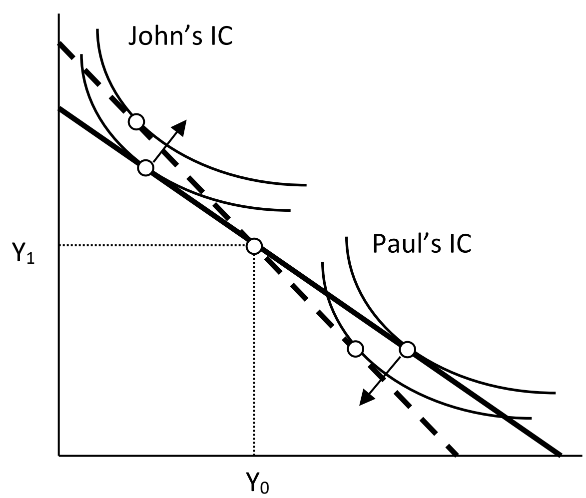
2604639 Finance Theories

Exercises and Solutions

Lecture 1

1. Two individuals are similar in every aspect except their preference toward current and future consumptions. Show graphically that higher interest rate will benefit the individual who prefers future to current consumptions and penalize the individual who prefers current to future consumptions.

Answer: Assume both individuals, John and Paul, have the same level of current and future incomes (Y_0, Y_1). But, John has more preference toward future consumption (flat IC), while Paul has more preference toward current consumption (steep IC). The bold budget line reflects the initial interest rate. Once the interest rate increases, the budget line become steeper (dashed line). As can be seen, comparing before and after the interest rate increases, John has become better off, while it is worse off for Paul.



2. Consider an investor with initial endowment IE_0 = \$16,000 and utility function $U = C_0 \cdot C_1$ facing an investment opportunity set (production possibility curve; PPC) described by the function;

$$Y_1 = 240 \times (16{,}000 - Y_0)^{1/2}$$

The interest rate for lending and borrowing is r = 20%. Find the optimal investment and consumption decision and show the result graphically.

Answer: Based on Fisher's Separation, the decision is divided into 2 independent steps, namely investment and consumption decisions. The first step is to find the optimal investment decision. The objective is to maximize the present value of total wealth. If INV is used to represent the amount invested, then $Y_0 = (16{,}000 - INV)$ will be left as current income and $240 \times INV^{1/2}$ will be the future income. The present value of total wealth after the investment decision is given by:

$$W = (16{,}000 - INV) + (240 \times INV^{1/2})/(1+0.2)$$

We want to choose INV, such that the value W is maximized. The first order condition is dW/dINV = 0.

$$dW/dINV = -1 + (240/1.2)(1/2)(1/INV^{1/2}) \quad (= 0)$$
$$= -1 + (100/INV^{1/2}) \quad (= 0)$$

By setting this first order derivative to zero, the solution is INV = \$10,000. Hence, after the investment decision the current income is \$6,000 and the future income is \$24,000. The present value of total wealth is 6,000 + 24,000/(1+0.2)) = \$26,000 (Is this a maximum or minimum?)

The second step is to find the optimal consumption decision. The objective is to maximize utility subject to a set of constraints (budget is used up).

Maximize: $U = C_0 \cdot C_1$

Subject to: $C_0 + C_1/(1+0.2) - 26{,}000 = 0$ **<u>or</u>** $T(C_0, C_1) = 0$

The lagrange equation is:

$$L = C_0 \cdot C_1 - \lambda[C_0 + C_1/(1+0.2) - 26{,}000]$$

The first order conditions are:

$$\partial L/\partial C_0 = C_1 - \lambda(1) = 0 \quad [1]$$
$$\partial L/\partial C_1 = C_0 - \lambda(1/(1+0.2)) = 0 \quad [2]$$
$$\partial L/\partial \lambda = -[C_0 + C_1/(1+0.2) - 26{,}000] = 0 \quad [3]$$

From [1] and [2], we get

$$C_1 = 1.2\, C_0 \quad [4]$$

Substitute [4] to [3]

$$[C_0 + C_0 - 26{,}000] = 0$$
$$C_0 = 26{,}000 / 2$$
$$= 13{,}000$$

Hence (from [4])

$$C_1 = 1.2(13{,}000)$$
$$= 15{,}600$$

There, starting from the initial endowment of \$16,000, the subject should put \$10,000 into current production. This will leave the subject with \$6,000 and \$24,000 in cash at current and future times, respectively. To support the current consumption of \$13,000, the subject has to borrow 13,000-6,000 = \$7,000 today. In the future, the subject will have to pay back the debt 7,000(1.20) = \$8,400. This will leave the subject with 24,000-84000 = \$15,600 to consume in the future.

