Scalarization of Dyadic Spectral Green's Functions and Network Formalism for Three-Dimensional Full-Wave Analysis of Planar Lines and Antennas

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***Abstract*— A novel and systematic method is presented for the complete determination of dyadic spectral Green's functions directly from Maxwell's equations. With the use of generalized scalarizations developed in this paper, four general and concise expressions for the spectral Green's functions for one-dimensionally inhomogeneous multilayer structures, excited by three-dimensional electric and magnetic current sources, are given in terms of modal amplitudes together with appropriate explicit singular terms at the source region. It is shown that Maxwell's equations in spectral-domain can be reduced, by using dyadic spectral eigenfunctions, to two sets of z-dependent inhomogeneous transmission-line equations for the modal amplitudes. One set of the transmission-line equations are due to the transverse current sources and the other set due to the vertical current sources. Utilizing these equations, network schematizations of the excitation, transmission and reflection processes of three-dimensional electromagnetic waves in one-dimensionally inhomogeneous multilayer structures are achieved in a full-wave manner. The determination of the spectral Green's functions becomes so simple that it is accomplished by the investigation of voltages and currents on the derived equivalent circuits. Examples of single- and multilayer structures are used to validate the general expressions and the equivalent circuits.**

I. Introduction

One-dimensionally inhomogeneous multilayer planar structures (dielectric or magnetic media), as shown in Fig. 1, now play an important role in microwave and millimeter-wave planar transmission lines and antennas [1], [2]. One of several useful methods for the analysis of such structures is the full-wave (or full-electromagnetic) spectral-domain technique, which has demonstrated excellent performance in electromagnetic simulation in both microwave and millimeter-wave bands [3]. In this technique, integral equations for unknown electric and magnetic current distributions on planar lines and antennas are first obtained by using dyadic spectral Green's functions for such planar structures. Important characteristic parameters of the planar lines and antennas (such as propagation constants, radiation impedances, etc.) can be derived from the solution of these integral equations.

Manuscript received October 21, 1992; revised December 22, 1993.

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IEEE Log Number 9404647.

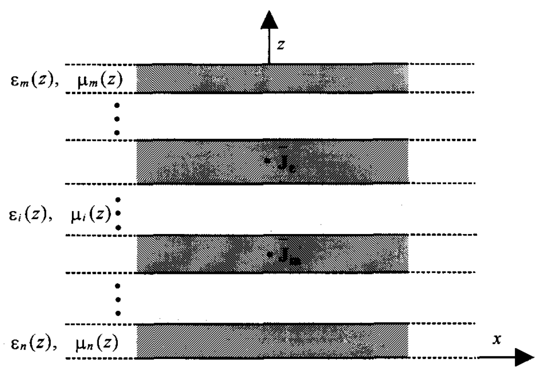

Fig. 1. One-dimensionally inhomogeneous multilayer structures (dielectric and magnetic media) excited by three-dimensional electric and magnetic point sources.

Hence, the dyadic spectral Green's functions are a determinant factor in considering whether the full-wave spectral-domain technique can be used to solve a practical problem of planar lines and antennas.

The spectral Green's functions for planar structures have been studied by a number of authors [4]–[16]. The methods used in the literature can be classified into three groups. One is called the vector potential method. In this method, the vector potential spectral Green's functions are first found and then the electric and magnetic dyadic spectral Green's functions are derived by means of vector operation [4]–[10]. As noted in [8], extending this procedure to problems of more than two-layer structures becomes too complicated and an iterative algorithm is proposed to take additional layers into consideration. The second method is known as the spectral domain impedance method [11], [12], which is a spectral-domain extension of the classic works on transverse field equations [17]. Hence, the method is limited to two-dimensional problems. Some separate procedures have been proposed in [13], [14] to deal with problems of the vertical exciting source, but no singular terms are observed at the source region. The third method is called the scattering superposition method [15], [16]. In this method, the free space spectral Green's functions are first found, and then the spectral Green's functions for multilayer structures are constructed by using scattering superposition. The procedure for using this method may

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become cumbersome because of the complex vector nature of electromagnetic waves.

In this paper, a novel and systematic method is presented for the complete determination of the dyadic spectral Green's functions directly from Maxwell's equations in the spectral-domain. In this approach, vector spectral eigenfunctions with unknown modal amplitudes for one-dimensionally inhomogeneous media are first defined and used to form dyadic spectral eigenfunctions (or generalized modal representations). Then, the spectral Green's functions are formally represented by the dyadic spectral eigenfunctions. Finally, inhomogeneous scalar equations for the unknown modal amplitudes are derived by substituting the spectral Green's functions into Maxwell's equations in spectral-domain form. By using this procedure, Maxwell's equations in spectral-domain form are reduced to two sets of z-dependent inhomogeneous transmission-line equations for the modal amplitudes. In other words, the present procedure provides a new way to scalarize Maxwell's equations as well as vector sources and boundary conditions for one-dimensionally inhomogeneous media. Four general and concise expressions for the spectral Green's functions are given in terms of the modal amplitudes together with appropriate explicit singular terms at the source region, which meet the recent needs of three-dimensional full-wave spectral-domain analysis of planar lines and antennas.

It is interesting to note that one set of transmission-line equations obtained describes the modal amplitudes of TE and TM modes excited by the transverse current source and another set of transmission-line equations describes the modal amplitudes of TE and TM modes excited by the vertical current source. Network schematizations of the excitation, transmission and reflection processes of three-dimensional electromagnetic waves in one-dimensionally inhomogeneous multilayer structures are achieved in a full-wave manner by introducing the equivalent circuits, which are composed of equivalent transmission lines, voltage and current sources. It is shown that both voltage and current sources are needed to represent a three-dimensional electromagnetic excitation source so that source conditions for the modal amplitudes are naturally included in the equivalent circuits. Therefore, the determination of the dyadic spectral Green's functions becomes simplified to the point that it can be accomplished by the investigation of voltage and current distributions on the equivalent circuits, using the conventional circuit theory for transmission lines.

The arrangement in this paper is as follows: In Section II, the basic equations governing the spectral Green's functions are given briefly. In Section III, the present procedures for the determination of the spectral Green's functions are described and the three-dimensional full-wave analogy of one-dimensionally inhomogeneous multilayer structures with transmission-lines is accomplished. In Section IV, two examples, a single-layer planar structure and a multilayer planar structure, are used to validate the general expressions and the results are compared with those given in the literature.

II. Basic Equations for Spectral Green's Functions

In this section, some of the well-known and important relations for the space-domain Green's functions are briefly given for the sake of convenience. These relations are extended to spectral-domain in order to obtain the basic equations for the spectral Green's functions for one-dimensionally inhomogeneous multilayer planar structures.

The electromagnetic fields in an inhomogeneous medium, excited by an electric current density source $\mathbf{J_e}(\mathbf{r})$ and a magnetic current density source $\mathbf{J_m}(\mathbf{r})$, are determined by Maxwell's equations

$$\nabla \times \mathbf{E}(\mathbf{r}) = -j\omega\mu_0\mu(\mathbf{r})\mathbf{H}(\mathbf{r}) - \mathbf{J_m}(\mathbf{r}), \tag{1a}$$

$$\nabla \times \mathbf{H}(\mathbf{r}) = j\omega\varepsilon_0\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r}) + \mathbf{J_e}(\mathbf{r}), \tag{1b}$$

$$\nabla \cdot [\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r})] = -\frac{\nabla \cdot \mathbf{J_e}(\mathbf{r})}{j\omega\varepsilon_0}, \tag{1c}$$

$$\nabla \cdot [\mu(\mathbf{r})\mathbf{H}(\mathbf{r})] = -\frac{\nabla \cdot \mathbf{J_m}(\mathbf{r})}{j\omega\mu_0} \tag{1d}$$

and by the boundary conditions, as well as the "radiation condition" at $|\mathbf{r}| \to \infty$.

