ELECTRIC VECTOR POTENTIAL DUE TO AN ELECTRIC LINE SOURCE ABOVE A LOSSY HALF-SPACE

Equivalence Principle Approach

IN THIS SECTION WE ## WISH TO CONSIDER AN EXAMPLE ILLUSTRATING THE APPLICATION OF THE EQUIVALENCE PRINCIPLE TO THE PROBLEM OF CALCULATING THE ELECTRIC VECTOR POTENTIAL DUE TO AN ELECTRIC LINE SOURCE IN ONE MEDIUM OF PROPERTIES (μ_a, ϵ_a), WHICH IS ABOVE ANOTHER MEDIUM CHARACTERIZED BY (μ_b, ϵ_b). THE LINE SOURCE IS Z-DIRECTED AND PARALLEL TO THE INTERFACE BETWEEN THE TWO HOMOGENEOUS MEDIA # AS SHOWN IN FIG. 1A. THE PERMITTIVITIES OF THE TWO MEDIA ARE CONSIDERED COMPLEX IN ORDER THAT EACH MEDIA MAY BE CONSIDERED AS GENERAL LINEAR ###### ELECTRO- ##### MAGNETIC MATERIAL.

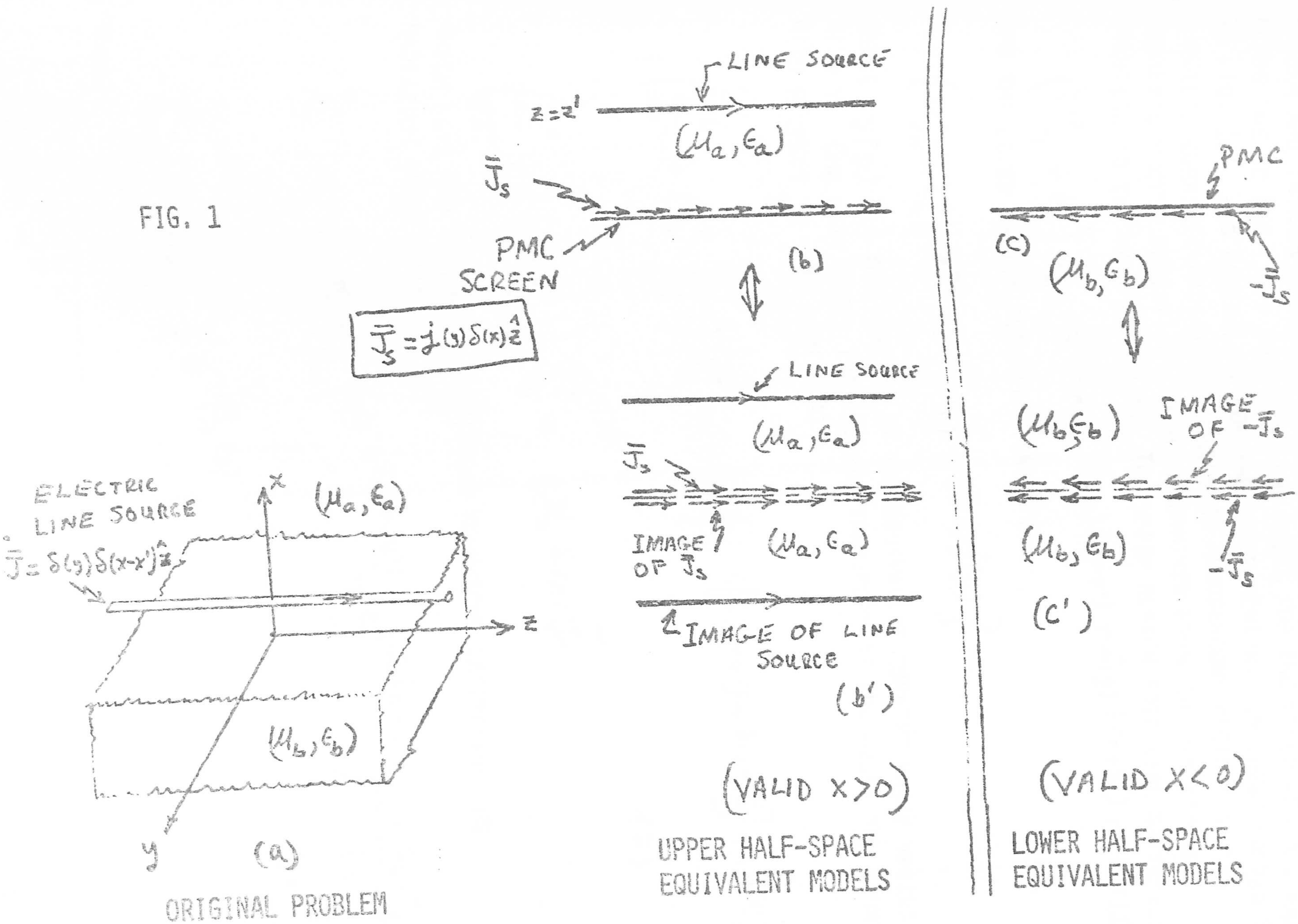
WE SOLVE THE PROBLEM BY CONSTRUCTING TWO EQUIVALENT MODELS, ONE VALID FOR THE UPPER HALF-SPACE AND THE OTHER FOR THE LOWER SPACE. ACCORDING TO THE EQUIVALENCE PRINCIPLE, WE MAY PLACE A PERFECTLY CONDUCTING MAGNETIC (PMC) SCREEN AT THE INTERFACE AND ON THE UPPER SIDE OF THIS SCREEN WE PLACE A SURFACE ELECTRIC CURRENT J_S HAVING VALUE