3. Discuss the implication of Fisher's Separation Theory to corporate financial managers.

The manager can make investment decision even without the knowledge of shareholder's consumption preference. What manager should focus is to maximize the firm's value. Furthermore, if investment decisions are held constant, financing decisions are irrelevant to the shareholders' wealth (note: this relies on the assumption that the capital market is perfect). Therefore, the managers should focus on the decision on the left-hand-side of the balance sheet.

4. Consider an investor with initial endowment IE_0 = \$32,000 and utility function $U = 2C_0^2 + 10C_0C_1$ facing an investment opportunity set (production possibility curve; PPC) described by the function;

$$\mathbf{Y_1 = 360 \times (32{,}000 - Y_0)^{1/2}}$$

The interest rate for lending and borrowing is r = 20%. Find the optimal investment and consumption decision and show the result graphically.

Answer: Based on Fisher's Separation, the decision is divided into 2 independent steps, namely investment and consumption decisions. The first step is to find the optimal investment decision. The objective is to maximize the present value of total wealth. If INV is used to represent the amount invested, then $Y_0 = (32{,}000 - INV)$ will be left as current income and $360 \times INV^{1/2}$ will be the future income. The present value of total wealth after the investment decision is given by:

$$W = (32{,}000 - INV) + (360 \times INV^{1/2})/(1+0.2).$$

We want to choose INV, such that the value W is maximized. The first order condition is $dW/dINV = 0$.

$$\begin{aligned} dW/dINV &= -1 + (360/1.2)(1/2)(1/INV^{1/2}) && (= 0) \\ &= -1 + (150/INV^{1/2}) && (= 0) \end{aligned}$$

By setting this first order derivative to zero, the solution is INV = \$22,500. Hence, after the investment decision the current income is \$9,500 and the future income is \$54,000. The present value of total wealth is 9,500 + 54,000/(1+0.2)) = \$54,500 (Is this a maximum or minimum?)

The second step is to find the optimal consumption decision. The objective is to maximize utility subject to a set of constraints (budget is used up).

$$\begin{aligned} &\text{Maximize:} && U = 2C_0^2 + 10C_0C_1 \\ &\text{Subject to:} && C_0 + C_1/(1+0.2) - 54{,}500 = 0 \quad \underline{\textbf{or}}\ T(C_0, C_1) = 0 \end{aligned}$$

The lagrange equation is:

$$L = 2C_0^2 + 10C_0C_1 - \lambda[C_0 + C_1/(1+0.2) - 54{,}500]$$

The first order conditions are:

$$\begin{aligned} \partial L/\partial C_0 &= 4C_0 + 10C_1 - \lambda(1) &= 0 \qquad [1] \\ \partial L/\partial C_1 &= 10C_0 - \lambda(1/(1+0.2)) &= 0 \qquad [2] \\ \partial L/\partial \lambda &= -[C_0 + C_1/(1+0.2) - 54{,}500] &= 0 \qquad [3] \end{aligned}$$

From [1] and [2], we get

$$C_1 = 0.8\, C_0 \qquad [4]$$

Substitute [4] to [3]

$$\begin{aligned} [C_0 + (0.8/1.2)C_0 - 54{,}500] &= 0 \\ C_0 &= (1.2/2)\, 54{,}500 \\ &= 32{,}700 \end{aligned}$$

