Derivative and Risk Mgt

I

- **Arbitrage** : make profit w/o risk of loss
- **Bond replication** :
  - Quoted as if par = \$100 , semi-coupon

EX

premium or discount depend on coupon rate

	Maturity	Coupon	Price
coupon-bearing bond: Bond-A	30 mos	10% (5% per period)	\$109
Bond-B	30 mos	3% (1.5% per period)	\$93
Bond-C (zero-coupon bond : no coupon payment or discount bond)	24 mos	0%	\$88.75 (price less than)

par : 100

	0	6M	12M	18M	24M	30M
Bond A	+580	(5	5	5	5	5+100)
Bond B	−600	(1.5	1.5	1.5	1.5	1.5+100)
Bond C	+1000	(			100	)
		2k	2k	2k	102k	0

24M 4% coupon bond

price = −0.58 (109) + 0.6 (93) − 1 (88.75)

= 96.17

if mispricing : 94.80 > 96.17 ⇒ short A, long B, short C

* Long 580 A, short 600 B , long 1000 C

II Interest rate

Convert from annual :

To semi-compounding : $(1+R_{annual}) = (1+\frac{R_{semi}}{2})^2 \Rightarrow R_{semi} = (\sqrt{(1+R_{annual})}-1)\times 2$ p.a.

To quarterly-compounding : $(1+R_{annual}) = (1+\frac{R_{quar}}{4})^4 \Rightarrow R_{quar} = ((1+R_{annual})^{1/4}-1)\times 4$ p.a.

To continuous-compounding : $(1+R_{annual}) = e^{R_{con.}} \Rightarrow R_{con.} = \ln(1+R_{annual})$

EX

Example 1 Suppose the 5-year interest rate is 2.83% semi-annual compound.

Q: Find the 5-year interest in continuous compound.

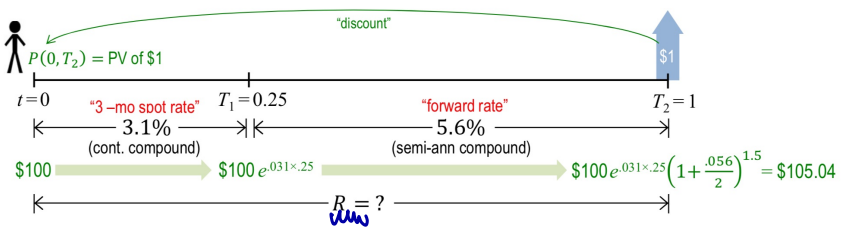
Ans :

$$\left(1+\frac{2.83\%}{2}\right)^{2\times 5} = e^{R_{con}\cdot 5}$$

$$R_{con} = \ln\left[\left(1+\frac{2.83\%}{2}\right)^2\right] = 2.81\% \text{ continuous compounding}$$

EX

Example 2



R_{annual} : $(1+R_{annual}) = (e^{3.1\%(0.25)})(1+\frac{5.6\%}{2})^{2(0.75)} \Rightarrow R_{annual} = 5.04\%$ annual compounding

$(1+\frac{R_{semi}}{2})^2 = \text{"—"} \Rightarrow R_{semi} = ((\text{"—"})^{1/2}-1)\times 2 = 4.978\%$ semi

$R_{quar} = ((\text{"—"})^{1/4}-1)\times 4 = 4.95\%$ quar

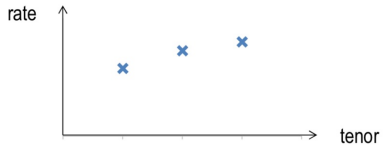
$e^{R_{con}\,1}$ $R_{conti} = \ln(\text{"—"}) = 4.92\%$ con

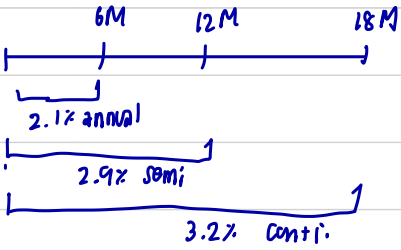
Ex

Example 3: Bond Valuation

- Given the following term structure of interest rate.

6-mo spot rate	2.1% (annual compound)
12-mo spot rate	2.9% (semi-annual compound)
18-mo spot rate	3.2% (continuous compound)





Price of X = $\frac{100}{(1+2.1\%)^{0.5}}$ = \$98.97

- Using this information, compute the price X of a 6-month zero-coupon bond and the price Y of a 9% coupon bond maturing in 18 months.

