MECH 539: Computational Aerodynamics

Final Project : Solve the Quasi One-Dimensional Euler Equations for Various Artificial Dissipation Schemes

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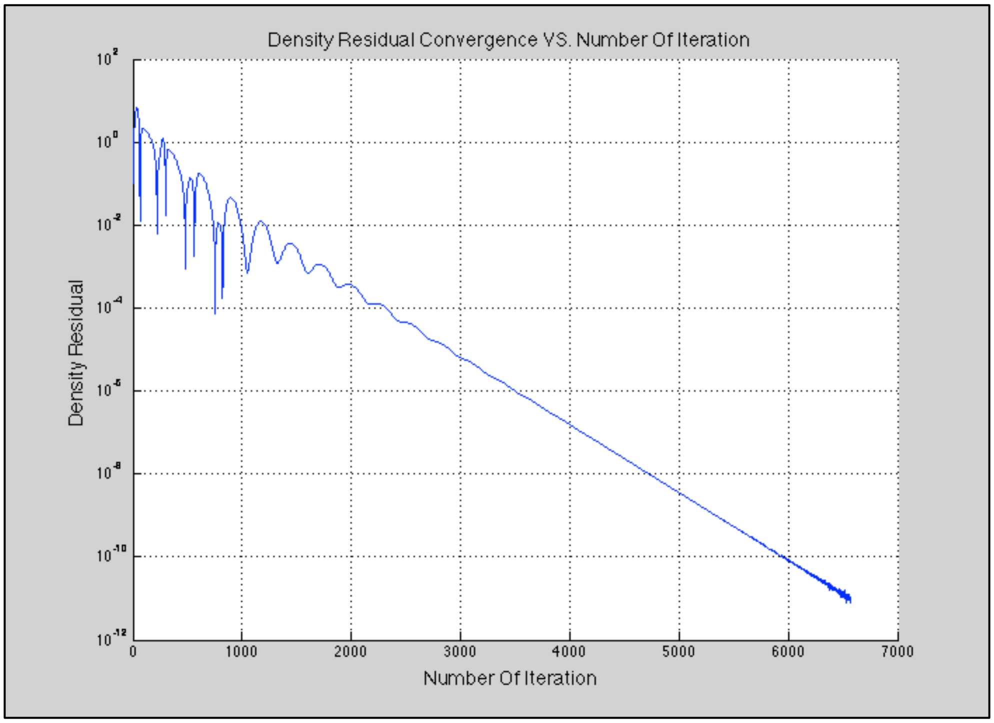
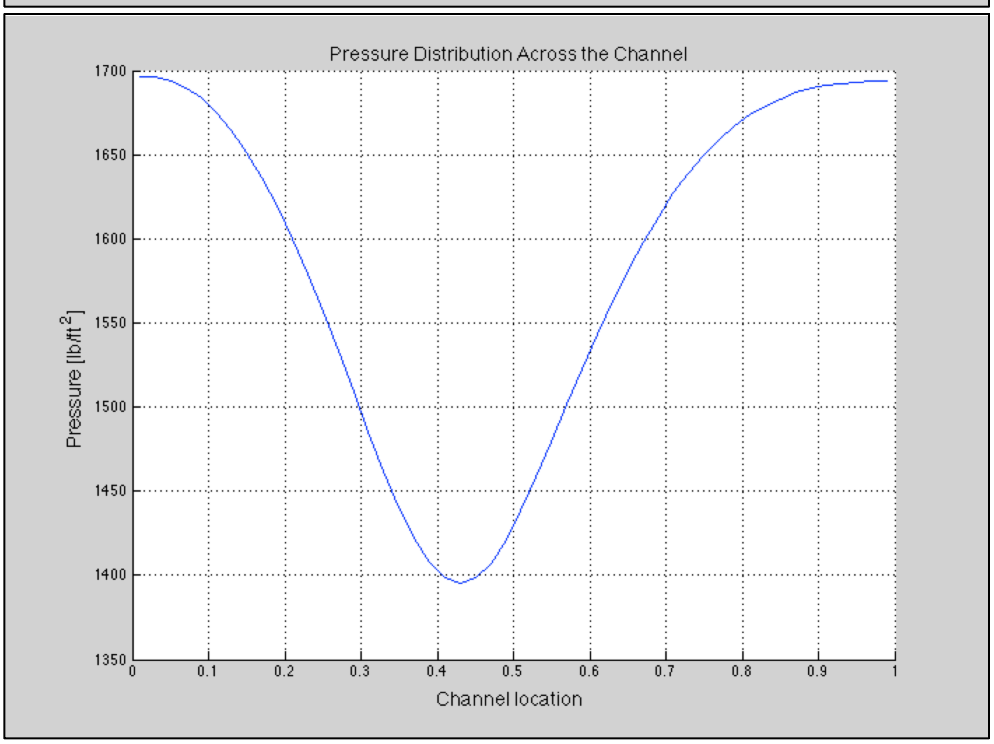
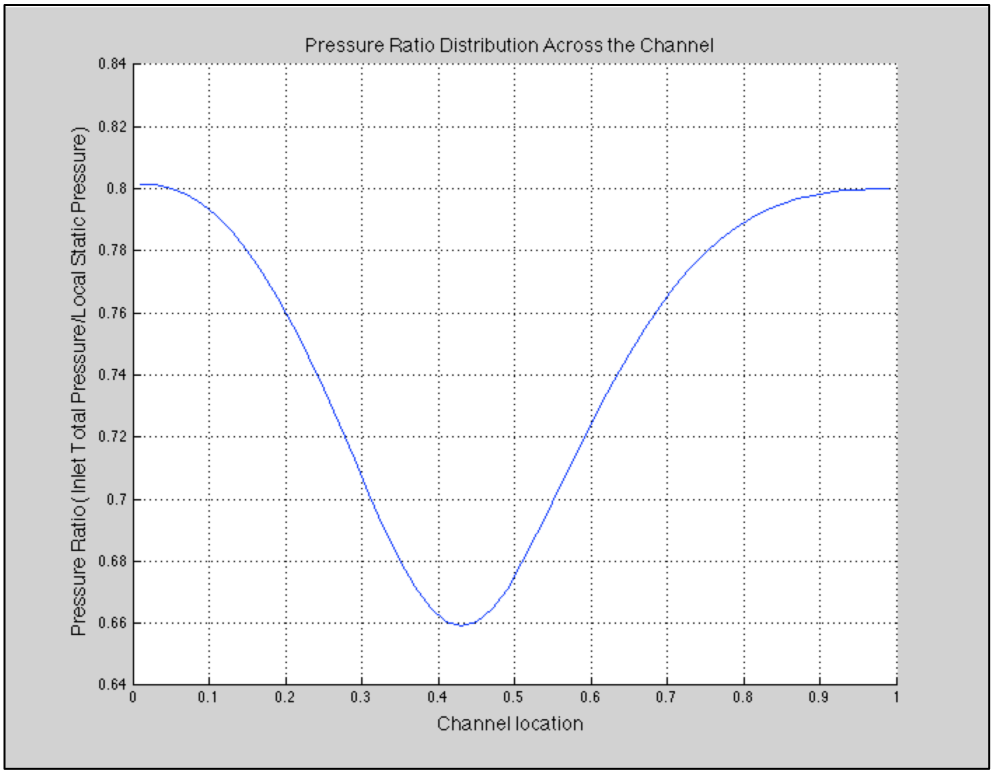
The tolerance of residual equation in all schemes was set to eps('single').

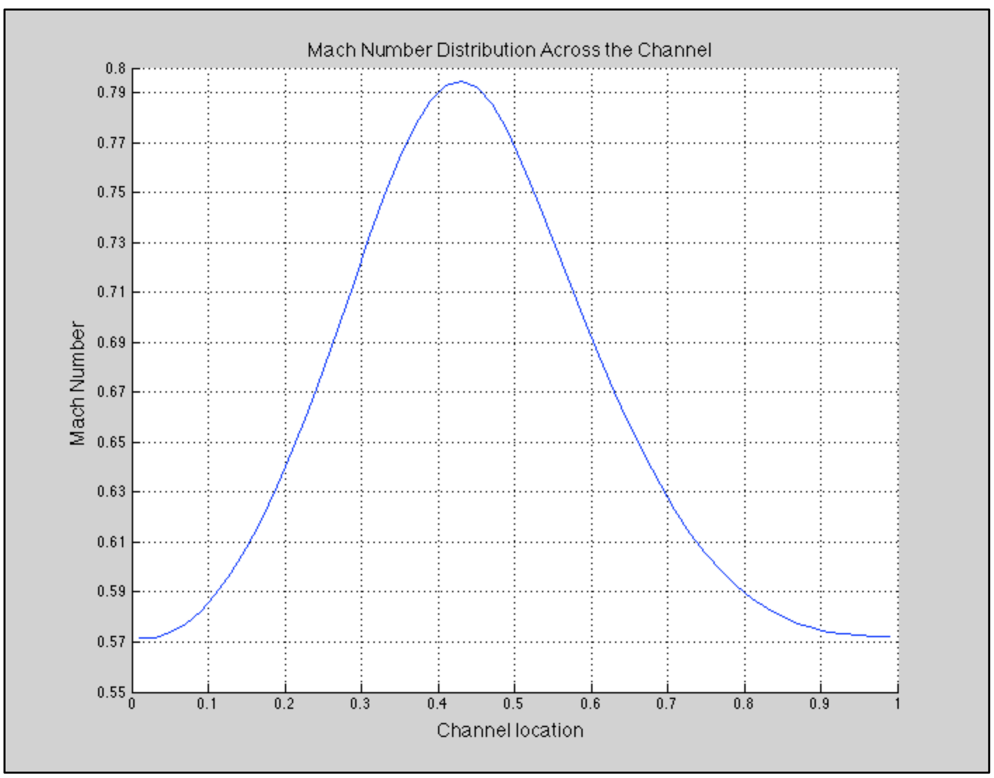
Question 1.

Quasi One-Dimensional Euler equation was solved using scalar dissipation scheme for the spatial discretization, and an Euler explicit scheme for the temporal discretization. Nozzle was discretized with 50 grid points. CFL value was set to 0.4 and epsilon value, ϵ, was set to 0.1.







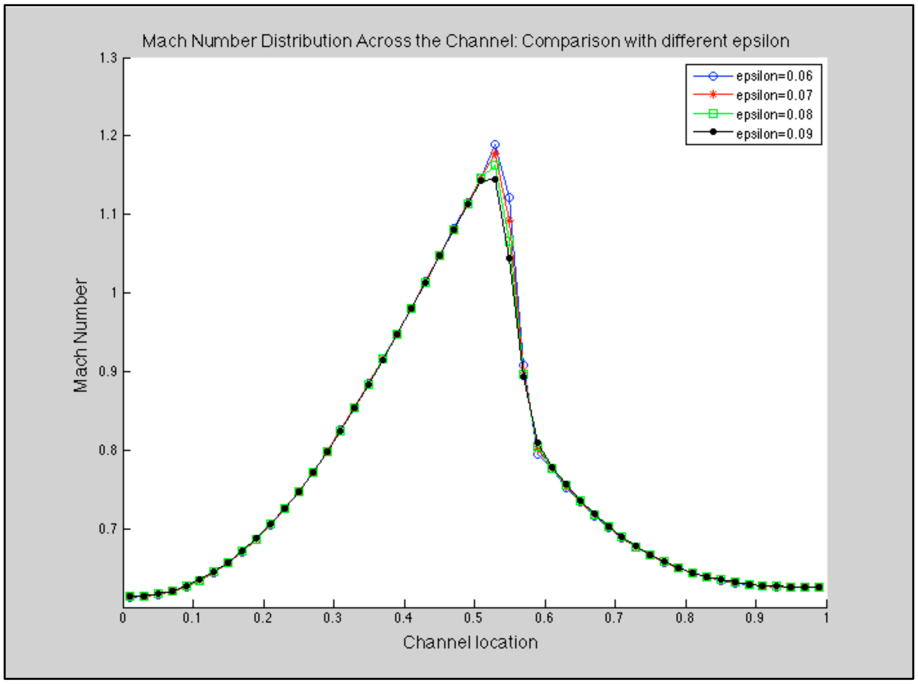


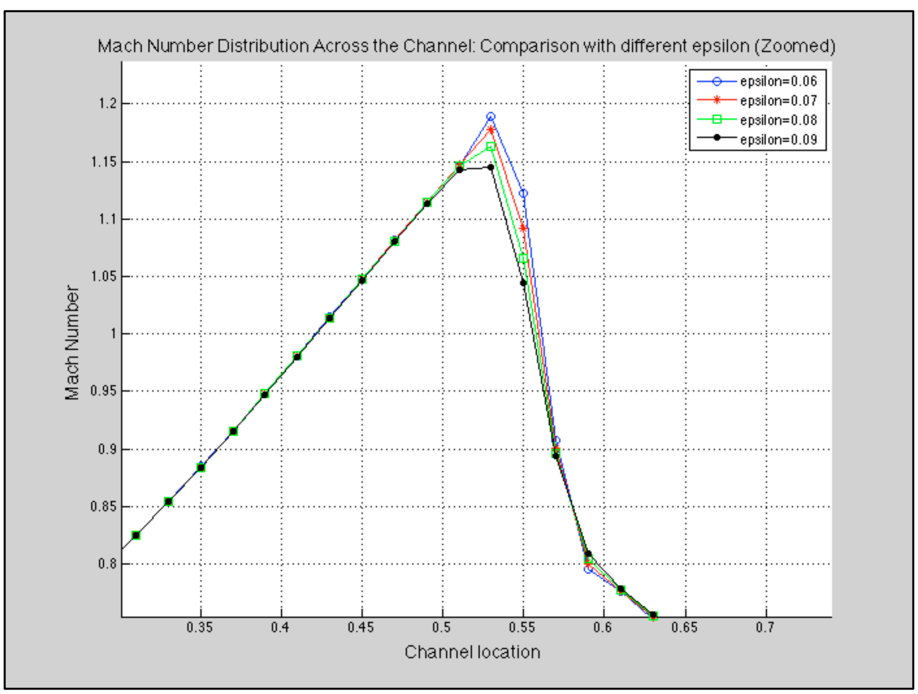
According to the plots above, the Mach number and pressure is inversely related. Since the flow is subsonic through entire nozzle, Mach number increases until the throat of the nozzle and decreases back again after the throat, while the pressure decreases until the throat of the nozzle and increase back after the throat.

Question 2.

Quasi One-Dimensional Euler equation was solved using scalar dissipation scheme for the spatial discretization, and an Euler explicit scheme for the temporal discretization. Nozzle was discretized with 50 grid points. CFL value was set to 0.4.

