10/15

Uniqueness Theorem is a more general case.

Notes # 8

Uniqueness

How do you prove solutions are unique.

Suppose there exists two solutions

$\nabla\times\mathbf{E}_1 = -j\omega\mu\mathbf{H}_1$	(1a)	$\nabla\times\mathbf{E}_2 = -j\omega\mu\mathbf{H}_2$	(1b)
$\nabla\times\mathbf{H}_1 = \mathbf{J} + j\omega\varepsilon\mathbf{E}_1$	(2a)	$\nabla\times\mathbf{H}_2 = \mathbf{J} + j\omega\varepsilon\mathbf{E}_2$	(2b)
$\nabla\cdot\mathbf{E}_1 = \frac{\rho}{\varepsilon}$	(3a)	$\nabla\cdot\mathbf{E}_2 = \frac{\rho}{\varepsilon}$	(3b)
$\nabla\cdot\mathbf{H}_1 = 0$	(4a)	$\nabla\cdot\mathbf{H}_2 = 0$	(4b)

which satisfy Maxwell's equations, the boundary conditions, and the radiation conditions.

↳ all fields go to zero.

EMP is linear theory

If the two solutions are subtracted from one another the result must satisfy the source free Maxwell equations. To show this, first subtract the above sets of equations 1b-1a, 2b-2a, etc.

$$\nabla\times(\mathbf{E}_2 - \mathbf{E}_1) = -j\omega\mu(\mathbf{H}_2 - \mathbf{H}_1),\ \text{etc} \qquad (5)$$

Then define $\mathbf{E}_2 - \mathbf{E}_1 = \mathbf{E}$ and $\mathbf{H}_2 - \mathbf{H}_1 = \mathbf{H}$ (6)

which gives:

$$\nabla\times\mathbf{E} = -j\omega\mu\mathbf{H} \qquad (7)$$

$(\nabla\times E)\cdot H^* = -j\omega\mu H\cdot H^*$
$= -j\omega\mu H^2$

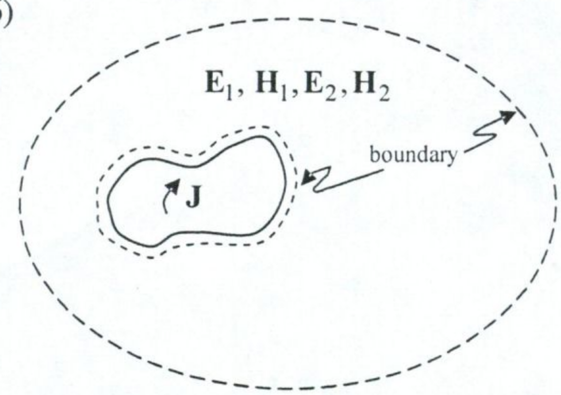
$$\nabla\times\mathbf{H} = (\sigma + j\omega\varepsilon)\mathbf{E} \qquad (8)$$

$\sigma - j\omega\varepsilon$

$(\nabla\times H)^*\cdot E = [(\sigma + j\omega\epsilon)E]^*\cdot E$
$= (\sigma - j\omega\epsilon)E^*\cdot E$
$= \sigma - j\omega\epsilon\, E^2$

$$\nabla\cdot\mathbf{E} = 0 \qquad (9)$$

$$\nabla\cdot\mathbf{H} = 0 \qquad (10)$$



Take the dot products of $\mathbf{H}^*$ with (7) and $\mathbf{E}$ with the conjugate of (8) and subtract. This gives

$$\nabla\cdot\left(\mathbf{E}\times\mathbf{H}^*\right) = -\sigma E^2 + j\omega\varepsilon E^2 - j\omega\mu H^2$$

where

$$\nabla\cdot\left(\mathbf{E}\times\mathbf{H}^*\right) = \mathbf{H}^*\cdot\nabla\times\mathbf{E} - \mathbf{E}\cdot\nabla\times\mathbf{H}^* \text{ has been used}$$

$(\nabla\times E)\cdot H^* - (\nabla\times H)^*\cdot E = -j\omega\mu H^2 - \sigma E^2 + j\omega\epsilon E^2$

$\nabla\cdot(E\times H^*) = j\omega\epsilon E^2 - j\omega\mu H^2 - \sigma E^2$

$\oint_S (E\times H^*)\, ds = j\omega\epsilon\int_V E^2 dv - j\omega\mu\int_V H^2 dv - \sigma\int_V E^2 dv$

Integrate over a volume V and apply the divergence theorem to the $L.H.S.$ of the result

$$\oint_S \left(\mathbf{E}\times\mathbf{H}^*\right)\cdot d\mathbf{s} = \int_V \sigma E^2 dv + j\omega\int_V \left(\varepsilon E^2 - \mu H^2\right) dv \qquad (11)$$

$$\oint_S \left(\mathbf{E}\times\mathbf{H}^*\right)\cdot d\mathbf{s} = \int_V \sigma E^2 dv + j\omega\int_V \left(\varepsilon E^2 - \mu H^2\right) dv \qquad (11)$$

