the formula for the weights in *G* is:

$$w_{\text{Min}}(I) = \frac{\sigma_{\text{J}}^2 - \text{Cov}(r_{\text{I}}, r_{\text{J}})}{\sigma_{\text{I}}^2 + \sigma_{\text{J}}^2 - 2\text{Cov}(r_{\text{I}}, r_{\text{J}})}$$

$$w_{\text{Min}}(J) = 1 - w_{\text{Min}}(I)$$

    Since all standard deviations are equal to 20%:

$$\text{Cov}(r_{\text{I}}, r_{\text{J}}) = \rho\sigma_{\text{I}}\sigma_{\text{J}} = 400\rho \text{ and } w_{\text{Min}}(I) = w_{\text{Min}}(J) = 0.5$$

    This intuitive result is an implication of a property of any efficient frontier, namely, that the covariances of the global minimum variance portfolio with all other assets on the frontier are identical and equal to its own variance. (Otherwise, additional diversification would further reduce the variance.) In this case, the standard deviation of *G*(*I*, *J*) reduces to:

$$\sigma_{\text{Min}}(G) = (200 \times [1 + \rho_{\text{IJ}}])^{1/2}$$

    This leads to the intuitive result that the desired addition would be the stock with the lowest correlation with Stock A, which is **Stock D**. The optimal portfolio is equally invested in Stock A and Stock D, and the standard deviation is **17.03%.**

18. No, the answer to Problem 17 would not change, at least if investors are not risk lovers. Risk-neutral investors would not care which portfolio they held since all portfolios have an expected return of 8%.

19. Yes, the answers to Problems 17 and 18 would change. The efficient frontier of risky assets is horizontal at 8%, so the optimal CAL runs from the risk-free rate through G. This implies risk-averse investors will just hold Treasury bills.

Topic 5

CHAPTER 8

Problem Sets

1. The advantage of the index model, compared to the Markowitz procedure, is the vastly reduced number of estimates required. In addition, the large number of estimates required for the Markowitz procedure can result in large aggregate estimation errors when implementing the procedure. The disadvantage of the index model arises from the model's assumption that return residuals are uncorrelated. This assumption will be incorrect if the index used omits a significant risk factor.

2. The trade-off entailed in departing from pure indexing in favor of an actively managed portfolio is between the probability (or the possibility) of superior performance against the certainty of additional management fees.

3. The answer to this question can be seen from the formulas for w^0 (Equation 8.24) and w^* (Equation 8.25). Other things held equal, w^0 is smaller the greater the residual variance of a candidate asset for inclusion in the portfolio. Further, we see that regardless of beta, when w^0 decreases, so does w^*. Therefore, other things equal, the greater the residual variance of an asset, the smaller its position in the optimal risky portfolio. That is, increased firm-specific risk reduces the extent to which an active investor will be willing to depart from an indexed portfolio.

4. The total risk premium equals: $\alpha + (\beta \times \text{Market risk premium})$. We call alpha a nonmarket return premium because it is the portion of the return premium that is independent of market performance.

   The Sharpe ratio indicates that a higher alpha makes a security more desirable. Alpha, the numerator of the Sharpe ratio, is a fixed number that is not affected by the standard deviation of returns, the denominator of the Sharpe ratio. Hence, an increase in alpha increases the Sharpe ratio. Since the portfolio alpha is the portfolio-weighted average of the securities' alphas, then, holding all other parameters fixed, an increase in a security's alpha results in an increase in the portfolio Sharpe ratio.

