On the development and evaluation of spatial domain Green Functions for multilayered structures with conductive sheets

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***Abstract*—This work focuses on Green Functions (GFs) of planar multilayered structures that may include arbitrary number of conductive sheets. The spectral domain GFs are derived through an automatized strategy based on the propagator matrix technique, while the spatial domain counterparts are evaluated numerically using a novel, efficient and error controlled numerical method. All the procedures remain general to accommodate for the tensorial nature of the conductivity of the sheets. Numerical examples of GFs for the Mixed Potential Integral Equation method are provided validating the proposed algorithms.**

***Index Terms*—integral equations, MPIE, multilayered structures, Green's Functions, conductive interfaces, tensorial conductivity.**

I. Introduction

Microwave and RF components, based on planar multilayered structures, can be integrated easily within printed circuits providing a cost effective performance. Recent developments in the fields of transformation optics [1], plasmonic antennas [2], strong light-matter interaction [3], solar antennas [4] include the addition of very thin metallizations among the stacked dielectric layers in order to achieve the desired electromagnetic performance. The classical numerical EM algorithms face difficulties in simulating such structures and therefore they have to be readdressed.

Among various computational EM techniques, an Integral Equation formulation linearized through a Method of Moments procedure (IE-MoM) offers competitive performance for modeling planar multilayered structures. Plenty IE-MoM formulations have been proposed in literature, the most popular include the one based on the electric field (EFIE), on the magnetic field (MFIE), or on the combination of both electric and magnetic field equations into a combined field equation (CFIE) and finally the Mixed Potential Integral Equation (MPIE). Specifically MPIE receives special attention, as it calls only for the evaluation of $1/R$ singularities of the scalar and vector potentials [5], and will be specially referenced throughout the current work.

One of the cornerstones of all the IE based methods is the derivation of the relevant Green Functions (GFs). In general, this is a tedious and time consuming task. In the specific case that the multilayered structure is transversally invariant, the GFs can be derived through their closed form spectral domain counterparts, by applying a double inverse Fourier Transform (IFT).

In this paper, a generalized procedure for deriving the spectral domain GFs for the EM fields and the mixed potentials GFs is developed in Section II. An efficient novel numerical algorithm for evaluating the spatial domain counterparts of the GFs developed in Section II up to machine precision is presented in Section III, providing the necessary input for a classical IE-MoM implementation. Finally, results for a technologically interesting structure are shown in Section IV, validating the method presented in this paper.

II. Development of Spectral Domain Green's Functions

The propagator matrix technique [5] is a powerful technique based on the propagation of the electric and magnetic fields through the different dielectric layers and matching to the appropriate boundary conditions through the terminating layers. The terminating boundary condition can be chosen among Sommerfeld radiation condition, PEC, PMC or surface impedance condition. The original development of the propagation matrix technique was targeting structures with multiple dielectric layers. The recent advances in the materials and the introduction of very thin, up to nanoscale or even atom level, conductive layers in the microwave components calls for the enhancement of this technique. Therefore, an appropriate boundary condition that connects the normal components of the electric and magnetic fields when a conductive sheet is present can be derived by enforcing the appropriate boundary conditions for the tangential fields. This results in the follow-

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ing connectivity matrix:

$$\begin{bmatrix} \tilde{G}^{zn(i)}_{EJi+1} \\ \dfrac{\partial \tilde{G}^{zn(i)}_{EJi+1}}{\partial z} \\ \tilde{G}^{zn(i)}_{HJi+1} \\ \dfrac{\partial \tilde{G}^{zn(i)}_{HJi+1}}{\partial z} \end{bmatrix} = \begin{bmatrix} \dfrac{\epsilon_i}{\epsilon_{i+1}} & \dfrac{-j\sigma^{(i)}_{uu}}{\omega\epsilon_{i+1}} & \dfrac{\mu_0\sigma^{(i)}_{uv}}{\epsilon_{i+1}} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \sigma^{(i)}_{vu} & j\omega\mu_0\sigma^{(i)}_{vv} & 1 \end{bmatrix} \begin{bmatrix} \tilde{G}^{zn(i)}_{EJi} \\ \dfrac{\partial \tilde{G}^{zn(i)}_{EJi}}{\partial z} \\ \tilde{G}^{zn(i)}_{HJi} \\ \dfrac{\partial \tilde{G}^{zn(i)}_{HJi}}{\partial z} \end{bmatrix} \tag{1}$$