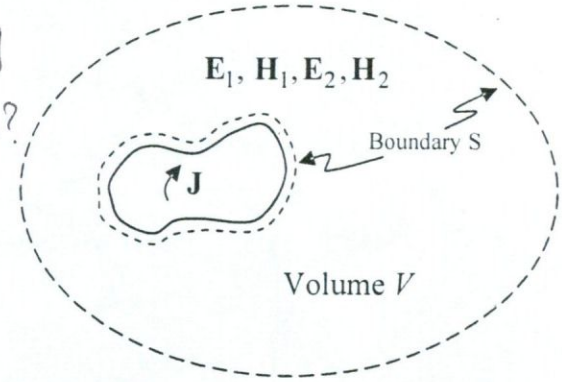
باؤنڈری پہ surface کا راج ہے نہ کہ field کا. اسلئے دونوں E_1, E_2 ایک ہوں گے باؤنڈری پہ جس کی وجہ سے

Note: The two solutions $E_{\substack{1\\2}}$, $H_{\substack{1\\2}}$ must be the same on the boundary (S) because both satisfy boundary conditions. isn't it stating the obvious?

یہ ہوگا

$\Rightarrow \mathbf{E} = \mathbf{H} = 0$ on S $\quad \Rightarrow \oint_S \left(\mathbf{E}\times\mathbf{H}^*\right)\cdot d\mathbf{s} = 0$

Since this is different



Since the $L.H.S.$ of (11) is zero, equate real and imaginary parts of the $R.H.S.$ to zero

- Real part:

$$\int \sigma E^2 dv = 0 \Rightarrow E^2 = 0 \text{ or } E = 0 \text{ in } \upsilon$$

- Imag part: Since $E^2 = 0$ from above then

$$j\omega\int_V \varepsilon E^2 dv = 0$$

and therefore

$$j\omega\int_V \mu H^2 dv = 0 \Rightarrow H^2 = 0 \text{ or } H = 0 \text{ in } V$$

Since $E = |\mathbf{E}|$ and $H = |\mathbf{H}|$ then

$$\mathbf{E}_2 - \mathbf{E}_1 = \mathbf{E} = 0 \text{ and } \quad \mathbf{H}_2 - \mathbf{H}_1 = \mathbf{H} = 0$$

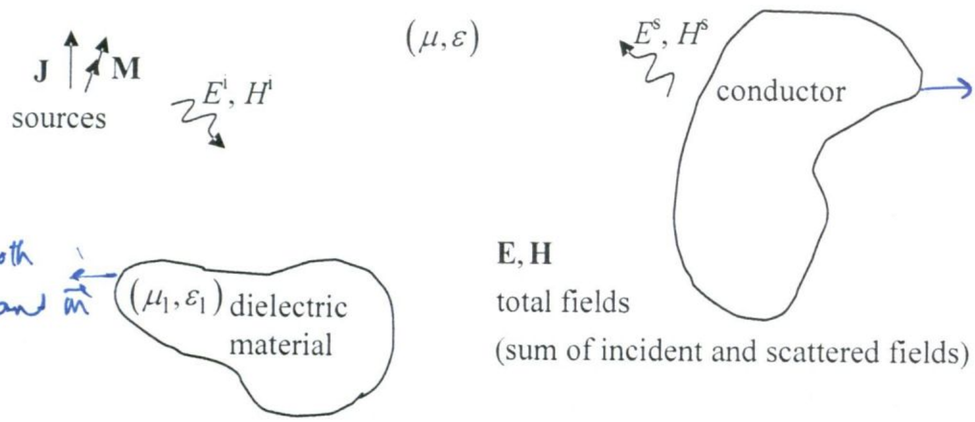
؟!؟ $\Rightarrow \mathbf{E}_2 = \mathbf{E}_1$ and $\mathbf{H}_2 = \mathbf{H}_1$ واہ بھئی

$\therefore$ If the fields in a region satisfy Maxwell's equations and the boundary conditions then they are unique – That is, the two solutions must be the same (are unique) in the volume contained by the boundary.

Object's presence around the antenna can be not derived.

