Array Antenna Pattern Modeling Methods That Include Mutual Coupling Effects

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Abstract—**Results from an investigation into methods of modeling the radiation patterns of phased arrays that include the effects of radiative mutual coupling are presented. The approaches are based either on the principle of pattern multiplication or the use of active element patterns. Theoretical derivations of the various active element pattern methods are also presented. In addition, a new method, the hybrid active element pattern method, is introduced which accurately predicts the patterns of small and medium-sized arrays of equally spaced elements. Example arrays of center-fed dipoles are analyzed to verify and illustrate the representations. The results are general, however, and can be applied to arrays of any type of element. The array patterns computed using both the classical pattern multiplication approach and the methods based on active element patterns are compared to those computed using accurate numerical codes based on the method of moments.**

I. Introduction

The analysis and design of phased array antennas are complicated by the presence of mutual coupling in the array environment. Several approaches have emerged that address mutual coupling effects in array analysis, including methods based on the principle of pattern multiplication, methods based on numerical techniques, and methods that employ active element patterns (the patterns of elements operating in the fully excited array) [1].

The pattern multiplication approach, referred to here as the *classical analysis method* because of its extensive use, applies to arrays consisting of "well-behaved" elements. "Well-behaved" elements have currents whose relative spatial distributions of magnitude and phase remain largely unchanged from element to element across the array, but whose feed point current magnitudes and phases can differ significantly from those expected for an isolated element. An array of half-wave dipoles is one example of such an array. The main effect of mutual coupling on a well-behaved element is to alter its active input impedance but not the shape of its current distribution; therefore, the relative patterns attributable to the individual elements can be assumed to be identical, and the "element pattern" can be factored out of the complete array pattern expression without introducing significant error into the pattern calculation. Often the element pattern closely matches that of an isolated element located in free space. Thus, the complete array pattern can be predicted accurately from only a knowledge of the element feed currents or, equivalently, the active element input impedances.

Classical analysis fails, however, for arrays that consist of elements whose individual element patterns are not identical. The principle of pattern multiplication cannot be applied to such arrays because the elements do not have a common element pattern. Examples include arrays of electrically large elements and arrays of dissimilar elements (elements that differ in either size, shape, or physical orientation). Numerical techniques such as the method of moments can be used to analyze some of these arrays. Numerical techniques directly solve for the current distributions on elements subject to the boundary conditions imposed by the array geometry. Although numerical techniques offer accuracy, they can be difficult to apply to large arrays. The method of moments requires the creation and solution of a large matrix equation, the size of which increases roughly as the square of the array size. This results in a computationally intensive approach, especially when patterns of a large electronically scanned array are desired for many scan directions. Also, there are some arrays for which there are presently no practical computer analysis codes that are widely available. Examples include arrays located in highly complex inhomogeneous media of large physical extent.

Active element pattern methods, which use the measured or computed patterns of individual elements in the array environment to calculate the pattern of the fully excited array, can often be employed when classical analysis and numerical techniques cannot. Active element patterns are either measured on a test range or are computed in some manner. The pattern data are then stored for later use in array pattern computations. Of course, if it is possible to compute the active element patterns using a given technique, then the fully excited array can usually be analyzed using the same technique. However, active element pattern methods usually permit faster pattern calculations than other methods once the element pattern data are available; therefore, active element pattern methods are often still preferable to other techniques.

Though the subject of mutual coupling is included in much of the array antenna literature, array analysis methods based on active element patterns are not as well documented as are the classical and numerical ap-

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proaches. The purpose of this paper is to derive several active element pattern methods, to point out subtle differences between the methods, and to compare the results obtained using active element pattern methods to those obtained using classical analysis and the method of moments. Such a fundamental investigation involves some basic material that is thought to be well understood. Therefore, the paper has some tutorial content. In addition to the theoretical discussion, some of the practical aspects of the various active element pattern methods are covered. Also, a new active element pattern method is introduced which yields accurate pattern calculations for arrays of equally spaced elements. This *hybrid active element pattern method* requires less computer memory and computation time than the other methods because it uses only a few element patterns.

