*Radio Science*, Volume 11, Number 7, pages 611-619, July 1976

# Electromagnetic penetration through an aperture in an infinite, planar screen separating two half spaces of different electromagnetic properties

*Chalmers M. Butler and Korada R. Umashankar*

*Department of Electrical Engineering, University of Mississippi, University, Mississippi 38677*

(Received January 19, 1976.)

Integro-differential equations are formulated for the general problem of electromagnetic diffraction by an aperture in a planar conducting screen of infinite extent separating two half spaces of different electromagnetic properties. With the aperture specialized to the case of an infinite slot, equations appropriate for TE and for TM illumination are deduced from those for the general aperture. These slot equations are solved numerically, and results are presented for several cases of interest.

## INTRODUCTION

Even though electromagnetic diffraction by an aperture in an infinite conducting screen has been the subject of much research over the years, almost all interest has been limited to the situation in which the screen resides in a homogeneous medium with little attention given to the special case of the screen separating half spaces of different electromagnetic characteristics. It is the purpose of this paper to investigate this special, but interesting, two-media case.

The two-media problem has been investigated for the special case of a narrow slot aperture by *Barakat* [1963] and by *Houlberg* [1967]. They develop equations for the slotted screen problem and present solution techniques applicable when the slot is narrow relative to the wavelengths in the two media; they give data for transmission and reflection coefficients. *Thomas* [1969] derives equations for a circular aperture and provides approximate formulas for transmission and backscatter coefficients with tabulated data valid for circles small relative to the wavelengths in the two media. Bethe theory has been extended to cover the two-media problem and polarizabilities have been calculated for four aperture shapes of practical importance (F. De Meulenaere and J. Van Bladel, personal communication, 1975).

In this paper, integro-differential equations are developed for the problem of an aperture of general shape in a perfectly conducting screen, of vanishing thickness and infinite extent, separating half spaces of different electromagnetic characteristics with illumination incident upon the aperture/screen in both half spaces. The only feasible way to solve the two-media equations is numerically, but, even with the relatively simple rectangular aperture, solving them this way would be very demanding upon computer storage. Therefore, in order to simplify the equations to be solved and yet to retain the essential features of the two-media problem, the aperture is specified to be an infinite slot of uniform width and the illumination is limited to the two cases in which the slot field is either transverse electric (TE) or transverse magnetic (TM) to the slot axis. The general aperture/screen equations for the two-media problem are specialized to the TE and TM slot cases and these reduced equations are solved numerically. The numerical solution procedures employed are valid for any reasonable slot widths, and representative data are presented for several cases of interest.

## FORMULATION

In the aperture $A$ and tangential to the aperture/screen plane, there exists some total electric field $\mathbf{E}_t^a$, which is, of course, unknown *a priori*. In terms of $\mathbf{E}_t^a$, the incident fields radiated by the specified sources, the electromagnetic properties of the two half spaces, and the aperture/screen geometry (Figure 1), one can write expressions for the magnetic field in each half space. The enforcement of continuity of transverse magnetic field through the aperture leads directly to the desired integro-differential equation for the problem under investigation.

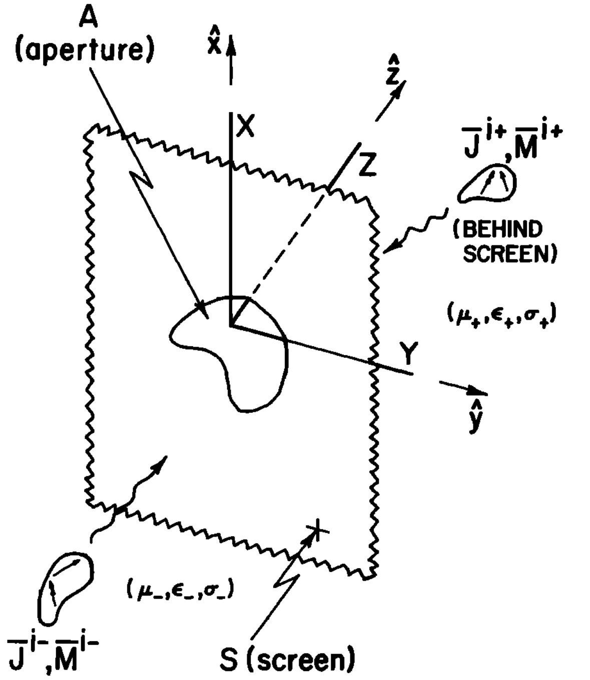


Fig. 1. Aperture in conducting screen separating two media of different electromagnetic properties.

Figure 2 depicts a step-by-step reduction of the left half-space ($z < 0$) problem to a simple equivalent problem in a form which readily suggests how one may develop an expression for the left half-space total magnetic field $\mathbf{H}^-$. This figure is only schematic and directions of vectors shown are not to be interpreted as actual directions. In Figure 2a, one sees the original problem while in Figure 2b the aperture-perforated screen is seen to be replaced by a perfectly conducting (shorted aperture) plane with the original tangential electric field $\mathbf{E}_t^a$ in the aperture restored at $z = 0^-$, $(x,y) \in A$, by an appropriate magnetic surface current $\mathbf{M}_s$ which is specified to have a value $-\mathbf{E}_t^a \times \hat{z}$, i.e., $\mathbf{M}_s = -\mathbf{E}_t^a \times \hat{z}$. Notice that this as-yet-undetermined magnetic current radiates in the presence of the conducting screen and that the left half-space sources $\mathbf{J}^{i-}$ and $\mathbf{M}^{i-}$ of the original problem in Figure 2a illuminate the shorted screen. Next, from image theory, one removes the conducting screen and arrives at the left half-space equivalent problem of Figure 2c.

