Engineering Analysis 2 Design Project

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1. Problem Statement

A crane is lifting two masses, with weights W_A and W_B, tied together. The angle made by each weight's cable (with tensions T_A and T_B) with the connection point is α. The crane exerts a force F, at angle β, and the total force the crane can exert is given by $F_{max} = 0.5 \cos(\beta)$ kN. The weights are proportional such that $\gamma W_A = W_B$ (drawing not to scale).

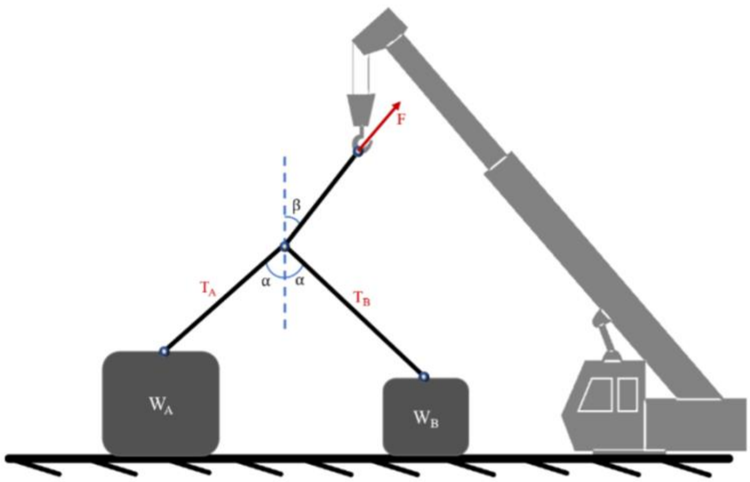


Figure 1: Diagram of the Problem

1. Express T_A and T_B in terms of F, α, and β, assuming F is such that the masses do not rise. (50 pts)
2. Express the angle β needed to lift both masses simultaneously, in terms of γ, W_A, and F. (50 pts)
3. If $\gamma = 1$, find the angle β needed to lift only a single mass. (25 pts)
4. Plot γ as a function of α and β, for $0 < \alpha < \pi/4$ and $0 < \beta < \pi/4$. (40 pts)
5. Plot F_{max} as a function of γ and W_A, if the crane is exerting its maximum power, for $1 < \gamma < 2$ and $1 < W_A < 10$ N. (35 pts)

