

Filtrage d'images fréquentiel

Cours n°3

Domaine de Fourier

■ Qu'est-ce que la TF d'une image ?

Transformée de Fourier discrète bidimensionnelle :

(x,y) sont les coordonnées
du *domaine spatial*

Directe :

$$F[u, v] = \sum_{x=0}^{w-1} \sum_{y=0}^{h-1} f[x, y] e^{-2i\pi(ux+vy)/wh}$$

(u,v) sont les coordonnées
du *domaine fréquentiel*

Inverse :

$$f[x, y] = \frac{1}{wh} \sum_{u=0}^{w-1} \sum_{v=0}^{h-1} F[u, v] e^{2i\pi(ux+vy)/wh}$$

Propriétés de la transformée de Fourier (1) :

ÉCRITURE SOUS FORME MODULE / PHASE

$$F[u, v] = \|F[u, v]\| e^{i\varphi[u, v]}$$

PÉRIODICITÉ

$$F[u, v] = F[u+w, v+h]$$

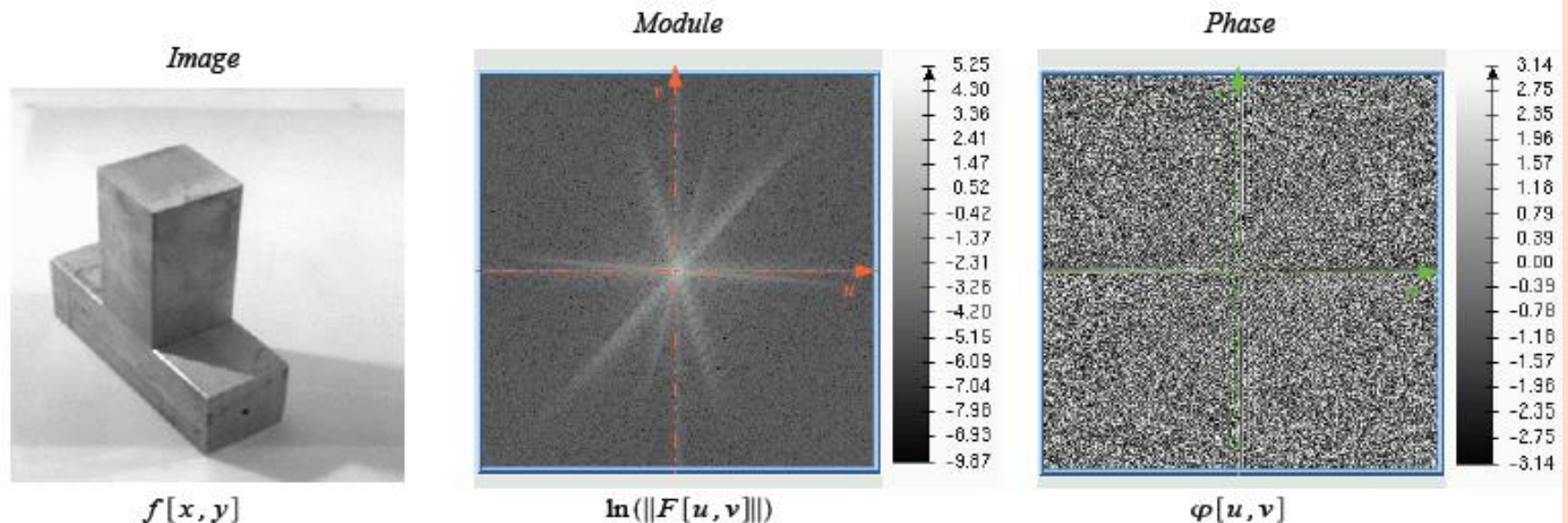
SYMÉTRIE

Si F est la transformée de Fourier d'une fonction réelle f :

$$F[u, v] = \overline{F[-u, -v]} \quad \text{et donc :} \quad \|F[u, v]\| = \|F[-u, -v]\| \quad \text{et} \quad \varphi[u, v] = -\varphi[-u, -v]$$

Source : Cours Manzanera, ENSTA

Domaine de Fourier



Propriétés de la transformée de Fourier (2) :

$$si \begin{cases} f[x, y] \xrightarrow{TF} F[u, v] \\ f_1[x, y] \rightarrow F_1[u, v] \\ f_2[x, y] \rightarrow F_2[u, v] \end{cases}$$

CORRESPONDANCE CONVOLUTION / PRODUIT

$$f_1[x, y] * f_2[x, y] \rightarrow F_1[u, v] \cdot F_2[u, v]$$

$$f_1[x, y] \cdot f_2[x, y] \rightarrow F_1[u, v] * F_2[u, v]$$

DÉRIVATION

$$\frac{\partial f[x, y]}{\partial x} \rightarrow iuF[u, v] \text{ et } \frac{\partial f[x, y]}{\partial y} \rightarrow ivF[u, v]$$

LINEARITÉ

$$a \cdot f_1[x, y] + b \cdot f_2[x, y] \rightarrow a \cdot F_1[u, v] + b \cdot F_2[u, v]$$

TRANSLATIONS SPATIALES / FRÉQUENTIELLES

$$f[x - x', y - y'] \rightarrow F[u, v] \cdot e^{-2i\pi(ux' + vy')/wh}$$

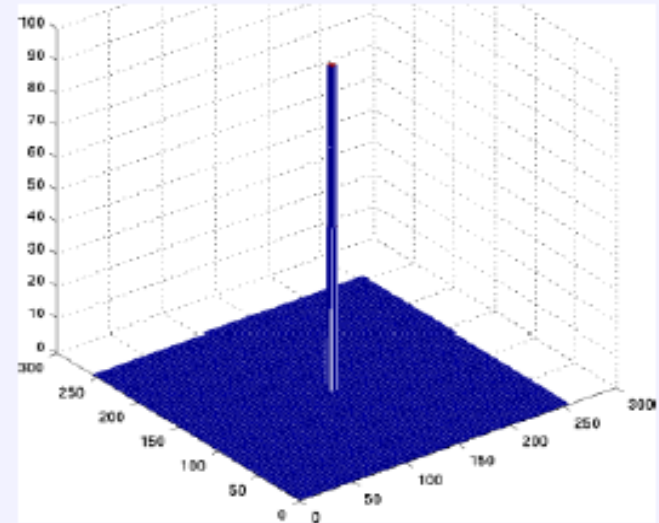
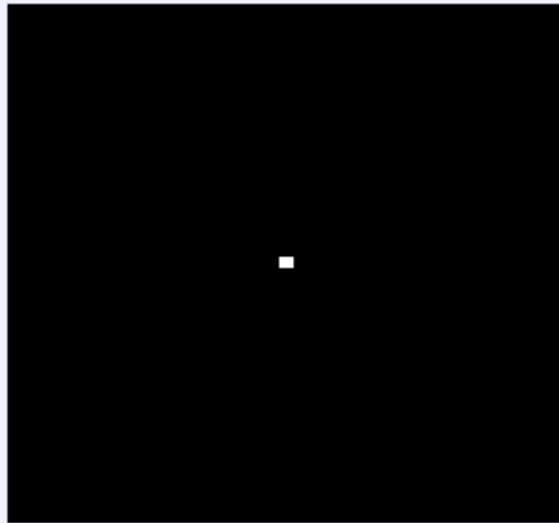
$$f[x, y] \cdot e^{2i\pi(u'x + v'y)/wh} \rightarrow F[u - u', v - v']$$

THÉORÈME DE PARSEVAL

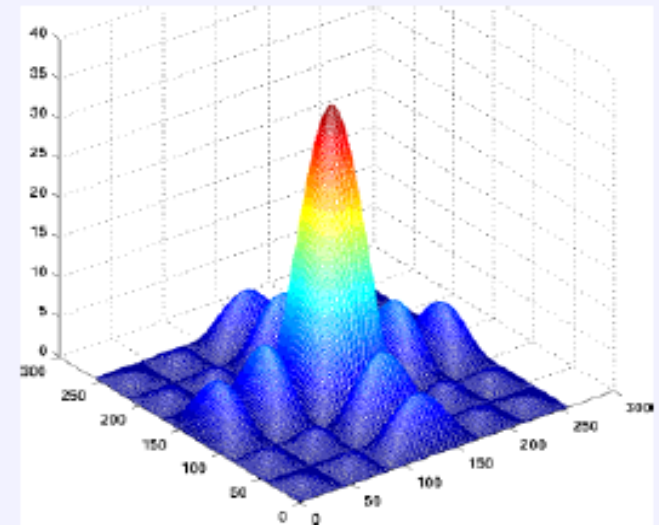
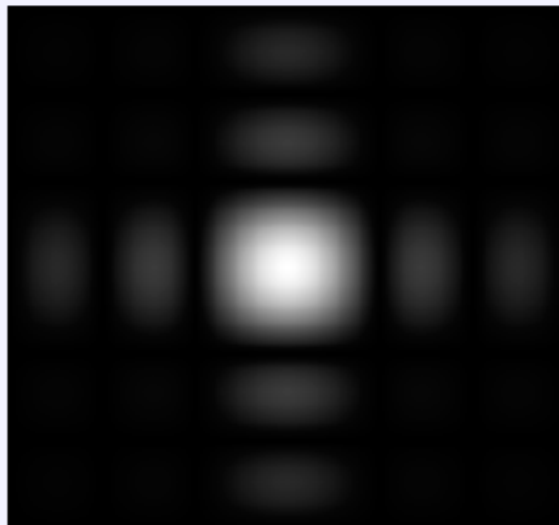
$$\sum_{x=0}^{w-1} \sum_{y=0}^{h-1} \|f[x, y]\|^2 = \frac{1}{wh} \sum_{u=0}^{w-1} \sum_{v=0}^{h-1} \|F[u, v]\|^2$$

Fonction Porte 2D : $x(t, u) = \text{Rect}\left(\frac{t}{T}\right) \cdot \text{Rect}\left(\frac{u}{T}\right)$

$x(t, u)$

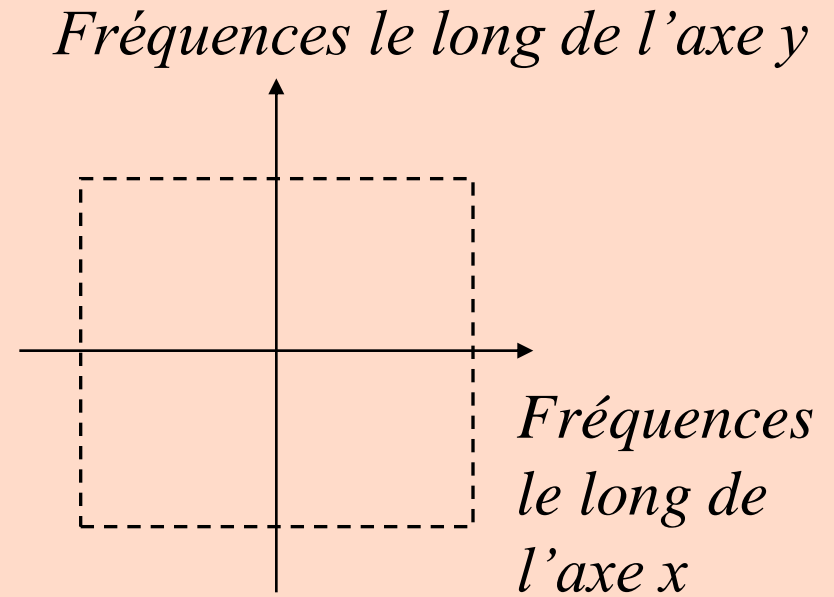
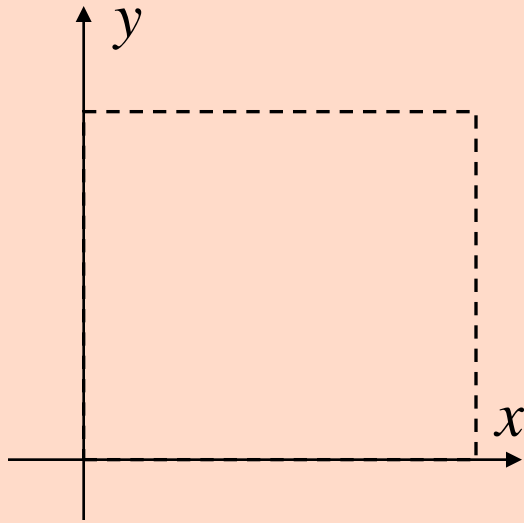