The total left half-space electromagnetic field $(\mathbf{E}^-, \mathbf{H}^-)$ is the sum of the field radiated by $\mathbf{M}_s$

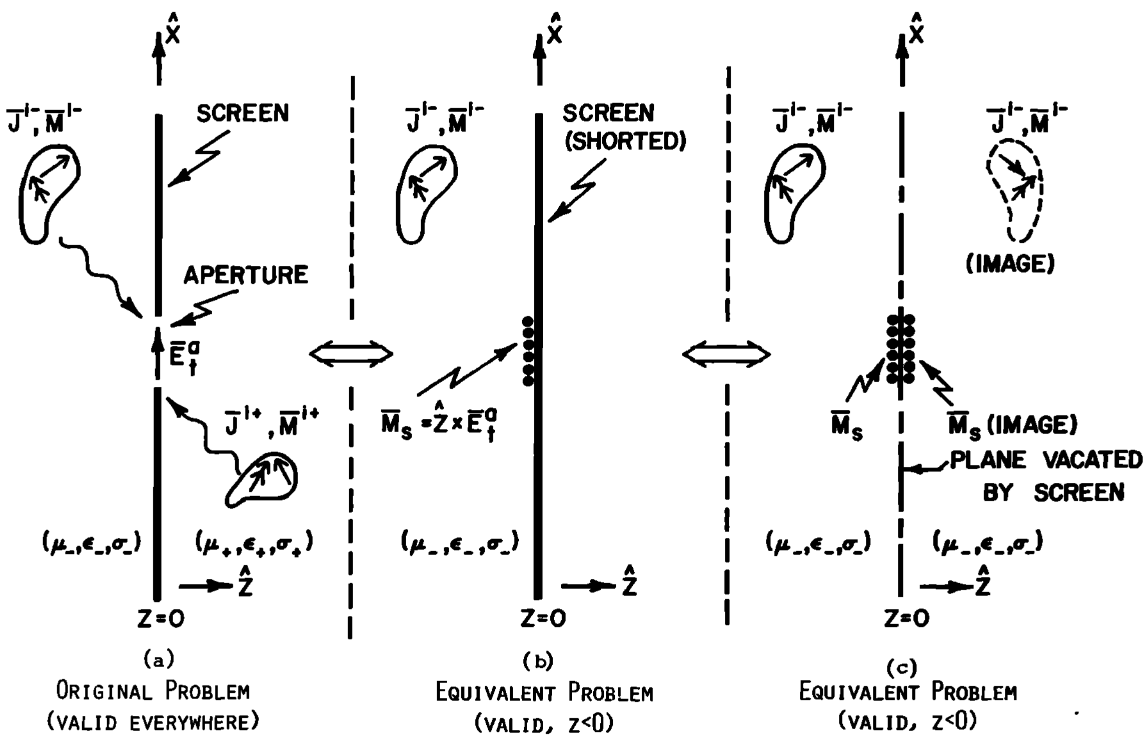


Fig. 2. Left half-space equivalent problems.

plus its image and that radiated by the left half-space sources $(\mathbf{J}^{i-},\mathbf{M}^{i-})$ plus their images, with all radiating in an infinite, homogeneous space characterized by $(\mu_-,\epsilon_-,\sigma_-)$ (Figure 2c). Because the field due to $(\mathbf{J}^{i-},\mathbf{M}^{i-})$ plus their images is that radiated by $(\mathbf{J}^{i-},\mathbf{M}^{i-})$ in the presence of the *shorted* screen, we denote this so-called short-circuit field by $(\mathbf{E}^{sc-},\mathbf{H}^{sc-})$.

One may develop the right half-space equivalences in a way identical to that discussed above for the left half space. $(\mathbf{E}^+,\mathbf{H}^+)$ and $(\mathbf{E}^{sc+},\mathbf{H}^{sc+})$ represent the total and short-circuit fields, respectively, in the right half space.

Aided by Figure 2c and an analogous equivalence for the right half space, one writes $\mathbf{H}^-$ and $\mathbf{H}^+$ as

$$\mathbf{H}^- = \mathbf{H}^{sc-} - j\frac{\omega}{k_-^2}(k_-^2\mathbf{F}^- + \nabla\nabla\cdot\mathbf{F}^-), \quad z<0 \tag{1}$$

and

$$\mathbf{H}^+ = \mathbf{H}^{sc+} + j\frac{\omega}{k_+^2}(k_+^2\mathbf{F}^+ + \nabla\nabla\cdot\mathbf{F}^+), \quad z>0 \tag{2}$$

where $\omega$ is the angular frequency of the surpressed harmonic variation in time $e^{j\omega t}$ and where

$$k_\pm^2 = \omega^2\mu_\pm\epsilon'_\pm = \omega^2\mu_\pm\epsilon_\pm - j\omega\mu_\pm\sigma_\pm \tag{3}$$

In (1), $\mathbf{F}^-$ $(\mathbf{F}^+)$ is the electric vector potential due to $\mathbf{M}_s$ $(-\mathbf{M}_s)$ plus its image:

$$\mathbf{F}^\pm(\mathbf{r}) = \frac{\epsilon'_\pm}{4\pi}\iint_A 2\mathbf{M}_s(\mathbf{r}')\frac{\exp(-jk_\pm|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|}\,ds' \tag{4}$$

where $\mathbf{r}$ locates a point $(x,y,z)$ in space and $\mathbf{r}' \in A$.