Using the standard vector analysis, the electric and magnetic fields in (1) can be expressed in terms of the current sources as

$$\mathbf{E}(\mathbf{r}) = \int_V \overline{\mathbf{G}}_{EJ}(\mathbf{r},\mathbf{r}') \cdot \mathbf{J_e}(\mathbf{r}')\, d\mathbf{r}' + \int_V \overline{\mathbf{G}}_{EM}(\mathbf{r},\mathbf{r}') \cdot \mathbf{J_m}(\mathbf{r}')\, d\mathbf{r}', \tag{2a}$$

$$\mathbf{H}(\mathbf{r}) = \int_V \overline{\mathbf{G}}_{HM}(\mathbf{r},\mathbf{r}') \cdot \mathbf{J_m}(\mathbf{r}')\, d\mathbf{r}' + \int_V \overline{\mathbf{G}}_{HJ}(\mathbf{r},\mathbf{r}') \cdot \mathbf{J_e}(\mathbf{r}')\, d\mathbf{r}' \tag{2b}$$

where the dyadic Green's functions satisfy the dyadic version of Maxwell's equations

$$\nabla \times \overline{\mathbf{G}}_{EJ}(\mathbf{r},\mathbf{r}') = -j\omega\mu_0\mu(\mathbf{r})\overline{\mathbf{G}}_{HJ}(\mathbf{r},\mathbf{r}'), \tag{3a}$$

$$\nabla \times \overline{\mathbf{G}}_{HJ}(\mathbf{r},\mathbf{r}') = j\omega\varepsilon_0\varepsilon(\mathbf{r})\overline{\mathbf{G}}_{EJ}(\mathbf{r},\mathbf{r}') + \overline{\mathbf{I}}\delta(\mathbf{r}-\mathbf{r}'), \tag{3b}$$

$$\nabla \cdot [\varepsilon(\mathbf{r})\overline{\mathbf{G}}_{EJ}(\mathbf{r},\mathbf{r}')] = -\frac{\nabla \cdot \overline{\mathbf{I}}\delta(\mathbf{r}-\mathbf{r}')}{j\omega\varepsilon_0}, \tag{3c}$$

$$\nabla \cdot [\mu(\mathbf{r})\overline{\mathbf{G}}_{HJ}(\mathbf{r},\mathbf{r}')] = 0 \tag{3d}$$

and

$$\nabla \times \overline{\mathbf{G}}_{EM}(\mathbf{r},\mathbf{r}') = -j\omega\mu_0\mu(\mathbf{r})\overline{\mathbf{G}}_{HM}(\mathbf{r},\mathbf{r}') - \overline{\mathbf{I}}\delta(\mathbf{r}-\mathbf{r}'), \tag{4a}$$

$$\nabla \times \overline{\mathbf{G}}_{HM}(\mathbf{r},\mathbf{r}') = j\omega\varepsilon_0\varepsilon(\mathbf{r})\overline{\mathbf{G}}_{EM}(\mathbf{r},\mathbf{r}'), \tag{4b}$$

$$\nabla \cdot [\mu(\mathbf{r})\overline{\mathbf{G}}_{HM}(\mathbf{r},\mathbf{r}')] = -\frac{\nabla \cdot \overline{\mathbf{I}}\delta(\mathbf{r}-\mathbf{r}')}{j\omega\mu_0} \tag{4c}$$

$$\nabla \cdot [\varepsilon(\mathbf{r})\overline{\mathbf{G}}_{EM}(\mathbf{r},\mathbf{r}')] = 0 \tag{4d}$$

where $\overline{\mathbf{I}}$ is a unit dyadic.

In this paper, only one-dimensionally inhomogeneous medium problems are considered and the z-axis of the chosen coordinate system is oriented perpendicularly to the boundaries of multilayer planar structures, as shown in Fig. 1. The inverse Fourier transform is defined as

$$F^{-1}\{\widetilde{\mathbf{A}}(\mathbf{k}_t, z, z')\} = \mathbf{A}(\rho, z, \rho', z') = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} \widetilde{\mathbf{A}}(\mathbf{k}_t, z, z')e^{-j\mathbf{k}_t\cdot(\rho-\rho')}\, d\mathbf{k}_t \tag{5}$$

where ρ is the transverse vector, $\mathbf{k}_t$ is the transverse vector wave number and is written as

$$\mathbf{k}_t = k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}} = k_t(\cos\alpha\hat{\mathbf{x}} + \sin\alpha\hat{\mathbf{y}}) = k_t\hat{\mathbf{k}}_t; k_t = |\mathbf{k}_t|. \tag{6}$$

From (5), $\widetilde{\nabla}$ operator in the spectral-domain form is given by

$$\widetilde{\nabla} = -j\mathbf{k}_t + \frac{d}{dz}\hat{\mathbf{z}}. \tag{7}$$

If the above Fourier transform relation (5) is applied to (2) and we set $\varepsilon(\mathbf{r}) = \varepsilon(z)$ and $\mu(\mathbf{r}) = \mu(z)$, the electric and magnetic fields can be written (using Parseval's theorem) as

$$\mathbf{E}(\mathbf{r}) = \int F^{-1}\{\widetilde{\widetilde{\mathbf{G}}}_{EJ}(\mathbf{k}_t, z, z')\cdot\widetilde{\mathbf{J}}^*_{\mathbf{e}}(\mathbf{k}_t, z')\}dz' + \int F^{-1}\{\widetilde{\widetilde{\mathbf{G}}}_{EM}(\mathbf{k}_t, z, z')\cdot\widetilde{\mathbf{J}}^*_{\mathbf{m}}(\mathbf{k}_t, z')\}dz', \tag{8a}$$

$$\mathbf{H}(\mathbf{r}) = \int F^{-1}\{\widetilde{\widetilde{\mathbf{G}}}_{HM}(\mathbf{k}_t, z, z')\cdot\widetilde{\mathbf{J}}^*_{\mathbf{m}}(\mathbf{k}_t, z')\}dz' + \int F^{-1}\{\widetilde{\widetilde{\mathbf{G}}}_{HJ}(\mathbf{k}_t, z, z')\cdot\widetilde{\mathbf{J}}^*_{\mathbf{e}}(\mathbf{k}_t, z')\}dz' \tag{8b}$$

where the spectral Green's functions satisfy the following dyadic version of Maxwell's equations in spectral-domain

$$\widetilde{\nabla}\times\widetilde{\widetilde{\mathbf{G}}}_{EJ}(\mathbf{k}_t, z, z') = -j\omega\mu_0\mu(z)\widetilde{\widetilde{\mathbf{G}}}_{HJ}(\mathbf{k}_t, z, z'), \tag{9a}$$

$$\widetilde{\nabla}\times\widetilde{\widetilde{\mathbf{G}}}_{HJ}(\mathbf{k}_t, z, z') = j\omega\varepsilon_0\varepsilon(z)\widetilde{\widetilde{\mathbf{G}}}_{EJ}(\mathbf{k}_t, z, z') + \bar{\mathbf{I}}\delta(z-z'), \tag{9b}$$

$$\widetilde{\nabla}\cdot[\varepsilon(z)\widetilde{\widetilde{\mathbf{G}}}_{EJ}(\mathbf{k}_t, z, z')] = -\frac{\widetilde{\nabla}\cdot\bar{\mathbf{I}}\delta(z-z')}{j\omega\varepsilon_0}, \tag{9c}$$

$$\widetilde{\nabla}\cdot[\mu(z)\widetilde{\widetilde{\mathbf{G}}}_{HJ}(\mathbf{k}_t, z, z')] = 0 \tag{9d}$$

and

$$\widetilde{\nabla}\times\widetilde{\widetilde{\mathbf{G}}}_{EM}(\mathbf{k}_t, z, z') = -j\omega\mu_0\mu(z)\widetilde{\widetilde{\mathbf{G}}}_{HM}(\mathbf{k}_t, z, z') - \bar{\mathbf{I}}\delta(z-z'), \tag{10a}$$

$$\widetilde{\nabla}\times\widetilde{\widetilde{\mathbf{G}}}_{HM}(\mathbf{k}_t, z, z') = j\omega\varepsilon_0\varepsilon(z)\widetilde{\widetilde{\mathbf{G}}}_{EM}(\mathbf{k}_t, z, z'), \tag{10b}$$

$$\widetilde{\nabla}\cdot[\varepsilon(z)\widetilde{\widetilde{\mathbf{G}}}_{EM}(\mathbf{k}_t, z, z')] = 0, \tag{10c}$$

$$\widetilde{\nabla}\cdot[\mu(z)\widetilde{\widetilde{\mathbf{G}}}_{HM}(\mathbf{k}_t, z, z')] = -\frac{\widetilde{\nabla}\cdot\bar{\mathbf{I}}\delta(z-z')}{j\omega\mu_0}. \tag{10d}$$

III. Scalarization of Dyadic Spectral Green's Function and Network Formalism

In this section, vector and dyadic spectral eigenfunctions for one-dimensionally inhomogeneous media are first defined. Then, the dyadic spectral Green's functions are scalarized so that they can be determined from two sets of z-dependent inhomogeneous transmission-line equations. Equivalent circuit models for one-dimensionally inhomogeneous multilayer planar structures, excited by three-dimensional electric and magnetic current sources, are given. Finally, some reciprocity relations of the spectral Green's functions are discussed.

