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Electrodynamics of quasi-two-dimensional electrons*

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For a system of electrons confined to a thin but finite layer, the nonlocal frequency-dependent dielectric tensor is constructed in the random-phase approximation and used in the Maxwell equations to study the electromagnetic properties. Retardation effects are negligible if the thickness of the electron layer is small compared with the wavelength of the electromagnetic wave.

I. INTRODUCTION

Quasi-two-dimensional electron systems have been studied intensively in recent years by both experimentalists and theorists. These systems are realized in the cases of electrons on the surface of liquid helium,[1] layered compounds,[2] and inversion and accumulation layers in semiconductor junctions.[3] The aim of this paper is the study of the electromagnetic properties in these quasi-two-dimensional systems. The thickness of the layer occupied by the electrons in the systems mentioned above is typically of the order of 10 Å. For a range of phenomena in these systems, the finite thickness is unimportant and a strictly two-dimensional model of the electron gas is used. Electrodynamics in such a system has been studied.[2,4-6] However, there are phenomena for which the small but finite thickness of the electron system cannot be neglected. Its effect on the electromagnetic properties, as well as the interaction effects of the electrons, will be the subject of study here.

Inversion layers in metal-oxide-semiconductor junctions provide examples of the systems to which the application of our theory is most appropriate. A constant electric field applied perpendicular to a metal-SiO_2-Si junction draws electrons from the bulk silicon and traps them in states confined to a small region on the silicon side of the oxide-semiconductor interface. Electron motion parallel to the interface is considered free in the effective-mass approximation. For a given momentum parallel to the interface, the energy levels of the electrons are quantized. (see Fig. 1.) As functions of the parallel momentum, the energy levels form parabolic bands. The energies and wave functions in the SiO_2-Si interface have been obtained by Stern and Howard,[7] taking the electron interaction in the Hartree approximation. For a typical surface density of electrons of about 10^{12} cm^{-2}, the lowest parabolic band is the only one partially occupied to a Fermi energy of about 5–10 meV. The first empty band lies about 10–20 meV above the lowest band. The ground state wave function is localized to within 50 Å of the interface. For the electron density in the range of experimental interest, the mean distance r_s between electrons (in units of the effective Bohr radius), which is a measure of the ratio of the Coulomb energy to the kinetic energy, ranges between 0.1 to 10. By contrast, electrons on the surface of liquid helium are restricted to a low-density regime ($r_s \sim 1000$) with the possibility of a Wigner lattice. Our theory applies only to reasonably high-density electrons.

Existence of the electron bands in the inversion layer has been confirmed by photoconductivity measurements,[8] absorption of infrared radiation,[9] and radiation of electrons excited by currents.[10] These experiments measure the excitation energy of raising one electron from the lowest band to the first or higher excited band. The excitation energy is *not* equal to the energy difference of the two bands calculated in the Hartree approximation by Stern,[7] where both energy levels of the electron are calculated under the influence of the *same* Hartree potential due to having all the electrons in the ground state. When an electron is excited from the occupied band, the Hartree potential is changed. The excitation energy should be calculated taking into account the self-consistent change of the Hartree potential. This polarization effect shifts the

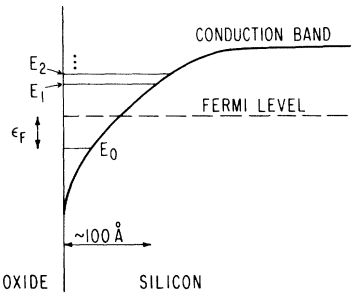

FIG. 1. Schematic energy levels of an inversion layer, shown for the (typical) case where only E_0 is occupied at zero temperature. The quantity ϵ_F is the two-dimensional Fermi energy for motion parallel to the interface.

excitation energy from the energy-level difference calculated by Stern. This effect was first discovered by Chen, Chen, and Burstein[11] using a slab model for the inversion layer, i.e., uniform density across the thickness of the slab. Allen, Tsui and Vinter[12] subsequently calculated the shift from the Hartree energy-band difference of Stern.[7]

By using the slab model, Chen *et al.*[11] also demonstrated the existence of retardation effects on the interband excitations. Our purpose is to investigate the electrodynamics in the inversion layer based on a more realistic model utilizing the electron wave functions as calculated by Stern. In Sec. II, the nonlocal dielectric tensor is expressed in terms of the electron subband energies and wave functions in the random phase approximation. In the limit of long wavelength but finite frequency, any dielectric screening process which involves the component of the electric field normal to the inversion layer couples directly to the density fluctuations.

In Sec. III, we investigate the longitudinal collective modes involving the density fluctuations, neglecting retardation. The poles of the density-density response function yield the collective modes, which consist of a low-frequency branch of plasma oscillations in the plane of the layer[4] and high-frequency interband excitation branches with energies shifted from the Hartree energies[12] due to polarization.[11] Although there are no new results in this section, it provides another view of the interband excitations and also facilitates the full solution with retardation.

The microscopic dielectric tensor obtained in Sec. II is used in Sec. IV to solve the Maxwell equations. With the plane geometry of the inversion layer, the solutions are of two types, transverse magnetic (TM) and transverse electric (TE) modes. Explicit solutions are given in the limit where the thickness of the inversion electron layer is much less than the wavelength of the electromagnetic wave or of the collective excitation. The TE modes have their electric field in the plane of the interface and consist of interband excitations only, at the unshifted Hartree energies. In the radiative region, their coupling to the electromagnetic wave is extremely weak, being of magnetic origin. Except for the case where the wave vector parallel to the interface is zero, the TM mode has an electric field component normal to the interface. The lowest branch of the TM mode is now the retarded two-dimensional plasma oscillation, a mixture of plasma oscillation in the plane of the interface and an electromagnetic wave.[5] When the wave-vector component parallel to the interface is of the same order of magnitude as that of the electromagnetic wave, the interband (high-frequency) excitations are approximately the same as the longitudinal case neglecting retardation effects. The retardation effect is small, of the order of the ratio of the thickness of the inversion layer to the wavelength of the electromagnetic wave. Unless the thickness of the inversion layer is made substantially larger than that typical in present systems, the retardation effect will be difficult to measure.

In Sec. V, our results are compared with those of Chen *et al.*[11] for the slab model. The dispersion relations of various polaritonlike modes of Chen *et al.* become difficult to distinguish from the dispersion relations of the electromagnetic wave and the longitudinal collective modes, in the limit where the slab is very thin compared with the wavelength of the electromagnetic wave. In that limit, the results of the slab model correspond qualitatively with our results.

II. NONLOCAL DIELECTRIC TENSOR

A. One-electron motion

Let the z axis be normal to the semiconductor-oxide interface. The insulating oxide occupies the half space $z<0$, with a dielectric constant ϵ_1. The semiconductor occupies the half space $z>0$, with a dielectric constant ϵ_2. For definiteness, we consider the case of an inversion layer formed on the semiconductor side of the interface from electrons in the conduction band. The electron motion is treated in the effective-mass approximation. For simplicity, the effective-mass tensor is taken to be diagonal with

$$m_x = m_y = m_t, \quad m_z = m_l. \tag{2.1}$$

The (100) surface of Si furnishes such an example.

