CHULALONGKORN BUSINESS SCHOOL

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2604-639

Financial economics

Finance Theories

Topic 1:

Investment Decisions under Certainty

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Agenda

1 Introduction to the Course

2 The Economic Theory of Choice

1st theory : utility theory

• Fisher's seperation theory

How firm should make investment decision on behalf of investors?

normative theory : try to give recommendation on how thing should be done

& policy related decision

involve maximization

use to solve

3 Optimal Consumption and Investment

Positive theory :
try to explain relation among economic variables

1. INTRODUCTION

1.1 Course Administration

1.2 Course Introduction

1.1 Course Administration

- The subject is divided into 2 parts
  - The first part
    - Asst Prof. Ruttachai Seelajaroen
    - 50% of the assessment (Midterm)
    - Contents: Investment analysis and asset pricing
  - The second part
    - Assoc Prof. Manapol Ekkayokkaya
    - 50% of the assessment (Final)
    - Contents: Corporate financial decisions

1.2 Course Introduction

Set up to do business, to take risk

- In 2604639 Sub-branch of economic Finance Theory, we will learn theories of finance that are related to corporate financial decisions.

  juristic person

  management makes decision instead of investors in the name of corporation

- Finance theories are theories that try to answer questions concerning the allocations of financial resources.

  e.g. invest in either project A or B or later

  - Two types of finance theories: Positive and Normative Theories

    deriving from standard or norm

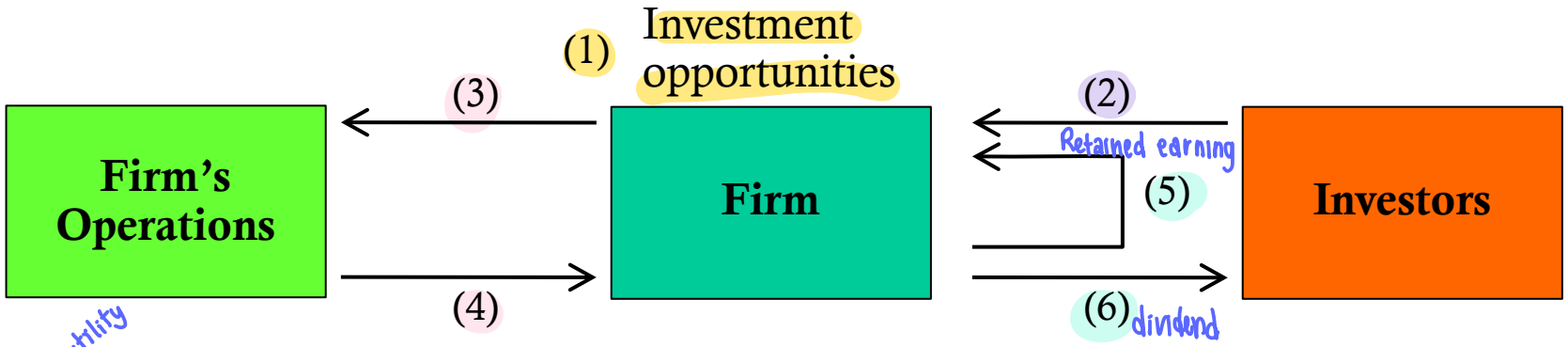
  - Compositions of a theory: Definitions, Assumptions and Hypotheses

    meanings of economic variables

  - To verify a positive theory, we test whether its hypotheses are consistent with the actual behavior in the market (i.e., empirical test).

    ซึ่งได้จากประสบการณ์หรือการทดลอง

Cash Flows and Financial Decisions of Firms



to maximize shareholders' utility

1st step → decision on raising asset

☆(1) Investment Decisions: A firm decides whether to invest in a project.

firm's side

2nd step → choose how to raise fund

☆(2) Financing Decisions: The firm decides the proportion between fund raised as debt and as equity.

BS
Asset | L
Eq

investors' side

(2) Investors decide how to allocate available cash between current consumption and/or capital markets.

(3)-(4) Cash is used by firm to acquire and operate the project. Then, cash is generated from operations.

(5)-(6) Firms decide how to allocate available cash between reinvestment or/and paying out dividend to investors.

Example: Theory of Demand

Description: Demand theory describes the way that changes in market price affect the quantity of a good or service demanded by consumers.

Type: Positive economics theory

define the economic variables first

1 **Definitions:** 1) Q_D = the quantity of the products that consumers are both willing and able to purchase at a given price level, and 2) P = price of the product

2 **Assumptions:** 1) All factors, except the price of the product, are held constant (all other things being equal, or *ceteris paribus*) 2) The product is a normal good 3) Customers are rational (prefer more to less)

constant

inferior goods
↳ consumers poorer, they consume more

↳ when consumers become wealthier → consume more

3 **Hypothesis:** As price of the product increases, the quantity demand will decrease.

positive theory → relation btw 2 variables

Expression in Mathematic Form: $Q_D = f(P)$

How can we test whether this is correct or not
↳ use empirical test ; use actual behavior in the market and test whether it is consistent with the hypothesis.

Forms of organization
- Sole ownership
- Partnership (general partner + limited partner)
- Corporation

1.2 Course Introduction

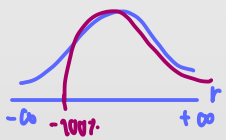
make it different from other forms of organization

- Two distinct features of a corporation compared to other forms of a business organization are;
  - The clear separation of ownership and control

    shareholder ⟷ management (can be different)
  - Limited liability

    creditors don't have right to go after shareholders' personal asset

    ↳ maximum loss = the whole investment



discrete $r_D = \frac{P_1}{P_0} - 1$ ไม่เกิน -100%

continuous $r_C = \ln\left(\frac{P_1}{P_0}\right) \to -\infty$ เข้าใกล้ 0

ln (ตัวเลข < 1) → -∞

ยิ่งเข้าใกล้ 0 ยิ่งเข้าใกล้ -∞

ทำไมจะเป็นต้องใช้ continuous เพราะจะ fit in กับ normal distribution มากกว่า

100		
90	-10%	ln(0.9) -ve
40	-60%	ln(0.4) -ve
10	-90%	ln(0.1) -ve
0	-100%	ln(0) -∞

- Questions we try to answer in this course
  - How investors make investment decisions?
  - How firms make investment and financing decisions?
  - How financial markets determine asset prices?
- Common features of any investment are "time" and "risk".

  time frame

  invest today

  return future

  don't know outcome

1.2 Course Introduction

- The course is divided into 2 parts.
  - The first part: Investment Analysis

    focusing on investor's point of view
    decide whether to invest in the business or not

    - How investors/individuals make investment decisions on financial securities issued by corporations.
    - Asset pricing theory
  - The second part: Corporate Financial Decisions
    - Investment decision
    - Financing decision

2. THE ECONOMIC THEORY OF CHOICE

2.1 Investment and Consumption Decisions

2.2 Elements of the Economic Theory of Choice

2.3 The Axioms of Choices

2.4 Total and Marginal Utility

2.5 The Indifference Curve

2.6 The Consumption Opportunity Set

2.7 The Optimal Consumption Choice

investment can only take place if there's saving → income > consumption ⇒ ∴ investment defer consumption to future

2.1 Investment and Consumption Decisions

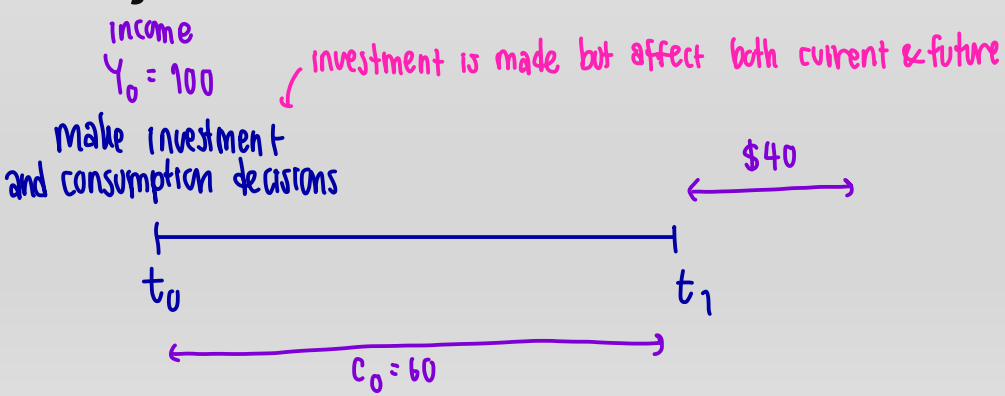
today : obtain assets
↑ financial assets, operating assets
give up resources

future : obtain principal + return

- In an investment, one pays money today (principal) to acquire income-generating assets and expects to earn investment return in the future (principal + return).
- An investment come from saving, which, in turn, exists because current consumption is less than current income.
- Essentially, an investment decision is the decision to sacrifice current consumption for future consumption.
- To understand how an investment decision is made, we need a framework to show how a choice (or tradeoff) between current versus future consumptions is made.

2.2 Elements of the Economic Theory of Choice

- The economic theory of choice or the utility theory attempts to establish empirical generalizations about the way in which individuals (the subject or the decision maker) make certain choice when facing a number of possible alternatives.
- To keep thing simple, we will assume;
  - Two time periods (2 points in time), t_0 and t_1 and
    know the outcome of investment for sure
  - Perfect certainty.



2.2 Elements of the Economic Theory of Choice

- The theory consists of 3 main elements
  - The object of choice: The level of current and future consumption or C_0 and C_1 (A combination of C_0 and C_1 is called a basket of consumption.)

    choosing btw

    budget $100 of consumption ($60, $40), ($50, $50), ...

    C_0 C_1

    opportunity set

    size depends on budget
  - The opportunity set: All possible combinations of C_0 and C_1 affordable by the subject. Affordability is constrained by his/her budget or income, Y_0 and Y_1.

    the one who make decision
  - Tastes or preferences: The subject's taste towards current and future consumptions.

    e.g. prefers $C_0 > C_1$
- The theory tries to explain how individuals allocate their limited resources between C_0 and C_1.

2.3 The Axiom of Choice

- The utility theory makes certain assumptions regarding individuals' behavior. These assumptions are called the axioms of choice.

  Rational investor : investor whose behavior is consistent with the assumption

- Let

  1 baskets of (c_0, c_1)

  x, y and z represent certain baskets of consumption

  prefer more

  $x \succ y$ means the subject prefers basket-x to basket-y.

  $x \sim y$ means the subject is indifferent between basket-x and basket-y.

  compare numarical manitude

  compare preference

  $x > y$ vs. $x \succ y$

2.3 The Axiom of Choice

E.g. $x = (\$70, \$90)$
$y = (\$40, \$60)$
subject can rank this
can make decision
must be always consistent

1) Comparability (or completeness): The subject has ability to rank his preference towards all possible baskets of consumption.

- He can always identify whether $x \succ y$, $y \succ x$ or $x \sim y$

2) Transitivity (or consistency): The subject is consistent in his preference rankings of consumption baskets.

- If $x \succ y$ and $y \succ z$, then $x \succ z$.
- If $x \sim y$ and $y \sim z$, then $x \sim z$.

2.3 The Axiom of Choice

3) Non-satiation of wants: The subject always prefers (i.e., to consume) more to less, or at worst would be indifferent.

same c_0 higher c_1 prefer to consume more

- If x = (10, 5) and y = (10, 8), then y ≻ x.

money addition to utility alway ⊕ but it is decreasing (rate of increasing)

4) Decreasing marginal utility: As more consumption takes place, the subject's satisfaction increases at a decreasing rate.

- The additional level of utility from an additional unit of consumption is called "marginal utility" or MU.
- This assumption implies diminishing marginal utility.

2.4 Total Utility

- The level of satisfaction obtained from a certain level of consumption is called "total utility".

utility function in mind

, has black box

- The theory assumes that an individual behaves as if he/she assigns numbers, called utility index, to all possible consumption baskets in a way that a larger number is assigned to a more preferred basket.
  - That is, if Utility[x] > Utility[y], it implies that $x \succ y$.
- Let's further assume that the utility numbers are generated through a function, called utility function, which reflects the individual's consumption preference.

black box to convert

(C_0, C_1) → Utility function $U(C_0, C_1)$ → Utility index

2.4 Total Utility

- A utility function is the function that converts baskets of consumption into utility numbers called “utility index”. This index is, then, used for preference ranking.
- The theory also assumes that the magnitude of the difference in utility index between 2 utility levels (ΔU) is meaningful.
  - This is known as cardinal utility.
- The utility theory is based on a set of assumptions. It is natural to ask what types of function are qualified for using as total utility function.

Utility Function, Utility Index and Preference Ranking

function that convert basket of consumptions into index
⇓
used to rank

(C_0, C_1)	$U = (C_0 \times C_1)^{1/2}$	
	# utils	Rank
(1, 2)	1.41	6
(1, 3)	1.73	5
(1, 4)	2.00	3.5
(2, 2)	2.00	3.5
(2, 3)	2.45	2
(2, 4)	2.83	1

utility keep increasing as we consume more ⇒ non-satiation of want ✓

same C_0

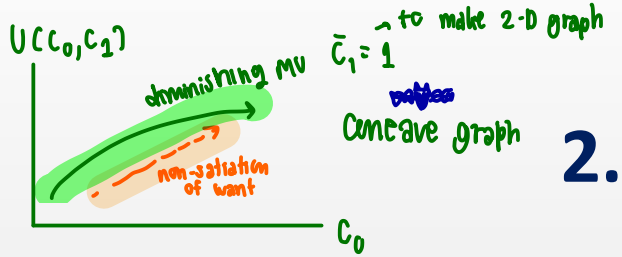
$MU_{C_1} = 0.45$

additional utility of increase C_1 by 1 unit
⇓
marginal utility of C_1 (future consumption

$MU_{C_1} = 0.38$

deminishing marginal utility
↓
obey axiom of choice

* Cobb-Douglas utility function



$\bar{C}_1 = 1$ → to make 2-D graph

Concave graph

2.4 Total Utility

- Properties of a qualified utility function
  - positive
  - Upward sloping (Non-satiable)
  - Concave shape (Diminishing marginal utility)
- A total utility curve is a graphical representation of a total utility function. It shows the relationship between levels of consumption and total utility.
- The slope of a utility function/curve is the marginal utility.

