Theory and Applications of Surface Waves.

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1. – Introduction.

Surface waves are closely connected with optics as well as microwaves. Attention was first drawn to them in connection with a purely optical phenomenon: in 1902, WOOD (1) had discovered anomalies in the spectrum of certain diffraction gratings, and five years later LORD RAYLEIGH (2) gave a tentative explanation in terms of surface waves propagating in a direction normal to the rulings. Since that time, the Wood anomalies have received renewed attention at intervals, notably by UGO FANO (3) in 1938 and, most recently, by HARVEY PALMER (4) and VICTOR TWERSKY (5).

When surface waves were first used in the microwave region, in connection with dielectric rods, leaky waveguides and Yagi arrays, it was not recognized that these devices have anything in common with Wood's anomalies. The first to examine surface waves deliberately was G. TORALDO DI FRANCIA (6) (1942) who performed a number of microwave experiments in Florence to prove their existence and illustrate the role they play in diffraction. Two years later, C. C. CUTLER of the Bell Telephone Laboratories observed them on a corrugated sheet and calculated some of their properties.

It is not easy to define a surface wave, and we will not attempt to give a rigorous definition until later on in section **3**. Let us specify merely that a surface wave is one which propagates along an interface between two media

(1) R. W. WOOD: *Phil. Mag.*, **4**, 396 (1902).
(2) LORD RAYLEIGH: *Phil. Mag.*, **14**, 60 (1907).
(3) U. FANO: *Ann. der Phys.*, **32**, 393 (1938); *Journ. Opt. Soc. Am.*, **31**, 213 (1941).
(4) C. H. PALMER: *Journ. Opt. Soc. Am.*, **42**, 269 (1952).
(5) V. TWERSKY: *Journ. Appl. Phys.*, **23**, 1099 (1952).
(6) G. TORALDO DI FRANCIA: *Ottica*, **7**, 197 (1942).

differing from each other either in their material constants, or in that one of them is continuous while the other is periodically structured, or in both. Because of the wave equation, we know that the Pythagorean relation

$$\Gamma^2 = \Gamma_n^2 + \Gamma_s^2 ,$$

holds in both media. The propagation constants Γ are of the form

$$\Gamma = \alpha + j\beta ,$$

where α is the attenuation, and β the phase constant. This relation is illustrated in fig. 1, with the hypothenuse always formed by the value of the propagation constant in the infinitely extended medium. The transverse propagation constants need not lie in the plane of the paper, since they themselves may be the resultant of a propagation constant normal to the direction of propagation as shown, and another that is perpendicular to the plane of the paper. This latter arises if, for example, the surface wave is confined in a channel formed by two metal walls parallel to the direction of propagation. The surface wave of fig. 1 is given by

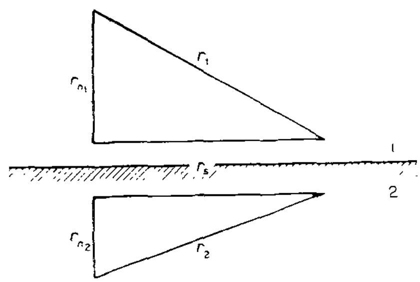

Fig. 1.

$$\exp\,[j\omega t - (\Gamma_n x + \Gamma_s z)] ,$$

and we note the important fact that the Pythagorean relation defines the surface propagation constant Γ_s as soon as the transverse characteristics are specified.

If there is no loss in the system, then Γ and Γ_s are both imaginary, and Γ_n is consequently a pure attenuation. The wave described by the expression

$$\exp\,[-\alpha_n x + j(\omega t - \beta_s z)$$

is then the simplest example of a surface wave along a plane interface. It is an *inhomogeneous* plane wave, i.e., the planes of constant phase (normal to the interface) and the planes of constant amplitude (parallel to the interface) do not coincide. Since

$$-\beta^2 = \alpha_n^2 - \beta_s^2 ,$$

it is clear that β_s is larger than β. In other words, the surface wave along the loss-less interface must be *slower* than the wave in the infinitely extended medium. In the language of the electrical engineer, this means that the interface acts like an inductive load. If losses are present, however, the surface phase velocity may exceed that in the adjacent media.

2. – A Survey of Surface Wave Types.

There are two sorts of surface waves: forced and free. These are terms usually encountered in the theory of resonance, and it is not clear what they should have to do with travelling waves. We will justify their usage in section **3**. For the time being — and indeed throughout this section —, we intend only a qualitative discussion. Forced waves, by analogy with a resonant cavity, are those which exist by virtue of a source located in finite space. They are *driven* by the source, and constitute part of its total field. Free waves, on the other hand, exist even in the absence of sources anywhere in finite space. They correspond to the self-oscillations of the closed cavity. As in the theory of resonance, the propagation constants Γ_s and Γ_n of the free waves depend on the geometry of the interface along which they propagate, and on the ε, μ and σ of the two adjacent media. The propagation constants of the forced waves, on the other hand, depend in addition on some characteristic of the source. As an example, consider the surface wave which arises in total internal reflection. Its propagation constant along the surface is given by

$$\beta_s = \omega\sqrt{\varepsilon_2\mu_2}\,\sin i\ ,$$

where ω is the frequency, and i the angle of incidence of the totally reflected ray in medium 2. The dependence on the angle of incidence shows the propagation constant to be a function of source location and the surface wave is therefore of the forced type. As another example, consider the wave along a thin dielectric slab placed on a metal sheet. Its propagation constant is given by

$$\beta_s \cong \omega\sqrt{\varepsilon_1\mu_1}\left[1 + \frac{\varepsilon_1\mu_1}{2}\,d^2\omega^2(1 - \varepsilon_1/\varepsilon_2)^2\right],$$

where subscripts 1 refer to air and 2 to the dielectric slab (thickness d). No source characteristics enter this expression, and we are dealing with a free wave.

The distinction of greatest use to the microwave engineer is that the forced waves are merely a partial aspect of the total source field, while the free waves are detached from the source, as it were, and propagate along the single surface as transmission line modes. They follow gentle bends and torsions in the surface as if they were waves inside a coaxial cable or a hollow waveguide. The theory of free surface waves has therefore much in common with ordinary waveguide theory; it extends our knowledge of energy transmission by conventional, shielded wave guides to open (unshielded) structures.

Several examples of forced surface waves are displayed in fig. 2. The best known is the surface wave which accompanies total internal reflection, and which represents the energy that penetrates into the medium with smaller index of refraction. The same picture applies to metallic reflection; in this case, the surface wave propagates in the medium with the higher index of refraction. A discussion of both cases can be found in MAX BORN's *Optik.* Total internal reflection in single as well as multiple layers has been analyzed in great detail by ARZELIÈRES (7).

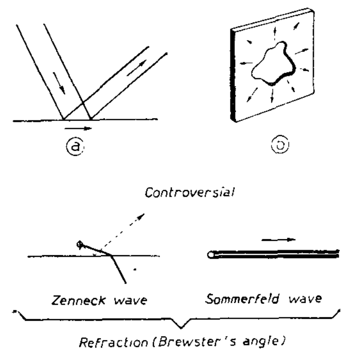

Fig. 2. – Types of surface waves I. Forced waves (continuous spectrum; radiation): *a*) metallic and internal reflection; *b*) aperture diffraction.

