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SURFACE WAVES IN A HOMOGENEOUS PLASMA SHARPLY BOUNDED BY A DIELECTRIC

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Abstract—The physics as well as the spectra and damping rates of high- and low-frequency surface waves in semi-infinite and cylindrical plasmas are studied. The boundary conditions (both macroscopic and microscopic) imposed on the plasma fluid/particles are discussed. The solution of the Vlasov–Maxwell set of equations for semi-infinite and cylindrical plasmas assuming mirror reflection of plasma particles at the boundary is given. The dispersion relations are derived and analysed. The dispersion properties obtained from the kinetic model are compared with those derived from the fluid model.

1. INTRODUCTION

IN THE past few years, great interest has been aroused in the study of the propagation of electromagnetic waves on the plasma boundary. The reason is that in reality all plasmas are in some fashion bounded. When studying the interaction of electromagnetic fields with plasma particles it is necessary to take into consideration not only the bulk or volume modes, but also the modes localized on the plasma surface, i.e. the so-called surface waves.

Surface modes may be important in connection with plasma diagnostics by laser light scattering of the particles or by examining the wave dispersive properties (ANICIN 1966a,b; ZHELYAZKOV *et al.*, 1967; SHIVAROVA *et al.*, 1975). Experimental measurements of the high-frequency surface wave dispersion can be used to estimate the radial inhomogeneity (AKAO and IDA, 1964; WASSINK and ESTIN, 1964) as well as the fluctuations (CRAWFORD *et al.*, 1963) in the plasma density. Since the surface wave electric field is maximum at the plasma–dielectric boundary (TRIVELPIECE and GOULD, 1959) it is possible to use it for the generation of plasmas (MOISAN *et al.*, 1974, 1975, 1976; KAMPMANN, 1976). Recently, it has been proposed that when a sufficiently intense laser beam interacts with a plasma, the plasma undergoes strong profile modification due to the light pressure (ESTABROOK *et al.*, 1975; DEGROOT and TULL, 1975; FORSLUND *et al.*, 1975, 1976; LEE *et al.*, 1977), so that (FEDOSEJEVS *et al.*, 1977) there is a transition region separating the low density plasma from the high density plasma near the critical density. For this reason it is of interest to study the types of waves, in particular the surface waves, which can exist on or near such a plasma density jump.

2. SURFACE WAVES IN AN UNMAGNETIZED PLASMA WITH A SHARP BOUNDARY

The concept of 'surface wave' was introduced in high-frequency electrodynamics. It represents a slow electromagnetic wave with a phase velocity

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$v_{ph} < c$ (c is the speed of light), propagating in a dielectric with permittivity $\varepsilon > 1$ and bounded by a vacuum ($\varepsilon_0 = 1$), i.e. in a dielectric waveguide (VANSHTEIN, 1957; KATSENELENBAUM, 1966). However, such electromagnetic waves are pseudo-surface waves to the effect that the wave electric (magnetic) field has volume character deep inside the dielectric and it decreases exponentially in the vacuum away from the dielectric.

It is well known (LANDAU and LIFSHITZ, 1960) that the propagation of pure surface wave (i.e. waves whose field decreases exponentially away from the interface) between two media is only possible when their permittivities have opposite signs (say $\varepsilon_1 > 0$ and $-|\varepsilon_2|$). In that case if the (y, z)-plane (Fig. 1) is the interface between the two media, the z-axis is the direction of wave propagation, and the y-axis is the direction of wave magnetic induction ($B_y \equiv B_t$), then the wave solutions which vanish at $x = \pm\infty$, are

$$\begin{aligned} B_1 &= B_0 \exp(ik_z z + \kappa_1 x), \quad \text{for} \quad x < 0, \\ B_2 &= B_0 \exp(ik_z z - \kappa_2 x), \quad \text{for} \quad x > 0, \end{aligned} \tag{1}$$

where

$$\kappa_1 = \left(k_z^2 - \frac{\omega^2}{c^2}\varepsilon_1\right)^{1/2} \quad \text{and} \quad \kappa_2 = \left(k_z^2 + \frac{\omega^2}{c^2}|\varepsilon_2|\right)^{1/2}.$$

Here ω is the wave frequency, k_z is the wave vector component along the interface S, and the real quantities κ_1 and κ_2 describe the wave field attenuation away from S (note that field attenuation is greater in the dielectric with negative permittivity).

Since in certain frequency domains the plasma can have a negative dielectric constant, propagation of true surface waves along a plasma–dielectric interface is possible. The investigation of surface wave propagation on a plasma–dielectric interface may be done in two ways:

(i) One supposes that the surface wave arises from a perturbation which propagates. In other words, we follow the time development of an initial perturbation. The finding of the dispersion relation reduces to solving the initial problem (GINZBURG and RUKHADZE, 1975) for a bounded plasma.

(ii) By calculating the indecies of refraction of an electromagnetic wave on the plasma–dielectric boundary one can obtain the condition for the propagation of a surface wave on the interface.

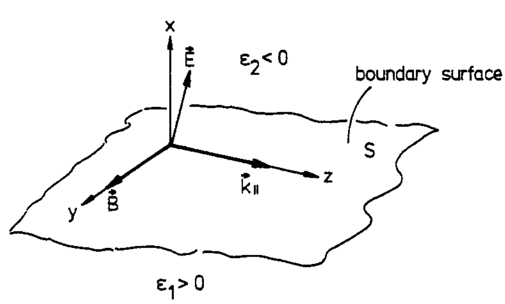

FIG. 1.—Wave field and wave vector configuration of a TM-surface wave on the interface between two dielectrics.

In the most theoretical works devoted to the study of surface wave propagation in a plasma the former method is used.

In order to get a dispersion relation for the surface waves it is necessary to apply the electromagnetic boundary conditions on the wave field at the interface between the two media, namely the continuity of the tangential components of the electric and magnetic fields. These conditions can be obtained by integrating Maxwell's equations across the interface, that is

$$\begin{aligned} B_{t1}\big|_{\text{surface}} &= B_{t2}\big|_{\text{surface}} \\ E_{t1}\big|_{\text{surface}} &= E_{t2}\big|_{\text{surface}} \quad (E_t \equiv E_z, \text{ see Fig. 1}). \end{aligned} \tag{2}$$

These equations are also valid at the boundary with a conductive medium on the condition that there is no surface current density on the interface. A condition equivalent to (2) is

$$Z_1\big|_{\text{surface}} = Z_2\big|_{\text{surface}}, \qquad z_{1,2} = \frac{E_{t1,2}}{iB_{t1,2}}, \tag{3}$$

that is, the surface impedances of the two media (plasma and dielectric) are equal. In the electrostatic limit ($v_{ph} \ll c$), one neglects the wave magnetic field, and the boundary conditions (2) reduce to the continuity of the tangential component of the electric field and the normal component of the displacement vector $\mathbf{D} = \varepsilon\mathbf{E}$, that is

$$\begin{aligned} E_{t1}\big|_{\text{surface}} &= E_{t2}\big|_{\text{surface}} \\ \varepsilon_1 E_{n1}\big|_{\text{surface}} &= \varepsilon_2 E_{n2}\big|_{\text{surface}}. \end{aligned} \tag{4}$$

These conditions are valid in the absence of surface electric charge density (which is related to the normal conduction current at the interface) (GOL'DSHTEIN and ZERNOV, 1971).

It follows from the boundary conditions that only a wave polarized in the (y, z)-plane (Fig. 1) can propagate as a true surface wave on the interface between two dielectric media which have dielectric constants with opposite signs. The wave magnetic field is parallel to the interface (LANDAU and LIFSHITZ, 1960), corresponding to a TM-wave.

In the second method of derivation of the surface wave dispersion relation on a plasma–vacuum interface one uses the fact (CLEMMOW and ELGIN, 1974) that a plane wave polarized with its magnetic vector parallel to the interface does not give rise to any reflected wave when the direction of propagation is at the Brewster angle α_B, given by $\tan \alpha_B = 1/n$, where n is the plasma refractive index: $n^2 = \varepsilon_{pl}$. Since the plasma dielectric constant can be negative (for $\omega < \omega_p$), n can be imaginary and consequently α_B can be complex. In the case $|n| > 1$, $\alpha_B = i\gamma$, where γ is a real number such that

$$|\tanh \gamma| = \left|\frac{1}{n}\right|, \tag{5}$$

a pure surface wave propagates along the plasma–vacuum interface and decays exponentially away from it. However, this manner of obtaining the dispersion relation is only formal, since there does not exist any real angle at which if an electromagnetic wave were to incident from the dielectric onto the plasma, it would propagate as a surface wave along the dielectric–plasma interface.