Motion of an electron in the inversion layer parallel to the interface is free. Motion perpendicular to the interface is taken to have been solved in the Hartree approximation,[7] yielding a set of discrete energy levels E_ν, $\nu = 0, 1, 2, \ldots$, with wave functions $\zeta_\nu(z)$. The large work function (~3 eV) from the semiconductor to the oxide makes the wave functions approximately zero for $z<0$. The wave function of an electron is a product of a plane wave with wave vector $\vec{k}$ parallel to the interface and $\zeta_\nu(z)$. The total single-particle energy is

$$E_{\nu k} = E_\nu + \hbar^2 k^2/2m_t. \tag{2.2}$$

We shall only be concerned with very low temperatures and assume that at $T=0$ only the lowest band ($\nu=0$) is occupied, with the two-dimensional Fermi surface with radius k_F related to the surface number density n_s by

$$k_F^2 = 2\pi n_s/s; \tag{2.3}$$

s being the number of valleys which provide the lowest occupied band.

We assume here that the oxide layer is thick enough for the presence of the metal to be ignored. It is simple to extend our calculation to the case of a finite-thickness oxide layer, as we shall do later.

B. Linear response in random-phase approximation

Consider first the irreducible part of the linear response,[13] i.e., the response to the *total* scalar and vector potentials rather than the external potentials. In the random-phase approximation, it is given by a bubble diagram, with the expression[14] at the exciting frequency ω of

$$P_{\alpha\beta}(\vec{r},\vec{r}';\omega)=\sum_{\nu p\nu' p'}\frac{f_{\nu' p'}-f_{\nu p}}{(E_{\nu' p'}-E_{\nu p})\hbar^{-1}-\omega}\times\langle \nu p|J_\alpha(\vec{r})|\nu' p'\rangle\langle \nu' p'|J_\beta(\vec{r}')|\nu p\rangle, \tag{2.4}$$

where $f_{\nu p}$ is the distribution function of the electronic state $\nu\vec{p}$ with wave function $|\nu p\rangle$. J_0 denotes the charge density and J_x, J_y, J_z denote the paramagnetic part of the current density. Although there are indications[15] that RPA may not be an adequate approximation for the electrons in the inversion layer, we use it to deduce qualitative aspects of the electrodynamics and return later to discuss how to extend the calculation beyond RPA.

Translational invariance parallel to the interface permits the Fourier transform in the x,y directions, yielding the irreducible polarization part:

$$P_{\alpha\beta}(\vec{k}\omega;z,z')=\sum_{\nu=0}^{\infty}F_\alpha(\nu z)\theta_{\alpha\beta}(\vec{k}\omega;\nu)F_\beta^*(\nu z'), \tag{2.5}$$

where

$$F_\alpha(\nu z)=\zeta_0(z)\zeta_\nu(z) \quad \text{if } \alpha=0,x,y, \tag{2.6a}$$

$$F_z(\nu z)=\frac{1}{2i}\left(\zeta_0(z)\frac{\partial\zeta_\nu}{\partial z}-\frac{\partial\zeta_0}{\partial z}\zeta_\nu(z)\right), \tag{2.6b}$$

and

$$\theta_{\alpha\beta}(\vec{k}\omega;\nu)=2s\int\frac{d^2p}{(2\pi)^2}\gamma_\alpha\left(\frac{e_\alpha e_\beta f_{0p+}}{(E_{0p+}-E_{\nu p-})\hbar^{-1}-\omega}-\frac{f_{0p-}}{(E_{\nu p+}-E_{0p-})\hbar^{-1}-\omega}\right)\gamma_\beta, \tag{2.7}$$

with

$$\vec{p}\pm=\vec{p}\pm\tfrac{1}{2}\vec{k}, \tag{2.8}$$

$$\gamma_0=-e, \tag{2.9a}$$

$$\gamma_x=-(e\hbar/m_t)p_x, \tag{2.9b}$$

$$\gamma_y=-(e\hbar/m_t)p_y, \tag{2.9c}$$

$$\gamma_z=-e\hbar/m_l, \tag{2.9d}$$

$$e_\alpha=1 \quad \text{if } x=0,x,y, \tag{2.10a}$$

$$e_\alpha=-1; \tag{2.10b}$$

e denotes the proton charge. In arriving at the expression (2.5) for the irreducible polarization, we have taken the zero-temperature limit and used $\nu=0$ as the only occupied band. Because of the equation of continuity, the density response functions are related to the current response functions. The RPA expressions satisfy these relations.

C. Dielectric tensor

If we take the electric field $\vec{E}(r,t)$ to be of the form

$$\vec{E}(\vec{r},t)=\vec{E}(\vec{k}\omega,z)e^{i\vec{k}\cdot\vec{r}_\parallel-i\omega t}, \tag{2.11}$$

where $\vec{r}_\parallel$ is the component of $\vec{r}$ parallel to the interface and $\vec{k}$ is a two-dimensional vector parallel to the interface, and the electric displacement to be of a similar form, then the dielectric tensor is given by

$$D_i(\vec{k}\omega;z)=\sum_j\int_{-\infty}^{+\infty}dz'\,\epsilon_{ij}(\vec{k}\omega;z,z')E_j(\vec{k}\omega;z'), \tag{2.12}$$

with $i,j=x,y,z$. We work in the gauge $\phi=0$, allowed because of the gauge invariance of the RPA.

Since the electrons are confined to $z>0$, in the oxide layer (for $z<0$, $z'<0$)

$$\epsilon_{ij}(k\omega;zz')=\epsilon_1\delta_{ij}\delta(z-z') \tag{2.13}$$

for $z<0$, $z'>0$ or $z>0$, $z'<0$,

$$\epsilon_{ij}(k\omega;zz')=0; \tag{2.14}$$

and in the semiconductor region, $z>0$, $z'>0$

$$\epsilon_{ij}(\vec{k}\omega;zz')=\epsilon_2\delta_{ij}\delta(z-z')-(4\pi n_s e^2/\omega^2 m_j)\delta_{ij}\delta(z-z')F_0(0z)-\frac{4\pi}{\hbar\omega^2}P_{ij}(\vec{k}\omega;zz'), \tag{2.15}$$

The first term is the background semiconductor dielectric constant and the second term is the diamagnetic term with density $n_s F_0(0z)$, where

$$F_0(0z)=\{\zeta_0(z)\}^2 \tag{2.16}$$

since only the lowest band is occupied. The third term is the paramagnetic current-current response.

Equation (2.12) then forms the constitutive equation to be used in conjunction with the Maxwell

equations. We can show that this procedure is equivalent to solving for the full density and current responses to *external* scalar and vector potentials of the system of electrons interacting via not only longitudinal Coulomb interactions, but also the propagating photon field, provided that some photon modifications of the vertices are neglected, which is certainly justified in the nonrelativistic regime.