Requiring that the magnetic field be continuous along any path through $A$,

$$\lim_{z\uparrow 0}\mathbf{H}^-\times\hat{z} = \lim_{z\downarrow 0}\mathbf{H}^+\times\hat{z}, \quad \text{in } A$$

and recognizing that $\mathbf{H}^{sc\pm}\times\hat{z} = 2\mathbf{H}^{i\pm}\times\hat{z}$ at $z = 0^\pm$, where $(\mathbf{E}^{i\pm},\mathbf{H}^{i\pm})$ is the incident field in the medium $(\mu_\pm,\epsilon_\pm,\sigma_\pm)$ due to $(\mathbf{J}^{i\pm},\mathbf{M}^{i\pm})$, one arrives at

$$[\mathbf{F}^+ + \mathbf{F}^- + \nabla_t\nabla_t\cdot(\mathbf{F}^+/k_+^2 + \mathbf{F}^-/k_-^2)]\times\hat{z}$$
$$= -j(2/\omega)(\mathbf{H}^{i-} - \mathbf{H}^{i+})\times\hat{z} \quad \text{in } A \tag{5}$$

Equation (5) is an integro-differential equation for the unknown magnetic current $\mathbf{M}_s$ in the aperture. The component of $\mathbf{M}_s$ which is normal to the aperture edge is zero along this edge and the component parallel to the edge is unbounded (according to the edge condition for $\mathbf{E}_t^a$) along the edge.

If the electromagnetic properties of the two half spaces were the same, (5) would reduce immediately to the familiar integro-differential equation for an aperture in a screen. For aperture shapes other than the circle, even this important traditional problem must be solved approximately by numerical techniques [*Mittra et al.*, 1973; *Lin et al.*, 1974]. Such numerical solution procedures demand very high computer storage and run times. Therefore, in order to retain the central features of the two-media problem but to reduce it to one for which numerical solution techniques are practical, we specialize the aperture to be an infinite slot of uniform width. Data which are representative of the behavior of quantities of interest in the general two-media problem are determined from solutions of the special case of the slot.

## INFINITE SLOT

For a slot of uniform width $w$ with its axis along the $y$ axis, we consider two cases of excitation in which the problem is two-dimensional: (i) an incident wave having only a $y$-component of magnetic field and having an electric field transverse to the slot axis, both being independent of $y$ (TE case); and (ii) an incident wave having only a $y$ component (negative) of electric field with the magnetic field transverse to the slot axis, both again independent of $y$ (TM case).

*TE case.* In the TE case, the only electric field which can exist in the slot is $x$-directed and independent of $y$. Thus, $\mathbf{M}_s = M_{s_y}(x)\hat{y}$ and the electric vector potentials have only $y$ components and are independent of $y$: $\mathbf{F}^\pm = F_y^\pm(x)\hat{y}$ for $z = 0$. In view of the properties above, (5) reduces in the TE case to

$$F_y^+(x) + F_y^-(x) = -j(2/\omega)[H_y^{i-}(x) - H_y^{i+}(x)] \quad \text{in slot} \tag{6}$$

Since $F_y^\pm$ are independent of $y$,

$$F_y^\pm(x) = (\epsilon'_\pm/2\pi)\int_{x'=-w/2}^{w/2} M_{s_y}(x')$$

$$\cdot \int_{y'=-\infty}^{\infty} \frac{\exp\{-jk_{\pm}[(x-x')^2+y'^2]^{1/2}\}}{[(x-x')^2+y'^2]^{1/2}}\, dy'\, dx' \tag{7}$$

which can be written

$$F_y^{\pm}(x) = \frac{\epsilon'_{\pm}}{2j} \int_{x'=-w/2}^{w/2} M_{s_y}(x')\, H_0^{(2)}(k_{\pm}|x-x'|)\, dx' \tag{8}$$

where $H_0^{(2)}(\xi)$ is the Hankel function of the second kind and zero order. Hence, one now has an integral equation for the unknown $M_{s_y}$ in the TE case:

$$\int_{x'=-w/2}^{w/2} M_{s_y}(x')\,[\epsilon'_{+} H_0^{(2)}(k_{+}|x-x'|) + \epsilon'_{-} H_0^{(2)}(k_{-}|x-x'|)]\, dx' = (4/\omega)[H_y^{i-}(x) - H_y^{i+}(x)],\quad x \in (-w/2, w/2) \tag{9}$$

*TM case.* In the TM case, the electric field in the slot is entirely $y$-directed and independent of $y$. Thus, $\mathbf{M}_s = M_{s_x}(x)\hat{x}$ and the electric vector potentials $\mathbf{F}^+$ and $\mathbf{F}^-$ are $x$-directed and are independent of $y$: $\mathbf{F}^{\pm} = F_x^{\pm}(x)\hat{x}$ at $z = 0$. Hence, in the TM case, (5) reduces to

$$(1/k_+^2)(\partial^2/\partial x^2 + k_+^2)\, F_x^+(x) + (1/k_-^2)(\partial^2/\partial x^2 + k_-^2) \cdot F_x^-(x) = -j(2/\omega)[H_x^{i-}(x) - H_x^{i+}(x)] \quad \text{in slot} \tag{10}$$

where $F_x^{\pm}(x)$ is the right-hand side of (7) with $M_{s_y}$ replaced by $M_{s_x}$. In view of (8), one sees that (10) reduces to the following integro-differential equation for the TM case:

$$(\partial^2/\partial x^2 + k_+^2) \int_{x'=-w/2}^{w/2} M_{s_x}(x')[(1/\mu_+) \cdot H_0^{(2)}(k_+|x-x'|)]\, dx' + (\partial^2/\partial x^2 + k_-^2) \int_{x'=-w/2}^{w/2} \cdot M_{s_x}(x')[(1/\mu_-) H_0^{(2)}(k_-|x-x'|)]\, dx' = 4\omega\,[H_x^{i-}(x) - H_x^{i+}(x)],\quad x \in (-w/2, w/2) \tag{11}$$