We now consider the linear theory of surface wave propagation, that is, we assume that all plasma parameters have the form $\phi = \phi_0 + \phi_1$, $\phi_1 \ll \phi_0$, where ϕ_0 is the equilibrium part and ϕ_1 is the perturbation part owing to the wave field. The oscillating quantities are assumed to behave sinusoidally as

$$\phi_1 \propto \exp(ik_z z - i\omega t). \tag{6}$$

Conditions (2) to (4) or (5) are sufficient for obtaining the dispersion relation of the surface waves propagating on the dielectric–plasma interface only in the case when one may neglect the effects of spatial dispersion. When we take into account the thermal motion of the plasma particles, it is necessary, besides the field boundary conditions (2) to (4) or (5) to satisfy one more condition determined by the behaviour of plasma particles at the boundary plane. In the kinetic model, this is the condition on the velocity distribution function $f(\mathbf{r}, \mathbf{v}, t)$ at the boundary surface. Actually, $f(\mathbf{r}, \mathbf{v}, t)$ is zero at the boundary and it increases away from the boundary surface into the plasma. Due to mathematical difficulties (REZVOV, 1970a; BARR and BOYD, 1972), one usually considers two 'idealize' boundary conditions for $f(\mathbf{r}, \mathbf{v}, t)$ (mirror and diffuse reflection) assuming a sharp boundary between the plasma and the dielectric, that is, one ignores the existence of a transition layer of inhomogeneous plasma with a thickness l. In this case one can only investigate the propagation of waves with wavelengths $\lambda \gg l$. As long as l is of the order of the Debye length r_D, the inequality $\lambda \gg l$ presupposes weakly damped waves for which $\lambda \gg r_D$. In the case of mirror reflection of the particles at the boundary, the perturbed part of the distribution function $f_1(\mathbf{r}, \mathbf{v}, t)$, associated with the waves, satisfies the following condition:

$$f_1(\mathbf{r}, \mathbf{v}_\parallel, v_n, t)\,|_{\text{surface}} = f_1(\mathbf{r}, \mathbf{v}_\parallel, -v_n, t)\,|_{\text{surface}}, \tag{7}$$

where $\mathbf{v}_\parallel$ and v_n are the tangential and normal (with respect to the interface) particle velocity components respectively (LANDAU, 1946; SILIN and RUKHADZE, 1961; ROMANOV, 1968). In the hydrodynamic approach, the mirror reflection condition is given by the equality:

$$v_n\,|_{\text{surface}} = 0, \tag{8}$$

where v_n is the normal component of the fluid velocity. The second idealize boundary condition for $f(\mathbf{r}, \mathbf{v}, t)$ (the condition for diffuse scattering) is discussed in detail by ROMANOV (1968), DRYAKHLUSHIN and ROMANOV (1968), HOPPS and WALDRON (1977). GORBATENKO and KURILKO (1964) and HOPPS and WALDRON (1977) used the generalized boundary condition

$$v_n f_1(\mathbf{r}, \mathbf{v}_\parallel, v_n, t)\,|_{\text{surface}} = \int_{v_n<0} d\mathbf{v}' P(\mathbf{v}\,|\,\mathbf{v}') f_1(\mathbf{r}, \mathbf{v}_\parallel', v_n', t) v_n'\,|_{\text{surface}} \tag{9}$$

where

$$P(\mathbf{v}\,|\,\mathbf{v}') = \mu\delta(v_\parallel' - v_\parallel)\delta(v_n' + v_n) + (1-\mu)P_d(\mathbf{v}) \tag{10}$$

is the probability that an incident particle of velocity $\mathbf{v}'$ is scattered with velocity $\mathbf{v}$. The first term in (10) represents specular reflection, $v_\parallel' \to v_\parallel$ and $v_n' \to -v_n$, in which the scattered particle has a complete memory of its incident velocity. The second term represents diffuse scattering, in which there is a complete loss of memory by the particle of its incident velocity. The probability $P_d(\mathbf{v})$ is a function characteristic of the boundary surface. The parameter μ can vary from 0 to 1 and it describes the relative contribution of specular reflection and diffuse scattering.

The propagation of surface waves along the plasma–dielectric boundary is possible in two frequency domains: high-frequency (at frequencies $\omega < \omega_{pe}$) and low-frequency (at $\omega < \omega_{pi}$); where

$$\omega_{p\alpha} = \left(\frac{4\pi n_0 e_\alpha^{\,2}}{m_\alpha}\right)^{1/2}$$

with $\alpha = \mathrm{e, i}$, is the electron/ion plasma frequency, n_0 is the equilibrium number density, and e_α and m_α ($m_e = m$, $m_i = M$) are the charge and mass of the particles of species α.

We shall consider surface wave propagation in a semi-infinite plasma, since this geometry gives the physical picture of the phenomenon, as well as in a cylindrical plasma column (with radius R), since in most experiments the geometry is cylindrical. (In a cylindrical geometry (with co-ordinates r, ϕ, z), the propagation of a TM-surface wave (E_r, B_ϕ, E_z) is along the z-axis.)

Let us consider briefly the physics of surface waves in a plasma. The first work which treats in detail (theoretically and experimentally) high-frequency surface waves in a plasma was by TRIVELPIECE and GOULD (1959). They emphasized the distinction between the waveguide modes of a cylindrical waveguide partially filled with a cold plasma (where principal effect on the waveguide modes is an up-shift of the cutoff frequency) and the long-wavelength space charge waves (surface waves) in a finite plasma. The surface waves are associated with perturbation currents on the plasma surface. The high-frequency surface waves in a cylindrical plasma are the surface analogue of the high-frequency bulk waves—the former have an electromechanical nature and they propagate with phase velocities $v_{ph} \ll c$ at frequencies much less than the waveguide cutoff frequency. On the other hand, the high-frequency surface oscillations in a semi-infinite plasma are the surface wave analogue of the longitudinal plasma oscillations. The characteristic frequency of the surface oscillations is lower than that of the bulk oscillations because a part of the wave field is outside the plasma and the effective force of interaction among the plasma particles is weaker (FUSE and ICHIMARU, 1975). The frequency of the surface oscillations can be obtained from a relation which is analogous to that for plasma oscillations, $\varepsilon(\omega) = 0$ (ROMANOV, 1964). The electric charges whose fluctuations give rise to surface oscillations are localized at the plasma–dielectric interface. These charges interact with each other as if they were in a homogeneous medium with an effective dielectric constant $\varepsilon_{eff} = \frac{1}{2}[\varepsilon_{pl}(\omega) + \varepsilon]$. The relation giving the frequency of the surface oscillations is then $\varepsilon_{eff} = 0$, from which we obtain for a cold plasma,

$$\omega_0 = \frac{\omega_{pe}}{\sqrt{2}} \tag{11a}$$

in a plasma bounded by vacuum, and

$$\omega_0 = \frac{\omega_{pe}}{\sqrt{1+\varepsilon}} \tag{11b}$$

in a plasma bounded by a dielectric (ε). The electric field of a surface wave propagating on the plasma–vacuum interface and the corresponding perturbation surface charges on the boundary are shown in Fig. 2. When the plasma has the shape of a column, the propagation of quasistatic surface waves ($v_{ph} \ll c$) at frequencies $\omega < \omega_0$ is possible (TRIVELPIECE and GOULD, 1959), and the

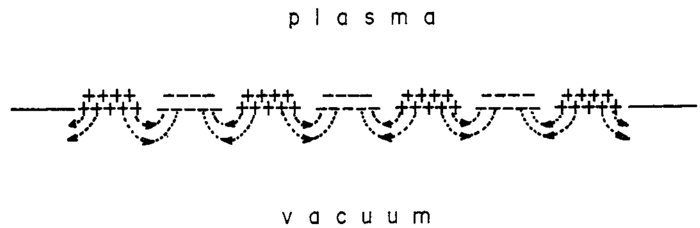

FIG. 2.—An electrostatic plasma surface wave on the plasma–vacuum boundary.

wavelength increases with the decrease of the frequency. With the increase of k_z, the wave field (E_z) and the wave energy become more concentrated at the dielectric–plasma boundary (CARLILE, 1964). The rate of surface wave field decay into the dielectric is the same as that for very slow electromagnetic waves in a dielectric waveguide (it is characterized by the wave number $k_\perp{}^0 = \kappa_1$), whereas the quantity

$$k_\perp = \left[k_z{}^2 - \frac{\omega^2}{c^2}\,\varepsilon(\omega)\right]^{1/2} \tag{12}$$

determines the surface wave electric field in the plasma and $k_\perp > k_\perp{}^0$ (TRIVELPIECE and GOULD, 1959). (In the electrostatic limit, the wave attenuation is the same in the plasma as in the dielectric, and is characterized by k_z.)

The use of a kinetic model in the study of surface waves gives a more precise idea for the electric field structure at the interface, namely, the surface wave electric field should be an effective value of the electric field of the bulk plasma waves at the boundary. The electrostatic wave investigation does not give an exact and complete picture for the physical nature of the waves. From the electromagnetic treatment of surface wave propagation in a plasma (ROMANOV 1964a, DIAMENT *et al.*, 1966; KAW and MCBRIDE, 1970: BARR and BOYD, 1972; CLEMMOW and ELGIN, 1974) one may see that the surface wave is neither pure longitudinal nor pure transverse. Since the perturbation field is both longitudinal and transverse the surface mode is a hybrid (mixed) mode (i.e. space charge plus transverse electromagnetic wave). However, as can be shown from (12), in the frequency domain

$$\frac{\omega^2}{c^2}\,|\varepsilon(\omega)| \ll k_z{}^2 \tag{13}$$

the wave magnetic field inside the plasma can be neglected (CLEMMOW and ELGIN, 1974) and the waves may be treated as potential (DIAMENT *et al.*, 1966). As long as $v_{ph} < (c/\sqrt{\varepsilon})$ in the whole frequency range ($(0, \omega_{pe}/\sqrt{1+\varepsilon})$ for electromagnetic surface waves in a cold plasma) of the wave propagation along the homogeneous plasma with sharp boundary, a surface wave cannot be radiated as an electromagnetic wave and vice versa, an electromagnetic wave cannot be coupled with any surface plasma wave. In other words, surface waves cannot be excited by an electromagnetic wave incident on the plasma boundary surface (ROMANOV, 1964(a,b,c); KAW and MCBRIDE, 1970; BARR and BOYD, 1972; CLEMMOW and ELGIN, 1974; KARPLYUK, 1976).