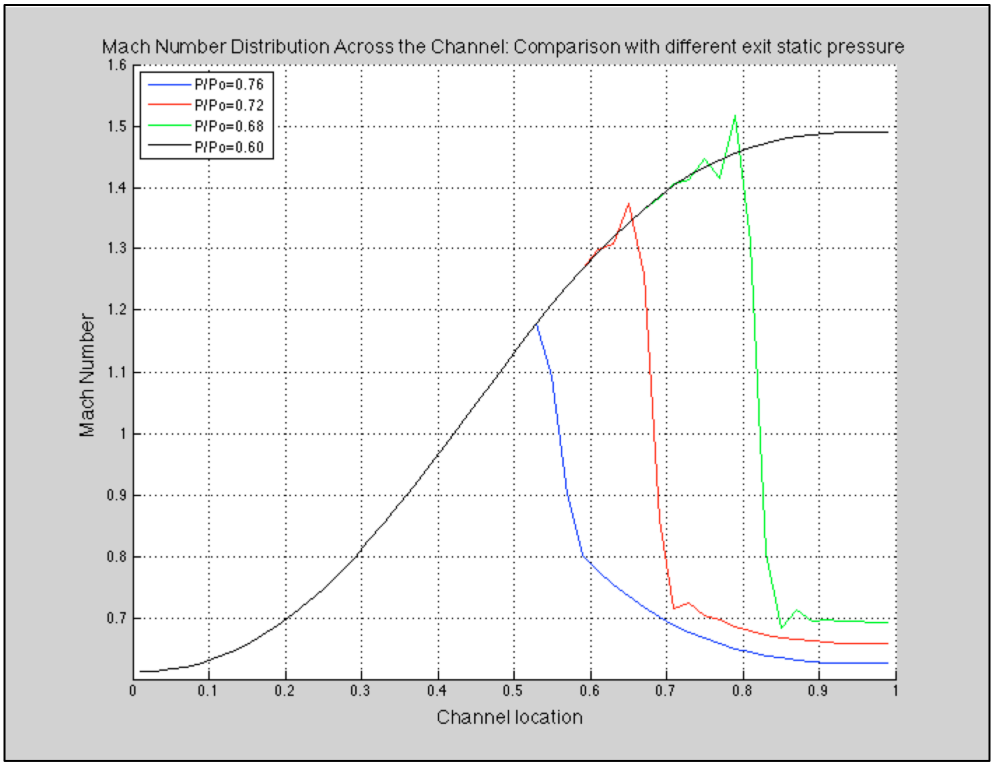
First, the epsilon value for pressure ratio of 0.76 was adjusted and the result was that higher epsilon value yielded results with larger dissipation error.



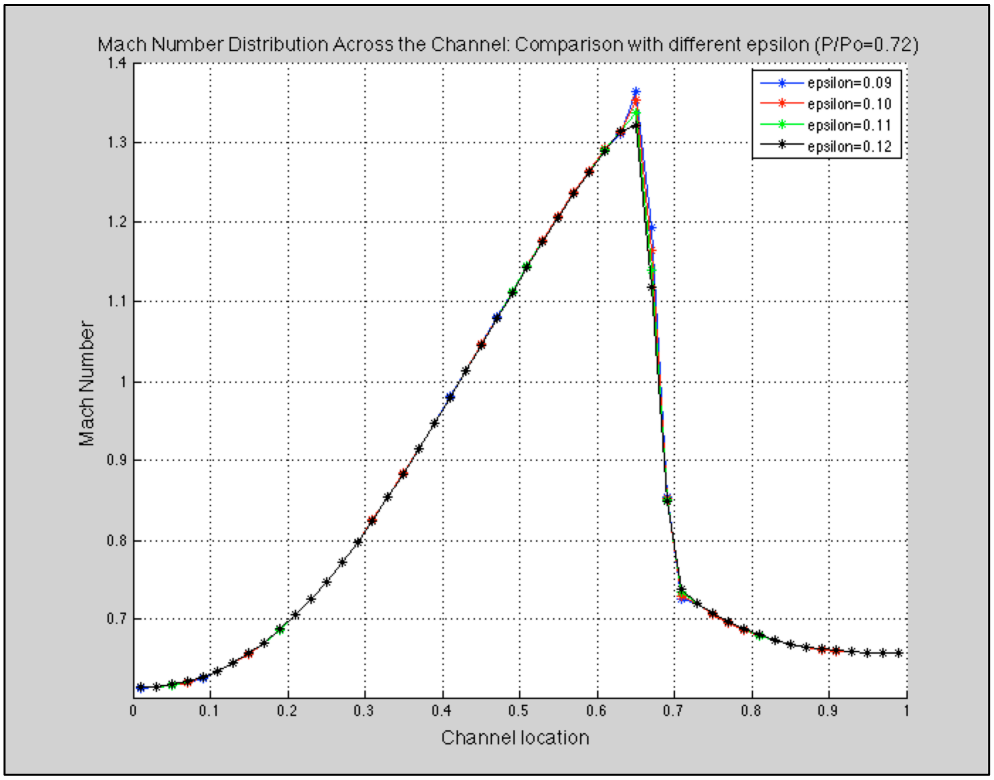


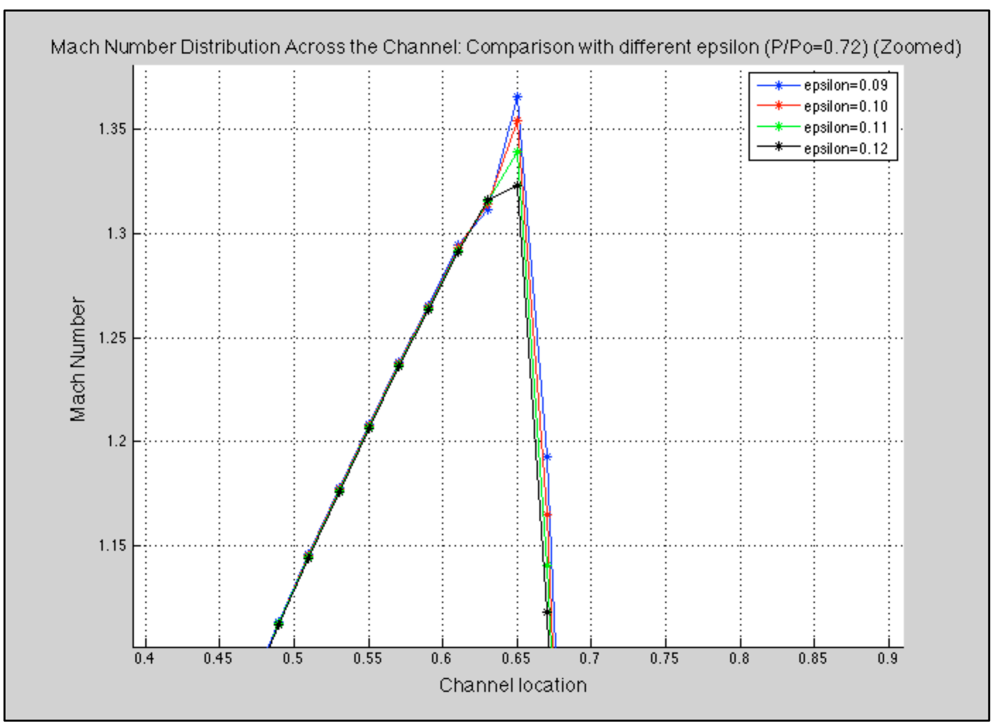
The above plots show the effect of epsilon value for the pressure ratio of 0.76. According to the plots, the epsilon value smaller than 0.07, the result is dissipative that the peak values of Mach number are smaller than they are supposed to be. But also, epsilon value larger than 0.07 introduced the dispersion error to the data that the peak value of Mach number increased larger non-linearly and also the next value of data was larger than it supposed to be so that the plot did not look smooth rather bit sharp at the shock. Therefore, epsilon value of 0.07 produced optimal result for the pressure ratio of 0.76.

And this epsilon value was tried for rest of cases.

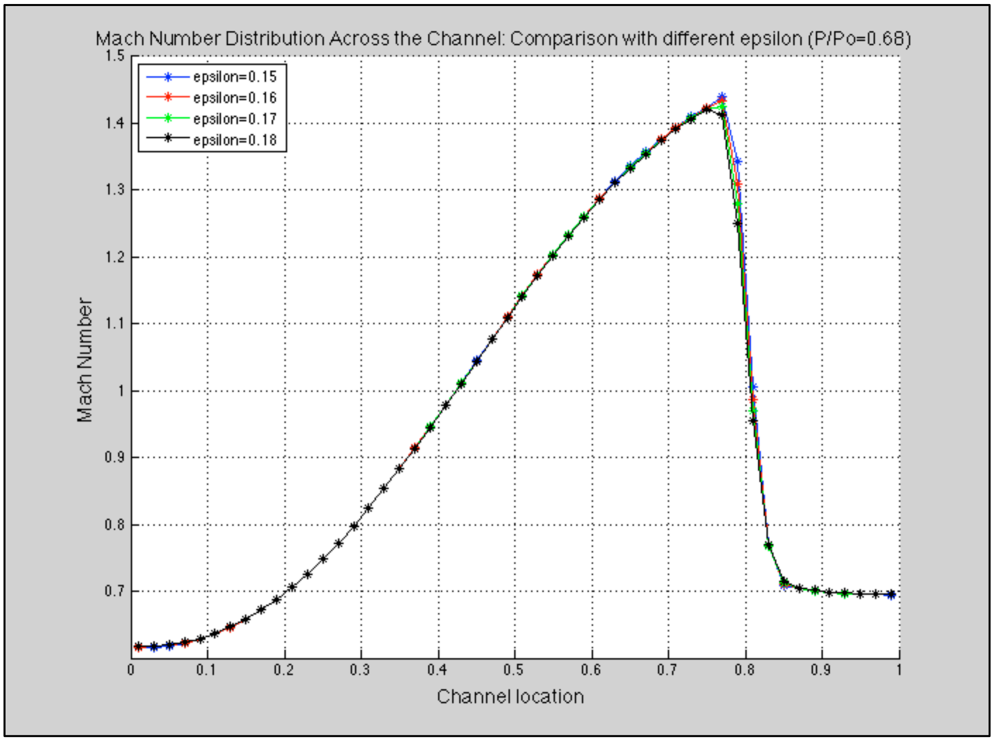


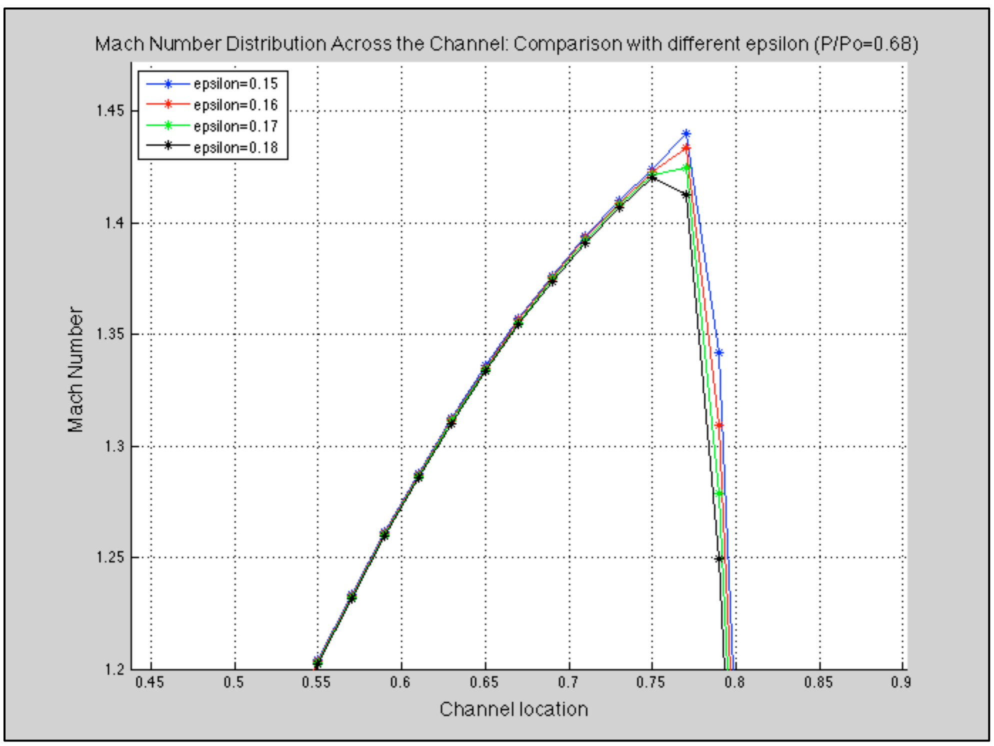
According to the plot above, the same epsilon value for lower pressure ratio cases produced the result with dispersion error, so the code was not stable. Thus, optimal epsilon value for each pressure ratio cases was investigated.



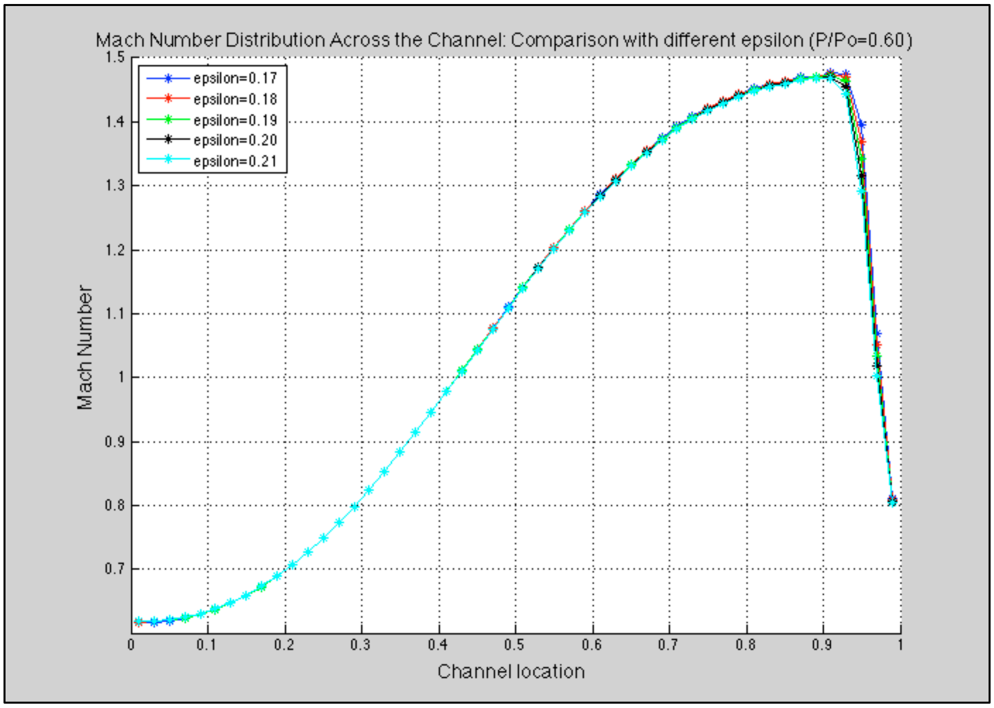


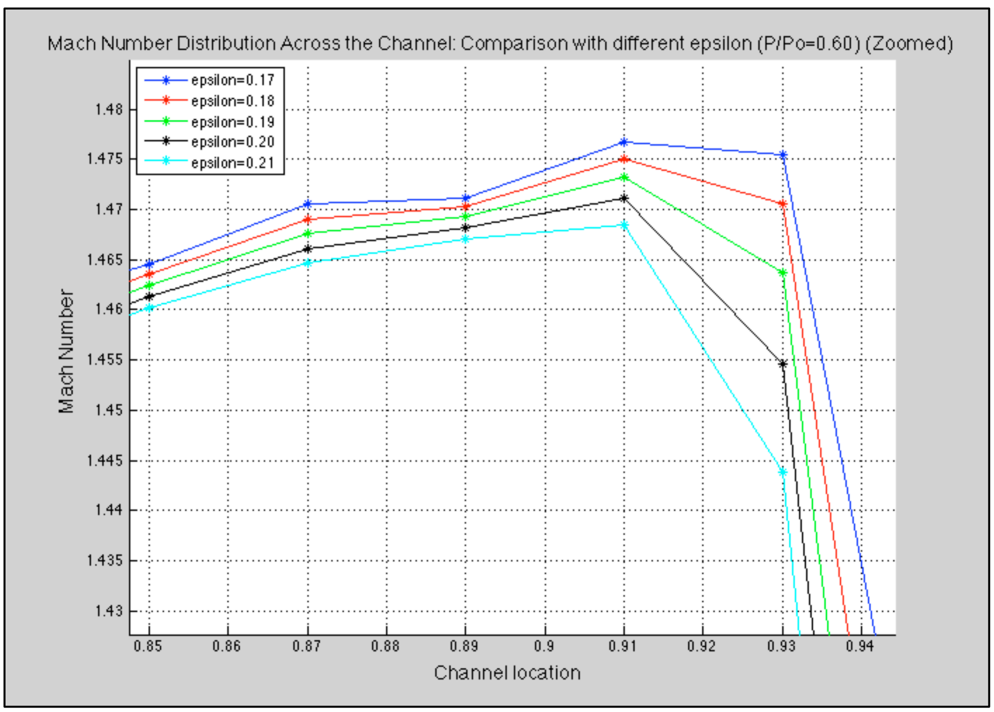
For the same analysis done above, the optimal epsilon value for the pressure ratio of 0.72 was 0.11.





