

Interest Rate and Term Structure

Interest Rates

- Interest rates are among the most closely watched variables in the economy. It is imperative that what exactly is meant by the phrase *interest rates* is understood.
- Interest rates, in other contexts, can be referred to as
 - Yield to maturity
 - Discount rate
 - Rate of return
 - Hurdle rate
- This chapter will discuss a concept known as *yield to maturity* (YTM) yield curve and term structure.



actual fed fund rate in the market
(lend/borrow among large commercial banks)
effective rate follows target rate by Fed

Yield to Maturity

- What is the “yield” or “yield to maturity”?

↑ hold bond until maturity and you obtain this rate

Computing Yield to Maturity

$$\hookrightarrow \frac{1000 \times 5}{2} = 25 \text{ semi-annually}$$

↑ face value
2 semi

- Investor buys 5% percent coupon (semi-annual payments) bond for \$951.90; bond matures in 3 years.
- What is the *yield to maturity* that the investor earn from holding this bond?

$$951.90 = \frac{25}{(1 + (i/2))^1} + \frac{25}{(1 + (i/2))^2} + \dots + \frac{1,025}{(1 + (i/2))^6}$$

face value
↓

FV: 1000		excel
PMT: 25		= rate(6, 25, -951.9, 1000)
PV: -951.90		= 3.4% semi-annual basis
N: 3 × 2 = 6		
i _{1/2} = 3.4% ⇒ i = 6.8%		

- Ans. 3.4% semi-annually, 6.8% annually

Common Yield Measures

- Expected Yield vs. Realized Yield

Expected Yield vs Realized Yield

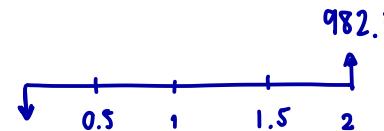
occur before transaction

- **Expected yield** refers to “the ex-ante or predicted yield for a given holding period” (normally, shorter than maturity). Thus investors must forecast 1) expected interest rate and bond price at the end of holding period
- For example, if the bond in the previous question was expected to sell at \$982.1 after two-year holding period, what will be the expected yield? ↳ N : 2x2

$$951.90 = \frac{25}{(1 + i/2)} + \cdots + \frac{25 + 982.1}{(1 + i/2)^4}$$

expected yield

- Ans 3.38% semi-annually, 6.76% annually
= rate(4, 25, -951.9, 982.1)



Expected Yield vs Realized Yield

occur after transaction

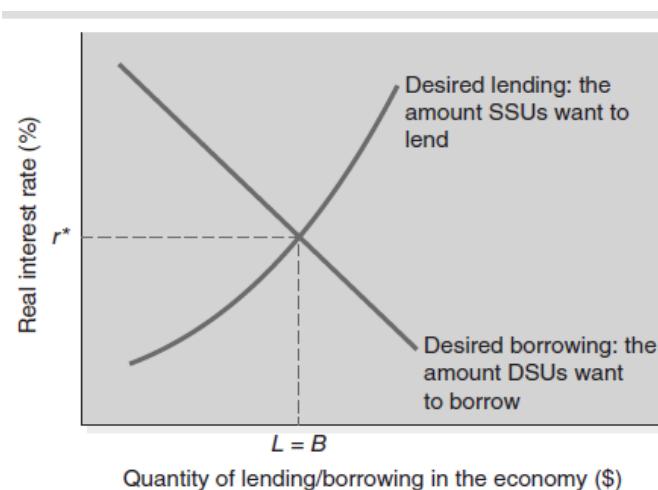
- **Realized yield** refers to “the ex-post or hindsight actual rate of return, given the cash flows **actually received** and their timing”.
- For example, if the bond in the previous question was eventually **sold at \$960** after two-year holding period, what will be the realized yield?

$$951.90 = \frac{25}{(1 + i/2)} + \cdots + \frac{25 + 960}{(1 + i/2)^4}$$

- Ans - 2.83% semi-annually → 5.66% annually

Why Do Interest Rates Change?

- We will examine the forces that move interest rates and the theories behind those movements. We discuss on
 - Supply and Demand in the Bond Market
 - Changes in Equilibrium Interest Rates



↳ explain the amount of funds related to change in interest rate

Loanable Funds Theory

- If competitive forces operate in financial sector, laws of supply and demand will bring rates into equilibrium.
- Equilibrium is temporary or dynamic: Any force that shifts supply or demand will tend to change interest rates.

Loanable Funds Theory

- lender* ↗ investor in bond
- Sources of funds available to invest in financial claims:
 - Consumer savings (household)
 - Business savings
 - Government budget surpluses *budget surplus income > spending*
 - Central Bank action *central bank purchases bond*
 - Uses of funds raised from issuing financial claims:
 - Consumer credit purchases *Borrower cannot issue bonds X*
 - Business investment → issue bond, largest borrower
 - Government budget deficits *budget deficit*

Determinants of Asset Demand

- An *asset* is a piece of property that is a store of value.
 - Assume that we face the question of whether to buy and hold an *asset* or whether to buy one asset rather than another, an individual may consider the following factors.
assume there's only 1 asset in economy "bond"
→ wealth ↑ → choose btw bond & consume
1. **Wealth:** total resources owned by individual, including all assets
 2. **Expected return:** return expected over the next period, on one asset relative to alternative assets
 3. **Risk:** degree of uncertainty associated with the return, on one asset relative to alternative assets
 4. **Liquidity:** the ease and speed with which an asset can be turned into cash, relative to alternative assets

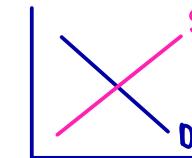
Determinants of Asset Demand

Quantity demanded of an asset is affected from...

1. **Wealth:** Holding everything else constant, an increase in wealth raises the quantity demanded of an asset
2. **Expected return:** An increase in an asset's expected return relative to that of an alternative asset, holding everything else unchanged, raises the quantity demanded of the asset
Note. This is not the same as "Expected interest rate" in the following slide.
3. **Risk:** Holding everything else constant, if an asset's risk rises relative to that of alternative assets, its quantity demanded will fall
4. **Liquidity:** The more liquid an asset is relative to alternative assets, holding everything else unchanged, the more desirable it is, and the greater will be the quantity demanded

Determinants of Asset Demand

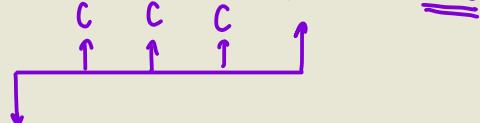
↑↑↑
↳ bond



Variable	Change in Variable	Change in Quantity Demanded at Each Bond Price	Shift in Demand Curve
Wealth	↑ people are wealthier	↑	
Expected interest rate	$\begin{array}{c} \text{return} \\ \downarrow \\ \text{current 3\%} \end{array}$ $\begin{array}{c} \text{market expect} \\ \downarrow \\ \text{4\%} \end{array}$ wait to get higher interest rate	↑	

Expected inflation

MUt expects higher inflation
fixed in amount
 $F + C$ not value

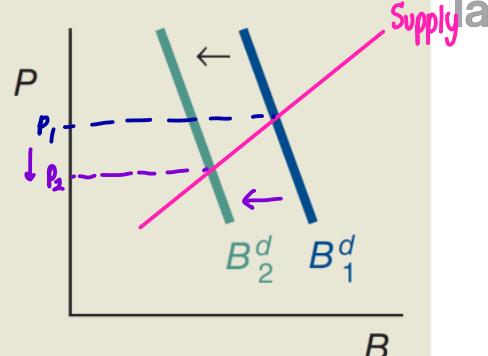


Riskiness of bonds relative to other assets

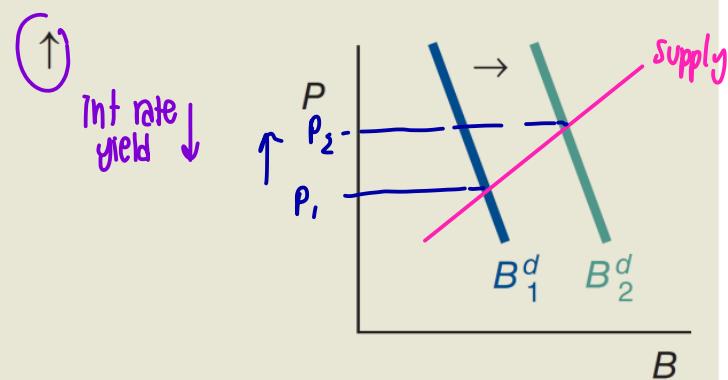
↓
e.g. credit spread

Liquidity of bonds
relative to other assets

as $\pi^e \uparrow \rightarrow$ int rate \uparrow



A supply and demand graph for bonds. The vertical axis is labeled P and the horizontal axis is labeled B . A pink upward-sloping curve is labeled "Supply". Two downward-sloping curves are labeled $B_d^d_2$ (green) and $B_d^d_1$ (blue). A horizontal dashed line intersects the green curve at price P_1 and the blue curve at price P_2 . An arrow points left from P_1 towards P_2 , indicating a decrease in price.



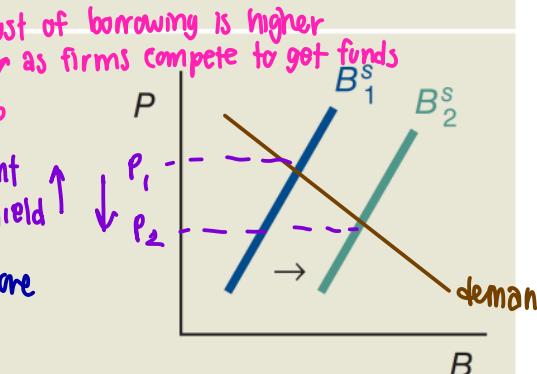
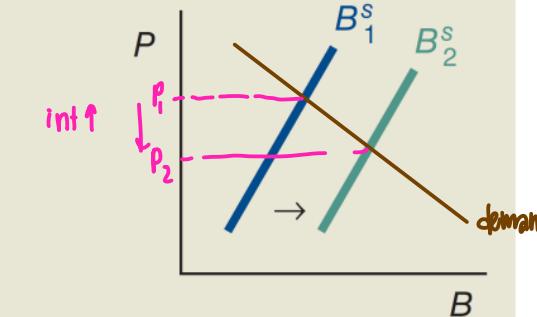
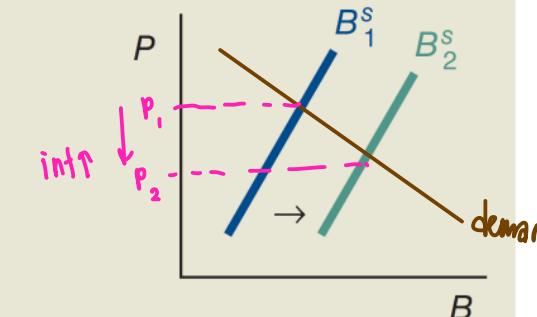
Note: Only increases in the variables are shown. The effect of decreases in the variables on the change in demand would be the opposite of those indicated in the remaining columns.

Determinants of Asset Supply

↑ issuers of bond

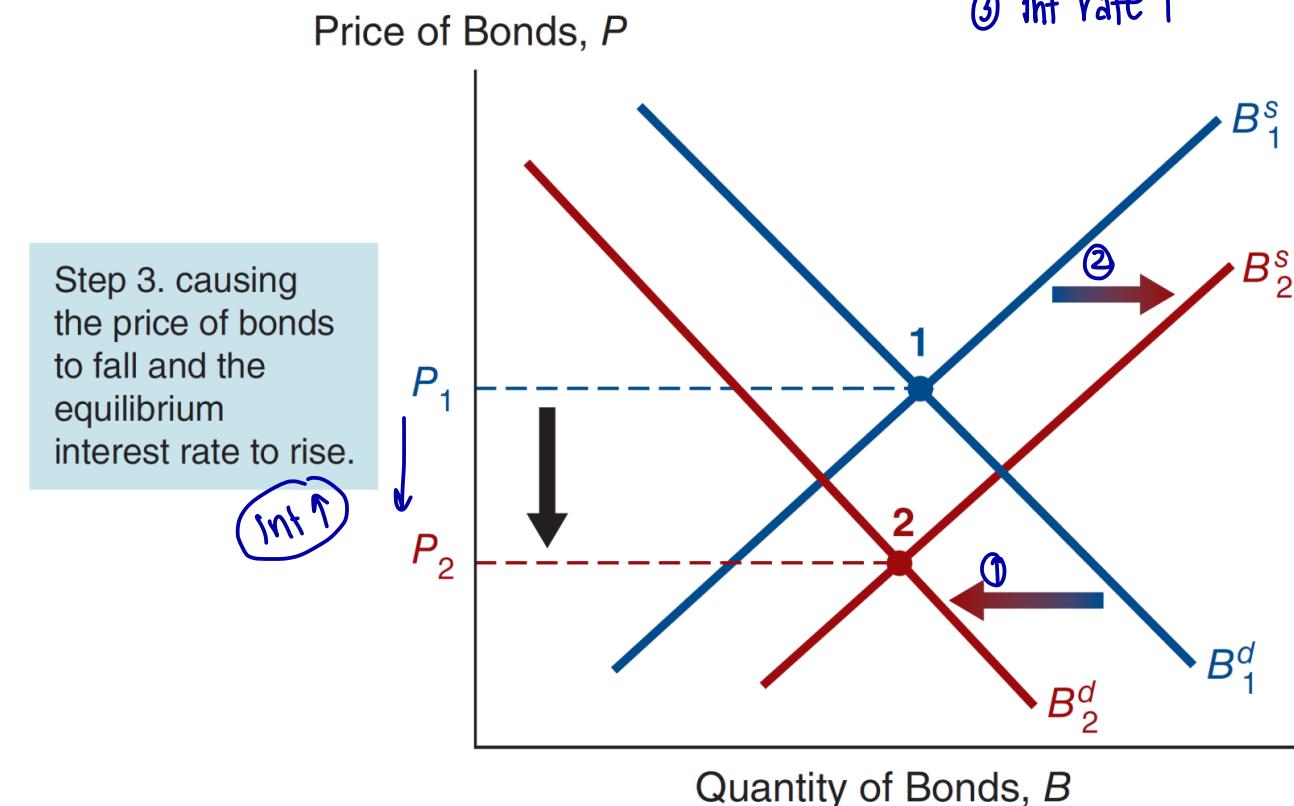
↳ business, government

- 1. Expected Profitability of Investment Opportunities:** In a business cycle expansion, the supply of bonds increases. Conversely, in a recession, when there are far fewer expected profitable investment opportunities, the supply of bonds falls.
- 2. Expected Inflation:** An increase in expected inflation causes the supply of bonds to increase.
- 3. Government Activities:** Higher government deficits increase the supply of bonds. Conversely, government surpluses decrease the supply of bonds.