Equivalence Principle

Suppose the sources radiate in the presence of some objects



all we need is J

both $\vec{J}$ and $\vec{m}$

Then we cannot use Green's function in free space.

$$\mathbf{A} = \frac{\mu}{4\pi} \int_{v'} \mathbf{J} \frac{e^{-jkR}}{R} dv' \qquad (1a)$$

$$\mathbf{F} = \frac{\varepsilon}{4\pi} \int_{v'} \mathbf{M} \frac{e^{-jkR}}{R} dv' \qquad (1b)$$

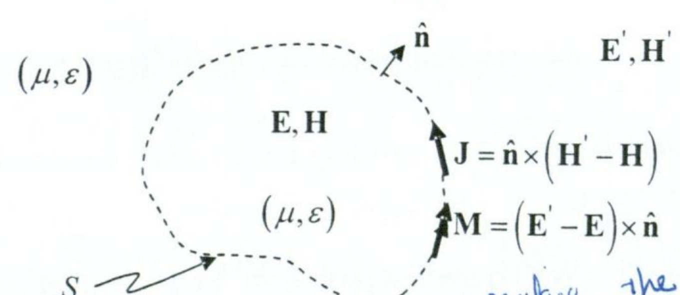
to find the fields in the problem because this particular vector potential $\mathbf{A}$ does not satisfy the boundary conditions created by the presence of the 2 bodies.

Construction of the equivalent problem:

Suppose we want to find fields inside a dashed region shown above. If we had been able to solve the original problem, then we would know the fields on the dashed surface and therefore we could construct the interior equivalent problem as follows:

How to find $\vec{J}$?

assume all E' & H' to be zero outside



→ tangential

notice the order

Requirement: maintain the field inside

Note:

(i) The medium (μ, ε) is now the same everywhere so (1a) and (1b) can be used.

(ii) $\mathbf{E}$ and $\mathbf{H}$ inside the dashed region remain the same as in the original problem.

(iii) Note that $\mathbf{E}'$ and $\mathbf{H}'$ are completely arbitrary. This is so because we only want to maintain the field inside S. If we change $\mathbf{E}'$ and $\mathbf{H}'$, $\mathbf{J}$ and $\mathbf{M}$ will change so as to maintain the interior $\mathbf{E}$ and $\mathbf{H}$.

$$\mathbf{J} = \hat{\mathbf{n}} \times (\mathbf{H}' - \mathbf{H})$$

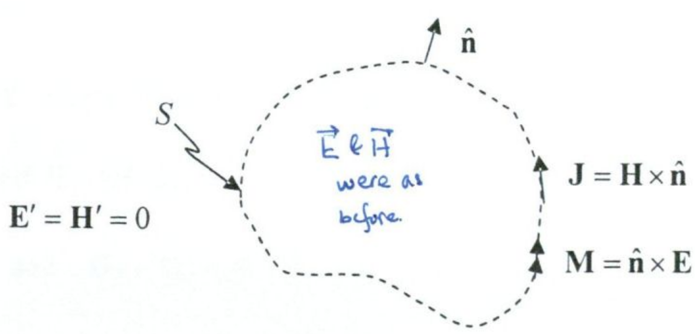
$$\mathbf{M} = (\mathbf{E}' - \mathbf{E}) \times \hat{\mathbf{n}}$$

$f(\mathbf{E}', \mathbf{H}', \mathbf{E}, \mathbf{H})$ arbitrary fixed

This means that $\mathbf{J}$ and $\mathbf{M}$ are not unique.

Since we can set $\mathbf{E}$ and $\mathbf{H}$ equal to anything, then in equivalence problems we always let $\mathbf{E}' = \mathbf{H}' = 0$

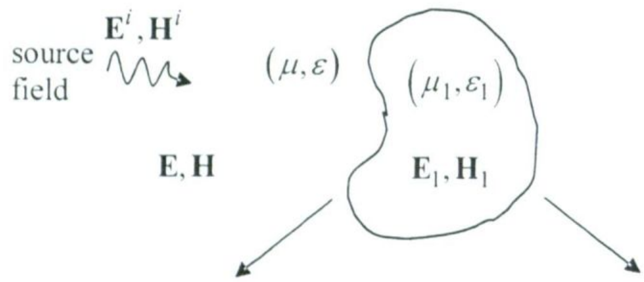
So if the fields in the original problem are known everywhere then the original problem (original sources, dielectric and conductor) can be replaced by



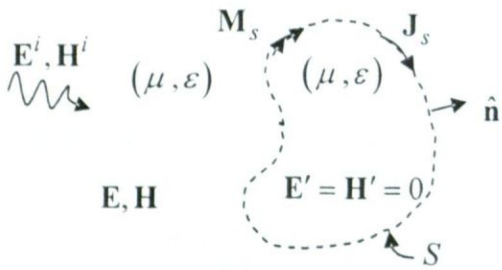
(μ, ε) (everywhere)