For the same analysis done above, the optimal epsilon value for the pressure ratio of 0.68 was 0.16.

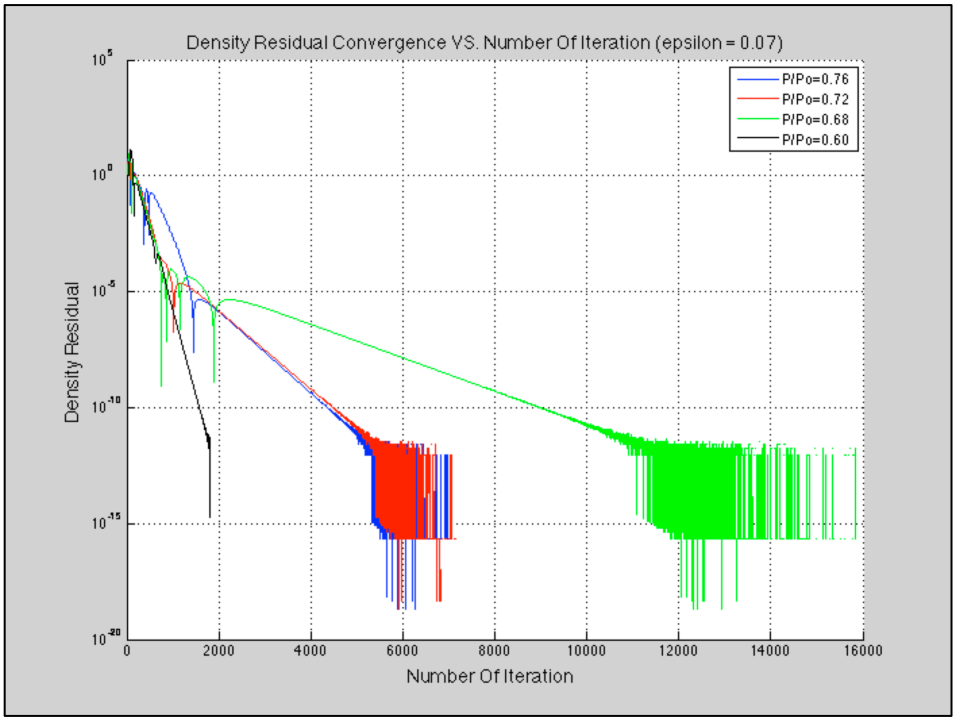


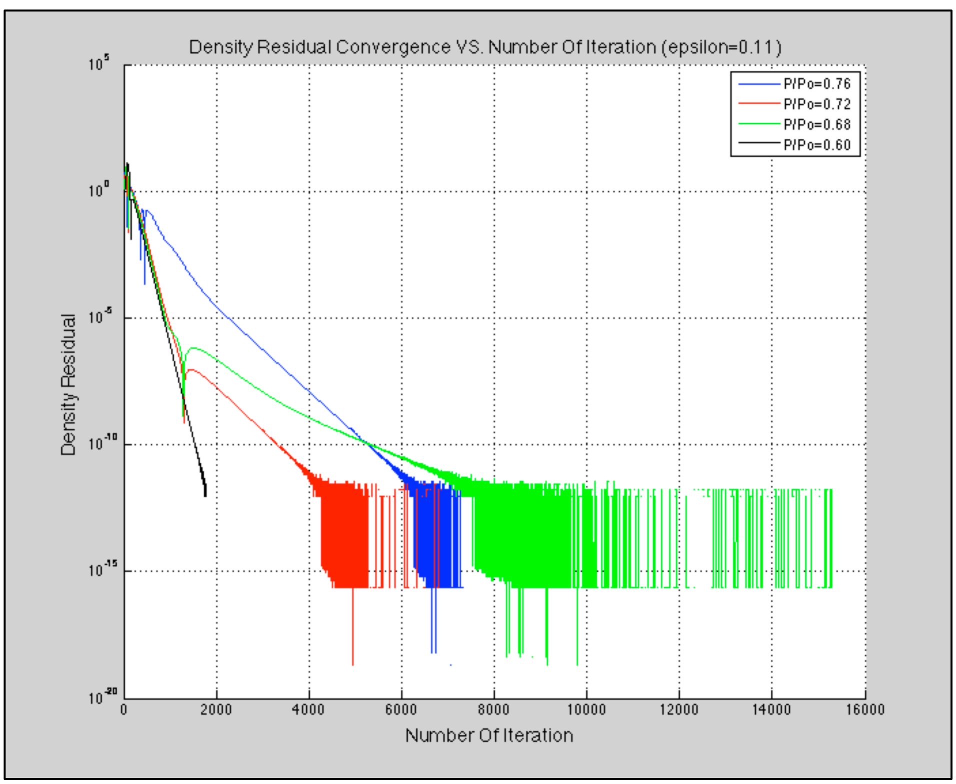


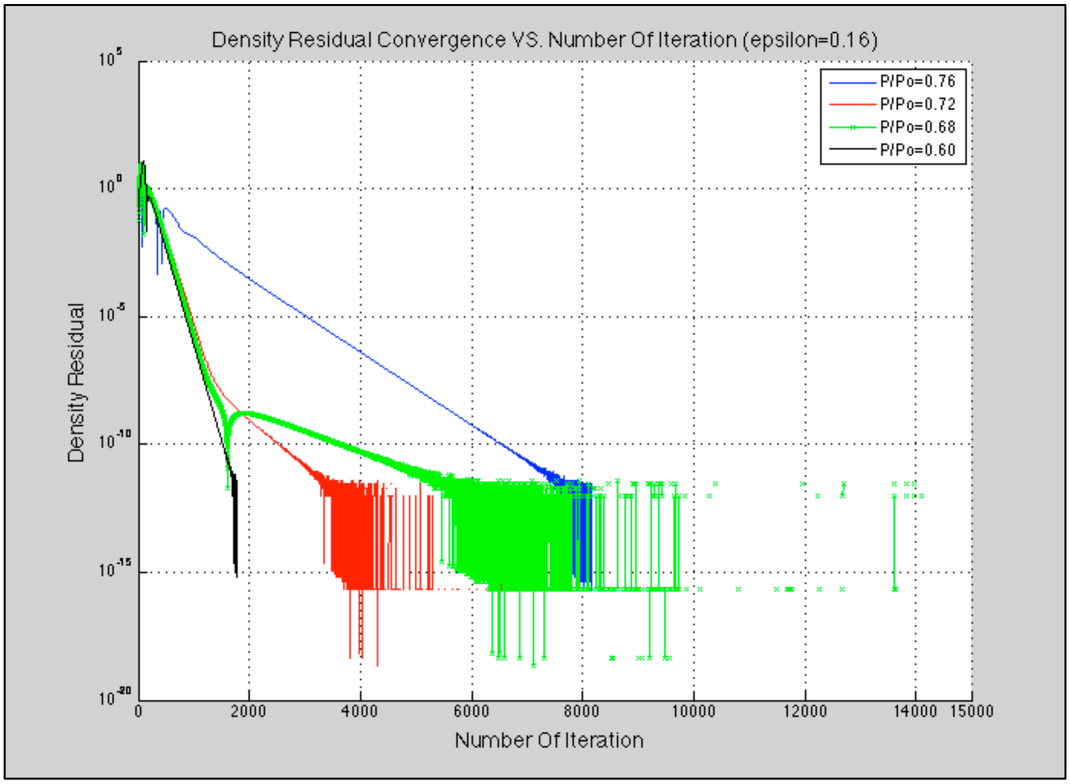
For the same analysis done above, the optimal epsilon value for the pressure ratio of 0.60 was 0.21.

Therefore, it could be concluded that there is optimal epsilon value for different pressure ratio cases that larger epsilon value introduces dissipation error to the data, while smaller epsilon value results the data with dispersion error. Moreover, for smaller pressure ratio case, optimal epsilon value was larger.

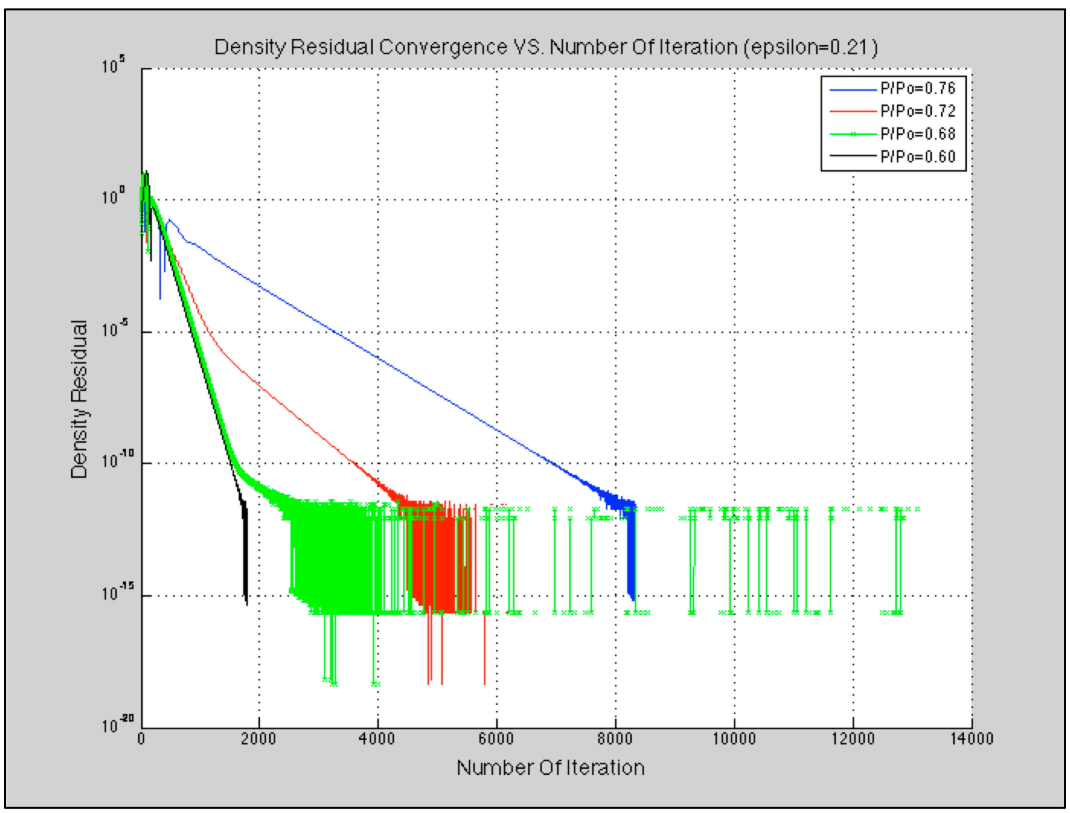
Residual Convergence





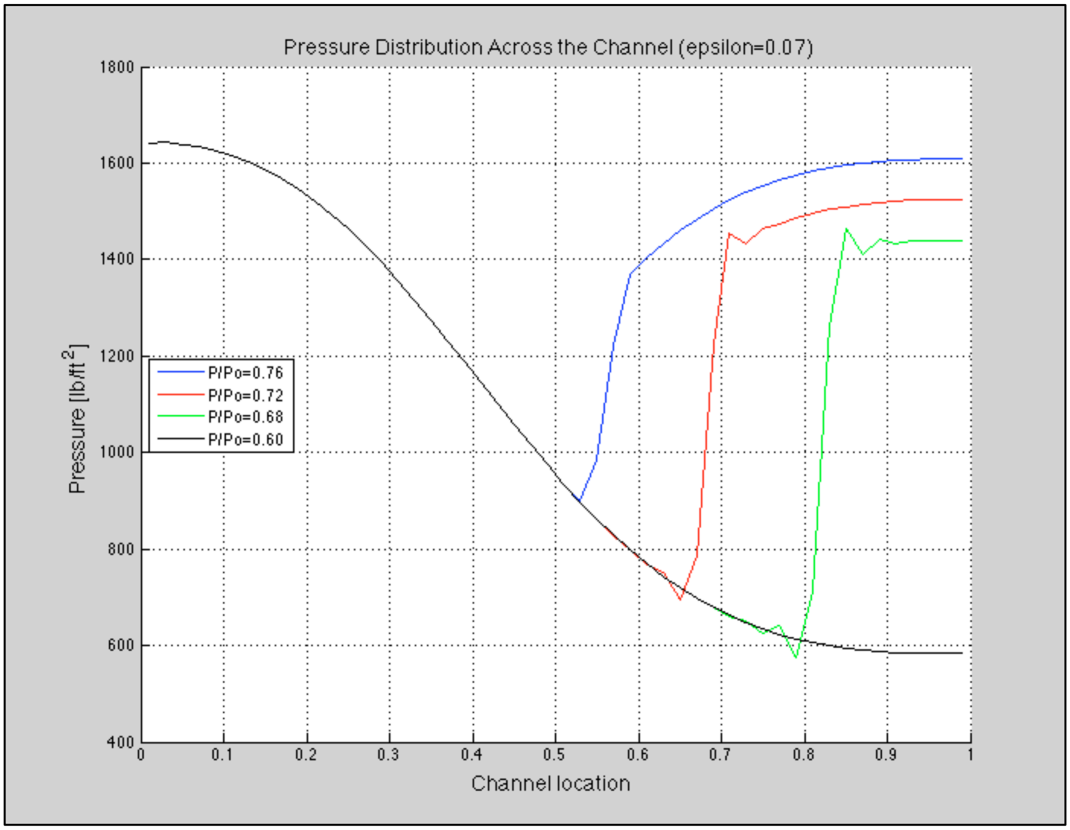


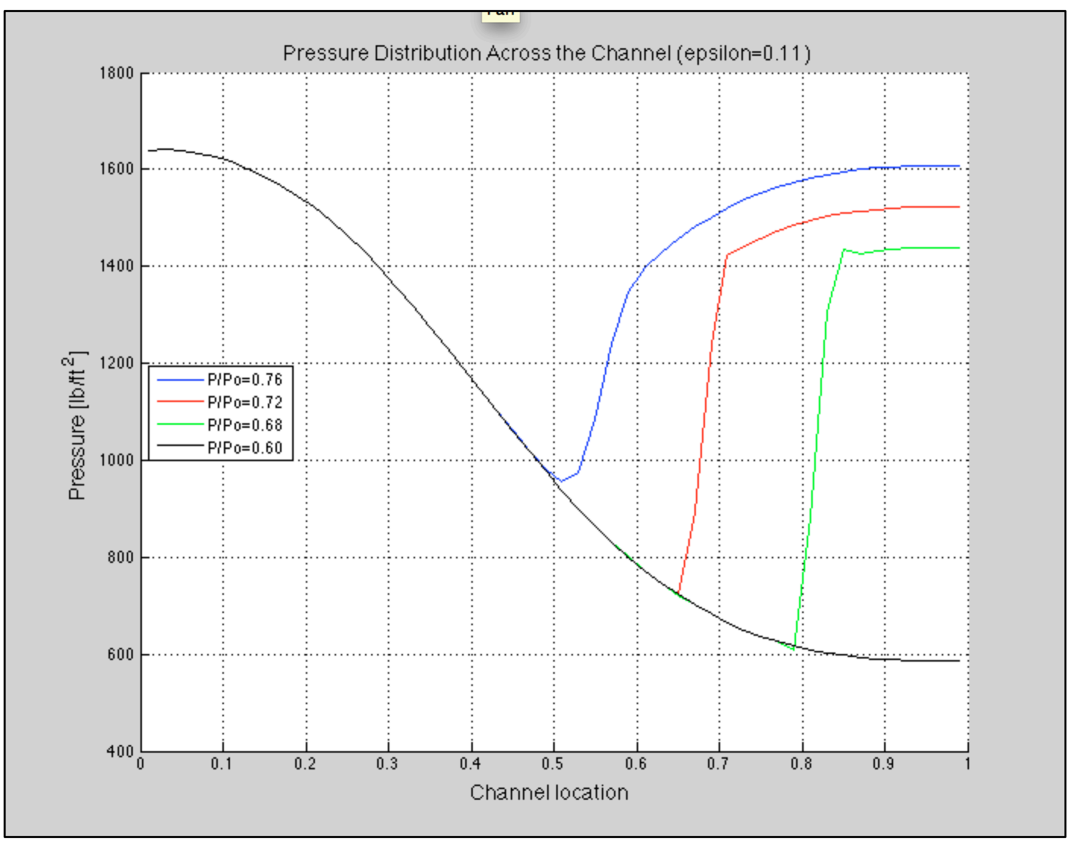
Density Residual Convergence VS. Number Of Iteration (epsilon=0.16)
P/Po=0.76
P/Po=0.72
P/Po=0.68
P/Po=0.60
Density Residual
Number Of Iteration

