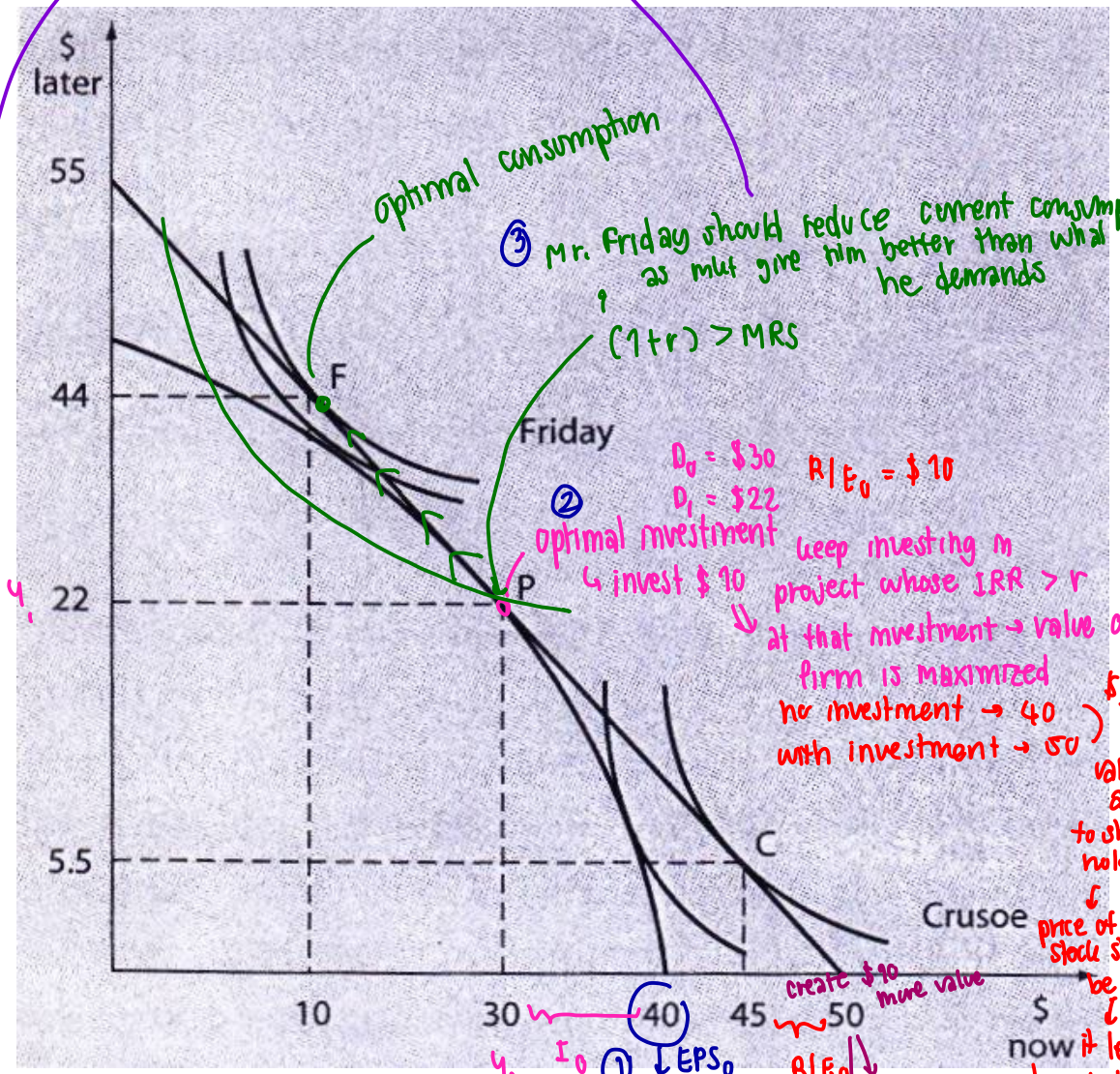
Exercise for Lecture 1

Quiz → 2 questions → 1 of them is this exercise
you have to be able to read graph

The graph shows: 1) production opportunity of a firm with two shareholders, Crusoe and Friday, 2) IC's of the two shareholders, and 3) the capital market opportunity line. Assume two periods with perfect certainty and perfect capital markets. All figures are stated on a per share basis. The management of the firm is considering how much of the $40 initial endowment (i.e., earnings per share from last year) should be paid out as dividend and kept as retained earnings.