D. Long-wavelength approximation

We are interested in the frequency-wave-vector region near light propagation, i.e., $\omega \sim ck$. Thus, in the irreducible polarization, we keep ω finite but take k to be much smaller than the Fermi radius k_F. Then, the polarization $\theta_{\alpha\beta}(k\omega; v)$ defined by Eq. (2.7) can be evaluated, yielding, by Eq. (2.5),

$$P_{00}(k=0,\omega;zz') = \sum_{\nu=1}^{\infty} F_0(\nu z) h_\nu(\omega) F_0(\nu z'), \tag{2.17}$$

$$\begin{aligned} P_{0z}(0\omega;zz') &= -\frac{\hbar}{m_t}\sum_{\nu=1}^{\infty} F_0(\nu z)\frac{\omega}{\Delta_\nu} h_\nu(\omega) F_z^*(\nu z') \\ &= +P_{z0}^*(\nu\omega, z'z), \end{aligned} \tag{2.18}$$

$$\begin{aligned} P_{xx}(0\omega;zz') &= \tfrac{1}{4} v_F^2 \sum_{\nu=1}^{\infty} F_x(\nu z) h_\nu(\omega) F_x^*(\nu z') \\ &= P_{yy}(0\omega;zz'), \end{aligned} \tag{2.19}$$

$$P_{zz}(0\omega;zz') = \left(\frac{\hbar}{m_t}\right)^2 \sum_{\nu=1}^{\infty} F_z(\nu z) h_\nu(\omega) F_z^*(\nu z'), \tag{2.20}$$

where v_F is the Fermi velocity k_F/m_t parallel to the interface,

$$h_\nu(\omega) = [2n_s e^2 \Delta_\nu/(\omega^2 - \Delta_\nu^2)] \tag{2.21}$$

and

$$\Delta_\nu = (E_\nu - E_0)/\hbar. \tag{2.22}$$

All other elements of $P_{\alpha\beta}$ vanish.

In a homogeneous electron gas, the quantity corresponding to P_{0z} vanishes for zero wave vector. In the inversion layer, the anisotropy makes P_{0z} finite. This result implies that an electromagnetic wave with a finite z-component of the electric field couples into the density fluctuations, i.e., the longitudinal collective modes. The screening processes will then be dominated by the density fluctuations.

The diamagnetic current term in the dielectric tensor, Eq. (2.15), which involves the position dependent density, can also be put in the same form as $P_{\alpha\beta}$ by means of the equality

$$F_0(0z)\delta(z-z') = \sum_{\nu=1}^{\infty} F_z(\nu z)\frac{2\hbar}{m_t \Delta_\nu} F_z^*(\nu z'). \tag{2.23}$$

From the Schrödinger equation for the wave function $\xi_\nu(z)$, it is easy to show that

$$\frac{d}{dz} F_z(\nu z) = i\frac{m_t \Delta_\nu}{\hbar} F_0(\nu z) \tag{2.24}$$

and hence,

$$F_z(\nu z) = \frac{i m_t \Delta_\nu}{\hbar}\int_0^z dt\, F_0(\nu t). \tag{2.25}$$

We start the proof of Eq. (2.23) from the right-hand side, which is equal to

$$\begin{aligned} &-2i\int_0^{z'} dt \sum_{\nu=0}^{\infty} F_z(\nu z) F_0(\nu t) \\ &\quad = -\int_0^{z'} dt \left[\xi_0(z)\left(\frac{\partial}{\partial z}\sum_{\nu=0}^{\infty}\xi_\nu(z)\xi_\nu(t)\right)\xi_0(t) \quad \xi_0(t)\right. \\ &\qquad \left. -\frac{\partial \xi_0}{\partial z}\left(\sum_{\nu=0}^{\infty}\xi_\nu(z)\xi_\nu(t)\right)\xi_0(t)\xi_0(t)\right] \\ &\quad = \xi_0(z)\delta(z-z')\xi_0(z'), \end{aligned} \tag{2.26}$$

using the completeness property of the wave functions $\xi_\nu(z)$.

Using Eq. (2.23) in Eq. (2.15), we obtain the dielectric tensor for the inversion layer in the form

$$\begin{aligned} \epsilon_{ij}(0\omega, zz') &= \epsilon_2 \delta_{ij}\delta(z-z') \\ &\quad + \delta_{ij}\sum_{\nu=0}^{\infty} F_j(\nu z)\alpha_j(\omega;\nu)F_j^*(\nu z'), \end{aligned} \tag{2.27}$$

where

$$\alpha_x(\omega;\nu) = \alpha_y(\omega;\nu) = -\frac{\Omega_t^2}{\omega^2}\left(1 + \frac{\Delta_\nu \epsilon_F/\hbar}{\omega^2 - \Delta_\nu^2}\right), \tag{2.28}$$

$$\alpha_z(\omega;\nu) = -\frac{\hbar}{m_t}\frac{2\Omega_l^2}{\Delta_\nu(\omega^2 - \Delta_\nu^2)}, \quad \nu \neq 0 \tag{2.29}$$

$$\alpha_z(\omega;\nu=0) = 0, \tag{2.30}$$

with

$$\Omega_t^2 = 4\pi n_s e^2/m_t, \tag{2.31}$$

$$\Omega_l^2 = 4\pi n_s e^2/m_l. \tag{2.32}$$

We note that the dielectric tensor is diagonal and that the position dependence of the nonlocal part is expressed in a separable form. Note also that Eqs. (2.23) and (2.20) combine to eliminate the ground-state diamagnetic term [Eq. (2.30)] in ϵ_{zz}. When the constitutive equation involving the dielectric tensor is substituted into the Maxwell equations, the latter become integro-differential equations. The separable form of the nonlocal part of the dielectric tensor makes the equations soluble.

III. COLLECTIVE MODES NEGLECTING RETARDATION

We first investigate the collective modes involving density fluctuations, neglecting all retardation effects. We examine the full density-density response function[16] $\pi(\vec{r},\vec{r}';\omega)$. The irreducible part of the density response, π^*, is given by Eq. (2.5):

$$\pi^*(\vec{k}\omega;zz') \equiv e^{-2}\hbar^{-1}P_{00}(\vec{k}\omega;z,z'). \tag{3.1}$$

Whereas π^* is the linear response to the *total* scalar potential, π is the linear response to the *external* scalar potential, related to π^* by the Dyson equation

$$\pi(\vec{k}\omega;zz') = \pi^*(\vec{k}\omega;zz') + \int dz_1 \int dz_2 \pi^*(\vec{k}\omega;zz_1) \times V(\vec{k};z_1z_2)\pi(\vec{k}\omega;z_2z'), \tag{3.2}$$

where $V(\vec{k};zz')$ is the effective Coulomb interaction between point charges in the inversion layer[5]:

$$V(\vec{k},zz') = \frac{2\pi e^2}{\epsilon_2 k}\left(e^{-k|z-z'|} + \frac{\epsilon_2-\epsilon_1}{\epsilon_2+\epsilon_1}e^{-k|z+z'|}\right) \tag{3.3}$$

including the effect of the image charges in the oxide region.

