Genetic Algorithm

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computer automation

Enthusiasm • Leadership • Innovation • Teamwork • Excellence

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Genetic Algorithm

- Created by Holland (1975)
- Optimization problems from the industries and engineering are very complex in nature
  - Hard to solve by conventional optimization techniques:
    - linear programming, mathematic programming, …
- In 1960s, a trend of imitating living beings to solve hard optimization problems
  - Simulate the natural evolutionary process of human beings
    - Stochastic optimization techniques
    - Evolutionary algorithms
  - Three main categories of evolutionary algorithms
    - GA, genetic algorithms
    - EP, evolutionary programming
    - ES, evolution strategies (solution search)

- Well known applications in area of industrial engineering
  - Scheduling and sequencing
  - Reliability design
  - Vehicle routing and scheduling
  - Group technology
  - Facility layout and location
  - Transportation
  - Machine Learning

General Structure of GA

- Stochastic search techniques based on the mechanism of natural selection and natural genetics
- Start with an initial set of random solutions
  - Population (a set of solution agents)
  - For simple problems
    - Each individual in the population is called a chromosome
    - Representing a solution to the problem
  - For complex problems
    - A set of chromosomes represents a solution
      - Called an organism
    - Each organism represents a solution to the complex problems

General Structure of GA

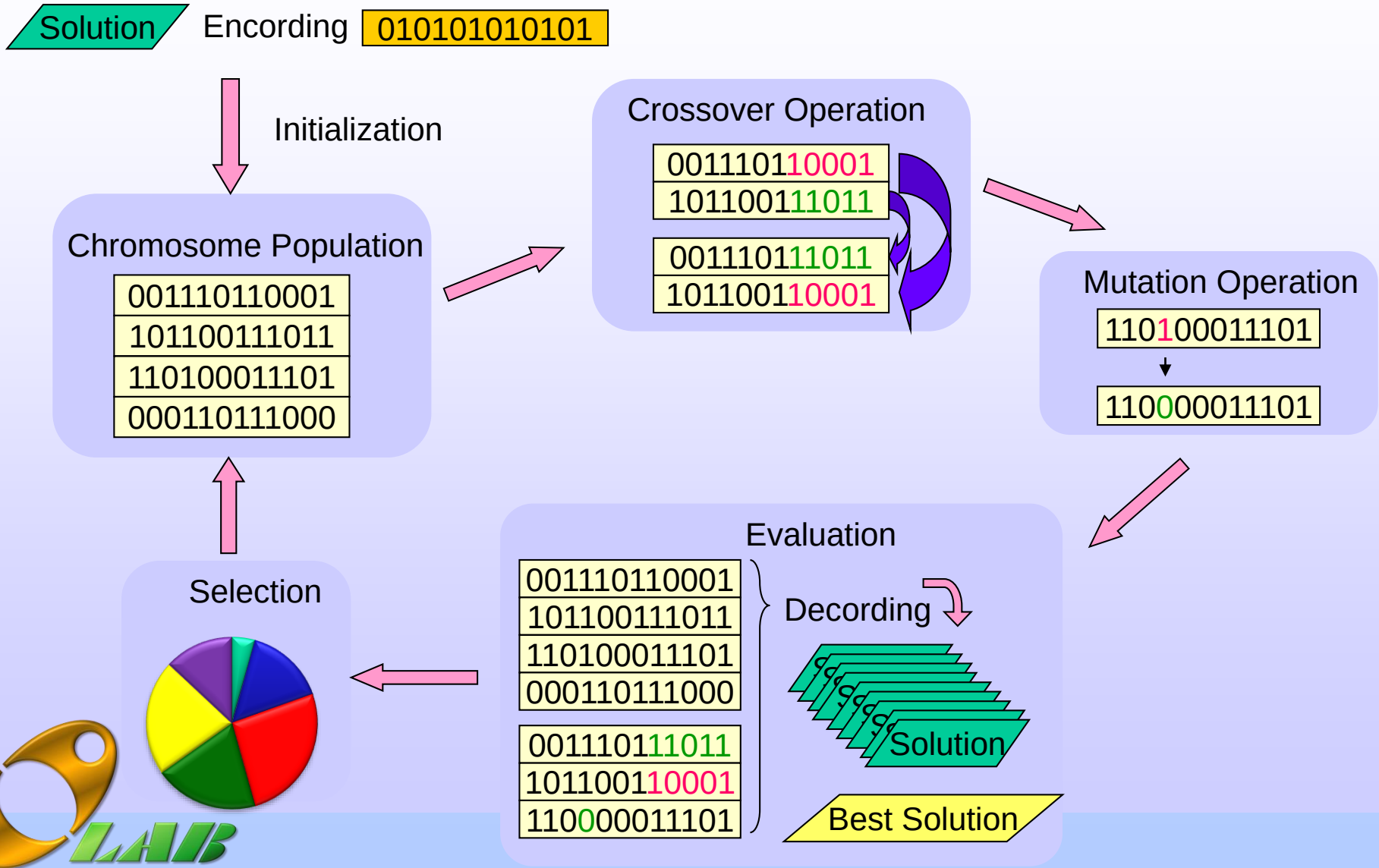
- Chromosome
  - A string of symbols
    - Typically is a binary bit string
  - Evolve through successive iterations, called generation
  - Evaluated by a measure of fitness of the single chromosome or the set of chromosomes (the organism)

- New chromosomes are called offspring
  - Formed by merging two chromosomes
    - By using a crossover operator
  - Formed by modifying a chromosome
    - By using a mutation operator
- New generation (population)
  - Formed by selecting some of the parents and offspring and rejecting others
    - Fitter chromosomes (high fitness) have higher probabilities of being selected
- The algorithm converges to the best chromosome
  - The optimum or suboptimal solution to the problem

Genetic Algorithm

- $P(t)$: **parents in current generation** t
- $C(t)$: **offspring in current generation** t

```
Begin
        t←0;
        initialize P(t)
        evaluate P(t);
        while( not termination condition ) do
                recombine P(t) to yield C(t)
                evaluate C(t);
                select P(t+1) from P(t) and C(t);
                t ← t+1;
        end
end
```


Solution
Encording
010101010101
Initialization
Chromosome Population
001110110001
101100111011
110100011101
000110111000
Crossover Operation
001110110001
101100111011
001110111011
101100110001
Mutation Operation
110100011101
110000011101
Evaluation
001110110001
101100111011
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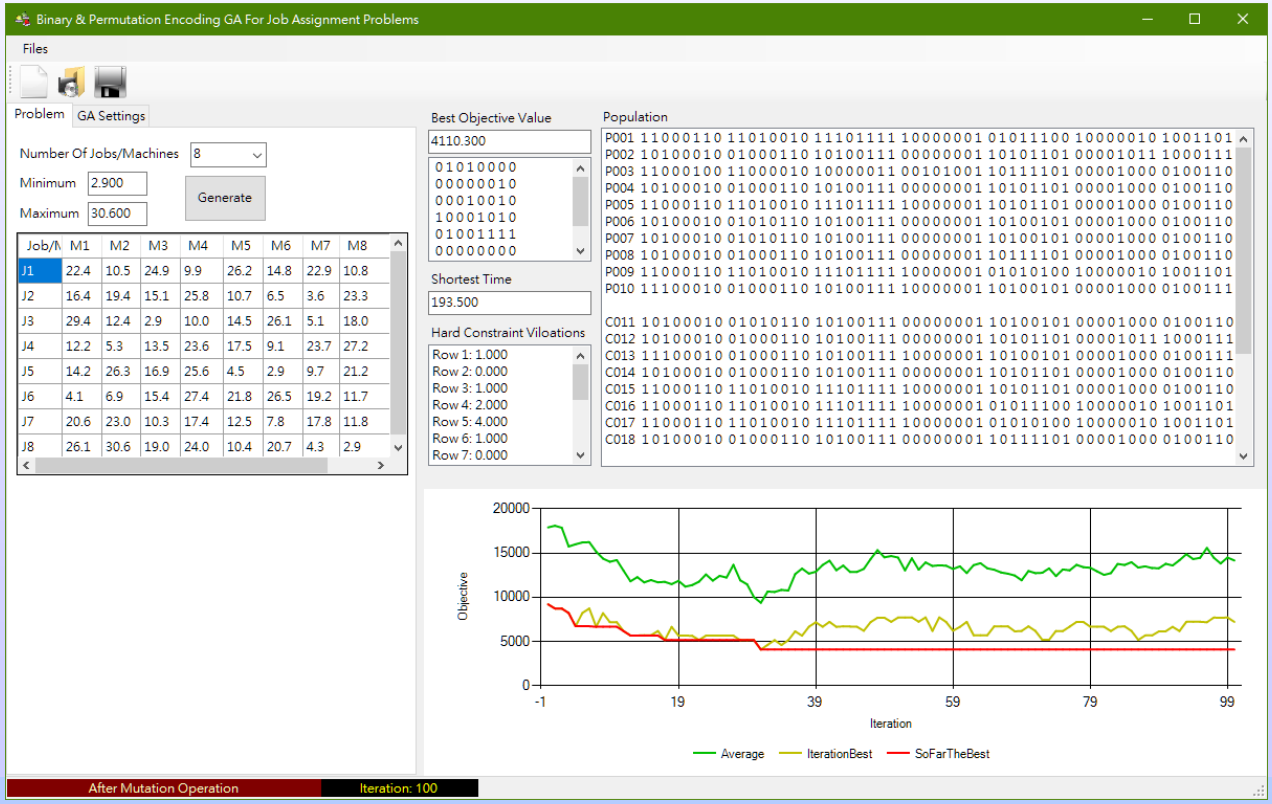
000110111000
001110111011
101100110001
110000011101
Decording
Solution
Best Solution
Selection

