Session 11
Capital Structure Theories

Objective

By the end of this session, students are expected to able to exhibit a critical understanding of the Capital Structure Irrelevance Propositions.

Introduction

Last session, we saw that there is a variety of financing choices (i.e., financial tactics) available and feasible to various types of firms. This pattern points towards the possibility that, for a given firm, different financing choices affects shareholders' wealth differently. In other words, the financing choice, e.g., debt-equity mix, that minimizes the cost of capital may well vary from firm to firm.

In this session, we will explore the theoretical argument that a financing choice does not affect shareholders' wealth.

Modigliani & Miller Capital Structure Irrelevance Propositions

In their seminal work, Modigliani and Miller (1958, AER, p. 261-297; 1963, AER, p. 433-443) show that the market value of any firm is *independent* of its capital structure. In other words, MM's (1958) theory posits that two identical firms will still have the same 'total' market value even if they differ in the leverage ratio or debt-equity mix. That is, leverage does not matter to valuation.

This argument relies on a number of strict assumptions (i.e., conditions) either explicitly or implicitly. In their argument of capital structure irrelevance, MM (1958) assumed the following (see also Copeland and Weston, 1988):

perfect capital mkt can have risk ; equity is risky ⇒ LHS, asset factory also risky

anyone, regardless risk, can borrow & lend at risk-free rate

(i) *Frictionless* capital markets; bond mkt, debt mkt
(ii) Individuals can borrow and lend at the *risk-free rate*; risk class
(iii) There are *<u>no</u> costs to bankruptcy*;
(iv) Firms issue only *risk-free debt* and *risky equity*; discount rate (k)
(v) All firms belong to the *same risk class*; at same systematic risk
(vi) The only form of tax is corporate tax (i.e., *<u>no</u> personal taxes*); investors do not pay income tax
(vii) All cash flow streams are perpetuities (i.e., *<u>no</u> growth*); constant
(viii) There is no adverse selection (thus, no signaling opportunities); and
(ix) There is no moral hazard.

Obviously, some of the assumptions above are simply ceteris paribus assumptions commonly required to make a model work.

Based these 9 assumptions, MM (1958) show that investors can create ***homemade leverage*** such that:

with debt, no debt

$$V_L = V_U \text{, if } \tau_c = 0 \text{.} \tag{1}$$

"no corporate tax" assumption

will never hold if there's no perfect capital mkt

Fisher's seperation → even there're assumptions: perfect mkt, certainty
is used by gov → want to see lots of investment to cover consumption in country

V_L is the value of a levered firm (i.e., a firm with debt in its capital structure), and V_U is the value of an unlevered firm (i.e., an all-equity firm). τ_c is the corporate tax rate.

Equation (1) is famously known as: ***Modigliani-Miller Proposition I***, which states that "the market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate ρ appropriate to its risk class". To provide proof for equation (1), MM (1958) resort to one of the very first 'arbitrage pricing' arguments in the finance discipline. argument discount rate = capitalization rate

$V_L = V_U$ → MM relies on arbitrage pricing → asset with same expected payoff must have same price otherwise we'll have in equilibrium condition → arbitrage happens

Let's start from Table 1, which shows what the value of *unlevered* and *levered* firms with the same net operating income (NOI) ought to be *in equilibrium*. To make our analysis nice and simple but without loss of generality, let us assume no taxes for now.

Suppose, LeverCo has 50% debt in its capital structure, and its equity is *observed* to be worth £500,000 (or, £50 per share as in the case of UnleverCo). Its expected cost of equity *must* be 14% $(70{,}000 \div 500{,}000)$.

same equilibrium for valuation

Table 1: *Equilibrium* expected firm values

	UnleverCo (no debt)	LeverCo (higher share price)
NOI (EBIT CF; net operating income (same income) same risk class)	£100,000	£100,000 (same risk)
less: $r_d D$	0 (int exp)	30,000 (mkt value of debt)
NI (should not change)	100,000 (for the asset (factory))	70,000 (for study)
ρ (discount rate is capitalization rate)	10%	10% (same factory)
NOI ÷ ρ (total firm value)	1,000,000 (mkt value equity)	1,000,000 (same amount of cash, same equilibrium, same return)
Market value of equity (given or as observed)	1,000,000	500,000
r_e (cost of equity)	10%	14%
r_f (risk-free rate)	6 ~~0~~%	6%
Market value of debt (given or as observed)	0 (bcs it's pure equity)	500,000

*debt is lower risky instrument than equity → use more debt lower equity → lower average cost of financing for the firm <WACC> ✗ by MM, real world

↑ average cost of financing will be the same in equilibrium <no mispricing>

Now, let's say (as most scholars did prior to 1958) investors are willing to pay a premium price for shares in LeverCo. The *mistaken reasoning* here is that since debt is a less-risky instrument (fair enough), replacing equity with debt should lower the average cost of financing the firm's operations.