$$J_S = \hat{x} \times \bar{H}(0) = \mathcal{J}(y)\, \delta(x)\, \hat{z}$$

WHERE $\bar{H}(0)$ IS THE TOTAL MAGNETIC FIELD AT THE INTERFACE IN FIG. 1A. THIS SURFACE ELECTRIC CURRENT SERVES TO CAUSE THE TRANSVERSE ######## MAGNETIC FIELD TO JUMP FROM ZERO AT $x = 0$ TO $\hat{x} \times \bar{H}(0)$ AT $x = 0^+$, THEREBY RESTORING THE BOUNDARY CONDITIONS OF THE ORIGINAL PROBLEM IN SO FAR AS THE #### UPPER HALF-SPACE IS CONCERNED. HENCE, THE EQUIVALENT MODEL OF FIG. 1B REPRESENTS THE ORIGINAL PROBLEM IN THE UPPER HALF-SPACE (OPEN); ONE SHOULD OBSERVE THAT THE ##### ORIGINAL LINE SOURCE IS RETAINED IN THIS EQUIVALENT MODEL. IN A SIMILAR MANNER, FIG. 1C IS THE APPROPRIATE MODEL FOR THE LOWER HALF-SPACE, IN WHICH THERE IS NO SOURCE SINCE THERE IS NONE IN THE LOWER REGION OF# THE ORIGINAL PROBLEM. NOTICE THAT THE EQUIVALENT SURFACE CURRENT OF THE LOWER REGION EQUIVALENT MODEL IS $-J_S$, THE REVERSAL IN DIRECTION BEING NECESSARY SO THAT THE TRANSVERSE MAGNETIC FIELD IN THE LOWER REGION JUMPS FROM ZERO AT $x = 0$ TO $\hat{x} \times \bar{H}(0)$ AT $x = 0^-$.

IN BOTH FIGS. 1B AND 1C, THE CURRENTS RADIATE IN THE PRESENCE OF THE ## PMC PLANAR SCREEN SO WE APPEAL TO IMAGE THEORY TO REMOVE THIS

FIG. 1



ELECTRIC LINE SOURCE
$\bar{J} = \delta(y)\delta(x-x')\hat{z}$
(μ_a, ϵ_a)
(μ_b, ϵ_b)
x
y
z
(a)
ORIGINAL PROBLEM
LINE SOURCE
$z = z'$
(μ_a, ϵ_a)
$\bar{J}_s$
PMC SCREEN
$\boxed{\bar{J}_s = j(y)\delta(x)\hat{z}}$
(b)
LINE SOURCE
(μ_a, ϵ_a)
$\bar{J}_s$
IMAGE OF $\bar{J}_s$
(μ_a, ϵ_a)
IMAGE OF LINE SOURCE
(b')
(VALID X>0)
UPPER HALF-SPACE EQUIVALENT MODELS
PMC
(c)
(μ_b, ϵ_b)
$-\bar{J}_s$
(μ_b, ϵ_b)
IMAGE OF $-\bar{J}_s$
(μ_b, ϵ_b)
$-\bar{J}_s$
(c')
(VALID X<0)
LOWER HALF-SPACE EQUIVALENT MODELS

SCREEN AND INTRODUCE APPROPRIATE IMAGE C URRENTS. THIS PROCEDURE LEADS ONE TO FIG. 1B', AN EQUIVALENT MODEL FOR THE UPPER REGION, AND TO FIG. 1C', AND EQUIVALENT FOR THE LOWER REGION. IN THESE LATTER TWO M ODELS, CURRENTS RADIATE IN UNBOUNDED HOMOGENEOUS REGIONS SO THE MAGNETIC VECTOR POTENTIAL CAN BE DETERMINED FROM THE PARTICULAR SOLUTION TO THE WAVE EQUATION. THE UPPER REGION POTENTIAL SATISFIES

$$\left(\nabla_{xy}^2 + k_a^2\right) A_z^a = -\mu_a \left\{ \delta(y)\,\delta(x-x') + \delta(y)\,\delta(x+x') + 2\,\delta(x)\,\mathcal{J}(y) \right\} \tag{1}$$

AND THE LOWER REGION POTENTIAL SATISFIES

$$\left(\nabla_{xy}^2 + k_b^2\right) A_z^b = -\mu_b \left\{ -2\,\delta(x)\,\mathcal{J}(y) \right\}$$

WHERE $\mathcal{J}(y)$ IS THE PORTION OF THE EQUIVALENT CURRENT WHICH VARIES WITH Y. ONE NEXT TAKES THE FOURIER TRANSFORM OF THE ABOVE TWO EQUATIONS WITH RESPECT TO Y TO ARRIVE AT

$$\left(\frac{d^2}{dx^2} + \left(k_a^2 - k_y^2\right)\right) \tilde{A}_z^a = -\mu_a \left\{ \left[\delta(x-x') + \delta(x+x')\right] + 2\,\delta(x)\,\tilde{\mathcal{J}} \right\} \tag{2}$$

$$\left(\frac{d^2}{dx^2} + \left(k_b^2 - k_y^2\right)\right) \tilde{A}_z^b = 2\mu_b\,\delta(x)\,\tilde{\mathcal{J}}$$

WHICH HAVE THE FOLLOWING PARTICULAR SOLUTIONS SATISFYING THE RADIATION CONDITION:

$$\tilde{A}_z^a = -j\,\mu_a \frac{1}{2\beta_a}\left(e^{-j\beta_a|x-x'|} + e^{-j\beta_a|x+x'|}\right) - j\,\frac{\mu_a}{\beta_a}\tilde{\mathcal{J}}\,e^{-j\beta_a|x|} \tag{3}$$

$$\tilde{A}_z^b = j\,\frac{\mu_b}{\beta_b}\,\tilde{\mathcal{J}}\,e^{-j\beta_b|x|}$$

WHERE

$$\beta_{\substack{a\\b}} = \sqrt{k_{\substack{a\\b}}^2 - k_y^2} \tag{4}$$

THE ELECTRIC FIELD IS CALCULATED FROM THE VECTOR POTENTIAL BY ~~MEA~~ MEANS OF

$$\bar{E} = -j\frac{\omega}{k^2}\left(k^2\bar{A} + \nabla\nabla\cdot\bar{A}\right) \tag{5}$$

WHICH, FOR THE COMPONENT OF THE FIELD PARALLEL TO THE INTERFACE, REDUCES TO

$$E_z^{\substack{a\\b}} = -j\omega A_z^{\substack{a\\b}} \tag{6}$$

FROM WHICH ONE CONCLUDES THAT THE ~~[illegible]~~ MAGNETIC VECTOR POTENTIAL (IN THE TRANSFORM DOMAIN) MUST SATISFY THE CONTINUITY CONDITION,

$$\tilde{A}_z^a\Big|_{x=0^+} = \tilde{A}_z^b\Big|_{x=0^-} \tag{7}$$

FROM EQS. (3) AND (7), ONE READILY DETERMINES THE TRANSFORM OF $\mathcal{J}$ TO BE

$$\tilde{\mathcal{J}} = -\mu_a \frac{\beta_b}{\mu_a\beta_b + \mu_b\beta_a} e^{-j\beta_a|x'|}, \quad x' > 0 \tag{8}$$

