2604-639
Finance Theories

Topic 2:
Investment Decisions under Uncertainty

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Agenda

what / how to present investment risk

1 Investment Risk

2 Utility Analysis Given Uncertainty

3 Attitude toward Risk

risk lover

risk neutral

* average investors are

risk averse

... .

1. INVESTMENT RISK

1.1 Fisher's Separation with Uncertainty

1.2 Investment Risk

1.3 Risk Investment Choices

1.1 Fisher's Separation with Uncertainty

investment decision & consumption decision are independent to each other. So management can make investment decision without no info about shareholder preference

- Under perfect capital markets and perfect certainty, Fisher's separation principle is obtained.

maximize firm value by invest in all projects that r irr > return from market

  - Result: The separation of the investment and consumption decisions
  - Implication: Corporations should use the "market value rule" when making investment or production decisions
- In this lecture, we will keep the perfect capital market assumption but allow for uncertainty.

deal with two dimensions – risk & return

1.1 Fisher's Separation with Uncertainty

don't know outcome for sure

high expected return comes along with high risk

not the best

- When the future outcomes are uncertain, corporate investment decisions create not only expected return, but also risk on financial securities issued by them.
- Although a specific investment decision may not be suitable to the risk-return preferences of all shareholders, they can undo the corporate decision by selling the security and rebalance their portfolios.
- In perfect capital markets, portfolio rebalancing bears no cost.
- Hence, the best way to satisfy all shareholders remains the same – that is to maximize the market value of the firm.

No risk → use risk-free rate to discount

with risk → — + risk premium to discount

fisher's seperation still hold

how mkt value of firm is determined ?

asset pricing

1.1 Fisher's Separation with Uncertainty

- This line of argument implies that the Fisher's Separation Principle still holds under uncertainty, given that the capital market is perfect.
- To apply the market value rule, we need a conceptual framework of how the market values of risky financial assets are determined.

→ mkt participants

How mkt price risky asset

- First, we need to understand how individuals make investment decisions when facing uncertainty.

risk

1.2 Investment Risk

- Risk is used to describe an investment where the actual future return is not known for certain, but for which an array of possible future returns and their probabilities are known (the probability distribution of returns is known).
- In a probability density graph, the x- and y-axis represent possible future returns and their probabilities, respectively.
- Probability distribution can either be discrete or continuous depending on the assumption on value of returns.
- Two main approaches can be used to derive the probability density of returns, subjective approach and historical (or actual distribution) approach.

more forward looking

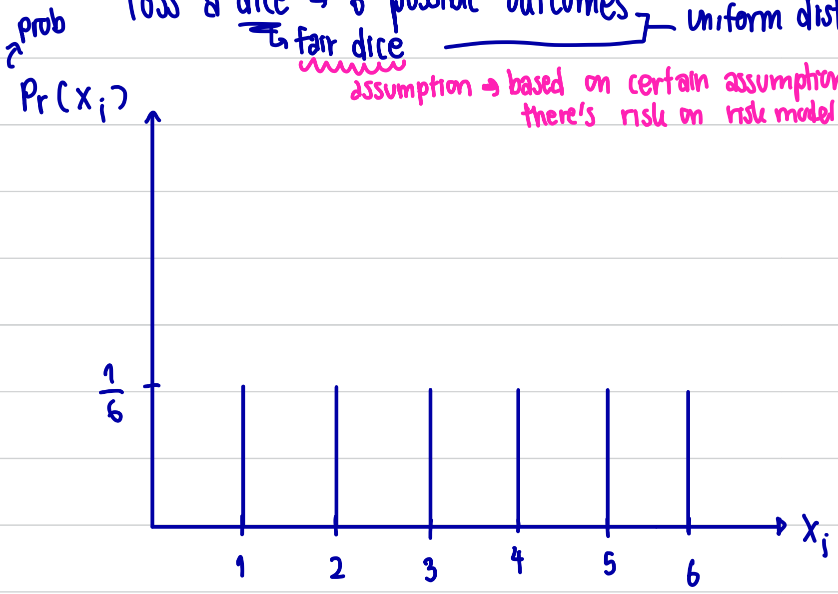
different analyst different outcome

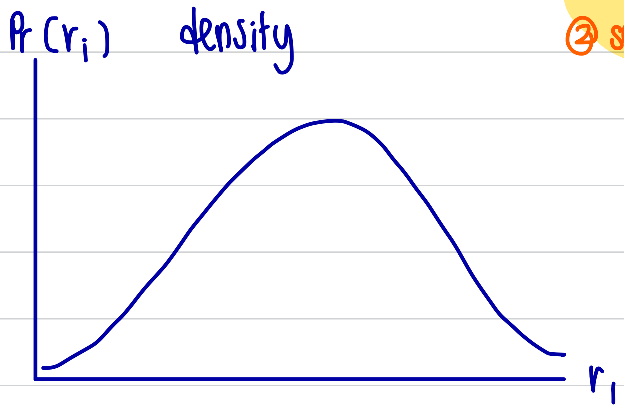
* used what happen in the ~~hast~~ past

use this in this subject

Toss a dice → 6 possible outcomes — uniform distribution → discrete $dist^n$ distribution vs. continuous dist
↳ fair dice   (in variable)   (in variable)
assumption → based on certain assumption
there's risk on risk model → "uncertainty"