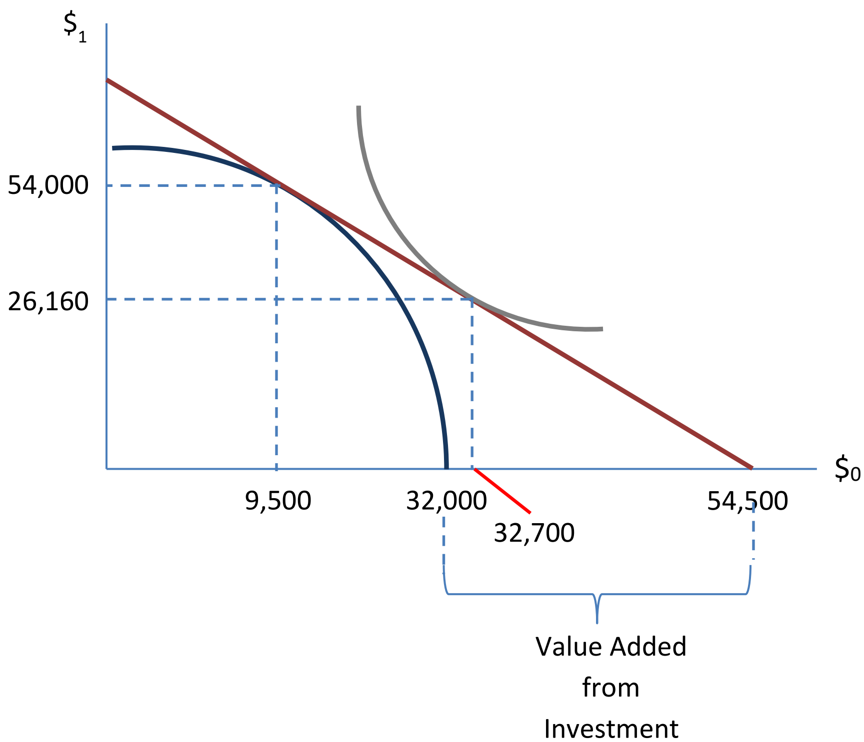
Hence (from [4])

$$\begin{aligned} C_1 &= 0.8(32{,}700) \\ &= 26{,}160 \end{aligned}$$

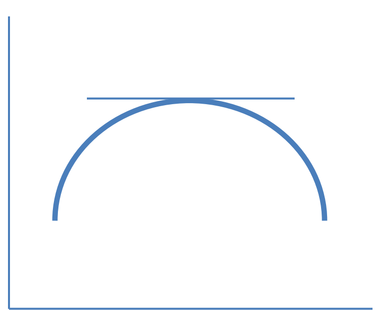
There, starting from the initial endowment of \$32,000, the subject should put \$22,500 into current production. This will leave the subject with \$9,500 and \$54,000 in cash at current and future times, respectively. To support the current consumption of \$32,700, the subject has to borrow 32,700-9,500 = \$23,200 today. In the future, the subject will have to pay back the debt 23,200(1.20) = \$27,840. This will leave the subject with 54,000-27,840 = \$26,160 to consume in the future.

The total NPV from all chosen project is the different between PV of total wealth after investment decision and the initial endowment. NPV = [9,500 + 54,000/(1+0.2)] – 32,000 = 54,500 – 32,000 = 22,500

PV of total consumption = 32,700 – 26,160/(1+0.2) = 54,500 (which is equal to PV of total wealth)



5. Verify slide 29: "The concavity assumption on total utility function (diminishing MRS) is sufficient to ensure that any point obeying MRS = (1+r) is the true maximum".



For a maximum, it is required that

$df(x)/dx = 0$ and

$d[df(x)/dx]/dx = d^2f(x)/dx^2 < 0$

$$
\begin{aligned}
\partial^2 L/\partial C_0^2 &= \partial^2 U/\partial C_0^2 - \partial^2 T/\partial C_0^2 \\
&= \partial MU_0/\partial C_0 - 0 \\
&= \partial MU_0/\partial C_0
\end{aligned}
$$

If the utility function exhibits diminishing marginal utility, $\partial MU_0/\partial C_0$ and $\partial MU_1/\partial C_1 < 0$. Hence, diminishing marginal utility ensures that the solution in questions 2 and 4 are truly maximum.

6. NPV is considered a superior capital budgeting technique compared to IRR and Payback Period because …..

Payback Period: Although Payback Period is quite simple to calculate and understand, it possesses one major drawback. The Payback Period does not take into consideration all free cash flows of a project. It only uses free cash flows up until the payback period and ignore free cash flows after that.