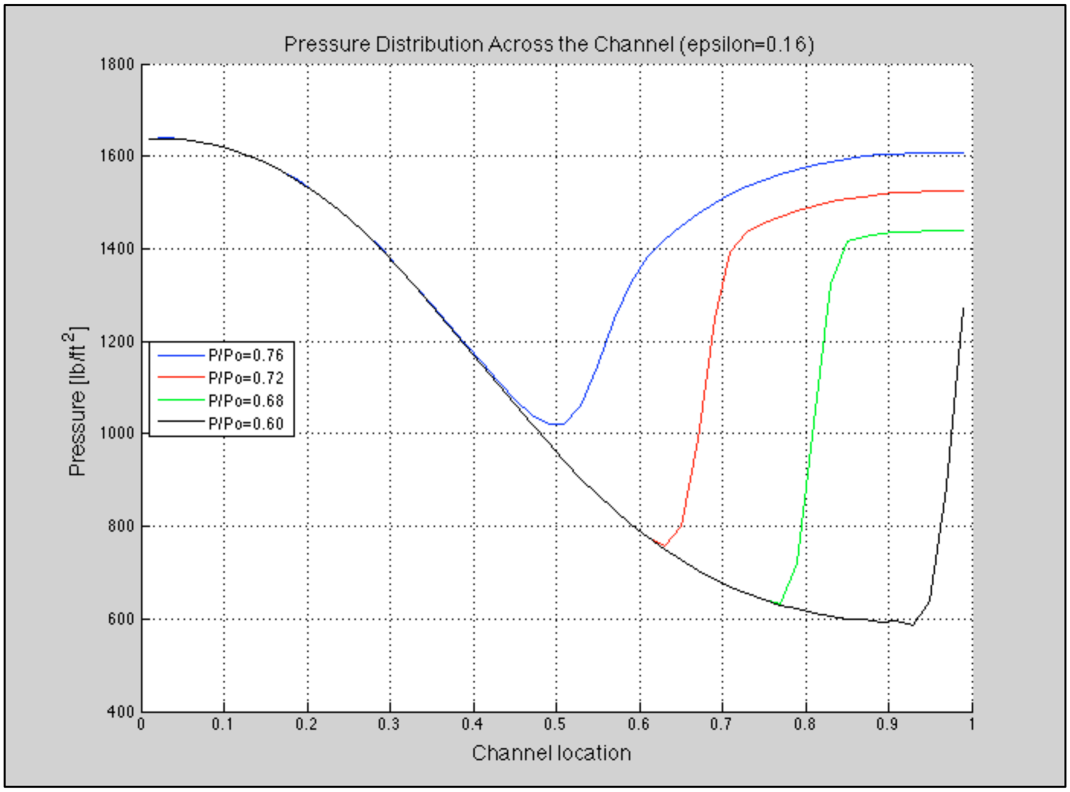


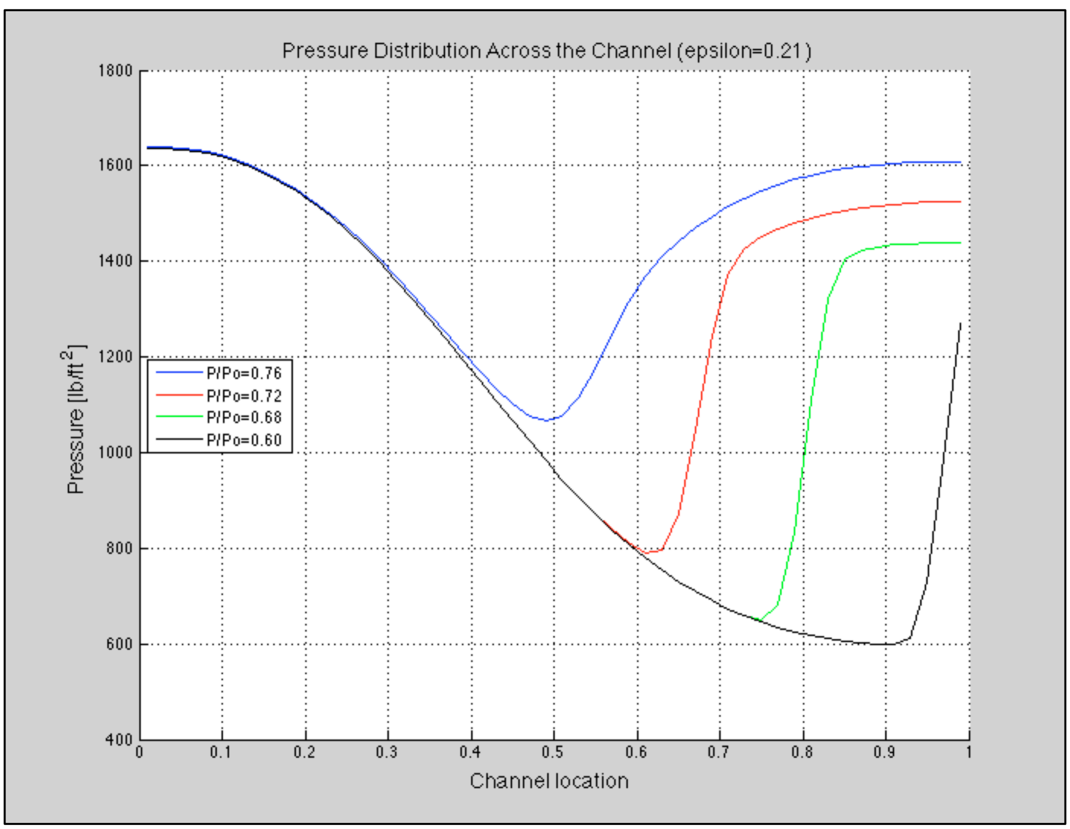
Density Residual Convergence VS. Number Of Iteration (epsilon=0.21)
P/Po=0.76
P/Po=0.72
P/Po=0.68
P/Po=0.60
Density Residual
Number Of Iteration

Pressure Distribution

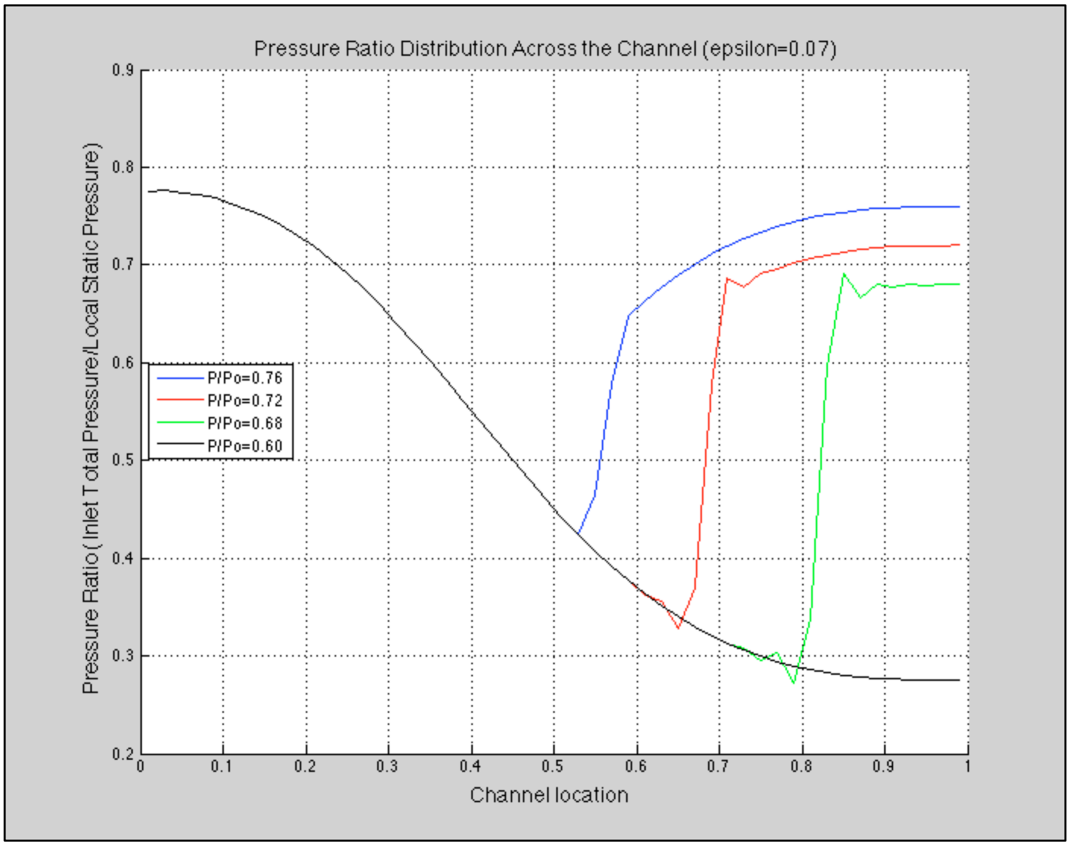


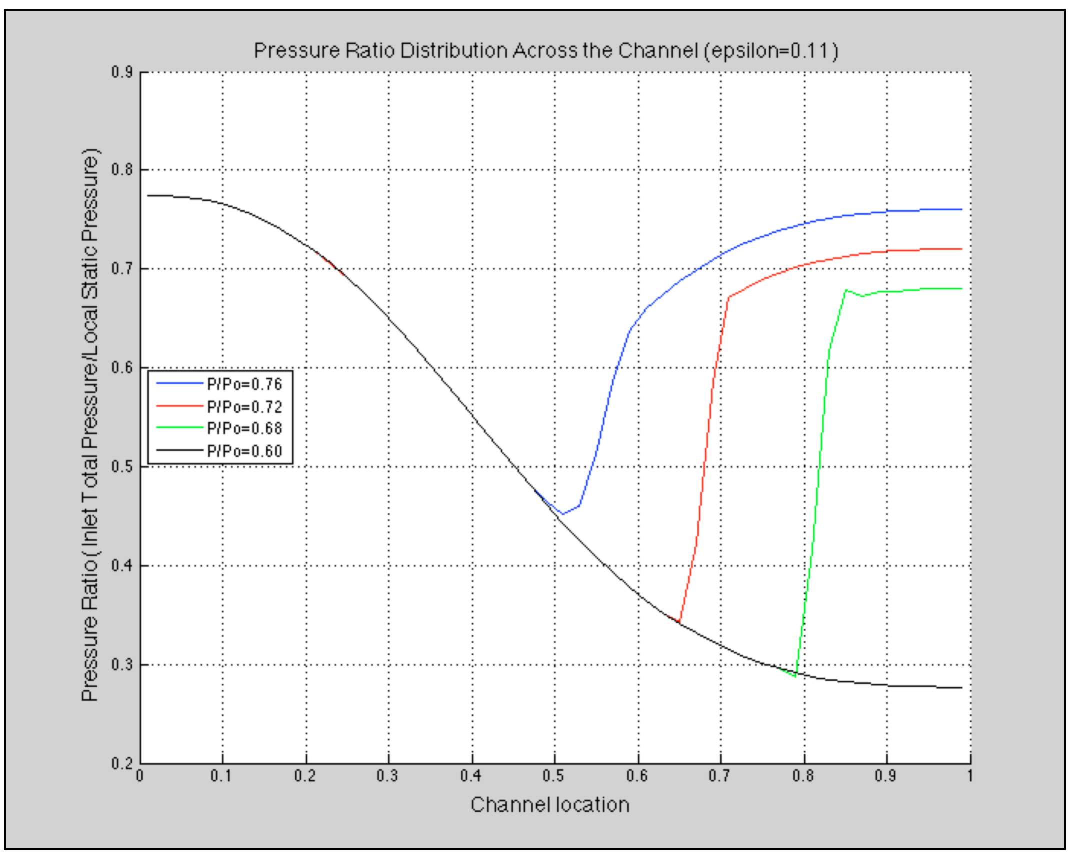


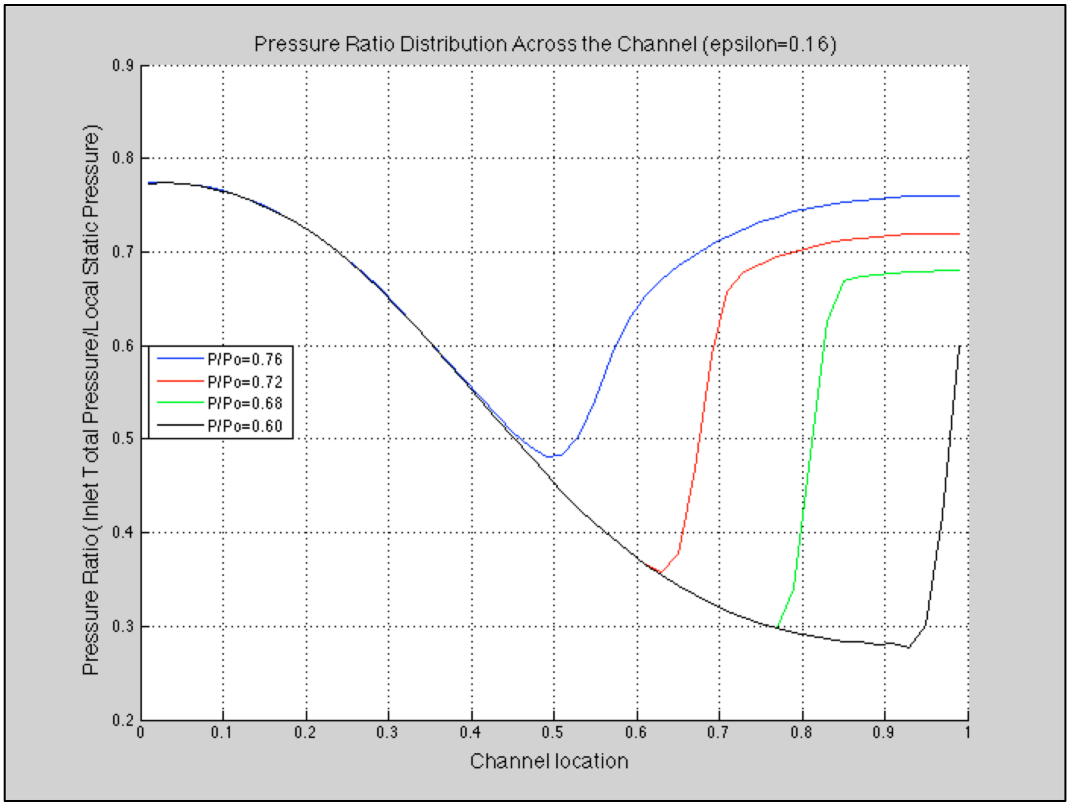

Pressure Distribution Across the Channel (epsilon=0.16)
Pressure [lb/ft2]
Channel location
P/Po=0.76
P/Po=0.72
P/Po=0.68
P/Po=0.60
1800
1600
1400
1200
1000
800
600
400
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