Surface waves arise on the shadow side of the screen in all cases of diffraction. Their existence is demonstrated most simply by use of the Fourier integral (8). A continuous spectrum of these waves runs in all directions in the aperture plane. No particular physical significance attaches to the individual surface wave, but the set as a whole represents the electromagnetic storage fields in the aperture region

Fig. 2 lists two further examples, concerning which there has been much controversy. Both involve the existence of surface waves along a conductor with loss. Both, too, are waves whose phase front is incident at Brewter's angle. When ZENNECK first proposed his wave some fifty years ago (9), he intended it to explain long-range radio propagation. It soon became apparent that ionospheric reflections constitute the true mechanism of long-range propagation. Experiments have consistently failed to show the existence of a ZENNECK wave in the field of ordinary radio antennas. Its theoretical existence, however, has been debated in innumerable articles, including several recent ones by T. KAHAN (10) who is a participant in this Symposium. The

(7) H. ARZELIÈRES: *Rev. d'Optique*, **27**, 205 (1948).
(8) H. G. BOOKER and P. C. CLEMMOW: *Journ. Inst. Elect. Engs.*, III, **97**, 11 (1950).
(9) J. A. STRATTON: *Electromagnetic Theory* (New York, 1941).
(10) T. KAHAN and G. ECKART: several articles in successive issues of the *Archiv d. Elektr. Übertr.*, **5**, (1951). See also articles by H. OTT in these issues.

controversy usually revolves around whether or not a surface wave component can be meaningfully separated out from the total field. The mathematical problem is quite complex, but the fundamental question, from our point of view, is whether the ZENNECK wave is forced or free. If it is forced, its separability or non-separability from the total field is a matter of formal manipulation and of little consequence. If it is free, hovewer, then it is always meaningful to separate it out and to attempt exciting it by sources more suitable than conventional radio antennas. For reasons which will be indicated in section **3**, I believe that the Sommerfeld and Zenneck waves are free modes. Preliminary experiments performed at our Laboratory indicate that the Sommerfeld wave can be excited on a lossy wire and will be guided by it around gradual bends. No experiments have as yet been performed with the Zenneck wave, but the two differ from each other only in being the cylindrical and plane version, respectively, of the same basic wave type, i.e., the *inhomogeneous* Brewster's angle wave.

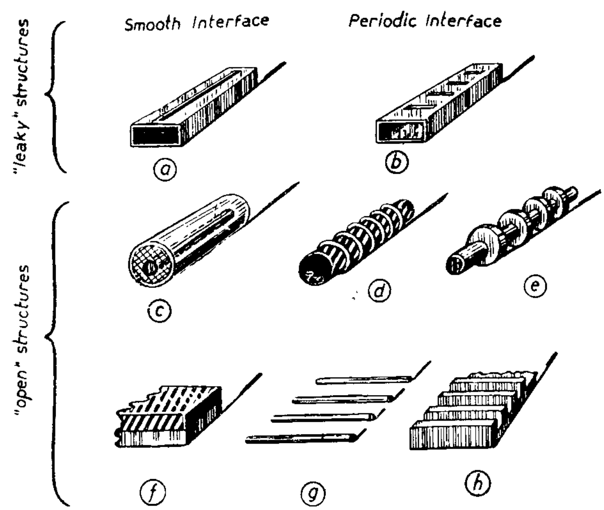

Fig. 3. – Types of surface waves II. Free modes (discrete spectrum; transmission).
a) channel guide; *b*) slotted guide; *c*) Goubau wire (dielectric rod); *d*) helix (rings); *e*) corrugated rod; *f*) slab (« ducts »); *g*) wire grid (Yagi); *h*) diffraction grating.

The chief structures capable of supporting free surface modes are shown in fig. 3. In the first column are the waveguides with smooth interface, in the second (and third) column are those with periodic interface. The first row is labeled « leaky » waveguides. This refers to conventional (shielded) waveguides which are coupled to space through a continuous or through periodic slots. Both of these types are familiar since the days of World War II. Detailed investigations of the long slot have been performed recently at Ohio State University (under Prof. V. RUMSEY), while the discrete slot arrays have been examined at the Hughes Aircraft Company and elsewhere in the United States, as well as abroad. The characteristics of the types are best calculated as a perturbation problem, starting with the undisturbed modes in a completely closed waveguide.

Our main interest in this paper centers on the truly open structures, displayed in rows 2 and 3. The rows differ only in that they show the cylindrical and plane version, respectively, of the same basic types. The first is a dielectric rod with a wire core. A large number of modes can be excited on this structure, and these have recently been calculated by a group working under Prof. BEAM at Northwestern University. When the dielectric mantle is thin, we have the now famous case of the GOUBAU [11] wire, the novel method of microwave transmission developed during the past few years. If, conversely, the conducting wire shrinks to zero, we have the ordinary delectric rod which has long been used as microwave antenna and, in optics, as a conductor of ordinary light into inaccessible regions inside the human body. Instead of the dielectric, a ferrite rod could be used. Calculations have been made by us which show that the free mode characteristics are about the same on both, with the ferrite rod having the advantage of smaller size at a given frequency.

The plane equivalent of the dielectric-covered wire is the slab shown in row 3. It may be backed by a metal wall, or surrounded on both sides by a dielectric with lower index of refraction. When the dielectric constant in the slab varies across its thickness, we have the configuration responsible for the « ducting » phenomenon in tropospheric propagation.

The second type of open waveguide is exemplified by the helix and the corrugated rod. Surface waves are propagated on these structures by virtue of the interface periodicity. Instead of a helix, we can use evenly spaced rings supported on a dielectric rod. Or, following MUELLER [12], we can sheathe the rod in fairly complicated patterns of conducting strips, obtaining a variety of surface wave propagation characteristics. An open waveguide with variable phase velocity can be built by inserting a ferrite core inside the helix and controlling the core permeability by means of a direct current sent through the helix wire.

The corrugated rod is usually an all-metal structure. It was first analyzed at our Laboratory [13]. The complementary construction, a corrugated dielectric rod, is most useful in the modified form in which beads of high dielectric constant are mounted on a low-dielectric connecting rod (cf. section **4**).

The plane equivalents of the helix and the corrugated rod are the wire grid and the diffraction grating, respectively. Surface waves on grids have been discussed by MACFARLANE [14]. Surface waves on diffraction gratings have already been mentioned as giving rise to Wood's anomalies.

Approximate equivalence relations hold between the smooth structures

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[11] G. GOUBAU: *Journ. Appl. Phys.*, **21**, 1119 (1950).
[12] G. E. MUELLER: *Proc. I.R.E.*, **40**, 71 (1952).
[13] W. ROTMAN: *Proc. I.R.E.*, **39**, 952 (1951).
[14] G. G. MACFARLANE: *Journ. Inst. Elect. Engs.*, III A, **93**, 1523 (1946).

and the periodic. Given a perfect conductor of radius r and a surrounding ferrite mantle of thickness t, the fundamental mode set up will have approximately the same characteristics as the fundamental mode on a sequence of rings, spaced a distance d apart and mounted on a ferrite core of radius r', provided the following relationship holds:

$$\left[\left(\frac{\mu_2}{\mu_1}-\frac{\varepsilon_1}{\varepsilon_2}\right)\frac{t}{2r}\right]_{\text{rod}}=\left[8\,\frac{\mu_2}{\mu_1}\left(\frac{r'}{d}\right)^2\right]_{\text{rings}},$$

where subscripts 1 refer to the outside medium (air), and subscripts 2 to the mantle and core, respectively. Similar relations exist if other structures are compared, such as a rod and a helix [15], or a dielectric tube and a corrugated rod. These relations have not been obtained by some unified way of reasoning, but by comparing the end results of calculations made for each separate structure. Occasionally it is possible to translate a structure capable of supporting TM waves directly into a complementary structure supporting TE waves of the same characteristics.