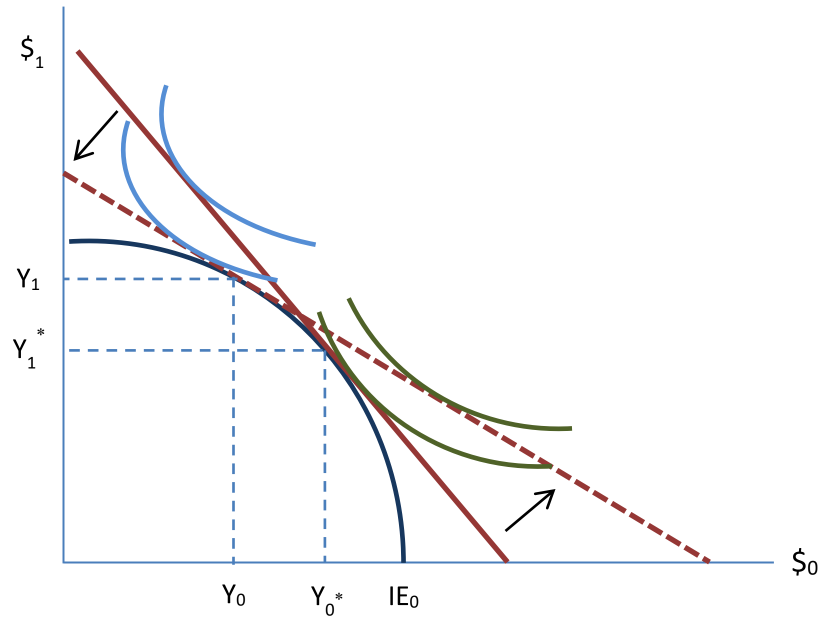
Internal Rate of Return: Although IRR possesses almost all properties of a good capital budgeting technique, it has some drawbacks. Furthermore, as its unit of measurement is % pa, we may wrongly feel that comparing IRRs across projects with different sizes and investment horizons is more appropriate than comparing NPV. Drawbacks of IRR include: 1) it may be impossible to calculate IRR for some projects, 2) there could be multiple IRRs for some projects, 3) IRR assumes that free cash flows received during the life of the project, could be reinvested that the rate of return equal to IRR, 4) in some cases, a project with higher IRR may not create value-added as much as a project with lower IRR, 5) IRR does not possess "Value Additivity Property" (i.e., when combining free cash flows from 2 or more projects, the IRR from the combined projects is NOT equal to the average IRR if each project. This means, we could not calculate IRRs of independent projects separately)

NPV: Advantages of NPV include; 1) NPV measure reflects value-added created by a project, which is consistent with the objective of a corporation to maximize value of the firm, 2) NPV takes into account all project's free cash flows and time value of money, 3) Projects with the same risk will be discounted at the same discount rates (unlike, IRR where projects are discounted at their own IRRs), 4) NPV possesses "Value Additivity Property" (i.e., when combining free cash flows from 2 or more projects, the NPV from the combined projects is equal to the sum of each project's NPV. This property allows us to calculate NPVs of independent projects separately)

7**. "If most of a firm's shareholders are retirees who depend their living on income distributed from the firm, it make more sense if the firm pays more dividend and forgo some investment opportunities. This will help shareholders to avoid borrowing money." In the context of the Fisher's Separation, do you agree with this statement? Explain.**

Disagree. According to Fisher's Separation, whether the firm should invest more or less in production is independent of shareholders' consumption preference. The firm should invest more as long as return on investment (IRR) is greater the cost of capital (interest rate, r). This is true even though it means less current dividend is paid to shareholders. And, shareholders should be happy even though they may have to borrow money to fulfill their current consumption. The shareholders are better off because the firm can make return higher than the shareholder's cost of borrowing.

7**. Show graphically how a decrease in interest rate will affect a firm's investment decision (holding investment opportunities constant) and their shareholders' utility. Explain/Comment the results**


$\$_1$
Y_1
Y_1^*
Y_0
Y_0^*
IE_0
$\$_0$

Lecture 2

1. You are planning to go overseas for a trip on which you plan to spend THB100,000 on shopping during the trip. Assumed that your utility from the trip depends on how much you spend on shopping (Y). The utility function is given by; U(Y) = ln Y.

a. If there is 30% chance that you will lose THB10,000 of your cash on the trip (i.e., the money is stolen during the trip), what is your E[*U*(Y)]?

E[*U*(Y)] = 0.7ln(100,000) + 0.3ln(90,000)
= 11.48132

b. If you can buy insurance against that loss of THB10,000 at an actuarially fair premium of THB3,000 will you buy the insurance?

U(100,000–3,000) = U(97,000)
= 11.48247

Since E[U(Y)] from not buying the insurance is lower than the utility of buying the insurance or U(978,000), I will buy the insurance.

c. What is the maximum premium you are willing to pay for the insurance?

