Unit: 2 Basic Probability

Course Content:

- Introduction
- Definition of Probability
- Experiment, Events
- Elementary Events, Independent Events, Mutually Exclusive Events
- Formula for finding Probability
- Union Probability, Joint Probability
- Addition Theorem of Probability
- Conditional Probability
- Probability Matrix

Basic Probability

- Definition: Probability is a numerical measurement of likelihood that an event has occurred.
- It is useful for occurrence and non-occurrence of an event.
- Value of probability is always assign on scale of 0 and 1.
- 0 probability means event is impossible to occur.(eg. Sun rises from west)
- 1 probability means event is almost certain to occur. (eg. Sun rises from east)
- Other probability between 0 and 1 represents degrees of likelihood that an event will occur.

Definition : Experiment:

An experiment is a process that generates a well defined outcome.

Definition: Event:

An event is an outcome of an experiment.

Experiment	Result
1. Toss a coin	Head and Tail
2. Roll a die	1,2,3,4,5,6
3. One day match	Win, loss and tie
4. Selection of some parts for quality control	Defective and Non defective
5. Toss two coins	HH, TT, HT, TH

Definition : Elementary Events:

Events that can not be broken down or decomposed into further events is known as elementary events.

Example: Suppose the experiment is to roll a die. The elementary events for this experiment are to get 1 or 2 or 3 …. Or 6. But rolling a die and getting an odd number is not an elementary event because the odd number can be further divided into 1, 3 and 5.

Definition: Sample Space:

The set of all possible experimental outcomes for an experiment is known as sample space.

An experimental outcome is also called sample point to identify as an element of sample space.

Ex. An experiment is to toss two coins then sample space is

S = { (1,1), (1,2), (1,3), (1,4),(1,5), (1,6), (2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6) (4,1), (4,2), (4,3), (4,4), (4,5),(4,6), (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), (6,1), (6,2),(6,3),(6,4),(6,5), (6,6) }

Here number of elements in a sample space is 36. It is denoted as n(S) = 36.

Definition: Independent Events:

Two or more events are said to be independent events if the occurrence or non occurrence of one of the event does not affect the occurrence or non occurrence of the other events.

Ex 1) To toss a coin and roll a die.

Definition: Mutually Exclusive Events:

Events are said to be mutually exclusive events if the happening of any one of them eliminates the happening of all the other events. i.e. no two or more of them can occur simultaneously in the same trial.

Ex. In throwing a die all the 6 faces numbered 1 to 6 are mutually exclusive since if any of these face comes, the possibility of others in the same trial ruled out.

*** Formula for finding Probability:**

Let n be the total number of all possible experimental cases of an event E and m be the total number of favourable experimental cases then the probability p of occurrence of the experiment E is given by,

$$P(E) = \frac{\text{Number of Favourable Cases}}{\text{Total Number of all possible Cases}} = \frac{m}{n}$$

Example 1: Two dice are thrown then what is the probability that –

(i) Both dice show same number (ii) The total of the number on both dice are 8 (iii) The total of the number on both dice are 10

Answer: In a random throw of two dice total number of possible cases are (n) = $6^2 = 36$

{ (1,1), (1,2), (1,3), (1,4),(1,5), (1,6), (2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6) (4,1), (4,2), (4,3), (4,4), (4,5),(4,6), (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), (6,1), (6,2),(6,3),(6,4),(6,5), (6,6) }

(i) Both dice show same number:

The favourable cases are { (1,1), (2,2), (3,3), (4,4), (5,5),(6,6)} Therefore m = 6.

Probability = $\frac{m}{n} = \frac{6}{36} = \frac{1}{6}$

(ii) The total of the number on both dice are 8

Favourable cases are { (2,6), (6,2), (3,5),(5,3), (4,4)} Therefore m = 5.

Probability = $\frac{5}{36}$

(iii) The total of the number on both dice are 10

Favourable cases are { (4,6), (6,4), (5,5)}. Therefore m = 3

Probability = $\frac{3}{36} = \frac{1}{12}$

Example 2: A letter from English alphabet is chosen at random. Calculate the probability that the letter so chosen is –

(i) A vowel (ii) Precedes L and a vowel (iii) Follows L and a vowel

Answer: Here total number of possible outcomes are (n) = 26

(i) The letter is vowel

Therefore favourable cases are a, e, i, o, u . Therefore m = 5

Probability = $\frac{m}{n} = \frac{5}{26}$

(ii) The letter precedes L and a vowel

Here favourable cases are a, e, i. Therefore m = 3

Probability = $\frac{m}{n} = \frac{3}{26}$

(iii) Follows L and a vowel

Here favourable cases are o, u. Therefore m = 2

Therefore probability = $\frac{m}{n} = \frac{2}{26} = \frac{1}{13}$

Example 3 : Let A, B and C are three mutually exclusive events associated with random experiment. Find P(A) given that P(B) = $\frac{3}{2}$ P(A) and P(C) = $\frac{1}{2}$ P(B)

Answer: Let P(A) = p then P(B) = $\frac{3}{2}$ p and P(C) = $\frac{1}{2}\,\frac{3}{2}$ p = $\frac{3}{4}$ p .