4.5% per period coupon paid every 6 month

Price of Y = $\frac{4.5}{(1+2.1\%)^{0.5}} + \frac{4.5}{(1+\frac{2.9\%}{2})^{2\cdot 1}} + 104.5e^{-3.2\%\cdot 1.5}$ = \$108.43

Yield → discount rate of all CFs

OTC (over-the-counter)

III Forward contracts

- Gain/loss or value of forward contract: $f_0 = 0$, $f_T(\text{long}) = S_T - F_0$, $f_T(\text{short}) = F_0 - S_T$
- Interest rate parity: Forward exchange rate is greater/lower than spot rate depends on differential btw foreign & domestic interest rate

  ⇓ if $i_{domestic} > i_{foreign}$ ⇒ forward price is traded at premium

  $i_{domestic} < i_{foreign}$ ⇒ forward is traded at discount

$f_T = S_T - F_0$ for long or $F_0 - S_T$ for short

- $f_0 = 0 = PV[S_T] - F_0 e^{-RT}$ if continuous compounding

  $F_0 = PV[S_T]e^{RT}$

depends on asset type of underlying:

① Non-income generating assets: $F_0 = S_0 e^{rT}$
- gold, zero-coupon bond, no-dividend stock

② Income-generating assets
- stock with lump-sum dividend: $F_0 = (S_0 - I_0)e^{rT}$ (PV of dividend)
- " —— " rate " —— ": $F_0 = (1 - \text{div}\%)\, S_0 e^{rT}$
- " —— " continuous " —— ": $F_0 = S_0 e^{(r-q)T}$
- foreign currency: $F_0 = S_0 e^{(r-q)T} = S_0 \frac{(1+r)^T}{(1+q)^T}$

  * Foreign exchange rate is equal to spot ex rate compounded by domestic int rate & discounted by foreign int rate

③ Assets with storage cost e.g. crude oil

$F_0 = (S_0 + U)e^{rT}$

EX

Example

		sell at this Bid		buy at this Ask
Spot USD : THB	=	34.45	—	35.50
		lend at		borrow at
1-yr USD rate	=	3.98%	—	4.01%
1-yr THB rate	=	1.75%	—	1.99%

Q: Which of the following quotes allow arbitrage? (Hint: Use trial and error)

			bid		ask
Dealer <A> :	1-yr fwd USD : THB	=	34.20	—	34.50
Dealer <B> :	1-yr fwd USD : THB	=	35.05	—	35.20
Dealer <C> :	1-yr fwd USD : THB	=	34.80	—	35.30

sell forward & buy spot

$F_0 = S_0 \frac{(1+r)^T}{(1+q)^T}$

(sell)
- Bid F_0 : to sell USD in the future

buy USD at $t=0$ → 1 USD = THB 35.5 buy borrow at 1.99% (home) and lend at 3.98% (foreign) → $\frac{35.5 \times (1.0199)}{1.0398} = 34.82$ (cost of 1 USD)

EX

		sell at	buy at
Spot GBP : THB	=	41.05	41.85
GBP : USD	=	1.2169	1.2422
GBP : EUR	=	1.1735	1.1875

1-yr spot int rate			
USD	=	4.62%	5.02% annual
GBP		2.18%	3.00%
THB		1.25%	1.66%
EUR		3.58%	3.99%

(sell)
Bid 1 USD in the future: borrow THB 41.85 to buy 1 GBP → sell 1 GBP at USD 1.2169

THB $\frac{41.85}{1.2169}$ to $\frac{1}{1.2169}$ GBP ← $\frac{1}{1.2169}$ GBP USD 1

cost of 1 USD in time $T = \frac{41.85}{1.2169} \times \frac{(1.0166)}{(1.0462)} = 33.498$ → short future (at 34.05) buy spot USD : THB

cost of sell 1 USD

IV Futures contract

- Exchange traded

- Mark-to-market everyday : the gains/losses are realized before the end of contract but daily even though you don't close your position

- margin account : need to deposit initial margin before start trading. <high volatility, high initial>
- Futures price tends to move with spot price, the final settlement price $F_T(T) = S_T$
- At maturity, $F_T = S_T$

V * Cost-of-Carry model :

① Stock, no div : $F_0 = S_0 e^{rT}$

② Stock, continuous div : $F_0 = S_0 e^{(r-q)T}$ (q: div rate, r: home int rate)

Foreign currency : $F_0 = S_0 e^{(r-q)T}$ (q: foreign int rate)

③ Commodity : $F_0 = S_0 e^{(r+u)T}$ (r: cost of fund, u: storage cost)

E.g., suppose $r = 0.78\%$, SET50 index = 949.89, and dividend rate of SET50 is 2.24%

Q: Find the price of SET50 futures that expires in 180 days.