The open waveguides of fig. 3 have much in common. The total field excited on them can always be split into two parts: the free modes which propagate as on a transmission line, and the remainder field which is simply the free-space radiation field of the source modified by the presence of the waveguide structure. The field of a dielectric rod fed by a hollow cylindrical waveguide, for example, consists of all the free waves on the rod, plus the spherical wave radiated from the mouth of the hollow waveguide and refracted by the dielectric rod which partially lies in its path. These two fields are orthogonal to each other.

In the theory of conventional (shielded) waveguides, we have one or more free modes which propagate transmission-line fashion, and in addition a set of evanescent modes excited in the vicinity of sources and of discontinuities in the line. These two fields are also orthogonal to each other. The analogy between open and closed waveguides is therefore clear: a discrete set of free modes propagates on both, and in addition there is a remainder field made up of an infinite set of evanescent modes in the closed guide, and of a continuous spectrum on the open guide. The first represents stored (reactive) energy, and the second a radiating wave.

On a lossless open structure, the transmission line modes do not attenuate in the direction of propagation, while the radiation field decreases inversely as the distance from the source. At large distances, therefore, the field of the transmission line modes predominates in the vicinity of the waveguide.

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(15) H. KADEN: *Archiv. d. Elektr. Übertr.*, **5**, 399 (1951).

In the presence of small losses, on the other hand, the transmission line modes attenuate at a slow exponential rate. This means that they will dominate the source field beyond a certain distance, as before, but ultimately they die out more rapidly than the radiated wave. The radiation field is then the only one to remain in the far field, and we see that Sommerfeld's radiation condition is fulfilled for all open waveguides other than the (fictitious) lossless structures.

3. – Theoretical Discussion.

Free modes on open waveguides are best analyzed in terms of the systematized approach which MARCUVITZ and SCHWINGER introduced for the shielded cylindrical waveguide [16]. They reduce the solution of the electromagnetic vector field problem to two subsidiary scalar problems: an eigenvalue problem in the plane transverse to the direction of propagation, which determines the transverse propagation constants and thereby, as we know from section **1**, the entire free mode characteristics; and a transmission line problem for each mode amplitude in the longitudinal direction, which determines the magnitude and phase of the free wave propagating down the guide and reflected by the load.

The second problem — that of the transmission line — is easily solved. There is only one essential difference between the conventional and the open line: an obstacle placed in a shielded guide gives rise to higher-order waves, all of which are free modes and either propagate or attenuate along the guide; on an open guide, hovewer, the higher order waves are free modes only in part. Thus only part of the scattered energy continues to be in guided form, and the rest is radiated off.

The first problem — determination of the eigenvalues — is easy or difficult depending on the nature of the boundary conditions. As we know from textbook treatments of electromagnetic theory, Maxwell's equations reduce to the scalar wave equation in the transverse plane. Since the equation is homogeneous, all of its solutions are free modes. They can be found in a variety of ways. The electrical engineer likes to note that the eigenvalue equation imposed by the boundary conditions is equivalent to a condition of transverse resonance. In other words, the impedance at the interface when looking up into medium 1, plus the impedance when looking down into medium 2, must add to zero. The eigenvalues correspond therefore to free oscillations in a two-dimensional resonant structure; this justifies our usage of the term

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[16] N. MARCUVITZ and J. SCHWINGER: *Journ. Appl. Phys.*, **22**, 806 (1951).

« free modes ». The impedance of a TM wave looking into air is given by

$$\frac{\Gamma_{n1}}{j\omega\varepsilon_1},$$

which is seen to be capacitive. To match it out, we require an inductive surface.

As an example, consider a dielectric slab on metal. The lowest eigenvalue of the transverse wave equation, subject to the condition that the tangential electric and magnetic fields must be continuous across the interface, is given

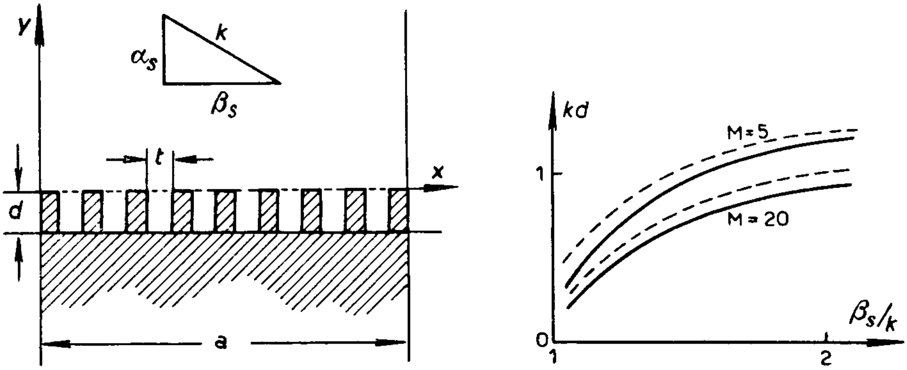


Fig. 4.

Above grooves:

$$H_z = \sum_{n=1}^{\infty} A_n \cos\frac{2n\pi x}{a} \exp\left[-\alpha_n y\right],$$

$$E_x = \frac{1}{i\omega\varepsilon}\sum_{n=1}^{\infty} \alpha_n A_n \cos\frac{2n\pi x}{a} \exp[-\alpha_n y],$$

Inside grooves:

$$H_z^p = B_0 \cos k(y+d) + \sum_{n=1}^{\infty} B_n \cos\frac{n\pi x'}{t} \exp[\beta_n y],$$

$$E_x^p = \frac{kB_0}{i\omega\varepsilon}\sin k(y+d) - \sum_{n=1}^{\infty} B_n\beta_n \cos\frac{n\pi x'}{t}\exp[\beta_n y].$$

Now

$$\int_0^a E_x H_z \,\mathrm{d}x = \sum_{p=1}^{M}\int_0^t E_x^p H_z^p \,\mathrm{d}x;$$

therefore

$$\boxed{\cot kd = \frac{2kt\left\{\frac{1}{\alpha_n a}\left[\int_0^a E^2 \cos_z \frac{2n\pi x}{a}\,\mathrm{d}x\right]^2 + \sum_{p=1}^{M}\sum_{n=1}^{\infty}\frac{t}{\beta_n t}\left[\int_0^t E_x^p \cos\frac{n\pi x}{t}\,\mathrm{d}x\right]^2\right\}}{\sum_{p=1}^{M}\left[\int_0^t E_x^p\,\mathrm{d}x\right]^2},}$$

by the following equation:

$$\alpha_{n1}\varepsilon_2/\varepsilon_1 = \sqrt{\beta_2^2-\beta_1^2-\alpha_{n1}^2}\ \mathrm{tg}\left(d\sqrt{\beta_2^2-\beta_1^2-\alpha_{n1}^2}\right)$$

The routine derivation of this expression is straightforward but lenghty. Instead, we write the resonance condition

$$\frac{\alpha_{n1}}{j\omega\varepsilon_1} = \frac{\beta_{n2}}{j\omega\varepsilon_2}\ \mathrm{tg}\,(d\beta_{n2})\ ,$$

and note that our equation follows immediately on making use of the Pythagorean relations in medium 1 and 2 so as to eliminate β_{n2}. The formula for β_s on a thin dielectric slab has already been stated in section **2**. It follows from our present equation by using the Pythagorean relation once more, and assuming $d \gg \beta_2$.

