# A Physics-Based Analytical Model for 2DEG Charge Density in AlGaN/GaN HEMT Devices

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***Abstract*—In this brief, we present a physics-based analytical model for 2-D electron gas density $n_s$ in AlGaN/GaN high-electron mobility transistors. The proposed model accounts for the interdependence between Fermi level $E_f$ and $n_s$. The model is developed by considering the variation of $E_f$, the first subband $E_0$, the second subband $E_1$, and $n_s$ with applied gate voltage $V_g$. The proposed model is in very good agreement with numerical calculations.**

***Index Terms*—AlGaN/GaN high-electron mobility transistor (HEMT), analytical models, two-dimensional electron gas (2DEG) charge density.**

## I. Introduction

AlGaN/GaN high-electron mobility transistor (HEMT) devices are considered to be very promising candidates for high-speed and high-power applications [1], [2]. These devices offer advantages such as high breakdown voltage, high charge density, and good electron mobility [3]–[5]. The formation of the 2-D electron gas (2DEG) in these devices is the heart of the device operation and has been studied in great detail in the literature. For accurate and fast simulation of circuits based on these devices, an analytical expression for 2DEG density $n_s$ is required. The expression should be also physics based, which obviates the use of a large number of empirical parameters.

To the best of the authors' knowledge, currently available models for $n_s$ are primarily based on numerical calculations, semiempirical model expressions, or simplifying approximations [6]–[11]. Models relying on numerical calculations will be slow because of the iterative nature. Semiempirical models are fast but do not provide needed insight into the device operation. They also tend to have a large number of empirical parameters, which have to be extracted from experimental data. In contrast to these approaches, we present a physics-based analytical model with minimal empirical parameters.

The difficulty in modeling the 2DEG charge density stems from the complicated variation of $E_f$ with $n_s$ in the quantum well. This relationship is given by [7]

$$n_s = DV_{\rm th}\left\{\ln\left[\exp\left(\frac{E_f - E_0}{V_{\rm th}}\right)+1\right] + \ln\left[\exp\left(\frac{E_f - E_1}{V_{\rm th}}\right)+1\right]\right\} \quad (1)$$

where $E_0 = \gamma_0 n_s^{2/3}$ and $E_1 = \gamma_1 n_s^{2/3}$ are the levels of the two lowest subbands, $E_f$ is the Fermi level expressed in volts, and $V_{\rm th} = kT/q$ is the thermal voltage. The definitions of the symbols used are given in Table I. Assuming that the AlGaN layer is completely ionized, we can write

$$n_s = \frac{\varepsilon}{qd}(V_{\rm go} - E_f) \quad (2)$$

where $V_{\rm go} = V_g - V_{\rm off}$, and $V_{\rm off}$ is the cutoff voltage. The calculations in this brief have been done for an $Al_{0.15}Ga_{0.85}/GaN$ structure and typical $V_{\rm off}$ of $-3$ V [6]. It is apparent from (1) and (2) that the variation of $n_s$ with respect to $V_g$ is a complicated transcendental function. Here, we propose simplifying assumptions in these equations, which allows us to develop a precise analytical expression for $n_s$ versus $V_g$ for the typical operating range of gate voltages and temperatures.

TABLE I
List of Symbols

| Symbol | Physical meaning |
|---|---|
| $n_s$ | Density of 2DEG |
| $d$ | Thickness of AlGaN layer 20 nm |
| $q$ | The electron charge |
| $\varepsilon$ | The permittivity of AlGaN |
| $D$ | Density of States 1.001e18 $m^{-2}V^{-1}$ [12] |
| $V_{off}$ | Cutoff voltage taken as -3V [6] |
| $k$ | Boltzmann's constant |
| $T$ | Temperature |
| $E_0$ | Position of first sub-band |
| $E_f$ | Position of Fermi level |
| $E_1$ | Position of second sub-band |
| $\gamma_0,\gamma_1$ | Experimentally determined parameters 2.12e-12 and 3.73e-12 $V.m^{4/3}$ respectively [13] |

## II. 2DEG Charge Density Model Development

Fig. 1 shows important quantities $E_f$, $E_0$, and $E_1$ as obtained from numerical calculations based on (1) and (2) for a wide range of gate voltages from close to $V_{\rm off}$ and up. It is interesting to note the relative positions of the subband levels compared with $E_f$ in this range. To accentuate the observation, we plot