A. Vector Spectral Eigenfunctions

For a source-free region, the electromagnetic fields in spectral-domain satisfy the following homogeneous Maxwell's equations

$$\widetilde{\nabla}\times\widetilde{\mathbf{E}}(\mathbf{k}_t, z) = -j\omega\mu_0\mu(z)\widetilde{\mathbf{H}}(\mathbf{k}_t, z), \tag{11a}$$

$$\widetilde{\nabla}\times\widetilde{\mathbf{H}}(\mathbf{k}_t, z) = j\omega\varepsilon_0\varepsilon(z)\widetilde{\mathbf{E}}(\mathbf{k}_t, z). \tag{11b}$$

The TE_z and TM_z eigenfunction representation of the electromagnetic fields in one-dimensionally inhomogeneous media is analogous to the decomposition of the fields in cylindrical waveguides [17–19]. Each plane wave in spectral domain can be considered as a vector eigenfunction with a corresponding wave number and a z-dependent modal amplitude. The vector spectral eigenfunctions are defined, for TE modes, as

$$\widetilde{\mathbf{M}}''(\mathbf{k}_t, z) = \frac{-1}{jk_t}\widetilde{\nabla}\times\hat{\mathbf{z}}V''(z) = V''(z)(\hat{\mathbf{k}}_t\times\hat{\mathbf{z}}), \tag{12a}$$

$$\widetilde{\mathbf{N}}''(\mathbf{k}_t, z) = \frac{-1}{j\omega\mu_0\mu(z)}\widetilde{\nabla}\times\widetilde{\mathbf{M}}''(\mathbf{k}_t, z) = I''(z)\hat{\mathbf{k}}_t - \frac{k_t}{\omega\mu_0\mu(z)}V''(z)\hat{\mathbf{z}} \tag{12b}$$

and for TM modes

$$\widetilde{\mathbf{M}}'(\mathbf{k}_t, z) = \frac{1}{jk_t}\widetilde{\nabla}\times\hat{\mathbf{z}}I'(z) = -I'(z)(\hat{\mathbf{k}}_t\times\hat{\mathbf{z}}), \tag{13a}$$

$$\widetilde{\mathbf{N}}'(\mathbf{k}_t, z) = \frac{1}{j\omega\varepsilon_0\varepsilon(z)}\widetilde{\nabla}\times\widetilde{\mathbf{M}}'(\mathbf{k}_t, z) = V'(z)\hat{\mathbf{k}}_t - \frac{k_t}{\omega\varepsilon_0\varepsilon(z)}I'(z)\hat{\mathbf{z}} \tag{13b}$$

where $\widetilde{\mathbf{M}}''$ and $\widetilde{\mathbf{N}}'$ are the electric eigen-fields of the modes, $\widetilde{\mathbf{N}}''$ and $\widetilde{\mathbf{M}}'$ are the magnetic eigen-fields of the modes. By substituting (12) and (13) into (11), it is easy to verify that modal amplitudes ("voltages" and "currents") V'', I'' and V', I' in the vector spectral eigenfunctions satisfy the following homogeneous transmission-line equations:

$$\frac{d}{dz}V(z) + jk_z(z)Z(z)I(z) = 0, \tag{14a}$$

$$\frac{d}{dz}I(z) + jk_z(z)Y(z)V(z) = 0, \tag{14b}$$

where the prime ($'$) and double prime ($''$) are omitted for convenience and

$$Z(z) = \frac{1}{Y(z)} = \begin{cases} Z'(z) = \dfrac{k_z(z)}{\omega\varepsilon_0\varepsilon(z)}, & \text{for } V' \text{ and } I' \\ Z''(z) = \dfrac{\omega\mu_0\mu(z)}{k_z(z)}, & \text{for } V'' \text{ and } I'' \end{cases} \tag{15a}$$

$$k_z^2(z) = \omega^2\mu_0\varepsilon_0\mu(z)\varepsilon(z) - k_t^2. \tag{15b}$$

The branch of the square root of $k_z^2(z)$ is chosen so that the modal coefficients in (14) satisfy the radiation condition.

Similarly, interchanging the source point z' and the field point z and applying reciprocity theorem, the vector spectral eigenfunctions in dependence on the variable z' are defined, for TE modes, as

$$\widetilde{\mathbf{M}}''(-\mathbf{k}_t, z') = -V''(z')(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}}), \tag{16a}$$

$$\widetilde{\mathbf{N}}''(-\mathbf{k}_t, z') = -I''(z')\hat{\mathbf{k}}_t - \frac{k_t}{\omega\mu_0\mu(z')}V''(z')\hat{\mathbf{z}} \tag{16b}$$

and for TM modes

$$\widetilde{\mathbf{M}}'(-\mathbf{k}_t, z') = I'(z')(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}}), \tag{17a}$$

$$\widetilde{\mathbf{N}}'(-\mathbf{k}_t, z') = -V'(z')\hat{\mathbf{k}}_t - \frac{k_t}{\omega\varepsilon_0\varepsilon(z')}I'(z')\hat{\mathbf{z}}. \tag{17b}$$

B. Dyadic Spectral Eigenfunctions (Generalized Modal Representations)

Dyadic spectral eigenfunctions for the dyadic version of Maxwell's equations (9) and (10) for a source-free region can be obtained by means of the combination of the vector spectral eigenfunctions with variables z and z', given in the above paragraph. First, the modal amplitudes with variables z and z' are combined and re-defined as

$$W(z, z') = V(z)V(z'), \tag{18a}$$

$$I_V(z, z') = I(z)V(z'), \tag{18b}$$

$$V_I(z, z') = V(z)I(z'), \tag{18c}$$

$$\Pi(z, z') = I(z)I(z'). \tag{18d}$$

By using the above definitions, the vector spectral eigenfunctions with variables z and z' can be combined to give compact forms of the dyadic spectral eigenfunction. In particular, the dyadic spectral eigenfunctions are written, for TE modes, as

$$\begin{aligned}(\widetilde{\mathbf{M}}\widetilde{\mathbf{M}})''(\mathbf{k}_t, z, z') &= \widetilde{\mathbf{M}}''(\mathbf{k}_t, z)\widetilde{\mathbf{M}}''(-\mathbf{k}_t, z') \\ &= [V''(z)(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}})][-V''(z')(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}})] \\ &= -W''(z, z')(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}})(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}}),\end{aligned} \tag{19a}$$

$$\begin{aligned}(\widetilde{\mathbf{M}}\widetilde{\mathbf{N}})''(\mathbf{k}_t, z, z') &= \widetilde{\mathbf{M}}''(\mathbf{k}_t, z)\widetilde{\mathbf{N}}''(-\mathbf{k}_t, z') \\ &= -V_I''(z, z')(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}})\hat{\mathbf{k}}_t - \frac{k_t}{\omega\mu_0\mu(z')}W'' \\ &\quad \times (z, z')(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}})\hat{\mathbf{z}},\end{aligned} \tag{19b}$$

$$\begin{aligned}(\widetilde{\mathbf{N}}\widetilde{\mathbf{M}})''(\mathbf{k}_t, z, z') &= \widetilde{\mathbf{N}}''(\mathbf{k}_t, z)\widetilde{\mathbf{M}}''(-\mathbf{k}_t, z') \\ &= -I_V''(z, z')\hat{\mathbf{k}}_t(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}}) + \frac{k_t}{\omega\mu_0\mu(z)}W'' \\ &\quad \times (z, z')\hat{\mathbf{z}}(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}}),\end{aligned} \tag{19c}$$

$$\begin{aligned}(\widetilde{\mathbf{N}}\widetilde{\mathbf{N}})''(\mathbf{k}_t, z, z') &= \widetilde{\mathbf{N}}''(\mathbf{k}_t, z)\widetilde{\mathbf{N}}''(-\mathbf{k}_t, z') \\ &= -\Pi''(z, z')\hat{\mathbf{k}}_t\hat{\mathbf{k}}_t + \frac{k_t}{\omega\mu_0\mu(z)}V_I''(z, z')\hat{\mathbf{z}}\hat{\mathbf{k}}_t, \\ &\quad - \frac{k_t}{\omega\mu_0\mu(z)}I_V''(z, z')\hat{\mathbf{k}}_t\hat{\mathbf{z}} \\ &\quad + \left(\frac{k_t}{\omega\mu_0}\right)^2 \frac{1}{\mu(z)\mu(z')}W''(z, z')\hat{\mathbf{z}}\hat{\mathbf{z}}\end{aligned} \tag{19d}$$

