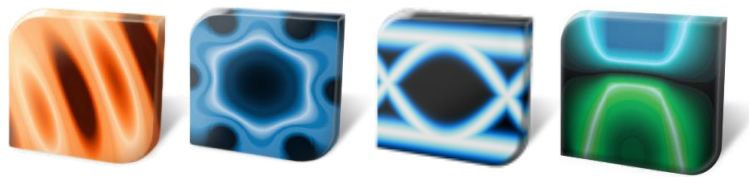


# Efficient Optical Modeling of Graphene

## in FDTD Solutions and MODE Solutions

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LUMERICAL SOLUTIONS INC.



# Outline

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## Introduction

- Lumerical's optical simulation products
- Physical properties of graphene

## New Graphene Modeling Workflow

- New surface conductivity model
- New surface geometric primitive
- Simple example: transmission through a graphene sheet

## Application Examples

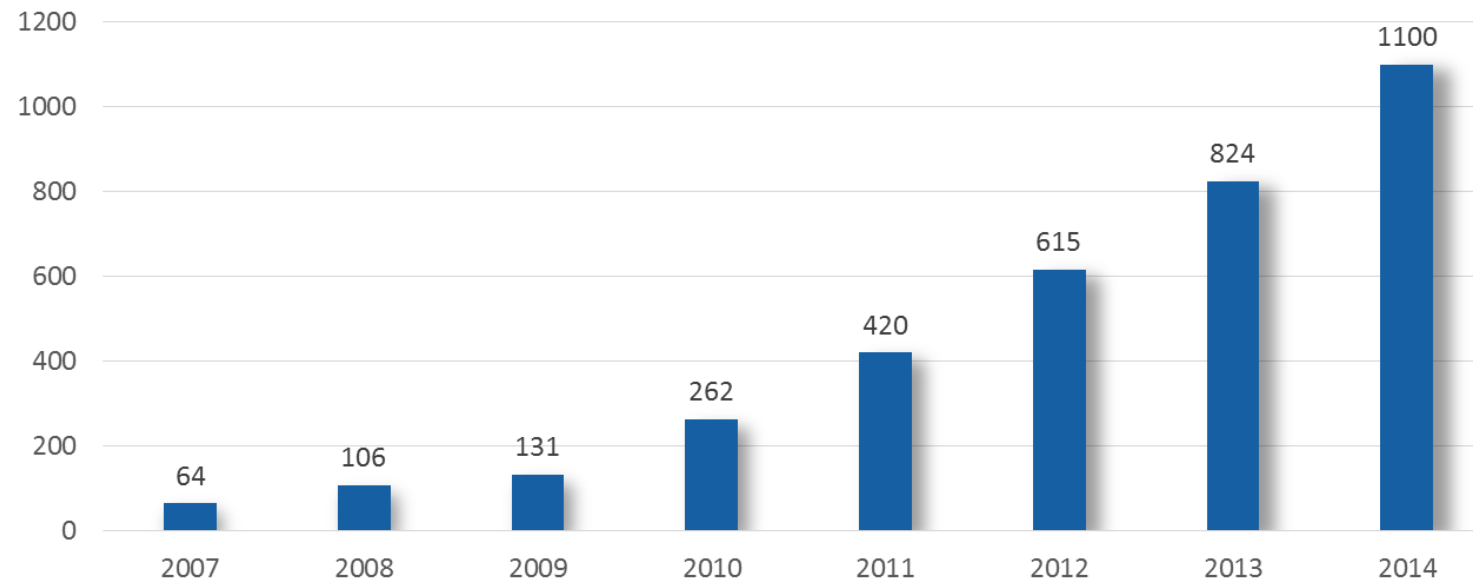
- Tunable THz metamaterial (FDTD)
- Graphene waveguide modulator (FDTD and MODE)
- Exciting a surface plasmon mode (FDTD and MODE)

## Summary

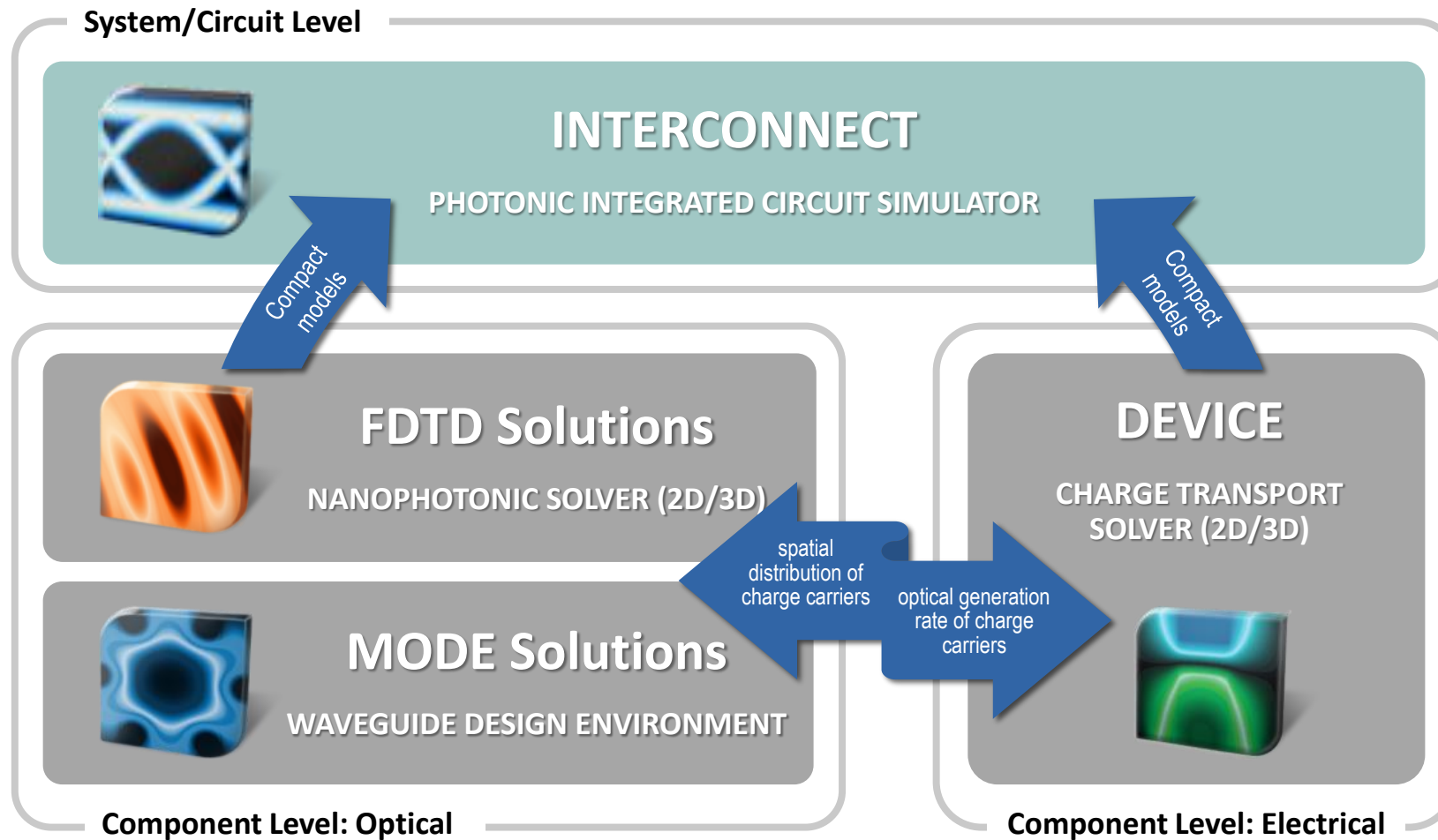
# Lumerical Solutions

Lumerical empowers R&D professionals with best-in-class design tools and services to support the creation of better photonic technologies

Patents and Publications Referencing Lumerical



# Our Products



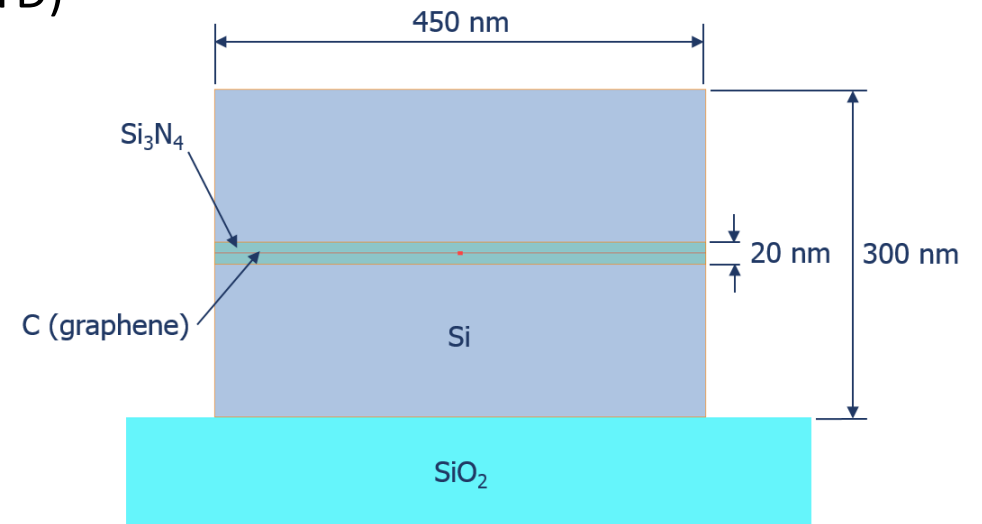
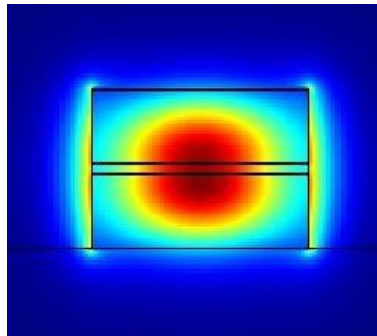
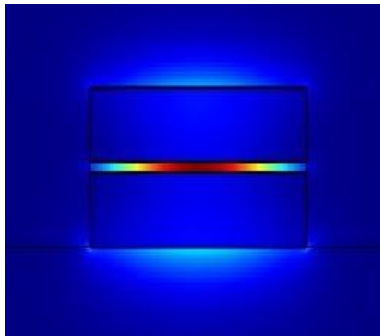
# Lumerical's Optical Solvers