$X(f, g)$

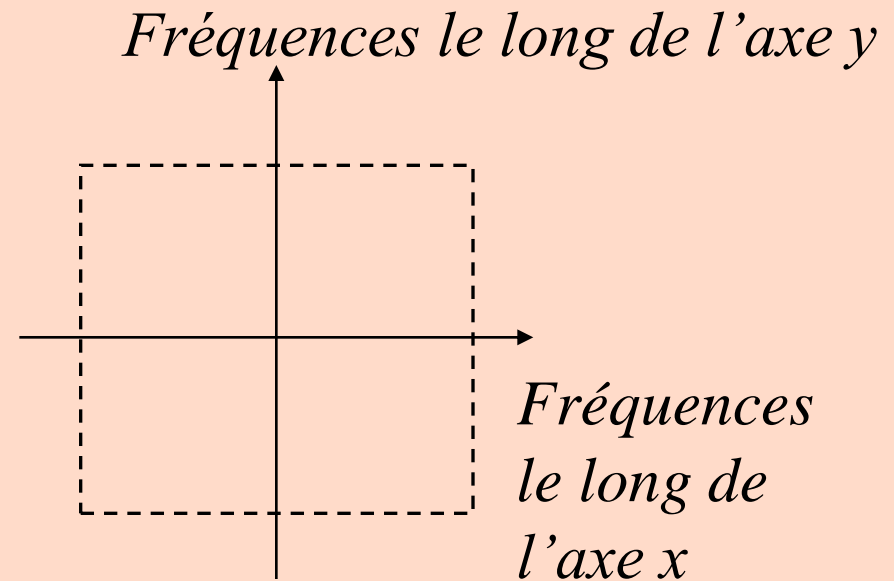
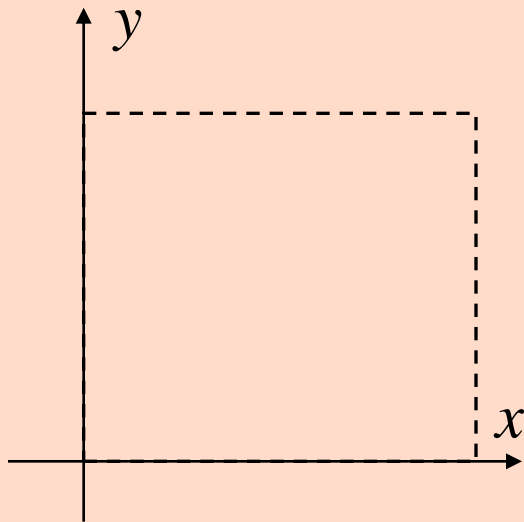


$X(f, g)$ sinc 2D (voir TD)

Domaine de Fourier



Domaine de Fourier



TF de cette
image ?

