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NEW NUMERICAL APPROACH IN THE ANALYSIS OF A THIN WIRE RADIATING OVER A LOSSY HALF-SPACE

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SUMMARY

The finite element formulation for a half-space electric field integral equation is described. Sommerfeld integrals, appearing in the kernel of the integral equation are calculated by means of exponential approximations. This approach shows advantages over the usual techniques. Obtained results are compared with other results available.

KEY WORDS: thin wire; radiation over lossy half-space; electric field integral equation; finite element formulation; Sommerfeld integrals; exponential approximation

1. INTRODUCTION

Radiation from a horizontal cylindrical antenna above a conducting half-space has drawn the attention of many prominent researchers during the last 80 years. Radiation analysis of such a structure has become of great practical importance, especially in the second half of the century, because of the possibility of direct application in the field of surface wave propagation, oceanography, geophysical explorations, submarine communications and detection. Analysis of such a problem is based on the extension of the theory of antenna radiation in free space. The effect of an imperfectly conducting half-space is considered by additional terms in the integral equation kernel which are expressed in the form of Sommerfeld integrals. It is convenient to consider two separate problems; seeking a method for solving the electric field integral equation. (Pocklington's equation), and Sommerfeld integrals calculation. The first problem is treated, relatively efficiently, for the case of an isolated antenna in free space, by various moment methods variants.[1–4] The most commonly used method is, in fact, the simplest variant of moment methods—subdomain collocation with finite differences for treating the second-order differential operator.[5,6] Because of its simplicity, this is the only technique which has been used for half-space problems, where Sommerfeld integrals imply further complexities and difficulties.

Sommerfeld integrals evaluation has drawn the attention of many prominent researchers in the last few decades and up till now has not been optimally solved. The main reason is the highly oscillating integrands which are very hard to evaluate accurately. Many approximate techniques have been developed, and they can be generally characterized as analytical and numerical. Good reviews are given in References 7 and 8. Significant researches in the last decade are given in References 9–11, and more recently in 12–14. So, the problem which still remains is to find an efficient technique for Sommerfeld integral evaluation, and a better technique for solving the corresponding integral equation (IE).

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In this paper Sommerfeld integrals are treated by a new approach, which uses exponential approximations, while Pocklington's equation is solved by a finite element method (FEM) variant adjusted to integral equations (IE), and the equivalent current distribution is obtained. Once the current distribution is determined, the evaluation of all other parameters of interest is straightforward. This variant of the finite element method is, in fact, an extension of the weak formulation of the solution, widely used for partial differential equations (PDE). The method should be essentially referred to as boundary element method (BEM) solution. It is important to point out that this approach avoids using finite differences in the integral equation kernel. Second-order differentiation over the IE kernel is, by this approach, replaced by trivial differentiation over basis and test functions outside the kernel. This is shown to provide more accurate and easier computation. In addition, the case of a horizontally oriented dipole antenna above a dissipative half-space is considered in the present paper, because such a problem is one of the most important in wire antenna applications. However, the suggested approach can readily be extended to the case of arbitrarily oriented wire antenna radiating over a lossy half-space.

2. THE MATHEMATICAL MODEL OF LINEAR ANTENNA RADIATING OVER LOSSY HALF-SPACE

The current distribution calculation along a thin horizontal cylindrical antenna radiating over an imperfectly conducting half-space (Figure 1) is an exterior or unbounded field problem, so it is convenient to model it by the corresponding boundary integral equation.

The electric field integral equation (EFIE) for the half-space problem is first analysed by integrating the contributions to the Hertz potential of an infinitesimal dipole over the entire length of the actual antenna.

Solutions of Maxwell's equations, with a harmonic time dependence, for a short dipole above a dissipative medium (Figure 2) subject to the appropriate boundary conditions can be obtained from the Hertz potential given in the form