① based on historical
② scenario → forecast

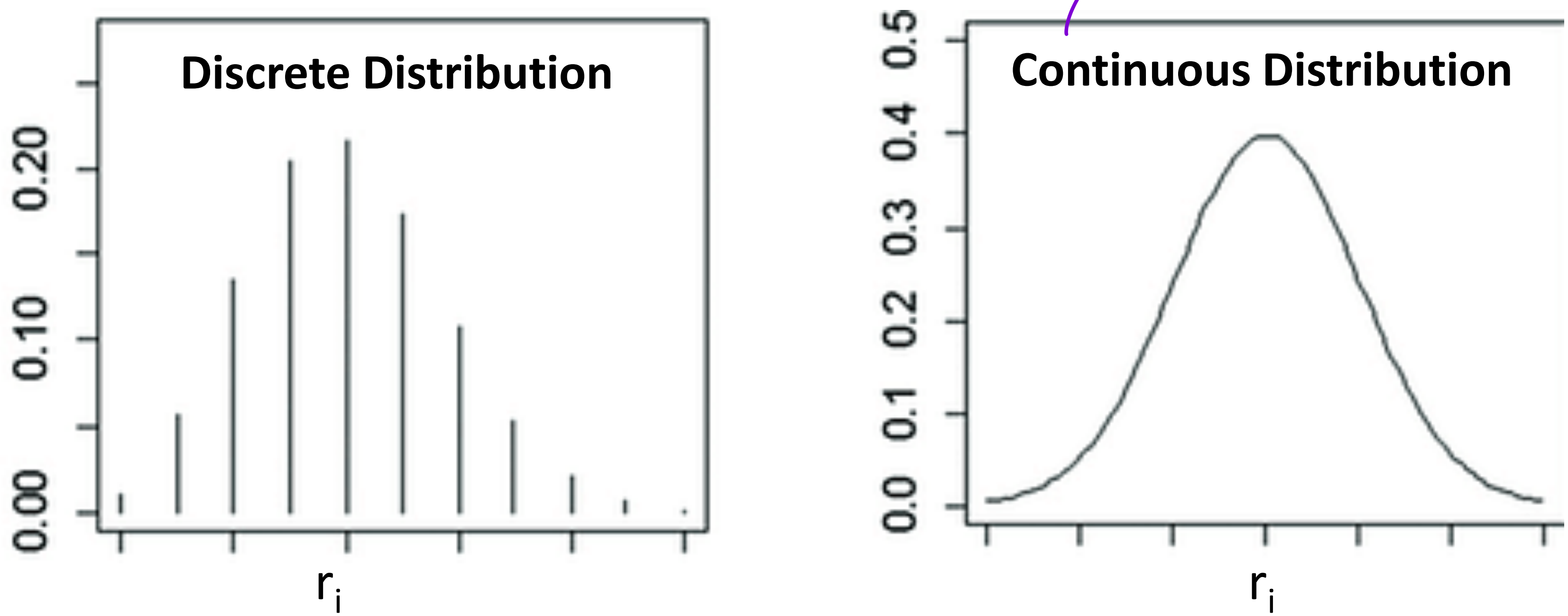




to use normal dist

Discrete vs Continuous Probability Distribution

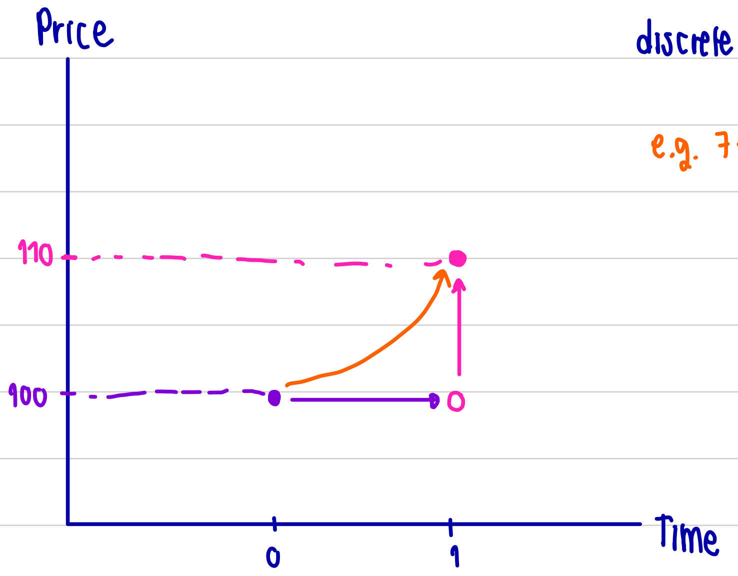
make this assumption



A discrete distribution is one in which the data can only take on certain values, for example integers. The term discrete refers to discrete in variable.

A continuous distribution is one in which data can take on any value within a specified range (which may be infinite). The term continuous refers to continuous in variable.

Time



discrete return $r_D = \frac{110-100}{100} = 10\%$

e.g. 7-11 obtain income all the time

$r_c = \ln\left(\frac{110}{100}\right) = \ln(1+r_D) < 10\%$

↳ continuous in time

$P_0 = 100$

next movement

± ฿ 1 discrete in variable

± ฿ 0.0000000001

tick size

if stock price fall to certain level the quoted price is incremental with certain number

make return discrete in reality

1.3 Risky Investment Choice

- An important question is how individuals choose an investment among risky alternatives.
- How do they rank alternative risky investments? How do they assign levels of preference to alternative risky investments?
- The following table shows probability distribution of returns of 3 securities, namely, A, B, and C.
  - For illustration purposes, we assume that risk can be fully captured by standard deviation of returns.

Probability Density of Returns on 3 Securities

distribution of return

y p_i	A		B		C	
	x r_i	$p_i r_i$	r_i	$p_i r_i$	r_i	$p_i r_i$
1/15	20	20/15	15	15/15	5	15/15
4/15	15	60/15	10	40/15	5	20/15
6/15	5	30/15	5	30/15	5	30/15
3/15	–5	–15/15	0	0/15	5	15/15
1/15	–15	–15/15	–10	–10/15	–5	–5/15
unit %	central value = mean $E[r_A]$	5.33	$E[r_B]$	5.00	$E[r_C]$	4.33
not on same footing $\%^2$	(dispersion from mean)^2 $Var[r_A]$	88.22	$Var[r_B]$	33.33	$Var[r_C]$	6.22
↓√ %	$SD[r_A]$	9.39	$SD[r_B]$	5.77	$SD[r_C]$	2.49

Which is your most preferred and least preferred choices?

What do you use to compare your preferences toward these 3 risky investments?

2. UTILITY ANALYSIS GIVEN UNCERTAINTY

2.1 The Maximum Expected Return Criterion

2.2 The St. Petersburg Paradox

2.3 Bernoulli's Solution

2.4 The Modern Theory of Utility

2.5 The Axioms of Choice

2.1 The Maximum Expected Return Criterion

- Based on the expected return criterion, the optimal decision is to invest in security with the highest E[R].
  - Hence, A ≻ B ≻ C based on expected return
  - The security that maximizes E[R] will also maximize expected future monetary outcomes or future wealth,
    current investment to maximize $E[W_1]$
    $E[W_1] = W_0(1+E[r_1])$ ⇒ not objective function
    expected investment outcome
- However, this does not reflect the actual choice made by many investors. For example, many investors may rank B ≻ A ≻ C.
- There must be something else other than $E[r_1]$ or $E[W_1]$ that affects the individual's choice.

2.1 The Maximum Expected Return Criterion

Investors are willing to pay < E[X]

If coin is fair: 0.5 H: +$10; 0.5 T: +$0

expected payoff $E[X] = \$5 \rightarrow P_0 < E[X]$ (maximum price)

On average, investors demand positive expected return

- Consider a risky investment with 50% chance of receiving \$10 and 50% chance of receiving nothing in the future.
- What is the maximum price you will pay for this investment?
  - The maximum price must reflect the "intrinsic value" (IV_0) of the investment.
  - In competitive market, the price will be driven toward IV_0.
- The game's expected payoff, $E[X] = 0.5(10) + 0.5(0) = \$5$
- Is \$5 the maximum price you will pay? (If you pay \$5, then, $E[r] = 0$.)
- Most people will not pay up to \$5 for this investment!! (implying that $IV_0 < E[X]$)

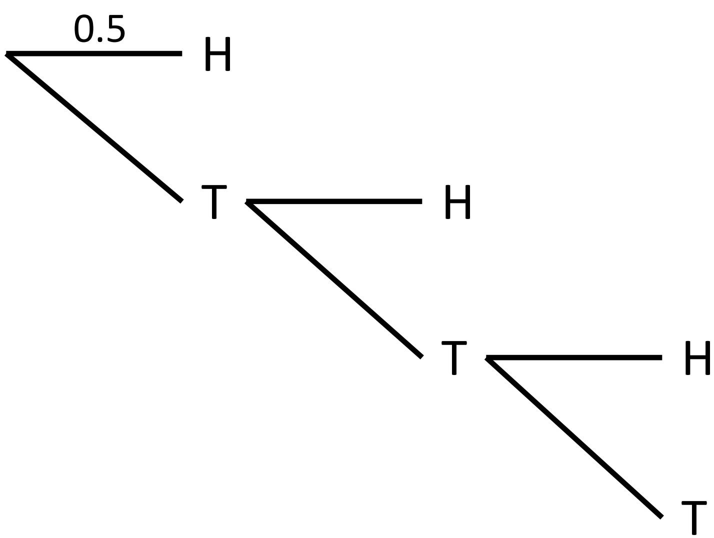
2.2 The St. Petersburg Paradox

- Consider the following game of chance. A coin is tossed until payoff depends on when 1st head appears the first head appears. If a head (H) first appears on the n^{th} flip, the player is paid $\$2^n$ and the game stops. payoff = $\$2^n$ → time when 1st head appear ⇓ expectated value is infinity
- What would be a fair price for one to pay for the opportunity to play such game?
- The principle of maximum expected return implies that an individual is only interested in expected return from an investment.
- It suggests that the game's expected value constitutes the maximum price (or fair price) one will pay for this game.
- Let's calculate the expected payoff.

Expected $-Payoff from the Tossing Coin Game

Let X_n = monetary payoffs if H appears on the n^{th} toss. X is an r.v.

# of toss (n)	Probability (p_n)	X_n
1	$(1/2)$	2^1
2	$(1/2)^2$	2^2
3	$(1/2)^3$	2^3
…	…	…
∞	$(1/2)^\infty$	2^∞



$$E[X] = \sum_{n=1}^{\infty} p_n X_n = \sum_{n=1}^{\infty} (1/2)^n 2^n = \infty$$

2.2 The St. Petersburg Paradox

- Let X_n = monetary payoffs if H appears on the n^{th} toss.
- The game could potentially go on forever, so the expected monetary payoffs is;

$$E[X] = \Sigma\, p_n X_n$$

$$= (1/2)^1 2^1 + (1/2)^2 2^2 + \ldots + (1/2)^\infty 2^\infty$$

$$= \infty$$

- How could we explain why most individuals will only pay a finite amount to enter the game? Competitive bid will not drive the price toward ∞ neither.
- This is problem is called the "St. Petersburg Paradox".

2.3 Bernoulli's Solution

why people want to pay for finite to play game, but $E[r] = \infty$

- Bernoulli, a Swiss mathematician, argues that the amount of money an individual is willing to pay for the game depends on the game's expected utility that he/she gained from game and not its expected money return.