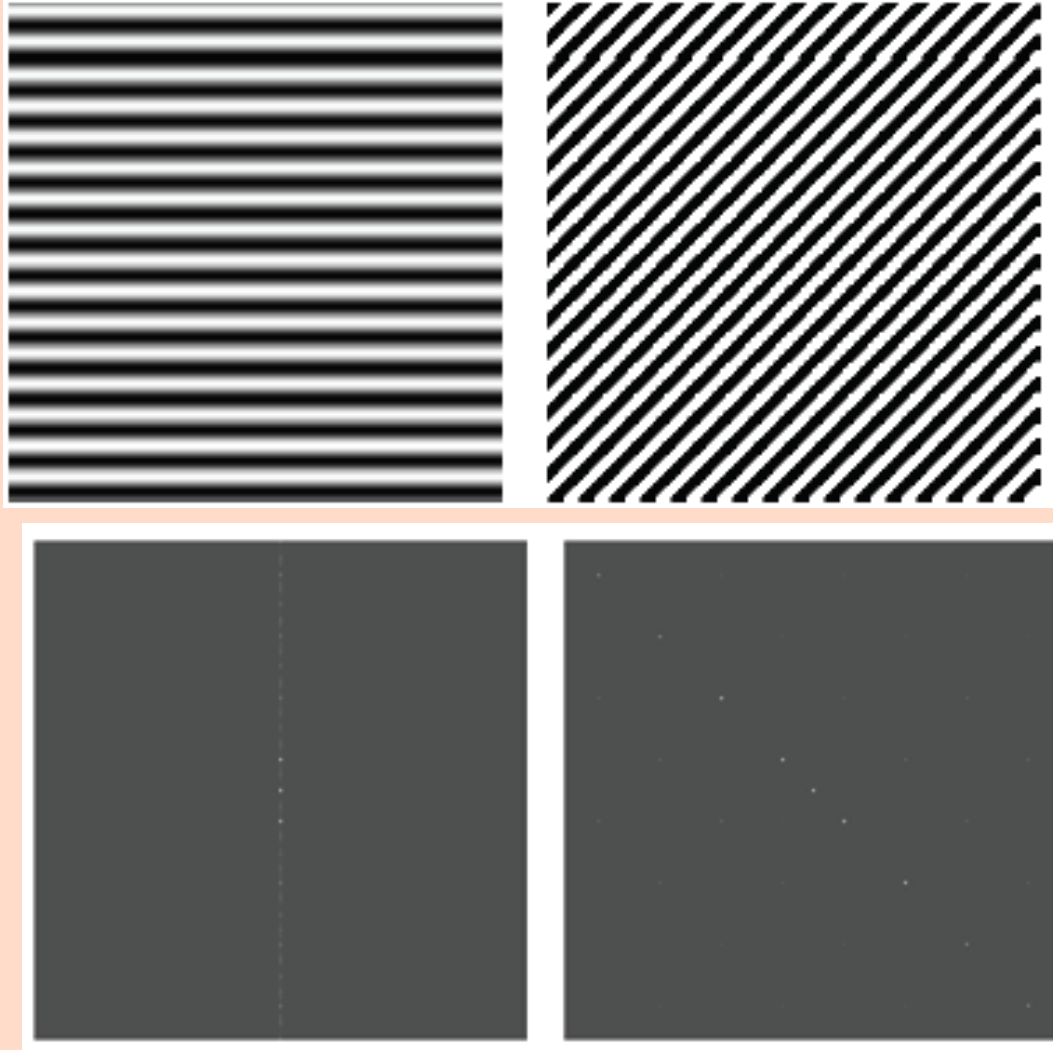
Domaine de Fourier



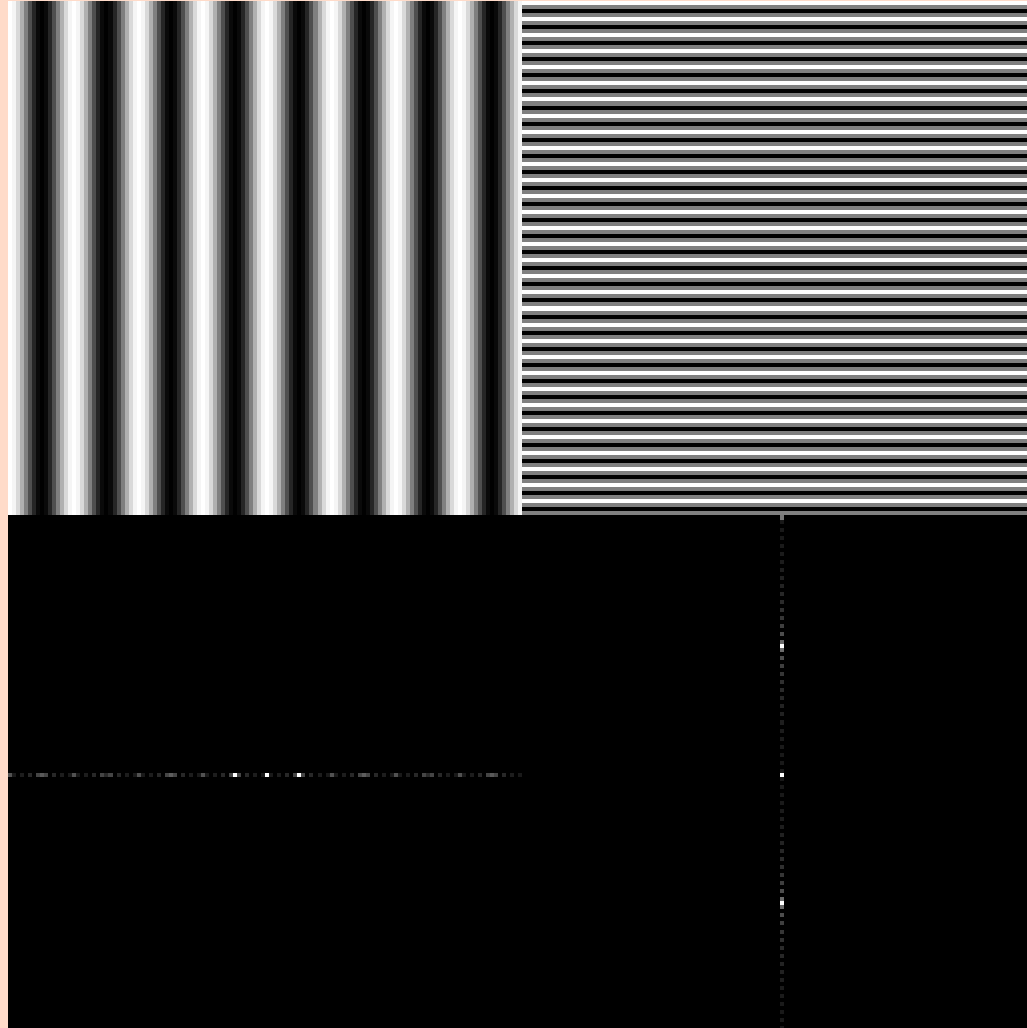
Domaine de Fourier



Domaine de Fourier



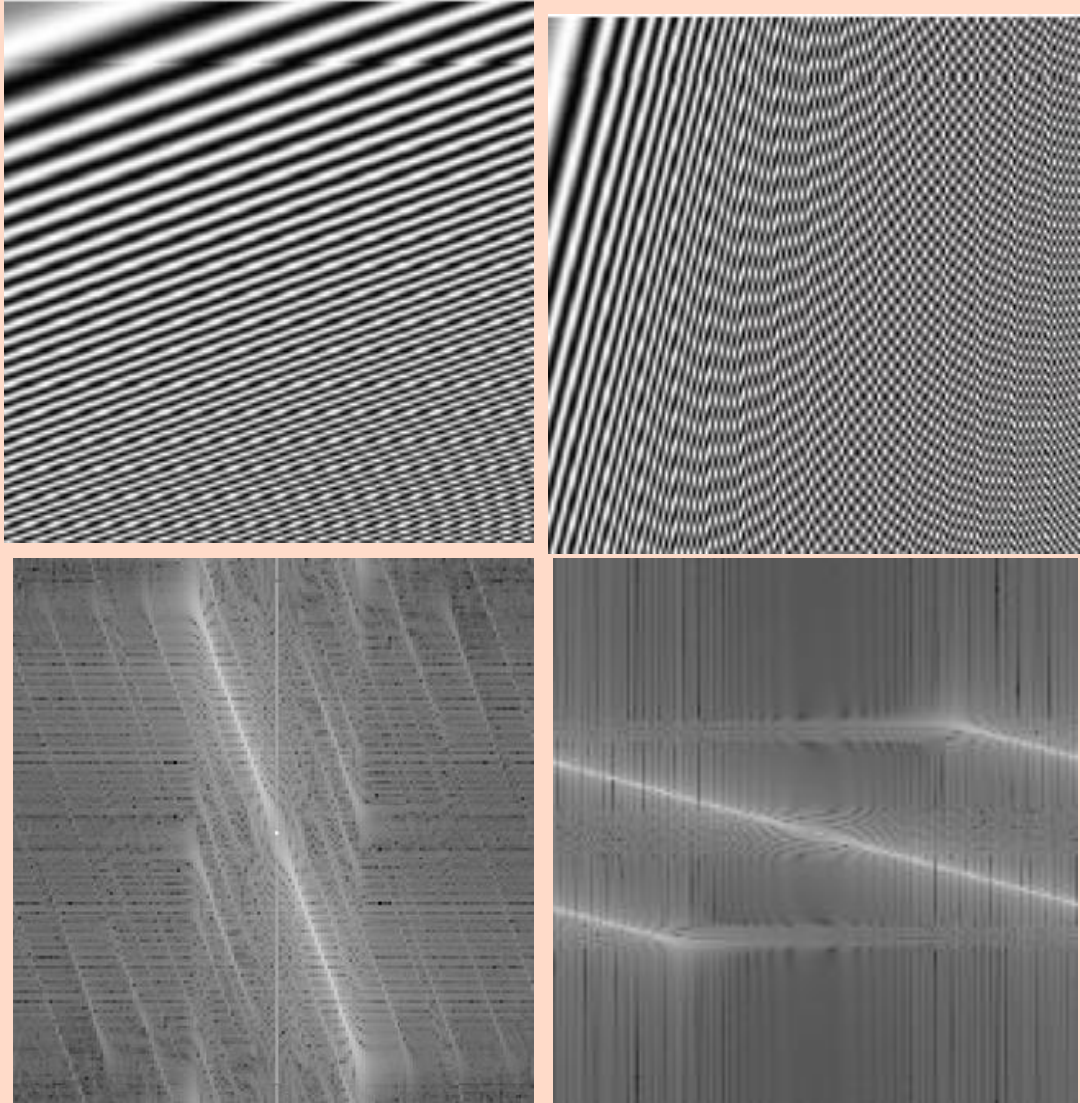
Domaine de Fourier



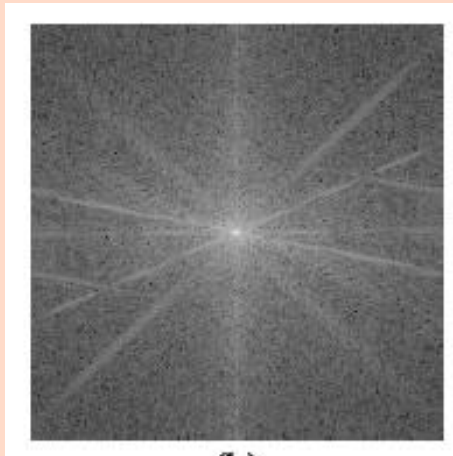
Source : <http://www.cs.unm.edu/~brayer/vision/fourier.html>

Domaine de Fourier

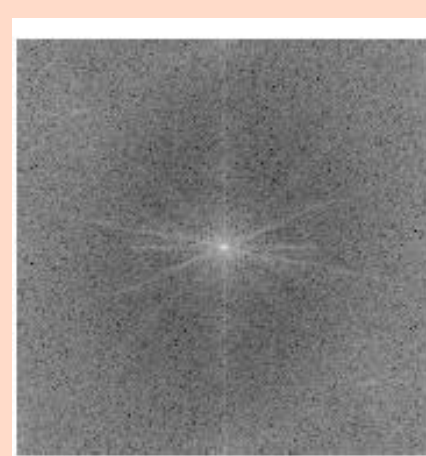
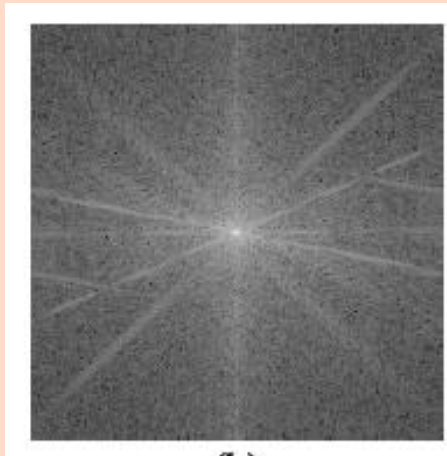
Source : Cours Manzanera, ENSTA



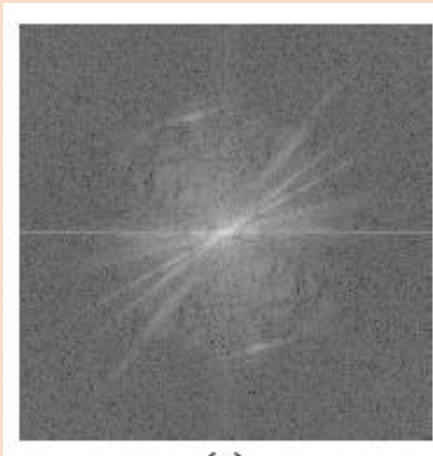
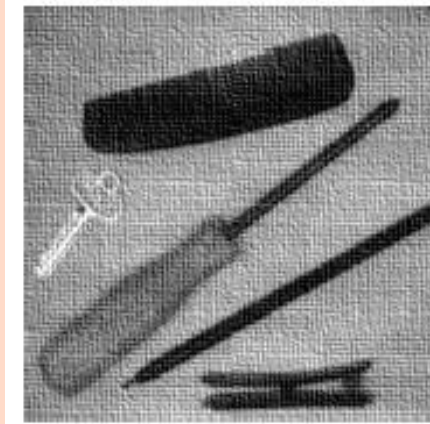
Domaine de Fourier



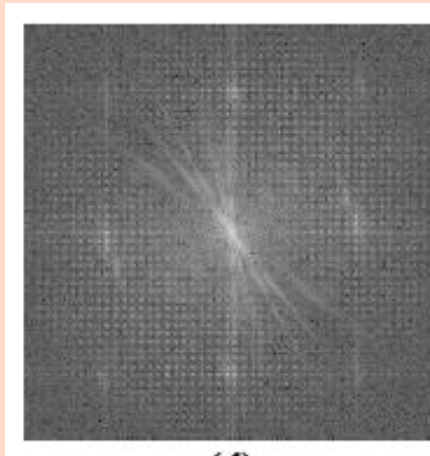
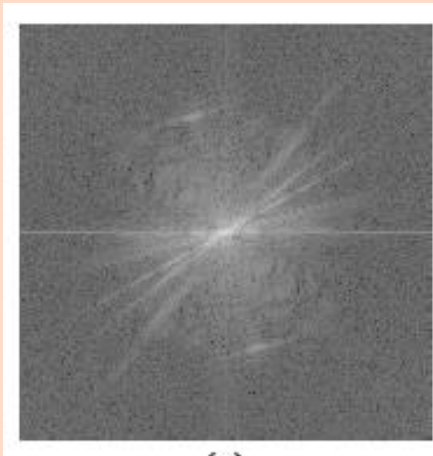
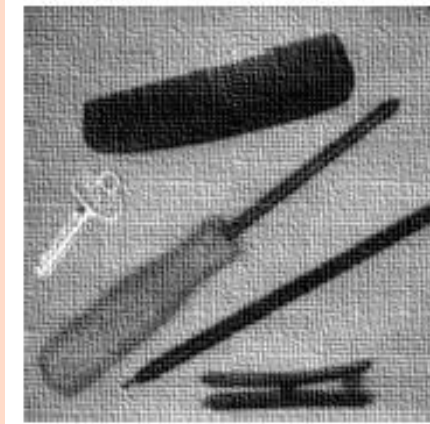
Domaine de Fourier



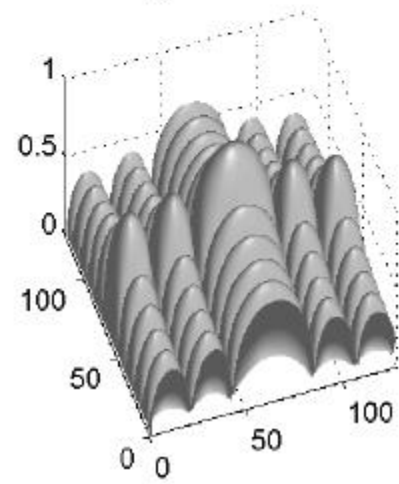
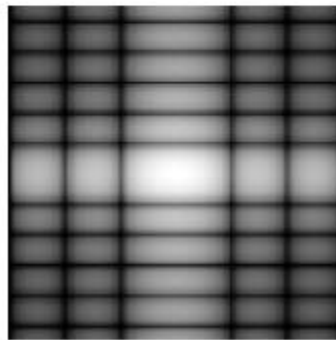
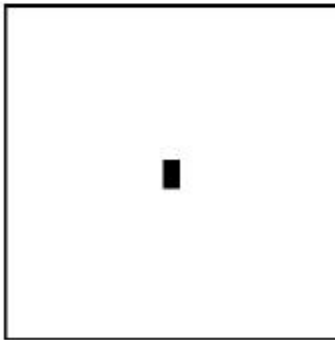
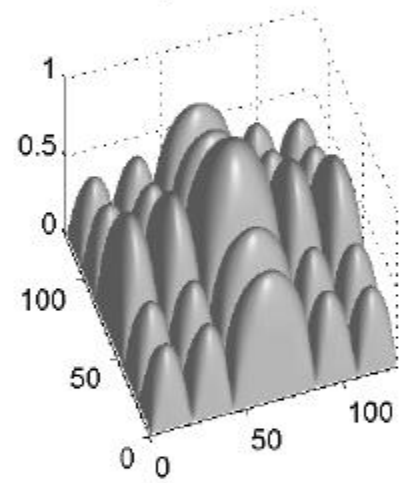
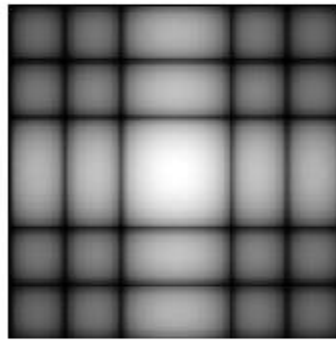
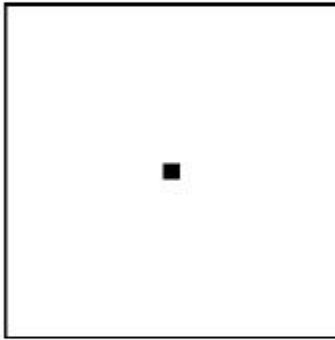
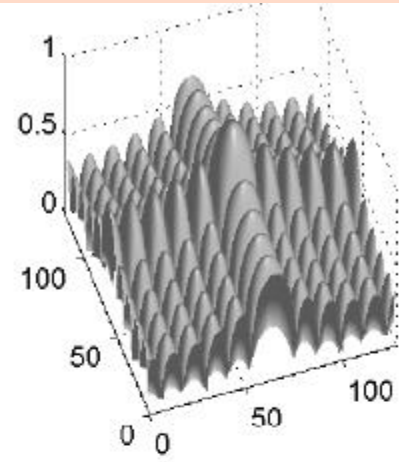
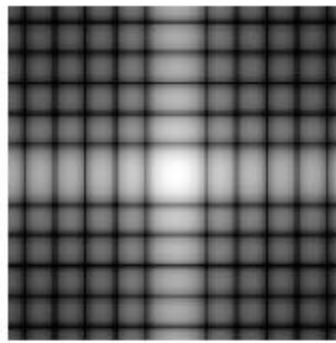
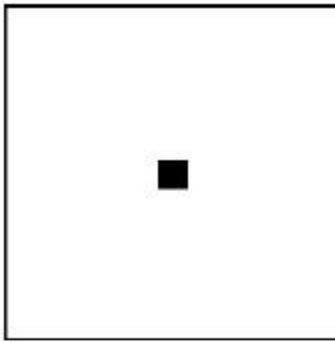
Domaine de Fourier