In a two-component nonisothermal plasma there also exist low-frequency surface waves (ROMANOV, 1965; KONDRATENKO, 1965; MCBRIDE and KAW, 1970), whose propagation is essentially determined by the ions. Since in the low-frequency domain, $v_{ph} \ll c$, the longitudinal nature of the waves is distinctly revealed and they are usually considered in the quasistatic approach as a surface wave analogue of the low-frequency bulk waves. The attenuation coefficient of a low-frequency surface wave into the plasma is (ROMANOV, 1965; KONDRATENKO, 1965; ZHELYAZKOV and NENOVSKI, 1973)

$$\kappa = k_z \left[1 - \frac{1}{k_z^2 r_{De}^2 \left(\frac{\omega_{pi}^2}{\omega^2} - 1 \right)} \right]^{1/2}, \tag{14}$$

where

$$r_{De} = \left(\frac{T_e}{4\pi n_0 e^2} \right)^{1/2}$$

is the electron Debye length. In order to be surface, the wave must have a frequency ω such that the quantity κ is real, i.e. it is necessary that the condition

$$b = 1 - \frac{1}{k_z^2 r_{De}^2 \left(\frac{\omega_{pi}^2}{\omega^2} - 1 \right)} > 0 \tag{15}$$

be fulfilled (ZHELYAZKOV and NENOVSKI, 1973). In the long wavelength limit, $k_z^2 r_{De}^2 \ll 1$, we get $\kappa \ll k_z$ (ROMANOV, 1965), i.e. the wave attenuation into the plasma is weak and the waves do not have a clear surface character. In this case the surface–wave–ion sound is practically indistinguishable from the bulk ion acoustic waves. In the short-wavelength limit, $k_z^2 r_{De}^2 \gg 1$, $\kappa \approx k_z$ (ROMANOV, 1965; ZHELYAZKOV and NENOVSKI, 1973) and the field penetrate depth into the plasma, as well as into the dielectric, is of the order of the wavelength. Then the surface character of the waves is clearly revealed and the surface wave frequency range is essentially different from that of the bulk waves.

In a hydrodynamic approach, the surface wave dispersion relation is the condition for the solvability of the set (2) or (4) and (8) in the case of warm plasmas. One may determine the fields into the plasma by solving the one- (two-) fluid equations and Maxwell's equations (or Poisson's equation when the waves are considered as electrostatic). Dispersion relations for high- and low-frequency surface waves using a fluid plasma model are derived by KAW and MCBRIDE (1970), DIAMENT *et al.* (1966), and by MCBRIDE and KAW (1970), KLEVANS and MITCHELL (1970) for semi-infinite plasma and plasma cylinder respectively. Since surface wave propagation using the hydrodynamic approach is discussed in detail by KONDRATENKO (1976) in our review, emphasis is placed on the kinetic theory of surface waves in a semi-infinite plasma and along a plasma column.

3. DERIVATION OF THE DISPERSION RELATIONS

Let us consider a two-component nonisothermal ($T_e \gg T_i$, $T_i \neq 0$) homogeneous isotropic collisionless sharply bounded plasma, which occupies the half-space $x>0$ (Fig. 1) and is bounded by a dielectric (vacuum) with a permittivity ε (for vacuum $\varepsilon \equiv \varepsilon_0 = 1$). The dynamics of the particles and fields in the plasma is governed by the linearized Vlasov and Maxwell's equations

$$\frac{\partial f_{1\alpha}}{\partial t} + \mathbf{v} \cdot \frac{\partial f_{1\alpha}}{\partial \mathbf{r}} + \frac{e_\alpha}{m_\alpha} \mathbf{E} \cdot \frac{\partial f_{0\alpha}}{\partial \mathbf{v}} = 0, \tag{16}$$

$$\begin{aligned} &\nabla \cdot \mathbf{D} = 0, \\ &\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \\ &\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}, \\ &\nabla \cdot \mathbf{B} = 0, \end{aligned} \tag{17}$$

where $\mathbf{D}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) + 4\pi \int_{-\infty}^{t} dt' \mathbf{j}(\mathbf{r}, t')$ and $\mathbf{j} = \sum_\alpha e_\alpha \int \mathbf{v} f_{1\alpha}\, dv$. The boundary condition for the functions $f_{1\alpha}$ is the condition (7) for specular reflection. Since the obvious method of solution of (16) would be Fourier transform with respect to x, it is necessary to extend the range of definition of $f_1(\mathbf{r}, \mathbf{v}, t)$ (as well as the field $\mathbf{E}$) into the nonphysical region $x<0$ in such a way that (16) is satisfied for $x>0$ and the boundary condition (7) is satisfied at $x=0$. Accordingly, we let

$$f_{1\alpha}(-x, \mathbf{r}_\parallel, v_x, \mathbf{v}_\parallel, t) = f_{1\alpha}(x, \mathbf{r}_\parallel, -v_x, \mathbf{v}_\parallel, t).$$

The distribution functions $f_{1\alpha}$ are invariant under the transformation $x \to -x$, $v_x \to -v_x$. In order to make equations (16) and (17) invariant under the same transformation the field $\mathbf{E}(\mathbf{r}, t)$ must be defined for $x<0$ in such a manner that E_x is an odd function of x and $\mathbf{E}_\parallel$ an even function of x, provided that $f_{0\alpha}$ are even functions of v_x.

Performing Fourier transform on the spatial dependence and Laplace transform with respect to time in equations (16), (17)

$$\int_{-\infty}^{\infty} d\mathbf{r} \exp(-i\mathbf{k} \cdot \mathbf{r}) \int_0^{\infty} dt \exp(i\omega t)$$

and neglecting the influence of the initial conditions, we obtain from (16)

$$f_{1\alpha}(\mathbf{k}, \mathbf{v}, \omega) = \frac{e_\alpha}{m_\alpha} \frac{\mathbf{E}(\omega, \mathbf{k}) \cdot \partial f_{0\alpha}(\mathbf{v})/\partial \mathbf{v}}{i(\omega - \mathbf{k} \cdot \mathbf{v})}, \tag{18}$$

where $k^2 = k_x^2 + k_\parallel^2$. The corresponding expression for current density is

$$\mathbf{j}(\omega, \mathbf{k}) = -i \sum_\alpha \frac{e_\alpha^2}{m_\alpha} \int d\mathbf{v} \frac{\mathbf{E}(\omega, \mathbf{k}) \cdot \partial f_{0\alpha}(\mathbf{v})/\partial \mathbf{v}}{\omega - \mathbf{k} \cdot \mathbf{v}}.$$

It follows from Maxwell's equations (17) that

$$\left[k^2 - \frac{\omega^2}{c^2} \varepsilon^{tr}(\omega, k)\right] \mathbf{E}^{tr}(\omega, \mathbf{k}) - \frac{\omega^2}{c^2} \varepsilon^l(\omega, k) \mathbf{E}^l(\omega, \mathbf{k}) = 2i\frac{\omega}{c} \mathbf{M}(x = +0), \tag{19}$$

where

$$\mathbf{M}=(0,0,B_y),$$

$$\mathbf{E}^l(\omega,\mathbf{k})=\frac{\mathbf{k}\cdot\mathbf{E}(\omega,\mathbf{k})}{k^2}\mathbf{k},\qquad \mathbf{E}^{tr}(\omega,\mathbf{k})=\mathbf{E}(\omega,\mathbf{k})-\mathbf{E}^l(\omega,\mathbf{k});$$

and

$$\varepsilon^l(\omega,k)=1+\sum_\alpha\frac{4\pi e_\alpha^2}{m_\alpha k^2}\int \mathrm{d}\mathbf{v}\frac{\mathbf{k}\cdot\partial f_{0\alpha}/\partial\mathbf{v}}{\omega-\mathbf{k}\cdot\mathbf{v}}$$
$$\varepsilon^{tr}(\omega,k)=1-\sum_\alpha\frac{4\pi e_\alpha^2}{m_\alpha\omega}\int \mathrm{d}\mathbf{v}\frac{f_{0\alpha}}{\omega-\mathbf{k}\cdot\mathbf{v}}$$

are accordingly the longitudinal and transverse plasma dielectric functions of an infinite plasma.

By performing an inverse Fourier transform with respect to x from (18) one may obtain the parallel component of the TM-wave electric field in the plasma:

$$\mathbf{E}_\parallel(x,\omega,\mathbf{k}_\parallel)=-\frac{i}{\pi}\frac{c}{\omega}\int_{-\infty}^{\infty}\left\{\mathrm{d}k_x e^{ik_x x}\frac{k_\parallel^2}{k^2\varepsilon^l(\omega,k)}-\frac{\omega^2}{c^2}\frac{k_x^2}{k^2\left(k^2-\frac{\omega^2}{c^2}\varepsilon^{tr}(\omega,k)\right)}\right\}\frac{\mathbf{k}_\parallel\cdot\mathbf{M}(x=+0)}{k_\parallel^2}\mathbf{k}_\parallel. \tag{19}$$

The solution of Maxwell's equations (17) in the dielectric ($x<0$) gives the following expression for $\mathbf{E}_\parallel(x,\omega,\mathbf{k}_\parallel)$:

$$\mathbf{E}_\parallel(x,\mathbf{k}_\parallel,\omega)=e^{k_\perp^0 x}\frac{i}{\varepsilon}\frac{c}{\omega}k_\perp^0\frac{\mathbf{k}_\parallel\cdot\mathbf{M}(x=-0)}{k_\parallel^2}\mathbf{k}_\parallel. \tag{20}$$

The boundary condition for the continuity of the parallel electric field at the interface $x=0$ yields the dispersion relation for the surface waves on a semi-infinite plasma:

$$\frac{1}{\pi}\left\{\int_{-\infty}^{\infty}\frac{\mathrm{d}k_x k_\parallel^2}{k^2\varepsilon^l(\omega,k)}+\int_{-\infty}^{\infty}\frac{\mathrm{d}k_x}{k^2}\frac{k_x^2}{\varepsilon^{tr}(\omega,k)-\frac{c^2k^2}{\omega^2}}\right\}=-\frac{k_\perp^0}{\varepsilon}, \tag{21}$$

where $k_\parallel\equiv k_z$. The derivation of the same dispersion relation in another way is also given in detail by GINZBURG and RUKHADZE (1975). Dispersion relation (21) was first obtained by ROMANOV (1964a). The same result is derived by BOULANGER and ASHBY (1971), BARR and BOYD (1972). CLEMMON and ELGIN (1974, 1975) obtained equation (21) by using the second method for finding the surface wave dispersion relation.