Since A, B and C are mutually exclusive events

P(A) + P(B) + P(C) = 1

$\therefore p + \frac{3}{2}p + \frac{3}{4}p = 1$

$\therefore \frac{4+6+3}{4}p = 1$

$\therefore \mathbf{p} = \frac{4}{13}$

Addition Theorem of Probability:

Let A and B are two non – disjoint events then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A and B are disjoint events than $P(A \cap B) = 0$ therefore

$P(A \cup B) = P(A) + P(B)$

Example 1: The probability that a student passes a JAVA test is 2/3 and the probability that he passes both JAVA and SM test is 14/45. The probability that he passes at least one test is 4/5. What is probability that he passes SM test?

Answer: Let us define following events:

A : Student passes JAVA test.

B: Student passes SM test.

We have given that, $P(A) = 2/3$, $P(A \cap B) = 14/45$ and $P(A \cup B) = 4/5$.

Therefore by addition theorem on probability,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore 4/5 = 2/3 + P(B) - 14/45$

$\therefore P(B) = 4/5 - 2/3 + 14/45$

$\therefore P(B) = (36 - 30 + 14) / 45$

$\therefore P(B) = 4/9$

Example 2: An integer is chosen at random from the first 200 digits. What is the probability that the integer is divisible by 6 or 8 ?

Answer: Here sample space S = { 1,2,3,….,198,199,200} .

∴ n = 200

Let us define two events:

A = Integer is divisible by 6 and B = Integer is divisible by 8.

For event A set of favourable cases are = { 6,12,18,….,198}

∴ Number of favourable cases = 198/6 = 33

∴ P(A) = 33/200

For event B set of favourable cases are = { 8,16,24,…,200}

∴ Number of favourable cases = 200/8 = 25

∴ P(B) = 25/200

Now L.C.M of 6 and 8 is 24. Hence a number is divisible by both 6 and 8 is divisible by 24.

Therefore favourable cases for A∩ B = { 24, 48, 72, …, 192}

∴ Number of favourable cases = 192/24 = 8

∴ P(A∩B) = 8/200

Now using addition theorem on probability,

P(AUB) = P(A) + P(B) – P(A∩B)

∴ P(AUB) = 33/200 + 25/200 – 8/200 = (33+25-8)/200 = 50/200

∴ P(AUB) = 1/4

Practice Example : A number is selected from the first 250 digits then what is the probability that the selected number is divisible by 6 or 9?

(Answer: 55/250)

P(A) = Probability of integer divisible by 6

Favourable Cases = { 6, 12, 18, ….246} m = 246/6 = 41

P(A) = 41/250

P(B) = Prob. Of integer divisible by 9 = { 9, 18, 27, …243} m = 243/9 = 27

P(B) = 27/250

P(A∩B) = Integer divisible by 18 (LCM of 6 and 9) = {18, 36, 54, …234} m= 13

13/250

By addition theorem (41+ 27 – 13)/ 250 = 55/250

Example: Consider P(A) = 0.10 , P(B) = 0.12 , P(C) = 0.21 , P(A∩C) = 0.05 and P(B∩C) = 0.03 then solve the following:

(i) P(AUC) (ii) P(BUC) (iii) P(A') (iv) P(C') (v) If A and B are independent events then find P(AUB)

Answer (i) P(AUC) = P(A) + P(C) – P(A∩C) = 0.10 + 0.21 – 0.05 **= 0.26**

(ii) P(BUC) = P(B)+P(C) – P(B∩C) = 0.12 + 0.21 – 0.03 = **0.3**

(iii) P(A') = 1 – P(A) = 1 – 0.10 **= 0.90**

(iv) P(C') = 1 – P(C) = 1 – 0.21 **= 0.79**

(v) If A and B are independent events therefore

P(AUB) = P(A)+P(B) = 0.10+0.12 **= 0.22**

Example: Two dice are rolled together. Find the probability of getting a doublet (Same number on both dice) or sum of faces as 4.

Solution:

When two dice are rolled together, there will be 6×6 = 36 outcomes.

{1, 2, 3, 4, 5 ,6} ×{ 1, 2, 3, 4, 5, 6} Let *S* be the sample space. Then *n* (*S*) = 36

Let *A* be the event of getting a doublet (or getting same number) and *B* be the event of getting face sum 4.