$F_0 = S_0 e^{(r-q)T} = 949.89\, e^{(0.78\% - 2.24\%)(\frac{180}{365})} = 943.08$

net gain/loss = $F_0 - S_0 e^{(r+u)T}$

* Hedging with futures:

buy spot: $P/L = S_T - S_0 e^{(r+u)T}$ gain/loss = $F_0 - S_T$ (F_T)

- During down-turn, spot ↓ → short futures
- Beta = $\frac{Cov(r_m, r_p)}{\sigma^2(r_m)}$
- To fully hedge against mkt movement (β_{target}) ⇒ $N^* = -\beta_A \frac{V_A}{V_F}$ (No. of futures; port size; futures price × multiplier of 1 contract), with β_{target} ⇒ $N^* = (\beta_A^* - \beta_A)\frac{V_A}{V_F}$
- NAV of port = $price_A \cdot \#A + price_B \cdot \#B + \dots$
- Port return = $w_A r_A + w_B r_B + \dots$
- Port var = $w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \underbrace{\sigma_A \sigma_B \rho_{AB}}_{\sigma_{AB}}$

VI Interest rate futures

* Treasury bond futures

- Clean price = dirty price (full) − accrued interest
- Invoice price = clean price + accrued interest
  = conversion factor (F_T at maturity (quote as if par = 100) × $\frac{\text{par value of bond}}{100}$) + $\left(\frac{\#\text{days btw previous coupon \& delivery}}{\#\text{days btw previous \& next coupon}} \times \text{interest}\right)$
  ↳ CF < 1 if delivered coupon rate < >
- Cheapest-to-deliver bond (CTD) → lowest cost of carry = $\underbrace{CF \times F_0 + AI_T}_{\text{invoice price}} - \underbrace{FV[S_0]}_{\text{price that you buy spot}} + FV[\text{coupons}]$

EX Today is 21/1/24, Bond A maturity: 15/11/48, par = 100K, c = 9%, coupon payment 15 May & Nov, CF = 0.7751, $r_f = 3.25\%$
delivery @ 30/9/24, F_0 = 135.5, clean price: 109.25 $\xrightarrow{\times 1000}$ 109.25K

- S_0 = full price = $109.25K + 2\%(100K)\left(\frac{67}{184}\right) = 109.98K$ at $t = 0$ (15 Nov 23 - 21 Jan 24)
- Cost of carry = $-109.98K \times e^{3.25\% \cdot \left(\frac{253}{365}\right)} + 2K \times e^{3.25\% \left(\frac{138}{365}\right)} + \left(0.7751 \times 135.5K + \left[2K \times \frac{138}{184}\right]\right)$ (15 May 24 - 30 Sep 24)

VII Swaps:

* Interest rate swap

- Swap value = $B_{fx}(0) - B_{fl}(0)$
- Floating rate is known in t−1 (settled in arrears) & change every period
  ↳ Floating leg pays (per period): if R is annually = notional × $[(1 + R_{annual})^{0.5} - 1]$
  semi = notional × $\left[\left(1 + \frac{R_{semi}}{2}\right)^{0.5 \times 2} - 1\right]$
  quarterly = notional × $\left[\left(1 + \frac{R_{quar}}{4}\right)^{0.5 \times 4} - 1\right]$
  conti. = notional × $\left[e^{R_{con} \times 0.5} - 1\right]$

Ex

Case 16-01-2024

On December 15th, 2023 Assume that the spot USD : THB exchange rate on this day is 35 baht per dollar. Zhixiang Luo, a derivative dealer, structures a new 2-year USD-THB currency swap with the following cash flows:

	฿-leg (fixed rate b)	$-leg (using floating treasury rate)
Jun 15th, 2024	฿35mil × b% / 2	$1mil × (6-mo rate on Dec 15th, 2018) / 2
Dec 15th, 2024	฿35mil × b% / 2	$1mil × (6-mo rate on Jun 15th, 2019) / 2
Jun 15th, 2025	฿35mil × b% / 2	$1mil × (6-mo rate on Dec 15th, 2019) / 2
Dec 15th, 2025	฿35mil × b% / 2 + $1mil	$1mil × (6-mo rate on Jun 15th, 2020) / 2 + $1mil

rates change as time passes as spot rates changes (time fraction also change

0	0.5	1	1.5	2
15/12/23	15/6/24	15/12/24	15/6/25	15/12/25

Given spot US int rate (annual)

Floating US int (semi)

$\Rightarrow (1+2.57\%)^{0.5} = \left(1+\frac{R_{semi}}{2}\right)^{2\times 0.5}$

2.57%	2.96%	3.19%	3.32%
$\left(\sqrt{1.0257}-1\right)\times 2$	$\left(\frac{1+2.96\%}{(1+2.57\%)^{0.5}}-1\right)\times 2$	$\left(\frac{(1+3.19\%)^{1.5}}{(1+2.96\%)^{1}}-1\right)\times 2$	$\left(\frac{(1+3.32\%)^{2}}{(1+3.19\%)^{1.5}}-1\right)\times 2$