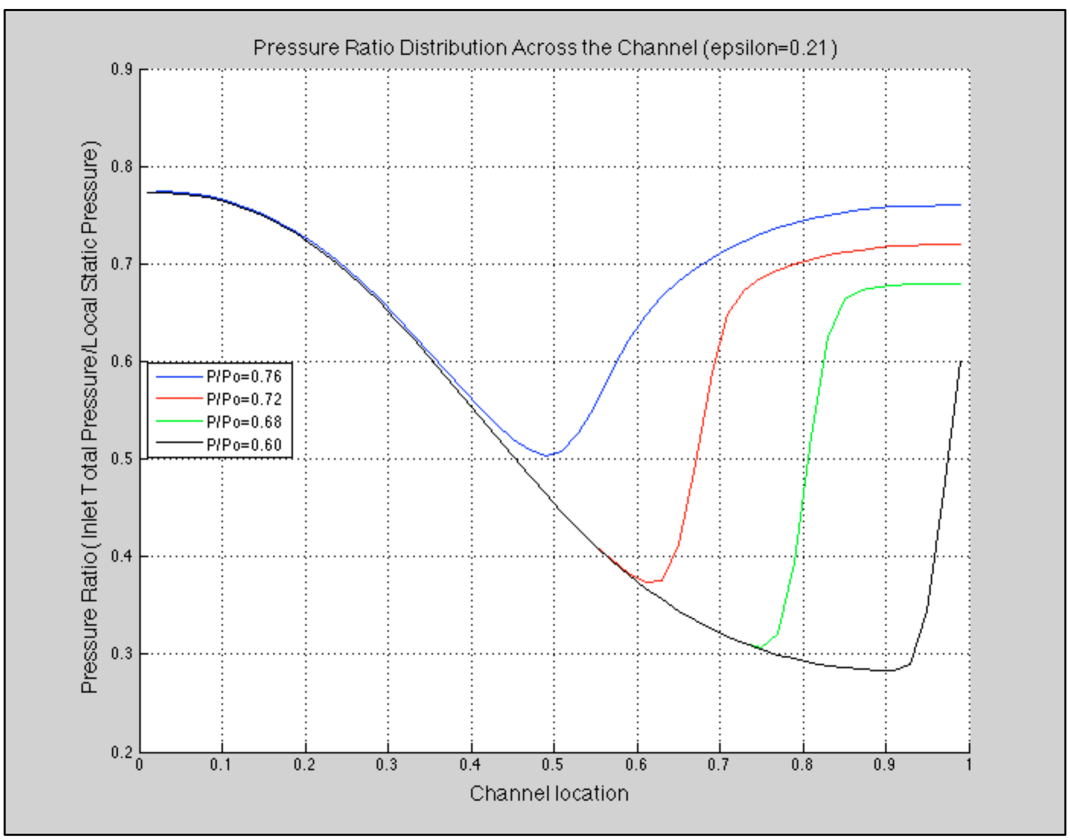

Pressure Distribution Across the Channel (epsilon=0.21)
Pressure [lb/ft2]
Channel location
P/Po=0.76
P/Po=0.72
P/Po=0.68
P/Po=0.60
1800
1600
1400
1200
1000
800
600
400
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Pressure Ratio Distribution

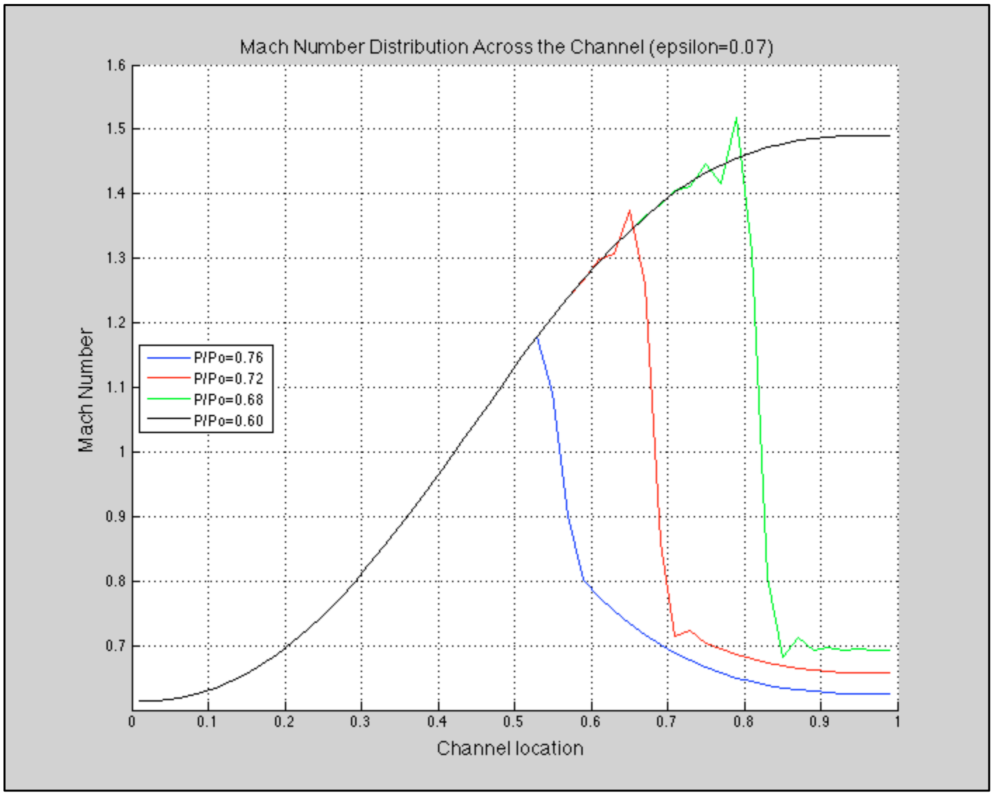


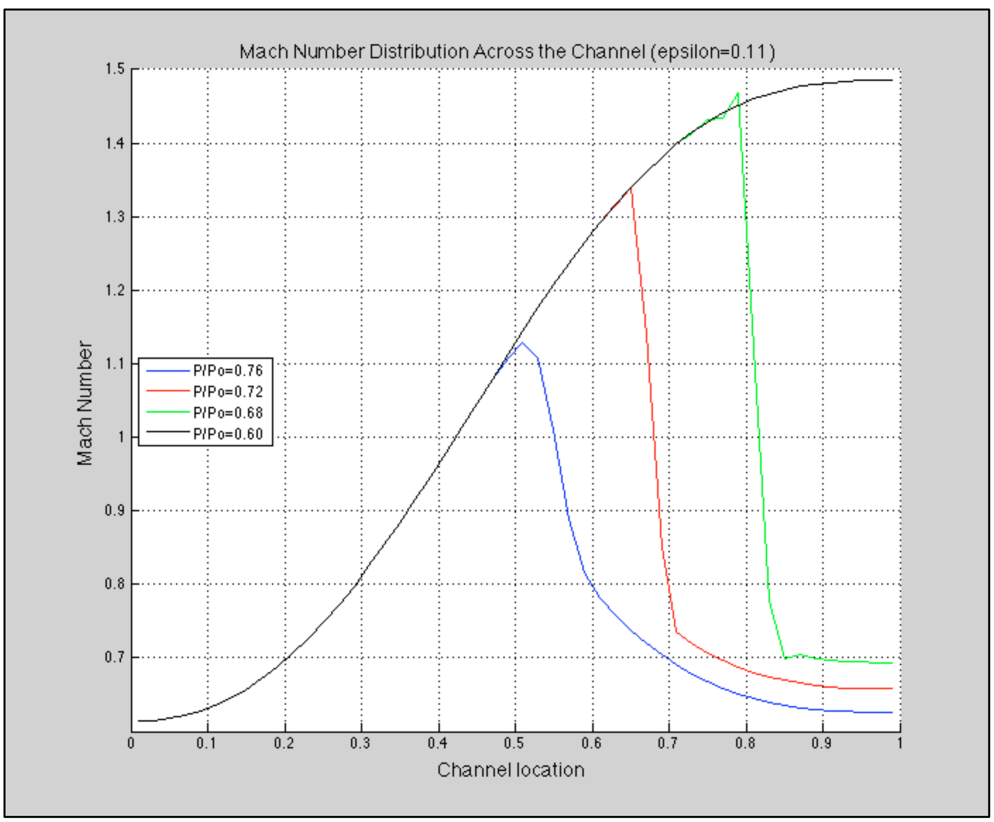


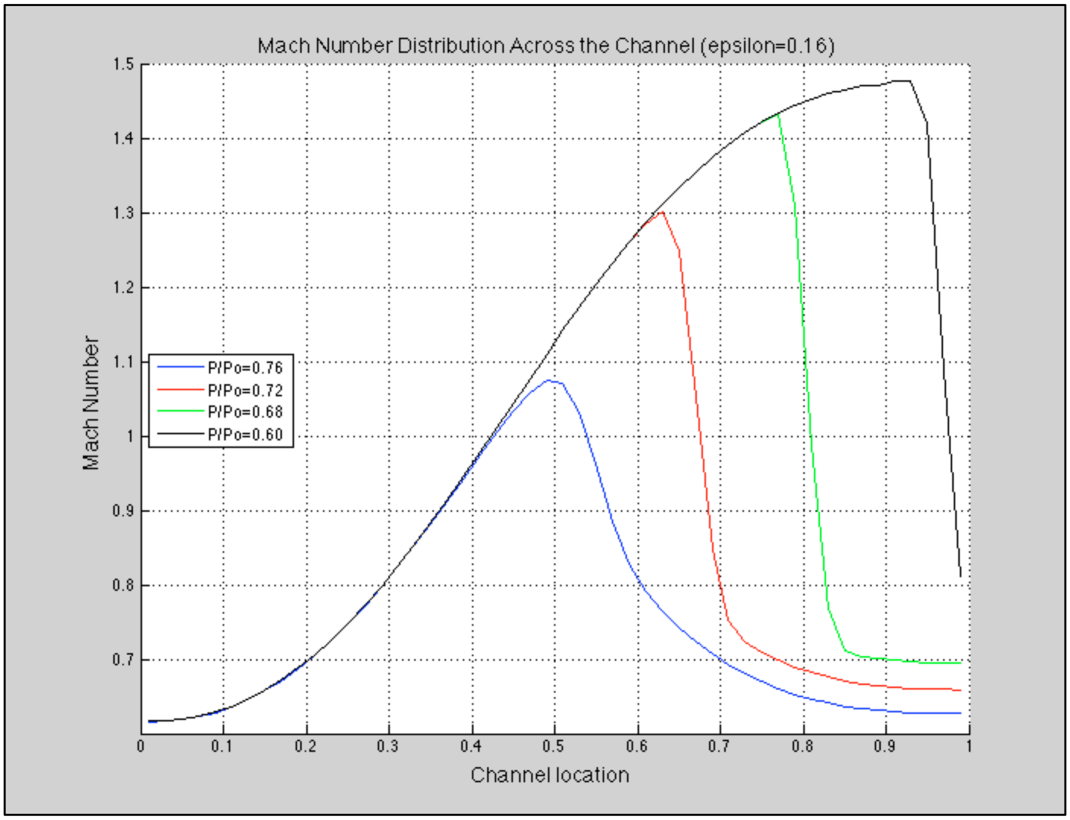

Pressure Ratio Distribution Across the Channel (epsilon=0.16)
Pressure Ratio(Inlet Total Pressure/Local Static Pressure)
Channel location
P/Po=0.76
P/Po=0.72
P/Po=0.68
P/Po=0.60
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Pressure Ratio Distribution Across the Channel (epsilon=0.21)
Pressure Ratio(Inlet Total Pressure/Local Static Pressure)
Channel location
P/Po=0.76
P/Po=0.72
P/Po=0.68
P/Po=0.60
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