What are the firm's optimal investment and dividend decisions? What is the value-added (i.e., NPV) the firm creates from that decision? Explain

Why is point P not the optimal consumption for Friday? Explain in terms of MRS, MRT or (1+r). How could Friday move from point P to his preferred basket of consumption?



decrease current consumption by 1 unit, Friday gets back 1+r to increase utility → Future consumption is what he get from mkt

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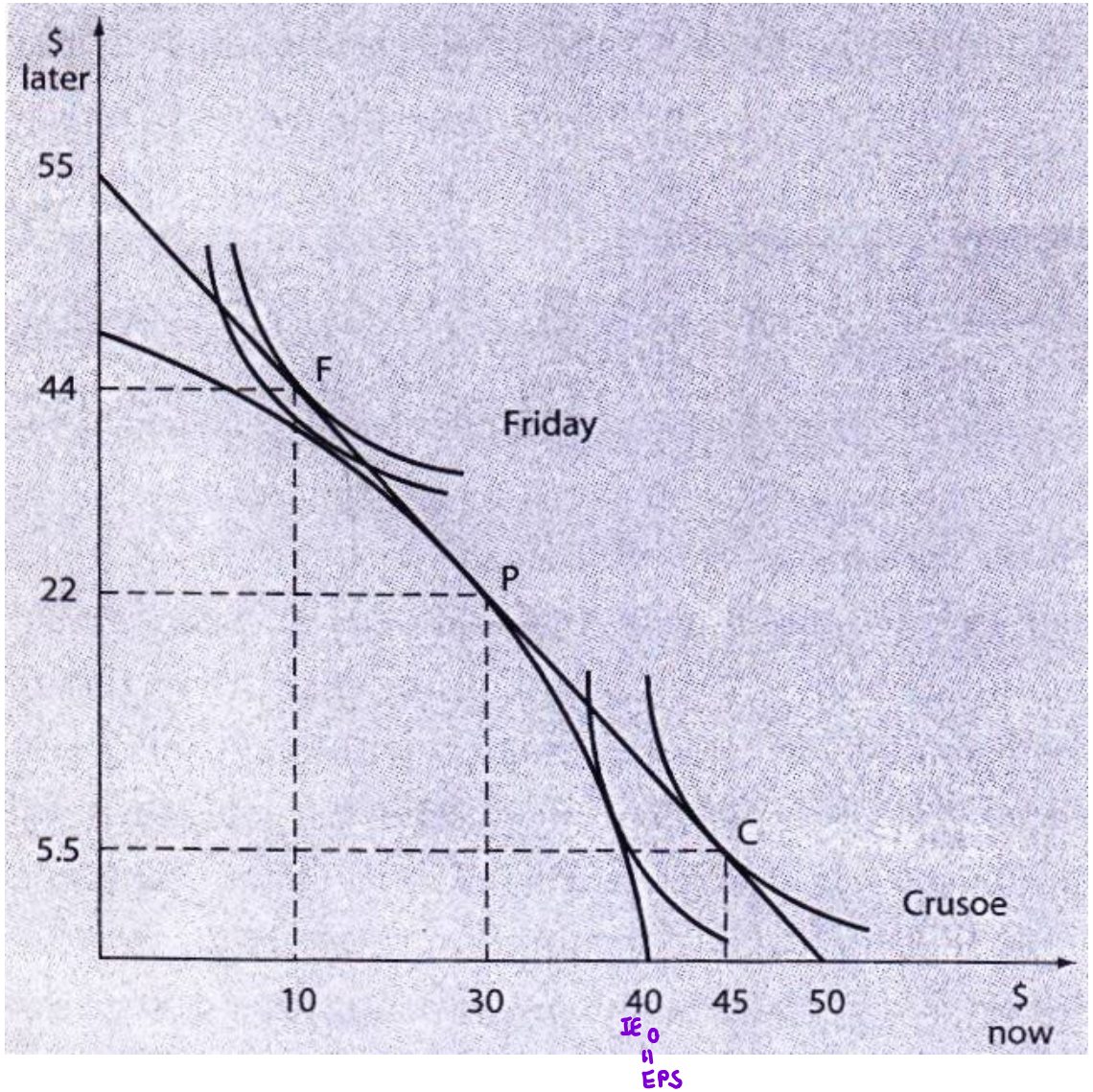
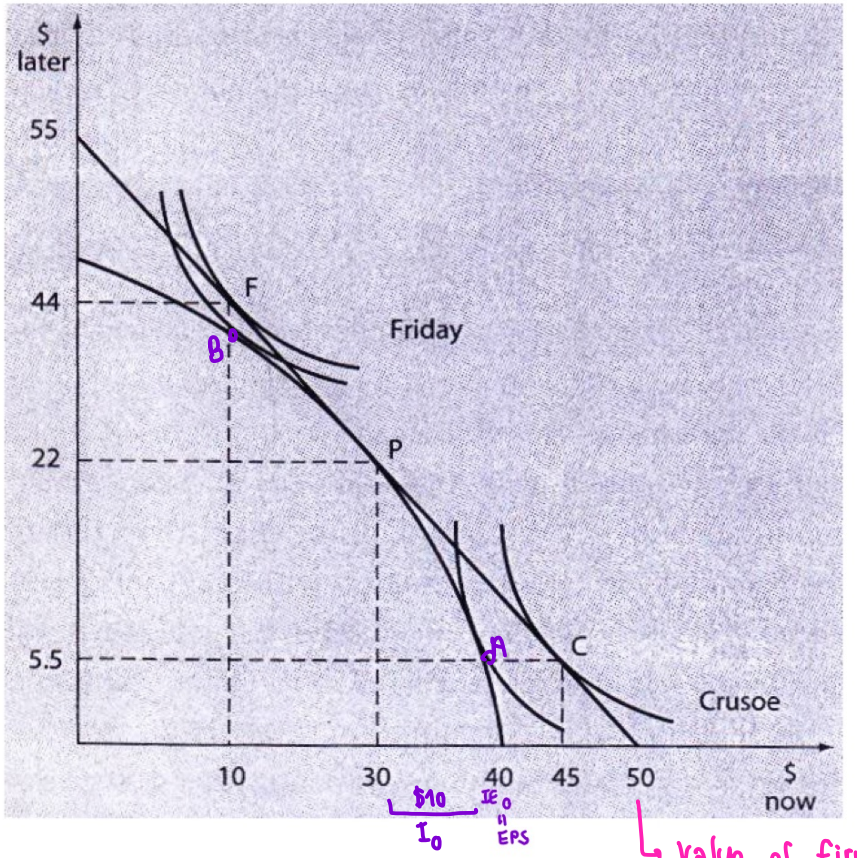