- Bernoulli's solution to the paradox is to argue that individuals do not assign the same value per dollar to all payoffs. assign satisfaction value to each dollar of payoff

$3^{rd} \rightarrow 2^3 \leftarrow Pr(\frac{1}{2})^3 \rightarrow$ less risky

$\vdots$

$10^{th} \rightarrow 2^{10} \leftarrow Pr(\frac{1}{2})^{10}$

risk aversion → dollar for lower round is less risky

or diminishing marginal utility

already has more dollar

not want another dollar that much

  - For example, each dollar to be received from the 10th flip is viewed as less valuable than each dollar from the 3rd flip.

  - Today we called this "diminishing marginal utility" or "risk aversion"

2.3 Bernoulli's Solution

- Bernoulli assumes that the utility of money is a logarithmic function of the size of the money prize (X).

$$U(X) = b \ln(X/a)$$

where U(X) is the utility function
a and b are positive constant

- Note that the function is chosen such that;
  - U(X) is an increasing function of monetary outcome – that is, dU/dX or $U'(X) > 0$. marginal utility is positive ; more money is prefered more
  - As X increases, U(X) increases at a decreasing rate – that is, d^2U/dX^2 or $U''(X) < 0$ (the function is bound). 2nd order of utility
diminishing marginal utility

2.3 Bernoulli's Solution

- The utility obtained if H appears after n tosses is

utility

$$U(X) = \cancel{b \ln (2n/a)} \quad b \ln (2^n / a)$$

- Now assume the principle of expected utility. The individual will pay at most an amount X_0 such that $U(X_0)$ is equal to the game's expected utility.

* We want to find X_0, such that $U[X_0] = E[U(X)]$

  - Note we ignore the discount factor which implies a very short investment period.

don't know outcome for sure

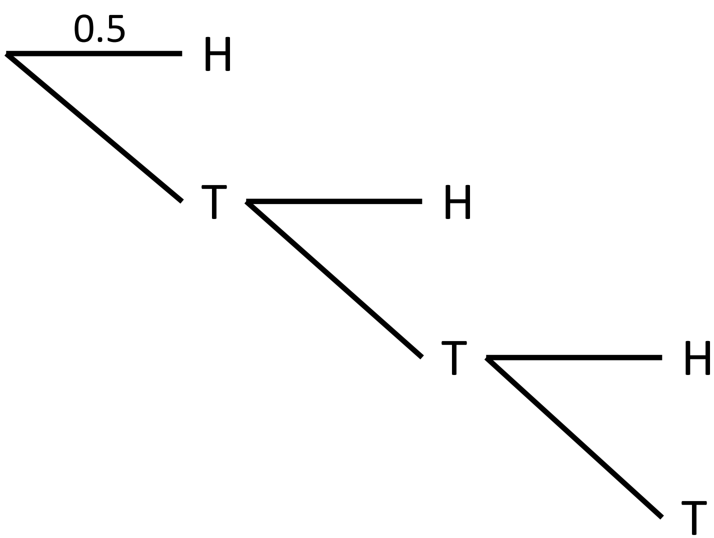
$U(X_0) = E[U(X)]$ → want to prove X_0 is finite number

↳ e.g. pay \$4 → utility ↓ → can pay game

Expected \$-Payoff from the Tossing Coin Game

Let X_n = monetary payoffs if H appears on the n^{th} toss. X is an r.v.

① convert payoff to utility



# of toss (n)	Probability (p_n)	X_n	$U(X_n)$
1	$(1/2)$	2^1 →	y
2	$(1/2)^2$	2^2 →	y
3	$(1/2)^3$	2^3 →	y
…	…	…	⋮
∞	$(1/2)^\infty$	2^∞ →	y

$$E[X] = \sum_{n=1}^{\infty} p_n X_n = \sum_{n=1}^{\infty} (1/2)^n 2^n = \infty$$

$\boxed{\text{then } \sum p_n y_n}$

2.3 Bernoulli's Solution

- The game expected utility or E[U(X)] is

$$E[U(X)] = \sum_{n=1}^{\infty} p_n \underbrace{U(X_n)}_{\text{replace by function}}$$

$$= \sum_{n=1}^{\infty} \frac{1}{2^n} \underbrace{\left[b \ln \frac{2^n}{a}\right]}_{b \ln 2^n - b \ln a}$$

$$= \left[\sum_{n=1}^{\infty} \frac{n}{2^n}\right] b \ln 2 - \left[\sum_{n=1}^{\infty} \frac{1}{2^n}\right] b \ln a$$

from $\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$ and $\sum_{n=1}^{\infty} \frac{n}{2^n} = 2$

$$E[U(X)] = 2\, b \ln 2 - b \ln a$$

$$= b \ln 2^2 - b \ln a$$

$$= b \ln \left(\frac{2^2}{a}\right)$$

$$= U(2^2)$$

$$= U(4) \; \text{CE} = E[U(X)]$$

2.3 Bernoulli's Solution

- Thus, an individual whose taste conforms to Bernoulli's utility function will pay $4 at most for the game.
- In other words, the individual is indifferent between a perfectly certain promise of a gain of $4 and the chance to play the game.
- certain amount of money that give you utility

  The amount $4 is called "certainty equivalent" or CE of the risky game.

  intrinsic value of game

  ↳ IU_0

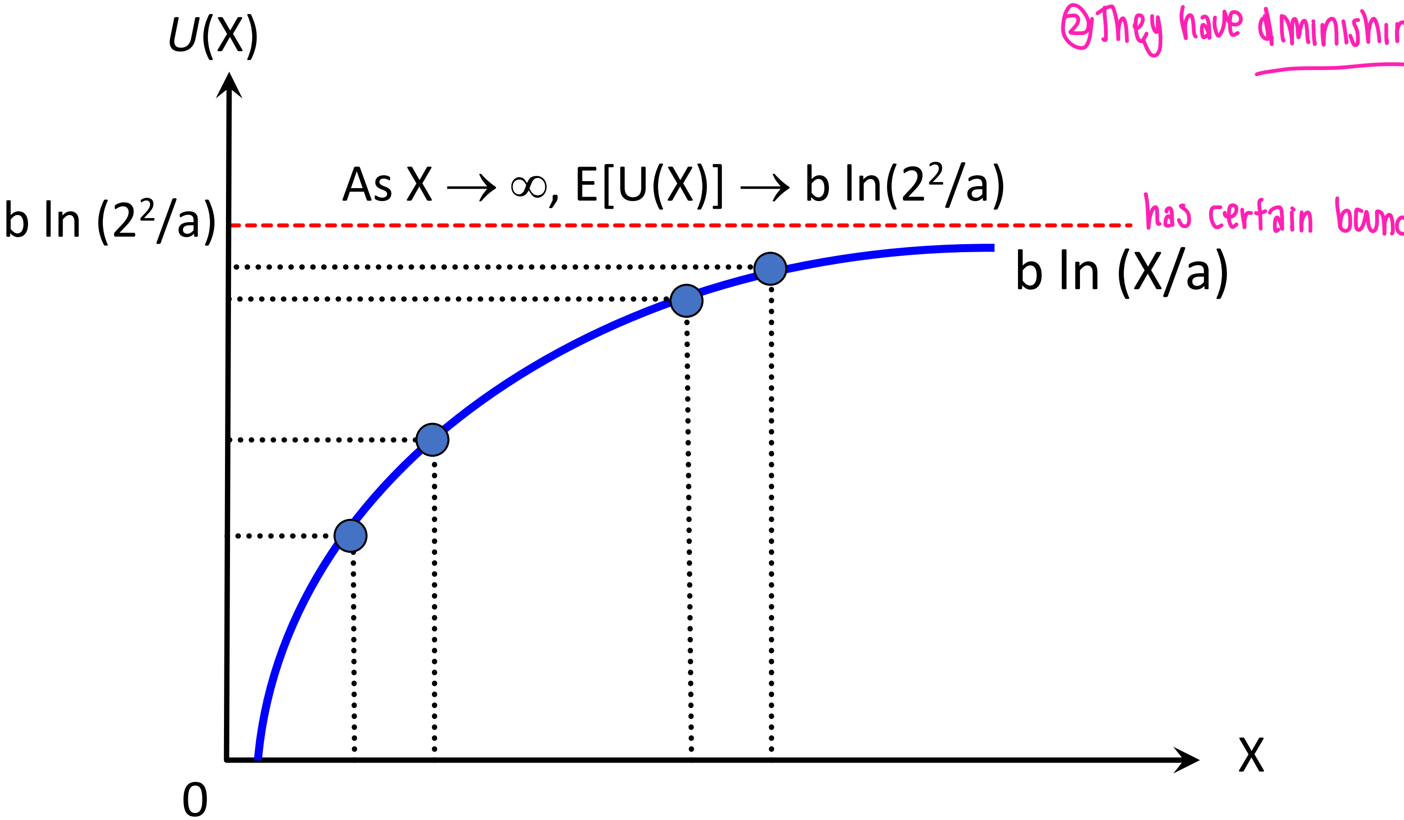
Total Utility Curve (U = b ln (X/a))

① Investors prefer more to less

② They have diminishing marginal utility

↓

risk aversion



The total utility reaches a finite number as the monetary outcome approaches infinity. This explains why an individual is willing to pay only a finite amount of money for the game.

