LECTURE NOTES

UNIT 4: HYPOTHESIS TESTING

COURSE CONNTENT

Introduction
Types of Hypothesis
Rejection region, Non rejection region
Type-I and Type-II error
Testing hypothesis about a population mean using z – statistics (sigma known)
Testing hypothesis about a population mean using t – statistics (sigma unknown)

Some useful notations:

1) Sample Mean : $\bar{x}$

2) Population Mean: μ

3) Sample standard deviation: s

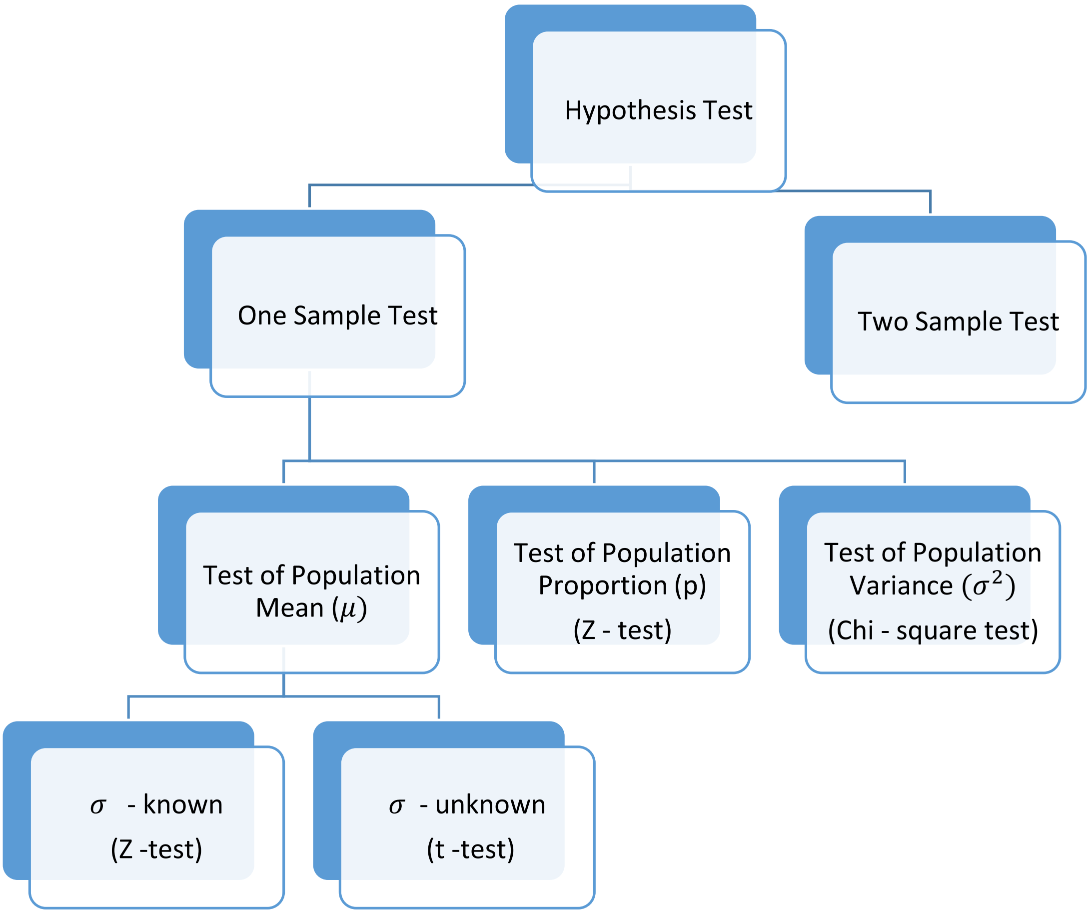
$$s = \sqrt{\frac{(x-\bar{x})^2}{n-1}}$$

4) Population standard deviation: σ

$$\sigma = \sqrt{\frac{(x-\mu)^2}{N}}$$

5) Sample size: n

6) Population size: N



Testing hypothesis about Population Mean (– known) using Z test :

Step 1:

Null Hypothesis (H_0) : μ = ____

Alternative Hypothesis (H_a) : $\mu \neq$ ____ (Two Tail test)

or $\mu >$ ____ or $\mu <$ ____ (One Tail test)