Variable	Change in Variable	Change in Quantity Supplied at Each Bond Price	Shift in Supply Curve
Profitability of investments in real project businesses see opportunity ↳ invest in new project → need money to borrow more → issue bonds more	↑	↑ cost of borrowing is higher as firms compete to get funds int yield ↑	 <p>A graph showing the bond market. The vertical axis is Price (P) and the horizontal axis is Bond (B). An upward-sloping supply curve shifts rightward from B_1^S to B_2^S. The initial equilibrium is at price P_1 and quantity supplied B_1^S. The new equilibrium is at a lower price P_2 and a higher quantity supplied B_2^S. A handwritten note indicates that the interest yield has increased.</p>
Expected inflation pay the same amount $P+C$ but lower value	↑	↑	 <p>A graph showing the bond market. The vertical axis is Price (P) and the horizontal axis is Bond (B). An upward-sloping supply curve shifts rightward from B_1^S to B_2^S. The initial equilibrium is at price P_1 and quantity supplied B_1^S. The new equilibrium is at a lower price P_2 and a higher quantity supplied B_2^S. A handwritten note indicates that the interest rate has increased.</p>
Government deficit has to borrow more ↳ issue more bond	↑	↑	 <p>A graph showing the bond market. The vertical axis is Price (P) and the horizontal axis is Bond (B). An upward-sloping supply curve shifts rightward from B_1^S to B_2^S. The initial equilibrium is at price P_1 and quantity supplied B_1^S. The new equilibrium is at a lower price P_2 and a higher quantity supplied B_2^S. A handwritten note indicates that the interest rate has increased.</p>

Note: Only increases in the variables are shown. The effect of decreases in the variables on the change in supply would be the opposite of those indicated in the remaining columns.

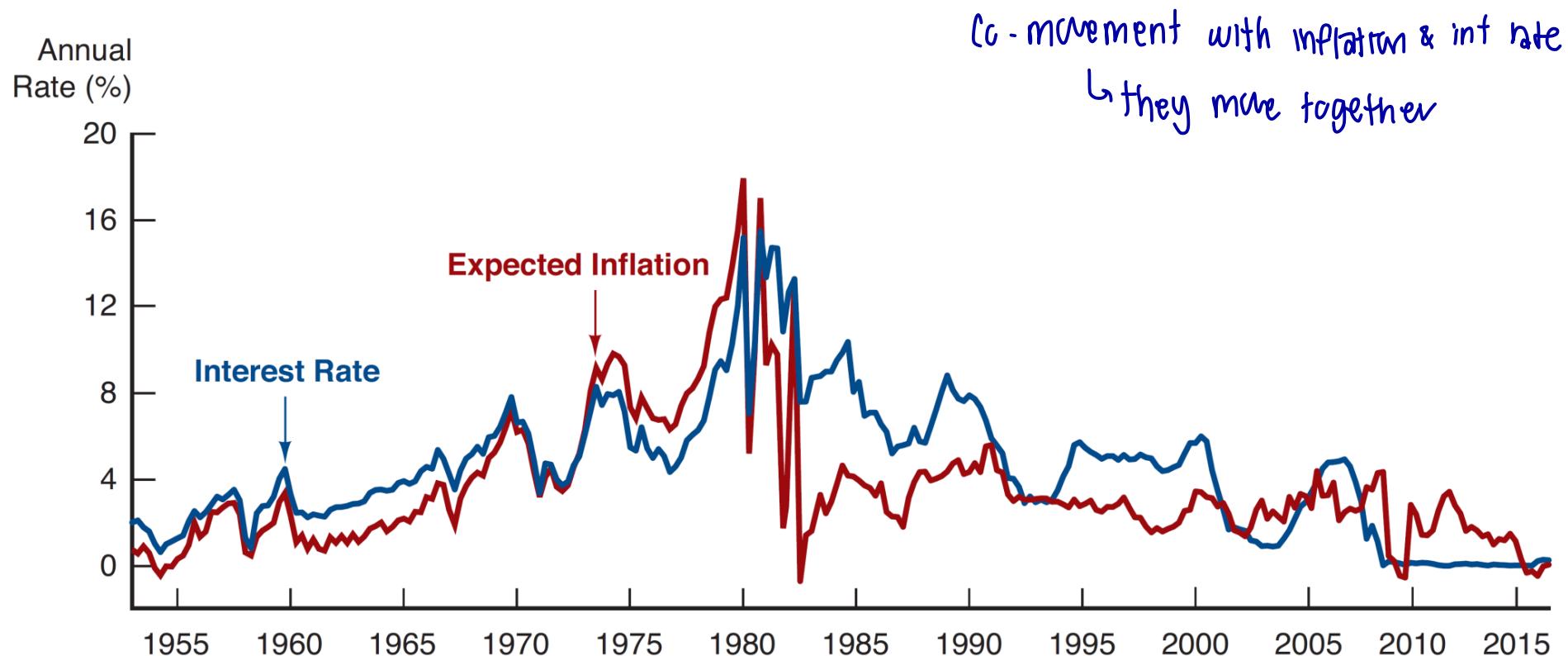
Response to Change in Expected Inflation



- ① Demand curve → shift left (purchasing power with given cr↓)
- ② Supply curve → shift right as it is good for them (pay same amount but lower value)
- ③ int rate ↑

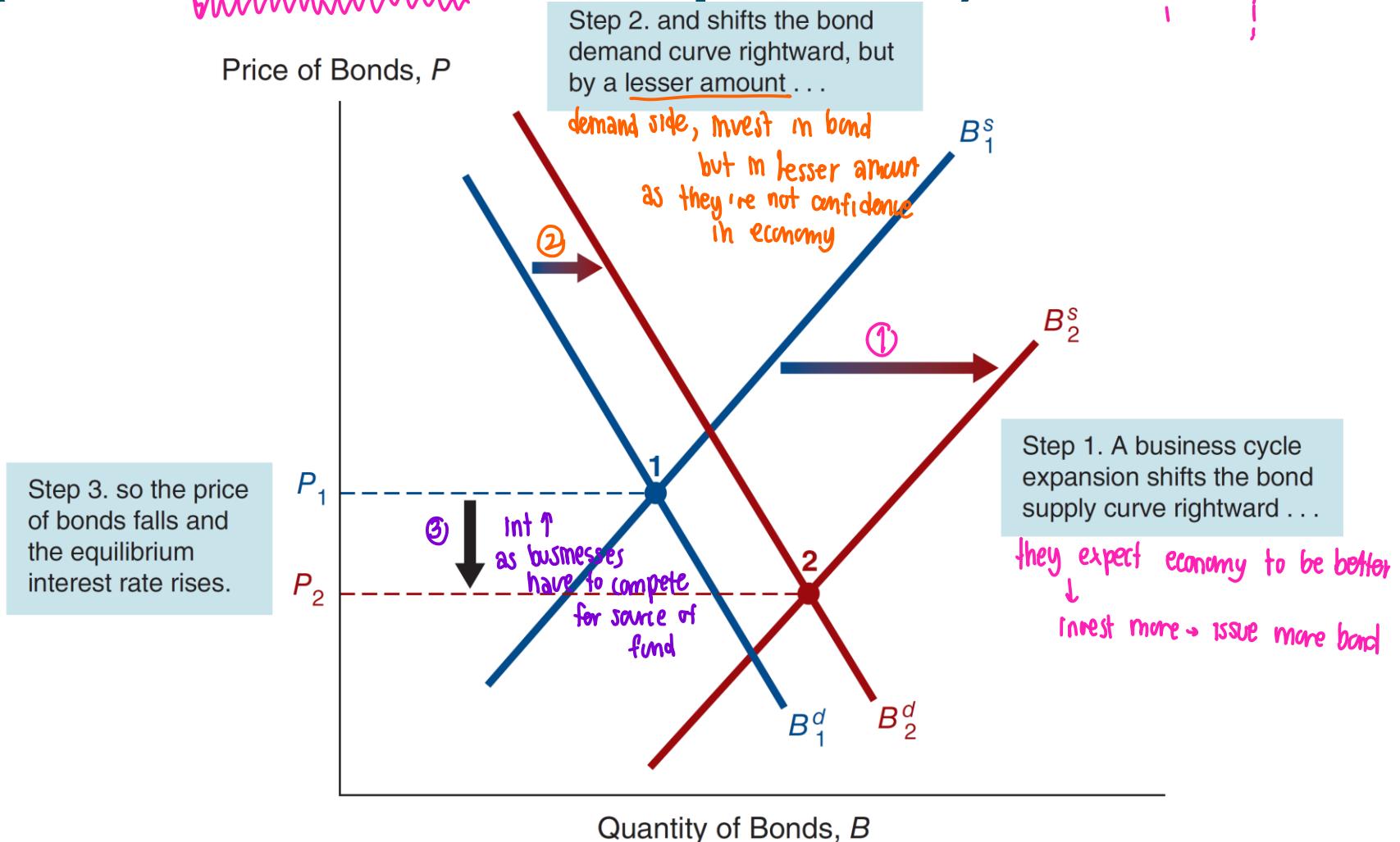
Step 2. and shifts the bond supply curve rightward ...

Expected Inflation and Interest Rates (Three-Month Treasury Bills), 1953–2016



Source: Expected inflation calculated using procedures outlined in Frederic S. Mishkin, "The Real Interest Rate: An Empirical Investigation," Carnegie-Rochester Conference Series on Public Policy 15 (1981): 151–200. These procedures involve estimating expected inflation as a function of past interest rates, inflation, and time trends. Nominal three-month Treasury bill rates from Federal Reserve Bank of St. Louis FRED database:
<https://fred.stlouisfed.org/series/TB3MS> and <https://fred.stlouisfed.org/series/CPIAUCSL>

Response to Business Cycle (from recession to expansion)



Risk and Term Structure

- We will examine the different rates that we observe for financial products.

Questions

- Do you think bonds with the same maturity offered by different issuers will have equal interest rates?

different level of riskiness | No
- Do you think bonds with maturity of 1, 5 or 10 years whether it will have equal interest rates?


Risk Structure of Interest Rates

Questions

- If you would purchase a bond from a **US government** ^{less risky} vs. Apple, what are expected yields from these issuers?
Apple is a business corporation
- If you would purchase a bond from a Johnson & Johnson ^{less risky} vs. Amazon, which one is riskier?
- If you would purchase a bond from a Netflix vs. Tesla, which one is riskier?

Credit rating → indicator we use as reference | fitch downgraded US from AAA to AA+

33522 US Equity Company Tree Ratings ▾ Alert Page 1/3 Credit Profile

United States of America govt

1) Bloomberg Default Risk | DRSK »
 2) Issuer Default Risk IG2

Moody's
 3) Outlook
 4) CC LT Foreign Bank Depst
 5) CC LT Foreign Curr Debt
 6) CC ST Foreign Bank Depst
 7) CC ST Foreign Curr Debt
 8) Long Term Rating
 9) LC Curr Issuer Rating
 10) FC Curr Issuer Rating
 11) Local Currency LT Debt

STABLE WR Aaa WR WR Aaa Aaa Aaa Aaa

12) Standard & Poor's
 13) Outlook
 14) Foreign Currency LT Debt
 15) Local Currency LT Debt
 16) Foreign Currency ST Debt
 17) Local Currency ST Debt

Fitch
 18) Outlook
 19) LT Issuer Default Rating
 20) LT LC Issuer Default
 21) Foreign Currency LT Debt
 22) Local Currency LT Debt
 23) ST Issuer Default Rating
 24) ST LC Issuer Default
 25) Sovereign Country Ceiling

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AAPL US Equity Company Tree Ratings ▾ Alert Page 1/2 Credit Profile

Apple Inc

1) Bloomberg Default Risk | DRSK »
 2) Issuer Default Risk IG1

3) Bloomberg Market Implied PD | MIPD »
 5 Year Issuer PD 0.018489

Moody's
 5) Outlook
 6) Issuer Rating
 7) Long Term Rating
 8) LT Corp Family Rating
 9) Senior Unsecured Debt
 10) Subordinated Debt
 11) Short Term

STABLE WR Aaa WR Aaa WR P-1

12) Standard & Poor's
 13) Outlook
 14) LT Foreign Issuer Credit
 15) LT Local Issuer Credit
 16) ST Foreign Issuer Credit
 17) ST Local Issuer Credit

18) Credit Benchmark Composites | CRDT »
 19) 6M Trend Upgraded
 20) Company Consensus* Not Subscrib...
 21) Company Band IG1
 22) Bank/Contributor Count 14
 23) Level of Agreement High
 24) Search Coverage Universe | CRSR »

*Premium Field

Bond Ratings by Moody's and S&P

Moody's Rating	S&P Rating	Description	Examples of Corporations with Bonds <u>Outstanding in 2016</u>
Aaa	AAA	Highest quality (lowest default risk)	Microsoft, J&J
Aa	AA	High quality	Apple, General Electric
A	A	Upper-medium grade	MetLife, Intel, Harley-Davidson
Baa	BBB	Medium grade	McDonalds, BofA, HP, FedEx, Southwest Airlines
Ba	BB	Lower-medium grade	Best Buy, American Airlines, Delta Airlines, United Airlines
B	B	Speculative	Netflix, Rite Aid, J.C. Penney
Caa	CCC,CC	Poor (high default risk)	Sears, Elizabeth Arden
C	D	Highly speculative	Halcon Resources, Seventy-Seven Energy

