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Radiation From a Horizontal Electric Dipole Buried in a Multilayer Earth [1]

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Field expressions for a horizontal electric dipole buried in a three-layer earth are derived with techniques developed for a two-layer earth. Parameters include the radio frequency, upper- and lower-layer depths, and the relative dielectric constant and conductivity or loss tangent of each layer. Analogies between multisection transmission lines and multilayer earth are discussed.

1. Statement of the Problem

The radiation fields from a horizontal electric dipole were studied initially for a two-layer Antarctic terrain (Biggs, 1968). The extension from a two- to a three-layer medium appears in figure 1, where the horizontal electric dipole is buried in the upper layer. The observation point is above the earth's surface. The propagation constants are

$$k_j^2 = \omega^2\mu_0\epsilon_j + i\omega\mu_0\sigma_j = k_0^2\{\epsilon_j' + i\epsilon_j''\}, \tag{1}$$

for the j-layer and a time dependence $\exp(-i\omega t)$ with conductivity σ_j and loss tangent ϵ_j'' for soil and ice, respectively.

The lossy dielectric representation is used for Antarctic media, where the layers may be ice at different temperatures, and the conductivity representation is used for soil, where layers may correspond to geological structures. Following the procedure in the two-layer terrain analysis, integral equations for the x and z components of the Hertz vector are obtained for the Sommerfeld (1926) representation of the electric field intensity components,

$$E_\theta = -30k_0^2 Idl \sin\theta\cos\theta\cos\phi \int_{-\infty}^{\infty} M(h)\,\frac{\gamma_1 k_0^2 M_{123}}{\gamma_0 k_1^2 + \gamma_1 k_0^2 M_{123}}\cdot\frac{e^{-\gamma_0 z + i\lambda p}}{\lambda}\,[H_0^{(1)}(\lambda p)e^{-i\lambda p}]d\lambda, \tag{2}$$

$$E_\phi = -i30k_0 Idl \sin\phi \int_{-\infty}^{\infty} N(h)\,\frac{1}{\gamma_0 + \gamma_1 N_{123}}\cdot e^{-\gamma_0 z + i\lambda p}[H_0^{(1)}(\lambda p)e^{-i\lambda p}]\lambda d\lambda, \tag{3}$$

where Idl is the dipole moment, and γ_j is, with $j = 0, 1, 2,$ or 3,

$$\gamma_j^2 = \lambda^2 - k_j^2. \tag{4}$$

The burial-depth functions $M(h)$ and $N(h)$ are (Wait, 1967)

$$M(h) = \frac{\gamma_1 k_2^2 \sinh\gamma_1(D_1 - h) + \gamma_2 k_1^2 \cosh\gamma_1(D_1 - h)}{\gamma_1 k_2^2 \sinh\gamma_1 D_1 + \gamma_2 k_1^2 \cosh\gamma_1 D_1}, \tag{5}$$

$$N(h) = \frac{\gamma_1 \cosh\gamma_1(D_1 - h) + \gamma_2 \sinh\gamma_1(D_1 - h)}{\gamma_1 \cosh\gamma_1 D_1 + \gamma_2 \sinh\gamma_1 D_1}, \tag{6}$$

and the layer-depth parameters M_{123} and N_{123} are

$$M_{123} = \frac{\gamma_2 k_1^2 M_{23} + \gamma_1 k_2^2 \tanh\gamma_1 D_1}{\gamma_1 k_2^2 + \gamma_2 k_1^2 M_{23}\tanh\gamma_1 D_1}, \tag{7}$$

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$$N_{123}=\frac{\gamma_2 N_{23}+\gamma_1 \tanh \gamma_1 D_1}{\gamma_1+\gamma_2 N_{23} \tanh \gamma_1 D_1}, \tag{8}$$

where M_{23} and N_{23} are

$$M_{23}=\frac{\gamma_3 k_2^2+\gamma_2 k_3^2 \tanh \gamma_2 D_2}{\gamma_2 k_3^2+\gamma_3 k_2^2 \tanh \gamma_2 D_2}, \tag{9}$$

$$N_{23}=\frac{\gamma_3+\gamma_2 \tanh \gamma_2 D_2}{\gamma_2+\gamma_3 \tanh \gamma_2 D_2}. \tag{10}$$