500000 + 6(10000)

Suppose investors are willing to accept LeverCo at £56 per share (instead of £50), which yields an expected cost of equity of 12.5% (70,000 ÷ 560,000). This implies LeverCo's total firm value of £1.06 mil while UnleverCo's total value is still £1 mil. This disequilibrium condition is shown in Table 2.

mkt value ↑ 60,000

why this go up not value of debt? (risk-free asset) → value of risk-free asset not change bc E(CF) is constant *assume no bankruptcy cost

Why is it a *dis*equilibrium condition?

expect future is good → pay higher future

price : $\frac{E[CF_t]}{(1+r)^t}$

→ not diff from table 1

Table 2: *Disequilibrium* expected firm values

fundamental for valuation

	UnleverCo	generate same CF	LeverCo
NOI	100,000	↔	100,000
less: r_dD	0		30,000
NI	100,000		70,000
ρ	10%		9.434%
NOI ÷ ρ (total firm value)	1,000,000	↔ disequilibrium → can do arbitrage	1,060,000
Market value of equity (given or as observed)	1,000,000		560,000
r_e (cost of equity)	10%		12.5%
r_f (risk-free rate)	6 ~~0~~%		6%
Market value of debt (given or as observed)	0		500,000

mispricing will not stay for long

mistake in believe

At least in the MM world, individual investors can exploit these valuations and make an *arbitrage* profit.

> Before going through the remaining tables, how would you go about exploiting the arbitrage opportunities arising from the overvaluation of LeverCo's stock?

Note that r_d and r_f in Tables 1 and 2 are the same (Why?)

Let's start from what we can see, i.e., LeverCo is *overvalued* relative to its fundamentals (being NOI). As a result, individual investors can make an arbitrage profit by taking and closing positions as shown in Tables 3 and 4 below:

Table 3: Position taking at start of year

Start of Year	CF	CF
Short 1% of LeverCo's stock	5,600	
Borrow @ 6% pa.	5,000	
Proceeds		10,600
Long 1% of UnleverCo's stock		(10,000)
Remaining proceeds		600
Long £600 worth of UnleverCo's stock		(600)
Cash holding at start of year		£0

As an individual investor is on average highly unlikely to buy an entire firm, let's work with some realistic portion, say, 1% of the firms' market cap.

At the beginning of the year, an investor makes the following transactions:

(I) Short (or, sell in advance) LeverCo's stock and receive the proceeds of £5,600 (i.e., 1% of LeverCo's market cap).

(II) Borrow £5,000 (or, 1% of LeverCo's debt) and promise to pay £300 in interest per year (i.e., @ 6% pa.).

So far, the investor has a cash inflow of £10,600 or 1% of LeverCo's total value.

(III) Use £10,000 of the cash inflow from (I) and (II) to long 1% stakes in the all-equity UnleverCo.

By taking actions in (I) and (II), the investor has borrowed on a personal account (i.e., created homemade leverage). By taking action in (III), the investor has constructed a new portfolio consisting of £10,000 worth of UnleverCo's stock, which belongs to the same risk class. So, the investor now owns a levered operating cash flow stream of the same systematic risk class that also represents 1% of LeverCo's stream without having to hold stock in LeverCo.

(IV) Since there is still £600 left from steps (I) to (III), the investor can buy more of UnleverCo's stock. Why not LeverCo's stock given that it offers higher expected return?

With this additional investment of £600 in UnleverCo's stock, the investor holds a portfolio worth £10,600 in total. With the borrowing in (II), the investor's leverage ratio is £5,000 ÷ £10,600, which is exactly the same as the leverage of, and has the same financial risk as, LeverCo in Table 2. Note again that the investor has constructed this portfolio using UnleverCo's stock, instead of LeverCo's stock.

(V) Hold the positions taken in steps (I) through (IV), say, for a year.

