MECH 539: Computational Aerodynamics

Project #5: Solve the Quasi One-Dimensional Euler Equations for a Supersonic Nozzle

Justin ChanWoo Yang
260368098
chanwoo.yang@mail.mcgill.ca

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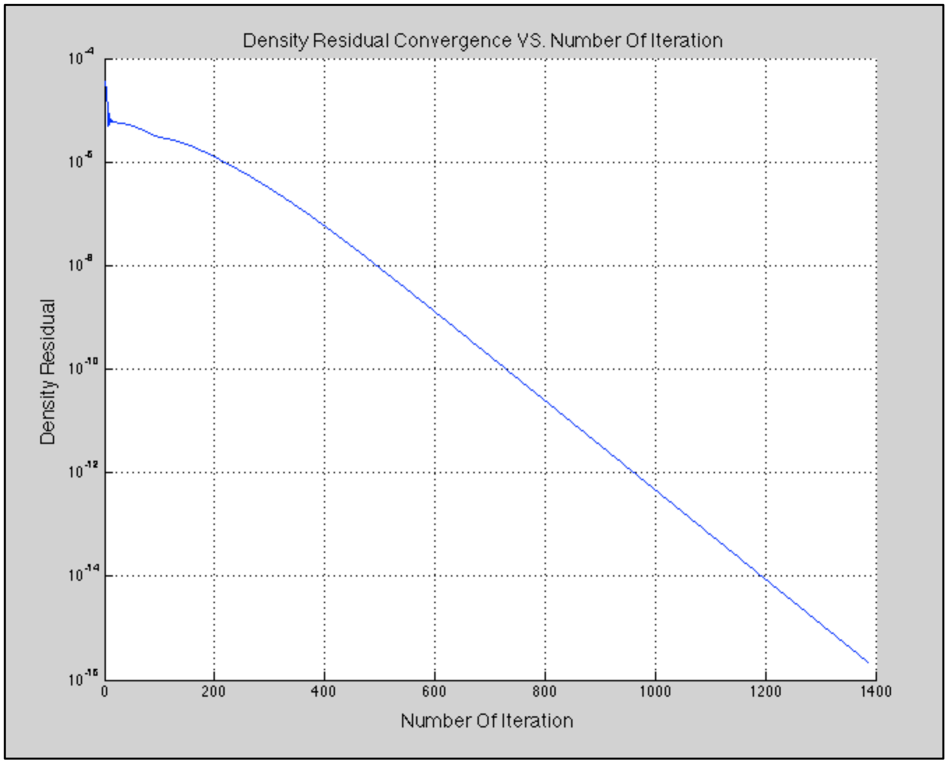
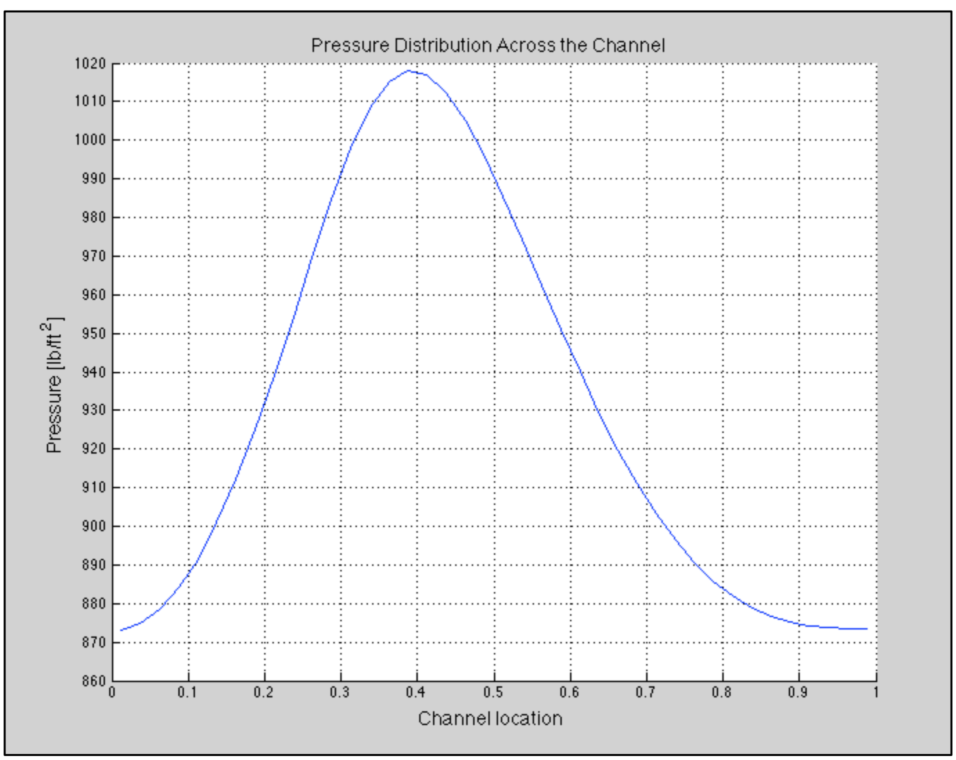
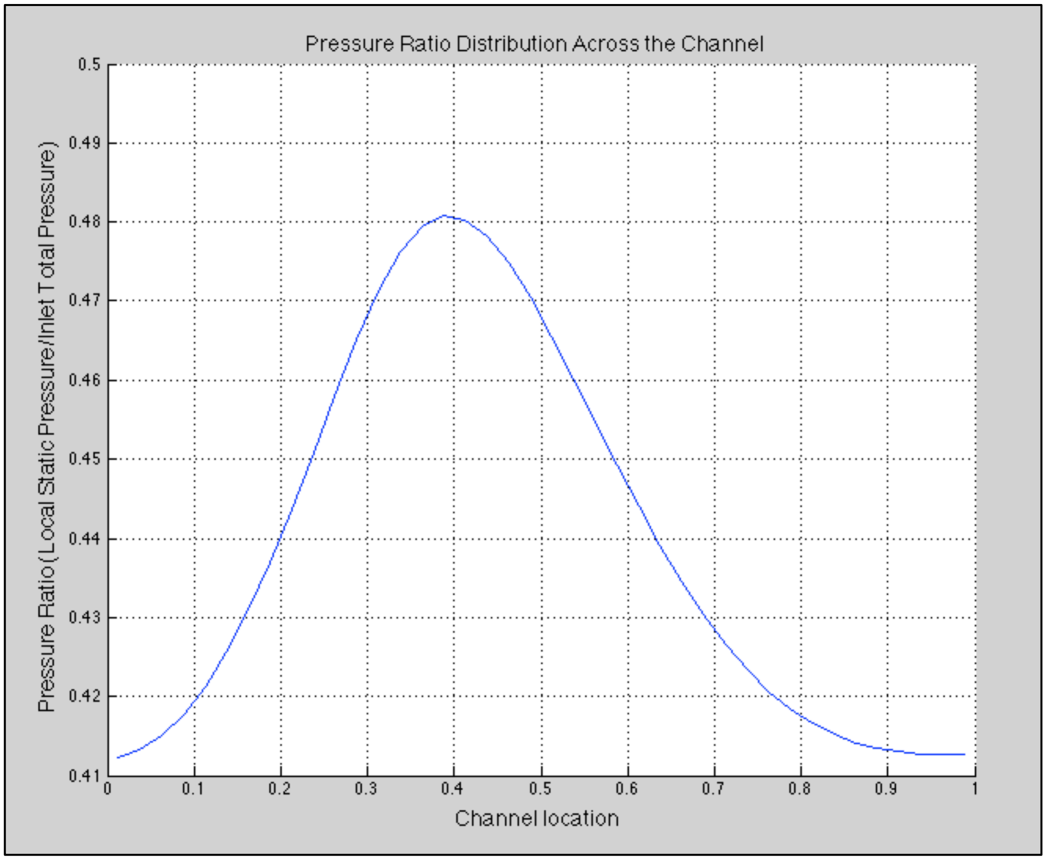
Question 1.
– 40 grid points were set