It is important to note that coupling occurs in the array environment through both radiation between elements and reflections in the feed network. Radiative coupling is usually referred to as mutual coupling. Both types of coupling can have significant effects on the element input impedances, the array gain, and the shape of the array pattern; however, only radiative coupling effects are considered here. Also, note that discussions are phrased in terms of transmitting arrays, but reciprocity permits direct application to receiving arrays.

II. The Classical Array Pattern Analysis Method

Array elements are excited either directly by a set of individual transmitter modules or, more typically, indirectly through a feed network. In either case the array excitation can be modeled as a set of Thevenin equivalent voltage sources with nonzero source impedances as shown in Fig. 1(a). This is the familiar *free-excitation model* discussed by Oliner and Malech [2]. If the generator impedances $\{Z_{gq}\}$ are zero, the model reduces to the *forced excitation model*. If a feed network is used, the equivalent generator impedances usually differ due to impedance mismatches and subsequent reflections in the network. On the other hand, if identical independent sources are used, as with an active array, the generator impedances can be considered identical. Frequently, the generator impedances are assumed to be identical regardless of the technique used to feed the array.

For analysis purposes the active element input impedance can be represented as an equivalent lumped impedance located at the feed terminals in place of the element, yielding the equivalent circuit shown in Fig. 1(b). Even if the elements are similar, their input impedances generally differ from that of an isolated element and from each other because of mutual coupling. This effect is caused by the difference in coupling environments presented to each element. For example, elements near the edges of an array experience a different coupling environment than do elements near the center. The active element impedances also depend on the array excitation and therefore vary with scan angle.

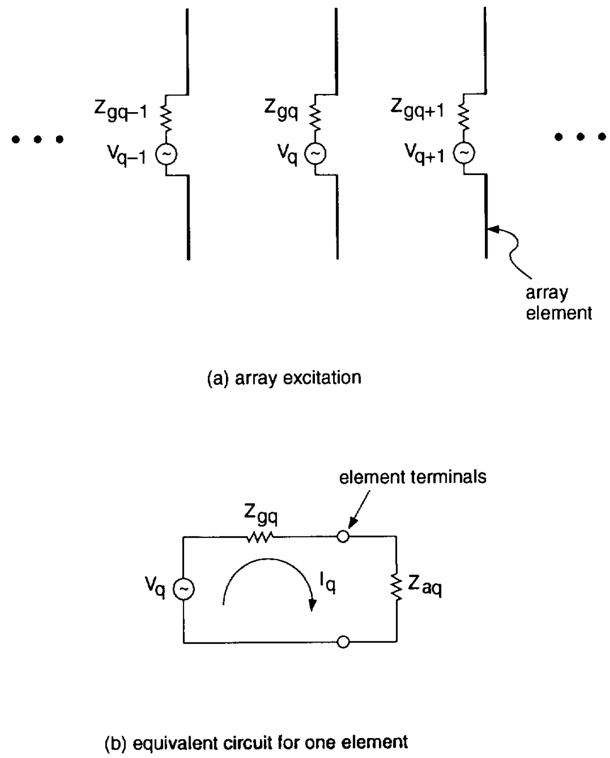

Fig. 1. Free excitation model for an array antenna. Excitations consist of source voltages $\{V_q\}$ with internal impedances $\{Z_{gq}\}$ as shown in (a). The equivalent circuit for a typical element is shown in (b).

The generator voltage values are set to achieve a desired array pattern. The resulting feed currents $\{I_q\}$, $q = 1, 2, \cdots, N_s$, where N_s is the number of voltage sources, are given by

$$I_q = \frac{V_q}{Z_{aq} + Z_{gq}}, \tag{1}$$

where Z_{aq}, as shown in Fig. 1(b), is the active input impedance of the qth element. If it is assumed that the generator impedances are nominally identical, then $Z_{gq} = Z_g$ for all q, where Z_g is a universal generator impedance. Because the active input impedances in a practical array are, in general, different for each element, the feed currents are not proportional to the generator voltages, nor do they share the same interelement phase relationships as the generator voltages.