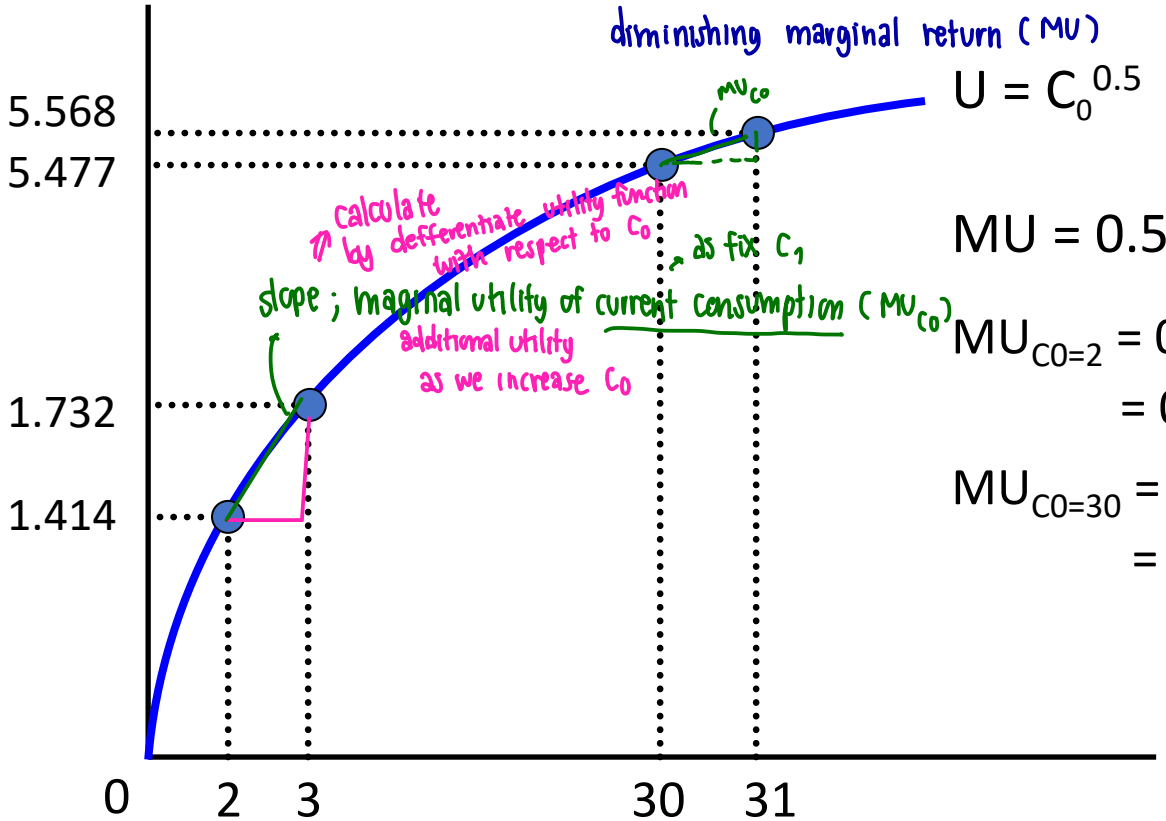
$$MU_0 = \Delta U/\Delta C_0 \ \ or \ \ \partial U/\partial C_0 \ \ or \ \ U'(C_0)$$

- MU_0 is the additional utilities obtained from one more unit of current consumption holding future consumption constant.

Total Utility Curve

Total Utility (U)

From $U = (C_0C_1)^{0.5}$ assuming C_1=\$1



$U = C_0^{0.5}$

$MU = 0.5/C_0^{0.5}$

$MU_{C0=2} = 0.5/2^{0.5}$
$= 0.354$ utils

$MU_{C0=30} = 0.5/30^{0.5}$
$= 0.091$ utils

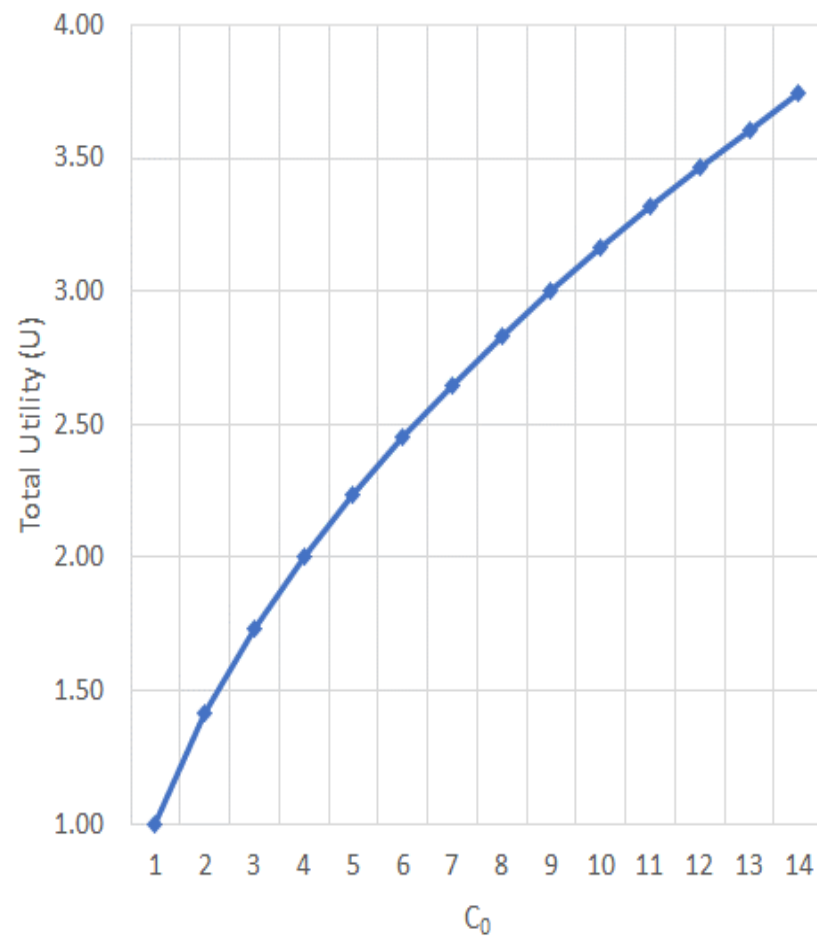
$MU_{C0=2} \approx 1.732 - 1.414$
≈ 0.318 utils

$MU_{C0=30} \approx 5.477 - 5.568$
≈ 0.091 utils

Example: Cobb-Douglas Utility Function

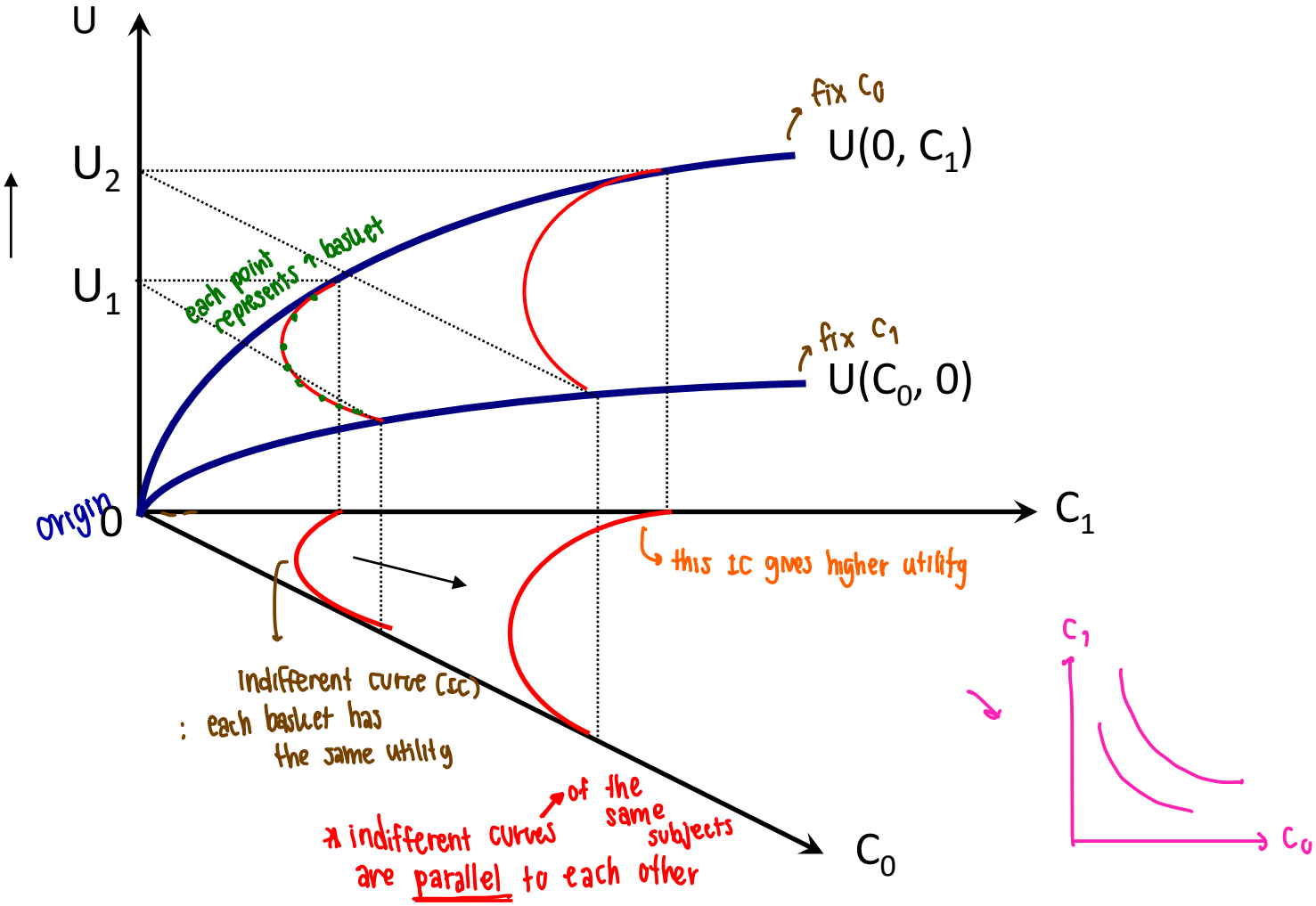
- Assume, the utility function of a person is $U(C_0, C_1) = (C_0 \times C_1)^{0.5}$. The table shows total utility from various levels of C_0, holding C_1 constant at \$1.

(C_0, C_1)	U	$\Delta U/\Delta C_0$	$\partial U/\partial C_0$
(1, 1)	1.000		
(2, 1)	1.414	0.414	0.354
(3, 1)	1.732	0.318	0.289
(4, 1)	2.000	0.268	0.250
(5, 1)	2.236	0.236	0.224
(6, 1)	2.449	0.213	0.204
(7, 1)	2.646	0.196	0.189
(8, 1)	2.828	0.183	0.177
…	…	…	…
(13, 1)	3.606	0.141	0.139
…	…	…	…
(30, 1)	5.477	0.108	0.091
(31, 1)	5.568	0.091	0.090



3D — C_0, C_1, level of utility (U)

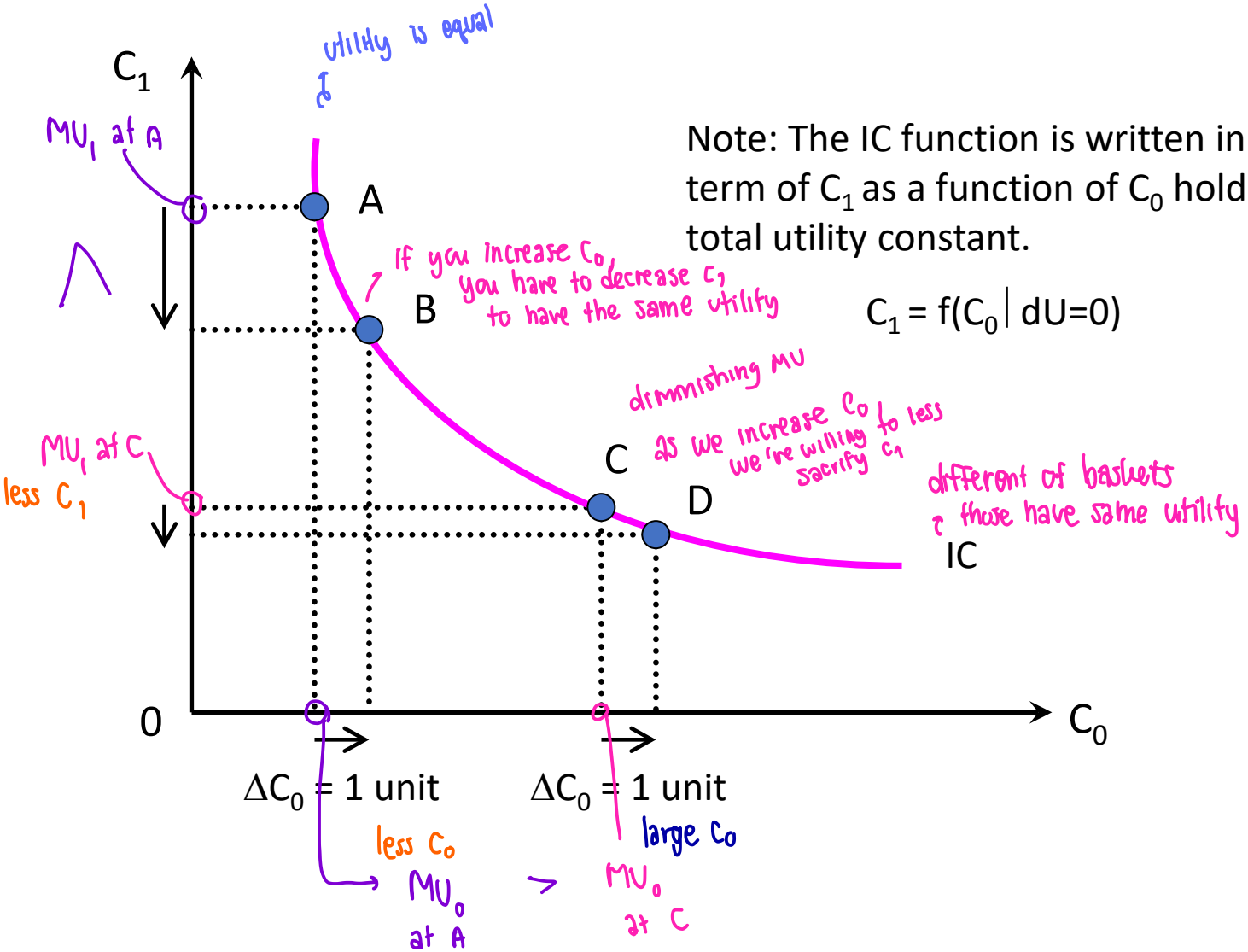
Total Utility "Surface" and Indifference Curves (Two Commodities)