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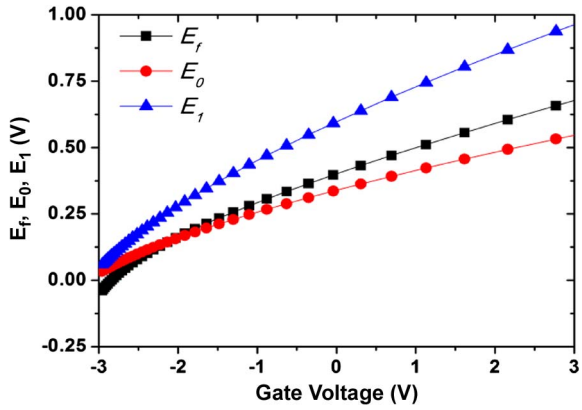


Fig. 1. Numerical calculations of $E_f$, $E_0$, and $E_1$ versus gate voltage. $T = 298$ K.

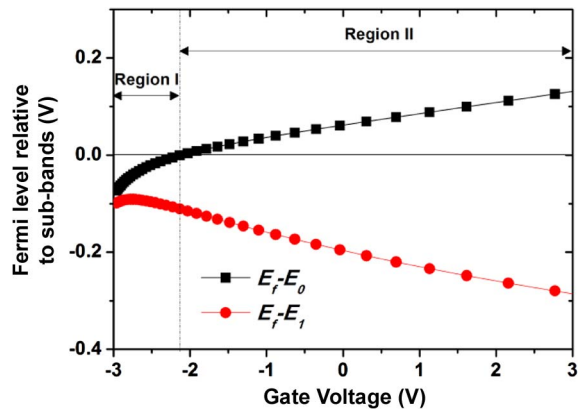


Fig. 2. Numerical calculations of $E_f - E_0$ and $E_f - E_1$ versus gate voltage. Regions I and II are marked in the figure.

$E_f - E_0$ and $E_f - E_1$ in Fig. 2 and find two observations: 1) Upper level $E_1$ is larger than $E_f$ for the complete range of the gate voltages. In addition, $E_1$ is significantly larger than $E_0$ over the same range. Hence, the contribution to $n_s$ from the second subband can be safely ignored. 2) Lower level $E_0$ has two distinct regions, namely, Region I, where $E_0$ is larger than $E_f$, corresponding to $V_{\rm go} < 0.9$ V, and Region II, where $E_0$ is lower than $E_f$ and $V_{\rm go} \geq 0.9$ V. We also note that, in the full range of gate voltages considered, except very close to cutoff, $E_f$ will be much smaller than $V_{\rm go}$.

Next, a model for $n_s$ is separately developed in Regions I and II. These models are then combined to give a unified model covering both regions.

### A. Charge Density Model in Region I

Using the aforementioned observation for Region I, (1) can be approximated as follows:

$$E_f^I \approx \gamma_0 \left(n_s^I\right)^{2/3} + V_{\rm th} \ln\left(\frac{n_s^I}{DV_{\rm th}}\right). \tag{3}$$

Equation (3) is obtained by neglecting the second term in (1) and replacing $E_0$ by $\gamma_0 (n_s^I)^{2/3}$. Next, we apply (2) to eliminate $n_s^I$ in (3) and expand the two terms on the right-hand side of (3) to first order in $E_f^I/V_{\rm go}$ (see above). Thus, we obtain the following analytical expression for $E_f^I$ versus gate voltage:

$$E_f^I = V_{\rm go} \frac{V_{\rm th} \ln(\beta V_{\rm go}) + \gamma_0 \left(\frac{C_g V_{\rm go}}{q}\right)^{2/3}}{V_{\rm go} + V_{\rm th} + \frac{2\gamma_0}{3}\left(\frac{C_g V_{\rm go}}{q}\right)^{2/3}} \tag{4}$$

where $C_g = \varepsilon/d$, and $\beta = C_g/(qDV_{\rm th})$. It is important to point out here that, for vertically scaled devices with $d < 5$ nm, the 2DEG distance from the interface, i.e., $\delta d$, will become comparable with $d$. In such cases, $d$ should be replaced by $d_{\rm eff} = d + \delta d$ in the model expressions. Combining (4) and (2), we find an analytical model for $n_s^I$ versus gate voltage in Region I, i.e.,

$$n_s^I = \frac{C_g V_{\rm go}}{q} \frac{V_{\rm go} + V_{\rm th}\left[1 - \ln(\beta V_{\rm go})\right] - \frac{\gamma_0}{3}\left(\frac{C_g V_{\rm go}}{q}\right)^{2/3}}{V_{\rm go} + V_{\rm th} + \frac{2\gamma_0}{3}\left(\frac{C_g V_{\rm go}}{q}\right)^{2/3}}. \tag{5}$$