WHICH, IN VIEW OF (3), LEADS TO

$$\tilde{A}_z^a = -j\frac{\mu_a}{2}\left(\frac{e^{-j\beta_a|x-x'|}}{\beta_a} + \frac{e^{-j\beta_a|x+x'|}}{\beta_a}\right)$$

$$+j\mu_a^2 \frac{\beta_b}{\mu_a\beta_b + \mu_b\beta_a}\frac{e^{-j\beta_a|x+x'|}}{\beta_a}, \quad \begin{matrix} x>0 \\ x'>0 \end{matrix} \tag{9}$$

THE TRANSFORM OF THE UPPER HALF-SPACE MAGNETIC VECTOR POTENTIAL,

ONE TAKES THE INVERSE TRANSFORM OF (9), WHICH IS

$$A_z^a = -j \frac{\mu_a}{4\pi} \int_{k_y=-\infty}^{\infty} \left(\frac{e^{-j\beta_a|x-x'|}}{\beta_a} + \frac{e^{-j\beta_a|x+x'|}}{\beta_a} \right) e^{jk_y y}\, dk_y$$

$$+ j \frac{\mu_a^2}{2\pi} \int_{k_y=-\infty}^{\infty} \frac{\beta_b}{\mu_a\beta_b + \mu_b\beta_a} \frac{e^{-j\beta_a|x+x'|}}{\beta_a} e^{jk_y y}\, dk_y \qquad (10)$$

THE FIRST INTEGRAL IS READILY RECONIZED TO BE PROPORTI@NAL TO TWO HANKEL FUNCTIONS, ## BUT THE SECOND MUST BE EVALUATED BY APPROPRIATE APPROXIMATE TECHNIQUES:

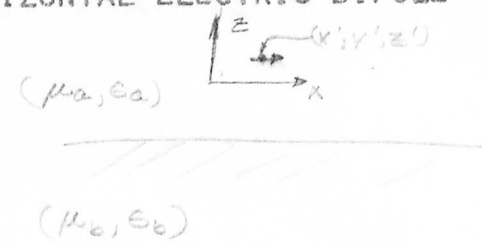
$$A_z^a = -j\frac{\mu_a}{4}\left\{ H_0^{(2)}\left(k_a\sqrt{(x-x')^2+y^2}\right) + H_0^{(2)}\left(k_a\sqrt{(x+x')^2+y^2}\right) \right\}$$

$$+ j \frac{\mu_a}{\pi} \int_{k_y=0}^{\infty} \frac{\beta_b}{\beta_b + \frac{\mu_b}{\mu_a}\beta_a} \frac{e^{-j\beta_a|x+x'|}}{\beta_a} \cos k_y y\, dk_y \qquad (11)$$

IN SUMMARY, THEN, WE # HAVE CALCULATED THE UPPER REGION VECTOR POTENTIAL BY MEANS OF THE ###### EQUIVALENCE PRINCIPLE.

3-D

MAGNETIC VECTOR POTENTIAL FOR A HORIZONTAL ELECTRIC DIPOLE ABOVE A LOSSY HALF-SPACE



BASIC EQUATIONS

$$\bar{J} = \delta(\bar{r}-\bar{r}')\hat{x}$$

$$(\nabla^2+k_a^2)A_x^a = -\mu_a\,\delta(\bar{r}-\bar{r}') \quad , z \geq -h \qquad (1a)$$

$$(\nabla^2+k_b^2)A_x^b = 0 \quad , z \leq -h \qquad (1b)$$

AND

$$(\nabla^2+k_a^2)A_z^a = 0 \quad , z \geq -h \qquad (2a)$$

$$(\nabla^2+k_b^2)A_z^b = 0 \quad , z \leq -h \qquad (2b)$$

PLUS RADIATION CONDITION

FIELD COMPONENTS PARALLEL TO THE INTERFACE ARE

$$H_x^{\substack{a\\b}} = \frac{1}{\mu_{\substack{a\\b}}}\frac{\partial}{\partial y}A_z^{\substack{a\\b}} \qquad E_x^{\substack{a\\b}} = -j\frac{\omega}{k^2_{\substack{a\\b}}}\left(k^2_{\substack{a\\b}}A_x^{\substack{a\\b}} + \frac{\partial^2}{\partial x^2}A_x^{\substack{a\\b}} + \frac{\partial^2}{\partial x\partial z}A_z^{\substack{a\\b}}\right)$$

$$H_y^{\substack{a\\b}} = \frac{1}{\mu_{\substack{a\\b}}}\left(\frac{\partial}{\partial z}A_x^{\substack{a\\b}} - \frac{\partial}{\partial x}A_z^{\substack{a\\b}}\right) \qquad E_y^{\substack{a\\b}} = -j\frac{\omega}{k^2_{\substack{a\\b}}}\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}A_x^{\substack{a\\b}} + \frac{\partial}{\partial z}A_z^{\substack{a\\b}}\right)$$

FROM THESE WE DEVELOP THE NEEDED ~~BOUNDARY~~ CONTINUITY CONDITION EXPRESSIONS IN THE TRANSFORM DOMAIN.

Transform in x and y

FROM CONTINUITY OF MAGNETIC FIELD (COMPONENTS PARALLEL TO THE INTERFACE), WE ARRIVE AT

$$\frac{1}{\mu_a}\tilde{A}_z^a = \frac{1}{\mu_b}\tilde{A}_z^b \quad , \qquad z=-h \tag{3a}$$

AND

$$\frac{1}{\mu_a}\left(\frac{\partial}{\partial z}\tilde{A}_x^a - jk_x\tilde{A}_z^a\right) = \frac{1}{\mu_b}\left(\frac{\partial}{\partial z}\tilde{A}_x^b - jk_x\tilde{A}_z^b\right), \quad z=-h \tag{3b}$$

IN THE TRANSFORM DOMAIN. IN A SIMILAR #### MANNER, CONTINUITY OF THE PARALLEL COMPONENTS AT THE INTERFACE OF THE ELECTRIC FIELD REQUIRES

$$\frac{1}{k_a^2}\left(k_a^2\tilde{A}_x^a - k_x^2\tilde{A}_x^a + jk_x\frac{\partial}{\partial z}\tilde{A}_z^a\right) = \frac{1}{k_b^2}\left(k_b^2\tilde{A}_x^b - k_x^2\tilde{A}_x^b + jk_x\frac{\partial}{\partial z}\tilde{A}_z^b\right),$$