2.5 The Indifference Curve

- Indifference curve (IC) is a locus of points representing different combination of C0 and C1, each of which provides the subject with the same level of utility.
- Properties of an IC derived from an eligible utility function
  - Downward sloping
  - Convex shape

Indifference Curve



Note: The IC function is written in term of C_1 as a function of C_0 holding total utility constant.

$$C_1 = f(C_0 \mid dU=0)$$

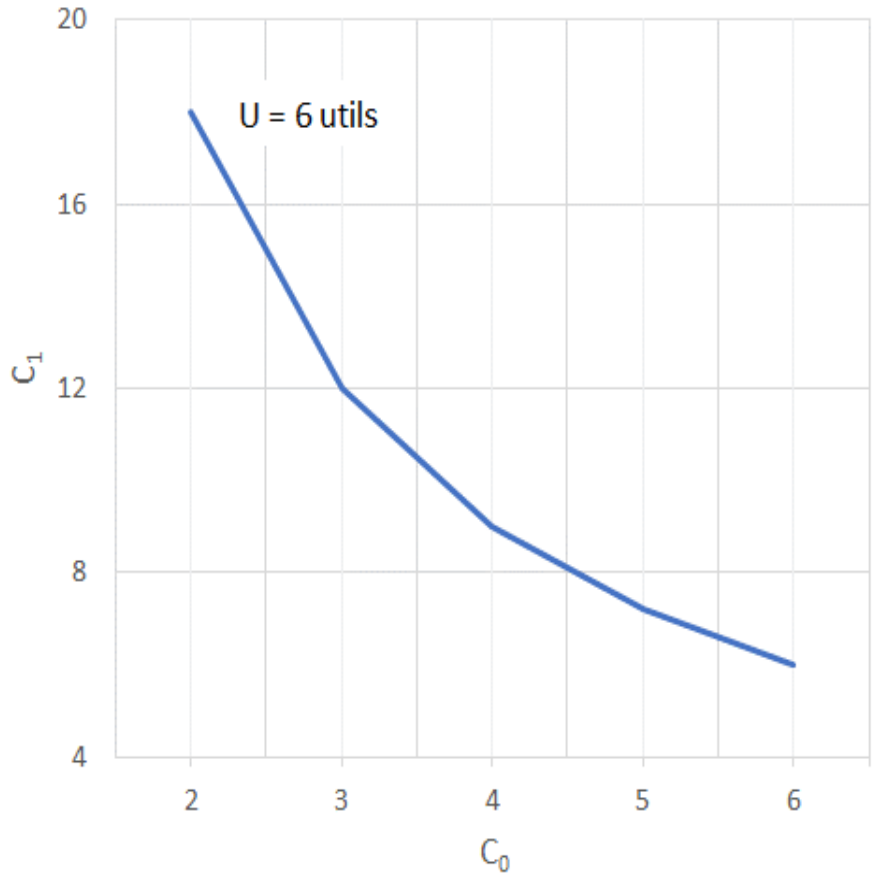
How to construct IC

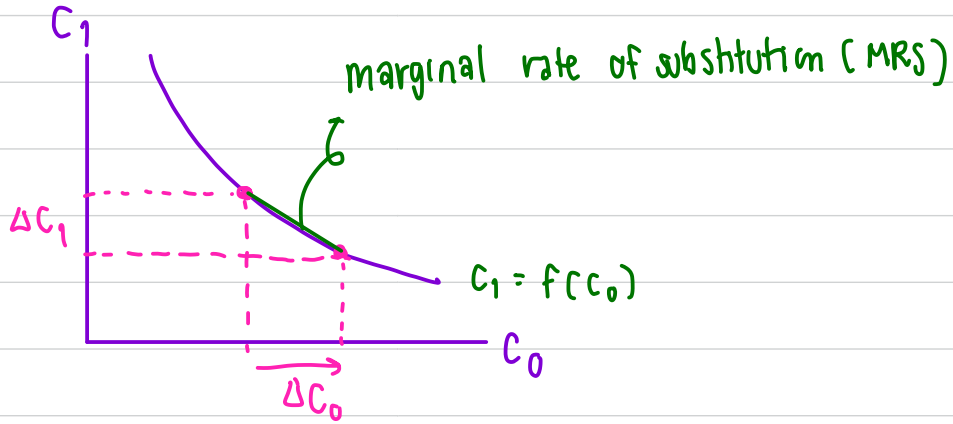
Example

- Assume, the utility function of a person is U(C0, C1) = $C_0^{0.5} \times C_1^{0.5}$. The table shows total utility from various levels of C0 and C1.

(C_0, C_1)	U
(2, 9)	4.24
(2, 18)	6.00
(3, 6)	4.24
(3, 9)	5.20
(3, 12)	6.00
(4, 6)	4.90
(4, 9)	6.00
(4, 12)	6.93
(5, 6)	5.48
(5, 7.2)	6.00
(5, 12)	7.75
(6, 6)	6.00
(6, 9)	7.35
(6, 12)	8.49

The IC derived for the total utility of 6 utils.





$$\left.\frac{\partial C_1}{\partial C_0}\right|_{dU=0}$$

2.5 The Indifference Curve

- The slope of the IC reflects marginal rate of substitution (MRS).
- MRS is the number of units of C_1 that must be given up per unit of C_0 added, in order to maintain a constant level of utility.

to make MRS ⊕

$$MRS = -(\partial C_1/\partial C_0)\Big|_{dU=0}$$

indifferent curve

- From $U = U(C_0, C_1)$, the total difference equation is;

IC

$$dU = \underbrace{(\partial U/\partial C_0)}_{\text{marginal utility of } C_0}\times dC_0 + \underbrace{(\partial U/\partial C_1)}_{\text{MU of } C_1}\times dC_1$$

C_0 เปลี่ยนไปกี่หน่วย

↳ Utility เปลี่ยนไปเท่าไรถ้า C_0 เปลี่ยนไป 1 หน่วย

Utility จะเปลี่ยนได้เพราะ C_0 & C_1

$$0 = MU_0\times dC_0 + MU_1\times dC_1$$

2.5 The Indifference Curve

- On the same IC, as C_0 and C_1 change, TU does not change (dU = 0).

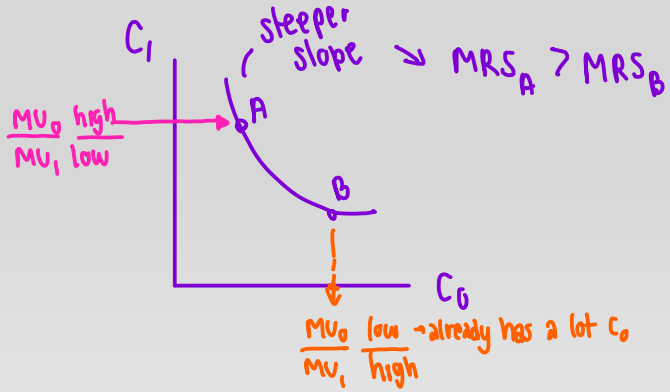
$$0 = MU_0 \times dC_0 + MU_1 \times dC_1$$

marginal utility of C_0 marginal utility of C_1

$$MU_0/MU_1 = -(dC_1/dC_0)\Big|_{dU=0}$$

$$= MRS$$

- As one moves from a point on the left to a point on the right of the same IC (i.e., from A to B and to C), MRS starts to drop. What is an economic reason for this?



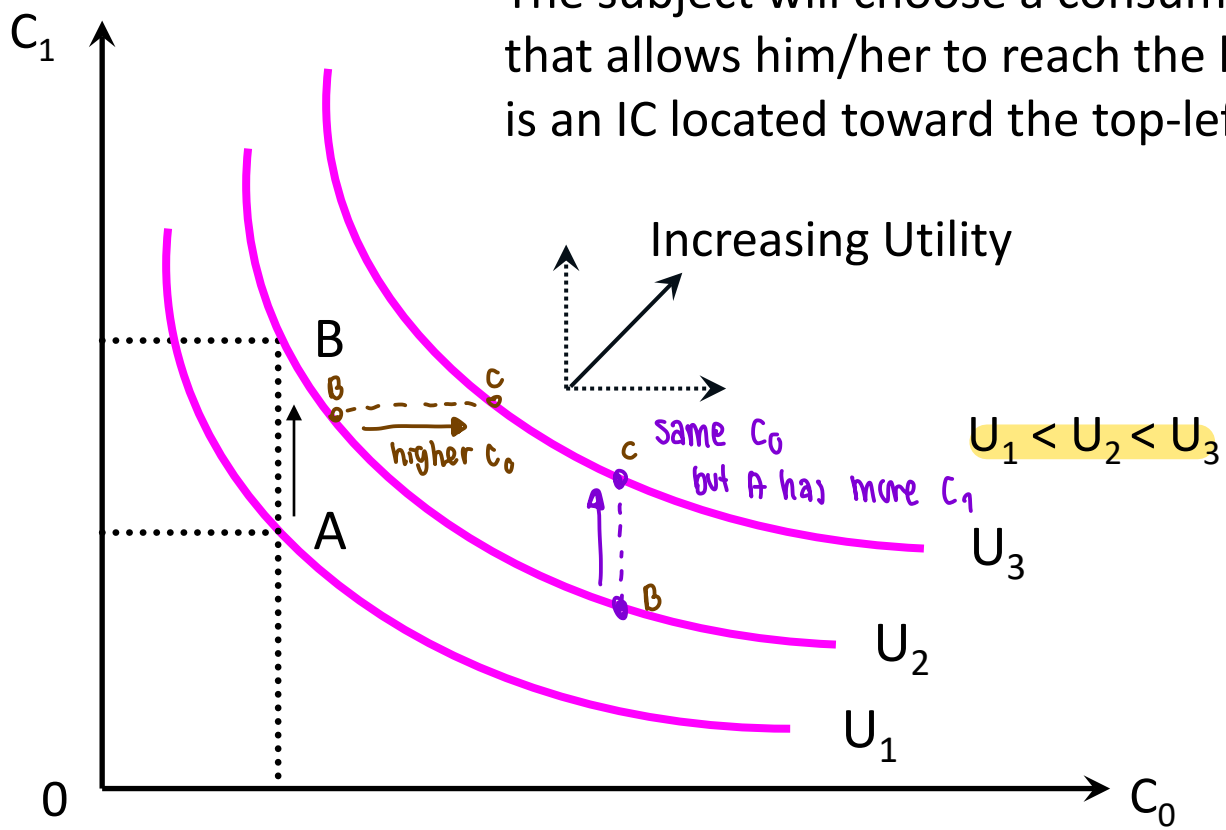
objective function: maximize level of utility

IC Map of an Individual

represent different U for a person

There are infinitely many ICs in the C_0-C_1 plane

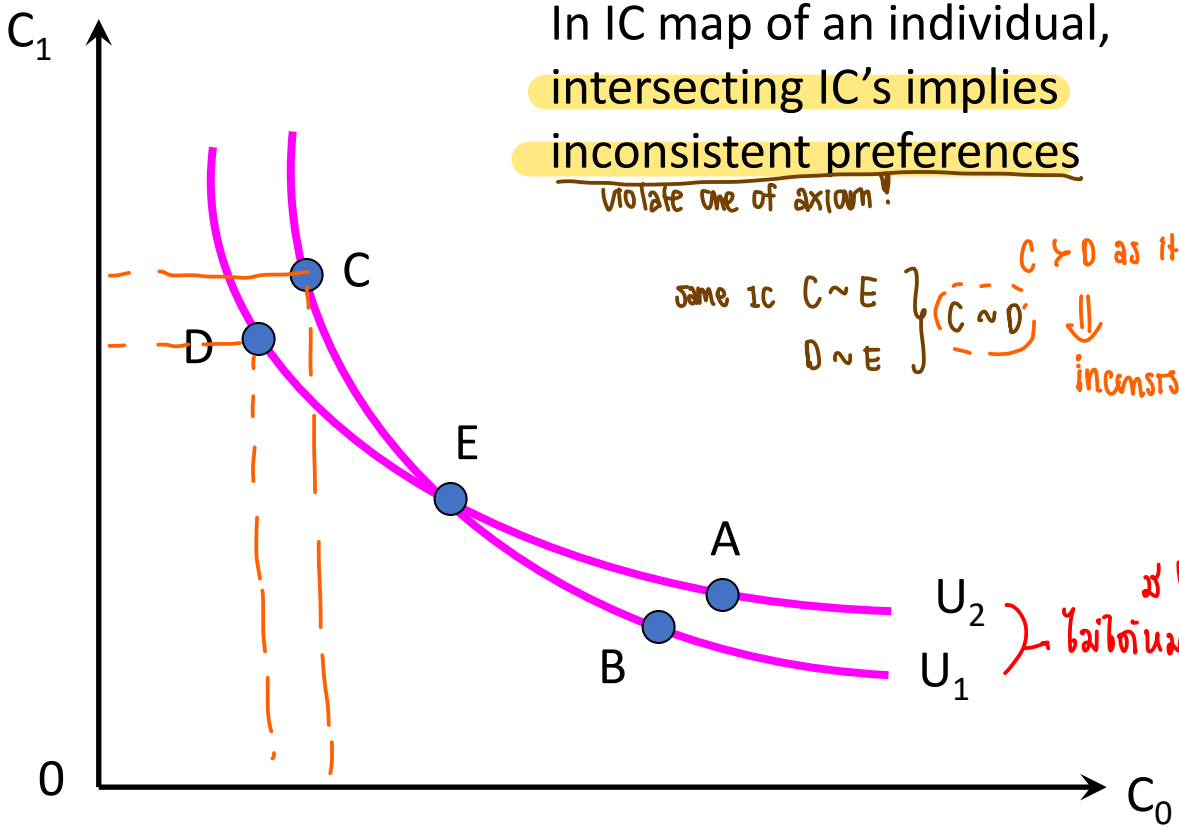
The subject will choose a consumption basket that allows him/her to reach the highest TU. That is an IC located toward the top-left direction.



