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MACM 316 Assignment 7: Report

Abstract

The purpose of this report is to investigate Numerical Standard Quadrature and Aitken's Delta Squared methods to compute the approximations of the following given integral:

$$I = \int_0^1 x^{-1} \sin(x^{-1} \log(x))\, \mathrm{d}x.$$

Discussion

For the first part of my code in *asn7.m* and defining the roots as $1 > a_1 > a_2 > a_3 > ... > 0$, the roots are computed using Standard normal Quadrature with the built-in function *fzero(fun, x_0)*. These roots are stored into a row vector variable called *a* of changeable size *n*. The approximations based on *n* are computed using the built-in function *integral(fun, min,max)*. The for *n = 100*, values oscillate between around -0.46, dipping above -0.45 and below -0.46 as seen in *Figure (1)*. The following method computes with the accuracy of 1 decimal place for smaller values of *n*.

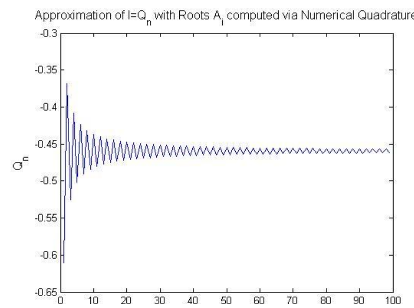

Figure (1) Approximation with n=100

In *Figure (2)*, a larger size *n* is chosen and as we can see from the graph, the approximation oscillates -0.4588 and -0.4599, thus is assumed that it converges to a value between the two numbers. The approximation is accurate to 2 decimal places using the above method of computation. Taking the average of the two points **-0.45935 as our first approximation.**

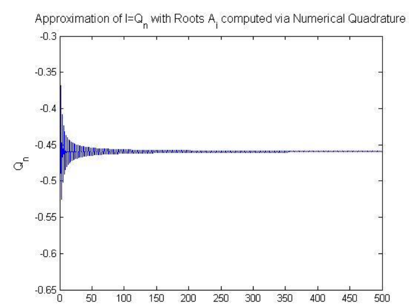

Figure (2) Approximation with n=500

Using computed roots found from above and the results for each approximation for *n*, the results above can be used to make quicker approximations via the *Aitkens Delta Squared* Method with the following :

$$\hat{Q}_n = Q_n - \frac{(Q_{n+1} - Q_n)^2}{Q_{n+2} - 2Q_{n+1} + Q_n}.$$

This results in approximations that converges completely to **-0.4594 as our second approximation**. Using *format long*, we can see that the approximation is accurate to 4 decimal places as the 5^{th} decimal place and continue to oscillate. ***Aitkens* method converges much faster and more robust** compared to that of *Standard Normal Quadrature* (See *Figure (3) and Figure (4)*. Let (1) be *Standard Normal Quadrature* and (2) be *Aitkens*, then the absolute and relative errors are:

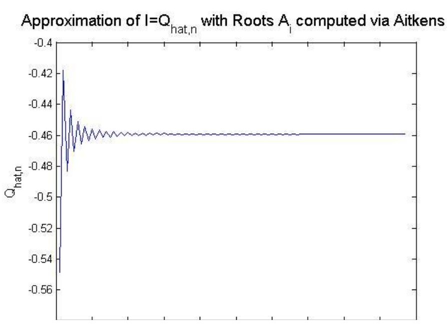

Figure (3) Approximation with n=100

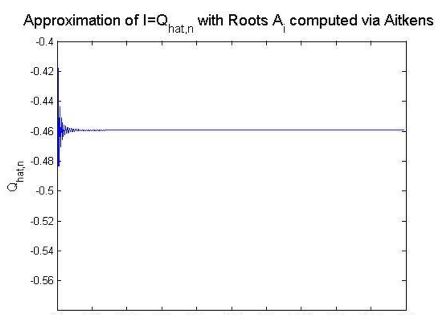

Figure (4) Approximation with n=500

Known Value: -0.5

Abs Errors
(1) 0.04065
(2) 0.0406
Relative Errors
(1) 0.0813
(2) 0.0812