2.4 The Modern Theory of Utility

How individual choose risky assets

They based their decision on expected utility not expected return

- Bernoulli shows that given the assumption of risk aversion, the expected utility approach can be used to show how individuals could derive the price of a risky asset.

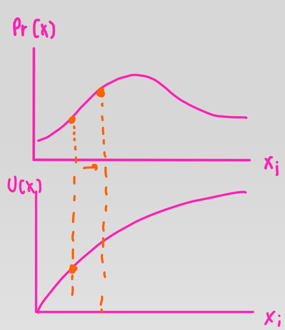
- The expected utility from a risky investment is the weighted sum of the utilities of each possible outcome, with weights equal to the probabilities of obtaining each outcome.

Discrete distribution1: $E[U(X)] = \sum_{i=1}^{n} p_i U(X_i)$

for each small incremental of outcome

Continuous distribution: $E[U(X)] = \int_{i=1}^{n} U(X_i) f(X_i) dX$

pdf



2.4 The Modern Theory of Utility

maximizing expected utility is always optimal solution under certain assumptions

- It was von Neumann and Morgenstern (VNM) who justify the use of the expected utility. They demonstrate that if the decision maker fulfills a number of reasonable consistency requirements, the expected utility hypothesis leads to optimal choice under conditions of uncertainty.
- The set consistency requirements is called the axioms of choice.
- Under the expected utility approach, the optimal choice of an individual faced with alternative risky investments from which he has to choose one is the investment which maximizes his expected utility.

2.5 The Axiom of Choice under Uncertainty

- Let,

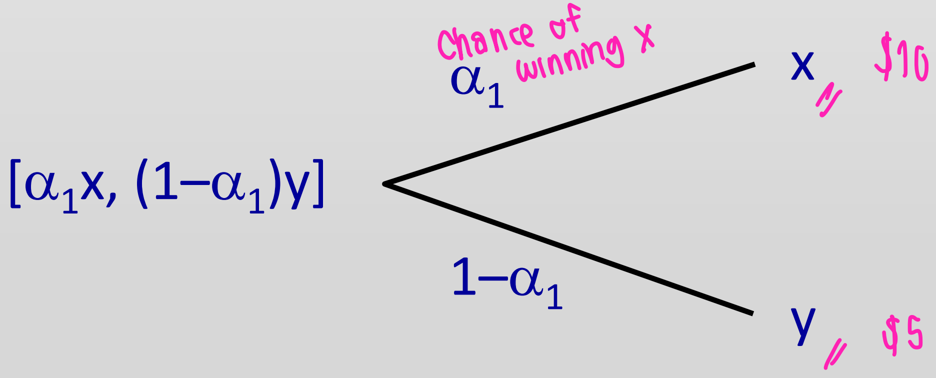
they're final payoff

outcome itself is not another gain

only one outcome can appear once a time

x, y and z represent certain elementary mutually exclusive outcomes

$[\alpha_1 x, (1-\alpha_1)y]$ is a game of chance where the probability of receiving x is α_1 and the probability of y is $(1-\alpha_1)$.



2.5 The Axiom of Choice under Uncertainty

1) Comparability (or completeness): An individual has ability to rank his preference towards all possible baskets of consumption.

- He can always identify whether x ≻ y, y ≻ x or x ~ y

สมบัติการถ่ายทอด นำหุ้นที่มีจุดร่วมมาเชื่อมเพื่อให้ compare กันได้

2) Transitivity (or consistency): An individual is consistent in his preference rankings of consumption baskets.

- If x ≻ y and y ≻ z, then x ≻ z
- If x ~ y and y ~ z, then x ~ z

3) Non-satiation of wants: An individual always prefers a certain outcome with a higher payoff.

prefer more to less

2.5 The Axiom of Choice under Uncertainty

4) Independence: An individual's attitudes toward particular
outcome prospects are not affected when these outcome prospects are combined with other prospects.

- If $x \sim y$ then $[\alpha_1 x, (1-\alpha_1)z] \sim [\alpha_1 y, (1-\alpha_1)z]$
- If $x \succ y$ then $[\alpha_1 x, (1-\alpha_1)z] \succ [\alpha_1 y, (1-\alpha_1)z]$

E.g. Camry $\prec$ Accord

$G1 \equiv [0.4 * \text{Vios}\ ;\ 0.6 * \text{Camry}]$

$G2 \equiv [0.4 * \text{Vios}\ ;\ 0.6 * \text{Accord}]$

$G2 \succ G1$

prob mao

can use this to rank

2.5 The Axiom of Choice under Uncertainty

- EX: Consider 2 investments,

  $G_1 \equiv [\alpha_1 \text{Camry}, (1-\alpha_1)\text{Vios}]$

  $G_2 \equiv [\alpha_1 \text{Accord}, (1-\alpha_1)\text{Vios}]$

  If Camry ~ Accord, then it must be the case that $G_1 \sim G_2$.

  If Camry $\succ$ Accord, then it must be the case that G1 $\succ$ G2.

When probabilities are the same b/w risky investments, individuals are only concerned with the value of elementary payoffs.

2.5 The Axiom of Choice under Uncertainty

5) Certainty Equivalent: For every game of chance, there is a value (called certainty equivalent; CE) such that the individual is indifferent between the game of chance and the CE (everything has its price)

outcome x, y, z

- If $x \succ y \succ z$, then there exists a unique α, such that $y \sim [\alpha x, (1-\alpha)z]$

Falls in btw best & worst outcome

game of chance → subject can always give price for game of chance

- That is, for every game of chance, $[\alpha x, (1-\alpha)z]$, an individual can always find a certain value, y, such that the expected utility from the game is equal to the utility of y. The value y is called "certainty equivalent"

E.g. PPJ

$$\text{PPJ}\begin{cases} 0.5 & 200 \\ 0.5 & 100 \end{cases} \quad 100 < CE < 200, \quad E[X] = 150$$

$P_0 = 120$ ↳ CE

2.5 The Axiom of Choice under Uncertainty

ทุกอย่างเหมือน prob ต่าง

the best

the worst

outcome btw x & z

6) **Ranking:** If $x \succ y \succ z$ and $x \succ u \succ z$, we can construct a game of chance, that is to find a unique probability α_i such that $y \sim [\alpha_1 x, (1-\alpha_1)z]$ and $u \sim [\alpha_2 x, (1-\alpha_2)z]$. It follows that if $\alpha_1 > \alpha_2$, then $y \succ u$ or if $\alpha_1 = \alpha_2$, then $y \sim u$.

⇓ same outcome

prob assigned to each outcome is different

the game that has more chance to win → we prefer this game more

⇓

use prob distribution

- **EX:** Given, Camry $\succ$ Vios

  $G_1 \equiv [\alpha_1 \text{Camry}, (1-\alpha_1)\text{Vios}]$

  $G_2 \equiv [\alpha_2 \text{Camry}, (1-\alpha_2)\text{Vios}]$.

  If $\alpha_1 = \alpha_2$, it must be the case that $G_1 \sim G_2$.

  If $\alpha_1 > \alpha_2$, it must be the case that $G_1 \succ G_2$.

- Axioms 3 and 5 imply that individuals are only concerned with elementary payoffs and their probabilities.

2.5 The Axiom of Choice under Uncertainty

- Given that an individual behaves consistently with the axioms of choice, VNM show that it is optimal for the individual to chooses the choice that maximize his/her E[U].
- VNM have developed the theory for cardinal (as opposed to ordinal) utility functions.
  - The Cardinal Utility approach assumes that utility is measurable, and the subject can express his satisfaction in cardinal or quantitative numbers, such as 1,2,3, and so on. Furthermore, the quantitative difference between two utility levels is meaningful.