The parameters M_{123} and M_{23} correspond to the parameters (Wait, 1958) for vertically polarized plane waves incident on a stratified medium, and N_{123} and N_{23} for horizontally polarized plane waves. When layers (2) and (3) are the same, they become parameters for a two-layer medium. For an m-layer medium, the parameters are

$$M_{12\ldots m}=\frac{\gamma_2 k_1^2 M_{2\ldots m}+\gamma_1 k_2^2 \tanh \gamma_1 D_1}{\gamma_1 k_2^2+\gamma_2 k_1^2 M_{2\ldots m} \tanh \gamma_1 D_1}, \tag{11}$$

$$\vdots$$

$$M_{(m-1)m}=\frac{\gamma_m k_{m-1}^2+\gamma_{m-1} k_m^2 \tanh \gamma_{m-1} D_{m-1}}{\gamma_{m-1} k_m^2+\gamma_m k_{m-1}^2 \tanh \gamma_{m-1} D_{m-1}}, \tag{12}$$

$$N_{12\ldots m}=\frac{\gamma_2 N_{2\ldots m}+\gamma_1 \tanh \gamma_1 D_1}{\gamma_1+\gamma_2 N_{2\ldots m} \tanh \gamma_1 D_1}, \tag{13}$$

$$N_{(m-1)m}=\frac{\gamma_m+\gamma_{m-1} \tanh \gamma_{m-1} D_{m-1}}{\gamma_{m-1}+\gamma_m \tanh \gamma_{m-1} D_{m-1}}. \tag{14}$$

When $k_j^2 \gg k_0^2$, the γ_j terms in (4) reduce to $-ik_j$, with M_{123} and N_{123}

$$M_{123} \simeq \frac{k_1 M_{23}-ik_2 \tan k_1 D_1}{k_2-ik_1 M_{23} \tan k_1 D_1}, \tag{15}$$

$$N_{123} \simeq \frac{k_2 N_{23}-ik_1 \tan k_1 D_1}{k_1-ik_2 N_{23} \tan k_1 D_1}, \tag{16}$$

with similar expressions for M_{23} and N_{23}.

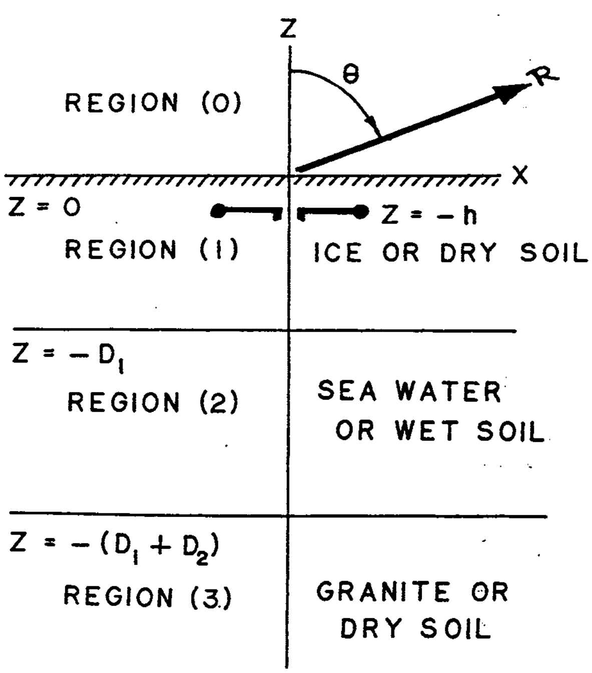

FIGURE 1. *Coordinate system for a 3-layer antarctic or soil medium with a buried horizontal dipole*

2. Radiation Field Expressions

The integration techniques for two-layer terrain are extended to the three-layer terrain. The saddle-point method is used for the space waves and the modified saddle-point method is used for the groundwave. The space wave expressions derived with the saddle-point method (Biggs and Swarm, 1968) are

$$E_\theta = i60k_0 IdlM(h)\frac{M_{123}\cos\phi\cos\theta\sqrt{n_1^2-\sin^2\theta}}{n_1^2\cos\theta+M_{123}\sqrt{n_1^2-\sin^2\theta}}\frac{e^{ik_0R}}{R}, \tag{17}$$

$$E_\phi = -i60k_0 IdlN(h)\frac{\sin\phi\cos\theta}{\cos\theta+N_{123}\sqrt{n_1^2-\sin^2\theta}}\frac{e^{ik_0R}}{R}, \tag{18}$$

with n_1^2 as the refractive index for the first layer. The groundwave expressions are

$$E_\theta = -60Idl\cos\phi\left\{\frac{n_1^2}{M_{123}\sqrt{n_1^2-\sin^2\theta}}\right\}\frac{e^{ik_0R+ik_0h\sqrt{n_1^2-\sin^2\theta}}}{R^2}, \tag{19}$$

$$E_\phi = 60Idl\sin\phi\left\{\frac{\sin^2\theta}{N_{123}^2(n_1^2-\sin^2\theta)}\right\}\frac{e^{ik_0R+ik_0h\sqrt{n_1^2-\sin^2\theta}}}{R^2}. \tag{20}$$