Because of the separable form of the irreducible polarization π^*, given by Eq. (2.5),

$$\pi^*(\vec{k}\omega;zz') = \sum_{\nu=0}^{\infty} F_0(\nu z)\pi_\nu^*(\vec{k}\omega)F_0(\nu z'), \tag{3.4}$$

with

$$\pi_\nu^*(\vec{k}\omega) \equiv \theta_{00}(\vec{k}\omega;v) \tag{3.5}$$

possessing the expression given by Eq. (2.7), the integral equation (3.3) is easily solved in terms of a matrix inversion. Let $\vec{F}(z)$ represent the row vector with elements $F_0(\nu z)$, $\nu = 0, 1.2, \ldots$, $\vec{F}^\dagger(z)$ the Hermitian conjugate and $\vec{\pi}^*(\vec{k}\omega)$ the diagonal matrix with elements $\delta_{\nu,\nu'}\pi_\nu^*(\vec{k}\omega)$. Evidently, $\pi(\vec{k}\omega,zz')$ has the same form as Eq. (3.4), which in matrix notation becomes

$$\pi(\vec{k}\omega;zz') = \vec{F}(z)\cdot\vec{\pi}(k\omega)\cdot\vec{F}^*(z'). \tag{3.6}$$

The integral equation (3.2) is reduced to a matrix equation

$$\vec{\pi}(\vec{k}\omega) = \vec{\pi}^*(\vec{k}\omega) + \vec{\pi}^*(\vec{k}\omega)\vec{V}(\vec{k})\vec{\pi}(\vec{k}\omega), \tag{3.7}$$

where the Coulomb interaction matrix has elements

$$V_{\nu\nu'}(\vec{k}) = \int dz \int dz' F_0(\nu z)V(\vec{k};zz')F_0(\nu'z'). \tag{3.8}$$

We need to carry out a matrix inversion to obtain

$$\vec{\pi} = \vec{\pi}^*(\vec{1} - \vec{V}\vec{\pi}^*)^{-1} \tag{3.9}$$

which shows that the collective modes are given by

$$\det(\vec{1} - \vec{V}\vec{\pi}^*) = 0. \tag{3.10}$$

Let us restrict our interest to wave vector k much smaller than the Fermi wavevector k_F. The spatial extent of the electron wave function $\zeta_\nu(z)$ is of the same order as $1/k_F$. Thus, in the Coulomb integral (3.8), we may take $kz \ll 1$ and keep only the leading term in a small wave vector expansion. It is convenient to express the Coulomb integral in terms of a quantity with the dimension of length:

$$V = (4\hbar e^2/\epsilon_2)L, \tag{3.11}$$

where

$$L_{00} \simeq \epsilon_2(\epsilon_1+\epsilon_2)^{-1}k^{-1}, \tag{3.12}$$

$$L_{\nu 0} = L_{0\nu} = -\frac{1}{2}\int\int dz\,dz' F_0(0z)|z-z'|F_0(\nu z') - \frac{1}{2}\frac{\epsilon_2-\epsilon_1}{\epsilon_2+\epsilon_1}\int dz\, F_0(\nu z)z \tag{3.13}$$

and

$$L_{\nu\nu'} \simeq -\frac{1}{2}\int\int dz\,dz'\,F_0(\nu z)|z-z'|F_0(\nu' z') = \int_0^\infty dz\left(\int_0^z dz_1 F_0(\nu z_1)\right)\left(\int_0^z dz_2 F_0(\nu' z_2)\right), \tag{3.14}$$

where ν and ν' are not zero. Equation (3.14) is identical to the length tensor $S_{\nu\nu'}$ of Allen *et al.*[12]

Typically, all the elements of $L_{\nu\nu'}$ except L_{00} are of the order 10 Å. Because of the disparity between L_{00} and $L_{\nu\nu'}$ for non zero ν and ν', we shall show shortly that one may neglect the off-diagonal elements $L_{0\nu}$, and $L_{\nu 0}$ and factorize the determinantal equation (3.10) into

$$1 - V_{00}\pi_0^* = 0 \tag{3.15}$$

and

$$\det(\delta_{\nu\nu'} - V_{\nu\nu}\pi_{\nu'}^*) = 0, \tag{3.16}$$

where, in the second determinant, ν, ν' are nonzero.

Consider first the ground-state term, (3.15). V_{00} is, from Eq. (3.11) and (3.12), just the Coulomb interaction for a two-dimensional electron gas in a medium with dielectric constant $\frac{1}{2}(\epsilon_1+\epsilon_2)$. $\pi_0^*(\vec{k}\omega)$ is just the polarization for the two-dimensional electron gas, already evaluated by Stern.[5] The root of Eq. (3.15) is the unretarded plasma oscillation with frequency

$$\omega_{2D}(k) = \left[\frac{2\pi n_s e^2}{m_t}\left(\frac{2}{\epsilon_1+\epsilon_2}\right)\right]^{1/2}k^{1/2} \tag{3.17}$$

which involves oscillations of charges parallel to the interface.

The collective modes from Eq. (3.16) involve ex-

citation of electrons from the lowest-occupied subband to higher-unoccupied bands, i.e., electron motion perpendicular to the interface. By Eqs. (2.17), (2.21), and (3.14), we obtain

$$V_{\nu\nu'}\pi^*_{\nu'} \equiv N_{\nu\nu'}(\omega) = \omega^2_{p\nu\nu'}/(\omega^2 - \Delta^2_{\nu'}), \tag{3.18}$$

where

$$\omega^2_{p\nu\nu'} = (8\pi n_s e^2/\hbar\epsilon_2)L_{\nu\nu'}\Delta_{\nu'} \tag{3.19}$$

and ν and ν' are nonzero. This result extends the two- and three-band calculations of Allen *et al.* to an arbitrary number of subbands. We note that for these excitations at long wavelength, the dielectric constant ϵ_1 of the oxide does not appear at all. The plasma frequency matrix $\omega^2_{p\nu\nu'}$ shifts the frequency of the interband resonance mode, as can be seen explicitly by the two-band example ($\nu = 0, 1$), where the collective mode frequency is given by

$$\omega^2_{c1} = \Delta^2_1 + \omega^2_{p11}, \tag{3.20}$$

which is the polarization effect noted by Chen *et al.*[11]

Neglecting the off-diagonal terms $V_{0\nu}$, the solution of Eq. (3.10) thus gives (i) A branch of two dimensional plasma oscillations involving motion of the electrons in the lowest suband only. (ii) Branches ω_{ci} of interband excitations with frequencies shifted from Δ_ν by the polarization effects in the z direction. For $k \ll k_F$, the lowest plasma branch and the high frequency ω_{ci} branches are well separated and the effects of the neglected off-diagonal terms $V_{0\nu}$ are indeed small. They become important only when the plasma branch rises sufficiently to cross the high-frequency branches, which occurs at wave vector k comparable to k_f, as has been verified by a two-band ($\nu = 0, 1$) calculation.

IV. RETARDATION EFFECTS

We now solve the full set of Maxwell equations

$$\operatorname{div}\vec{D} = 0, \tag{4.1a}$$

$$\operatorname{div}\vec{B} = 0, \tag{4.1b}$$

$$\operatorname{curl}\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}, \tag{4.1c}$$

$$\operatorname{curl}\vec{B} = \frac{1}{c}\frac{\partial\vec{D}}{\partial t}. \tag{4.1d}$$

The fields are put in the form of Eq. (2.11) with the constitutive equation (2.12) connecting the electric displacement to the electric field. For convenience, we let the x axis be along the direction of the wave vector $\vec{k}$. Then the Maxwell equations separate into two disconnected sets, one involving E_x, E_z, B_y and one involving only E_y, B_x, B_z. The former set yields the transverse magnetic (TM) modes with the magnetic field perpendicular to both the wave vector $\vec{k}$ and the normal to the interface (the z axis). The latter set yields the transverse electric (TE) modes. We treat them separately.