It is often assumed that all of the elements in an array have the same element pattern, usually that of an isolated element in free space, regardless of mutual coupling effects. If this assumption is valid, the principle of pattern multiplication applies and the array pattern $E(\theta, \phi)$ is computed using the expression

$$E(\theta, \phi) = g_{\text{isol}}(\theta, \phi) \sum_{q=1}^{N_s} I_q e^{jk\hat{r} \cdot r_q}, \tag{2}$$

where $\boldsymbol{g}_{\mathrm{isol}}(\theta, \phi)$ is the pattern of an isolated array element, k is the free-space propagation constant, $\hat{\boldsymbol{r}}$ is the unit radial vector from the coordinate origin in the observation direction (θ, ϕ), and $\boldsymbol{r}_q$ is a position vector from the origin to the center of the qth element; boldface type indicates a vector quantity. This method has been widely applied in array analysis; therefore, it is referred to here as the *classical array analysis method*.

The classical analysis method, characterized by the use of the principle of pattern multiplication in (2), assumes that the element current distributions (for arrays of wire elements) remain identical within a complex multiplicative constant and do not vary from element to element; that is, all of the current distributions have the same relative spatial variation along each element. A similar formulation applies to aperture elements (such as horns), in which it is assumed that the aperture field distributions over the elements are identical in shape. Clearly, this assumption is not valid in many cases, such as for arrays of electrically large elements and arrays of dissimilar elements. Nevertheless, patterns computed using classical analysis are accurate enough for many arrays encountered in practice, especially arrays of parallel half-wave dipole elements that are not too closely spaced.

II. Exact Active Element Pattern Methods

In cases where mutual coupling effects are significant and neither classical analysis nor numerical methods can be applied, *active element pattern methods* can be employed to evaluate the array pattern without approximation. In these methods a unique active element pattern is computed or measured for each element that contains all of the mutual coupling effects associated with the surrounding array environment and the excitation of the element. Two forms of the active element pattern method are presented in this section.

A. The Unit-Excitation Active Element Pattern Method

The integral equation that applies to both radiation and scattering problems is [1]

$$E_{\mathrm{inc}}(s) = \int_{\mathrm{array}} J(s')K(s, s')\, ds', \tag{3}$$

where $J(s')$ is the unknown current distribution along the antenna wires, $K(s, s')$ is a kernel determined by the geometry of the problem, and $E_{\mathrm{inc}}(s)$ is the incident excitation field on the antenna surface. In general the integral equation in (3) should be expressed in terms of vector quantities, but it reduces to this scalar form for wire antennas. For the case of a transmitting array of dipole elements, $E_{\mathrm{inc}}(s)$ is the incident field produced along the wire surfaces when feed voltages are applied to the input terminals of the array elements.

If the array is located in a linear medium, the total incident excitation field is a sum of contributions from each applied feed voltage:

$$E_{\mathrm{inc}}(s) = \sum_{q=1}^{N_s} V_q e_{\mathrm{inc}}^q(s), \tag{4}$$

where V_q is the complex-valued feed voltage applied to the qth element, N_s is the number of voltage sources applied to the array, and $e_{\mathrm{inc}}^q(s)$ is the excitation field produced on the wire surfaces when the feed voltage applied to the qth element is unity. The decomposition of the incident excitation field shown in (4) suggests that each field component $e_{\mathrm{inc}}^q(s)$ induces part of the current distribution on each element in the array. The corresponding *unit-excitation current distribution*, $J_q(s')$, is defined such that

$$e_{\mathrm{inc}}^q(s) = \int_{\mathrm{array}} J_q(s')K(s, s')\, ds', \tag{5}$$

where $J_q(s')$ is the current distribution induced on the array elements when a unity feed voltage is applied to the qth element. Note that the quantity $J_q(s')$ encompasses not only the current excited on the qth element, but also the current parasitically excited on the other elements.

