

MACM 316 Assignment 5: Report

Synopsis

The goal of this report is to investigate the distribution of the roots of all binary polynomials of a given maximum degree d . In other words, we will be computing and plotting

$R_d = \{r \in \mathbb{C} : r \text{ is a root of a binary polynomial of degree } n \leq d \text{ with coefficients } a_n, a_{n-1}, \dots, a_0 \in \{-1, 1\}\}.$

Discussion

After several runs of my code, there were at total d roots for a d -degree binary polynomials of the 2^{d+1} different permutations of the polynomial coefficients.

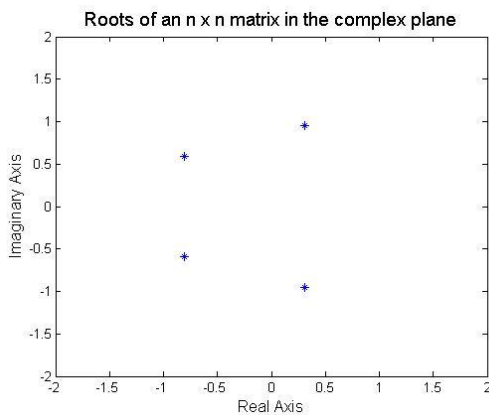


Figure 1. Roots of 4th degree polynomial

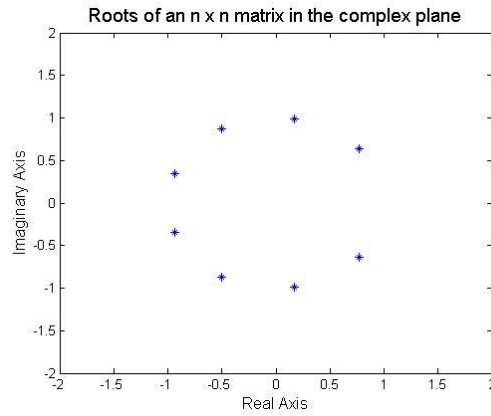


Figure 2. Roots of 8th degree polynomial

In Figure 1, The roots of the 4th degree polynomial first appear to be arbitrary, however, the degree increases in Figure 2, the roots of the polynomials start settling between $\{-1, 1\}$ on both the y and x-axis. As mentioned above, in figure 1, there is a total of 4 roots for a 4th degree polynomial and in figure 2, there is a total of 8 roots for an 8th degree polynomial, and in Figure 3, 12 roots for a 12th degree polynomial

The roots of the polynomials consistently form a circle with the radius of 1 on the plot in Figure 3. By definition, a circle with a radius of one is called the unit circle. The unit circle becomes apparent as the degree of d becomes larger as in Figure 4. From the results of *RandPolyRoots.m* which computes the roots of random coefficients of numerous matrices which also forms the unit circle when plotted in a robust manner, I can conclude that plotting the roots of any polynomial will result in the unit circle.

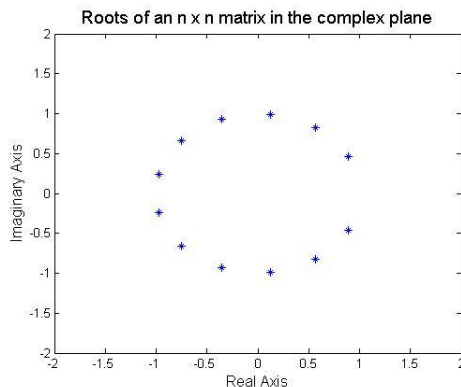


Figure 3. Roots of 12th degree polynomial

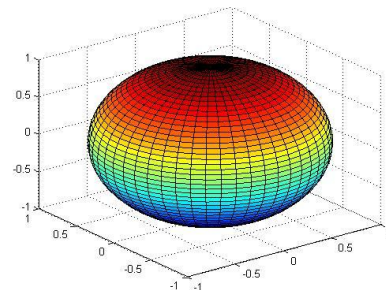


Figure 4. Unit Circle from Roots of Polynomial