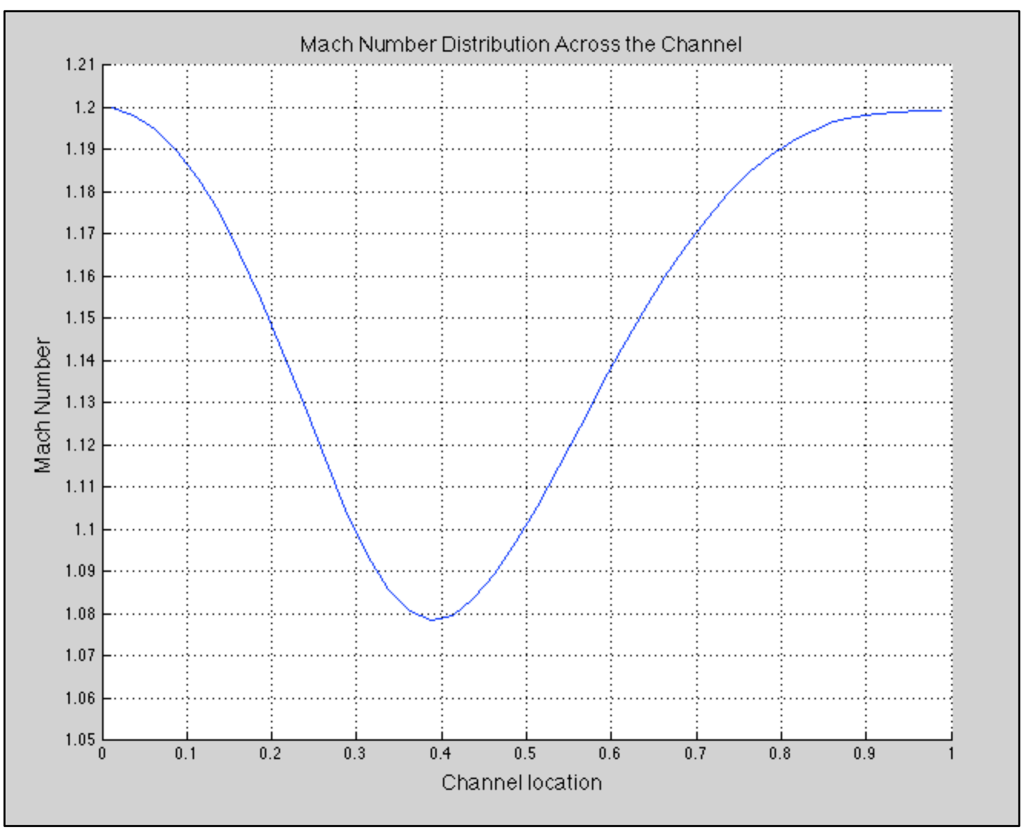
<u>Scalar Dissipation Scheme</u>

- CFL value was set to 0.9 and epsilon was set to 0.1



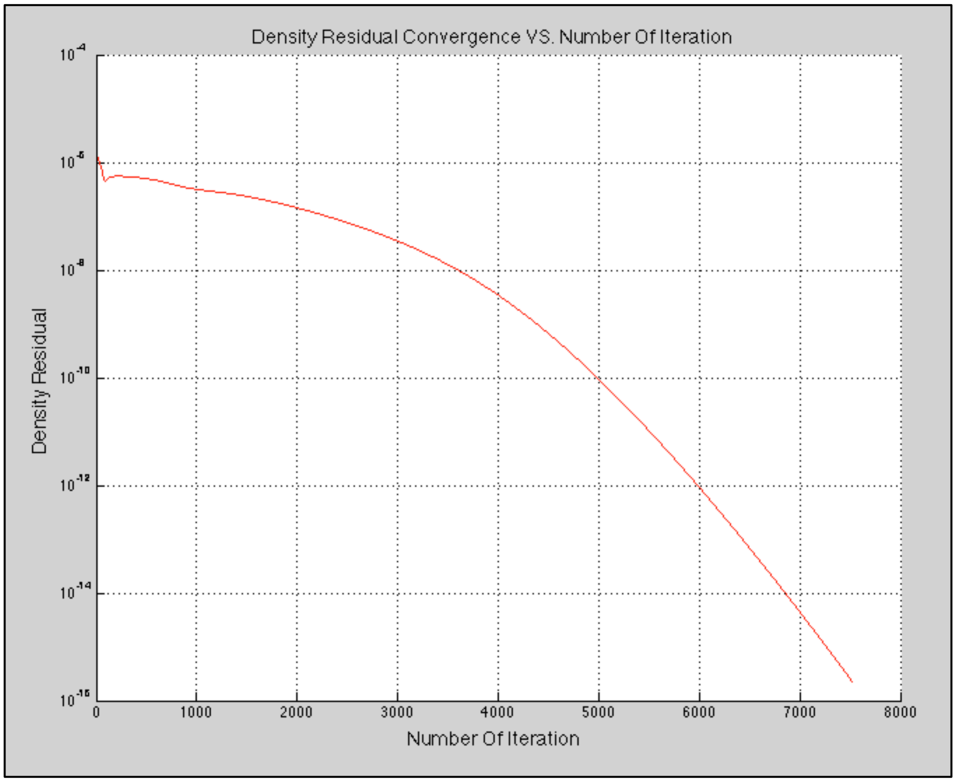


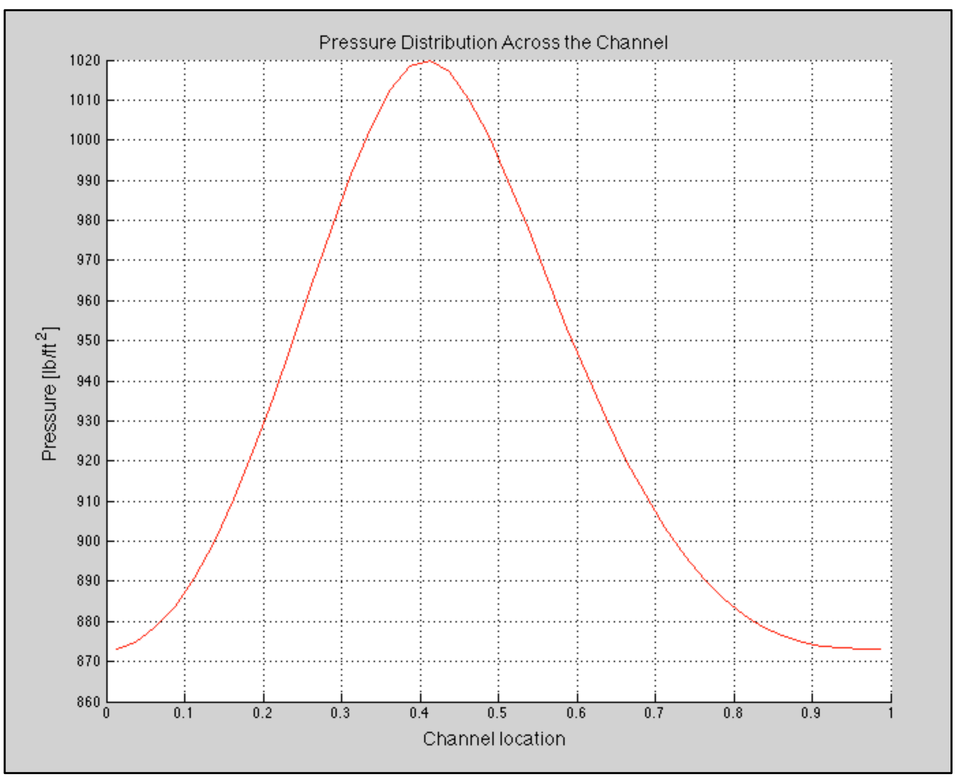

Pressure Ratio Distribution Across the Channel
Pressure Ratio (Local Static Pressure/Inlet Total Pressure)
Channel location
0.5
0.49
0.48
0.47
0.46
0.45
0.44
0.43
0.42
0.41
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