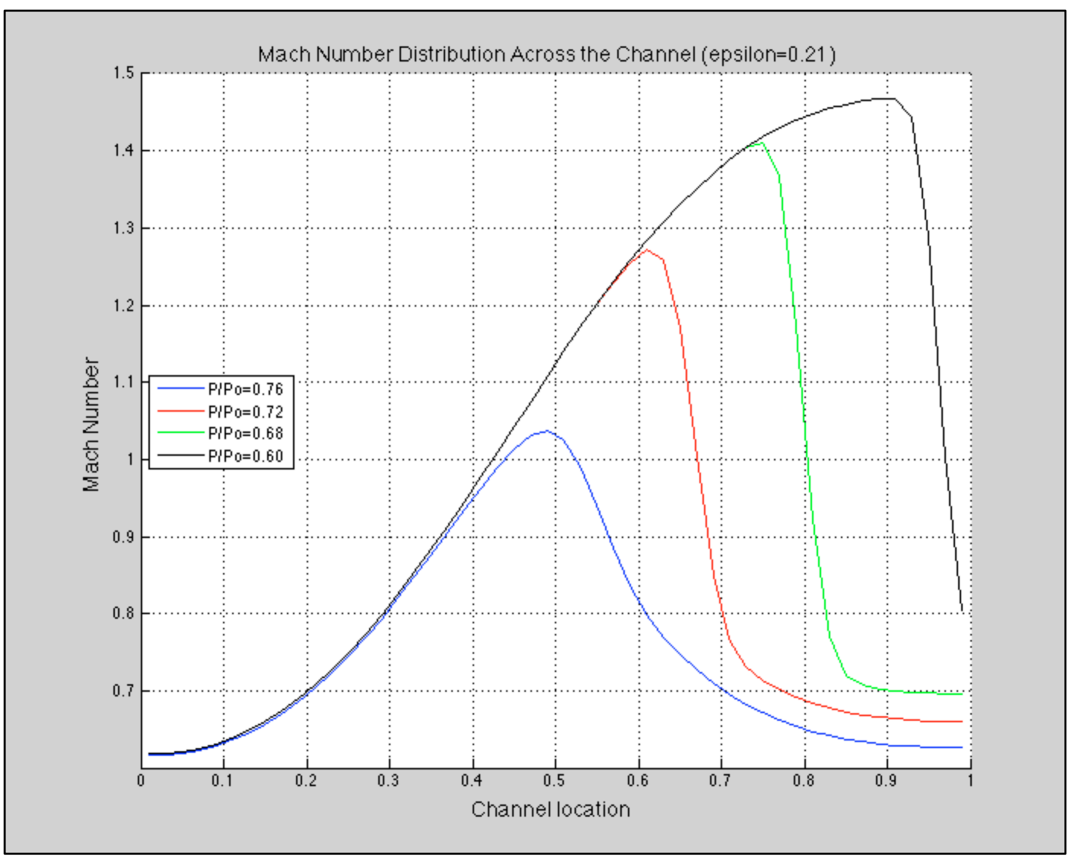
<u>Mach Number Distribution</u>







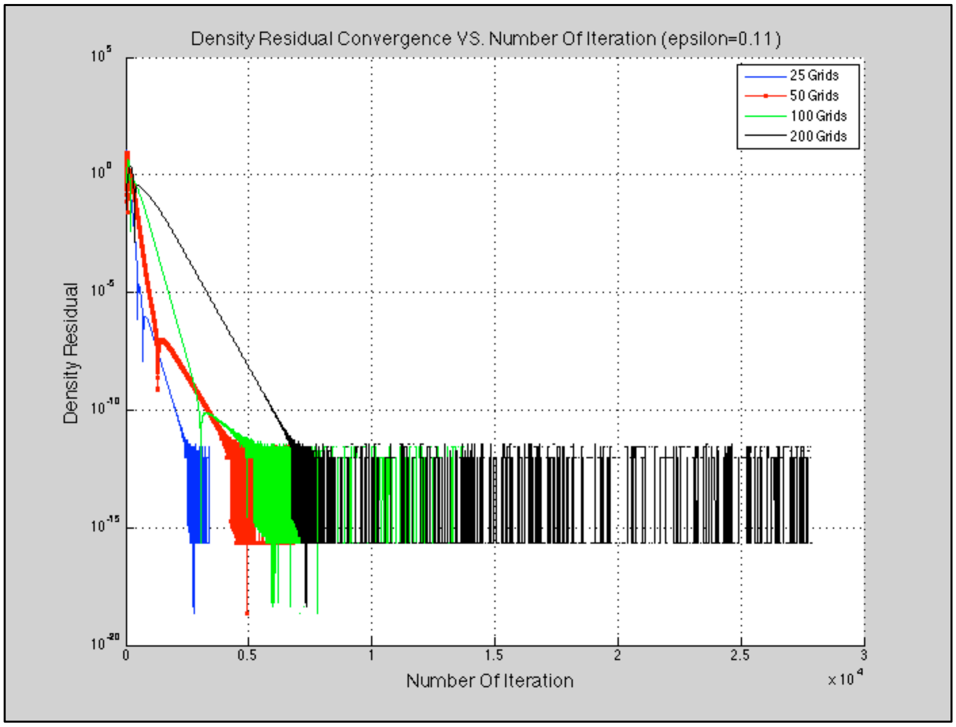
Mach Number Distribution Across the Channel (epsilon=0.16)
Mach Number
Channel location
P/Po=0.76
P/Po=0.72
P/Po=0.68
P/Po=0.60

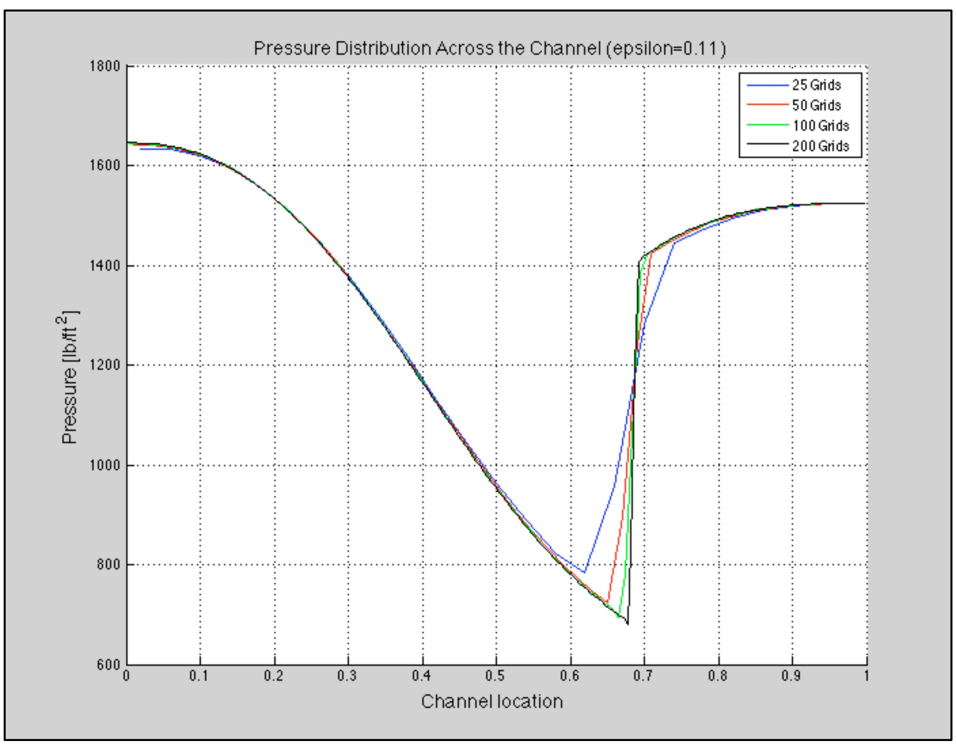


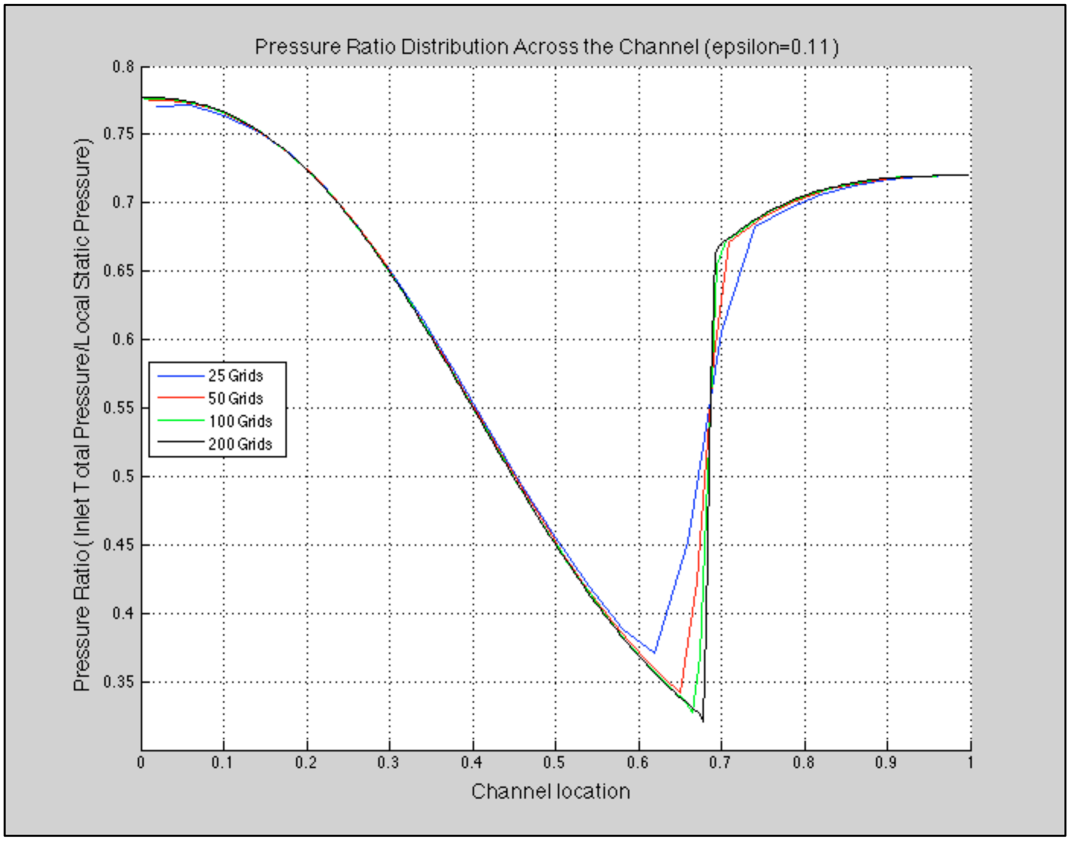
Mach Number Distribution Across the Channel (epsilon=0.21)
Mach Number
Channel location
P/Po=0.76
P/Po=0.72
P/Po=0.68
P/Po=0.60

Question 3.

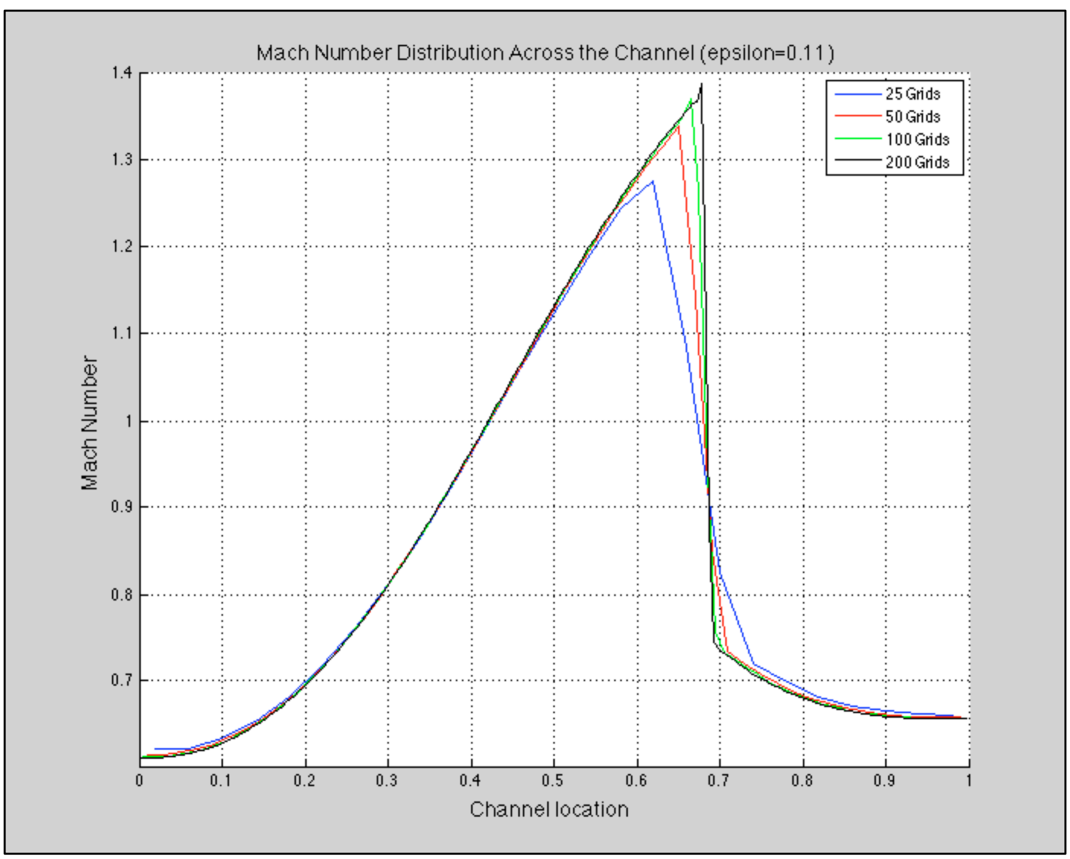
Scalar dissipation scheme was used for the spatial discretization, and an Euler explicit scheme was used for the temporal discretization.
The exit pressure ratio was set to 0.72 and CFL value was set to 0.4. As reported by 'Question 2', the epsilon value for this case was fixed to 0.11. The nozzle was discretized with different grid sizes to study the effect of grid size.





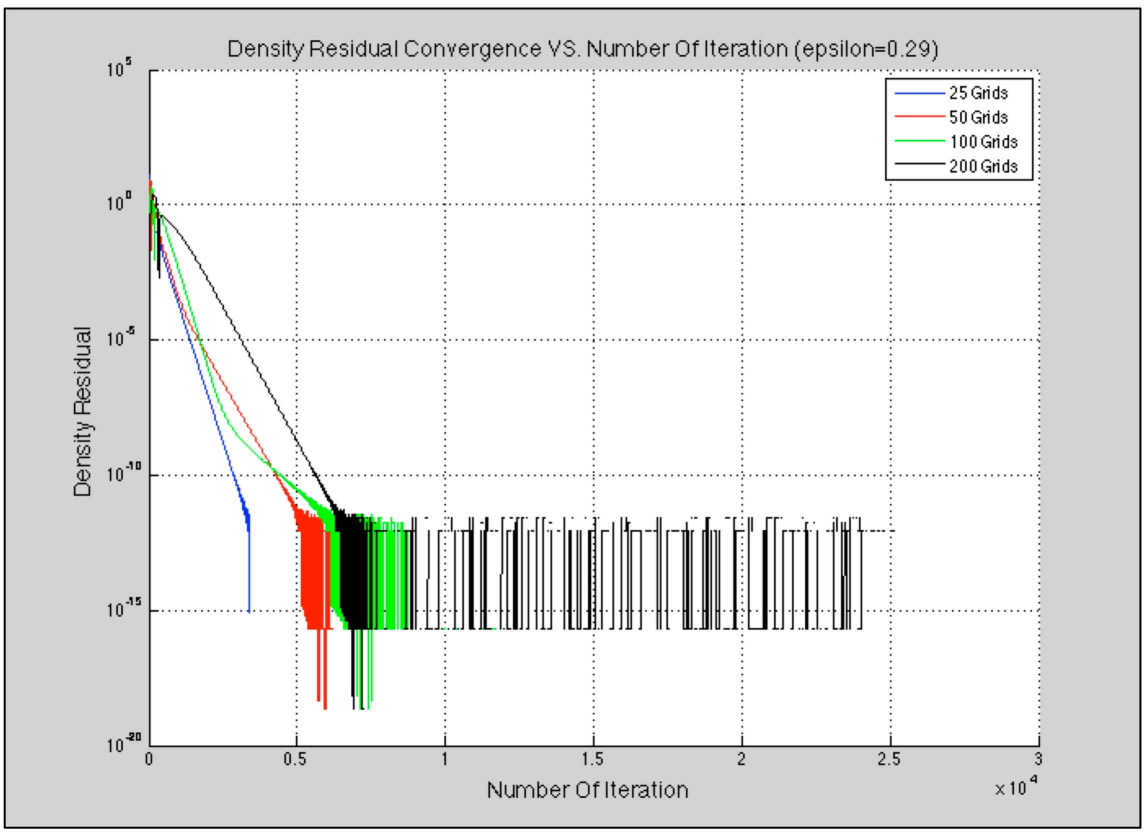


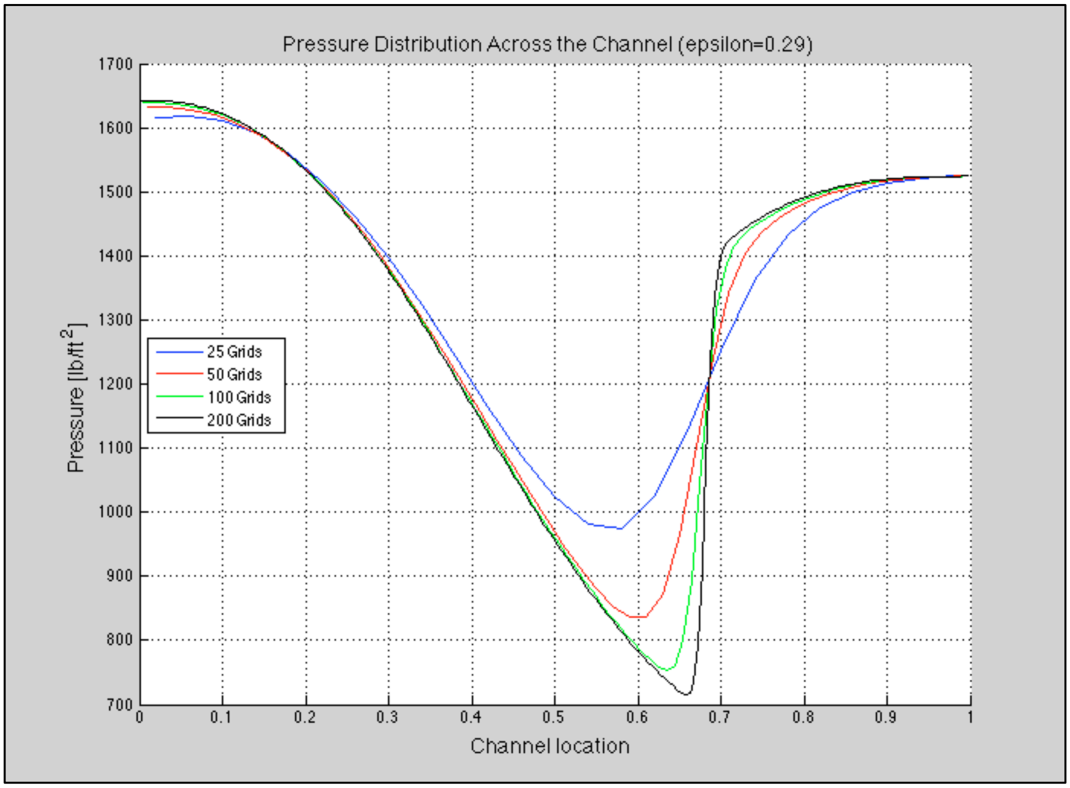
Pressure Ratio Distribution Across the Channel (epsilon=0.11)
Pressure Ratio (Inlet Total Pressure/Local Static Pressure)
Channel location
25 Grids
50 Grids
100 Grids
200 Grids