A version of (5) can be written for each voltage source used to excite the array. Using (5) in (4) yields

$$E_{\mathrm{inc}}(s) = \int_{\mathrm{array}} \sum_{q=1}^{N_s} V_q J_q(s')K(s, s')\, \boldsymbol{ds}'. \tag{6}$$

Comparison of (6) with (3) gives

$$J(s') = \sum_{q=1}^{N_s} V_q J_q(s'). \tag{7}$$

This relationship implies that the total current distribution on a fully excited array is the sum of the individual unit-excitation current distribution components, which correspond to the unit excitations of individual array elements, scaled by the complex-valued feed voltages $\{V_q\}$. Since the radiation pattern of an antenna is computed from the total current distribution along the antenna wires, the pattern radiated by a fully excited phased array can be expressed using a superposition of the radiation patterns arising from their corresponding current components in (7) as

$$\boldsymbol{E}(\theta, \phi) = \sum_{q=1}^{N_s} V_q \boldsymbol{g}_u^q(\theta, \phi), \tag{8}$$

where $\boldsymbol{g}_u^q(\theta, \phi)$ is the field produced by the qth unit-excitation current distribution component, $J_q(s')$. The quantity $\boldsymbol{g}_u^q(\theta, \phi)$ is called the *unit-excitation active element pattern*; it represents the pattern radiated by the array when the qth element is excited by a unit voltage with its associated generator impedance Z_{gq}, and the other elements are loaded by their respective generator impedances $\{Z_{gq}\}$. The set $\{\boldsymbol{g}_u^q(\theta, \phi)\}$ contains all of the effects of radiative mutual coupling, and therefore (8) is an exact expression for the array pattern (to the extent that the set $\{\boldsymbol{g}_u^q(\theta, \phi)\}$ can be determined exactly).

The result shown in (8) is significant because it demonstrates that the array pattern can be computed for any set of feed voltages $\{V_q\}$ from a single set of unit-excitation active element patterns $\{g_u^q(\theta, \phi)\}$, which only have to be computed or measured once for each element. The $\{g_u^q(\theta, \phi)\}$ remain constant for a fixed array geometry, frequency, and set of generator impedances. Though the array pattern expression in (8) was derived for arrays of wire elements, it applies equally well to arrays comprised of any other element type, including dissimilar elements. Also note that (8) applies to arrays located in inhomogeneous media, as long as the media are linear.

In contrast to the isolated element pattern $g_{\text{isol}}(\theta, \phi)$ used in the classical analysis method, the active element pattern $g_u^q(\theta, \phi)$ represents the pattern radiated by the entire array when only one element is directly excited and the other elements are parasitically excited by the active element. For example, the isolated element pattern of a single dipole in an array of parallel dipoles is omnidirectional in both amplitude and phase in the H-plane. The active element pattern of the same dipole, however, is the superposition of the omnidirectional patterns of the active element and the parasitically excited elements, which produces a pattern that is not omnidirectional but instead is highly dependent upon the geometry of the array.

B. The Phase-Adjusted Unit-Excitation Active Element Pattern Method

The array pattern expression of (8) does not explicitly show the dependence of the pattern on the array geometry. This is in contrast to the classical array pattern expression of (2), which contains the spatial phase term $\exp(jk\hat{r} \cdot r_q)$. In (8), the spatial phase information attributable to the element locations is implicitly contained in the active element patterns. A different active element pattern, referred to here as the *phase-adjusted unit-excitation active element pattern*, $g_p^q(\theta, \phi)$, is created by extracting the spatial phase information using

$$g_p^q(\theta, \phi) = g_u^q(\theta, \phi) e^{-jk\hat{r} \cdot r_q}, \tag{9}$$

which, after substituting (9) into (8), leads to the array pattern expression

$$E(\theta, \phi) = \sum_{q=1}^{N_s} V_q g_p^q(\theta, \phi) e^{jk\hat{r} \cdot r_q}. \tag{10}$$

The phase-adjusted active element pattern approach has appeared in the literature without derivation; for example, see [3], [4], and [5].