GA Solver Preview

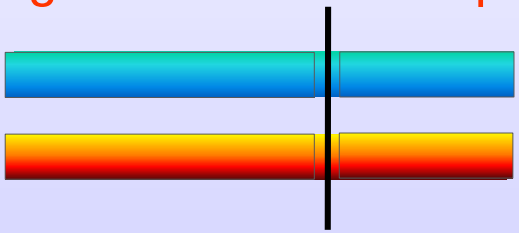
Job assignment optimization problem: N jobs to be assigned to N machines
A job can only be assigned to one machine; A machine can only process one job
A linear programming problem → should be solved by hard computing method
0-1 variables are used subject to the above assignment requirements
X_{ij}= 1, job i is assigned to machine j



- **Initialization**
  - **Usually, random initialization**
- **Recombination**
  - **Usually, via crossover and mutation operations**
- **Genetic operations**
  - **(1) Crossover and Mutation**
  - **(2) Selection**
    - Mimic the process of heredity of genes
    - Mimic the Darwinian evolution

Crossover operation

- **Main genetic operation**
  - **Operate on two chromosomes to generate offspring by combining parent's features**
  - **Simple way**
    - Choose a random cut-point and generate offspring by combining the left segment of one parent with the right segment of the other parent



  - **Determine the performance of GA**
- **Crossover rate, P_c**
  - **Allow only P_c x pop_size chromosomes to perform crossover operation**

Mutation Operation

- Produce random changes in various chromosomes
- Simple way
  - Alter one or more genes
- Mutation rate, P_m
  - The percentage of the total number of genes to be operated by the mutation
  - Control the rate at which new genes are introduced into the population

Solution Search

- Problems do not have priori information about the sequence of steps leading to a solution
  - Search is required
- Search
  - Blind strategy
    - Use no information about the problem
  - Heuristic strategy
    - Use additional information to guide the search along with the best search directions
- Two important issues in solution search
  - Exploitation (探究)
    - For the nearest best solution
  - Exploration (探索)
    - For the possible best solution

Solution Searches

- Hill-climbing search (Steepest descent search)
  - Exploit the best solution for possible improvement, ignoring the exploration of the search space
- Random search
  - Explore the search space ignoring the exploitation of the promising regions

GA vs. Conventional Optimization and Search Procedures

- Work with a coding of solution set
  - Not the solutions themselves
- Solution search from a population of solutions
  - Not a single solution
- Uses payoff information (fitness function)
  - Not derivatives or auxiliary knowledge
- Use the probabilistic transition rules
  - Not deterministic rules

Solution Searches

- **Genetic Search**
  - **A class of general-purpose search method**
  - **Combine directed and stochastic search**
  - **Initialization**
    - Explore all solution space
  - **Crossover operation**
    - Explore the neighborhood of each chromosome
    - One can not always use simple operations for all kinds of Problem
      - Use information of the problem domain to design the operations
      - Otherwise, it is a blind search

- **Traditional Optimization Search (hard computing)**
  - **Point-to-point**
  - **Generate a deterministic sequence of computation based on the gradient or higher-order derivatives of objective function**
  - **The point is improved along the deepest descending or ascending direction**
  - **Risk of falling in local optima**

Genetic search

- **Population-to-population**
  - Not point-to-point (single-to-single)
- **Multiple directional search by a population of potential solutions**
- **Escape from local optima**
- **The population undergoes a simulated evolution**
  - The relatively good solutions are reproduced
  - The relatively bad solutions die
  - Use probabilistic transition rules

GA Optimization Method

- **Previous Approach (Classic Approach)**
  - **Used as a generic tool for many difficult-to-solve problem**
    - Chromosomes: Fixed-length binary strings
    - Simple binary crossover and mutation operators
    - Domain-independent
  - **Emphasis on the robust adaptive systems**
  - **Difficult to apply directly and successfully into difficult-to-solve problem**

GA Meta-Heuristic

- **Current Approach**
  - **GA is used as a meta-heuristics**
    - Nonstandard implementations are created for particular problem
  - **Various data structure for nature representation**
  - **Various sets of genetic operators applied to these data structures**

Advantages of GA

- Have little mathematical requirements about the optimization problem
  - Can handle any kind of objective functions and constraints (linear or nonlinear)
  - Discrete, continuous, or mixed solution space
- Genetic operators are very effective in performing global search
  - Traditional one (hard computing) performs local search by a stepwise procedure
- Flexibility to hybridize with domain-dependent heuristics to make an efficient implementation for a specific problem

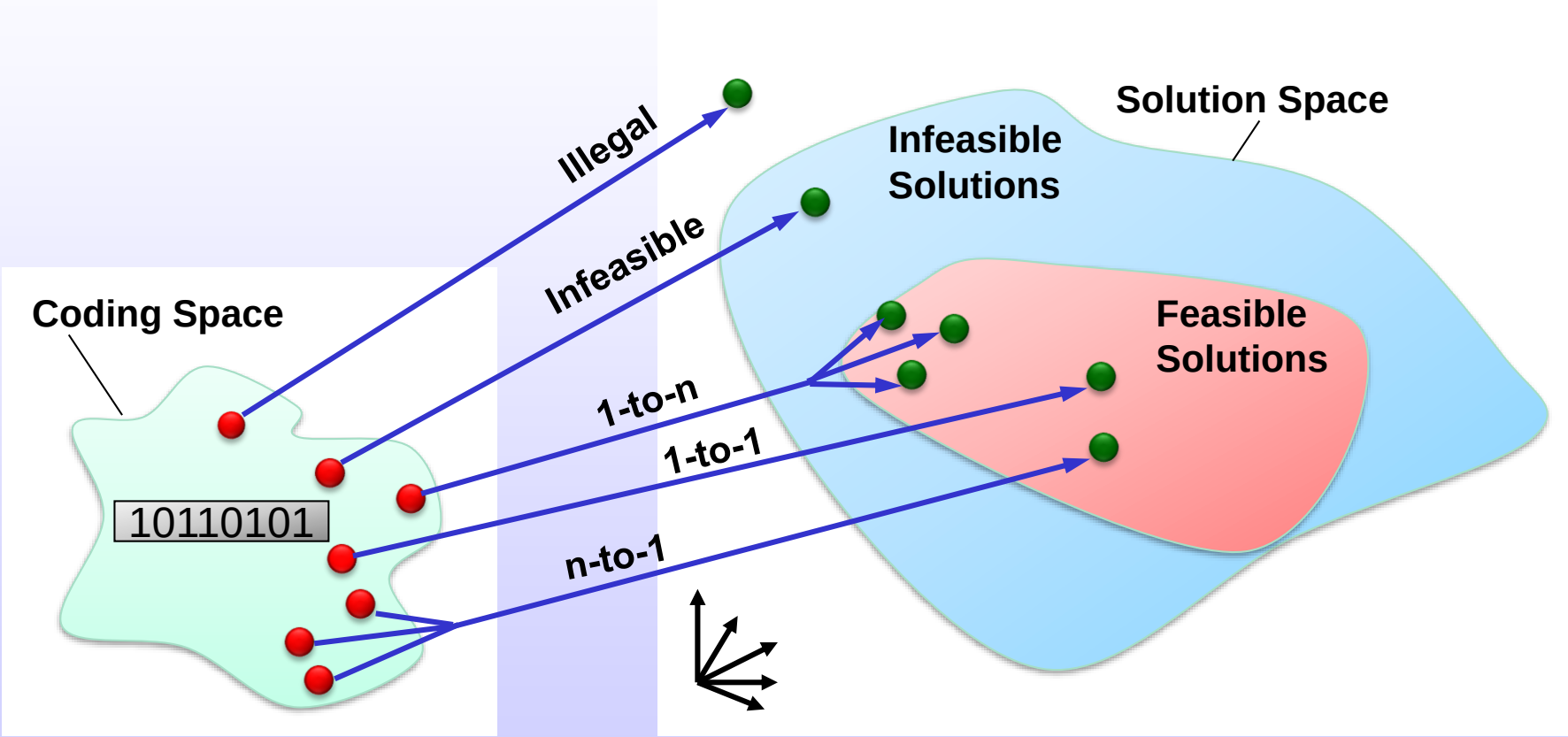
Vocabulary of GA

- Biologic organism
  - One or more chromosomes specify the complete organism
  - Genotype (基因型)→ the complete set of chromosomes
  - Phenotype(顯形) →the organism
- Chromosome
  - Comprises a number of genes
- Gene
  - Represents a particular feature of human being
  - Encodes a particular feature of the organism and the location

Encoding Problem

- **Coding Space vs. Solution Space**
  - **Coding space**
    - Genetic operations work on
  - **Solution space**
    - Where the evaluation operation works on
    - Where the selection operation works on

- **Critical issues**
  - **Legality of a chromosome**
    - Whether a chromosome represents a solution?
      - Problem: No such a solution exists
  - **Feasibility of a chromosome**
    - A solution decoded from a chromosome is a feasible solution?
      - Problem: The solution is infeasible
  - **Uniqueness of mapping**
    - Whether a chromosome represents only one solution and the solutions is represented by the only one chromosome?
      - Problem: A chromosome represents many solutions
      - Problem: Several chromosomes represent the same solution



Strategies

- **Illegality**
  - **Resulted from crossover and mutation operations**
  - **Rejecting method**
  - **Penalizing method**
  - **Repair method**
    - Convert an illegal chromosome to a legal one
    - Better than rejecting and penalizing strategies
- **Infeasibility**
  - **Penalty methods**
  - **Violation amounts are penalized and added to the objective function to degrade the fitness**
- **Uniqueness**
  - **1-to-n**
    - Hard to deal with; should avoid it
  - **N-to-1**
    - Inefficient GA

Encoding Types

- Binary Representation and Encoding
- Integer Representation and Encoding
- Real Representation and Encoding
- Symbol Representation and Encoding

Binary Representation and Encoding

- **Can be applied for various type of optimization problems**
  - **Binary Optimization Problem**
    - Directly use binary encoding
  - **Integer optimization problem**
    - Find the required number of digitals for each integer variable
    - Encode the binary representation for the integer
    - If negative integers are involved, digital shifting operation is conducted
    - One special case of real number optimization problem
  - **Real number optimization Problem**
    - Compute the required numbers of digits to cover the range of each real variable, including the required decimal precision