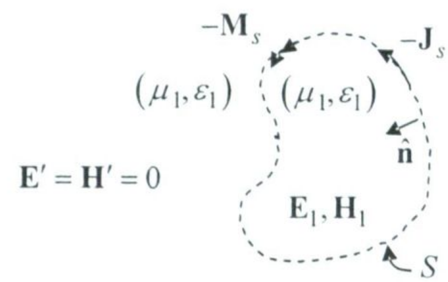
Example

equivalence: dielectric body



Exterior problem (see pg 4-11) Interior Problem





Exterior problem:

$$\mathbf{E} = \mathbf{E}_{scat} + \mathbf{E}_i$$

$$\mathbf{E} = -\frac{j\omega\mu}{4\pi k^2}\left(k^2 + \nabla\nabla\cdot\right)\underbrace{\int_S \mathbf{J}_s(\mathbf{r}')\frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}ds'}_{\overline{A}}$$

$$-\nabla\times\underbrace{\int_S \mathbf{M}_s(\mathbf{r}')\frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}ds'}_{\overline{F}}$$

is known $\leftarrow$ $+\mathbf{E}^i$

$$\mathbf{H} = \mathbf{H}_{scat} + \mathbf{H}_i$$

$$\mathbf{H} = \nabla\times\underbrace{\int_S \frac{\mathbf{J}_s(\mathbf{r}')e^{-jk|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}ds'}_{\vec{A}}$$

$$-\frac{j\omega\varepsilon}{4\pi k^2}\left(k^2 + \nabla\nabla\cdot\right)\underbrace{\int_S \frac{\mathbf{M}_s(\mathbf{r}')e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}ds'}_{\overline{F}}$$

$$+\mathbf{H}^i$$

$\hat{n}\times(\vec{H}_1 - \vec{H}_2) = \vec{J}_s \neq 0$ since we have a dielectric

Interior problem:

$$\mathbf{E}_1 = \frac{-j\omega\mu_1}{4\pi k_1^2}\left(k_1^2 + \nabla\nabla\cdot\right)\int_S \frac{-\mathbf{J}_s(\mathbf{r}')e^{-jk_1|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}ds'$$

$$-\nabla\times\int_S \frac{-\mathbf{M}_s(\mathbf{r}')e^{-jk_1|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}ds'$$

$$\mathbf{H}_1 = \nabla\times\int_S \frac{-\mathbf{J}_s(\mathbf{r}')e^{-jk_1|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}ds'$$

$$\frac{-j\omega\varepsilon_1}{4\pi k_1^2}\left(k^2 + \nabla\nabla\cdot\right)\int_S \frac{-\mathbf{M}_s(\mathbf{r}')e^{-jk_1|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}ds'$$

Note: $\mathbf{J}_s$ (inside) $= -\mathbf{J}_s$ (outside) and $\mathbf{M}_s$ (inside) $= -\mathbf{M}_s$ (outside) because the tangential components of $\mathbf{E}$ and $\mathbf{H}$ are continuous across the boundary and the $\hat{\mathbf{n}}'s$ are in opposite directions.
$\underset{out}{\hat{\mathbf{n}}} \times \left(\mathbf{H}^{outside} - \mathbf{H}^{inside}\right) = 0 \rightarrow \underset{out}{\hat{\mathbf{n}}} \times \mathbf{H}^{outside} = \underset{out}{\hat{\mathbf{n}}} \times \mathbf{H}^{inside}$ but $\underset{out}{\hat{\mathbf{n}}} \times \mathbf{H}^{inside} = -\underset{out}{\hat{\mathbf{n}}} \times \mathbf{H}^{inside} = \underset{in}{\hat{\mathbf{n}}} \times \mathbf{H}^{inside}$
requires that $\mathbf{J}_s$ (inside) $= -\mathbf{J}_s$ (outside)

2-equation 2 unknowns.

$$\hat{\mathbf{n}}\times\left(\mathbf{E}-\mathbf{E}_1\right)\underset{\mathbf{r}=\mathbf{s}}{=}0$$ Dielectric

$$\frac{j\omega\mu}{4\pi k^2}\hat{\mathbf{n}}\times\left(k^2+\nabla\nabla\cdot\right)\iint_S\frac{\mathbf{J}_s(\mathbf{s}')e^{-jk|\mathbf{s}-\mathbf{s}'|}}{|\mathbf{s}-\mathbf{s}'|}ds'$$

$$+\frac{j\omega\mu_1}{4\pi k_1^2}\hat{\mathbf{n}}\times\left(k_1^2+\nabla\nabla\cdot\right)\iint_S\frac{\mathbf{J}_s(\mathbf{s}')e^{-jk_1|\mathbf{s}-\mathbf{s}'|}}{|\mathbf{s}-\mathbf{s}'|}ds'$$