which connects the normal components of the fields in the layer i below the conductive interface (i) with the ones above it in the layer $i+1$. Magnetic currents have been dropped in the current development. The conductivity of the sheet can be tensorial, i.e. $\bar{\bar{\sigma}} = \sigma_{uu}\hat{u}\hat{u} + \sigma_{uv}\hat{u}\hat{v} + \sigma_{vu}\hat{v}\hat{u} + \sigma_{vv}\hat{v}\hat{v}$, $\hat{u}$ and $\hat{v}$ being the rotated spectral domain coordinates [6], which reduces to the scalar case if $\sigma_{uu} = \sigma_{vv} = \sigma$ and $\sigma_{uv} = \sigma_{vu} = 0$. From (1) it becomes obvious that in the case of an interface with scalar conductivity, the TM and TE components of the EM fields remain uncoupled, which is not any more the case if the conductivity is tensorial.

The connection of the GFs for the EM fields with the GFs for the vector and scalar potential in the spectral domain can be resolved in a unique way by enforcing the Lorenz Gauge and the Sommerfeld choice for the potentials [6], [7]. Consequently, the spectral domain GFs for a MPIE become available and in order to serve as input for a IE-MoM code, their spatial domain counterparts have to be efficiently calculated. This task is discussed in Section III.

III. Evaluation of Spatial Domain Green's Functions

The spatial domain GFs are connected to the spectral domain ones through the generalized Sommerfeld integrals [8]:

$$G(\rho, z, z') = S_n[\tilde{G}] \triangleq \int_0^\infty J_n(k_\rho\rho)k_\rho^{n+1}\tilde{G}(k_\rho, z, z')dk_\rho, \tag{2}$$

where J_n is the n^{th} order Bessel function of the first kind, ρ the horizontal distance between source and observation point and z and z' their vertical positions, respectively. $\tilde{G}$ can be chosen among the components of the spectral domain GFs either for the fields or for the potentials, which are available in closed form based on the analysis of the previous section.

The numerical evaluation of (2) is not trivial due to the semi-infinite integration domain, the high oscillation of the Bessel function and finally due to the poles and the branch cuts that the spectral domain GFs $\tilde{G}$ introduce. Traditionally, the integration path is deformed in the complex k_ρ plane in order to avoid the branch point and the singularities [9]. If the position of the singular points vary from very close to the branch point to very far away from it, a typical situation if graphene layers at THz are considered [10], then the deformed path integration becomes inefficient.

In order to improve the efficiency and the accuracy of the numerical evaluation of (2), an integration on the real axis is proposed. As a first step, the poles of $\tilde{G}$ have to be identified, extracted and added back in the spatial domain analytically. The identification of the location of the poles is not always easy, especially if plasmon modes are propagating. However, this task is performed only once in the beginning of the EM analysis.

Afterwards, the integration domain is split between the first part that includes the branch point and the tail that inherits the semi-infinite integration domain. The tail of the Sommerfeld integral is numerically evaluated through the Weighted Averages (WA) technique [11], while the remaining singularity-free first part can be integrated efficiently only if the branch point is properly treated. As the higher order derivatives of $\tilde{G}$ are not bounded [12] at the branch point, standard quadrature rules fail. This difficulty can be overcome by employing advanced quadrature rules, that are able to handle such singularities. In the current work, the Double Exponential rule (DE) [13] is used, enhanced with an error estimator [14] that guarantees the accuracy of the numerical results. Finally, if all the methods presented in Sections II and III are properly combined, the spatial domain GFs for the scalar and vector potential of multilayered structures can be evaluated, as shown is Section IV.

IV. Numerical Example

In order to validate the method, an example of high technological impact is selected. This is a graphene layer without a magnetic bias separating two semi-infinite media as shown in Fig. 1. This example is important as it can support plasmon modes in the THz region and its dyadic GFs for the fields have been developed [10], [15] using different approaches than the one proposed in this paper.

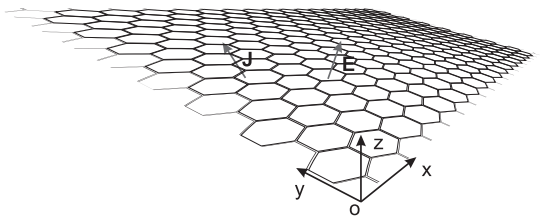

Fig. 1. A single graphene layer embedded in free space at the position $z = 0$.