and for TM modes

$$\begin{aligned}(\widetilde{\mathbf{M}}\widetilde{\mathbf{M}})'(\mathbf{k}_t, z, z') &= \widetilde{\mathbf{M}}'(\mathbf{k}_t, z)\widetilde{\mathbf{M}}'(-\mathbf{k}_t, z') \\ &= -\Pi'(z, z')(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}})(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}}),\end{aligned} \tag{20a}$$

$$\begin{aligned}(\widetilde{\mathbf{M}}\widetilde{\mathbf{N}})'(\mathbf{k}_t, z, z') &= \widetilde{\mathbf{M}}'(\mathbf{k}_t, z)\widetilde{\mathbf{N}}'(-\mathbf{k}_t, z') \\ &= I_V'(z, z')(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}})\hat{\mathbf{k}}_t \\ &\quad + \frac{k_t}{\omega\varepsilon_0\varepsilon(z')}\Pi'(z, z')(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}})\hat{\mathbf{z}},\end{aligned} \tag{20b}$$

$$\begin{aligned}(\widetilde{\mathbf{N}}\widetilde{\mathbf{M}})'(\mathbf{k}_t, z, z') &= \widetilde{\mathbf{N}}'(\mathbf{k}_t, z)\widetilde{\mathbf{M}}'(-\mathbf{k}_t, z') \\ &= V_I'(z, z')\hat{\mathbf{k}}_t(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}}) \\ &\quad - \frac{k_t}{\omega\varepsilon_0\varepsilon(z)}\Pi'(z, z')\hat{\mathbf{z}}(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}}),\end{aligned} \tag{20c}$$

$$\begin{aligned}(\widetilde{\mathbf{N}}\widetilde{\mathbf{N}})'(\mathbf{k}_t, z, z') &= \widetilde{\mathbf{N}}'(\mathbf{k}_t, z)\widetilde{\mathbf{N}}'(-\mathbf{k}_t, z') \\ &= -W'(z, z')\hat{\mathbf{k}}_t\hat{\mathbf{k}}_t + \frac{k_t}{\omega\varepsilon_0\varepsilon(z)}I_V'(z, z')\hat{\mathbf{z}}\hat{\mathbf{k}}_t \\ &\quad - \frac{k_t}{\omega\varepsilon_0\varepsilon(z')}V_I'(z, z')\hat{\mathbf{k}}_t\hat{\mathbf{z}} \\ &\quad + \left(\frac{k_t}{\omega\varepsilon_0}\right)^2 \frac{1}{\varepsilon(z)\varepsilon(z')}\Pi'(z, z')\hat{\mathbf{z}}\hat{\mathbf{z}}\end{aligned} \tag{20d}$$

where $(\widetilde{\mathbf{M}}\widetilde{\mathbf{M}})''$ and $(\widetilde{\mathbf{N}}\widetilde{\mathbf{N}})'$ denote the electric fields excited by electric current sources, $(\widetilde{\mathbf{M}}\widetilde{\mathbf{N}})''$ and $(\widetilde{\mathbf{N}}\widetilde{\mathbf{M}})'$ the electric fields excited by magnetic current sources; $(\widetilde{\mathbf{M}}\widetilde{\mathbf{M}})'$ and $(\widetilde{\mathbf{N}}\widetilde{\mathbf{N}})''$ denote the magnetic fields excited by magnetic current sources, $(\widetilde{\mathbf{N}}\widetilde{\mathbf{M}})''$ and $(\widetilde{\mathbf{M}}\widetilde{\mathbf{N}})'$ the magnetic fields excited by electric current sources. In other words, $(\widetilde{\mathbf{M}}\widetilde{\mathbf{M}})''$ and $(\widetilde{\mathbf{N}}\widetilde{\mathbf{N}})'$ are the dyadic spectral eigenfunctions for $\widetilde{\overline{\mathbf{G}}}_{EJ}$, $(\widetilde{\mathbf{M}}\widetilde{\mathbf{N}})''$ and $(\widetilde{\mathbf{N}}\widetilde{\mathbf{M}})'$ for $\widetilde{\overline{\mathbf{G}}}_{EM}$, $(\widetilde{\mathbf{M}}\widetilde{\mathbf{M}})'$ and $(\widetilde{\mathbf{N}}\widetilde{\mathbf{N}})''$ for $\widetilde{\overline{\mathbf{G}}}_{HM}$, and $(\widetilde{\mathbf{N}}\widetilde{\mathbf{M}})''$ and $(\widetilde{\mathbf{M}}\widetilde{\mathbf{N}})'$ for $\widetilde{\overline{\mathbf{G}}}_{HJ}$.

C. Scalarization of Spectral Green's Functions and Network Formalism

It is seen, from section IIIA, that the problems of solving the homogeneous vector Maxwell's equations (11) are reduced to problems of solving the homogeneous scalar equations (14) by means of the vector spectral eigenfunctions. In the following, it will be shown that by using the dyadic spectral eigenfunctions defined above, such a scalarization technique can be extended so that problems of solving the inhomogeneous *dyadic* Maxwell's equations, (9) and (10), are reduced to problems of solving inhomogeneous *scalar* equations. The dyadic version of Maxwell's equations (9) are first considered here. Since $\widetilde{\overline{\mathbf{G}}}_{HJ}$ is solenoidal, $\widetilde{\overline{\mathbf{G}}}_{HJ}$ can be expanded completely in terms of the dyadic spectral eigenfunctions $(\widetilde{\mathbf{N}}\widetilde{\mathbf{M}})''$ and $(\widetilde{\mathbf{M}}\widetilde{\mathbf{N}})'$. It is written as

$$\begin{aligned}\widetilde{\overline{\mathbf{G}}}_{HJ}(\mathbf{k}_t, z, z') &= (\widetilde{\mathbf{N}}\widetilde{\mathbf{M}})''(\mathbf{k}_t, z, z') + (\widetilde{\mathbf{M}}\widetilde{\mathbf{N}})'(\mathbf{k}_t. z. z') \\ &= -I_V''(z, z')\hat{\mathbf{k}}_t(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}}) \\ &\quad + \frac{k_t}{\omega\mu_0\mu(z)}W''(z. z')\hat{\mathbf{z}}(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}}), \\ &\quad + I_V'(z. z')(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}})\hat{\mathbf{k}}_t \\ &\quad + \frac{k_t}{\omega\varepsilon_0\varepsilon(z')}\Pi'(z. z')(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}})\hat{\mathbf{z}}.\end{aligned} \tag{21a}$$

By substituting (21a) into (9) and using the dyadic operation of the dyadic Green's functions at the source region [20], $\tilde{\overline{\mathbf{G}}}_{EJ}$ can be expressed as

$$\begin{aligned}
&\tilde{\overline{\mathbf{G}}}_{EJ}(\mathbf{k}_t, z, z') \\
&\quad = \frac{1}{j\omega\varepsilon_0\varepsilon(z)}[\tilde{\nabla} \times \tilde{\overline{\mathbf{G}}}_{HJ}(\mathbf{k}_t, z, z') - \overline{\mathbf{I}}\delta(z - z')] \\
&\quad = -W''(z, z')(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}})(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}}) - W'(z, z')\hat{\mathbf{k}}_t\hat{\mathbf{k}}_t \\
&\qquad + \frac{k_t}{\omega\varepsilon_0\varepsilon(z)} I_V'(z, z')\hat{\mathbf{z}}\hat{\mathbf{k}}_t - \frac{k_t}{\omega\varepsilon_0\varepsilon(z')} V_I'(z, z')\hat{\mathbf{k}}_t\hat{\mathbf{z}} \\
&\qquad + \left(\frac{k_t}{\omega\varepsilon_0}\right)^2 \frac{1}{\varepsilon(z)\varepsilon(z')}\Pi'(z, z')\hat{\mathbf{z}}\hat{\mathbf{z}} \\
&\qquad - \frac{\delta(z - z')}{j\omega\varepsilon_0\varepsilon(z)}\hat{\mathbf{z}}\hat{\mathbf{z}} \\
&\quad = (\widetilde{\mathbf{M}}\widetilde{\mathbf{M}})''(\mathbf{k}_t, z, z') + (\widetilde{\mathbf{N}}\widetilde{\mathbf{N}})'(\mathbf{k}_t, z, z') \\
&\qquad - \frac{\delta(z - z')}{j\omega\varepsilon_0\varepsilon(z)}\hat{\mathbf{z}}\hat{\mathbf{z}}
\end{aligned} \tag{21b}$$