$(1+2.96\%)^{1} = (1+2.57\%)^{0.5}\left(1+\frac{F_{6m,6M}}{2}\right)^{0.5\times 2}$

Floating leg (US)

USD 1 M × $\frac{2.55\%}{2}$ ← doesn't change even time passes

USD 1M × ___ + 1M

↓ discount by $(1+2.57\%)^{0.5}$

sum = $B_{fl}(0)$

Fixed leg

35M × $\frac{R_{semi}}{2}$ ← fixed rate

VIII Intro to options

VIII Intro to options, e.g. American Call Option (exp. Dec 2023) to buy 100 shares of MSFT at $360/share

- Call option (right to buy) on an underlying asset — can be exercised anytime before maturity (C_t) — European
- expiration / maturity
- contract size / multiplier
- underlying asset
- exercise (K) / strike price

① Option premium (C_t or c_t) fluctuates along with underlying share price (S_t) → move in same direction

② $C_t \leq S_t$ — rarely exists in efficient mkt → So, $C_t \geq S_t - K$

③ C_t must not be cheaper than $S_t - K$; If $C_t < S_t - K$ then long call can make arbitrage by immediately exercise the option → payoff = $S_t - K$ → net arbitrage profit = $S_t - K - C_t$

④ Intrinsic value of Option = $(S_t - K)^+$ → If $S_t > K$ in-the-money, if $S_t < K$ out-of-money (tells min price of option)

⑤ C_t (option premium) = intrinsic value + time value (0 or positive)

⑥ Time value depends on many factors: ▸ volatility of underlying asset (high volatility → high time value, uncertainty), ▸ far from maturity

↳ ▸ deep ITM or OTM → near 0 : long call < high S_t / ITM, low S_t OTM >, long short < high S_t / OTM, low S_t / ITM >

↳ time value at maturity = 0 , if becomes 0 before maturity → signal to exercise option

⑦ If Time value > 0 , option price > pay-off from exercising → option should not be exercised!

⑧ Whenever option's price = intrinsic value (time value = 0) → long position will exercise option (same as selling option)

⑨ Most traders don't exercise but trade for profit & close positions before maturity.

★ There is leverage effect on option : return % = $((S_t - K)^+ / \text{option price}) - 1$ → high profit or loss all (<0)

★ Intrinsic value for American = $(S_t - K)^+$, European = $(S_t - Ke^{-r(T-t)})^+$, price/value/premium (C_t) = IV + time value

★ Usually $C_t \geq c_t$ but if no dividend, $C_t = c_t$

★ Put option : IV = $(K - S_t)^+$, $K > S_t$ ITM, $K < S_t$ OTM

X Binomial model: only two possible scenarios next period (price ↑ or ↓)

↳ becomes more realistic by reduce range of time step → to almost zero "Black-Scholes model"
→ multi-period model ↔ geometric grounding motion

* Up-factor (u) = S_u/S_0, down-factor (d) = S_d/S_0

[if stock has volatility σ and Δt is small → $u = e^{\sigma\sqrt{\Delta t}}$, $d = e^{-\sigma\sqrt{\Delta t}}$

σ low $u \approx 1.00..$, $d \approx 0.999..$

* Should never use real-world prob to compute option price! bcs everyone is risk averse
↳ using real-world prob can cause arbitrage

call
- If option price is too low / underprice: long call option & short share → long put option & long share
- If option price is overprice: short option & long underlying → short put option & short underlying

put

(no arbitrage price)

* Risk-neutral pricing formula

fake prob (q) = $\frac{e^{r\Delta t} - d}{u-d} = \frac{(1+r)^{\Delta t} - d}{u-d} = \frac{(1+\frac{r_{semi}}{2})^{2\Delta t} - d}{u-d}$ (annual)

v - call option price, p - put option price

delta (h) = $\frac{V_u - V_d}{S_u - S_d}$ (option price) for any type of option

Ex: if S_0 (price now) = \$100, $u = 1.2$, $d = 0.7$, $r_f = 5\%$ cont. compound → $q = \frac{e^{5\%(1)} - 1.7}{1.2 - 1.7} = 70\%$

Option price (right → left)

$h(\text{put}) = -0.35$

$S_0 = \$100$

$q = 70\%$: $S_1 = 120$, $V_1 = (0.7 \times 39)e^{-5\%(1)}$, $P_1 = (0.3 \times 21)e^{-5\%(1)} = 6$