$$+\hat{\mathbf{n}}\times\nabla\times\int_S\frac{\mathbf{M}_s(\mathbf{s}')e^{-jk|\mathbf{s}-\mathbf{s}'|}}{4\pi|\mathbf{s}-\mathbf{s}'|}ds'+\hat{\mathbf{n}}\times\nabla\times\int_S\frac{\mathbf{M}_s(\mathbf{s}')e^{-jk_1|\mathbf{s}-\mathbf{s}'|}}{|\mathbf{s}-\mathbf{s}'|}ds'$$

$$=\hat{\mathbf{n}}\times\mathbf{E}^i \qquad \text{(a)}$$

For dielectric ~~Cons~~ $\vec{J}_s$ and $\vec{M}_s$ are not 'really' there. Just there for finding out ~~M~~ $\vec{E}$ and $\vec{H}$. That's why it is called Equivalence Principle

continuity of magnetic field across surface

$$\hat{\mathbf{n}}\times\left(\mathbf{H}-\mathbf{H}_1\right)=0$$

$$\hat{\mathbf{n}}\times\nabla\times\frac{1}{4\pi}\int_S\frac{\mathbf{J}_s(\mathbf{s}')e^{-jk|\mathbf{s}-\mathbf{s}'|}}{|\mathbf{s}-\mathbf{s}'|}ds'+\hat{\mathbf{n}}\times\nabla\times\frac{1}{4\pi}\int_S\frac{\mathbf{J}_s(\mathbf{s}')e^{-jk_1|\mathbf{s}-\mathbf{s}'|}}{|\mathbf{s}-\mathbf{s}'|}ds'$$

$$\frac{-j\omega\varepsilon}{4\pi k^2}\hat{\mathbf{n}}\times\left(k^2+\nabla\nabla\cdot\right)\iint_S\frac{\mathbf{M}_s(\mathbf{s}')e^{-jk|\mathbf{s}-\mathbf{s}'|}}{|\mathbf{s}-\mathbf{s}'|}ds'$$

$$\frac{-j\omega\varepsilon_1}{4\pi k_1^2}\hat{\mathbf{n}}\times\left(k_1^2+\nabla\nabla\cdot\right)\iint_S\frac{\mathbf{M}_s(\mathbf{s}')e^{-jk_1|\mathbf{s}-\mathbf{s}'|}}{|\mathbf{s}-\mathbf{s}'|}ds'$$

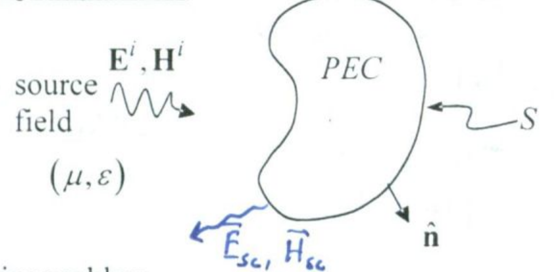
$$=-\hat{\mathbf{n}}\times\mathbf{H}^i\left(=-\hat{\mathbf{n}}\times\frac{\mathbf{k}\times\mathbf{E}^i}{\eta}\text{(if a plane wave)}\right) \qquad \text{(b)}$$

∴ (a) & (b) are 2 eqns with 2 unknowns. There is no analytical solution. The method of moments (MoM) numerical method is used to obtain an approximate solution for $\mathbf{J}_s$ and $\mathbf{M}_s$.

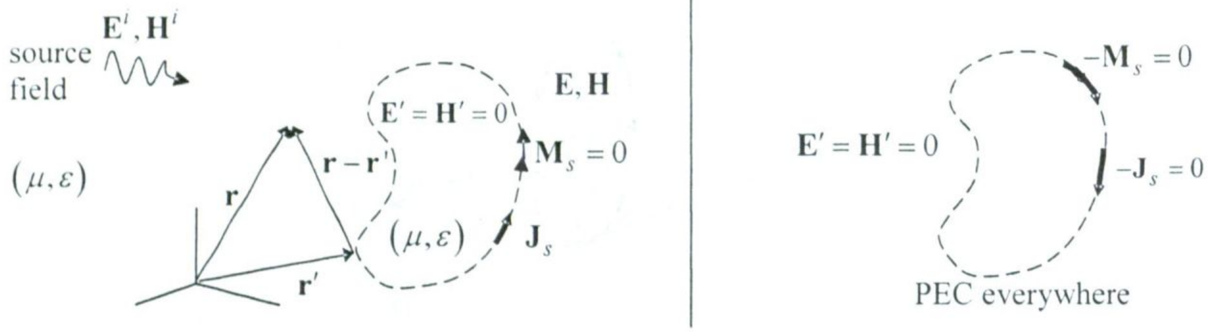
So $M_s = 0$

$E_{tan} = 0$

Example: Equivalence principle for a PEC



Exterior problem | Interior problem



The electric field due to the source $(\mathbf{J}_s)$ and the incident field $(\mathbf{E}^i)$ make up the exterior field

$$\mathbf{E} = \mathbf{E}^{scat} + \mathbf{E}^i = -\frac{j\omega}{k^2}\left(k^2\mathbf{A} + \nabla\nabla\cdot\mathbf{A}\right) + \mathbf{E}^i$$

