CS224w:
Social and Information Network Analysis

Assignment Submission Fill in and include this cover sheet with each of your assignments. Assignments are due at 9:00 am. All students (SCPD and non-SCPD) must submit their homeworks via GradeScope (`http://www.gradescope.com`). Students can typeset or scan their homeworks. Make sure that you answer each question on a separate page. That is, one answer per page regardless of the answer length. Students also need to upload their code at `http://snap.stanford.edu/submit`. Put all the code for a single question into a single file and upload it. Please do not put any code in your GradeScope submissions.

Late Day Policy Each student will have a total of *two* free late periods. *One late period expires at the start of each class.* (Homeworks are usually due on Thursdays, which means the first late periods expires on the following Tuesday at 9:00am.) Once these late periods are exhausted, any assignments turned in late will be penalized 50% per late period. However, no assignment will be accepted more than *one* late period after its due date.

Honor Code We strongly encourage students to form study groups. Students may discuss and work on homework problems in groups. However, each student must write down their solutions independently i.e., each student must understand the solution well enough in order to reconstruct it by him/herself. Students should clearly mention the names of all the other students who were part of their discussion group. Using code or solutions obtained from the web (github/google/previous year solutions etc.) is considered an honor code violation. We check all the submissions for plagiarism. We take the honor code very seriously and expect students to do the same.

Your name: Devon Zuegel
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Discussion Group: ______

I acknowledge and accept the Honor Code.

(Signed) Devon Kristine Zuegel

Devon Zuegel // 8 Oct 2015 // CS 224W

Problem Set 1

Problem 1 :: Network characteristics (30 points)

Problem 1, Part A

> Code in `hw1p1a.py` + `myutil.py`.

Problem 1, Part B

In any given social group, your friends are on average more popular than you are, due to a sampling bias called The Friendship Paradox. This happens because "people with more friends are more likely to be your friend in the first place; that is, they have a higher propensity to make friends in the first place" (Wikipedia).

The same is true for this node network, where a direct neighbor of any given node has a higher expected degree than it. By increasing the probability that new nodes are connected to a neighbor node rather than just any random node, we are creating the graph through **preferential attachment**.

As a result, the degrees follow a power law distribution more closely for lower values of `prob` (which corresponds to higher probabilities that new nodes connect to a random existing node `n`'s neighbor rather than `n` itself).

Problem 1, Part C

> Code in `hw1p1c.py` + `myutil.py`.

If a large bin contains an outlier or a large number of nodes, we would expect to see a larger alpha value.

Output from running `hw1p1a.py`:

```
xmin  = 1
alpha = 2.02223810403
```

These values aren't super surprising. The slope at $10^0 = 1$ of the log graph (generated by running the code found in `hw1p1a.py`) in part a is basically ∞, straight straight up and down vertically, which suggests that the the minimum value is `1`.

Our $\alpha = 2.0222\ldots$ also makes sense, since the power law reading explained that any alpha whose value is `> 1` has "bins" of increasing size but decreasing number of nodes they contain.

Problem 1, Part D

> Code in `hw1p1d.py` + `myutil.py`.

In part a, earlier nodes tend to have a higher degree because they have more opportunities to connect to many nodes. Meanwhile, in this part we see that most nodes have approximately the same degree along a normal distribution. This is because each node is connected (on average) to 10% of the other nodes at any given time, rather than new nodes being randomly connected to any already-added nodes (which is what gave old nodes the advantage in part a).

Problem 2 :: Who is the most central actor? (30 points)

Problem 2, Part A

20 actors with the highest degree centrality:

```
RANK  NAME                   DEGREE            NUM FILMS   MAIN GENRE
#1    Davis, Mark (V)        0.0449180703564   540         Adult
#2    Sanders, Alex (I)      0.0349490088232   467         Adult
#3    North, Peter (I)       0.0343187807952   460         Adult
#4    Marcus, Mr.            0.0334593789389   435         Adult
#5    Tedeschi, Tony         0.0321416294259   364         Adult
#6    Dough, Jon             0.0317978686834   300         Adult
#7    Stone, Lee (II)        0.0312249341125   403         Adult
#8    Voyeur, Vince          0.0305374126275   370         Adult
#9    Lawrence, Joel (II)    0.0286467285436   315         Adult
#10   Steele, Lexington     0.028245674344     429         Adult
#11   Ashley, Jay           0.0280737939727    309         Adult
#12   Boy, T.T.             0.0272143921164    336         Adult
#13   Jeremy, Ron           0.0269852182881    280         Adult
#14   Cannon, Chris (III)   0.0269852182881    287         Adult
#15   Bune, Tyce            0.0265268706314    267         Adult
#16   Hanks, Tom            0.0261831098889    75          Family
#17   Michaels, Sean        0.0258393491463    252         Adult
#18   Stone, Kyle           0.0257820556892    278         Adult
#19   Hardman, Dave         0.0250945342042    319         Adult
#20   Surewood, Brian       0.0245215996333    244         Adult
```