## NUMERICAL SOLUTION

Knowledge of the equivalent slot magnetic current enables one to calculate all quantities of interest in either the TE or the TM case. Therefore, in this section, techniques for solving the two equations, (9) and (11), are presented. The integral equation (9) for the TE case introduces no special difficulties and it can be solved numerically quite readily by means of collocation with the magnetic current $M_{s_y}$ approximated by a linear combination of pulses [*Harrington*, 1968]. On the other hand, the integro-differential equation (11) for the TM case warrants careful scrutiny. It cannot be converted to a Hallén-type equation, as is often done in thin-wire analyses, because there are two distinct harmonic differential operators (since $k_+ \neq k_-$) on the left-hand side; the presence of the different $k$'s also precludes the use of the so-called reaction matching technique [*Mittra*, 1973]. There is no reduced kernel as in wire theory available as an approximation of the Hankel functions in (11) so interchange of differentiation and integration, with subsequent application of collocation, would lead to difficulties. However, this equation can be solved numerically by means of a scheme utilizing piecewise linear testing and almost any reasonable subdomain basis set for approximating the magnetic current. This solution procedure is discussed below.

The interval of the slot $(-w/2, w/2)$ is partitioned into $2M$ subintervals each of length $\Delta = w/(2M)$. We define testing functions (triangles) [*Harrington*, 1968],

$$\Lambda_m^l(x) = \begin{cases} \dfrac{\Delta - |x - x_m|}{\Delta}, & x \in (x_{m-1}, x_{m+1}) \\[2ex] 0, & x \notin (x_{m-1}, x_{m+1}) \end{cases} \tag{12}$$

Testing (11) [*Wilton and Butler*, 1976] by $\Lambda_m^l$ of (12) and integrating by parts twice yields

$$(1/4\omega)\{(1/\Delta\mu_+)[f(k_+x_{m+1}) - 2f(k_+x_m) + f(k_+x_{m-1})] + (1/\Delta\mu_-)[f(k_-x_{m+1}) - 2f(k_-x_m) + f(k_-x_{m-1})] + \int_{x=x_{m-1}}^{x_{m+1}} [(k_+^2/\mu_+)f(k_+x) + (k_-^2/\mu_-)f(k_-x)] \cdot \Lambda_m^l(x)\, dx\} = \int_{x=x_{m-1}}^{x_{m+1}} [H_x^{i-}(x) - H_x^{i+}(x)] \cdot \Lambda_m^l(x)\, dx,\quad m = 0, \pm 1, \pm 2, \ldots, \pm(M-1) \tag{13}$$

where

$$f(kx) = \int_{x'=-w/2}^{w/2} M_{s_x}(x')\, H_0^{(2)}(k|x - x'|)\, dx' \tag{14}$$

Even though the numerical solution can be based upon (13), there is an approximation which one may employ to greatly simplify certain computations. In particular, when $\Delta$ is sufficiently small, i.e., $\Delta k_- \ll 1$ and $\Delta k_+ \ll 1$ in the present application,

$$\int_{x=x_{m-1}}^{x_{m+1}} u(x)\, \Lambda_m^l(x)\, dx \doteq \Delta u(x_m) \tag{15}$$

Subject to this good approximation, (13) becomes

$$\begin{aligned}&(1/4\omega)\big[(1/\Delta\mu_+)\{f(k_+x_{m+1}) - 2[1-(k_+\Delta)^2/2]\\ &\cdot f(k_+x_m) + f(k_+x_{m-1})\} + (1/\Delta\mu_-)\{f(k_-x_{m+1})\\ &- 2[1-(k_-\Delta)^2/2]\, f(k_-x_m) + f(k_-x_{m-1})\}\big]\\ &\doteq \Delta\,[H_x^{i-}(x_m) - H_x^{i+}(x_m)],\\ &\quad m = 0, \pm 1, \pm 2, \ldots, \pm(M-1)\end{aligned} \tag{16}$$

Even though (16) is the finite-difference approximation of (11), it was obtained by means of piecewise linear testing of (11) subject to the good approximation of (15). As pointed out by *Wilton and Butler* [1976], (16) enjoys all the advantages of a weighted averaging solution procedure plus the simplicity of a collocation scheme. For example, if $\{\Lambda_n^l\}$ of (12) were employed for representation of $M_{s_x}$, then, subject to (15), (16) would lead to a Galerkin solution.

A number of basis sets would be suitable for approximation of $M_{s_x}$, but, due to its simplicity and since it yields results as efficiently as any other, we select the set comprising pulses [*Harrington*, 1968] for approximating $M_{s_x}$:

$$M_{s_x}(x) = \sum_{n=-N}^{N} M_n p_n(x) \tag{17}$$

where $p_n(x) = 1$, $x \in (x_n - \Delta/2, x_n + \Delta/2)$, and $p_n(x) = 0$ outside this interval. The weighting coefficients $\{M_n\}$ are the unknown quantities to be determined. Half pulses are placed at each end of the interval $(-w/2, w/2)$ and are set equal to zero in order to ensure satisfaction of the boundary condition $M_{s_x}(-w/2) = M_{s_x}(w/2) = 0$. With this current approximation introduced into (16), the following system of linear algebraic equations can be constructed:

$$\sum_{n=-N}^{N} M_n Y_{mn} = I_m, \quad m = 0, \pm 1, \pm 2, \ldots, \pm N \tag{18}$$

where

$$I_m = \Delta\,[H_x^{i-}(x_m) - H_x^{i+}(x_m)] \tag{19}$$

and

$$\begin{aligned}Y_{mn} = \frac{1}{4\omega\Delta}\Bigg\{\int_{x'=-\Delta/2}^{\Delta/2}\Bigg[&\frac{1}{\mu_+}\{H_0^{(2)}[k_+|(m-n+1)\Delta\\ &- x'|] - 2[1-(k_+\Delta)^2/2]\, H_0^{(2)}[k_+|(m-n)\Delta\\ &- x'|] + H_0^{(2)}[k_+|(m-n-1)\Delta - x'|]\}\\ &+ (1/\mu_-)\{H_0^{(2)}[k_-|(m-n+1)\Delta - x'|]\\ &- 2[1-(k_-\Delta)^2/2]\, H_0^{(2)}[k_-|(m-n)\Delta - x'|]\\ &+ H_0^{(2)}[k_-|(m-n-1)\Delta - x'|]\}\Bigg]dx'\Bigg\}\end{aligned} \tag{20}$$

with

$$x_m = m\Delta, \quad m = 0, \pm 1, \pm 2, \ldots, \pm N \tag{21}$$

$$x_n = n\Delta, \quad n = 0, \pm 1, \pm 2, \ldots, \pm N \tag{22}$$

The admittance elements $Y_{mn}$ of (20) can be calculated easily, and the linear system of (18) can be solved by standard means to obtain $M_n$, knowledge of which leads to an approximation for $M_{s_x}$ by means of (17).

## RESULTS AND CONCLUSIONS

The equivalent slot magnetic current is presented in Figures 3-11 for the TE and TM cases of interest. It is emphasized that the magnetic current in these figures, $M_{s_y}$ of (9) and $M_{s_x}$ of (11), is the surface current on the left half-space side of the shorted screen which, when radiating in the presence of the shorted screen, produces the total field less the short-circuit field in the region $z < 0$, i.e., $(\mathbf{E}^- - \mathbf{E}^{sc-}, \mathbf{H}^- - \mathbf{H}^{sc-})$. This surface magnetic current $\mathbf{M}_s$ is equal in complex magnitude to $\mathbf{E}_t^a$, the total transverse electric field which exists in the aperture; in the TE case, $E_{t_x}^a = M_{s_y}$, and, in the TM case, $E_{t_y}^a = M_{s_x}$. Also, all data presented are normalized with respect to the left half-space intrinsic impedance $\eta_-$ and the appropriate component of the incident magnetic field evaluated at the center of the slot $\mathbf{H}^i(0)$, where $\mathbf{H}^i$ is defined for convenience to be the magnetic field incident from the left less that incident from the right $(\mathbf{H}^i = \mathbf{H}^{i-} - \mathbf{H}^{i+})$. The media occupying the half spaces are

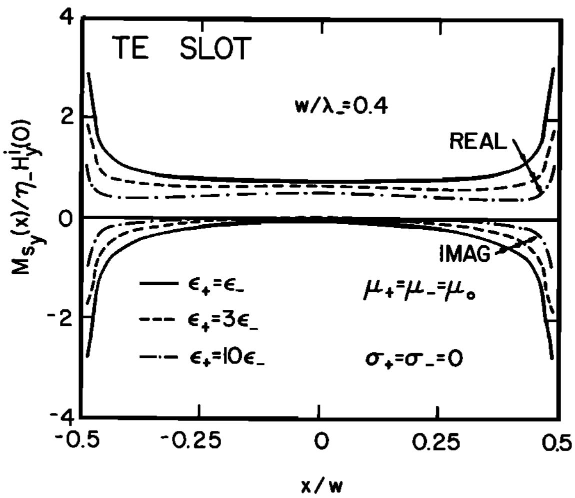


Fig. 3. TE equivalent magnetic current in 0.4-wavelength slot for different right half-space permittivities.

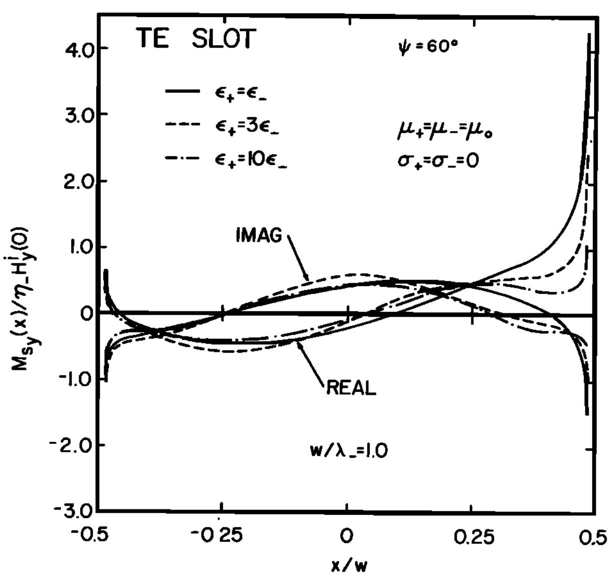


Fig. 5. TE equivalent magnetic current in 1.0-wavelength slot for different right half-space permittivities (60 deg incidence).

lossless ($\sigma_{\pm} = 0$) with $\mu_{\pm} = \mu_0$, and, also, slot widths are measured in numbers of left half-space wavelengths $\lambda_-$ $(=2\pi/k_-)$.

