Surface-Integral Equations for Electromagnetic Scattering from Impenetrable and Penetrable Sheets

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Abstract

We discuss integral equations for electromagnetic-scattering problems involving open (in general, non-planar), thin, impenetrable and penetrable sheets, characterized by two different reflectivities for their two faces. A unified formulation is presented for both such systems, in terms of coupled surface-integral equations, for two tangential surface-polarization currents. Numerical results for some relevant representative scatterers are given.

1. Introduction

There is a significant interest in scattering from thin layers of materials with combined dielectric and magnetic properties. The reflection and transmission properties of such systems are of relevance in the context of electromagnetic-signature reduction, frequency-selective surfaces, radome and antenna design, for example.

The calculation of fields, scattered or radiated by systems of thin layers, becomes significantly less computationally intense if the layers can be modeled as infinitesimally thin structures, described in terms of a set of boundary conditions. In these cases, the computational problem reduces to solving a system of *surface*- integral equations.

Several types of boundary conditions have been developed for thin, layered sheets. For impenetrable sheets (typically, thin layers placed over perfectly conducting surfaces), impedance-boundary conditions [1], and the corresponding integral equations for *closed* surfaces, are well established [2, 3]. Other types of boundary conditions and the corresponding integral equations apply to penetrable sheets. In this case, there is no distinction between closed and open surfaces, since both are described by transmission-boundary conditions. Thin, penetrable sheets of dielectric material have been analyzed in terms of the "impedance- (or resistive-) sheet approximation" [4]. The electromagnetic dual of the resistive sheet is a magnetically conductive sheet [5], supporting only a magnetic current. A combination of resistive and magnetic sheets, simulating a thin layer with combined dielectric and magnetic properties, has been considered in [6], in the context of surface-curvature-corrected boundary conditions, and was subsequently re-introduced as a "combined electrically resistive/magnetically conductive sheet" in [7].

The formulations listed above are applicable to "symmetric" sheets, i.e., sheets characterized by identical reflection coefficients on their two sides. Some other developments, such as the approach of [8], apply also to flat "asymmetric" sheets, with their two sides characterized by two different reflection coefficients. In fact, such systems have been treated mostly in the context of higher-order boundary conditions, introduced by Karp and Karal [9], and extensively analyzed in a number of recently published articles (e.g., see [10, 11, 12]).

The majority of research done in the area of thin sheets has been devoted to the problem of boundary conditions, rather than to the integral equations following from them. In principle, such integral equations can be obtained from the conventional, closed-surface integral-field representations. However, their derivation, especially for sheets described by "asymmetric" boundary conditions, may require evaluation of nontrivial limits. Such integral equations have been formulated only for some special geometries (e.g., flat plates [8]).

In this paper, we concentrate on a case of a layered sheet, but with *different* reflection properties characterizing its two faces, and described by leading, zero-order boundary conditions. Such boundary conditions constitute the most general algebraic relations involving only tangential components of the fields, but not their derivatives. However, our main purpose is not to study the boundary conditions, but rather to provide a simple and practical set of corresponding surface-*integral equations* [13]. Our equations are based on an intuitive, but rigorous, formulation of the zero-order sheet-boundary conditions, in the form of a generalized "Ohm's law," relating fields to effective electric and magnetic tangential-surface currents, $\mathbf{J}_T$ and $\mathbf{M}_T$. These currents are then used in a surface-integral representation for the fields, and thus become the unknowns in the integral equations. The resulting equations for $\mathbf{J}_T$ and $\mathbf{M}_T$ are not more complicated than the familiar electric-field and magnetic-field equations: they involve the same integral operators, but coupled in a nontrivial way. In spite of their simplicity and their intuitive physical interpretation, they are applicable to a broad range of electromagnetic-scattering problems, including the following:

(a) An *impenetrable sheet*, characterized by *two different surface impedances* (and thus two different reflection coefficients) on its two sides. An example of a realization of such a sheet is a perfectly conducting surface, coated on both sides with thin layers of different material properties, shown in Figure 1a.

(b) A *penetrable sheet*, with its two faces characterized by *two different reflection coefficients*. An example of such a sheet is a stack

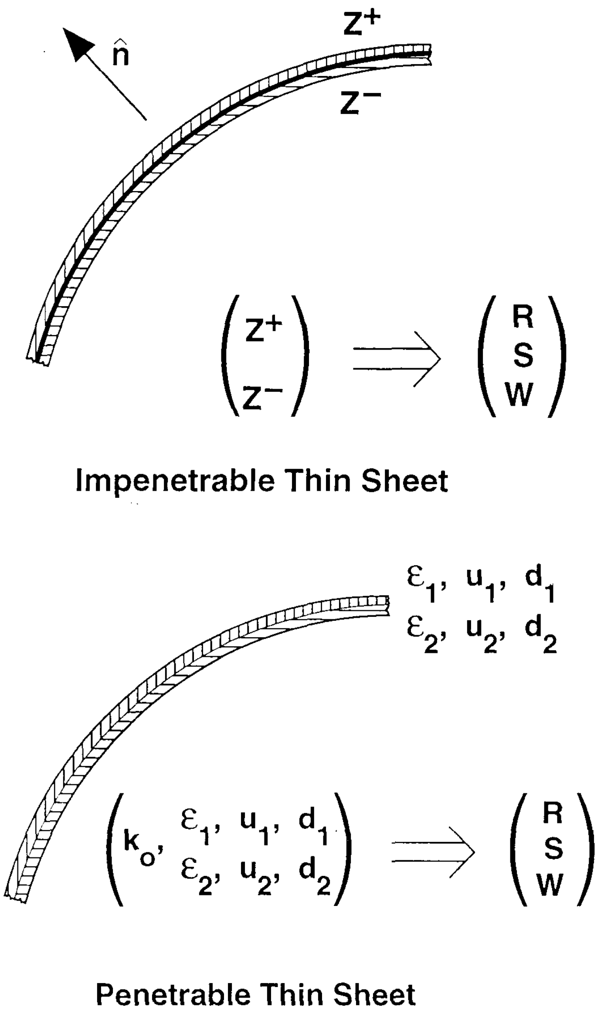

Figure 1. Typical systems described by the thin-sheet "Ohm's laws" boundary conditions of Equation (1): (a) a perfectly conducting sheet sandwiched between two dielectric/magnetic layers, characterized by impedances Z^+ and Z^-; and (b) a penetrable system of two thin dielectric/magnetic layers.

of thin layers, with different dielectric and magnetic properties (shown in Figure 1b, for a system of two layers).

In the course of the paper, we also discuss how the boundary conditions and the corresponding integral equations reduce to the special cases of perfectly conducting and resistive sheets, impenetrable (impedance) sheets with two equal face impedances, and penetrable ("electrically resistive/magnetically conductive") sheets *with identical reflection coefficients on both sides*.

2. General form of the boundary conditions

Our formulation of surface-integral equations, presented in Section 6, is based on a set of non-derivative boundary conditions, for a general penetrable or impenetrable stack of layers, simulated by an effectively infinitely thin sheet. Such boundary conditions can be thought of as the lowest-order term in the general framework of the higher-order boundary conditions [9].

The boundary conditions (discussed in more detail in [13] and in Sections 3 and 4) we shall be using, in the context of the surface-integral representation, can be written in the following compact "Ohm's-law" form[1]:

$$\langle \mathbf{E}_T(\mathbf{r}) \rangle = R_T(\mathbf{r})\mathbf{J}_T(\mathbf{r}) + W_T(\mathbf{r})\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{M}_T(\mathbf{r}), \tag{1a}$$

$$\langle \mathbf{H}_T(\mathbf{r}) \rangle = S_T(\mathbf{r})\mathbf{M}_T(\mathbf{r}) + W_T(\mathbf{r})\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{J}_T(\mathbf{r}). \tag{1b}$$

This relates the average tangential fields,

$$\langle \mathbf{E}_T(\mathbf{r}) \rangle = \frac{1}{2}\left[\mathbf{E}_T^+(\mathbf{r}) + \mathbf{E}_T^-(\mathbf{r})\right], \tag{2a}$$

$$\langle \mathbf{H}_T(\mathbf{r}) \rangle = \frac{1}{2}\left[\mathbf{H}_T^+(\mathbf{r}) + \mathbf{H}_T^-(\mathbf{r})\right], \tag{2b}$$

to the currents, defined as the fields' jumps,

$$\mathbf{J}_T(\mathbf{r}) = \hat{\mathbf{n}}(\mathbf{r}) \times \left[\mathbf{H}_T^+(\mathbf{r}) - \mathbf{H}_T^-(\mathbf{r})\right], \tag{3a}$$

$$\mathbf{M}_T(\mathbf{r}) = -\hat{\mathbf{n}}(\mathbf{r}) \times \left[\mathbf{E}_T^+(\mathbf{r}) - \mathbf{E}_T^-(\mathbf{r})\right]. \tag{3b}$$

Here, the superscripts "+" and "-" refer to the two faces of the surface, and the unit-normal vector, $\hat{\mathbf{n}}(\mathbf{r})$, is oriented from the "-" to the "+" face.