where the modal amplitudes satisfy two sets of inhomogeneous transmission-line equations:

$$\frac{d}{dz}W(z, z') + jk_z(z)Z(z)I_V(z, z') = 0, \tag{22a}$$

$$\frac{d}{dz}I_V(z, z') + jk_z(z)Y(z)W(z, z') = \delta(z - z'), \tag{22b}$$

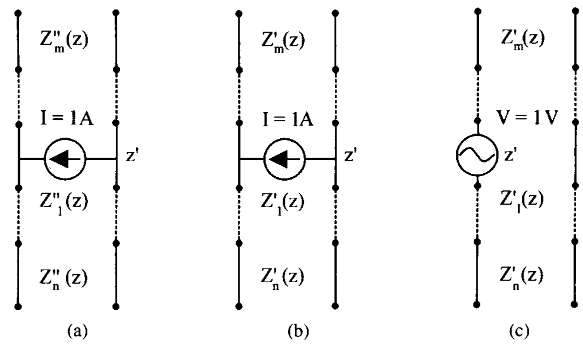
and

$$\frac{d}{dz}\Pi(z, z') + jk_z(z)Y(z)V_I(z, z') = 0, \tag{23a}$$

$$\frac{d}{dz}V_I(z, z') + jk_z(z)Z(z)\Pi(z, z') = \delta(z - z') \tag{23b}$$

where the definitions of Z, Y and k_z are the same as in (15).

Based on (22) and (23), network schematizations of excitation, transmission and reflection processes of three-dimensional electromagnetic waves in one-dimensionally inhomogeneous multilayer structures can be achieved by introducing three equivalent circuits, as shown in Fig. 2. The modal amplitudes are analogous with voltages and currents on the equivalent circuits. It is seen that both TE and TM modes are excited by a transverse electric current source, but only TM modes are excited by a vertical electric current source. Since the source conditions of TM modes for a transverse electric current source are different from the those for a vertical electric current source, a current source and a voltage source are used, respectively, in two TM mode equivalent circuits. All the source and boundary conditions for determining the modal amplitudes are included in the equivalent circuits. In other words, the scalar continuous conditions of currents and voltages on the equivalent circuits are equivalent to the vector source and boundary conditions of three-dimensional electromagnetic fields derived from Maxwell's equations by using vector analysis. Hence, the determination of the modal amplitudes becomes so simple that it is accomplished by the investigation of voltages and currents on the equivalent circuits by using a pure circuit theory approach.



TE modes (a)	*TM* modes (b)	*TM* modes (c)
Analogous relations: $W'' = V_{trans}$ $I_V'' = I_{trans}$	$W' = V_{trans}$ $I_V' = I_{trans}$	$\Pi' = I_{trans}$ $V_I' = V_{trans}$
Boundary conditions: Ground plate: $W'' = 0$ Diele. /mag. media: $W_i'' = W_j''$ $I_{Vi}'' = I_{Vj}''$	$W' = 0$ $W_i' = W_j'$ $I_{Vi}' = I_{Vj}'$	$\frac{d}{dz}\Pi' = 0$ $\Pi_i' = \Pi_j'$ $V_{Ii}' = V_{Ij}'$
Source conditions: $W''(z'^+) = W''(z'^-)$ $I_V''(z'^+) - I_V''(z'^-) = 1$	$W'(z'^+) = W'(z'^-)$ $I_V'(z'^+) - I_V'(z'^-) = 1$	$\Pi'(z'^+) = \Pi'(z'^-)$ $V_I'(z'^+) - V_I'(z'^-) = 1$

Fig. 2. Equivalent TE and TM circuits models for one-dimensionally inhomogeneous multilayer planar structures excited by a three-dimensional electric point source. (a) TE modes for transverse electric point source excitation. (b) TM modes for transverse electric point source excitation. (c) TM modes for vertical electric point source excitation.

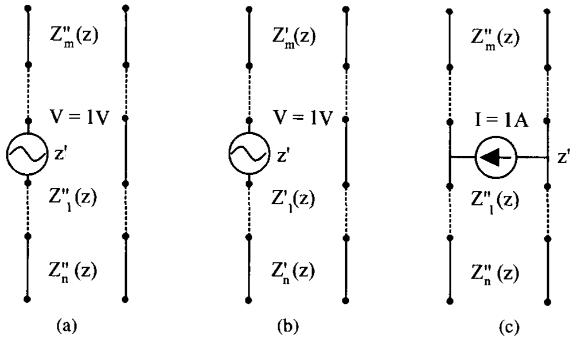
By using similar procedures, the spectral dyadic Green's functions $\tilde{\overline{\mathbf{G}}}_{EM}$ and $\tilde{\overline{\mathbf{G}}}_{HM}$ in (10) are given by

$$\begin{aligned}
&\tilde{\overline{\mathbf{G}}}_{EM}(\mathbf{k}_t, z, z') \\
&\quad = (\widetilde{\mathbf{M}}\widetilde{\mathbf{N}})''(\mathbf{k}_t, z, z') + (\widetilde{\mathbf{N}}\widetilde{\mathbf{M}})'(\mathbf{k}_t, z, z') \\
&\quad = -V_I''(z, z')(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}})\hat{\mathbf{k}}_t - \frac{k_t}{\omega\mu_0\mu(z')}W''(z, z')(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}})\hat{\mathbf{z}} \\
&\qquad + V_I'(z, z')\hat{\mathbf{k}}_t(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}}) - \frac{k_t}{\omega\varepsilon_0\varepsilon(z)}\Pi'(z, z')\hat{\mathbf{z}}(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}})
\end{aligned} \tag{24a}$$

and

$$\begin{aligned}
&\tilde{\overline{\mathbf{G}}}_{HM}(\mathbf{k}_t, z, z') \\
&\quad = -\frac{1}{j\omega\mu_0\mu(z)}[\tilde{\nabla} \times \tilde{\overline{\mathbf{G}}}_{EM}(\mathbf{k}_t, z, z') + \overline{\mathbf{I}}\delta(z - z')] \\
&\quad = -\Pi'(z, z')(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}})(\hat{\mathbf{k}}_t \times \hat{\mathbf{z}}) - \Pi''(z, z')\hat{\mathbf{k}}_t\hat{\mathbf{k}}_t \\
&\qquad + \frac{k_t}{\omega\mu_0\mu(z)}V_I''(z, z')\hat{\mathbf{z}}\hat{\mathbf{k}}_t - \frac{k_t}{\omega\mu_0\mu(z')}I_V''(z, z')\hat{\mathbf{k}}_t\hat{\mathbf{z}} \\
&\qquad + \left(\frac{k_t}{\omega\mu_0}\right)^2 \frac{1}{\mu(z)\mu(z')}W''(z, z')\hat{\mathbf{z}}\hat{\mathbf{z}} - \frac{\delta(z - z')}{j\omega\mu_0\mu(z')}\hat{\mathbf{z}}\hat{\mathbf{z}} \\
&\quad = (\widetilde{\mathbf{M}}\widetilde{\mathbf{M}})'(\mathbf{k}_t, z, z') + (\widetilde{\mathbf{N}}\widetilde{\mathbf{N}})''(\mathbf{k}_t, z, z') - \frac{\delta(z - z')}{j\omega\mu_0\mu(z)}\hat{\mathbf{z}}\hat{\mathbf{z}}
\end{aligned} \tag{24b}$$

where the modal amplitudes also satisfy two sets of inhomogeneous transmission-line equations (22) and (23), but



TE modes (a)	*TM* modes (b)	*TE* modes (c)
Analogous relations: $\Pi'' = I_{trans}$ $V_I'' = V_{trans}$	$\Pi' = I_{trans}$ $V_I' = V_{trans}$	$W'' = V_{trans}$ $I_V'' = I_{trans}$
Boundary conditions: Ground plate: $\frac{d}{dz}\Pi'' = 0$ Diele./mag. media: $\Pi_i'' = \Pi_j''$ $V_{Ii}'' = V_{Ij}''$	$\frac{d}{dz}\Pi' = 0$ $\Pi_i' = \Pi_j'$ $V_{Ii}' = V_{Ij}'$	$W'' = 0$ $W_i'' = W_j''$ $I_{Vi}'' = I_{Vj}''$
Source condition: $\Pi''(z'^+) = \Pi''(z'^-)$ $V_I''(z'^+) - V_I''(z'^-) = 1$	$\Pi'(z'^+) = \Pi'(z'^-)$ $V_I'(z'^+) - V_I'(z'^-) = 1$	$W''(z'^+) = W''(z'^-)$ $I_V''(z'^+) - I_V''(z'^-) = 1$

Fig. 3. Equivalent TE and TM circuits models for one-dimensionally inhomogeneous multilayer planar structures excited by a three-dimensional magnetic point source. (a) TE modes for transverse magnetic point source excitation. (b) TM modes for transverse magnetic point source excitation. (c) TE modes for vertical magnetic point source excitation.

the modal amplitudes due to the transverse component of a magnetic current source are determined by (23) and the modal amplitudes due to the vertical component of a magnetic current source are determined by (22). It is seen that both TE and TM modes are excited by a transverse magnetic current source, but only TE modes are excited by a vertical magnetic current source. Three equivalent circuits for these modes are shown in Fig. 3. All the source and boundary conditions for determining the modal amplitudes are included in the equivalent circuits.