Step 2: Find Observed or Calculated Value

$$Z_{cal} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

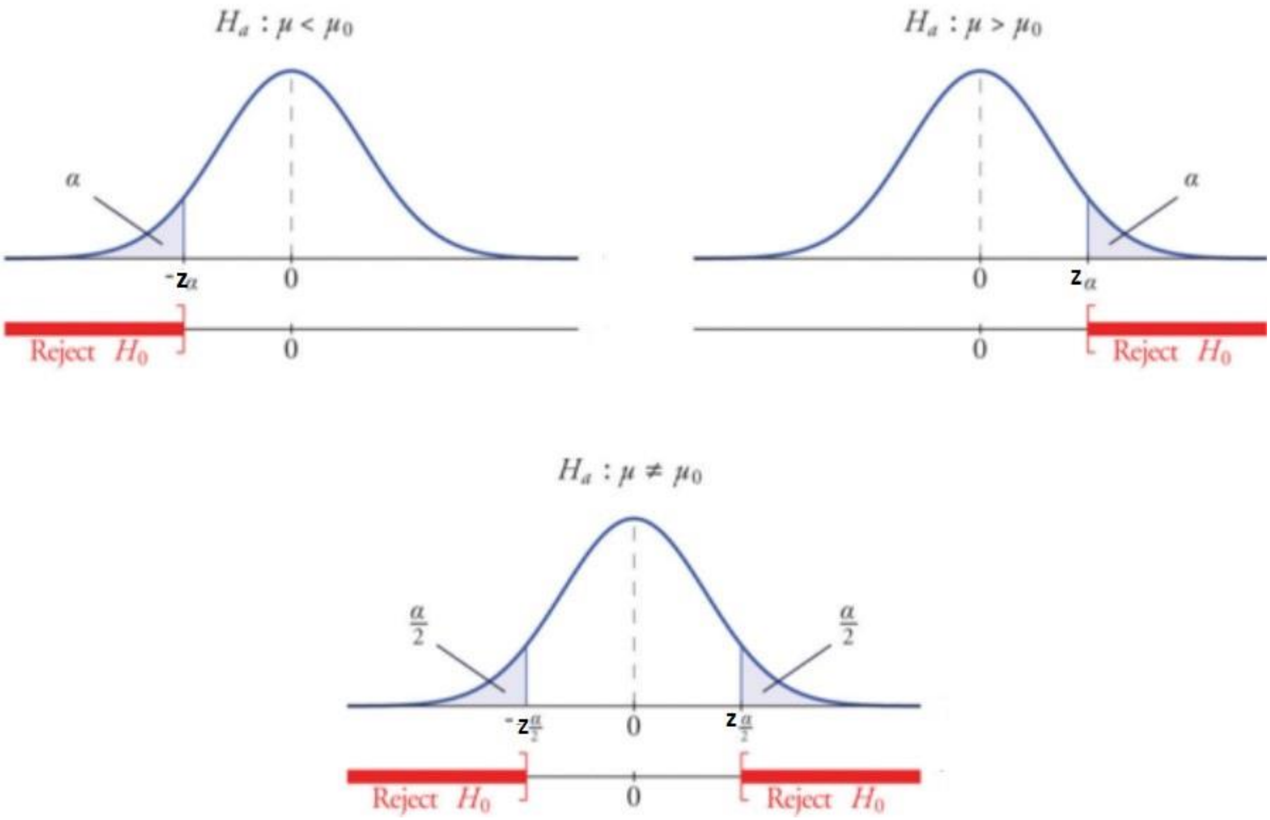
Step 3: Find Tabulated Value

- α is given in the question (If α is not given then consider $\alpha = 0.05$)

One Tail test	Two Tail Test
Null Hypothesis (H_0) : μ = ____ Alternative Hypothesis (H_a) : $\mu >$ ____ or μ < ____	Null Hypothesis (H_0) : μ = ____ Alternative Hypothesis (H_a) : $\mu \neq$ ____
Probability = 0.5 – α	Probability = 0.5 – ($\frac{\alpha}{2}$)

From Z – table , Z_{tab} = _____	From Z – table , Z_{tab} = _____

Step 4: Compare Z_{cal} and Z_{tab}



Step 5: Conclusion

If Null hypothesis is in accepted area, then Null hypothesis is accepted.

If Null hypothesis is in rejected area, then Null hypothesis is rejected.

Example 1:

Analyse the following hypothesis:

$H_0 : \mu = 25$
$H_a : \mu \neq 25$
Sample Mean = 28.1
Sample Size= 57
Population Standard Deviation= 8.46
Significant Level = 0.01

Solution :

Here sample mean $(\bar{x})$ = 28.1

Sample size (n) = 57

Population standard deviation (σ) = 8.46

Significant Level (α) = 0.01

Step 1:

Here H_0 : μ = 25
H_a : $\mu \neq 25$

Step 2: Find Observed or Calculated Value

$$Z_{cal} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$\therefore Z_{cal} = \frac{(28.1-25)}{\frac{8.46}{\sqrt{57}}} = \frac{3.100}{\frac{8.46}{7.550}} = \frac{3.100}{1.121} = 2.765$$

Step 3: Find Tabulated Value

Here α = 0.01

Here we have to apply two tailed test

Therefore,

Probability = 0.5 –($\frac{\alpha}{2}$) = 0.5 – $\frac{0.01}{2}$ = 0.5 – 0.05 = 0.4950

From Z – table,

$Z_{tab} = \pm$ 2.575

Step 4 : Compare Z_{cal} and Z_{tab}

Image (Curve)

Step 5: Conclusion

Here $Z_{cal} > Z_{tab}$. Also Z_{cal} is in rejected region.