Inconsistent Preferences

In IC map of an individual, intersecting IC's implies inconsistent preferences

violate one of axiom!



same IC C ~ E
D ~ E
(C ~ D)

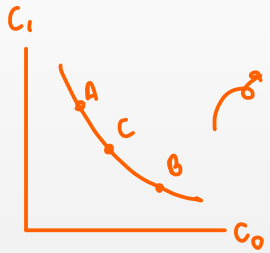
C ≻ D as it has more C_0 & C_1

⇓

inconsistent!!!

มี basket E ร่วมกัน

ไม่ได้แสดงว่า $U_1 = U_2$



must choose baskets within budget constraint

income level

$(Y_0, Y_1) = (100, 120)$

$r_f = 10\%$

2.6 The Consumption Opportunity Set

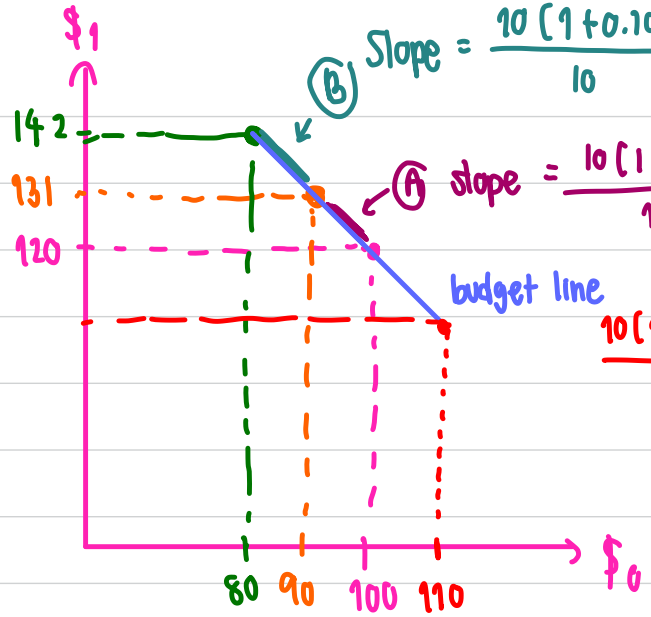
feasible consumption set

- From non-satiation of wants, an individual would try to maximize his/her total utility – that is, to reach the most upper-right IC. However, the ability to do so is restricted by his/her own wealth or budget.
- The first step of making the consumption decision is to lay down all affordable baskets of consumption. Then, the next step is to choose the basket that maximize utility.
- All possible baskets of consumption that are attainable to the subject is called the consumption opportunity set.
- How to determine the consumption opportunity set?

income level

$(Y_0, Y_1) = (100, 120)$

$r_f = 10\%$

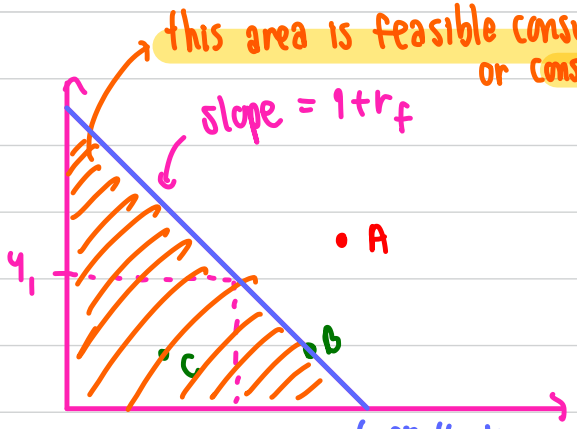


(B) Slope $= \frac{10(1+0.10)}{10} = 1 + r_f$

(A) slope $= \frac{10(1+0.10)}{10} = 1 + r_f$

borrow \$10 to consume today

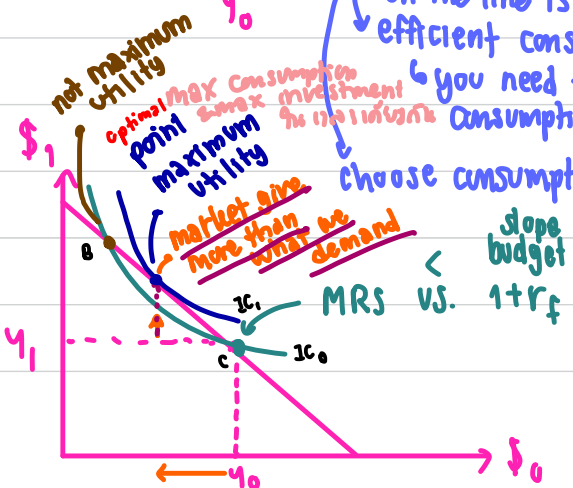
$\frac{10(1+0.10)}{10} = 1 + r_f$



on the line is efficient consumption

↳ you need to decrease one of consumption to increase another

choose consumption basket on this line



slope of budget line

MRS < vs. $1 + r_f$

Utility of $IC_0 < IC_1$ & at B MRS $> 1 + r_f$

↓

B is not the optimal point

axiom of choice → prefer more to less

2.6 The Consumption Opportunity Set

- Assumptions
  - The current and future incomes (Y_0 and Y_1) are known.
  - There exists a capital market where one can exchange current income for future income and vice versa (lending and borrowing).

    either lend or borrow money

    used as reference point

    ideal situation

    efficient capital market
    ↳ possible

  - Perfect capital market

    รวมทั้ง bank & capital market

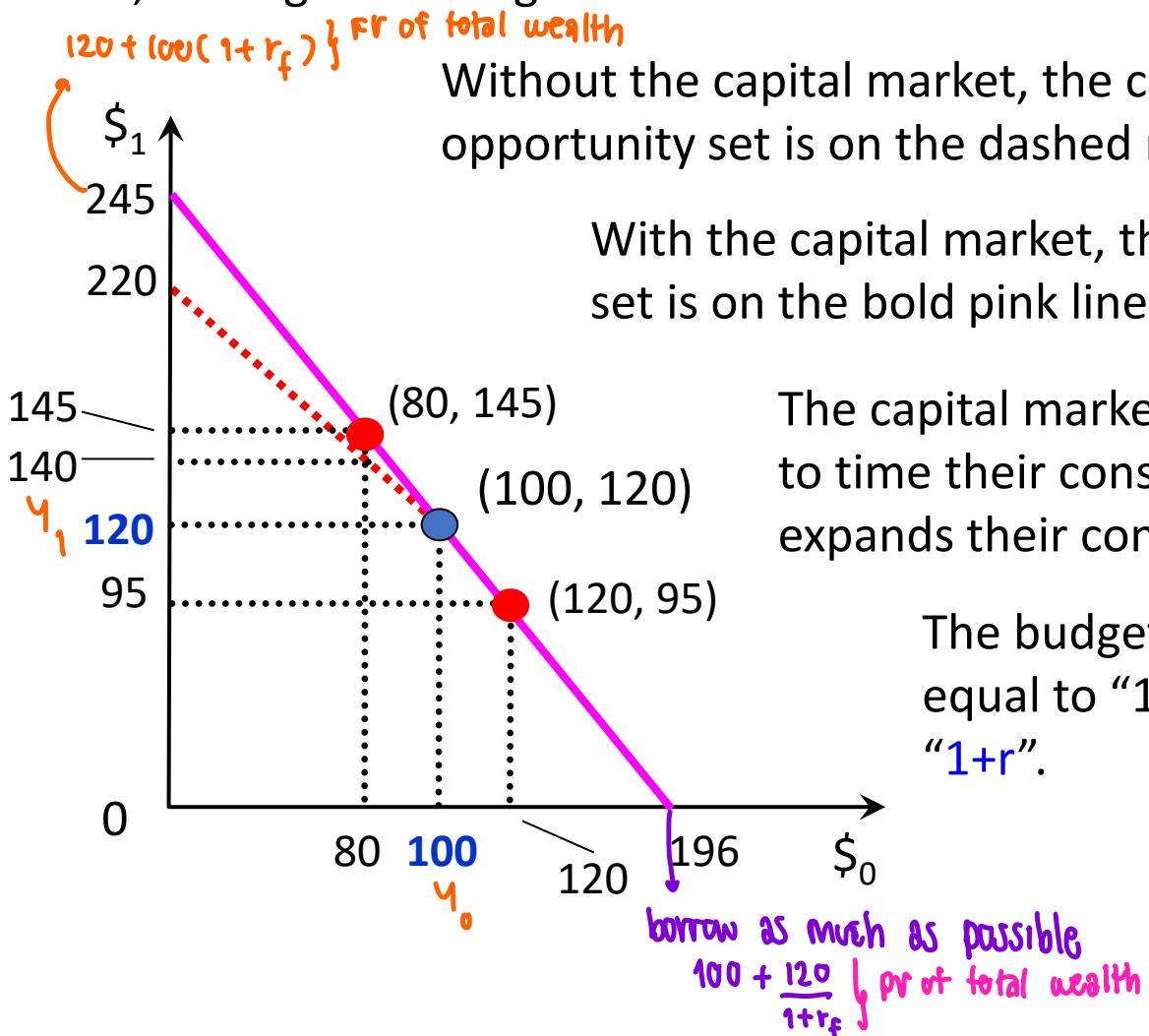
    ทุกคนได้รู้ข้อมูล

    - Equal and costless access to information ✓ no information assymetry
    - All participants are price takers → each of us are very small
      ↳ alone will not affect interest rate
    - No transaction costs and taxes (frictionless)

      no taxes, no fee

Constructing the Consumption Opportunity Set

An individual with the current and future incomes of (Y_0, Y_1) = ($100, $120). The market interest rate is 25%. Assume that he will use up his wealth in his life-time, leaving no heritage and debt.



Without the capital market, the consumption opportunity set is on the dashed red-line

With the capital market, the consumption opportunity set is on the bold pink line, called the budget line.

The capital market not only allows individuals to time their consumption patterns, but also expands their consumption opportunity set.

The budget line is linear with the slope equal to "1 + market interest rate" or "1+r".

Mr. A (100, 120)
Mr. B (150, 200)

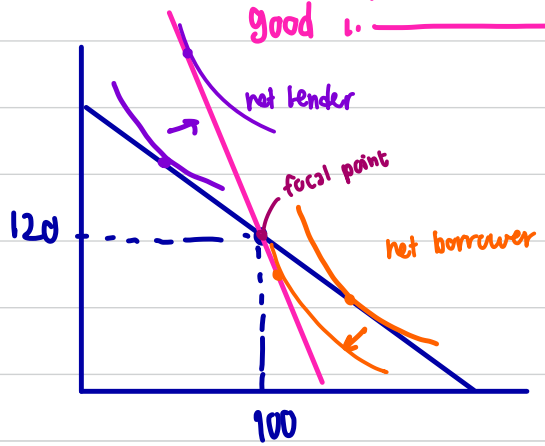
face the same r_f

Increase in r_f ; $r_f = 25\% \rightarrow 40\%$

bad for person who prefer C_0 ⇒ net borrower
good " —— " C_1 ⇒ net lender

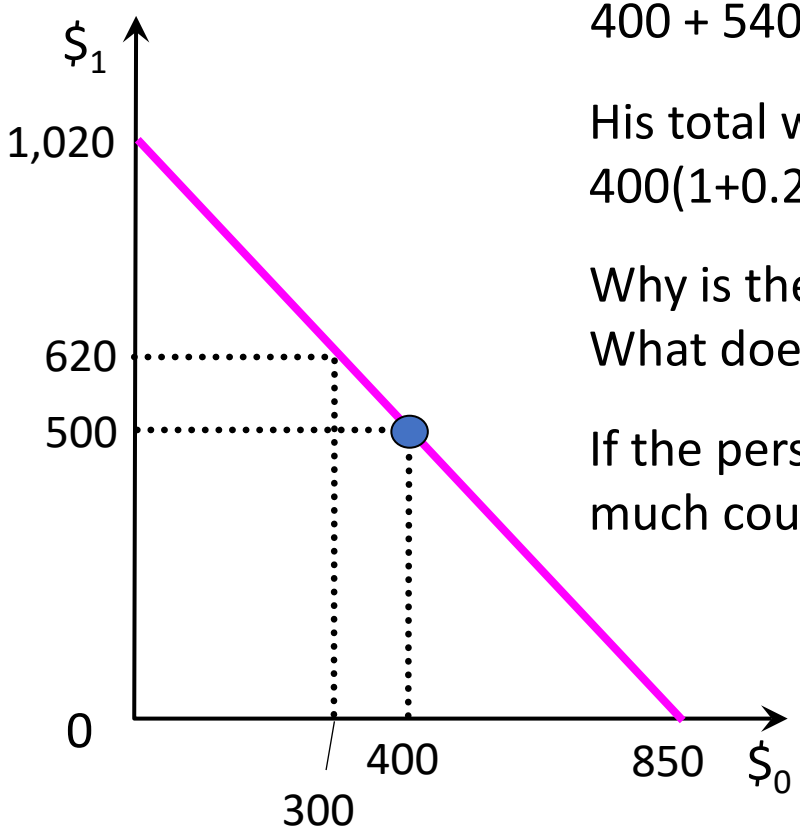
Utility of net borrower ↓

↑ Utility of net lender ↑



Constructing the Consumption Opportunity Set (2)

Construct the consumption opportunity set for a person whose current and future incomes of (Y_0, Y_1) = (\$400, \$540). The market interest rate is 20%.



