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Asymptotic Theory for Dipole Radiation in the Presence of a Lossy Slab Lying on a Conducting Half-Space

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Abstract—**The basic theory for dipole radiation in the presence of a two-layer half-space is outlined with special reference to its use as a model for studying radio propagation through and over heavily vegetated terrain. The source dipole may be located above or below the top surface of the slab. The dipole orientation is either vertical or horizontal. The asymptotic derivations for the field expressions are carried out without making the usual assumption that the refractive index of the uppermost layer is large compared with unity. The final results exhibit the expected inverse square dependence of the field on horizontal range.**

Introduction

FOR SHORT-RANGE communication (i.e., <50 km), the earth's boundary and its immediate environment determine the total path loss between transmitter and receiver. The influence of the local environment is particularly noticeable in the case of terrain that is heavily obscured by dense vegatation. There have been some attempts to describe this situation in terms of a single half-space model with semiempirical modifications [1], [2] introduced to account for the inhomogeneities.

In this paper we discuss the theory for an idealized slab model that has already shown some success [3]–[6] in predicting received field strengths for propagation over terrain covered with dense vegetation. This same model has also been applied to electromagnetic investigations of other environments, such as in the polar ice caps and in deep basement rock layers [7]–[9].

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Vertical Dipole Excitation

We consider first a vertical electric diple located at a height h over a two-layer conducting half-space. The situation is illustrated in Fig. 1 where we have used a cylindrical coordinate system (ρ, ϕ, z) with the dipole of moment Ids located at $z=h$ on the axis. It is assumed that free space, with properties ϵ_0 and μ_0, occupies the region $z>0$, while the two-layer medium occupies $z<0$. From $0>z>-D$, the electrical properties are σ_1, ϵ_1, and μ_1, and for $-D>z>-\infty$, the electrical properties are σ_2, ϵ_2, and μ_2.

The electromagnetic fields at the point P, in the free-space region, may be derived from a Hertz vector that has only a z component Π_{0z}. A straightforward analysis [8] gives the following exact result, for a time factor exp $(i\omega t)$,

$$\Pi_{0z} = \frac{Ids}{4\pi i\epsilon_0\omega}\left[\frac{e^{-ik_0R_0}}{R_0} - \frac{e^{-ik_0R}}{R} + F\right] \tag{1}$$

where $R_0=[(z-h)^2+\rho^2]^{1/2}$, $R=[(z+h)^2+\rho^2]^{1/2}$, $k_0=(\mu_0\epsilon_0)^{1/2}\omega=\omega/c$, and F is a contour integral. The latter is given by

$$F = \int_{-\infty}^{+\infty}\frac{K_0}{K_0+Z_1}H_0^{(2)}(\lambda\rho)\cdot\exp\left[-u_0(h+z)\right]\frac{\lambda}{u_0}d\lambda, \tag{2}$$

where

$$Z_1 = K_1\frac{K_2+K_1\tanh u_1 D}{K_1+K_2\tanh u_1D}, \tag{3}$$

$$K_1 = \frac{u_1}{\sigma_1+i\epsilon_1\omega},\quad K_2 = \frac{u_2}{\sigma_2+i\epsilon_2\omega},\quad K_0 = \frac{u_0}{i\epsilon_0\omega},$$

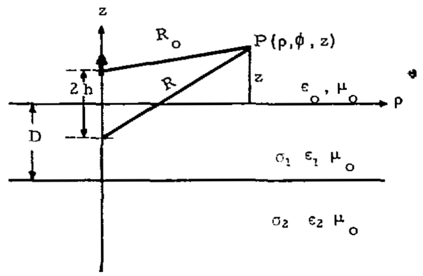

Fig. 1. Vertical electric dipole over a two-layer half-space.

$$u_1 = (\lambda^2 - k_1^2)^{1/2}, \; u_2 = (\lambda^2 - k_2^2)^{1/2}, \; u_0 = (\lambda^2 - k_0^2)^{1/2},$$
$$k_1^2 = - i\mu_0\omega(\sigma_1 + i\epsilon_1\omega) \text{ and } k_2^2 = - i\mu_0\omega(\sigma_2 + i\epsilon_2\omega),$$

and $H_0^{(2)}(\lambda\rho)$ is a Hankel function of the second kind of order zero and argument $\lambda\rho$. The contour of integration runs along the real axis of λ from $-\infty$ to $+\infty$.