## MODE Solutions

- Finite difference eigenmode solver (FDE)
- Bidirectional eigenmode expansion solver (EME)
- 2.5-D variational finite difference time domain solver (varFDTD)

## FDTD Solutions

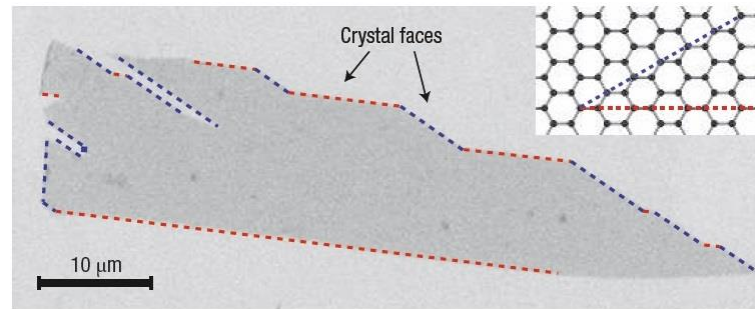
- 2-D and 3-D finite difference time domain solver (FDTD)



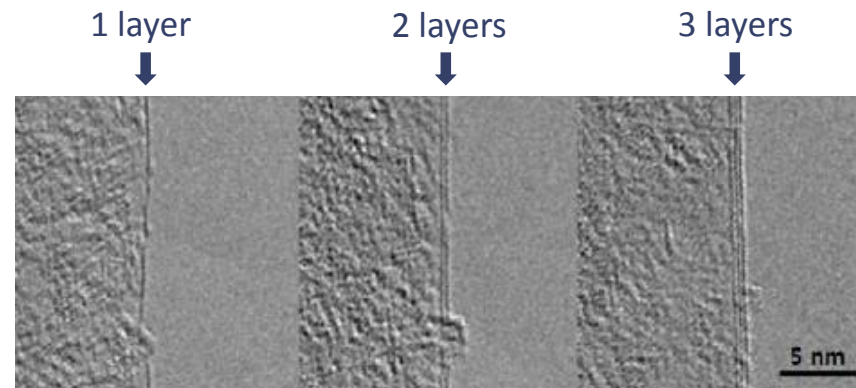
# Graphene's Physics

## What makes graphene different from other optical materials?

- It is a truly 2-D material:



[1] A.K. Geim and K.S. Novoselov, "The rise of graphene," *Nature Materials*, vol. 6, pp. 183-191, 2007.



[2] S. Bae *et al.*, "Roll-to-roll production of 30-inch graphene films for transparent electrodes," *Nature Nanotechnology*, vol. 5, pp. 574-578, 2010.

# Graphene's Physics

Graphene is usually modeled using a surface conductivity:

$$\sigma(\omega, \Gamma, \mu_c, T) = -\frac{ie^2(\omega + i2\Gamma)}{\pi\hbar^2} \left[ \frac{1}{(\omega + i2\Gamma)^2} \int_0^\infty \epsilon \left( \frac{\partial f_d(\epsilon)}{\partial \epsilon} - \frac{\partial f_d(-\epsilon)}{\partial \epsilon} \right) d\epsilon - \int_0^\infty \frac{f_d(-\epsilon) - f_d(\epsilon)}{(\omega + i2\Gamma)^2 - 4(\epsilon/\hbar)^2} d\epsilon \right]$$

$f_d(\epsilon) = (e^{(\epsilon - \mu_c)/(k_B T)} + 1)^{-1}$

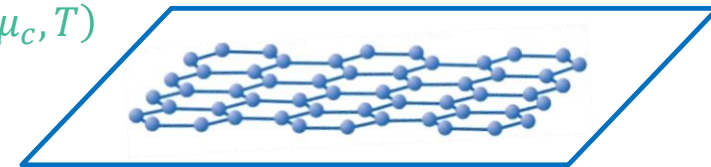
↑↑  
intrabandinterband

[3] G. Hanson, "Dyadic Green's functions and guided surface waves for a surface conductivity model of graphene", *Journal of Applied Physics*, vol. 103 (64302), 2008.

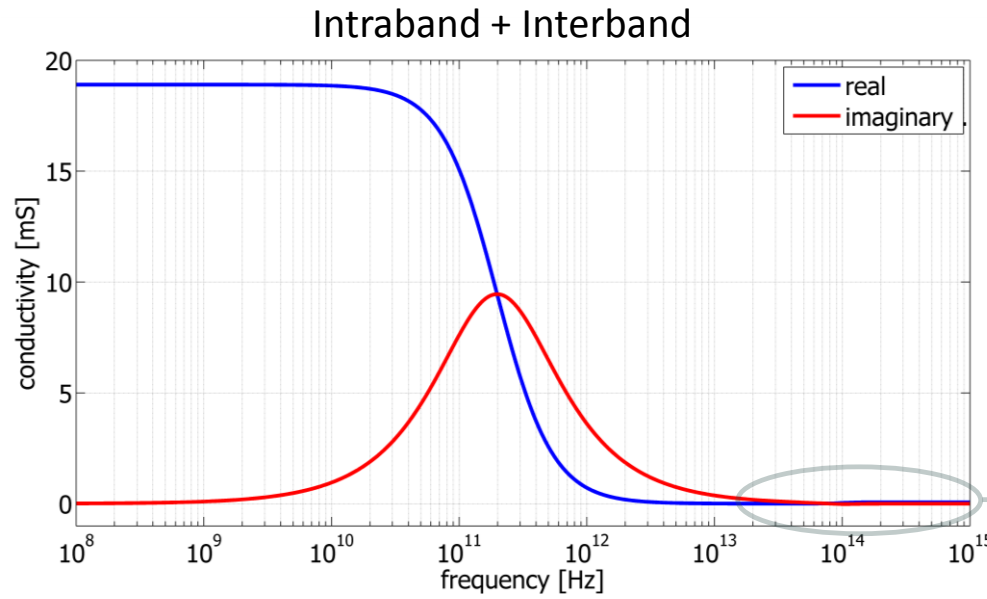
**The new workflow is based on a surface conductivity model**

- Two new components have been added to the UI in FDTD and MODE:
  - a new **surface conductivity model**,
  - and a **new surface primitive**.

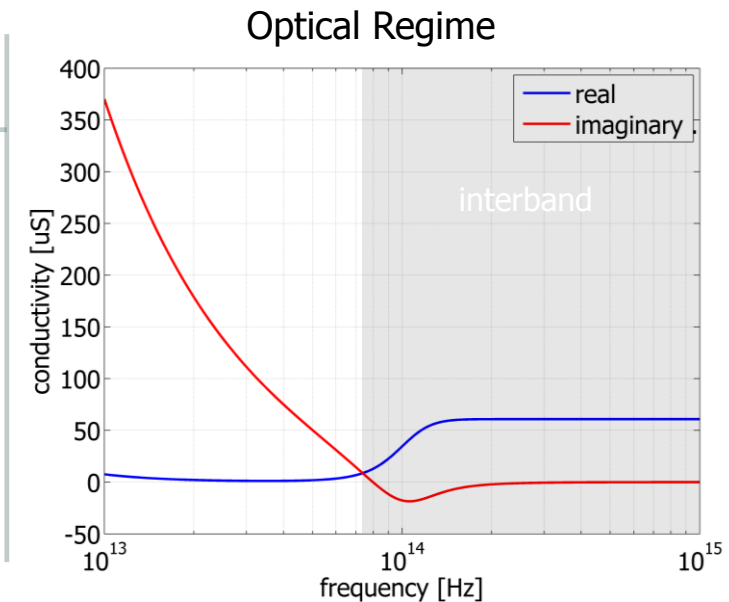
$\sigma(\omega, \Gamma, \mu_c, T)$



# Surface Conductivity of Graphene



$$\begin{aligned}\Gamma &= 0.41 \text{ meV} \\ \mu_c &= 0.2 \text{ eV} \\ T &= 300 \text{ K}\end{aligned}$$



## Notes

- Intraband term dominates at low frequencies.
- Interband term dominates at optical frequencies.
- Interband term must be obtained numerically.