We have to solve for Y such that
U(Y) = ln(Y) = 11.48132
Y = e11.48132
= 98,888.88
Maximum insurance premium = 100,000 – 98,888.88 = 3,111.11

2. When there is bad news that increases the uncertainty (or risk) of future prospects of the economy, stock prices generally drop. This causes negative return to portfolio investor. This observation is inconsistent with the notion of "high risk, higher return", rather it is consistent with "high risk, low return". Do you agree? Explain.

Guideline: Two points of discussion here. (1) why price drop, (2) whether the fact that existing holders of the stock earn negative return violates the notion of "high risk, high return". Here the difference between actual return and expected return must be pointed out.

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1. What is the difference between risk and uncertainty?

Answer: Both risk and uncertainty refer to situations where there are more than one possible outcomes and therefore the actual future outcome is not known for sure. However, risk and uncertainty are different in that: risk is used to describe the situation where (1) all possible outcomes are known and (2) the probability of each outcome is also known. (In other words, risk is the situation that the probability distribution of outcomes is known.) On the other hand, uncertainty is the situation where we only know all possible outcomes but not their probabilities, or where not all possible outcomes are known.

2. Will a risk averter enter speculative activities?

Answer: By definition, risk averter is a person who will undertake a risky investment if and only if there is large enough positive risk premium. Therefore, risk averters will enter speculative activities only if the expected returns from such activities are positive.

3. Can a person who purchase a lottery ticket be classified as risk averter?

In general, the expected return for a lottery buyer is negative, as an agent (i.e., the government lottery office) who issues the lottery makes profit on average. Therefore, a lottery buyer should be classified as a risk lover. (Note that some lottery buyers might strongly believe that certain numbers will turn out as winners. They, therefore, buy these numbers, because they believe that the numbers have positive expected return. In this case, they are using ***subjective*** rather than ***objective*** probability. The claim that lottery has negative expected return is based on objective probability.)

4. How could we draw the total utility lines for (1) an individual who is willing to buy a lottery ticket; and (2) an individuals who is willing to purchase a car insurance policy? Can the same person enter both transactions? If your answer is yes, how do you explain this contradiction?

Insurance buyers.

Assume: W_0 is the value of the car at the beginning of the period and it is also future value of the car if there is no accident (assume no time value of money)

W_1 is the future value of the car if there is an accident = $W_0 - X$

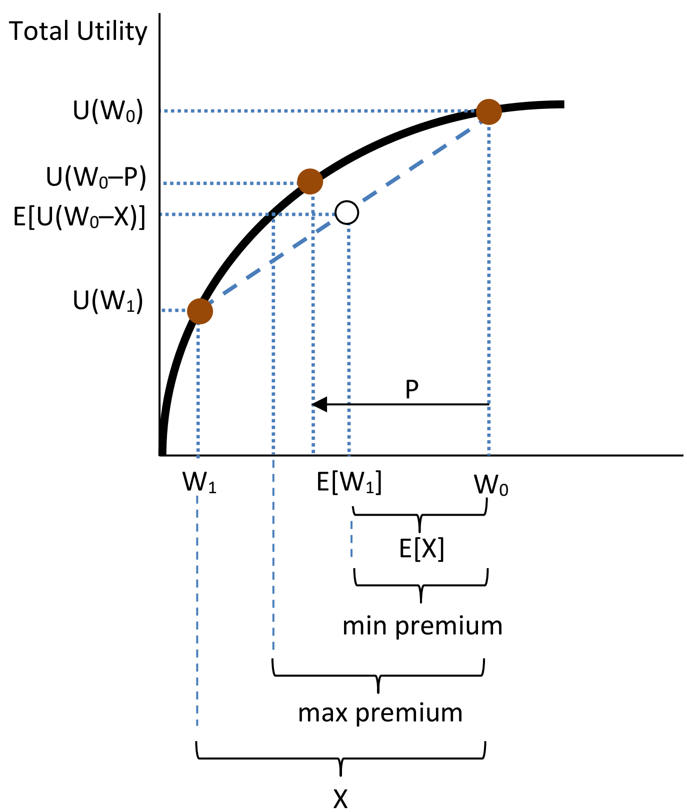
X is the cost of fixing the car if there is an accident (Note that both W_1 and X are random variables as we do not know what will happen

E[X] is the expected cost of fixing the car (weighted between \$0 if there is no accident and \$X if there is.)