5. a. To optimize this portfolio, one would need:

   $n = 60$ estimates of means

   $n = 60$ estimates of variances

   $$\frac{n^2 - n}{2} = 1,770 \text{ estimates of covariances}$$

   Therefore, in total: $\dfrac{n^2 + 3n}{2} =$ **1,890** estimates

   b. In a single index model: $r_i - r_f = \alpha_i + \beta_i (r_M - r_f) + e_i$

   Equivalently, using excess returns: $R_i = \alpha_i + \beta_i R_M + e_i$

The variance of the rate of return can be decomposed into the components:

(l) The variance due to the common market factor: $\beta_i^2\sigma_M^2$

(2) The variance due to firm specific unanticipated events: $\sigma^2(e_i)$

In this model: $\text{Cov}(r_i, r_j) = \beta_i \beta_j \sigma$

The number of parameter estimates is:

n = 60 estimates of the mean $E(r_i)$

n = 60 estimates of the sensitivity coefficient β_i

n = 60 estimates of the firm-specific variance $\sigma^2(e_i)$

1 estimate of the market mean $E(r_M)$

1 estimate of the market variance σ_M^2

Therefore, in total, **182** estimates.

The single index model reduces the total number of required estimates from 1,890 to 182. In general, the number of parameter estimates is reduced from:

$$\left(\frac{n^2 + 3n}{2}\right) \text{ to } (3n + 2)$$

6. a. The standard deviation of each individual stock is given by:

$$\sigma_i = (\beta_i^2\sigma_M^2 + \sigma^2[e_i])^{1/2}$$

Since $\beta_A = 0.8$, $\beta_B = 1.2$, $\sigma(e_A) = 30\%$, $\sigma(e_B) = 40\%$, and $\sigma_M = 22\%$, we get:

$\sigma_A = (0.8^2 \times 22^2 + 30^2)^{1/2} =$ **34.78%**

$\sigma_B = (1.2^2 \times 22^2 + 40^2)^{1/2} =$ **47.93%**

b. The expected rate of return on a portfolio is the weighted average of the expected returns of the individual securities:

$E(r_P) = w_A \times E(r_A) + w_B \times E(r_B) + w_f \times r_f$

$E(r_P) = (0.30 \times 13\%) + (0.45 \times 18\%) + (0.25 \times 8\%) =$ **14%**

The beta of a portfolio is similarly a weighted average of the betas of the individual securities:

$\beta_P = w_A \times \beta_A + w_B \times \beta_B + w_f \times \beta_f$

$\beta_P = (0.30 \times 0.8) + (0.45 \times 1.2) + (0.25 \times 0.0) =$ **0.78**

The variance of this portfolio is:

$$\sigma_P^2 = \beta_P^2 \sigma_M^2 + \sigma^2(e_P)$$

where $\beta_P^2 \sigma_M^2$ is the systematic component and $\sigma^2(e_P)$ is the nonsystematic component.

Since the residuals (e_i) are uncorrelated, the nonsystematic variance is:

$$\sigma^2(e_P) = w_A^2 \times \sigma^2(e_A) + w_B^2 \times \sigma^2(e_B) + w_f^2 \times \sigma^2(e_f)$$

$$= (0.30^2 \times 30^2) + (0.45^2 \times 40^2) + (0.25^2 \times 0) = 405$$

where $\sigma^2(e_A)$ and $\sigma^2(e_B)$ are the firm-specific (nonsystematic) variances of Stocks A and B, and $\sigma^2(e_f)$, the nonsystematic variance of T-bills, is zero. The residual standard deviation of the portfolio is thus:

$\sigma(e_P) = (405)^{1/2} =$ **20.12%**

The total variance of the portfolio is then:

$$\sigma_P^2 = (0.78^2 \times 22^2) + 405 = 699.47 \rightarrow \sigma_P = 0.2645$$

The total standard deviation is **26.45%.**

7. a. The two figures depict the stocks' security characteristic lines (SCL). Stock A has higher firm-specific risk because the deviations of the observations from the SCL are larger for Stock A than for Stock B. Deviations are measured by the vertical distance of each observation from the SCL.

b. Beta is the slope of the SCL, which is the measure of systematic risk. The SCL for Stock B is steeper; hence Stock B's systematic risk is greater.

