Chapter 1

2

2.1

I think that by "arc", the author is referring to path. (Spivak uses arc to refer to one-one path.) In that case, this is easy.

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3.1

The center of a group G is the set of all elements z such that $z \cdot g = g \cdot z$.

If two path classes gives rises to the same isomorphism if and only if $\gamma'\gamma^{-1}$ belongs to the center of the group $\pi(X, x)$.

3.2

We claim the following: this is equivalent to:

Fix a point x; if for all values of y, and all path class γ and γ' connecting x and y, $\gamma'\gamma^{-1}$ be an element of the center of the group.

Basically this is an extension of the result from problem 3.1. The only difference is that we can simply state the equivalence for a fixed x and this would be equivalent for any point in X too.

3.3

Suppose $f \sim g$, then $f \cdot \bar{g} = g \cdot \bar{g} = \mathscr{E}_x \in \pi(X, x)$.

Suppose $f \cdot \bar{g} = \mathscr{E}_x$, then $f \cdot \bar{g}g = \mathscr{E}_x \cdot g \implies f \cdot \mathscr{E}_y = g \implies f = g$.

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