Applying the strategy described in Section II, the scalar and vector potential spectral domain GFs can be derived. For demonstration purposes, the zx and zz components of the dyadic GF for the vector potential A are hereby presented. If the source and observation points are assumed to be on the opposite sides of the graphene layer, $\tilde{G}^{zx}_A$ and $\tilde{G}^{zz}_A$ are given by (3) and (4), respectively:

$$\tilde{G}^{zx}_A = -je^{jk_{z2}z}e^{-jk_{z1}z'}\tilde{J}_x k_x\mu_0 \frac{1}{k_\rho^2}\frac{2k_{z2}D^{TM} + \epsilon_2 k_{z1}D^{TE}}{D^{TM}D^{TE}} \tag{3}$$

$$\tilde{G}^{zz}_A = -je^{jk_{z2}z}e^{-jk_{z1}z'}\tilde{J}_z\mu_0\frac{\epsilon_2}{D^{TM}}, \tag{4}$$

where the TM and TE poles are given by:

$$D^{TM}(\sigma) = k_{z1}k_{z2}\frac{\sigma}{\omega} + \epsilon_2 k_{z1} + \epsilon_1 k_{z2} \tag{5a}$$

$$D^{TE}(\sigma) = k_{z1} + k_{z2} + \mu_0\sigma\omega \tag{5b}$$

The spatial domain counterparts of $\tilde{G}_A^{zx}$ and $\tilde{G}_A^{zz}$ have been calculated using the techniques described in Section III and are plotted for the $z' = z = 0$ vertical positions of the source and the observation points. This is the worst situation in terms of numerical evaluations as (3) and (4) do not decay exponentially for large values of k_ρ. The results, where the two semi-infinite media are considered of the same dielectric e_0, are shown in Fig. 2 and Fig. 3, respectively for different excitation frequencies and horizontal distances ρ:

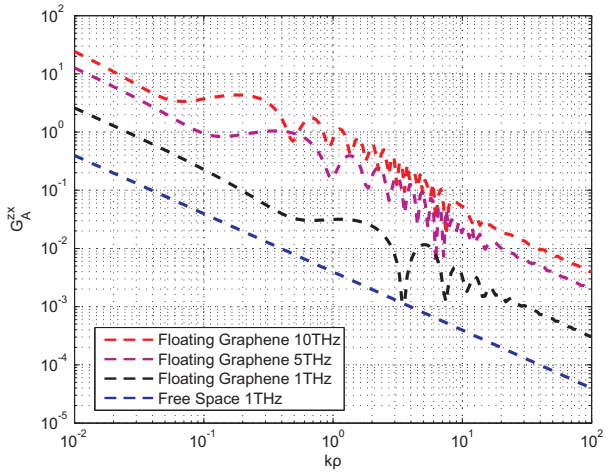

Fig. 2. $|G_A^{zx}|$ of floating graphene sheet for different horizontal distances ρ and frequencies. The source and observation are located on the graphene sheet but on different sides.

The conductivity of graphene is estimated through the Kubo formula [16] at 300K with a chemical potential of 0.2eV and relaxation time of 1ps. The behavior of a horizontal and a vertical dipole in the cases of Fig. 2 and Fig. 3 radiating without the presence of the conductive sheet is also shown. The effect of the oscillations in the presence of graphene are due to the propagation of the plasmon mode, which after some wavelengths vanishes due to the high losses that exhibits and a behavior close to the free space one is retrieved.

V. Conclusion

A general purpose technique for deriving the spectral and spatial domain GFs for the vector and scalar potentials has been developed for structures with stacked dielectrics separated by conductive sheets with either scalar or tensorial description of their conductivity. The hereby calculated spatial domain GFs serve as an input for a MPIE numerical code towards the efficient EM modeling of such structures.

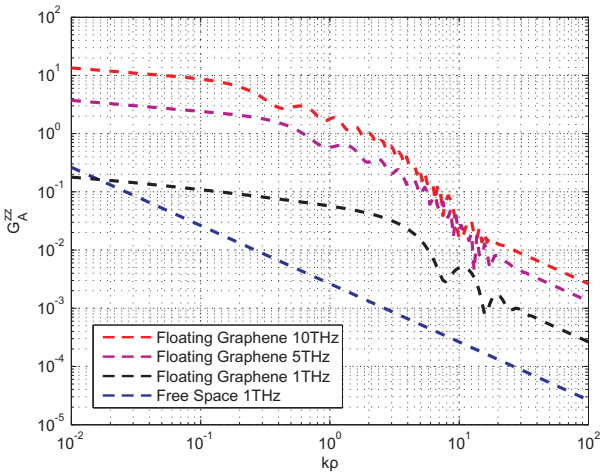

Fig. 3. $|G_A^{zz}|$ of floating graphene sheet for different horizontal distances ρ and frequencies. The source and observation are located on the graphene sheet but on different sides.

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