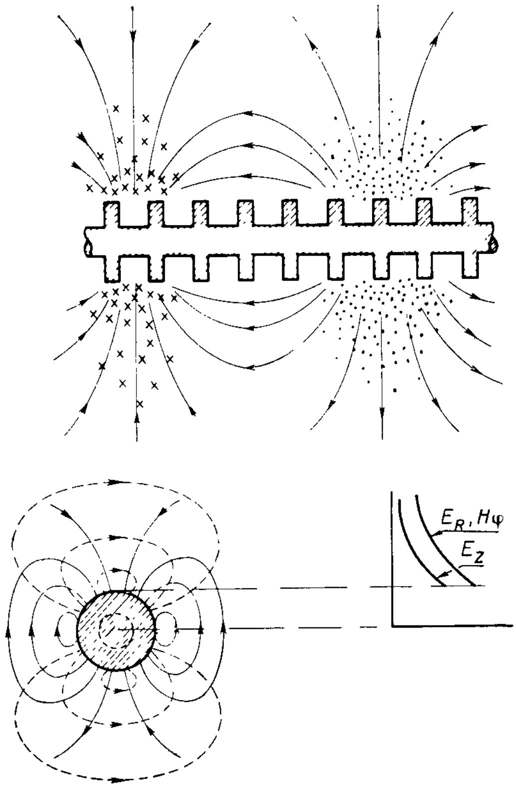
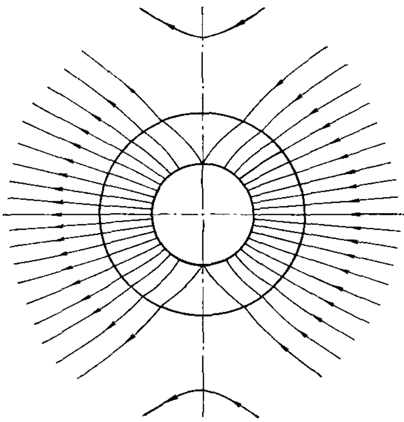


Fig. 5. – Fields on corrugated rod.

Fig. 4 displays the elegant variational method, due to SCHWINGER, which permits an accurate determination of the eigenvalues in fairly complicated regions. The dominant wave travelling inside the grooves is the simple TEM mode. The power flow downward into the grooves is equated with the power flowing up. If we write this expression in terms of the unknown tangential electric field at the interface, we can recast it in a form such that a first order error in the trial field produces only a positive second order variation in kd (k is the phase constant in free space). This establishes a lower bound on kd, while analogous use of a trial magnetic field at the interface establishes an upper bound. Calculations performed by W. LUCKE at the Stanford Research Institute have shown that the propagation constant is closely determined after a single try.

The situation gets more complicated when the dielectric constant or the permeability vary in the transverse direction. This occurs in atmospheric ducts, for example, and obliges one to solve not the simple wave equation with constant coefficient but Schrödinger's equation of quantum mechanics. Every problem of electromagnetic waveguiding on transversely inhomogeneous structures has a potential-wall analogue, and well known approximations such as the WKB method apply to the one branch of physics as well as the other.

Our knowledge of the transverse and surface propagation constants allows



HE_{11}; $c = 0{,}5$; $b/\sigma = 0{,}5$; $\lambda_g/\lambda = 0{,}732$

Fig. 6 *a*.

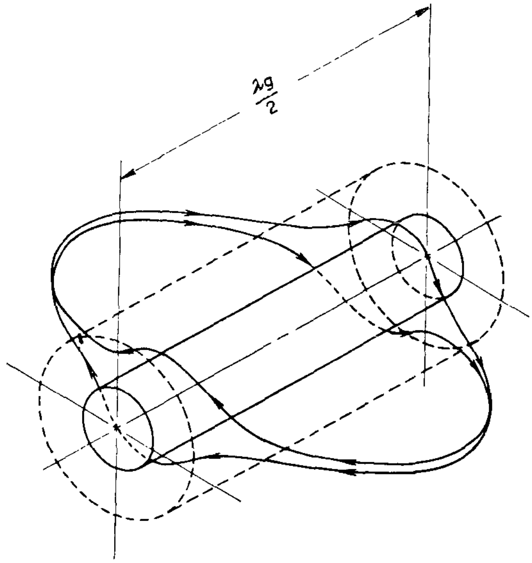


Fig. 6 *b*.

us to construct the mode configuration. Fig. 5 illustrates the corrugated rod. Only the lowest order wave propagates on this structure, all the others being sharply attenuated by the copper losses. An interesting mode is shown on figs. 6*a* (electric field) and 6*b* (magnetic field) (*). This is a so-called hybrid wave, having both electric and magnetic longitudinal components. It propagates on a dielectric covered (GOUBAU) wire, and has no cut-off frequency, just as the fundamental TM mode examined by GOUBAU. All other modes are cut-off when the radius or the dielectric thickness are not large enough. Two more illustrations ([13]) are given in figs. 7 and 8, showing comparison between experimental and theoretical results for dielectric-filled corrugations and a spirally grooved copper rod. These were made at our Laboratory.

We mentioned at the end of section 2 that the free modes on an open waveguide do not form a complete set of eigenvalues, a situation unlike that in

(*) By kind permission of Prof. BEAM of Northwestern University, and the U.S. Army Signal Corps under whose contract these figures were prepared.

shielded waveguides, where the eigenvalues are infinite in number and therefore constitute a complete set. Consider an open cylindrical guide, for example. Its eigenvalues are given by the resonance condition

$$\frac{\mu_1 \Gamma_{n1}}{\Gamma_1^2} \frac{H_0^{(1)}(-j\,\Gamma_{n1} r)}{H_1^{(1)}(-j\,\Gamma_{n1} r)} = \frac{\mu_2 \Gamma_{n2}}{\Gamma_2^2} \frac{J_0(-j\Gamma_{n2} r)}{J_1(-j\Gamma_{n2} r)}\,,$$

where r is the radius of the rod (medium 2), embedded in medium 1. $H_0(-j\Gamma_{n2} r)$ increases exponentially when the imaginary part of its argument (i.e., the transverse attenuation outside the guide) is negative. To satisfy the radiation condition, then, it is necessary to restrict ourselves to solutions with non-negative α_{n1}. This immediately restricts the roots to a finite number.

Application of the Pythagorean relations in the two media to the eigenvalue equation shows, furthermore, that no roots exist if Γ_2 is less than Γ_1. This means that no free modes can be excited in a region with dielectric constant less than that of the surrounding medium, such as an ionized layer in air.

In a shielded waveguide, a propagating mode becomes evanescent below cut-off. ADLER has shown ([17]), however, that no evanescent modes can exist on an open guide, so that a propagating wave simply ceases to exist at cut-off frequency. This is merely another indication of the fact that the part of the solution which was formerly represented by the set of evanescent modes is still missing. In terms of the Green's function associated with the boundary conditions, what we have determined so far is its discrete spectrum, while its continuous spectrum, representing the radiating remainder field mentioned in section **2**, is yet to be calculated. This task is best performed in the plane of the complex propagation constant, or one of its simple transforms.

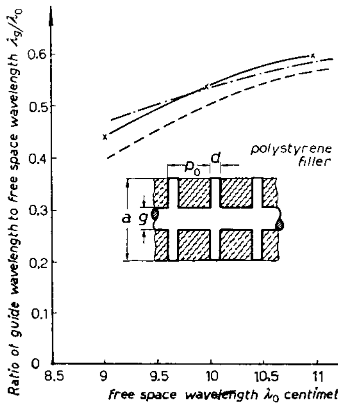


Fig. 7.

Dimensions: $a = 0{,}960$ in.; $p_0 = 0{,}245$ in.; $g/a = 0{,}209$; $d/p_0 = 0{,}25$; For Polystyr.: $\varepsilon = 2{,}56$
———x Measured values; corrugations filled with dielectric.
- - - - - Theoretical values; corrugations filled with dielectric.
- — - — Measured values; unfilled corrugations.