The method of evaluating the integral in (1) has been discussed in the references cited above. For present purposes, we will obtain an asymptotic approximation that is appropriate when the upper layer is a lossy dielectric and the lower layer is a relatively good conductor. The integration contour and the pole singularities are indicated in Fig. 2.

The integral has branch points at $\lambda = \pm k_0$ and $\lambda = \pm k_2$. For present purposes, it is convenient to draw branch cuts from k_0 and k_2 to $-i\infty$ in the manner indicated. A study of the problem indicates that for a lossy slab lying on a relatively highly conducting substrate, the predominant pole, denoted by k_s in Fig. 2, may be either proper or improper.

In order to evaluate the integral, the contour is deformed around the proper pole(s) and branch lines as indicated in Fig. 2. In the present case, the principal contribution is the branch line integration around k_0 and we designate this quantity by $F^{(0)}$. It may be written

$$F^{(0)} = \frac{1}{i\epsilon_0\omega}\int_{k_0}^{k_0-i\infty}\left[\frac{\exp\left[-u_0^{(+)}(h+z)\right]}{K_0^{(+)}+Z_1^{(+)}} - \frac{\exp\left[-u_0^{(-)}(h+z)\right]}{K_0^{(-)}+Z_1^{(-)}}\right]H_0^{(2)}(\lambda\rho)\lambda d\lambda \quad (4)$$

where subscripts (+) and (−) designate the value of the functions evaluated on the right- and left-hand sides, respectively, of the branch line. Now, it may be verified readily that $Z_1^{(-)} = Z_1^{(+)} = Z_1$ for this branch line. Also, by changing to the variable s by the substitution $\lambda = k_0 - is^2$, it follows that

$$H_0^{(2)}(\lambda\rho) \cong \left(\frac{2i}{\pi\lambda\rho}\right)^{1/2} e^{-i\lambda\rho} \cong \left(\frac{2i}{\pi k_0\rho}\right)^{1/2} e^{-ik_0\rho}e^{-s^2\rho} \quad (5)$$

is an excellent approximation when $k_0\rho \gg 1$ for all values of s from 0 to ∞. As a result of the exponential damping factor, we note that

$$u_0^{(+)} = \left[(\lambda - k_0)(\lambda + k_0)\right]^{1/2} \cong (-is^2)^{1/2}(2k_0)^{1/2} = e^{-i\pi/4}s(2k_0)^{1/2}$$

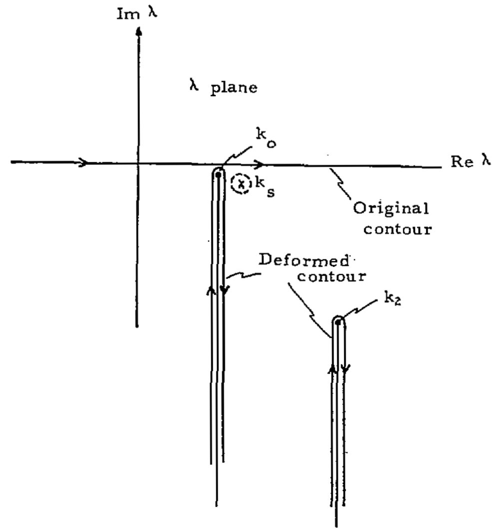

Fig. 2. Integration contours in the complex λ plane.

when the argument of s^2 is defined as 0 for this (+) side of the branch line. Then, in a similar manner, it is found that $u_0^{(-)} \cong -e^{-i\pi/4}s(2k_0)^{1/2}$ when the argument of s^2 is taken to be 2π.

