Problem set 7

CHEM6343 : Graduate Quantum Mechanics

November 2, 2020

Problem 1. Harmonic Oscillator Perturbation

Consider a 1 dimensional Harmonic Oscillator with frequency ω_0 and charge q. Let $|\phi_n\rangle$ be the eigenstates and E_n the eigenvalues of the Harmonic oscillator in the absence of an electric field.

For $t < 0$ the oscillator is in its ground state; at t=0 an electric field is activated for a time duration τ giving a perturbation

$$W(t) = \begin{cases} -q\mathcal{E}X & \text{if } 0 \leq t \leq \tau \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $\mathcal{E}$ is the field amplitude and X is the position variable. Let $\mathcal{P}_{n,0}$ be the probability that oscillator has transitioned to the state $|\phi_n\rangle$ after the pulse.

(a) Calculate $\mathcal{P}_{1,0}$ using first order perturbation theory. How does $\mathcal{P}_{1,0}$ vary with τ for fixed ω_0? **(b)** Show that to obtain a non-zero $\mathcal{P}_{2,0}$ requires, at minimum, a second-order perturbation theory calculation. **(c)** Write out the perturbation theory diagrams for each component of the first and second order terms.

Problem 2. Hydrogen Atom Perturbation A hydrogen atom is placed in a time-dependent electric field $\vec{E}(t) = E(t)\vec{e}_z$ that is oriented along the z-axis. Calculate all matrix elements of the perturbation $W = eE(t)z$ between the ground state (n=1) and the (quadruply degenerate) first excited state (n=2).

HINT: USE THE SYMMETRY OF THE PROBLEM - there is only 1 integral that you actually need to evaluate.

Problem 3. Two-level System Consider a perturbation to a two-level system (basis: $|a\rangle, |b\rangle$)with matrix elements

$$W_{a,b} = W_{b,a} = \frac{\alpha}{\sqrt{\pi}\tau} e^{-(t/\tau)^2} \tag{2}$$

and $W_{a,a} = W_{b,b} = 0$, where $\alpha, \tau > 0$.

(a) Calculate $\mathcal{P}_{b\leftarrow a}(t)$ at $t = \inf$ if the initial wave function is in state a at $t = -\inf$.

(b) In the limit that $\tau \to 0$, $W_{a,b} = \alpha\delta(t)$. Compute the $\tau \to 0$ limit of your expression in part (a).

(c) Calculate $\mathcal{P}_{b\leftarrow a}(t)$ at $t = \inf$ using $W_{a,b} = \alpha\delta(t)$ if the initial wave function is in state a at $t = -\inf$. Compare to your expression in part (b).

(d) Now consider the opposite limit: $\omega_0\tau >> 1$. What is the limit of your expression in part (a)?

Problem 4. Rabi Flopping Solve Griffiths Problem 11.9.

Problem 5. Time Ordered Exponentials Solve Griffiths Problem 11.23.

NOTE: This problem is challenging and will take time. Don't wait for the last minute...