The results presented in this and the previous sections have ramifications in the execution and interpretation of active element pattern measurements. The phase-adjusted active element pattern differs from the unit-excitation element pattern in that the latter is referenced to the origin of the array coordinate system. In the phase-adjusted pattern this spatial translation is accounted for by multiplying $g_p^q(\theta, \phi)$ by the spatial phase term. The implication of this in pattern measurements is that when the array under test is fixed in location and the elements are excited individually in turn, the resulting set of measured patterns are unit-excitation active element patterns. Thus, (8) should be used to compute array patterns from the measured pattern data.

Because the spatial phase term has been extracted from the phase-adjusted active element pattern, one might infer that (10) could be used to compute patterns for arbitrary array geometries without having to compute new element patterns. This, of course, is not true, because the phase-adjusted element patterns change whenever the array geometry changes. Although little seems to have been gained by introducing the phase-adjusted element pattern, it will be shown in the next section that the concept is useful in the development of approximate array analysis methods.

IV. Approximate Active Element Pattern Methods

A. The Average Active Element Pattern Method

As the number of elements in an equally spaced array of similar elements increases, the phase-adjusted active element patterns of the interior elements (those elements located away from the edges of the array) become more and more alike. For a large array the set $\{g_p^q(\theta, \phi)\}$ in (10) can be approximated by the active element pattern of a "typical" interior element, allowing an *average active element pattern*, $g_{\text{av}}(\theta, \phi)$, to be factored out:

$$E(\theta, \phi) = g_{\text{av}}(\theta, \phi) \sum_{q=1}^{N_s} V_q e^{jk\hat{r} \cdot r_q}. \tag{11}$$

Array pattern expressions similar to (11) have appeared in the literature [3]–[5].

In the limit of an array comprised of an infinite number of elements, (11) yields an exact array pattern expression since the phase-adjusted active element patterns of all of the elements in such an array are identical. Array analysis methods that exploit this result and model large arrays as infinite arrays are common [2], [6]. Though the average element pattern approach can produce accurate results for very large, equally spaced arrays, it usually fails for small arrays in which mutual coupling effects cause significant variation in active input impedances and element patterns across the array.

B. The Hybrid Active Element Pattern Method

The analysis of arrays that contain a moderate number of elements (tens to hundreds of elements) often presents a special analysis problem. The unit-excitation and phase-adjusted methods become impractical as the number of array elements increases because active element pattern data are required for each element. On the other hand, the average element pattern method is accurate only for large arrays. A *hybrid active element pattern analysis method* is introduced here to analyze medium-sized, equally spaced arrays accurately and efficiently.

The derivation of the hybrid method begins with the phase-adjusted active element pattern expression of (10). The array elements are divided into an *interior* element group and an *edge* element group. The edge elements are typically taken to be the first few active elements on each end of a linear array, or the first few rows and columns of active elements on the sides of a planar array. If any passive "guard" elements are present in the array, they are not included in either group. The number of interior elements is N_i and the number of edge elements is N_e. The total number of active array elements is equal to the number of excitation sources; thus $N_s = N_i + N_e$. The array pattern is computed using phase-adjusted element patterns as

$$E(\theta, \phi) = \sum_{n=1}^{N_i} V_n g_p^n(\theta, \phi) e^{jk\hat{r}\cdot r_n} + \sum_{m=1}^{N_e} V_m g_p^m(\theta, \phi) e^{jk\hat{r}\cdot r_m}, \quad (12)$$

where the n subscript refers to interior elements and the m subscript refers to edge elements. If any passive elements are present, their effects are included in the active element patterns.