In setting up the integral for the complete field, we choose a Green's function appropriate to the source. By use of the Fourier transform, an integral ex-

([17]) R. B. ADLER: *Proc. I.R.E.*, **40**, 339 (1952).

pression for the field at any point can then be found. The integrand has poles and branch points in the Γ-plane. By deforming the contour in a convenient manner, the integral is split into the sum of residues from the poles, plus a remainder term which is evaluated asymptotically by the saddle point method ([18-21]). The poles occur when the denominator of the integrand is zero, and it turns out that their location coincides with the eigenvalues of the transverse wave equation. « Poles » and « free modes » are coextensive terms, therefore. The remainder field depends entirely on the source distribution, except that its branch points always occur at Γ_1 and Γ_2. In some instances, the remainder integral can be split further, one term being identifiable as the local storage field of the source, while the other is evaluated in the far field, as before.

By placing the branch cuts along arcs of rectangular hyperbolae starting at the branch points and reaching to infinity along the real axis, a further observation can be made ([18]). No contribution is made to the remainder field unless the branch cut belongs to the outermost medium, i.e., the one which reaches to infinity. A dielectric rod, for example, appears to have two branch cuts at first, one starting at Γ_1 and the other at Γ_2. The first, however, produces an integrand which is an even function of both Γ_{n1} and Γ_{n2}, and thus can be shown to vanish. The second branch cut, evaluated at large distances, gives the radiation field of the dielectric rod. If the rod is surrounded by a metal shield, this second branch cut disappears, too. In its stead, there appears a collection of poles, densely spaced along a line slightly below the former branch cut if the shield is several wavelengths in diameter, or less densely spaced and further removed from the former cut if the shield is of smaller radius. When shield and rod radius coincide, we have an ordinary

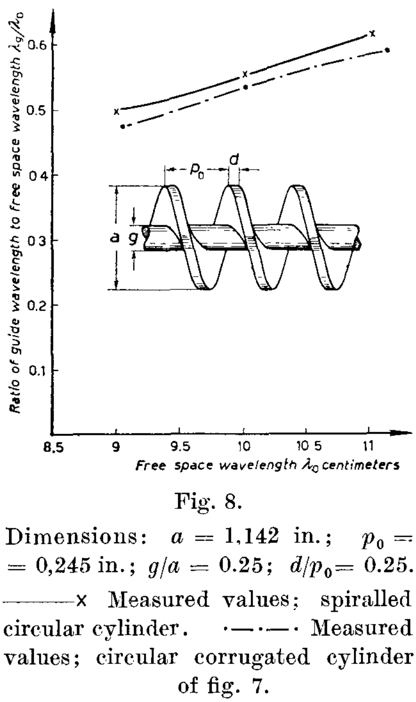


Fig. 8.

Dimensions: $a = 1{,}142$ in.; $p_0 = 0{,}245$ in.; $g/a = 0.25$; $d/p_0 = 0.25$. ——x Measured values; spiralled circular cylinder. ·—·—· Measured values; circular corrugated cylinder of fig. 7.

([18]) G. M. Roe: *Thesis* Univ. of Minnesota (1947) (unpublished).
([19]) C. T. Tai: *Journ. Appl. Phys.*, **22**, 405 (1951).
([20]) H. Ott: *Zeits. f. angew. Phys.*, **3**, 123 (1951).
([21]) R. M. Whitmer: *Journ. Appl. Phys.*, **23**, 949 (1952).

dielectric filled waveguide, and the poles lie along the real and imaginary axis (evanescent and propagation modes, respectively).

By analizing cylindrical structures in terms of their poles and branch cuts, we can make a continuous transition from the hollow cylindrical waveguide with perfectly conducting walls to the single wire conductor, including all types of open waveguide (with smooth interface) on the way. We begin with the dielectric-fillled cylindrical waveguide, and allow its walls to be real metal instead of a perfect conductor. The number of poles suddenly stops being infinite; in standard X-band guide, for example, there are some 10^{30} of them. This number is still incredibly large, but the important point is that the set of free modes is no longer complete. We expect to find a (negligible) contribution from the branch cut corresponding to the lossy metal, since this medium extends out to infinity. As the conductivity of the outer medium is further reduced, the Γ_1 branch point moves in from its high values in metal, until with zero conductivity outside it comes to lie on the imaginary axis. This represents the dielectric rod in air; all of its poles lie along the imaginary axis between the two branch points, in agreement with our earlier assertion to the effect that evanescent modes cannot exist on the open waveguide. If now the conductivity of the inner medium is allowed to rise, the inside branch point Γ_2 moves off the imaginary axis. When the inside has become an ordinary wire, Γ_2 will be found at the point occupied by Γ_1 in the case of the cylindrical waveguide with metal walls. A number of higher-mode poles cluster near Γ_1. *One* pole, however, remains near the imaginary axis, and this corresponds to the Sommerfeld lossy wire mode mentioned in section **2**. When the wire becomes a perfect conductor, this pole falls right into Γ_2. Physically speaking, this means that the mode has become fictitious in that its input impedance is infinite.

The pole and branch cut method just described enables us to find the relative amplitude of the set of surface waves excited by a given source, as well as of the remainder field. A rival method has recently been developed by GOUBAU [22] whereby we can compute the surface wave amplitudes, though not the remainder field (except in some instances). It by-passes integration in the Γ-plane through clever use of the reciprocity theorem. Applications of this method to practical problems have yet to be made.

4. – Surface Wave Experiments.

(a) *Measurements.* – The properties of surface waves are measured much like those of any guided mode. Probes must be designed with chokes to pre-

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([22]) G. GOUBAU: *Proc. I.R.E.*, **40**, 865 (1952).

vent the probe arm from picking up the remainder field through which it invariably passes. Fig. 9 shows a typical setup for measurements along the corrugated rod. The termination is first matched into space, resulting in a standing wave ratio close to unity. The field along the line is found to vary rapidly in phase and amplitude throughout the first few wavelengths from the source, as the direct radiation from the feed and the surface wave interfere constructively or destructively with each other. With an efficient transition, the source field dies out rapidly, and the amplitude is fairly constant from then on, modified only by a slight exponential decline due to heat losses in the copper. In the lower part of fig. 9 we see a disk large enough to reflect essentially all the power along the line. This provides us with the moveable short required for conventional transmission line measurements of input impedance and of the effect of discontinuities placed along the rod. These discontinuities, as we mentioned in section **3**, reflect the energy incident upon them only in part as a guided wave, the rest being radiated off in place of the evanescent waves which would arise in a shielded waveguide. This implies that discontinuities must be treated as *six*-terminal networks, not four. The effect of a small obstacle is much like that of a high-impedance slot in waveguide walls, and the equivalent network parameters are determined in the same way.

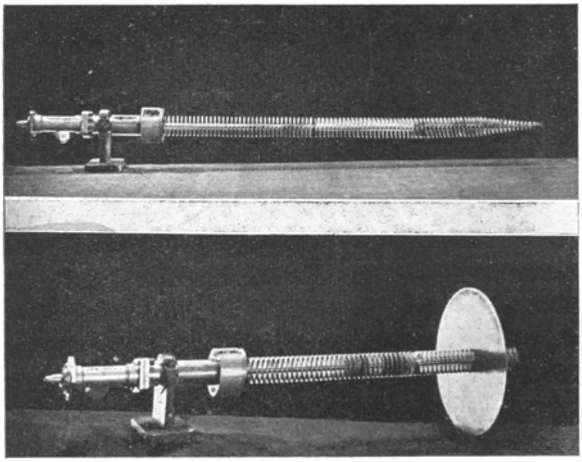


Fig. 9.