What are the firm's optimal investment (1.1) and dividend decisions? What is the value-added (1.2) (i.e., NPV) the firm creates from that decision? Explain

Firm should invest in real production until MRT = $1+r_f$

1.1.1) Firm should invest \$10 per share to reach optimal investment at point P. The optimal investment is point P because, at point P, the marginal rate of transformation (MRT or $1+IRR_P$) is equal to return from capital market ($1+r_f$).

- At point A, $MRT > 1+r_f$. Thus, firm should keep investing as cost of borrowing (r_f) still lower than MRT ($1+IRR_i$).
- At point B, $MRT < 1+r_f$. Firm should not invest beyond P as firm's return from investment in project (IRR_i) is lower than the cost of borrowing (r_f)

1.1.2) Current dividend paid to shareholders = \$30 per share, D_1 = \$22 per share

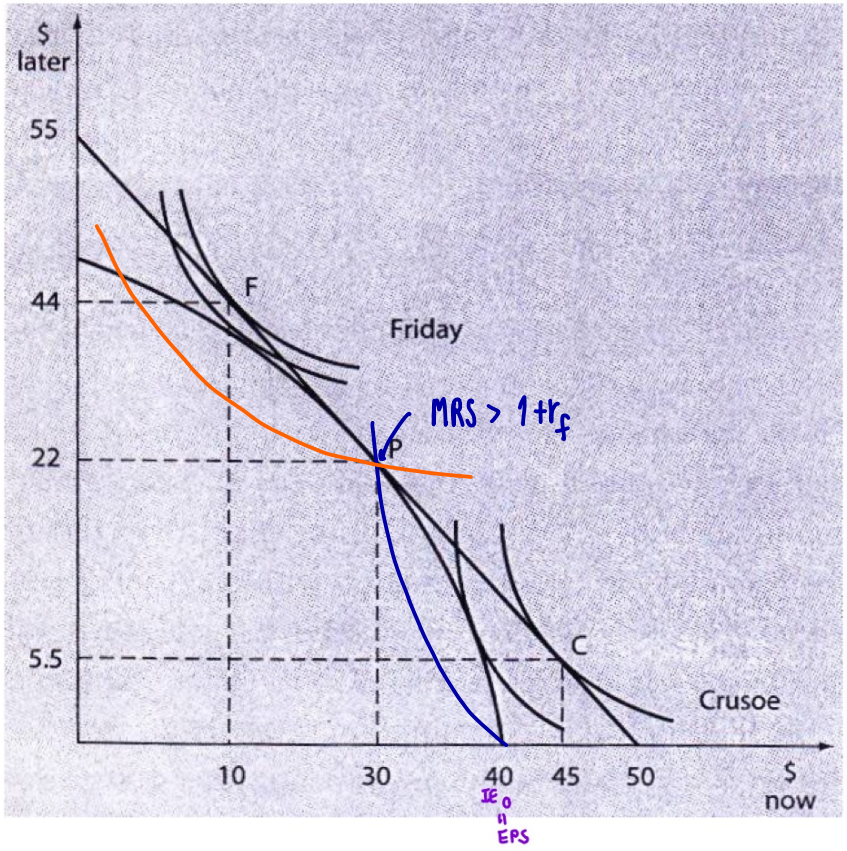
1.2) The valued added to the firm/shareholders is \$50 - \$40 = \$10 per share

Value of firm = \$50 = \$30 + \$22/(1+10%) + \$20 | $\frac{\Delta Y}{\Delta X} = 1+r_f \Rightarrow \frac{55}{50} = 1+r_f \Rightarrow r_f = 10\%$

* According to Fisher's Seperation Theorem, investment decision can be made seperately and independently from consumption decisions. So, to maximize shareholders' utility, managers should maximize the firm's value by investing in real production until last project has $IRR_P = r_f$.

* Point P is not the optimal consumption of Crusoe (net borrower).

This is because, at point P, $MRS < 1+r_f$



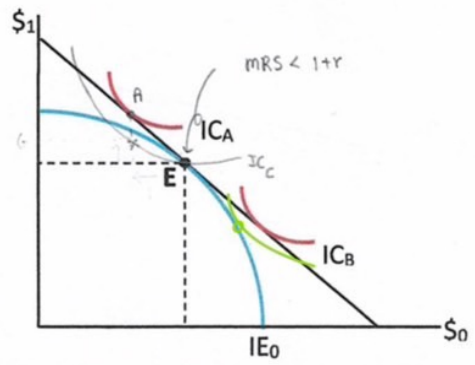
Why is point P not the optimal consumption for ~~Friday~~ Crusoe? Explain in terms of MRS, MRT or (1+r). How could ~~Friday~~ Crusoe move from point P to his preferred basket of consumption?

Fisher's seperation theorem → consumption decision can be made separately from investment decision. ✓ independently &

* Point P is not the optimal consumption for Crusoe because MRS_c is greater than $1+r_f$. This means that what Crusoe demands is greater than r_f (return) from capital market. Also, with perfect capital market assumption, the cost of borrowing is equal to the return on lending in the capital market. Thus, Crusoe will increase his current consumption by borrowing money until $MRS_c = 1+r_f$ at point C, which is the optimal consu tion for Crusoe.

