TP2 Roots of Equations

1 Explain and develop a computer program to find roots of the equation,

$$bx + c = 0$$

2 Explain and develop a computer program to find roots of the equation,

$$ax^2 + bx + c = 0$$

3 Develop computer programs to compute the following sums with loops and then with mathematical formula for $n = 10, 20, 50$.

a $1 + 2 + \cdots + n = \dfrac{n(n+1)}{2}$

b $\dfrac{1}{1 \cdot 2} + \dfrac{1}{2 \cdot 3} + \cdots + \dfrac{1}{n(n+1)} = 1 - \dfrac{1}{n+1}$

4 Develop computer programs to compute the sum $\displaystyle\sum_{k=1}^{n} \frac{1}{k!}$.

5 Develop a computer program to determine the exponential function which can be written in infinite series form

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

then compute the value of e^x for $x = 0.1, 0.5, 1$ with polynomial approximation of degree 5.

6 Develop a computer program to compute the value of polynomial $p(x) = \displaystyle\sum_{k=0}^{n} a_k x^k$ at $x = x_0$. Use your program to compute $p(0.1)$ if $p(x) = 1 - 3x^2 + 2x^3 - x^5 + 4x^6$.

7 Let $f(x) = (x-1)^{10}, p = 1$ and $p_n = 1 + \dfrac{1}{n}$. Show that $|f(p_n)| < 10^{-3}$ whenever $n > 1$ but that $|p - p_n| < 10^{-3}$ requires that $n > 1000$.

8 Let (p_n) be the sequence defined by $p_n = \displaystyle\sum_{k=1}^{n} \frac{1}{k}$. Show that (p_n) diverges even though $\displaystyle\lim_{n\to\infty} (p_n - p_{n-1}) = 0$.

9 Use the Bisection method to find p_3 for $f(x) = \sqrt{x} - \cos x$ on $[0, 1]$.

10 Use the Bisection method to find solutions accurate to within 10^{-2} for $x^3 - 7x^2 + 14x - 6 = 0$ on each interval.

a $[0, 1]$

b $[1, 3.2]$

c $[3.2, 4]$

11 Use the Bisection method to find solutions accurate to within 10^{-5} for the following problems.

- **a** $x - 2^{-x} = 0$ for $0 \leq x \leq 1$
- **b** $e^x - x^2 + 3x - 2 = 0$ for $0 \leq x \leq 1$
- **c** $2x\cos(2x) - (x+1)^2 = 0$ for $-3 \leq x \leq -2$ and $-1 \leq x \leq 0$
- **d** $x\cos(x) - 2x^2 + 3x - 1 = 0$ for $0.2 \leq x \leq 0.3$ and $1.2 \leq x \leq 1.3$

12 Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^4 - 3x^2 - 3 = 0$ on $[1, 2]$. Use $p_0 = 1$.

13 Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^3 - x - 1 = 0$ on $[1, 2]$. Use $p_0 = 1$.

14 For each of the following equations, use the given interval or determine an interval $[a, b]$ on which fixed-point iteration will converge. Estimate the number of iterations necessary to obtain approximations accurate to within 10^{-5}, and perform the calculations.

- **a** $2 + \sin x - x = 0$ use $[2, 3]$
- **b** $x^3 - 2x - 5 = 0$ use $[2, 3]$
- **c** $3x^2 - e^x = 0$
- **d** $x - \cos x = 0$

15 Let $f(x) = x^2 - 6$ and $p_0 = 1$. Use Newton's method to find p_2.

16 Let $f(x) = -x^3 - \cos x$ and $p_0 = -1$. Use Newton's method to find P_2.

17 Write a general purpose algorithm for the method of False Position.

- **a** Describe your algorithm objective?
- **b** Describe required input parameter(s)?
- **c** Describe output value(s)?
- **d** Write your algorithm body here.
- **e** Implement your developed algorithm with Python, or any other programming language.

18 Let $f(x) = x^2 - 6$. With $p_0 = 3$ and $p_1 = 2$, find p_3.

- **a** Use the secant Method.
- **b** Use the method of False Position.
- **c** Which of (a) or (b) is closer to $\sqrt{6}$?

19 Let $f(x) = -x^3 - \cos x$. With $p_0 = 0$ and $p_1 = 0$, find p_3.

- **a** Use the Secant method.
- **b** Use the method of False Position.

20 Find solutions accurate to within 10^{-5} for the problem $e^x - 3x^2 = 0$ for $0 \leq x \leq 1$ and $3 \leq x \leq 5$ using

- **a** Newton's method.
- **b** Secant method.
- **c** The method of False Position.

21 The four-degree polynomial $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$ has two real zeros, one in $[-1, 0]$ and the other in $[0, 1]$. Attempt to approximate these zeros to within 10^{-6} using the

- **a** Method of False Position
- **b** Secant method
- **c** Newton's method

Use the endpoints of each interval as the initial approximations in (a) and (b) and the midpoints as the initial approximation in (c).