Now $\nabla\times\vec{F}$ since $\vec{M}_s \approx 0$

Since

$$\mathbf{n}\times\left(\mathbf{E} - \cancelto{0}{\mathbf{E}_1}\right)_{\mathbf{r}=\mathbf{s}} = 0$$

$$\hat{\mathbf{n}}\times\left(\mathbf{E}^{scat} + \mathbf{E}^i\right) = 0$$

$$\hat{\mathbf{n}}\times\mathbf{E}^{scat} = -\hat{\mathbf{n}}\times\mathbf{E}^i$$

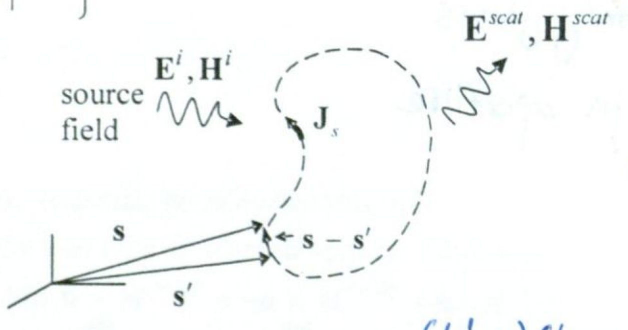
The complete integral equation is the Electric field integral equation:

$$\frac{-j\omega\mu}{4\pi k^2}\hat{\mathbf{n}}\times\left\{\left(k^2 + \nabla\nabla\cdot\right)\iint_s \mathbf{J}_s(\mathbf{s}')\frac{e^{-jk|\mathbf{s}-\mathbf{s}'|}}{|\mathbf{s}-\mathbf{s}'|}ds'\right\} = -\hat{\mathbf{n}}\times\mathbf{E}^i(s)$$

only unknown

There is only a single unknown since $\vec{M}_s = 0$

$\vec{J}_s$ is actual current here for PEC case

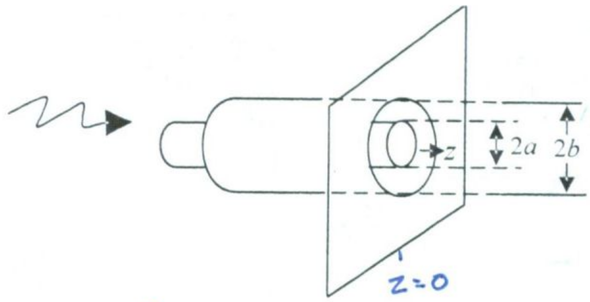


$\delta(\rho' - a)\,\delta($

$|\vec{r} - \vec{r}'| = \sqrt{(\vec{r}-\vec{r}')\cdot(\vec{r}-\vec{r}')}$

$= r^2 + r'^2 - 2\vec{r}\cdot\vec{r}'$

Sect 3.6 – Approximate solution for a coaxial line opening into a conducting plane $(b << \lambda)$



Interior Field

$$\mathbf{E}^{\text{int}} = \mathbf{E}^{+} + \mathbf{E}^{-}$$

$$\doteq -\frac{V_0}{\rho \ln(b/a)} e^{-jkz}\hat{\boldsymbol{\rho}} - \overset{\overset{=1 \text{ for o.c}}{\uparrow}}{\Gamma} \frac{V_0}{\rho \ln(b/a)} e^{jkz}\hat{\boldsymbol{\rho}}$$

(assuming only TEM mode and neglecting higher order modes)

