SUPERRESOLUTION BY DIFFRACTION OF SUBWAVES

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Illuminating an object with subwaves results in diffracted homogeneous waves which contain information on spatial frequencies of the object spectrum beyond the classical resolution limit. A report is given on some experiments with simple grating-like objects showing how this information can be utilized to form magnified images with superresolution .

1. INTRODUCTION

Optical subwaves † are electromagnetic waves of optical frequencies the wavelength and phase velocities of which are smaller than those of homogeneous waves of the same frequency in the medium concerned. Such waves can be forced upon the medium by the boundary conditions, e.g. in case of total reflection, diffraction by a grating with small period, or propagation in a medium with certain periodic boundary structures [2]. "The price of such small wavelength (or rapid phase changes) along one direction, however, is complete attenuation in another (perpendicular) direction. Compressing phase velocity along one axis loses amplitude at right angles [3]!" But notwithstanding their limited penetration depth these waves might open up two new ways for exceeding the classical resolution limit as recent interference and diffraction experiments with subwaves [4, 5] have shown:
1) Evanescent waves corresponding to Fourier components of the object spectrum with spatial periods less than λ can be recorded by interference with a homogeneous wave or another subwave [6]. A method for recovering the information recorded in this way has still to be worked out (e.g. by illumination with a subwave).
2) Illumination of the object with a subwave leads to diffracted homogeneous waves which contain information about the higher spatial frequencies of the object [7]. Only this method will be illustrated in this paper by some model experiments with subwaves generated by total reflection.

† We prefer the name "subwaves" instead of "evanescent waves" or "surface waves" for following reasons:
1. As a generic term it should comprise all wave-types with subnormal wavelength and phase velocity e.g. also waves of the "slow wave" type as known in the microwave field [1].
2. The evanescence of the waves is a consequence of the impressed field distribution with subnormal phase velocity and wavelength.
3. It is rather the subnormal wavelength and phase velocity than the evanescence of these waves which are of importance for the following experiments.
4. The waves do not only propagate on the surface proper but extend over a finite volume of the medium.

2. PRINCIPLE AND PERFORMANCE OF THE EXPERIMENTS

As pointed out in previous papers [7, 8] for an object with limited spectral bandwidth the method comprises three principal steps (see fig. 1).

a) Transformation of the higher spatial frequencies of the object $U(k_a)$ to homogeneous waves $U(k_b)$ by illuminating the object with a subwave k_s:

$$k_b = k_a - k_s \quad .$$

b) Magnification by a factor M leads to the spectrum $U(k_b/M)$.

c) Shifting this spectrum by k_s/M results in the magnified image of the object $U(k_a/M)$:

$$k_b/M + k_s/M = k_a/M .$$

The transformation a) is the essential step of the method which has been verified experimentally [8]. It is based on our experimental results [4, 5] which show that the diffraction of subwaves can be described by a modified form of the grating equation, as far as the diffraction angles are concerned. It should be noted that illumination by a subwave can be interpreted as oblique illumination with an angle of incidence φ,

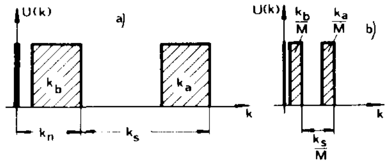

Fig. 1. Scheme of the method in spatial frequency representation. a) The spectrum of the object $U(k_a)$ is shifted by illumination with subwave k_s, resulting in the spectrum $U(k_b)$. b) Magnification and shifting of this spectrum by k_s/M leads to the magnified image $U(k_a/M)$.

where $\sin\varphi > 1$. Objects with greater spectral bandwidth have to be illuminated successively with homogeneous waves of different directions and subwaves of different wavelengths. The coherent composition of the different partial spectra together with the appropriate frequency shifts corresponding to step c) can be accomplished holographically, the use of focused image holograms according to Hohberg [9] being particularly advantageous. For a more detailed discussion of the fundamentals of superresolution experiments see [10].

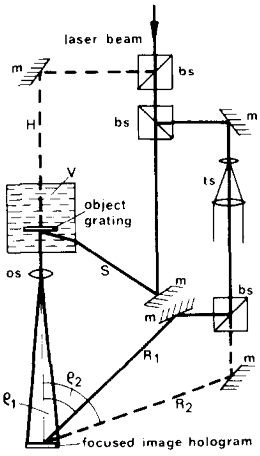

Fig. 2. Experimental arrangement: bs, beam splitter; m, mirror; os, optical system; ts, telescopic system; H, homogeneous illuminating wave; S, wave for subwave illumination; R_1, reference wave with S; R_2, reference wave with H.