22 The accumulated value of a savings account based on regular periodic payments can be determined from the annuity due equation,

$$A = \frac{P}{i}[(1+i)^n - 1].$$

In this equation, A is the amount in the account, P is the amount regularly deposited, and i is the rate of interest per period for the n deposit periods. An engineer would like to have a savings account valued at $\$\ 750,000$ upon retirement in 20 years and can afford to put $\$\ 1500$ per month toward this goal. What is the minimal interest rate at which this amount can be invested, assuming that the interest is compounded monthly?

23 Write a general purpose algorithm for Müller's method.

- **a** Describe your algorithm objective?
- **b** Describe required input parameter(s)?
- **c** Describe output value(s)?
- **d** Write your algorithm body here.
- **e** Implement your developed algorithm with Python, or any other programming language.

24 Use each of the following methods to find a solution in $[0.1, 1]$ accurate to within 10^{-4} for

$$600x^4 - 550x^3 + 200x^2 - 20x - 1 = 0.$$

- **a** Bisection method
- **b** Newton's method
- **c** Secant method
- **d** method of False Position
- **e** Müller's method.

25 Write a general purpose algorithm for a method that can find all real and complex zeros of a given polynomial.

- **a** Describe your algorithm objective?
- **b** Describe required input parameter(s)?
- **c** Describe output value(s)?
- **d** Write your algorithm body here.
- **e** Implement your developed algorithm with Python, or any other programming language.

26 Find approximations to within 10^{-5} to all the zeros of each of the following polynomials

- **a** $-20 + 16x + x^2 - 4x^3 + x^4$
- **b** $x^4 + 5x^3 - 9x^2 - 85x - 136$
- **c** $x^5 + 11x^4 - 21x^3 - 10x^2 - 21x - 5$

using Müller's and Lagurre's method.

27 Find approximations to within 10^{-10} to all the zeros of each of the following Lagurre polynomials using Müller's and Lagurre's method.

n	$L_n(x)$
0	1
1	$-x+1$
2	$\frac{1}{2}(x^2 - 4x + 2)$
3	$\frac{1}{6}(-x^3 + 9x^2 - 18x + 6)$
4	$\frac{1}{24}(x^4 - 16x^3 + 72x^2 - 96x + 24)$
5	$\frac{1}{720}(x^6 - 36x^5 + 450x^4 - 2400x^3 + 5400x^2 - 4320x + 720)$
n	$\frac{1}{n!}((-x)^n + n^2(-x)^{n-1} + \cdots + n(n!)(-x) + n!)$

28 Find approximations to within 10^{-10} to all the zeros of each of the following Legendre polynomials using Müller's and Lagurre's method.

n	$L_n(x)$
0	1
1	x
2	$-\frac{1}{3} + x^2$
3	$-\frac{3}{5}x + x^3$
4	$\frac{3}{35} - \frac{6}{7}x^2 + x^4$
5	$\frac{5}{21}x - \frac{10}{9}x^3 + x^5$
6	$-\frac{5}{231} + \frac{5}{11}x^2 - \frac{15}{11}x^4 + x^6$
7	$-\frac{35}{429}x + \frac{105}{143}x^3 - \frac{21}{13}x^5 + x^7$
8	$\frac{7}{1287} - \frac{28}{143}x^2 + \frac{14}{13}x^4 - \frac{28}{15}x^6 + x^8$
9	$\frac{63}{2431}x - \frac{84}{221}x^3 + \frac{126}{85}x^5 - \frac{36}{17}x^7 + x^9$
10	$-\frac{63}{46189} + \frac{315}{4199}x^2 - \frac{210}{323}x^4 + \frac{630}{323}x^6 - \frac{45}{19}x^8 + x^{10}$

29 Find approximations to within 10^{-10} to all the zeros of each of the following probabilist's Hermite polynomials using Müller's and Lagurre's method.

n	$He_n(x)$
0	1
1	x
2	$x^2 - 1$
3	$x^3 - 3x$
4	$x^4 - 6x^2 + 3$
5	$x^5 - 10x^3 + 15x$
6	$x^6 - 15x^4 + 45x^2 - 15$
7	$x^7 - 21x^5 + 105x^3 - 105x$
8	$x^8 - 28x^6 + 210x^4 - 420x^2 + 105$
9	$x^9 - 36x^7 + 378x^5 - 1260x^3 + 945x$
10	$x^{10} - 45x^8 + 630x^6 - 3150x^4 + 4725x^2 - 945$

30 Find approximations to within 10^{-10} to all the zeros of each of the following probabilist's Hermite polynomials using Müller's and Lagurre's method.

n	$H_n(x)$
0	1
1	$2x$
2	$4x^2 - 2$
3	$8x^3 - 12x$
4	$16x^4 - 48x^2 + 12$
5	$32x^5 - 160x^3 + 120x$
6	$64x^6 - 480x^4 + 720x^2 - 120$
7	$128x^7 - 1344x^5 + 3360x^3 - 1680x$
8	$256x^8 - 3584x^6 + 13440x^4 - 13440x^2 + 1680$
9	$512x^9 - 9216x^7 + 48384x^5 - 80640x^3 + 30240x$
10	$1024x^{10} - 23040x^8 + 161280x^6 - 403200x^4 + 302400x^2 - 30240$