His total wealth measured in present value:
400 + 540/(1+0.2) = \$850

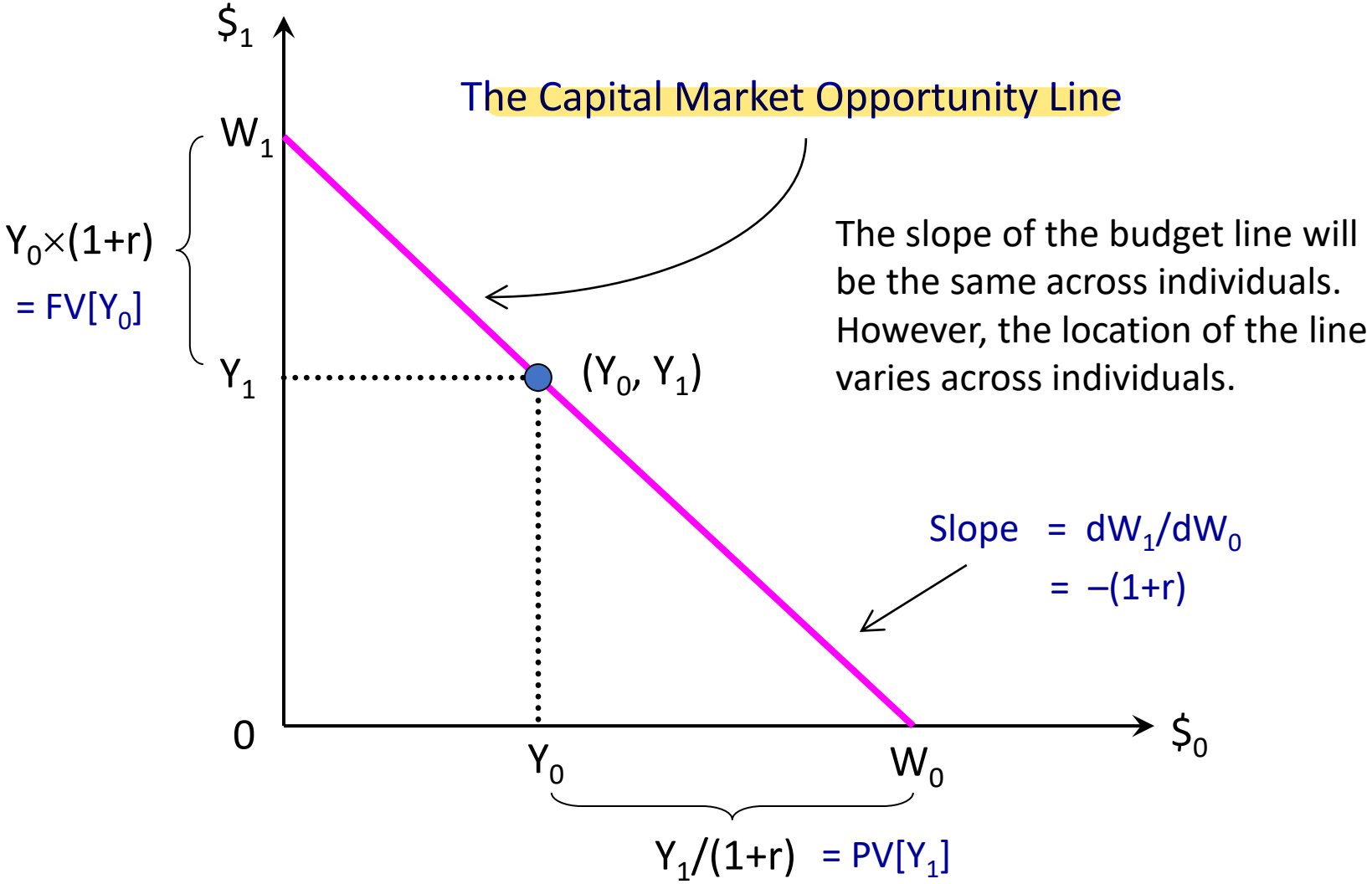
His total wealth measured in future value:
400(1+0.2) + 540 = \$1,020

Why is the budget line linear?
What does the slope of the budget line represent?

If the person only consumes \$300 today, how much could he consume in the future?

What will happen to the budget line if;
[1] there is a decrease in interest rate, or
[2] his future income increases to \$600?

The Consumption Opportunity Set



2.7 The Optimal Consumption Choice

- From non-satiation of wants, an individual will only consider baskets of consumption that lie on the budget line only.
- These baskets constitute the efficient consumption set.
- Baskets above the line are not attainable, while baskets under the line are feasible but inefficient.
- The solution for the optimal consumption can be shown in graphical form as follows.

  [1] Start with income (Y_0, Y_1)

  [2] Identify the budget line

  [3] choose the basket on the line that maximize total utility

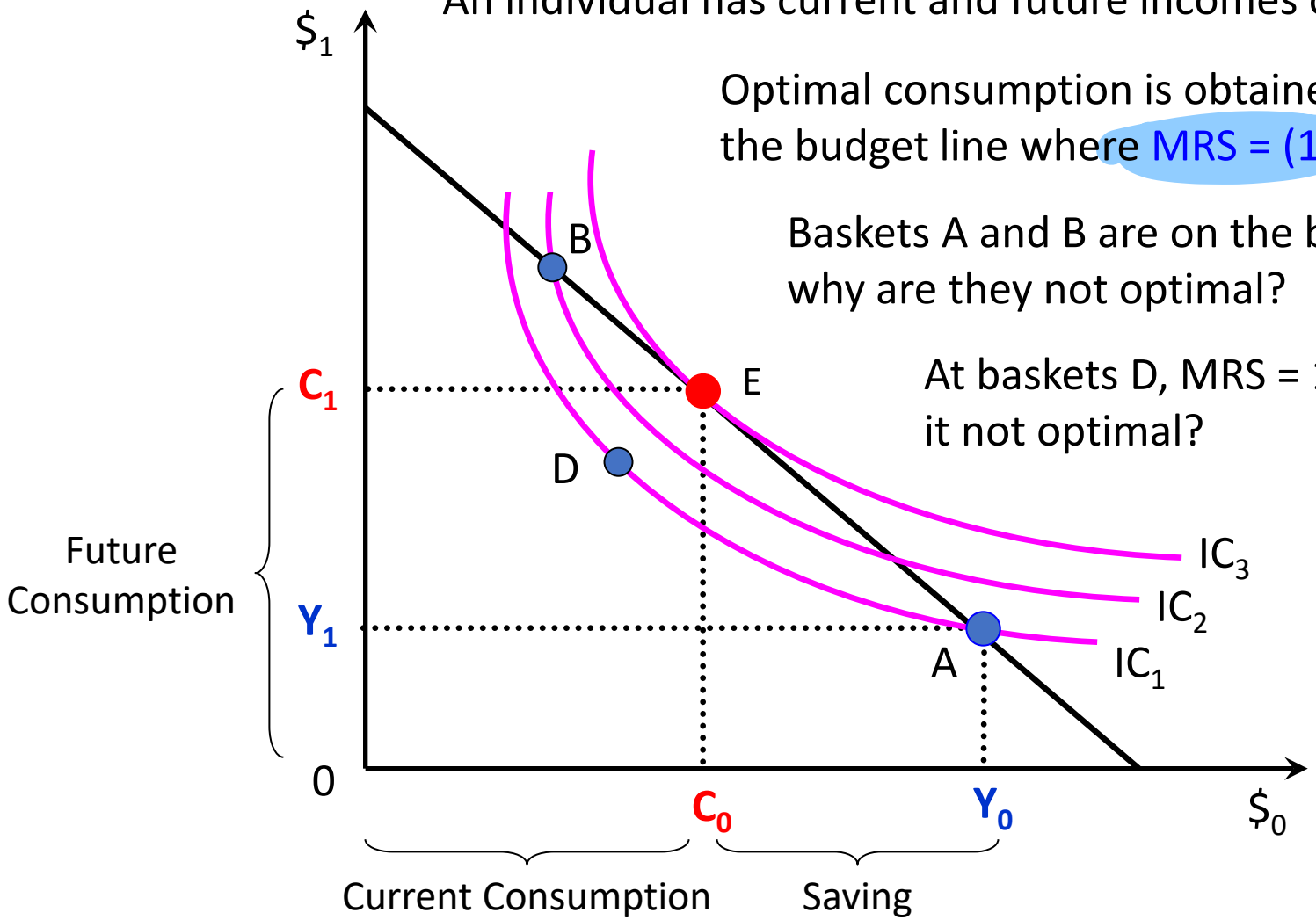
Optimal Investment-Consumption Decision

An individual has current and future incomes of Y_0 and Y_1.

Optimal consumption is obtained along the budget line where MRS = (1+r).

Baskets A and B are on the budget line, why are they not optimal?

At baskets D, MRS = 1+r, why is it not optimal?



2.7 The Optimal Consumption Choice

- To find solution in mathematical form, we set up the objective function

$$\text{Maximize } U(C_0, C_1) \text{ wrt. } C_0 \text{ and } C_1$$

$$\text{Subject to } T(C_0, C_1) = 0 \qquad \text{[budget constraint]}$$

where $T(C_0, C_1) = [Y_0 + Y_1/(1+r)] - [C_0 + C_1/(1+r)]$

("lifetime income = lifetime consumption")

- The Lagrange equation

$$L = U(C_0, C_1) - \lambda T(C_0, C_1)$$

2.7 The Optimal Consumption Choice

- The first order conditions (FOC)

$$\partial L/\partial C_0 = MU_0 - \lambda(\partial T/\partial C_0) = 0$$

$$\partial L/\partial C_1 = MU_1 - \lambda(\partial T/\partial C_1) = 0$$

$$\partial L/\partial \lambda = T(C_0, C_1) = 0$$

- The optimal consumption is obtained at the basket that

$$MU_0/MU_1 = (\partial T/\partial C_0) / (\partial T/\partial C_1)$$

slope of IC slope of budget line

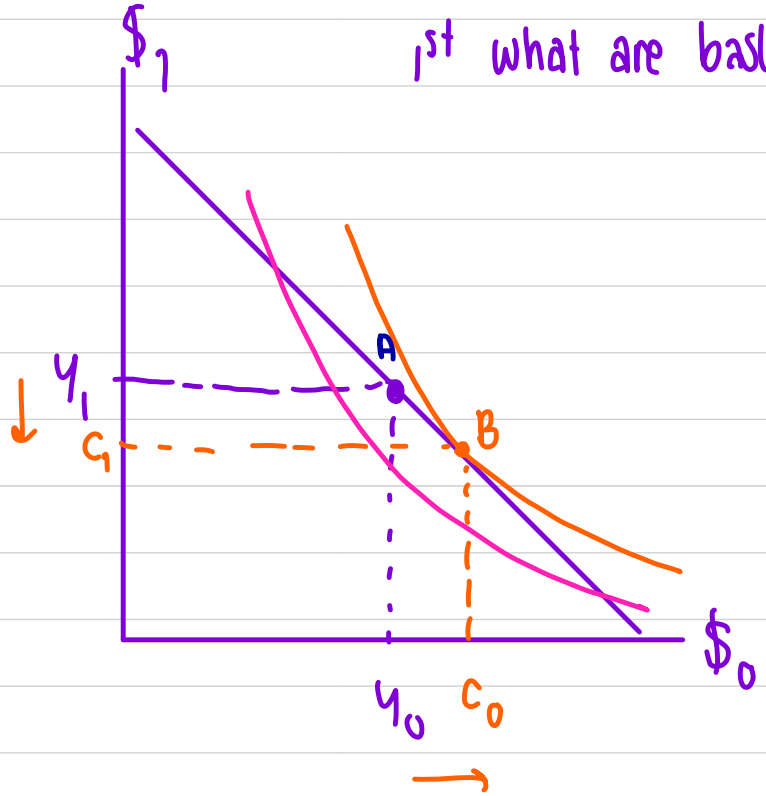
$$MRS = (1+r)$$

tengent

- The first order conditions (FOC): The concavity assumption on total utility (diminishing MRS) is sufficient to ensure that the point where MRS = (1+r) is the true maximum.

Optimal consumption ?

1st what are baskets we can afford with budget



investment in real project production

3. OPTIMAL INVESTMENT-CONSUMPTION DECISIONS

represents opportunities to invest in real productive assets

3.1 The Production Possibility Curve (PPC)

3.2 The Optimal Investment-Consumption Decision without Capital Market

3.3 The Optimal Investment-Consumption Decision with Capital Market

3.4 The Case of a Corporation

3.5 Limitations

3.1 The Production Possibility Curve

- Let's consider investments in real production rather than in financial assets (i.e., lending).
- The production possibility curve (PPC) represents possible levels of outputs from different levels of investment, holding available resources and technology constant.
- The slope of the PPC reflects the marginal rate of transformation (MRT) between current resources and future products. We will assume diminishing MRT.

$1 + IRR_i$

slope is not constant

Constructing the Production Possibility Curve

$1E_0$

An investor has $1,000 available today (initial endowment). The table shows his investment opportunities. He wants to decide how to allocate his current endowment between C_0 and investment in real productive assets.