? white line

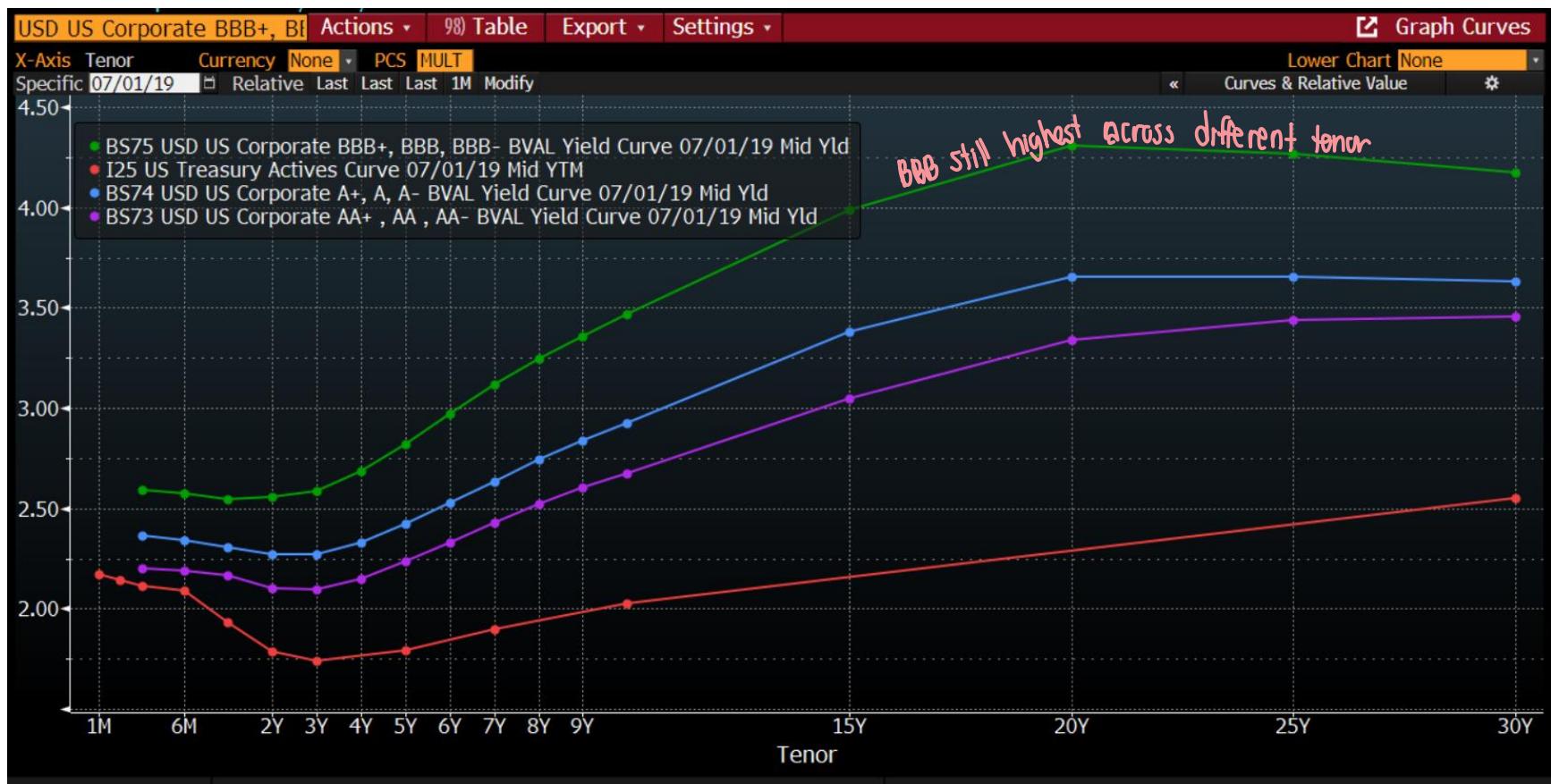
5-Year Yield of Treasury vs. A vs. BBB

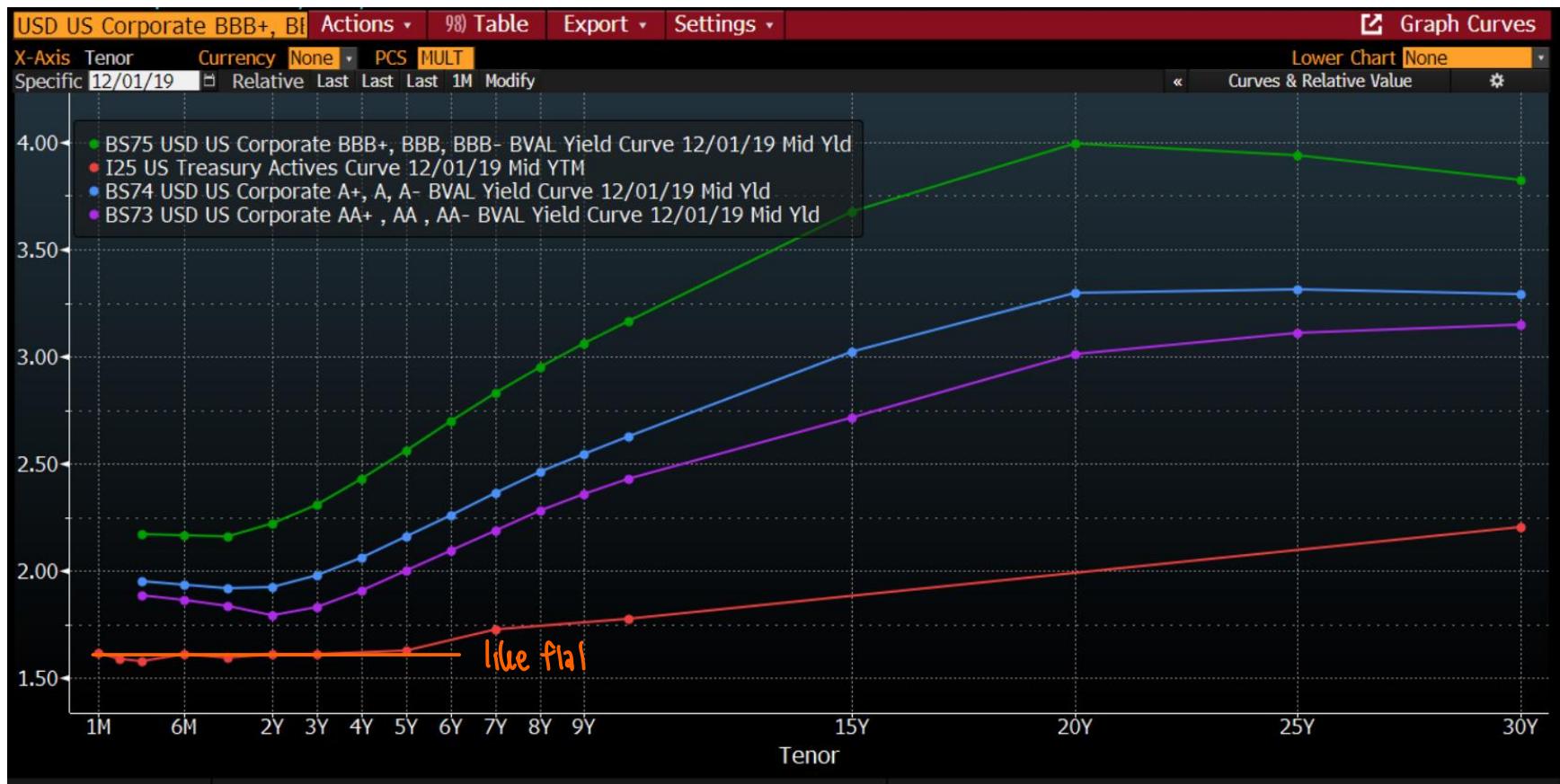




* In term of riskiness, still the same

more risky bond offer higher rate

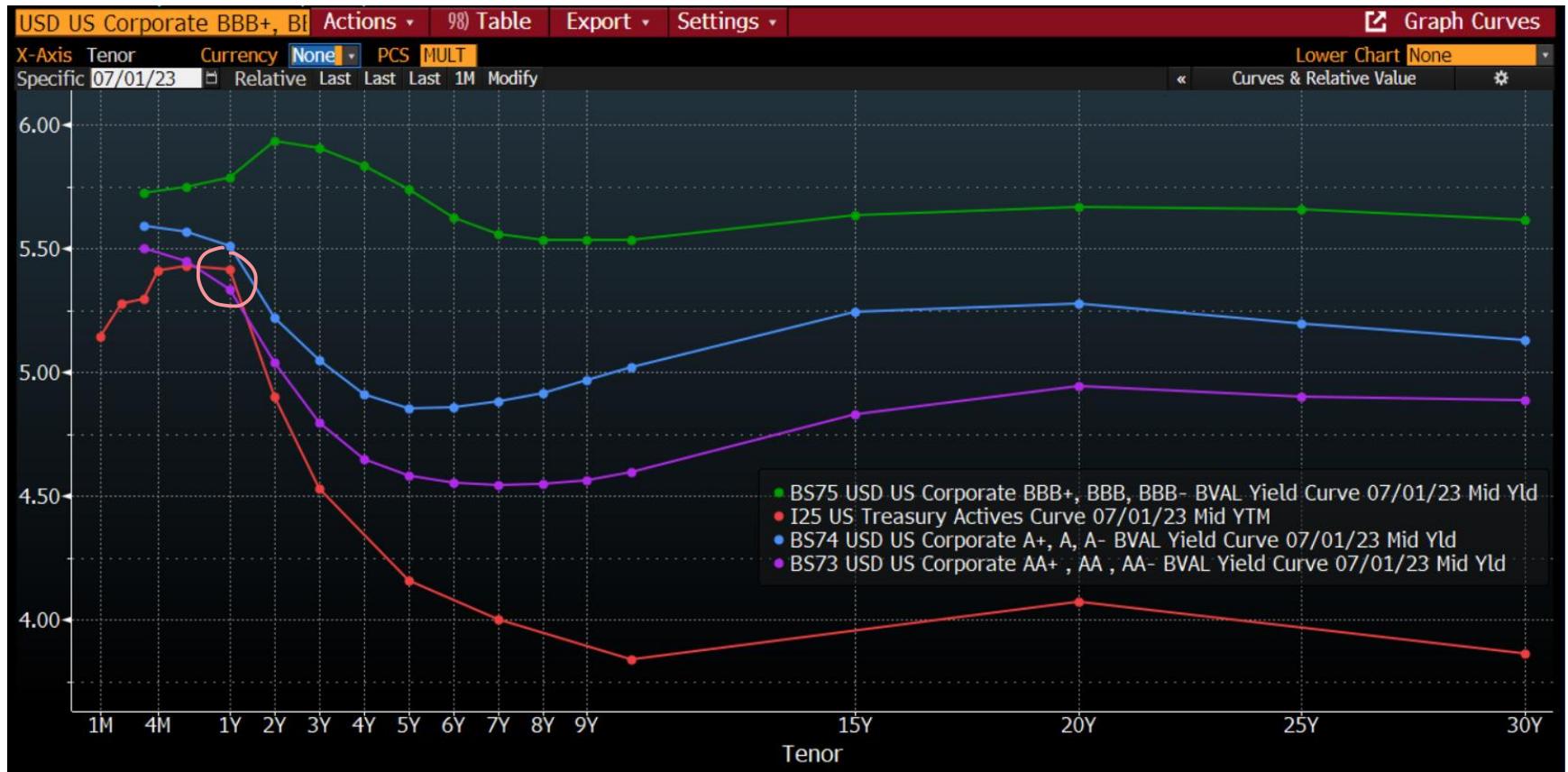




credit / risk premium

In term of riskiness, spread btw each type of bond decrease \Rightarrow insurer firm paid higher credit spread
 as we don't know when business would be back to normal

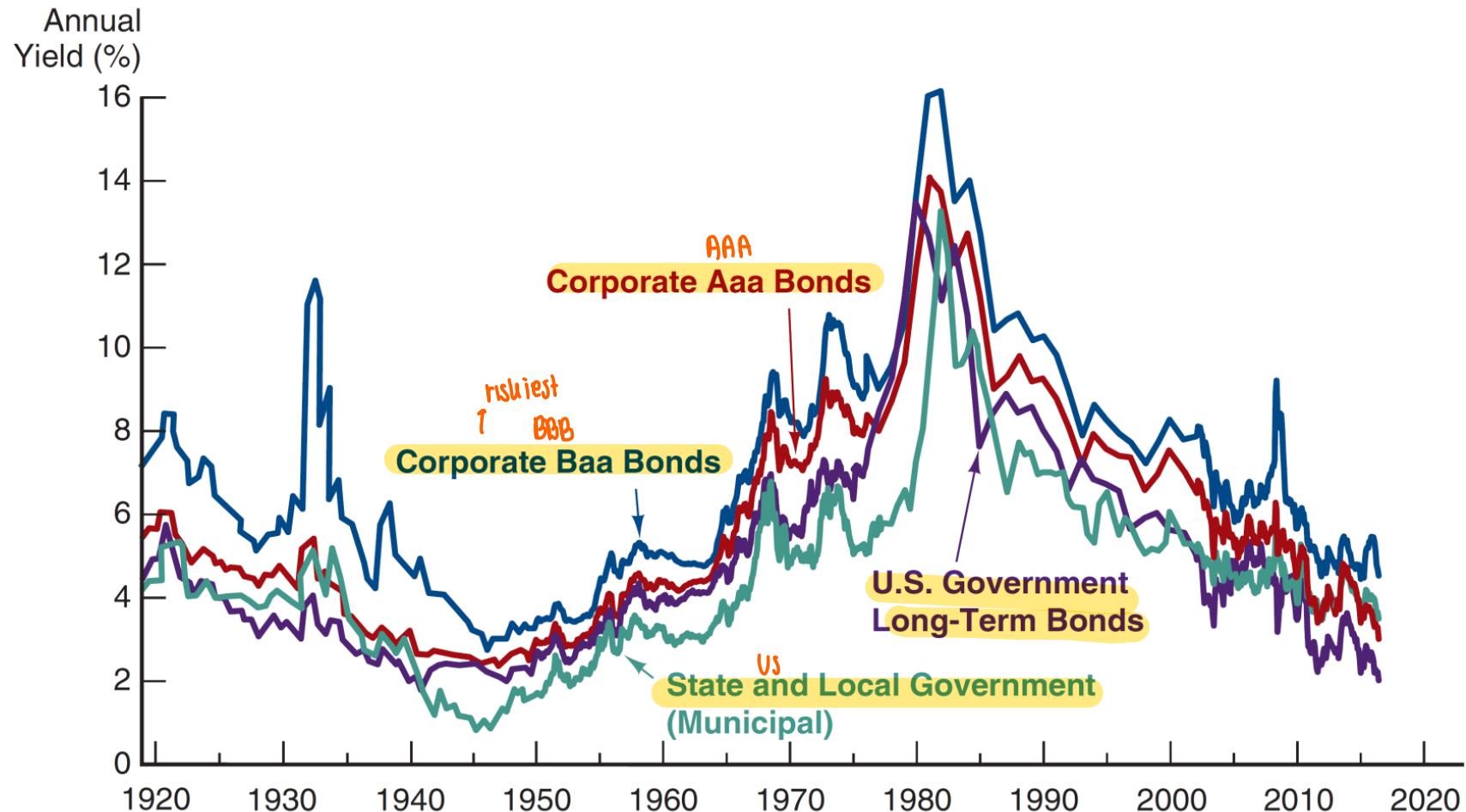
\Rightarrow 2022
 \Rightarrow Russia-Ukraine war \Rightarrow inflation \uparrow \rightarrow int rate \uparrow
 \Rightarrow 2023
 market worries about war & inflation



Risk Structure of Interest Rates

- Let's see the yields for several categories of long-term bonds over the last 90 years.
- Note several aspects regarding these rates, related to different bond categories and how this has changed through time.