### B. Charge Density Model in Region II

In Region II, observations 1) and 2) applied to (1) result in the following approximate expression for $n_s^{\rm II}$:

$$n_s^{\rm II} \approx D\left(E_f^{\rm II} - E_0\right). \tag{6}$$

Combining this with (2) using the same approximation as was used for Region I, we now obtain

$$E_f^{\rm II} = V_{\rm go} \frac{\beta V_{\rm th} V_{\rm go} + \gamma_0 \left(\frac{C_g V_{\rm go}}{q}\right)^{2/3}}{(1 + \beta V_{\rm th}) V_{\rm go} + \frac{2\gamma_0}{3}\left(\frac{C_g V_{\rm go}}{q}\right)^{2/3}} \tag{7}$$

$$n_s^{\rm II} = \frac{C_g V_{\rm go}}{q} \frac{V_{\rm go} - \frac{\gamma_0}{3}\left(\frac{C_g V_{\rm go}}{q}\right)^{2/3}}{(1 + \beta V_{\rm th}) V_{\rm go} + \frac{2\gamma_0}{3}\left(\frac{C_g V_{\rm go}}{q}\right)^{2/3}}. \tag{8}$$

### C. Unified Model for Charge Density

In order to find a unifying expression for $E_f$ and $n_s$ applicable for the full range of gate voltages, we observe that the respective expressions for Regions I and II differ only in one term in the numerator and one term in denominator, both with a factor of $V_{\rm th}$. Hence, both the regional models are expected to provide a fairly reasonable approximation for most of the voltage range considered, except near cutoff. In fact, similar accuracy can be obtained by removing all terms with factor $V_{\rm th}$ in (4), (5), (7), and (8), resulting in approximate unified expressions for $E_f$ and $n_s$. However, to improve accuracy, we propose to use a unified form that combines the two regional models. Considering specifically the expression for $n_s$, we can write

$$n_s = \frac{C_g V_{\rm go}}{q} \frac{V_{\rm go} + V_{\rm th}\left[1 - \ln(\beta V_{\rm gon})\right] - \frac{\gamma_0}{3}\left(\frac{C_g V_{\rm go}}{q}\right)^{2/3}}{V_{\rm go}\left(1 + \frac{V_{\rm th}}{V_{\rm god}}\right) + \frac{2\gamma_0}{3}\left(\frac{C_g V_{\rm go}}{q}\right)^{2/3}}. \tag{9}$$

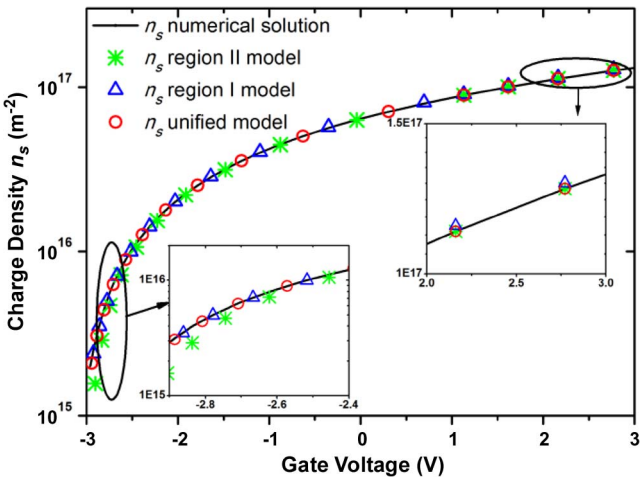


Fig. 3. Comparison of numerical calculations and the unified and regional analytical models of $n_s$ versus gate voltage. $T = 298$ K.