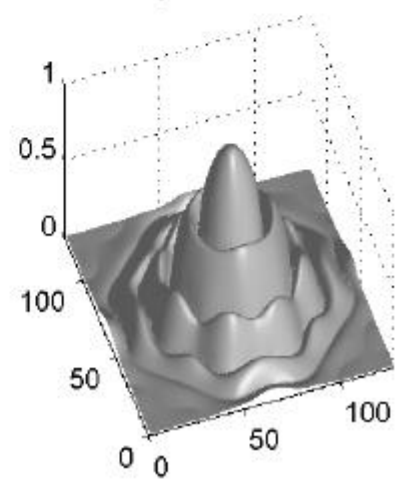
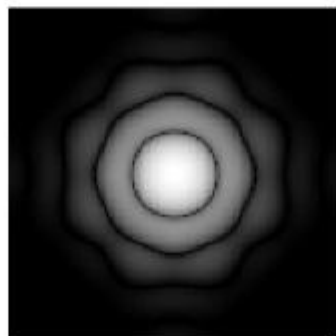
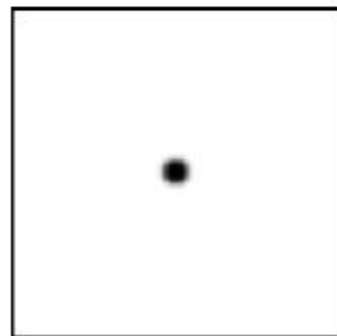
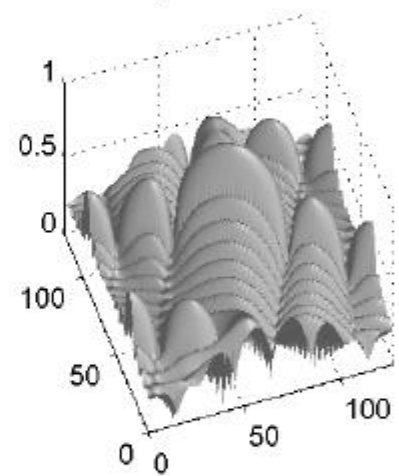
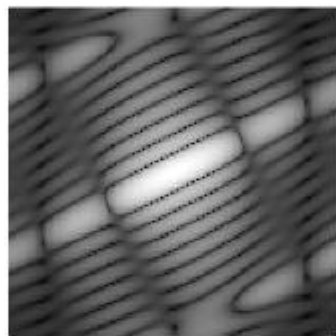
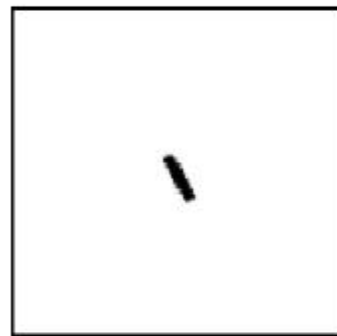
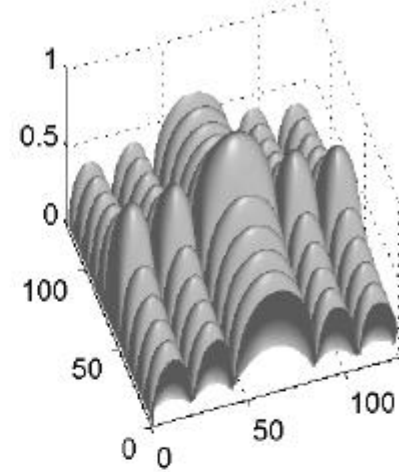
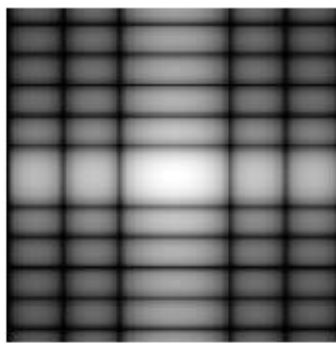
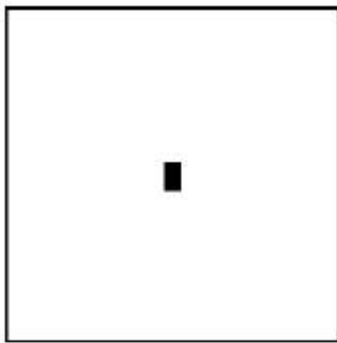


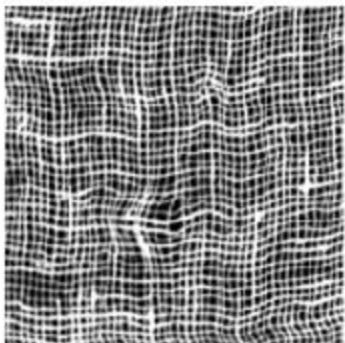
Domaine de Fourier



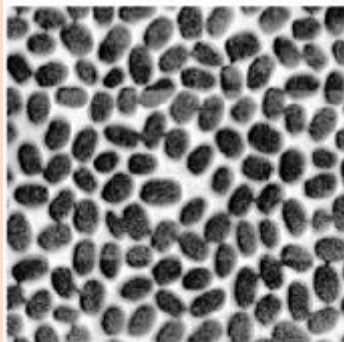
Source : Cours Manzanera, ENSTA



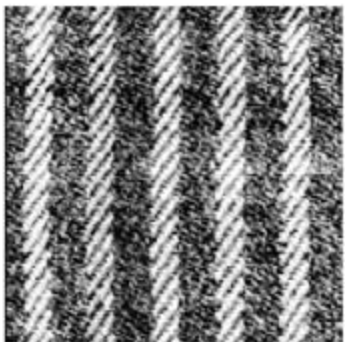




1



2



3



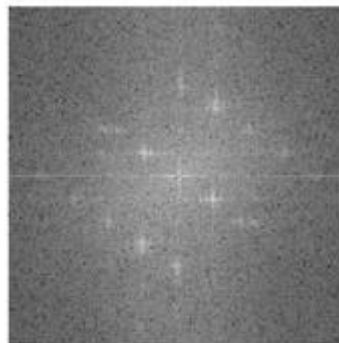
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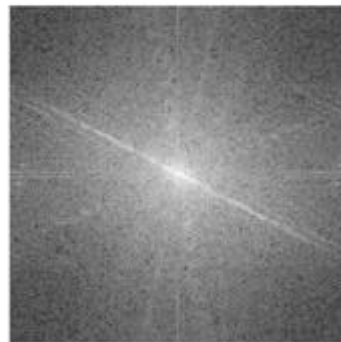
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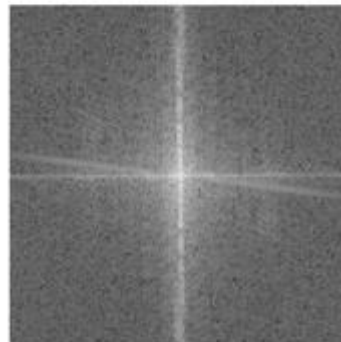
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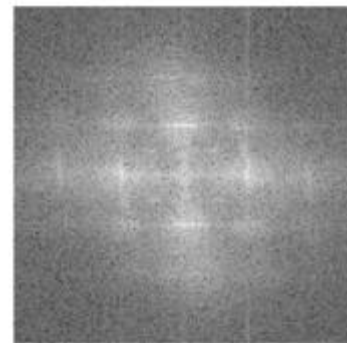
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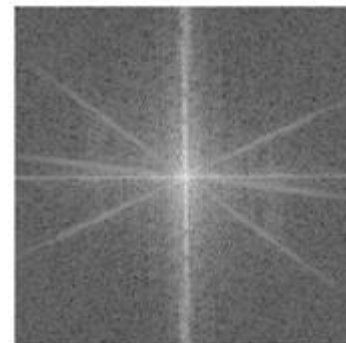
C



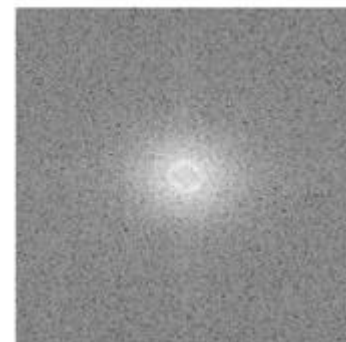
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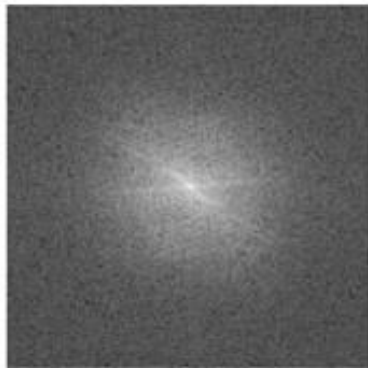
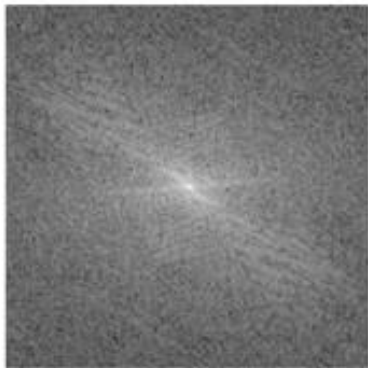
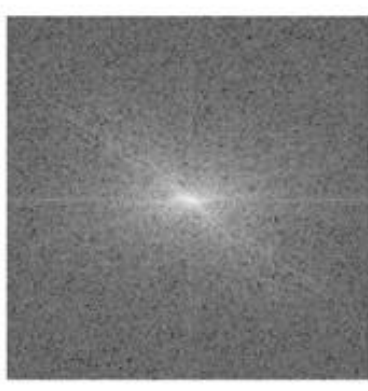
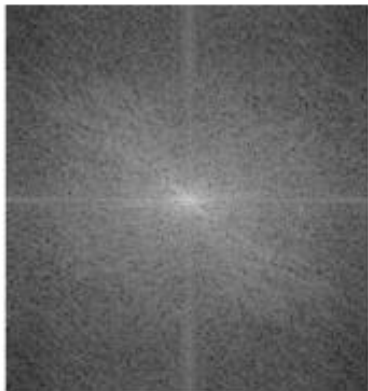
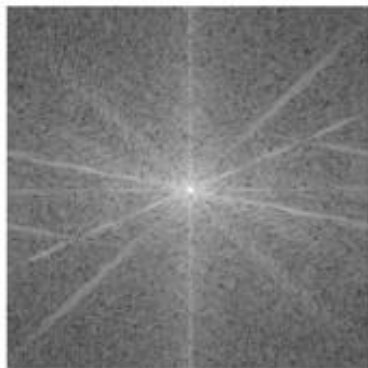
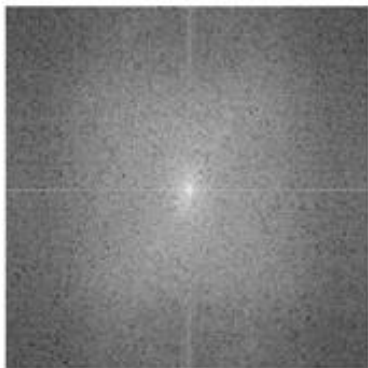
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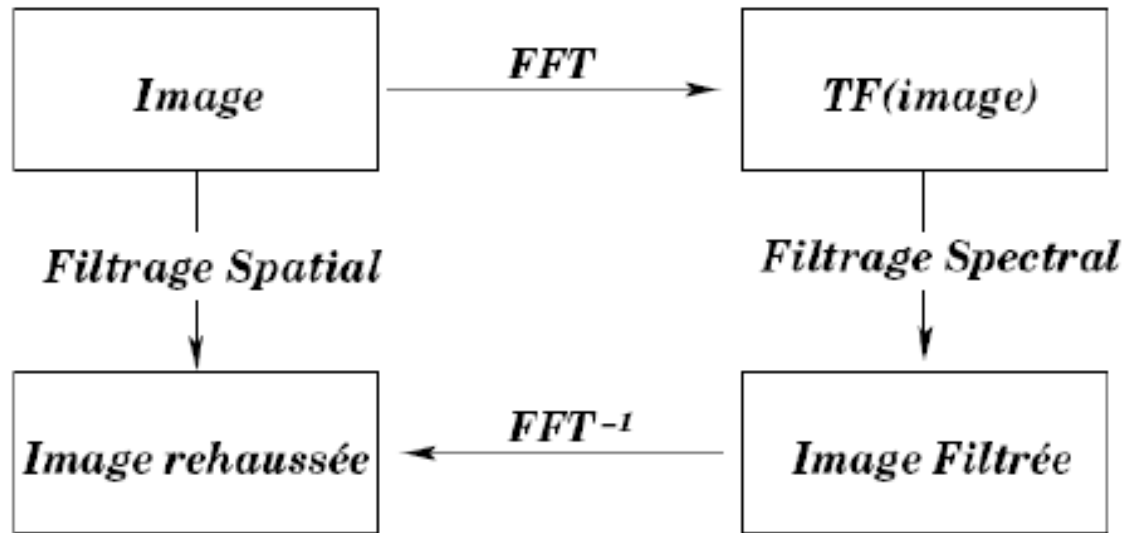
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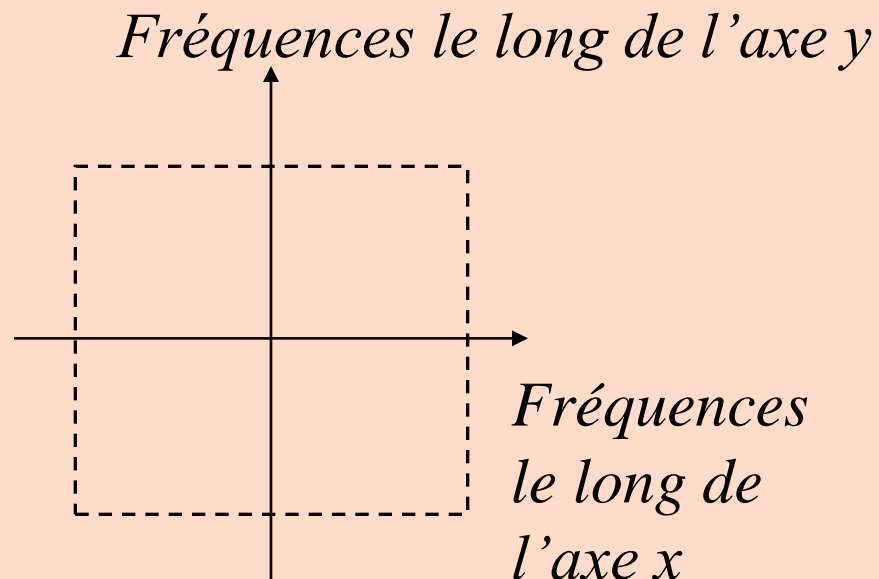
F