Then *A* = {(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)}

Then, n (A) = 6

Therefore P(A) = $\frac{n(A)}{n(S)} = \frac{6}{36}$

B = {(1,3),(2,2),(3,1)}

Then, n (B) = 3

Therefore P(B) = $\frac{n(B)}{n(S)} = \frac{3}{36}$

Now $A \cap B$ = {(2,2)} . So n ($A \cap B$) = 1.

Therefore P(A ∩ B) = $\frac{n(A \cap B)}{n(S)} = \frac{1}{36}$

Now, by addition theorem of probability,

P(A ∪ B) = P(A) + P(B) – P(A∩B)

= 6/36 + 3/36 – 1/36 = 8/36 = 2/9

Hence, the required probability is 2/9.

Example: Two dice are thrown then what is the probability that –

(i) Both dice show different numbers (ii) The total of the number on both dice are between 2 and 12 (iii) The total of the number on both dice are 9 (iv)The total of the number on both dice is 13 (v) The total of the number on both dice are from 2 to 12.

Example: In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the students is selected at random. Find the probability that

(i) The student opted for NCC but not NSS.

(ii) The student opted for NSS but not NCC.

Solution:

Here number of possible outcomes n(S) = 50

Let us define two events:

A : Student opted for NCC , therefore n(A) = 28

B: Student opted for NSS , therefore n(B) = 30

And n(A ∩ B) = 18

Therefore p(A) = $\frac{28}{50}$

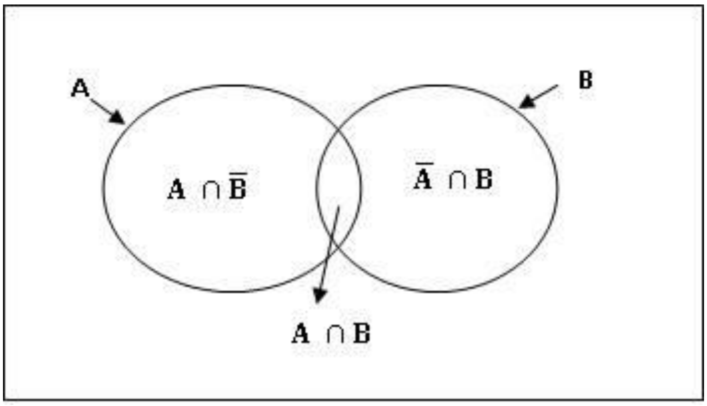
p(B) = $\frac{30}{50}$ and p(A ∩ B) = $\frac{18}{50}$

(i) Probability of the students opted for NCC but not NSS

p(A ∩ $\bar{B}$) = $P(A) - P(A \cap B)$ = 28/50 – 18/50 =1/5

(ii) Probability of the students opted for NSS but not NCC.

p($\bar{A}$ ∩ B) = $P(B) - P(A \cap B)$ = 30/50 – 18/50 = 6/25



Example: A die is thrown twice. Let A be the event, 'First die shows 5' and B be the event, 'second die shows 5'. Find P(A∪B) .

Solution:

Sample space n(S) = 36

Let A be the event that the first die shows the number "5"

A = {(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)}

$$n(A) = 6$$

$$P(A) = n(A)/n(S)$$

$$P(A) = 6/36$$

Let B be the event that the second die shows the number "5"

B = {(1, 5) (2, 5) (3, 5) (4, 5) (5, 5), (6, 5)}

$$n(B) = 6$$

$$P(B) = n(B)/n(S)$$

$$P(B) = 6/36$$

$A \cap B = \{5, 5\}$

$n(A \cap B) = 1$

$P(A \cap B) = n(A \cap B)/n(S)$

$$P(A \cap B) = 1/36$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= (6/36) + (6/36) - (1/36)$$

$$= (12 - 1)/36$$

$$= 11/36$$

Example : If A and B are mutually exclusive events P(A) = 3/8 and P(B) = 1/8, then find (i) P(A') (ii) P(A U B) (iii) P(A' ∩ B) (iv) P(A' U B')

Solution :

(i) P(A')

$P(A') = 1 - P(A) = 1 - (3/8)$

$P(A') = 5/8$

(ii) P(A U B)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Since A and B are mutually exclusive events, $P(A \cap B) = 0$.