Finally, it is pointed out that the dyadic spectral Green's functions given above satisfy certain reciprocity relations. In view of (18), (22), and (23), the relations between the modal amplitudes due to a transverse point-source and those due to a vertical point-source are written as

$$V_I(z,z') = \frac{-1}{jk_z(z')Z(z')}\frac{d}{dz'}W(z,z'), \tag{25a}$$

$$I_V(z,z') = \frac{-1}{jk_z(z')Z(z')}\frac{d}{dz'}\Pi(z,z'), \tag{25b}$$

$$\Pi(z,z') = \frac{-1}{[jk_z(z)Z(z)][jk_z(z')Z(z')]}\frac{d^2}{dz\,dz'}W(z,z'). \tag{25c}$$

The reciprocity relations for the modal amplitudes are given by

$$W(z,z') = W(z',z), \tag{26a}$$

$$\Pi(z,z') = \Pi(z',z), \tag{26b}$$

$$I_V(z,z') = -V_I(z',z). \tag{26c}$$

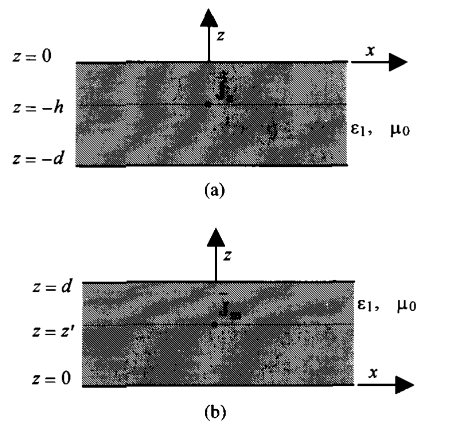


Fig. 4. Single-layer structures. (a) Excited by three-dimensional electric point sources. (b) Excited by three-dimensional magnetic point sources.

From (26) it is easy to see that the dyadic spectral Green's functions satisfy the following reciprocity relations

$$\tilde{\overline{\mathbf{G}}}_{EJ}(\mathbf{k}_t,z,z') = \widetilde{\overline{\mathbf{G}}^{\mathbf{T}}_{EJ}}(\mathbf{k}_t,z',z), \tag{27a}$$

$$\tilde{\overline{\mathbf{G}}}_{HM}(\mathbf{k}_t,z,z') = \widetilde{\overline{\mathbf{G}}^{\mathbf{T}}_{HM}}(\mathbf{k}_t,z',z), \tag{27b}$$

$$\tilde{\overline{\mathbf{G}}}_{HJ}(\mathbf{k}_t,z,z') = -\widetilde{\overline{\mathbf{G}}^{\mathbf{T}}_{EM}}(\mathbf{k}_t,z',z) \tag{27c}$$

where superscript "T" denotes the transposed dyadic. The above reciprocity relations are the representation of the Lorentz reciprocity theorem in spectral domain for one-dimensionally inhomogeneous planar structures with the geometric symmetry about z-axis.

IV. Examples

Two examples, a single-layer structure and a multilayer structure, are used to validate the general expressions and the equivalent circuits obtained in the above section. It will be shown that various expressions, derived by using different approaches in the literature, can be obtained by means of the circuit theory for transmission lines.

A. Single-Layer Structures

In this example, the modal amplitudes in the spectral Green's functions (21) and (24) will be determined by solving two sets of inhomogeneous transmission-line equations (22) and (23). For the case of a three-dimensional electric current source excitation, the geometric parameters used in [14] are plotted in Fig. 4(a). Allowing $k_z(z) = -j\gamma(z)$, the modal amplitudes $W''(z,h)$, $W'(z,h)$ and $\Pi'(z,h)$ are given by

$$W''(z,h) = \begin{cases} V''(\Omega=0)e^{-\gamma_2 z}, & z \geq 0 \\ V''(\Omega=z), & -h \leq z \leq 0 \\ V''(\Omega=-h)\dfrac{\mathrm{sh}\gamma_1(z+d)}{\mathrm{sh}\gamma_1(z-d)}, & -d \leq z \leq -h \end{cases} \tag{28}$$

$$W'(z,h) = \begin{cases} V'(\Omega=0)e^{-\gamma_2 z}, & z \geq 0 \\ V'(\Omega=z), & -h \leq z \leq 0 \\ V'(\Omega=-h)\dfrac{\mathrm{sh}\gamma_1(z+d)}{\mathrm{sh}\ \gamma_1(z-d)}, & -d \leq z \leq -h \end{cases} \tag{29}$$

$$\Pi'(z,h) = \begin{cases} I'(\Omega=0)e^{-\gamma_2 z}, & z \geq 0 \\ I'(\Omega=z), & -h \leq z \leq 0 \\ I'(\Omega=-h)\dfrac{\mathrm{ch}\gamma_1(z+d)}{\mathrm{ch}\gamma_1(z-d)}, & -d \leq z \leq -h \end{cases} \tag{30}$$

where

$$V''(\Omega) = A''[(-\gamma_2/\gamma_1)\mathrm{sh}(\gamma_1\Omega) + \mathrm{ch}(\gamma_1\Omega)], \tag{31a}$$
$$V'(\Omega) = A'[(-\gamma_1/\varepsilon_1\gamma_2)\mathrm{sh}(\gamma_1\Omega) + \mathrm{ch}(\gamma_1\Omega)], \tag{31b}$$
$$I'(\Omega) = B'[(-\varepsilon_1\gamma_2/\gamma_1)\mathrm{sh}(\gamma_1\Omega) + \mathrm{ch}(\gamma_1\Omega)], \tag{31c}$$
$$A'' = j\omega\mu_0 \frac{\mathrm{sh}\gamma_1(d-h)}{\gamma_1 \mathrm{sh}\ (\gamma_1 d) + \gamma_2 \mathrm{ch}(\gamma_1 d)}, \tag{31d}$$
$$A' = \frac{\gamma_1\gamma_2}{j\omega\varepsilon_0} \frac{\mathrm{sh}\gamma_1(d-h)}{\gamma_1\mathrm{sh}(\gamma_1 d) + \varepsilon_1\gamma_2\mathrm{ch}(\gamma_1 d)}, \tag{31e}$$
$$B' = j\omega\varepsilon_0\varepsilon_1 \frac{\mathrm{ch}\gamma_1(d-h)}{\gamma_1\mathrm{sh}(\gamma_1 d) + \varepsilon_1\gamma_2\mathrm{ch}(\gamma_1 d)}, \tag{31f}$$
$$\gamma_1^2 = k_t^2 - \omega^2\mu_0\varepsilon_0\varepsilon_1, \tag{31g}$$
$$\gamma_2^2 = k_t^2 - \omega^2\mu_0\varepsilon_0. \tag{31g}$$

The other modal amplitudes I_V'', I_V' and V_I' in (21) can easily be obtained by means of (22a) and (23a). It is easy to prove that except for a singular term at the source region, the electric spectral Green's function obtained here is equivalent to the one given in [14]. The singular term in (21b) is equivalent to that given in [9], [10], where the singular term was determined by using a "principal volume."