The R^2 (or squared correlation coefficient) of the SCL is the ratio of the explained variance of the stock's return to total variance, and the total variance is the sum of the explained variance plus the unexplained variance (the stock's residual variance):

$$R^2 = \frac{\beta_i^2 \sigma_M^2}{\beta_i^2 \sigma_M^2 + \sigma^2(e_i)}$$

c. Since the explained variance for Stock B is greater than for Stock A (the explained variance is $\beta_B^2 \sigma_M^2$, which is greater since its beta is higher), *and* its residual variance $\sigma^2(e_B)$ is smaller, its R^2 is higher than Stock A's.

d. Alpha is the intercept of the SCL with the expected return axis. Stock A has a small positive alpha, whereas Stock B has a negative alpha; hence, Stock A's alpha is larger.

e. The correlation coefficient is simply the square root of R2, so Stock B's correlation with the market is higher.

9. The standard deviation of each stock can be derived from the following equation for R^2:

$$R_i^2 = \frac{\beta_i^2 \sigma_M^2}{\sigma_i^2} = \frac{Explainedvariance}{Totalvariance}$$

Therefore:

$$\sigma_A^2 = \frac{\beta_A^2 \sigma_M^2}{R_A^2} = \frac{0.7^2 \times .20^2}{0.20} = .098$$

$$\sigma_A = .3130 = \mathbf{31.30\%}$$

For stock B:

$$\sigma_B^2 = \frac{\beta_B^2 \sigma_M^2}{R_B^2} = \frac{1.2^2 \times .20^2}{0.12} = .048$$

$$\sigma_B = .6928 = \mathbf{69.26\%}$$

10. The systematic risk for A is:

$$\beta_A^2 \times \sigma_M^2 = 0.70^2 \times 20^2 = \mathbf{196}$$

The firm-specific risk of A (the residual variance) is the difference between A's total risk and its systematic risk:

$$980 - 196 = \mathbf{784}$$

The systematic risk for B is:

$$\beta_B^2 \times \sigma_M^2 = 1.20^2 \times 20^2 = \mathbf{576}$$

B's firm-specific risk (residual variance) is:

$$4{,}800 - 576 = \mathbf{4{,}224}$$

11. The covariance between the returns of A and B is (residuals are uncorrelated):

$$\text{Cov}(r_A, r_B) = \beta_A \beta_B \sigma_M^2 = 0.70 \times 1.20 \times .04 = \mathbf{0.0336}$$

The correlation coefficient between the returns of A and B is (standard deviation from Problem 9):

$$\rho_{AB} = \frac{\text{Cov}(r_A r_B)}{\sigma_A \sigma_B} = \frac{.0336}{.3130 \times .6928} = \mathbf{0.1549}$$

12. Note that the correlation is the square root of R^2: $\rho = \sqrt{R^2}$

$$\text{Cov}(r_{A,}r_M) = \rho\sigma_A\sigma_M = 0.20^{1/2} \times 31.30 \times 20 = 280$$

$$\text{Cov}(r_{B,}r_M) = \rho\sigma_B\sigma_M = 0.12^{1/2} \times 69.28 \times 20 = 480$$

13. For portfolio P we can compute:

$$R_i^2 = \frac{\beta_i^2\sigma_M^2}{\sigma_i^2} = \frac{\text{Explained variance}}{\text{Total variance}}$$

Therefore:

$$\sigma_A^2 = \frac{\beta_A^2\sigma_M^2}{R_A^2} = \frac{0.7^2 \times .20^2}{0.20} = .098$$

$\sigma_A = .3130$ **= 31.30%**

For stock B:

$$\sigma_B^2 = \frac{\beta_B^2\sigma_M^2}{R_B^2} = \frac{1.2^2 \times .20^2}{0.12} = .048$$

$\sigma_B = .6928$ **= 69.26%**

$\sigma_P = ([0.6^2 \times 980] + [0.4^2 \times 4800] + [2 \times 0.4 \times 0.6 \times 336])^{1/2} = \mathbf{1282.08}^{1/2} = \mathbf{35.81\%}$