A. TM modes (p polarization)

Elimination of the magnetic field B_y from the Maxwell equations yields the wave equations

$$-\frac{d^2}{dz^2}E_x + ik\frac{dE_z}{dz} = \frac{\omega^2}{c^2}D_x, \tag{4.2a}$$

$$ik\frac{dE_x}{dz} + k^2E_z = \frac{\omega^2}{c^2}D_z. \tag{4.2b}$$

The solution in the oxide region ($z < 0$) is straightforward. Let us concentrate on the seimconductor region ($z > 0$). It is convenient to define

$$k_z^2 = (\omega/c)^2\epsilon_2 - k^2. \tag{4.3}$$

In the absence of the nonlocal terms in the dielectric tensor (2.27), due to the polarization effects of the electrons in the inversion layer, k_z would be the component of the wave vector normal to the interface. $k_z^2 > 0$ describes propagating modes and $k_z^2 < 0$ describes bound modes which attenuate exponentially from the interface $z = 0$ into the semiconductor region.

We have found it convenient to solve the wave equations (4.2) by eliminating E_z. The constitutive equation is used with the dielectric tensor given in the long-wave approximation (2.27). Thus, from Eq. (4.2b),

$$E_z = \frac{ik}{k_z^2}\frac{dE_x}{dz} - \frac{\omega^2}{k_z^2c^2}\sum_\nu F_z(\nu z)\alpha_z(\omega;\nu) \times \int_0^\infty F_z^*(\nu z')E_z(z')dz'. \tag{4.4}$$

The nonlocal polarization effects of the electrons in the inversion layer induce electric field components of the form $F_j(\nu z)$. The evaluation of their coefficients is best handled by matrix algebra. We define vectors $\vec{X}$ and $\vec{Z}$ with elements

$$X_\nu = \int_0^\infty dz F_x^*(\nu z)E_x(z), \tag{4.5}$$

$$Z_\nu = \int_0^\infty dz F_z^*(\nu z)E_z(z), \tag{4.6}$$

and matrices $\vec{\alpha}_z$, $\vec{\Delta}$ with elements $\alpha_z(\omega;\nu)\delta_{\nu\nu'}$ and $\Delta_\nu\delta_{\nu\nu'}$.

To obtain $\vec{Z}$ in terms of $\vec{X}$, we multiply Eq. (4.4) by $F_z^*(\nu z)$ and integrate. The first term on the right is

$$\int_0^\infty dz \frac{ik}{k_z^2} F_z^*(\nu z) \frac{dE_x}{dz} = \frac{km_1\Delta_\nu}{k_z^2\hbar} X_\nu \tag{4.7}$$

using Eq. (2.24). Thus, from Eq. (4.4) we have

$$\vec{Z} = -\frac{km_1}{k_z^2\hbar}\vec{\Delta}\left(\vec{I} - \frac{\omega^2\epsilon_2}{c^2k_z^2}\vec{N}\right)^{-1}\vec{X}, \tag{4.8}$$

where $\vec{I}$ is the unit matrix and $\vec{N}$ is the matrix with elements defined by Eq. (3.18), using the equality

$$\int_0^\infty dz\, F_z^*(\nu z)F_z(\nu' z) = \left(\frac{m_1}{\hbar}\right)^2 \Delta_\nu L_{\nu\nu'}\Delta_{\nu'} \tag{4.9}$$

$L_{\nu\nu'}$ being defined in Eq. (3.14).

Substitution of Eq. (4.4) into Eq. (4.1a) yields a wave equation for E_x:

$$\frac{d^2E_x}{dz^2} + k_z^2E_x = \sum_{\nu\nu'} F_x(\nu z)B_{\nu\nu'} \int_0^\infty dz\, F_x^*(\nu' z')E_x(z'), \tag{4.10}$$

where $B_{\nu\nu'}$ are the elements of the matrix $\vec{B}$ given by

$$\vec{B} = -\frac{k_z^2}{\epsilon_2}\left[\vec{\alpha}_x + \left(\frac{km_1}{k_z^2\hbar}\right)^2 \vec{\Delta}\vec{\alpha}_z\vec{\Delta}\left(\vec{I} - \frac{\omega^2\epsilon_2}{c^2k_z^2}\vec{N}\right)^{-1}\right]. \tag{4.11}$$

The integro-differential equation (4.10) can be transformed into an integral equation with the help of the Green's function given by

$$\left(\frac{d^2}{dz^2} + k_z^2\right)G(z,z') = \delta(z-z'). \tag{4.12}$$

This Green's function expresses the propagation of the electromagnetic wave in the background semiconductor without the inversion layer electrons and satisfies boundary conditions at $z = \pm\infty$ and at various interfaces. It is evaluated in the Appendix for the cases of two semi-infinite media and of a transmission line.[9,17] The formal solution for the electric field is

$$E_x(z) = E_x^{ext}(z) + \sum_{\nu\nu'} \int_0^\infty dz'\, G(z,z') \times F_x(\nu z')B_{\nu\nu'}X_{\nu'}, \tag{4.13}$$

including the possibility of excitation by an external field $\vec{E}^{ext}$. The coefficients X_ν are obtained by multiplying the equation with $F_x^*(\nu z)$ and integrating, yielding a matrix equation

$$\vec{\Lambda}\vec{X} = \vec{X}^{ext} \tag{4.14}$$

where

$$X_\nu^{ext} = \int_0^\infty dz\, F_x^*(\nu z)E_x^{ext}(z). \tag{4.15}$$

The matrix $\vec{\Lambda}$ is given by

$$\vec{\Lambda} = \vec{I} - \vec{G}\cdot\vec{B} \tag{4.16}$$

and $\vec{G}$ is the matrix with elements

$$G_{\nu\nu'} = \int_0^\infty dz \int_0^\infty dz'\, F_x^*(\nu z)G(z,z')F_x(\nu z'). \tag{4.17}$$

Thus, the solution of the Maxwell equations has been reduced to the inversion of the matrix equation (4.15).