Risk Structure of Interest Rates



Risk Structure of Long Bonds in U.S.

- Two important features of the interest-rate behavior of bonds.
 - Rates on different bond categories change from one year to the next.
 - Spreads on different bond categories change from one year to the next.

Factors Affecting Risk Structure of Interest Rates

To further examine these features, we will look at three specific risk factors.

- Default Risk
- Liquidity
- Income Tax Considerations

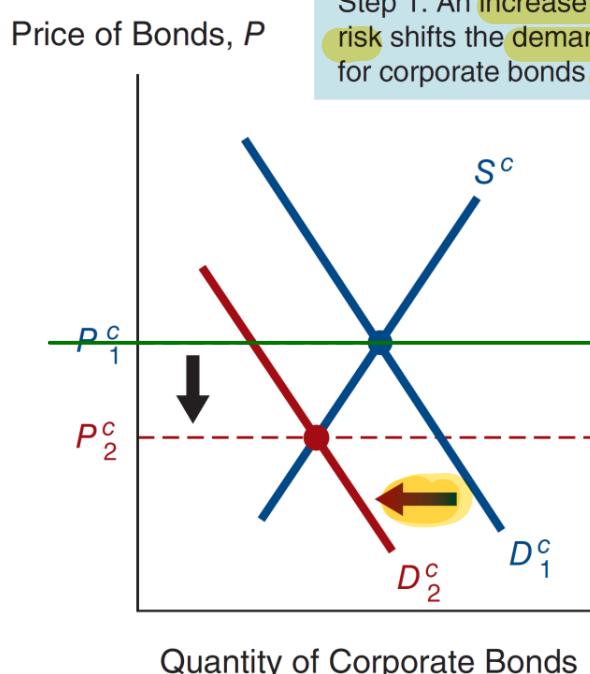
Default Risk Factor

- **Risk of default**, which occurs when the issuer of the bond is unable or unwilling to make interest payments when promised.
- U.S. Treasury bonds have been considered to have no default risk.
- Bonds like these with no default risk are called default-free bonds. But are these bonds *truly default-free bonds?*

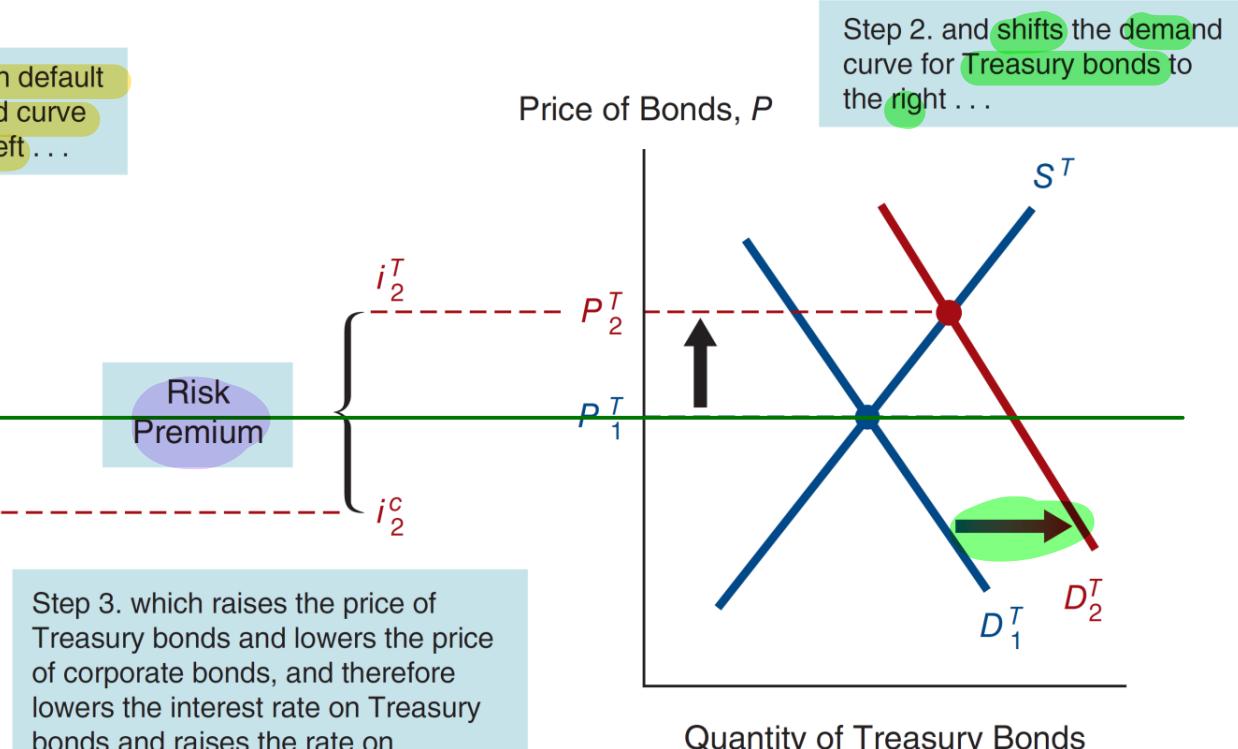
Default Risk Factor

- The spread between the interest rates on bonds with default risk and default-free bonds, called the **risk premium**, indicates how much additional interest people must earn in order to be willing to hold that risky bond.
- A bond with default risk will always have a positive risk premium, and an increase in its default risk will raise the risk premium.

Default Risk Factor



Step 3. which raises the price of Treasury bonds and lowers the price of corporate bonds, and therefore lowers the interest rate on Treasury bonds and raises the rate on corporate bonds, thereby increasing the spread between the interest rates on corporate versus Treasury bonds.



Risk free

Increase in Default on Corporate Bonds

- Corporate Bond Market
 1. R^e on corporate bonds \downarrow , $D^c \downarrow$, D^c shifts left
 2. Risk of corporate bonds \uparrow , $D^c \downarrow$, D^c shifts left
 3. $P^c \downarrow$, $i^c \uparrow$
- Treasury Bond Market
 1. Relative R^e on Treasury bonds \uparrow , $D^T \uparrow$, D^T shifts right
 2. Relative risk of Treasury bonds \downarrow , $D^T \uparrow$, D^T shifts right $P^T \uparrow$, $i^T \downarrow$
- Outcome
 - Risk premium, $i^c - i^T$, rises

Case: The Global Financial Crisis and the Baa-Treasury Spread

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- Starting in 2007, the subprime mortgage market collapsed, leading to large losses for financial institutions.
- Because of the questions raised about the quality of Baa bonds, the demand for lower-credit bonds fell, and a “flight- to-quality” followed (demand for T-securities increased).

Case: The Global Financial Crisis and the Baa-Treasury Spread

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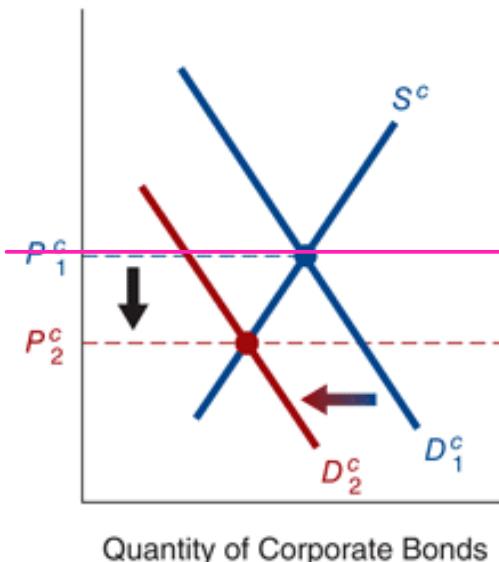


Liquidity Factor

- Another attribute of a bond that influences its interest rate is its liquidity
- Liquid asset is one that can be quickly and cheaply converted into cash if the need arises.
- The more liquid an asset is, the more desirable it is (higher demand), holding everything else constant.
- It is normal that treasury bonds are more liquid than corporate bonds.

Liquidity Factor

Price of Bonds, P

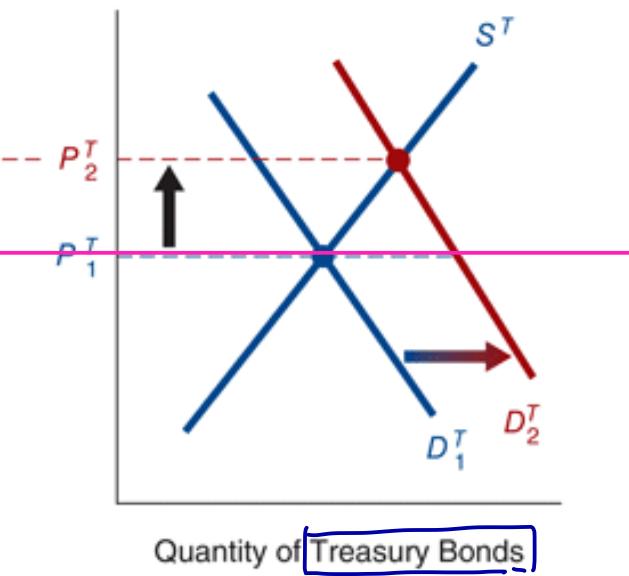


(a) Corporate bond market

diff btw spread
of govt bond
and risky bond

Risk Premium

Price of Bonds, P



(b) Default-free (U.S. Treasury) bond market

More liquid

Response to an Increase in Liquidity on Corporate Bonds

The Master of Science in Finance * Chulalongkorn University

Corporate Bond Becomes Less Liquid

- Corporate Bond Market
 1. Liquidity of corporate bonds \downarrow , $D^c \downarrow$, D^c shifts left
 2. $P^c \downarrow$, $i^c \uparrow$
- Treasury Bond Market
 1. Relatively more liquid Treasury bonds, $D^T \uparrow$, D^T shifts right
 2. $P^T \uparrow$, $i^T \downarrow$
- Outcome
 - Risk premium, $i^c - i^T$, rises
- Risk premium reflects not only corporate bonds' default risk but also lower liquidity

Liquidity Factor

- The differences between interest rates on corporate bonds and Treasury bonds (that is, the risk premiums) reflect not only the corporate bond's default risk but its liquidity too. This is why a risk premium is sometimes called a *risk and liquidity premium*.

Income Tax Considerations

- This is generally true for the municipal bonds (munis). Interest on municipal bonds is exempt from federal income tax.
 - Munis and taxable corporates are similar except for the taxation of interest.
 - The yield on municipals equals the yield on taxables times one minus the marginal tax rate.
$$i_m = i_t (1-t)$$
 - The yield on municipals equals the yield on taxables times one minus the marginal tax rate.

Income Tax Considerations

- An investor has the choice of a treasury bond with a yield of 5% or a municipal bond yielding 4%. (Assume issued at par)
 $\text{Treasury Bond } \$50 \rightarrow \$50 (1 - 0.05) = \$47.50$
 $\text{Municipal Bond } \$40 \rightarrow \$40 (1 - 0.04) = \$38.40$
The municipal bond is more tax-free.
- If the investor has a marginal tax rate of 30%, which bond should he/she select?
 $\text{Treasury Bond } \$50 \rightarrow \$50 (1 - 0.30) = \$35$
 $\text{Municipal Bond } \$40 \rightarrow \$40 (1 - 0.04) = \$38.40$
The municipal bond is more tax-free.
- What if the government bond offers the yield of 6%?

Term to maturity = time to maturity

time

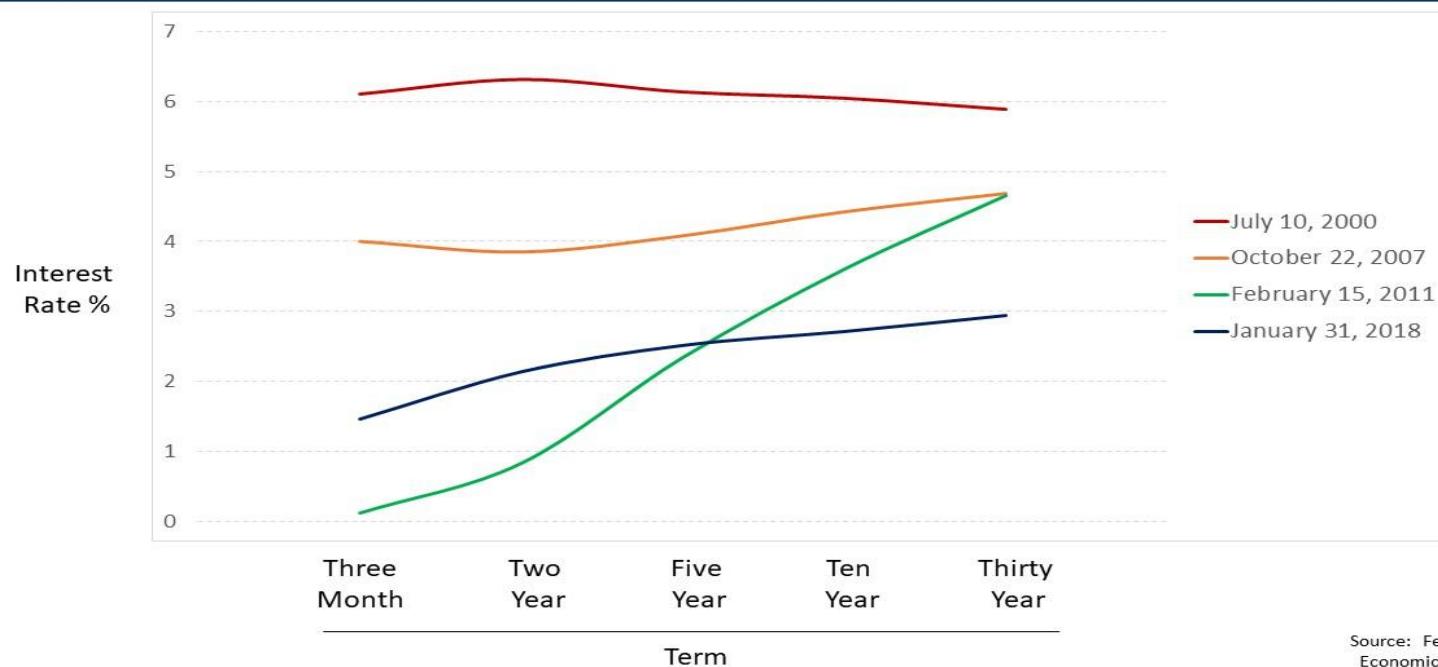
Term Structure of Interest Rates

- Now that we understand risk, liquidity, and taxes, we turn to another important influence on interest rates—maturity.
- Bonds with different maturities tend to have different required rates, all else equal.

Term Structure of Interest Rates

- What is the 3-month rate? The two-year rate? How can we explain the shape of yield curve?