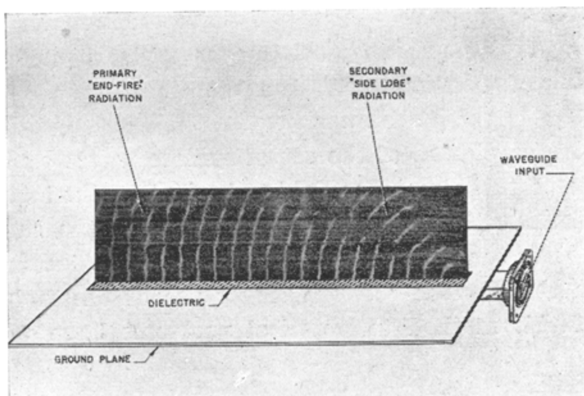


Fig. 10.

Fig. 10 shows phase contours obtained with the phase and amplitude plotter described in our paper on « Recent Developments in Microwave Diffraction » elsewhere in this volume. The direct radiation from the source is plainly visible, as are the phase fronts of the guided wave which lie at approximately ninety degrees to the amplitude contours. The three open waveguides of fig. 11 are the corrugated rod, the corrugated strip, and the grooved spiral rod. Binomial transformers effect a smooth transition from the waveguide to the strip and back into waveguide again. The corrugation depth is less than one-quarter of a wavelength, giving an inductive strip as required by theory. Structures of this sort are inherently narro - wband, but periodic variations in groove depth ameliorate this condition if desired. Theory is sufficiently far advanced to account for the effect on the mode characteristics of groove spacing, depth and thickness, as well as of the strip width.

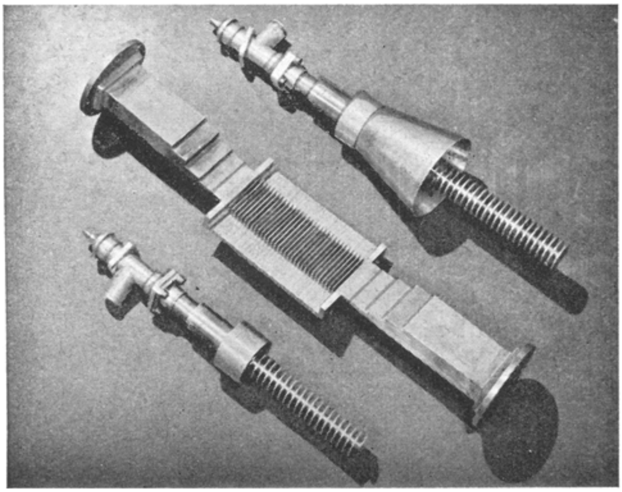


Fig. 11.

If a horn is placed near a dielectric sheet (fig. 12), some of the energy emerging from it is trapped by the sheet in the form of surface waves. This is much like what occurs in atmospheric ducting, except that the dielectric constant is a function of thickness in that case. WHITMER ([21]) has calculated the trapping coefficient (i.e., the relative energy abstracted by the surface modes) assuming a line source embedded in the slab. The coefficients for the several modes depend, of course, on the particular source assumed. It can be stated as a gene-

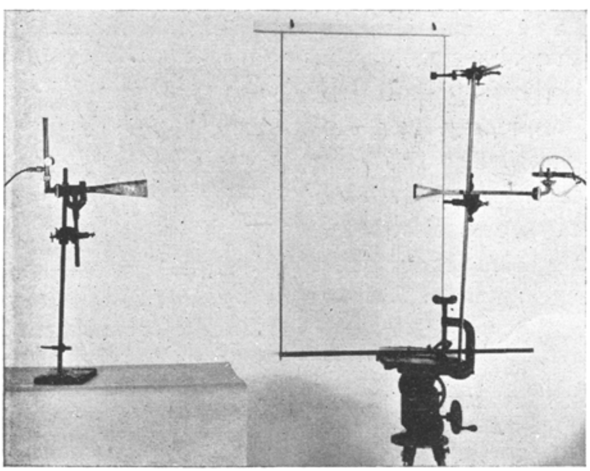


Fig. 12.

rally valid approximation for most sources, however, that the amplitude of the n-th mode is rather small until the frequency reaches cut-off for the $(n+1)$–st mode, whereupon it rises rapidly to its high frequency (large slab thickness) asymptotic value.

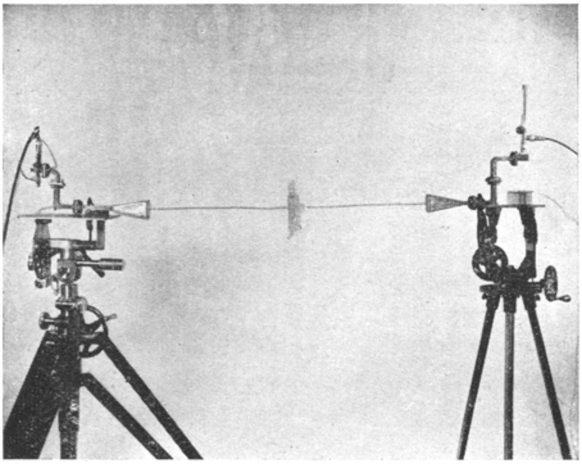


Fig. 13.

Fig. 13 shows an experimental setup for the Sommerfeld wave on a single wire. The material used is nichrome, which is somewhat lossier than copper but has the advantage of not oxydizing in air. The experiment was intended to establish the existence of this wave concerning which there has been some doubt. For this purpose, an oxydized layer had to be carefully avoided since it represents a dielectric mantle capable of supporting a Goubau-type mode. The small device in the center is a tinfoil aperture supported on a block of polystyrene foam; its presence has no noticeable effect on the guided field which is closely enough bound to the wire so that 95% of its energy passes through the hole. Bends along the wire cause some losses, but not more than on other open waveguide structures.

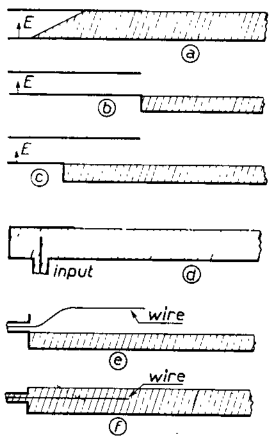


Fig. 14. – *a*) Waveguide to channel transition; *b*) Horn to channel transition; *c*) Modified horn to channel transition; *d*) Dipole-reflector input; *e*) Overhead wire exciter; *f*) Recessed wire exciter.

(b) *Excitation.* – The excitation of surface waves presents special problems. We generally desire to couple as much as possible of the energy from the source into some particular free mode. Fig. 14 shows six possible transitions, all of which have been tried at our Laboratory (by W. Rotman), and elsewhere. It is possible to obtain transfer efficiencies of over 90%, so that only 10% of the energy is lost in direct radiation from the source. A perfect match could be achieved by providing a source whose phase and amplitude distribution across a transverse plane is exactly like that of the mode to be excited. This would require sources extending to infinity, but the loss incurred by using finite size (e.g., a large horn aperture) need not be large ([11]). The source aperture can, furthermore, be virtual instead of real. In other words, we can use one of the sources in fig. 14 despite the fact that the phase and amplitude distribution is quite incorrect at the feed point itself. Within a couple of wavelengths or more from the feed, the field will nonetheless approximate the desired distribution in a transverse plane. The one thing to note is that the feed must always provide one with a strong inhomogeneous wave component, since surface waves are themselves inhomogeneous. It so happens that most electromagnetic sources have strongly inhomogeneous near fields, and almost any kind of source will therefore excite at least a weak component of surface wave, provided merely that it furnishes the proper polarization. The horn of fig. 12 is fed by a TE_{01} waveguide, and excites trapped TE modes in the dielectric slab if placed in such a way that its aperture is within a wavelength or two from the surface. When placed further away, the field emerging from the horn begins to approximate an ordinary plane wave, and lacks the inhomogeneous component necessary for surface wave excitation.