For an equally spaced array, the phase-adjusted element patterns of the interior elements are often similar enough that (12) can be rewritten

$$E(\theta, \phi) = g_{\text{av}}^i(\theta, \phi) \sum_{n=1}^{N_i} V_n e^{jk\hat{r}\cdot r_n} + \sum_{m=1}^{N_e} V_m g_p^m(\theta, \phi) e^{jk\hat{r}\cdot r_m}, \quad (13)$$

where $g_{\text{av}}^i(\theta, \phi)$ is the average active element pattern of the interior elements. An alternative form for the array pattern can be found by using (9) in (13) to obtain the expression

$$E(\theta, \phi) = g_{\text{av}}^i(\theta, \phi) \sum_{n=1}^{N_i} V_n e^{jk\hat{r}\cdot r_n} + \sum_{m=1}^{N_e} V_m g_u^m(\theta, \phi). \quad (14)$$

Either (13) or (14) can be used to approximate the pattern of a medium-sized, equally spaced array. Only $N_e + 1$ element patterns must be stored in memory in order to use the hybrid method. This is much less than the $N_e + N_i$ patterns that must be stored in order to use one of the exact methods of Section III.

V. Results of Computer Experiments

A seven-element array of center-fed parallel dipoles was used as a testbed to demonstrate the various array analysis methods discussed above. A dipole array was used because such arrays can be fully analyzed, including mutual coupling, using numerical analysis codes based on the method of moments. The geometry of the array is shown in Fig. 2. The dipole elements are 0.476λ long and are spaced 0.4λ apart. Each element is fed by its own individual voltage generator, and all generator impedances are assumed to be 50 Ω. The electromagnetic surface patch (ESP) code [7] was used for all moment-method calculations.

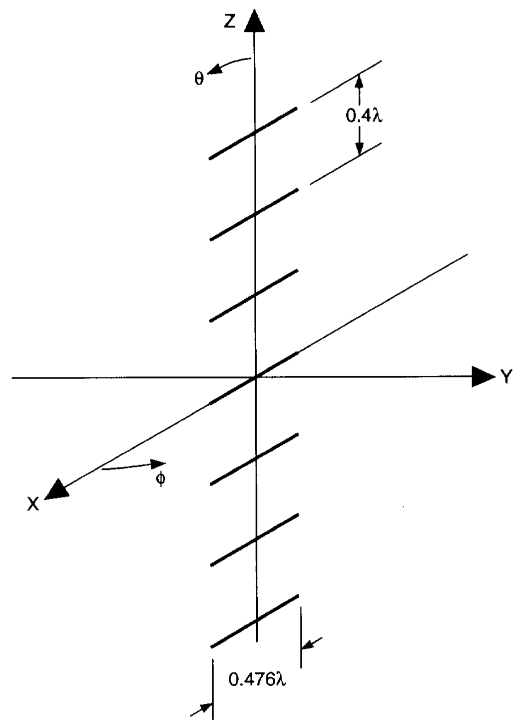

Fig. 2. Geometry of the testbed array. The array consists of seven parallel center-fed dipole elements uniformly spaced along the z axis. All elements are oriented parallel to the x axis.

The computed patterns for the seven-element dipole array scanned to 0°, 60°, and 90° with respect to the array axis are shown in Fig. 3. The solid curves are from the classical array pattern analysis method of (2). The dashed curves are the patterns computed using ESP. The voltage excitations are of uniform magnitude and have relative phases adjusted to steer the main beam to the desired direction. Note the very good agreement between the two pattern sets. This good agreement was achieved because the values of the feed currents $\{I_q\}$ used in the classical analysis computations were obtained by first solving for the terminal currents using ESP. Also, the nominally half-wave dipole elements used in this example are very "well-behaved" elements; that is, their element patterns differ little from that of an isolated element in free space, and their current distributions vary smoothly and slowly with position along the element.