$E[W_1]$ the expected value of the car in the future if no insurance is purchased

Note that, $E[W_1] = W_0 - E[X]$

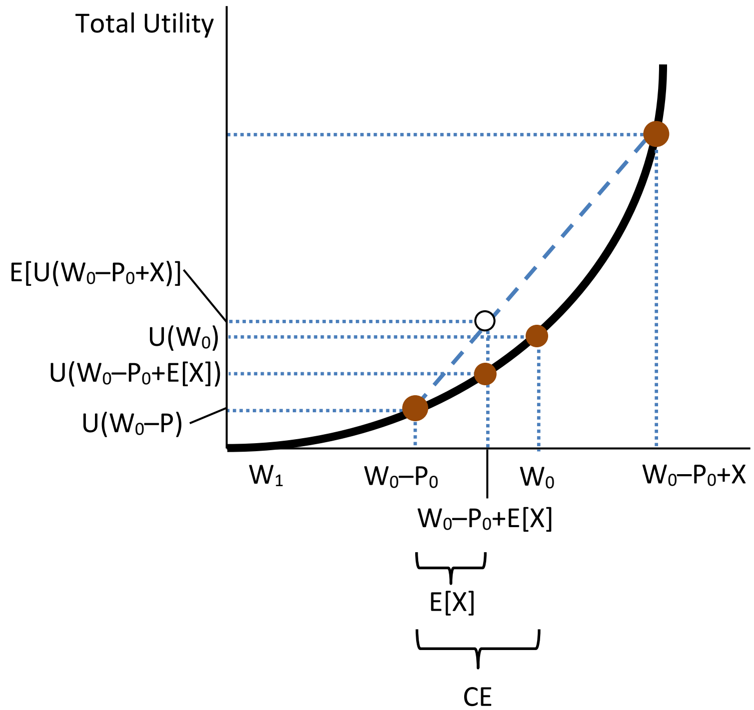
P is the insurance premium



If the car owner buys a car insurance, the cost of fixing the car will be covered. But, he has to pay an insurance premium. With insurance, the net future value of the car is known for sure, and it is equal to W_0–P. It must be noted that for an insurance company to survive, the premium must be greater than the expected cost of fixing the car, that is, $P > E[X]$. For a car owner to buy insurance, it must be the case that $U(W_0–P) \geq E[U(W_0–X)]$. This can be true only if the total utility curve is located on the left hand side of the straight line connecting the possible payoffs if there is no insurance.

Lottery buyers.

Assume: W_0 is the level of wealth if not purchasing the lottery
X_H is the payoff from the gamble if gamblers' win
X_L is the payoff from the gamble if gamblers' lose, which is $0
$E[X]$ is the expected pay-off from the gamble, i.e., $E[X] = \alpha X_H + (1–\alpha)X_L$
α is the odd of winning
P_0 is the cost of entering the gamble (i.e., price of the lottery ticket)
$W_0–P_0+E[X]$ is the expected wealth after purchasing the lottery



Risk lover is defined as a person whose

$$E[U(W_0–P_0+X)] > U(W_0–P_0+E[X])$$

This above inequality holds even when the lottery is a fair game, $P_0 = E[X]$. That is, $W_0–P_0+ E[X] = W_0$, and

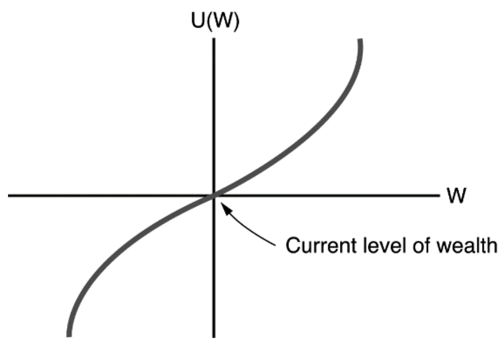
$$E[U(W_0–P_0+X)] > U(W_0)$$

In other words, risk lovers enjoy the risky game even though the price paid is equal to the game expected payoff. However, lottery is a gamble, not a fair game, which means $P_0 > E[X]$. Therefore, in this case, $W_0–P_0+E[X] < W_0$.

$$E[U(W_0–P_0+X)] \quad > \quad U(W_0) \quad > \quad U(W_0–P_0+E[X])$$

Hence, at the wealth level $W_0 – P_0 + E[X]$, the utility of that level of certain wealth is lower than the expected utility from the lottery. This implies that the total utility curve is located underneath or on the RHS of the straight line connecting the two possible outcomes. It can be seen that the certain wealth level (CE) that make the gambler to be indifferent between CE and the gamble is higher than the game's expected payoffs, CE > E[X]. The CE reflects the maximum ticket price that the gambler is willing to pay.