From the Appendix, the Green's function is shown to be of the form

$$G(z,z') = [2ik_z(1+\lambda\sigma)]^{-1} \times (e^{ik_z|z-z'|} + \lambda e^{ik_z|z+z'|} - \sigma e^{-ik_z|z+z'|} - \lambda\sigma e^{-ik_z|z-z'|}), \tag{4.18}$$

where $\lambda(k\omega)$ and $\sigma(k\omega)$ are the coefficients depening on the particular geometry of the layers. For bound states where $k_z^2 < 0$, the replacement is understood that

$$-ik_z \to \beta_2 = [k^2 - (\omega/c)^2\epsilon_2]^{1/2}. \tag{4.19}$$

The evaluation of the integrals $G_{\nu\nu'}$ is carried out in the long-wave approximation $k_z z \ll 1$, as in Sec. III, since the coordinate z is limited by the wave function $\xi_\nu(z)$ in the form factor $F_x(\nu z)$. Keeping the leading term in the small k_z expansion,

$$G_{00} = (1+\lambda)(1-\sigma)[2ik_z(1+\lambda\sigma)]^{-1}, \tag{4.20}$$

$$G_{\nu 0} = G_{0\nu} = \frac{1}{2}\int_0^\infty dz \int_0^\infty dz'\, F_0(0z)\,|z-z'|\,F_0(\nu z') - \frac{1}{2}\frac{\lambda+\sigma}{1+\lambda\sigma}\int_0^\infty dz\, F_0(\nu z)z, \tag{4.21}$$

$$G_{\nu\nu'} = -L_{\nu\nu'}, \tag{4.22}$$

where ν and ν' are nonzero integers, and $L_{\nu\nu'}$ are given in Eq. (3.14). Thus, $G_{\nu\nu'}$ are independent of the geometry of the layers and of the retardation effects of the electromagnetic wave in lowest order.

The collective modes are given by

$$\det\vec{\Lambda} = 0 \tag{4.23}$$

In the matrix $\vec{B}$, the matrix elements $B_{\nu 0}$ and $B_{0\nu}$ are zero. For the same reason as in Sec. III, i.e., because of the disparity between the magnitudes of G_{00} and $G_{\nu\nu'}$, we may neglect the elements $G_{\nu 0}$ and decouple the determinantal equation into

$$\Lambda_{00} = 0 \tag{4.24}$$

and

$$\det(\Lambda_{\nu\nu'}) = 0, \tag{4.25}$$

where ν and ν' are nonzero. We examine separately these two solutions.

1. Two-dimensional plasma oscillations

Equation (4.24), with the help of Eqs. (4.11) and (4.20) reduces to

$$\omega^2=\frac{2\pi n_s e^2}{m_t\epsilon_2}\,\frac{(1+\lambda)(1-\sigma)}{1+\lambda\sigma}\,\beta_2. \tag{4.26}$$

These modes are the two-dimensional plasma oscillations including the effects of retardation and the geometry of the layers.[5,6,12,18] Since the plasmons are bound modes, occurring only for $k_z^2<0$, we have used the replacement of $-ik_z$ by β_2 of Eq. (4.19).

Consider first the simple geometry case of the semiconductor occupying $z>0$ and the oxide $z<0$. Then, from Eq. (A5)

$$\sigma=0, \tag{4.27}$$

$$\lambda=\left(\frac{\beta_1\epsilon_2}{\beta_2\epsilon_1}-1\right)\left(\frac{\beta_1\epsilon_2}{\beta_2\epsilon_1}+1\right)^{-1} \tag{4.28}$$

the classical reflection coefficient, with β_1 defined in a similar way to Eq. (4.19) except replacing ϵ_2 by ϵ_1, the dielectric constant of the oxide layer. The frequency of the plasmon reduces to Stern's expression[5] if we put $\epsilon_1=\epsilon_2$. For large wave vector k, i.e.,

$$k_F^2\gg k^2\gg(\omega^2/c^2)\epsilon_2>(\omega^2/c_2)\epsilon_1, \tag{4.29}$$

the plasmon frequency approaches the longitudinal plasma frequency Eq. (3.17). For small k, such that

$$k^2\ll\omega_{2D}^2\epsilon_2/c^2. \tag{4.30}$$

the retardation effect dominates the plasmon branch and

$$\omega\sim ck\epsilon_2^{-1/2}. \tag{4.31}$$

The frequency approaches the light line of the semiconductor from below.

A physical picture of the two dimensional plasmon mode can be constructed. The electrons remain in the lowest subband and oscillate parallel to the interface, giving rise to a diamagnetic current. The resultant electric field has only x and z components, does not excite the electrons to higher bands, i.e.,

$$\int_0^\infty dz\,\zeta_\nu(z)E_x(z)\zeta_0(z)=X_\nu=0,\quad \nu\neq 0 \tag{4.32}$$

and depends sensitively on the surrounding media (through σ and λ). For example, for the transmission line geometry, where the semiconductor occupies $0<z<t_2$, the oxide $-t_1<z<0$ and both are bounded by perfect conductors, Eq. (4.26) becomes

$$\omega^2=\frac{4\pi n_s e^2}{m_t\epsilon_2}\,\beta_2\left(\coth\beta_2 t_2+\frac{\epsilon_1\beta_2}{\epsilon_2\beta_1}\coth\beta_1 t_1\right)^{-1}. \tag{4.33}$$

In the limit of a very thin oxide layer, $\beta_1 t_1\ll 1$, the plasma frequency becomes

$$\omega\simeq(4\pi n_s e^2/m_t)k(t_1/\epsilon_1)^{1/2}. \tag{4.34}$$

The frequency is considerably depressed since the electric field is forced to be nearly normal to the interface in the inversion layer.

2. High-frequency modes involving interband transitions

Now we consider the zeros of Eq. (4.25). The contribution of the polarization α_x to the elements $B_{\nu\nu'}$ for ν, ν' nonzero in Eq. (4.11) is negligible compared with the α_z term provided,

$$k>k_F\Delta_\nu/m_t c^2.$$

Subsitution of Eqs. (4.11) and (4.22) into (4.16) reduces Eq. (4.25), after some matrix manipulation, to

$$\det\left[\left(\vec{I}-\frac{\omega^2\epsilon_2}{c^2k_z^2}\vec{N}\right)^{-1}\left(\vec{I}-\vec{N}\right)\right]=0. \tag{4.35}$$

The roots of $\det(\vec{I}-\vec{N})=0$ are precisely the high frequency ω_{ci} modes given by Eq. (3.16) and (3.18) which we found in Sec. III neglecting retardation. These frequencies cross the semiconductor light line ($\beta_2=0$) and are independent of k provided that $k\ll k_F$ and not so close to zero as to violate the inequality (4.34). In the extremely small region of k near zero ($k\leq k_F\Delta_\nu/m_t c^2$), the frequencies tend to Δ_ν, the interband transition frequencies. A qualitative plot of these frequencies versus wave vector is shown in Fig. 2 for the two semi-infinite media case. We can now justify the neglect of the $\Lambda_{\nu 0}$ coupling of the low-frequency plasma branch

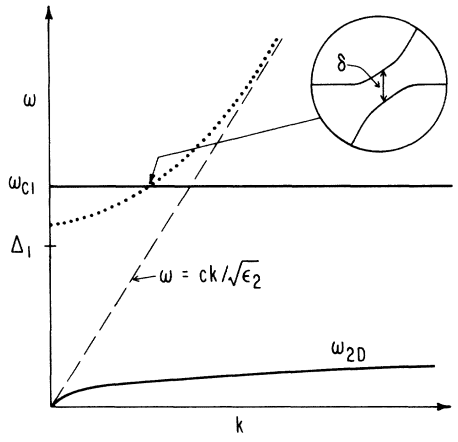

FIG. 2. Transverse magnetic (TM) collective modes. For simplicity only one interband excitation ω_{c1} is shown. The curve labeled ω_{2D} is the intraband two-dimensional plasmon. Also shown in the inset is the small splitting δ where a transmission line mode (dotted line) would otherwise intersect ω_{c1}.

and these high-frequency interband transition modes, since for $k \ll k_F$ they are far apart.