Therefore Null Hypothesis is rejected.

Example 2 : Test the following hypothesis:

H_0 : μ = 7.48
H_a : μ < 7.48
Sample Mean = 6.91
Sample Size= 24
Population Standard Deviation= 1.21
Significant Level = 0.01

Solution:

Here,

Here sample mean ($\bar{x}$) = 6.91

Sample size (n) = 24

Population standard deviation (σ) = 1.21

Significant Level (α) = 0.01

Step 1:

Here $H_0 : \mu = 7.48$
$H_a : \mu < 7.48$

Step 2: Find Observed or Calculated Value

$$Z_{cal} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$\therefore Z_{cal} = \frac{(6.91 - 7.48)}{\frac{1.21}{\sqrt{24}}} = \frac{-0.570}{\frac{1.21}{4.899}} = \frac{-0.570}{0.2457} = -2.308$$

Step 3: Find Tabulated Value

Here $\alpha = 0.01$

Here we have to apply one tailed test

Therefore,

Probability = 0.5 – α = 0.5 – 0.01 = 0.4900

Therefore, $Z_{tab} = -2.33$

(Z_{cal} and Z_{tab} always have same sign in one tailed test)

Step 4 : Compare Z_{cal} and Z_{tab}

Curve

Step 5: Conclusion

Here $Z_{cal} < Z_{tab}$. Also Z_{cal} is in accepted region.

Therefore Null Hypothesis is accepted.

$\therefore \mu = 7.48$

Example 3:

Test hypothesis for the given data:

$H_0 : \mu = 1200$
$H_a : \mu > 1200$
Sample Mean = 1215
Sample Size= 113

Population Standard Deviation= 100
Significant Level = 0.10

Solution:

Here,

Here sample mean ($\bar{x}$) = 1215

Sample size (n) = 113

Population standard deviation (σ) = 100

Significant Level (α) = 0.10

Step 1:

Here H_0 : μ = 1200
H_a : μ > 1200

Step 2: Find Observed or Calculated Value

$$Z_{cal} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$\therefore Z_{cal} = \frac{(1215-1200)}{\frac{100}{\sqrt{113}}} = \frac{15}{\frac{100}{10.630}} = \frac{15}{9.407} = 1.595$$

Step 3: Find Tabulated Value

Here α = 0.10

Here we have to apply one tailed test

Therefore,

Probability = 0.5 – α = 0.5 – 0.10 = 0.4000

Therefore, Z_{tab} = 1.28

(Z_{cal} and Z_{tab} always have same sign in one tailed test)

Step 4 : Compare Z_{cal} and Z_{tab}

Curve

Step 5: Conclusion

Here $Z_{cal} > Z_{tab}$. Also Z_{cal} is in rejected region.

Therefore Null Hypothesis is rejected.

$\therefore \mu > 1200$

Example 4:

Consider the following sample data:

3	4	4	5	4	4	3	4
5	4	4	4	5	4	4	4
4	4	4	5	5	4	3	5
5	5	4	4	4	4	4	4

The population standard deviation of this data is 0.57 with 0.05 significant levels. These are the ratings of different managers from the different areas of the market. Then test whether the rating of the managers are significantly lower than 4.30 or not.

Solution:

Here sample mean $(\bar{x}) = \frac{\sum x}{n} = \frac{133}{32} = 4.156$

Sample size (n) = 32

Population standard deviation (σ) = 0.574

Significant Level (α) = 0.01

Step 1:

Here $H_0 : \mu = 4.30$
$H_a : \mu < 4.30$

Step 2: Find Observed or Calculated Value

$Z_{cal} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

$\therefore Z_{cal} = \frac{(4.156-4.30)}{\frac{0.574}{\sqrt{32}}} = \frac{-0.144}{\frac{0.574}{5.657}} = \frac{-0.144}{0.101} = -1.426$

Step 3: Find Tabulated Value

Here $\alpha = 0.05$

Here we have to apply one tailed test

Therefore,

Probability = 0.5 – α = 0.5 – 0.05 = 0.4500

Therefore, $Z_{tab} = -1.64$

(Z_{cal} and Z_{tab} always have same sign in one tailed test)

Step 4 : Compare Z_{cal} and Z_{tab}

Curve

Step 5: Conclusion

Here $Z_{cal} < Z_{tab}$. Also Z_{cal} is in accepted region.