AND

$$z=-h \tag{4a}$$

$$\frac{1}{k_a^2}\left(jk_x\tilde{A}_x^a + \frac{\partial}{\partial z}\tilde{A}_z^a\right) = \frac{1}{k_b^2}\left(jk_x\tilde{A}_x^b + \frac{\partial}{\partial z}\tilde{A}_z^b\right), \quad z=-h \tag{4b}$$

AT THIS POINT IT IS CONVENIENT TO FORM VARIOUS COMBINATIONS OF (3) AND (4) TO OBTAIN A MORE USEFUL SET OF FOUR ##### LINEARLY ######### INDEPENDENT EQUATIONS. FIRST, WE COMBINE (3A) AND (3B) TO OBTAIN

$$\frac{1}{\mu_a}\frac{\partial}{\partial z}\tilde{A}_x^a = \frac{1}{\mu_b}\frac{\partial}{\partial z}\tilde{A}_x^b \quad , \qquad z=-h \tag{5}$$

FROM (4A) AND (4B), ONE CAN ARRIVE AT

$$\tilde{A}_x^a = \tilde{A}_x^b \quad , \qquad z = -h \tag{6}$$

AND SUBSTITUTION OF (6) INTO (4B) LEADS TO

$$\frac{1}{k_a^2}\frac{\partial}{\partial z}\tilde{A}_z^a - \frac{1}{k_b^2}\frac{\partial}{\partial z}\tilde{A}_z^b = jk_x\left(\frac{1}{k_b^2} - \frac{1}{k_a^2}\right)\tilde{A}_x^a \;, \quad z = -h \tag{7}$$

AFTER FORMING THE SOLUTIONS FOR THE TWO COMPONENTS OF THE VECTOR POTENTIAL, ONE APPEALS TO (5) AND (6) TO DETERMINE THE ARBITRARY CONSTANTS ASSOCIATED WITH THE X COMPONENT OF THE POTENTIAL, AND, FROM KNOWLEDGE OF THIS COMPONENT, PLUS (3A) AND (7), ONE CAN DETERMINE THE Z COMPONNET OF THE POTENTIAL.

AS BEFORB, WE CONSTRUCT SOLUTIONS IN THE TRANSFROM DOMAIN IN EACH HALF-SPACE:

$$\tilde{A}_x^a = B^a e^{-j\beta_a(z+h)} - \mu_a \frac{j}{4\pi} e^{-j(k_x x' + k_y y')} \frac{e^{-j\beta_a|z-z'|}}{\beta_a} \;, \quad z \geq -h \tag{8a}$$

$$A_x^a = -\frac{j\mu_a}{4\pi}\frac{e^{-j(k_x x' + k_y y')}}{\beta_a}\left(e^{-j\beta_a|z-z'|} + \Gamma e^{-j\beta_a(z+z')}\right)$$

$$\tilde{A}_x^b = B^b e^{j\beta_b(z+h)} \quad , \qquad z \leq -h \tag{8b}$$

OBSERVE THAT IN THE UPPER REGION THE SOLUTION HAS BOTH A PARTICULAR AND A HOMOGENEOUS SOLUTION, WHILE IN THE LOWER REGION THERE IS ONLY A HOMOGENEOUS PART. NOTE FURTHER THAT THE TWO EXPRESSIONS SATISFY THE RADIATION CONDITION. NOW, WE DETERMINE THE CONSTANTS FROM THE CONDITIONS EMBODIED IN EQS. (5) AND (6); THESE CONSTRAINTS IMPOSE THE FOLLOWING RELATIONSHIPS:

$$\beta_{\substack{a\\b}} = \sqrt{k_{\substack{a\\b}}^2 - k_x^2 - k_y^2}$$

$$\begin{bmatrix} 1 & -1 \\ -\frac{\beta_a}{\beta_b}\frac{\mu_b}{\mu_a} & -1 \end{bmatrix} \begin{bmatrix} B^a \\ B^b \end{bmatrix} = j\frac{\mu_a}{4\pi} e^{-j(k_x x'+k_y y')} \frac{e^{-j\beta_a(z'+h)}}{\beta_a} \begin{bmatrix} 1 \\ \frac{\beta_a}{\beta_b}\frac{\mu_b}{\mu_a} \end{bmatrix}$$

FROM THIS MATRIX EQUATION, ONE OBTAINS

$$B^a = j\frac{\mu_a}{4\pi}\left(\frac{\beta_b-\beta_a\frac{\mu_b}{\mu_a}}{\beta_b+\beta_a\frac{\mu_b}{\mu_a}}\right) e^{-j(k_x x'+k_y y')} \frac{e^{-j\beta_a(z'+h)}}{\beta_a} \tag{9}$$

$$\frac{\beta_b-\beta_a\left(\frac{\mu_b}{\mu_a}\right)}{\beta_b+\beta_a\left(\frac{\mu_b}{\mu_a}\right)} = \frac{\beta_b+\beta_a\left(\frac{\mu_b}{\mu_a}\right)-2\beta_a\left(\frac{\mu_b}{\mu_a}\right)}{\beta_b+\beta_a\left(\frac{\mu_b}{\mu_a}\right)} = 1-\frac{2\beta_a\left(\frac{\mu_b}{\mu_a}\right)}{\beta_b+\beta_a\left(\frac{\mu_b}{\mu_a}\right)} \tag{10}$$

WHICH ENABLES ~~####~~ HIM TO EXPRESS THE TRANSFORM OF A_X^A AS

$$\tilde{A}_x^a = -j\frac{\mu_a}{4\pi} e^{-j(k_x x'+k_y y')}\left[\frac{e^{-j\beta_a|z-z'|}}{\beta_a} - \frac{e^{-j\beta_a|z+z'+2h|}}{\beta_a}\right]$$

$$-j\frac{\cancel{\mu_a}}{\cancel{4\pi}_{2}}\left(\frac{2\beta_a\left[\frac{\mu_b}{\mu_a}\right]}{\beta_b+\beta_a\left[\frac{\mu_b}{\mu_a}\right]}\right) e^{-j(k_x x'+k_y y')} \frac{e^{-j\beta_a|z+z'+2h|}}{\cancel{\beta_a}} \tag{11}$$