3. ATTITUDE TOWARD RISK

3.1 Attitude toward Risk

3.2 Measuring Degrees of Risk Aversion

3.3 Portfolio Selection with Partial Information

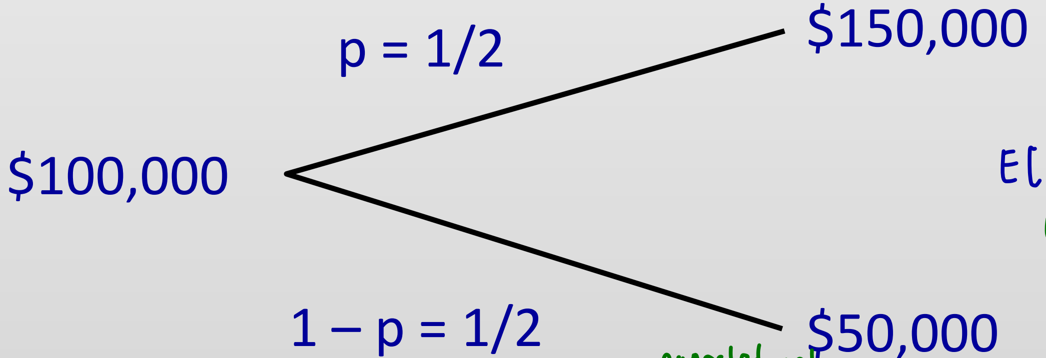
objective function to make decision

$\max \; E[U(x)] \rightarrow$ optimal decision

3.1 Attitude toward Risk

- It will be useful to distinguish between three classes of investors, namely, risk averter, risk lover and risk neutral.
- Consider the following actuarially fair game of chance.

& symmetric with equal prob for each outcome



$= \frac{1}{2} 150k + \frac{1}{2} 50k$

$E[x] = \$100,000$

expected payoff

expected payoff - price of participation = 0

expected return

- A fair game of chance is the one in which the expected monetary prize equals the price of participation. That is $E[r] = 0$.

$E[x] = \$100,000$ $\$100,000$

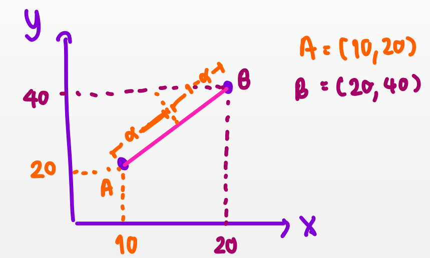
if you feel indifferent btw certain $100,000 and E[x] ⇒ risk neutral

· expected return = 0% as the price of participation = expected price (fair game)

expected return > 0% ⇒ risk averse because $U(P_0)$ > $E[U(x)]$

< 0% → risk lover; utility from that payoff; expected utility that derived from game

3.1 Attitude toward Risk

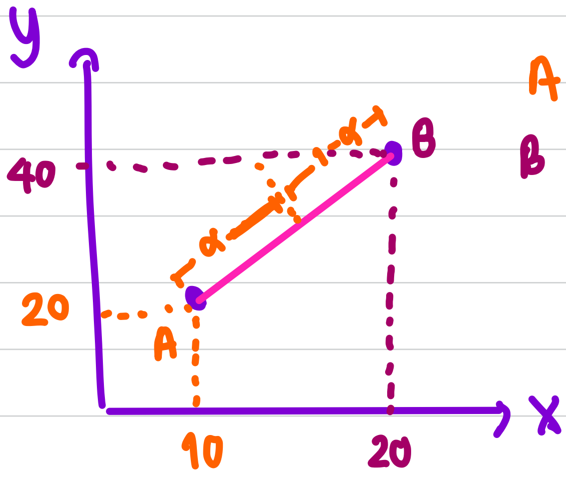


- Risk averse investors will reject a fair game, because the expected utility from the game is less than the utility from the price paid to enter the game.

$$U(E[X]) > E[U(X)]$$

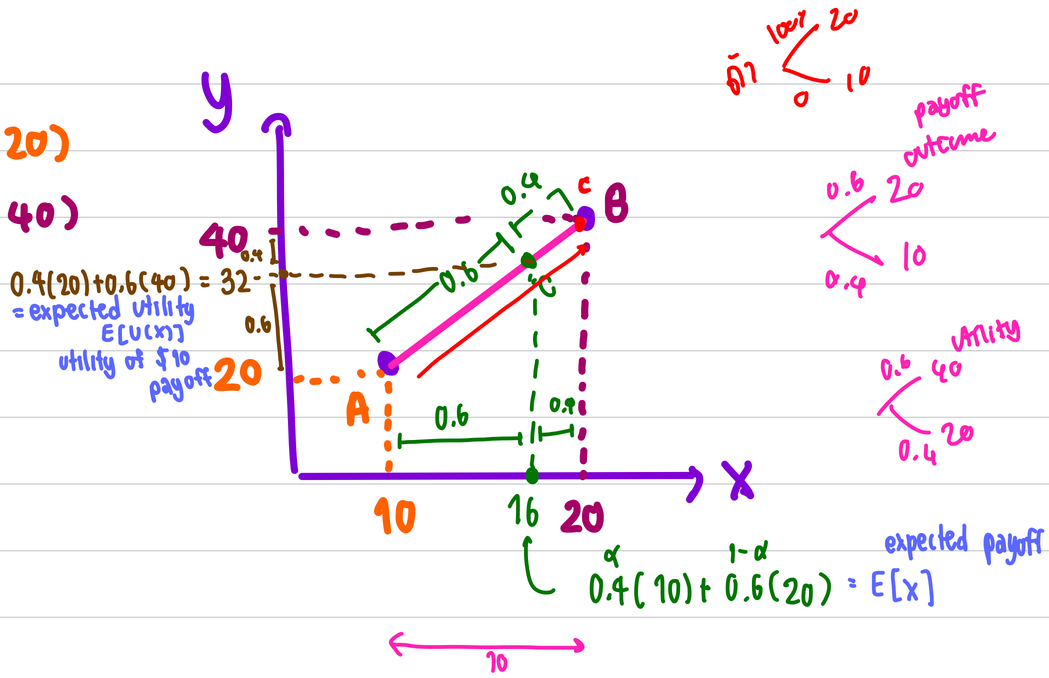
- A risk averter will prefer a perfectly certain return to an uncertain one with equal expected value. A risk averter could be a speculator, but not a gambler.

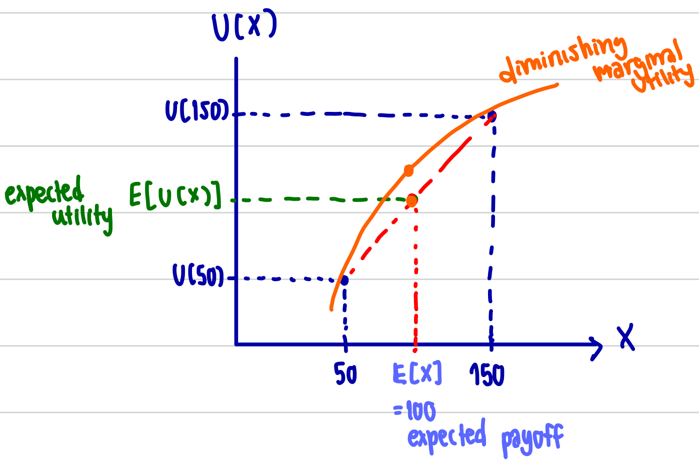
- The total utility function for risk averters is upward sloping and concave.
  - $U'(X) > 0$ means more is preferred to less.
  - $U''(X) < 0$ means risk aversion



A = (10,20)

B = (20,40)





