MECH 539: Computational Aerodynamics
Department of Mechanical Engineering, McGill University

Project #1: Numerical Methods for One-Dimensional Wave Equation
Due 19th. February, 2013

Solve the one-dimensional wave equation

$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial u}{\partial x} = 0.$$

Use the following schemes upwind, Lax, Lax–Wendroff, Leap-Frog, and MacCormack, for the initial condition

$$u = \frac{1}{2}\left(1 + \tanh[250(x - 20)]\right), \qquad 0 \leq x \leq 40$$

and exact Dirichlet boundary conditions. Choose initially 41 grid point mesh with $\Delta x = 1$, and compute to $t = 10$.

1. Solve this problem for all three methods for $\Delta t = 1.0$ and $\Delta t = 0.5$, and compare graphically with the exact stationary solution. Discuss the differences between the various solutions.

2. Perform a grid study in space and time with two of the schemes of your choice. [Note: Use grids of successive refinement starting with the 41 grid point mesh as the coarsest grid.

3. Select two of the schemes and derive its stability condition.

4. Select one of the schemes and demonstrate that its consistent.