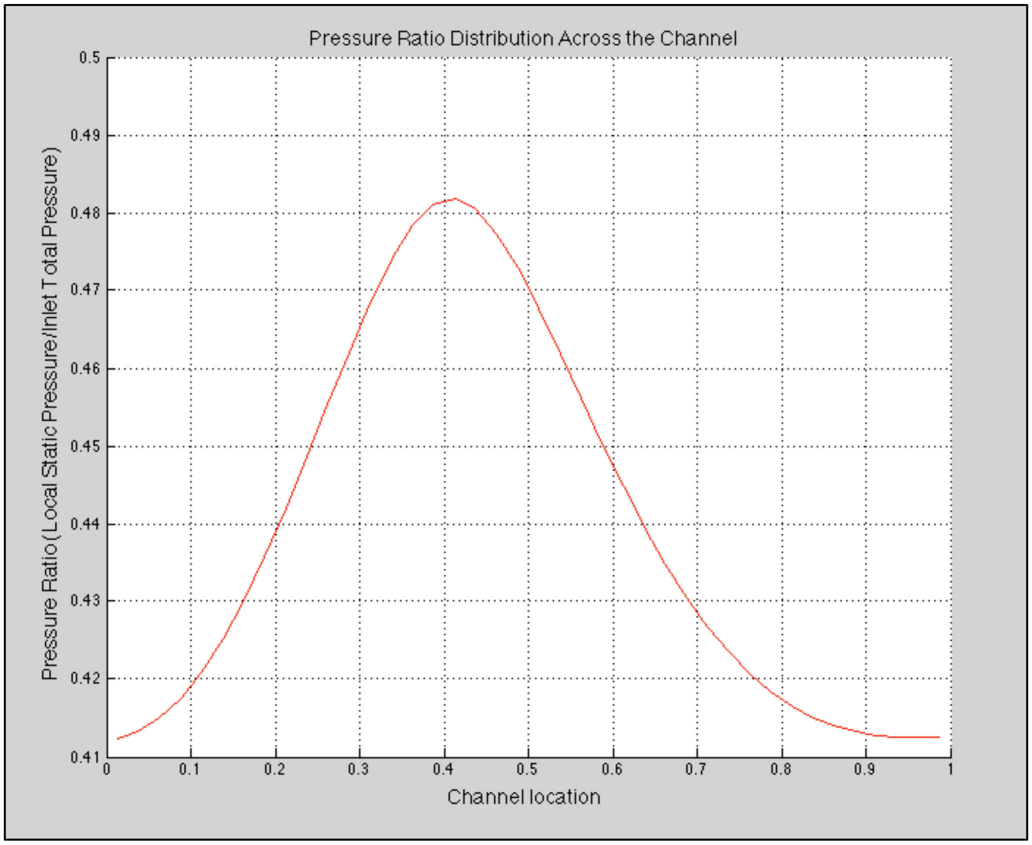

Mach Number Distribution Across the Channel
Mach Number
Channel location
1.21
1.2
1.19
1.18
1.17
1.16
1.15
1.14
1.13
1.12
1.11
1.1
1.09
1.08
1.07
1.06
1.05
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

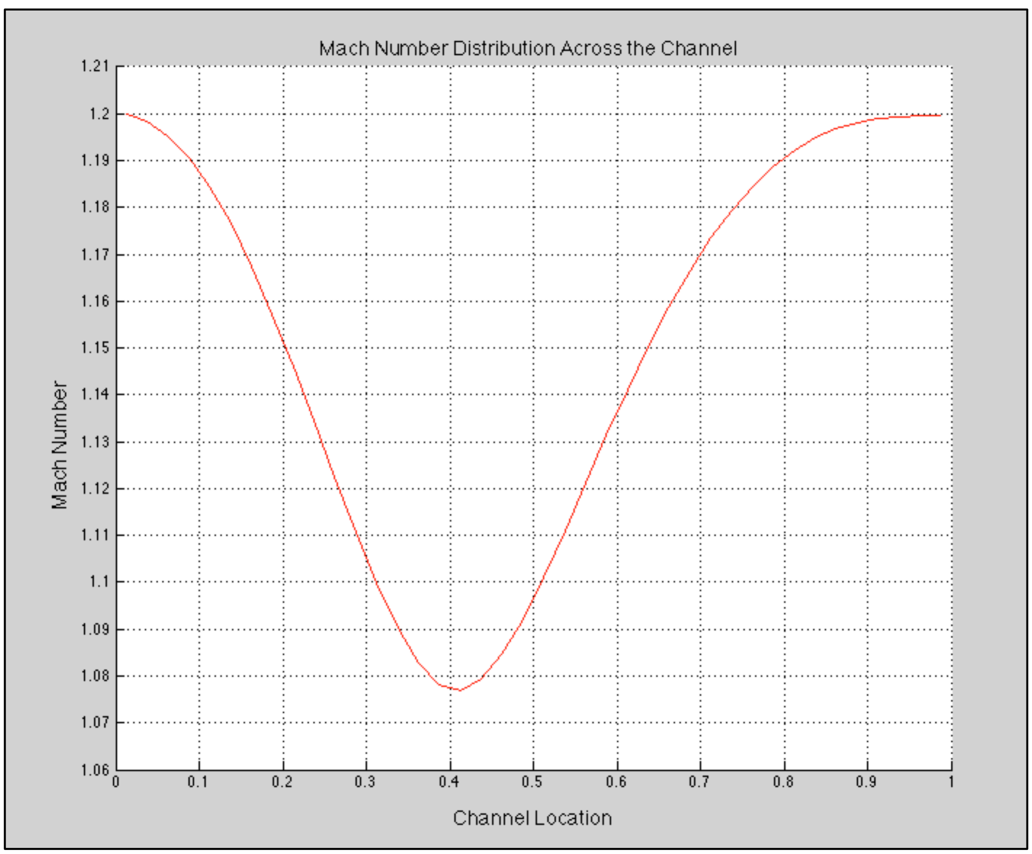
Steger Warming Scheme

- CFL value was set to 0.1



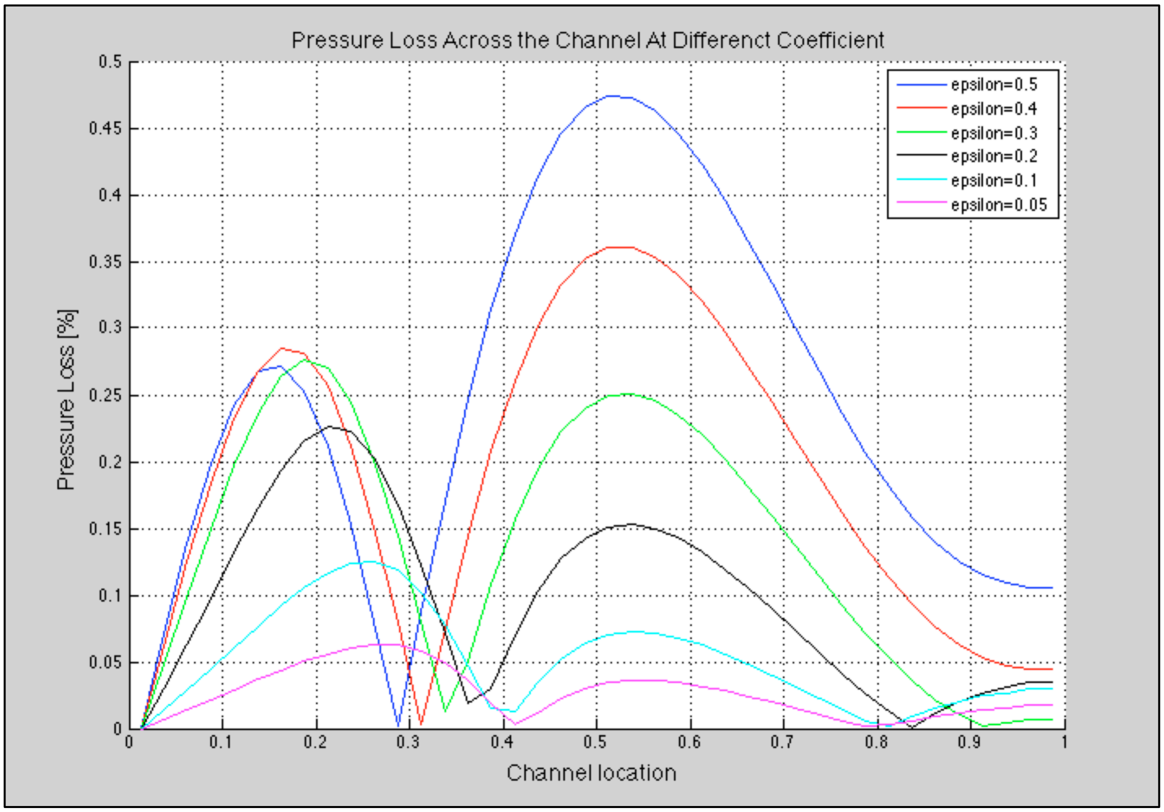



Pressure Ratio Distribution Across the Channel
Pressure Ratio (Local Static Pressure/Inlet Total Pressure)
0.5
0.49
0.48
0.47
0.46
0.45
0.44
0.43
0.42
0.41
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Channel location