Table 4: Position Closing at End of Year

After 1 Year		CF	CF
Unwind long positions on			
(1)	UnleverCo	10,000	
(2)	Additional UnleverCo	600	10,600
Return on long positions on			
(3)	UnleverCo (£10,000 × 10%)	1,000	
(4)	Additional UnleverCo (£600 × 10%)	60	1,060
Pay off borrowing			
(5)	Principal	(5,000)	
(6)	Interest @ 6% pa.	(300)	(5,300)
Close short position on LeverCo			
(7)	Stock	(5,600)	
(8)	Expected return (£5,600 × 12.5%)	(700)	(6,300)
[Expected] net position After 1 year			£60

At year end, the investor closes her *short* position on LeverCo's stock using the proceeds from unwinding her *long* positions:

(VI) Unwind her initial long position in UnleverCo's stock and get back £10,000, and the additional long position in UnleverCo's stock and get back £600.

Why isn't there any <u>capital</u> gain on UnleverCo's and LeverCo's stock?

The investor receives return of £1,000 (£10,000 × 10%) from holding the initial long position in UnLeverCo's stock and £60 (£600 × 10%) from the additional investment.

The total proceeds from unwinding the long positions are <u>£11,660</u> (10,000 + 600 + 1,000 + 60).

(VII) Pay off the £5,000 debt plus £300 interest, totaling to a cash outflow of £5,300.

(VIII) Using £5,600 of the proceeds from unwinding her long positions as in (VI) to buy and deliver the LeverCo's stock to close the short position created in (I). The investor also has to compensate the buying party to the short-sell contract for the opportunity cost of holding short-sell securities. This compensation is the £700 expected return on LeverCo's stock (item 8 in Table 4). In this case, it would be the remittance of the known cash dividends. From closing her short position, the investor therefore pays out £6,300 in total.

Note that the net return on the new and levered portfolio of 1% of UnleverCo's stock is £700 [(3) – (6), or £1000 – £300], which is <u>identical</u> to the £700 expected return on 1% of LeverCo's stock [£5,600 × 12.5%]. With this, the investor also gets to earn an additional return of £60 (on UnleverCo's stock – item 4 in Table 4) without having to bear any extra risk at all.

From steps (I) through (VIII), an investor can expect to have a *net* position of £60. This expected profit is *risk-free* since an investor has perfectly hedged her position. It is the expectation of this £60 arbitrage profit that drives investors to *short* LeverCo's stock and to *long* UnleverCo's stock at time 0 (i.e., the start of the year) until equilibrium is restored (i.e., until LeverCo's share price decreases such that the arbitrage profit disappears, or from Table 2 back to Table 1).

What would happen if r_e for LeverCo was set *too high* (e.g., 18%), or if investors put a discount on leverage?

Now, we have seen the arbitrage proof for the irrelevance theorem. We can also see that the theorem is unlikely to hold in reality.

So, does it have any importance for practitioners at all? What could be the main implications of *Proposition I*? What can we learn from it?

First, if a firm decides to do the thing (e.g., issue debt and repurchase equity) investors can already do on their own (e.g., create homemade leverage), it will <u>not</u> increase value.

In the case of a capital structure decision, the borrowing decision hence does not affect the expected return on the firm's assets (ρ).

shdr can diversify risk on their own

Secondly, understanding the conditions under which leverage is irrelevant helps managers disentangle what is and what is not important about real-world financing choices. This implies that managers need to know how to adjust their firm's capital structure in response to the market conditions (i.e., frictions).

› several unlisted firms issue ST bond.

exploit (to adapt to multi condition)

From the first implication, it can be seen that an investor who holds *all* of the debt and equity in a levered firm would be entitled to *all* of the firm's operating income. Consequently, the expected return on this investor's portfolio would be equal to ρ, which is and should be equal to a *weighted* average of the expected returns on individual constituent holdings. Thus, ρ on *all* of the firm's securities (i.e., debt and equity) can be expressed as:

1% equity 2% debt
700 + 300
?

expected return

$$\rho = (\overset{0.5}{w_d} \cdot \overset{6\%}{r_d}) + (w_e \cdot r_e) = \left(\frac{\mathrm{D}}{\mathrm{D+E}} \cdot r_d\right) + \left(\frac{\mathrm{E}}{\mathrm{D+E}} \cdot r_e\right). \tag{2}$$

Does equation (2) look familiar? Have you heard of the weighted average cost of capital or WACC?

required return on equity
or expected return on equity

With some rearrangement, equation (2) can be written as:

equilibrium return on equity

observed
all at the same time

$$r_e = \rho + \frac{\mathrm{D}}{\mathrm{E}}(\rho - r_d). \tag{3}$$

equilibrium condition

↱ D is not risk-free anymore

↳ if D/E ↗ →

E(r_e) increases linearly with D/E

Given the assumptions, equation (3) says that *the expected return on equity in a levered firm increases in proportion to, or linearly with, the D/E ratio.* Note that the D/E ratio is expressed in *market value*, and as a result, r_e should be referred to as E(r_e). This prediction is the ***Modigliani-Miller Proposition II.*** Of course, if the firm has no debt, then $\rho = \mathrm{E}(r_e)$, which is the case for UnleverCo in Table 1.