There're 4 possible project

Project	invest today Investment Outlay	Net Cash Receipts at End of Year	Internal Rate of Return (IRR)	
C	500	600	20%	3rd
A	100	200	100%	1st
D	300	315	5%	4th
B	100	150	50%	2nd

The first project to consider should be the one that gives the highest return on investment. Therefore, it is useful to rearrange the projects in accordance with their returns on investment.

ranking by IRR

slope of PPC = 1+IRR

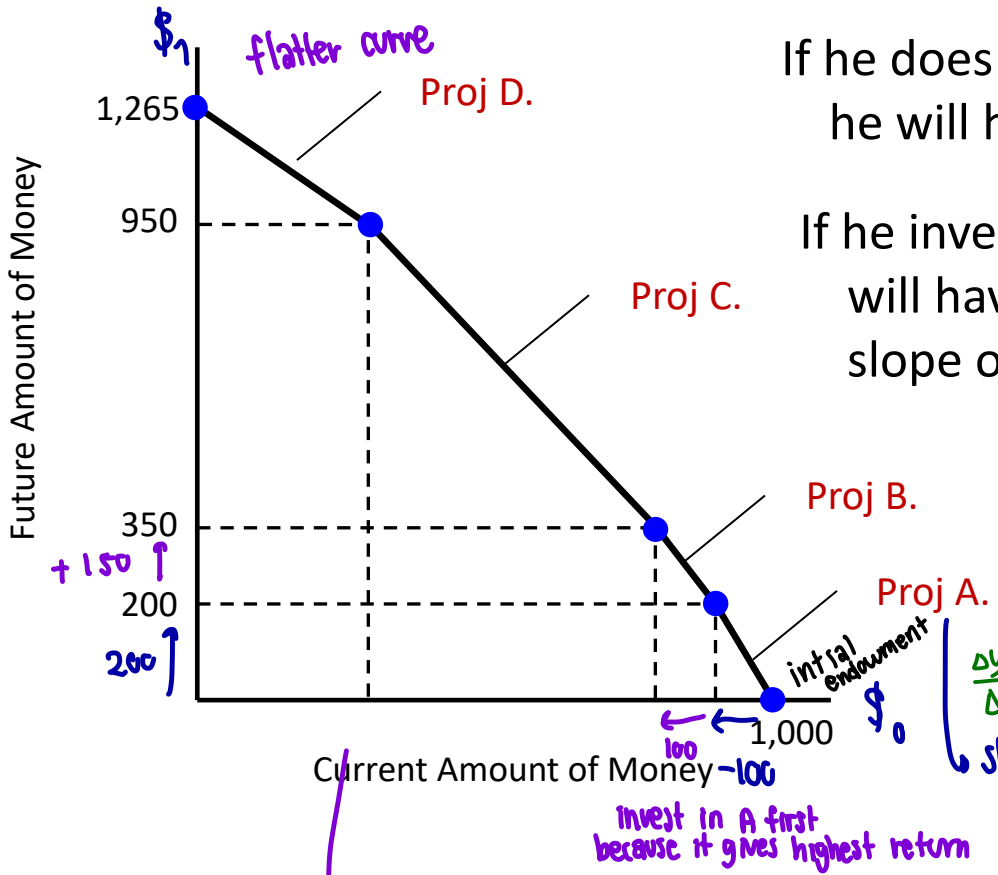
marginal rate of transformation

assume there's no capital mkt

↳ can only invest in real production

Project	Investment Outlay	Net Cash Receipts	IRR
A	100	200	100%
B	100	150	50%
C	500	600	20%
D	300	315	5%

Information in columns 2 and 3 can be used to construct the PPC to represent his consumption-investment opportunities.



If he does not invest in production at all, he will have C_0 = \$1,000 and C_1 = 0.

If he invests 1 project, he will choose A. He will have C_0 = \$900 and C_1 = 200. The slope of the PPC for Proj.A = $1+IRR_A$ = 2.

If he invests 2 projects, he will choose A and B. He will have C_0 = \$800 and C_1 = 350. The slope of the PPC for Pro. B = $1+IRR_B$ = 1.5.

$\frac{\Delta y}{\Delta x}$

slope = $\frac{\overbrace{100(1+IRR)}^{\text{return}}}{100}$ = (1+IRR)

↳ IE

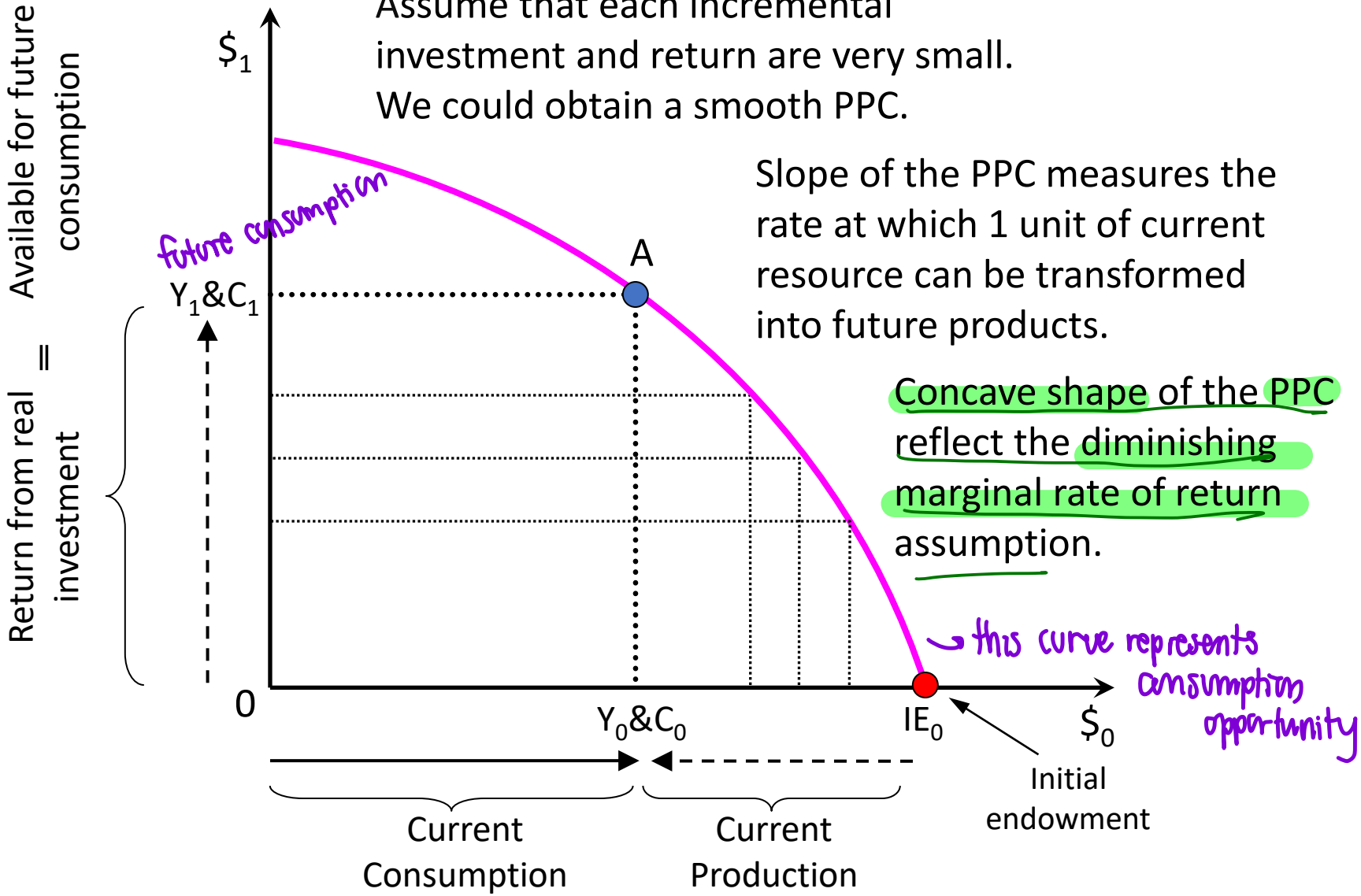
assuming that PPC is smooth

The Production Possibility Curve (PPC)

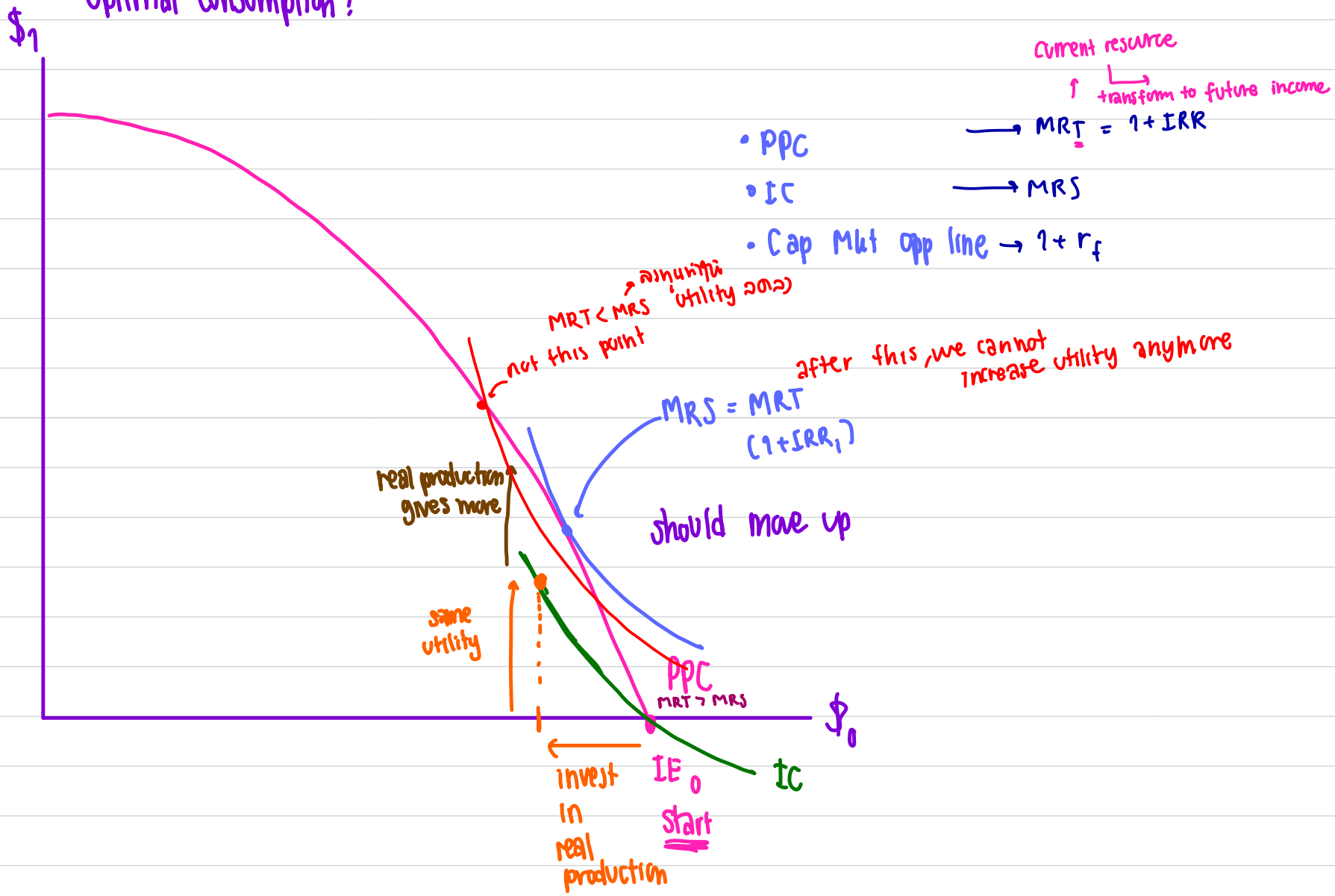
Assume that each incremental investment and return are very small. We could obtain a smooth PPC.

Slope of the PPC measures the rate at which 1 unit of current resource can be transformed into future products.

Concave shape of the PPC reflect the diminishing marginal rate of return assumption.



Optimal consumption ?



3.2 Optimal Inv-Cons Dec. without Capital Market

- An individual starts with an initial endowment IE_0. He is deciding between C_0 and C_1. If $C_0 < IE_0$, the excess resource could be invested in real production to increase C_1.
- He should start by examining his PPC and choose the optimal investment and consumption point on the PPC.
- As shown in the next slide, the optimal decision is obtained on the PPC where the slope of the PPC is equal to the slope of the IC.

$$MRS = MRT$$

- Note that MRT = 1+IRR.