# New Surface Conductivity Model

2015b Release  
FDTD & MODE

Material Database

Material List

Name	Mesh Order	Color	Type	Last modified
Al2O3 - Palik	2		Sampled data	2008-07-04 04:49 PM
Au (Gold) - CRC	2		Sampled data	2008-07-04 04:50 PM
Au (Gold) - Johnson and C...	2		Sampled data	2009-06-23 11:58 AM
Au (Gold) - Palik	2		Sampled data	2008-09-08 05:19 PM
C (graphene) - Falkovsky (...)	2		Analytic mate...	2013-11-07 11:47 AM
C (graphene) - broadband	2		Graphene	2015-04-14 10:13 AM
Cr (Chromium) - CRC	2		Sampled data	2008-08-13 02:57 PM
Cr (Chromium) - Palik	2		Sampled data	2008-07-04 04:49 PM

Material Properties

Anisotropy: None

tolerance	0.005
max coefficients	30

scattering rate (eV)	0.00041
chemical potential (eV)	0.2
temperature (K)	300

model parameters

Show advanced settings

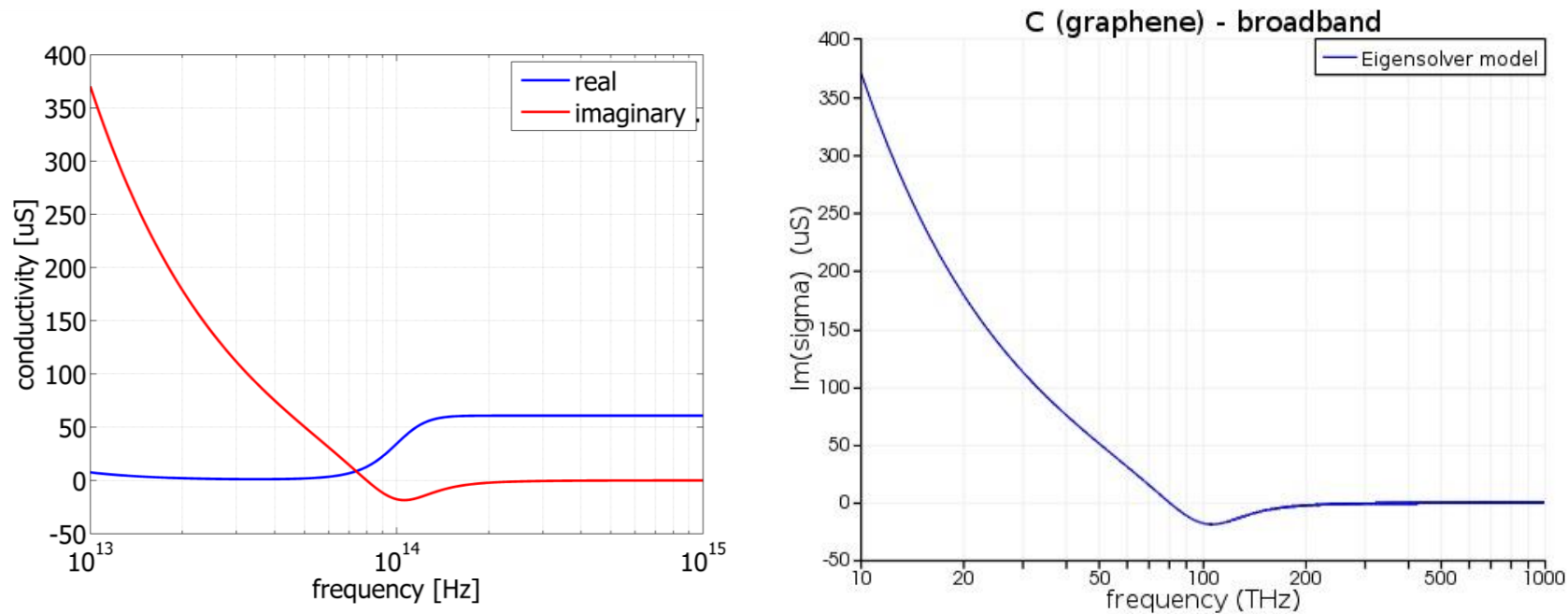
Go to Material Explorer

OK Cancel

(n,k) Material  
Analytic material  
Chi2  
Chi3 Raman Kerr  
Chi3/Chi2  
Conductive  
Debye  
Dielectric  
Four-Level Two-Electron (Version 1.0.0)  
**Graphene**  
Kerr nonlinear  
Lorentz  
Magnetic Electric Lorentz (Version 1.0.0)  
PEC  
Paramagnetic  
Plasma  
Sampled data  
Sellmeier  
np Density

new model

# Generating Conductivity Plots



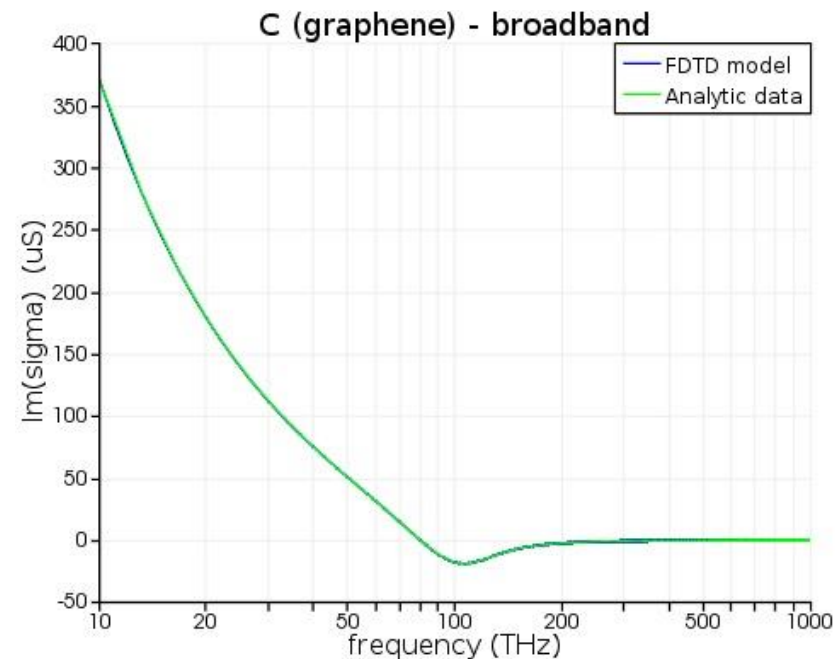
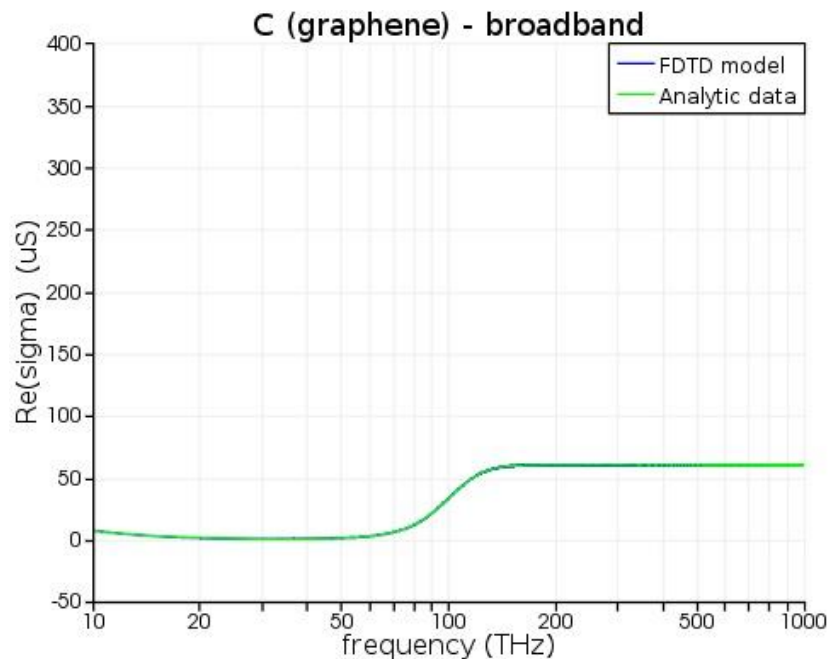
## Notes

- Brief demonstration: how to generate surface conductivity plots.
- Interband integral is computed numerically.