$$P(A \cup B) = (3/8) + (1/8)$$

$$= (3 + 1)/8$$

$$= 4/8$$

$$P(A \cup B) = 1/2$$

(iii) $P(A' \cap B)$

$$P(A' \cap B) = P(B) - P(A \cap B)$$

$$= (1/8) - 0$$

$$P(A' \cap B) = 1/8$$

(iv) $P(A' \cup B')$

$$P(A' \cup B') = P(A \cap B)'$$

$$P(A \cap B)' = 1 - P(A \cap B)$$

$$= 1 - 0$$

$$P(A \cap B)' = 1$$

(A U B) ' = A' ∩ B'

(A ∩ B)' = A' U B'

Example1: According to a research survey, approximately 67 % of US people have Cable TV. 74 % of the US people have wireless TV. Suppose 55% of the US people have cable TV and wireless TV both. A US people is randomly selected then –

(i) What is the probability that a person has no cable TV?
(ii) Find Probability that a person has cable TV or Wireless TV

(iii) Find probability that a person has neither cable TV nor wireless TV
(iv) Find probability that a person has Cable TV or Wireless TV but not both

Let us define two events:

A: Person has cable TV so P(A) = 67/100 = 0.67

B: Person has wireless TV P(B) = 74/100 = 0.74

$A \cap B$: Person has both Cable TV and Wireless TV $P(A \cap B) = 55/100 = 0.55$

(i) $P(A')$ (ii) $P(A \cup B)$ (iii) $P(A \cup B)'$ (iv) $P(A \cup B) - P(A \cap B)$

Example : According to a survey 54 % of IT companies follow writing test for employee hiring process and 44 % IT companies follow practical viva test for employee hiring process. Assume that 35 % of IT companies follow both the process than –

(i) What is the probability that randomly selected company follow either writing test or viva test?

(ii) What is the probability that selected company follow writing or viva test but not both?

(iii) What is the probability that selected company follow neither writing nor viva test?

(iv) What is the probability that selected company does not follow writing test?

A: Company follows writing test

B : Company follows viva test

MCQ

1)If A and B are disjoint events than $P(A \cup B)$ = ____

a) $P(A \cap B)$ b) $P(A) - P(B)$ c) $P(A) + P(B)$ d) None of these

2)If P (A) = 0.10 than P (A') = ____

a) 0.80 b) 0.90 c) 0.10 d) 1

3)If A and B are disjoint events than P (A ∩ B) = _______

a) 0 b) 1 c) 0.5 d) 0.25

4)If A and B are disjoint events and P (A) = 0.21 and P (B) = 0.37 than P (A ∪ B) = _____

a) 0.16 b) 0.58 c) 0 d) 1

5) Which of the following is not an elementary event?

a) Toss a coin and getting Head b) Toss a coin and getting Tail c) Roll a die and getting 1 d) Roll a die and getting odd number.

6) Suppose any letter from English Alphabet is randomly selected than what is the probability that randomly selected alphabet is a vowel?

a) 1 b) 0 c) $\frac{5}{26}$ d) $\frac{21}{26}$

7) Suppose any letter from English Alphabet is randomly selected than what is the probability that randomly selected alphabet is not a vowel?

a) 1 b) 0 c) $\frac{5}{26}$ d) $\frac{21}{26}$

8) For any two events A and B, P(A) = 0.12 , P(B) = 0.24 and P(A ∩ B) = 0.08 than P(A ∪ B) = _____

a) 0.20 b) 0.44 c) 0.28 d) 0.4

9) Two dice are thrown than what is the probability that sum on both dice is 1?

a) $\frac{1}{36}$ b) $\frac{2}{5}$ c) 0 d) 1

CONDITIONAL PROBABILITY:

Let A and B are two non - disjoint events then conditional probability is denoted by p(A|B). This expression is read as the probability of A will occur given that B is known to have occurred.

Ex. A person owns ford car given that he owns Maruti car.

Formula for Conditional Probability:

P(A |B) = $\frac{P(A \cap B)}{P(B)}$

And P(B | A) = $\frac{P(A \cap B)}{P(A)}$

Example : From a city population, the probability of selecting –

(i) A male or smoker is $\frac{7}{10}$

(ii) A male smoker is $\frac{2}{5}$ **and**

(iii) A male, if a smoker is already selected is $\frac{2}{3}$**.**

Calculate the probability of selecting –

(a) A non-smoker

(b) A male

(c) A smoker, if a male is first selected

Solution:

Define the following events:

A: Male is selected

B: A smoker is selected

It is given that

P (A U B) $= \frac{7}{10}$, $P(A \cap B) = \frac{2}{5}$ **and P(A | B) =** $\frac{2}{3}$

(a) The probability of selecting a non- smoker:

$P(\overline{B}) = 1 - P(B)$

$= 1 - \frac{P(A \cap B)}{P(A|B)}$

$= 1 - \left(\frac{2/5}{2/3}\right)$

$= 1 - \left(\frac{2}{5} \times \frac{3}{2}\right)$

= 1 – 3/5

Therefore, P($\overline{B}$) = 2/ 5

Also P(B) = 3/5

(b) A male

P(A U B) = P(A) + P(B) - P(A ∩ B)