Using the approximations indicated above, (4) is transformed into

$$F^{(0)} \cong \int_0^\infty \left\{\frac{\exp\left[-e^{-i\pi/4}(2k_0)^{1/2}s(h+z)\right]}{e^{-i\pi/4}s(2k_0)^{1/2} + ik_0\Delta} + \frac{\exp\left[e^{-i\pi/4}(2k_0)^{1/2}s(h+z)\right]}{e^{-i\pi/4}s(2k_0)^{1/2} - ik_0\Delta}\right\} e^{-s^2\rho} \times \left(\frac{2i}{\pi k_0\rho}\right)^{1/2} e^{-ik_0\rho}k_0(-i2s)ds, \quad (6)$$

where $\Delta = Z_1/\eta_0$ evaluated at $\lambda = k_0$ and $\eta_0 = (\mu_0/\epsilon_0)^{1/2} = 120\pi$. Since we have already assumed that the important values of s are near zero, (6) may be further simplified by retaining only two terms in the expansion of the exponential functions. The denominators are approximated in a similar fashion. Then we easily find that

$$F^{(0)} \cong 2\left[(1 + ik_0\Delta h)(1 + ik_0\Delta z)/(ik_0\Delta^2\rho^2)\right] \cdot \exp(-ik_0\rho). \quad (7)$$

Here we have used the identity

$$\int_0^\infty s^2 \exp(-s^2\rho)ds = \pi^{1/2}/(4\rho^{3/2}).$$

Neglected terms in (7) vary as ρ^{-3}, ρ^{-4}, $\cdots$, etc. Actually, (7) is an adequate approximation to the branch line integration, provided $|k_0\rho\Delta^2| \gg 1$, and $(z+h) \ll \rho$.

The branch line integration associated with k_2 is carried out in a manner similar to that for k_0 but now the contribution is proportional to $\rho^{-2}\exp(-ik_2\rho)$ which is heavily damped since k_2 has an appreciable imaginary part when the lower medium is finitely conducting. Therefore, in what follows, we assume that this contribution to the integral F is negligible.

The other possible contribution is the residue of the pole k_s if the latter be on the proper Riemann sheet. Under present circumstances, the pole is proper if Re $(1-\Delta^2)^{1/2}>1$ and Im $(1-\Delta^2)^{1/2}<0$. The pole contribution is proportional to $(k_0\rho)^{-1/2}$ exp $[-ik_0(1-\Delta^2)^{1/2}\rho]$ which is heavily damped if $-(k_0\rho)$ Im $(1-\Delta^2)^{1/2}\gg 1$. We assume this is the case in what follows.

Using (7), and performing the required operations on the z component of the Hertz vector, we find that the vertical electric field at the point P is given by

$$E_{0z} \cong \frac{i\mu_0\omega I ds}{2\pi\rho} e^{-ik_0\rho} W(p) f(h) f(z), \tag{8}$$

where

$$W(p) \cong -\frac{1}{2p}, \qquad p = -ik_0\rho\Delta^2/2,$$

$$f(h) \cong 1 + ik_0 h\Delta, \qquad f(z) \cong 1 + ik_0 z\Delta.$$

The dimensionless quantity $W(p)$ is analogous to the ground-wave attenuation function for propagation over a homogeneous half-space, and thus p may be described as a numerical distance parameter. In order that this simple form of $W(p)$ be valid, it is required that $|p|\gg 1$. Also, the height-gain functions $f(h)$ and $f(z)$ are valid only if both h and $z \ll \rho$. A detailed discussion of the function $W(p)$ under more general conditions has recently been given by King and Schlak [10].

The height-gain functions given above are to be used when both z and h are positive. When z or h is negative, we use instead the easily proved result that

$$f(z) \cong (k_0/k_1)^2 \left[\frac{e^{u_1 z} + \hat{r}_{\parallel} e^{-2u_1 D} e^{-u_1 z}}{1 + \hat{r}_{\parallel} e^{-2u_1 D}}\right]_{\lambda = k_0}, \tag{9}$$

where

$$\hat{r}_{\parallel} = \frac{K_2 - K_1}{K_2 + K_1}.$$

If Re $2[k_0^2 - k_1^2]^{1/2} D \gg 1$, the "depth-gain" function is well approximated by the simple exponential form

$$f(z) \cong (k_0/k_1)^2 \exp\left[i(k_1^2 - k_0^2)^{1/2} z\right].$$

Horizontal Dipole Excitation

The next case to be considered is the excitation of the same two-layer structure by a horizontal electric dipole. The situation is illustrated in Fig. 3 where we have indicated that the dipole is oriented in the x direction. The formal solution of the problem may be obtained in a straightforward manner if it is assumed at the outset that the Hertz vector has both an x and a z component. For the situation where z and $h>0$, it is found that [8]

$$\Pi_{0x} = \frac{I ds}{4\pi i\epsilon_0\omega}\left[\frac{e^{-ik_0 R_0}}{R_0} + \frac{1}{2}\int_{-\infty}^{+\infty} r_{\perp} e^{-u_0(z+h)} \frac{\lambda}{u_0} H_0^{(2)}(\lambda\rho) d\lambda\right] \tag{10}$$