- Every actor on that list (except Tom Hanks) has been at well over 200 films. As such, they've simply worked with lots of people.
- Every actor on that list (again except for Tom Hanks) mostly stars in adult films.

Problem 2, Part B

20 actors with the highest betweenness centrality:

```
RANK  NAME                    BETWEENNESS      NUM FILMS
#1    Jeremy, Ron             9748544.2189     280
#2    Chan, Jackie (I)        4716909.32165    59
#3    Cruz, Penelope          4330663.26451    46
#4    Shahlavi, Darren        4295502.79784    16
#5    Del Rosario, Monsour    4267099.43969    20
#6    Depardieu, Gerard       4037356.14719    56
```

```
#7    Bachchan, Amitabh       2570247.12237    35
#8    Jackson, Samuel L.      2539613.88751    97
#9    Soualem, Zinedine       2368164.44674    65
#10   Del Rio, Olivia         2316387.53485    84
#11   Jaenicke, Hannes        2136980.21405    66
#12   Hayek, Salma            2117389.70142    44
#13   Pele                    2098484.5328     10
#14   Knaup, Herbert          2062584.64127    50
#15   Goldberg, Whoopi        2051621.39925    109
#16   Roth, Cecilia           2019247.01694    23
#17   Bellucci, Monica        2006220.95681    43
#18   Hanks, Tom              1977252.23099    75
#19   August, Pernilla        1937362.14452    31
#20   Kier, Udo               1919260.77495    69
```

- While the actors with high degree centrality were all extremely prolific, the actors on this list are nearly all very well-respected in multiple genres. Vertices that have a high probability to occur on a randomly chosen shortest path between two randomly chosen vertices have a high betweenness, and since these actors are all so well-respected in multiple genres it makes sense that they are a connection point for usually disparate groups.
- The actors on this list tend to be involved in dramas, which as we can see from the actor graph tend to be more spread out (as compared to the fantasy folks who are all clumped together).
- The only actors found on both lists are “Jeremy, Ron” and “Hanks, Tom”.
- Betweenness centrality tends to follow a power law distribution, which is reflected even here, where we have only the top 20: the top ranked actor “Jeremy, Ron” has nearly twice the betweenness score as the #2 ranked actor “Chan, Jackie (I)”. Meanwhile the #2-#6 ranked actors' scores are nearly twice that of #7-20 (and probably beyond).

Problem 2, Part C

20 ACTORS WITH THE HIGHEST CLOSENESS CENTRALITY:

```
RANK  NAME                    BETWEENNESS      NUM FILMS
#1    Jackson, Samuel L.       0.309265198363    97
#2    Goldberg, Whoopi         0.307760125544    109
#3    Berry, Halle             0.305904621694    63
#4    Diaz, Cameron            0.305668902471    59
#5    Hanks, Tom               0.305230575521    75
#6    Stiller, Ben             0.304719006966    66
#7    Myers, Mike (I)          0.30261104754     58
#8    Douglas, Michael (I)     0.302605801071    41
#9    Lopez, Jennifer (I)      0.301216670981    68
#10   De Niro, Robert          0.300708095722    51
#11   Willis, Bruce (I)        0.300485487036    52
#12   Cruise, Tom              0.300407910363    46
#13   Hopper, Dennis           0.299336294569    106
#14   Kidman, Nicole           0.298767545361    54
#15   Smith, Will (I)          0.298552906161    57
#16   Washington, Denzel       0.298547799463    49
```

```
#17   Travolta, John           0.298512057465    63
#18   Madonna (I)              0.298358974359    61
#19   Schwarzenegger, Arnold   0.297743129595    70
#20   Hoffman, Dustin          0.29758068641     56
```

- All of the actors on this list are “A-list celebrities”. They are not as prolific as those on the first list and respected in as many different genres as the second, but they are the most famous. Thus while they don’t have incredibly high degree nor do they connect disparate groups, they are highly sought after and have likely all acted alongside another performer who does have those other characteristics. They are in the center of things rather than on the fringe.
- “Hanks, Tom” is the only actor to show up on all three lists (and the intersection of the first and third), while only “Jackson, Samuel L.” and “Goldberg, Whoopi” join him on both the second and third lists.