$$\Pi = e_x \Pi_x + e_z \Pi_z \tag{1}$$

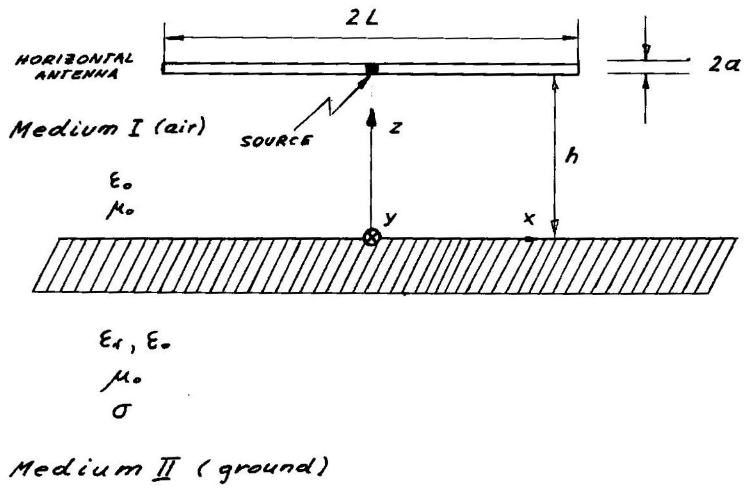

Figure 1. Horizontal antenna over imperfectly conducting ground

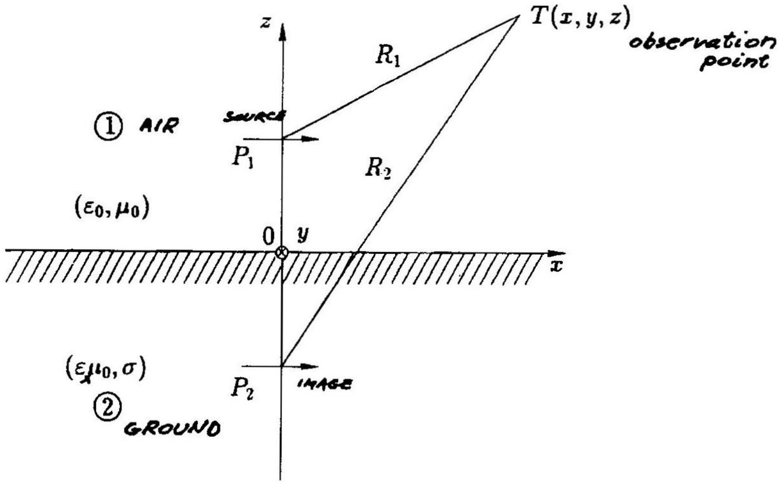

Figure 2. Hertzian dipole above dissipative half-space

which satisfies the vectorial inhomogeneous Helmholtz equation in medium 1, and the vectorial homogeneous Helmholtz equation in medium 2.
In scalar form, the equations in the two media are as follows.

Medium 1:

$$\nabla^2 \Pi_{1x} + k_1^2 \Pi_{1x} = -p\delta(r) \tag{2}$$

$$\nabla^2 \Pi_{1z} + k_1^2 \Pi_{1z} = 0 \tag{3}$$

where $\delta(r)$ is a delta function and p is the classical Hertzian dipole moment

$$p = \frac{I\Delta x'}{j\omega\varepsilon_0}$$

and ε_0 is the permittivity of vacuum, ω is the applied frequency, I is the current along dipole, $\Delta x'$ is the dipole length and k_1 is the phase constant of medium 1 (free space).

Mediums 2:

$$\nabla^2 \Pi_{2x} + k_2^2 \Pi_{2x} = 0 \tag{4}$$

$$\nabla^2 \Pi_{2z} + k_2^2 \Pi_{2z} = 0 \tag{5}$$

where k_2 is the phase constant of medium 2 (lossy ground). The boundary conditions at interface $z = 0$ become

$$k_1^2 \Pi_{1x} = k_2^2 \Pi_{2x} \tag{6}$$

$$\frac{\partial \Pi_{1x}}{\partial x} + \frac{\partial \Pi_{1z}}{\partial z} = \frac{\partial \Pi_{2x}}{\partial x} + \frac{\partial \Pi_{2z}}{\partial z} \tag{7}$$

$$k_1^2 \Pi_{1z} = k_2^2 \Pi_{2z} \tag{8}$$

$$k_1^2 \frac{\partial \Pi_{1x}}{\partial z} = k_2^2 \frac{\partial \Pi_{2x}}{\partial z} \tag{9}$$

In this paper the solution in the free space is of interest. The solution for the Hertz potential in medium 1 for a delta-function source is given by[6,7,15]

$$\Pi_{1x} = \frac{1}{4\pi j\omega\varepsilon}\{g_0(x,x') - g_i(x,x') + U_{11}\} \tag{10}$$

$$\Pi_{1z} = \frac{1}{j4\pi\omega\varepsilon}\frac{\partial W_{11}}{\partial x} \tag{11}$$

where $g_0(x,x')$ denotes the free-space Green function in the form

$$g_0(x,x') = \frac{\mathrm{e}^{-jk_1R_1}}{R_1} \tag{12}$$

$g_i(x,x')$ derives from image theory and is given by

$$g_i(x,x') = \frac{\mathrm{e}^{-jk_1R_2}}{R_2} \tag{13}$$

and R_1 and R_2 (Figure 1) are distances from the source to the observation point, respectively.