For these high-frequency modes, the electron motion involved is normal to the interface due to interband excitation. The resultant electric field has both x and z components with

$$\int_0^\infty dz\, \zeta_0(z) E_x(z) \zeta_0(z) = 0 \tag{4.36}$$

decoupled from the two-dimensional plasma oscillation. The broken translational symmetry allows the coupling of the transverse electromagnetic wave to the longitudinal density response which, therefore, dominates the screening process, shifting the interband excitation frequencies and making the retardation effect negligible. The modes are insensitive to the dielectric property of the oxide layer.

For $k_z^2 < 0$, these modes are truly bound modes; for $k_z^2 > 0$, they are radiative. We shall consider their effect on the reflectivity presently. For a transmission line geometry,[9,17] the propagating cavity modes in the absence of the inversion layer electrons are given by (see the Appendix)

$$\mu \equiv \tan k_z t_2 + \frac{\epsilon_2 k_{1z}}{\epsilon_1 k_z} \tan k_{1z} t_1 = 0, \tag{4.37}$$

where

$$k_{1z}^2 = (\omega^2/c^2)\epsilon_1 - k^2. \tag{4.38}$$

To the leading order in the expansion in powers of k/k_F, the high-frequency modes are independent of the geometry. In order to investigate the coupling of the high-frequency modes to the cavity modes, we must retain the next order terms in $G_{\nu\nu'}$. Then, to Eq. (4.22) we must add

$$\delta G_{\nu\nu'} = k_z z_\nu z_{\nu'}/\mu, \tag{4.39}$$

where

$$z_\nu = \int_0^\infty dz\, F_0(\nu z) z. \tag{4.40}$$

In the secular determinant in Eq. (4.35), to the term $\vec{I} - \vec{N}$ must be added elements,

$$\frac{k^2}{\mu k_z} \frac{8\pi n_s e^2}{\hbar \epsilon_2} \frac{\Delta_\nu z_\nu z_{\nu'}}{\omega^2 - \Delta_{\nu'}^2}. \tag{4.41}$$

These determine the splitting of the interband modes and the cavity modes where they would otherwise cross. The size of the splitting is roughly (see Fig. 2)

$$\delta \sim \omega_{c1} z_1 (t_2 L_{11})^{-1/2} \sim 0.01 \omega_{c1} \tag{4.42}$$

using the thickness of the silicon layer $t_2 \sim 0.02$ cm, $L_{11} \sim 2.5$ Å, and $z_1 \sim 15$ Å. δ is much less than the width of the ω_{c1} resonance, which is about $0.2\omega_{c1}$.

3. *Reflectivity*

For the configuration of two semi-infinite media, consider the reflection of an electromagnetic wave incident from the semiconductor side at an angle θ_i to the normal. The exciting electric field is

$$E_x^{\text{ext}} = (e^{-ik_z z} + \lambda e^{ik_z z}) e^{ikx - i\omega t} \tag{4.43}$$

with

$$k_z = (\omega/c)\epsilon_2^{1/2} \cos\theta_i, \tag{4.44}$$

$$k = (\omega/c)\epsilon_2^{1/2} \sin\theta_i. \tag{4.45}$$

The first term on the right-hand side of Eq. (4.43) is the incident wave with unit amplitude and the second term is the reflection from the interface in the absence of the inversion layer electrons. This field should be put in Eqs (4.13) and (4.14) to obtain the actual electric field, which contains a reflected wave of the form $E' e^{ik_z z - i\omega t}$ for large and positive z, where

$$E' = \lambda + \frac{1}{2ik_z} \int_0^\infty dz' (e^{-ik_z z'} + \lambda e^{ik_z z'}) \times \sum_{\nu\nu'\nu''} F_x(\nu z') B_{\nu\nu'} \Lambda^{-1}_{\nu'\nu''} X^{\text{ext}}_{\nu''}, \tag{4.46}$$

where

$$X_\nu^{\text{ext}} = \int dz\, F_0(\nu z)(e^{-ik_z z} + \lambda e^{ik_z z}). \tag{4.47}$$

Using the expression (4.47) on the integrals in (4.46) yields the matrix expression

$$E' = \lambda + (1/2ik_z) \vec{X}^{\text{ext}} \cdot \vec{B} \circ \vec{\Lambda}^{-1} \cdot \vec{X}^{\text{ext}}. \tag{4.48}$$

The reflectivity is simply

$$R = |E'|^2. \tag{4.49}$$

The first term of the reflection coefficient E' in Eq. (4.48) is just the reflection term of the bare semiconductor-oxide boundary, and the second term is due to the inversion layer electrons. The second term can be expressed as a sum of contributions from various modes worked out Sec. IV A 2. For a propagating electromagnetic wave, $k < (\omega/c)\sqrt{\epsilon_2}$. For this region, there is no resonance in the intraband plasmon branch, and the reflection behavior is similar to the usual plasmon contribution. The interband modes, however, can resonate with the propagating electromagnetic wave and absorb its energy, thus greatly reducing the reflectivity. This behavior is explicitly demonstrated by a two-subband model, for which

$$E' = \lambda - \frac{i(1+\lambda) k_z \Omega_p^2}{\omega^2 + i k_z \Omega_p^2} + \frac{i(1-\lambda)^2 k^2}{2k_z} \frac{z_1^2}{L_{11}} \frac{\omega_{c1}^2 - \Delta_1^2}{\omega^2 - \omega_{c1}^2}. \tag{4.50}$$

The second term on the right-hand side comes from the intraband plasmon, with

$$\Omega_p^2=(2\pi n_s e^2/m_t\epsilon_2)(1+\lambda). \tag{4.51}$$

The third term comes from the interband excitation, with resonance at ω_{c1}, given by Eq. (3.20).

B. TE modes (*s* polarization)

For the TE modes,[19] the electric field is parallel to the interface and is along the y axis. If the magnetic field is eliminated from the Maxwell equations (4.1), the resulting wave equation is

$$\frac{d^2E_y}{dz^2}-k^2E_y=-(\omega^2/c^2)D_y. \tag{4.52}$$

Solution in the oxide region ($z<0$) is straightforward and has to be matched with the solution in the semiconductor region ($z>0$), given by

$$\frac{d^2E_y}{dz^2}+k_z^2E_y=\sum_\nu F_y(\nu z)\left(-\frac{\omega^2}{c^2}\alpha_y(\omega;\nu)\right)\times\int_0^\infty dz' F_y^*(\nu z')E_y(z'), \tag{4.53}$$

using the constitutive equation (2.27). This equationis solved in the same way as Eq. (4.10). We simply have to replace the matrix B by the diagonal matrix $-(\omega^2/c^2)\alpha_y$, and the Green's function by one appropriate to the TE mode (see the Appendix).