Domaine de Fourier

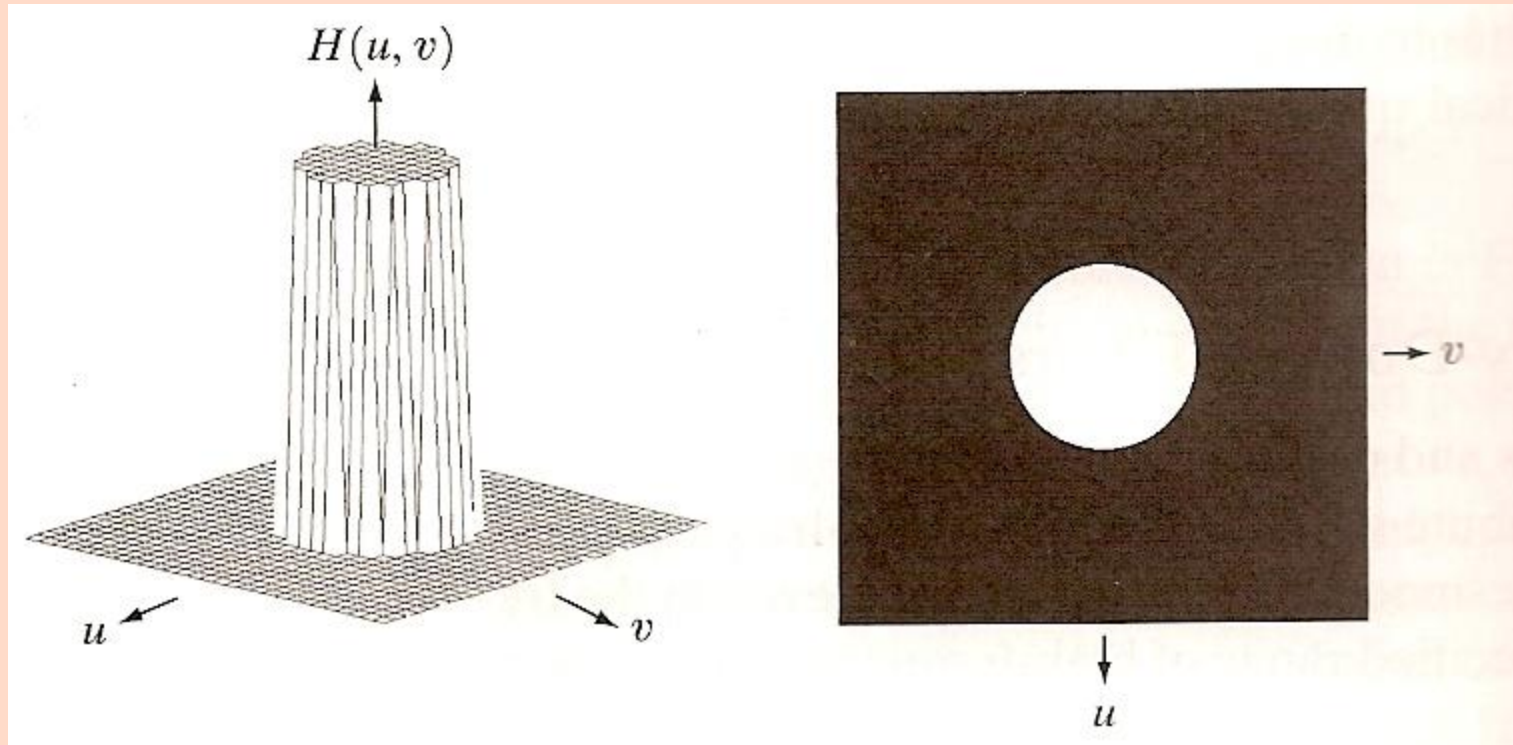


Quel serait le gabarit d'un filtre passe-bas idéal ?