** ที่มาของ firm decission ⇒ MRT

2. The figure above shows PPC of a firm, CML and the shareholders' ICs. Assume perfect certainty, perfect capital market and 2 periods. ***[2 points]***



Ans $MRS_E < 1+r$

1 3/4

ต้อง compare MRS

2.1 Why is point E not the optimal consumption for shareholder A? Explain in terms of MRS, MRT or (1+r). How could shareholder A move from point E to his preferred basket of consumption?

Point E is not the optimal consumption for shareholder A because MRS_A is less than 1+r, meaning that what we get back from capital market is higher than what do we want to maintain the same level of utility (MRS). To move from point E to shareholder A's preferred market consumption, we should reduce consumption today and put money in the capital market that allow to increase future consumption.

1

2.2 How does the existence of capital market improve the welfare (i.e., utility) of the shareholders?

(optimal consumption)

Following the concept prefer more to less, shareholdes will try to acheive IC that located on the upper right hand side to increase total utility. In addion from 2.1, it can be seen that shareholder A will move from point E to his preferred basket of consumption because capital market provides a better return than MRS. As a result, total utility increase and IC curve shift upper.

3/4

× MRT

1. Mr. Chan is an investor whose behavior is consistent with the axiom of choice. He is considering 2 stocks, namely, ADM and CMS. The possible payoffs for the 2 stocks are as follows. ***[1.5 points]***

   - ADM($600, $400; 0.6)
   - CMS($600, $300; 0.5)

1.1 Rank the 2 stocks based on Mr.Chan's preference. Show your work (i.e., explain step-by-step.) ***[1 point]***

- Assume : B ($ 600, $300, 0.6)

1) Compare ABS with B => Based on more preferred to less and independent axiom, conclusion is ADM ≻ B

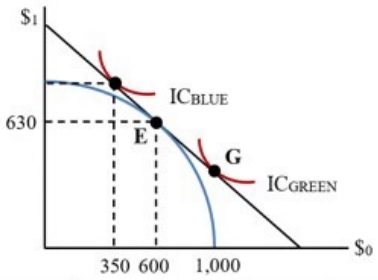
2) Compare B with CMS => Based on more prefferred to less and ranking axiom, conclusion is B ≻ CMS

3) Compare ABS with CMS => Based on consistency axiom

ADM ≻ B B ≻ CMS therefore ADM ≻ CMS

Ans ADM ≻ CMS

Q1

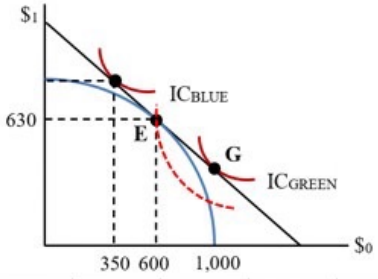


r = 20%

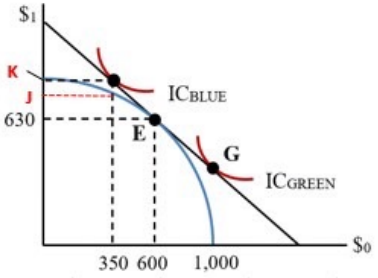
I_0 = 400

D_0 = 600

D_1 = 630



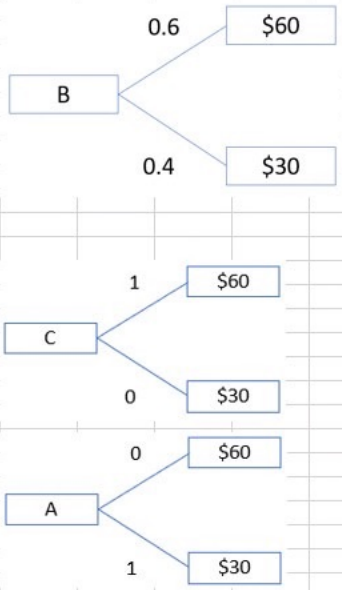
As can be seen from the graph, at basket E, MRS > 1+r. This means, the amount of C_1 that Mr.Green is willing to sacrifice to increase C_0 by the holding total utility constant is greater than the amount of C_1 that he has to sacrifice if he were to borrow the money to increase C_0. By moving downward along the capital marekt opportunity line from E to G (i.e., increase C_0 and decrease C_1), his total utility will increase.