Friedman and Savage [1948] show that it is possible to explain both gambling and insurance if an individual has a utility function such as that shown in the figure below. The individual is risk averse to decreases in wealth because his utility function is concave below his current wealth. Therefore, he will be willing to buy insurance against losses. At the same time he will be willing to buy a lottery ticket which offers him a (small) probability of enormous gains in wealth because his utility function is convex above his current wealth.



Gambling and insurance

Alternative ways to explain this seemingly contradicting behavior are;

- Individuals use subjective probability (i.e., gut feeling) rather than objective probability when it comes to gambling
- When gambling small amount of money that has no significant negative effect on total wealth, the individual become risk lover. But when the stake is high (i.e., losing a car) they become risk averse. (as per Friedman and Savage [1948])
- Individuals are inconsistent in their decisions.

5. Try the example on how to construct a utility function in CWS, p. 50-52.

6. Question 3.3 and 3.8 from CWS, p.70.

3.3 (a)

$$
\begin{aligned}
E[U(W)] &= .5\ln(4,000) + .5\ln(6,000) \\
&= .5(8.29405) + .5(8.699515) \\
&= 8.4967825 \\
e^{\ln W} &= W \\
e^{8.4967825} &= \$4,898.98 = W
\end{aligned}
$$

Therefore, the individual would be indifferent between the gamble and $4,898.98 for sure. This amounts to a risk premium of $101.02. Therefore, he would not buy insurance for $125.

(b) The second gamble, given his first loss, is $4,000 plus or minus $1,000. Its expected utility is

$$\begin{aligned} E[U(W)] &= .5\ln(3,000) + .5\ln(5,000) \\ &= .5(8.006368) + .5(8.517193) = 8.26178 \\ e^{\ln W} &= e^{8.26178} = \$3,872.98 = W \end{aligned}$$

Now the individual would be willing to pay up to $127.02 for insurance. Since insurance costs only $125, he will buy it.

3.8 First, we have to compute the expected utility of the individual's risk.

$$\begin{aligned} E(U(W)) &= \sum p_i U(W_i) \\ &= .1U(1) + .1U(50,000) + .8U(100,000) \\ &= .1(0) + .1(10.81978) + .8(11.51293) \\ &= 10.292322 \end{aligned}$$

Next, what level of wealth would make him indifferent to the risk?

$$\begin{aligned} \ln W &= 10.292322 \\ W &= e^{10.292322} \\ W &= 29,505 \end{aligned}$$

The maximum insurance premium is

$$\begin{aligned} \text{Risk premium} &= \text{E (W)} - \text{certainty equivalent} \\ &= \$85,000.1 - \$29,505 \\ &= \$55,495.1 \end{aligned}$$

7. Show that if U(W) is a utility function of an individual, and V(X) is a positive linear transformation of U(W) then both U(W) and V(X) have the same ARA and RRA measures.

From ARA and RRA measures for U(W);

$$A(W) = -U''(W)/U'(W)$$
$$R(W) = R\times A(W)$$

Let $V(W) = a + bU(W)$, where a and b are positive constant.

$V'(W) = bU'(W)$

$V''(W) = bU''(W)$

A(W) for V(W) is: $A(W) = -bU''(W)/bU'(W) = -U''(W)/U'(W)$ [= A(W) for U(W)]

R(W) for V(W) is: $R(W) = R\times A(W) = R\times U(W)$ [= R(W) for U(W)]

Hence A(W) and R(W) of V(W) are the same as those of U(W).

9 Some social commentators criticize that the stock market is a casino for rich people. Do you agree with their comment? Why/Why not?

Lecture 3

1. What is the relationship (if any) of the FSD and SSD rules to investment's expected return? What is their relationship to the variance of the distribution of returns?

<u>Answer:</u> Given the same investment feasibility set, SSD efficient set is a subset of FSD efficient set. This is because FSD only assumes that investors prefer more to less. SSD assumes that not only investors prefer more to less, but they are also risk averse.

A necessary condition (but not sufficient) for investment F to be preferred to G under FSD is that $E[r_F] > E[r_G]$. This is the same for SSD that is a necessary condition (but not sufficient) for investment F to be preferred to G under SSD is that $E[r_F] > E[r_G]$. However, we can derive similar conclusion regarding variance.