Unit-excitation active element patterns for the testbed array were computed using ESP. The element patterns were calculated by specifying a unit feed voltage and a 50-Ω generator impedance at the input terminals of the active element. The inactive elements were terminated

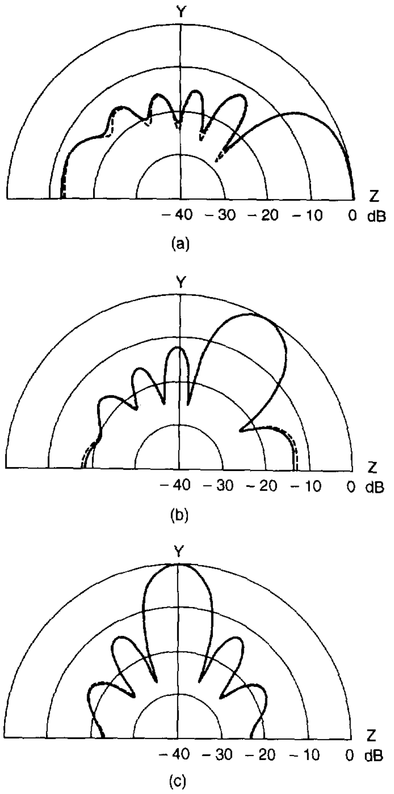

Fig. 3. Patterns of a seven-element array of parallel 0.476λ-long dipoles spaced 0.4λ apart scanned to (a) endfire, (b) 30° off broadside, (c) broadside. The solid curves are the patterns computed using the classical analysis method and the dashed curves are those computed using either ESP or the unit-excitation active element pattern method.

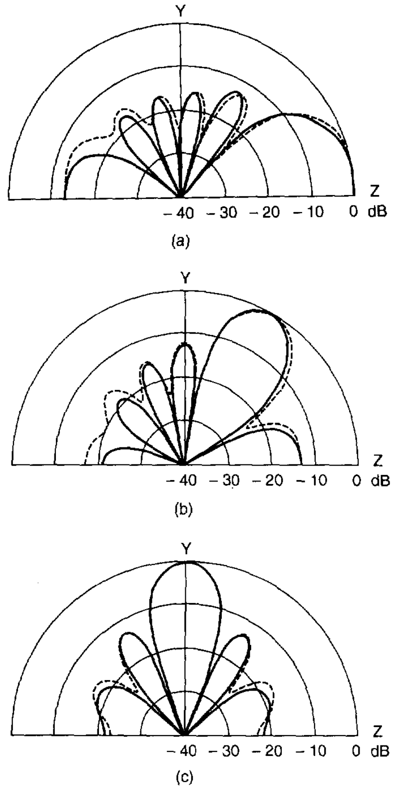

Fig. 4. Patterns of a seven-element array of parallel 0.476λ-long dipoles spaced 0.4λ apart scanned to (a) endfire, (b) 30° off broadside, (c) broadside. The solid curves are the patterns computed using the average active element pattern method and the dashed curves are those computed using either ESP or the unit-excitation active element pattern method.

with 50-Ω loads. The patterns computed using the unit-excitation active element pattern method of (8) exactly match the patterns computed using ESP. This is not surprising since the unit-excitation element pattern method is exact (to the extent that the active element patterns are exact) and because the element patterns were computed using ESP by employing the same wire segmentation as that used in the ESP analysis of the fully excited array.

Patterns computed using the average active element pattern method of (11) are compared to those computed using ESP in Fig. 4. The unit-excitation active element pattern of the center array element was selected to represent the average element pattern. As expected, the average element pattern method yields poor pattern predictions because of the small array size. The reason for these poor results is that the summation in (11) is the coupling-free array factor in which the applied feed voltages $\{V_q\}$ embody none of the effects of mutual coupling. Thus, patterns computed using the average element pattern method are the "ideal" array factor combined with an average element pattern.

Array patterns computed using the hybrid active element pattern method are compared to those computed using ESP in Fig. 5. The two elements on each end of the array (element numbers 1, 2, 6, and 7) were considered to be "edge" elements for this analysis and thus were assumed to have unique element patterns. The element patterns of the three interior elements were all assumed to equal that of the center element. The patterns computed using the hybrid element pattern method closely match those predicted using ESP. These results show that pattern degradation in medium-sized, equally spaced arrays is largely traceable to the influence of the edge elements. An array of seven parallel 0.952λ dipoles spaced 0.4λ apart was also analyzed using the hybrid element pattern method. Though the results are not shown here, the computed pattern closely matched that computed using ESP, confirming the accuracy of the hybrid method when applied to arrays comprised of electrically large elements.