P(A) = P(A U B) – P(B) + P(A ∩ B)

Therefore P(A) = 7/10 – 3/5 +2/5

P(A) = 1/2

(c) **A smoker if a male is first selected**

P(B |A) = $\frac{P(A \cap B)}{P(A)}$ = ($\frac{2/5}{1/2}$) = ($\frac{2}{5} \times \frac{2}{1}$) = 4/5

Example: Given that P(A) = 0.10 , P(B) = 0.12 , P(C) = 0.21 , P(A ∩ C) = 0.05 and P(B ∩ C) = 0.03 then solve the following :

(i) P (A ∪ C) (ii) P(B ∪ C) (iii) P(A | C) (iv) P(C | A) (v) P(B | C) (vi) P (C | B)

Probability Matrices:

Probability Matrix displays Marginal (Individual) probabilities and Intersection Probability.

Example 1: Consider the following data of HR company:

	Male	**Female**
Manager	**8**	**3**
Assistant	**31**	**13**
Technician	**52**	**17**
Clerk	**9**	**22**

Then-

(i) Calculate probability of randomly selected employee is Male.
(ii) Calculate probability of randomly selected employee is Manager.
(iii)Calculate probability of randomly selected employee is Male or Manager.

(iv) Calculate probability of randomly selected employee is assistant or female. **(v) Find the probability that randomly selected employee is a Male and a Manager.** **(vi) Find the probability that randomly selected employee is Technician and Clerk.**
(vii) Find the probability that randomly selected employee is Technician or Clerk.

Solution:

	Male	Female	Total
Manager	8	3	11
Assistant	31	13	44
Technician	52	17	69
Clerk	9	22	31
Total	100	55	155

		E	F	
		Male	Female	Total
A	Manager	8/155 =0.05161	3/155 =0.01935	11/155 = 0.07096 = P(A)
B	Assistant	0.2	0.08387	0.28387 = P(B)
C	Technician	0.33548	0.10967	0.4451 =P(C)
D	Clerk	0.05806	0.14193	0.2 = P(D)
	Total	0.64516 = P(E)	0.35483 = P(F)	1

(i) Calculate probability of randomly selected employee is Male.

$P(E) = 0.64516$

(ii) Calculate probability of randomly selected employee is Manager.

$P(A) = 0.07096$

(iii) Calculate probability of randomly selected employee is Male or Manager.

$P(E \cup A) = P(E) + P(A) – P (E \cap A) = 0.64516 + 0.07096 – 0.05161 = 0.66451$

(iv) Calculate probability of randomly selected employee is assistant or female.

$P(B \cup F) = P(B) + P(F) – P(B \cap F) = 0.28387 + 0.35483 - 0.08387 = 0.55483$

(v) Find the probability that randomly selected employee is a Male and a Manager.

P (E ∩ A) = 0.05161

(vi) Find the probability that randomly selected employee is Technician and Clerk.

P(C ∩ D) = 0

(vii) Find the probability that randomly selected employee is Technician or Clerk

P (C ∪ D) = 0.4451 + 0.2 = 0.6451

Example 2:

Consider the following probability matrix:

	D	E	F
A	5	8	12
B	10	6	4
C	8	2	5

Calculate following probability equations:

(i) P(A|E) (ii) P(B|D) (iii) P(E|C) (iv) P(A') (v) P(B') (vi) P(C |F) (vii) P(E | A)

Solution:

	D	E	F	Total
A	5	8	12	25
B	10	6	4	20
C	8	2	5	15
Total	23	16	21	60

	D	E	F	Total
A	5/60 = 0.08333	8/60 = 0.1333	12/60 = 0.2	25/60 = 0.41666
B	0.1666	0.1	0.0666	0.3333
C	0.1333	0.0333	0.08333	0.25
Total	0.3833	0.2666	0.35	1

(i) P (A | E) = $\frac{P(A \cap E)}{P(E)} = \frac{0.1333}{0.2666}$ = 0.5 (ii) P(B|D) = $\frac{P(B \cap D)}{P(D)} = \frac{0.1666}{0.3833}$ = 0.4347

(iii) P(E|C) = $\frac{P(E \cap C)}{P(C)} = \frac{0.0333}{0.25}$ = 0.13332

(iv) P(A') = 1 – P(A) = 1 – 0.41666 = 0.5833

Example 3: Consider the contingency table for the feedback:

	YES	NO	NO OPINION
A	15	12	8
B	11	17	19
C	21	32	27
D	18	13	12

(i) Find the probability of "A" given that feedback was YES.