# New Surface Conductivity Model in FDTD

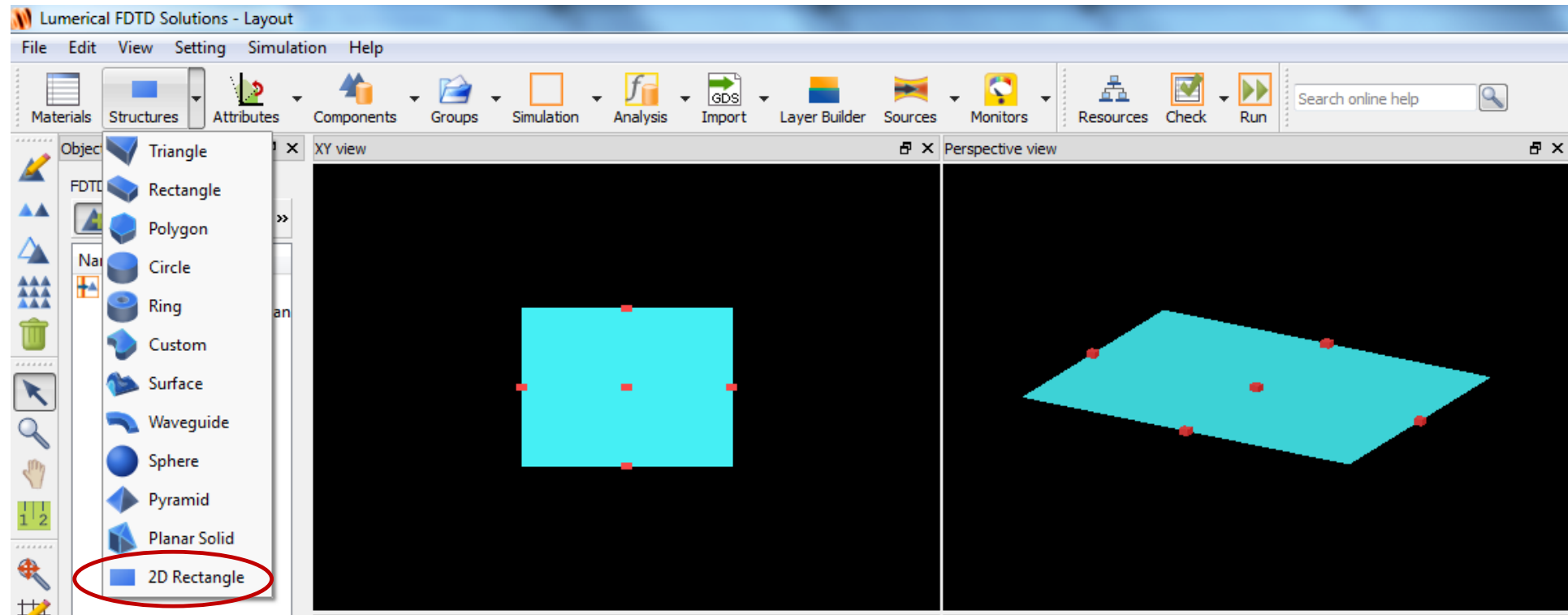
## FDTD Solutions needs a multi-coefficient material (MCM) model

- FDTD Solutions creates a multi-coefficient model of the surface conductivity.
- This model can be compared with the surface conductivity data.



# New Surface Primitive

FDTD 2015B Release

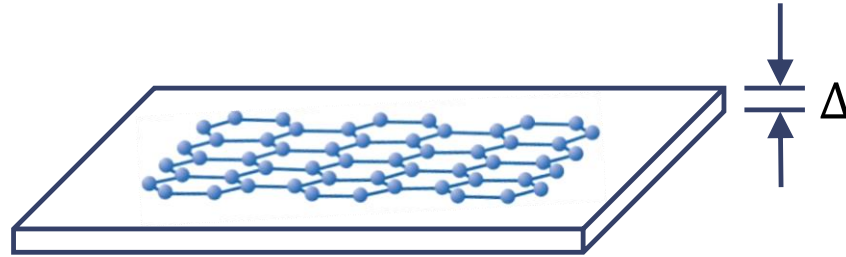


- The new 2D rectangle was specifically created for modeling graphene.
- The surface normal vector must be parallel to one of the three coordinate axes.

# New Graphene Modeling Workflow

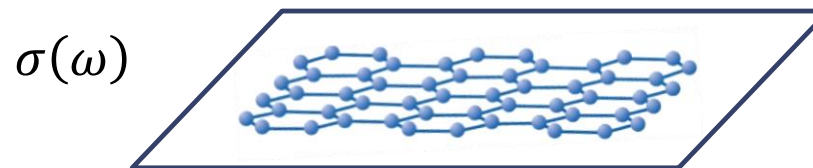
A surface conductivity can be turned into a volumetric permittivity:

$$\varepsilon(\omega) = \varepsilon_0 \varepsilon_r + i \frac{\sigma(\omega)}{\omega \Delta}$$



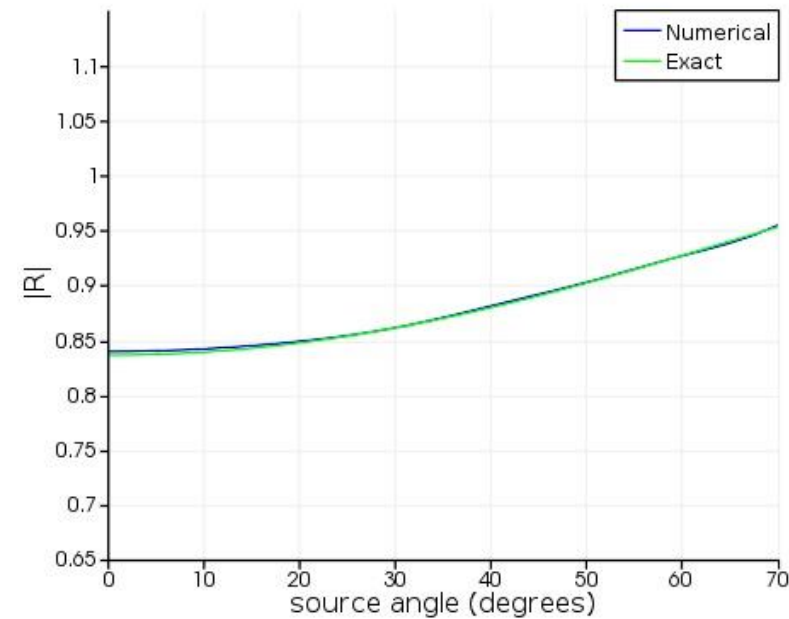
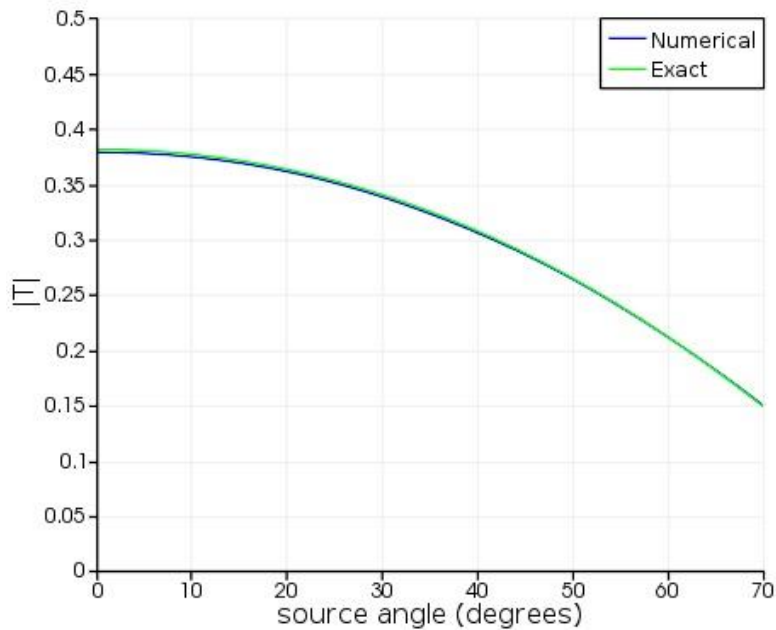
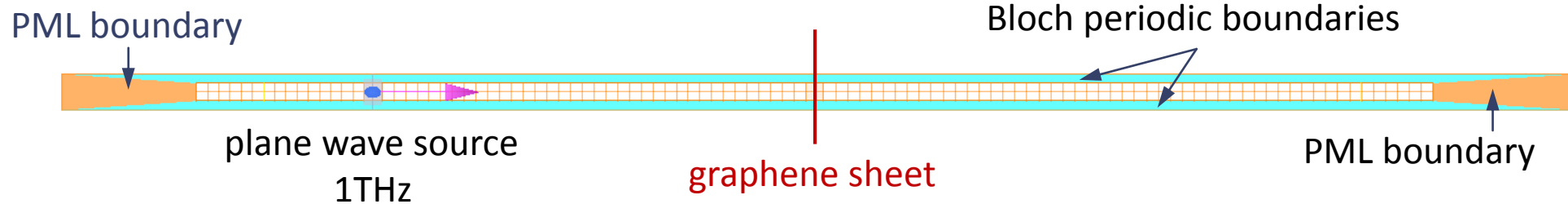
- A volumetric permittivity model is currently employed in FDTD and MODE.

**The new workflow is based on a surface conductivity model:**



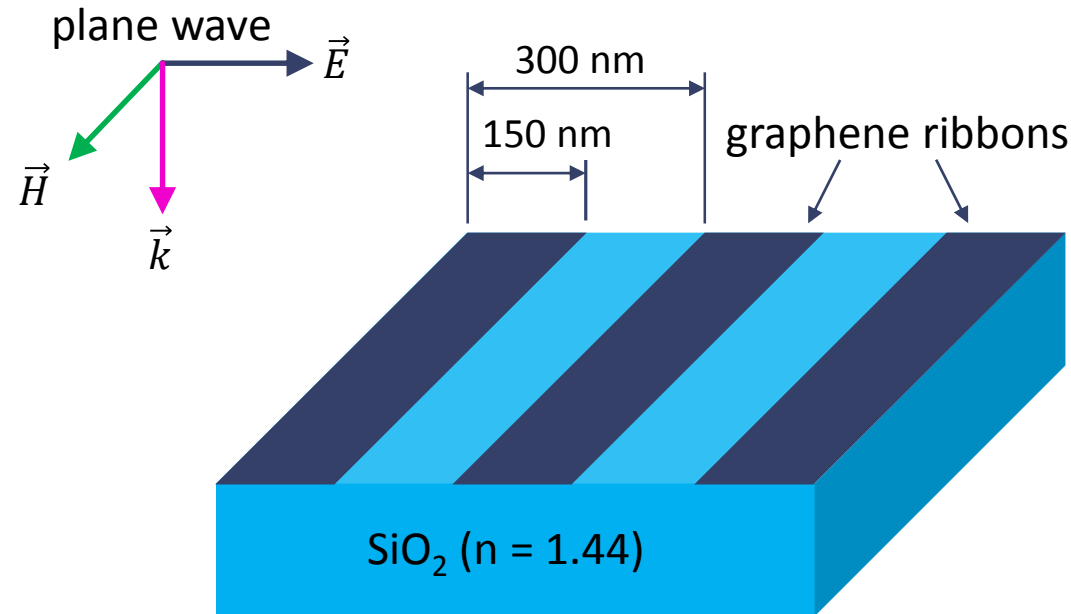
- New UI components: a surface conductivity model, and a surface primitive.