In the electrostatic limit, with

$$\frac{\omega^2}{c^2}|\varepsilon(\omega)|\ll k_z^2,$$

where $\varepsilon(\omega)=1-\frac{\omega_{pe}^2}{\omega^2}$ is the dielectric constant of a cold plasma, it follows from

(21) the dispersion relation of longitudinal surface waves in an infinite plasma:

$$\frac{k_z}{\pi}\int\frac{\mathrm{d}k_x}{k^2\varepsilon^l(\omega,k)}=-\frac{1}{\varepsilon}. \tag{22}$$

Equation (22) is derived also in different ways (directly in electrostatic limit) by GUERNSEY (1965), SHIVAROVA (1974), ATANASSOV *et al.* (1974), FUSE and ICHIMARU (1975) using Vlasov and Poisson's equations.

When we neglect the plasma spatial dispersion, from (21) one obtains the well-known dispersion relation of surface waves on a semi-infinite cold plasma:

$$\frac{k_\perp}{\varepsilon(\omega)}=-\frac{k_\perp^0}{\varepsilon}. \tag{23}$$

We note that the dispersion relation of surface waves in hot semi-infinite plasma bounded by a dielectric, with diffuse scattering at the boundary is obtained by GORBATENKO and KURILKO (1964), ROMANOV (1968), HOPPS and WALDRON (1977).

The kinetic treatment of surface wave propagation along a hot plasma column is so far done only for specular reflection (7) of the plasma particles at the boundary surface $r=R$. The problem was first solved by KONDRATENKO (1972, 1976) by using series expansions in Fourier-Bessel and Dini's series for the fields and currents in the plasma. Since the dispersion relation obtained by him and used by ANIČIN and BABOVIĆ (1975) for determining the collisionless surface wave damping is not a wholly correct self-consistent solution of Vlasov and Maxwell's equations (the contribution of the integral

$$I=\int_0^R \mathrm{d}r\,\psi^-(r)J_0(r\xi_n), \tag{24}$$

where $\psi^-(r)=f_{1\alpha}(r,v_r)-f_{1\alpha}(r,-v_r)$, ξ_n are the roots of the equation $J_1(R\xi)=0$ and J_n are Bessel functions of the nth order, has been neglected) we propose here another method for finding the exact dispersion relation (ATANASSOV *et al.* 1976).

Let us examine the propagation of axi-symmetric electromagnetic (TM) waves $\propto \exp(-i\omega t+ik_z z)$ along a plasma cylinder with a radius R bounded by a dielectric with permittivity ε. The starting equations are (16) and (17) which will be solved with the boundary condition (7) ($v_n\equiv v_r$) in cylindrical co-ordinates (r,ϕ,z). It is convenient to introduce the functions $\psi_\alpha^\pm(r,v_r)=f_{1\alpha}(r,v_r)\pm f_{1\alpha}(r,-v_r)$, rewriting (16) and boundary condition (7) in the form

$$\begin{aligned}\frac{\mathrm{d}\psi_\alpha^+}{\mathrm{d}r}-i\beta\psi_\alpha^-&=-2e_\alpha f_{0\alpha}{}'E_r\\ \frac{\mathrm{d}\psi_\alpha^-}{\mathrm{d}r}+i\beta\psi_\alpha^+&=-2e_\alpha\frac{v_z}{v_r}f_{0\alpha}{}'E_z\end{aligned} \tag{25}$$

and

$$\psi_\alpha^-(r=R,v_r)=0, \tag{26}$$

where $\beta=\dfrac{\omega-k_z v_z}{v_r}$.

We shall seek the solutions of the set (25) with boundary condition (26) in the form of plane waves

$$\begin{Bmatrix} \psi_\alpha^-(r) \\ E_r(r) \end{Bmatrix} = \sum_{n=-\infty}^{\infty} \begin{Bmatrix} \psi_{\alpha n}^- \\ E_{rn} \end{Bmatrix} \sin \kappa_n r$$
$$\begin{Bmatrix} \psi_\alpha^+(r) \\ E_z(r) \end{Bmatrix} = \sum_{n=-\infty}^{\infty} \begin{Bmatrix} \psi_{\alpha n}^+ \\ E_{zn} \end{Bmatrix} \cos \kappa_n r \tag{27}$$

when $\kappa_n = \pi n/R$. From (25) we obtain the following expressions for the coefficients $\psi_{\alpha n}^{\mp}$:

$$\psi_{\alpha n}^- = -2e_\alpha \frac{f_{0\alpha}'}{\beta^2 - \kappa_n^2}\left(i\beta E_{rn} - \kappa_n \frac{v_z}{v_r} E_{zn}\right)$$
$$\psi_{\alpha n}^+ = -2e_\alpha \frac{f_{0\alpha}'}{\beta^2 - \kappa_n^2}\left(\kappa_n E_{rn} + i\beta \frac{v_z}{v_r} E_{zn}\right) \tag{28}$$

in terms of which we define both the current density

$$j_{r,z}(r) = \sum_\alpha e_\alpha \int_{-\infty}^{\infty} \mathrm{d}v_z \int_0^{\infty} \mathrm{d}v_r \psi_\alpha^{\mp}(r) v_{r,z} \tag{29}$$

and the displacement vector

$$D_{r,z}(r) = E_{r,z}(r) + \frac{4\pi i}{\omega} j_{r,z}(r). \tag{30}$$

The latter with $\mathbf{k}_n = (\kappa_n, k_z)$ is

$$D_r(r) = \sum_{n=-\infty}^{\infty} \sin \kappa_n r \left\{\left[\varepsilon^{\mathrm{tr}}(\omega, k_n) + \frac{\kappa_n^2}{k_n^2}(\varepsilon^l(\omega, k_n) - \varepsilon^{\mathrm{tr}}(\omega, k_n))\right] E_{rn} + i\frac{\kappa_n k_z}{k_n^2}(\varepsilon^l(\omega, k_n) - \varepsilon^{\mathrm{tr}}(\omega, k_n)) E_{zn}\right\}$$
$$D_z(r) = \sum_{n=-\infty}^{\infty} \cos \kappa_n r \left\{-i\frac{\kappa_n k_z}{k_n^2}(\varepsilon^l(\omega, k_n) - \varepsilon^{\mathrm{tr}}(\omega, k_n)) E_{rn} + \left[\varepsilon^{\mathrm{tr}}(\omega, k_n) + \frac{k_z^2}{k_n^2}(\varepsilon^l(\omega, k_n) - \varepsilon^{\mathrm{tr}}(\omega, k_n))\right] E_{zn}\right\}. \tag{31}$$

Inserting expressions (31) into Maxwell's equations

$$ik_z E_r(r) - \frac{\mathrm{d}E_z(r)}{\mathrm{d}r} = \frac{i}{k_z}\frac{\omega^2}{c^2} D_r(r)$$
$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}[rD_r(r)] + ik_z D_z(r) = 0 \tag{32}$$
$$B_\phi(r) = \frac{\omega}{k_z c} D_r(r)$$

we determine the plasma self-consistent field. The first equation gives the relation between the Fourier amplitudes E_{rn} and E_{zn}

$$E_{rn} = -\frac{\kappa_n}{ik_z}\frac{\Delta_n^{tr}+\frac{\omega^2}{c^2}\varepsilon^l(\omega, k_n)}{\Delta_n^{tr}-\frac{\omega^2}{c^2}\frac{\kappa_n^2}{k_z^2}\varepsilon^l(\omega, k_n)}E_{zn}, \tag{33}$$

where $\Delta_n^{tr} = k_n^2 - \frac{\omega^2}{c^2}\varepsilon^{tr}(\omega, k_n)$. Further, from (31) and (33) we express the Fourier amplitudes D_{rn} and D_{zn} in terms of E_{zn}

$$\begin{aligned} D_{rn} &= i\frac{\kappa_n}{k_z}\frac{k_n^2\varepsilon^l(\omega, k_n)}{\Delta_n^{tr}-\frac{\omega^2}{c^2}\frac{\kappa_n^2}{k_n^2}\varepsilon^l(\omega, k_n)}E_{zn} \\ D_{zn} &= \frac{k_z^2-\frac{\omega^2}{c^2}\varepsilon^{tr}(\omega, k_n)}{k_z^2}\frac{k_n^2\varepsilon^l(\omega, k_n}{\Delta_n^{tr}-\frac{\omega^2}{c^2}\frac{\kappa_n^2}{k_z^2}\varepsilon^l(\omega, k_n)}E_{zn} \end{aligned} \tag{34}$$

from which one can see that when the spatial dispersion in ε^{tr} is neglected, the equality

$$-ik_z\frac{dD_z(r)}{dr} = k_\perp^2 D_r(r) \tag{35}$$

with $k_\perp = \left[k_z^2 - \frac{\omega^2}{c^2}\varepsilon^{tr}(\omega)\right]^{1/2}$ is valid. On the basis of (32), (35) and (34) we determine the plasma impedance at the cylinder surface $Z_{pl} = (E_z(R))/(iB_\phi(R))$ and after equating it with the dielectric impedance one obtains the dispersion relation of the electromagnetic surface waves propagating along a hot plasma column:

$$\frac{1}{k_\perp}\sum_{n=-\infty}^{\infty}(-1)^n\left\{\frac{k_z^2}{k_n^2}\frac{k_n^2-(\omega^2/c^2)\varepsilon^{tr}(\omega)}{\varepsilon^l(\omega, k_n)} - \frac{\omega^2}{c^2}\frac{\kappa_n^2}{k_n^2}\right\}\times A_n = -\frac{k_\perp^0}{\varepsilon}\frac{K_0(k_\perp^0 R)}{K_1(k_\perp^0 R)}, \tag{36}$$

where

$$A_n \equiv \frac{1}{RI_1(k_\perp R)}\int_0^R dr I_0(k_\perp r)\cos\kappa_n r \tag{37}$$

and I_n and K_n are the modified Bessel functions of the first and second kind. In electrostatic limit dispersion relation (36) reduces to the following equation

$$1+\varepsilon\frac{K_1(k_z R)}{K_0(k_z R)}\sum_{n=-\infty}^{\infty}(-1)^n\frac{A_n}{\varepsilon^l(\omega, k_n)} = 0. \tag{38}$$