Problem 3 :: Foodie Madness (40 points)

Notes on Matrix Multiplication & Dot Products

If we multiply $\mathbf{x}^T$ (a $1 \times n$ matrix) with any n-dimensional vector $\mathbf{y}$ (viewed as an $n \times 1$ matrix), we end up with a matrix multiplication equivalent to the familiar dot product of $\mathbf{x} \cdot \mathbf{y}$:

$$\mathbf{x}^T\mathbf{y} = \begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_n \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix} = x_1y_1 + x_2y_2 + x_3y_3 + \ldots + x_ny_n = \mathbf{x} \cdot \mathbf{y}$$

.

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} a_x & a_y & a_z \end{bmatrix}$$

$$a_x = a_1x_1 + a_2x_2 + a_3x_3$$

$$a_y = a_1y_1 + a_2y_2 + a_3y_3$$

$$a_z = a_1z_1 + a_2z_2 + a_3z_3$$

Problem 3, Part A

(a) $v_i(k)$ = total amt of food consumed by chef $\boxed{i}$ at end of event with $\boxed{k}$ rounds

A = adjacency matrix that captures chefs' mutual agreemts ($\boxed{n} \times \boxed{n}$ matrix of 1s and 0s)

n = # of chefs

$Q_j[i]$ = quantity of food left on chef $\boxed{i}$'s table at the end of round $\boxed{j}$

Round 0

each chef makes q_i & leaves it on their table $\Big\}$ $Q_0 = [q_1, q_2, \dots, q_{n-1}, q_n]$

Round 1

each chef eats p of the food on their table $\Big\}$ pQ_0 consumed

then shares the remainder with their $\boxed{d}$ friends $\Big\}$ $Q_1 = AQ_0\left(\frac{1-p}{d}\right)$

Round 2

pQ_1 consumed $\longrightarrow$ $Q_2 = AQ_1\left(\frac{1-p}{d}\right)$

... and so on ...

$$V_1 = \boxed{pQ_0} = p[q_1, q_2, \dots, q_{n-1}, q_n]$$

$$V_2 = pQ_0 + pQ_1 = p\left[q_1 + \left(\frac{1-p}{d}\right)pq_1, \dots, q_n + \left(\frac{1-p}{d}\right)pq_n\right]$$

$$= \left(\frac{pd+d+1-p}{d}\right)[q_1, q_2, \dots, q_{n-1}, q_n]$$

$$= \boxed{\left(\frac{pd+d+1-p}{d}\right) Q_0}$$

Problem 3, Part B, C, & D

$\boxed{3.}$ $\boxed{\text{Foodie Madness}}$ (cont.)

(b) Want: $F(k) = [f_1(k), f_2(k), \ldots, f_n(k)]$ where $f_i(k)$ = amt of food left on chef $\boxed{i}$'s table at the end of round $\boxed{k}$

$\hookrightarrow Q_k$ in part (a), so I'm going w/ that

$$\left.\begin{aligned} Q_0 &= [q_1, q_2, \ldots, q_n] = X^0 Q_0 \\ Q_1 &= A\left(\tfrac{1-p}{d}\right) Q_0 = X^1 Q_0 \\ Q_2 &= A\left(\tfrac{1-p}{d}\right) Q_1 = X^2 Q_0 \\ Q_3 &= A\left(\tfrac{1-p}{d}\right) Q_2 = X^3 Q_0 \end{aligned}\right\} \quad \text{Let } X = A\left(\tfrac{1-p}{d}\right) \quad \boxed{Q_k = \left[A\left(\tfrac{1-p}{d}\right)\right]^k Q_0} = f(k)$$