Real Number Encoded as Binary Genes

Encoding

$l \le x \le u;$

n : number of desired digits in decimal part

m : required length of the binary string for x_i

$2^{m-1} < (u-l) \times 10^n \le 2^m - 1$

Binary string encording : m is calculated first

$b = Encording(x, l, u, m) = B\left(\left\lceil (x-l)(2^m - 1)/(u-l) \right\rceil, m\right)$

$B(i, m)$: Binary expression for integer i in m bits

$Example: -3 \le x = -2.6880 \le 12.1, n = 4;$

$(12.1 - (-3)) \times 10^4 = 151000$

$2^{17} = 131072 < 151000 \le 2^{18} = 262144; m = 18$

$b = Encording(-2.688, -3, 12.1, 18) =$

$B\left(\left\lceil (-2.588 + 3)(2^{18} - 1)/(12.1 + 3) \right\rceil, 18\right) =$

$B(\lceil 5416.46 \rceil, 18) = B(5417, 18) = 000001010100101001$

Decoding

Decording from binary string encording:

$$x = Decoding(b, l, u, m) = l + D(b)\frac{u-l}{2^m - 1}$$

$D(b)$: Decimal value of a binary number b

$$Example: x = Decoding(b, l, u, m)$$

$$Decoding(000001010100101001, -3, 12.1, 18)$$

$$x = -3.0 + D(000001010100101001)\frac{12.1+3}{2^{18}-1}$$

$$= -3.0 + 5417 \times \frac{12.1+3}{2^{18}-1} = -2.6879$$

Crossover Operations for Binary Representation

- **Binary Representation**
  - One-point
  - Two-point
  - N-point

01110101000100101100101010001
10010111010001010001010000100

01110101000100101001010001000
10010111010001010100101010001

01110101000101000100100010000
10010111010010100100100101010

01110101010010101001000100
10010111001010001010010101

Mutation Operation for Binary Representation

- **Mutation Operator**
  - **Determine the number of genes to be mutated**
  - **Randomly determine the positions of these genes in the whole population**
  - **Duplicate the parents whose genes are selected as proto-children**
  - **Alter the genes' values to resulting new offspring**

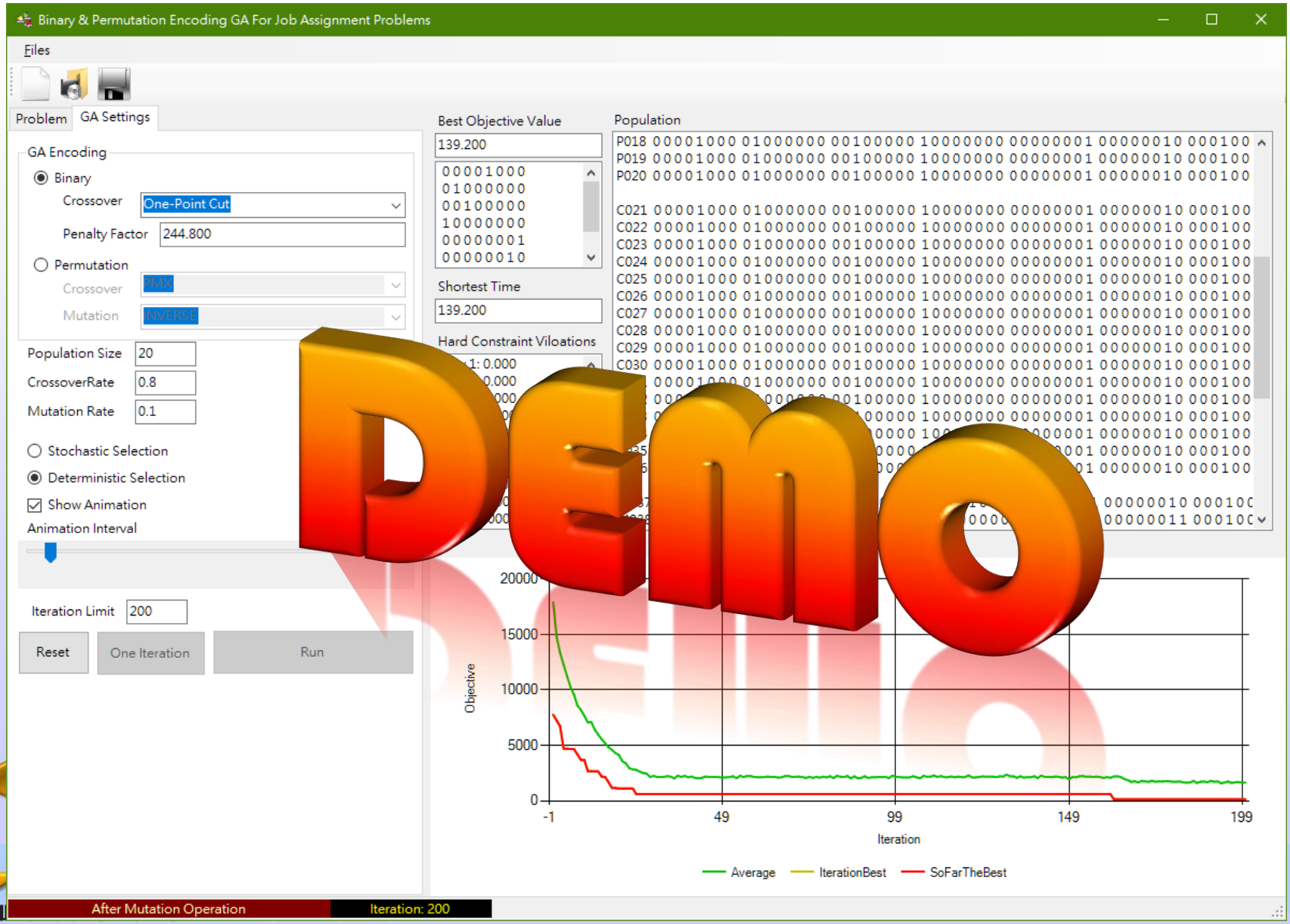
```
01110101000101000100
10001000100010010100
11010101111101010101
10101010001000100010
11010101000010001000
11101010001000100100
11110010101010010100
11101001010010100101
```

P_m=0.04
Pop x *length* = 8x20 =160
Number of mutated genes = 0.04x160= 6

Offspring

```
01110101100101000100
10101110000000110010
11010100000010001000
11101001010010100001
```

Binary Encoding GA for Job Assignment



Integer Representation and Encoding

- For sequencing and grouping optimization problems
- Directly use integer symbols to encode the chromosomes
  - E.g.;
    - [5 3 2 1 6 8 7 4 9] : for permutation encoding
    - [2 2 1 3 3 1 1 2 4] : for object grouping problems

Crossover Operations for Integer Encoding

- **Permutation representation (E.g., TSPs, JSPs, SAPs)**
  - **Canonical Approach:**
    - Extension of two-point or multi point crossover
    - Repairing procedures are embedded
    - Based on blind random mechanism
      - No guarantee that offspring is better
    - Operators
      - Partial-mapped Crossover (PMX)
      - Order Crossover (OX)
      - Position-based Crossover
      - Order-based Crossover
      - Cycle Crossover (CX)
      - Subtour Exchange
  - **Heuristic Approach**
    - Problem-dependent Heuristic Crossovers

TSPs: Traveling Salesman Problem; JSPs: Job shop Scheduling Problems; SAPs: Single line Assembly Problems

Partial-mapped Crossover PMX

- Proposed by Goldberg and Lingle
  - Step 1: Select two positions to obtain a substring (mapping section)
  - Step 2: Exchange the substring of parents to produce proto-children
  - Step 3: Determine the mapping relationship between two mapping sections
  - Step 4: Legalize offspring with the mapping relationship

Parent 1	1	2	3	4	5	6	7	8	9
Parent 2	6	4	5	7	9	8	3	1	2

Proto-child 1	1	2	5	7	9	8	7	8	9
Proto-child 2	6	4	3	4	5	6	3	1	2

5	7	9	8
3	4	5	6

3←→5←→9
4←→7
6←→8

Offspring 1	1	2	5	7	9	8	4	6	3
Offspring 2	8	7	3	4	5	6	9	1	2

Order Crossover (OX)

- Step 1: Select a substring from parent 1
- Step 2: Copying the substring into the corresponding position for proto-child 1
- Step 3: Mark out the genes in the parent 2
- Step 4: Add the unmarked genes left in parent2 sequentially into the void positions at proto-child 1 to complete offspring 1
- Step 5: Repeat the same steps starting from parent 2 and against parent 1 to produce offspring 2

Parent 1	1	2	3	4	5	6	7	8	9
Proto-child 1			3	4	5	6			
Parent 2	6	4	5	7	9	8	3	1	2

Offspring 1	7	9	3	4	5	6	8	1	2

Parent 1	1	2	3	4	5	6	7	8	9
Proto-child 2			5	7	9	8			
Parent 2	6	4	5	7	9	8	3	1	2

Offspring 2	1	2	5	7	9	8	3	4	6

Position-based Crossover

- Step1: Randomly select a set of positions
- Step2: Coping genes at these positions to the same position to produce proto-child 1
- Step 3: Mark out the genes of these position in parent 2
- Step 4: Add the unmarked genes in parent2 sequentially into the void positions at proto-child 1 to complete offspring 1
- Step 5: Repeat the same steps starting from parent 2 and against parent 1 to produce offspring 2

Parent 1	1	2	3	4	5	6	7	8	9
Proto-child 1		2			5	6			9
Parent 2	6	4	5	7	9	8	3	1	2

Offspring 1	4	2	7	8	5	6	3	1	9

Parent 1	1	2	3	4	5	6	7	8	9
Proto-child 2		4			9	8			2
Parent 2	6	4	5	7	9	8	3	1	2

Offspring 2	1	4	3	5	9	8	6	7	2

Order-based Crossover

- A variation of position-based crossover
- The order of genes in the selected position in parent 1 is imposed on the corresponding genes in parent 2