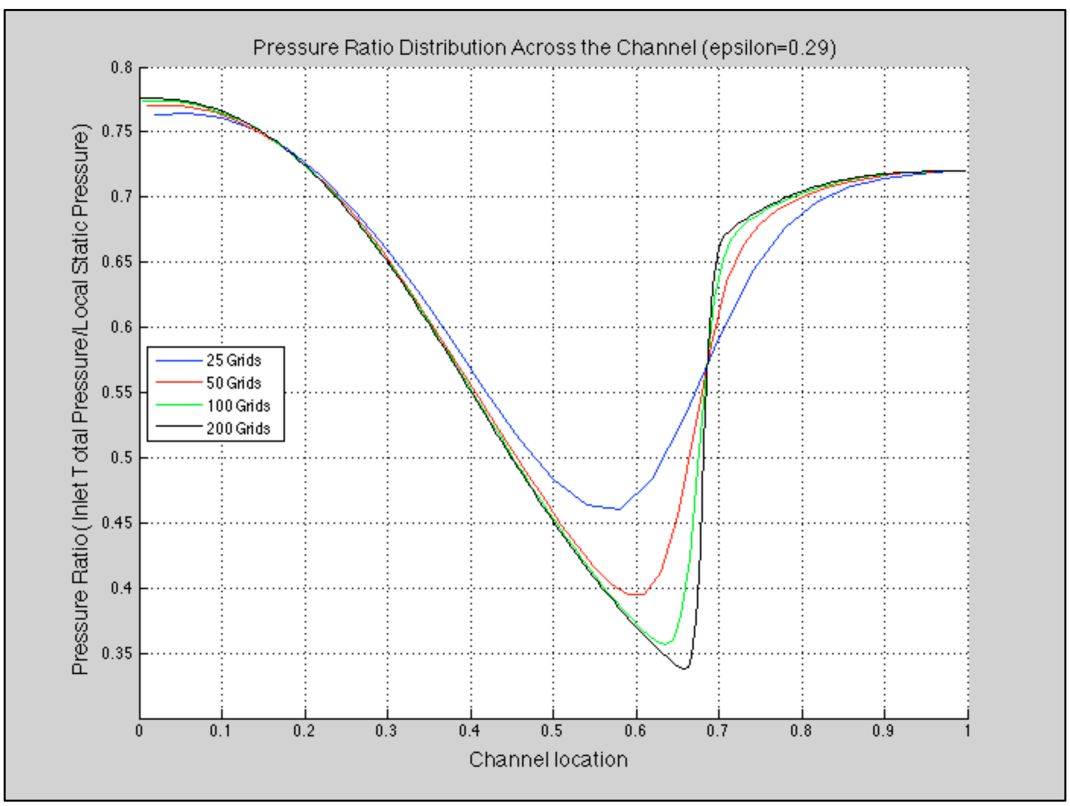
Mach Number Distribution Across the Channel (epsilon=0.11)
Mach Number
Channel location
25 Grids
50 Grids
100 Grids
200 Grids

The shock did exist at the diverging section of the channel. The location of the shock stayed the same no matter what the grid size was, but the accuracy of data was varied with grid sizes. From the plots above, it was noticeable that dissipation was occurred for data of 25 grid points, while sudden slight increase of data at the shock for data of 100 and 200 grids due to the dispersion error. Thus, the discrepancy of data between different grid points could be interpreted that the fixed epsilon value of 0.11 introduced dissipation error to data of 25 grid points, and introduced dispersion error to data of 100 and 200 grid points. In other words, it could be interpreted in that the epsilon value, set as 0.11, was optimal set for 50 grid points because at 50 grid points, the code introduced neither dispersion error nor the dissipation error and there is optimal value of epsilon also for different grid points. To verify this, higher epsilon value, 0.29, was tried.

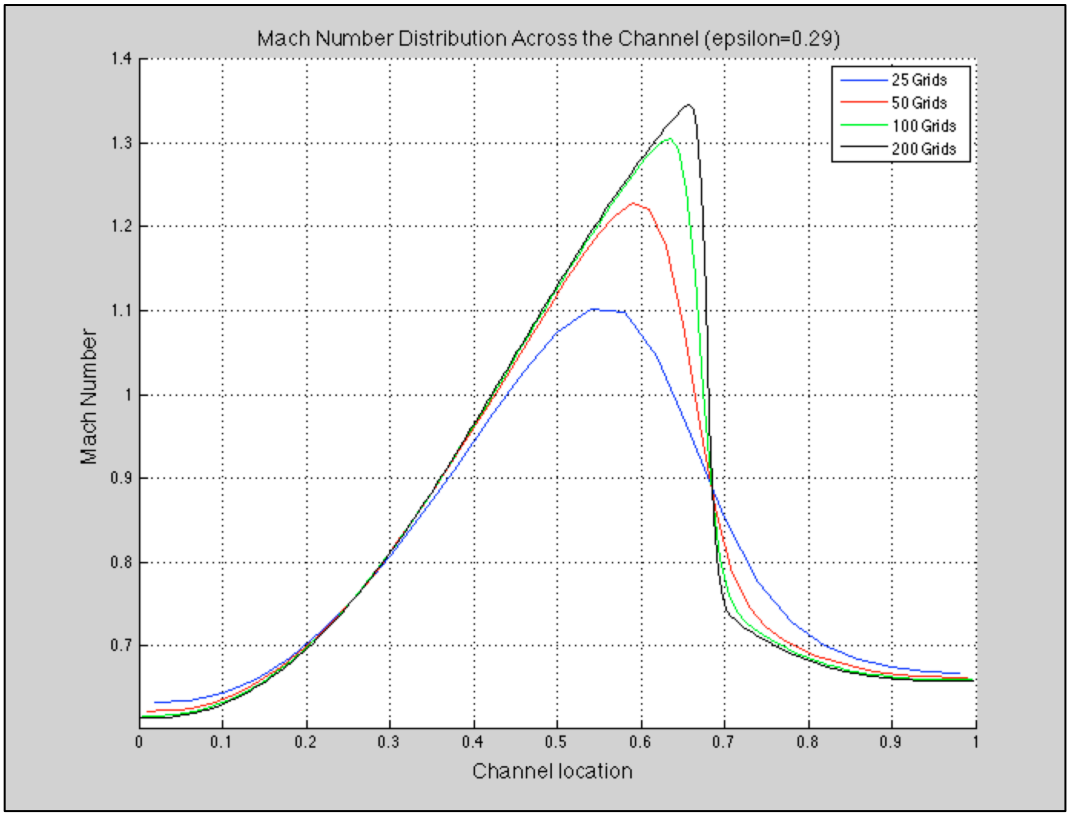




Pressure Distribution Across the Channel (epsilon=0.29)
Pressure [lb/ft2]
Channel location
25 Grids
50 Grids
100 Grids
200 Grids
1700
1600
1500
1400
1300
1200
1100
1000
900
800
700
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



Pressure Ratio Distribution Across the Channel (epsilon=0.29)
Pressure Ratio (Inlet Total Pressure/Local Static Pressure)
Channel location
25 Grids
50 Grids
100 Grids
200 Grids
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



The data was re-produced with epsilon value of 0.29. According to the new pressure distribution plot and Mach number distribution plot, it became clear that there is optimal epsilon value for different grid size. First, as the grid points got smaller, the data was more affected by the dissipation error. Also, the peak Mach number of 200 grid points with epsilon value of 0.29, which is about 1.35, was very close to the peak Mach number of 50 grid points with epsilon value of 0.11. Moreover, the minimum pressure ratio of 200 grid points with epsilon value of 0.29, which is about 0.33, was also very close to the minimum pressure ratio of 50 grid points with epsilon value of 0.11. These were occurred at the shock location and the location of 200 grid points with epsilon value of 0.29 and that of 50 grid points with epsilon value of 0.11 were the same. Therefore, the location of shock remained same, despite of varying the grid points.

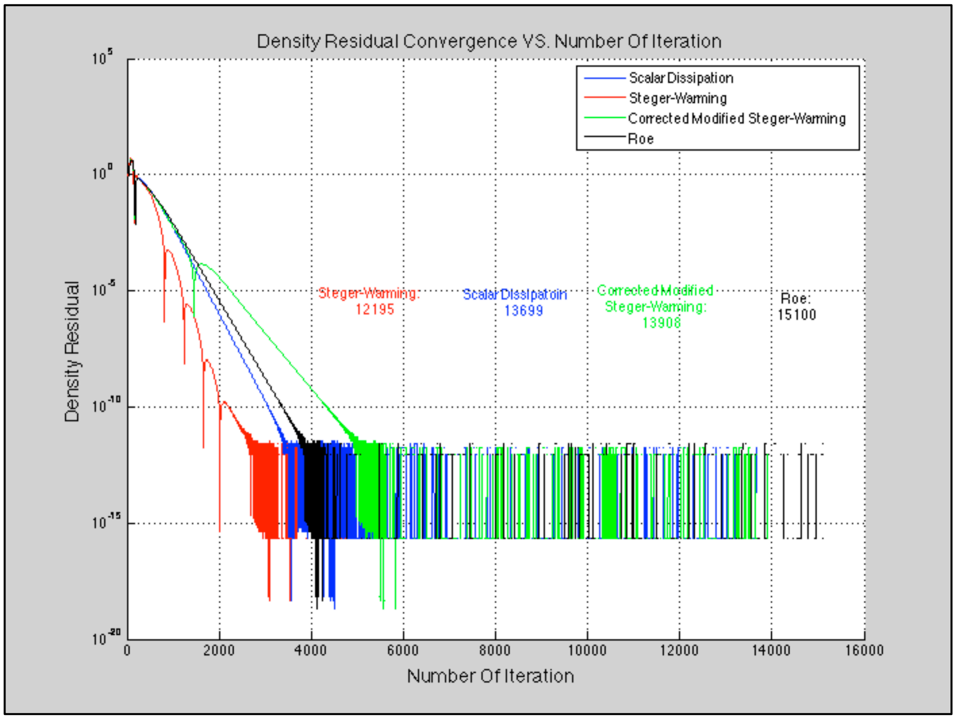
Meanwhile, the number of iterations varied with the grid points. Precisely, larger number of iterations was required, as the grid point was bigger. At epsilon value of 0.11, required number of iterations for different grid points are follow:

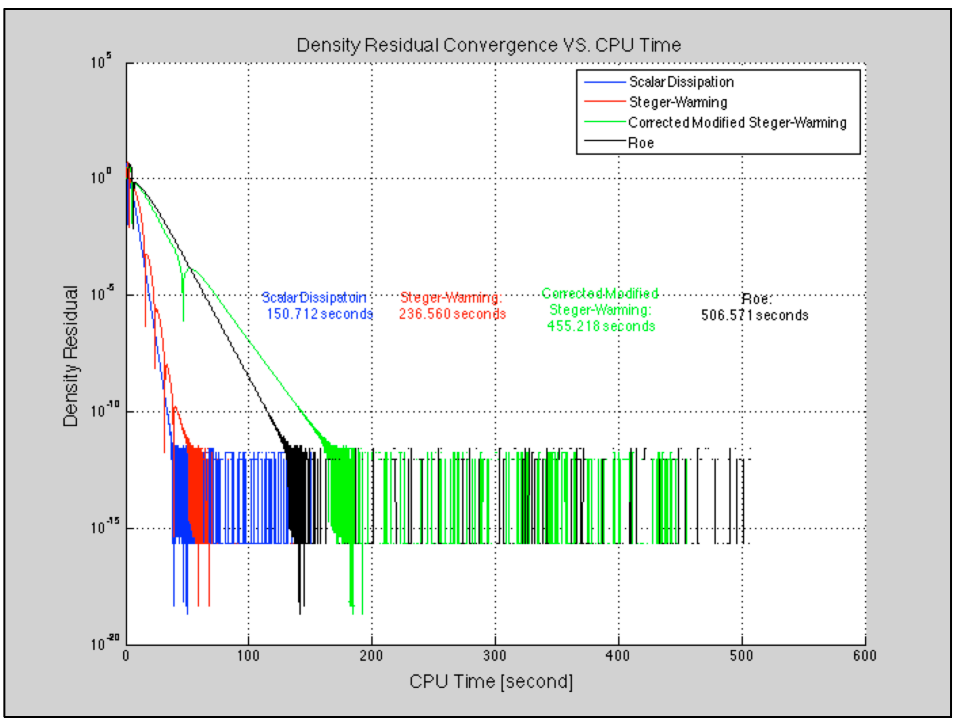
- 25 grid points: 3435 iterations
- 50 grid points: 6882 iterations
- 100 grid points: 13872 iterations
- 200 grid points: 27887 iterations

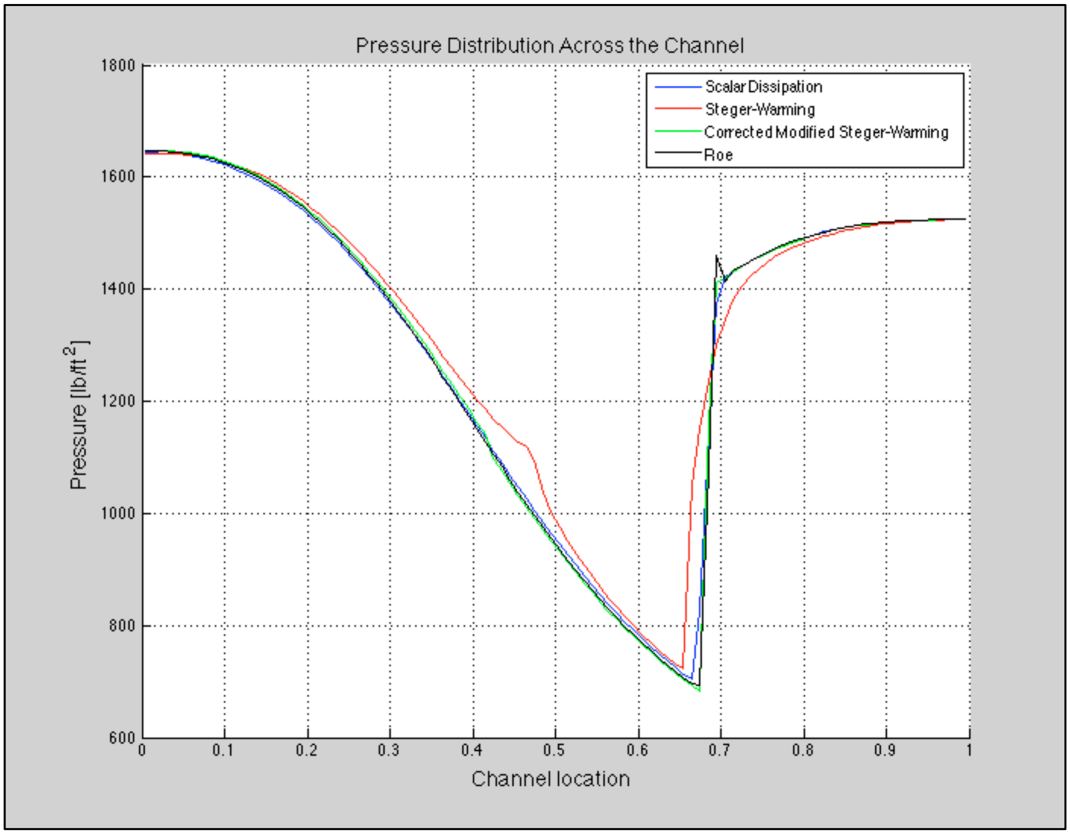
It seemed that the number of iterations is exponentially proportional to the grid points.

