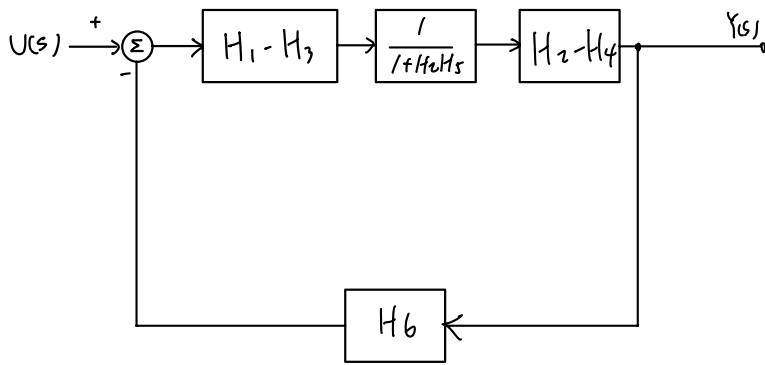


1.



$$\text{Sol: } G_1 = \frac{(H_1 - H_3)}{1 + H_2 H_5} \cdot (H_2 - H_4)$$

$$L_1 = \frac{(H_1 - H_3)(H_2 - H_4)}{1 + H_2 H_5} \cdot H_6$$

$$\Delta = \left| + \frac{(H_1 - H_3)(H_2 - H_4)}{1 + H_2 H_5} H_6 + 0 \right|$$

$$\Delta_1 = 1 - 0 = 1$$

$$\Rightarrow G(s) = \frac{Y(s)}{U(s)} = \frac{\frac{H_1 - H_3}{1 + H_2 H_5} \cdot (H_2 - H_4)}{\left| + \frac{(H_1 - H_3)(H_2 - H_4)}{1 + H_2 H_5} H_6 \right|} = \frac{(H_1 - H_3)(H_2 - H_4)}{1 + H_2 H_5 + (H_1 - H_3)(H_2 - H_4)H_6}$$

$$= \frac{H_1 H_2 - H_1 H_4 - H_3 H_2 + H_3 H_4}{1 + H_2 H_5 + H_1 H_2 H_6 - H_1 H_4 H_6 - H_3 H_2 H_6 + H_3 H_4 H_6} \quad \#$$

2.

①

$$\omega_n = \sqrt{\frac{K_{PD} K_{VCO}}{NC}} = \sqrt{\frac{10^{-4} \cdot 2\pi \cdot 10^9}{32 \cdot \frac{1}{32 \cdot (2\pi)^2 \cdot 10^9}}} = \sqrt{10^5 \times (2\pi)^2 \cdot 10^9}$$

$$\Rightarrow 2\pi \cdot 10^7 = \sqrt{\frac{10^{-4} \cdot 2\pi \cdot 10^9}{32 \cdot C}} \Rightarrow C = \frac{\frac{10^{-4}}{2\pi} \cdot 2\pi \cdot 10^9}{(2\pi \cdot 10^7)^2 \cdot 32} = \frac{1}{(2\pi)^2 \cdot 10^9 \cdot 32}$$

$$\zeta = \frac{\omega_n}{2} RC$$

$$\Rightarrow \zeta = 1 \Rightarrow 1 = \frac{2\pi \cdot 10^7}{2} \cdot R \cdot \frac{1}{(2\pi)^2 \cdot 32 \cdot 10^9} \Rightarrow R = 128\pi \cdot 10^2 \Omega$$

$$\Rightarrow \zeta = 0.2 \Rightarrow 0.2 = \frac{2\pi \cdot 10^7}{2} \cdot R \cdot \frac{1}{(2\pi)^2 \cdot 32 \cdot 10^9} \Rightarrow R = 2.56\pi \cdot 10^3 \Omega$$

