Financial Econometrics
Problem Set 2: Due on Sunday November 12, 2023

1. Consider each of the following statements is true, is false, or has not sufficient information to make a conclusion.
    a. Let $Z1$ and $Z2$ be valid instruments. A linear combination of the two instruments are also a valid instrument.
    b. Measurement error in a regressor can cause endogenous problem for the OLS estimator.
    c. MLE is a consistent estimator for linear regression model.
    d. If the error term is not mean-independent of a regressor, then the OLS estimator is not consistent.
    e. OLS estimator is inconsistent when serial correlation is present.
    f. Newey-West Standard Error can be used when data contain heteroskedasticity.
    g. ARMA process is weak stationary
    h. Random Walk process is weak stationary
    i. Under the Random Effect assumption, OLS is consistent.
    j. OLS estimator is inconsistent in with the presence of ARCH.

2. In which of the following situations, can OLS be a consistent estimator? Explain.
    a. Fixed Effect
    b. Random Effect
    c. Omitted variable
    d. Simultaneous equations
    e. Measurement error

3. Let X be a discrete random variable with the following p.m.f.

$$P\{X = x\} = \frac{\lambda^x \exp(-\lambda)}{x!}$$

Then we say that X is distributed with Poisson Distribution with parameter λ. This is a well-known distribution proposed by a French Mathematician, Poisson (1837), and is used to express the probability of the number of events occurring in a given fixed interval of time, provided that each event happens independently of the others. Note that X can take values 0, 1, 2, …., and all positive integers. For example, X may be the number that you receive a phone call in a day, given that the chance that one person calls you doesn't affect the chance that the others call you.
    a. Suppose that you know that your data $X_1, \dots, X_n \sim iid\ Poisson(\lambda)$ but you don't know λ and want to estimate it by MLE. Write down the likelihood function and log-likelihood function of observing the data set.
    b. Prove that $\hat{\lambda}_{MLE} = \bar{X}_n$.

4. Suppose that we want to study the effect of policy interest rate, denoted by int, on inflation rate as denoted by inf. Suppose that we use the finite distributed lag model and get the following result.

Dependent Variable inf				
Regressors	_con	int	$l.int$	$l2.int$
Coefficient	1.633	0.479	-0.157	-0.323
Std. Error	0.65	0.22	0.004	0.023

a. What is the immediate impact propensity of the policy rate on inflation?
b. Suppose that the change in interest rate is temporary, what are the effect of policy rate change on inflation rate after one, two, and three periods?
c. What is the long-run multiplier of the policy rate on inflation?
d. Suppose that the central bank set interest rate by also looking at contemporaneous inflation rate. Do you think we can use OLS to estimate this model? If not, what should be a proper estimator to use?

5. Suppose that we have a sample of working-age Thai people with college degrees, and we want to study factors contributing to the probability that they invest in the Stock Exchange of Thailand. Let SET be a dummy variable taking value 1 if a person invests in SET; inc is the person's annual income in units of thousand Baht; $exper$ is the person's working experience in years; age is the person's age in years; and fin is a dummy variable taking 1 if the person graduated with a finance-related degree. Suppose that we use the Logit model to estimate and get the following result:

regress SET on all independent x

Dependent variable: SET					
Regressors	Constant	inc	$exper$	age	fin
$\hat{\beta}$	-2.74	0.0112	0.0079	-0.0014	1.280
$se(\hat{\beta})$	(0.12)	(0.001)	(0.003)	(0.028)	(0.56)

$x\hat{\beta} = -2.74 + 0.0112(120) + 0.0079(7) - 0.0014(30) + 1.280$
$= -0.1027$

$\Pr\{y=1|x\} = \frac{e^{-0.1027}}{1+e^{-0.1027}} = 47.43\%$

a. What is the probability predicted by the estimated model that a 30 years old person with a Finance degree and 7 years of work experience who earn 10,000 Baht a month would invest in SET?
b. For a person ages 25 years old with 2 years of work experience earning 120,000 Baht a year, what is the marginal effect of a finance degree on the probability that he invests in SET?
c. For a person ages 25 years old with a finance degree and 2 years of work experience earning 120,000 Baht a year, what is the effect of income level on the probability that he invests in SET?
d. Now suppose that the given result is from Probit instead of Logit. What is the probability predicted by the estimated model that a 30 years old person with a Finance degree and 7 years of work experience who earns 10,000 Baht a month would invest in SET?