The three complex parameters, R_T, S_T, and W_T, have the physical interpretation of the electric and magnetic resistivities[2], and a "cross-resistivity," relating the electric current to the magnetic field, and vice versa. We note that the coplanar electric and magnetic currents, $\mathbf{J}_T$ and $\mathbf{M}_T$, are only effective currents, representing nontrivial spatial-current distributions within the sheet. In this sense, for example, a surface-magnetic current is equivalent to a double electric-current sheet.

Interestingly, the "Ohm's law" form of the boundary conditions, involving *three parameters* R_T, S_T, and W_T, is consistent with the scattering properties of a thin penetrable layer, which are also determined by *three parameters*: two reflection coefficients ($\mathcal{R}^{\pm}$ for the two faces), and one transmission coefficient ($\mathcal{T}$).[3]

The scattering properties of a thin impenetrable layer are determined by two independent parameters: R_T and S_T, or $\mathcal{R}^+$ and $\mathcal{R}^-$, or the two impedances, Z^+ and Z^-. In this case, impenetrability ($\mathcal{T} = 0$) implies a relation among R_T, S_T, and W_T.

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[1]The form of the boundary conditions, Equation 1, may be also arrived at by writing the most-general linear relation between the tangential-field components on two sides of a sheet, consistent with duality and reciprocity.

[2]Because of the units in which it is expressed, $(\mathrm{m}\Omega)^{-1}$, the parameter S_T is conventionally named the "magnetic conductivity." However, in our context it is more consistent to refer to all the parameters, R_T, S_T, and W_T, as resistivities.

[3]It follows from reciprocity that the transmission coefficients for the two faces are identical.

3. Boundary conditions for impenetrable thin-layer systems

For an open impenetrable surface, the conventional form of the impedance-boundary conditions is [1]

$$\mathbf{E}_T^{\pm}(\mathbf{r}) \mp Z^{\pm}(\mathbf{r})\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{H}_T^{\pm}(\mathbf{r}) = 0, \tag{4}$$

where, in general, the surface impedances $Z^+(\mathbf{r})$ and $Z^-(\mathbf{r})$ may be different.

The values $Z^{\pm}(\mathbf{r})$ characterize reflection properties of the "$\pm$" sheet faces. They can be determined experimentally through a suitable set of measurements of the reflection coefficients,

$$\mathcal{R}^{\pm}(\theta) = \frac{1 - Z^{\pm}/Z_0 \cos\theta}{1 + Z^{\pm}/Z_0 \cos\theta}, \tag{5a}$$

$$\mathcal{T}(\theta) = 0, \tag{5b}$$

where $Z_0 = \sqrt{\mu_0/\varepsilon_0}$, and θ is the angle between the direction of the incident-wave vector and the normal to the appropriate face (i.e., $\hat{\mathbf{n}}$ for the "+" face, and $-\hat{\mathbf{n}}$ for the "-" face).[4]

An example of a physical system, described by the boundary conditions of Equation (4), is a thin, perfectly conducting sheet, sandwiched between two dielectric/magnetic layers, characterized by respective material properties $\varepsilon^{\pm}$, $\mu^{\pm}$, and thicknesses $d^{\pm}$. The impedances of the "$\pm$" sheet faces are then

$$Z^{\pm} = -i\sqrt{\frac{\mu^{\pm}}{\varepsilon^{\pm}}}\tan\left(\sqrt{\varepsilon^{\pm}\mu^{\pm}/(\varepsilon_0\mu_0)}\,k_0 d^{\pm}\right),$$

where k_0 is the free-space wave number.

In the case of $Z^+ + Z^- \neq 0$, we may rewrite the impedance-boundary conditions, Equation (4), in the equivalent form of "Ohm's law" Equation (1), in which the three "resistivities," R_T, S_T, and W_T, are related to the sheet impedances (and admittances $Y^{\pm} = 1/Z^{\pm}$) through

$$R_T(\mathbf{r}) = \frac{1}{Y^+(\mathbf{r}) + Y^-(\mathbf{r})}, \quad S_T(\mathbf{r}) = \frac{1}{Z^+(\mathbf{r}) + Z^-(\mathbf{r})},$$

$$W_T(\mathbf{r}) = \frac{1}{2}\frac{Z^+(\mathbf{r}) - Z^-(\mathbf{r})}{Z^+(\mathbf{r}) + Z^-(\mathbf{r})}, \text{ for } Z^+(\mathbf{r}) + Z^-(\mathbf{r}) \neq 0. \tag{6}$$

(In order to bring the boundary conditions, Equation (4), to the form of Equation (1), with the resistivities defined by Equation (6), we simply add and subtract Equations (4) to obtain the relations between the tangential components of the sums and differences of the fields. The algebra requires assuming that $Z^+ + Z^- \neq 0$.)

It follows from the expressions in Equation (6) that the resistivities are not independent: they satisfy the constraint

$$4\left[R_T(\mathbf{r})S_T(\mathbf{r}) + W_T^2(\mathbf{r})\right] = 1. \tag{7}$$

[4]All expressions for reflection and transmission coefficients in this paper refer to the TE polarization (electric field in the sheet plane). The results for the TM polarization can be obtained by duality.

The above condition can be given the following interpretation. When the constraint of Equation (7) is added to the boundary conditions in Equation (1), the sheet they describe becomes impenetrable. Mathematically, it is only in this case that the boundary conditions in Equation (1) (which have the form of *transmission* boundary conditions) decouple into two sets of independent boundary conditions, Equation (4), for the two faces. This property will become evident in the next section, in the context of our discussion of penetrable sheets.

We also note that the "cross-resistivity" W_T is responsible for different reflection properties of the two faces: it vanishes for a surface with two identical face impedances, Z^+ and Z^-.

4. Boundary conditions for penetrable thin-layer systems

A thin, penetrable sheet can be characterized, in general, by two reflection coefficients, $\mathcal{R}^+(\theta)$ and $\mathcal{R}^-(\theta)$, corresponding to the two sides of the sheet, and by one transmission coefficient, $\mathcal{T}(\theta)$. These three coefficients can be determined explicitly from Maxwell's equations. A general multilayer stack of planar-dielectric and magnetic sheets is analyzed in Appendix A. It follows from this analysis that, in analogy to the case of an impenetrable sheet, the boundary conditions can also be written in the "Ohm's law" form of Equation (1), where the effective resistivities, R_T, S_T, and W_T, are related to the reflection and transmission coefficients by

$$\mathcal{R}^{\pm}(\theta) = -\frac{2R_T\cos\theta - 2S_T/\cos\theta \pm 4W_T}{4\left(R_T S_T + W_T^2\right) + 1 + 2R_T\cos\theta + 2S_T/\cos\theta}, \tag{8a}$$

$$\mathcal{T}(\theta) = \frac{4\left(R_T S_T + W_T^2\right) - 1}{4\left(R_T S_T + W_T^2\right) + 1 + 2R_T\cos\theta + 2S_T/\cos\theta}. \tag{8b}$$

Alternatively, the resistivities, R_T, S_T, and W_T, can be regarded as phenomenological parameters, which can be determined experimentally through a suitable set of measurements of the reflection and transmission coefficients. More-explicit expressions for the resistivities, in terms of the reflection and transmission coefficients at *normal incidence*, are

$$\left.\begin{matrix} R_T \\ S_T \end{matrix}\right\} = -\frac{1}{2}\frac{\left[1 \mp \mathcal{R}^+(0)\right]\left[1 \mp \mathcal{R}^-(0)\right] - \mathcal{T}^2(0)}{\mathcal{R}^+(0)\mathcal{R}^-(0) - \left[1 - \mathcal{T}(0)\right]^2}, \tag{9a}$$

$$W_T = \frac{1}{2}\frac{\mathcal{R}^+(0) - \mathcal{R}^-(0)}{\mathcal{R}^+(0)\mathcal{R}^-(0) - \left[1 - \mathcal{T}(0)\right]^2}. \tag{9b}$$

It follows, from Equation (8b), that the transmission coefficient vanishes when the resistivities satisfy the condition of Equation (7). In this case, the penetrable sheet becomes equivalent to an impenetrable impedance sheet.