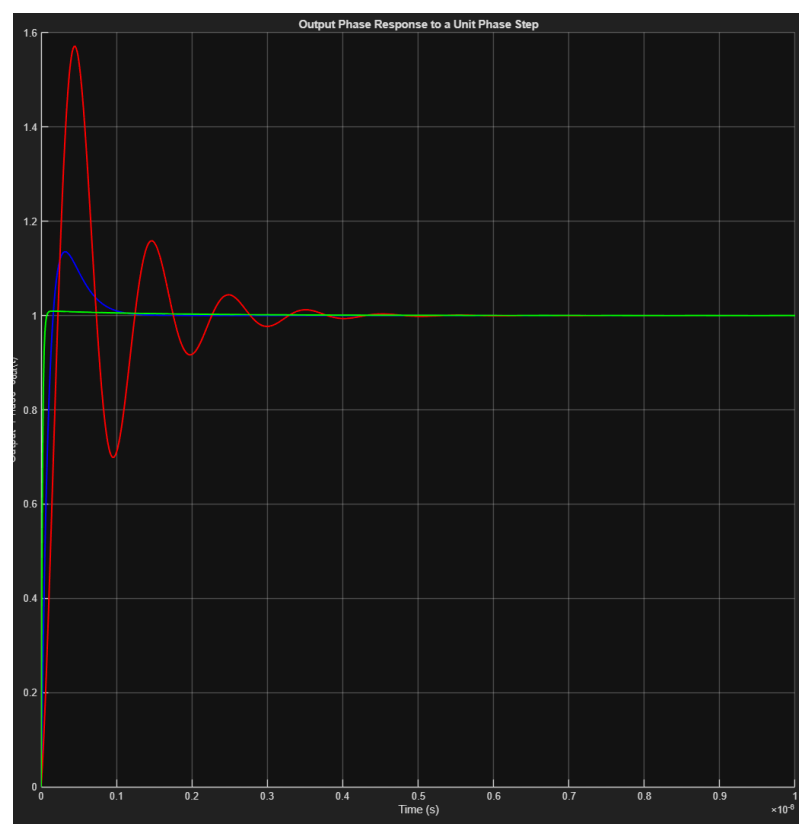
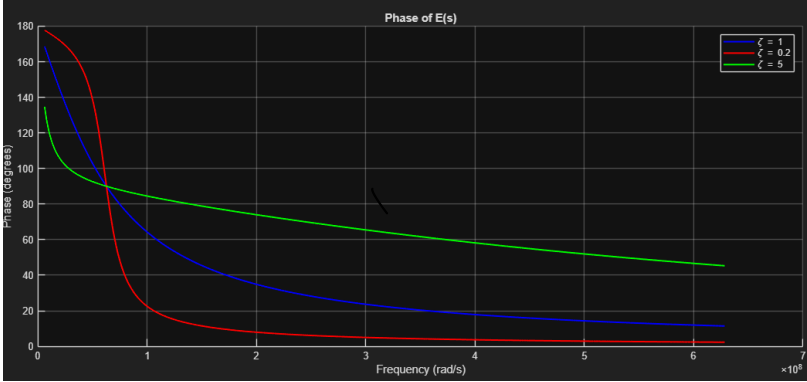
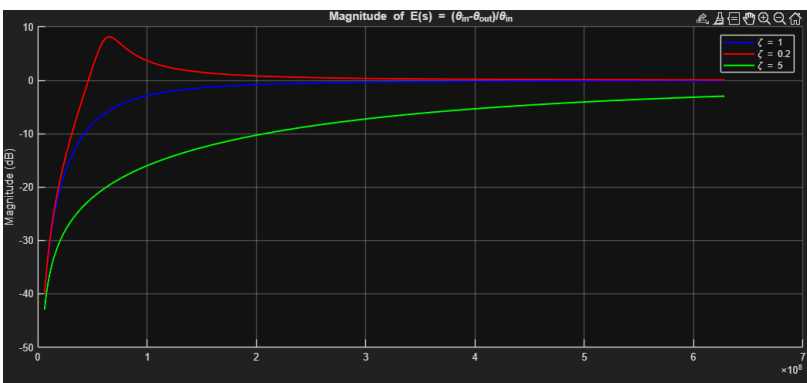
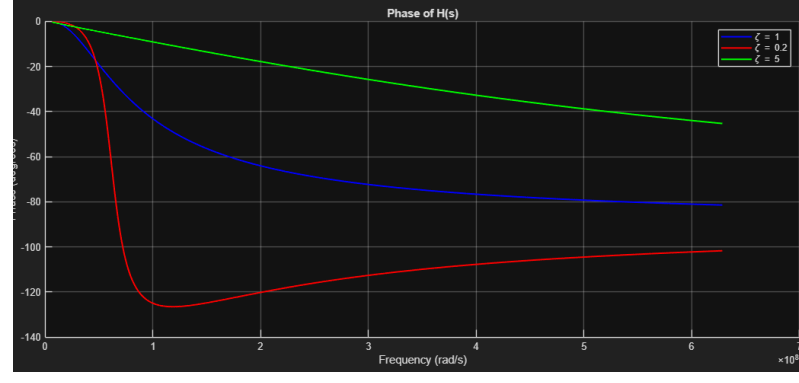