THE INVERSE TRANSFORM OF (11) IS HANDLED EXACTLY AS WE DID THE TRANSFORM OF THE VECTOR POTENTIAL FOR THE VERTICAL DIPOLE:

$$A_x^a(x,y,z) = \frac{\mu_a}{4\pi}\frac{e^{-jk_aR}}{R} - \frac{\mu_a}{4\pi}\frac{e^{-jk_aR_i}}{R_i}$$

$$-j\frac{\mu_a}{2\pi}\int_{\rho=0}^{\infty}\rho J_0(r\rho)\frac{e^{-j\beta_a|z+z'+2h|}}{\beta_a+\left(\frac{\mu_a}{\mu_b}\right)\beta_b}\,d\rho \qquad (12)$$

$+$

NOW WE WISH TO CALCULATE THE VERTICAL ~~COMPONENT~~ COMPONENT OF THE VECTOR POTENTIAL DUE TO THE HORIZONTALLY DIRECTED DIPOLE. AS SEEN FROM (2) THE VERTICAL COMPONENT MUST SATISFY HOMOGENEOUS DE'S IN BOTH HALF-SPACES SO ONLY COMPLEMENTARY SOLUTIONS ARE ACCEPTABLE. IN THE TRANSFORM DOMAIN, THESE HOMOGENEOUS SOLUTIONS (RADIATION CONDITION HAS BEEN IMPOSED) ARE

$$\tilde{A}_z^a = D^a e^{-j\beta_a(z+h)} \quad , \quad z \geq -h \qquad (13a)$$

$$\tilde{A}_z^b = D^b e^{j\beta_b(z+h)} \quad , \quad z \leq -h \qquad (13b)$$

FROM (3A), WE REQUIRE

$$\frac{D^a}{\mu_a} = \frac{D^b}{\mu_b} \Rightarrow D^b = \frac{\mu_b}{\mu_a}D^a$$

AND FROM (7) WE HAVE THE FOLLOWING ~~CON~~ CONDITION ON D^A:

$$\frac{1}{k_a^2}\left(-j\beta_a D^a\right) - \frac{1}{k_b^2}\left(j\beta_b\right)\left(\frac{\mu_b}{\mu_a}\right)D^a = jk_x\left(\frac{1}{k_b^2} - \frac{1}{k_a^2}\right)\tilde{A}_x^a\bigg|_{z=-h}$$

OR

$$D^a = -j\frac{\mu_a}{4\pi}k_x\left(\frac{k_b^2 - k_a^2}{k_b^2\beta_a + \frac{\mu_b}{\mu_a}k_a^2\beta_b}\right)\left(\frac{2}{\beta_a + \beta_b\left(\frac{\mu_b}{\mu_a}\right)}\right)e^{-j(k_x x' + k_y y')}e^{-j\beta_a|z'+h|}$$

HENCE, WE FIND THAT

$$\tilde{A}_z^a = -j\frac{\mu_a}{4\pi}\left(\frac{k_b^2 - k_a^2}{k_b^2\beta_b + \frac{\mu_b}{\mu_a}k_a^2\beta_b}\right)\left(\frac{2k_x}{\beta_a + \frac{\mu_b}{\mu_a}\beta_b}\right)e^{-j(k_x x' + k_y y')}e^{-j\beta_a|z+z'+2h|} \tag{14}$$

WHICH, AFTER FOURIER INVERSION, YIELDS

$$A_z^a(x,y,z) = -j\frac{\mu_a}{(2\pi)^2}\int_{k_x=-\infty}^{\infty}\int_{k_y=-\infty}^{\infty}\left(\frac{k_b^2 - k_a^2}{k_b^2\beta_a + \frac{\mu_b}{\mu_a}k_a^2\beta_b}\right)\left(\frac{k_x}{\beta_a + \frac{\mu_b}{\mu_a}\beta_b}\right)e^{j[(x-x')k_x + (y-y')k_y]}e^{-j\beta_a|z+z'+2h|}\,dk_y\,dk_x \tag{15}$$

WE FOLLOW THE PROCEDURE DISCUSSED ABOVE AND MAKE THE SAME SUBSTITUTION OF INTEGRATION VARIABLES TO ARRIVE AT

$$A_z^a = -j\frac{\mu_a}{2\pi}\int_{\rho=0}^{\infty}\left(\frac{k_b^2-k_a^2}{k_b^2\beta_a+\frac{\mu_b}{\mu_a}k_a^2\beta_b}\right)\frac{e^{-j\beta_a|z+z'+2h|}}{\beta_a+\frac{\mu_b}{\mu_a}\beta_b}\rho^2\frac{1}{2\pi}\int_{\theta=0}^{2\pi}\cos\theta\, e^{jr\rho\cos(\theta-\varphi)}\,d\theta\, d\rho$$

WHICH, ######## IN VIEW OF

$$\frac{1}{2\pi}\int_{\theta=0}^{2\pi}\cos\theta\, e^{jr\rho\cos(\theta-\varphi)}\,d\theta = j\cos\varphi\, J_1(r\rho), \qquad (16)$$

CAN BE ######### CONVERTED TO

$$A_z^a = \frac{\mu_a}{2\pi}\cos\varphi\int_{\rho=0}^{\infty}\rho^2 J_1(r\rho)\left(\frac{k_b^2-k_a^2}{k_b^2\beta_a+\frac{\mu_b}{\mu_a}k_a^2\beta_b}\right)\frac{e^{-j\beta_a|z+z'+2h|}}{\beta_a+\frac{\mu_b}{\mu_a}\beta_b}\,d\rho \qquad (17)$$

