

MACM 316 Assignment 8 Report

Euler's method gives an approximation to the solution of ordinary differential equations (ODE), given an initial value with the following conditions

$$\begin{aligned}y'(t) &= f(t, y(t)) \\ y(t_0) &= y_0\end{aligned}$$

Using the above, we choose a value h as the step-size and we get the following for approximations at time t_n and $t_{n+1} = t_n + h$. Combining the above, we get the following equation

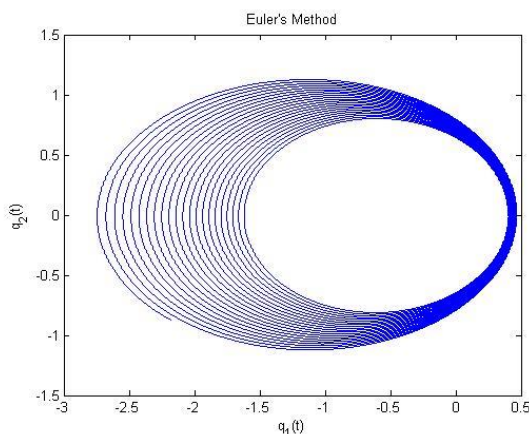
$$y_{n+1} = y_n + hf(t_n, y_t)$$

which is used to solve the system of two planets, $q(t) = \{q_1(t), q_2(t)\}$ and $p(t) = \{p_1(t), p_2(t)\}$ in *Figure(1)*.

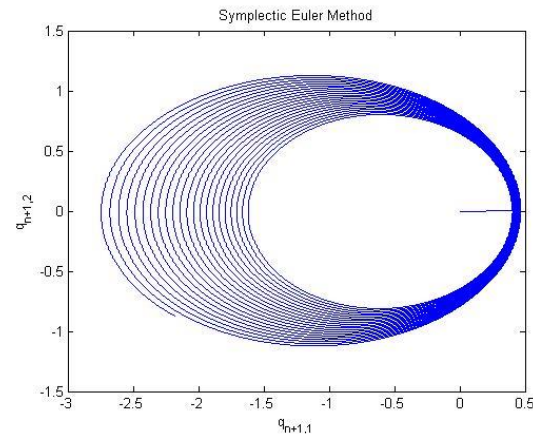
Using Euler's method, *Figure(1)/Figure(2)* is the plot of $q_2(t)$ vs $q_1(t)$, resulting in a elliptic spiral to the left of the origin. The scattered values on the left converge to a fixed value on the right; $q_1(t)$ converges to -2.1846 where as $q_2(t)$ converges to -0.8685.

The angular momentum $A(t)$ where $t=0$ is 0.8 and increases to 0.8941. A positive angular momentum means that the values will spiral clockwise, which is conserved in *Figure(1)*. $A(t)$ increases over time and as a result, the values become more dense at t_n . As seen in *Figure(1)*, the negative Hamiltonian value of -0.5000 to -0.3112 is preserved in the plot.

Following Euler's method is the Symplectic Euler Method, which I used to again, approximate the solutions to the ordinary differential equations given. Both *Figure(1)* and *Figure(2)* look the same appearance-wise, the direction of the values on the plot is counter-clockwise thus we will look at the values for $A(t)$ and $H(t)$. $A(t)$ where $t=0$ is 0.8, and also increases to 0.8941. As suspected, $H(t)$ values are also the same, increases from -0.5000 to -0.3112.



Figure(1) Euler's Method



Figure(2) Symplectic Euler Method