The object used for the experiments were absorption gratings‡ generated by interference within a photographic layer (Agfa-Gevaert 8 E 75). Since the size of the AgBr-particles in the unexposed layer is small as compared with the wavelength we may use an average value of the refractive index. Furthermore, since the complex refractive index of the layer for $\lambda = 633$ nm is $m = n - ik' = 1.63 - 1.2 \times 10^{-3}i$, i.e. $k' \ll n$, the interference structure can - with good approximation - be calculated neglecting the absorption of the layer and putting $m = 1.63$ [11, p. 615]. After processing the resulting object structure can be considered as an absorption grating within a medium of the refractive index 1.53 [4, 5].

The experimental arrangement for taking the focused image holograms is shown schematically in fig. 2. The object grating is placed within the glass vessel V filled with CH_2I_2 and is imaged onto the hologram plate by the optical system os with magnification M. First it is illuminated with the subwave deriving from the waves S‡‡ ; in this case the reference wave R_1 is used. In a second exposure the object is illuminated with the homogeneous wave H and the reference wave R_2 is used. The angle ρ_2 has to be choosen in such a way that on reconstruction with the reference wave R_2 the frequency spectrum resulting from the subwave illumination is shifted relatively to that of the second exposure by k_s/M. Supposing thin holograms and using the grating equation, one obtains

$$\sin\rho_2 = \sin\rho_1 + k_s/M\,k_0 \,. \tag{1}$$

The reconstruction of the focused image hologram thus obtained can be performed with white light under a microscope. The method will be illustrated by a very simple example, an object-grating with period $a = 380$ nm, $k_a = 2630\ \mathrm{mm}^{-1}$. By illuminating it with a subwave (wavelength $\lambda_s = 400$ nm, wave number $k_s = 2500\ \mathrm{mm}^{-1}$) one obtains a homogeneous diffracted wave under a diffraction angle $\varphi_H = \arcsin[(k_a - k_s)/k_n]$ which is $2^o43'$ in CH_2I_2 and $4^o43'$ in air. In the image plane of the optical system ($M = 10$) which we suppose to be telescopic the diffracted wave impinges with an angle of incidence $\varphi_i = \arcsin[(k_a - k_s)/M k_0] = 28'$. By superimposing the reference wave R_1

‡ More exactly: absorbing Ag-particles with a grating-like distribution.

‡‡ The experimental technique is described in [4, 5].

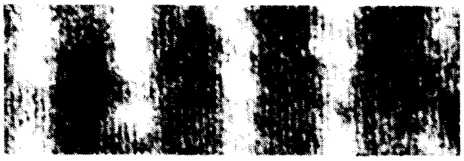

Fig. 3. Microphotograph of the magnified image of object I reconstructed from the focused image hologram. Grating with a_1 = 0.38 μm; a_2 = 4.6 μm; M = 10.

(incidence angle ρ_1 = 36°) it is holographically recorded, producing a grating with period h_1 = = 1.09 μm. The zero order spectrum resulting from illuminating the object with the homogeneous wave H is a plane wave with incidence angle 0°. It is recorded by interference with the reference wave R_2, with ρ_2 = 48°15' [from eq. (1)]. After processing the hologram is reconstructed with wave R_2 producing simultaneously the zero order wave (φ_0 = 0°) and the shifted sideband-wave under an angle φ_s = 9°35'‡. The interference of these two waves leads to the magnified image of the object-grating with period b^* = = $\lambda/\sin\varphi_s$ = 3.8 μm, quite analogous to the formation of the secondary image by interference of the waves emanating from the primary image according to Abbe's theory [11, p. 419]. The description given here is very much simplified for the purpose of elucidation; for a more detailed discussion of some of the problems involved see [12].

3. EXPERIMENTAL RESULTS

By the method described many experiments have been performed; some typical results only will be dealt with here.

Object I: Interference fringes generated by interference of a homogeneous wave with a subwave according to [4, p. 601-603]; the spatial frequency measured parallel to the grating plane was k_{a_1} = 2630 mm^{-1}. Superimposed was a second grating with k_{a_2} = 216 mm^{-1}. The optical system had a numerical aperture of 0.18, the magnification was M = 10. A micro-photograph of the magnified image obtained by reconstruction of the focused image hologram with white light under a microscope is shown in fig. 3. In a second experiment with this object a plane wave was used as zero order spectrum. Also in this case a magnified image of the object, though of inferior quality, was obtained. This shows that not only a single frequency (k_{a_1}) but also the sum and difference frequencies $k_{a_1} + pk_{a_2}$ ($p = \pm 1, \pm 2 \ldots$), present in the object-grating, were actually transformed by subwave illumination.