Parent 1	1	2	3	4	5	6	7	8	9
Proto-child 1	2		5		6				9
Parent 2	6	4	5	7	9	8	3	1	2

Parent 1	1	2	3	4	5	6	7	8	9
Proto-child 2		6			5	9			2
Parent 2	6	4	5	7	9	8	3	1	2

Offspring 1	2	4	5	7	6	8	3	1	9
Offspring 2	1	6	3	4	5	9	7	8	2

Cycle Crossover (CX)

- Step1: Find the cycle defined by the genes between parents
- Step2: Copy the genes in the cycle to a child with the corresponding positions of parent 1
- Step3: Mark out the genes of these position in parent 2
- Step 4: Add the unmarked genes in parent2 sequentially into the void positions at proto-child 1 to complete offspring 1
- Step 5: Repeat the same steps starting from parent 2 and against parent 1 to produce offspring 2

Parent 1	1	2	3	4	5	6	7	8	9
Parent 2	6	4	5	7	9	8	3	1	2

Cycle 1: 1-6-8-1
Cycle 2: 2-4-7-3-5-9-2

Proto-child 1	1					6		8	
Parent 2	6	4	5	7	9	8	3	1	2

Parent 1	1	2	3	4	5	6	7	8	9
Proto-Child 2	6					8		1	

Offspring 1	1	4	5	7	8	6	3	8	2
Offspring 2	6	2	3	4	5	8	7	1	9

Subtour Exchange Crossover

- **Step1: Define a subtour length**
- **Setp2: Select subtour from parents that contains common cities**
- **Step3: Exchange subtour to produce two offspring**

Parent 1	1	2	3	4	5	6	7	8	9
Parent 2	6	4	5	7	9	8	3	1	2

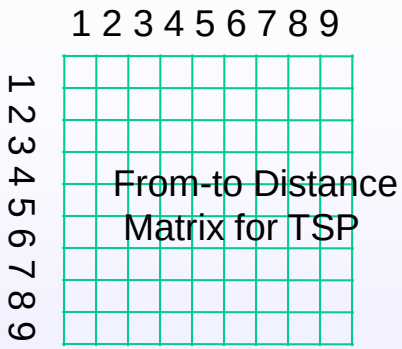
Subtour Length = 4;
Subtour1: 4-5-6-7

Offspring 1	1	2	3	6	4	5	7	8	9
Offspring 2	4	5	6	7	9	8	3	1	2

Heuristic Crossover

- Problem dependent
- Follows the goal of maximizing the fitness
- For example: TSP
  - Step 1: For a pair of parents, pick a random city for the start
  - Step2: Choose the shortest edge leading from the current city which does not lead to cycle.
    - If two edges lead to a cycle, choose a random city to continue
  - Step3: If the tour is completed, stop; otherwise, goto step 2

Parent 1	1	2	3	4	5	6	7	8	9
Parent 2	6	4	5	7	9	8	3	1	2



3 (for offspring 1)
3-shortest(3-4, 3-2, 3-1, 3-8)
3-8
3-8-shortest(8-9,8-7)
3-8-7
3-8-7-shortest(7-6,7-9,7-5)
3-8-7-9
3-8-7-9-shortest(9-1)
3-8-7-9-1
3-8-7-9-1-shortest(1-2)
3-8-7-9-1-2
3-8-7-9-1-2-shortest(2-6)

3-8-7-9-1-2-6
3-8-7-9-1-2-6-shortest(6-5,6-4)
3-8-7-9-1-2-6-5
3-8-7-9-1-2-6-5-shortest(5-4)
3-8-7-9-1-2-6-5-4

Offspring 1	3	8	7	9	1	2	6	5	4

7 (for offspring 2)
7-shortest(7-9, 7-8, 7-6,)
…

Offspring 2	7	6	4	3	1	2	9	8	5

Mutation Operators

- **Inversion Mutation**
  - Select two positions on a chromosome to invert the substring between these two positions

Parent	1	2	3	4	5	6	7	8	9
Child	1	2	6	5	4	3	7	8	9

- **Insertion Mutation**
  - Select a gene and insert it in a random position

Parent	1	2	3	4	5	6	7	8	9
Child	1	2	4	5	6	3	7	8	9

- **Displacement Mutation**
  - Select a subtour and insert it in a random position

Parent	1	2	3	4	5	6	7	8	9
Child	1	2	6	7	8	3	4	5	9

Reciprocal Exchange Mutation

- Select two positions to swap the genes

Parent	1	2	3	4	5	6	7	8	9
Child	1	2	8	4	5	6	7	3	9

Heuristic Mutation

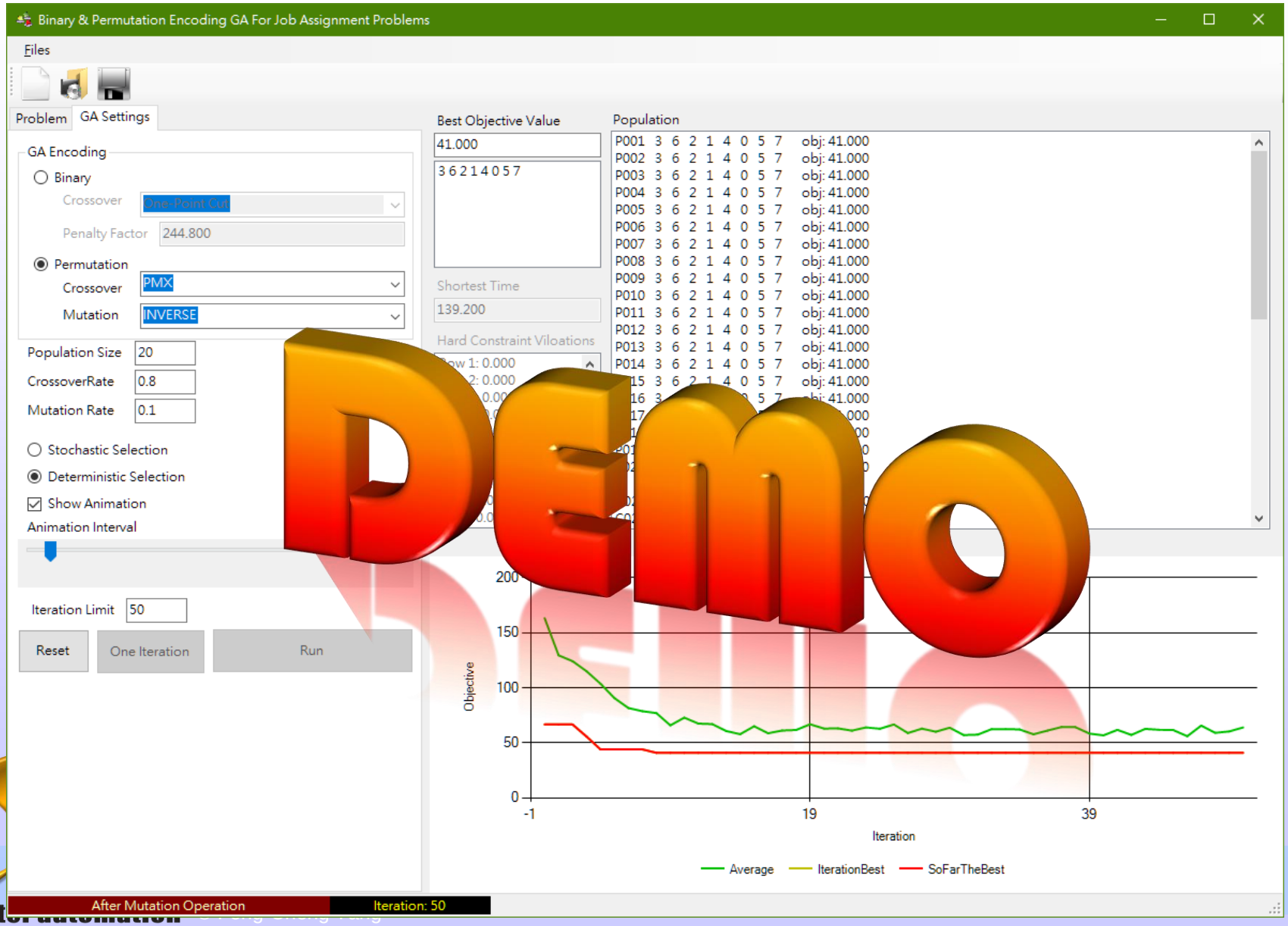
- Step1: pick up *c* genes
- Step2: generate neighbors from all possible permutations of the selected genes
- Step3: evaluate all neighbors and select the best one as offspring

Parent	1	2	3	4	5	6	7	8	9

Neighbor 1	1	2	3	4	7	6	5	8	9
Neighbor 2	1	2	5	4	7	6	3	8	9
Neighbor 3	1	2	5	4	3	6	7	8	9
Neighbor 4	1	2	7	4	5	6	3	8	9
Neighbor 5	1	2	7	4	3	6	5	8	9

Children

Permutation Encoding GA for Job Assignment



Real Number Representation

- Directly use a serial of real numbers to represent genes of a chromosome which represents a solution
  - Each variable should have upper and lower bounds
- Crossover Operators
  - One point or two-point cuts to select segments of genes to be crossovered
  - Apply arithmetical operator on these genes

$$\mathbf{x}_1' = \alpha \mathbf{x}_1 + \beta \mathbf{x}_2$$

$$\mathbf{x}_2' = \beta \mathbf{x}_1 + \alpha \mathbf{x}_2$$

- Convex crossover: $\alpha+\beta=1.0$, $\alpha,\beta \geqq 0.0$
- Affine crossover: $\alpha+\beta=1.0$
- Linear crossover: $\alpha,\beta \in R$

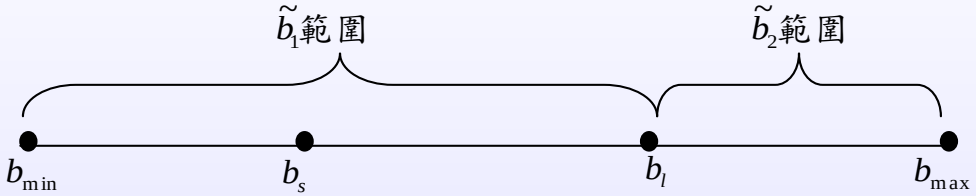
CALab Proposed Five Crossover Operators for Real Representation