# Plane Wave Transmission / Reflection



# Tunable THz Metamaterial

Graphene ribbons deposited on a dielectric substrate:

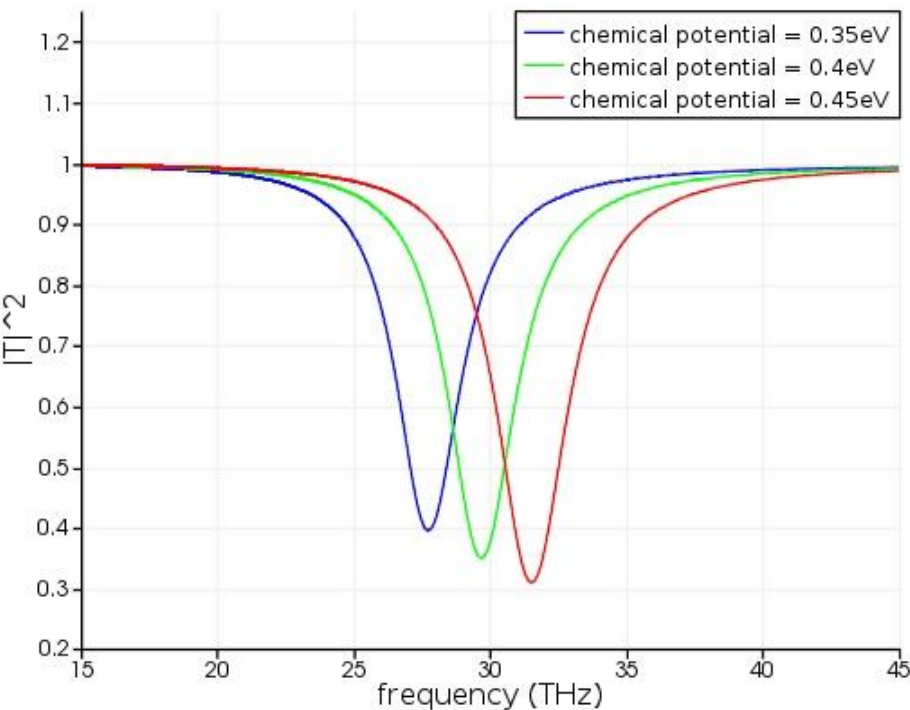


- Chemical potential of the ribbons can be tuned using a voltage source.
- The FDTD solver can produce broadband results thanks to the MCM model.

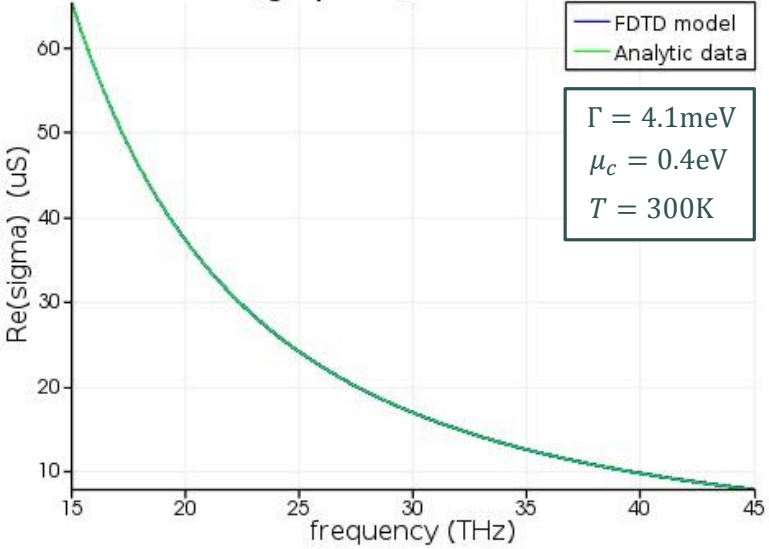
[4] H. Chu and C. Gan, "Active plasmonic switching at mid-infrared wavelengths with graphene ribbon arrays," *Applied Physics Letters*, vol. 102 (231107), 2013.

# Tunable THz Metamaterial

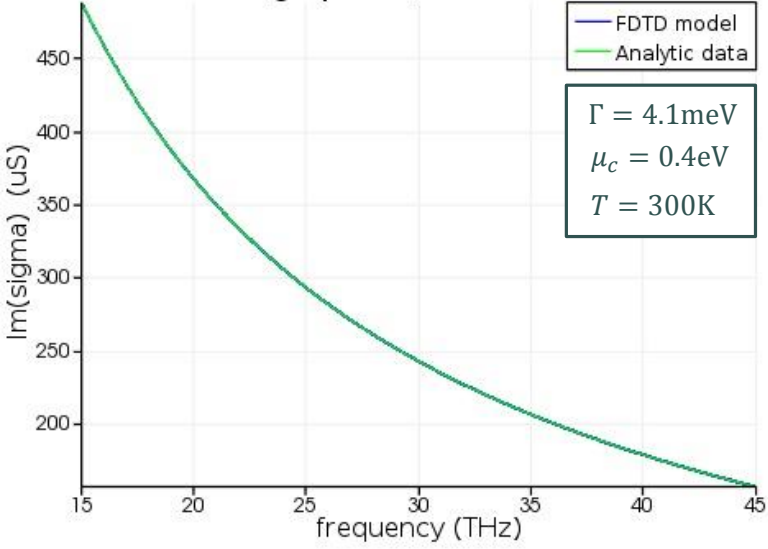
Transmitted Power Without Substrate



C (graphene) - broadband



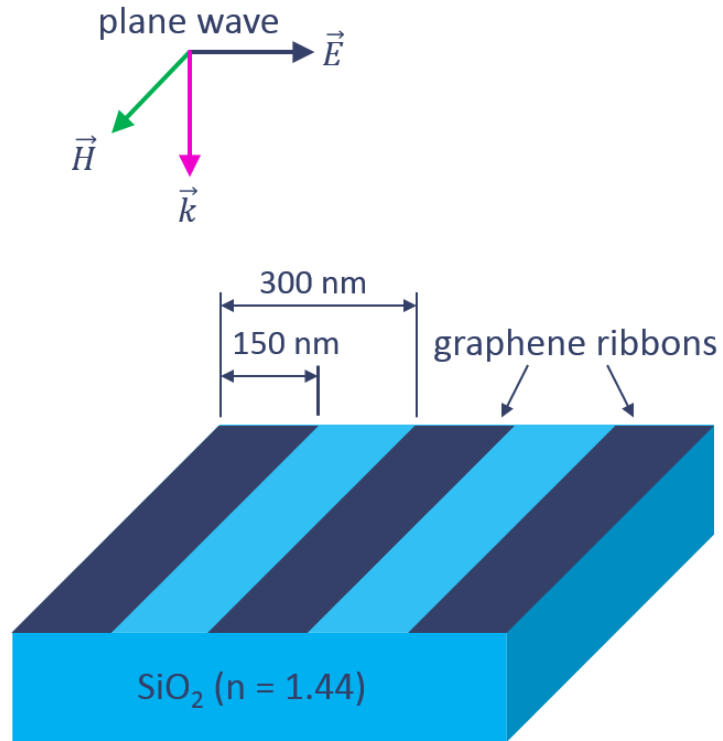
C (graphene) - broadband



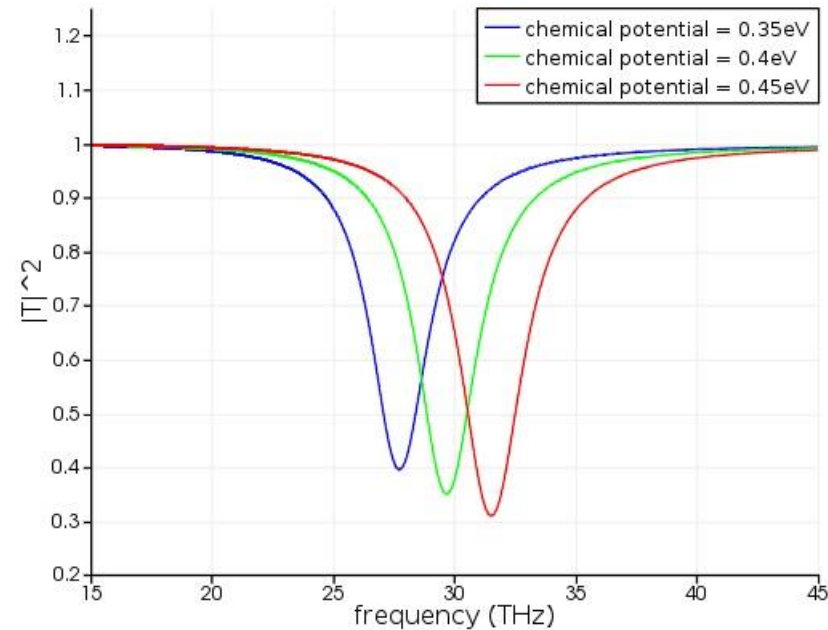
method	min mesh size	time
new	2.5 nm	67s
old	0.1 nm	1h+7min