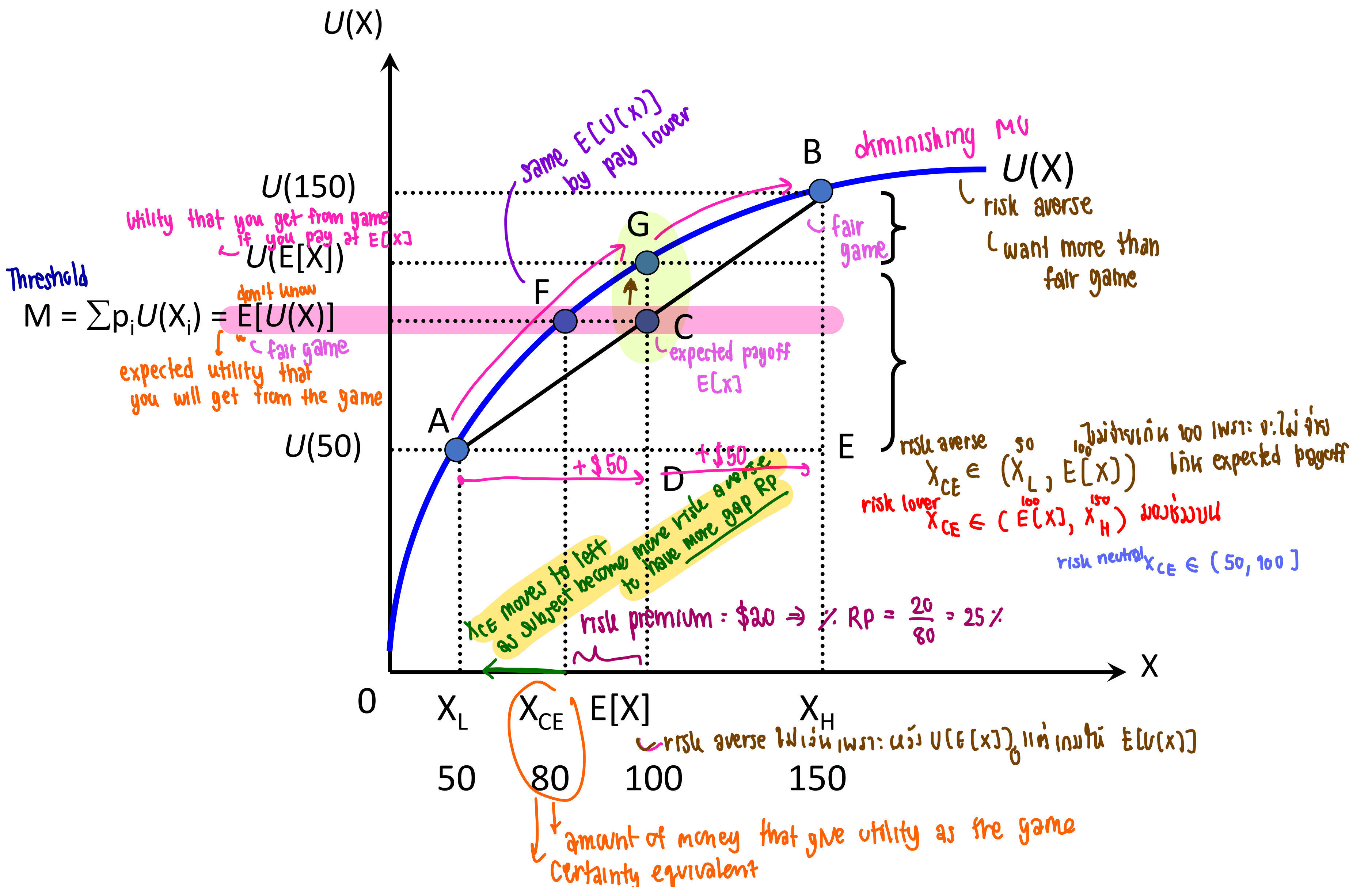
$U(100) > E[U(x)]$ (vs.)

$E[x]$

risk averters will not play fair game

because their expected utility from game < utility of expected payoff

Total Utility Curve of a Risk Averter



Describe utility curve of risk averter

3.1 Attitude toward Risk

- Certainty equivalent (CE) is the level of price (or certain payoff) that an investor is willing to pay to participate the fair game. That is, the investor is indifferent between taking the risky investment or taking the certain payoff.
- For risk averters, "CE < E[X]", as premium is needed to induce them into a risky investment. From the graph, CE = 80,000.

$$\% \text{ RP} = (E[X] / CE) - 1$$

$$= (100/80) - 1 = 0.25 \text{ or } 25\%$$

- As an investor becomes more averse to risk, what happen to his CE? Will he be willing to take more or less risk?

3.1 Attitude toward Risk

the game is played or not

- Risk neutral investors are indifferent whether or not a fair game is undertaken.

what he loss = what he gain from game

$$U(E[X]) = E[U(X)]$$

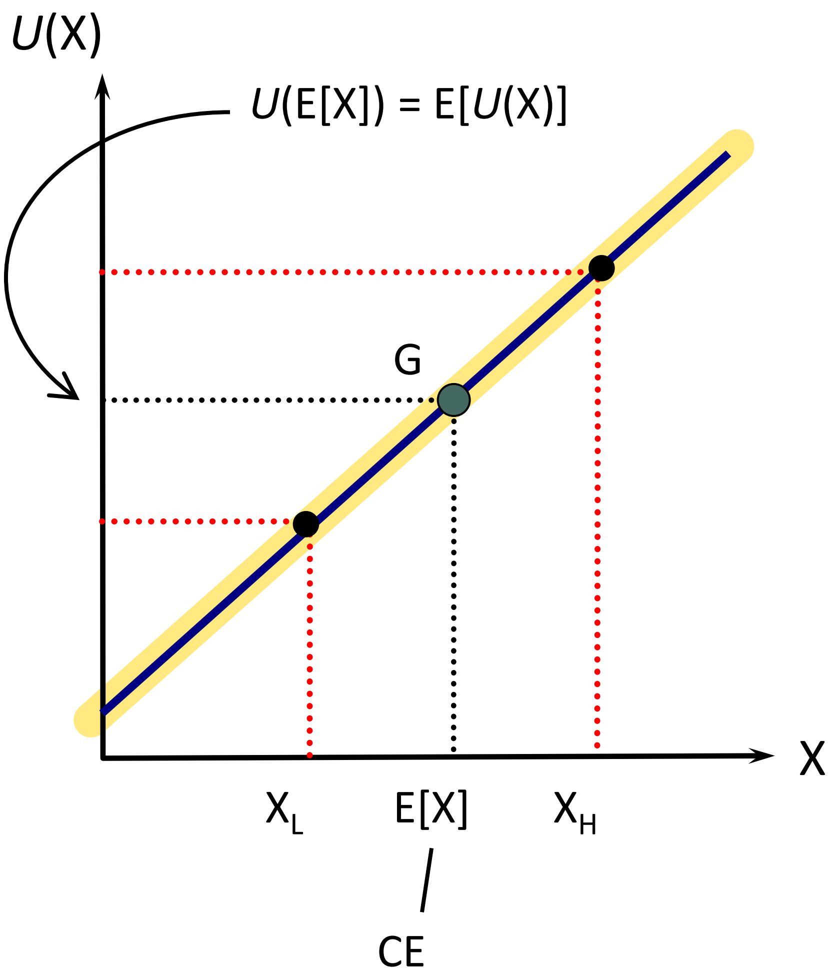
- Risk seeking investors (or risk lover) will always take a fair game.

$$U(E[X]) < E[U(X)]$$

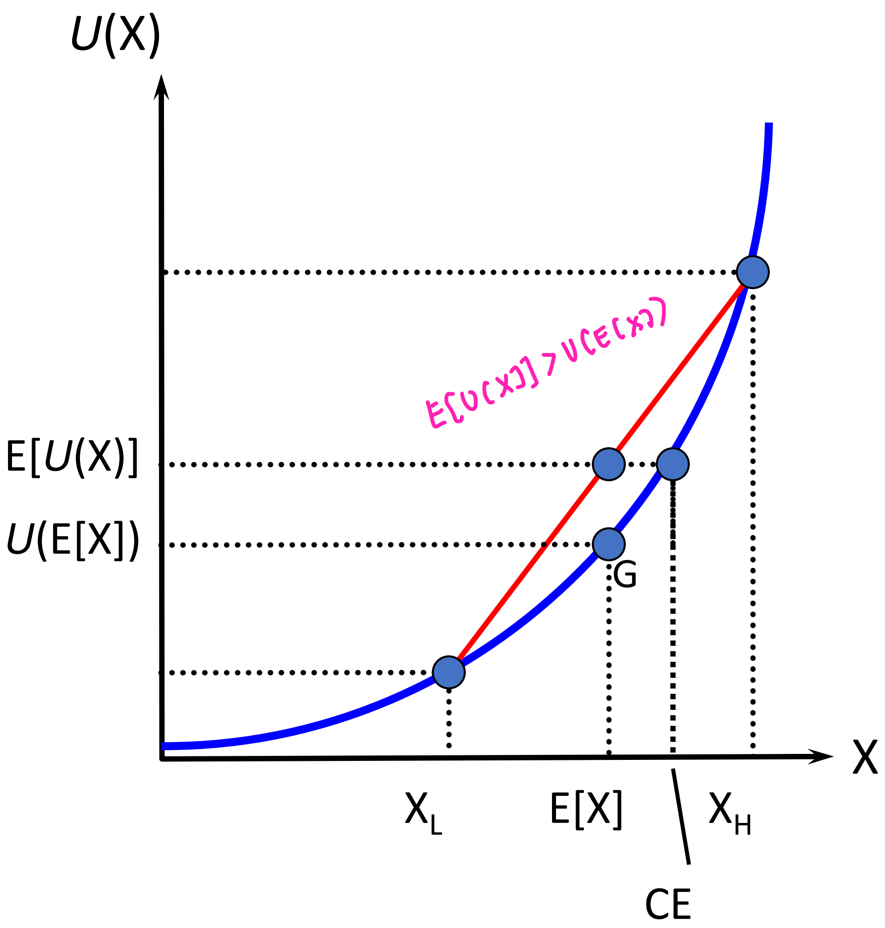
  - Risk seekers are willing to pay premium (i.e., willing to accept negative risk premium) to take risky investments.
  - A risk lover by nature is a gambler.