These first two terms from (10) form a dipole radiating over a perfectly conducting half-space. The effect of the imperfectly conducting half-space is taken into account by means of 'attenuation' terms in the form of Sommerfeld integrals. These integrals derive from the boundary-value problem defined by the Helmholtz equations and appropriate boundary conditions. Sommerfeld integrals U_{11} and W_{11} are given by

$$U_{11} = 2\int_0^{\infty}\frac{\mathrm{e}^{-2\mu_1(z+h)}}{\mu_1+\mu_2}J_0(\lambda\rho)\lambda\,\mathrm{d}\lambda \tag{14}$$

$$W_{11} = 2\int_0^{\infty}\frac{(\mu_1-\mu_2)\mathrm{e}^{-2\mu_1(z+h)}}{k_2^2\mu_1+k_1^2\mu_2}J_0(\lambda\rho)\lambda\,\mathrm{d}\lambda \tag{15}$$

where

$$\mu_1 = (\lambda^2 - k_1^2)^{1/2}$$

$$\mu_2 = (\lambda^2 - k_2^2)^{1/2}$$

and λ is the integration variable while h is the distance from the interface to the source.

Now, in order to utilize these results to obtain the Hertz potential components of a finite antenna, one must consider an infinitesimal dipole of length $\mathrm{d}x'$. Then the contrbution of the tangential electric field due to the infinitesimal Hertz potential components can be expressed in the form[16]

$$\mathrm{d}E_x = \left(\frac{\partial^2}{\partial x^2} + k_1^2\right)\mathrm{d}\Pi_{1x} + \frac{\partial^2}{\partial x\partial z}\mathrm{d}\Pi_{1z} \tag{16}$$

Integrating all the contributions to the Hertz potential from the source antenna and its image, Figure 2, it follows:

$$\Pi_{1x} = \frac{1}{\mathrm{j}4\pi\omega\varepsilon}\int_{-L}^{L}\{g_0(x,x') - g_i(x,x') + U_{11}\}I(x')\,\mathrm{d}x' \tag{17}$$

$$\Pi_{1z} = \frac{1}{\mathrm{j}4\pi\omega\varepsilon}\int_{-L}^{L}\frac{\partial W_{11}}{\partial x}I(x')\,\mathrm{d}x' \tag{18}$$

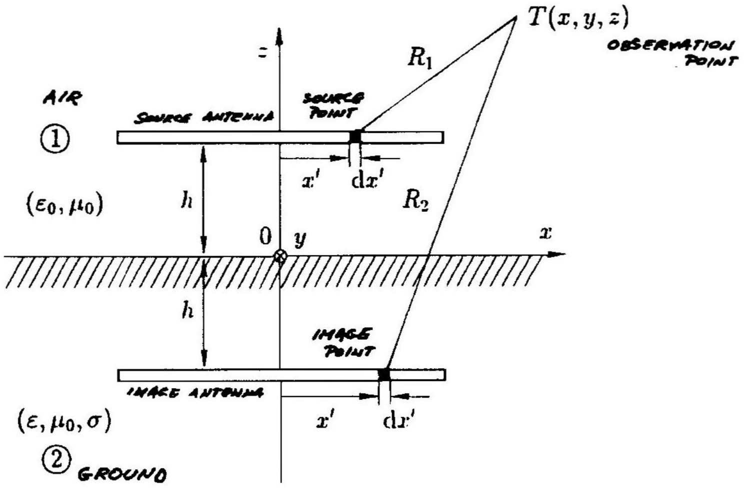

Figure 3. Horizontal antenna over imperfectly conducting ground with source and observation points in air

where L is a half of the antenna length and $I(x')$ is the unknown current which is to be determined.