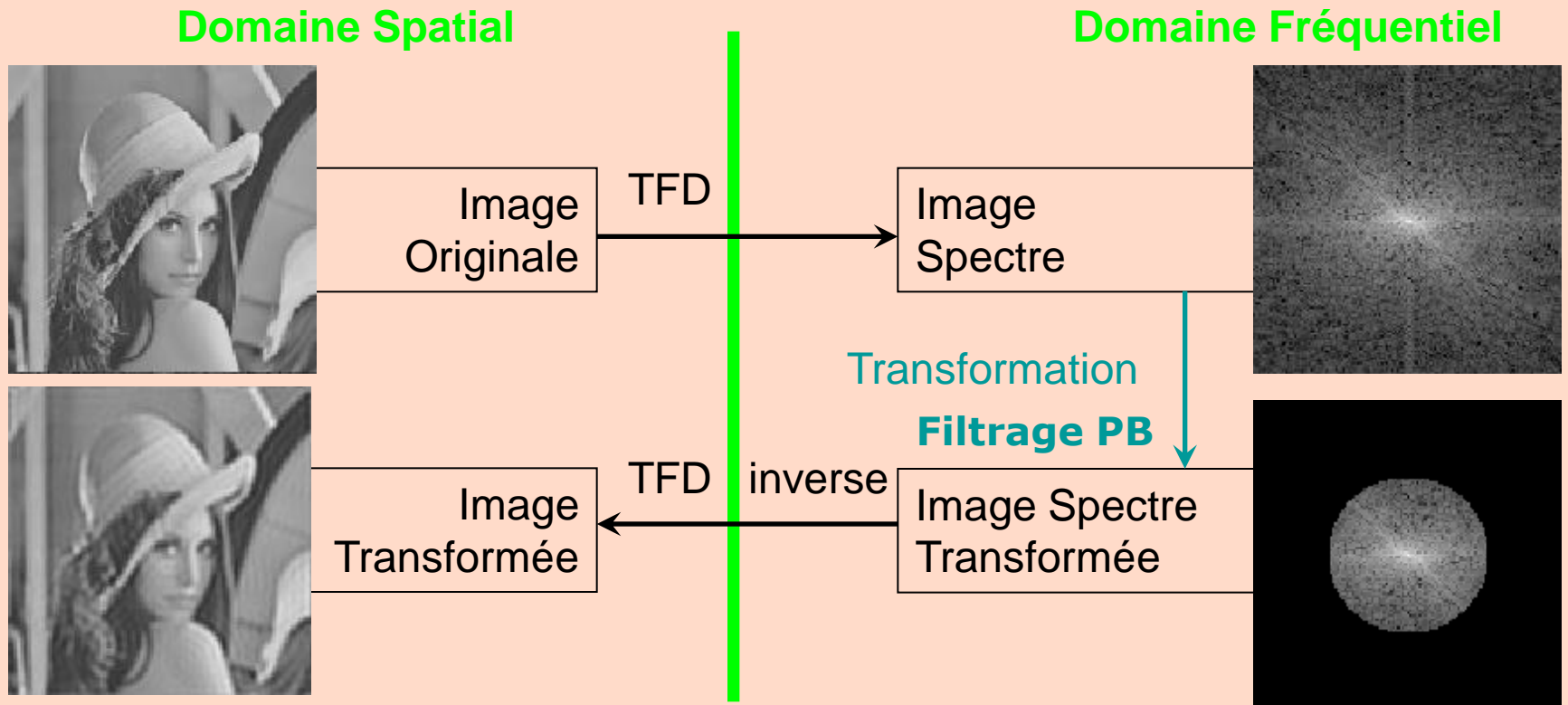
Domaine de Fourier

■ Filtre passe-bas idéal



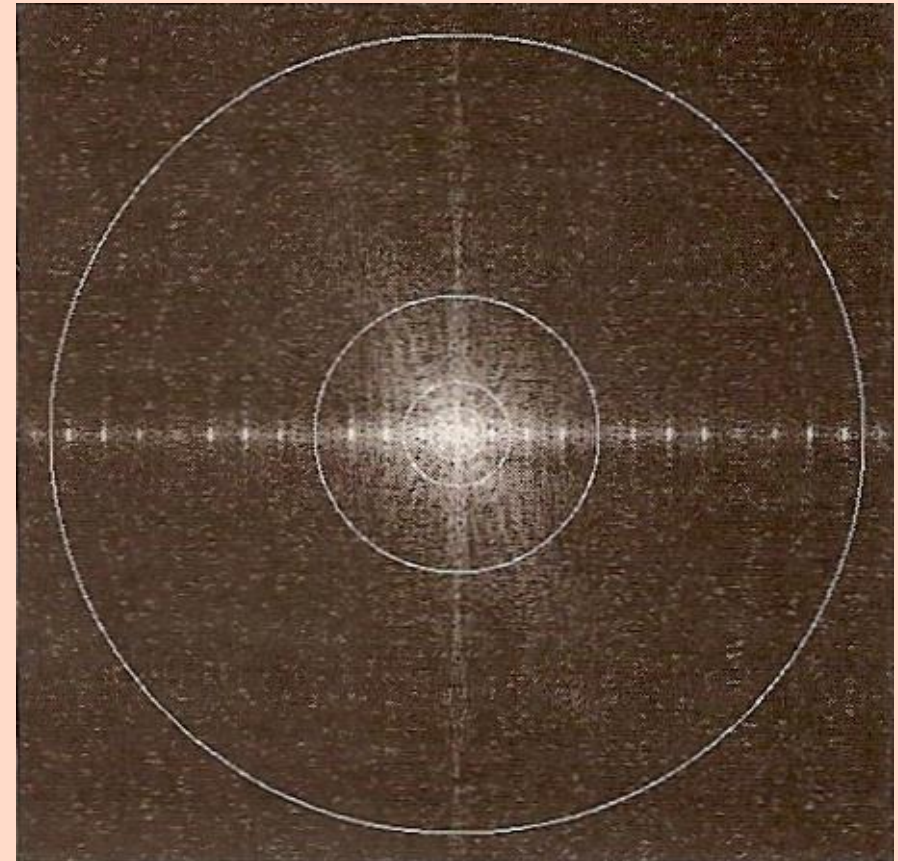
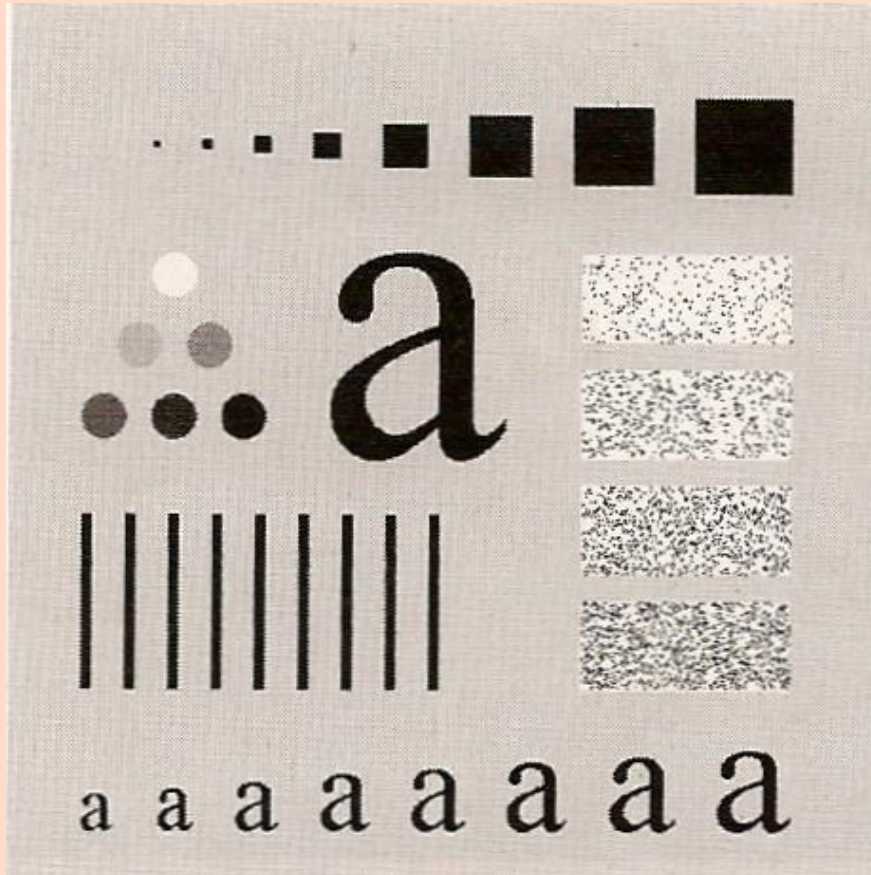
Source : Gonzalez & Wood

Chaîne de filtrage fréquentiel



Domaine de Fourier

■ Image (500x500) et son spectre



Source : Gonzalez & Wood

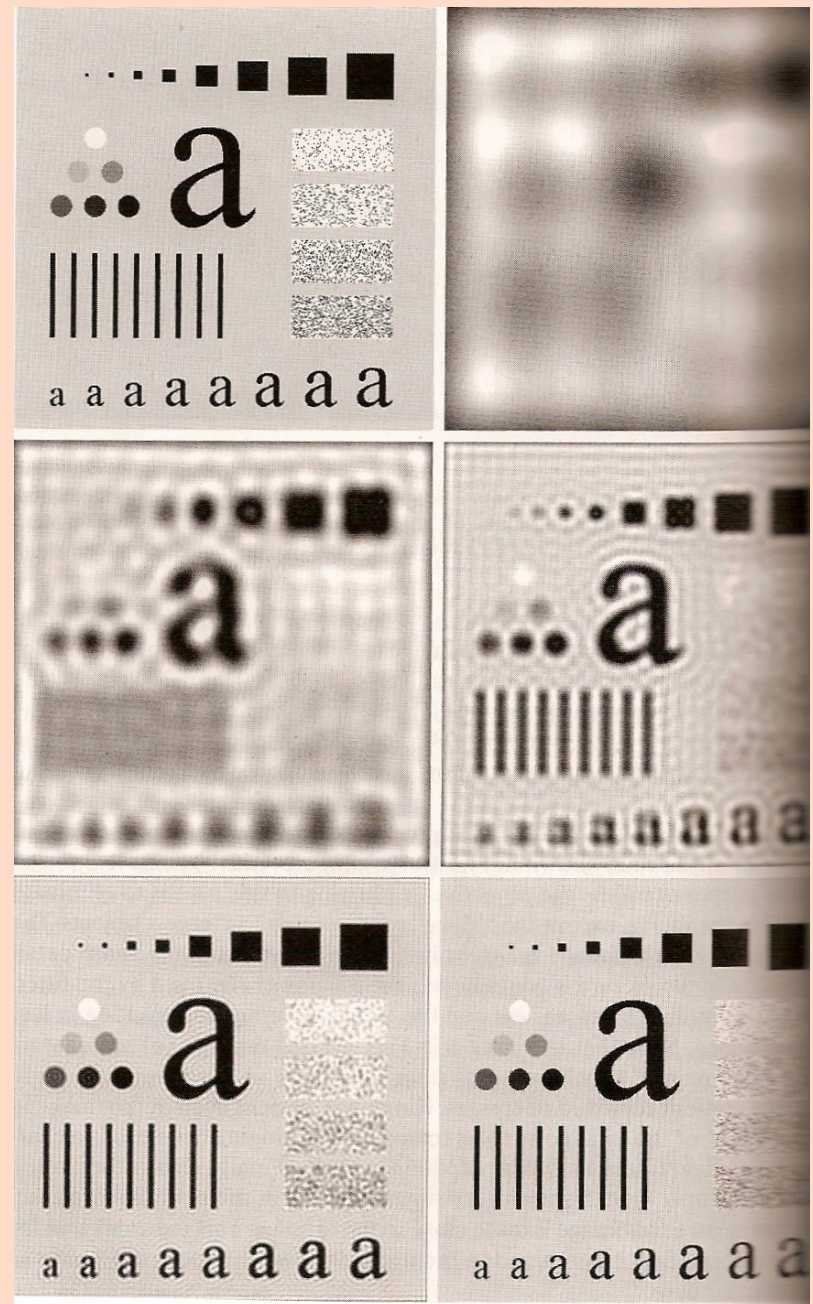
Rayons : 5, 15, 30, 80, 230

Puissance conservée : 92% → 99,5%

Domaine de Fourier

■ Application du filtre passe-bas idéal

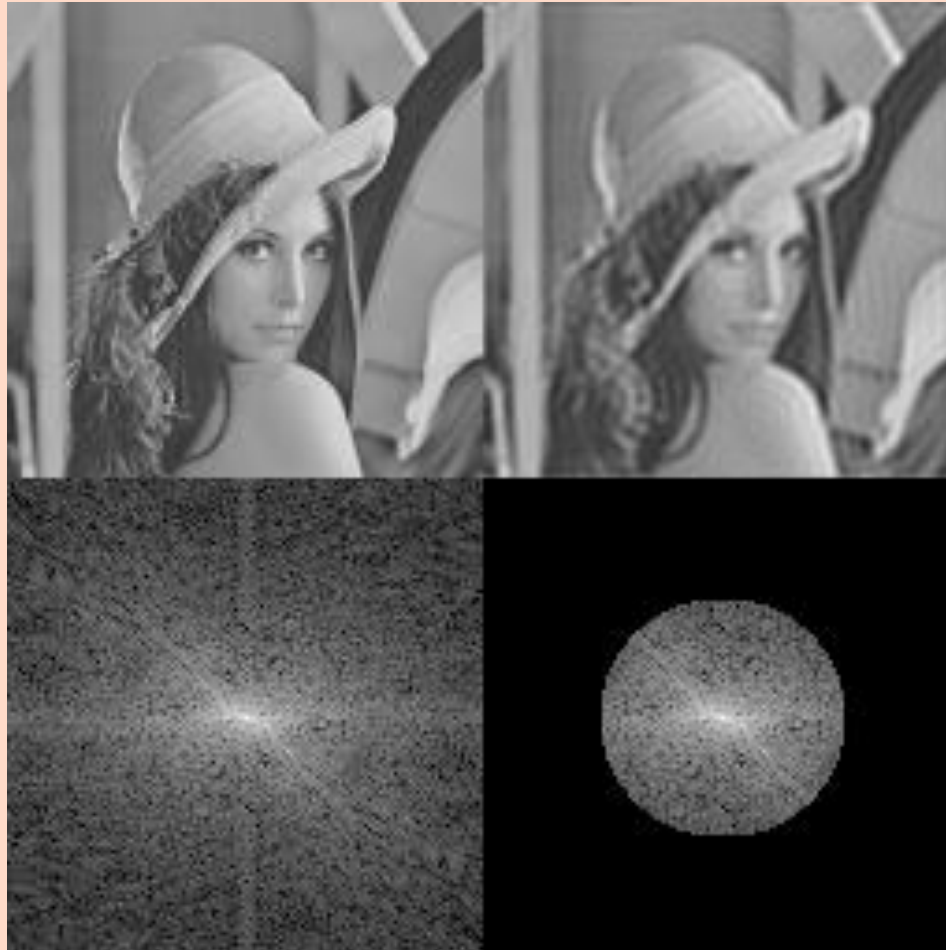
Phénomène de Gibbs



Source : Gonzalez & Wood

Domaine de Fourier

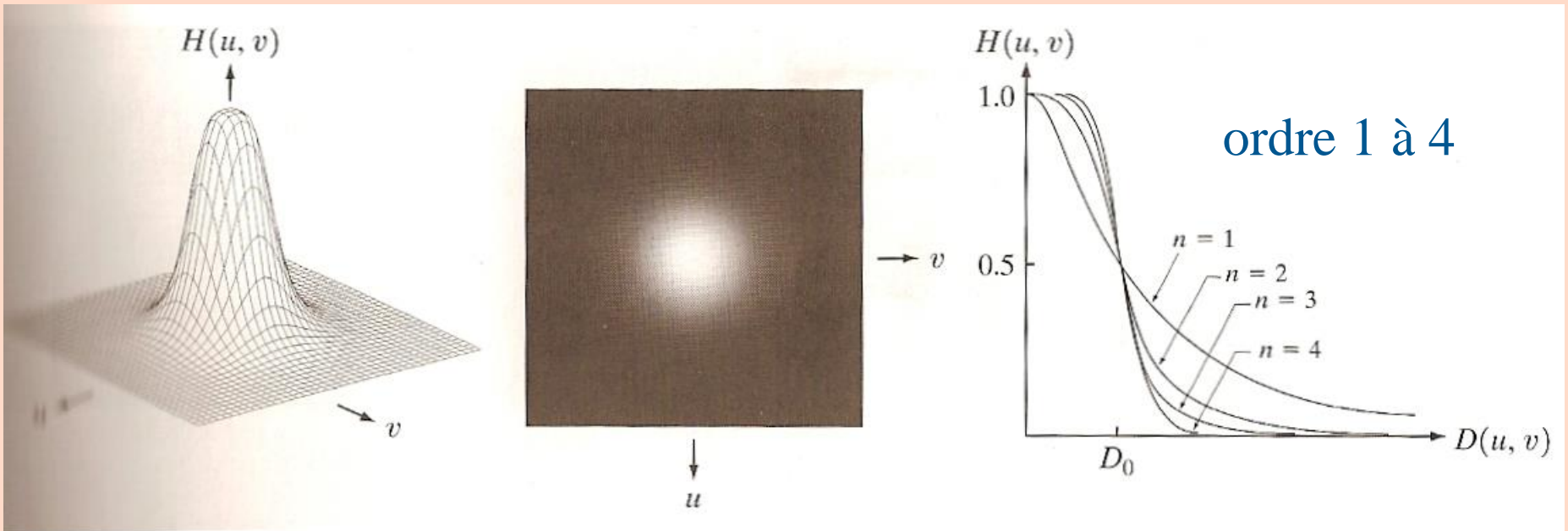
- Image Lena et application du passe-bas idéal