2. Theory Manual

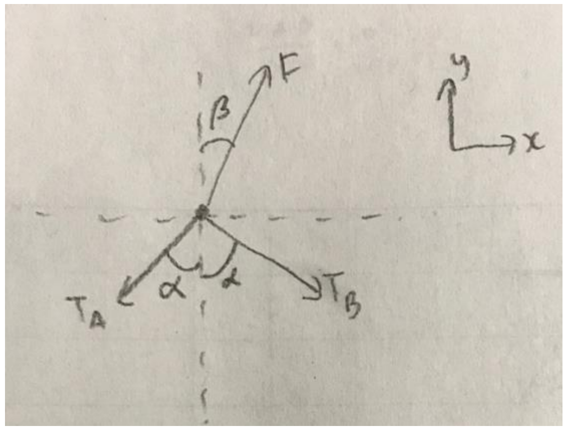


Figure 2: FBD of F, T_A and T_B at equilibrium

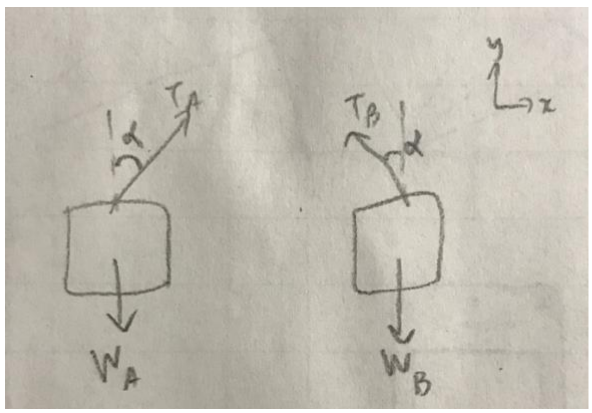


Figure 3: FBD of tension forces at equilibrium with their attached weights

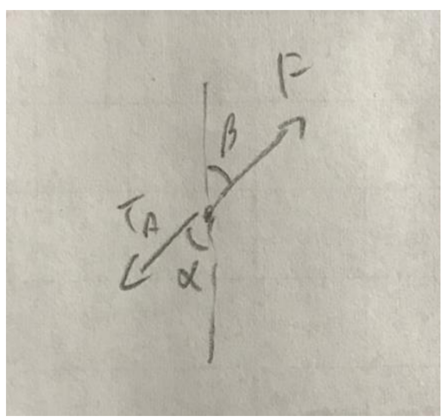


Figure 4: FBD of F at equilibrium with T_A

Variables used in the theory manual are equivalent to those given in the problem statement, namely weights W_A and W_B, angles α and β (which correspond to their labels on Figure 1), tension forces T_A and T_B, crane force F and F_max, and constant γ. Note that subscript is expressed with an underscore unless in an equation. Equations are numbered in the format …(#). Assume angles are in radians and forces in newtons.

Problem 1

The force F as well as the two tension forces T_A and T_B are in equilibrium as the two masses are in constant velocity (of zero). Thus we can refer to Figure 2 (which happens to contain all the variables we need for this problem) and set up the equilibrium equations for forces in the x direction and y direction, respectively.

$$F \sin\beta + T_B \sin\alpha - T_A \sin\alpha = 0 \ldots (1)$$

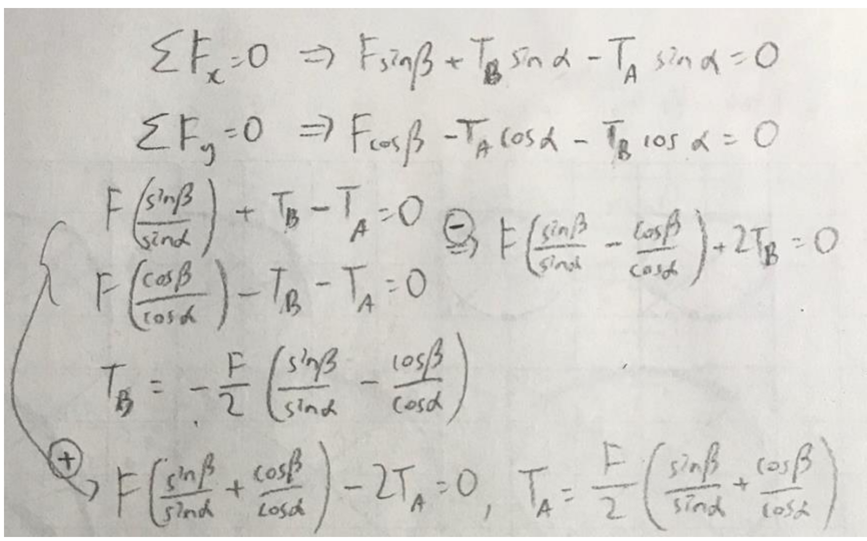
$$F \cos\beta - T_A \cos\alpha - T_A \cos\alpha = 0 \ldots (2)$$

Manipulating the two equations produces the following expressions for T_A and T_B:

$$T_A = \frac{F}{2}\left(\frac{\sin\beta}{\sin\alpha} + \frac{\cos\beta}{\cos\alpha}\right) \text{ (N)} \ldots (3)$$

$$T_B = -\frac{F}{2}\left(\frac{\sin\beta}{\sin\alpha} - \frac{\cos\beta}{\cos\alpha}\right) \text{ (N)} \ldots (4)$$

Detailed calculations and work are as follows:



$$\Sigma F_x = 0 \Rightarrow F\sin\beta + T_B \sin\alpha - T_A \sin\alpha = 0$$

$$\Sigma F_y = 0 \Rightarrow F\cos\beta - T_A \cos\alpha - T_B \cos\alpha = 0$$

$$F\left(\frac{\sin\beta}{\sin\alpha}\right) + T_B - T_A = 0 \quad \ominus \Rightarrow F\left(\frac{\sin\beta}{\sin\alpha} - \frac{\cos\beta}{\cos\alpha}\right) + 2T_B = 0$$

$$F\left(\frac{\cos\beta}{\cos\alpha}\right) - T_B - T_A = 0$$

$$T_B = -\frac{F}{2}\left(\frac{\sin\beta}{\sin\alpha} - \frac{\cos\beta}{\cos\alpha}\right)$$

$$\oplus \Rightarrow F\left(\frac{\sin\beta}{\sin\alpha} + \frac{\cos\beta}{\cos\alpha}\right) - 2T_A = 0, \quad T_A = \frac{F}{2}\left(\frac{\sin\beta}{\sin\alpha} + \frac{\cos\beta}{\cos\alpha}\right)$$