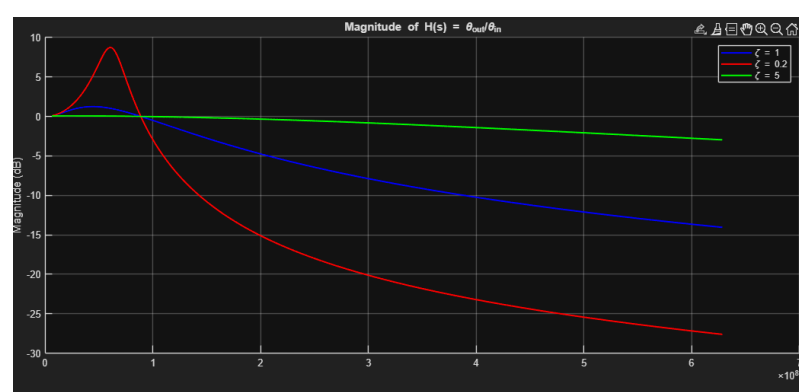
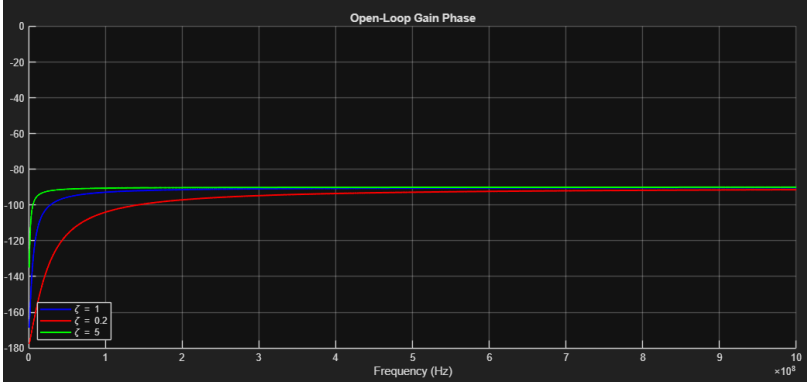
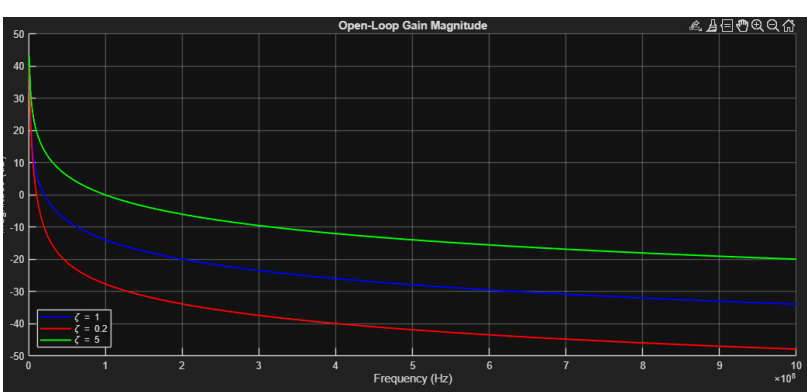
$$\Rightarrow \zeta = 5 \Rightarrow R = 6.4\pi \cdot 10^4 \Omega$$