Source : <http://www.cs.unm.edu/~brayer/vision/fourier.html>

Domaine de Fourier

■ Filtre passe-bas de Butterworth

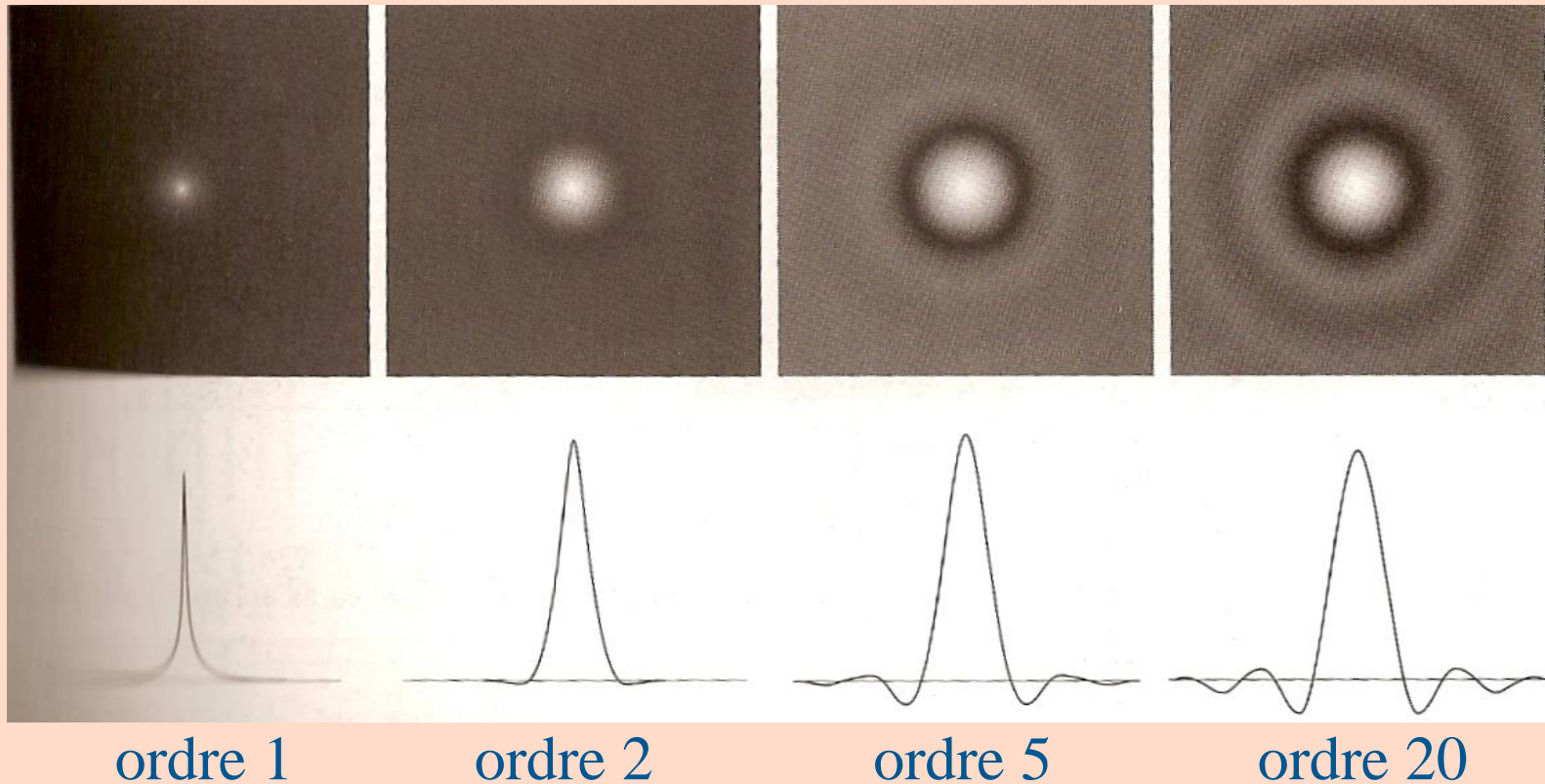


Fonction de transfert

Source : Gonzalez & Wood

Domaine de Fourier

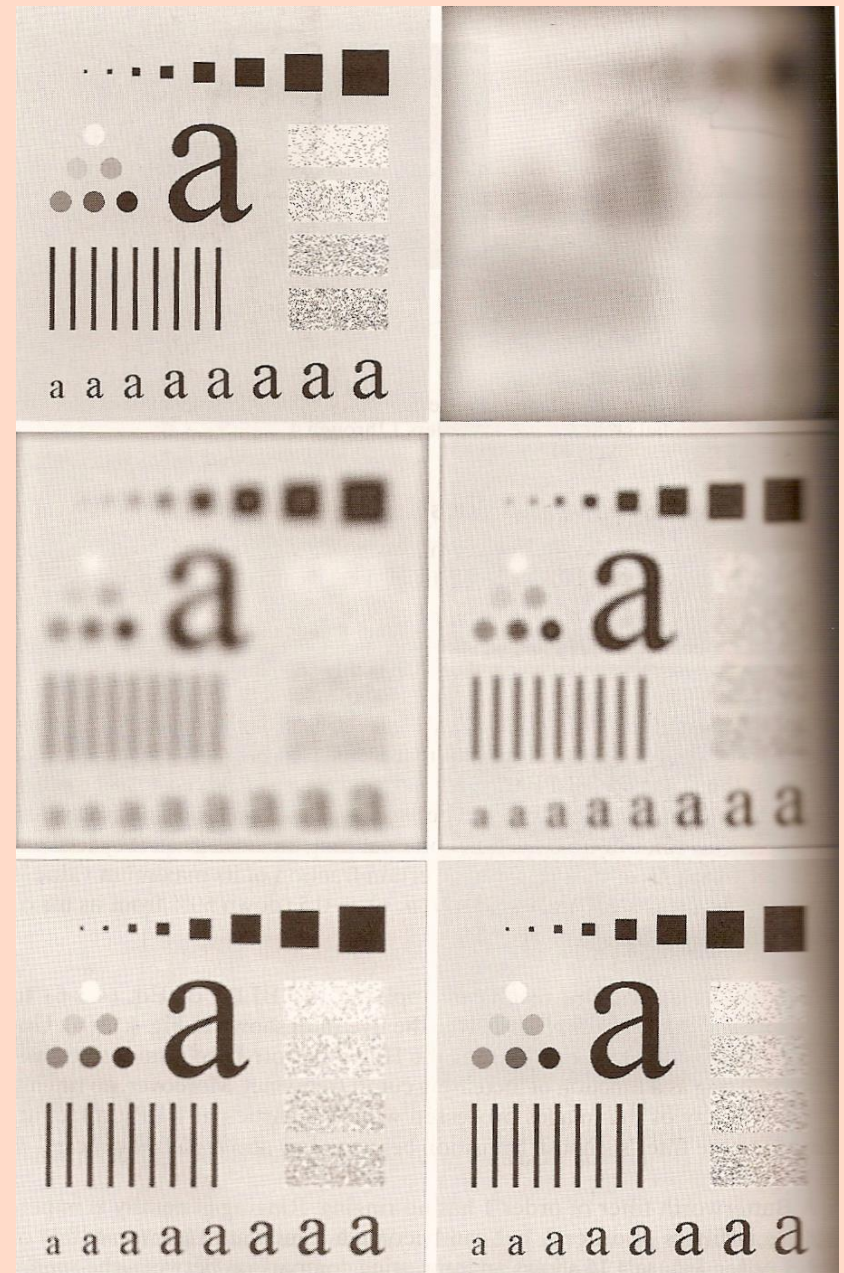
■ Filtre passe-bas de Butterworth



Source : Gonzalez & Wood

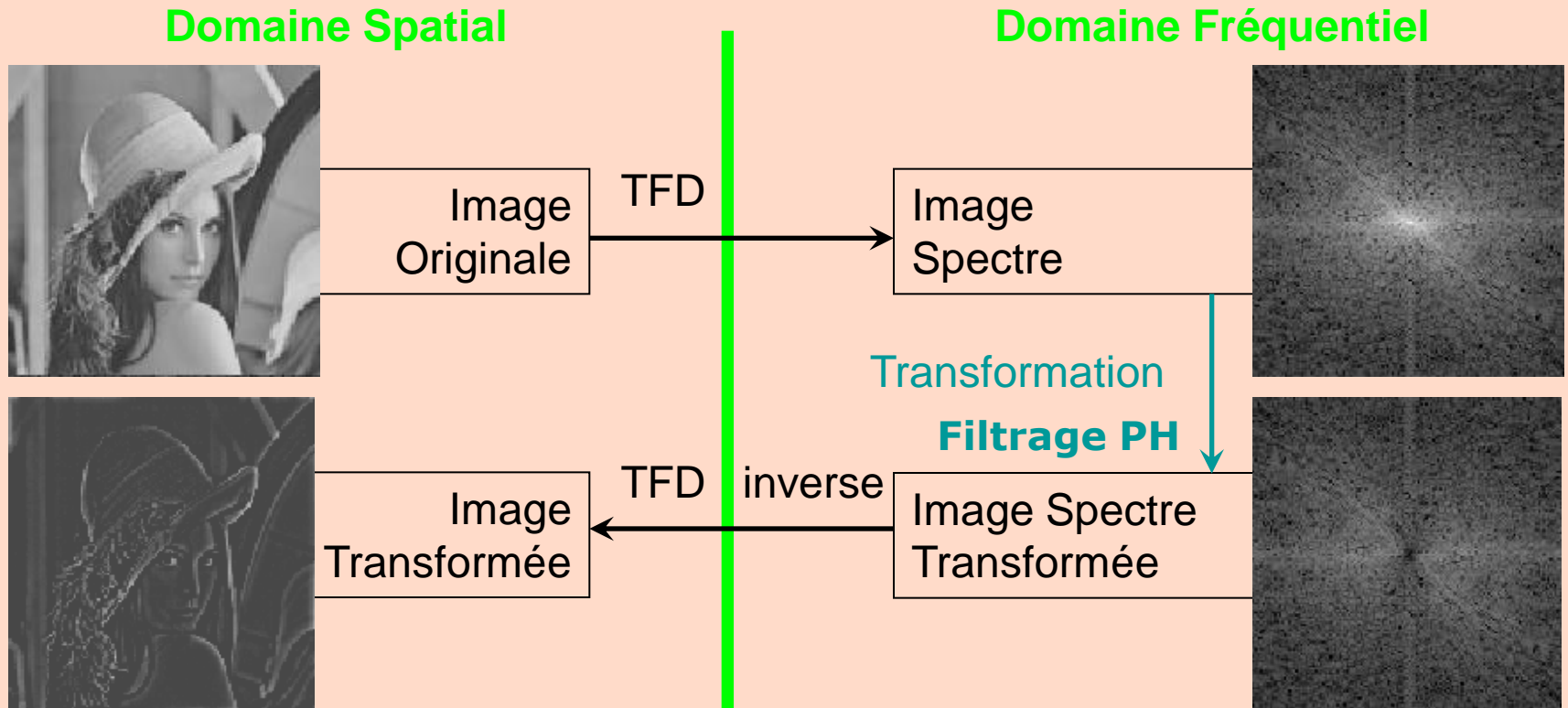
Domaine de Fourier

■ Application du filtre
passe-bas de
Butterworth (ordre 2)