VI. Conclusions

A full spectrum of array analysis methods has been presented, and the properties of the methods are summarized in Table I. There is a trade-off in the array analysis methods. As the number of array elements increases, the

TABLE I
SUMMARY OF ARRAY ANALYSIS METHODS THAT INCLUDE THE EFFECTS OF MUTUAL COUPLING

Analysis Method	Equation Number	Level of Approximation	Advantages	Disadvantages
Classical method	(2)	Approx.	Fast computation time Requires very little computer memory	Not accurate for arrays of large/ dissimilar elements
Unit-excitation AEP method	(8)	Exact	Exact for any configuration of any type of element in any scan direction	Requires large amount of computer time/memory Obtaining element pattern data is time-consuming
Phase-adjusted AEP method	(10)	Exact	Same as unit-excitation method	Same as unit-excitation method
Average AEP method	(11)	Approx.	Fast computation time Requires very little computer memory	Requires equal element spacing Requires similar elements Only accurate for large arrays
Hybrid AEP method	(13), (14)	Approx.	Requires little computer time/memory Incorporates edge-element effects	Requires equal element spacing Requires similar elements

Note: AEP = active element pattern.

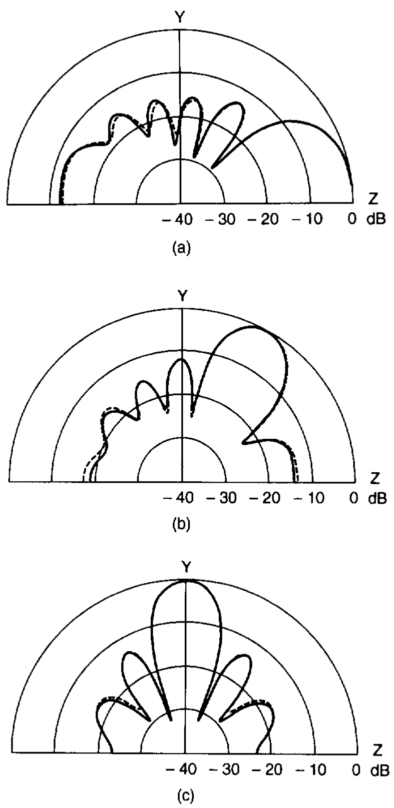

Fig. 5. Patterns of a seven-element array of parallel 0.476λ-long dipoles spaced 0.4λ spart scanned to (a) endfire, (b) 30° off broadside, (c) broadside. The solid curves are the patterns computed using the hybrid active element pattern method and the dashed curves are those computed using either ESP or the unit-excitation active element pattern method.

exact methods of (8) and (10) increase in difficulty because element pattern data are required for each element; thus, these techniques are more suitable to small arrays. For large arrays the average active element pattern method (11) becomes accurate. For medium-sized arrays the hybrid active element pattern method of either (13) or (14) often suffices. The exact number of elements for which each method is suitable depends on the element type, interelement spacing, level of acceptable accuracy, and availability of computing resources.

Note that when the hybrid active element pattern method was applied to the testbed array to generate the array patterns shown in Fig. 5, the two elements on each end of the array were considered to be "edge" elements. The analysis could have been performed by considering only the single elements at each end of the array (element numbers 1 and 7) to be edge elements, most likely resulting in predicted patterns that were less accurate than those shown in Fig. 5. Obviously, there is a trade-off when applying the hybrid method between considering a large number of elements to be edge elements and thus obtaining greater accuracy, or considering only a few elements to be edge elements and thus saving computer memory space and the time spent gathering edge-element pattern data. Further research could establish guidelines governing how many elements to consider "edge" elements when applying the hybrid active element pattern method.

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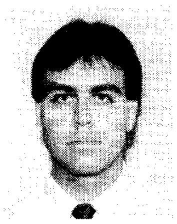

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