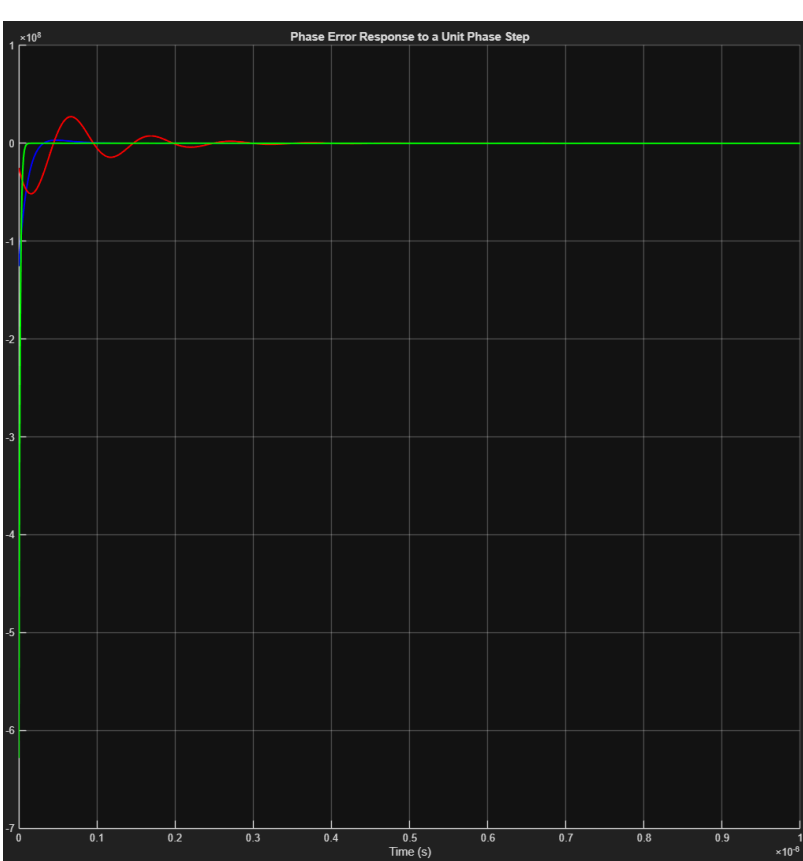
$$\text{Transfer Function: } \frac{K_{PD} K_{VCO} R(s + \frac{1}{RC})}{s^2 + \left(\frac{K_{PD} K_{VCO} R}{N}\right)s + \frac{K_{PD} K_{VCO}}{NC}} = \frac{\frac{10^{-4} \cdot 2\pi \cdot 10^9 R(s + \frac{1}{RC})}{s^2 + \frac{10^5 R}{32}s + \frac{(2\pi)^2 \cdot 10^9 \cdot 32}{32}}$$