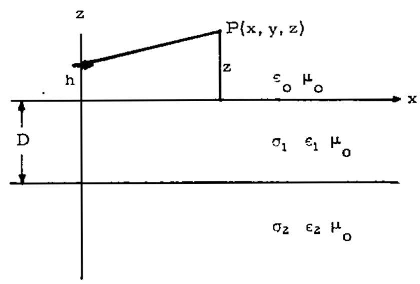

Fig. 3. Horizontal electric dipole over a two-layer half-space.

and

$$\Pi_{0z} = \frac{I ds}{4\pi i\epsilon_0\omega}\frac{\partial}{\partial x}\int_{-\infty}^{+\infty}\frac{(r_{\parallel}+r_{\perp})}{2\lambda} e^{-u_0(z+h)} H_0^{(2)}(\lambda\rho) d\lambda, \tag{11}$$

where

$$r_{\parallel} = \frac{K_0 - Z_1}{K_0 + Z_1} \quad \text{and} \quad r_{\perp} = \frac{N_0 - Y_1}{N_0 + Y_1}$$

in which

$$Y_1 = N_1 \frac{N_2 + N_1 \tanh u_1 D}{N_1 + N_2 \tanh u_1 D},$$

and $N_0 = u_0/(i\mu_0\omega)$, $N_1 = u_1/(i\mu_0\omega)$, and $N_2 = u_2/(i\mu_0\omega)$. The other parameters in (10) and (11) were defined previously.

If the source dipole is lowered into the middle layer such that h is a negative quantity, we obtain the following expressions for the components of the Hertz vector in the upper half-space $z>0$:

$$\Pi_{0x} = \frac{I ds}{4\pi i\epsilon_0\omega}\int_{-\infty}^{\infty}\frac{(1+r_{\perp})}{2} g_{\perp}(h) e^{-u_0 z}\frac{\lambda}{u_0} H_0^{(2)}(\lambda\rho) d\lambda \tag{12}$$

and

$$\Pi_{0z} = \frac{I ds}{4\pi i\epsilon_0\omega}\frac{\partial}{\partial x}\int_{-\infty}^{+\infty}\left[\frac{(r_{\parallel}-1)}{2\lambda} g_{\parallel}(h) + \frac{(1+r_{\perp})}{2\lambda} g_{\perp}(h)\right] e^{-u_0 z}\frac{\lambda}{u_0} H_0^{(2)}(\lambda\rho) d\lambda \tag{13}$$

where

$$g_{\parallel}(h) = e^{-u_1|h|}\left[\frac{1 - \hat{r}_{\parallel} e^{-2u_1(D-|h|)}}{1 - \hat{r}_{\parallel} e^{-2u_1 D}}\right], \tag{14}$$

$$g_{\perp}(h) = e^{-u_1|h|}\left[\frac{1 + \hat{r}_{\perp} e^{-2u_1(D-|h|)}}{1 + \hat{r}_{\perp} e^{-2u_1 D}}\right], \tag{15}$$

in which

$$\hat{r}_{\parallel} = \frac{K_2 - K_1}{K_2 + K_1} \quad \text{and} \quad \hat{r}_{\perp} = \frac{N_2 - N_1}{N_2 + N_1}.$$

Actually, the expressions given by (12) and (13) were derived by Biggs and Swarm [9] but our results do not agree with theirs. Agreement could be achieved only if the square bracket terms in (14) and (15) were both set equal to one. The presence of these factors is required in order to satisfy the boundary conditions that implicitly account for multiple reflection phenomena within the middle layer. Clearly, this effect cannot be neglected unless $|u_1 D| \gg 1$, which is violated in many cases of practical interest.