30%: $S_1 = 70$, $V_1 = 0$, $P_1 = (0.7 \times 21 + 0.3 \times 56)e^{-5\%(1)} = 30$

$S_2 = 144$: $V_2 = (S_T - K)^+ = (144 - 105)^+ = 39$, $P_2 = (K - S_T)^+ = (105 - 144)^+ = 0$

$S_2 = 84$: $V_2 = 0$, $P_2 = 21$

$S_2 = 49$: $V_2 = 0$, $P_2 = 56$

left → right

$P_0 = \frac{0.7 \times 6 + 0.3 \times 70}{e^{5\% \times (1)}} = 12.56$

$h(\text{put}) = -0.48$

h for put = $\frac{P_u - P_d}{84 - 49} = \frac{21 - 56}{84 - 49} = -1$ delta for put option is negative

* NOTE:
For American option: $P_0 = \max\{P_0, \text{intrinsic value}\}$
P_0 from calculation; intrinsic value: if this is greater, then exercise now
call: $(S_0 - K)^+$
put: $(K - S_0)^+$

XIV Risk Management

↗ process of measuring risks & using tools to adjust risk to desired level

measures: Volatility, Beta (sensitivity to mkt movement), Duration (sensitivity to int rate), Greeks (delta, gamma, vega, etc.)

Tools: diversification, futures contract, int rate derivatives, options

Types of risks:

① Market risk: any type of risk came from price fluctuation → widely-used measure is Value-at-Risk (VaR)
↳ e.g. exchange rate fluctuation for imexport business, int rate → bond price, oil price for airline

② Credit risk: some bond will be downgraded e.g. corporate bond default

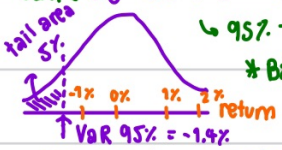
③ Liquidity risk: lack of a liquid mkt to trade certain bond

④ Operational risk: record-keeping system crashing, employee strike, human error

95%, -1.65
99%, -2.33

VaR: e.g. invest \$1 mm, 95% VaR sure that will not lose more than -\$14K (-1.4%)

↳ 95% - VaR = 5th percentile on part's return distribution



* Based on normal dist: 95% VaR = $\mu_p - 1.65\sigma_p$ (0%)

$\sigma_p^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + w_C^2\sigma_C^2 + 2w_Aw_B\rho_{AB}\sigma_A\sigma_B + 2w_Aw_C\rho_{AC}\sigma_A\sigma_C + 2w_Bw_C\rho_{BC}\sigma_B\sigma_C$

$\mu_p = w_A\mu_A + w_B\mu_B + w_C\mu_C$

$\rho_{AB} = \frac{\sigma_{AB}}{\sigma_A\sigma_B}$

don't have to be consecutive day (independent)

• 1-yr VaR from samples of 1-day returns
① 1-yr VaR = (1-day VaR) × $\sqrt{252}$ ② Historical simulation with bootstrapping

XV Credit risk

Expected loss (EL) = loss given default × prob of default × exposure at default
(1 - recovery rate) (depends on time horizon) (money you lend)

Credit risk that people default on you

Given rating, cumulative default prob (PD) $Q(T) = P(\tau \le T)$

Ex $Q(2) = P(\tau \le 2) = 0.49\%$, $P(2 < \tau \le 3) = Q(3) - Q(2) = 0.91\% - 0.49\% = 0.42\%$, $P(\tau > 5) = 1 - Q(5)$
chance that default within 2 yrs ↳ survive more than 2 yrs but default before yr 3

survive more than 5 yrs

* Survival prob = 1 - PD * T unit in year!!! 6 mo = 0.5 year

* PD Estimation: ① use mkt data → historical default rate (real world), bond prices (risk-neutral)

② Model: Intensity-based: $Q(T) = 1 - e^{-\lambda T}$; λ is default intensity (any positive number)
e.g. $\lambda = 25$ bps = 0.25%

↳ structural model (Merton's) → use mkt value of firm's asset, volatility of firm asset, r_f, firm's debt
(Black, Scholes; option pricing theory; volatile)

Shdr E = 0

* Stock is call option on firm's asset → if mkt value of asset not enough to payback → default ↗

* PD = $1 - N(d_2)$ distance to default

☆ Properties of stock options:

* Effect of variables on option pricing

	Futures / forward	European call option price (value) c	$\leq$	American C	European put option price (value) p	$\leq$	American P
① Underlying price (S_0) ↑	↑ delta 1 price impact	↑ higher intrinsic value ↑			↓ lower intrinsic value ↓		
② Exercise / Strike price (K) ↑	–	↓ lower " ——— " ↓			↑ higher " ——— "		
③ Time-to-maturity (T) ↑	↑ longer opportunity cost / funding ↑	↑ if r is ⊕ ↑ higher time value			?		↑ higher time value
④ Volatility / Implied vol (σ) ↑	–	↑ ——— higher time value ——— ↑					
⑤ Risk-free rate (r) ↑	↑ higher opportunity / funding cost	↑ higher forward price ↑			↓ higher forward price of underlying		
⑥ PV of dividend (D) ↑ paid life of option	↓ lowers the share price ——— ↓				↑ lower the share price		