U.S. Treasury Yield Curves



Source: Federal Reserve
Economic Data (FRED)

expectation
(liquidity premium)

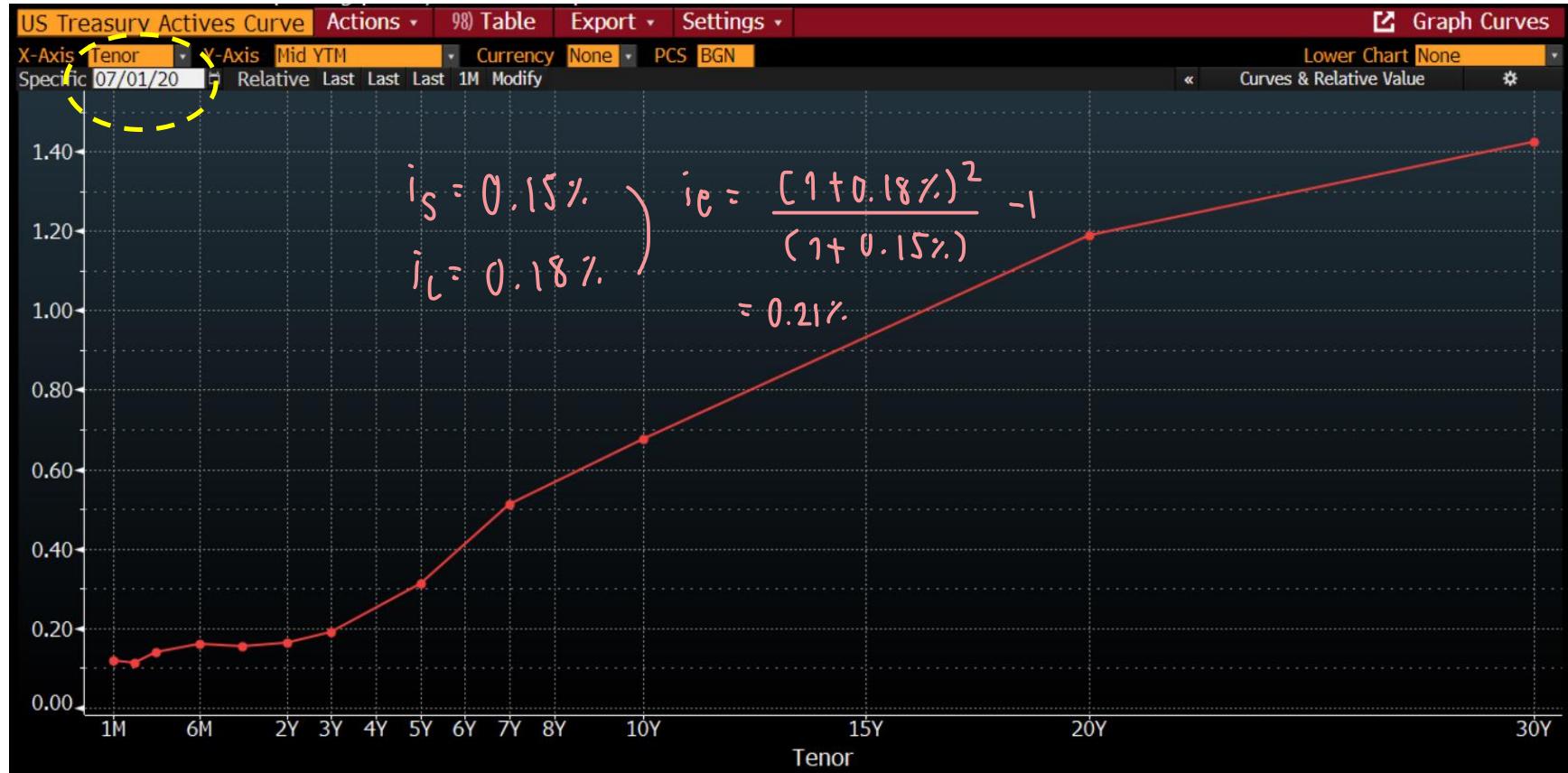
US Treasury Yield Curve

Diagram illustrating the relationship between spot interest rates (i_s), forward interest rates (i_e), and the expected future rate (i_f). The diagram shows two time periods: 1Y (blue) and 2Y (orange). i_s is the proxy for 1Y. i_e is expected to occur 1 yr from now. i_f is the forward int rate in the future.

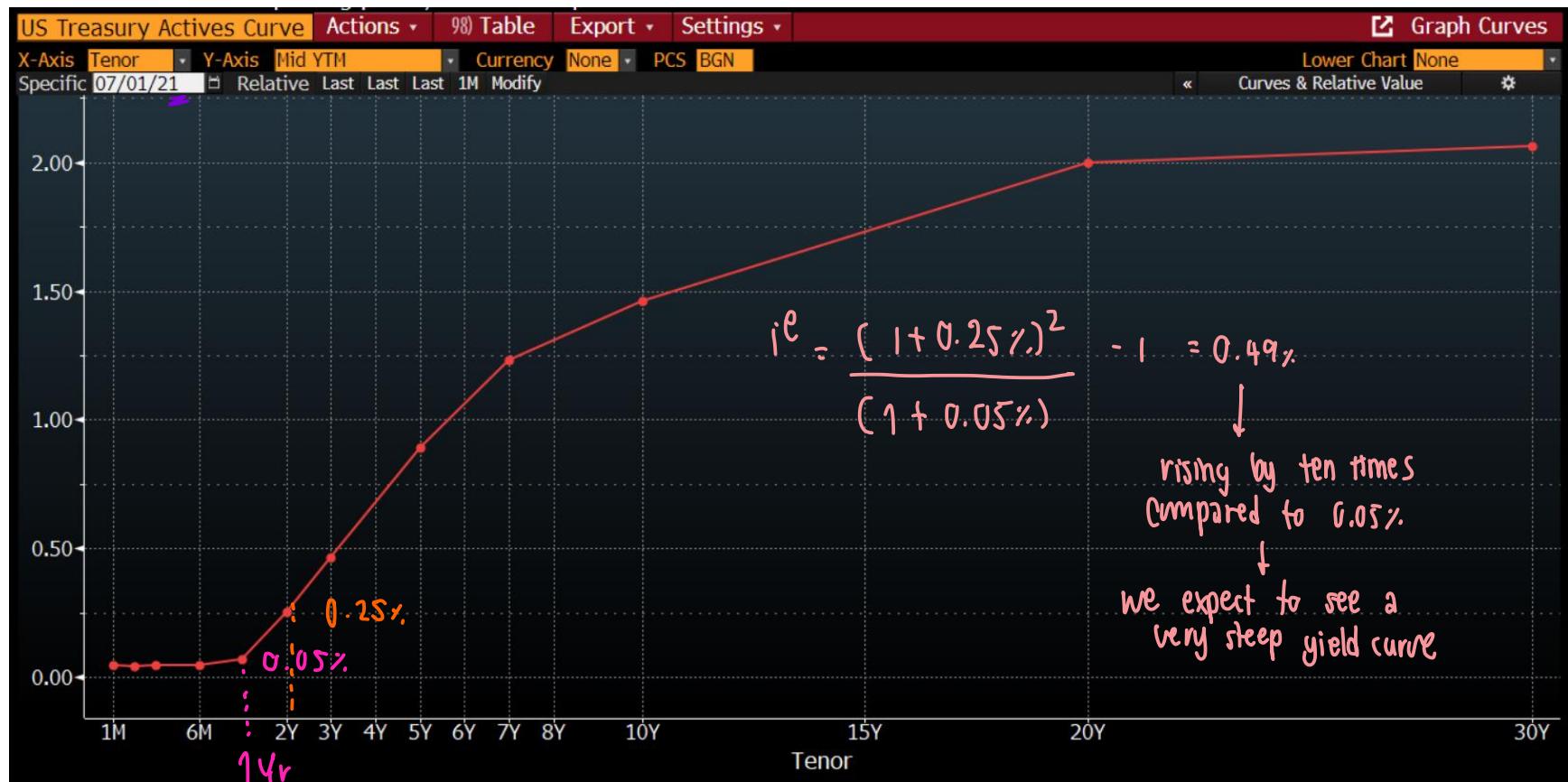
$$(1+i_s)(1+i_e) = (1+i_f)^2$$

$$i_e = \frac{(1+i_f)^2}{(1+i_s)} - 1$$

After COVID-19 occurred



US Treasury Yield Curve

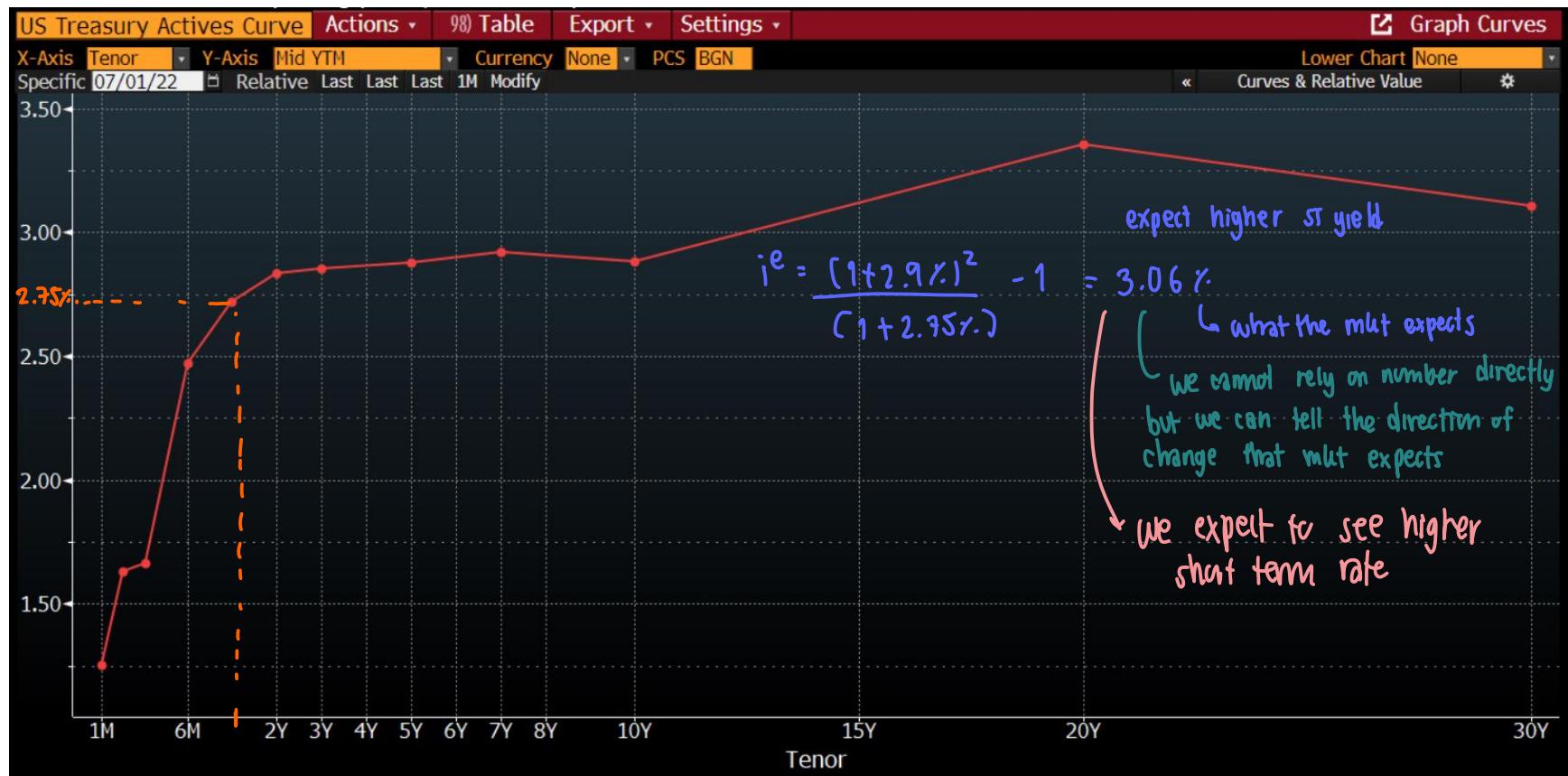
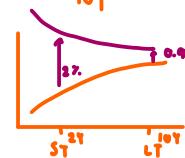


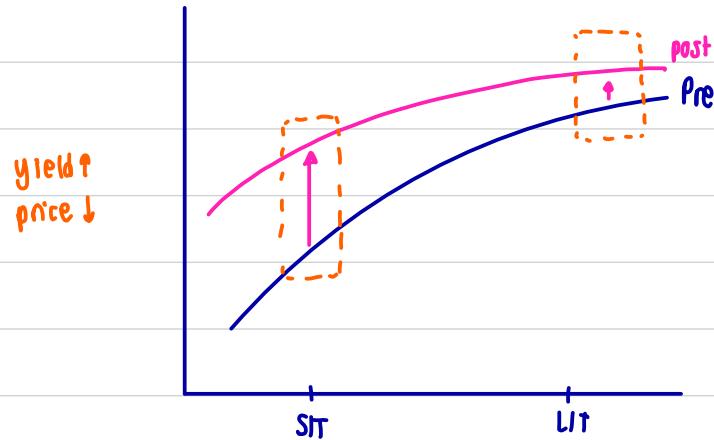
Regarding LT rate:

$$i_{2Y} = 2.9\% \xrightarrow{+2\%} 4.9\%$$

$$i_{10Y} = 2.95\% \xrightarrow{+0.9\%} 3.85\%$$

ST rate rose more than LT rate
 $\Delta S/T > \Delta L/T$





$\Delta SIT > \Delta LIT$
yield curve becomes flatter

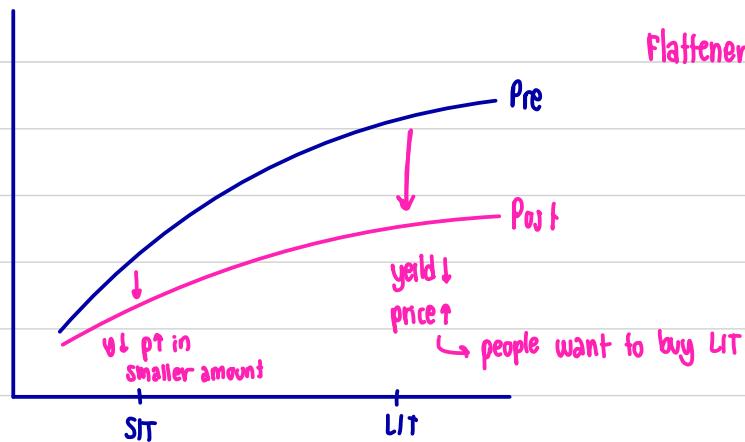
yield ↑
price ↓

In this case,
LIT term's price ↓ but not much
as price ↓ in SIT

↳ They sell SIT rather than LIT

mlut expects that fed may increase the policy rate

↳ slowdown economy or inflation ↑



people want to buy LIT more because

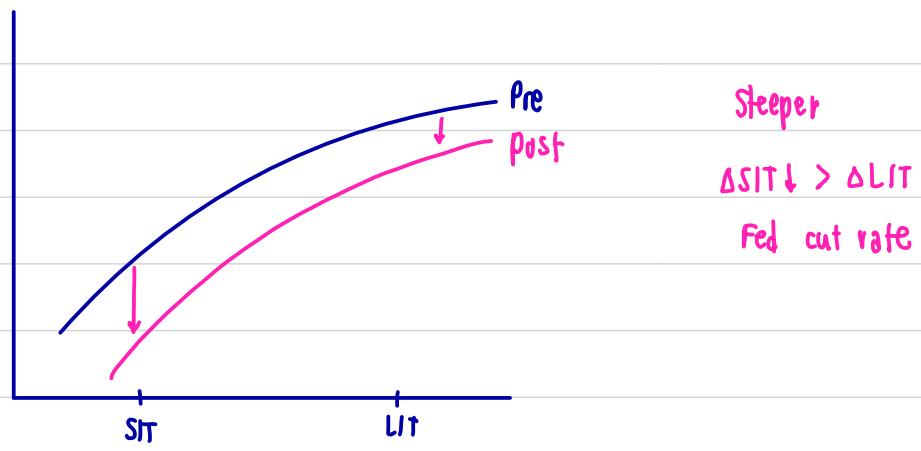
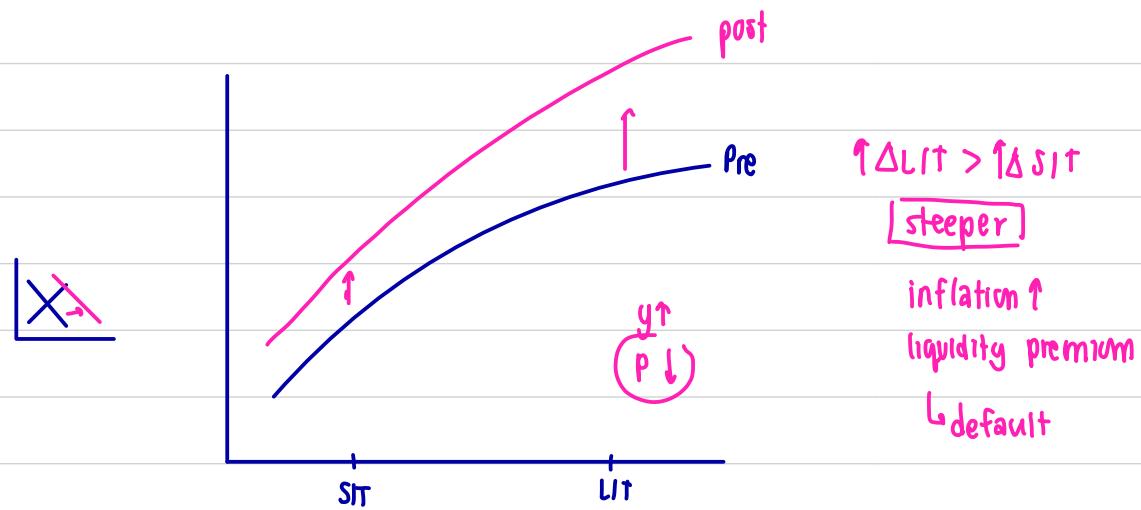
- inflation ↓ → demand in bond will rise → probably LIT bond
- recession: extreme case

that investor want to lock in rate

thus LIT yield >
SIT yield

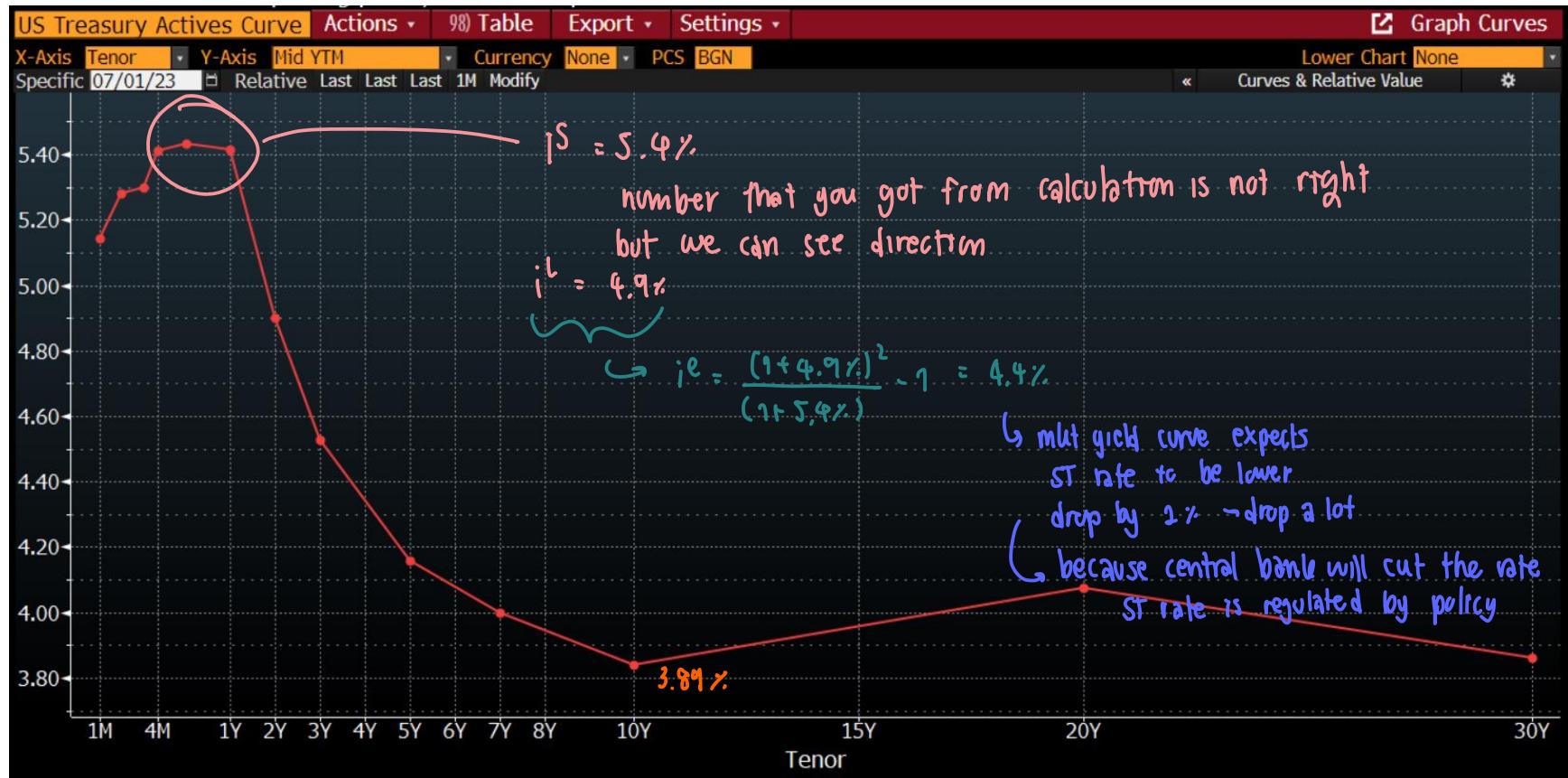
SIT rate is depend on policy made by central bank

↳ FED may want to encourage economy a little bit → yield drop a little



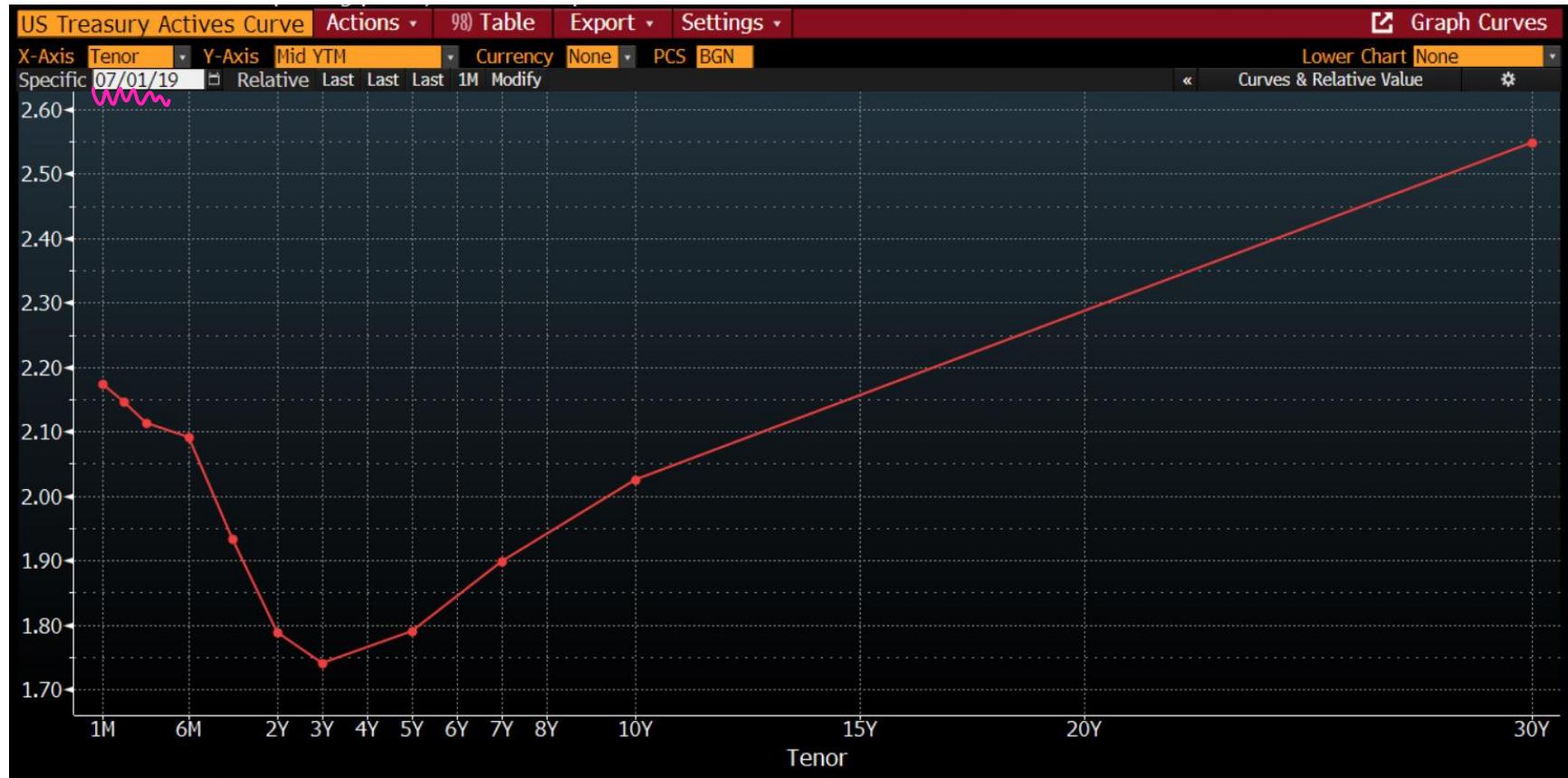
US Treasury Yield Curve

→ Inverted yield curve may caused by too fast rising policy rate by FED



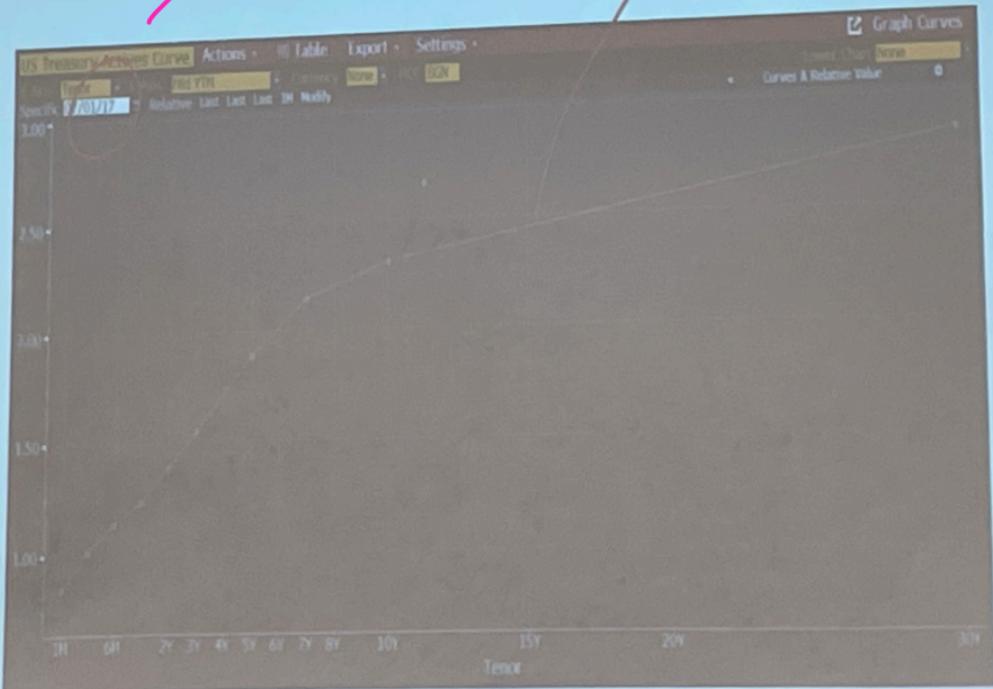
US Treasury Yield Curve

Shape looks weird



US Treasury Yield Curve

Upward slope
 $\nearrow = \text{Normal Yield Curve}$



Questions

- How can we explain the behaviors of the yield curve?
Short maturity should be lower yield → most of the time but not all the time
- What can be the determinants that influence the shape and level of yield curve (i.e. interest rate)?