2. Consider the following five investment options:

The following table shows cumulative probability of 5 investments.

r	A	B	C	D	E
5	0.00	0.00	0.50	0.00	0.00
6	0.00	0.00	0.75	0.00	0.25
8	0.00	0.00	1.00	0.00	0.25
10	0.50	0.33	1.00	0.00	0.25
12	0.50	0.33	1.00	0.50	0.25
15	0.50	0.66	1.00	0.50	0.25
20	1.00	0.66	1.00	1.00	0.25
30	1.00	1.00	1.00	1.00	0.25
40	1.00	1.00	1.00	1.00	1.00

To identify FSD efficient investments, differences in cumulative probability, [G(r)–F(r)], are calculated and shown below.

		Drop C	Drop A		Drop C			Drop C	Drop C	
r	A-B	A-C	A-D	A-E	B-C	B-D	B-E	C-D	C-E	D-E
5	0.00	-0.50	0.00	0.00	-0.50	0.00	0.00	0.50	0.50	0.00
6	0.00	-0.75	0.00	-0.25	-0.75	0.00	-0.25	0.75	0.50	-0.25
8	0.00	-1.00	0.00	-0.25	-1.00	0.00	-0.25	1.00	0.75	-0.25
10	0.17	-0.50	0.50	0.25	-0.67	0.33	0.08	1.00	0.75	-0.25
12	0.17	-0.50	0.00	0.25	-0.67	-0.17	0.08	0.50	0.75	0.25
15	-0.16	-0.50	0.00	0.25	-0.34	0.16	0.41	0.50	0.75	0.25
20	0.34	0.00	0.00	0.75	-0.34	-0.34	0.41	0.00	0.75	0.75
30	0.00	0.00	0.00	0.75	0.00	0.00	0.75	0.00	0.75	0.75
40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Under FSD, the dominant investments are B, D and E.

To identify SSD efficient investments, cumulative differences in cumulative probability, [∫G(r)–F(r)], are calculated and shown below.

	Drop A	Drop C	Drop A		Drop C			Drop C	Drop C	
r	A-B	A-C	A-D	A-E	B-C	B-D	B-E	C-D	C-E	D-E
	0	0	0	0	0	0	0	0	0	0
5	0.00	-0.50	0.00	0.00	-0.50	0.00	0.00	0.50	0.50	0.00
6	0.00	-1.25	0.00	-0.25	-1.25	0.00	-0.25	1.25	1.00	-0.25
8	0.00	-2.25	0.00	-0.50	-2.25	0.00	-0.50	2.25	1.75	-0.50
10	0.17	-2.75	0.50	-0.25	-2.92	0.33	-0.42	3.25	2.50	-0.75
12	0.34	-3.25	0.50	0.00	-3.59	0.16	-0.34	3.75	3.25	-0.50
15	0.18	-3.75	0.50	0.25	-3.93	0.32	0.07	4.25	4.00	-0.25
20	0.52	-3.75	0.50	1.00	-4.27	-0.02	0.48	4.25	4.75	0.50
30	0.52	-3.75	0.50	1.75	-4.27	-0.02	1.23	4.25	5.50	1.25
40	0.52	-3.75	0.50	1.75	-4.27	-0.02	1.23	4.25	5.50	1.25

Under SSD, the dominant investments are B, D and E

3. The FSD rule is better than SSD criterion since it does not require strong assumptions regarding investors' preferences." Appraise.

<u>Answer:</u> As mentioned in question 1, FSD only relies on one assumption that people prefer more to less. But, SSD assumes that people prefer more to less and are risk averse. FSD has an advantage of relying on less assumptions, and therefore, it is application to a larger group of people. However, on main drawback of FSD is that the criteria is not strong. Therefore, FSD classification usually leaves large number of securities in the efficient investment set. This by itself may defeat the purpose of classification.

4. Which investment is in FSD and SSD

<u>Answer:</u> From the table below, the second last column indicates that FSD cannot classify investments F and G apart. The last column indicates that investment F is preferred to G under the SSD.

r	F	G	F-G	Cumu(F-G)
0.5	0.00	0.19	-0.19	-0.19
1	0.25	0.19	0.06	-0.13
1.5	0.25	0.38	-0.13	-0.25
2	0.50	0.38	0.13	-0.13
2.5	0.50	0.63	-0.13	-0.25
3.5	0.50	0.81	-0.31	-0.56
4.5	0.50	1.00	-0.50	-1.06
9	0.75	1.00	-0.25	-1.31
10	1.00	1.00	0.00	-1.31