$\beta_P = (0.6 \times 0.7) + (0.4 \times 1.2) = \mathbf{0.90}$

$$\sigma^2(e_P) = \sigma_P^2 - \beta_P^2\sigma_M^2 = 1282.08 - (0.90^2 \times 400) = 958.08$$

$\text{Cov}(r_{P,}r_M) = \beta_P\,\sigma_M^2 = 0.90 \times 400 = 360$

This same result can also be attained using the covariances of the individual stocks with the market:

$\text{Cov}(r_{P,}r_M) = \text{Cov}(0.6r_A + 0.4r_B, r_M) = 0.6 \times \text{Cov}(r_A, r_M) + 0.4 \times \text{Cov}(r_B,r_M)$

$= (0.6 \times 280) + (0.4 \times 480) = \mathbf{360}$

14. Note that the variance of T-bills is zero, and the covariance of T-bills with any asset is zero (calculations from previous Problems 9–13). Therefore, for portfolio Q:

$$\sigma_Q = (w_P^2\sigma_P^2 + w_M^2\sigma_M^2 + 2 \times w_P \times w_M \times \text{Cov}[r_P, r_M])^{1/2}$$

$$= \left([0.5^2 \times 1{,}282.08] + [0.3^2 \times 400] + [2 \times 0.5 \times 0.3 \times 360]\right)^{1/2} = \mathbf{21.55\%}$$

$$\beta_Q = w_P\beta_P + w_M\beta_M = (0.5 \times 0.90) + (0.3 \times 1) + (0.20 \times 0) = \mathbf{0.75}$$

$$\sigma^2(e_Q) = \sigma_Q^2 - \beta_Q^2\sigma_M^2 = 464.52 - (0.75^2 \times 400) = \mathbf{239.52}$$

$$\text{Cov}(r_Q, r_M) = \beta_Q \sigma_M^2 = 0.75 \times 400 = \mathbf{300}$$

Topic 6

CHAPTER 9

Problem Sets

3. a. False. $\beta = 0$ implies $E(r) = r_f$, not zero.

   b. False. Investors require a risk premium only for bearing systematic (undiversifiable or market) risk. Total volatility, as measured by the standard deviation, includes diversifiable risk.

   c. False. Your portfolio should be invested 75% in the market portfolio and 25% in T-bills. Then: $\beta_P = (0.75 \times 1) + (0.25 \times 0) = 0.75$

7. Correct answer is choice a. Beta is a measure of systematic risk. Since only systematic risk is rewarded, it is safe to conclude that the expected return will be higher for Kaskin's stock than for Quinn's stock.

8. a. The CAPM states that the appropriate discount rate for the project is:

   $r_f + \beta \times (E[r_M] - r_f) = 0.08 + (1.8 \times [0.16 - 0.08]) = 0.224$ or 22.4%

   Using this discount rate:

$$\text{NPV} = -\$40 + \sum_{t=1}^{10} \frac{\$15}{1.224^t} = -\$40 + (\$15 \times \text{Annuity factor [22.4\%, 10 years]}) = \mathbf{\$18.09}$$

   b. The internal rate of return (IRR) for the project is 35.73%:

$$\text{NPV} = -\$40 + \sum_{t=1}^{10} \frac{\$15}{\text{IRR}^t} = 0 \rightarrow \text{IRR} = .3573 \text{ or } 35.73\%$$

   Recall from your introductory finance class that NPV is positive if IRR > discount rate (or, equivalently, hurdle rate). The highest value that beta can take before the hurdle rate exceeds the IRR is determined by:

   $0.3573\ 0.08 + \beta \times (0.16 - 0.08) \rightarrow \beta = 0.2773/0.08 = \mathbf{3.47}$