(c) Let $m_i(k)$ = amt of food consumed in round $\boxed{k}$ by chef $\boxed{i}$

$$m_i(k) = f_i(k-1)$$

$$\begin{aligned} V_i(k) &= m_i(1) + m_i(2) + \ldots + m_i(k) + pq_i \\ &= f_i(0) + f_i(1) + \ldots + f_i(k-1) \\ &= \sum_{j=0}^{k-1} \left[A\tfrac{(1-p)}{d}\right]^j Q_0 \end{aligned}$$

$$\boxed{V(k) = \sum_{j=0}^{k} \left[A\left(\tfrac{1-p}{d}\right)\right]^j Q_0 + pQ_0 = Q_0 \cdot \sum_{j=0}^{k} \left(A\left(\tfrac{1-p}{d}\right)\right)^j + pQ_0}$$

(d) $V(k) = Q_0 \sum_{j=0}^{k} \left[A\left(\tfrac{1-p}{d}\right)\right]^j + pQ_0$ $\qquad$ Want: $V(\infty)$

$$\left[\sum_{x=0}^{\infty} A^n = (1-A)^{-1} \text{ where } A[i][j] < 1 \text{ for all } i,j\right.$$

Let $X = A\left(\frac{1-p}{d}\right)$

$$\begin{aligned} V(\infty) &= Q_0 \sum_{j=0}^{\infty} X^j + pQ_0 \\ &= Q_0 (1-X)^{-1} + pQ_0 \\ &= \boxed{Q_0 \left(1 - A\left(\tfrac{1-p}{d}\right)\right)^{-1} + pQ_0} \end{aligned}$$

Problem 3, Part E

```
def rank(mylist):
    indices = list(range(len(mylist)))
    indices.sort(key=lambda x: mylist[x], reverse=True)
    ranked = [0] * len(indices)
    for new_i, old_i in enumerate(indices):   ranked[old_i] = new_i + 1
    return ranked
```

Scores after `k = 1` rounds:

```
Q1       = [ 0.6, 0.55, 0.6, 0.55, 0.6, 0.45, 0.6, 0.45]
rankings = [ 1, 5, 2, 6, 3, 7, 4, 8 ]
```

… after `k = 2` rounds:

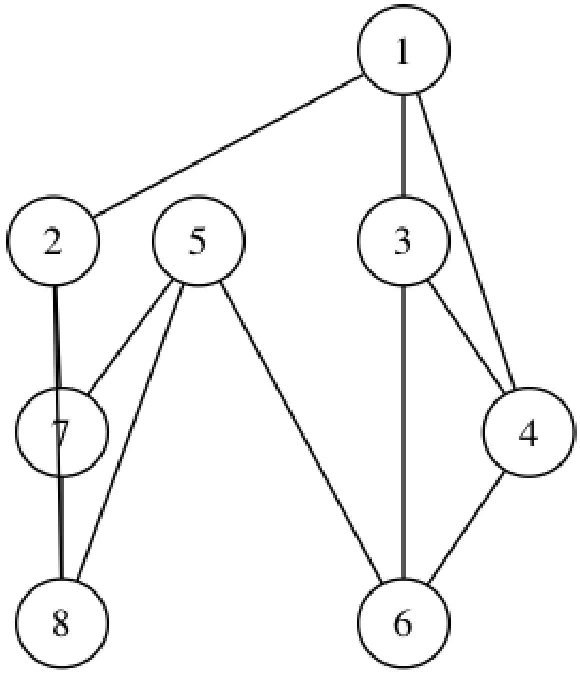
```
Q2       = [ 1.166, 1.1, 1.133, 1.1, 1.1, 1.033, 1.133, 1.033]
rankings = [ 1, 4, 2, 5, 6, 7, 3, 8 ]
```

… after `k = 3` rounds:

```
Q3       = [ 1.4388, 1.3805, 1.4166, 1.3805, 1.3833, 1.2972, 1.4055, 1.2972]
rankings = [1, 5, 2, 6, 4, 7, 3, 8]
```

… after `k = ∞` rounds:

```
Qinf     = [ 10.08, 10.01, 10.04, 10.01, 10.0, 9.95, 10.04, 9.95]
rankings = [ 1, 4, 2, 5, 6, 7, 3, 8 ]
```



Chef graph

Problem 3, Part F

- Part c changes slightly in that what `v(k)` is now the value of what was previously `v(k + 1)`.
- Part d does not change, because `∞ + x = ∞` for any finite `x`. The amount of food eaten at each consecutive round converges to `0` as `k → ∞`.