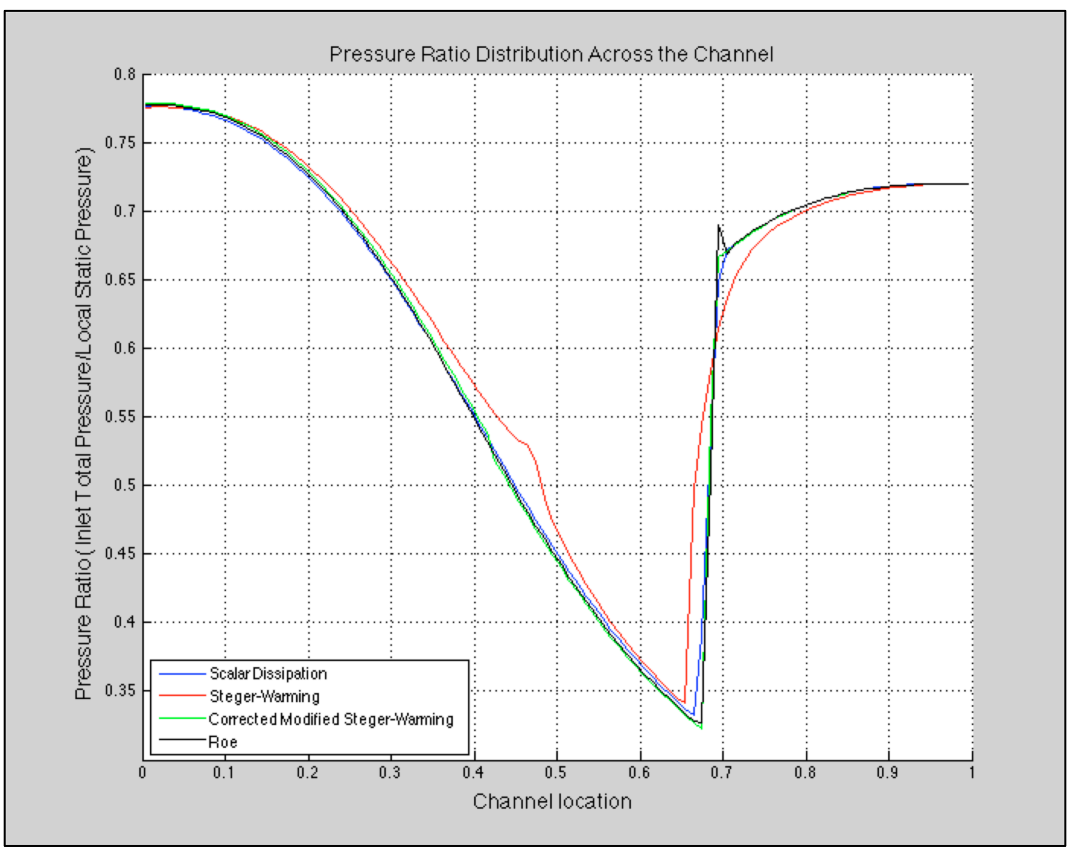
Question 4.

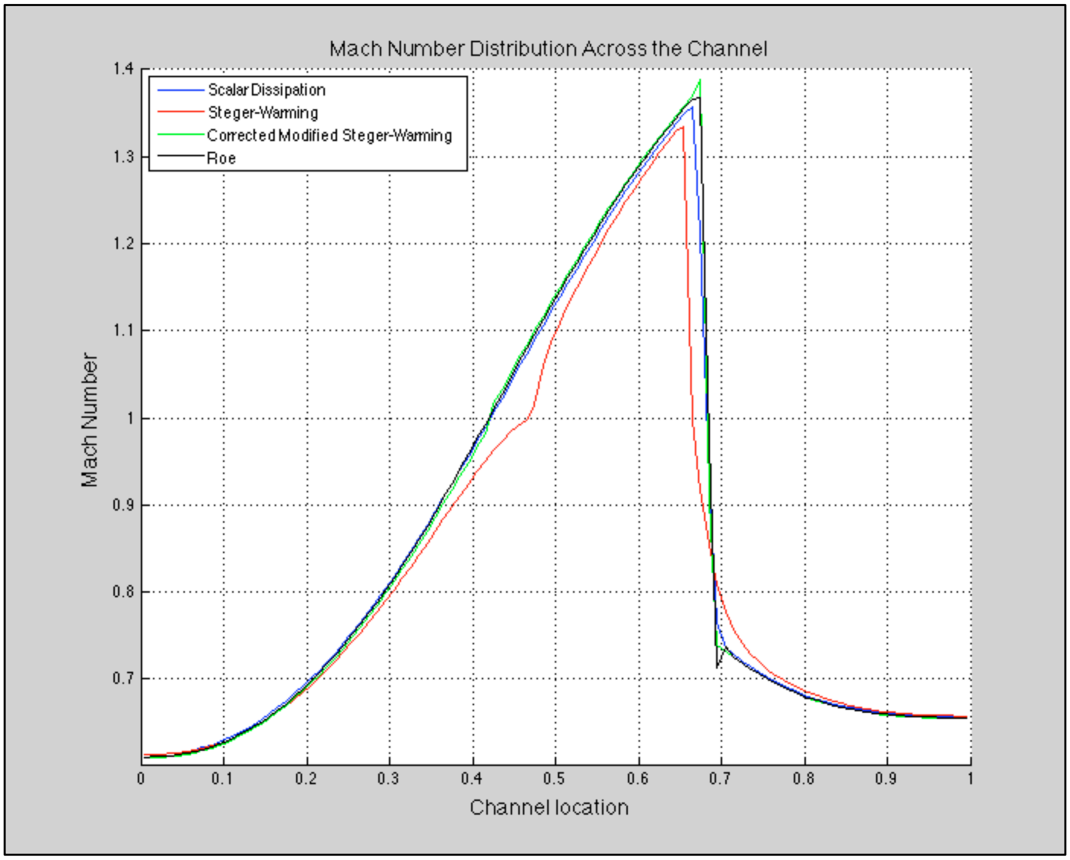
Euler explicit scheme was used for the temporal discretization.
The exit pressure ratio was set to 0.72 and CFL value was set to 0.4. As reported by 'Question 2' and 'Question3', the epsilon value for this case was adjusted to 0.13 as an estimation of optimal epsilon value. The nozzle was discretized with 100 grid points.






Pressure Distribution Across the Channel
Scalar Dissipation
Steger-Warming
Corrected Modified Steger-Warming
Roe
Pressure [lb/ft^2]
1800
1600
1400
1200
1000
800
600
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Channel location


Pressure Ratio Distribution Across the Channel
Pressure Ratio (Inlet Total Pressure/Local Static Pressure)
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
Scalar Dissipation
Steger-Warming
Corrected Modified Steger-Warming
Roe
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Channel location

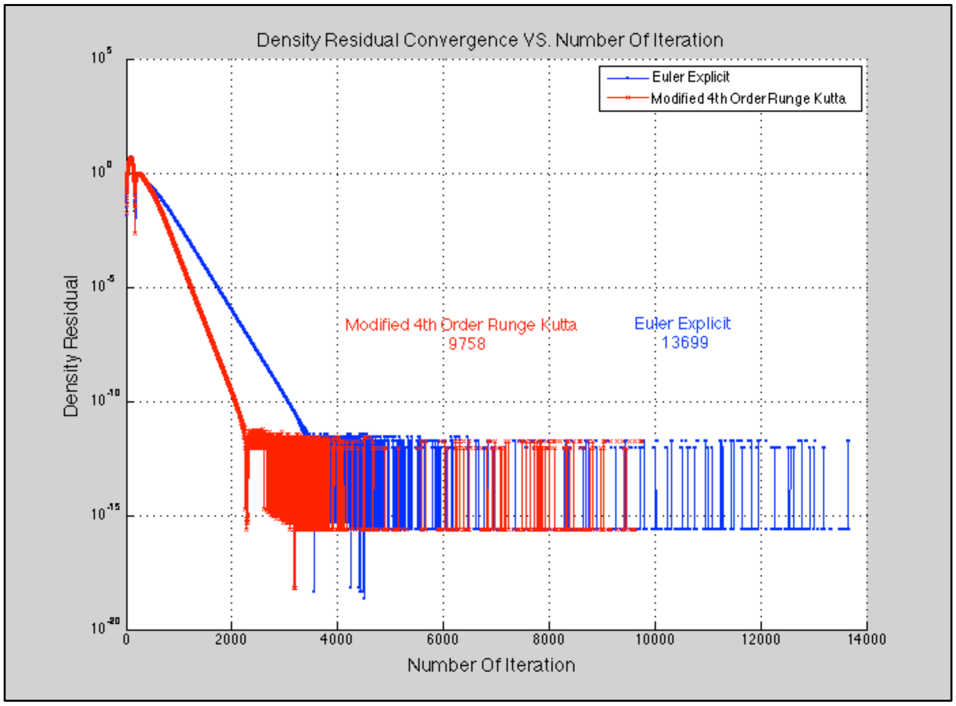


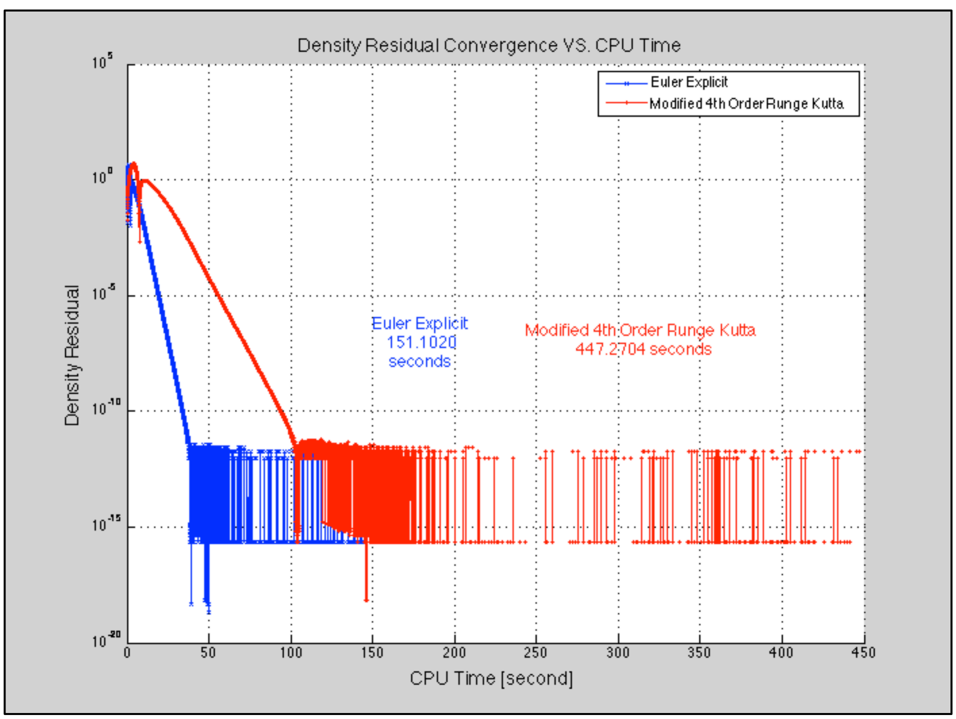
For the residual convergence, Steger-Warming scheme required the least number of iteration, next was Scalar Dissipation scheme, and then the Corrected Modified Steger-Warming scheme. And Roe scheme required the most number of iteration. Also for the residual convergence as a function of CPU time, Scalar Dissipation scheme was the fastest scheme, next was Steger-Warming, and then the next was Corrected Modified Steger-Warming scheme. Finally the Roe scheme was the slowest scheme among four schemes. It is because each scheme requires different memory sizes. For example, Roe scheme is very expensive because evaluation of $\hat{A}$ matrix requires a lot of memories, and Corrected Modified Steger-Warming scheme is also expensive because it needs to access both Steger-Warming scheme and Modified Steger-Warming scheme at the same time.

All four schemes were able to produce the pressure distribution plot and Mach number distribution plot. The shock locations were all the same, but the accuracy, again, was the factor that discrepancy arises. According to the plots, Scalar Dissipation scheme was affected by neither dispersion nor dissipation error, while data produced by Steger-Warming was suffered from the relatively large dissipation error, and the other two schemes were suffered from the dispersion error. Thus, it could be interpreted that there is also optimal epsilon value for different scheme, and furthermore, the schemes are very sensitive to the epsilon value.

Question 5.

Scalar dissipation scheme was used for the spatial discretization.
The exit pressure ratio was set to 0.72 and CFL value was set to 0.4. As reported by 'Question 2' and 'Question3', the epsilon value for this case was adjusted to 0.13 as an estimation of optimal epsilon value. The nozzle was discretized with 100 grid points. Euler Explicit and Jameson's Modified 4th Order Runge Kutta were used to study the effect of temporal discretization scheme.





The fourth order Runge Kutta with Scalar Dissipation scheme took less number of iteration than the Euler Explicit with Scalar Dissipation scheme to converge the data to the solution. In CPU time, however, Euler Explicit scheme took much less time than the fourth order Runge Kutta scheme. It was because there are several steps of equations to compute data for single iteration in Runge Kutta scheme, while only single equation for single iteration in Euler Explicit scheme. Therefore, since Runge Kutta scheme compute results several times more than Euler Explicit does during each iterations, it requires less iterations to converge to the solution. But, both methods use the same Scalar Dissipation scheme for the equation, Runge Kutta takes much more time than Euler Explicit for single iteration.
In addition, the computation of Δt is different between Euler Explicit and Modified 4th Order Runge Kutta:

```
if scheme == 1                    %Euler Explicit
    dt = CFL*dx/uCurrent ;
elseif scheme == 2                %Modified 4th Order Runge Kutta
    dt = CFL*((2*sqrt(2))/(abs(uCurrent)/dx+cCurrent*sqrt(1/(dx^2))));
end
```

So, this also could affect how scheme converges the data well at each iterations.