A simple example of a physical system described by the boundary conditions of Equation (1), with the resistivities of Equations (8), is a thin dielectric/magnetic layer, characterized by material parameters ε and μ, and thickness d. In this case, the respective values of the parameters are (see Appendix A)

$$R_T = \frac{i}{2}\sqrt{\frac{\mu}{\varepsilon}}\cot\left(\frac{1}{2}\sqrt{\varepsilon\mu/(\varepsilon_0\mu_0)}\,k_0 d\right),$$

$$S_T = \frac{i}{2}\sqrt{\frac{\varepsilon}{\mu}}\cot\left(\frac{1}{2}\sqrt{\varepsilon\mu/(\varepsilon_0\mu_0)}k_0 d\right),$$

$$W_T = 0. \tag{10}$$

In this special case of a single layer, $\mathcal{R}^+(0) = \mathcal{R}^-(0)$, and the cross resistivity $W_T = 0$. The boundary conditions, Equation (1), then reduce to the simple form of electrically-resistive/magnetically-conductive boundary conditions, discussed in [7]:

$$\langle \mathbf{E}_T(\mathbf{r})\rangle = R_T(\mathbf{r})\mathbf{J}_T(\mathbf{r}), \tag{11a}$$

$$\langle \mathbf{H}_T(\mathbf{r})\rangle = S_T(\mathbf{r})\mathbf{M}_T(\mathbf{r}). \tag{11b}$$

In general, however, a penetrable sheet, consisting of more than one layer, cannot be described by the boundary conditions of Equation (11), because the "cross-resistivity" parameter, W_T, does not vanish.

In Appendix A, we derive the relations between the tangential components of the fields $\mathbf{E}_T^+(\mathbf{r})$, $\mathbf{E}_T^-(\mathbf{r})$, $\mathbf{H}_T^+(\mathbf{r})$, and $\mathbf{H}_T^-(\mathbf{r})$, on two sides of a stack of layers. From this simple analysis, the reflection and transmission coefficients (equivalently, resistivities R_T, S_T, and W_T) can be constructed in terms of material parameters, thicknesses of the individual layers, and the frequency. We also provide explicit formulas for these coefficients for a single layer, and for a system of two layers.

5. Validity of the thin-layer boundary conditions for impenetrable and penetrable sheets

The requirements for the validity of thin-layer boundary conditions, Equation (1), for both penetrable and impenetrable layers, are similar to those for the impedance-boundary condition discussed in a number of sources, e.g., in [1]. A more-detailed analysis of the applicability of the thin-layer boundary conditions to a multilayer-penetrable stack is given in Appendix A.

Physically, the zero-order thin-layer boundary conditions for impenetrable (impedance) and penetrable sheets hold, provided the curvature of the surface is small compared to the wavelength (so that it can be locally described as an infinite plane), and provided the layer materials are such that the incident wave refracts in the medium approximately in the direction normal to the local-tangent plane. In this physical situation, the tangential components of the wave vector have a negligible effect on both the wave's magnitude and phase.

A well-known and important practical value of the thin-layer boundary conditions is that often they provide a quite-accurate description of physical systems, for which either it is very difficult to establish the rigorous range of applicability, or for which, from a purely mathematical point of view, they may appear not to be valid.

6. Construction of integral equations

We derive the surface-integral equations for open, thin-layer electromagnetic-scattering problems in three steps.

• We adopt an integral representation (analogous to the Stratton-Chu [14] representation) of the fields outside an *open* surface, Σ, in terms of two tangential-surface currents, $\mathbf{J}_T(\mathbf{r})$ and $\mathbf{M}_T(\mathbf{r})$:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{(in)}(\mathbf{r}) + ik_0\int_\Sigma dS(\mathbf{r}')\left[Z_0\tilde{G}(\mathbf{r}-\mathbf{r}')\mathbf{J}_T(\mathbf{r}') - \Gamma(\mathbf{r}-\mathbf{r}')\times\mathbf{M}_T(\mathbf{r}')\right]$$

$$\mathbf{H}(\mathbf{r}) = \mathbf{H}^{(in)}(\mathbf{r}) + ik_0\int_\Sigma dS(\mathbf{r}')\left[Y_0\tilde{G}(\mathbf{r}-\mathbf{r}')\mathbf{M}_T(\mathbf{r}') + \Gamma(\mathbf{r}-\mathbf{r}')\times\mathbf{J}_T(\mathbf{r}')\right] \tag{12}$$

where $\mathbf{E}^{(in)}$ and $\mathbf{H}^{(in)}$ are the incident fields, $Y_0 = 1/Z_0$, $\tilde{G}(\mathbf{r}) = \left(1 + k_0^{-2}\nabla\otimes\nabla\right)G(\mathbf{r})$ is the free-space dyadic Green's function. $\Gamma(\mathbf{r}) = -ik_0^{-1}\nabla G(\mathbf{r})$ is the vector Green's function, expressed in terms of the *scalar* Green's function: $G(r) = e^{ik_0 r}/4\pi r$ in three dimensions, and $G(r) = \frac{i}{4}H_0^{(1)}(k_0 r)$ in two dimensions.

• We find the values of the field components just above $\left(\mathbf{E}^+, \mathbf{H}^+\right)$ and just below $\left(\mathbf{E}^-, \mathbf{H}^-\right)$ the surface, resulting from the assumed integral representation of Equation (12),

$$\mathbf{E}_T^{\pm}(\mathbf{r}) = \mathbf{E}_T^{(in)}(\mathbf{r}) \pm \frac{1}{2}\hat{\mathbf{n}}(\mathbf{r})\times\mathbf{M}_T(\mathbf{r}) + ik_0\int_\Sigma dS(\mathbf{r}')\left[Z_0\tilde{G}(\mathbf{r}-\mathbf{r}')\mathbf{J}_T(\mathbf{r}') - \Gamma_{PV}(\mathbf{r}-\mathbf{r}')\times\mathbf{M}_T(\mathbf{r}')\right] \tag{13a}$$

$$\mathbf{H}_T^{\pm}(\mathbf{r}) = \mathbf{H}_T^{(in)}(\mathbf{r}) \mp \frac{1}{2}\hat{\mathbf{n}}(\mathbf{r})\times\mathbf{J}_T(\mathbf{r}) + ik_0\int_\Sigma dS(\mathbf{r}')\left[Y_0\tilde{G}(\mathbf{r}-\mathbf{r}')\mathbf{M}_T(\mathbf{r}') + \Gamma_{PV}(\mathbf{r}-\mathbf{r}')\times\mathbf{J}_T(\mathbf{r}')\right] \tag{13b}$$

where Γ_{PV} is the principal-value part of the vector Green's function. According to Equations (13), the discontinuities of the tangential components of $\mathbf{E}$ and $\mathbf{H}$ across the surface Σ are consistent with Equations (3).

• We substitute the expressions in Equations (13), for the field components on the upper and lower faces of the sheet, into Equation (1), representing the boundary conditions. As a result, we obtain the desired integral equations for two cases:

6.1 Surface-integral equations for impenetrable sheets with $Z^+ + Z^- \neq 0$ and for penetrable sheets

$$\mathbf{E}_T^{(in)}(\mathbf{r}) - R_T(\mathbf{r})\mathbf{J}_T(\mathbf{r}) - W_T(\mathbf{r})\hat{\mathbf{n}}(\mathbf{r})\times\mathbf{M}_T(\mathbf{r}) + ik_0\int_\Sigma dS(\mathbf{r}')\left[Z_0\tilde{G}(\mathbf{r}-\mathbf{r}')\mathbf{J}_T(\mathbf{r}') - \Gamma_{PV}(\mathbf{r}-\mathbf{r}')\times\mathbf{M}_T(\mathbf{r}')\right] = 0 \tag{14a}$$

$$\mathbf{H}_T^{(in)}(\mathbf{r}) - S_T(\mathbf{r})M_T(\mathbf{r}) - W_T(\mathbf{r})\hat{\mathbf{n}}(\mathbf{r})\times\mathbf{J}_T(\mathbf{r}) + ik_0\int_\Sigma dS(\mathbf{r}')\left[Y_0\tilde{G}(\mathbf{r}-\mathbf{r}')\mathbf{M}_T(\mathbf{r}') + \Gamma_{PV}(\mathbf{r}-\mathbf{r}')\times\mathbf{J}_T(\mathbf{r}')\right] = 0 \tag{14b}$$

The above equations constitute *a set of coupled integral equations* of the second kind for the unknown electric ($\mathbf{J}_T$) and magnetic ($\mathbf{M}_T$) surface currents.