$$\zeta = 1, \text{ pole} \Rightarrow \frac{-128\pi \cdot 10^2 \cdot \frac{10^5}{32} \pm \sqrt{(4\pi \cdot 10^2)^2 - 4(2\pi \cdot 10^7)^2}}{2} = -2\pi \times 10^7 \text{ (critical damped)}$$

$$\zeta = 0.5, \text{ poles} \Rightarrow s = -\zeta \omega_n \pm \omega_n \sqrt{\zeta^2 - 1} = -\frac{K_{PD} K_{VCO} R}{N} \pm \sqrt{\frac{K_{PD} K_{VCO}}{NC}} \cdot \sqrt{0.04 - 1} = \frac{2.56\pi \cdot 10^8}{64} \pm j\sqrt{0.96 \cdot 10^5 (2\pi)^2 \cdot 10^9} = 4\pi \times 10^6 \pm j6.16 \times 10^7$$

$$\zeta = 5, \text{ poles} \Rightarrow s = \frac{-10^5 \cdot 6.4\pi \cdot 10^4}{64} \pm \sqrt{10^5 (2\pi)^2 \cdot 10^9 \cdot 24} = -6.34 \times 10^6 \text{ or } -6.22 \times 10^8 \text{ (overdamped)}$$





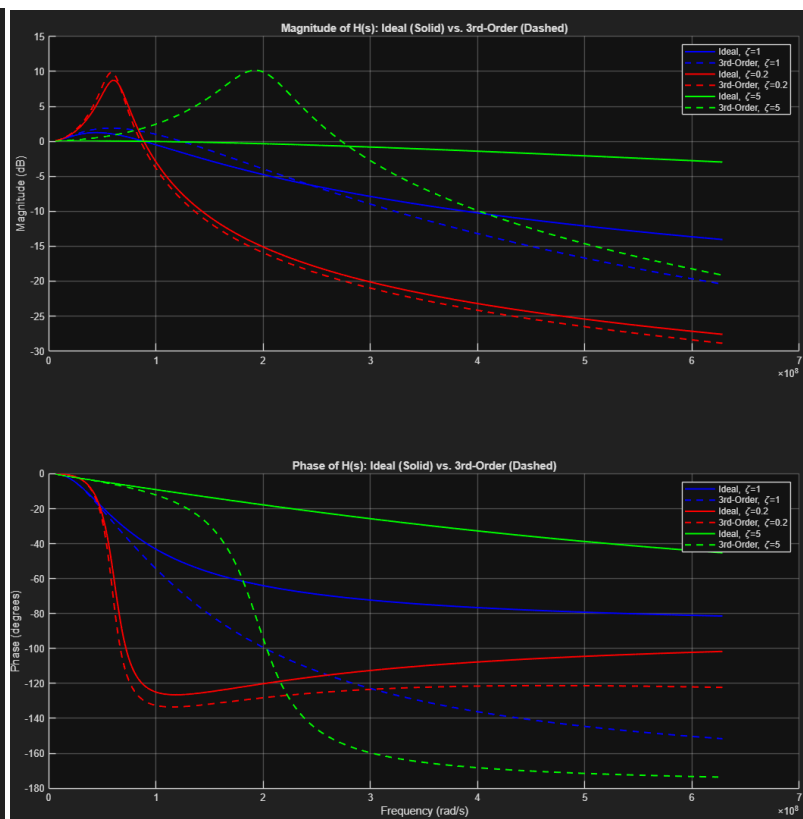
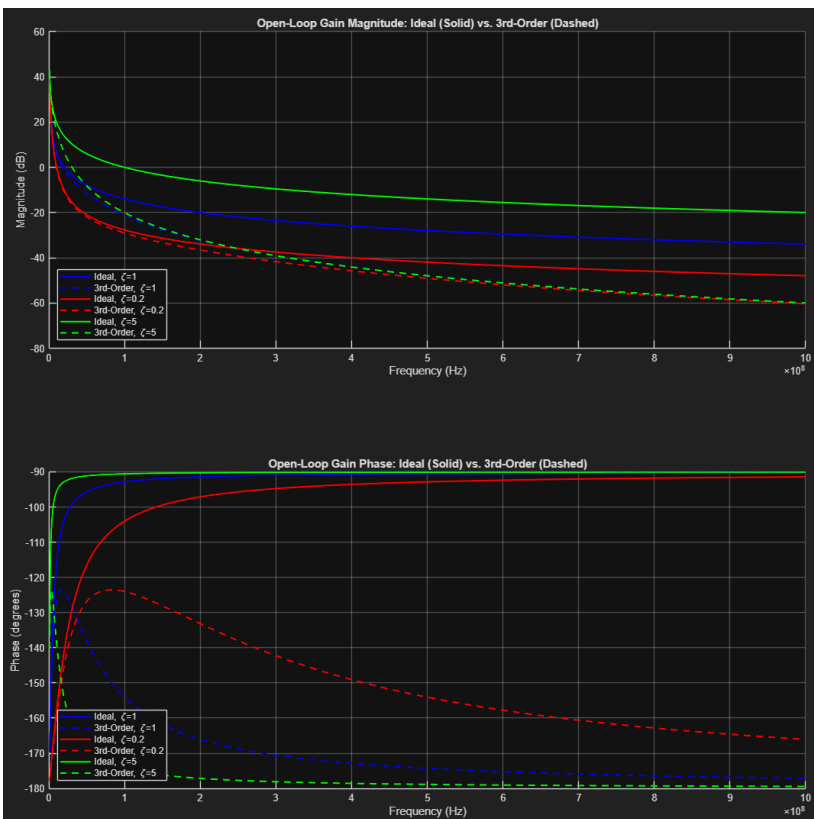
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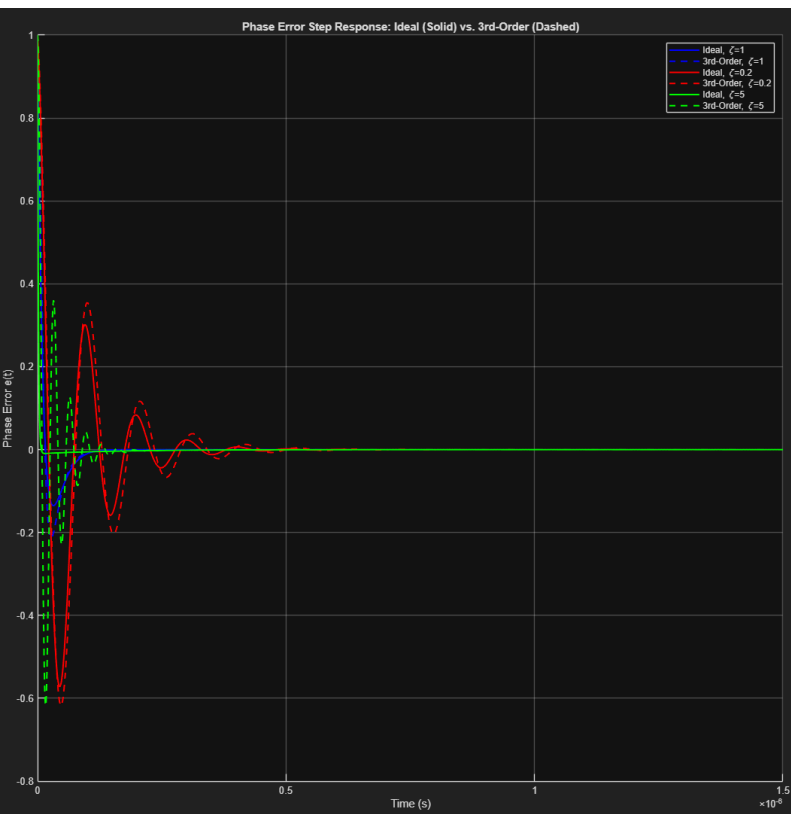
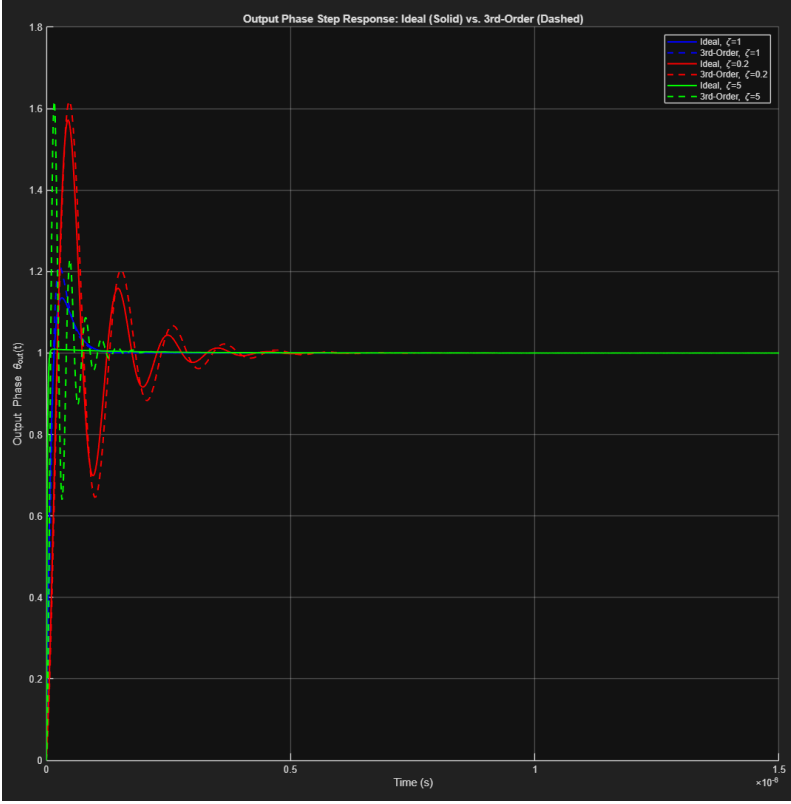
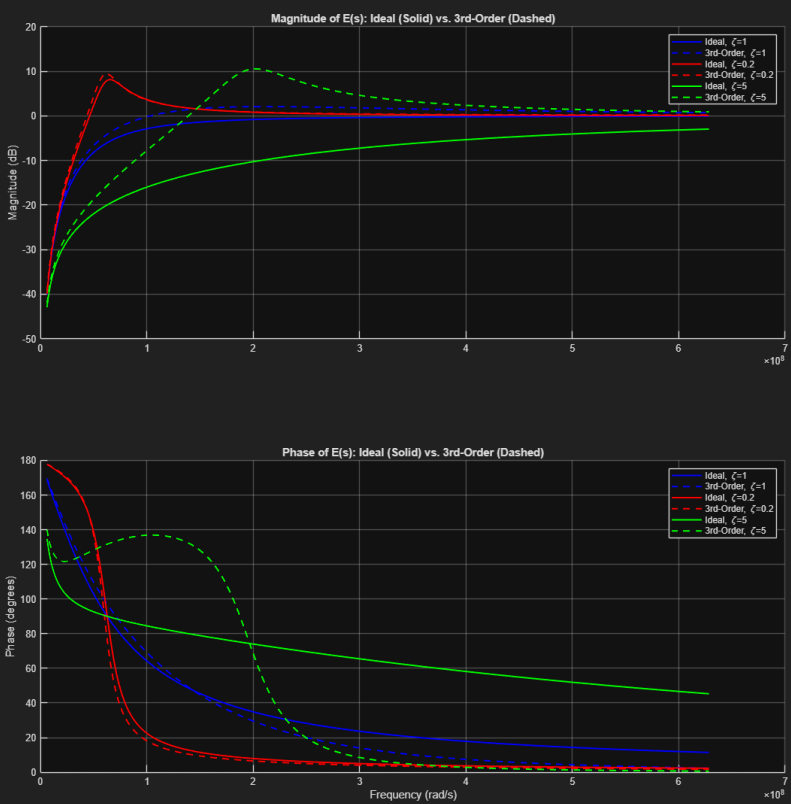
--- PART A: Open-Loop Analysis & Phase Margin ---
For zeta = 1.0, Phase Margin = 76.35 degrees
For zeta = 0.2, Phase Margin = 22.60 degrees
For zeta = 5.0, Phase Margin = 89.43 degrees