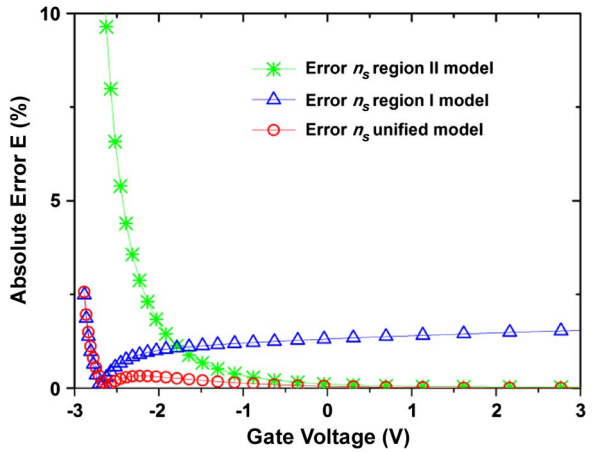


Fig. 4. Percentage absolute error between numerical calculations and analytical models of $n_s$ versus gate voltage.

Here, $V_{\rm gon}$ and $V_{\rm god}$ are functions of $V_{\rm go}$ given by the interpolation expression

$$V_{\rm gox} = \frac{V_{\rm go}\alpha_x}{\sqrt{V_{\rm go}^2 + \alpha_x^2}} \tag{10}$$

where $\alpha_n = e/\beta$, and $\alpha_d = 1/\beta$. We observe that (9) has the proper limiting form corresponding to that of (5) for Region I and (8) for Region II. A similar unified expression can be also easily established for $E_f$ by applying (9) in (2).

## III. Results and Discussions

The model developed for $n_s$ is compared with numerical calculations in Fig. 3, where the two regional models are indicated along with the unified model at room temperature. It is apparent from this figure that all three models agree quite well with the simulations for most of the gate voltage range considered, but the unified model has higher precision everywhere with typically less than 1% error for $V_{\rm go} > 0.1$ V, as shown in Fig. 4. Since the assumption that $E_f^I/V_{\rm go} \ll 1$ does not apply very close to cutoff, the error tends to steeply increase in this region, as indicated in Fig. 4. Region I model actually performs quite well also in Region II, with an error of less that 2% in that region. Region II model performs relatively poorly in Region I, with a rapidly increasing error for $V_{\rm go} < 1$ V.

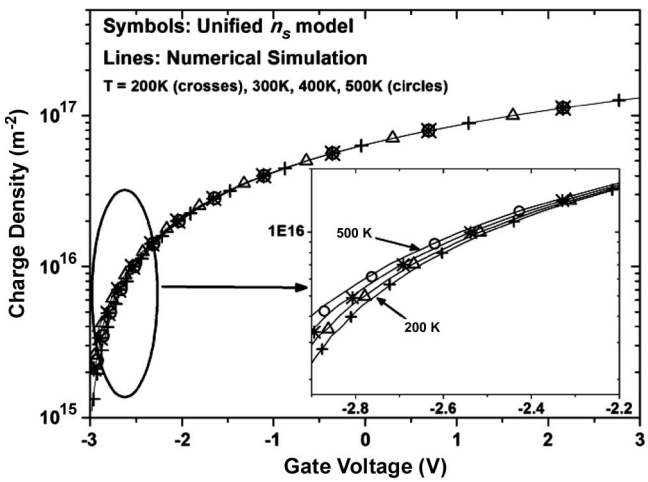


Fig. 5. Comparison of numerical calculations and the unified analytical model of $n_s$ versus gate voltage for $T = 200$, 300, 400, and 500 K.

Since GaN devices often operate at high power, it is also of interest to assess the precision of the present model at elevated temperatures. In Fig. 5, the unified model for $n_s$ is compared with numerical calculations at different temperatures up to 500 K. It is apparent from the results that the model agrees quite well with the numerical calculations throughout this temperature range, although the error tends to increase somewhat near cutoff with increasing temperature. The increase in error is found to be almost linear with increasing temperature for bias point $V_{\rm go} = 0.2$ V. However, it can be argued that high temperatures due to self-heating will occur only well above cutoff, where the error is very small.

## IV. Conclusion

We have presented a physics-based fully analytical model for the dependence of 2DEG density $n_s$ on gate bias. The model is developed by simplifying the basic device equations in different regions of operation and combining them. The proposed model does not have any empirical or fitting parameters. The modeling results are in excellent agreement with numerical calculations for a wide range of temperatures. This model can be used to develop physics-based compact models of GaN HEMT devices.

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