For the case of the magnetic current source excitation, the geometric parameters used in [7] are plotted in Fig. 4(b). The modal amplitudes Π'' and Π' are written as

$$\Pi''(z,z') = \begin{cases} I''(\Omega=d)e^{-jk_2(z-d)}, & z \geq d \\ I''(\Omega=z), & z' \leq z \leq d \\ I''(\Omega=z')\dfrac{\cos k_1 z}{\cos k_1 z'}, & 0 \leq z \leq z' \end{cases} \tag{32}$$

$$\Pi'(z,z') = \begin{cases} I'(\Omega=d)e^{-jk_2(z-d)}, & z \geq d \\ I'(\Omega=z), & z' \leq z \leq d \\ I'(\Omega=z')\dfrac{\cos k_1 z}{\cos k_1 z'}, & 0 \leq z \leq z' \end{cases} \tag{33}$$

where

$$I''(\Omega) = (B_1''\sin k_1\Omega + B_2''\cos k_1\Omega)\cos k_1 z', \tag{34a}$$
$$I'(\Omega) = (B_1'\sin k_1\Omega + B_2'\cos k_1\Omega)\cos k_1 z', \tag{34b}$$
$$B_1'' = -\frac{jk_1}{\omega\mu_0}, \tag{34c}$$
$$B_2'' = \frac{k_1}{\omega\mu_0} \frac{k_2\cos k_1 d + jk_1\sin k_1 d}{k_1\cos k_1 d + jk_2\sin k_1 d}, \tag{34d}$$
$$B_1' = -\frac{j\omega\varepsilon_0\varepsilon_1}{k_1}, \tag{34e}$$
$$B_2' = \frac{\omega\varepsilon_0\varepsilon_1}{k_1} \frac{k_1\cos k_1 d + jk_2\varepsilon_1\sin k_1 d}{k_2\varepsilon_1\cos k_1 d + jk_1\sin k_1 d}, \tag{34f}$$
$$k_1^2 = \omega^2\mu_0\varepsilon_0\varepsilon_1 - k_t^2, \tag{34g}$$
$$k_2^2 = \omega^2\mu_0\varepsilon_0 - k_t^2. \tag{34h}$$

By substituting modal coefficients Π'' Π' into (24) and letting $z = z' = 0$, it is easy to prove that the results obtained here are equivalent to the ones given in [7].

B. Multilayer Structures

Since the modal amplitudes in the spectral Green's functions are analogous with voltages and currents on the equivalent circuits, they can be obtained by means of the conventional circuit theory for transmission lines. In the following, it will be shown that various results for multilayer structures, given in the literature, can be obtained by using such circuit theory.

Standing-Wave Approach: In this approach, the modal amplitudes are represented by the standing-wave voltages and currents on the equivalent transmission lines. From the circuit theory for transmission lines, the standing-wave voltages and currents on the transmission lines can be obtained by using the two-port network matrices which are composed of the impedances/admittances of the transmission lines, (such as impedance matrices, admittance matrices, voltage-current transmission ($ABCD$) matrices, etc.) For example, the standing-wave voltage and current at the source point $z = z'$ in Fig. 2(a) are given by

$$W''(z=z'^+) = W''(z=z'^-) = \frac{1}{Y_u'' + Y_d''}, \tag{35a}$$
$$I_V''(z=z'^+) = \frac{Y_u''}{Y_u'' + Y_d''}, \tag{35b}$$
$$I_V''(z=z'^-) = \frac{-Y_d''}{Y_u'' + Y_d''} \tag{35c}$$

where Y_u'' and Y_d'' are the admittances looking up and down at $z = z'$ into the equivalent circuit. The voltage and current at any point on the equivalent transmission line can be derived by using $ABCD$ matrices of the line. For the case of two-dimensional problems, it is seen that the results obtained here are equivalent to those derived by using spectral domain impedance approach [11], [12].

Traveling-Wave Approach: In this approach, the modal amplitudes in the spectral Green's functions are represented by the traveling-wave voltages and currents on the equivalent transmission lines. The traveling-wave voltages and currents on the transmission lines in Figs. 2 and 3 can be obtained by using the two-port network matrices which are composed of voltage and current reflection and transmission coefficients of the lines, (such as scattering matrices and wave-amplitude transmission matrices). In the following, the modal amplitudes for multilayer homogeneous structures are determined by using voltage wave-amplitude transmission matrices.

The modal amplitudes W and I_V for multilayer planar structures are first considered. The geometrical configuration of the problem is shown in Fig. 5, where the prime (′) and the double prime (″) are omitted for convenience. The general solutions for the modal amplitudes can be written as

$$W(z,z') = \begin{cases} a_i^+ e^{-jk_i z} + a_i^- e^{jk_i z}, & z \geq z' \\ b_j^+ e^{jk_j z} + b_j^- e^{-jk_j z}, & z \leq z' \end{cases} \tag{36a}$$

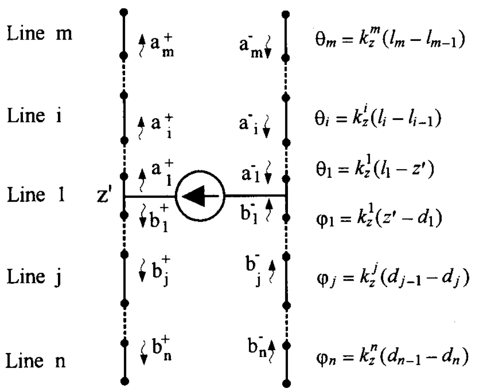


Fig. 5. Geometric configuration of the problem.

$$I_V(z,z') = \begin{cases} \left(\dfrac{a_i^+}{Z_i}\right)e^{-jk_i z} + \left(\dfrac{-a_i^-}{Z_i}\right)e^{jk_i z}, & z \geq z' \\ \left(\dfrac{-b_j^+}{Z_j}\right)e^{jk_j z} + \left(\dfrac{b_j^-}{Z_j}\right)e^{-jk_j z}, & z \leq z' \end{cases} \tag{36b}$$

where a_i^+, b_j^+ and a_i^-, b_j^- are incident and reflected voltage wave amplitudes on lines i and j. Using the transmission matrices $\overline{\mathbf{U}}$ and $\overline{\mathbf{V}}$ of order 2×2 (defined in the Appendix). The voltage wave amplitudes a_i and b_j on lines i and j can be expressed, in terms of the voltage wave amplitudes a_1 and b_1 on line 1 (or the source region), as

$$\begin{bmatrix} a_i^+ \\ a_i^- \end{bmatrix} = \overline{\mathbf{U}}_{(I-1)} \begin{bmatrix} a_1^+ \\ a_1^- \end{bmatrix}, \tag{37a}$$

$$\begin{bmatrix} b_j^+ \\ b_j^- \end{bmatrix} = \overline{\mathbf{V}}_{(J-1)} \begin{bmatrix} b_1^+ \\ b_1^- \end{bmatrix}. \tag{37b}$$

By enforcing the source conditions at $z = z'$, a_1 and b_1 are given by

$$a_1^+ = \frac{Z_1(1+\Gamma_1^d)}{2(1-\Gamma_1^u\Gamma_1^d)}, \tag{38a}$$

$$a_1^- = a_1^+\Gamma_1^u, \tag{38b}$$

$$b_1^+ = \frac{Z_1(1+\Gamma_1^u)}{2(1-\Gamma_1^u\Gamma_1^d)}, \tag{38c}$$

$$b_1^- = b_1^+\Gamma_1^d \tag{38b}$$

where Γ_1^u and Γ_1^d are the voltage reflection coefficients looking upward and downward at $z = z'$ into the equivalent circuits. These coefficients can be determined by employing transmission matrices $\overline{\mathbf{A}}_M$ and $\overline{\mathbf{B}}_N$ of order 2×2, defined in the appendix, and by enforcing the termination conditions for the incident and reflected voltages. For the case when the top or bottom line is infinitely long (or free space for planar structures), the reflected voltage $a_{(m+1)}$ or $b_{(n+1)}$ vanishes, and we obtain

$$\Gamma_1^u = A_{21}^M / A_{11}^M, \tag{39a}$$

$$\Gamma_1^d = B_{21}^N / B_{11}^N \tag{39b}$$

where A_{ij}^M and B_{ij}^N are elements of the transmission matrices $\overline{\mathbf{A}}_M$ and $\overline{\mathbf{B}}_N$. For the case when the top or bottom line is grounded, the relations between the incident and reflected voltage wave amplitudes on line m or line n are given by

$$a_m^+ e^{-j\vartheta_m} = a_m^- e^{j\vartheta_m}, \tag{40a}$$

$$b_n^+ e^{j\varphi_n} = b_n^- e^{-j\varphi_n} \tag{40b}$$

and we obtain

$$\Gamma_1^u = \frac{A_{21}^{(M-1)} - A_{22}^{(M-1)} e^{-j2\vartheta_m}}{A_{11}^{(M-1)} - A_{12}^{(M-1)} e^{-j2\vartheta_m}}, \tag{41a}$$

$$\Gamma_1^d = \frac{B_{21}^{(N-1)} - B_{22}^{(N-1)} e^{j2\varphi_n}}{B_{11}^{(N-1)} - B_{12}^{(N-1)} e^{j2\varphi_n}}. \tag{41b}$$