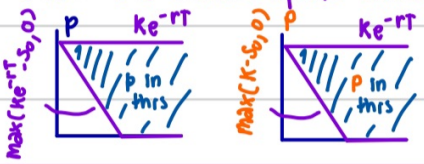
* Find whether there's arbitrage opportunity:

- Lower bound for c, no dividends: $c \geq \max(S_0 - \underbrace{Ke^{-rT}}_{\text{PV of strike price}}, 0)$
- Lower bound for p, " ——— ": $p \geq \max(Ke^{-rT} - S_0, 0)$

Bounds for c / C option (no div)

* Put-call parity (no div): $c + Ke^{-rT} = p + S_0$

* American call on non-div stock should never be exercised early (lose time value)
* Puts have limited upside unlike call option

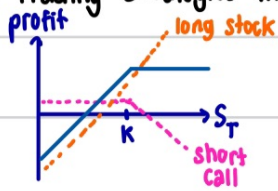


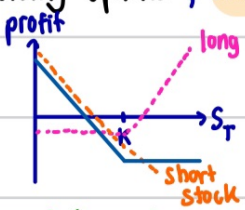
Impact of div on lower bounds to option price

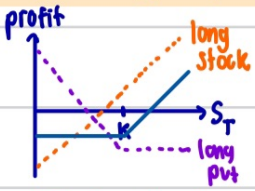
$c \geq S_0 - D - Ke^{-rT}$

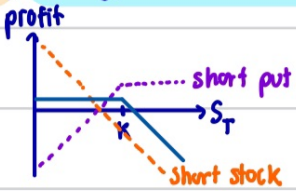
$p \geq D + Ke^{-rT} - S_0$

☆ Trading Strategies involving options; Two or more same type (spread), diff type (combination)

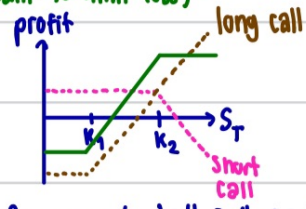




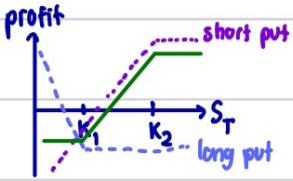




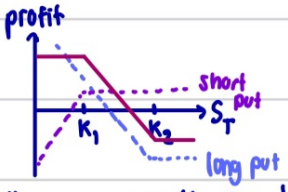
▸ Bull Spread using calls (want to limit loss)



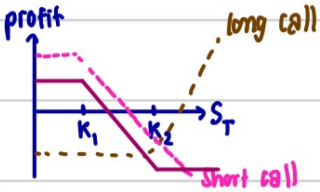
▸ Bull Spread using puts



▸ Bear Spread puts

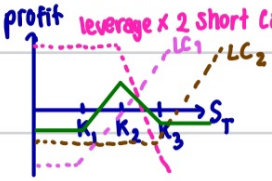


▸ Bear Spread Calls



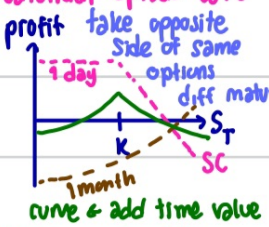
* Box spread: bull call spread + bear put spread → all options worth PV diff btw K (European)

▸ Butterfly spread calls



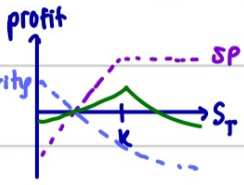
▸ Butterfly spread puts

▸ Calendar Spread calls

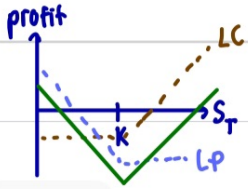


curve ← add time value

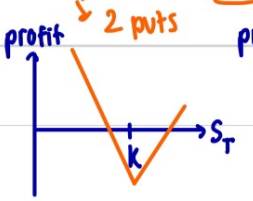
▸ Calendar spread puts

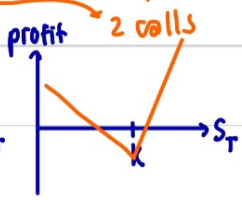


▸ Straddle combination

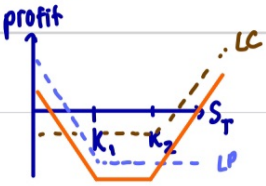


▸ Strip & Strap (leverage-up 1 side)





▸ Strangle combination



2604643 Derivatives and Risk Management

There are 3 questions (page 6 to 8). Answer all the questions. The maximum possible points are indicated in each question. The full score is 60 points and the passing score is 30 points.