6.2 Surface-integral equations for an impenetrable sheet with $Z^+ + Z^- = 0$

In this special case of a surface characterized by two impedances $Z^+ + Z^- = 0$ (this case includes, for example, the perfectly conducting surface, i.e., the case of $Z^\pm = 0$), the impedance-boundary conditions of Equation (4), combined with the expressions of Equation (3) for the discontinuities of the fields, reduce to the following relations between the currents and the fields:

$$\mathbf{M}_T(\mathbf{r}) = -Z(\mathbf{r})\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{J}_T(\mathbf{r}), \tag{15a}$$

$$\langle \mathbf{E}_T(\mathbf{r})\rangle = Z(\mathbf{r})\hat{\mathbf{n}}(\mathbf{r}) \times \langle \mathbf{H}_T(\mathbf{r})\rangle. \tag{15b}$$

By substituting the expressions of Equations (13), for the fields on the upper and lower faces of the sheet, into the boundary condition of Equation (15b), we obtain *a single equation*

$$\begin{aligned}\mathbf{E}_T^{(in)}(\mathbf{r}) = &-ik_0\int_\Sigma dS(\mathbf{r}')\left[Z_0\tilde{G}(\mathbf{r}-\mathbf{r}')\mathbf{J}_T(\mathbf{r}') - \mathbf{\Gamma}_{PV}(\mathbf{r}-\mathbf{r}') \times \mathbf{M}_T(\mathbf{r}')\right] \\ &+ Z(\mathbf{r})\hat{\mathbf{n}}(\mathbf{r}) \times \\ &\left\{\mathbf{H}_T^{(in)}(\mathbf{r}) + ik_0\int_\Sigma dS(\mathbf{r}')\left[Y_0\tilde{G}(\mathbf{r}-\mathbf{r}')\mathbf{M}_T(\mathbf{r}') + \mathbf{\Gamma}_{PV}(\mathbf{r}-\mathbf{r}') \times \mathbf{J}_T(\mathbf{r}')\right]\right\}\end{aligned} \tag{16}$$

By using Equations (15) we may convert the above equation into a single integral equation, either for the unknown electric current, $\mathbf{J}_T(\mathbf{r})$, or for the unknown magnetic current, $\mathbf{M}_T(\mathbf{r})$.

The scattered-electric field and the radar cross section for both the above cases, 6.1 and 6.2, can be obtained by using the asymptotic ($\mathbf{r} \to \infty$) form of the Green's function in the integral representation of Equations (12). Its explicit form is

$$\begin{aligned}&\mathbf{E}^{(sc)}(\mathbf{r}) = \\ &g_\infty(r)ik_0 e^{ik_0 r}\int_\Sigma dS(\mathbf{r}')e^{-ik_0\hat{\mathbf{r}}\cdot\mathbf{r}'}\left\{-Z_0\hat{\mathbf{r}} \times \left[\hat{\mathbf{r}} \times \mathbf{J}_T(\mathbf{r}')\right] - \hat{\mathbf{r}} \times \mathbf{M}_T(\mathbf{r}')\right\}\end{aligned} \tag{17}$$

where $g_\infty(r) = \dfrac{e^{ik_0 r}}{4\pi r}$ in three dimensions, and $g_\infty(r) = \dfrac{i\,e^{ik_0 r}}{2\sqrt{2\pi k_0 r}}e^{-i\pi/4}$ in two dimensions.

7. Decoupled surface-integral equations

In several special cases, the system of Equations (14) decouples into separate equations for the electric and magnetic currents, $\mathbf{J}_T(\mathbf{r})$ and $\mathbf{M}_T(\mathbf{r})$.

First, one of the currents may vanish identically. This happens for the perfect electric conductor ($S_T = \infty$, and hence $M_T = 0$), and for the perfect magnetic conductor ($R_T = \infty$, and $J_T = 0$).

Second, if both currents are non-vanishing, the equations decouple only when the reflection coefficients of the two faces are identical (so that the "cross-resistivity" W_T vanishes) *and* the surface is flat, so that the vector-Green's function, Γ_{PV}, does not contribute to Equations (14).

8. Examples and discussion

In this section, we discuss the form of the integral Equations (14), for some special cases involving both planar and non-planar targets, together with some of their applications.

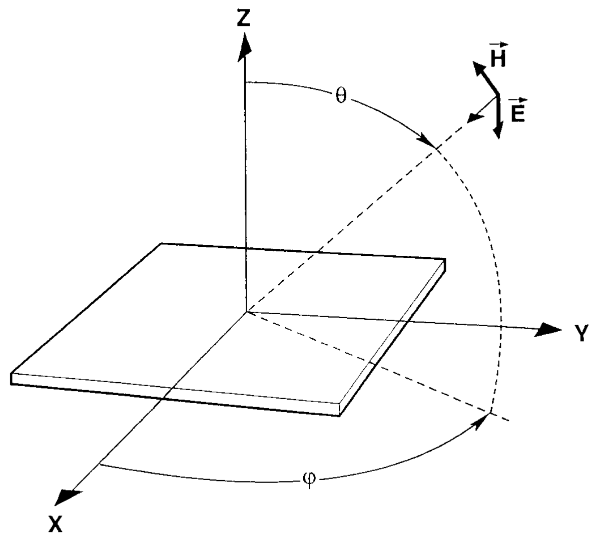

Figure 2. Scattering by a square plate.

As a simple planar target, we have chosen a $1\lambda \times 1\lambda$ square plate, located in the (x,y) plane (Figure 2), and characterized by various representative material properties.

8.1 A perfectly-conducting sheet

Integral equations for flat or curved sheets, in the limit $\varepsilon \to \infty$, $\mu = \mu_0$, can be obtained from Equations (14) by setting $R_T = 0$, $W_T = 0$, and $S_T = \infty$ in them; or from Equation (16) by setting $Z = 0$. In either case, the well-known electric-field integral equation results:

$$\mathbf{E}_T^{(in)}(\mathbf{r}) + ik_0 Z_0\int_\Sigma dS(\mathbf{r}')\tilde{G}(\mathbf{r}-\mathbf{r}')\mathbf{J}_T(\mathbf{r}') = 0, \tag{18a}$$

$$\mathbf{M}_T(\mathbf{r}) = 0. \tag{18b}$$

8.2 A resistive sheet

By setting $W_T = 0$, $S_T = \infty$, and $R_T = R \neq 0$ in Equations (14), we obtain the well-known set of equations for a thin-dielectric layer or a resistive sheet [4]:

$$\mathbf{E}_T^{(in)}(\mathbf{r}) - R\mathbf{J}_T(\mathbf{r}) + ik_0 Z_0\int_\Sigma dS(\mathbf{r}')\tilde{G}(\mathbf{r}-\mathbf{r}')\mathbf{J}_T(\mathbf{r}') = 0, \tag{19a}$$

$$\mathbf{M}_T(\mathbf{r}) = 0. \tag{19b}$$

These equations are also valid for planar as well as curved sheets.