(ii) Find the probability of "C" given that feedback was NO.

(iii) Find probability that event "C" occurred or feedback NO OPINION generate. **(iv) Find probability of getting NO feedback and event "B".**

(v) Find probability of getting YES feed back given that "D" has already occurred.

(vi) Find probability of " No Opinion" given that "A" is already occurred.

Solution:

	E	F	G	
	YES	NO	NO OPINION	TOTAL
A	15	12	8	35
B	11	17	19	47
C	21	32	27	80
D	18	13	12	43
TOTAL	65	74	66	205

(i) P(A | E) (ii) P (C | F) (iii) P (C U G) (iv) P (F ∩ B) (v) P (E | D) (vi) P(G | A)

Example : Consider the following different business sector data according to geographical location survey of 200 employees.

	LOCATIONS			
	(A)NORTH EAST	(B)SOUTH WEST	(C)MID WEST	(D)WEST
FINANCE (E)	24	10	8	14
MANUFACTURE (F)	30	6	22	12
COMMUNICATION (G)	28	18	12	16

Assume that one employee is randomly selected then –

(i) What is the probability that selected employee is from west location or working in manufacture section? P(D U E)

(ii) What is the probability that selected employee is from South West or West location?

P(B U D)

(iii) What is the probability that selected employee is from west location and working in finance sector? P(D ∩ E)

(iv) What is the probability that selected employee is not form North East location? P(A')

Example 1 :Consider the following probability matrix:

	D	**E**	**F**
A	**6**	**10**	**4**
B	**12**	**4**	**4**
C	**2**	**6**	**12**

Calculate following probability equations:

(i) P(A|E) (ii) P(B|D) (iii) P(E|C) (iv) P(A') (v) P(B') (vi) P(C |F) (vii) P(E | A)

Example 2:Consider the following probability matrix:

	D	E	F
A	9	12	4
B	10	8	7
C	6	5	14

Calculate following probability equations:

1) P(A’) 2) P(B’) 3) P(C’) 4) P(A | E) 5)P (B | C) 6) P(C U F)

7) P(AU D)

	D	E	F	Total
A	9	12	4	25
B	10	8	7	25
C	6	5	14	25
Total	25	25	25	75

	D	E	F	Total
A	9/75=0.12	12/75 =0.16	4/75 =0.0533	25/75 =0.3333
B	0.1333	0.10666	0.0933	0.3333
C	0.08	0.0666	0.18666	0.3333
Total	0.3333	0.3333	0.3333	1

$P(A \cup D) = P(A) + P(D) – P(A\cap D)$

$= 0.3333 + 0.3333 – 0.12$

0.5466

Example 1 : Consider the following table :

	C	D	Total
A	41	8	49
B	45	6	51
Total	86	14	100

	C	D	Total
A	**0.41**	**0.08**	**0.49**
B	**0.45**	**0.06**	**0.51**
Total	**0.86**	**0.14**	**1**

Evaluate : (1) P (A | C) 2) P(B | D) 3) P(C | B) 4) P(D | A)

5) P(A ∪ D) 6) P(B ∪ C) 7) P(B ∪ D)

P (A | C) = $\frac{P(A \cap C)}{P(C)}$ = 0.41/ 0.86 = 0.4767

Example: Consider the following data:

	C	D	E
A	12	25	8
B	2	5	6

Evaluate:1) P(A ∪ C) 2) P (B ∪ D) 3) P (A ∪ E) 4) P (A | E)

5) P (B | E) 6) P (D | B) 7) P (E | B)

	C	D	E	
A	12	25	8	45
B	2	5	6	13
	14	30	14	58

P(A U C) = P(A) + P(C) – P (A ∩ C) = 45/58 + 14/ 58 – 12/58

= (45 + 14 – 12)/58 = 0.810

Example1: A random integer is chosen from 1 to 100. What is the probability that selected integer is divisible by 2 or 3?

Solution: A: Integer is divisible by 2

{ 2 , 4, ….., 100} = 50

P(A) = 50/100 = 0.50

B : Integer divisible by 3

{ 3, 6, 9, …., 99} = 33

P(B) = 33/100 = 0.33

A ∩ B : Integer is divisible by 2 and 3 (LCM of 2 and 3 = 6)

{ 6, 12, 18, …., 96} = 16

P (A ∩ B) = 16/100 = 0.16

P(A U B) = 0.50 + 0.33 – 0.16 = 0.67

Example2: Consider the following data:

	C	D	E
A	13	26	9
B	3	6	7

Calculate: 1) P(A U B) 2) P (B | E) 3) P(B U D) 4) P(A U E)