6. Suppose a researcher has a sample of 200 monthly observations on the stochastic process $\{Y_t\}$ and wants to fit the data with an ARMA model. In order to select the model, he computes the ACF and PACF as follows:

k	1	2	3	4	5	6	7	8	9
ACF	0.83	0.71	0.60	0.45	0.44	0.35	0.29	0.20	0.11
PACF	0.83	0.16	-0.09	0.05	0.04	-0.05	0.01	0.10	-0.03

a The researcher decides not to use the MA model to fit the data. Based on the given correlogram, why do you think he makes such decision?
b Suppose that the researcher suspects that AR(2) may be better. So, he runs AR(2) and get the following result:

$$Y_t = 0.74Y_{t-1} + \hat{\rho}_2 Y_{t-2} + e_t$$

What is the numerical value of $\hat{\rho}_2$ in the above equation?

⑤ LPM : prob is linear function

$$Pr\{y=1 \mid x\} = \beta_0 + \beta_1 \text{inc} + \beta_2 \text{exper} + \beta_3 \text{age} + \beta_4 \text{fin}$$

PROBIT :

$$Pr\{y=1 \mid x\} = \Phi(\beta_0 + \beta_1 \text{inc} + \beta_2 \text{exper} + \beta_3 \text{age} + \beta_4 \text{fin})$$

LOGIT :

$$Pr\{y=1 \mid x\} = \frac{e^{\beta_0 + \beta_1 \text{inc} + \beta_2 \text{exper} + \beta_3 \text{age} + \beta_4 \text{fin}}}{1 + e^{\beta_0 + \beta_1 \text{inc} + \beta_2 \text{exper} + \beta_3 \text{age} + \beta_4 \text{fin}}}$$

c Conduct a hypothesis testing to see if he would prefer AR(1) or AR(2). State the null and alternative hypotheses, test statistics, distribution of the test statistics, critical value at 5% significance level, p-value, and the conclusion of the test whether AR(1) or AR(2) is preferred.

7. A researcher uses a dataset that comprises a sample of 545 full-time working males who have completed their schooling by 1980. This is a balanced panel covering the period from years 1980 to 1987. He does panel data analysis and gets the following results

Dependent Variable: ln(wage)

Regressors	Between	Fixed Effect	Pooled OLS	Random Effect
Constant	0.490	-	-0.034	-0.104
	(0.221)		(0.065)	(0.111)
Schooling	0.095	-	0.099	0.101
	(0.011)		(0.005)	(0.009)
Experience	-0.050	0.116	0.089	0.112
	(0.050)	(0.008)	(0.010)	(0.008)
Experience2	0.0051	-0.0043	-0.0028	-0.0041
	(0.0032)	(0.0006)	(0.0007)	(0.0006)
Union	0.274	0.081	0.180	0.106
Member	(0.047)	(0.019)	(0.017)	(0.018)
Married	0.145	0.045	0.108	0.063
	(0.041)	(0.018)	(0.016)	(0.017)
Black	-0.139	-	-0.144	-0.144
	(0.049)		(0.024)	(0.048)
Hispanic	0.005	-	0.016	0.020
	(0.043)		(0.021)	(0.043)
Public Sector	-0.056	0.035	0.004	0.030
	(0.109)	(0.039)	(0.037)	(0.036)

where *Wage* indicate earning in unit of dollars per month; *Schooling* is years of education, *Experience* is working experience in unit of years, *Union member* is a dummy variable taking value 1 if the worker is a member of a union; *Married* is a dummy variable taking value 1 if the worker is married; *Black* is a dummy variable taking value 1 if the worker is African American; *Hispanic* is a dummy variable taking value 1 if the worker is Hispanic; and *Public Sector* is a dummy variable taking value 1 if the worker works in the public sector. Standard errors are displayed in parentheses.

a. Explain why the researcher does not present the resulting coefficients of *Schooling, Black*, and *Hispanic* estimated by the Fixed Effect estimator?
b. Suppose somebody questions that married male workers have higher wage than the others. Use the result from the Fixed Effect estimator, how would you test this hypothesis? State the null hypothesis, alternative hypothesis, test-statistics, distribution of the test statistics, 5% critical value, and the p-value.
c. Construct a 95% confidence interval for the effect of getting married on wage.
d. Suppose that the researcher carries out the Hausman Test and get the test statistic equal to 7.90. State the distribution of the Hausman Test Statistics, the critical value at 5% significance level, whether you can reject the null hypothesis, and your conclusion about Random Effect model.
e. Do you think the Between estimator is consistent if you reject the null hypothesis of the Hausman test? Why?

f. Do you think the Pooled OLS estimator is consistent if you can reject the null hypothesis of the Hausman test? Why?

9. Let $Y_t = 0.9Y_{t-1} + e_t + 0.5e_{t-1} + 0.2e_{t-2}$, where $\{e_t\}$ is the White-Noise process. Compute the covariance $cov(Y_t, Y_{t+k})$ for $k = 1, 2, 3, 4$ in terms of variance of Y, and variance of e, i.e. σ_Y^2, σ_e^2 respecitively.