8.3 A penetrable plate with equal face-reflection coefficients

Integral equations for a penetrable dielectric/magnetic plate with identical face-reflection coefficients can be obtained by setting $W_T = 0$ in Equations (14) (as required from relation (9b). For a *planar* sheet, contributions of the vector Green's function vanish, and the equations *decouple* and become

$$\mathbf{E}_T^{(in)}(\mathbf{r}) - R_T\mathbf{J}_T(\mathbf{r}) + ik_0 Z_0\int_\Sigma dS(\mathbf{r}')\tilde{G}(\mathbf{r}-\mathbf{r}')\mathbf{J}_T(\mathbf{r}') = 0, \tag{20a}$$

$$\mathbf{H}_T^{(in)}(\mathbf{r}) - S_T\mathbf{M}_T(\mathbf{r}) + ik_0 Y_0\int_\Sigma dS(\mathbf{r}')\tilde{G}(\mathbf{r}-\mathbf{r}')\mathbf{M}_T(\mathbf{r}') = 0. \tag{20b}$$

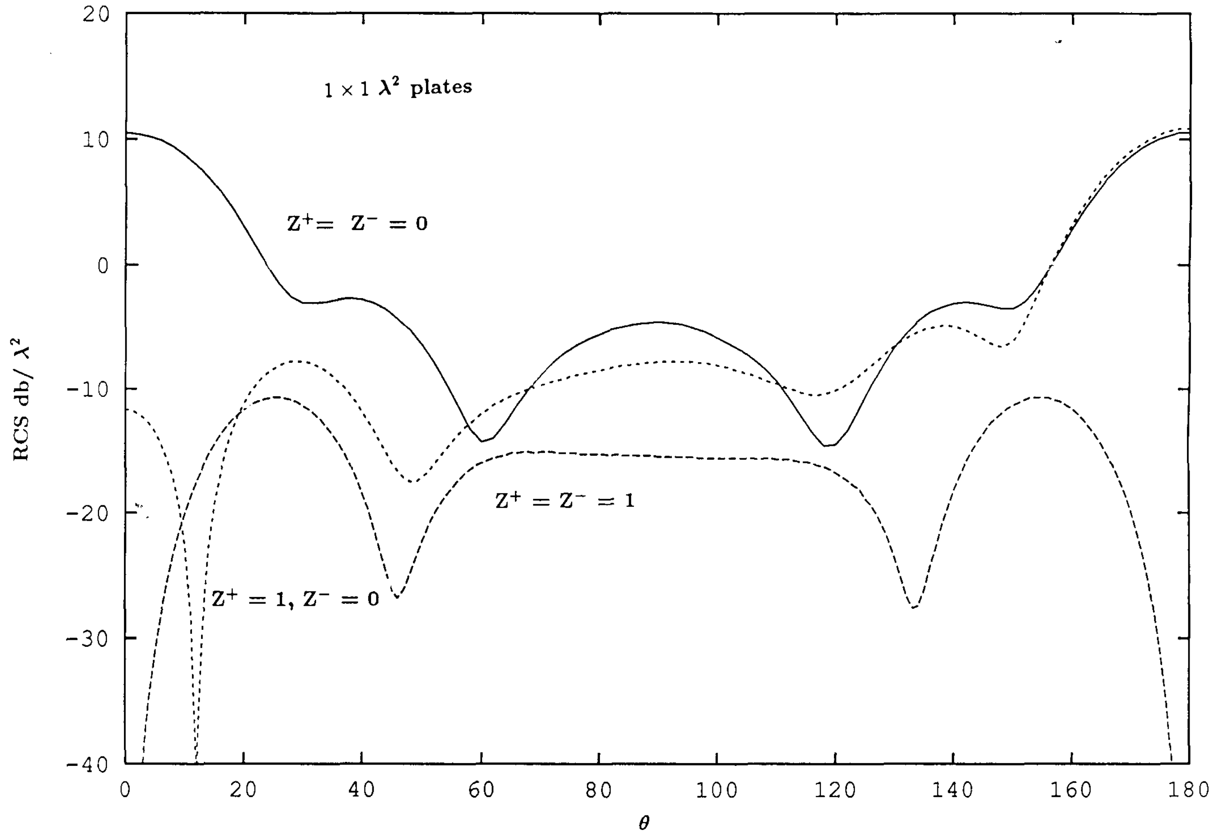

Figure 3. A comparison of the radar cross sections of a $1\lambda \times 1\lambda$ plate with different material properties: a perfectly conducting plate ($Z^+ = Z^- = 0$); an impedance plate with $Z^+ / Z_0 = Z^- / Z_0 = 1$ or, equivalently, an electrically resistive/magnetically conductive plate with $R / Z_0 = S / Y_0 = 1/2$; and an impedance plate with an upper-face impedance of $Z^+ / Z_0 = 1$, a perfectly conducting ($Z^- = 0$) lower face, and a electrically resistive/magnetically conductive plate with $S / Z_0 = 1/2$ and $R = 0$. The incident plane wave is horizontally polarized.

8.4 An impenetrable plate with equal face-reflection coefficients

For a planar sheet with two identical face impedances, the Equations (14) applicable to this case also decouple, and become

$$\mathbf{E}_T^{(in)}(\mathbf{r}) - \frac{Z}{2}\mathbf{J}_T(\mathbf{r}) + ik_0 Z_0 \int_\Sigma dS(\mathbf{r}')\widetilde{G}(\mathbf{r}-\mathbf{r}')\mathbf{J}_T(\mathbf{r}') = 0, \qquad (21a)$$

$$\mathbf{H}_T^{(in)}(\mathbf{r}) - \frac{1}{2Z}\mathbf{M}_T(\mathbf{r}) + ik_0 Y_0 \int_\Sigma dS(\mathbf{r}')\widetilde{G}(\mathbf{r}-\mathbf{r}')\mathbf{M}_T(\mathbf{r}') = 0. \qquad (21b)$$

As we already mentioned in Section 4, Equations (20) and (21) have the same mathematical structure. Hence, an electrically resistive/magnetically conductive sheet with $R = Z/2$ and $S = 1/2Z$ has the same scattering properties as an impenetrable sheet with two identical face impedances (see Figure 3). This statement is valid for planar as well as arbitrarily shaped sheets. However, one more comment is in order here. From the observation that both Equations (21a) and (21b) have the same mathematical structure as the integral equation for the electric current (19a) for a resistive sheet, it follows that the resistive-sheet integral equation can be used to calculate scattering from a *planar* impedance sheet with two identical face impedances. For the impedance sheet, however, both electric, $\mathbf{J}_T$, and magnetic, $\mathbf{M}_T$, currents contribute to Equation (17). Hence, the scattered field produced by the impedance sheet with two identical face impedances is, in general, quite different than that produced by the resistive sheet, alone. An exception to the above statement is the scattering of a TM plane wave at grazing ($\theta = 90°$) incidence where, because $\mathbf{H}_T^{(in)} = 0$, Equation (21b) has the solution $\mathbf{M}_T = 0$. Numerical examples presented in Figure 4 illustrate this point: the radar cross sections for the cases (c) and (d) are identical for the grazing incidence angle, and the difference between them increases gradually as θ departs from 90°.

We also note that, while solving a scattering problem involving a planar-impedance surface with identical face impedances requires solving two uncoupled integral equations, which have the same mathematical structure as that describing the resistive sheet alone, in general, non-planar surfaces with two identical face impedances (e.g., corner reflectors) cannot be modeled with the resistive-sheet integral equation (19a), due to the presence of the coupling terms in Equations (14).

Also, our computation of the radar cross section for a square flat plate, with two face impedances which are identical and equal to 1 at normal incidence, is consistent with the Weston theorem