An interesting phenomena is found when the upper layer has a lower refractive index than the integrated refractive index of all lower layers. The integrated refractive indices for three and m layers are

$$n_{23}=\frac{n_2}{M_{23}}, \qquad n_{2\ldots m}=\frac{n_2}{M_{2\ldots m}}, \tag{21}$$

When upper-layer depth D_1 is almost zero, the pole location is the value of λ satisfying Sommerfeld's denominator set equal to zero,

$$\gamma_0k_2^2+\gamma_2k_0^2M_{23}=0, \qquad \lambda_p=+k_2\sqrt{\frac{n_2^2-M_{23}^2}{n_2^4-M_{23}^2}}, \tag{22}$$

and as D_1 increases to D_2, (22) changes to

$$\gamma_0k_1^2+\gamma_1k_0^2M_{123}=0, \qquad \lambda_p=+k_1\sqrt{\frac{n_1^2-M_{123}^2}{n_1^4-M_{123}^2}}, \tag{23}$$

and as D_1 approaches infinity, M_{123} approaches unity, and

$$\gamma_0k_1^2+\gamma_1k_0^2=0, \qquad \lambda_p=+k_1\frac{1}{\sqrt{n_1^2+1}}, \tag{24}$$

The pole location in (22) may be on either side of the branch cut $+k_0$ on the complex λ-plane in figure 2, which also shows the integration path and other branch cuts. Pole loci for two-layer media in figure 3 move counterclockwise with increasing D_1 for an upper layer of ice at different ice temperatures above soil. The integrated soil conductivity is 0.01 mho/m and the signal frequency is 10.0 kHz. In crossing the $+k_0$ branch cut, normalized to 1.0, the "virtual" poles become real poles. When real poles are trapped by deformation of the integration path, a residue term is created,

$$E_\theta = 120k_0Idl\cos\phi\, M_{123}\frac{n_1^5(n_1^2-1)}{(n_1^4-M_{123}^2)^2}\sqrt{\frac{k_0}{\lambda_p}}\cdot\sqrt{\pi x_0^2/2}\,e^{-x_0^2/2}\left\{\frac{e^{ik_0z\{n_1^2-M_{123}h\}\sqrt{\frac{n_1^2-1}{n_1^4-M_{123}^2}}+ik_0R}}{R}\right., \tag{25}$$

where $x_0^2/2$ is the numerical distance p, equal to

$$x_0^2/2=ik_0R(M_{123}/n_1)^2/2=p. \tag{26}$$

For practical distances, except at VHF, the pole is too close to the saddle point for this method to be adequate. The modified saddle-point method (Bn˜os, 1965; Wait, 1966) adds and subtracts the pole singularity to form two integrals, one

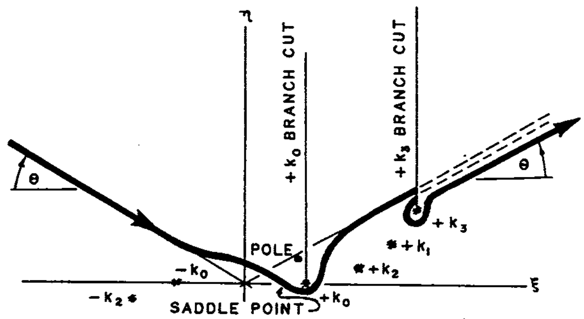

FIGURE 2. *Path of integration for the saddle-point method with branch cuts, pole, and saddle point.*

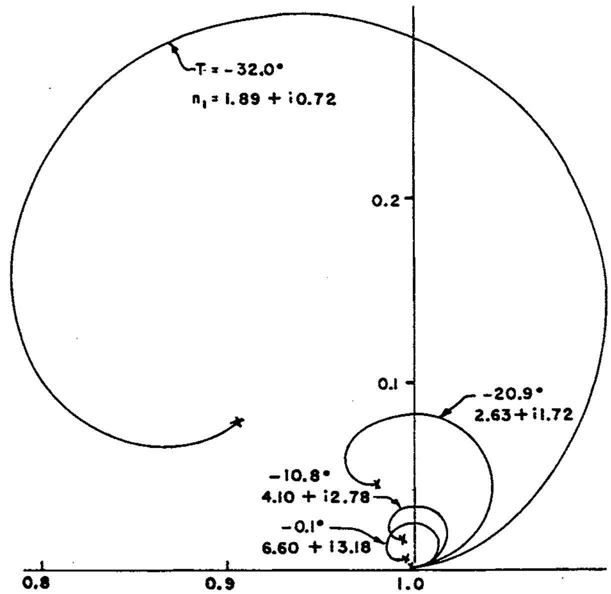

FIGURE 3. *Pole loci for increasing ice-layer depth* D_1 *at different temperatures above conducting soil.*