Risk-free

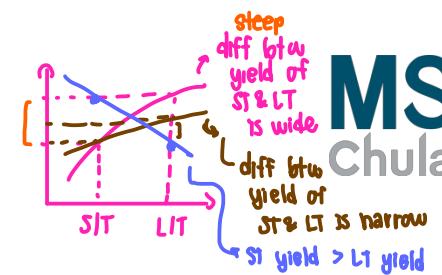
U.S. Govt Bonds Yield History

↳ graphs move together

Bloomberg terminal



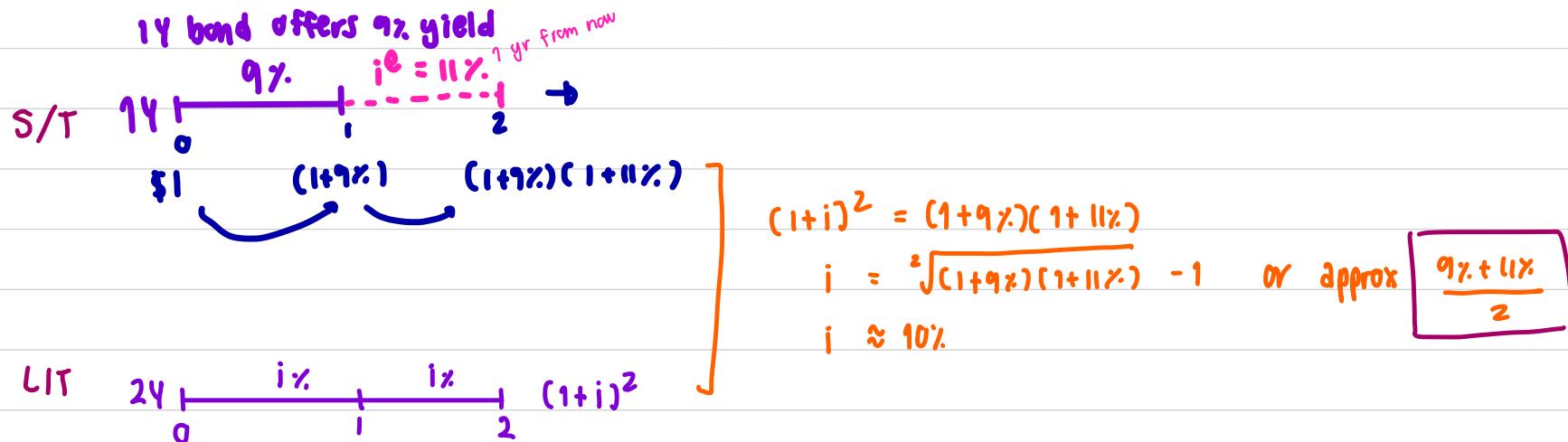
U.S. Govt Bonds Yield History



Three Facts about Term Structure

Interest rates on U.S. government bonds with different maturities on previous slide depicts that...

1. Interest rates for different maturities move together.
2. Yield curves tend to have steep upward slope when short rates are low and a downward slope when short rates are high.
3. Yield curve is typically upward sloping.



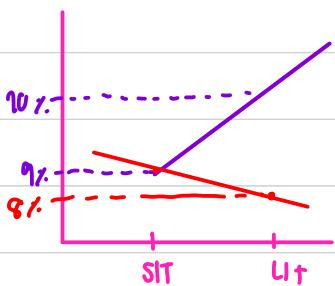
expectation theory \rightarrow L/T rate should be equal to average of S/T rates

* You can find expected mt rate (i^e) we can observe this

$$i^{LT} = \frac{i^{ST} + i^e}{2}$$

$$10\% = \frac{9\% + i^e}{2}$$

$i^e = 11\%$ we do not observe i^e in reality



* We observe upward slope if mt expects higher int rate

* ↓ ↓ ↓ downward slope " " " lower ↓ ↓ ↓

e.g. $8\% < \frac{9\% + 7\%}{2}$

L/T S/T

Three Theories of Term Structure

How can we explain the shape of yield curve?

1. Expectations Theory
2. Market Segmentation Theory
3. Liquidity Premium Theory

Expectations Theory

- **Expectation theory** states that the interest rate on a long-term bond will equal an average of the short term interest rates that people expect to occur over the life of the long-term bond.
- **Key Assumption:** Bonds of different maturities are perfect substitutes
- **Implication:** The expected return (R^e) on bonds of different maturities are equal

Expectations Theory

- The important point of this theory is that if the *Expectations Theory* is correct, your *expected* wealth is the same (at the start) for both strategies.
- Expected return from strategy 1

$$(1 + i_2)(1 + i_{t+1}^e) - 1 = 1 + i_t + i_{t+1}^e + i_t(i_{t+1}^e) - 1$$

- Since $i_t(i_{t+1}^e)$ is extremely small, expected return is approximately

$$i_t + i_{t+1}^e$$

Expectations Theory

- Expected return from strategy 2

$$(1 + i_{2t})(1 + i_{2t}) - 1 = 1 + 2(i_{2t}) + (i_{2t})^2 - 1$$

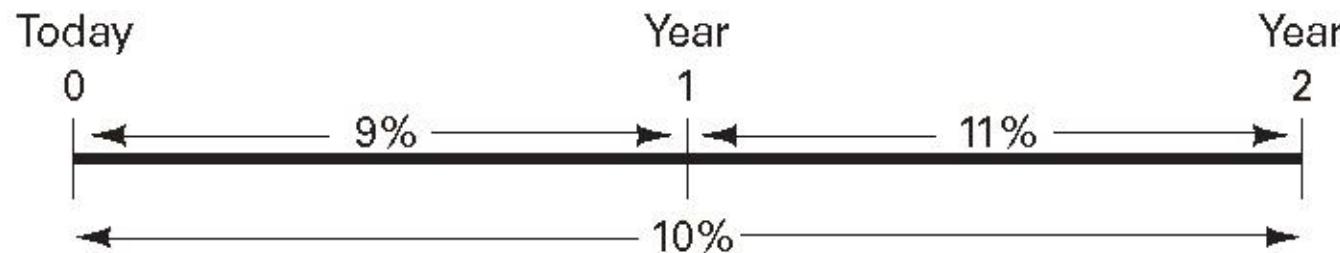
- Since $(i_{2t})^2$ is extremely small, expected return is approximately $2(i_{2t})$
- Expected returns of two strategies are equal

$$i_{2t} = \frac{i_t + i_{t+1}^e}{2}$$

- Note that this is an approximation only. The actual calculation must be based on geometric mean.
- i_{2t} is also called the *forward rate* which is the rates of return required in the future to make the yields of different investment strategies the same

Example: Expectations Theory

- Let's try an example with actual numbers
- The current interest rate on a one-year bond is 9% and you expect the interest rate on one-year bond next year to be 11%. What interest rate must a two-year bond have?



- Answer 9.995% or ~10%.

More generally for n -period bond...

$$i_{nt} = \frac{i_t + i_{t+1} + i_{t+2} + \dots + i_{t+(n-1)}}{n}$$

- This equation simply states that the “**interest rate on a long-term bond equals the average of short rates expected to occur over life of the long-term bond**”.

More generally for n -period bond...

A numerical example: Suppose we are given the following information:

Year	Actual	Expected
	One Year	One Year
2019	8.5%	
2020		9.5%
2021		11.00%
2022		11.75%

- Then the expected rate on a two-year security should be 8.99%. The expected rate on a three-year security should be 9.662%. The expected rate on a four-year security should be 10.18%.

More generally for *n*-period bond...

- Another numerical example
 - One-year interest rate over the next five years are expected to be 5%, 6%, 7%, 8%, and 9%
- Interest rate on two-year bond today:

$$(5\% + 6\%)/2 = 5.5\%$$

- Interest rate for five-year bond today:
$$(5\% + 6\% + 7\% + 8\% + 9\%)/5 = 7\%$$
- Interest rate for one- to five-year bonds today:
$$5\%, 5.5\%, 6\%, 6.5\% \text{ and } 7\%$$

Expectations Theory Summary

- To summarize:
 1. When short rates are expected to *rise in future*, average of future short rates = i_{nt} is above today's short rate; therefore yield curve is *upward sloping*.
 2. When short rates expected to *stay same* in future, average of future short rates same as today's, and yield curve is *flat*.
 3. Only when short rates expected to *fall* will yield curve be *downward sloping!!!*

Expectations Theory Summary

- Pure expectations theory explains fact 1 — that short and long rates move together
 1. Short rate rises are persistent
 2. If $i_t \uparrow$ today, i_{t+1}^e, i_{t+2}^e etc. $\uparrow \Rightarrow$ average of future rates $\uparrow \Rightarrow i_{nt} \uparrow$
 3. Therefore: $i_t \uparrow \Rightarrow i_{nt} \uparrow$
(i.e., short and long rates move together)

Expectations Theory Summary

- Explains fact 2—that yield curves tend to have steep slope when short rates are low and downward slope when short rates are high
 1. When short rates are low, they are expected to rise to normal level, and long rate = average of future short rates will be well above today's short rate; yield curve will have steep upward slope.
 2. When short rates are high, they will be expected to fall in future, and long rate will be below current short rate; yield curve will have downward slope.

Expectations Theory Summary

- Doesn't explain fact 3—that yield curve usually has upward slope
 - However, short rates are as likely to fall as rise, so average of expected future short rates will not usually be higher than current short rate: therefore, yield curve will be usually flat rather than upward sloping..

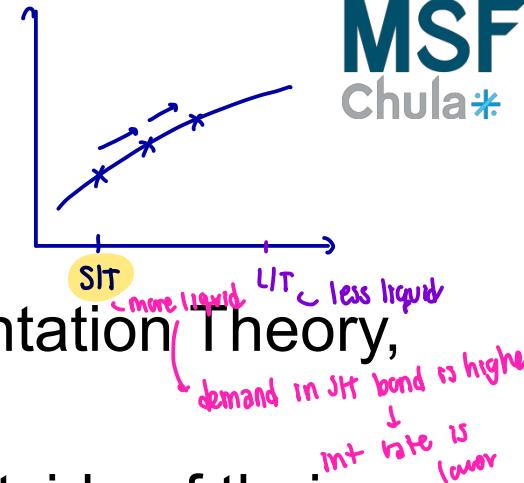
Market Segmentation Theory



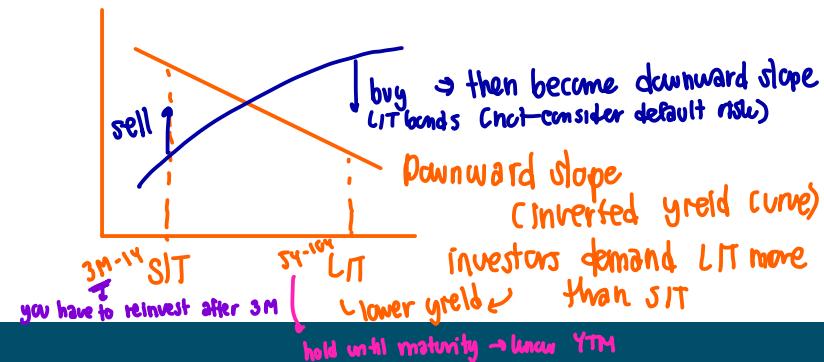
- **Market Segmentation Theory** states that markets for different-maturity bonds are completely separate and segmented. Market participants have strong preferences for securities of particular maturity and buy and sell securities consistent with their maturity preferences.
(they have their own segment, they will not invest in other segment)
- **Key Assumption:** Bonds of different maturities are not substitutes at all
- **Implication:** Markets are completely segmented; interest rate at each maturity are determined separately. If market participants do not trade outside their maturity preferences, then discontinuities and spikes are possible in the yield curve

Market Segmentation Theory

- Later, an extension of the Market Segmentation Theory, is the **Preferred Habitat Theory** (PH).
- PH allows market participants to trade outside of their preferred maturity if adequately compensated for the additional risk.
- PH allows for humps or twists in the yield curve, but limits the discontinuities possible under Market Segmentation Theory. PH is consistent with a smooth yield curve.



* Suppose institutional investors expect recession in 1Y
lock in the rate for L/T
they will buy more L/T bonds
and sell SIT bonds



Market Segmentation Theory

- Explains fact 3—that yield curve is usually upward sloping
 - People typically prefer short holding periods and thus have higher demand for short-term bonds, which have higher prices and lower interest rates than long bonds
- Does not explain fact 1 or fact 2 because its assumes long-term and short-term rates are determined independently.

, extension of expectation theory

Liquidity Premium Theory

- **Liquidity Premium Theory** states that interest rate on a long-term bond will equal to average of short-term interest rates expected over the life of the long-term bond *plus* liquidity premium.
- Long-term securities have greater risk and investors require greater premiums to give up liquidity. (*Why greater risk?*)
- **Key Assumption:** Bonds of different maturities are substitutes, but are not perfect substitutes
- **Implication:** Modifies Pure Expectations Theory with features of Market Segmentation Theory

Liquidity Premium Theory

- Investors prefer short-term rather than long-term bonds. This implies that investors must be paid positive liquidity premium, i_{nt} , to hold long term bonds.
- Results in following modification of Expectations Theory, where ℓ_{nt} is the liquidity premium added to the equation.

$$i_{nt} = \frac{i_t + i_{t+1}^e + i_{t+2}^e + \dots + i_{t+(n-1)}^e}{n} + l_{nt}$$

$i_{SIT} = 3\%$, $i^e = 4\%$
 $i^{LIT} = 3.5\%$

$i^{LIT} = \frac{i^{SIT} + i^e}{2} + LP = \frac{3\% + 4\%}{2} + 0.5\% = 3.5\%$

$i^{LIT} = \frac{i^{SIT} + i^e}{2} + LP = \frac{3\% + 4\%}{2} + 0.5\% = 4\%$
 suppose i^e remain the same then i^{LIT} should be 3%.

liquidity premium causes upward slope

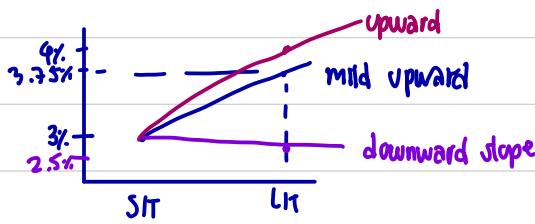
SIT $3\% \quad 4\%$
 LIT $3.5\% \quad 4\%$

$i^{LIT} = \frac{i^{SIT} + i^e}{2} + LP = \frac{3\% + 4\%}{2} + 0.5\% = 4\%$
 make 4C become even steeper

Inverted yield curve

$$\text{--- } 3\% \text{ --- } 2.5\% \text{ --- } , \quad i^{\text{LT}} = \frac{3\% + 2.5\%}{2} + 0.5\% = 3.75\%$$

+ + +



$$\text{--- } 3\% \text{ --- } 1\% \text{ --- } , \quad i^{\text{LT}} = \frac{3\% + 1\%}{2} + 0.5\% = 2.5\%$$

+ + +

slope can change
due to
 ① change in liquidity premium
 ② S/T i.e.