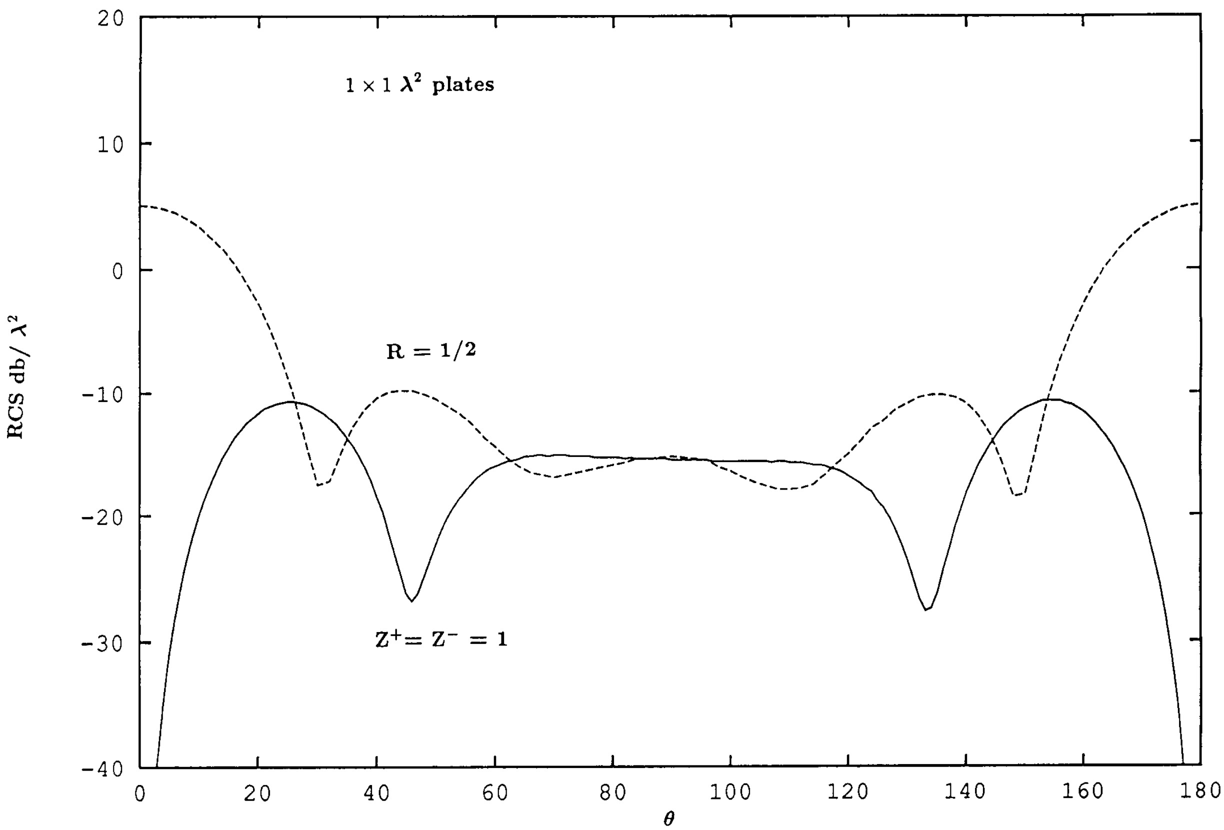

Figure 4. A comparison of the radar cross sections of a $1\lambda \times 1\lambda$ plate with different material properties: an impedance plate with $Z^+ / Z_0 = Z^- / Z_0 = 1$; and an electrically resistive plate with $R / Z_0 = 1/2$ and $S = \infty$. The incident plane wave is horizontally polarized.

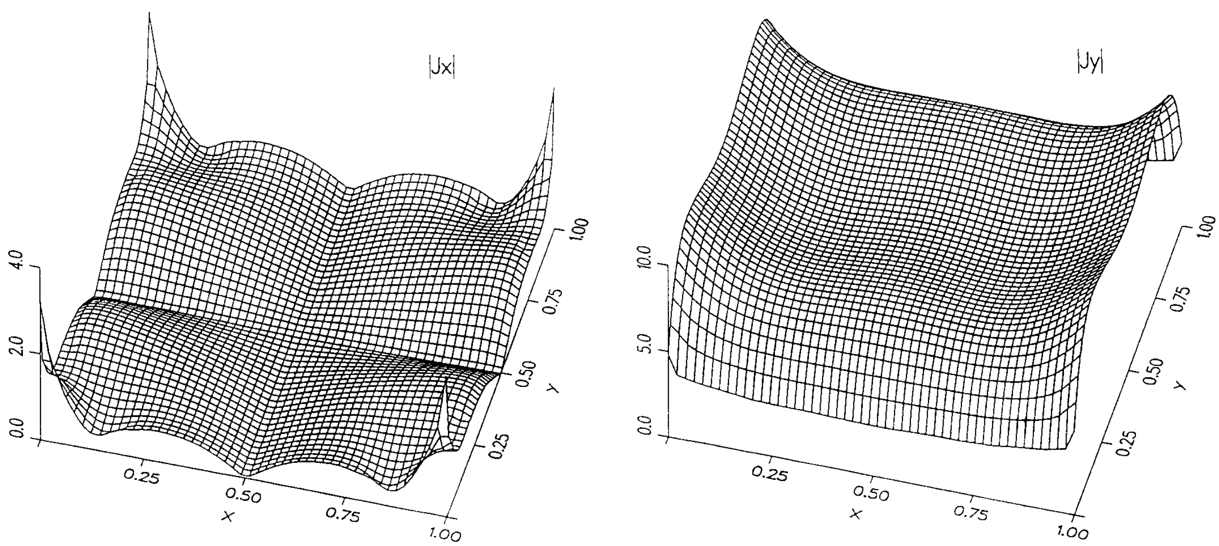

Figure 5. The absolute values of (a) the x and (b) the y components of the electric and magnetic currents, computed for a $1\lambda \times 1\lambda$ impedance plate with $Z^+ / Z_0 = Z^- / Z_0 = 1$ at normal incidence, with the incident electric field along the y axis. In this case, $J_x(x,y) = M_y(y,x)$ is an odd function of $(x - 1/2)$ and $(y - 1/2)$, while $J_y(x,y) = M_x(y,x)$ is an even function of these variables. The backscattering radar cross section is identically zero, because of cancellations of the electric and magnetic current contributions.

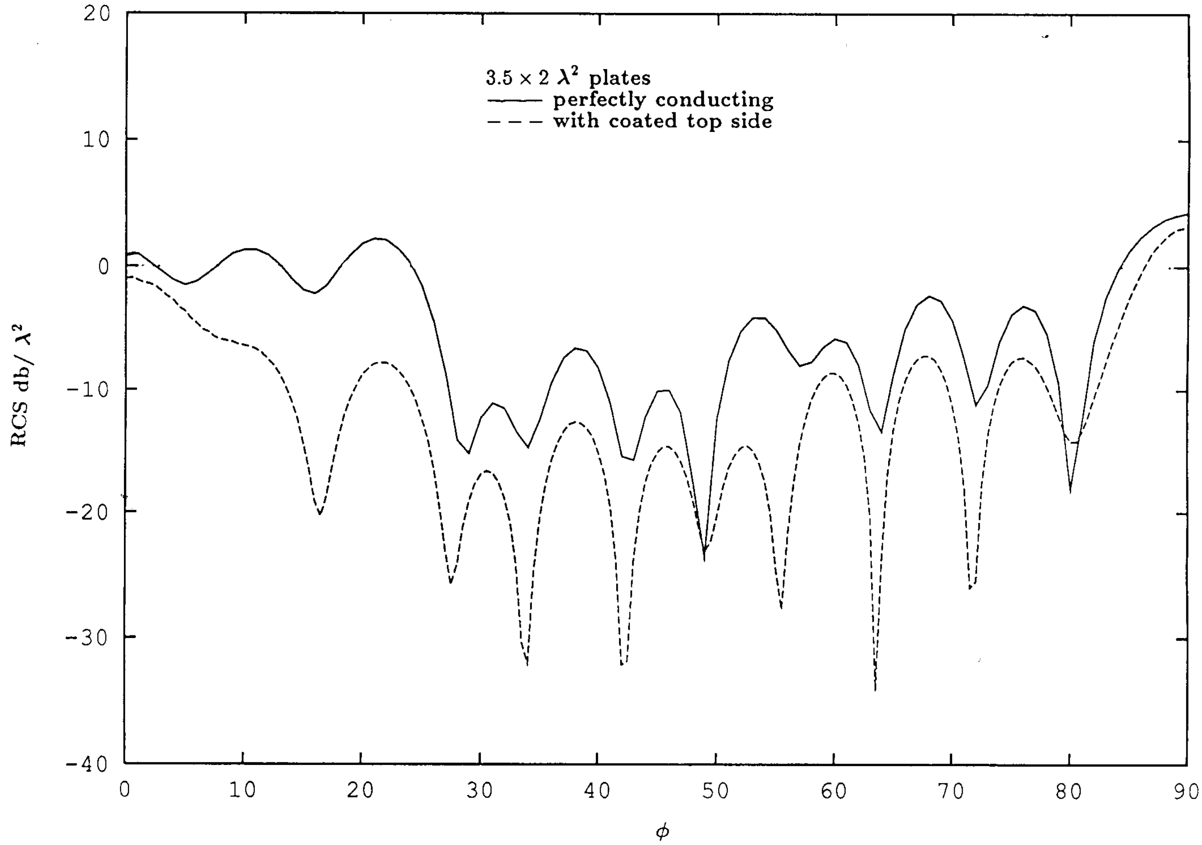

Figure 6. The radar cross sections of a 7.0"×4.0" ($3.5\lambda \times 2\lambda$ at 5.9 GHz) plate: (a) A perfectly conducting plate; and (b) A perfectly conducting plate, coated on one side with a material layer of thickness $d = 0.001025$ m and with $\varepsilon / \varepsilon_0 = 20.0 + i1.4$, and $\mu / \mu_0 = 1.7 + i2.0$. The incident plane wave, at 5.9 GHz, is horizontally polarized. The elevation angle is 10°. This calculation is in very good agreement with the data measured by the Electromagnetic Code Consortium [16] (however, the data have not yet been released officially).