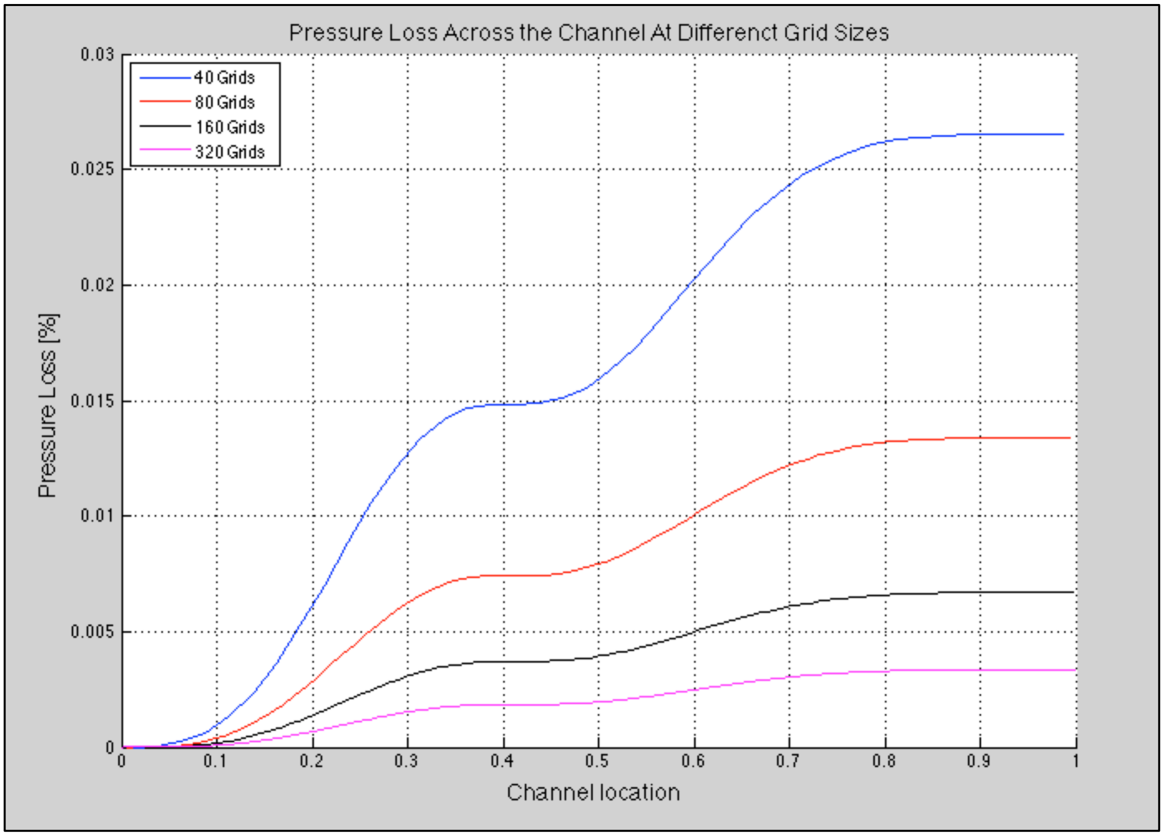

Mach Number Distribution Across the Channel
Mach Number
1.21
1.2
1.19
1.18
1.17
1.16
1.15
1.14
1.13
1.12
1.11
1.1
1.09
1.08
1.07
1.06
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Channel Location

Question 2.

CFL value was fixed at 0.7. The higher CFL value made the code to diverge for the epsilon value at around 0.5 or 0.4. And value of coefficient of scalar dissipation scheme was varied to analyze it effect on the total pressure loss.



For the constant CFL value, the decrease in value of epsilon, coefficient for the scalar dissipation scheme, reduces the total pressure loss along the channel. Also, along the channel, the pressure loss is the minimum at near the throat of the channel, so the lower epsilon value leads this minimum point to be located much closer to the throat location.
In Addition, from epsilon value of 0.5 to 0.3, the outlet total pressure loss decreased as the epsilon was decreased, and after decreasing the epsilon value below 0.4, the pressure loss at outlet stayed below 0.05%.

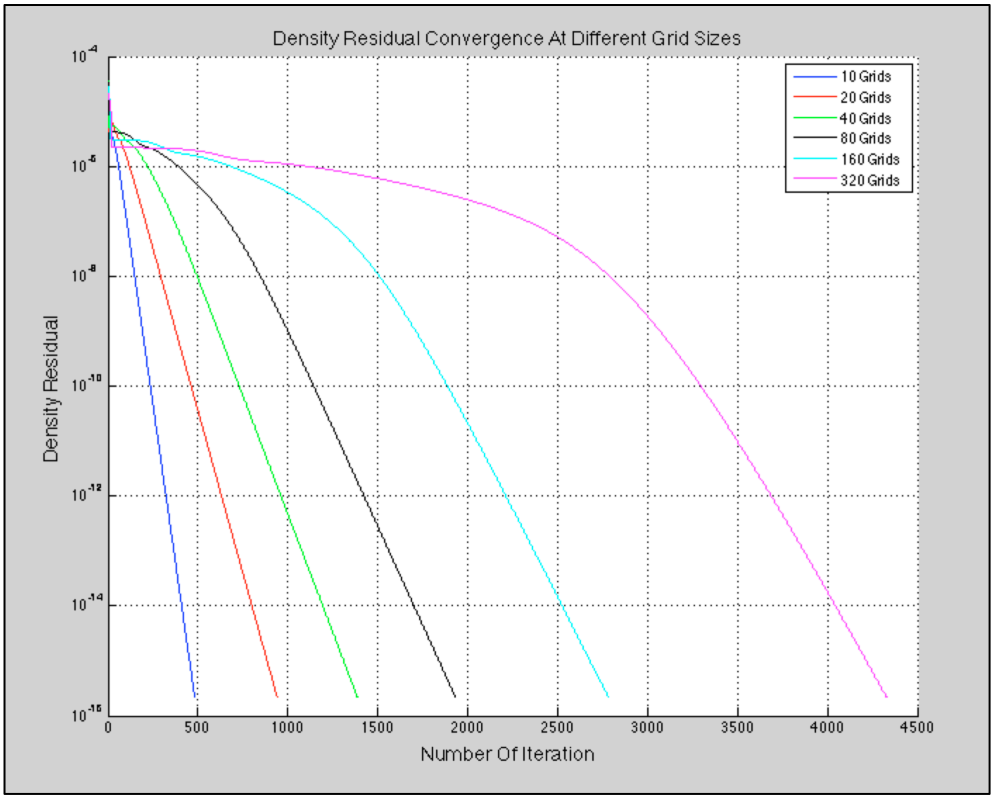


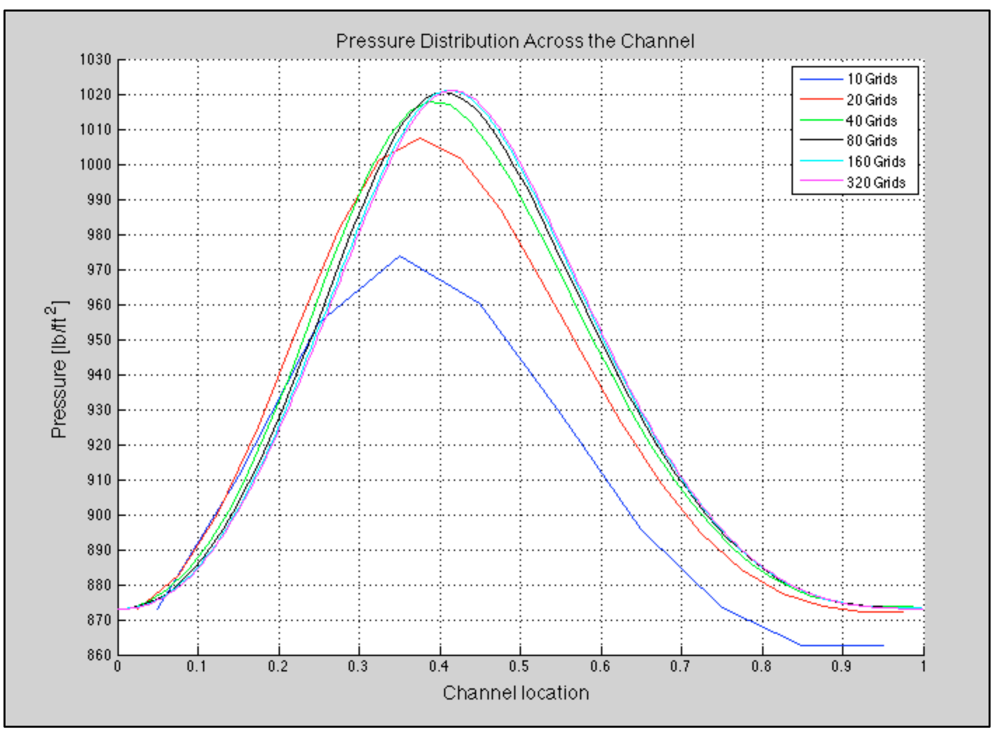
And, this is the pressure loss across the channel plot at different grid sizes for Steger Warming scheme. The CFL value was set at 0.1.