Problem 1 The following table shows the spot interest rates in continuous compound:

1-month rate = 1.94%	4-month rate = 2.29%	7-month rate = 2.36%	10-month rate = 2.41%
2-month rate = 2.15%	5-month rate = 2.31%	8-month rate = 2.38%	11-month rate = 2.42%
3-month rate = 2.25%	6-month rate = 2.34%	9-month rate = 2.40%	12-month rate = 2.43%

a) (10 points) Compute the 6-month interest rate, 6 months forward (answer in annual compounding)

b) (10 points) Compute the 3-month interest rate, 9 months forward (answer in quarterly compound)

~~FV of \$1 = FV of \$1~~

~~a) $e^{0.0243} = e^{0.0234 \times 6/12} \times e^{r \times 6/12}$~~

~~$r = 0.0252 = 2.52\%$ pa~~

$e^{2.43\% \cdot 1} = e^{2.34\% \cdot 0.5} \times (1 + F_{6m,6m})^{0.5}$

$F_{6m,6m} = \left(\frac{e^{2.43\%}}{e^{2.34\% \cdot 0.5}}\right)^2 - 1 = 2.55\%$ annual compounding

b) $e^{2.43\% \cdot 1} = e^{2.4\% \cdot \frac{9}{12}} \times \left(1 + \frac{F_{3m,9m}}{4}\right)^{4 \times 3/12}$

$F_{3m,9m} = \left(\left(\frac{e^{2.43\% \cdot 1}}{e^{2.4\% \cdot 9/12}}\right) - 1\right) \times 4 = 2.53\%$ quarterly com

Problem 2 Describe the main differences between the Forward Rate Agreements (FRA) and Interest Rate Swaps (IRS). Also, give 2 examples of the usefulness of the FRAs and the IRSs. (20 points)

Differences between Forward Rate Agreements (FRA) and Interest Rate Swaps (IRS)

Forward Rate Agreements (FRA):
1. **Definition:** An FRA is a contract between two parties to exchange interest payments on a notional principal amount for a future period based on an agreed-upon interest rate.
2. **Duration:** Typically short-term, usually up to one year.
3. **Payments:** Only a single payment is made at the settlement date, which is the difference between the agreed-upon interest rate and the actual interest rate.
4. **Purpose:** Primarily used to hedge against interest rate risk for a short-term period.

Interest Rate Swaps (IRS):
1. **Definition:** An IRS is a contract between two parties to exchange cash flows of interest payments on a notional principal amount over a set period, typically one party pays a fixed rate while the other pays a floating rate.
2. **Duration:** Usually long-term, ranging from one year to 30 years.
3. **Payments:** Multiple periodic payments are made throughout the life of the contract.
4. **Purpose:** Used for both hedging against interest rate risk and for speculative purposes over a longer term.

Examples of Usefulness

Forward Rate Agreements (FRAs):
1. **Hedging Interest Rate Exposure:**
   - **Example:** A company expects to take out a loan in three months and is concerned that interest rates might rise. By entering into an FRA, the company locks in the interest rate today for the future loan, thus protecting itself from the risk of rising interest rates.
2. **Speculating on Interest Rate Movements:**
   - **Example:** An investor believes that interest rates will fall in the next six months. They enter into an FRA to receive the fixed rate and pay the floating rate. If interest rates fall, the investor profits from the difference.

Interest Rate Swaps (IRSs):
1. **Managing Long-term Interest Rate Exposure:**
   - **Example:** A company has a long-term floating rate debt but prefers to have fixed interest payments for budgeting purposes. By entering into an IRS, the company swaps its floating rate payments for fixed rate payments, thus stabilizing its interest expenses.
2. **Arbitraging Interest Rate Differentials:**
   - **Example:** Two companies, one with a comparative advantage in borrowing at fixed rates and the other at floating rates, enter into an IRS to swap their interest payments. Both companies benefit from the swap by obtaining a lower effective borrowing rate than they could achieve individually.

In summary, FRAs are used for short-term interest rate risk management and speculative purposes, involving a single payment at settlement. In contrast, IRSs are used for long-term interest rate risk management and can involve multiple periodic payments throughout the life of the contract.