Similarly, modal amplitudes Π and V_I can be written as

$$\Pi(z,z') = \begin{cases} c_i^+ e^{-jk_i z} + c_i^- e^{jk_i z}, & z \geq z' \\ d_j^+ e^{jk_j z} + d_j^- e^{-jk_j z}, & z \leq z' \end{cases} \tag{42a}$$

$$V_I(z,z') = \begin{cases} \left(\dfrac{c_i^+}{Y_i}\right)e^{-jk_i z} + \left(\dfrac{-c_i^-}{Y_i}\right)e^{jk_i z}, & z \geq z' \\ \left(\dfrac{-d_j^+}{Y_j}\right)e^{jk_j z} + \left(\dfrac{d_j^-}{Y_j}\right)e^{-jk_j z}, & z \leq z' \end{cases} \tag{42b}$$

where c_i^+, d_j^+ and c_i^-, d_j^- are incident and reflected current wave amplitudes on lines i and j. They can be expressed, in terms of the current wave amplitudes c_1 and d_1 on line 1 (or the source region), as

$$\begin{bmatrix} c_i^+ \\ -c_i^- \end{bmatrix} = \left(\frac{Y_i}{Y_1}\right) \overline{\mathbf{U}}_{(I-1)} \begin{bmatrix} c_1^+ \\ -c_1^- \end{bmatrix}, \tag{43a}$$

$$\begin{bmatrix} -d_j^+ \\ d_j^- \end{bmatrix} = \left(\frac{Y_j}{Y_1}\right) \overline{\mathbf{V}}_{(J-1)} \begin{bmatrix} -d_1^+ \\ d_1^- \end{bmatrix}, \tag{43b}$$

$$\overline{\mathbf{U}}_I = \prod_{h=1}^{i} \frac{1}{T_{(i+2-h)(i+1-h)}} \times \begin{bmatrix} e^{j\vartheta_{(i+1-h)}} & R_{(i+2-h)(i+1-h)} e^{j\vartheta_{(i+1-h)}} \\ R_{(i+2-h)(i+1-h)} e^{-j\vartheta_{(i+1-h)}} & e^{-j\vartheta_{(i+1-h)}} \end{bmatrix} \tag{46a}$$

$$\overline{\mathbf{V}}_J = \prod_{h=1}^{j} \frac{1}{T_{(j+2-h)(j+1-h)}} \times \left[e^{-j\varphi_{(j+1-h)}} \quad R_{(j+2-h)(j+1-h)} e^{-j\varphi_{(j+1-h)}} R_{(j+2-h)(j+1-h)} e^{j\varphi_{(j+1-h)}} \quad e^{j\varphi_{(j+1-h)}} \right] \tag{46b}$$

$$c_1^+ = \frac{Y_1(1-\Gamma_1^d)}{2(1-\Gamma_1^u\Gamma_1^d)}, \tag{44a}$$
$$c_1^- = -c_1^+\Gamma_1^u, \tag{44b}$$
$$d_1^+ = \frac{Y_1(1-\Gamma_1^u)}{2(1-\Gamma_1^u\Gamma_1^d)}, \tag{44c}$$
$$d_1^- = -d_1^+\Gamma_1^d \tag{44d}$$

where the relation for the voltage and current reflection coefficients on a transmission line is used. So far the traveling-wave representation of the modal amplitudes for multilayer planar structures has been obtained by using voltage-wave-amplitude transmission matrices. From the above derivation, it is seen that only the source conditions at $z = z'$ and the termination conditions in the equivalent circuits are used to specify the general solutions (36) and (42). Since the transmission matrices are employed, the continuous conditions at all other interfaces of the equivalent transmission lines are automatically satisfied. For the electric spectral Green's functions, it can be proved that the results obtained here are equivalent to that given in [15] for two-layer structures and equivalent to that given in [16] for multilayer structures if the electric current source is located in free space.

V. Conclusion

This paper presents a novel and systematic method to scalarize dyadic spectral Green's functions for one-dimensionally inhomogeneous multilayer planar structures, excited by electric and magnetic current sources. It is shown that the modal amplitudes in the spectral Green's functions satisfy two sets of inhomogeneous transmission-line equations. The determination of the modal amplitudes becomes so simplified that it is accomplished by using a pure circuit theory approach. Three-dimensional full-wave spectral analysis of planar transmission lines and antennas in one-dimensionally inhomogeneous multilayer planar structures can be formulated in a simple fashion by using the spectral Green's functions given in this paper.

Appendix

The geometric configuration of the problem is shown in Fig. 5. The voltage wave amplitudes a_{i+1} and b_{j+1} on line $(i+1)$ and line $(j+1)$ can be expressed, in terms of the voltage wave amplitudes a_1 and b_1 on line 1 (or the source region), as

$$\begin{bmatrix} a_{i+1}^+ \\ a_{i+1}^- \end{bmatrix} = \overline{\mathbf{U}}_I \begin{bmatrix} a_1^+ \\ a_1^- \end{bmatrix}, \tag{45a}$$
$$\begin{bmatrix} b_{j+1}^+ \\ b_{j+1}^- \end{bmatrix} = \overline{\mathbf{V}}_J \begin{bmatrix} b_1^+ \\ b_1^- \end{bmatrix} \tag{45b}$$

where (46a), (see bottom of the previous page) is the voltage-wave-amplitude transmission matrix of order 2×2 above the source region, (see (46b) at the bottom of the previous page) is the voltage-wave-amplitude transmission matrix of order 2×2 below the source region, and

$$T_{(h+1)h} = \frac{2Z_h}{Z_{(h+1)} + Z_h}, \tag{47a}$$
$$R_{(h+1)h} = \frac{Z_h - Z_{(h+1)}}{Z_{(h+1)} + Z_h} \tag{47b}$$

are the voltage transmission and reflection coefficients from line $(h+1)$ to line h, where Z_h is the characteristic impedance of line h, as defined in (15).

The voltage wave amplitudes a_1 and b_1 on line 1 (or the source region) can also be expressed, in terms of the voltage wave amplitudes a_{i+1} and b_{j+1} on line $(i+1)$ and line $(j+1)$, as

$$\begin{bmatrix} a_1^+ \\ a_1^- \end{bmatrix} = \overline{\mathbf{A}}_I \begin{bmatrix} a_{i+1}^+ \\ a_{i+1}^- \end{bmatrix}, \tag{48a}$$
$$\begin{bmatrix} b_1^+ \\ b_1^- \end{bmatrix} = \overline{\mathbf{B}}_J \begin{bmatrix} b_{j+1}^+ \\ b_{j+1}^- \end{bmatrix} \tag{48b}$$

where

$$\overline{\mathbf{A}}_I = \prod_{h=1}^{i} \frac{1}{T_{h(h+1)}} \times \begin{bmatrix} e^{j\vartheta_h} & R_{h(h+1)}e^{j\vartheta_h} \\ R_{h(h+1)}e^{-j\vartheta_h} & e^{-j\vartheta_h} \end{bmatrix} \tag{49a}$$

is the voltage-wave-amplitude transmission matrix of order 2×2 above the source region,

$$\overline{\mathbf{B}}_J = \prod_{h=1}^{j} \frac{1}{T_{h(h+1)}} \times \begin{bmatrix} e^{-j\varphi_h} & R_{h(h+1)}e^{-j\varphi_h} \\ R_{h(h+1)}e^{j\varphi_h} & e^{j\varphi_h} \end{bmatrix} \tag{49b}$$

is the voltage-wave-amplitude transmission matrix of order 2×2 below the source region, and

$$T_{h(h+1)} = \frac{2Z_{(h+1)}}{Z_{(h+1)} + Z_h}, \tag{50a}$$
$$R_{h(h+1)} = \frac{Z_{(h+1)} - Z_h}{Z_{(h+1)} + Z_h} \tag{50b}$$

are the voltage transmission and reflection coefficients from line h to line $(h+1)$.

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