Figure 5: Work for Problem 1

Problem 2

We now use Figure 3 which contains the free body diagrams for the weights. Let us assume equilibrium when lifting the masses simultaneously (both masses are going to have the same constant velocity when being lifted). Using the relation γW_A = W_B as well as the equilibrium equations derived from Figure 3

$$T_A \cos \alpha - W_A = 0 \ldots (5)$$

$$T_B \cos \alpha - W_B = 0 \ldots (6)$$

And the equilibrium equations (1) and (2) since the force pulling the masses is still in equilibrium with the tension forces allows us to attain the final expression for β which is as follows:

$$\beta = \cos^{-1} \frac{W_A(1+\gamma)}{F} \text{ (rad)} \ldots (7)$$

Detailed calculations and work are as follows:

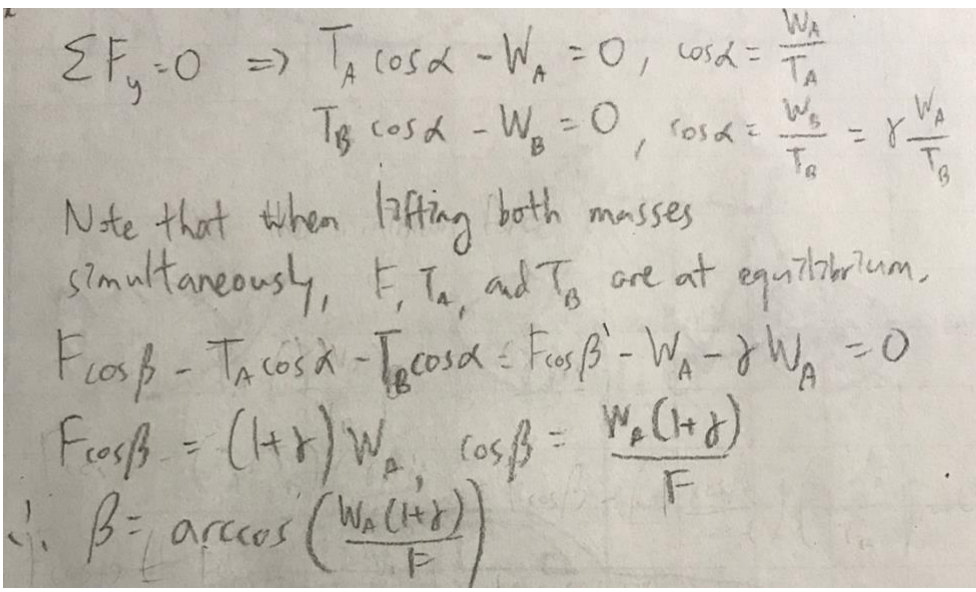


Figure 6: Work for Problem 2

Problem 3

γ being 1 means that W_A and W_B are equal in value and thus we can pick either of the two weights to proceed with the solution. Picking W_A for convenience, we can draw the FBD that isolates F and T_A since the one with T_A and W_A doesn't really help with finding the value of β. The equilibrium equations for Figure 4 are as follows:

$$T_A \sin \alpha = F \sin \beta \ldots (8)$$

$$T_A \cos \alpha = F \cos \beta \ldots (9)$$

Manipulating the two equations produces the final result of

$$\beta = \alpha \text{ (rad)} \ldots (10)$$

Below is the detailed work for this problem.

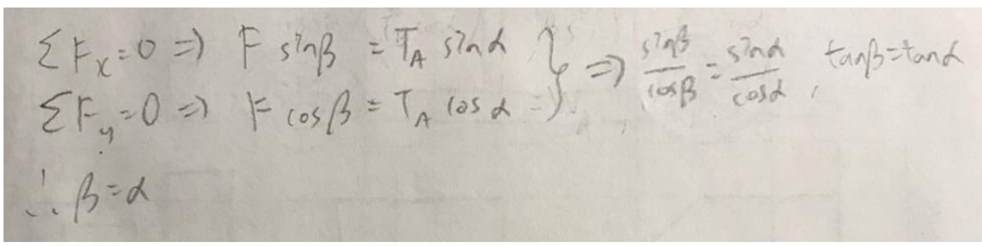


Figure 7: Work for Problem 3

Problem 4

The crane force and the two tension forces are in equilibrium again, and so are the tension forces and each of the respective weights. We use equations 5 and 6 as well as the expressions for the tension forces in equations 3 and 4 to attain the following expression for γ:

$$\gamma = \frac{\sin(\alpha - \beta)}{\sin(\alpha + \beta)} \ldots (11)$$

Detailed work is as follows.

