Chapter 1

2

2.1

I think that by "arc", the author is referring to path. (Spivak uses arc to refer to one-one path.) In that case, this is easy.

3

3.1

The center of a group G is the set of all elements z such that $z \cdot g = g \cdot z$.

If two path classes gives rises to the same isomorphism if and only if $\gamma'\gamma^{-1}$ belongs to the center of the group $\pi(X, x)$.

3.2

We claim the following: this is equivalent to:

Fix a point x; if for all values of y, and all path class γ and γ' connecting x and y, $\gamma'\gamma^{-1}$ be an element of the center of the group.

Basically this is an extension of the result from problem 3.1. The only difference is that we can simply state the equivalence for a fixed x and this would be equivalent for any point in X too.

3.3

Suppose $f \sim g$, then $f \cdot \bar{g} = g \cdot \bar{g} = \mathscr{E}_x \in \pi(X, x)$.

Suppose $f \cdot \bar{g} = \mathscr{E}_x$, then $f \cdot \bar{g}g = \mathscr{E}_x \cdot g \implies f \cdot \mathscr{E}_y = g \implies f = g$.

4

TODO 4.1

4.2

Recall that X is Hausdorff if and only if the diagonal $\Delta = \{x \times x | x \in X\}$ is closed.

Given two maps $f, g \colon X \to Y$, then the set $A = \{x \in X | f(x) = g(x)\}$ is closed if Y is Hausdorff. This can be proved by making use of the above observation, since $A = (f \times g)^{-1}(\Delta)$.

Let $f\colon X \to X$ be a retraction mapping from X onto Z and $i\colon X \to X$ be the identity map. Then the points where both these map are equal is clearly the set Z which will become closed.

TODO 4.3

4.4

Suppose A be a retract of X and $r\colon X \to A$ be the corresponding retraction. Then define a function $\phi\colon X \times Y \to A \times Y$ by $\phi(x, y) = (r(x), y)$. This function is clearly continuous and thus forms a retraction mapping from $X \times Y$ onto $A \times Y$.

Suppose that there is a retraction $\phi\colon X \times Y \to A \times Y$. Define a retraction $r\colon X \times A$ by first picking a point $y_0 \in Y$ and by making $r(x) = \pi_1 \circ \phi(x, y_0)$. Once again one can verify that this mapping is a retraction.

4.5

This is easy. Compose the two retraction mappings to get the required retraction.

4.6

Without loss of generality, one can assume that the $(n-1)$ dimensional sphere S^{n-1} is the unit-sphere centered at the origin and the point x_0 is the origin.

Let $X = \mathbb{R}^n - \{0\}$, then define the function $\phi\colon X \times I \to X$ by

$\phi(x, t) = (1-t)x + t \cdot \frac{x}{\|x\|}$.

One can verify that this is indeed the required deformation retract.

4.7

The center circle.

4.8