Now, the scattered tangential electric field along a thin perfectly conducting wire, of length $2L$, and radius $2a$, horizontally located above a conducting half-space (Figure 3), is given by

$$E_x^s(x,z) = \left[\frac{\partial^2}{\partial x^2} + k_1^2\right]\Pi_x + \frac{\partial^2}{\partial x \partial z}\Pi_z \tag{19}$$

After substituting (17) and (18) into (19) and taking into account the well-known boundary condition that the total tangential electric field vanishes along the antenna surface:

$$E_x(x,a)^i + E_x(x,a)^s = 0 \tag{20}$$

one finally obtains the Pocklington integral equation for an antenna above a lossy ground:

$$E_x^i = -\frac{1}{j4\pi\omega\varepsilon}\int_{-L}^{L}\left\{\left[\frac{\partial^2}{\partial x^2} + k_1^2\right][g_0(x,x') - g_i(x,x') + U_{11}] + \frac{\partial^2}{\partial x \partial z}[\partial W_{11}]\right\} I(x')\,\mathrm{d}x' \tag{21}$$

where E_x^i denotes the incident electric field. In the special case of an isolated cylindrical antenna in free space, only the g_0 term remains, while g_i, U_{11} and W_{11} vanish.

On the other hand, if the more demanding case of an arbitrarily oriented wire antenna of arbitrary geometry over a lossy half-space is considered, then one must deal with the general curved wire electric field integral equation such as the one given in References 17 or 18.

3. THE WEAK FINITE ELEMENT FORMULATION OF THE ELECTRIC FIELD INTEGRAL EQUATION—BOUNDARY ELEMENT METHOD SOLUTION

The finite element method (FEM) has historically been developed primarily as a tool for solving partial differential equations (PDE), and many classical references exist in that field. The method is much less developed as a technique for solving integral equations (IE). The most significant works in that field are by Jeng and Wexler,[19,20] McDonald *et al.*,[21] and Silvester and Chan.[22,23] In References 19–21 only static field problems are considered. The solution employs a variational

FEM scheme, which is not shown to be convenient for modelling dynamic field problems, such as electromagnetic radiation or scattering.

On the other hand, Silvester and Chan[22,23] have solved Hallen's and Pocklington's integral equations, respectively, using a projective technique. This technique may be called, according to the analogy with solving PDE, the strong finite element formulation of the integral equation. If one uses this formulation for solving Pocklington's equation, analytical differentiation over the kernel must be performed. The basic difficulty of this approach, even in the case of free space, is in the appearance of near-singular integrals in the double integral calculations. In the case of a conducting half-space, such an approach is completely useless because of further complexities caused by Sommerfeld integrals.

Due to these obvious difficulties, as mentioned before, conducting half-space problems are usually modelled by means of a combination of the collocation technique and finite differences.

In this paper, in order to avoid these problems, the Pocklington integral equation is modelled in a completely new way, using the weak formulation of the problem. The weak formulation of the solution of (21) should be referred to as the boundary element method (BEM) solution and can be outlined in a few steps.

It is first convenient to use an operator form of (21), which is symbolically written as

$$KI = Y \tag{22}$$

where K is a linear operator, and I is the unknown function to be found for a given excitation Y.

The unknown current $I(x)$ is then expanded into a finite sum of linearly independent basis functions $\{N_i\}$ with unknown complex coefficients a_i, i.e.

$$I \cong I_n = \sum_{i=1}^{n} \alpha_i N_i \tag{23}$$

Substituting (23) into (22) yields

$$KI \cong KI_n = \sum_{i=1}^{n} \alpha_i KN_i = Y_n = P_n(Y) \tag{24}$$

where $P_n(Y)$ is a so-called projection operator.[24] Now the residual R_n is to be formed,

$$R_n = KI_n - Y = P_n(Y) - Y \tag{25}$$

Finally, according to the definition of the scalar product of functions in Hilbert function space the error R_n is weighted to zero with respect to certain weighting functions $\{W_j\}$, i.e.

$$\langle R_n, W_j \rangle = 0, \quad j = 1, 2, \ldots, n \tag{26}$$

where the expression in brackets denotes

$$\langle R_n, W_j \rangle = \int_{\Omega} R_n W_j^* \, \mathrm{d}\Omega \tag{27}$$

and Ω is the domain of interest. Since the operator K is linear, one obtains a system of algebraic equations and by choosing $W_j = N_j$, the Galerkin–Bubnov procedure yields

$$\sum_{i=1}^{n} \alpha_i \langle KN_i, N_j \rangle = \langle Y, N_j \rangle, \quad j = 1, 2, \ldots, n \tag{28}$$

