Linear Algebra and its Applications

HW#9

Note for HW#8 Problem 2:

$$A=\begin{bmatrix}1 & 0 & 2\\ 1 & 1 & 4\\ 1 & -1 & 0\end{bmatrix}.$$

(c) The pseudoinverse of A is defined as A^{+} such that $A^{+}Ax=x_r$

(e) Since A is not full-rank, the right inverse of A. does not exit.

1. Apply the Gram-Schmidt process to $a=[1, 1, 0]^{T}$, $b=[1, 0, 1]^{T}$ and $c=[0, 1, 1]^{T}$ and write the result in the form $A=QR$.
2. Find the best parabola: $y = C + Dt + Et^2$ fit to the following measurements:

   $y = 2$ at $t = -1$,

   $y = 0$ at $t = 0$,

   $y = 3$ at $t = 1$,

   $y = 5$ at $t = 2$.

   Find your approximate solution by QR factorization and plot a figure in the Excel to illustrate the observations with best-fit parabola using Excel.
3. Find the Fourier coefficients a_0, a_1, b_1, a_2, b_2 of the step function $y(x)$ which equals -1 on the interval $0\le x\le 3\pi/2$ and 4 on the remaining interval $3\pi/2<x\le 2\pi$. Plot $y(x)$ and the Fourier series on the same figure (you may use Excel to create the figure).
4. Find the closest degree-2 polynomial function $C + Dx + Ex^2$ to fit $f(x)=\cos x$ over $0\le x\le 2\pi$ by:

   (1) solving the normal equation

   (2) minimizing the least square and

   (3) the Legendre polynomials.

   Plot the two functions on the same figure (you may use Excel to create the figure).

   Note that:

   $\int x\cos x dx = \cos x + x\sin x + C$;

   $\int x^n \cos x dx = x^n \sin x + nx^{n-1}\cos x - n(n-1)\int x^{n-2}\cos x dx$;

   $\int x\sin x dx = \sin x - x\cos x + C$;

   $\int x^n \sin x dx = -x^n \cos x + nx^{n-1}\sin x - n(n-1)\int x^{n-2}\sin x dx$.