Total Utility Curves for Risk Neutral

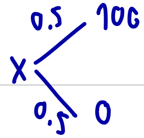
minimum risk premium = 0



Total Utility Curves for Risk Lover

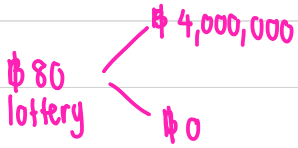


- A gambler → risk lover ↗ also rational
  มอง reward สูงสุด
  accept negative expected return
- A speculator → risk averse
  expected return is positive

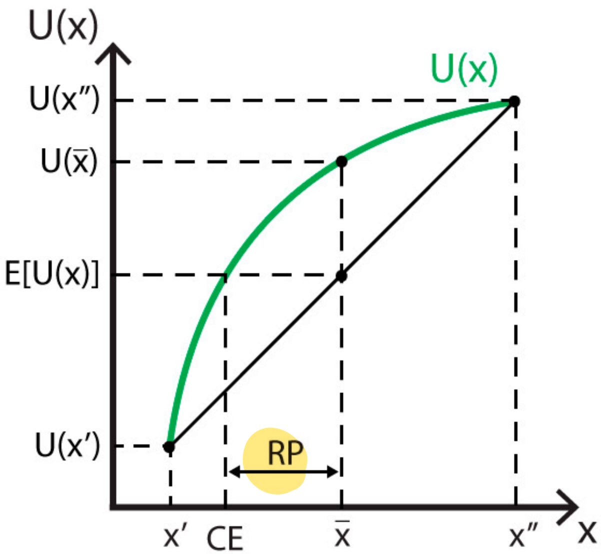


$E[x] = 50$ $\boxed{P < 100}$

- Government lottery → gamble
  ↳ poor people
- Stock market → market to speculate



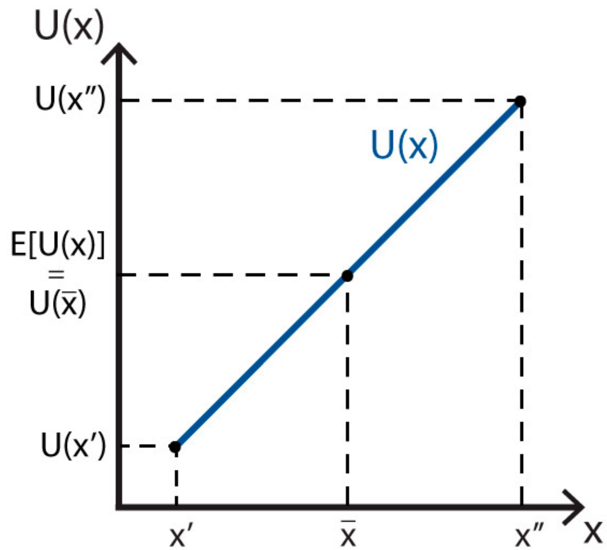
$E[x] = 60$



Risk averse individual

$E[U(x)] < U(\bar{x})$

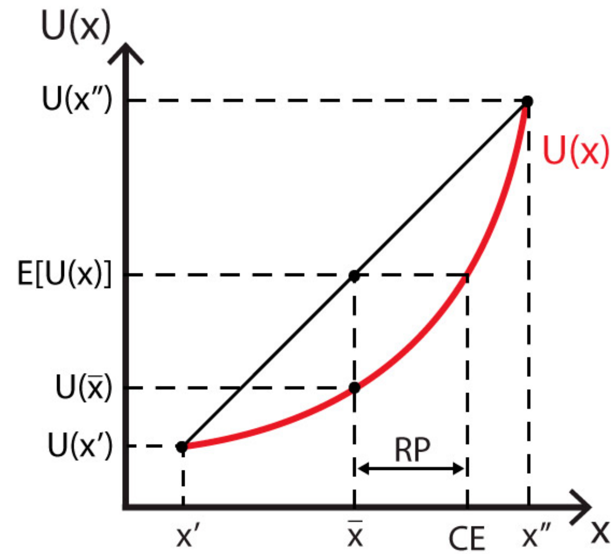
$CE < \bar{x}$



Risk neutral individual

$E[U(x)] = U(\bar{x})$

$CE = \bar{x}$



Risk loving individual

$E[U(x)] > U(\bar{x})$

$CE > \bar{x}$

3.2 Measuring Degrees of Risk Aversion

- We usually assume that an average investor is risk averse, because it is common to observe that an average person tends to refuse a fair (but risky) game.
- It is also believed that the level/degree of risk aversion is a function of the investor's current wealth level.

  is difference among individuals
- For example, an investor may be willing to invest more money in risk assets (i.e., becomes less risk averse) as his wealth increases.

  ↳ his degree of risk aversion change
- How could we quantify an investor's degree of risk aversion?

3.2 Measuring Degrees of Risk Aversion

- Assume a risk averse investor with the current wealth of W_0.
- Offer him a fair game X≡[0.5*(-$H), 0.5*($H)]. This game has E[X] = 0, where X's are the possible payoffs $H or –$H.
- As a risk averter, he will refuse to play the game unless;

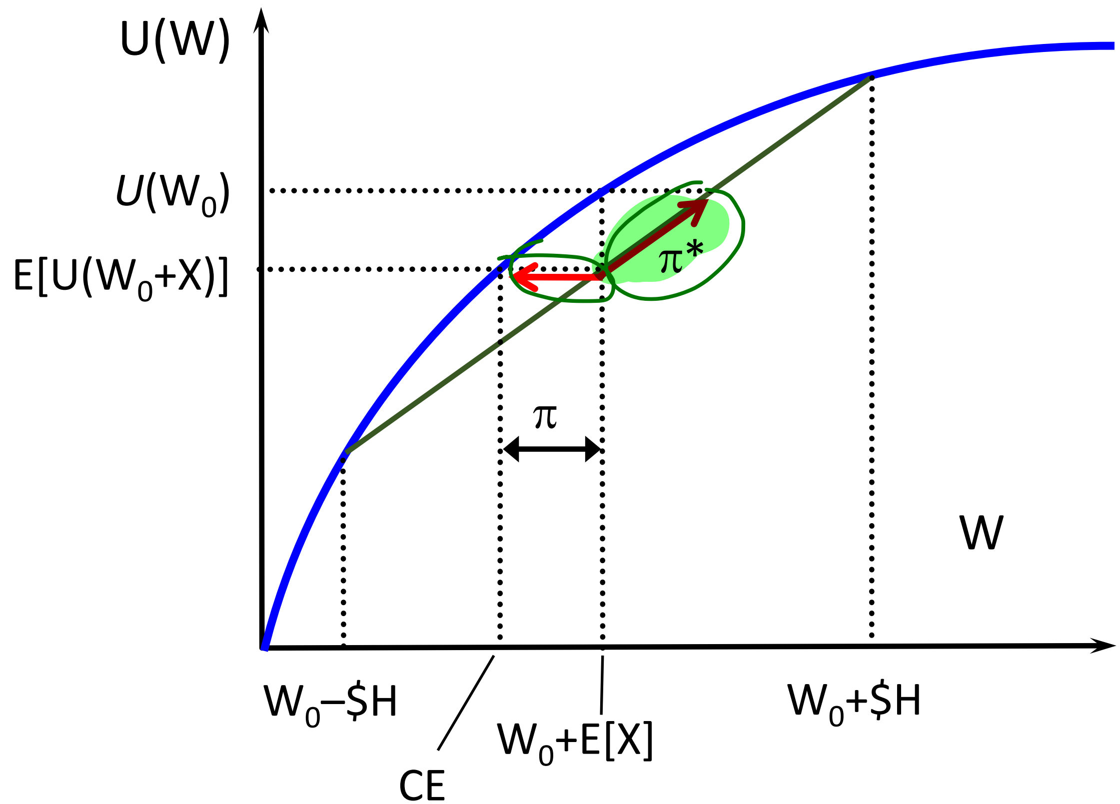
  offer today to pursuade person to pay game