$\frac{\mu_a}{\mu_b}$ 4/15/84

WHERE ONE RECALLS THAT

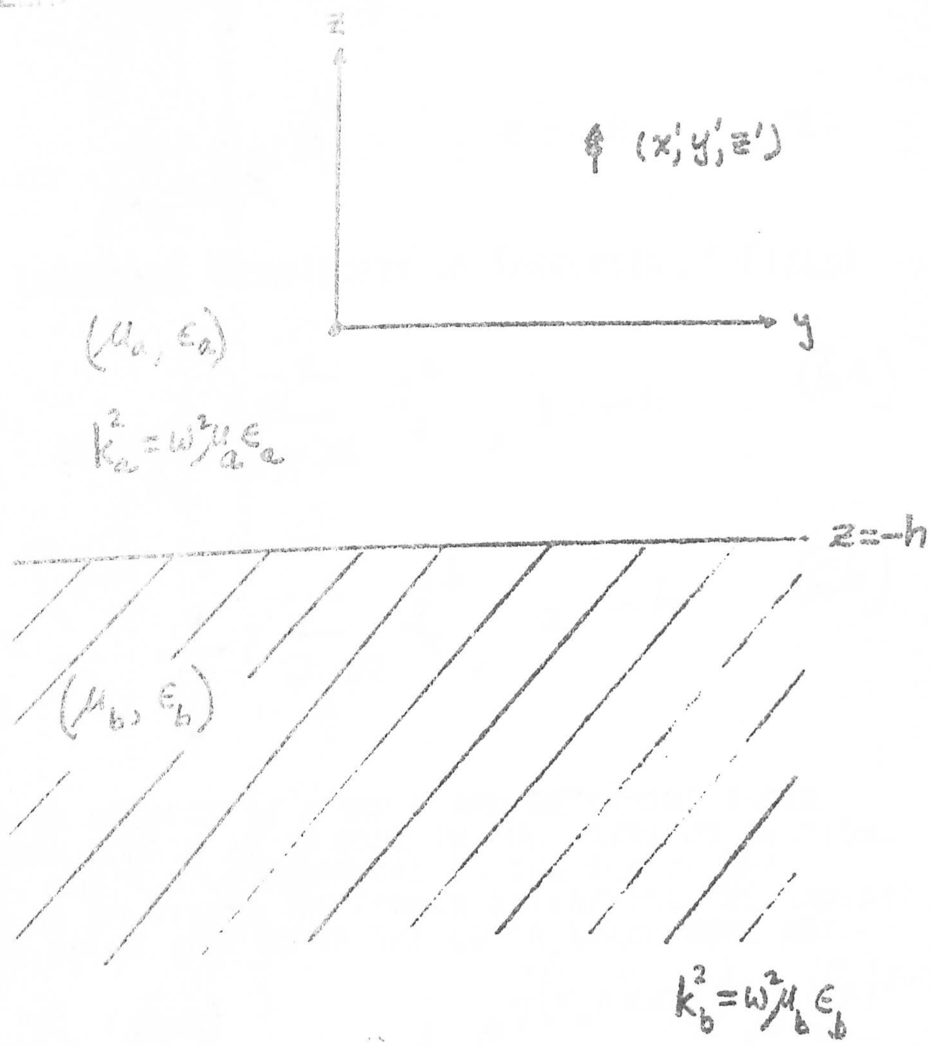
$$\cos\varphi = \frac{x-x'}{r}\ ; \quad r=\sqrt{(x-x')^2+(y-y')^2} \qquad (18a)$$

$$\beta_{\substack{a\\b}} = \sqrt{k_{\substack{a\\b}}^2-\rho^2} \qquad (18b)$$

THIS IS THE VERTICAL COMPONENT OF POTENTIAL DUE TO A HORIZONTALLY

MAGNETIC VECTOR POTENTIAL FOR VERTICAL ELECTRIC DIPOLE ABOVE A LOSSY HALF-SPACE (GREEN'S FUNCTION FOR TWO-SPACE PROBLEM)

3-D



NOTE

$\epsilon = \underset{\sim}{\epsilon} - j\frac{\sigma}{\omega}$

$\underset{\sim}{\epsilon}$ - PERMITTIVITY

BASIC EQUATIONS

$$\bar{J} = \delta(\bar{r} - \bar{r}')\hat{z} \quad ; \quad z' > -h$$

$$(\nabla^2 + k_a^2) A_z^a = -\mu_a \delta(\bar{r} - \bar{r}') \quad , \quad z \geq -h \qquad (1)$$

$$(\nabla^2 + k_b^2) A_z^b = 0 \quad , \quad z \leq -h \qquad (2)$$

PLUS RADIATION CONDITION

$$H_x^{\substack{a\\b}} = \frac{1}{\mu_{\substack{a\\b}}} \frac{\partial}{\partial y} A_z^{\substack{a\\b}} \quad (3a) \qquad E_x^{\substack{a\\b}} = -j\frac{\omega}{k_{\substack{a\\b}}^2} \frac{\partial^2}{\partial x \partial z} A_z^{\substack{a\\b}} \quad (4a)$$

$$H_y^{\substack{a\\b}} = -\frac{1}{\mu_{\substack{a\\b}}} \frac{\partial}{\partial x} A_z^{\substack{a\\b}} \quad (3b) \qquad E_y^{\substack{a\\b}} = -j\frac{\omega}{k_{\substack{a\\b}}^2} \frac{\partial^2}{\partial y \partial z} A_z^{\substack{a\\b}} \quad (4b)$$

BOUNDARY CONDITIONS AT INTERFACE (CONTINUITY

$$\frac{1}{\mu_a}\frac{\partial}{\partial y}A_z^a = \frac{1}{\mu_b}\frac{\partial}{\partial y}A_z^b \quad , z=-h \qquad (5a)$$

$$\frac{1}{\mu_a}\frac{\partial}{\partial x}A_z^a = \frac{1}{\mu_b}\frac{\partial}{\partial x}A_z^b \quad , z=-h \qquad (5b)$$

BOUNDARY CONDITIONS AT INTERFACE (CONTINUITY OF TANGENTIAL E-FIELD)

$$\frac{1}{k_a^2}\frac{\partial^2}{\partial x\partial z}A_z^a = \frac{1}{k_b^2}\frac{\partial^2}{\partial x\partial z}A_z^b \quad , z=-h \qquad (6a)$$

$$\frac{1}{k_a^2}\frac{\partial^2}{\partial y\partial z}A_z^a = \frac{1}{k_b^2}\frac{\partial^2}{\partial y\partial z}A_z^b \quad , z=-h \qquad (6b)$$

TRANSFORM (1) AND (2) WITH RESPECT TO X AND Y AND DETERMINE THEIR SOLUTIONS IN THE TRANSFORM DOMAIN AS IS DONE IN THE PREVIOUS SECTION. IN THE UPPER HALF SPACE, THE VECTOR POTENTIAL IS THE SUM OF THE HOMOGENEOUS AND PARTICULAR SOLUTION, THE FORMER SATISFYING THE RADIATION CONDITION AND THE LATTER BEING THAT AS IF THE LOWER SPACE WERE NOT PRESENT.