[15]. This states that, due to the invariance of the system with respect to rotation around the z axis by $\phi = 90°$, the cross section at normal incidence should identically vanish. This "black-body behavior" of the $Z^+ = Z^- = 1$ plate is not due to the vanishing of the currents, but rather to the destructive interference of their respective contributions to the scattered field, given by Equation (17). Figure 5 shows that the components of the electric and magnetic currents, in this case, are comparable and quite large, especially near the plate edges.

8.5 An impenetrable plate with different face-reflection coefficients

This case is realized by an impedance plate with different face impedances. As an example, we consider a plate with the upper face characterized by $Z^+ \neq 0$, and the lower face perfectly conducting, $Z^- = 0$.

The corresponding integral equations (14) become

$$\mathbf{E}_T^{(in)}(\mathbf{r}) \qquad -\frac{1}{2}\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{M}_T(\mathbf{r}) + ik_0 Z_0 \int_\Sigma dS(\mathbf{r}')\tilde{G}(\mathbf{r}-\mathbf{r}')\mathbf{J}_T(\mathbf{r}') = 0, \quad (22a)$$

$$\mathbf{H}_T^{(in)}(\mathbf{r}) - \frac{1}{Z^+}\mathbf{M}_T(\mathbf{r}) - \frac{1}{2}\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{J}_T(\mathbf{r}) + ik_0 Y_0 \int_\Sigma dS(\mathbf{r}')\tilde{G}(\mathbf{r}-\mathbf{r}')\mathbf{M}_T(\mathbf{r}') = 0. \quad (22b)$$

The radar cross section corresponding to the above case, computed with $Z^+ / Z_0 = 1$ and $Z^- = 0$, is presented in Figure 3.

When the plane wave illuminates the $Z^+ / Z_0 = 1$ face at normal incidence, the cross section is considerably smaller than that for the perfectly conducting, $Z^- = 0$, face. But, as the Weston theorem does not apply, it does not vanish identically.

As another example, we have considered the data measured by the Electromagnetic Code Consortium [16] for a set of benchmark cases, involving flat plates coated with thin material layers on one side, at a near-grazing (10°) elevation angle. In our analysis, we have used Equations (22), with impedances of $Z^+ / Z_0 = 0.3425 - i0.157$, and $Z^- = 0$. These correspond to a perfectly conducting plate, coated on one side with a layer of thickness $d = 0.001025$ m, and with $\varepsilon / \varepsilon_0 = 20.0 + i1.4$, and $\mu / \mu_0 = 1.7 + i2.0$. The incident plane wave, at 5.9 GHz and a 10° elevation angle, is horizontally (TM) polarized. The plate dimensions are 7.0"×4.0" ($3.5\lambda \times 2.0\lambda$).

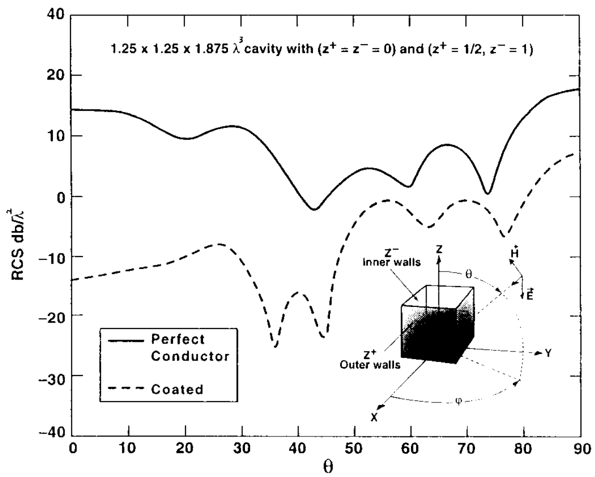

Figure 7. A comparison of radar cross sections for a $1.25 \times 1.25 \times 1.875\lambda^3$ rectangular cavity, illuminated with a vertically polarized incident wave, as a function of the azimuth angle, θ, computed for (a) all perfectly conducting walls; and (b) walls coated with RAM with $Z^- / Z_0 = 1$ on the inner sides, and with $Z^+ / Z_0 = 0.5$ on the outer sides. The longer cavity side is parallel to the z axis, and its closed bottom is in the (x,y) plane.

Although the data have not been released officially at this point, we have been allowed to mention that our calculation, based on Equations (22), and presented in Figure 6, is in very good agreement with the measurements[5].

8.6 A penetrable plate with different face-reflection coefficients

In this case (the most general for a penetrable sheet), the Equations (14) reduce, for a flat plate, to

$$\mathbf{E}_T^{(in)}(\mathbf{r}) - R_T(\mathbf{r})\mathbf{J}_T(\mathbf{r}) - W_T(\mathbf{r})\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{M}_T(\mathbf{r}) + ik_0 Z_0 \int_\Sigma dS(\mathbf{r}')\widetilde{G}(\mathbf{r}-\mathbf{r}')\mathbf{J}_T(\mathbf{r}') = 0, \quad (23a)$$

$$\mathbf{H}_T^{(in)}(\mathbf{r}) - S_T(\mathbf{r})\mathbf{M}_T(\mathbf{r}) - W_T(\mathbf{r})\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{J}_T(\mathbf{r}) + ik_0 Y_0 \int_\Sigma dS(\mathbf{r}')\widetilde{G}(\mathbf{r}-\mathbf{r}')\mathbf{M}_T(\mathbf{r}') = 0. \quad (23b)$$

In our example of a $1\lambda \times 1\lambda$ plate, and for not-too-small reflection coefficients (typically, $|\mathcal{R}^\pm| \geq 0.1$), the backscattering-cross sections at normal incidence are fairly independent of the value of the transmission coefficient. In contrast, the forward-scattering cross sections of an impenetrable (impedance) plate, and of a plate with a transmission coefficient of the order of 0.5 may differ by as much as 10 dB (actually, the cross section for the impenetrable plate is larger, because of the diffraction effects).

Finally, as the last example, we consider a scattering problem involving a non-planar target.

8.7 A rectangular cavity with different RAM coating on inner- and outer-wall sides

In this case, we use the most-general form of the integral equations (14), with the parameters R_T, S_T, and W_T, computed from the values of the wall impedances, according to Equations (6).

The curves shown in Figure 7 represent radar cross sections for a $1.25 \times 1.25 \times 1.875\lambda^3$ rectangular cavity, illuminated with a vertically polarized incident wave. These are shown as functions of the azimuth angle, θ, computed for (a) all walls perfectly conducting, and (b) walls coated with RAM, with $Z^- / Z_0 = 1$ on the inner sides, and with $Z^+ / Z_0 = 0.5$ on the outer sides. The longer cavity side is parallel to the z axis, and its closed bottom is in the (x,y) plane. As seen from Figure 7, the coating has the strongest effect on the cross section when the incident wave illuminates the absorbing $(Z^- / Z_0 = 1)$ interior of the cavity. In this case (i.e., for $\theta = 0°$), the coating reduces the cross section by nearly 30 dB. If a side of the cavity with $Z^+ / Z_0 = 0.5$ is illuminated ($\theta = 90°$), the cross section is reduced by only 10 dB.

Appendix A
General zero-order boundary conditions for a thin multi-layer stack

For completeness, we present here the main results of an analysis of an arbitrary system of layers, characterized by given electric permittivities and magnetic permeabilities.

We consider a system of infinite planar layers, illuminated with a plane wave incident at an angle θ with respect to the normal to the layers, and with the electric field in the layers' plane. This model is applicable to physical systems for which the curvature radius of the layered surface is large compared to the wavelength, and the considered domain is not too close (on the wavelength scale) to the surface edges (if present).

By solving the resulting one-dimensional Maxwell's equations, the reflection and transmission coefficients of the system can be expressed as

$$\mathcal{R}^\pm(\theta) = \frac{\pm[Q_{11}(\theta) - Q_{22}(\theta)] + Q_{12}(\theta)\cos\theta - Q_{21}(\theta)/\cos\theta}{Q_{11}(\theta) + Q_{22}(\theta) + Q_{12}(\theta)\cos\theta + Q_{21}(\theta)/\cos\theta}, \quad (A1a)$$

$$\mathcal{T}(\theta) = 2e^{-ik_0 d\cos\theta}\left[Q_{11}(\theta) + Q_{22}(\theta) + Q_{12}(\theta)\cos\theta + Q_{21}(\theta)/\cos\theta\right]^{-1} \quad (A1b)$$

in terms of elements, $Q_{ij}(\theta)$, of a complex, 2×2 matrix $Q(\theta)$, which provides a linear relationship between tangential components of the fields on the upper and lower faces of the stack. The phase factor in Equation (A1a) (in which d is the total stack thickness) takes into account the fact that the transmission coefficient is defined in terms of the fields on the *upper* side of the stack.