--- PART B: Closed-Loop Pole Values ---
For zeta = 1.0, Poles are: -6.283e+07 and -6.283e+07
For zeta = 0.2, Poles are: -1.257e+07 and -1.257e+07
                        Imaginary part: +/- 6.156e+07 j
For zeta = 5.0, Poles are: -6.347e+06 and -6.220e+08
  
```

The under damped case ($\zeta = 0.2$) shows peaking in the frequency response and overshoot in the transient response, while the critically damped ($\zeta = 1$) and overdamped ($\zeta = 5$) cases exhibit no peaking.

3.





```

--- Analyzing for ideal zeta = 1.0 ---
Poles of 3rd-Order System:
1.0e+08 *

-1.4757 + 0.5307i
-1.4757 - 0.5307i
-0.5043 + 0.0000i

--- Analyzing for ideal zeta = 0.2 ---
Poles of 3rd-Order System:
1.0e+09 *

-1.7069 + 0.0000i
-0.0105 + 0.0594i
-0.0105 - 0.0594i

--- Analyzing for ideal zeta = 5.0 ---
Poles of 3rd-Order System:
1.0e+08 *

-0.3138 + 1.9518i
-0.3138 - 1.9518i
-0.0635 + 0.0000i
  
```

Answer: C_z introduces a third pole into the system. This pole directly reduces the system's phase margin. The lower phase margin makes the system less stable, especially in the transient response. For every ζ value, the third-order system has more overshoot and ringing than ideal second-order system. However, C_z harms stability, the purpose of C_z is provide better filtering.