- 大值分隔兩段式(Large Value Divided, LVD)
- 小值分隔兩段式(Small Value Divided, SVD)
- 中段和任一端段式(Middle and One End Segments, MOES)
- 兩端兩段式(Two End Segments, TES)
- 中段往復兩段式 (Forward and Backward Middle Segment, FBMS)

五種實數交配演算的交配值落點範圍

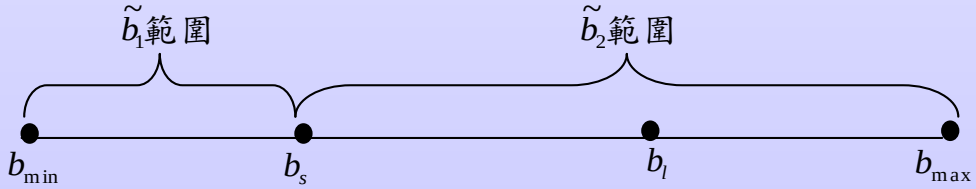
LVD 大值分隔兩段式

$$\tilde{b}_1 = \alpha b_{\min} + (1-\alpha) b_l \qquad \tilde{b}_2 = \alpha b_l + (1-\alpha) b_{\max}$$



SVD 小值分隔兩段式

$$\tilde{b}_1 = \alpha b_{\min} + (1-\alpha) b_s \qquad \tilde{b}_2 = \alpha b_s + (1-\alpha) b_{\max}$$



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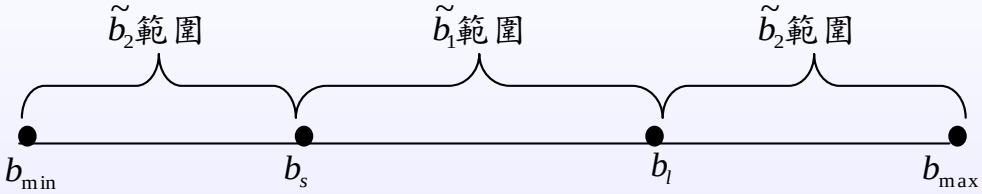
MOES 中段和任一端段式

$$\tilde{b}_1 = \alpha b_s + (1-\alpha) b_l \qquad \tilde{b}_2 = \begin{cases} \alpha b_l + (1-\alpha) b_{\max}, & \text{if } \sim \mathrm{U}(0,1) > 0.5 \\ \alpha b_{\min} + (1-\alpha) b_s, & \text{otherwise} \end{cases}$$

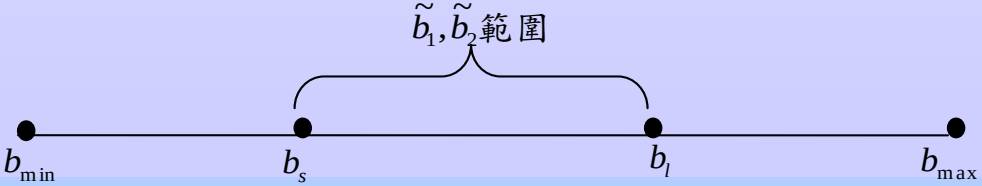


TES 兩端兩段式

$$\tilde{b}_1 = \alpha b_{\min} + (1-\alpha) b_s \qquad \tilde{b}_2 = \alpha b_l + (1-\alpha) b_{\max}$$

FBMS 中段往復兩段式

$$\tilde{b}_1 = \alpha b_s + (1-\alpha) b_l \qquad \tilde{b}_2 = \alpha b_l + (1-\alpha) b_s$$



Mutation Operation for Real Number Representation

Dynamic Mutation (Uniform Mutation)

- **For fine tuning for high precision**

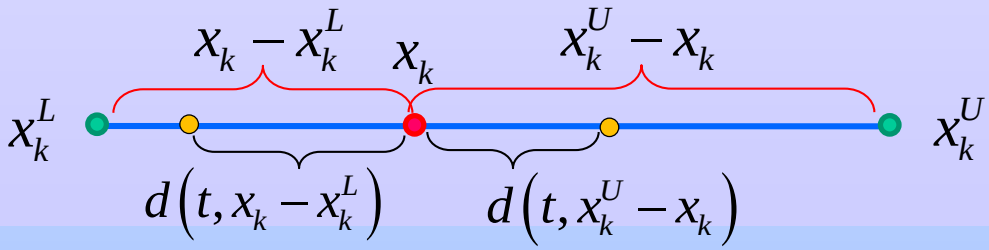
$$\boldsymbol{x} = [x_1 x_2 \cdots x_n],\ \boldsymbol{x}' = [x_1 x_2 \cdots x_k' \cdots x_n]$$

$$x_k' = \begin{cases} x_k + d\left(t, x_k^U - x_k\right) \\ x_k - d\left(t, x_k - x_k^L\right) \end{cases},\ d(t, y) = \sim U(0,1) \cdot y \cdot \left(1 - \frac{t}{T}\right)^b$$