Object II: A crossed grating with spatial frequency k_{a_1} = 2630 mm^{-1} and, perpendicular to it, k_{a_2} = 194 mm^{-1}. The reconstruction from the focused image hologram (M = 10) resulted in a magnified image with spatial frequencies 262 mm^{-1} and 19.9 mm^{-1}, in good agreement with the theoretical values.

Object III: A modulated grating of equal carrier frequency (2630 mm^{-1}), generated by taking a hologram of a diffuse object (a black and white pattern P) with subwave reference according to [8, fig. 1a]. The magnified reconstructed image showed a slightly modulated grating with the carrier frequency as expected.

Object IV: A grating of spatial frequency k_{a_1} = 5220 mm^{-1}, generated by standing subwaves according to [4, p. 604]. This spatial frequency lies beyond the resolution limit for homogeneous waves (here : 4830 mm^{-1}). For taking the first focused image hologram in this experiment we turned the object grating (fig. 2) by such an angle ψ that the diffracted wave resulting from the subwave illumination crossed the front wall of the vessel V perpendicularly. To calculate the angle ρ_2 it is then necessary to add $(n_1 \sin\psi)/M$ on the right-hand side of eq. (1), with n_1 = refractive index of CH_2I_2. By reconstructing the focused image hologram a magnified (M = 20) image of the object was obtained which, of course, in this case is nothing else but two-beam interference.

Object V: A modulated grating of carrier frequency 5330 mm^{-1} with two-dimensional bandwidth of about 60 × 60 mm^{-2}, obtained by taking a subwave hologram of the pattern P according to [8, fig. 1b]. Now, the additional term of eq. (1) as given above is exactly valid only for the central frequency of the object spectrum; because of the narrow bandwidth, however, the resulting distortion should be small. To check if this complex spectrum was transmitted by subwave illumination, we used a plane wave for the second exposure of the focused image hologram and

‡ In principle there may also be further diffracted sideband-waves corresponding to the -1. and to higher diffraction orders from the grating with period h_1. By proper choice of the holographic technique and the subsequent imaging method the effects of these as well as of the undiffracted reconstruction wave can be supressed.

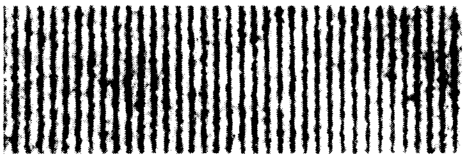

Fig. 4. Microphotograph of the magnified image of object V reconstructed from the focused image hologram: subwave hologram of a black and white pattern with carrier frequency k_a = 5330 mm^{-1}, a = 0.188 μm. M = 20. The very small modulation (60 mm^{-1}) is scarcely perceptible.

made a photograph of the reconstructed image (cf. fig. 4). If this plate was illuminated with a laser beam we could reconstruct an image of the pattern P under a small diffraction angle corresponding to the reduced carrier frequency (fig. 5a). The original pattern (fig. 5b) and the reconstructed image from the subwave hologram (fig. 5c) are shown for comparison. It should be noted that there are 4 intermediate steps between fig. 5b and fig. 5a: 1) Subwave hologram SH of the pattern P. 2) Focused image hologram FH of SH. 3) Photograph I of the reconstructed image from FH. 4) Photograph of the reconstructed image from I. With this object (a - priori - information!) it was also possible to obtain directly a magnified image in the image plane of os (fig. 2) by superimposing a plane wave with the spectrum, resulting from the subwave illumination, shifted by the appropriate angle. Again the pattern P could be reconstructed from the magnified image.

4. CONCLUDING REMARKS

One could perhaps think that the resolution obtained might be a mere consequence of some kind of immersion in CH_2I_2. But it should be noted that the object-gratings actually are in the gelatine layer with refractive index n = 1.53, whereas with respect to immersion Abbe [13] stated (-in English translation-): "If a microscope of the future intended to utilize the high refractivity of the diamond in this respect - which certainly would be inconceivable for other reasons - then it would be necessary to embed all objects of the microscopic investigation inside the diamond without any intermediate substance". With the method proposed it would not be necessary to enclose the object inside the diamond but only to put it onto the surface of the diamond. On the other hand the method is limited to plane or very thin objects because of the small penetration depth of the subwaves. The significance of the experiments described does not lie so much in the resolution obtained which is limited by the wavelength of subwaves generated by total reflection, but in the proof that it is possible to exceed the classical resolution limit by subwave illumination. Now it seems interesting to look for other methods of generating subwaves with shorter wavelengths, methods similar to the slow wave techniques of microwaves being of particular interest.

Fig. 5. Black and white pattern: a) reconstructed from the magnified image of the object V; b) original pattern; c) reconstructed directly from the object V (subwave hologram SH).

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