incident TEM mode

reflection coefficient at an open circuit $\simeq +1$

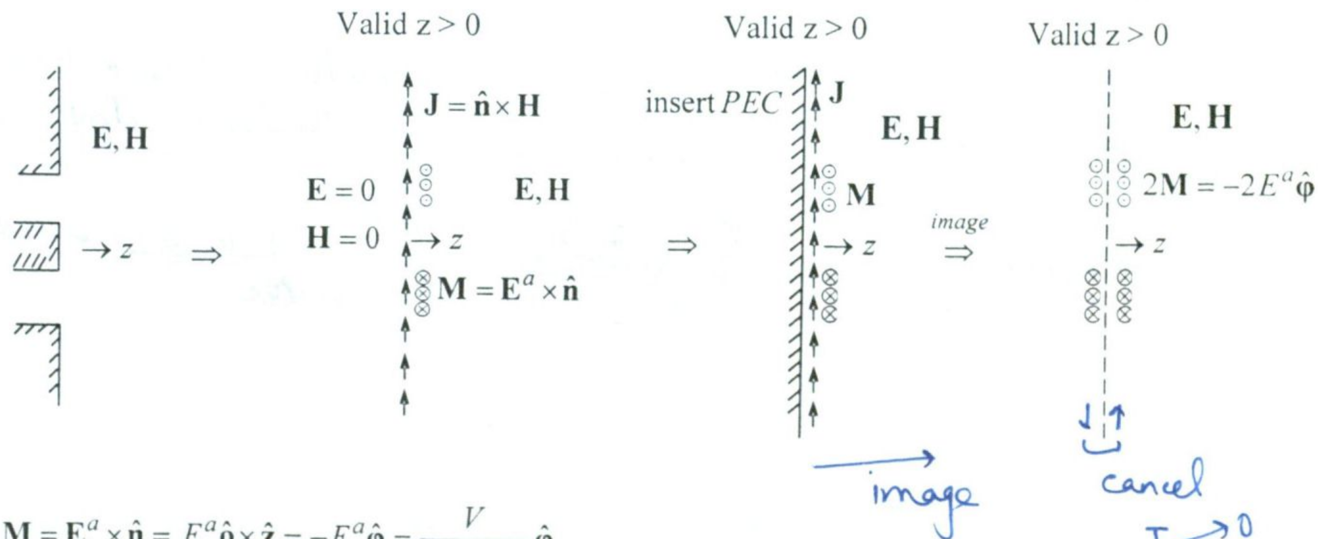
why not PMC.

- In the aperture $(z = 0)$

$$\mathbf{E}^{\text{int}} = \mathbf{E}^{a} \simeq -\frac{2V_0}{\rho \ln(b/a)}\hat{\boldsymbol{\rho}} \qquad \left(-\frac{V}{\rho \ln(b/a)}\hat{\boldsymbol{\rho}} \text{ in Harrington} \right)$$

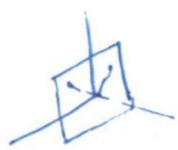
Try PMC

We get infinite sheet of current.



$$\mathbf{M} = \mathbf{E}^{a} \times \hat{\mathbf{n}} = E^{a}\hat{\boldsymbol{\rho}} \times \hat{\mathbf{z}} = -E^{a}\hat{\boldsymbol{\varphi}} = \frac{V}{\rho \ln b/a}\hat{\boldsymbol{\varphi}}$$

$$\mathbf{E} = -\frac{1}{\varepsilon}\nabla \times \mathbf{F} = -\nabla \times \frac{1}{4\pi} \int_{\rho'=a}^{b} \int_{\phi'=0}^{2\pi} 2\mathbf{M}(\rho') \frac{e^{-jkR}}{R} \rho' d\rho' d\phi'$$



x_1'

$\nabla \times F$

dz

$\int M$

$$\mathbf{E} = -\nabla \times \frac{1}{4\pi} \int_{\rho'=a}^{b} \int_{\phi'=0}^{2\pi} \frac{2V}{\cancel{\rho'} \ln b/a} \hat{\boldsymbol{\phi}}' \frac{e^{-jkR}}{R} \cancel{\rho'} d\phi' d\rho'$$

$$\hat{\boldsymbol{\phi}}' = (\hat{\boldsymbol{\phi}}' \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} + (\hat{\boldsymbol{\phi}}' \cdot \hat{\boldsymbol{\theta}})\hat{\boldsymbol{\theta}} + (\hat{\boldsymbol{\phi}}' \cdot \hat{\boldsymbol{\phi}})\hat{\boldsymbol{\phi}}$$

$$= \sin\theta \sin(\phi - \phi')\hat{\mathbf{r}} + \cos\theta \sin(\phi - \phi')\hat{\boldsymbol{\theta}} + \cos(\phi - \phi')\hat{\boldsymbol{\phi}}$$

(from notes #5)

The magnetic current is independent of ϕ so the field will be ϕ independent $\Rightarrow$ choose $\phi = 0$

$$\hat{\boldsymbol{\phi}}' = -\sin\theta \sin\phi' \hat{\mathbf{r}} - \cos\theta \sin\phi' \hat{\boldsymbol{\theta}} + \cos\phi' \hat{\boldsymbol{\phi}}$$

$$\frac{e^{-jkR}}{R} = \frac{e^{-jk(r - \rho' \cos\phi' \sin\theta)}}{r}$$

$$\mathbf{E} = -\frac{2V}{4\pi \ln(b/a)} \nabla \times \frac{e^{-jkr}}{r} \int_{\rho'=a}^{b} \left\{ \int_{\phi'=0}^{2\pi} e^{jk\rho' \cos\phi' \sin\theta} \times \left[\underbrace{-\sin\theta \sin\phi' \hat{\mathbf{r}}}_{I_r} \underbrace{- \cos\theta \sin\phi' \hat{\boldsymbol{\theta}}}_{I_\theta} \underbrace{+ \cos\phi' \hat{\boldsymbol{\phi}}}_{I_\phi} \right] d\phi' \right\} d\rho'$$