17. Since the stock's beta is equal to 1.2, its expected rate of return is:

   $0.06 + 1.2 \times (0.16 - 0.06) = 18\%$

$$E(r) = \frac{D_1 + P_1 - P_0}{P_0} \rightarrow 0.18 = \frac{P_1 - \$50 + \$6}{\$50} \rightarrow P_1 = \mathbf{\$53}$$

18. The series of $1,000 payments is a perpetuity. If beta is 0.5, the cash flow should be discounted at the rate:

   $0.06 + (0.5 \times [0.16 - 0.06]) = 0.11 = 11\%$

   PV = $1,000/0.11 = $9,090.91

If, however, beta is equal to 1, then the investment should yield 16%, and the price paid for the firm should be:

$$PV = \$1{,}000/0.16 = \$6{,}250$$

The difference, **\$2,840.91**, is the amount I will overpay if I erroneously assume that beta is 0.5 rather than 1.

19. Using the SML: $0.04 = 0.06 + \beta \times (0.16 - 0.06) \Rightarrow \beta = -0.02/0.10 = \mathbf{-0.2}$

20. $r_1 = 19\%$; $r_2 = 16\%$; $\beta_1 = 1.5$; $\beta_2 = 1$

a. To determine which investor was a better selector of individual stocks we look at abnormal return, which is the ex-post alpha; that is, the abnormal return is the difference between the actual return and that predicted by the SML. Without information about the parameters of this equation (risk-free rate and market rate of return) we cannot determine which investor was more accurate.

b. If $r_f = 6\%$ and $r_M = 14\%$, then (using the notation alpha for the abnormal return):

$$\alpha_1 = 0.19 - [0.06 + 1.5 \times (0.14 - 0.06)] = 0.19 - 0.18 = 1\%$$

$$\alpha_2 = 0.16 - [0.06 + 1 \times (0.14 - 0.06)] = 0.16 - 0.14 = \mathbf{2\%}$$

Here, the second investor has the larger abnormal return and thus appears to be the superior stock selector. By making better predictions, the second investor appears to have tilted his portfolio toward underpriced stocks.

c. If $r_f = 3\%$ and $r_M = 15\%$, then:

$$\alpha_1 = 0.19 - (0.03 + 1.5 \times [0.15 - 0.03]) = 0.19 - 0.21 = -2\%$$

$$\alpha_2 = 0.16 - (0.03 + 1 \times [0.15 - 0.03]) = 0.16 - 0.15 = \mathbf{1\%}$$

The second investor appear to be the superior stock selector; the first investor's predictions appear to have negative value.

23. a. $E(r_P) = r_f + \beta_P \times [E(r_M) - r_f] = 5\% + 0.8\ (15\% - 5\%) = 13\%$

$\alpha = 14\% - 13\% = \mathbf{1\%}$

You should invest in this fund because alpha is positive.

b. The passive portfolio with the same beta as the fund should be invested 80% in the market-index portfolio and 20% in the money market account. For this portfolio:

$$E(r_P) = (0.8 \times 15\%) + (0.2 \times 5\%) = \mathbf{13\%}$$

$$14\% - 13\% = 1\% = \alpha$$

CFA Questions

11. a. McKay should borrow funds and invest those funds proportionately in Murray's existing portfolio (i.e., buy more risky assets on margin). In addition to increased expected return, the alternative portfolio on the capital market line will also have increased risk, which is caused by the higher proportion of risky assets in the total portfolio.

    b. McKay should substitute low-beta stocks for high-beta stocks to reduce the overall beta of York's portfolio. By reducing the portfolio beta, McKay will reduce the systematic risk of the portfolio and, therefore, reduce its volatility relative to the market. The security market line (SML) suggests such action (i.e., moving down the SML), even though reducing beta may result in a slight loss of portfolio efficiency unless full diversification is maintained. York's primary objective, however, is not to maintain efficiency but to reduce risk exposure; reducing portfolio beta meets that objective. Since York does not want to engage in borrowing or lending, McKay cannot reduce risk by selling (portfolio) equities and using the proceeds to buy risk-free assets.