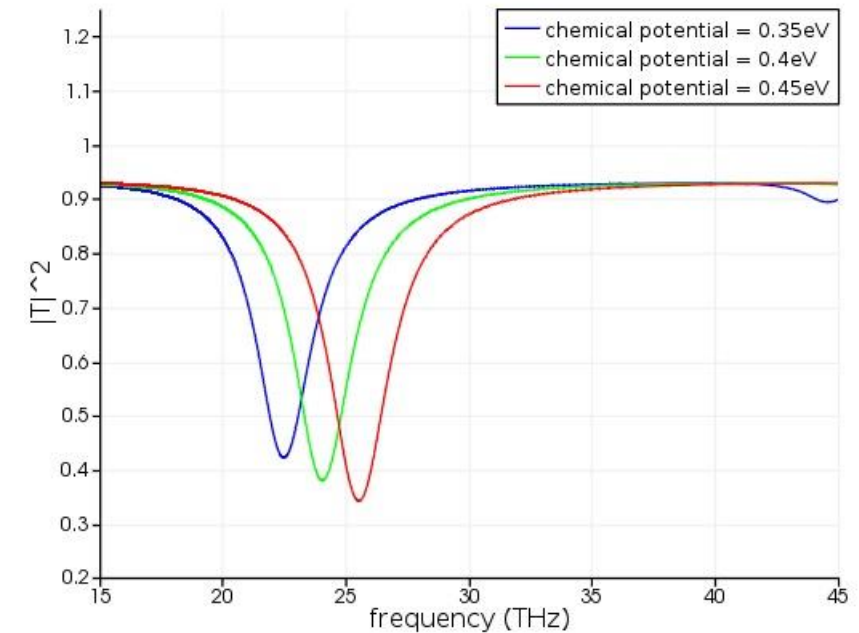
# Tunable THz Metamaterial



Transmitted Power Without Substrate

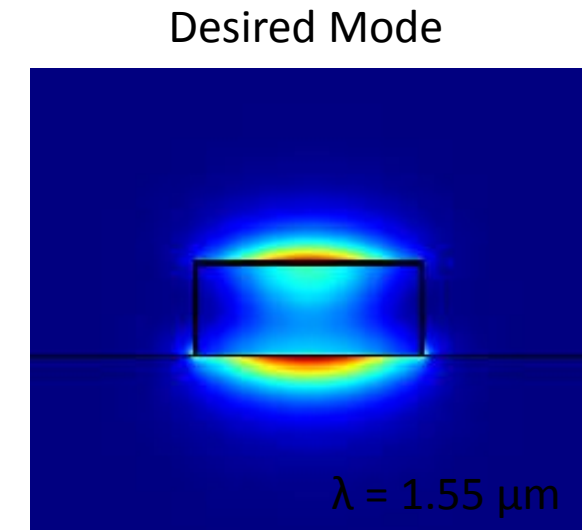
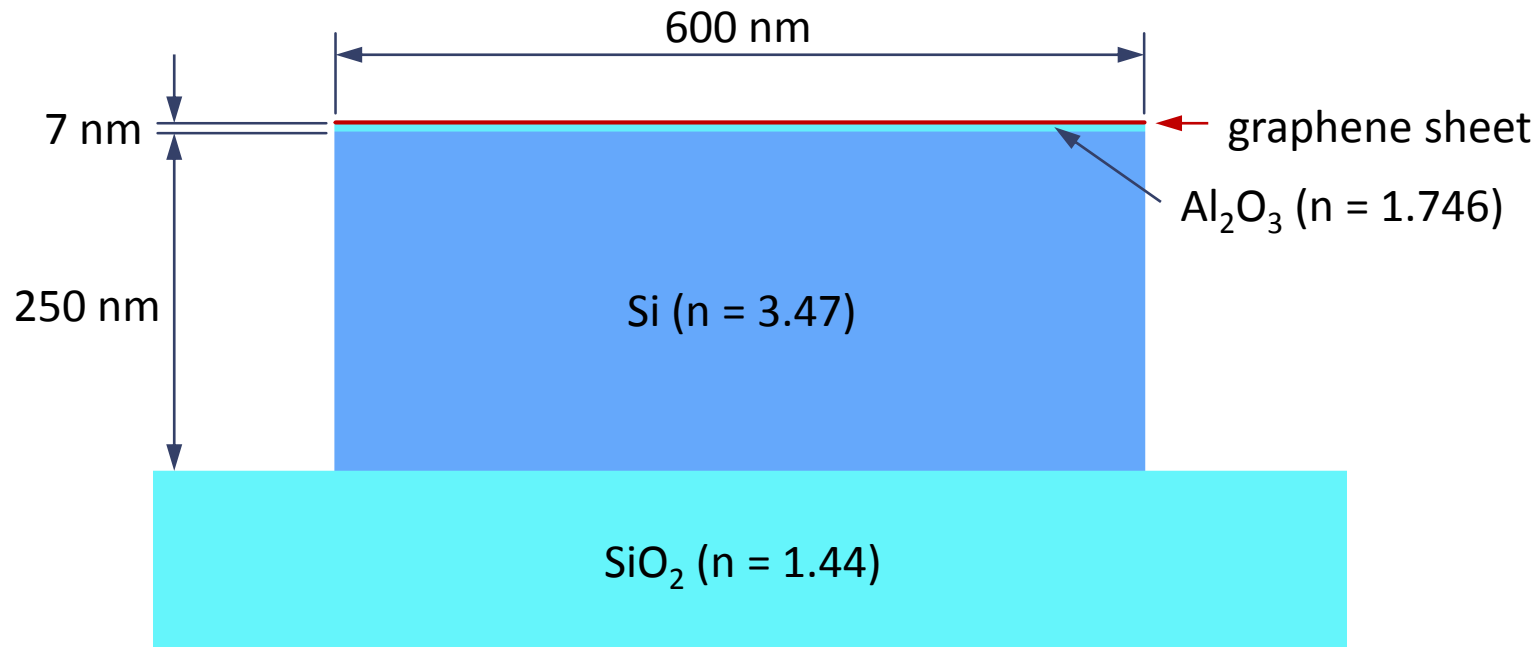


Transmitted Power With Substrate



# Electro-Optical Modulator

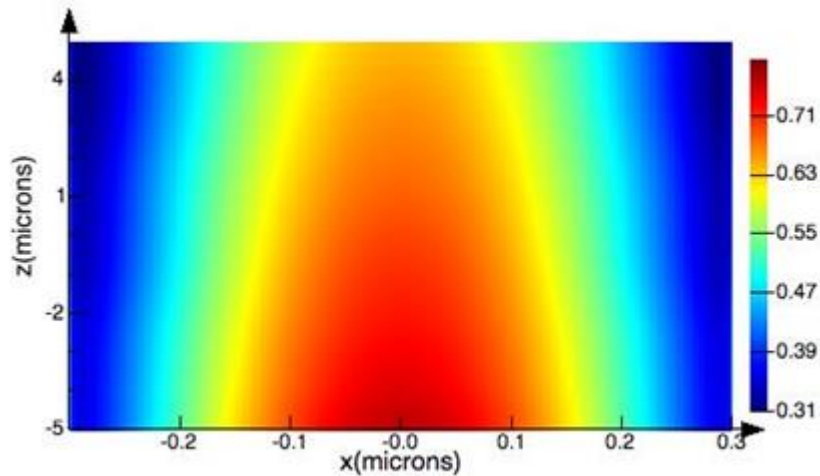
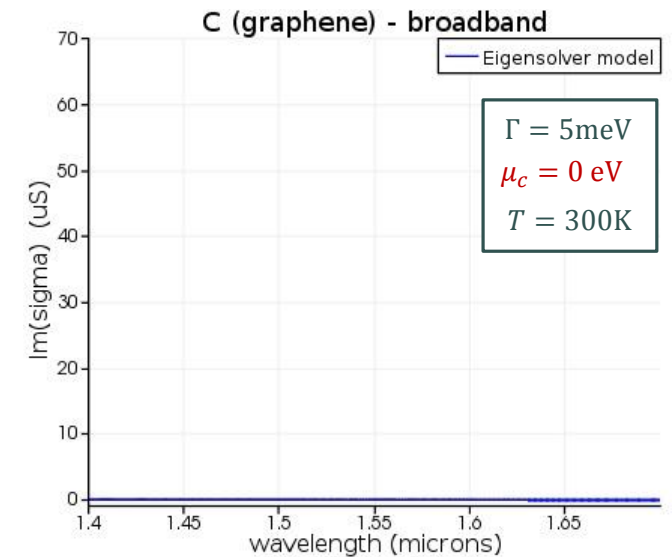
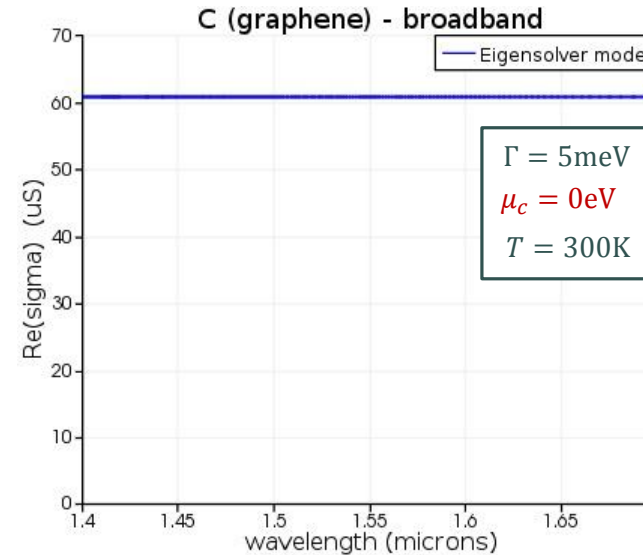
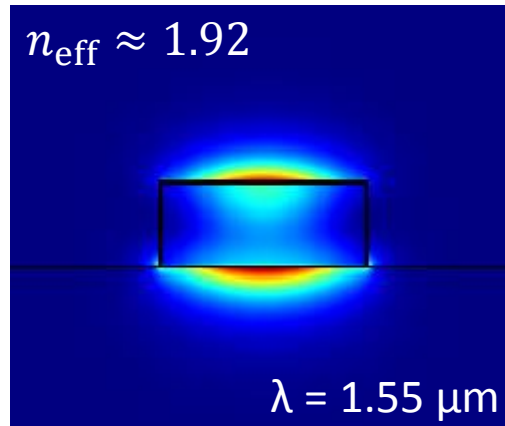
Based on the following waveguide configuration:



- Chemical potential of the graphene layer can be tuned using a voltage source.
- The shown waveguide mode was obtained using the FDE solver.