  [1] we also offer him $π in cash if he takes the game, or

  [2] improve the odd of +$H from 0.5 to 0.5+π*.
- A person with higher degree of risk aversion will demand higher $π or π*.

  degree of risk aversion measure
- Kenneth Arrow and John Pratt proposed 2 alternative measures of risk aversion, Absolute Risk Aversion (ARA) and Relative Risk Aversion (RRA)

Total Utility Curve



3.2 Measuring Degrees of Risk Aversion

ask the subject what is the price of π

- Absolute Risk Aversion (ARA)

$$ARA(W) = -U''(W)/U'(W)$$

where W measures the investor's income level

- In ARA, we ask, what value of π^* would the investor need to accept a game of chance of $\pm\$X$.
- The higher the π^*, the higher the degree of ARA.
- In general, ARA depends on W. More specifically, it seems likely that ARA(W) decreases when W goes up.

3.2 Measuring Degrees of Risk Aversion

- Relative Risk Aversion (RRA)

$$RRA(W) = -W \times [U''(W)/U'(W)]$$

- While ARA describes an investor's attitude towards absolute bets of +/- \$X, RRA describes his attitude towards relative bets of +/- kX, where k is a fraction of total income.

willingness to take risk

- In RRA, we ask what value of π^* would you need to accept a bet of +/-1% of your wealth.

- Since the coefficient of RRA describes aversion to risk over bets that are expressed relative to wealth, it is more plausible to assume that investors have constant RRA.

3.2 Measuring Degrees of Risk Aversion

- From $ARA(W) = -U''(W)/U'(W)$,
  - $U'(W) = \partial U(W)/\partial W = MU$ (rate of change in total utility) > 0
  - $U''(W) = \partial(\partial U(W)/\partial W)/\partial W = \partial MU/\partial W$ (rate of change in MU)
  - Since we assume more is preferred to less, $U'(W) > 0$. The sign of $A(W)$ depends on the sign of $U''(W)$
  - If we also assume diminishing marginal utility or risk aversion, we have $U''(W) < 0$. In this case, we always have $A(W) > 0$.

Changes in ARA as the Level of Wealth Changes

Condition	Implication	Property of A(W)
Increasing ARA	As $W_0 \uparrow$, more \$-RP is required (or hold fewer \$ in risky assets)	$A'(W) > 0$
Constant ARA	As $W_0 \uparrow$, the same \$-RP is required (or hold the same \$ in risky assets)	$A'(W) = 0$
Decreasing ARA	As $W_0 \uparrow$, less \$-RP is required (or hold more \$ in risky assets)	$A'(W) < 0$

Changes in RRA as the Level of Wealth Changes

Condition	Implication	Property of R(W)
Increasing RRA	As $W_0\uparrow$, more %-RP is required (or $\downarrow$ % of wealth invested in risky assets)	$R'(W) > 0$
Constant RRA	As $W_0\uparrow$, same %-RP is required (or same % of wealth invested in risky assets)	$R'(W) = 0$
Decreasing RRA	As $W_0\uparrow$, less %-RP is required (or $\uparrow$ % of wealth invested in risky assets)	$R'(W) < 0$

Example

- Logarithmic utility functions: $U = a + b \ln(W)$ where $b > 0$
    - $U'(W) = b/W$ and $U''(W) = -b/W^2$ [Concave fn]
    - $A(W) = 1/W$ [decreasing ARA]
    - $R(W) = 1$ [constant RRA]

- Power utility functions: $U = W^b/b$ where $0 < b < 1$
    - $U'(W) = W^{b-1}$ and $U''(W) = (b-1)W^{b-2}$ [Concave fn]
    - $A(W) = (1-b)/W$ [decreasing ARA]
    - $R(W) = (1-b)$ [constant RRA]

Example

it relies on some assumptions that not reflect actual people

- Quadratic utility function: $U = a + bW + cW^2$ where $c < 0$ and $b > -2cW$
  - $U'(W) = b+2cW$ and $U''(W) = 2c$ [Concave fn]
  - $A(W) = (-2c) / (b+2cW)$ [increasing ARA]
  - $R(W) = (-2cW) / (b+2cW)$ [increasing RRA]

as he become wealthier → he becomes more and more averse in absolute term

100 → $R_f = 50$, Risky $= 50$

⇓

200 → $R_f = 160$, Risky $= 40$

unrealistic

3.3 Portfolio Selection with Partial Information

Step 1 find utility of each investment

Step 2 → expected utility

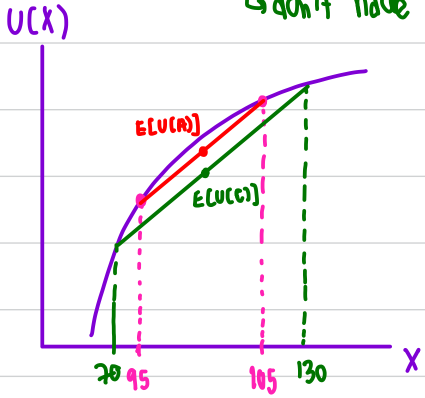
- Consider the following investments

Pr	W_A	Pr	W_B	Pr	W_C
1/2	payoff from A 95 $U(x_i) = 100 + 2(95)^{0.5} = 119.5$	1/2	85 $U(x) = 100 + 2(85)^{0.5} = 118.439$	1/2	70 $U(x) = 116.733$
1/2	105 $U(x_i) = 100 + 2(105)^{0.5} = 120.5$	1/4	100 $U(x) = 120$	1/2	130 $U(x) = 110.954$
		1/4	140 $U(x) = 123.664$		
$E[W_A] = 100$		**$E[W_B] = 102.50$**		**$E[W_C] = 100$**	

$p_i \times U(x_i)$ expected utility 119.994 120.136 113.844

- Case 1: The individual utility function is known where $U(W) = 100 + 2W^{0.5}$ utility function
  - The expected utility implies that B is preferred. $B \succ A \succ C$

instead of three
make choice among two stocks is easier for individual to choose
↳ don't have to know utility function under risk averse



2.7 The Optimal Consumption Choice

- Case 2: We only have partial information that the individual is a risk averter.
  - In this case we can only reduce the opportunity set by eliminating those inferior options
    - Feasible set = A, B and C
    - Efficient set = A and B identify efficient investment
    - Inferior investment = C not efficient
  - We cannot determine whether A is preferred to B or vise versa, unless we know the exact utility function.

Exercises

1. What is the difference between risk and uncertainty?

2. Will a risk averter enter speculative activities?

3. Can a person who purchase a lottery ticket be classified as risk averter?

4. How could we draw the total utility lines for (1) an individual who is willing to buy a lottery ticket; and (2) an individuals who is willing to purchase a car insurance policy? Can the same person enter both transactions? If your answer is yes, how do you explain this contradiction?

5. Show that if U(W) is a utility function of an individual, and V(X) is a positive linear transformation of U(W) then both U(W) and V(X) have the same ARA and RRA measures.

6. Some social commentators criticize that the stock market is a casino for rich people. Do you agree with their comment? Why/Why not?

Exercises

8. In Bodie, Kane and Markus, the utility function is written as:

$$U = E[r] - (1/2)A\sigma^2$$

where U = utility (here, assumed utility is a function of return)

E[r] = expected return on asset

A = coefficient of risk aversion ($A>0$)

σ^2 = variance of returns on asset

½ = a scaling factor

Both E[r] and σ^2 are constant terms.

What type of attitude toward risk does this function assume?

1

	Risk	Uncertainty
All possible outcome	✓	~
Probability of each outcome	✓	X

we know the distribution of outcome