Figures 3 and 4 display the behavior of the slot magnetic current, or total electric field transverse to the screen, in 0.4- and 1.0-wavelength slots, respectively, for the TE case with $H_y^i$ constant over the slot. One observes two major effects associated with the change in $\epsilon_+$, the right half-space permittivity: first, the magnitude of the slot current reduces with increasing $\epsilon$, and, second, the edge singularity manifests itself closer and closer to the screen edge ($x = \pm w/2$) as $\epsilon_+$ increases. In Figure 5 $M_{s_y}$ is shown for the TE case when the excitation is a non-normally incident plane wave in the left half space, traveling along a path at $\Psi = 60°$ above the $z$ axis of Figure 1.

Turning attention to results in the TM cases, we mention that Figure 6 shows data for $M_{s_x}$ in a slotted screen separating half spaces of the same electromagnetic properties with constant $H_x^i$; these data are provided here for reference in subsequent dis-

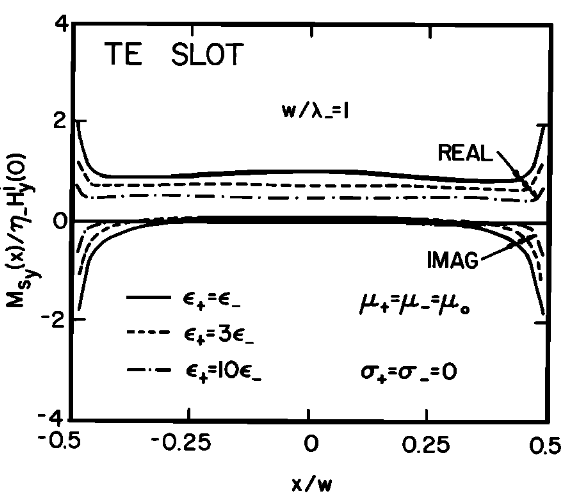


Fig. 4. TE equivalent magnetic current in 1.0-wavelength slot for different right half-space permittivities.

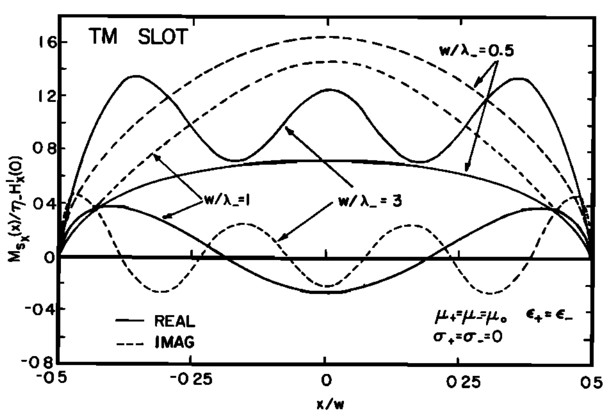


Fig. 6. TM equivalent magnetic current in 0.5-, 1.0-, and 3.0-wavelength slots in screen separating half-spaces of the same media.

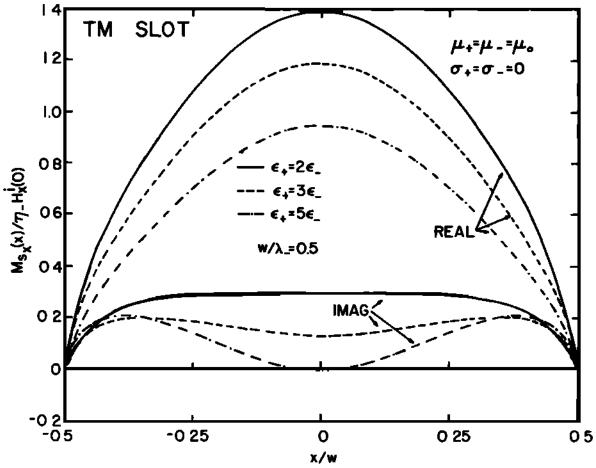


Fig. 7. TM equivalent magnetic current in 0.5-wavelength slot for different right half-space permittivities.

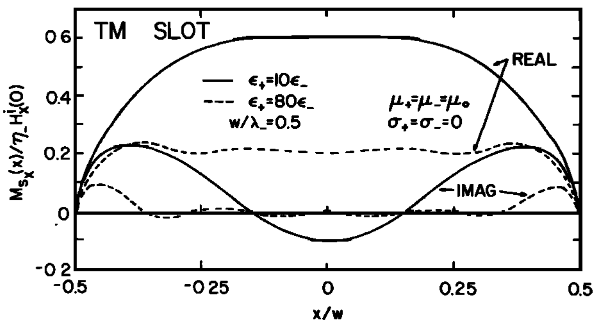


Fig. 8. TM equivalent magnetic current in 0.5-wavelength slot in screen separating half spaces of high dielectric contrast.

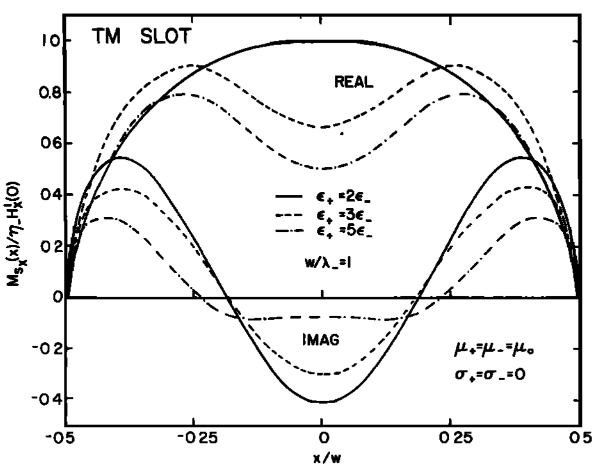


Fig. 9. TM equivalent magnetic current in 1.0-wavelength slot for different right half-space permittivities.