Therefore Null Hypothesis is accepted.

$\therefore \mu = 4.3$

Example 5: A manufacturer company produce a machinery part. Suppose average strength of this part is 5 pound per million. The company takes a random sample of 42 parts. The sample mean of strength of the part is 5.06 .Assume that population standard deviation is 0.28 then test whether average strength of part is 5 pounds per million or not.

(Do it yourself)

Testing hypothesis about Population Mean (– unknown) using t - test :

Step 1:

Null Hypothesis (H_0) : μ = ____

Alternative Hypothesis (H_a) : $\mu \neq$ ____ (Two Tailed test)

or $\mu >$ ____ or $\mu <$ ____ (One Tailed test)

Step 2: Find Observed or Calculated Value

$$t_{cal} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Step 3: Find Tabulated Value

- α is given in the question (If α is not given then consider $\alpha = 0.05$)

One Tail test	Two Tail Test
Null Hypothesis (H_0) : μ = ____ Alternative Hypothesis (H_a) : $\mu >$ ____ or $\mu <$ ____	Null Hypothesis (H_0) : μ = ____ Alternative Hypothesis (H_a) : $\mu \neq$ ____
Level = α Degree of freedom (df) = n – 1 = ___	Level = $\frac{\alpha}{2}$ Degree of freedom (df) = n – 1 = ___
From t – table , t_{tab} = ____	From t – table , t_{tab} = ____

Step 4: Compare t_{cal} and t_{tab} :

Curve

Step 5: Conclusion

If t_{cal} is in accepted area, then null hypothesis is accepted. Otherwise null hypothesis is rejected.

Example 1:

A random sample of 20 products is taken. The sample mean of this product is 16.45 and sample standard deviation is 3.59. Assume that the data is normally distributed with 5 % of significant level than analyse that whether population mean is 16 or not.

Solution:

Here,

Here sample mean $(\bar{x})$ = 16.45

Sample size (n) = 20

Sample standard deviation (s) = 3.59

Significant Level (α) = 5% = $\frac{5}{100}$ = 0.05

Step 1: Identify hypothesis

Here $H_0 : \mu = 16$
$H_a : \mu \neq 16$

Step 2: Find Observed or Calculated Value

$$t_{cal} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{(16.45 - 16)}{\frac{3.59}{\sqrt{20}}} = \frac{0.450}{\frac{3.59}{4.472}} = \frac{0.450}{0.803} = 0.560$$

Step 3: Find Tabulated Value

Here we have to apply two tailed test.

Therefore,

Level = $\frac{\alpha}{2} = \frac{0.05}{2}$ = 0.025

And degree of freedom (df) = n – 1 = 20 – 1 = 19

Therefore, $t_{tab} = \pm 2.093$

Step 4: Compare t_{cal} and t_{tab}

Curve

Step 5: Conclusion

Here $t_{cal} < t_{tab}$. Also t_{cal} is in accepted area.

Therefore, null hypothesis is accepted.

$\therefore \mu = 16$

Example 2: A random sample of 51 products is taken, Average of this sample is 58.42 and sample variance is 25.68 with 1% significant level. Then determine whether population mean is lower than 60 or not.

Solution:

Here,

Here sample mean $(\bar{x})$ = 58.42

Sample size (n) = 51

Sample variance (s^2) = 25.68

Sample standard deviation (s) = 5.068

Significant Level (α) = 1% = $\frac{1}{100}$ = 0.01

Step 1: Identify hypothesis

Here $H_0 : \mu = 60$
$H_a : \mu < 60$

Step 2: Find Observed or Calculated Value

$t_{cal} = \frac{\overline{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{(58.60 - 60)}{\frac{5.068}{\sqrt{51}}} = \frac{-1.580}{\frac{5.068}{7.141}} = \frac{-1.580}{0.710} = -2.225$

Step 3: Find Tabulated Value

Here we have to apply one tailed test.

Therefore,

Level = α = 0.01

And degree of freedom (df) = n – 1 = 51 – 1 = 50

Therefore, t_{tab} = - 2.403

Step 4: Compare t_{cal} and t_{tab}

Curve

Step 5: Conclusion

Here $t_{cal} < t_{tab}$. Also t_{cal} is in accepted area.

Therefore, null hypothesis is accepted.

$\therefore \mu = 60$

Example 3:

Consider the following data taken from a random sample:

1200, 1175, 1080, 1275, 1201, 1387, 1090, 1280, 1400, 1287, 1225

Use 5 % of significant level to determine whether population mean is greater than 1160 or not.