For Mr.Blue, the excess D_0 is \$250. If he asks the firm to invest the excess D_0 in real production, the future payoff is \$(J-630). If he invests the excess D_0 in the capital market, the future payoffs is \$(K-630). He should invest the excess dividend by himself.

Note that if the firm were to invest beyond point E (moving upward along the PCC), the IRR of the following projects will be less than the market interest rate (i.e., MRT < 1+r). As the market interest rate is higher, Mr.Blue should invest the excess dividend in the capital market rather than real production.

Q2



From Certain Equivalent: If $x \succ y \succ z$, then there exists a unique α, such that $y \sim [\alpha x, (1-\alpha)z]$.

From more is preferred to less, \$60 ≻ \$30.

Hence the certain vale y (or P_B) must line within the (\$30, \$60) such that $P_B \sim$ [0.6*\$60, 0.4*\$30]. Hence the price range is \$30 < P_B < \$60.

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One may also use the ranking axioms and construct bridging securities to reach the same conclusion.

Asset C gives a certain \$60 payoff, hence the P_C = \$60.
Asset A gives a certain \$30 payoff, hence the P_A = \$30.

From more is preferred to less and ranking axiom, C ≻ B and B ≻ A. Hence, $P_A < P_B < P_C$ or \$30 < P_B < \$60.

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One can also argue that;
1) If P_B = 60, the buyer will at best breakeven and has a chance to make a loss. If P_B > 60, the buyer will make a sure loss. Hence, for rational buyer, P_B < \$60.
2) If P_B = 30, the (short) seller will at best breakeven and has a chance to make a loss. If P_B < 30, the (short) seller will make a sure loss. Hence, for rational seller, P_B > \$30.

Hence, \$30 < P_B < \$60.

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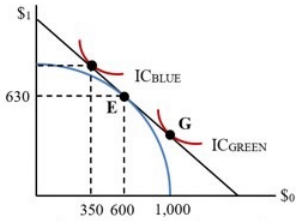
Note: Students must explain why no one will buy at a price ≥ \$60, and no one will sell at a price ≤ \$30.

Q1



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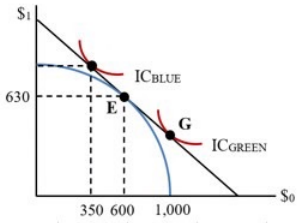


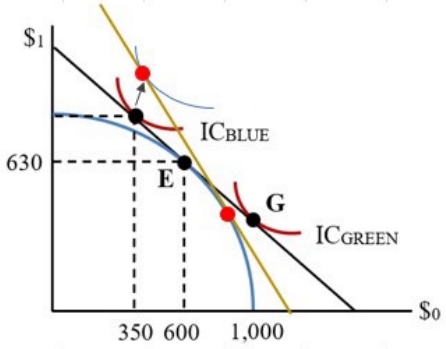
Today, Mr. Blue will receive the current dividned of $600 and he will invest 600-350 = $250 in the capital market to earn 20% return. In the future he will receive the future dividend of $630 and the payoff from the investment of 250(1+0.2) = $300. Therefore, his optimal consumption basket is (C0, C1) = (350, 930).
He achieves this by lending part of the current dividend in the capital market.

Without investment in real production, the firm's current value is $1,000/share.

With the optimal investment decision, shareholder receive the dividends of $600 today and $630 in the future. The present value of the firm is 600 + 630/(1+0.2) = $1,125/share.

Therefore, the value added is 1,125-1,000 = $125/share

Q2



As can be seen from the graph, an increase in interest rate causes the capital market line to be steeper. The new optimal investment decision now move along the old PPC toward the right hand side, implying lower investment in real production. As for Mr. Blue, his IC will shift toward the upper-right-hand direction, implying higher level of utility than before.

The logic behind lower investment is that with higher interest rate the firm will have less projects with IRR ≥ r. The logic behind higher utility for Mr. Blue is that, he is a net lender, a higher interest rate gives him more $ return in the future.