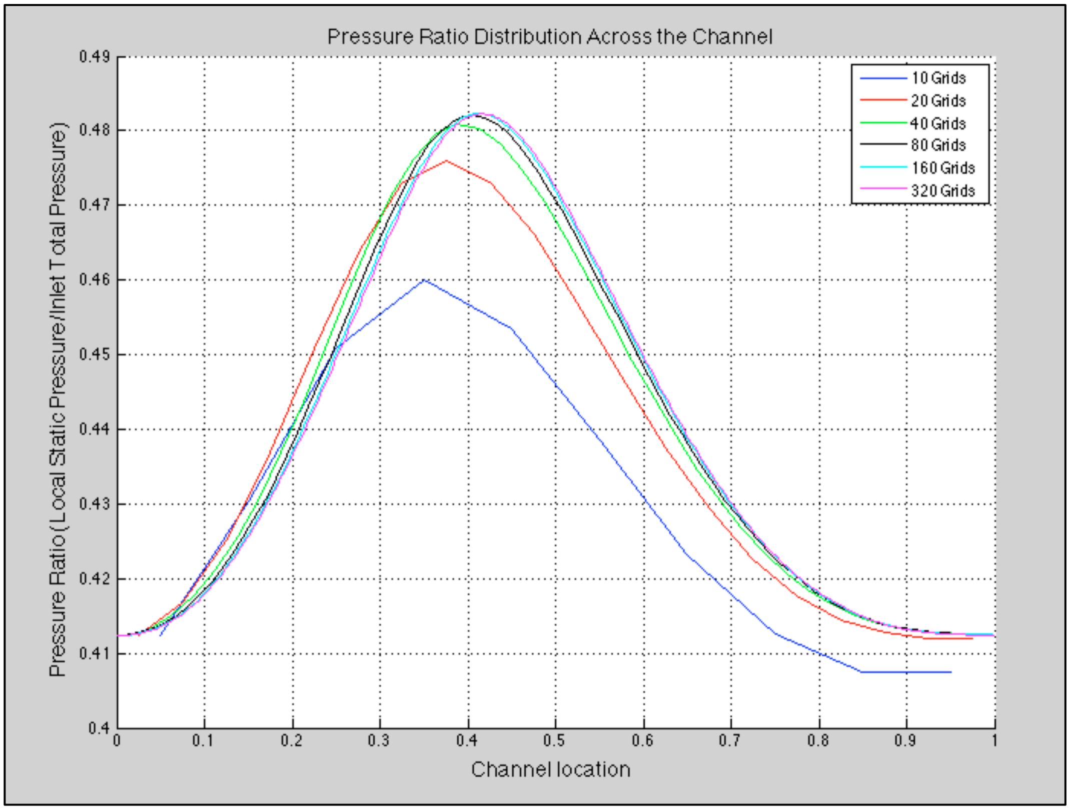
Question 3.

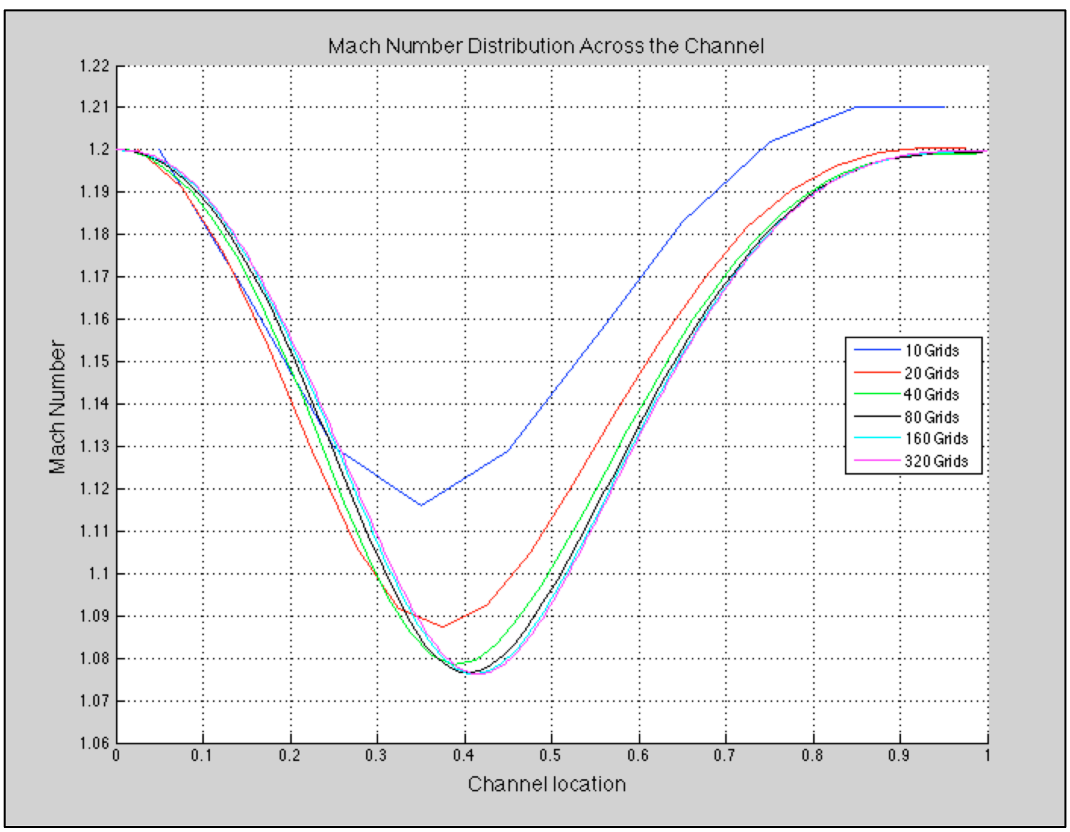
<u>Scalar Dissipation Scheme</u>

- CFL value was set to 0.9 and epsilon was set to 0.1





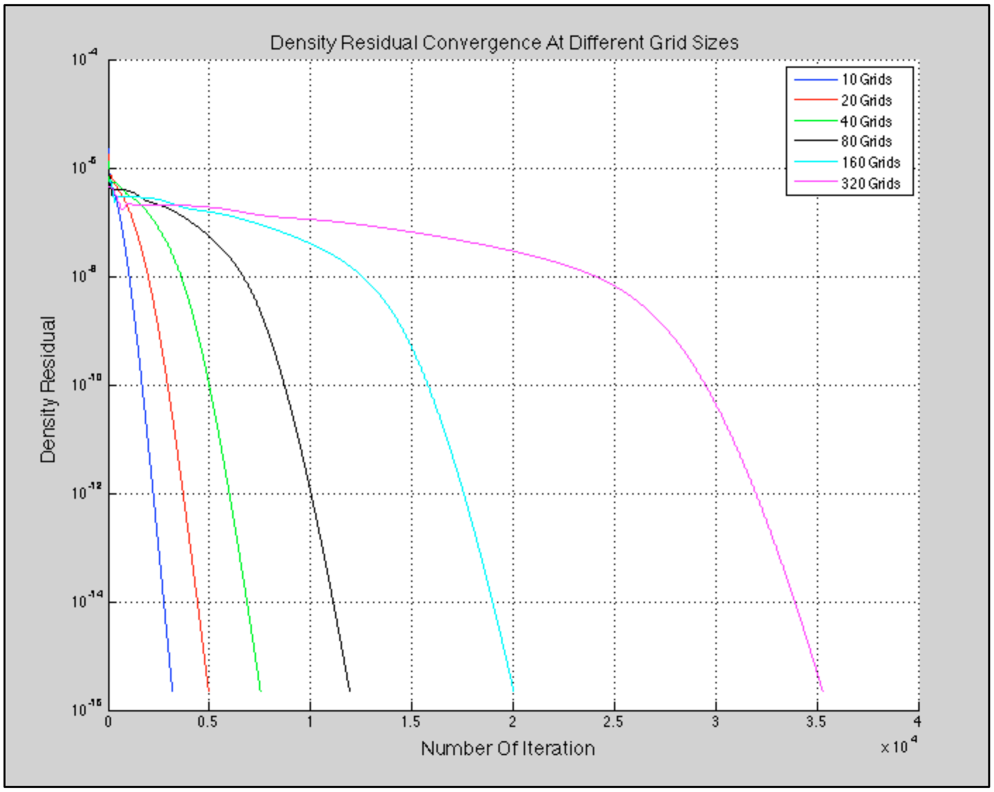

Pressure Ratio Distribution Across the Channel
Pressure Ratio (Local Static Pressure/Inlet Total Pressure)
Channel location
10 Grids
20 Grids
40 Grids
80 Grids
160 Grids
320 Grids