analytic at the pole location and the other with the effect of the singularity. The analytic integral is evaluated with the saddle-point method and the other is expressed in the form of the complementary error function (erfc). When applied to (2), the result is

$$E_\theta = i60k_0Idl\cos\phi\left\{M_{123}\frac{n_1^5(n_1^2-1)}{(n_1^4-M_{123}^2)^2}\right\}\sqrt{\frac{k_0}{\lambda_p}}\cdot\{F(x_0)\}\,\frac{e^{ik_0z\{n_1^2-M_{123}h\}\sqrt{\frac{n_1^2-1}{n_1^4-M_{123}^2}}+ik_0R}}{R}, \tag{27}$$

where $F(x_0)$, the groundwave attenuation function, is

$$F(x_0) = 1 + i\sqrt{\pi x_0^2/2}\,e^{-\frac{1}{2}x_0^2}\operatorname{erfc}(ix_0/\sqrt{2}). \tag{28}$$

Oscillations in amplitude and phase of $F(x_0)$ for phases of $\frac{1}{2}x_0^2$ or p from 0° to −90° are encountered in stratified media if the upper layer has a smaller refractive index than the integrated refractive index of the lower layers (Wait, 1958; Biggs and Swarm, 1965; King and Schlak, 1967).

A similar expression for E_ϕ is obtained for horizontal polarization where $F(y_0)$ has the same functional form as $F(x_0)$.

$$E_\phi = -i60k_0Idl\sin\phi\,F(y_0)\,\frac{e^{ik_0R}}{R}, \tag{29}$$

but the argument y_0 has the form,

$$y_0^2/2 = ik_0Rn_1^2N_{123}^2/2. \tag{30}$$

3. Transmission Line Analogies

Similarities in multilayer results and those for transmission lines are seen if (17) and (18) are written in terms of R_v and R_h,

$$E_\theta = -i30k_0IdlM(h)\cos\phi\cos\theta\,[1+R_v]\,\frac{e^{ik_0R}}{R}, \tag{31}$$

$$E_\phi = -i30k_0IdlN(h)\sin\phi[1+R_h]\,\frac{e^{ik_0R}}{R}, \tag{32}$$

where the reflection coefficients R_v and R_h are for multilayer media,

$$R_v = \frac{M\sqrt{n_1^2 - \sin^2\theta} - n_1^2 \cos\theta}{M\sqrt{n_1^2 - \sin^2\theta} + n_1^2 \cos\theta}, \tag{33}$$

$$R_h = \frac{\cos\theta - N\sqrt{n_1^2 - \sin^2\theta}}{\cos\theta + N\sqrt{n_1^2 - \sin^2\theta}}, \tag{34}$$

and written in the form of load impedances Z_L and Z_0 (for air),

$$R_v = \frac{Z_{Lv} - Z_{0v}}{Z_{Lv} + Z_{0v}}, \qquad R_h = \frac{Z_{Lh} - Z_{0h}}{Z_{Lh} + Z_{0h}}, \tag{35}$$

where Z_{Lv} and Z_{0v} are the impedances for *TM* modes,

$$Z_{Lv} = M\eta_1 \sqrt{1 - \frac{\sin^2\theta}{n_1^2}}, \qquad Z_{0v} = \eta_0 \cos\theta, \tag{36}$$

and Z_{Lh} and Z_{0h} are the impedances for *TE* modes,

$$Z_{Lh} = \frac{\eta_1}{N\sqrt{1 - \frac{\sin^2\theta}{n_1^2}}}, \qquad Z_{0h} = \frac{\eta_0}{\cos\theta}, \tag{37}$$

to find expressions of plane waves incident on multilayer media. For vertical incidence, the expressions are found for *TEM* modes in a multisection transmission line. For other angles of incidence, *TE* or *TM* modes are found if $\sin\theta/n_1$ is replaced by f_c/f, where f_c is the cutoff frequency. These similarities permit waveguide measurements and Smith Charts to be used in predicting the radiation fields of horizontal dipoles in anisotropic and stratified media.

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