[5]On the other hand, the calculation for the TE polarization, for the same near-grazing-elevation angle, based on the formulation presented here, deviates from the data significantly. We have been able to improve the agreement by supplementing the boundary conditions of Equation (1) with "Ohm's law" for the normal-current components [17]. Such normal-current-component contributions can also be considered as higher-order corrections to the thin-layer approximations of Equations (1), equivalent to the higher-order impedance-boundary conditions [9]. In the case of a coated, perfectly conducting sheet, their effect is negligible for the TM polarization, but quite significant for the TE polarization.

The matrix $Q(\theta)$ (the "transfer matrix") is constructed as a product of transfer matrices, corresponding to individual layers. For N layers, numbered from the bottom to the top of the stack, we have

$$Q(\theta) = Q_N(\theta)Q_{N-1}(\theta)\cdots Q_2(\theta)Q_1(\theta), \tag{A2}$$

where

$$Q_n(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & \eta_n(\theta) \end{pmatrix} \times \begin{pmatrix} \cos[K_n(\theta)d_n] & -i\sin[K_n(\theta)d_n] \\ -i\sin[K_n(\theta)d_n] & \cos[K_n(\theta)d_n] \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1/\eta_n(\theta) \end{pmatrix} \tag{A3}$$

In the last expression,

$$\eta_n(\theta) = \sqrt{1 - \frac{\varepsilon_0\mu_0}{\varepsilon_n\mu_n}\sin^2\theta}\sqrt{\frac{\mu_0\varepsilon_n}{\varepsilon_0\mu_n}}. \tag{A4}$$

d_n is the nth layer's thickness, and

$$K_n(\theta) = \sqrt{\varepsilon_n\mu_n/(\varepsilon_0\mu_0) - \sin^2\theta}\,k_0 \tag{A5}$$

is the normal component of the wave vector in the nth layer. As follows from their construction, the matrices $Q_n(\theta)$, as well as their product, have unit determinants: $\det Q_n(\theta) = \det Q(\theta) = 1$, i.e., they can be specified by only three complex parameters.

For layers of arbitrary thicknesses and material parameters, the transfer matrix, and therefore the reflection and transmission coefficients of Equation (A1), have a complicated θ dependence. However, in the limit in which the θ dependence of the transfer matrix can be neglected, Equation (A1) gives the same angular dependence of the reflection and transmission coefficients as Equation (8). This fact implies that the layered sheet can be described by the boundary conditions of Equation (1), that there exists a one-to-one relationship among the three independent elements of Q and the resistivities R_T, S_T, and W_T.[6] That relationship, which can be derived by comparing Equation (A1) with Equation (8), becomes particularly simple in the *thin layer limit* $k_0 d << 1$, in which the phase correction $\exp(-ik_0 d\cos\theta)$ in Equation (A1) is negligible:

$$\begin{pmatrix} W_T & R_T \\ S_T & -W_T \end{pmatrix} = \frac{1}{2}(Q-1)^{-1}(Q+1) = \frac{1}{Q_{11}+Q_{22}-2}\begin{pmatrix} \frac{1}{2}(Q_{11}-Q_{22}) & Q_{12} \\ Q_{21} & -\frac{1}{2}(Q_{11}-Q_{22}) \end{pmatrix} \tag{A6}$$

(we used here the fact that if $\det Q = 1$, then $(Q-1)^{-1}(Q+1)$ is traceless).

To summarize, the considered stack of layers can be described in terms of the boundary conditions of Equations (1), with the resistivities given by Equation (A6), in the limit where

[6]Under more-relaxed conditions on layer thicknesses and material properties, higher-order (derivative) boundary conditions can be obtained [11].

$$k_0 d << 1 \quad \text{(thin-layer limit)}, \tag{A7}$$

and, for every layer,

$$\left.\begin{array}{l} |\varepsilon_n\mu_n|/(\varepsilon_0\mu_0) >> 1, \\ kd_n << \sqrt{|\varepsilon_n\mu_n|/(\varepsilon_0\mu_0)} \end{array}\right\} \text{(small-refraction-angle limit)} \tag{A8}$$

The last two conditions imply that the wave inside the layered medium propagates approximately in the direction normal to the sheet, so that the θ dependence in Equations (A4) and (A5) can be neglected. In other words, the tangential components of the wave vector have a negligible effect, both on the wave's magnitude and phase. The relative errors in the solution, corresponding to the two conditions of Equations (A7) and (A8), are

$$O(k_0 d),$$

$$O\left(\frac{\varepsilon_0\mu_0}{\varepsilon_n\mu_n}\right),$$

$$O\left(\frac{k_0 d_n}{\sqrt{\varepsilon_n\mu_n/(\varepsilon_0\mu_0)}}\right). \tag{A9}$$

The conditions of Equation (A8) do *not* preclude large phase shifts in the layers: the phase shift $\delta_n \equiv \sqrt{\varepsilon_n\mu_n/(\varepsilon_0\mu_0)}K_0 d_n$ may be large, provide it satisfies the constraint $|\delta_n| << |\varepsilon_n\mu_n|/(\varepsilon_0\mu_0)$.

Two layers

For a system of two layers, characterized by the parameters $\varepsilon_1, \mu_1, d_1$ and $\varepsilon_2, \mu_2, d_2$, we find, from Equations (A2) and (A3), the transfer matrix

$$Q = \begin{pmatrix} c_1c_2 - \eta_1\eta_2^{-1}s_1s_2 & -i\eta_1^{-1}s_1c_2 - i\eta_2^{-1}c_1s_2 \\ -i\eta_1s_1c_2 - i\eta_2c_1s_2 & c_1c_2 - \eta_1^{-1}\eta_2s_1s_2 \end{pmatrix}, \tag{A10}$$

where $c_n \equiv \cos(\delta_n)$, $s_n \equiv \sin(\delta_n)$, and, in the thin-layer and small-refraction-angle limits, $\eta_n = \sqrt{(\mu_0\varepsilon_n)/(\varepsilon_0\mu_n)}$, and $\delta_n = \sqrt{\varepsilon_n\mu_n/(\varepsilon_0\mu_0)}k_0 d_n$, $n = 1,2$. We can check that the condition $\det Q = 1$ is satisfied. We also find that $Q_{11} = Q_{22}$, and the two reflection coefficients are equal, if and only if $\eta_1 = \eta_2$ (provided $s_1s_2 \neq 0$).

Single layer

For a single layer, of thickness d and material parameters ε and μ, the resulting transfer matrix, Q, is

$$Q = \begin{pmatrix} \cos\delta & -i\sqrt{\mu/\varepsilon}\sin\delta \\ -i\sqrt{\varepsilon/\mu}\sin\delta & \cos\delta \end{pmatrix}, \tag{A11}$$

with $\delta \equiv \sqrt{\varepsilon\mu/(\varepsilon_0\mu_0)}K_0 d$.

If the conditions of Equations (A7) and (A8) hold, we can use the relation (A6) to obtain the resistivities:

$$R_T = \frac{i}{2}\sqrt{\frac{\mu}{\varepsilon}}\cot\delta,\quad S_T = \frac{i}{2}\sqrt{\frac{\varepsilon}{\mu}}\cot\delta,\quad W_T = 0. \tag{A12}$$

If, instead, the phase shift is small ($|\delta| << 1$), but the phase correction in Equation (A1b) is *not* negligible, we find, *for normal incidence*,

$$R_T = \frac{iZ_0}{k_0 d(\varepsilon / \varepsilon_0 - 1)}, \quad S_T = \frac{iY_0}{k_0 d(\mu / \mu_0 - 1)}, \quad W_T = 0. \quad \text{(A13)}$$

However, if the conditions (A8) are not satisfied (for example, when ε and μ are not sufficiently large), the reflection and transmission coefficients obtained from the transfer equations have additional θ dependencies, *not* reproduced by the boundary conditions of Equation (1) (and the resulting reflection and transmission coefficients of Equations (8)). In other words, the boundary conditions of Equation (1), with the resistivities of Equations (A13), are *not* equivalent to the original scattering problem: they reproduce $\mathcal{R}^{\pm}(\theta)$ and $\mathcal{T}(\theta)$ only at $\theta = 0$.

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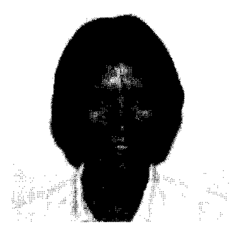

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