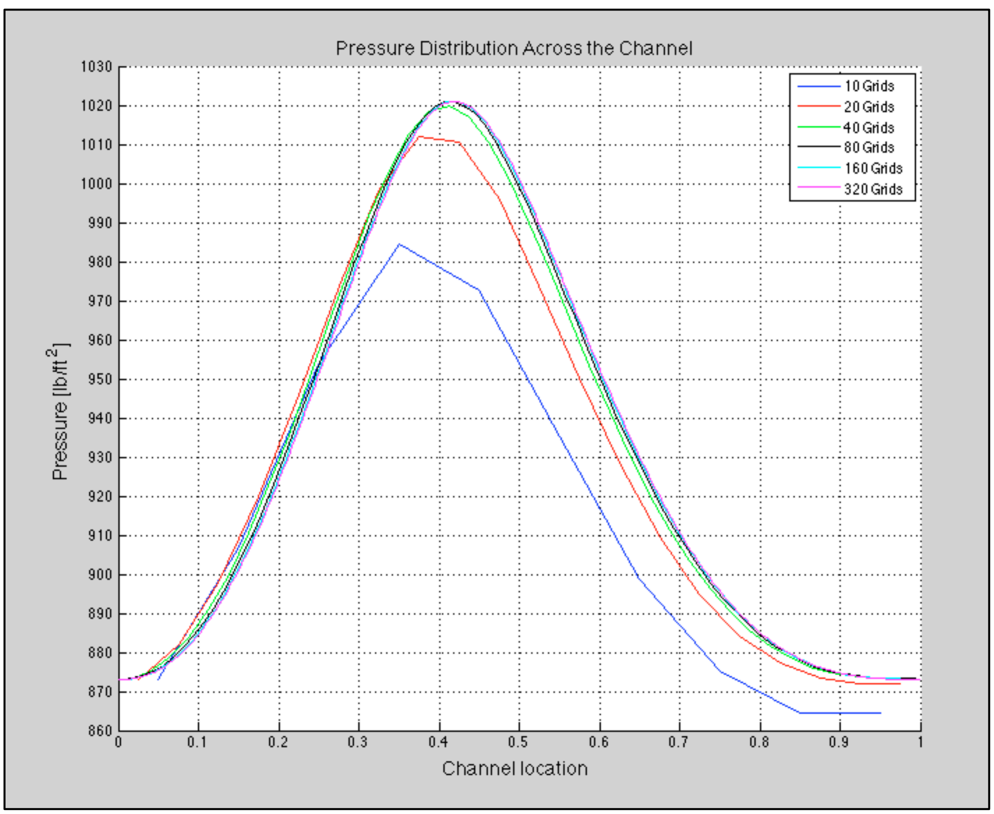

Mach Number Distribution Across the Channel
Mach Number
Channel location
10 Grids
20 Grids
40 Grids
80 Grids
160 Grids
320 Grids

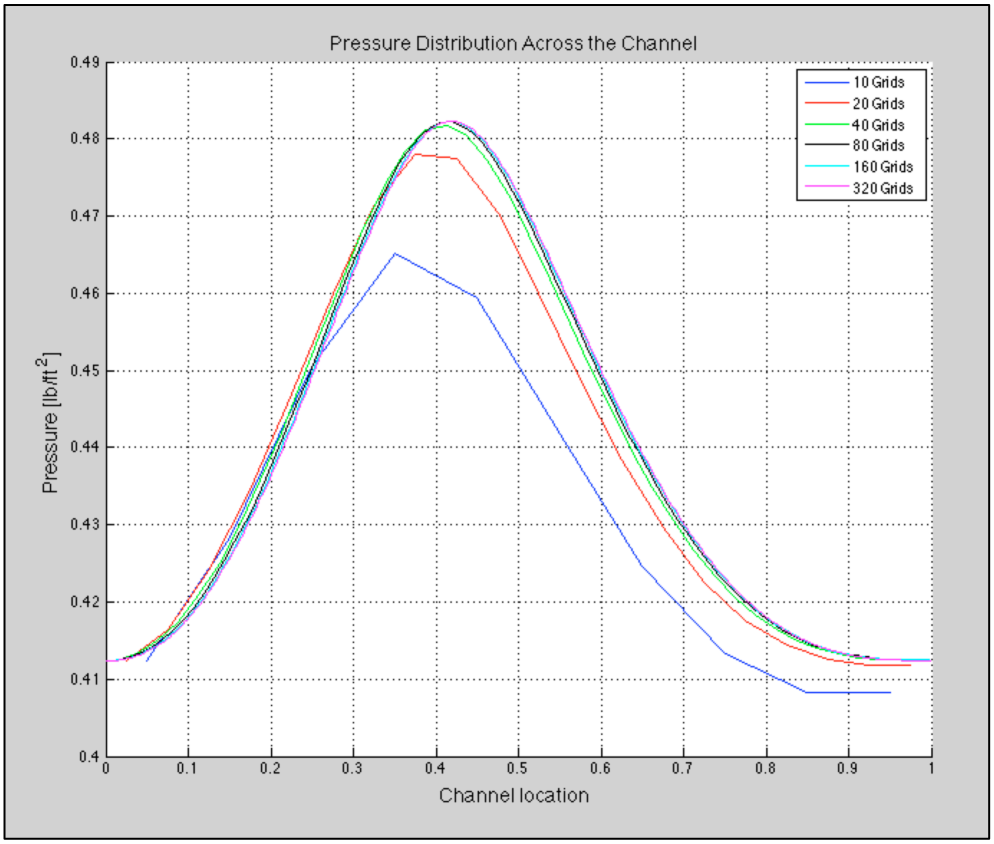
First, for the density residual convergence, the higher grid size took more number of iterations to converge to the solution. For the pressure distribution across the channel, lower grid sizes produced the incorrect results that the location of maximum pressure was behind from where it was supposed to be, since the maximum pressure should be located at the throat, which is the channel location of 0.4. Also, the magnitude of maximum pressure was much smaller than the correct value. Thus, the higher grid sizes produced the more accurate result.
For the Mach number distribution, since the inlet Mach number is supersonic, the Mach number at the throat should be close to sonic Mach number. But, lower grid sizes produced the incorrect results that location of minimum Mach number was also behind from where it was supposed to be. Also, the magnitude of Mach number was larger than the correct value. Therefore, the higher grid size produced the accurate result.