Topic 7

CHAPTER 10

Problem Sets

1. The revised estimate of the expected rate of return on the stock would be the old estimate plus the sum of the products of the unexpected change in each factor (industrial production and inflation) times the respective sensitivity coefficient:

   Revised estimate = 12% + (1 × [5% − 3%] + 0.5 × [8% − 5%]) = **15.5%**

   Note that the IP change is (5% − 3%), and the IR change is: (8% − 5%).

2. The APT factors must correlate with major sources of uncertainty, that is, sources of uncertainty that are of concern to many investors. Researchers might investigate factors that correlate with uncertainty in consumption and investment opportunities. GDP, the inflation rate, and interest rates are among the factors that can be expected to determine risk premiums. Researchers will try to avoid factors that have significant correlation with each other.

   Industrial production (IP) is a good indicator of changes in the business cycle. Thus, IP is a candidate for a factor that is highly correlated with uncertainties that have to do with investment and consumption opportunities in the economy.

4. Equation 10.8 applies here:

   $E(r_p) = r_f + \beta_{P1}\,(E[r_1] - r_f) + \beta_{P2}\,(E[r_2] - r_f)$

   We need to find the risk premium (RP) for each of the two factors:

   $RP_1 = (E[r_1] - r_f)$ and $RP_2 = (E[r_2] - r_f)$

   To do so, solve the following system of two equations with two unknowns:

   $0.31 = 0.06 + (1.5 \times RP_1) + (2.0 \times RP_2)$

   $0.27 = 0.06 + (2.2 \times RP_1) + (-0.2 \times RP_2)$

   The solution to this set of equations is

   $RP_1 = 10\%$ and $RP_2 = 5\%$

   Thus, the expected return-beta relationship is

   $\boldsymbol{E(r_P) = 6\% + (\beta_{P1} \times 10\%) + (\beta_{P2} \times 5\%)}$

5. The expected return for portfolio *F* equals the risk-free rate since its beta equals 0.

   For portfolio *A*, the ratio of risk premium to beta is (12 − 6)/1.2 = 5

   For portfolio *E*, the ratio is lower at (8 − 6)/0.6 = 3.33

This implies that an arbitrage opportunity exists. For instance, you can create a portfolio *G* with beta equal to 0.6 (the same as E's) by combining portfolio A and portfolio F in equal weights. The expected return and beta for portfolio G are then:

$E(r_G) = (0.5 \times 12\%) + (0.5 \times 6\%) = 9\%$

$\beta_G = (0.5 \times 1.2) + (0.5 \times 0\%) = 0.6$

Comparing portfolio G to portfolio E, G has the same beta and higher return. Therefore, an arbitrage opportunity exists by buying portfolio G and selling an equal amount of portfolio E. The profit for this arbitrage will be

$r_G - r_E = (9\% + [0.6 \times F]) - (8\% + [0.6 \times F]) = \mathbf{1\%}$

That is, 1% of the funds (long or short) in each portfolio.

9. a. A long position in a portfolio (P) composed of portfolios A and B will offer an expected return–beta trade-off lying on a straight line between points A and B. Therefore, we can choose weights such that $\beta_P = \beta_C$ but with expected return higher than that of portfolio C. Hence, combining P with a short position in C will create an arbitrage portfolio with zero investment, zero beta, and positive rate of return.

   b. The argument in Part a leads to the proposition that the coefficient of β^2 must be zero to preclude arbitrage opportunities.

11. The APT *required* (i.e., equilibrium) rate of return on the stock based on r_f and the factor betas is

    Required $E(r) = 6\% + (1 \times 6\%) + (0.5 \times 2\%) + (0.75 \times 4\%) = \mathbf{16\%}$

According to the equation for the return on the stock, the expected return on the stock is 15% (because the *expected* surprises on all factors are zero by definition). Because the (actually) expected return based on risk is less than the required return, we conclude that the stock is overpriced.

CFA Questions