In the case of a cold electron plasma, as $\varepsilon^l = \varepsilon^{tr} = 1-(\omega_{pe}^2/\omega^2)$, from (36) we get the well-known dispersion relation (FEINBERG and GORBATENKO, 1959; AKAO and IDA, 1964):

$$\frac{k_\perp}{\varepsilon(\omega)}\frac{I_0(k_\perp R)}{I_1(k_\perp R)}=-\frac{k_\perp^0}{\varepsilon}\frac{K_0(k_\perp^0 R)}{K_1(k_\perp^0 R)}. \tag{39}$$

Taking into account that in the case of a thick cylinder $k_\perp R>1$, the asymptotic value of (37) is given by the formula

$$A_n\to(-1)^n\frac{k_\perp}{R(k_\perp{}^2+\kappa_n{}^2)}+O\left[\frac{1}{(k_\perp R)^2}\right], \tag{40}$$

from (36) with $R\to\infty$ one may obtain the dispersion relation of surface waves in hot semi-finite plasma (21).

The value of A_n (37) may be calculated in the limits of thick ($k_\perp R>1$) and thin ($k_\perp R<1$) cylinder:

$$A_n=\begin{cases}\dfrac{(-1)^n}{k_\perp{}^2+\kappa_n{}^2}\dfrac{k_\perp}{R}\left[1+\dfrac{k_\perp}{k_\perp{}^2+\kappa_n{}^2}\dfrac{1}{R}\dfrac{I_0(k_\perp R)}{I_1(k_\perp R)}\right] & \text{at}\quad k_\perp R>1\\[2ex] \left.\begin{array}{ll}\dfrac{(-1)^n}{k_\perp{}^2+\kappa_n{}^2}\dfrac{k_\perp}{R}\left(1-\dfrac{3}{4}\dfrac{k_\perp{}^2}{k_\perp{}^2+\kappa_n{}^2}\right) & \text{when}\quad n\neq 0\\[4ex] \dfrac{2}{k_\perp R} & \text{when}\quad n=0\end{array}\right\} & \text{at}\quad k_\perp R<1.\end{cases} \tag{41}$$

We emphasize that the basic features in the derivation of the exact dispersion relation of the surface waves in a cylindrical plasma column bounded by a dielectric, are making Fourier expansions in $\cos\kappa_n r$ and $\sin\kappa_n r$ for the components of the electric field into the plasma and neglecting the spatial dispersion in the transverse plasma dielectric function.

Within the framework of the kinetic approach one may obtain expressions for the dispersion and the time (γ_ω) and spatial ($\gamma_{\mathbf{k}}$) damping rate of the surface waves in a homogeneous hot plasma with sharp boundary by solving the wave dispersion relations ((21) and (36) for semi-infinite plasma and plasma cylinder correspondingly) assuming Maxwellian distributions for the particles

$$f_{0\alpha}(\mathbf{v})=n_{0\alpha}\left(\frac{m_\alpha}{2\pi T_\alpha}\right)^{3/2}\exp\left(-\frac{m_\alpha}{2T_\alpha}\mathbf{v}^2\right)$$

and replacing ω with $\omega+i\gamma$ (at $|\gamma|/\omega\ll 1$).

4. HIGH-FREQUENCY SURFACE WAVES IN SEMI-INFINITE PLASMA AND PLASMA CYLINDER

The exact analysis of the general dispersion relation (21) of the surface waves is a semi-infinite hot plasma bounded by a dielectric is associated with a number of mathematical difficulties, because of that we present here only the results for the dispersion and damping rate of high-frequency surface waves obtained in the limits of different approximations. On one hand the approximations which should be made are related to the analysis of the dispersion relation separately in the regions where the waves are longitudinal (electrostatic approximation) (region I in Fig. 3) and transverse-longitudinal (region II in Fig. 3). (Figure 3 shows the dispersion curve of high-frequency surface waves in cold semi-infinite plasma

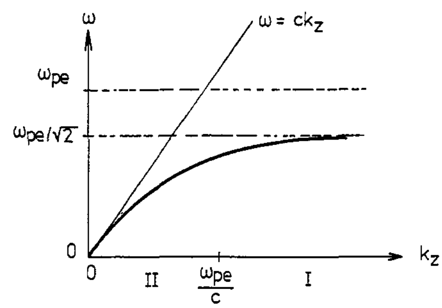

FIG. 3.—Dispersion curve of high-frequency surface waves on the cold plasma–vacuum interface.

bounded by vacuum (SCHUMANN, 1950, ECONOMOU, 1969). On the other hand, in analysing the dispersion relation every author accounts for the different accuracy, the contribution of the various ranges of the arguments

$$x_\alpha = \frac{\omega}{\sqrt{2}|\mathbf{k}|v_{T\alpha}}$$

($v_{T\alpha} = (T_\alpha/m_\alpha)^{1/2}$ are the particles' thermal velocities) of the function

$$Z(x_\alpha) = 2i \exp(-x_\alpha^2) \int_{-\infty}^{ix_\alpha} dt \exp(-t^2) \tag{42}$$

in the expressions for plasma dispersion functions. Figure 4 illustrates the ranges of the variable k_x related to the corresponding ranges of x_α, where the asymptotic

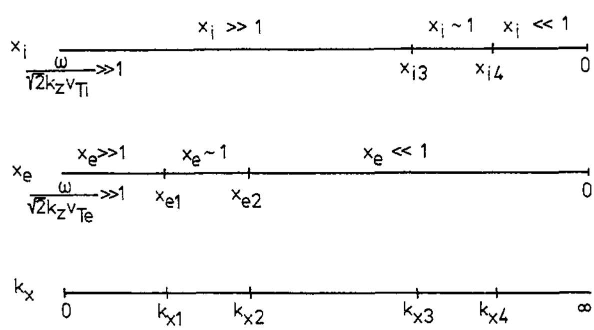

FIG. 4.—Relations among the variables x_i, x_e and k_x used in calculating the spectrum and damping rate of high-frequency surface waves.

and table values of the function $Z(x_\alpha)$ (SHIVAROVA, 1974) are applicable. BARR and BOYD (1972) by using only the asymptote of $Z(x_\alpha)$ for $x \gg 1$, in the limiting case $\omega_{pe}{}^2 v_{Te}{}^2/(\omega^2 c^2) \ll 1$, have been obtained the following expression for the spectrum of high-frequency surface waves in the whole frequency domain of their propagation

$$\omega^2 k_z^2 = \left[k_z^2 + \frac{1}{3v_{Te}^2}\frac{\omega^2}{\omega_{pe}^2}(\omega_{pe}^2-\omega^2)\right]^{1/2} \times\left\{\omega^2\left(k_z^2+\frac{\omega_{pe}^2-\omega^2}{c^2}\right)^{1/2} - (\omega_{pe}^2-\omega^2)\left(k_z^2-\frac{\omega^2}{c^2}\right)^{1/2}\right\}. \quad (43)$$

This result can be compared directly with the wave spectra obtained by KAW and MCBRIDE (1970), CLEMMOW and ELGIN (1974) by means of a fluid plasma model. One may see that the results found on the basis of both kinetic and hydrodynamic description, coincide in the electromagnetic region (II in Fig. 3) and diverge in the electrostatic one (I in Fig. 3) as the difference reveals in the terms which take into account the electron thermal motion.

In electrostatic limit $v_{ph} \ll c$ (region I in Fig. 3) the spectrum and damping rate of high-frequency surface waves in semi-infinite bounded by vacuum plasma are determined by the formulae:

$$\operatorname{Re} D(\omega, k_z) = 0, \quad (44)$$

$$\gamma_\omega = -\frac{\operatorname{Im} D(\omega, k)}{\dfrac{\partial \operatorname{Re} D(\omega, k_z)}{\partial \omega}}, \quad (45)$$

where $\operatorname{Re} D(\omega, k_z)$ and $\operatorname{Im} D(\omega, k_z)$ are the real and imaginary part of the dispersion relation (22), respectively. Bearing in mind that in the regions of applicability of $Z(x_\alpha)$'s asymptotes $\operatorname{Re} \varepsilon^l(\omega, k) \gg \operatorname{Im} \varepsilon^l(\omega, k)$, there were obtained by ALIEV *et al.* (1972), SHIVAROVA (1974) the following results:

$$\omega = \frac{\omega_{pe}}{\sqrt{2}}\left\{1 + C_1 k_z r_{De}\left[1 + O\left(\frac{v_{Ti}}{v_{Te}}\right)\right]\right\}, \quad (46)$$

$$\gamma = -\frac{\omega_{pe}}{\sqrt{2}} k_z r_{De} C_2 \left[1 - O\left(\frac{v_{Ti}}{v_{Te}}\right)\right], \quad (47)$$

where $C_1 = 1.22$ and $C_2 = 0.176$. These values of the coefficients $C_{1,2}$ are obtained by taking into account the actual contributions of all the regions of the argument x_α (of the function $Z(x_\alpha)$) in the integrating procedure. Because of approximations made in analysing equation (22), the values for $C_{1,2}$ obtained by GUERNSEY (1969), ROMANOV (1964a,c), CHENG and HARRIS (1969), TRONG (1972), GINZBURG and RUKHADZE (1975), FUSE and ICHIMARU (1975) discern from those indicated here. When we neglect the plasma spatial dispersion from expression (45) one obtains the frequency (11a) of surface oscillations of a cold semi-infinite bounded by vacuum plasma. The influence of thermal electron motion on the dispersion of electrostatic high-frequency surface waves in semi-infinite plasma manifests itself in broadening of the wave frequency range ($\omega > \omega_{pe}/\sqrt{2}$ or $\omega > \omega_{pe}/\sqrt{1+\varepsilon}$ in the case of plasma bounded by dielectric; compare with Fig. 3). In contrast to the bulk waves, the term taking into account the electron thermal motion in surface wave spectrum contains the first power of k_z. While the Landau damping for bulk waves is exponentially small, the surface wave damping rate owing to the thermal electron motion is considerable even though for small values of k_z. This is due to the fact that the surface wave field exponentially decaying into the plasma depth (ATANASSOV *et al.*, 1974).