Steger Warming Scheme

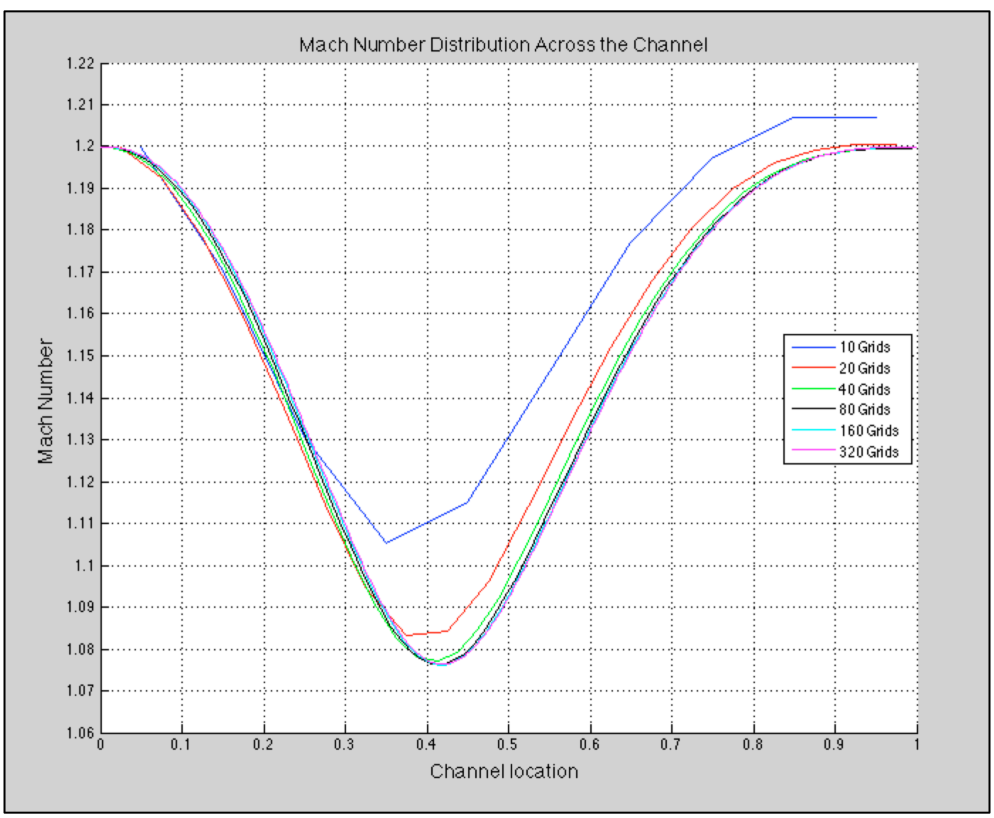
- CFL value was set to 0.1







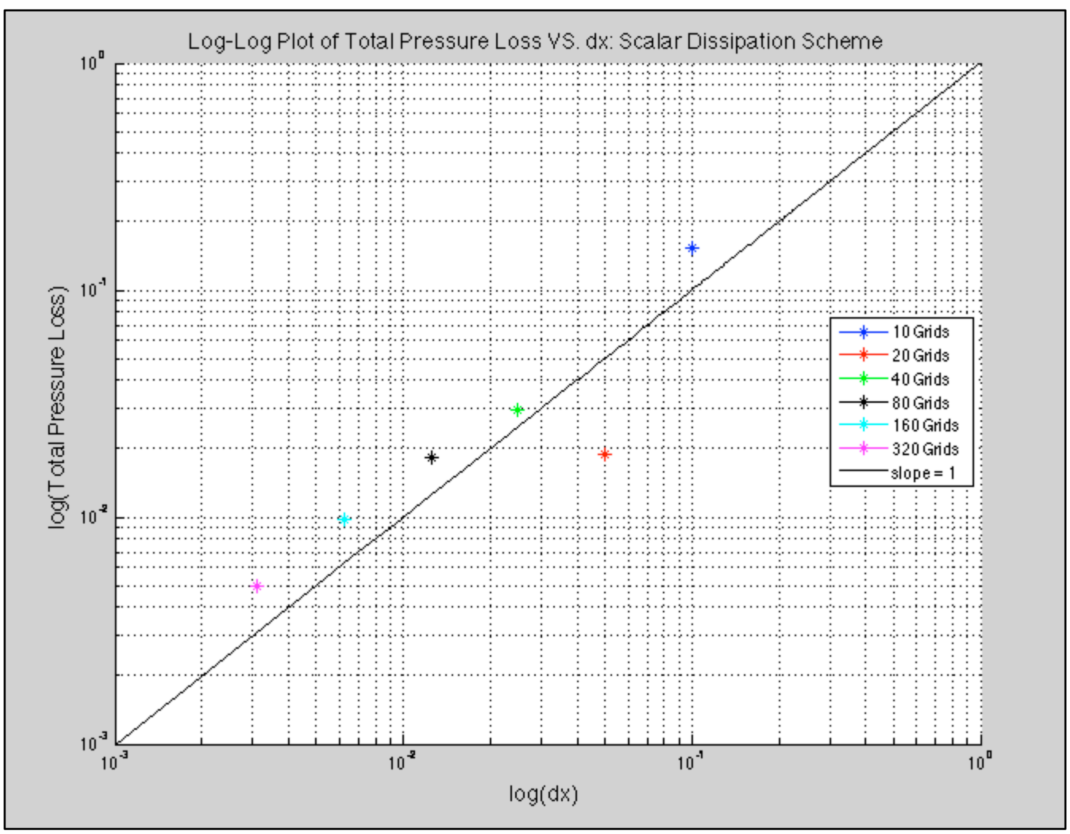
Pressure Distribution Across the Channel
Pressure [lb/ft^2]
Channel location
10 Grids
20 Grids
40 Grids
80 Grids
160 Grids
320 Grids



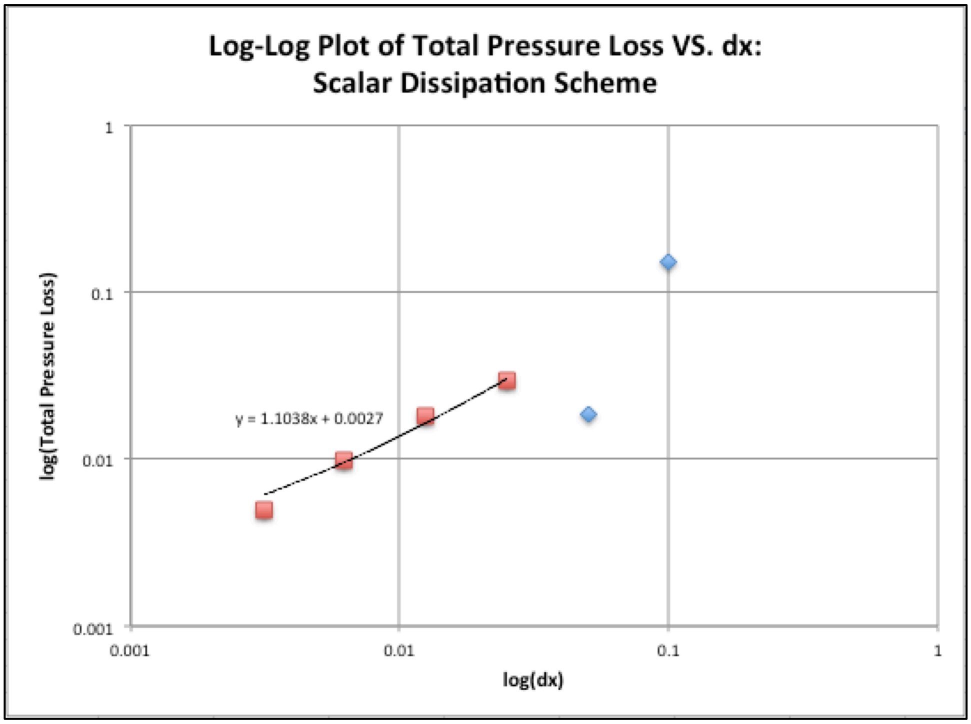
Mach Number Distribution Across the Channel
Mach Number
Channel location
10 Grids
20 Grids
40 Grids
80 Grids
160 Grids
320 Grids

The results of Steger Warming scheme were very similar to those of Scalar Dissipation scheme.
For the density residual convergence, the higher grid size took more number of iterations to converge to the solution, but more number of iteration was required for Steger Warming scheme than the Scalar Dissipation scheme to converge to the solution. For the pressure distribution across the channel, lower grid sizes produced the incorrect results that the location of maximum pressure was behind from where it was supposed to be, since the maximum pressure should be located at the throat, which is the channel location of 0.4. Also, the magnitude of maximum pressure was much smaller than the correct value. Thus, the higher grid sizes produced the more accurate result.
For the Mach number distribution, since the inlet Mach number is supersonic, the Mach number at the throat should be close to sonic Mach number. But, lower grid sizes produced the incorrect results that location of minimum Mach number was also behind from where it was supposed to be. Also, the magnitude of Mach number was larger than the correct value. Therefore, the higher grid size produced the accurate result.

Lastly, log-log plot of percentage of the total pressure loss versus dx was plotted for both Scalar Dissipation scheme and Steger Warming scheme