The field components in the space $z>0$ may be obtained in the usual manner by performing derivative operations on the Hertz vector. Of special interest is the vertical electric field E_{0z} which is obtained from

$$E_{0z} = k_0^2 \Pi_{0z} + \frac{\partial}{\partial z}\left(\frac{\partial \Pi_{0x}}{\partial x} + \frac{\partial \Pi_{0z}}{\partial z}\right). \tag{16}$$

After cancellation of terms we get, for the case $h<0$,

$$E_{0z} = \frac{Ids}{4\pi i \epsilon_0 \omega} \frac{\partial}{\partial x} \int_{-\infty}^{+\infty} \frac{r_{\parallel} - 1}{2} \lambda g_{\parallel}(h) e^{-u_0 z} H_0^{(2)}(\lambda\rho) d\lambda, \tag{17}$$

which does not depend on $r_{\perp}$.

The lateral wave contribution to (17) may be written in a form analogous to (8) if we invoke the same assumptions. Thus, for example, if $h=0$,

$$E_{0z} \cong \frac{i\mu_0 \omega Ids}{2\pi\rho} e^{-ik_0\rho} W(p) f(z) \Delta \cos\phi, \tag{18}$$

where $\cos\phi = x/\rho$. The only difference here is that a multiplier $\Delta \cos\phi$ appears which is to be expected on physical grounds since Δ is a measure of the wave tilt and $\cos\phi$ is the expected radiation pattern in the horizontal plane.

In addition to the "vertically polarized" lateral wave being radiated from the horizontal electric dipole, there is a horizontally polarized component. This has a maximum in the broadside direction. Asymptotically, we find that for $z>0$ and for $h=0$,

$$E_{0\phi} \cong \frac{i\mu_0 \omega Ids}{2\pi\rho} e^{-ik_0\rho} W(p_m) f_m(z) \sin\phi, \tag{19}$$

where

$$W(p_m) \cong -\frac{1}{2p_m}, \qquad p_m \cong -ik_0\rho\Omega^2/2,$$

$$f_m(z) \cong 1 + ik_0\Omega z, \qquad \sin\phi = y/\rho,$$

in which

$$\Omega = \eta_0 Y_1 \big|_{\lambda=k_0}.$$

It is interesting to compare the vertical electric field E_{0z} for a vertical dipole located just above the upper surface and the maximum horizontal electric fields $E_{0\phi}$ (at $\phi=\pi/2$) from the horizontal dipole. For the case $z=0$, these are given by

$$\begin{bmatrix} E_{0z} \\ E_{0\phi} \end{bmatrix} \cong \frac{\eta_0 Ids}{2\pi\rho^2} \begin{bmatrix} \Delta^{-2} \\ \Omega^{-2} \end{bmatrix}. \tag{20}$$

This may be rewritten in the equivalent form

$$\begin{bmatrix} E_{0z} \\ E_{0\phi} \end{bmatrix} \cong \frac{60Ids}{\rho^2}\left(1 - \frac{k_0^2}{k_1^2}\right)^{-1} \begin{bmatrix} (k_1/k_0)^2 & Q_v^{-2} \\ (k_0/k_1)^2 & Q_h^2 \end{bmatrix} \tag{21}$$

where

$$Q_v = \frac{Z_1}{K_1}\bigg|_{\lambda=k_0} \quad \text{and} \quad Q_h = \frac{N_1}{Y_1}\bigg|_{\lambda=k_0}.$$

The stratification factors Q_v and Q_h are normalized to be unity if D, the thickness of the middle layer, is effectively infinite. It is evident that for this limiting case, E_{0z} is greater than $E_{0\phi}$ by a factor $(k_1/k_0)^4$.

Concluding Remarks

The results indicate that for VHF propagation over jungle-covered terrain, the resultant field varies roughly as inverse distance squared whether or not the source is located above or below the top of the jungle. This conclusion is not influenced by the orientation of the source dipole but, on the other hand, the excitation of the lateral wave is dependent on both the antenna orientations and the thickness of the jungle cover. A detailed numerical study of these interrelationships would be useful. Other worthwhile extensions of the theory might account for the horizontal and vertical inhomogeneities within the slab itself.

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