If we relax part of assumption (iv) and allow firms to issue risky debt, lenders will require a higher interest rate as firms borrow more (i.e., as the firm borrows more, default risk increases). When this occurs, *Proposition II* predicts a *slower* increase in E(r_e). Hence, the more debt in the capital structure, the slower the increase in E(r_e) with further borrowing.

∵ +Δ[D/E] causes +Δ[E(r_d)]

* we will never meet buy-side analyst but we see sell-side analyst

What could be an economic intuition for a slower increase in E(r_e)? Debtholders absorbing part of the increase in default risk due to greater borrowing?

from assumption, equity is risky, risky

P/L
Constant EBIT
- int exp
profit

↱ irrelevance with firm value

Taken together, *Proposition I* says that leverage has no effect on shareholders' wealth. *Proposition II* then says that the *expected* return on equity increases with D/E, i.e., leverage is *relevant* to the return shareholders require for holding equity in the firm.

* debt is assumed risk-free

int exp become larger as firm borrows more

So, how can shareholders be indifferent between holding levered equity and unlevered equity as predicted by *Proposition I*? What could be the intuition here? Could it be that more debt leads to greater default risk, which in turn makes

depend on face value of debt ↑
int-expense ↑
⇓
prob of EBIT that can cover int expense ↓
(EBIT fall short of int expense ↑)

shdr bear greater risk ← default ↑
require higher return

residual claims riskier? Though risk-free, debt still imposes *fixed obligations* on the firm's net operating income.

$+\Delta[E(r_e)] = f[+\Delta(\text{risk})]$

Any increase in expected return on equity simply reflects an increase in risk (i.e., default risk) due to fixed obligations, and a resulting increase in the *required* rate of return on equity such that ρ remains unchanged. As the less risky instrument (debt) replaces some of the more risky instrument (equity), the cost of the more risky instrument rises reflecting the increase in default risk brought about by the increase in the weight of the less risky instrument. In other words, it is a balancing act. From Table 1, we saw that, in equilibrium, E(r_e) for the *levered* firm must be 14% if E(r_d) is 6%. Now, let's plug these figures into equation (2): systematic risk ρ → make equilibrium required rate fluctuate?

- is risk-adjusted discount rate for an asset

$$\rho = 6\%(50\%) + 14\%(50\%) = 10\% .$$

equity becomes more risky as debt ↑

This confirms that UnleverCo's ρ or WACC remains at 10% despite a change in its capital structure (i.e., from zero debt to 50% debt financing). In equilibrium, this balancing will take place, regardless of whether debt is risk-free or risky.

ρ doesn't affected by change in a firm's capital structure

Another useful way to think about the irrelevance of leverage to ρ or total firm value is to consider what ρ is supposed to reflect, or what drives it. The practice of using the WACC is a mean to estimating a proxy for ρ, which is unobservable in reality. The WACC is not ρ (capitalization rate). By definition, ρ is a function of systematic risk, i.e., underlying business risk, which is exogenous to firms. As a result, ρ cannot be affected by D/E. If D/E is irrelevant to ρ, it also follows that *debt maturity is irrelevant* – at the least in the MM world.

* How much risk of not getting their money back would a rational lender willing bear in practice?

Implicit in the proof so far is the assumption of no taxes (either corporate or personal). As shown in assumption (vi) above, however, the MM irrelevance argument allows ***corporate tax***.

↳ if great prob to default → need collateral

int expense is tax deductable → tax benefit / saving from debt financing

What happens to the value of an unlevered firm and a levered firm when there are corporate taxes?

When D/E = 0, $\tau_c > 0$ leads to a wealth transfer from shareholders to the government. Or, simply:

$$V_U = \frac{\mathrm{E}(NOI)(1-\tau_c)}{\rho} .$$

For a levered firm (i.e., when D/E > 0), debt reduces the government's tax claim on the firm's earnings available to shareholders. In other words, MM argue that:

e.g. $\tau_c = 20\%$ we can save 20% × B

value of bond

$$V_L = V_U + \tau_c B , \tag{4}$$

PV of expected interest expenses

↳ tax deductable of value of debt

B is the market value of bond.