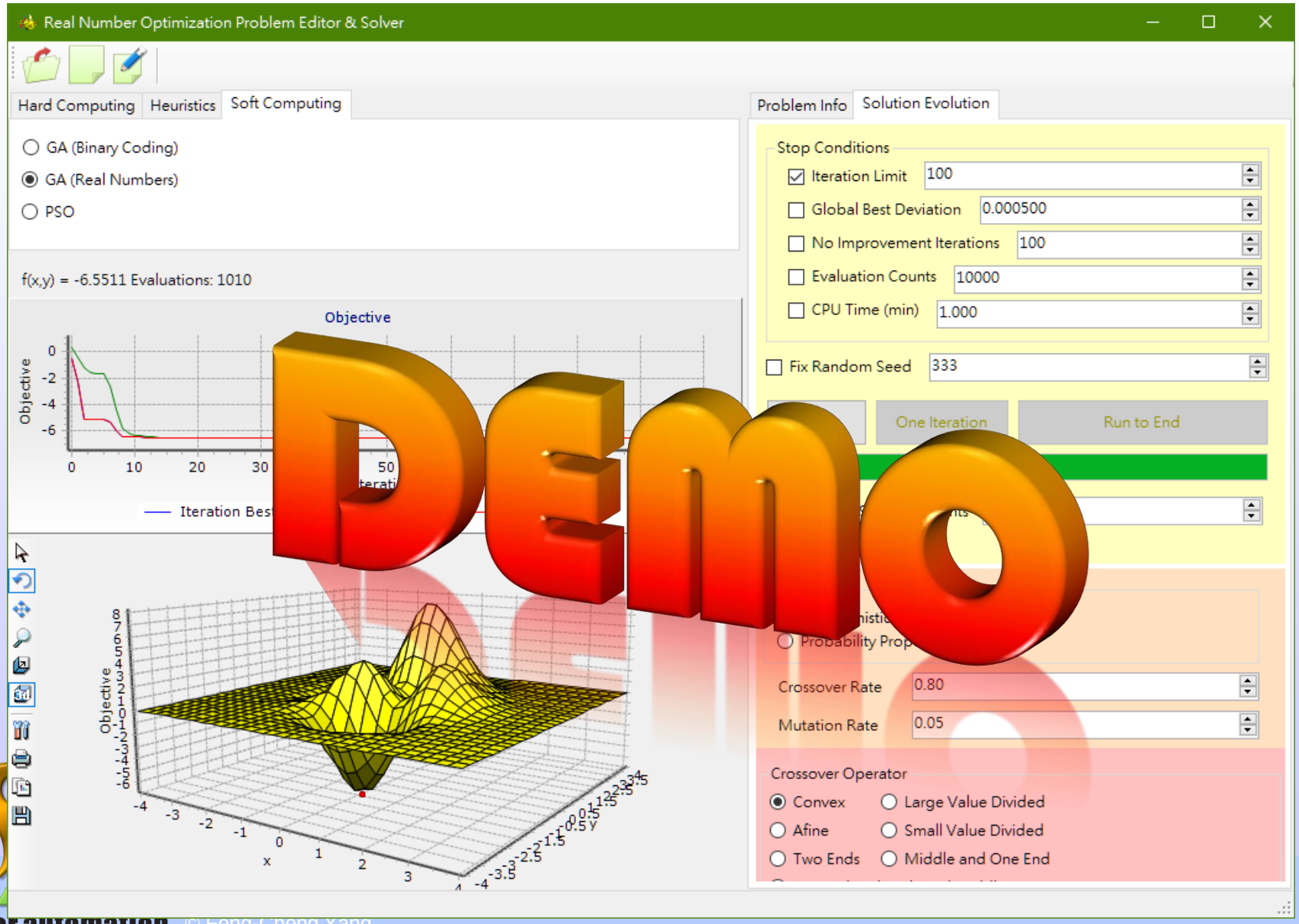
t: iteration index

T: total number of iterations

b: parameter determines the degree of nonuniformity



Real Number Encoding GA for Continuous Optimization Problems



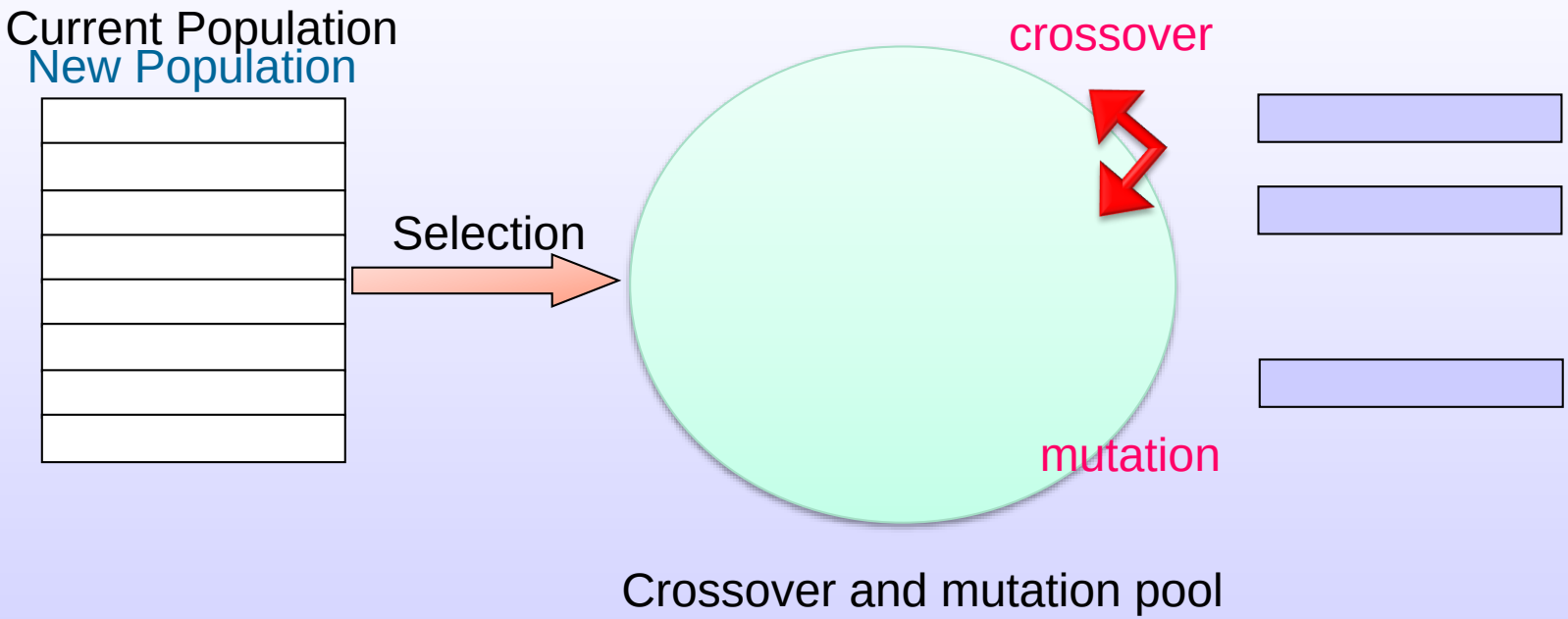
Selection Operation (篩選)

- Darwinian natural selection
- A critical step in GA:
  - A inappropriate selection:
    - Search will terminate prematurely
    - Progress will be slower than necessary
- Low selection pressure ← at the start
  - Favor for a wide exploration of search space
- High selection pressure ← at the end
  - To exploit the most promising region

Genetic Selection

- **Regular sampling space for selection**
  - **Size:** *pop_size*
  - **Selection targets: parents**
  - **Holland's original GA**
    - Randomly select chromosomes from the current generation
    - Parents are replaced by their offspring at once
    - Replacement Strategies
      - select parents for reproduction & offspring replaces parents

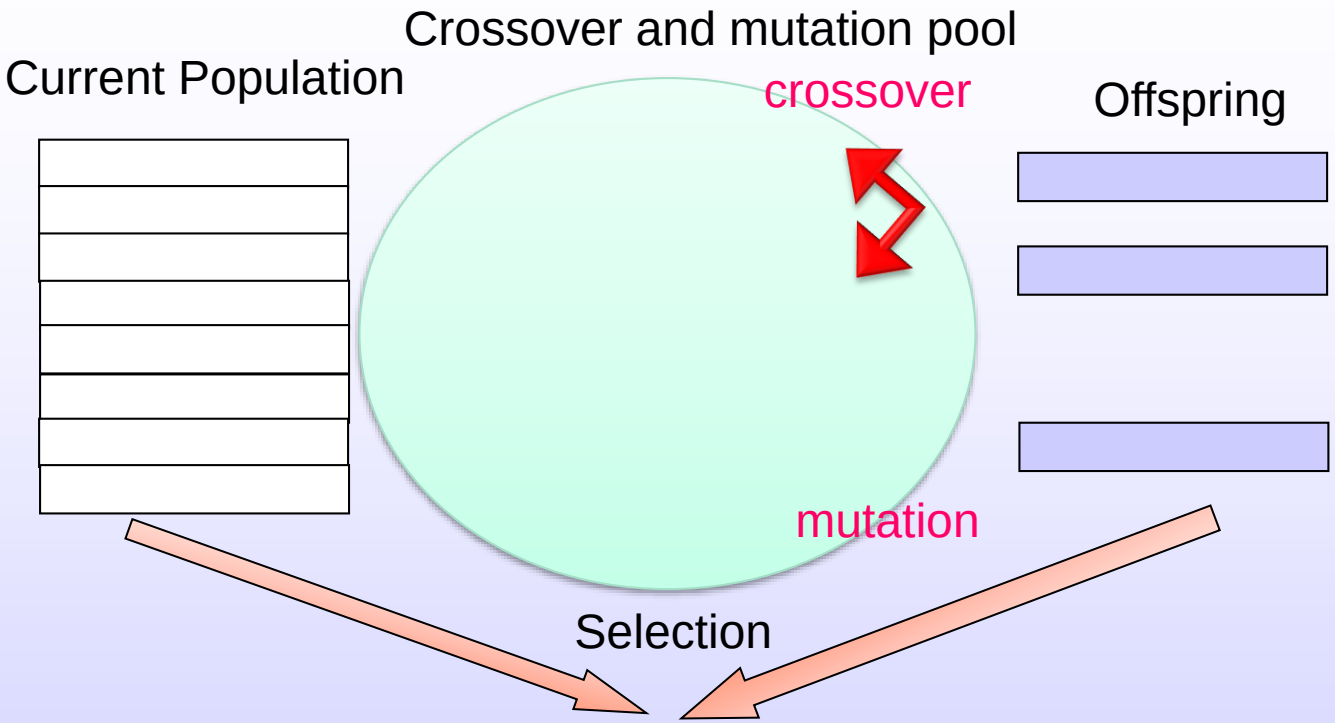
Holland's Selection and Replacement



Enlarged sampling space

- **Enlarged sampling space for selection**
  - **Size:** *pop_size* **+** *off_size*
  - **Selection targets: all parents and offspring**
  - **Both parents and offspring have the same chance of competing for survival**
  - *pop_size* **best out of offspring and parents are selected (***off_size* **+** *pop_size***)**

Enlarged sampling space



Best Sampling Mechanism

- **Stochastic Sampling**
  - **Number of copies of each chromosome is based on its survival probability**
    - Either 0, 1, or multiple copies are selected for each chromosome
- **Deterministic Sampling**
  - **The best *pop_size* chromosomes are selected from the sampling space**
    - Either 0 or 1 copy is selected for each chromosome
- **Mixed Sampling**

Stochastic Sampling

- **Probability proportionate selection**
  - **Roulette wheel selection**
  - **Selection probability for each chromosome proportional to the fitness value**
  - **Selection process is spinning the roulette wheel** *pop_size* **times**