Problem 4 Give an example of a portfolio that is delta-neutral. Also, briefly explain the purpose of delta hedging. (10 points)

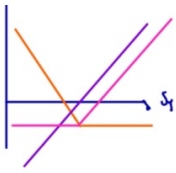
----- END OF DERIVATIVES AND RISK MANAGEMENT EXAMINATION -----

- Delta is the sensitivity of portfolio to the change in price of underlying assets
- Example of neutral delta port :
  - Long 100 shares of stock with delta = $100 \times 1 = 100$
  - short call options
    - suppose delta of 1 call option is 0.5
    - then, we need to short $\frac{100}{0.5} = 200$ call option to obtain neutral delta

      $100 - (200 \times 0.5) = 0$

- Purpose of delta hedging is to manage risk by lower the sensitivity of portfolio to underlying asset price. Moreover, it helps to stabilize/protect the portfolio value to remain the same despite change in underlying asset price.

Problem 3 John is an option trader. He considers the following two strategies:



- **Strategy A** : Protective Put (Long one stock and long one put option)
- **Strategy B** : Buy a Call

Assume that all options involved are European, have the same strike price, and matures at the same time. Also assume that the put-call parity holds at all times. Suppose also that the risk-free rate is negligible (i.e., assume that the risk-free rate is zero). John feels that Strategy A and Strategy B are very similar. He thinks that:

$c + K_e = p + S_T$

I. At time zero, both strategies require the same amount of *initial outlay*.
II. The *profit* at any time before maturity is the same for both strategies.
III. At maturity, the profit graphs for both strategies show the same maximum loss.
IV. At maturity, the profit graphs for both strategies show the same breakeven point.

Mary is another option trader. She thinks that John's statements are not entirely correct. Mary agrees with Statements III-IV, but disagrees with Statements I-II.

Do you think John is right, or Mary is right? Or do you think that both John and Mary are partially wrong? Among Statements I-IV, please identify which ones you think are correct, and which ones you think are incorrect. Give reasons to support your belief. (20 points)

which ones you think are incorrect. Give reasons to support your belief. (20 points)

▸ Statement I is not true. The initial outlay of strategy A is probably more expensive than strategy B. This is because, the call option premium that investors is lower or equal to spot stock price minus strike price. Thus, strategy B, with the need to pay for spot stock price, is more expensive.

▸ Statement II is true :

- The payoff of 2 strategies are the same according to the graph.
- As call-put parity holds, call premium must equal to put premium for the options on same underlying asset and same strike & maturity

▸ Statement III is true, both strategies have same minimum loss at call or put premium.

▸ Statement IV is true as both strategies need stock to rise by at least call or put premium to breakeven.

2) give at least 4 factors that affect **EU call option premium**. Summarize in a table and explain for each factor

Factors affecting call's price	
1. Underlying's price (+) $S_t \uparrow \rightarrow C \uparrow$	
2. Dividend (−) $D \rightarrow S_t \downarrow \rightarrow C \downarrow$	
3. Strike price (−) $K \uparrow \rightarrow C \downarrow$	
?? 4. Time-to-maturity (+) not the case for EU! เพราะฉะนั้นอาจจะ: Time Value ปัจจุบัน: → C↑	
5. Risk-free rate (+) $r \uparrow \rightarrow$ PV of $K \downarrow \rightarrow C \uparrow$	
** 6. Volatility (+) volatility มัน un $Vol \uparrow \rightarrow C \uparrow$	

→ The higher underlying's price, the higher call option premium.
Since call option give you the chance to buy underlying assets at pre-specified price.

→ The higher dividend, the lower call option premium.
Dividend announcement have the effect of reducing the stock price on ex-dividend date. This is bad for the value of call option.

→ The lower strike price, the higher call option premium.
Call option becomes more valuable when the strike price is low. Call option holder can buy underlying assets cheaper than the market price.

→ The longer TTM, the higher call option premium.
the owner of the long-life option has all exercise opportunities open to the owner of the short-life option.
↳ แต่อาจจะไม่ใช้กับ European Call option นะ

→ The higher rf, the higher call option premium.
Higher rf drive present value of K down. Therefore, it will increase call option premium.

→ The higher volatility of underlying assets, the higher call option premium.
As volatility increases, the chance that stock will do very well or very poorly increase. So, the value of call option increase as well.

at maturity; $C_T = S_T - K$

Along the way; $C_t = S_t - \frac{K}{(1+r)^n}$ ↓

↑ ↑

Problem 2 Explain the relationship between the forward exchange rate, the spot exchange rate, the foreign interest rate, and the domestic interest rate. If possible, use an equation to help explain the relationship. (10 points)

fwd rate has to be non-arbitrage

Forward exchange rate = spot exchange rate $\times \frac{(1+r_h)^T}{(1+r_f)^T}$

Forward rate is traded at discount if $r_h < r_f$

premium $>$