$$E_z(x, k_z, \omega) = -\frac{ik_z}{\pi}\int_{-\infty}^{\infty} \frac{dk_x \exp(ik_x x)}{|\mathbf{k}|^2 \varepsilon^l(\omega, k)} E_x(x=0, k_z, \omega) \tag{48}$$

contains Fourier harmonics with $k_x \in (0, \infty)$, i.e. not only weakly damped oscillations, $kr_{De} < 1$, but also relatively highly damped short-wavelength oscillations, $kr_{De} > 1$ (ROMANOV, 1964a, GORBATENKO and KURILKO, 1964, GUERNSEY, 1969). A cause for the strong damping of the surface waves is the Cherenkov dissipation of energy by the plasma electrons and, namely, the short-wavelength components of surface wave field Fourier expansion are those waves which interact most effectively with the electrons (GORBATENKO and KURILKO, 1964, ROMANOV, 1968). The specific damping manifests itself most strongly in the region $\varepsilon(\omega) \approx -\varepsilon$. In contrast to the bulk waves, the group velocity of the surface waves in electrostatic limit

$$v_{gr} \equiv \frac{\partial\omega}{\partial k_z} \approx \frac{\sqrt{2}v_{Te}}{1+\varepsilon}$$

practically does not depend on the wave vector.

In the limiting case

$$\frac{c^2 k_z^2}{|\varepsilon(\omega)|} \ll \omega_{pe}^2,$$

i.e. for the extremely long-wavelength surface waves when the transverse character of the waves reveals (region II in Fig. 3), from dispersion relation (21) ROMANOV (1964a) gets the following expression:

$$\omega = \frac{ck_z}{\sqrt{\varepsilon}}\left\{1 - \frac{c^2 k_z^2}{2\omega_{pe}^2} - \frac{1}{\sqrt{2\pi\varepsilon}}\frac{c}{v_{Te}}\left(\frac{ck_z}{\omega_{pe}}\right)^3 \ln 2\varepsilon \left(\frac{v_{Te}}{c}\right)^2 \left(\frac{\omega_{pe}}{ck_z}\right)^2\right\}. \tag{49}$$

Here the wave dispersion is determined mostly from the behaviour of $\varepsilon^{tr}(\omega, k)$, because of that the wave spectrum is not practically influenced by the thermal electron motion, but the damping rate depends on $\varepsilon^l(\omega, k)$.

According to ROMANOV (1968) other mechanisms of particle reflection at the boundary plane (for instance diffuse reflection) do not produce qualitatively new results for surface wave propagation. With diffuse scattering of the electrons and weak spatial dispersion, the wave damping rate depends linearly on the electron thermal velocity (MIROSHNICHENKO, 1966), and is somewhat greater than in the case of pure specular reflection. The greater damping is explained (REZVOV, 1970b) by the stronger interaction of electrons with the plasma boundary. The electrons lose the entire energy transferred to them by the surface wave to the boundary surface, whereas in the mirror reflection this transfer of energy is considerably weaker. In a recent paper, however, HOPPS and WALDRON (1977), who considered the same problem using a new approach, stated that the strong Landau damping of the high-frequency surface mode in the case of specular reflection is not as severe for a boundary with diffuse scattering. In any case the short-wavelength surface waves remain highly damped even with the inclusion of diffuse scattering. The wave dispersion of the surface mode with the generalized boundary condition (9) is calculated by GORBATENKO and KURILKO (1964), REZVOV (1970c; 1971a; 1975, 1976a,b).

The study of the propagation of surface modes on the basis of microscopic theory (Fermi statistics at arbitrary values of the chemical potential) is examined by REZVOV (1971b,c).

Since in investigating the wave propagation along a plasma column, one usually uses the approximations of thick or thin cylinder, as well as treats the waves as either electrostatic or electromagnetic, it is necessary to point out the meaning and limits of the applicability of these approximations (SHIVAROVA and ZHELYAZKOV, 1977). As long as the high-frequency surface waves exist at frequencies $\omega < \omega_{pe}$, from condition (13) for the validity of the electrostatic approximation, it follows that the electrostatic limit may be used when

$$\frac{\omega_{pe}}{ck_z} < 1 \tag{50}$$

whereas at

$$\frac{\omega_{pe}}{ck_z} > 1 \tag{51}$$

the surface modes are transverse–longitudinal and it is necessary to treat them electromagnetically. In the case of a thick cylinder, $k_\perp R > 1$; when $(\omega_{pe}R)/c < 1$, according to (50) the electrostatic approximation is valid for $k_z R > 1$. As $(\omega_{pe}R)/c > 1$ the electrostatic treatment is applicable if $k_z R > (\omega_{pe}/c)R$; while if $k_z R < (\omega_{pe}/c)R$, it is necessary to consider the waves electromagnetically. In the case of a thin cylinder, $k_\perp R < 1$; as $(\omega_{pe}R)/c < 1$ in accordance with (50) it follows that the electrostatic approximation is valid for $(\omega_{pe}R)/c < k_z R < 1$ whereas if $k_z R < (\omega_{pe}R)/c$ it is necessary (see (51)) to use an electromagnetic approach. The above conditions are illustrated in Fig. 5.

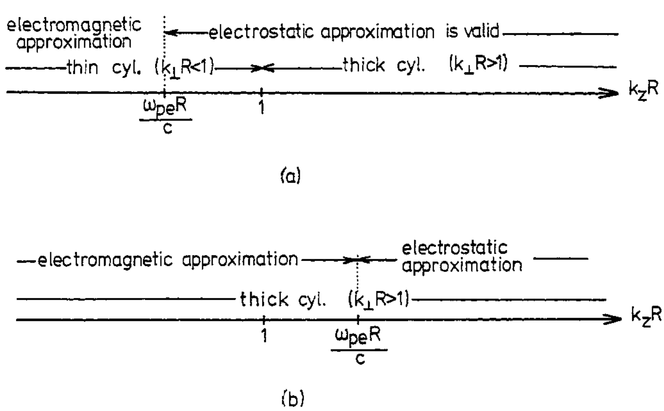

FIG. 5.—Scheme for the applicability of the electrostatic/electromagnetic approximations in cylindrical plasmas.

Here we shall not discuss the results on the spectra of the surface waves (obtained on the basis of a macroscopic plasma model) in cylindrical plasmas. These results merely supplement the pioneer work by TRIVELPIECE and GOULD

(1959). We only note that analytical expressions for the spectra and damping rates, due to the collisions, for the cases of thick (Fig. 5b) and thin (Fig. 5a) cylinder with electromagnetic consideration are obtained by KONDRATENKO (1976), SHIVAROVA and ZHELYAZKOV (1977). Furthermore, the influence of the thermal electron motion on the dispersion of the longitudinal waves in warm plasma is calculated (in the hydrodynamical approach) by SHIVAROVA and ZHELYAZKOV (1977).

The kinetic dispersion relation (36) of the high-frequency surface modes propagating along a hot plasma column is analysed in (ATANASSOV *et al.*, 1976). The spatial dispersion is taken into account only in the imaginary part of (36), in order to determine the Cherenkov's wave damping rate. In that case, dispersion relation (36) can be presented in the form

$$D(\omega, k_z) \equiv \frac{k_\perp}{\varepsilon(\omega)} \frac{I_0(k_\perp R)}{I_1(k_\perp R)} + \frac{k_\perp^0}{\varepsilon} \frac{K_0(k_\perp^0 R)}{K_1(k_\perp^0 R)} - id = 0, \tag{52}$$

where the real part of (52) is the dispersion relation of the waves in a cold plasma column (39) and the imaginary part

$$d = \frac{1}{k_\perp} \sum_{n=-\infty}^{\infty} (-1)^n \frac{k_z^2}{k_n^2} \left[k_n^2 - \frac{\omega^2}{c^2} \varepsilon^{tr}(\omega) \right] \frac{\operatorname{Im} \varepsilon^l(\omega, k_n)}{|\varepsilon^l(\omega, k_n)|^2} A_n \tag{53}$$

determines the wave damping owing to the electron thermal motion.