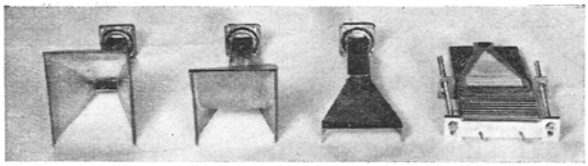
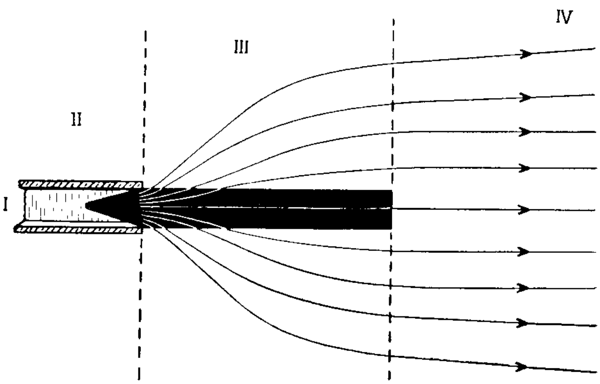


Fig. 15.

Three horns for launching waves on a corrugated strip are shown in fig. 15 The uncovered bottom surface is on the right, with the grooves starting near the mouth and deepening gradually toward the aperture. The first horn on the left has a large enough aperture for efficient excitation of the surface wave. Its flare angle is a bit too steep, however, and as a result higher order modes appear in the horn, whose energy is lost in the radiation field. The second horn is exponential, so as to throw the radiated beam off at an angle to the corrugated strip. This enables us to measure the mode properties relatively unperturbed by the source field. Once again, however, the flare is too steep for efficient power transfer to the surface. In the third horn we go to the opposite extreme of allowing no flare, and thus no higher modes. This construction is the most efficient of the three, but it is not optimum, as the aperture is now too small to simulate the required field distribution over the entire transverse plane. The best solution undoubtedly lies in a compromise between the three, i.e., a horn with gentle exponential flare. This, however, has not yet been tried.

(c) *Radiation.* – When a hollow waveguide is opened by a transverse cut, some of the energy is reflected back into the guide at the discontinuity, but most of it leaks into space to establish a radiation pattern which is calculated by integrating the field distribution over the aperture. To be quite accurate, this integration must be performed not only over the physical aperture, but over the outside wall of the waveguide as well, because currents flow there which are not negligible in the vicinity of the aperture. If the waveguide is terminated in a transverse baffle, the integration is performed over the entire plane.



16. – End-fire antennas.

An open waveguide cut to finite length will also radiate. It therefore constitutes an antenna, and we ask ourselves how to calculate its radiation pattern. Fig. 16 shows a dielectric rod fed from a circular waveguide. The spectra in regions II and IV are wholly continuous (radiation), discrete in region I (guiding), and mixed in region III. A major portion of the energy from the hollow waveguide is transferred into free modes along the rod; some of it, however, emerges as a remainder field which spills into region II and accompanies the free modes in region III. At the end of the rod, the discontinuity sets up small reflected waves in the form of the incident free modes, and also radiates a remainder field off into space. The conventional way of finding the far field pattern of the rod as a whole is to integrate the tangential electric field over its entire surface. This creates some difficulty for physical intuition, since the main contribution to the tangential fields comes from the free modes which constitute precisely the *non*-radiating part of the total field. To avoid this apparent paradox, we can integrate instead over the transverse plane separating regions III and IV. Schelkunoff's Equivalence Theorem guarantees that the result of either integration is the same. As the length of the rod is increased, the gain goes up – though not monotonically, as the direct source field is at times in phase with the main beam and at times out of phase. Beyond a certain length, the gain increases no further: the contribution of the source field, which decreases inversely as the distance from the feed, has now become negligible in the transverse aperture plane when compared to the amplitude of the free mode field, and no further changes in the aperture distribution, and hence in the far field pattern, can be expected.

If the dielectric rod or slab is partially clad with metal, as in the channel guides investigated at Ohio State University and by W. ROTMAN at our Laboratory, the surface wave velocity may exceed that in air. The main beam then emerges at an angle given approximately by arc cos (β_1/β_s), and it is physically more meaningful to extend the integration over the entire length of the antenna rather than a transverse plane.

Although the surface wave antenna lends itself best for end-fire applications, it can also be made to produce a broadside beam. The simplest example is given by the helix, one of whose basic modes radiates in that fashion. We could also place radiating discontinuities along the surface, or, better still, use a method due to MUELLER ([12]) who loaded a dielectric rod with periodic disks of high dielectric constant. The disks bind their surface wave so closely that its amplitude is negligible compared to the amplitude of the surface wave on the rod itself; the disks are then spaced in such a way that the in-between sections of dielectric rod radiate in phase.

The field distribution of an antenna and its radiation pattern are related as a pair of Fourier transforms. It is possible, therefore, to obtain all sorts of beam shapes by controlling the amplitude distribution in the desired manner, and by speeding or slowing up the surface wave to obtain the desired phase distribution. It has been shown by R. C. SPENCER ([23]) that if the amplitude distribution is expanded as a power series, the corresponding radiation pattern becomes a series of ascending-order derivatives of the diffraction pattern for a constant-amplitude source. The odd derivatives vanish if the amplitude along the surface is an even function with respect to the aperture center; as a result, the pattern is of the type shown in fig. 17*a*, with major and minor lobes separated by deep nulls. If, however, the amplitude distribution is asymmetric, then the odd derivatives reach maxima whenever the even derivatives are zero, and vice versa. As a result, the pattern can have no nulls and looks as sketched in fig. 17*b*. The surface wave antenna is therefore capable of producing a shaped beam, for example of the *csc*^{2} type so frequently used in radar. A detailed analysis of antenna beam shaping has been made by A. S. DUNBAR of the Stanford Research Institute ([24]).

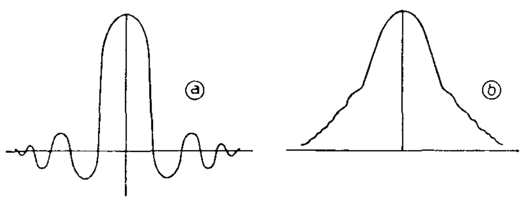


Fig. 17.

Another practical application of surface wave antennas lies in the con-

([23]) S. SILVER: *Microwave Anntenna Theory and Design*, Radiation Laboratory Series Vol. 12 (New York, 1949).

([24]) A. S. DUNBAR: *Journ. Appl. Phys.*, **23**, 847 (1952).

struction of flush-mounted arrays, such as the one illustrated in fig. 18. This combination of the broadside array effect with the individual end-fire antennas results in very sharp beams and low side lobes.