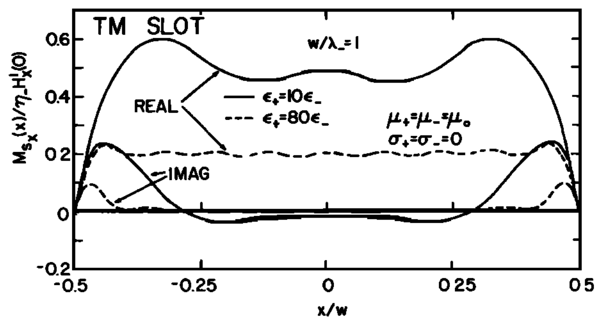


Fig. 10. TM equivalent magnetic current in 1.0-wavelength slot in screen separating half spaces of high dielectric contrast.

cussion of the two-media problem. From Figures 7-10, one observes that not only do changes in magnitude of $M_{s_x}$ occur with changes in $\epsilon_+$, but also significant modifications are introduced in its distribution or shape, especially with high dielectric contrast between the two media (Figures 8 and 10). As one expects from the fact that the kernel of (11) comprises two Hankel functions, one with argument $k_-|x-x'|$ and the other with argument $k_+|x-x'|$, he recognizes that the distribution of $M_{s_x}$ becomes similar to that of a wider slot (larger $w/\lambda_-$) with increases in $\epsilon_+$. Such similarities are apparent from comparisons of curves in Figures 7-10 with those in Figure 6. Figure 11 illustrates the behavior of $M_{s_x}$ due to non-normally ($\Psi = 60°$)

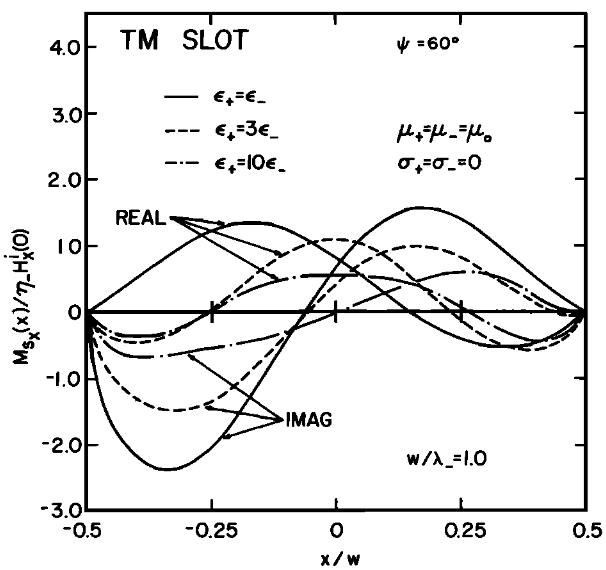


Fig. 11. TM equivalent magnetic current in 1.0-wavelength slot for different right half-space permittivities (60 deg incidence).

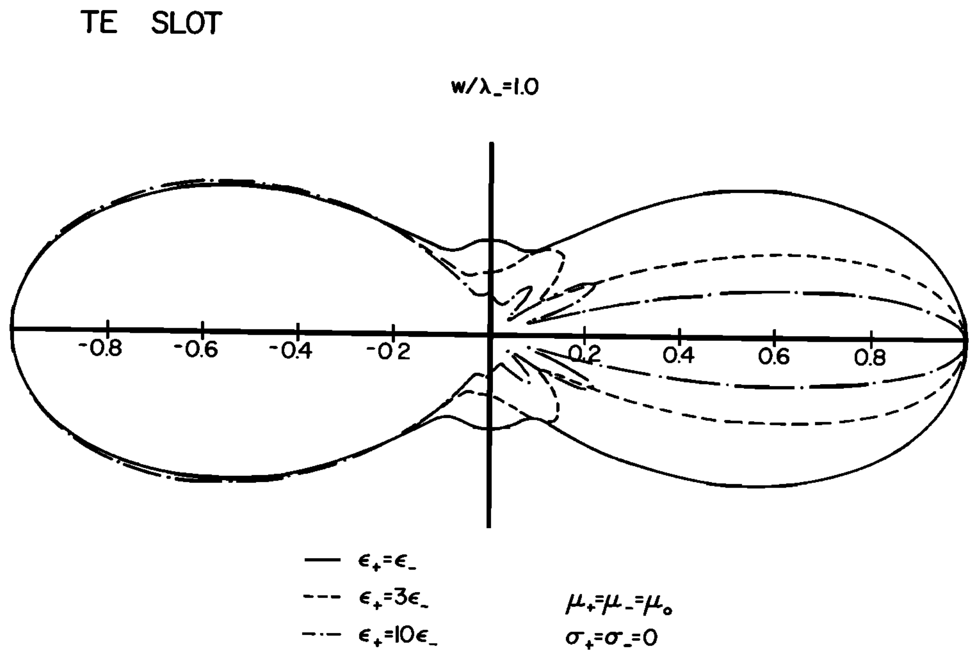


Fig. 12. Far magnetic field due to presence of TE-excited slot in screen for different right half-space permittivities.