Investment-Consumption Decision without Capital Market

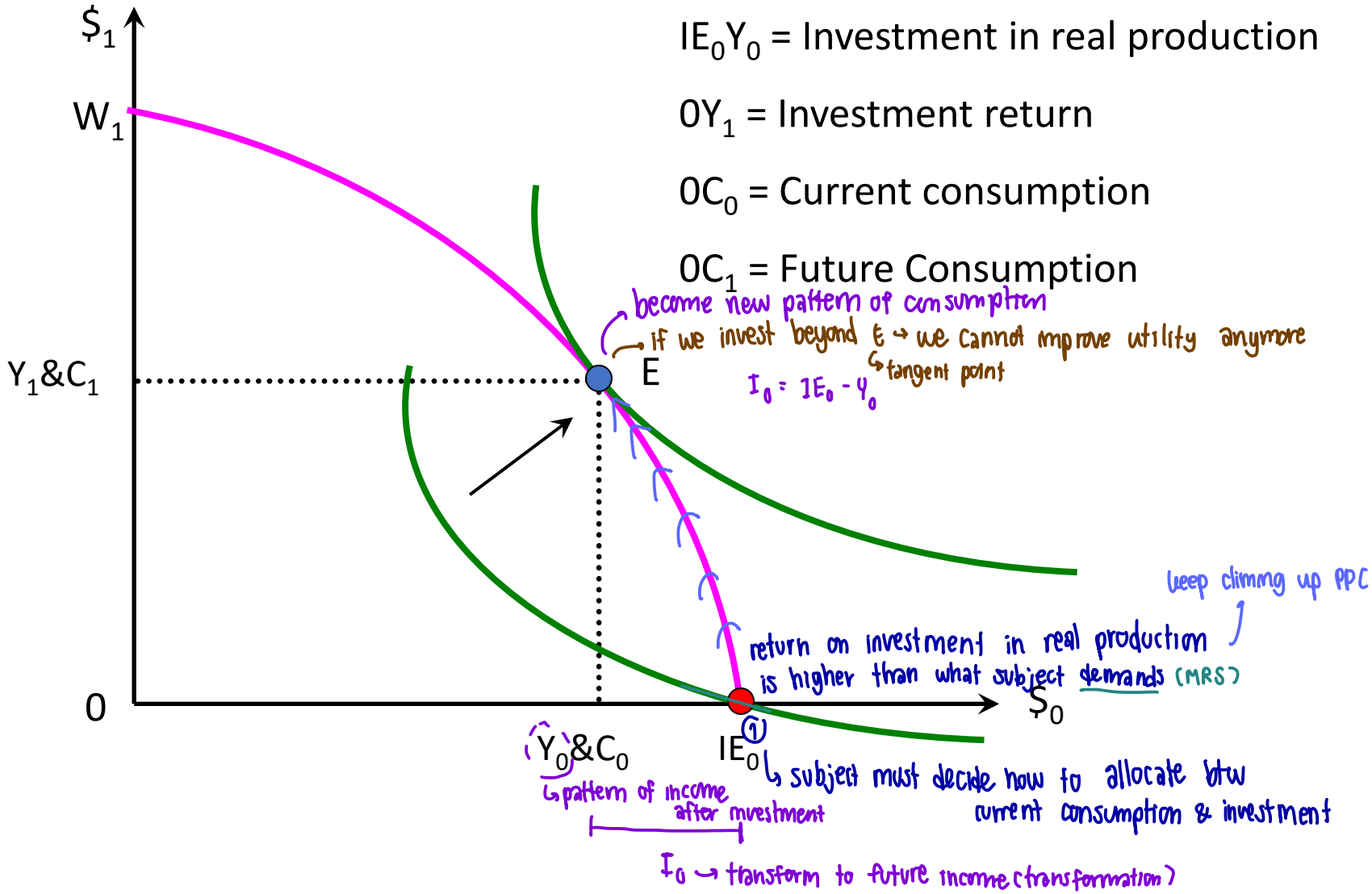
IE_0 = Initial endowment

IE_0Y_0 = Investment in real production

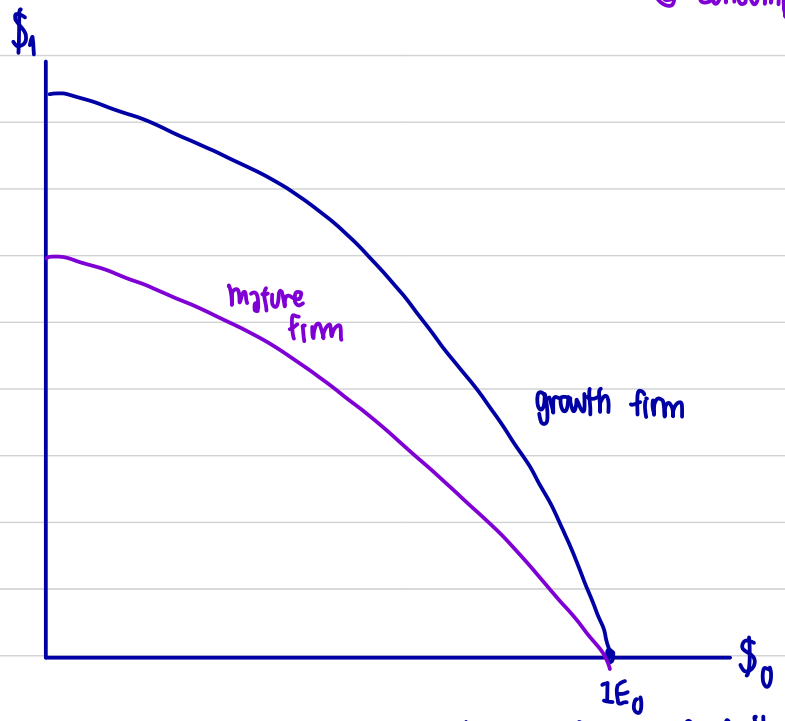
$0Y_1$ = Investment return

$0C_0$ = Current consumption

$0C_1$ = Future Consumption

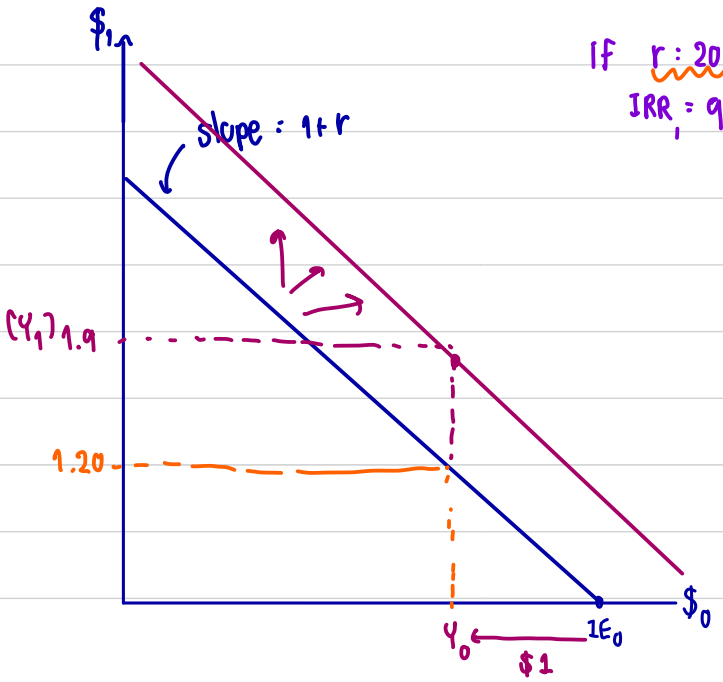


You have to make 2 decisions
① investment decision — ① in real production ; MRT diminishing
② consumption decision — ② capital mkt ; 1 + r constant



* Objective is to maximize utility ⇒ think of investment first → give the full picture of what we can afford
↳ create opportunity to consume
⇓
invest until 1+IRR = 1+r
of last project

If r : 20%
IRR1 : 90%



3.3 Optimal Inv-Cons Dec. with Capital Market

- With the capital market, the individual can choose whether to invest his resources in real production or in capital markets (lending).
- How could investment (real vs financial) and consumption be decided simultaneously when one decision affect the other decision?
- It turns out that the individual should make the investment decision that maximizes his wealth first. This will allow him to maximize the consumption opportunity set.
- With the largest possible consumption opportunity set, he could then make the optimal consumption decision.

3.3 Optimal Inv-Cons Dec. with Capital Market

- Starting with IE_0;

  [1] Make the production decision that maximizes wealth.

  – Let I_0 = the amount invested in real production and Y_1 is the future outcome from the investment. This will leave the individual with $IE_0 – I_0$ today. Let call this Y_0 (= $IE_0 – I_0$). After the investment decision, the individual will have Y_0 left today and Y_1 in the future.

  [2] Make the consumption decision that maximizes utility.

  – With (Y_0, Y_1), the individual could make use of the capital market to expand the consumption opportunity set and then choose the optimal consumption basket.

[1] Optimal Investment Decision

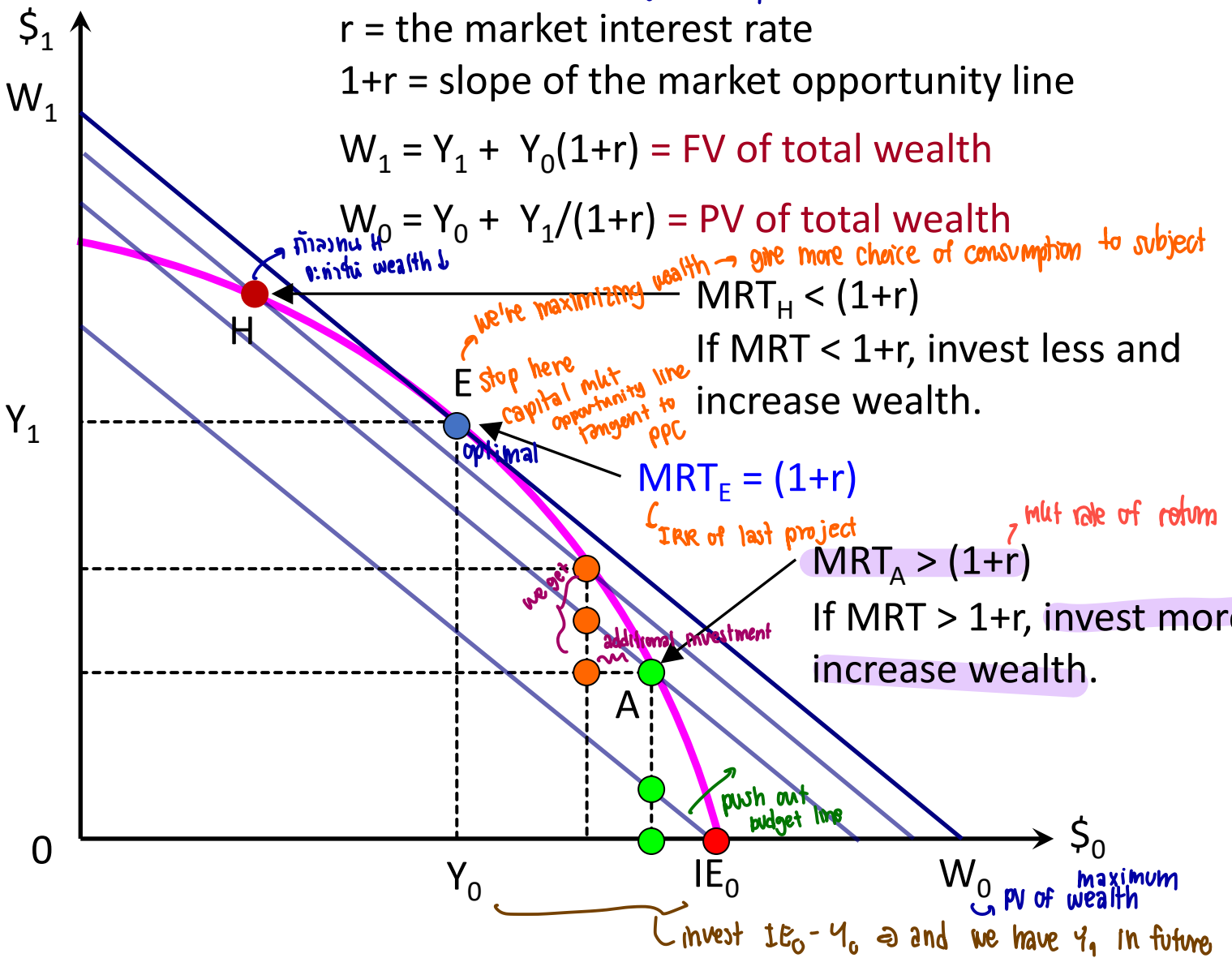
เราจะ IE_0, Y_0, Y_1, r

r = the market interest rate

1+r = slope of the market opportunity line

$W_1 = Y_1 + Y_0(1+r)$ = FV of total wealth

$W_0 = Y_0 + Y_1/(1+r)$ = PV of total wealth



$MRT_H < (1+r)$

If MRT < 1+r, invest less and increase wealth.

$MRT_E = (1+r)$

IRR of last project

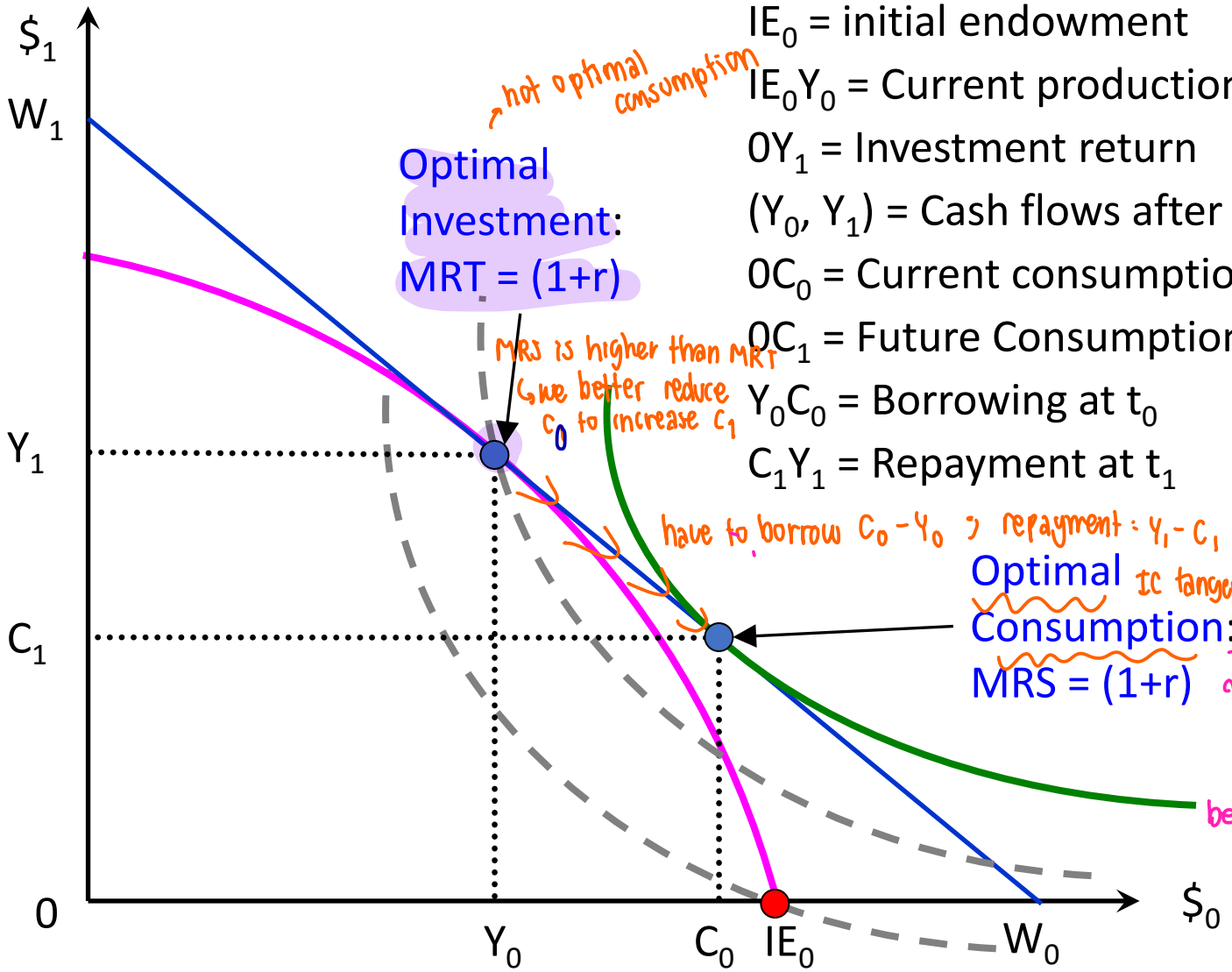
mkt rate of return

$MRT_A > (1+r)$

If MRT > 1+r, invest more and increase wealth.

bring in consumption preference

[2] Optimal Consumption Decision



IE_0 = initial endowment

IE_0Y_0 = Current production

$0Y_1$ = Investment return

(Y_0, Y_1) = Cash flows after investment

$0C_0$ = Current consumption

$0C_1$ = Future Consumption

Y_0C_0 = Borrowing at t_0

C_1Y_1 = Repayment at t_1

ทำไมต้องลงทุนก่อน แล้วค่อยกู้ ทำไมไม่ จด IE0 ไป C0 เลย

because return on investment (irr) is greater than mkt rate of return (r)

กู้มาลงทุนดีกว่า เอาเงิน IE0 ไปลงทุนเมื่อได้ return สูงกว่า กู้มา consume

low borrowing cost

3.3 Optimal Inv-Cons Dec. with Capital Market

* Subject should make investment decision first → until MRT = 1+r (not including consumption preference)

⇓

then make consumption decision

↳ consume until MRS = 1+r

borrow / lend

- The graphical solution implies that, regardless of consumption preference, if "MRT > (1+r)", the individual should invest in real production more, and vice versa.