In the spectrum of a thick cylinder, $k_\perp R > 1$ (Fig. 5b) the wave spectrum and the spatial damping rate are given by

$$k_z = \frac{\omega}{c} \left(\frac{\varepsilon\, |\varepsilon(\omega)|}{|\varepsilon(\omega)| - \varepsilon} \right)^{1/2} \left\{ 1 + \frac{c}{2R\omega} \frac{1}{(|\varepsilon(\omega)| - \varepsilon)^{1/2}} \right\} \tag{54}$$

$$\gamma_{\mathbf{k}} = 2\sqrt{\frac{2}{\pi}} \left(\frac{\omega_{pe}}{\omega} \right)^2 \frac{v_{Te}}{c} \frac{\omega}{c} \varepsilon^{5/2} \frac{|\varepsilon(\omega)|^{1/2}}{[|\varepsilon(\omega)| + \varepsilon][|\varepsilon(\omega)| - \varepsilon]^2}$$
$$\times \left[1 + 4 \frac{c}{\omega R} \frac{|\varepsilon(\omega)|}{[|\varepsilon(\omega)| - \varepsilon]^{1/2}} \left(\frac{v_{Te}}{c} \right)^2 \right] \tag{55}$$

on the condition that $|\varepsilon(\omega)| > \varepsilon$. In obtaining (55), we have used the relation

$$\lim_{\Delta \to +0} \Delta \sum_{n=0}^{\infty} f(n\Delta) = \int_0^\infty \mathrm{d}x f(x) \tag{56}$$

with $\Delta = \dfrac{\pi v_{Te}}{R\omega} \ll 1$, and used the respective asymptotic values of the function $Z(x_\alpha)$ at small ($x_\alpha \ll 1$) and large ($x_\alpha \gg 1$) values of its argument and replaced $|\varepsilon^l(x_\alpha)|^2$ by its value in the range of maximum wave damping ($x_\alpha \sim 1$), assuming $\operatorname{Re} \varepsilon^l \gg \operatorname{Im} \varepsilon^l$. It follows from (54) and (55) that at a fixed frequency ω the wavelength in the plasma cylinder is shorter than it would be in a semi-infinite plasma, and the finite plasma size increases the wave damping.

In the case of a thin cylinder, $k_\perp R < 1$, (Fig. 5a) the phase velocity and the spatial damping rate of the high-frequency surface waves are (ATANASSOV *et al.*, 1976)

$$v_{ph} \equiv \frac{\omega}{k} = \frac{\omega_{pe} R}{\sqrt{\varepsilon}} \left(\ln \frac{1}{\sqrt{k_z R}} \right)^{1/2} \tag{57}$$

$$\gamma_k = \frac{\varepsilon}{\sqrt{2\pi} R} \frac{v_{Te}}{v_{ph}} \left(\frac{\omega}{\omega_{pe}} \right)^2 \frac{1}{\ln (1/k_z R)} \ln \frac{\omega_{pe}}{\omega}. \tag{58}$$

The latter expression is obtained at frequencies $\omega \ll \omega_{pe}$ by using the same approximations as in obtaining formula (55). It is interesting to note that because of the approximations used in analysing dispersion relation (36) formulae (57) and (58) coincide with the results found by KONDRATENKO (1972, 1976).

The derivation of more exact analytical expressions for high-frequency surface wave propagation in a plasma column is possible by investigating the electrostatic limit (38) of the dispersion relation (36) (SHIVAROVA and ZHELYAZKOV, 1978a). The surface waves have longitudinal character in a wide frequency domain as $(\omega_{pe} R)/c < 1$ (Fig. 5a) — an inequality which is satisfied for gas-discharge plasmas at low pressures.

The wave spectrum in long-wavelength approximation ($v_{ph} \gg v_{Te}$) (i.e. weak plasma spatial dispersion), obtained by calculating the infinite series in the real part of the dispersion relation (38) with the help of asymptotes of $Z(x_\alpha)$ for $x_\alpha \gg 1$ and $x_\alpha \ll 1$ (Fig. 4), in the case of thick ($k_z R > 1$) and thin ($k_z R < 1$) cylinder has the form (SHIVAROVA and ZHELYAZKOV, 1978a):

$$\omega = \frac{\omega_{pe}}{\sqrt{1+B_0}} \left\{ 1 + \frac{1}{2} |G_0| \left[\left(\frac{3}{B_0} \right)^{1/2} + \frac{B_0}{1+B_0} + 3 \frac{r_{De}}{R} \frac{(1+B_0)}{|G_0|} \right] k_z r_{De} \right\} \tag{59}$$

where $B_0 = |G_0| \dfrac{I_0(k_z R)}{I_1(k_z R)}$ with

$$G_0 = -\varepsilon \frac{K_1(k_z R)}{K_0(K_z R)} = \begin{cases} -\varepsilon \left(1 + \dfrac{1}{2k_z R} \right) & \text{as } k_z R > 1 \\ -\dfrac{\varepsilon}{k_z R} \left[\ln \left(\dfrac{1}{k_z R} \right) \right]^{-1} & \text{as } k_z R < 1. \end{cases} \tag{60}$$

The thermal corrections to the wave spectrum in a cold plasma obtained by means of a kinetic model are greater than those found on the basis of a fluid model (SHIVAROVA *et al.*, 1975).

The damping rate of longitudinal surface waves, owing to the electron thermal motion, is (SHIVAROVA and ZHELYAZKOV, 1978a)

$$\gamma = -\frac{\omega_{pe}}{\sqrt{1+B_0}} \frac{|G_0| B_0}{\sqrt{2\pi}(1+B_0)} k_z r_{De} \times \left\{ \frac{3}{\sqrt{e}} \frac{(1+B_0)^{3/2}}{(3+4B_0)^2} + \frac{1}{(1+B_0)^{1/2}} \left(\ln \sqrt{2+B_0} - \frac{1}{2} \frac{1+B_0}{2+B_0} \right) \right\}, \tag{61}$$

where $|G_0|$ and B_0 have the above mentioned values for the cases of thick and thin cylinder respectively (see (60)). One may see that the influence of electron thermal motion on the wave spectrum and damping rate is greater for a thick cylinder. The dependence of geometrical factors on k_zR provides a monotonic variation of the frequency spectrum and the wave damping rate with the variation of the wavelength and the radius of the plasma column.

5. LOW-FREQUENCY SURFACE WAVES IN SEMI-INFINITE PLASMA AND PLASMA CYLINDER

Here, as before, we shall consider only the results for the propagation of low-frequency surface waves obtained on the basis of kinetic dispersion relations (22) and (38) assuming Maxwellian distributions for the particles of a nonisothermal plasma. The wave frequency range is determined by the following inequalities:

$$v_{T1} \ll v_{ph} \equiv \frac{\omega}{k_z} \ll v_{Te}. \tag{62}$$

In the exact analysis of the dispersion relation in the low-frequency domain it is important to take into account the contributions of all values of the arguments

$$x_\alpha = \frac{\omega}{\sqrt{2}\,|\mathbf{k}|\,v_{T\alpha}}$$

of function $Z(x_\alpha)$ determining the form of $\varepsilon^l(\omega, k)$. Figure 6 illustrates the

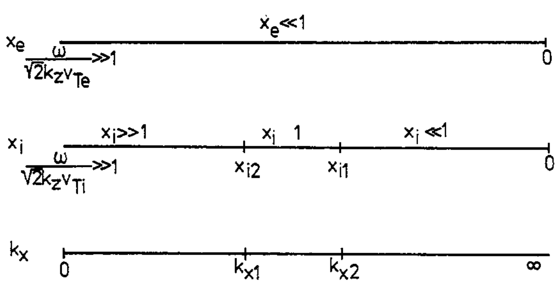

FIG. 6.—Relations among the variables x_e, x_i and k_x used in calculating the spectrum and damping rate of low-frequency surface waves.

domain for variable k_x in dispersion relation (22) confronted with the ranges of x_α, in which both the asymptotic and table values of function $Z(x_\alpha)$ are applicable (ATANASSOV *et al.*, 1974).

It follows from (62) that the thermal motion of the electrons specifies the conditions for propagation of the low-frequency surface modes. In the case of a semi-infinite plasma (with cold ions) from dispersion relation (22) one obtains the following wave dispersion (ROMANOV, 1964b, 1965; KONDRATENKO, 1965; ATANASSOV *et al.*, 1974; GINZBURG and RUKHADZE, 1975):

$$\omega_0{}^2 = \frac{\omega_{pi}{}^2}{1 + \dfrac{1}{2k_z{}^2 r_{De}{}^2} + \sqrt{\varepsilon^2 + \dfrac{1}{4k_z{}^4 r_{De}{}^4}}} \tag{63}$$

graphically presented (for bounded by vacuum plasma) in Fig. 7 (MCBRIDE and KAW, 1970). In long-wavelength limit ($k_z{}^2 r_{De}{}^2 \ll 1$) the dispersion of the low-

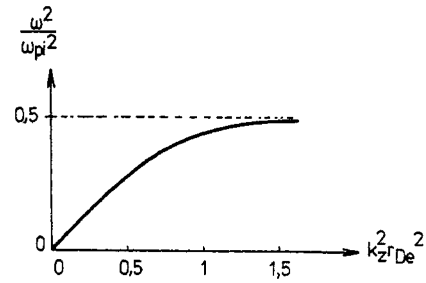

FIG. 7.—Dispersion curve of low-frequency surface waves on the nonisothermal plasma–vacuum interface.

frequency surface modes is very close to that of the bulk ion acoustic waves

$$\left(v_{ph} \sim v_s = \sqrt{\frac{T_e}{M}}\right)$$

and at very short wavelengths the wave spectrum reduces to localized oscillations at frequency $\omega = \omega_{pi}/\sqrt{2}$ in the case of plasma bounded by vacuum or $\omega = \omega_{pi}/\sqrt{1+\varepsilon}$ as it is bounded by dielectric. The derivation of a formula for the damping rate of the low-frequency surface waves is associated with the calculation of the integral

$$I = -\left(\frac{2}{\pi}\right)^{1/2} k_z \frac{\omega}{v_{Te}} \frac{\omega_{pe}^2}{v_{Te}^2} \frac{1}{\left(\frac{\omega_{pe}^2}{\omega^2} - 1\right)^2} \int_0^\infty \frac{\mathrm{d}k_x}{\sqrt{k_x^2 + k_z^2}[k_x^2 + bk_z^2]^2}. \tag{64}$$