Roulette wheel selection

$$p_k = \frac{f_k}{\sum_{i=1}^{pop\_size+off\_size} f_i}, \; f_k > 0$$

$$q_k = \sum_{i=1}^{k} p_i, k \in \{1, 2, \ldots, pos\_size + off\_size\}$$

for $i = 1$ to pop_size

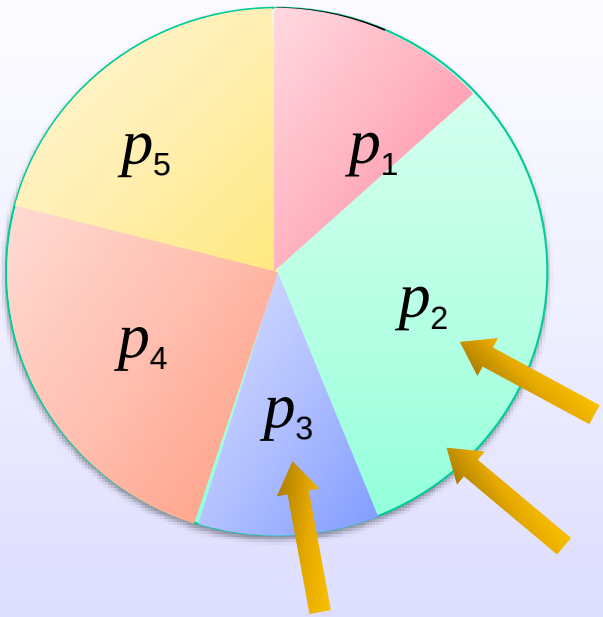
$q \leftarrow U(0,1)$

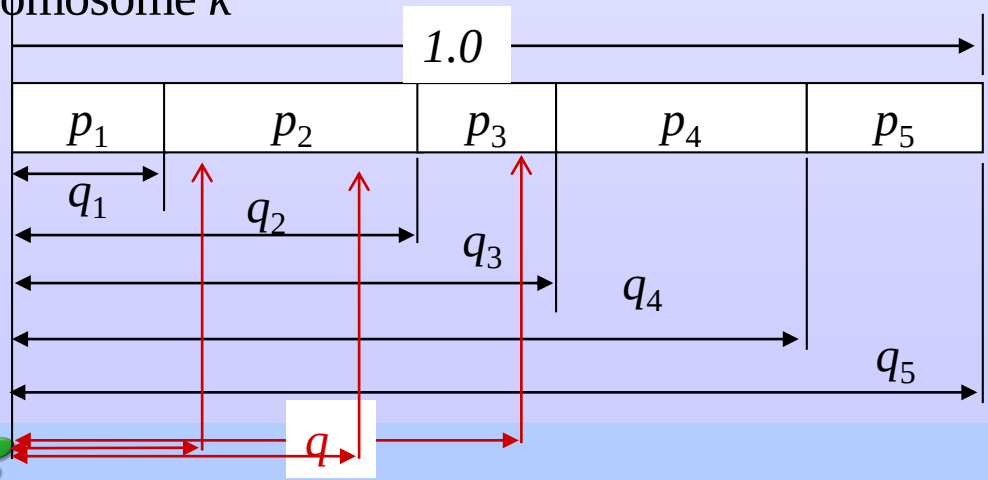
$k \rightarrow 1$

while $(q > q_k)\, k \leftarrow k + 1$

select chromosome k

enfor

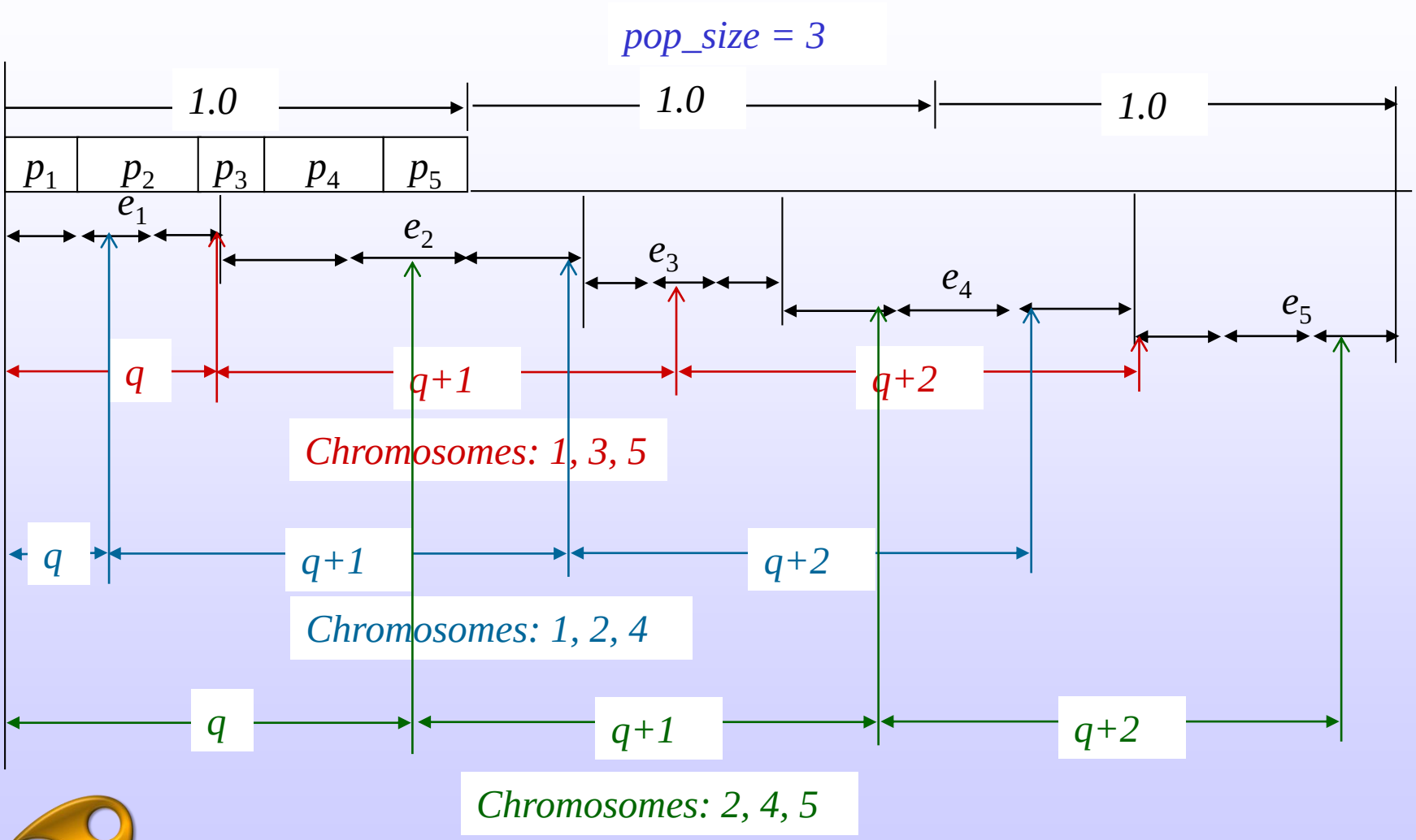




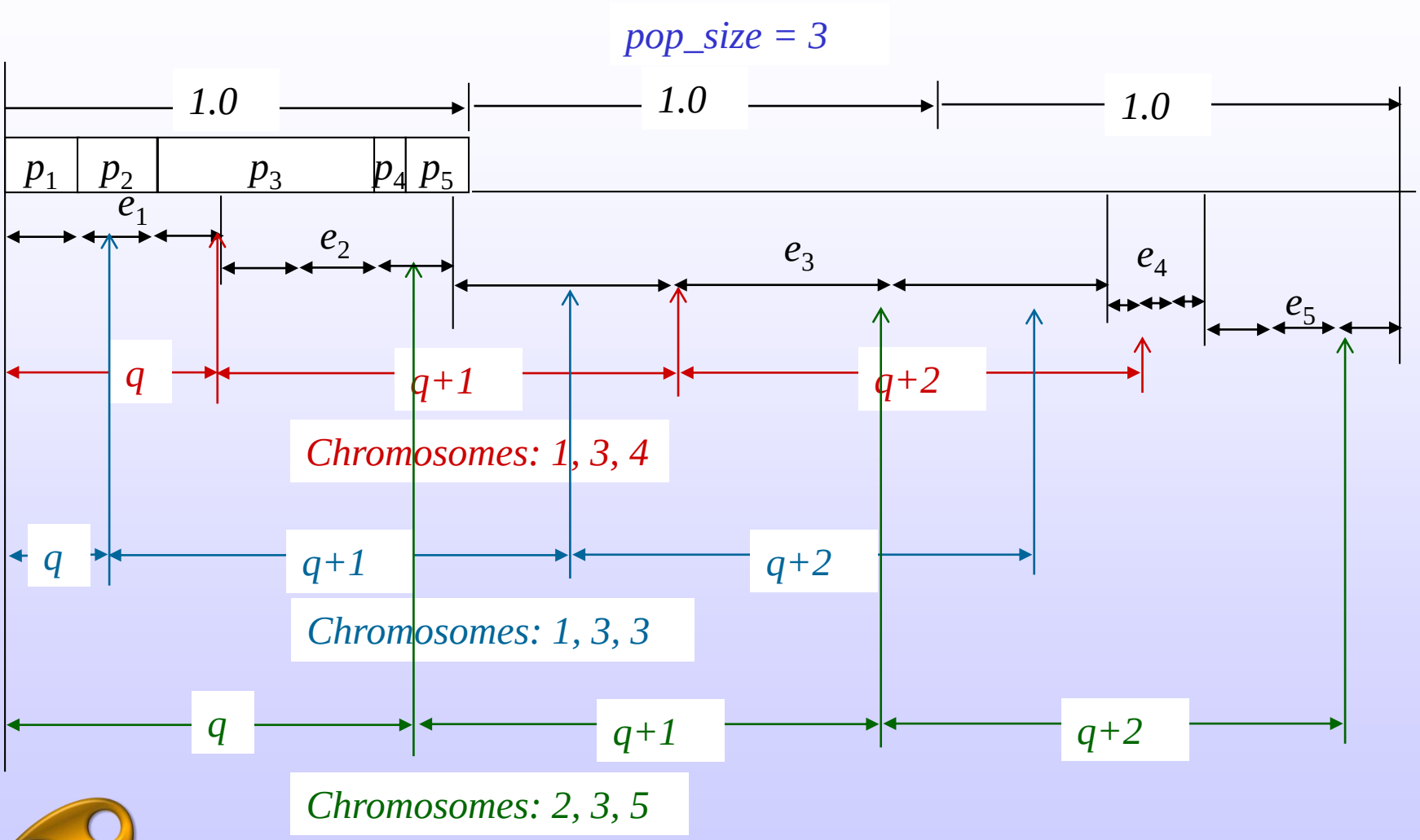
- **Alternative**
  - **Only one roulette wheel turn to get** *pop_size* **chromosomes**

$$e_k = p_k \times pop\_size, \forall k = 1,2,\ldots, pop\_size + off\_size$$
$$sum \leftarrow 0$$
$$q \leftarrow U(0,1)$$
$$count \leftarrow 0$$
$$\textbf{for } k = 1 \textbf{ to } (pos\_size + off\_size)$$
$$\quad sum \leftarrow sum + e_k$$
$$\quad \textbf{while } (sum > q \wedge count < pop\_size)$$
$$\quad\quad \text{select chromosome } k$$
$$\quad\quad count \leftarrow count + 1$$
$$\quad\quad q \leftarrow q + 1$$
$$\quad \textbf{end while}$$
$$\textbf{end for}$$

Demo



Sample (multiple)



Deterministic Sampling

- Best *pop_size* chromosomes are selected from the sampling space
- Truncation Selection
  - Threshold T is defined, T% best chromosomes are selected
  - Each receives 100/T copies
- Block Selection
  - The *pop_size*/s best chromosomes get s copies

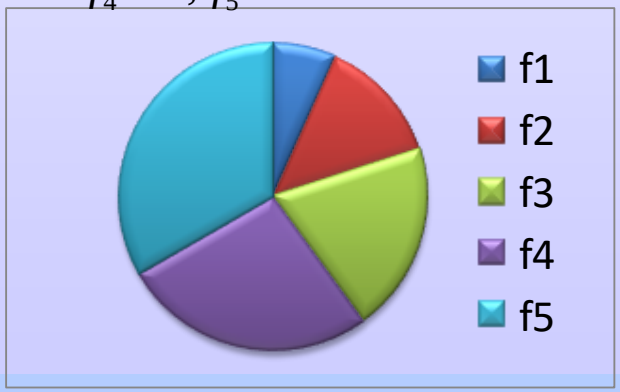
Mixed Sampling

- Contain both random and deterministic features
- Tournament selection
  - Randomly choose *tournament_size* chromosomes and pick the best one
    - Binary tournament: *tournament_size* = 2
- Stochastic tournament selection
  - Roulette wheel method but select two successive chromosomes and pick the best one