[5] M. Liu, X. Yin, E. Ulin-Avila, B. Geng, T. Zentgraf, L. Ju, F. Wang and X. Zhang, "A graphene-based broadband optical modulator", *Nature Letters*, vol. 474, pp. 64-67, 2011.

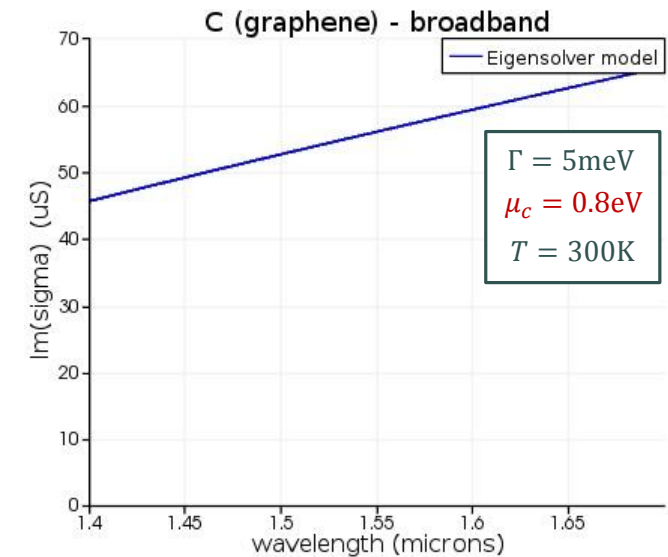
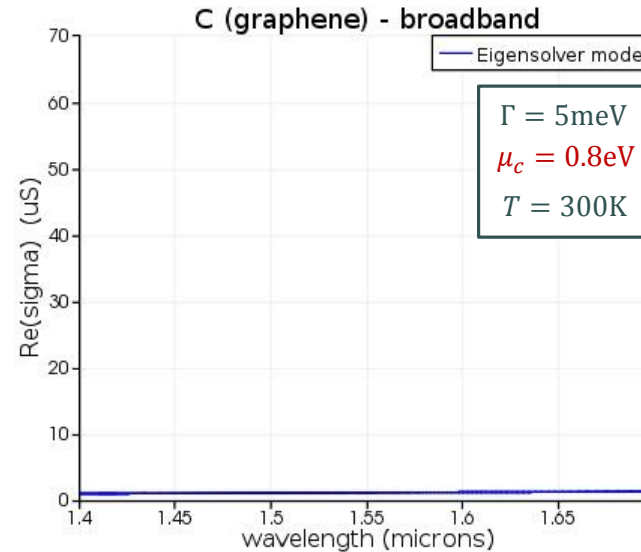
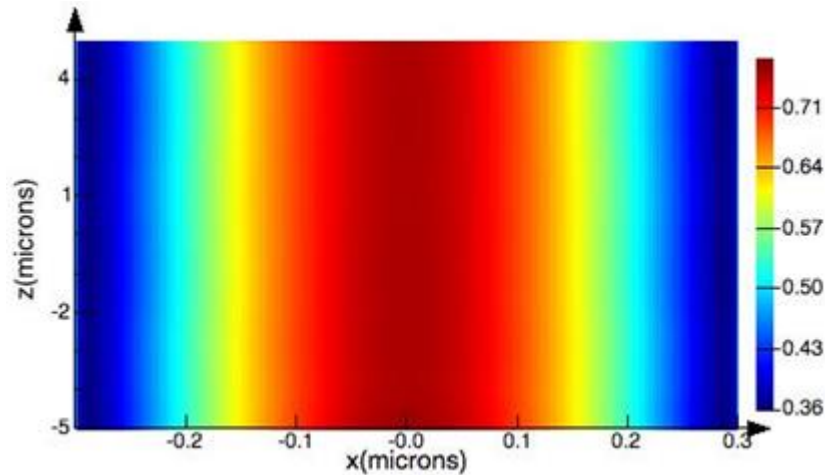
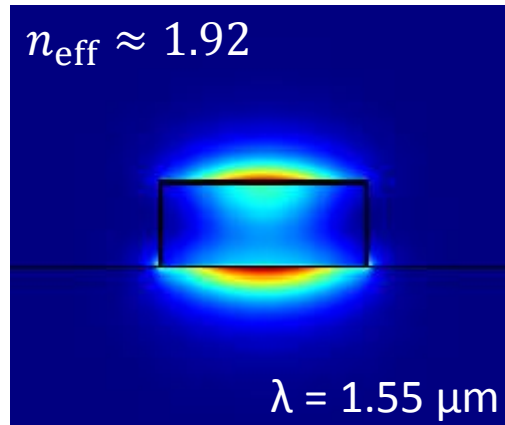
# Electro-Optical Modulator – Off State



← **Loss = 0.148 dB/ $\mu\text{m}$**

- When the chemical potential is zero, the loss is high enough to attenuate the waveguide mode significantly.

# Electro-Optical Modulator – On State



← **Loss = 0.0032 dB/ $\mu\text{m}$**

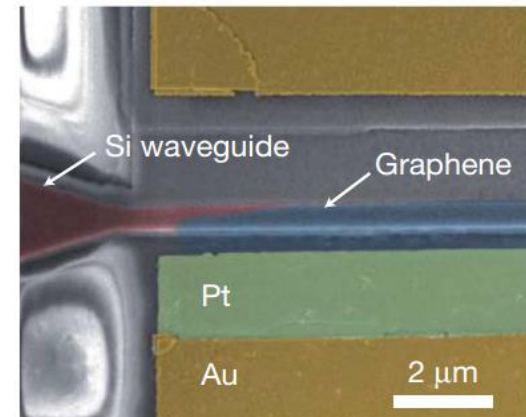
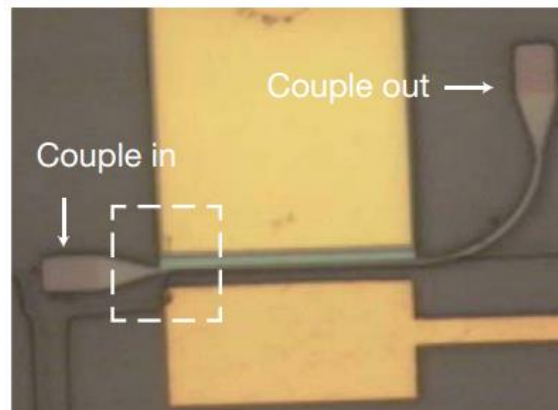
- When the chemical potential is 0.8 eV, the loss is low and there is virtually no attenuation of the waveguide mode.

# Electro-Optical Modulator

## FDTD Speed-up:

method	time
new	39 min
old	2h+15min

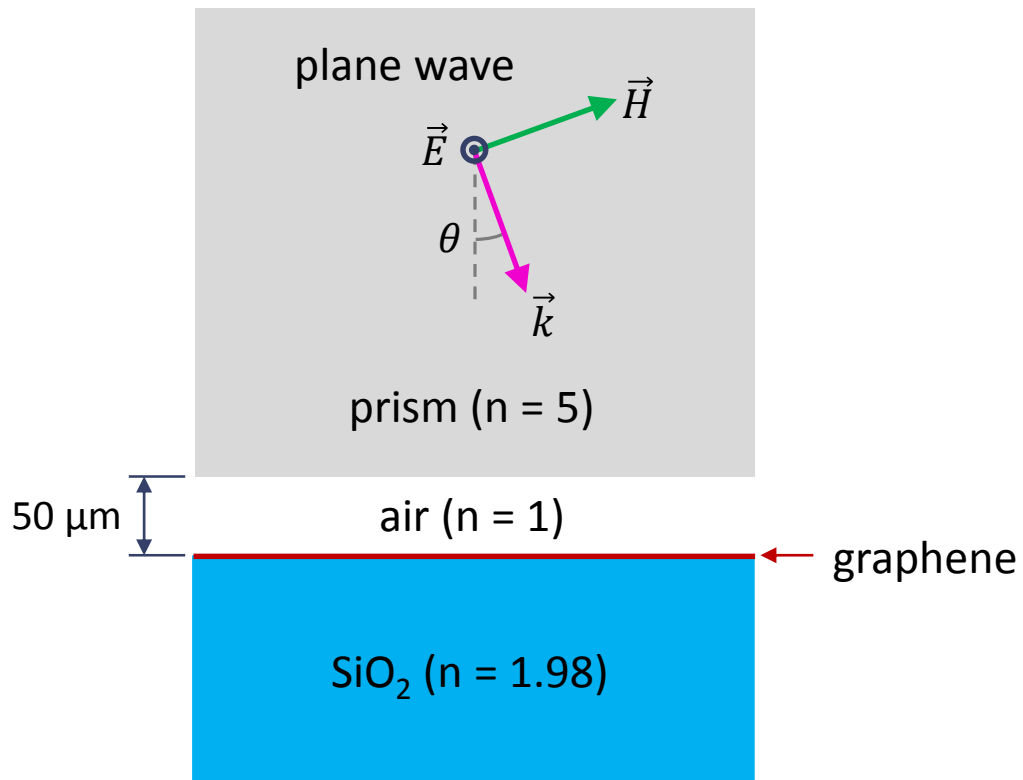
## Prototype:



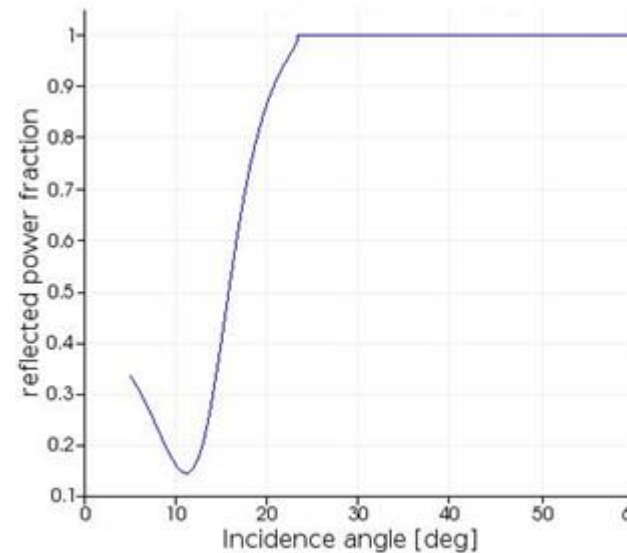
[5] M. Liu, X. Yin, E. Ulin-Avila, B. Geng, T. Zentgraf, L. Ju, F. Wang and X. Zhang , "A graphene-based broadband optical modulator", *Nature Letters*, vol. 474, pp. 64-67, 2011.

# Exciting a Surface Plasmon Mode

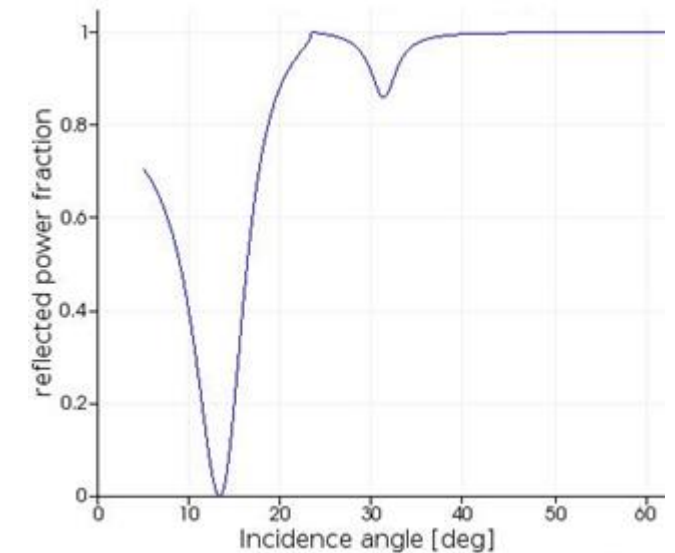
Light at 0.5 THz incident from a prism:



Without Graphene Sheet



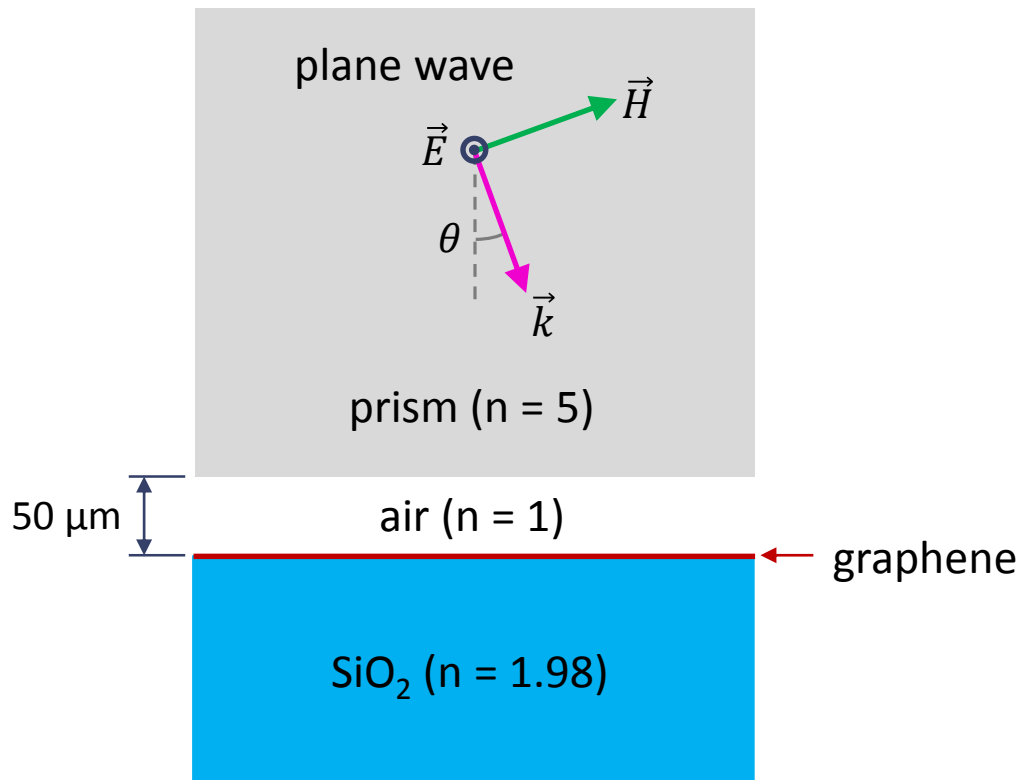
With Graphene Sheet



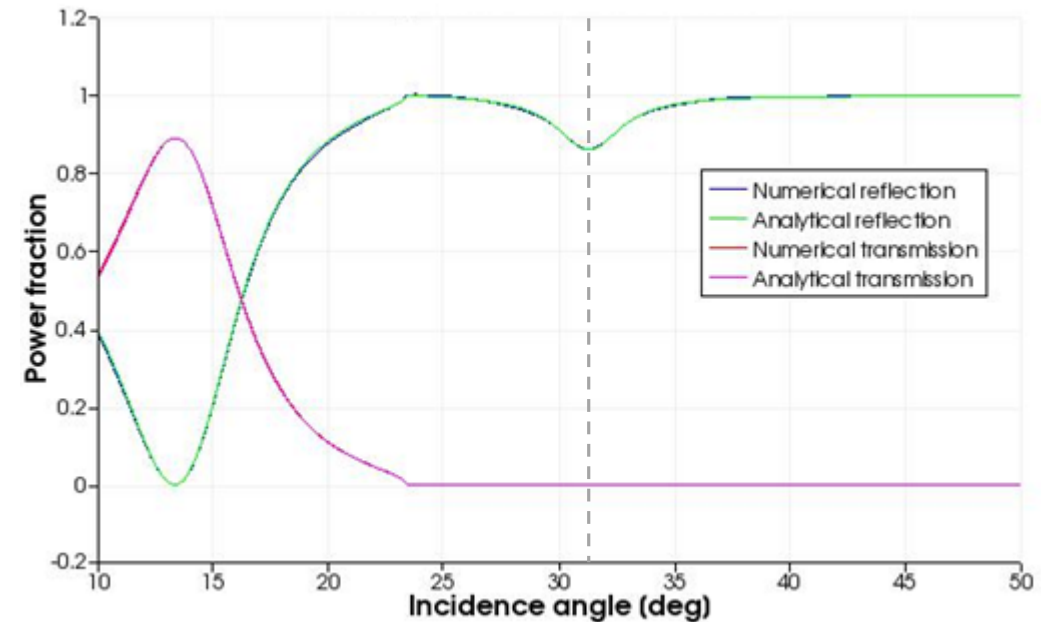
- At  $\theta \approx 31^\circ$ , a surface plasmon (SP) is excited.
- Exact results obtained with the transfer matrix method.

# Exciting a Surface Plasmon Mode

Light at 0.5 THz incident from a prism:



Numerical vs. Analytic



- The SP mode can also be modeled using the FDE solver:

$$n_{\text{eff}} = 2.5625 + i0.1202 \quad \theta = \sin^{-1}(\text{Re}(n_{\text{eff}})/5) \approx 30.8^\circ$$

# Summary

---

- **Improved graphene workflow available in FDTD and MODE (2015B)**
- **Graphene is now treated as a 2-D material**
  - A new surface conductivity model is being introduced.
  - A new geometric primitive specifically for graphene.
- **Treating graphene as a 2-D material leads to faster simulations**



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