Here b is given by (15) and it is related to the imaginary part (see formula (45)) of dispersion relation (22). The general expression for low-frequency surface wave damping rate due to the electron thermal motion is (ATANASSOV *et al.*, 1974):

$$\gamma = -\frac{1}{2}\left(\frac{2}{\pi}\frac{m}{M}\right)^{1/2} \frac{\frac{\omega_0^4}{\omega_{pi}^4}\left(\frac{\omega_{pi}^2}{\omega_0^2} - 1\right)^{3/2}}{2k_z^2 r_{De}^2\left(\frac{\omega_{pi}^2}{\omega_0^2} - 1\right) - 1} \left\{\left[k_z^2 r_{De}^2\left(\frac{\omega_{pi}^2}{\omega_0^2} - 1\right) - 1\right]^{1/2}\right.$$

$$\left. - \left[k_z^2 r_{De}^2\left(\frac{\omega_{pi}^2}{\omega_0^2} - 1\right) - 2\right] \arctan\left[k_z^2 r_{De}^2\left(\frac{\omega_{pi}^2}{\omega_0^2} - 1\right) - 1\right]^{-1/2}\right\} \omega_{pi}. \tag{65}$$

The damping rate, obtained in (ABU-ASALI *et al.*, 1975; GINZBURG and RUKHADZE, 1975), is greater than (65) since in calculating the expression (64) the authors have neglected the energy transfer between the thermal plasma electrons and short-wavelength Fourier components, $kr_{De} \gg 1$ (in the wave Fourier components, together with the waves which supply energy to the electrons, there exist also waves which gain energy at the expense of the particles; but the former is the prevailing process). Expressions (63) and (65) are valid if only $|\mathrm{Re}\, \varepsilon^l(\omega, k)|^2 \gg |\mathrm{Im}\, \varepsilon^l(\omega, k)|^2$. This inequality imposes certain restrictions on the wavelength of the long-wavelength surface ion acoustic waves (ALIEV *et al.*, 1972; TRONG, 1972; ATANASSOV *et al.*, 1974):

$$k_z^4 r_{De}^4 > \frac{1}{\varepsilon^2}\left(\frac{m}{M}\right)^{1/2}. \tag{66}$$

The effect of ion thermal motion on surface wave propagation is considered both in long-wavelength ($k_z r_{De} < 1$) and short-wavelength ($k_z r_{De} > 1$) approximations. In the case of a semi-bounded by vacuum plasma, the following results have been obtained (TRONG, 1972; ATANASSOV *et al.*, 1974):

$$\omega = k_z v_s\left\{1 - \frac{1}{2}k_z^2 r_{De}^2 + O\left(\frac{T_i}{T_e}\right) + O\left[(k_z r_{De})^4\left(\frac{T_i}{T_e}\right)^{1/2}\right]\right\} \quad k_z r_{De} < 1 \tag{67}$$

$$\gamma = -k_z v_s\left\{\left(\frac{\pi}{8}\frac{m}{M}\right)^{1/2} + O\left[\frac{T_i}{T_e}(k_z r_{De})^{-4}\left(\frac{m}{M}\right)^{1/2}\right]\right\} \tag{68}$$

$$\omega = \frac{\omega_{pi}}{\sqrt{2}}\left(1 - \frac{1}{8k_z^2 r_{De}^2} + 1.22 k_z r_{Di}\right) \quad k_z r_{De} > 1 \tag{69}$$

$$\gamma = -\frac{\omega_{pi}}{\sqrt{2}}\left[\frac{1}{6\pi^{1/2}}\left(\frac{m}{M}\right)^{1/2}\frac{1}{k_z^3 r_{De}^3} + 0.176\, k_z r_{Di}\right]. \tag{70}$$

Aside from the numerical coefficients, expression (70) has been found by ROMANOV (1965), and the damping rate (68) by FUSE and ICHIMARU (1975). Formula (70) contains two terms causing the damping of the surface waves. The former term is related to the mechanism of energy transfer between the wave and chaotic moving electrons, and the latter term is analogous to expression (47) and it is determined by the mechanism of wave–ion energy exchange (ROMANOV 1965, MCBRIDE and KAW 1970). As one would expect, the ion thermal motion influences on the spectrum and damping rate of short wavelength low-frequency (ion) waves, whereas the spectrum and damping rate of the long wavelength low-frequency (ion acoustic) modes are mainly determined by the electron thermal motion. The terms of γ due to the thermal electron motion coincide, in structure, with the corresponding terms in the damping rates of the bulk ion acoustic and ion plasma waves (ATANASSOV *et al.*, 1974).

The spectra and damping rates of low-frequency surface waves both for thick ($k_z R > 1$) and thin ($k_z R < 1$) plasma cylinder one may obtain from dispersion relation (38) in the frequency domain (62) (SHIVAROVA and ZHELYAZKOV, 1968b). In order to be surface the wave should satisfy the condition (15), i.e. in the case of

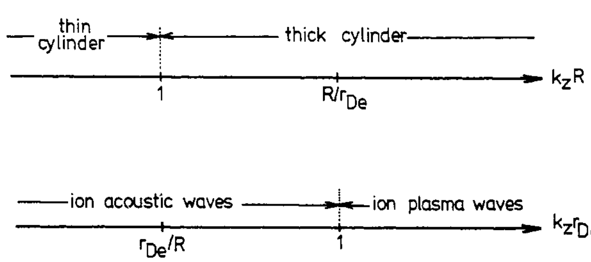

FIG. 8.—Scheme for the possibility of propagation of ion acoustic/ion plasma waves in cylindrical nonisothermal plasmas.

thick cylinder, $k_zR > 1$, surface waves exist at $\kappa R \gg 1$, (κ given by (14)) as in that region both short-wavelength ($k_z r_{De} > 1$) and long-wavelength ($k_z r_{De} < 1$) modes propagate (Fig. 8). When $\kappa R \ll 1$ one may expect the propagation of surface ion acoustic waves along a thin cylinder, $k_zR < 1$. By using the asymptotes of the function $Z(x_\alpha)$ (Fig. 6) one gets the following expressions for the spectra and damping rates of short and long wavelength low-frequency surface waves propagating along a thick plasma column ($k_zR > 1$) bounded by vacuum:

$$\omega = \frac{\omega_{pi}}{\sqrt{2}}\left\{1 - \frac{1}{8}\left(\frac{1}{k_z^2 r_{De}^2} + \frac{4}{k_zR}\right) + k_z R_{Di}\left[1.60 + \left(\frac{5}{k_z^2 r_{De}^2} + \frac{2.25}{k_zR}\right)k_z R_{Di}\right]\right\} \quad (71)$$

$$\gamma = -\frac{\omega_{pi}}{\sqrt{2}}\left\{\left(\frac{1}{\pi}\frac{m}{M}\right)^{1/2}\frac{1}{2}\frac{1}{k_z r_{De}}\left(\frac{1}{3}\frac{1}{k_z^2 r_{De}^2} + \frac{1}{32}\frac{1}{k_zR}\right) + k_z r_{Di}\left(0.145 + 0.538\frac{k_z^2 R_{Di}^2}{k_zR}\right)\right\} \quad (72)$$

$$\omega = k_z v_s\left\{1 - \frac{1}{2}k_z^2 r_{De}^2 - \frac{1}{2}k_z^4 r_{De}^4\left(1 + \frac{2}{k_zR}\right) + k_z r_{Di}\left[k_z^4 r_{De}^4 + \frac{3}{2}\frac{r_{Di}}{r_{De}}\frac{1}{k_z r_{De}}\left(1 + \frac{1}{k_zR}\right)\right]\right\} \quad (73)$$

$$\gamma = -k_z v_s\left\{\frac{1}{2}\left(\frac{\pi}{2}\frac{m}{M}\right)^{1/2}(1 - 2k_z^2 r_{De}^2)\left(1 + \frac{1}{4k_zR}\right\} + O\left[\frac{T_i}{T_e}(k_z r_{De})^{-4}\frac{m}{M}\right]\right\}. \quad (74)$$

The spectra and damping rates due to the electron thermal motion have been obtained by calculating the infinite series in dispersion relation (38), while the terms taking into account the ion contribution in the wave damping rates have been determined on the basis of asymptotic formula (56). The finite plasma radius increases the wave number and the wave damping rate with respect to those in semi-infinite plasma, and also increases the influence of the ion thermal motion on the wave spectrum and damping rate.

The propagation of surface ion acoustic waves along a thin plasma cylinder ($k_zR < 1$) is described by the formulae

$$\omega = k_z v_z\left[1 - \frac{r_{De}^2}{R^2}\frac{1}{\ln(1/k_zR)}\right]\left[1 + \frac{1}{2}\frac{k_z r_{Di}}{k_zR\ln(1/k_zR)}(1 + 6k_z r_{De})\right] \quad (75)$$

$$\gamma = -k_z v_s\left[\frac{1}{4}\left(\frac{\pi}{2}\frac{m}{M}\right)^{1/2}\frac{R^2}{r_{De}^2}\ln\frac{1}{k_zR} + 0.245\frac{k_z r_{De} r_{Di}}{R\ln(1/k_zR)}\right]. \quad (76)$$

One may see from (76) that for a thin plasma cylinder the wave damping rate owing to the electron thermal motion is greater than that for a thick plasma column.

6. CONCLUDING REMARKS

The high- and low-frequency surface waves on sharply bounded plasmas considered here are the simplest waves on a plasma–dielectric interface. The results reviewed in this paper should be extended in two directions: (1) by including magnetic fields and (2) by taking into account the inhomogeneous plasma layer near the boundary. One expects that the magnetic field does not change the general results of the kinetic theory presented, but will allow a wider variety of waves to be considered, and will give rise to additional mechanism of wave dissipation.

The influence of plasma inhomogeneity on the surface wave spectra and damping rates, and the investigations of some new effects associated with it will be subject of a subsequent review paper.

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