$$I_\phi = \int_{\phi'=0}^{2\pi} \cos\phi' e^{jk\rho' \cos\phi' \sin\theta} d\phi' = j2\pi J_1(k\rho' \sin\theta)$$ from (9.1.21) A&S

$$\doteq j2\pi \frac{k\rho'}{2} \sin\theta$$ (when $\rho' << \lambda$) from (9.1.7) A&S

$$= j\pi k\rho' \sin\theta$$

$$I_r = -\sin\theta \int_{\phi'=0}^{2\pi} \sin\phi' e^{jk\rho' \cos\phi' \sin\theta} d\phi' = -\sin\theta \frac{2\pi^{1/2}}{\left(\frac{k\rho' \sin\theta}{2}\right)^{1/2}} J_{1/2}(k\rho' \sin\theta)$$ (9.1.20)A&S

$$= -\sin\theta \frac{2\pi^{1/2}}{\left(\frac{k\rho' \sin\theta}{2}\right)^{1/2}} \left[\left(\frac{2k\rho' \sin\theta}{\pi}\right) j_0(k\rho' \sin\theta) \right]$$

$$= -4\sin\theta \, j_0(k\rho' \sin\theta)$$

$$= -4\sin\theta \frac{\sin(k\rho' \sin\theta)}{\underbrace{k\rho' \sin\theta}_{\simeq 1 (\rho' << \lambda)}}$$ (10.1.11) A&S

$$= -4\sin\theta$$

$$I_\theta = -\cos\theta \int_{\phi'=0}^{2\pi} \sin\phi' e^{jk\rho'\cos\phi'\sin\theta} d\phi' = -4\cos\theta$$

$$\mathbf{E} = -\frac{V}{2\pi \ln\left(b/a\right)} \nabla \times \frac{e^{-jkr}}{r} \left\{ j\pi k \sin\theta \int_{\rho'=a}^{b} \rho' d\rho' \hat{\boldsymbol{\varphi}} \right.$$

$$\left. -4\left(\sin\theta \hat{\mathbf{r}} + \cos\theta \hat{\boldsymbol{\theta}}\right) \int_{\rho'=a}^{b} d\rho' \right\}$$

$$= -\frac{V}{2\pi \ln\left(b/a\right)} \nabla \times \frac{e^{-jkr}}{r} \left[\frac{j\pi k \sin\theta}{2} \left(b^2 - a^2\right) \hat{\boldsymbol{\varphi}} \right.$$

$$\left. -4(b-a)\left(\sin\theta \hat{\mathbf{r}} + \cos\theta \hat{\boldsymbol{\theta}}\right) \right]$$

$$= -\frac{V}{2\pi \ln\left(b/a\right)} \left\{ \frac{j\pi k}{2} \left(b^2 - a^2\right) \left[\frac{e}{r}^{-jkr} \left(\frac{1}{r \sin} \theta \frac{\partial}{\partial\theta} \sin^2\theta \right) \hat{\mathbf{r}} \right. \right.$$

$$\left. -\frac{1}{r} \sin\theta \frac{\partial}{\partial r} e^{-jkr} \hat{\boldsymbol{\theta}} \right]$$

$$+4(b-a) \left[\frac{e^{-jkr}}{r} \left(\frac{1}{r} \frac{\partial}{\partial\theta} \sin\theta \right) \right] \hat{\boldsymbol{\varphi}}$$

$$+4(b-a) \left[\frac{e^{-jkr}}{r} \cos\theta \left(\frac{1}{r} \frac{\partial}{\partial r} e^{-jkr} \right) \right] \hat{\boldsymbol{\varphi}}$$

$$= -j \frac{V}{4 \ln b/a} k \left(b^2 - a^2\right) \left[-\frac{1}{r} \sin\theta (-jk) e^{-jkr} \hat{\boldsymbol{\theta}} \right]$$

$$E_\theta = \frac{Vk^2}{4 \ln b/a} \left(b^2 - a^2\right) \frac{e^{-jkr}}{r} \sin\theta$$

quite similar to what we got in normally.

$$H_\phi = \frac{Vk^2}{4\eta \ln b/a} \left(b^2 - a^2\right) \frac{e^{-jkr}}{r} \sin\theta \rightarrow \text{eqn 3-20 with } \frac{k^2}{4\eta} \rightarrow \frac{\omega\varepsilon\pi}{2\lambda}$$