One final comment regarding the radiation field of unshielded waveguides: It is always orthogonal to the free modes, but there is a difference depending on whether the guide is dissipative or not. If there is no loss, the orthogonality relation

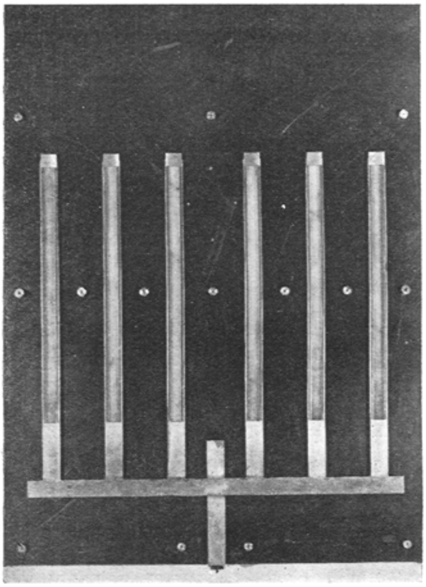


Fig. 18.

$$\int_{\Sigma} E_s \wedge H_r^* \, d\sigma = 0,$$

holds in any transverse plane Σ, where E_s is the electric surface wave field and H_r^* the conjugate magnetic radiation field (due to the residues and branch cut, respectively). This relation implies that the power in the radiation field is entirely separate from the power in the guided field, so that the source couples energy into two parallel circuits with no mutual coupling between them. If, on the other hand, the structure is dissipative, as for example the Sommerfeld wire or the Beverage wave antenna over a lossy earth, the orthogonality relation is weakend to read

$$\int_{\Sigma} E_s \wedge H_r \, d\sigma = 0.$$

The guided and radiated electromagnetic fields are still represented by the residues and branch cut, respectively, but their *power* can no longer be separated. Due to the presence of coupling terms, the total power delivered by the source is no longer the sum of the power in the surface waves and the power in the remainder field. The open waveguide is seen to be related to space like a cylindrical transmission line to a spherical one, with mutual coupling between them.

5. – Conclusion: Comparison with Optics.

By a suitable scaling of all dimensions, we may expect to find the surface wave phenomena just described for the microwave region to exist in optics as well. There will be two important differences, however, quite apart from the usual ones which stem from the fact that optical sources are random in phase and polarization:

(*a*) The plastic tube which conducts light into inaccessible regions within the human body supports ten thousands of modes, not just a few as in the microwave case. In the presence of so many poles, the branch cut contribution is negligible. It can be shown by means of the saddle point method that the group of modes as a whole behaves according to the laws of geometric optics, i.e., the totality of modes constitutes an equivalent wave which bounces down the tube in a series of total internal reflections. The amount of energy carried outside the tube diameter is quite negligible.

(*b*) The property of metals changes with increasing frequency. In the microwave region, a good conductor has a dielectric constant which is wholly imaginary; the input impedance is therefore half resistive and half inductive. This is the condition for the existence of the Zenneck wave on a flat sheet, a wave apparently so hard to excite that it has not been observed as yet (some authorities, as we have noted, deny its existence altogether). At optical frequencies, however, the dielectric constant of metals is essentially negative, which makes the surface purely inductive. This means that optical surface waves are as easily excited on metal as microwaves are on corrugated sheets or in dielectric slabs. It follows that we may expect a difference between optical and microwave diffraction phenomena. As we saw in section 2, forced surface waves are set up on the shadow side of a diffracting screen. These waves cannot be expected to transfer any appreciable energy from the diffracted beam into the Zenneck wave. In the optical case, however, the metal is inductive and a good deal of energy might go into the free surface mode. As a result, the field intensity near the screen ought to be greater with ordinary light than with microwaves, an effect which has never been studied.

Two papers have recently appeared which examine diffraction gratings from the microwave point of view ([25,26]). The Wood anomalies themselves have been re-investigated in great detail by PALMER ([4]). In order to improve older theories, and to explain the new experimental facts brought to light

([25]) E. M. T. JONES: *Proc. I. R. E.*, **40**, 721 (1952).
([26]) A. BOIVIN: *J. Opt. Soc. Am.*, **42**, 60 (1952).

by this work, TWERSKY (5) has used rigorous electromagnetic techniques, many of which were developed in recent years for application to microwave problems. We see, therefore, that a theory which began some fifty years ago with a purely optical discovery is now, half a century later and much improved by microwave practice, being successfully re-applied to the very phenomenon from which it took its start.

Acknowledgments.

All work on surface waves at the Air Force Cambridge Research Center has been performed under the direction of Dr. R. C. SPENCER, Chief of the Antenna Laboratory, and of Mr. R. M. BARRETT, Chief of the Airborne Antenna Branch. The author is also indebted to Dr. G. GOUBAU of the U.S. Signal Corps Engineering Laboratories and Dr. A. A. OLINER of the Polytechnic Institute of Brooklyn for a number of helpful discussions on the general nature or surface waves.

INTERVENTI E DISCUSSIONI

— G. TORALDO DI FRANCIA:

a) One of Mr. ZUCKER's remarks reminds me of a very peculiar experiment, where evanescent waves are involved. When a plane wave is incident on a grating, we have a set of waves diffracted by reflection and a set of waves diffracted by transmission. When we increase the angle of incidence, any two corresponding waves of the two sets disappear at the same time, becoming evanescent waves. But if the grating is placed on the boudary of two media having different refractive indices, it can very well happen that one of the reflected waves disappears before the corresponding transmitted wave. In this case the experiment shows that as soon as the former wave disappears, the latter becomes much more intensive. That is what I called « total transmission » because of its close analogy with total reflection.

b) What precisely is your feeling about the existence of the Zenneck wave?

— F. J. ZUCKER:

My « feeling » is that the Zenneck wave should exist as a « free wave » to the extent that we can speak of free waves at all in connection with lossy systems. Let us call it a quasi-free mode to indicate that power must be supplied by the source to compensate for the ohmic losses. But it is still a solution of the transverse resonance problem, and thus not a forced wave.

I have tried to point out in the above paper that *all* types of surface waves must ultimately die out in order to satisfy the radiation condition at infinity. The Zenneck wave is no exception. As OTT has shown (1), interference between the Zenneck and the space wave « eats up » the former at a certain distance. This may occur fairly close to the source if we use a dipole, which is a most inefficient exciter of the Zenneck wave. The range may, however, be much extended if we assume a long vertical wire driven with the phase and amplitude distribution of the Zenneck wave itself (2). The effective excitation of the Zenneck wave over large distances is therefore contingent on finding a suitable source. An ordinary radio antenna is certainly *not* suitable, and chances of finding a good exciter that is technically realizable are probably poor.

— A. C. S. VAN HEEL:

With waves in the earth considerable difficulties arise, when one tries to explain certain phenomena. Especially the relatively large intensity of the so-called Fermat-wave, occurring when explosion waves are reflected against layers with high wave velocity, is hard to understand. Could we expect some help from microwave theory and experiments, or perhaps from analogies?

— F. J. ZUCKER:

Yes. Acoustic surface waves have been observed by Dr. KOCK of the Bell Telephone Laboratories along a disc-on-rod structure similar to the one shown here.

Surface waves have also been observed, in connection with explosions along the interface between two liquids of unequal density (3). I don't know, however, to what extent their possible role in earthquakes has been investigated.

There is, finally, some evidence that surface waves of the de Broglie type can exist in the potential wall along a crystal surface. The anomalies which they cause in molecular beam diffraction are known as the « dellen » effect (4).

— J. SIMON:

L'onde de choc incidente sous une incidence très oblique n'est plus réflechie en onde de choc (phénomène de Mach) (5); il ne semble pas qu'il y ait des ondes de choc de surface.

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(1) H. OTT: *Arch.d. Elektr.*, Uebertragung, successive articles in the winter and spring issues of 1952.
(2) G. GOUBAU: *Zeits. f. Angew. Phys.*, Zenneck Festheft (Spring, 1951).
(3) O. VON SCHMIDT: *Phys. Zeits.*, **39**, 868 (1938).
(4) S. FRISCH: *Zeits. f. Physik*, **84**, 443 (1933).
(5) R. J. SEEGER and H. POLACHECK: *Jour. Appl. Phys.*, **22**, 640 (1951).