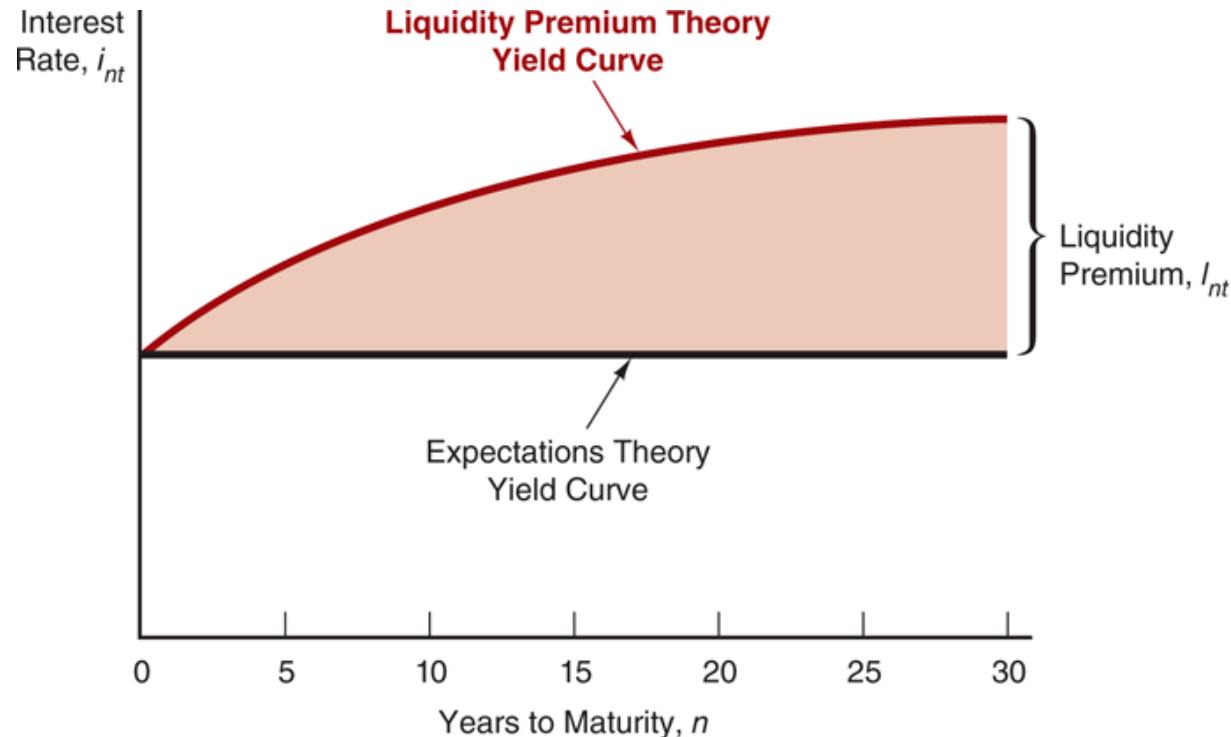
↑
main factor that changes S/T i.e.
is central bank policy rate

↓
in market we reflect this
by change S/T rate

Fed cut rate little bit → see mild upward / flat slope
 a lot → downward slope
 ↓
because of recession

Liquidity Premium Theory

- We can also see this graphically...

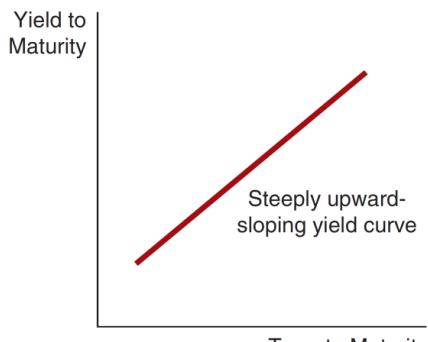


Liquidity Premium Theory Summary

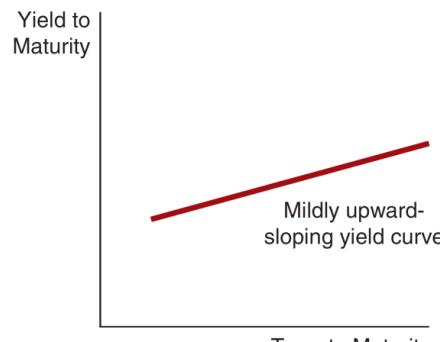
- Explains All 3 Facts
 - Explains fact 3—that, liquidity premium explains why the yield curve slopes upward most of the time. Yet the liquidity premium can change over time
 - Explains fact 1 and fact 2 using same explanations as pure expectations theory because it has average of future short rates as determinant of long rate

Liquidity Premium Theory Summary

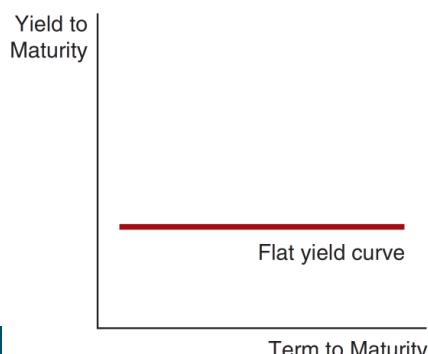
- Market's Expectations of Future Short-Term Interest Rates According to the Liquidity Premium Theory



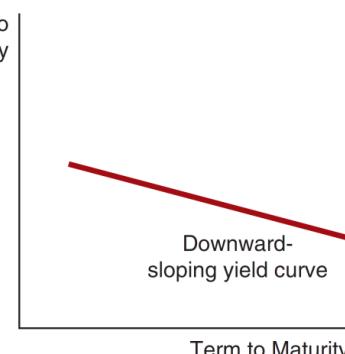
(a) Future short-term interest rates expected to rise



(b) Future short-term interest rates expected to stay the same



(c) Future short-term interest rates expected to fall moderately



(d) Future short-term interest rates expected to fall sharply

expectation
(liquidity premium)

2Y proxy for ST

i_s i_e

forward int rate in the future
expected to occur 1 yr from now

$$(1+i_s)(1+i_e) = \frac{(1+i_2)^2}{(1+i_s)} - 1$$

US Treasury Yield Curve



US Treasury Yield Curve (10 minus 2 Yrs)

LIT

SIT

↳ common indicator as proxy for short term
should have positive number



US Treasury Yield Curve (10 minus 2 Yrs)

★ 10-Year Treasury Constant Maturity Minus 2-Year Treasury Constant Maturity

(T10Y2Y)

[DOWNLOAD !\[\]\(009314918bc0adc32b10a70ab31c37ab_img.jpg\)](#)

Observation:
2023-01-11: -0.66
(+ more)
Updated: Jul 10, 2023

Units:
Percent,
Not Seasonally
Adjusted

Frequency:
Daily

1Y | 5Y | 10Y | Max

2014-07-22

to 2023-01-11

[EDIT GRAPH !\[\]\(d6c68046fbda1ce8c8b81a3f14aae2a7_img.jpg\)](#)

FRED  — 10-Year Treasury Constant Maturity Minus 2-Year Treasury Constant Maturity



US Treasury Yield Curve (10 minus 2 Yrs)

★ 10-Year Treasury Constant Maturity Minus 2-Year Treasury Constant Maturity
(T10Y2Y)

[DOWNLOAD](#)

Observation:
2023-01-11: -0.66
(+ more)
Updated: Jul 10, 2023

Units:
Percent,
Not Seasonally
Adjusted

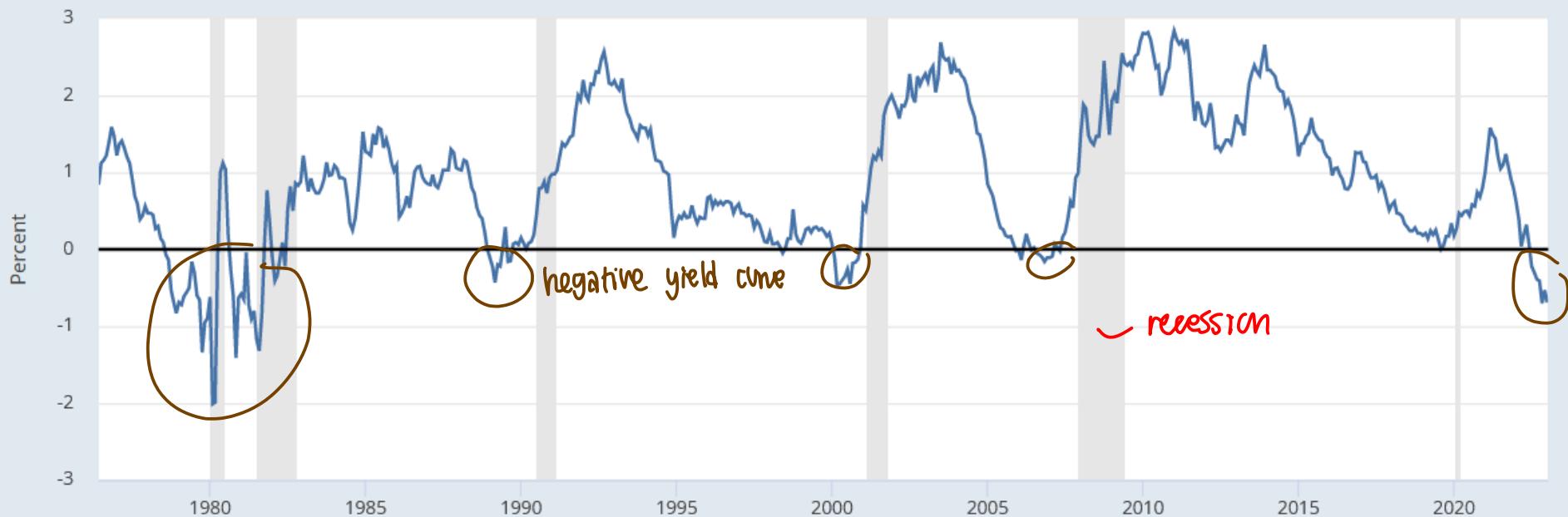
Frequency:
Daily

1Y | 5Y | 10Y | Max

1976-06-01 to 2023-01-11

[EDIT GRAPH](#)

FRED — 10-Year Treasury Constant Maturity Minus 2-Year Treasury Constant Maturity



How Yield Curve is Important?

- Slope of the yield curve is important in managing financial intermediaries.
- They borrow funds in financial markets from surplus spending units, and after intermediation, lend the funds to businesses and consumers.
- An upward-sloping is generally favorable because they borrow most of their funds in the short term (transaction accounts and time deposits) and lend the funds at longer maturities, such as consumer loans, automobile loans, and home mortgages. The more steeply the yield curve slopes upward, the wider the *spread* between the borrowing and lending rates and the greater the profit for the financial intermediaries.
- At the beginning of a period of economic expansion, interest rates tend to be low and the yield curve is upward sloping.

4C

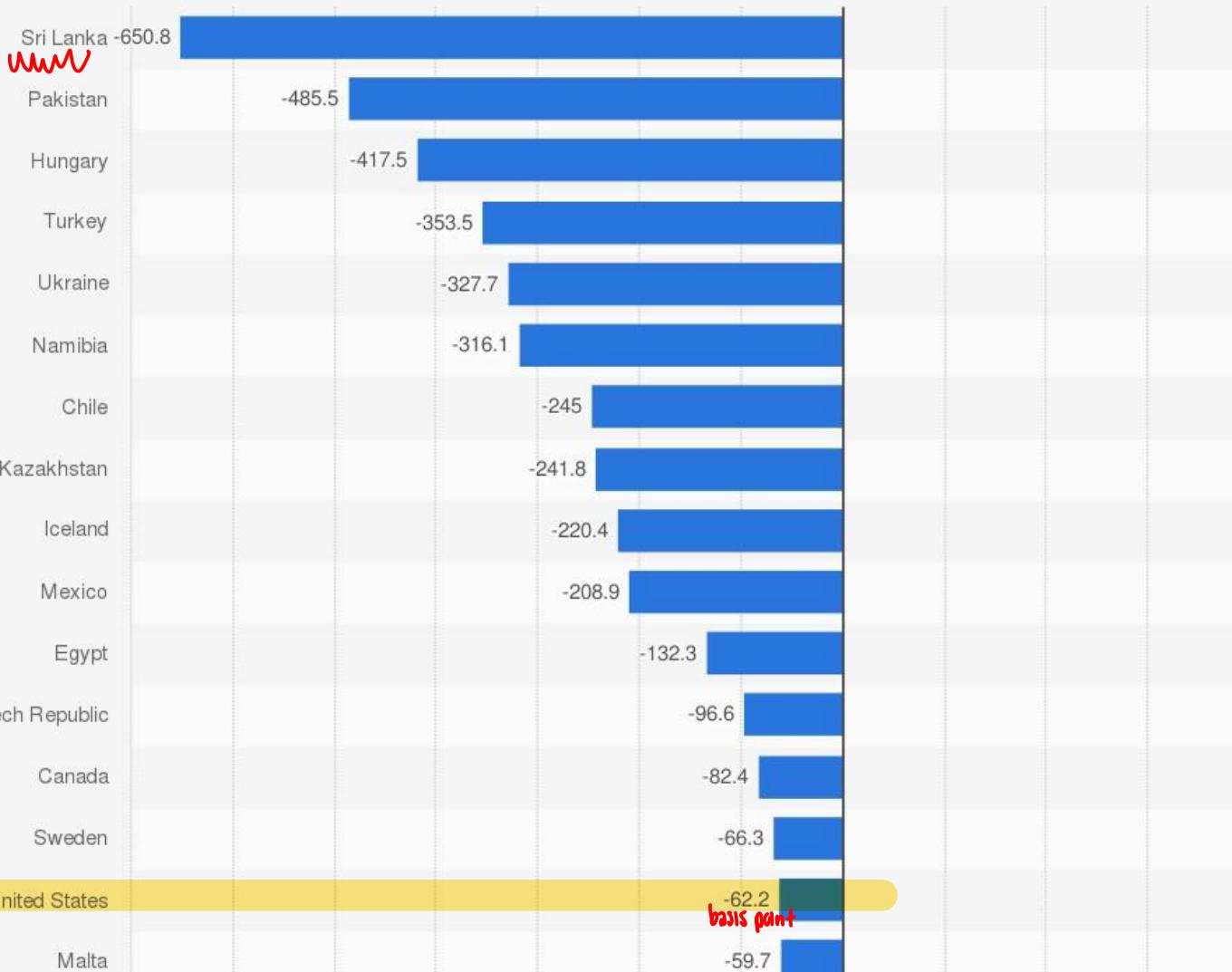






**Selected ten-year minus two-year government bond yield spreads worldwide as of
April 25, 2023, by country (in basis points)**

↳ experience negative yield curve



Source

Website (worldgovernmentbonds.com)
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Additional Information:

Worldwide; April 25, 2023