5)P(E | A) 6) P (B ∩ C) 6) P(D | B) 7) P(B | D)

	C	D	E	Total
A	13	26	9	48
B	3	6	7	16
Total	16	32	16	64

	C	D	E	Total
A	0.203	0.406	0.1406	0.75
B	0.0468	0.093	0.1093	0.25
Total	0.25	0.50	0.25	1

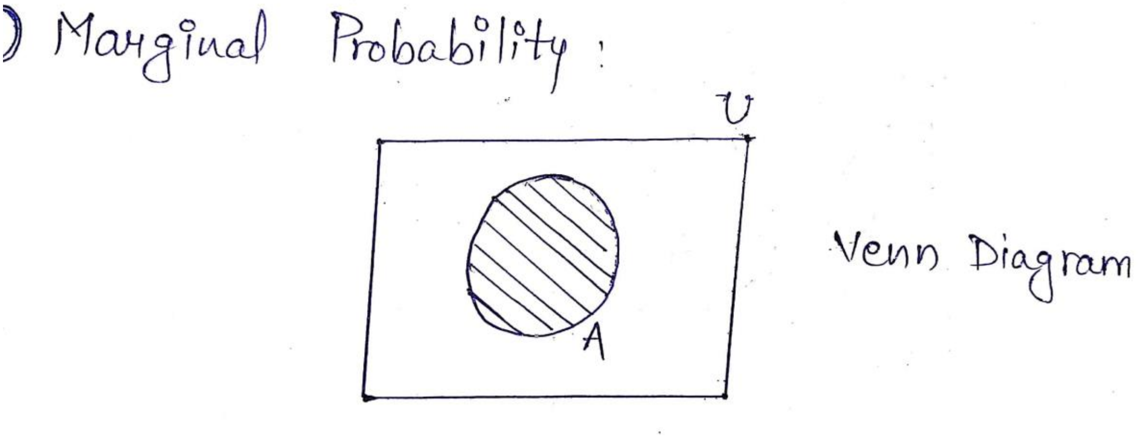
1) P(A U B) = P (A) + P(B) = 0.75 + 0.25 = 1

2) P(B | E) = P(B ∩ E) / P(E) = 0.1093 / 0.25 = 0.4372

Venn Diagrams

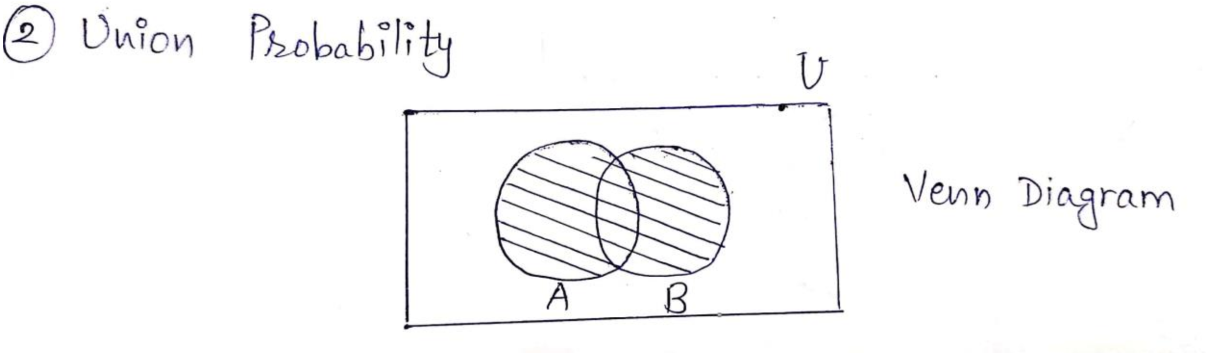
1) Marginal Probability:

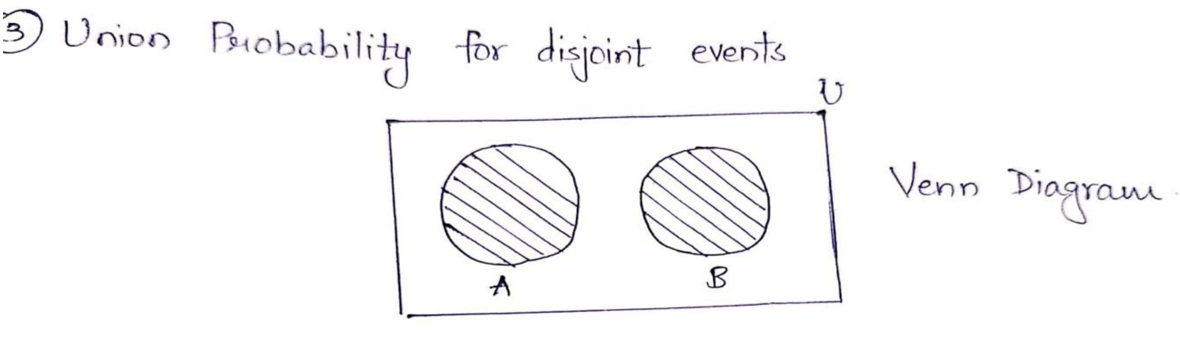
Let A be any event than P(A) denotes the marginal probability of event A.