Thus, the normal modes are given by

$$\det[\vec{I}+(\omega^2/c^2)\vec{G}\cdot\vec{\alpha}_y]=0. \tag{4.54}$$

Again, we can approximately partition the determinant into $\nu=0$ and $\nu\neq 0$ terms. For the case of the two semi-infinite media, the Green's function has the same form as the TM mode case, except

$$\lambda=(k_{z2}-k_{z1})/(k_{z2}+k_{z1}), \tag{4.55}$$

where k_{z2} is the k_z in Eq. (4.3) for the semiconductor region and k_{z1} is an analogous quantity for the insulator region.

The intraband normal mode would exist if the $\nu=0$, $\nu'=0$ element of Eq. (4.54) vanishes, i.e., if

$$1+\frac{\omega^2}{c^2}G_{00}\alpha_y(0)=1+\frac{\Omega_t^2}{(\beta_1+\beta_2)c^2} \tag{4.56}$$

vanishes [when the replacement (4.10) is effected], an impossibility for real frequencies ω and wave vectors k. The absence of the corresponding plasmon in the TE mode is understandable since the electric field is completely transverse.

The determinant on the right-hand side of Eq. (4.54) has poles at the unshifted interband frequencies Δ_ν and has zeros shifted from Δ_ν by a minute amount, of the order $\hbar\Delta_\nu^2/m_tc^2$. The zeros correspond to normal modes. In the case of reflection, these modes are extremely weakly coupled to the electromagnetic wave since it is the magnetic field rather than the electric field which excites these modes. The coupling strength of the TE modes relative to the TM modes is again of the order $\hbar\Delta_1/m_tc^2$, i.e., about 10^{-9}. It is not possible experimentally to align an electric field $\vec{E}$ exactly parallel to the surface. Consequently, the slightest component of an electric field normal to the interface will drive the TM resonances at the shifted frequencies ω_{ci}, overwhelming the TE resonances.

V. DISCUSSION

Chen *et al.*[11] used a slab model of the inversion layer with constant electron density and obtained apparently more modes than we did in Sec. IV. Although the dielectric tensor we used is very nonlocal, in the limit of a very thin inversion layer the form of the dielectric tensor of the slab model is qualitatively correct. The terms $G_{\nu\nu'}$ in Eq. (4.22) which involve propagation of the electromagnetic wave inside the inversion layer cannot be reproduced except very approximately in the slab model. In the slab model, if the thickness of the inversion layer is negligible compared with the wavelength of the excitation, then the multitude of modes which Chen *et al.* obtained coalesce and correspond to just those in Sec. IV. Our formulation may also be applied to the thick inversion layer case if the long-wave approximation in Eq. (2.17) and Eqs. (4.20)–(4.22) is not used.

Finally, we have shown that retardation and boundary effects are negligible for the high-frequency modes ω_{ci}. Consequently, any improvement on the values for ω_{ci} must come from a calculation of the density polarization beyond the random phase approximation. Corrections to the RPA bubble consist of (i) exchange and correlation corrections to the electron self-energy (ii) vertex corrections, which are just the electron-hole attraction. Estimates by Vinter[15] indicate that these two corrections are large but of opposite sign. Therefore, great care must be taken in going beyond RPA in including both types of correction. One guarantee is to make the self-energy and vertex satisfy the Ward identity[20] (i.e., to take diagrams which form a conserving approximation[21]). We hope to deal with this problem in a later work.

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APPENDIX: ELECTROMAGNETIC GREEN'S FUNCTIONS

We present here a brief derivation of the Green's function

$$\left(\frac{d^2}{dz^2}+k_x^2\right)G(k_x;z,z')=\delta(z-z');\quad 0\le z\le t_2 \tag{A1}$$

subject to homogeneous boundary conditions

$$G(0,z')-\alpha\frac{dG}{d_z}(0,z')=0, \tag{A2}$$

$$G(t_2,z')-\alpha'\frac{d}{dz}G(t_2,z')=0. \tag{A2}$$

Application of standard methods[22] leads to

$$G(k_x;zz')=[(2ik_x)(1+\lambda\sigma)]^{-1}\times(e^{ik_x|z-z'|}+\lambda e^{ik_x|z+z'|}-\sigma e^{-ik_x|z+z'|}-\sigma\lambda e^{-ik_x|z-z'|}), \tag{A3a}$$

$$\lambda(k_x)=-(1+ik_x\alpha)(1-ik_x\alpha)^{-1}, \tag{A3b}$$

$$\sigma(k_x)=(1-ik_x\alpha')(1+ik_x\alpha')^{-1}, \tag{A3c}$$

which can be continued to the nonpropagating sector $k_x^2<0$ by Eq. (4.19). We now turn to calculation of λ and σ for particular geometries.

1. Two semi-infinite media

A dielectric with constant $\epsilon_2(\epsilon_1)$ occupies $z>0$ (<0). The requirement of outgoing waves at $z\to+\infty$ demands $\sigma\equiv 0$, while λ depends upon the particular mode (TE or TM).

For TM (p polarization) modes we require continuity of $E_x(0)$, $D_z(0)$. Equation (4.2b) and the vanishing of $\xi_\nu(z=0)=0$ lead to

$$\left(E_x-\frac{ik_{z1}\epsilon_2}{k_x^2\epsilon_1}\frac{dE_x}{dz}\right)_{z\to 0^+}=0, \tag{A4}$$

where k_{z1} is defined by Eq. (4.38). Thus Eqs. (A2) and (A3b) demand

$$\alpha=(ik_{z1}\epsilon_2/k_x^2\epsilon_1),\qquad \lambda=[(k_{z1}\epsilon_2/k_x\epsilon_1)-1][(k_{z1}\epsilon_2/k_x\epsilon_1)+1]^{-1} \tag{A5}$$

just the usual reflection coefficient[19] in the absence of the inversion layer.

A similar argument for TE (s polarization) modes, with $E_y(0)$, $dE_y(0)/dz$ continuous, gives

$$\alpha=ik_{z1}^{-1},\qquad \lambda=[k_x/k_{z1})-1][(k_x/k_{z1})+1]^{-1} \tag{A6}$$

the classical S-reflection coefficient.[19]

2. Wave guide configuration

Consider the oxide bounded by a perfect conductor at $z=-t_1$ while the semiconductor is similarly confined at $z=+t_2$. For simplicity we restrict attention to TM modes, in which case the vanishing of $E_x(z=t_2)$ requires $\alpha'=0$, so that

$$\sigma=e^{2ik_xt_2}. \tag{A7}$$

Continuity of $E_x(0)$, $D_z(0)$ leads, by arguments similar to those giving Eq. (A5), to

$$\alpha=-(ik_{Z1}\epsilon_2/k_x^2\epsilon_1)\tan(k_{Z1}t_1),\qquad \lambda=[1+(k_{Z1}\epsilon_2/k_Z\epsilon_1)\tan(k_{Z1}t_1)]\times[1-(k_{Z1}\epsilon_2/k_Z\epsilon_1)\tan k_{Z1}t_1)]^{-1}. \tag{A8}$$

Finally, the dispersion relation for wave-guide modes is given from Eq. (A3a) as $1+\sigma\lambda=0$, satisfied by

$$(k_{Z1}/\epsilon_1)\tan(k_{Z1}t_1)+(k_Z/\epsilon_2)\tan k_Zt_2=0. \tag{A9}$$

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