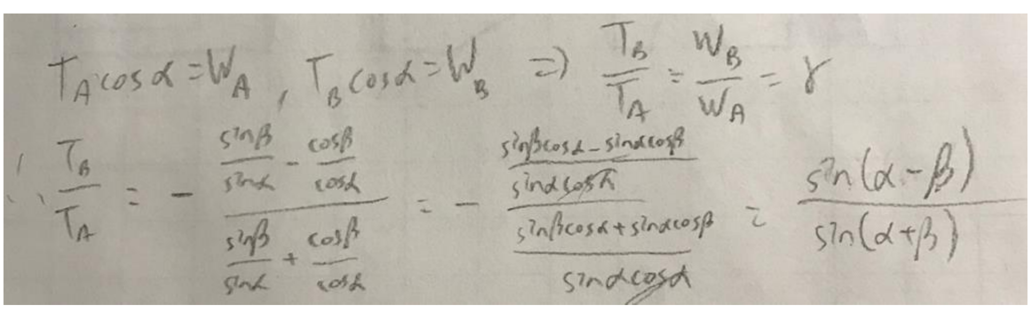


Figure 8: Work for Problem 4

Problem 5

The crane force and the two tension forces are in equilibrium again, and so are the tension forces and each of the respective weights. We use F_max instead of F, and using the expression for F_max as well as the definition of β found in equation 7 to attain the following expression for F_max:

$$F_{max} = \sqrt{0.5W_A(1+\gamma)}\ (N) \ldots (12)$$

Detailed work is as follows:

$$F_{max} = 0.5 \cos\left(\arccos\left(\frac{W_A(1+\gamma)}{F_{max}}\right)\right) = 0.5\,\frac{W_A(1+\gamma)}{F_{max}}$$

$$F_{max} = \sqrt{0.5W_A(1+\gamma)}$$

Figure 9: Work for Problem 5

3. Programmer Manual

Variable	Description
a	Vector of values of alpha from 0 to π/4 in radians -> x coordinate of meshgrid of a and b
b	Vector of values of beta from 0 to π/4 in radians -> y coordinate of meshgrid of a and b
gamma	Array of values of gamma computed from a and b
g	Vector of values of gamma from 1 to 2 in radians -> x coordinate of meshgrid of g and wa
wa	Vector of values of W_A from 1 to 10 in newtons -> y coordinate of meshgrid of g and wa
fmax	Array of values of fmax computed from g and wa

Function	Description
linspace	Returns vector of evenly spaced points defined by inputs
meshgrid	Returns 2d coordinates based on input vectors
sin	Returns sin value of input
figure	Creates new figure window
surf	Creates 3d colored surface plot based on inputs
title	Creates title of plot
xlabel, ylabel, zlabel	Creates axis labels for plot
sqrt	Returns square root of input value

Outline

- Create vectors a and b
- Create meshgrid based on a and b
- Compute gamma
- Plot a, b, and gamma on same set of axes
- Create title and labels for the plot
- Create vectors g and wa
- Create meshgrid based on g and wa
- Compute fmax
- Plot g, wa, and fmax on same set of axes
- Create title and labels for the plot

4. Results and Analysis

Problem 1:

$$T_A = \frac{F}{2}\left(\frac{\sin\beta}{\sin\alpha} + \frac{\cos\beta}{\cos\alpha}\right)\ (N)\ \ldots (3)$$

$$T_B = -\frac{F}{2}\left(\frac{\sin\beta}{\sin\alpha} - \frac{\cos\beta}{\cos\alpha}\right)\ (N)\ \ldots (4)$$

This result makes sense because T_B is in the same x direction as F, which means that it doesn't require as much magnitude to balance out with F, while T_A needs to balance out F in both the x and y directions. The expression putting T_B at a higher magnitude than T_A thus makes sense. The coefficient F/2 also makes sense because the components of T_A and T_B need to sum up to that of F, looking at Figure 2.

Problem 2:

$$\beta = \cos^{-1}\frac{W_A(1+\gamma)}{F}\ (rad)\ \ldots (7)$$

This result makes sense because beta is an angle, so it makes sense to have the inverse cosine function in the expression. Having gamma multiplied to W_A also makes sense because that constitutes W_B. F being involved in the equation also makes sense because beta is the angle F forms with the y axis, as seen in Figure 2.