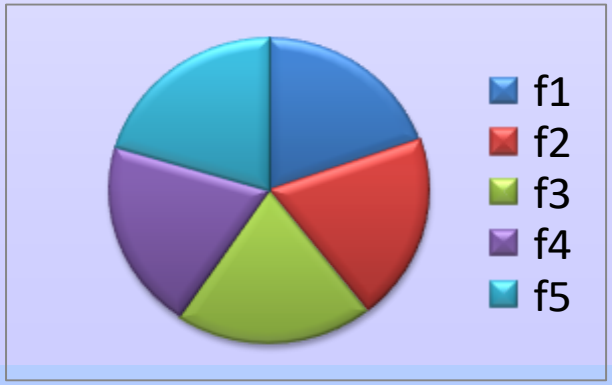
Selection Probability (fitness)

- **Proportional Selection Approach**
  - **Selection probability is proportional to fitness**
  - **Best practice requirement:**
    - Earlier: require low selection pressure; few super chromosomes are recognized
    - Later: require high selection pressure; to exploit the promising solutions

$f_1 = 1,\ f_2 = 2,\ f_3 = 3,$
$f_4 = 4,\ f_5 = 5$



$f_1 = 101,\ f_2 = 102,\ f_3 = 103,$
$f_4 = 104,\ f_5 = 105$



- **Improve methods**
  - **Scaling mechanism**
    - Maps raw objective values to positive real values as theirs fitnesses to determine the survival probability
  - **Ranking mechanism**
    - Ignores the actual objective values
    - Uses a ranking of chromosomes to determine the survival probability

Fitness Scaling

- Maintain a reasonable differential between relative fitness ratings of chromosomes
- Prevent a rapid takeover by some super chromosomes to limit competition early, but to stimulate it later

$$Scaled\ fitness: \tilde{f}_k = g(f_k)$$

- Transformation function $g()$: linear scaling sigma truncation, power law scaling, logarithmic scaling, ...
  - Static scaling
  - Dynamic Scaling
    - According to the scatter situation of fitness values
    - According to the number of generations

Scaling

- **Linear Scaling** $\tilde{f}_k = a \cdot f_k + b$
  - **Parameters a and b are selected to let average chromosome receive one offspring and the best receives a specified number of copies (usually two)**
- **Dynamic linear scaling**

$$\tilde{f}_k = a \cdot f_k + b_t$$

$$b_t = -f_{\min}$$

Sigma Truncation

- Improve linear scaling
- Deal with the negative values
- To incorporate the problem dependent information into the mapping

$$\tilde{f}_k = f_k - \left(f_{avg} - c \cdot \sigma \right)$$

- c is a small integer, σ is the population standard deviation, f_{avg} is the average raw fitness

- **Power Law Scaling**

$$\tilde{f}_k = f_k^{\alpha}$$

  - **The value α is problem-dependent, *α = 1.005* is suggested**
  - **When *α* increases, the gap between the best and worst chromosomes increased**
- **Another power law scaling**

$$\tilde{f}_k = \left(a \cdot f_k + b\right)^{\alpha}$$

- **Logarithmic scaling**
  - **For mapping the objective function of minimization problem**

$$\tilde{f}_k = b - \log(f_k),\ b > \log(f_k)$$

- **Windowing**
  - **Use a moving baseline technique**
  - **Maintain constant selection pressure**

$\tilde{f}_k = f_k - f_w$, w: the window size $2 \sim 10$,

f_w: the worst value observed in the w most recent generations

- **Normalizing**
  - **One kind of dynamic scaling**
  - **For maximization problems**

$$\tilde{f}_k = \frac{f_k - f_{\min} + \gamma}{f_{\max} - f_{\min} + \gamma}, \; f_{\max} : \textit{the best raw fitness}$$

  - **γ is a small positive real number**
    - To prevent zero division
- **Boltzmann Selection**

$$\tilde{f}_k = e^{f_k / T}$$

  - **Selection pressure is low when T is high**

Ranking

- Sort the population from the best to the worst
- Assign probability according to the ranking but not its raw fitness
- Linear Ranking

$$p_k = q_{\max} - (k-1) \cdot \frac{q_{\max} - q_{\min}}{pop\_size - 1};$$

$q_{\max}, q_{\min}$: the probability for the best and worst chromosomes

- Exponential Ranking

$$(1)\ p_k = q - (1-q)^{k-1}$$

Large q, stronger selective pressure

$$(2)\ p_k = q^{k-1}$$

q is typically 0.99; the best 1, the worest q^{pop_size-1}

Objective Values to Fitness

o_k : objective value of chromosome k

f_k : fitness of chromosome k

b : minimum fitness

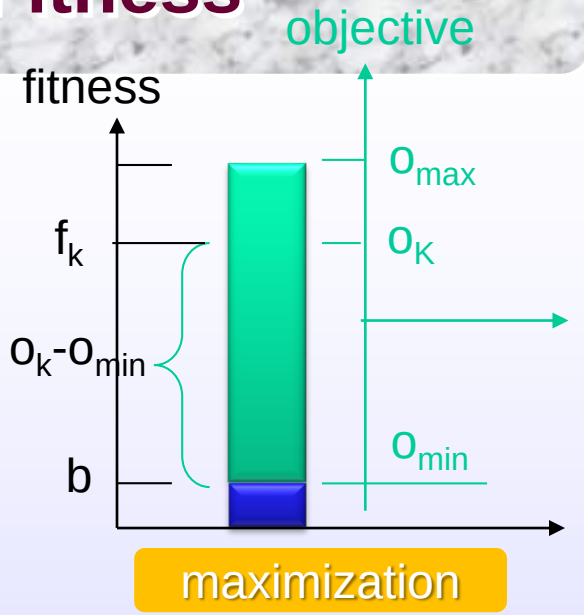
α : least fitness fraction, $0 < \alpha < 0.5$

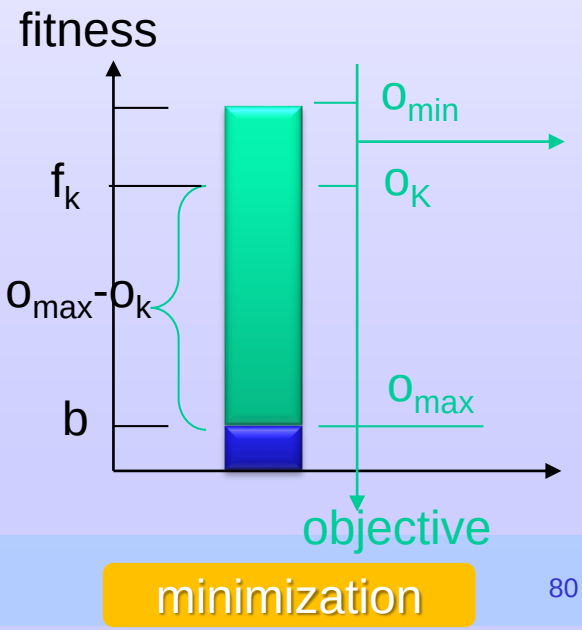
$$o_{\max} = \max_{\forall k}\{o_k\}$$

$$o_{\min} = \min_{\forall k}\{o_k\}$$

$$b = \max\{\alpha(o_{\max} - o_{\min}), 10^{-5}\}$$

$$f_k = b + \begin{cases} o_k - o_{\min}, \text{for maximization problem} \\ o_{\max} - o_k, \text{for minimization problem} \end{cases}$$





GA Applications

- Optimization Problems
  - Multi-objective Optimization Problem
  - Stochastic Optimization Problem
  - Nonlinear Goal Programming
  - Interval Programming
  - Combinatorial Optimization Problem

Application Problems

- Reliability Optimization Problem
  - Network Reliability Design
  - Tree-Based Network Reliability and LAN Design
  - Multiobjective Reliability Design
- Word-Matching Problem
- Traveling Salesman Problem
- Quadratic Assignment Problem
- Film-Copy Deliverer Problem
- Location-Allocation Problem
- Obstacle Location-Allocation Problem

- Set-Covering Problem
  - Airline Crew Scheduling Problem
- Bin-Packing Problem
- Knapsack Problem
  - Binary Representation Approach
  - Order (permutation) Representation Approach
  - Variable-Length Representation Approach

- Minimum Spanning Tree Problem
  - Quadratic MSTP
  - Degree-Constrained MSTP
  - Bicriteria MSTP
- Fuzzy Optimization Problem
  - Fuzzy Linear Programming
  - Fuzzy Nonlinear Programming
  - Fuzzy Nonlinear Mixed-Integer Goal Programming
  - Fuzzy Multiobjective Integer Programming

- **Scheduling Problem**
  - **Job-Shop Scheduling Problem**
    - IP Model
    - LP Model
    - Graph Model
  - **Flow-shop Sequencing Problem**
  - **Grouped Job Scheduling Problem**
  - **Resource-Constrained Project Scheduling Problem**
  - **Single-Machine Scheduling Problem**
  - **Earliness and Tardiness Machine Scheduling Problem**
  - **Parallel Machine Scheduling Problem**
  - **Multiprocessor Scheduling**

- Transportation Problem
  - Linear TP
  - Bicriterial LP
  - Bicriteria Solid Transportation Problem
  - Fuzzy Multicriteria Solid Transportation Problem
- Advanced Transportation Problem
  - Multiobjective Transportation Problem
  - Fixed-Charge Transportation Problem
  - Capacitated Plant Location Problem
  - Bicriteria Transportation Problem with Fuzzy Coefficients

- Network Design and Routing Problem
  - Shortest path Problem
  - Adaptive Network Routing
  - Centralized Network Design Problem
  - Computer Network Expansion Problem
  - Multistage Process Planning
  - M/G/s Queuing Facility Location Problem
- Manufacturing Cell Design Problem
- Facility Layout Design Problem
  - Single-Row MLP
  - Multiple-Row MLP
  - Fuzzy Facility Layout Problem
- Fuzzy Vehicle Routing and Scheduling Problem
- Production Plan Problem