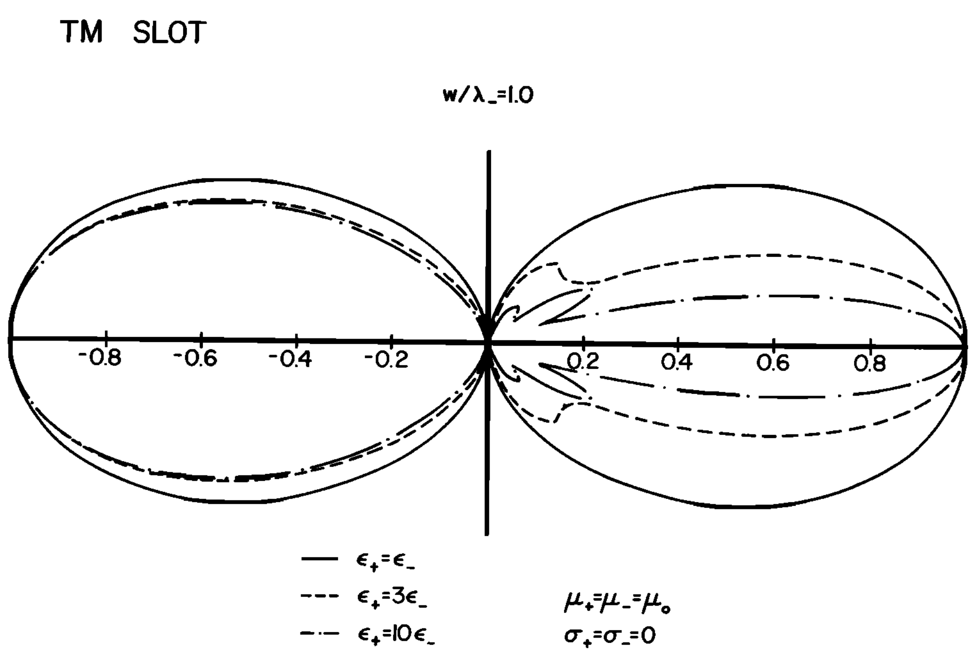


Fig. 13. Far electric field due to presence of TM-excited slot in screen for different right half-space permittivities.

incident TM excitation ($H_x^{i+} = 0$) with different values of $\epsilon_+$.

For a large aperture in a screen separating half spaces of the same media, it is well known that the field in the aperture at locations remote from the aperture/screen edge(s) is essentially equal to the incident field evaluated at such points. Of course, *large* in the present context means that the aperture dimensions are large relative to the single wavelength ($\lambda_+ = \lambda_-$) in the same-media problem. In the two-media problem, however, with different wavelengths in the two half spaces, the size of an aperture must be judged on the basis of both $\lambda_+$ and $\lambda_-$. In either the same- or the two-media aperture/screen problem, though, the deviation of the total field from the incident field in the aperture is dependent upon the distance in terms of wavelengths that the constituent contributions to the scattered field from locations on the screen must travel to the point in question. In the absence of the conducting screen entirely and with a normally incident plane wave in the left half space, the ratio of the total electric field to the incident electric field at the interface of two dielectric half spaces is readily determined to be $2/[1 + (\epsilon_+/\epsilon_-)^{1/2}]$. In the slot with the screen present at this interface, the ratio of the total electric field $\mathbf{E}_t^a$ ($=\mathbf{M}_s \times \hat{z}$) to the incident electric field $\mathbf{E}^{i-}$ ($=\eta_- \mathbf{H}^{i-} \times \hat{z}$) is $E_{t_x}^a/E_x^{i-} = M_{s_y}/(\eta_- H_y^{i-})$ for the TE case and is $E_{t_y}^a/E_y^{i-} = M_{s_x}/(\eta_- H_x^{i-})$ for the TM case. From an inspection of the data displayed in the figures, one finds the field ratios in the neighborhood of $x = 0$ to be nearly equal, particularly so when the dielectric contrast is high ($\epsilon_+/\epsilon_- = 10, 80$). These observations lead one to conclude that, with sufficiently high contrast, the electric field near the center of a moderately wide slot excited by a normally incident plane wave is approximately equal to the value one would calculate there for the two-media problem with the screen removed.

The final two Figures (12 and 13) show the far fields on the two sides of the screen. To a minor degree, the right half-space patterns differ from those of the left half space because of the different slot distributions for different dielectric contrasts, but the major differences are due to the different electrical widths ($w/\lambda_-$ or $w/\lambda_+$) of the slot in the two media.

In addition to those presented here, data are available for lossy media and it is pointed out that the analysis of this paper can be extended directly to enable one to solve the problem of multiple, parallel slots in a screen separating different media.

## REFERENCES


Barakat, R. (1963), Diffraction of plane waves by a slit between two different media, *J. Opt. Soc. Amer., 53,* 1231–1243.

Harrington, R. F. (1968), *Field Computation by Moment Methods,* Macmillan, New York.

Houlberg, K. (1967), Diffraction by a narrow slit in the interface between two media, *Can. J. Phys., 45,* 57–81.

Lin, J. L., Curtis, W. L., and M. C. Vincent (1974), On the field distribution of an aperture, *IEEE Trans. Antennas Propagat., AP-22,* 608–610.

Mittra, R. (Ed.) (1973), *Computer Techniques for Electromagnetics,* Pergamon, New York.

Mittra, R., Y. Rahmat-Samii, D. V. Jamnejad, and W. A. Davis (1973), A new look at the thin-plate scattering problem, *Radio Sci., 8,* 869–875.

Thomas, D. P. (1969), Electromagnetic diffraction by a circular aperture in a plane screen between different media, *Can. J. Phys., 47,* 921–930.

Wilton, D. R., and C. M. Butler (1976), Efficient numerical techniques for solving Pocklington's equation and their relationships to other methods, *IEEE Trans. Antennas Propagat., AP-24,* 83–86.