Problem 3:

$$\beta = \alpha\ (rad)\ \ldots (10)$$

This result makes sense because then T_A and T_B lie on the same line in terms of direction, which would make the pulling of the crane a lot easier and efficient to execute. Both angles being acute angles also makes sense because beta being substantially larger than alpha to the point that it becomes obtuse, for instance, would make the pulling a lot harder to operate.

Problem 4:

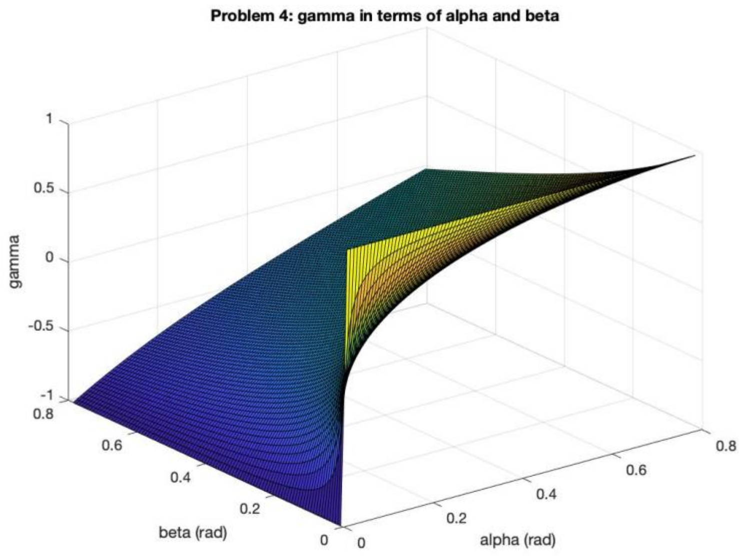


Figure 10: Result of Problem 4

$\gamma = \frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)}$ is the equation for the plot above, and the equation does seem to match up with the plot because gamma shows an upward trend as alpha increases from 0 to $\pi/4$. One thing to note is that there are negative values of gamma being plotted on the graph. In theory, by definition of gamma, there shouldn't be negative values because gamma is the coefficient between two weights. However, in the problem (and specifically Figure 3) we defined W_A and W_B in the downward y direction. Having a negative value of gamma, and thus a negative value of W_B may imply that, for certain combinations of the angles alpha and beta, there is an upward force acting on the mass of weight B.

Problem 5:

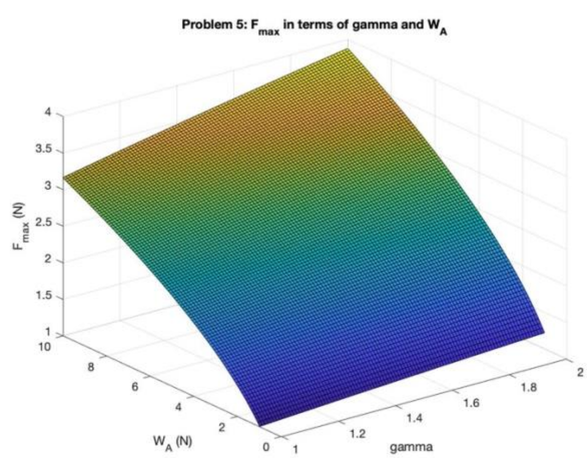


Figure 11: Result of Problem 5

$F_{max} = \sqrt{0.5W_A(1+\gamma)}$ is the equation for the above plot, and it makes sense because it appears to be displaying a square root – like shape of growth. The values being nonnegative also makes sense because we need a positive value of force to pull the masses up. As the weight of A increases we see an increase in the maximum force (same for gamma, but at a less noticeable degree) and this makes sense because the heavier the objects get, the greater the force the crane can – and should – be able to exert.

5. Appendix

```
a = linspace(0,pi/4,100);
b = linspace(0,pi/4,100);
[a,b] = meshgrid(a,b);

gamma = sin(a-b)./sin(a+b);
figure
surf(a,b,gamma)
title('Problem 4: gamma in terms of alpha and beta')
xlabel('alpha (rad)')
ylabel('beta (rad)')
zlabel('gamma')

g = linspace(1,2,100);
wa = linspace(1,10,100);
[g,wa] = meshgrid(g,wa);
fmax = sqrt(0.5.*wa.*(1+g));

figure
surf(g,wa,fmax)
title('Problem 5: F_m_a_x in terms of gamma and W_A')
xlabel('gamma')
ylabel('W_A (N)')
zlabel('F_m_a_x (N)')
```