$$\tilde{A}_z^a = C^a e^{-j\beta_a(z+h)} - \mu_a \frac{j}{4\pi} e^{-j(k_x x' + k_y y')} \frac{e^{-j\beta_a|z-z'|}}{\beta_a} ,$$

$$z \geq -h \qquad (7a)$$

IN THE LOWER HALF-SPACE, WE HAVE ONLY A HOMOGENEOUS SOLUTION WHICH MUST, OF COURSE SATISFY THE RADIATION CONDITION.

$$\tilde{A}_z^b = C^b e^{j\beta_b(z+h)} \quad , z \leq -h \qquad (7b)$$

$$\beta_{\substack{a\\b}} = \sqrt{k_{\substack{a\\b}}^2 - k_x^2 - k_y^2}$$

THE BOUNDARY CONDITIONS IN THE TRANSFORM DOMAIN REDUCE FROM FOUR (5A), (5B), (6A), AND (6B). FROM (5) ONE OBTAINS

$$\frac{1}{\mu_a}(jk_y)\tilde{A}_z^a = \frac{1}{\mu_b}(jk_y)\tilde{A}_z^b \quad , z=-h \qquad (5a')$$

OR

$$\frac{1}{\mu_a}\tilde{A}_z^a = \frac{1}{\mu_b}\tilde{A}_z^b \quad , z=-h \qquad (8)$$

AND FROM (6) HE ARRIVES AT

$$\frac{1}{k_a^2}\frac{\partial}{\partial z}\tilde{A}_z^a = \frac{1}{k_b^2}\frac{\partial}{\partial z}\tilde{A}_z^b \quad , z=-h \qquad (9)$$

IN ORDER TO ENFORCE (9), ONE MUST HAVE THE DERIVATIVE OF (7):

$$\frac{\partial}{\partial z}\tilde{A}_z^a = -j\beta_a C^a e^{-j\beta_a(z+h)} + \frac{\mu_a}{4\pi} e^{-j(k_x x' + k_y y')} e^{-j\beta_a(z'-z)} \quad ,$$

$$-h \le z < z'$$

$$\frac{\partial}{\partial z}\tilde{A}_z^b = j\beta_b C^b e^{j\beta_b(z+h)} \quad , \qquad z \le -h$$

EQUATIONS (8) AND (9), SUBJECT TO (7), ENABLE ONE TO OBTAIN A SET OF TWO SIMULTANEOUS EQUATIONS FROM WHICH ONE CAN DETERMINE THE CONSTANTS C^A AND C^B:

$$\begin{bmatrix} -j\dfrac{\beta_a}{k_a^2} & -j\dfrac{\beta_b}{k_b^2} \\ \dfrac{1}{\mu_a} & -\dfrac{1}{\mu_b} \end{bmatrix} \begin{bmatrix} C^a \\ C^b \end{bmatrix} = \frac{\mu_a}{4\pi} e^{-j(k_x x' + k_y y')} \frac{e^{-j\beta_a(z'+h)}}{\beta_a} \begin{bmatrix} -\dfrac{\beta_a}{k_a^2} \\ j\dfrac{1}{\mu_a} \end{bmatrix}$$

FROM THE ABOVE EQUATION, C^A IS FOUND TO BE

$$C^a = j\frac{\mu_a}{4\pi}\left[\frac{1-\gamma}{1+\gamma}\right] e^{-j(k_x x' + k_y y')} \frac{e^{-j\beta_a(z'+h)}}{\beta_a} \tag{10}$$

WHERE

$$\gamma = \frac{\mu_a}{\mu_b}\frac{\beta_a}{\beta_b}\frac{k_b^2}{k_a^2} = \frac{\epsilon_b}{\epsilon_a}\frac{\beta_a}{\beta_b} \tag{11}$$

AS A MATTER OF INTEREST, WE NOTE THAT

$$\frac{1-\gamma}{1+\gamma} = \frac{1+\gamma-2\gamma}{1+\gamma} = 1 - 2\frac{\gamma}{1+\gamma} \tag{12}$$

KNOWING C^A, ONE READILY DETERMINES FROM (7) THE TRANSFORM OF THE UPPER HALF-SPACE VECTOR POTENTIAL:

$$\tilde{A}_z^a = j\frac{\mu_a}{4\pi} e^{-j(k_x x' + k_y y')} \frac{e^{-j\beta_a(z+z'+2h)}}{\beta_a} - j\frac{\mu_a}{4\pi} e^{-j(k_x x' + k_y y')} \frac{e^{-j\beta_a|z-z'|}}{\beta_a}$$

$$-j\frac{\mu_a}{4\pi}\frac{2\gamma}{1+\gamma} e^{-j(k_x x' + k_y y')} \frac{e^{-j\beta_a(z+z'+2h)}}{\beta_a} \tag{13}$$

THE INVERSE TRANSFORM OF (13) IS DETERMINED AS IS DONE IN THE PREVIOUS SECTION AND IS FOUND TO BE

$$A_z^a(x,y,z) = \frac{\mu_a}{4\pi}\frac{e^{-jk_a R}}{R} - \frac{\mu_a}{4\pi}\frac{e^{-jk_a R_i}}{R_i}$$

$$-j\mu_a \frac{1}{(2\pi)^2}\int_{k_x=-\infty}^{\infty}\int_{k_y=-\infty}^{\infty}\frac{\gamma}{1+\gamma}\frac{e^{-j\beta_a|z+z'+2h|}}{\beta_a}e^{j[(x-x')k_x+(y-y')k_y]}dk_y dk_x \qquad (14)$$

WHERE

$$R=\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{1/2} \qquad (15a)$$

$$R_i=\left[(x-x')^2+(y-y')^2+(z+z'+2h)^2\right]^{1/2} \qquad (15b)$$

BY EMPLOYING THE SAME COORDINATE TRANSFORMATION AND RECOGNIZING AN INTEGRAL REPRESENTATION OF THE BESSEL FUNCTION AS DISCUSSED EARLIER, THE FINAL EXPRESSION FOR THE UPPER HALF-SPACE VECTOR POTENTIAL IS ACHIEVED:

$$A_z^a(x,y,z) = \frac{\mu_a}{4\pi}\frac{e^{-jk_a R}}{R} - \frac{\mu_a}{4\pi}\frac{e^{-jk_a R_i}}{R_i}$$

$$-j\frac{\mu_a}{2\pi}\int_{\rho=0}^{\infty}\rho\, J_0(r\rho)\frac{\gamma}{1+\gamma}\frac{e^{-j\beta_a|z+z'+2h|}}{\beta_a}d\rho \qquad (16)$$