To understand equation (4), first, we can write:

$$V_U = \frac{\mathrm{E}(FCF)}{\rho} = \frac{\mathrm{E}(NOI)(1-\tau_c)}{\rho}, \tag{5}$$

FCF = *after*-tax perpetual free cash flow,
NOI = net Operating Income.

$$\mathrm{E}(NOI)(1-\tau_c) = (\mathrm{Rev} - \mathrm{CashEx} - \mathrm{depn})(1-\tau_c) + \cancel{\mathrm{depn}} - \cancel{I}.$$

'Rev' denotes Revenue; 'CashEx' is cash expenses; 'depn' is depreciation; and 'I' denotes replacement investment.

Why do "depn" and "I" cancel each other out?

Now, let *the firm issue debt*. The after-tax cash flows must then be split up between shareholders and debt holders. Shareholders receive: $\mathrm{E}(NI) + \mathrm{depn} - I$, which, the net cash flow after interest, taxes and replacement investment. Debt holders receive: r_dD, which is, interest on debt. For a levered firm, its cash flows are therefore:

$$\mathrm{E}(NI) + \cancel{\mathrm{depn}} - \cancel{I} + r_dD = (\mathrm{Rev} - \mathrm{CashEx} - \mathrm{depn} - r_dD)(1 - \tau_c) + \cancel{\mathrm{depn}} - \cancel{I} + r_dD$$

By rearranging, we have:

$$\begin{aligned} \mathrm{E}(NI) + r_dD &= (\mathrm{Rev} - \mathrm{CashEx} - \mathrm{depn})(1 - \tau_c) - \cancel{r_dD} + r_dD\tau_c + \cancel{r_dD} \\ &= \mathrm{E}(NOI)(1 - \tau_c) + r_dD\tau_c \end{aligned} \tag{6}$$

Note that the first right-hand-side term in equation (6) is exactly the same as the numerator of equation (5), or the after-tax *FCF* stream for the *unlevered* firm in the *same risk class* [assumption (v)]. Since the stream is also a perpetual stream [assumption (vii)], ρ from equation (5) is the appropriate discount rate for $\mathrm{E}(NOI)(1 - \tau_c)$ in equation (6). The second right-hand-side term in equation (6), $r_dD\tau_c$, by assumption (iv), is *risk-free*. Thus, $r_dD\tau_c$ is discounted at the *before*-tax risk-free rate of return, r_f.

The value of a levered firm is therefore:

$$V_L = \frac{\mathrm{E}(NOI)(1-\tau_c)}{\rho} + \frac{r_dD\tau_c}{r_f}. \tag{7}$$

Why is r_f a before-tax rate? *Is* ρ a before-tax rate as well?

Because r_dD is by definition a *perpetual* stream, the value of debt is $B = r_dD/r_f$. Given equation (5), we can now rewrite equation (7) as:

$$V_L = V_U + \tau_cB, \tag{8}$$

which is precisely equation (4).

The value of a *levered* firm is therefore equal to the value of an *unlevered* firm (V_U) plus the present value of the *tax shields* provided by debt ($\tau_c B$). Note that when $\tau_c = 0$, equation (4) becomes equation (1). Equations (5) through (8) therefore effectively provide proof for equation (1).

The second right-hand-side term in Equation (6), i.e., $r_d D \tau_c$, can be rearranged as $[(r_d \cdot \tau_c) \cdot D]$. Since the term $(r_d \cdot \tau_c)$ is the tax shield on the interest expense, the after-tax r_d is thus $r_d(1-\tau_c)$. When $\tau_c > 0$, equation (2) or WACC thus becomes:

Equity inv become more risky as firm borrow more

$$\rho = \left(\frac{\mathrm{D}}{\mathrm{D+E}} \cdot r_d \overbrace{(1-\tau_c)}^{\text{tax adjustment}}\right) + \left(\frac{\mathrm{E}}{\mathrm{D+E}} \cdot r_e\right). \tag{9}$$

Of course, in equilibrium r_d and r_e must be defined as E(r_d) and E(r_e), respectively. Doing so is very important especially for practitioners. In practice, the cost of capital must be estimated at the margin, and the term "at the margin" refers to the rate of return real-world investors "expect" on their marginal investment in the firm, e.g., when the firm seeks to raise funds.

Recommended Readings

Modigliani, F., and Miller M.H., 1958. The cost of capital, corporation finance and the theory of investment. *American Economic Review* 48, 261-297.

Modigliani, F., and Miller M.H., 1963. Corporate taxes and the cost of capital: A correction. *American Economic Review* 53, 433-443.