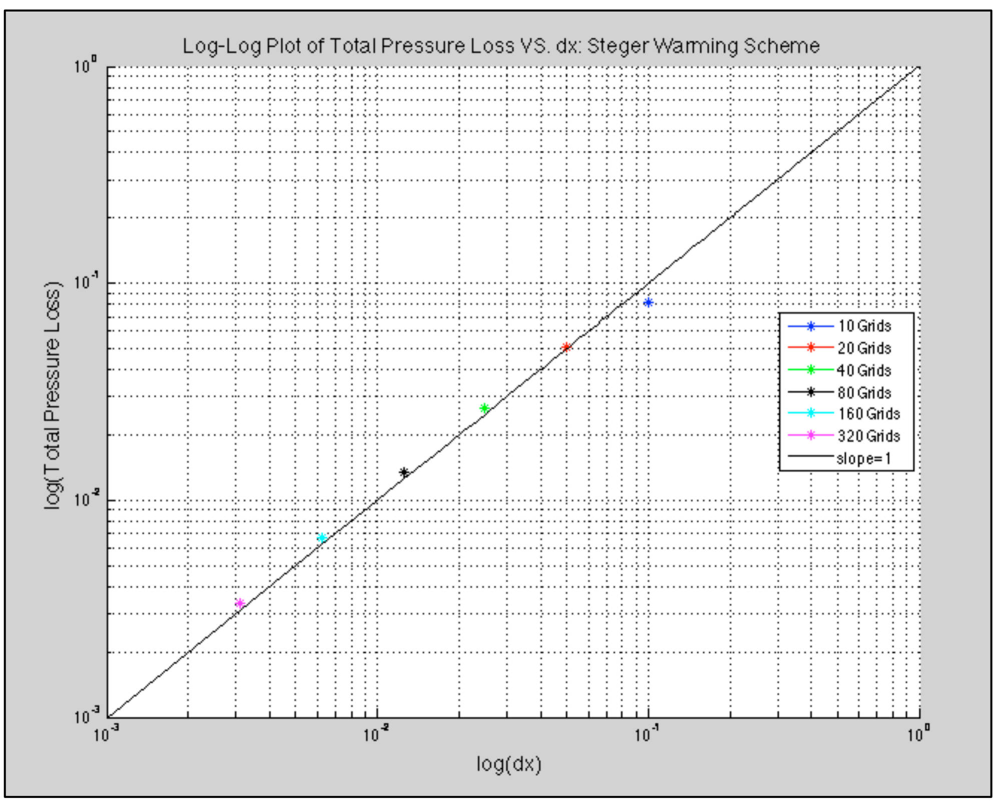


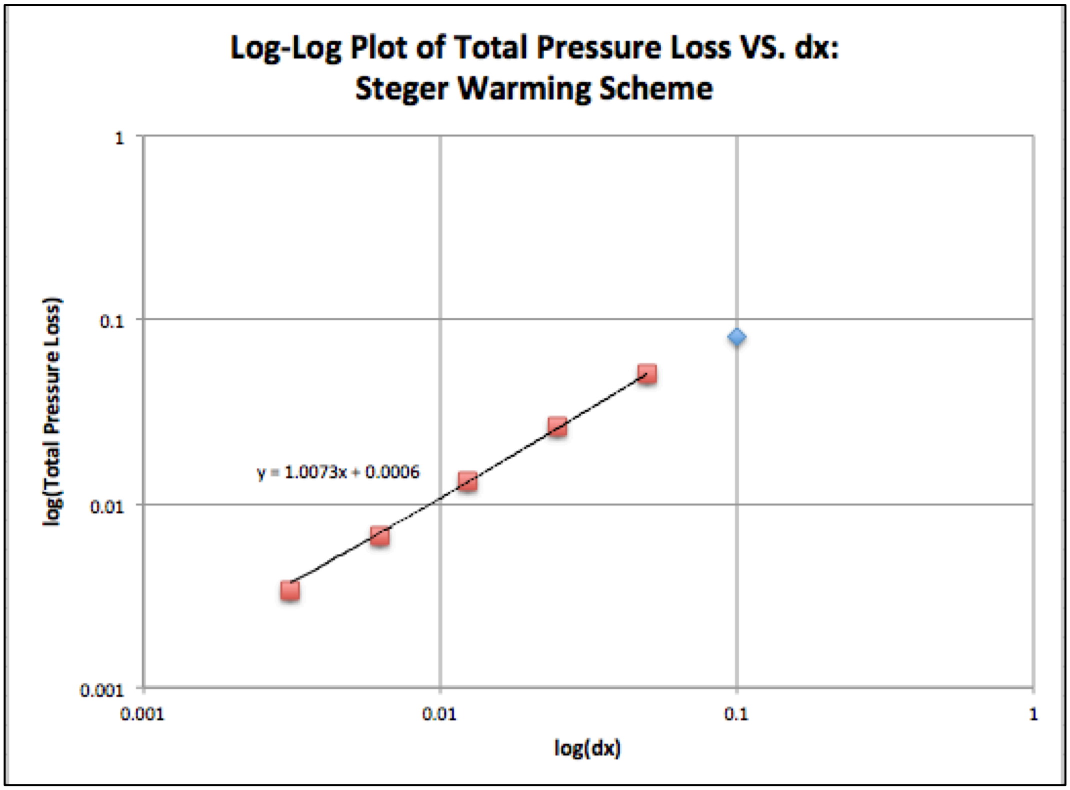
The slope of plots of Scalar Dissipation scheme was close to 1. In the excel, the approximate slope was calculated.



The slope of the plot was obtained as 1.1038, which is close to 1.

Same procedure for the Steger Warming scheme data.





The slope of plots of Steger Warming scheme was obtained as 1.0073, which is almost equal to 1.

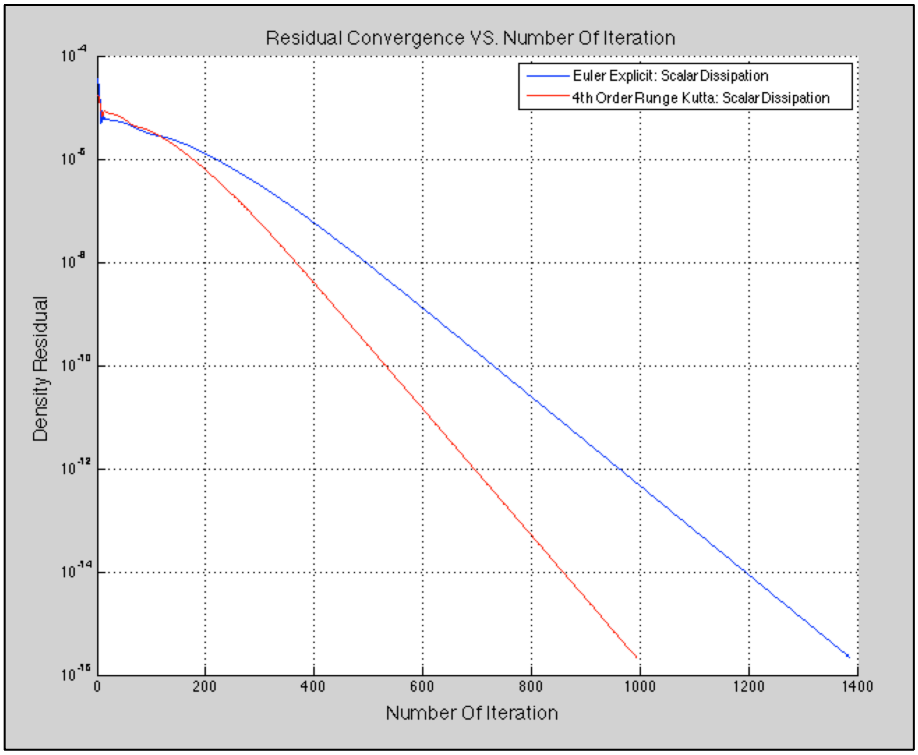
Since both schemes yielded the slope very close to 1, both methods provided the correct order of accuracy.

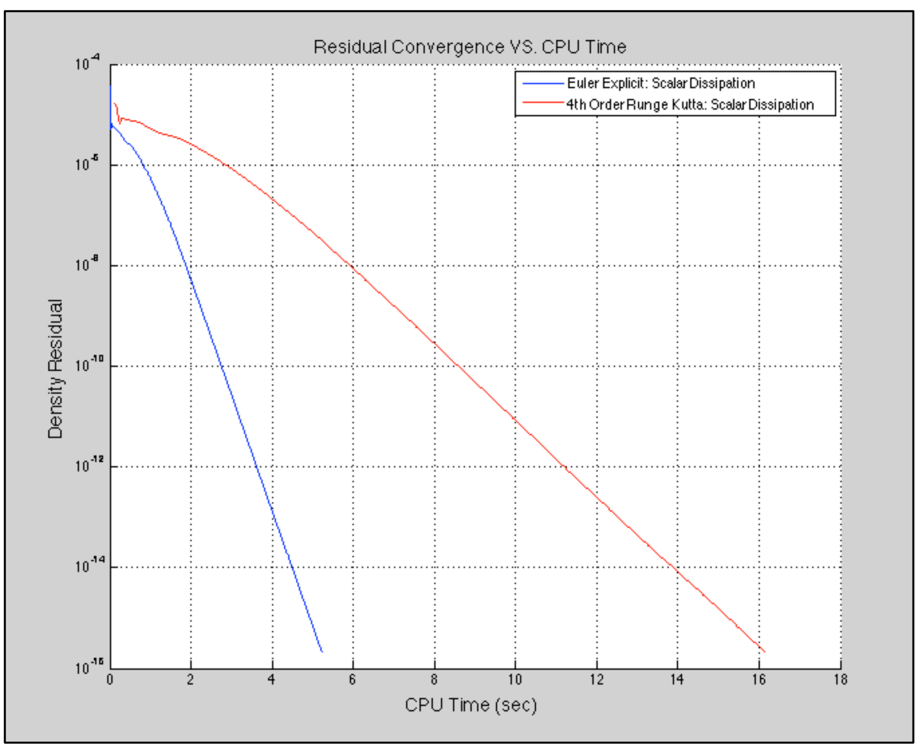
Question 4.

- Steger Warming scheme cases were solved in question1, 2, and 3

Question 5.

– 40 grids were set. CFL was set to 0.9 and epsilon was set to 0.1





The fourth order Runge Kutta with Scalar Dissipation scheme took less number of iteration than the Euler Explicit with Scalar Dissipation scheme to converge the data to the solution. In CPU time, however, Euler Explicit scheme took much less time than the fourth order Runge Kutta scheme. It was because there are several steps of equations to compute data for single iteration in Runge Kutta scheme, while only single equation for single iteration in Euler Explicit scheme. Therefore, since Runge Kutta scheme compute results several times more than Euler Explicit does during each iterations, it requires less iterations to converge to the solution. But, both methods use the same Scalar Dissipation scheme for the equation, Runge Kutta takes much more time than Euler Explicit for single iteration.
In addition, the time taken for Runge Kutta with Steger Warming scheme to converge the data to the solution for 40 grids was much longer than other scheme that no results was converged even much after 40 minutes of run.