WHERE

$$\beta_{\substack{a\\b}} = \sqrt{k_{\substack{a\\b}}^2 - \rho^2} \qquad (17a)$$

$$r = (x-x')^2 + (y-y')^2 \qquad (17b)$$

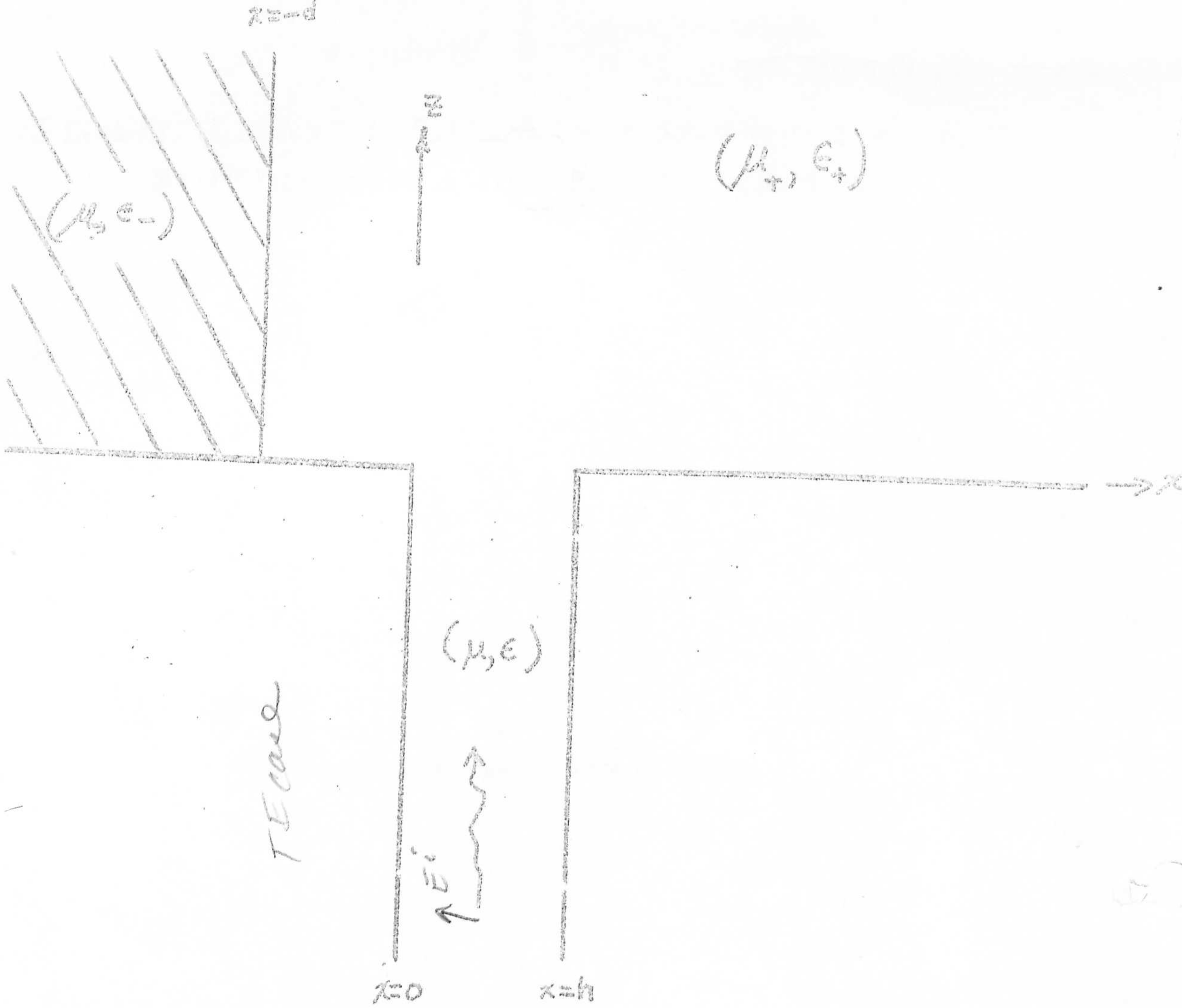
$$\frac{\gamma}{1+\gamma} = \frac{\epsilon_b \beta_a}{\epsilon_a \beta_b + \epsilon_b \beta_a} \qquad (17c)$$

FROM THE FORM OF (17c) AND THAT OF THE INTEGRAND OF (16), ONE OBSERVES THAT THE INTEGRAND OF THE LAST TERM IN (16) IS REGULAR.

$$\frac{\gamma}{1+\gamma} = \frac{1+\gamma-1}{1+\gamma} = 1 - \frac{1}{1+\gamma} \qquad (18)$$

$$A_z^a(x,y,z) = \frac{\mu_a}{4\pi}\frac{e^{-jk_a R}}{R} + \frac{\mu_a}{4\pi}\frac{e^{-jk_a R_i}}{R_i}$$

$$+j\frac{\mu_a}{2\pi}\int_{\rho=0}^{\infty} \rho\, J_0(r\rho)\frac{1}{1+\gamma}\frac{e^{-j\beta_a|z+z'+2h|}}{\beta_a}\,d\rho \qquad (19)$$



$$\frac{\omega \epsilon_+}{2}\int_{x'=0}^{h} E_x(x',0)\left\{H_0^{(2)}(k_+|x-x'|) - H_0^{(2)}(k_+|x+x'+2d|) + \frac{4}{\pi}\epsilon_-\int_{\xi=0}^{\infty}\frac{e^{-j\beta_+(x+x'+2d)}}{\epsilon_-\beta_+ + \epsilon_+\beta_-}\,d\xi\right\}dx'$$

$$+ f_a(E_x, x) = \frac{2V}{\zeta h}, \quad \beta_\pm = \sqrt{k_\pm^2 - \xi^2}$$