2) Union Probability:

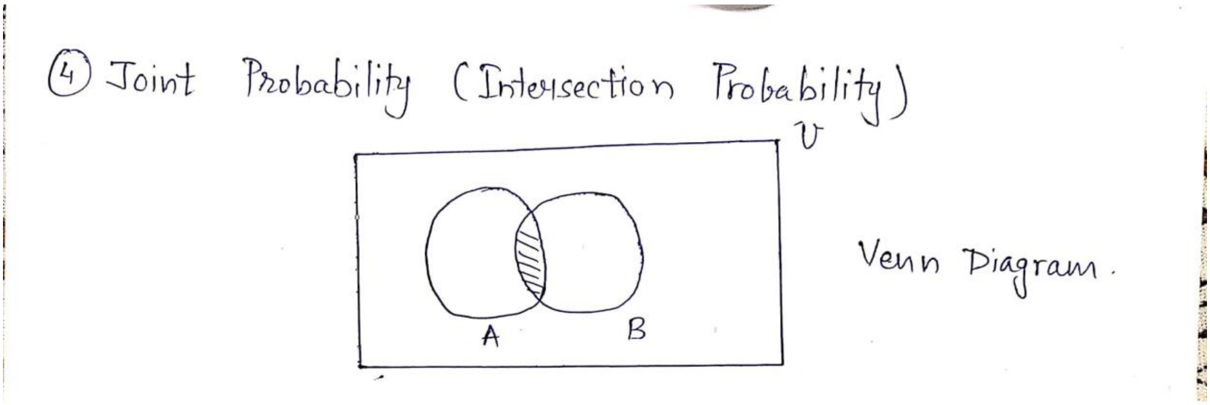
Let A and B are two events than union probability is denoted by P(A ∪ B) described as probability of event A or event B occurs.





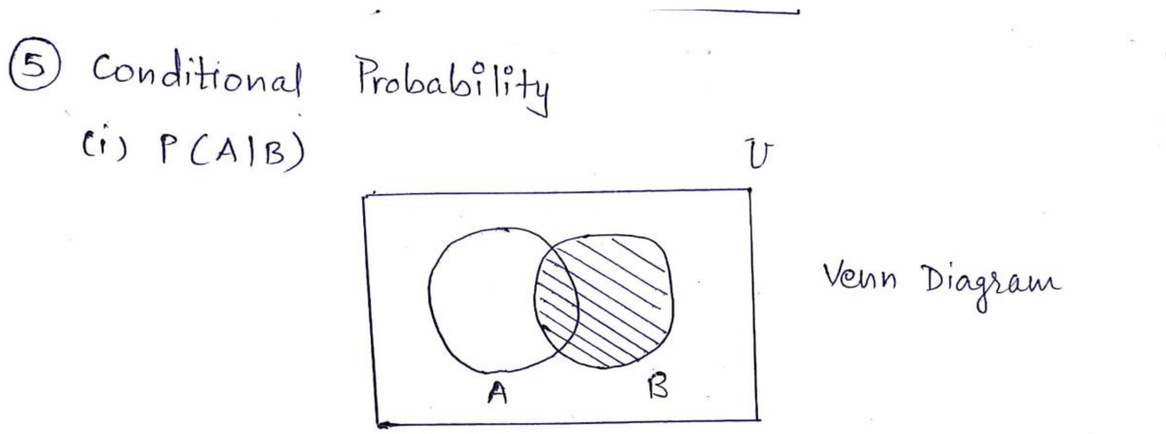
3) Joint Probability (Intersection Probability)

Let A and B are two events than joint probability is denoted by $P(A \cap B)$ described as probability of event A and event B occurred.



4) Conditional Probability:

Let A and B are two events than conditional probability is denoted by $P(A \mid B)$ described as probability of event A is occurring given that B is already occurred.



Short Questions:

1) If a die rolled 3 times than how many different outcomes are possible? $(6)^3 = 216$

2) If 4 coins are tossed than how many different outcomes are possible? $(2)^4 = 16$

3) Suppose in a lottery 6 numbers are drown from digits 0 to 9 with replacement (digits can be reuse) then how many different serial numbers are possible for the lottery?

$(10)^6$ = 10,00,000