- To find the optimal investment-consumption decision in mathematical form; → to use calculus to find max U
  - The PPC should be written in the form (note, IE_0 is a constant):

$$Y_1 = f(I_0) = f(IE_0 - Y_0) = f(Y_0)$$

  - The Utility function should be written in the form:

$$U = U(C_0, C_1)$$

3.3 Optimal Inv-Cons Dec. with Capital Market

[1] Find the optimal investment on the PPC where W_0 is maximized.

$$\text{Maximize} \quad W_0 = Y_0 + f(Y_0)/(1+r) \quad \text{wrt. } Y_0$$

The solution is found where MRT = (1+r). ← optimal investment

[2] Using Y_0 and Y_1 from step [1] to find the optimal consumption

$$\text{Maximize} \quad U(C_0, C_1) \quad \text{wrt. } C_0 \text{ and } C_1$$

$$\text{Subject to} \quad [Y_0 + Y_1/(1+r)] - [C_0 + C_1/(1+r)] = 0$$

The solution is found where the MRS = (1+r). ← optimal consumption

↓ depend on mkt

*when we make inv decision → don't concern about consumption preference → to maximize firm value and expand the choices for shareholders

3.4 The Case of a Corporation

obj. maximize shareholders' utility

- In a corporation, shareholders delegate the investment decision-making to the management. It is not immediately clear how managers could construct their investment decisions to satisfy all shareholders whose utilities are driven from consumption.
  - Shareholders' consumption preferences are not known
  - Preferences differ across different shareholders
- Is it possible to make one investment decision that satisfies all shareholders?

According to Fisher Seperation Theorem, investment decisions can be made seperately and independently from consumption decisions. Therefore, to maximize shareholders' utility, the management should maximize the value of firm by investing in real production until MRT = 1+r, which is the optimal investment.

This increase the opportunities for shareholders

3.4 The Case of a Corporation

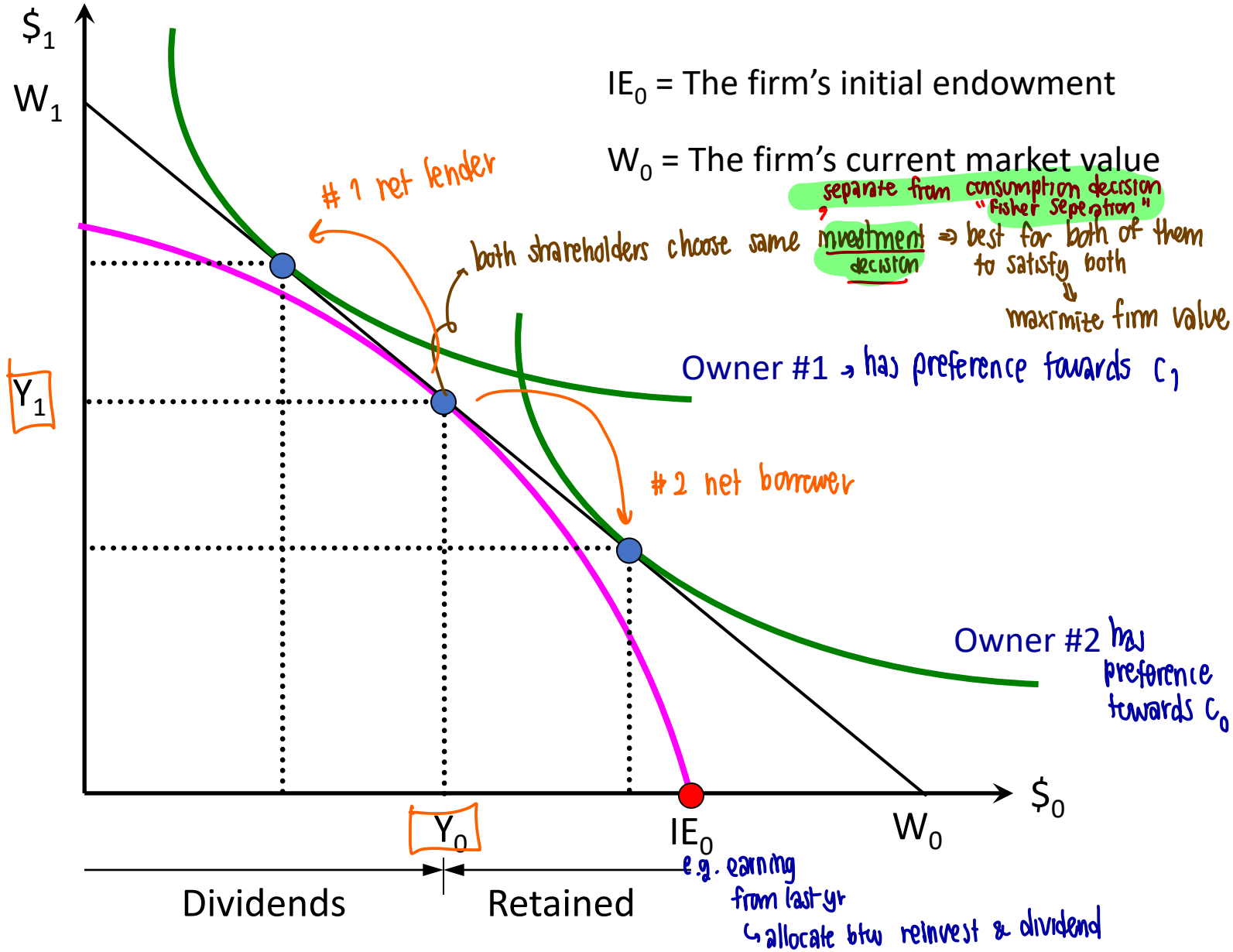
- As shown in the previous section, investment decisions can be made separately and independently from consumption decisions. This is the principle of separation called the Fisher Separation Theorem.
- Managers can maximize shareholder's utility by maximizing firm's value (i.e., W_0). This is achieved by investing in all projects whose IRR $\geq$ r. No specific information about the owner's consumption preference is required.
- This decision criterion is known as the market value criterion.
- Shareholders could make use of the capital market if the timing of cash flows from the firm does not suit their consumptions.

3.4 The Case of a Corporation

- The principle of separation is still applicable even when there are multiple shareholders.
- The figure below assumes that the firm is endowed with IE_0 and the management is deciding on how much to reinvest and how much to payout as dividend. The firm's investment opportunity set is presented by the PPC.
- Regardless of their consumption preferences, both sets of shareholders prefer the management to make the same investment decision.

2 shareholders → 1 firm has same PPC

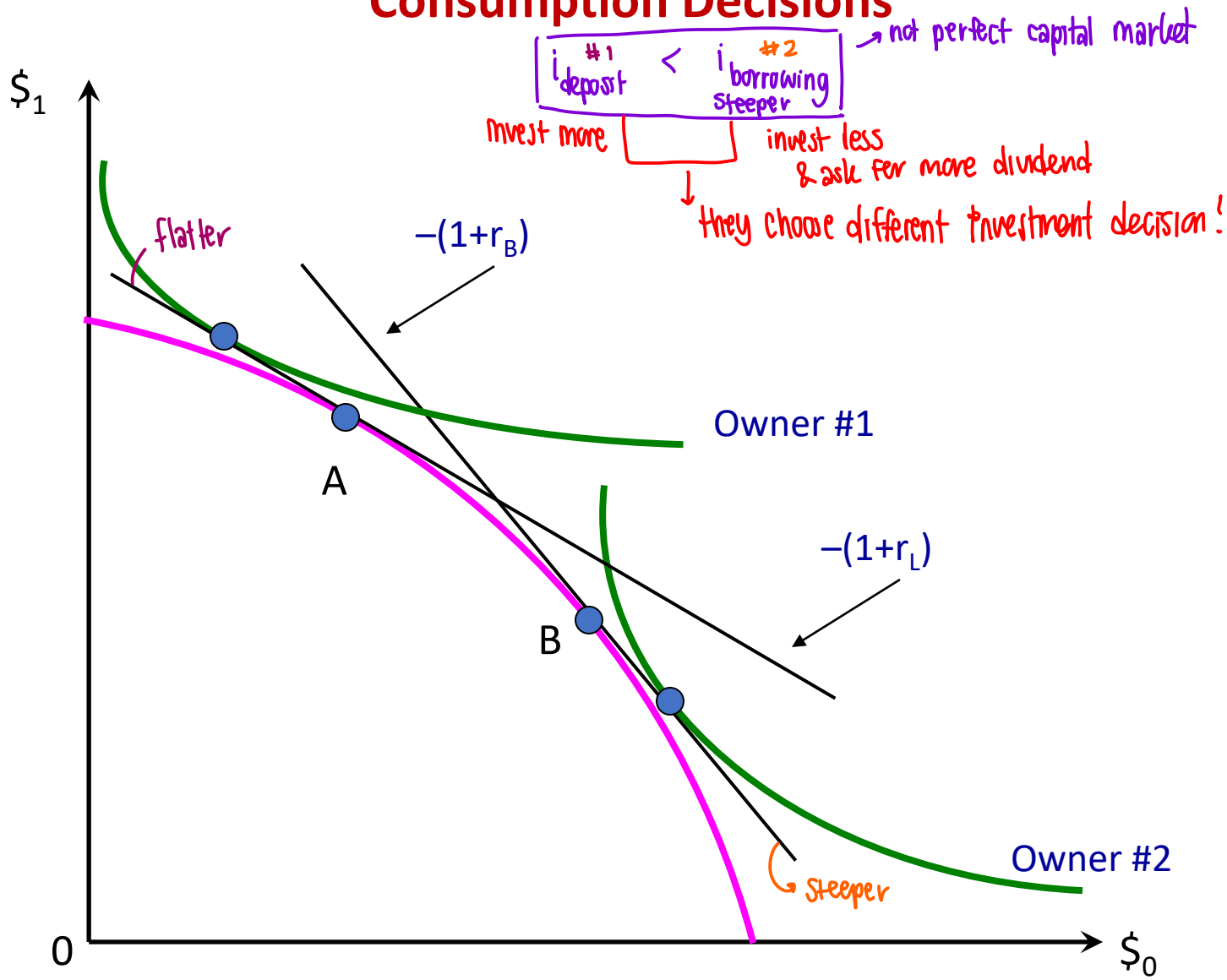
Corporate Investment Decision



3.5 Limitation

- The analyses so far have assumed that
  - There is no uncertainty regarding future outcomes of real investment. perfect certainty
  - The capital market is perfect.
- When capital market is not perfect, the Fisher's Separation Theorem breaks down.
- Examples of market imperfection include:
  - Transaction cost
  - Information asymmetry and agency problem

The Breakdown of the Separation between Investment and Consumption Decisions



relax certainty

don't know investment outcome for sure

3.6 Introducing Uncertainty

- The analysis so far assumes that investment outcomes are known with certainty. Therefore, the risk-free rate is the appropriate opportunity cost of capital.

  for sure

- However, corporations always make investment decisions without knowing the future outcomes for sure.
- Therefore, we need to adjust our framework by incorporating investment risk. That is, the utility theory need to be adjusted such that it could analysis how investors response to risk.

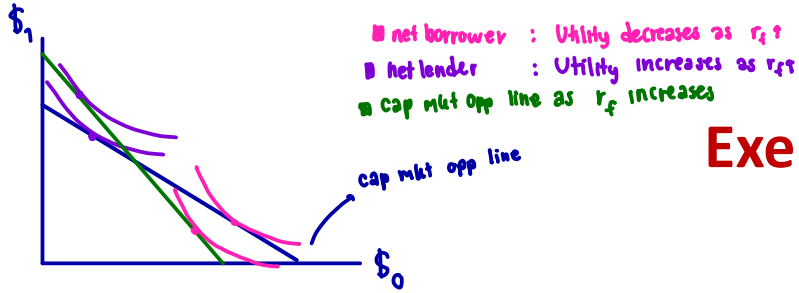
uncertainty vs. risk not the same in finance

is real

but cannot be measure

make assumption to measure it

to control it



Exercises

1. Two individuals are similar in every aspect except their preference toward current and future consumptions. Show graphically that an increase in interest rate will benefit the individual who prefers future to current consumptions and penalize the individual who prefers current to future consumptions.

→ need calculus → ไม่ออกสอบ → เน้น graph

2. Consider an investor with initial endowment IE_0 = \$16,000 and utility function $U = C_0 \cdot C_1$ facing an investment opportunity set (production possibility curve; PPC) described by the function;

$$Y_1 = 240 \times (16{,}000 - Y_0)^{1/2}$$

The interest rate for lending and borrowing is r = 20%. Find the optimal investment and consumption decision and show the result graphically.

3. Discuss the implication of Fisher's Separation Theory to corporate financial managers.

4. Verify slide 29: "The concavity assumption on total utility function (diminishing MRS) is sufficient to ensure that any point obeying MRS = (1+r) is the true maximum".

Exercises

5. Consider an investor with initial endowment IE_0 = \$32,000 and utility function $U = 2C_0^2+10C_0C_1$ facing an investment opportunity set (production possibility curve; PPC) described by the function;

$$Y_1 = 360\times(32{,}000 - Y_0)^{1/2}$$

The interest rate for lending and borrowing is r = 20%.

(a) How much will the investor invest in production?

(b) What is the NPV of the investments chosen be the investor?

(c) What is the present value of his total consumption in both periods?

(d) Does the investor borrow or lend in the capital market? Give a numerical answer.

(e) What is the optimal consumption for the two periods? Show the result graphically.