Source : Gonzalez & Wood

Filtrage passe-haut

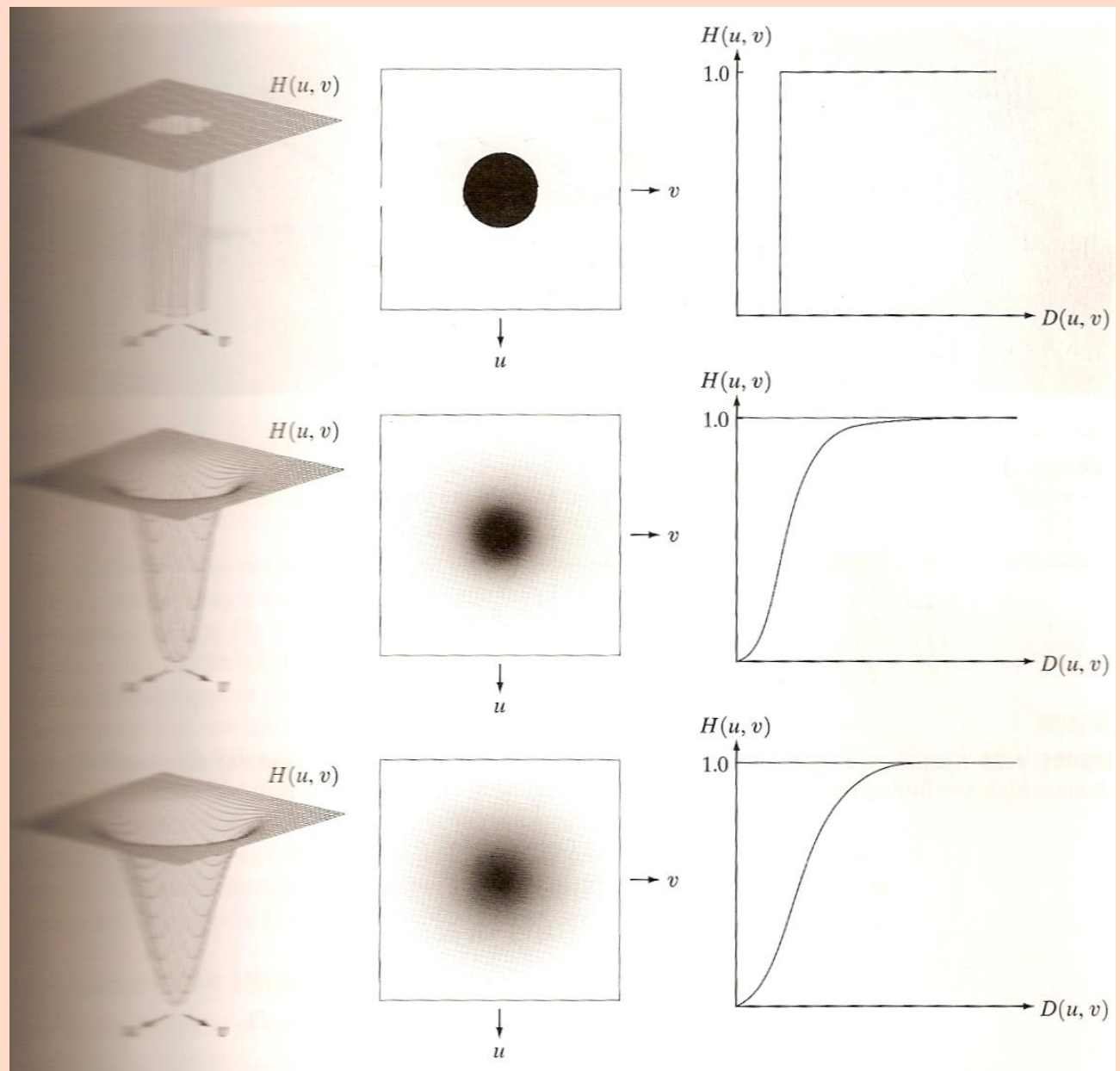


Domaine de Fourier

■ Filtres idéal
passe-haut

Butterworth

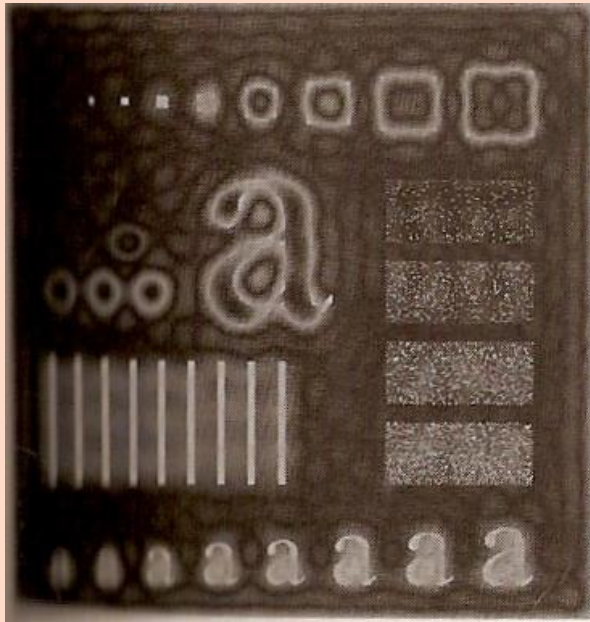
Gaussian



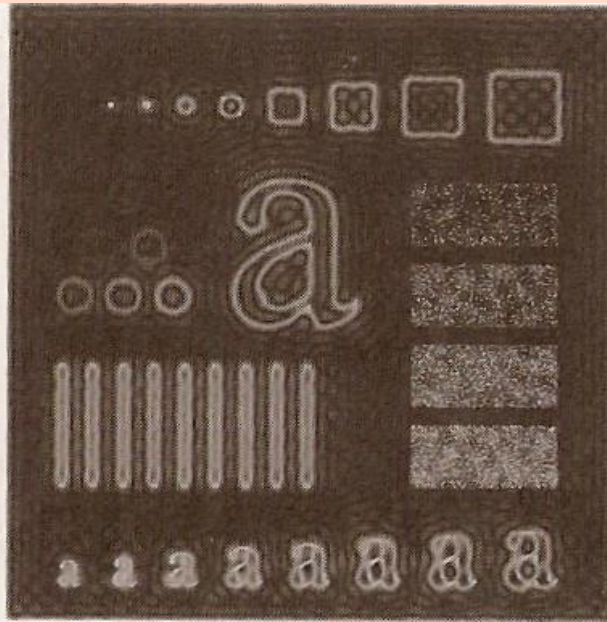
Source : Gonzalez & Wood

Domaine de Fourier

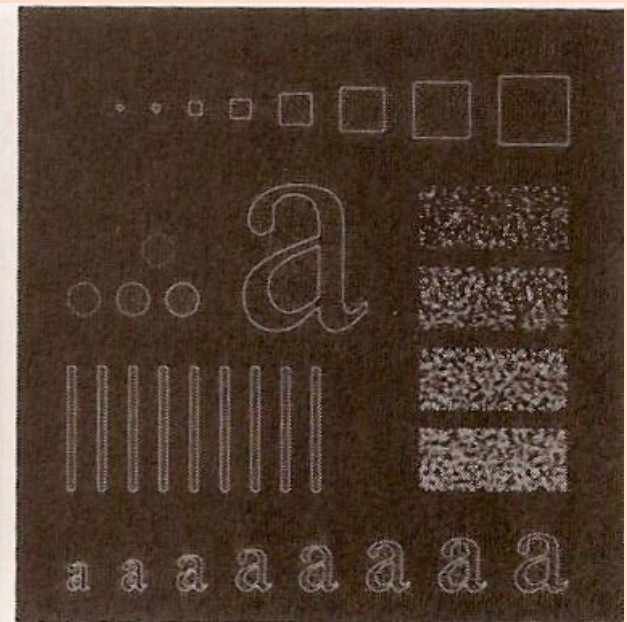
■ Application du filtre passe-haut idéal



Rayon 15



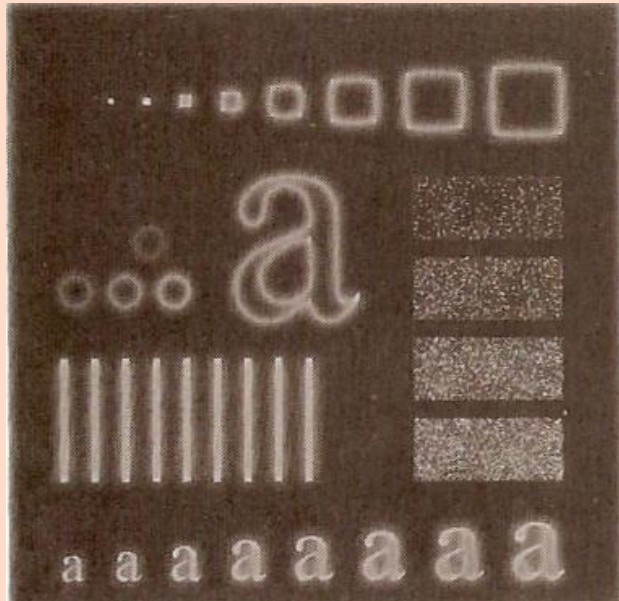
Rayon 30



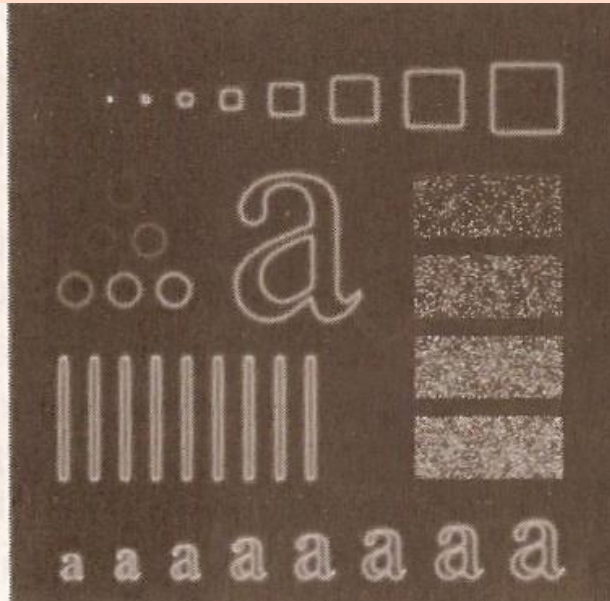
Rayon 80

Domaine de Fourier

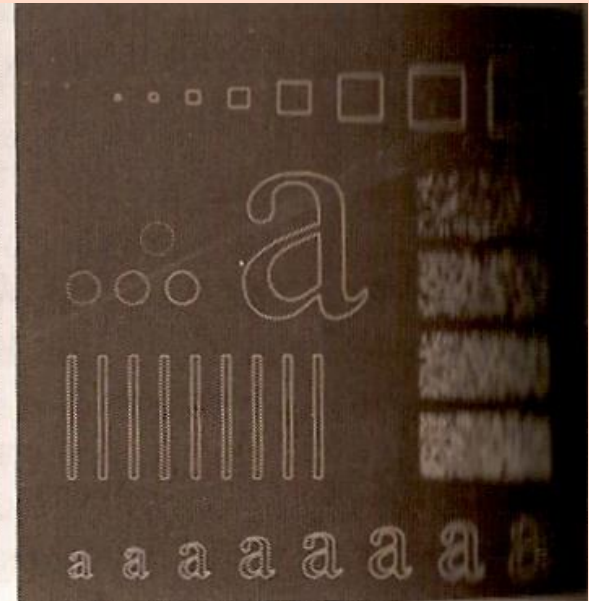
■ Application du filtre passe-haut de Butterworth (ordre 2)



Rayon 15



Rayon 30



Rayon 80

Références

- Cours B Nazarian, Imagerie numérique, Centre IRMf La Timone
- Cours R Zapata, Vision, LIRMM, Univ. Montpellier
- Cours C. Fernandez-Maloigne, Vision artificielle, IRCOM-SIC, Univ. Poitiers
- Cours S. Miguët, Techniques avancées en imagerie, LIRIS, Univ Lyon 2
- Cours A. Dieterlen, Traitement d'images