Equation (28) is the strong formulation of the Galerkin–Bubnov technique for solving integral equation (21). Using the property of integral equation kernel symmetry, and taking into account the boundary conditions for current at the free ends of the thin wire, after integration by parts it follows:

$$\sum_{j=1}^{n}\sum_{i=1}^{n}\alpha_i\left\{-\int_{-L}^{L}\frac{\mathrm{d}N_j(x)}{\mathrm{d}x}\int_{-L}^{L}\frac{\mathrm{d}N_i(x')}{\mathrm{d}x'}g_{\mathrm{H}}(x,x')\,\mathrm{d}x\,\mathrm{d}x\right.$$
$$+k_1^2\int_{-L}^{L}N_j(x)\int_{-L}^{L}N_i(x')g_{\mathrm{H}}(x,x')\,\mathrm{d}x'\,\mathrm{d}x$$
$$-\int_{-L}^{L}\frac{\mathrm{d}N_j(x)}{\mathrm{d}x}\int_{-L}^{L}N_i(x')g_{\mathrm{V}}(x,x')\,\mathrm{d}x'\,dx$$
$$=\sum_{j=1}^{n}\frac{4\pi\omega\varepsilon}{j}\int_{-L}^{L}E_x^i(x)\,N_j(x)\,\mathrm{d}x \tag{29}$$

where

$$g_{\mathrm{H}}(x,x') = g_0(x,x') - g_{\mathrm{i}}(x,x') + U_{11}$$

and

$$g_{\mathrm{V}}(x,x') = \frac{\partial^2 W_{11}}{\partial x\,\partial z}$$

Equation (29) can be called (according the analogy to PDE solutions) the weak formulation of the Galerkin–Bubnov solution of the integral equation (21). This formulation is convenient for an integral finite element technique implementation—the boundary element technique. The advantage of such a formulation is obvious. The problematic second-order differential operator is replaced by trivial derivatives over basis and test (weight) functions which one chooses at will. The only requirement which is to be satisfied by bases and weights is that they must be chosen from the class of order-one differentiable functions.

In addition, boundary conditions are subsequently incorporated into the global matrix of the linear equation system, which is a significant advantage over moment methods; namely, in moment methods, basis and test functions are chosen in a way to satisfy boundary conditions.

According to the well-known finite element algorithm, the domain of integration is divided into subdomains—segments called finite (or boundary) elements. These elements are connected at nodes. Global basis functions are assigned to nodes, and shape functions are assigned to elements. A global matrix is assembled from finite element matrices, as is the right side vector of the system of linear equations. So the system of equations yields

$$\sum_{j=1}^{M}\sum_{i=1}^{M}[a]_{ji}^{e}\,\{\alpha\}_i^e = \sum_{j=1}^{M}\{b\}_j^e \tag{30}$$

where

$$[a]_{ji}^{e} = -\int_{\Delta l_j}\int_{\Delta l_i}\sum_{m=1}^{n_e}\sum_{k=1}^{n_e}\frac{\mathrm{d}N_{mj}^e(x)}{\mathrm{d}x}\frac{\mathrm{d}N_{ki}^e(x')}{\mathrm{d}x'}g_{\mathrm{H}}(x,x')\,\mathrm{d}x'\,\mathrm{d}x$$
$$+k_1^2\int_{\Delta l_j}\int_{\Delta l_i}\sum_{m=1}^{n_e}\sum_{k=1}^{n_e}N_{mj}^e(x)\,N_{ki}^e(x')\,g_{\mathrm{H}}(x,x')\,\mathrm{d}x'\,\mathrm{d}x$$
$$-\int_{\Delta l_j}\int_{\Delta l_i}\sum_{m=1}^{n_e}\sum_{k=1}^{n_e}\frac{\mathrm{d}N_{mj}^e(x)}{\mathrm{d}x}N_{ki}^e(x')\,g_{\mathrm{V}}(x,x')\,\mathrm{d}x'\,\mathrm{d}x \tag{31}$$

and

$$\{b\}_j^e = \frac{4\pi\omega\varepsilon}{j} \int_{\Delta l_j} \sum_{m=1}^{n_e} E_x^i(x) N_{mj}^e(x)\,\mathrm{d}x \tag{32}$$

where $[a]_{ji}^e$ is the finite element matrix, $\{b\}_j^e$ the finite element right-hand side vector, $\{a\}_i^e$ the finite element solution vector $N_{mj,ki}$ the shape functions over jth and ith finite elements, respectively, M the total number of finite elements over domain of interest, n_e the total number of local nodes per element and $\Delta l_{i,j}$ the width of ith and jth finite elements, respectively. With respect to the fact that functions $N(x)$ are required to be of class c^1 (once differentiable) a convenient choice for the shape functions over the finite elements is the family of Lagrange's polynomials given by

$$L_i(x) = \prod_{j=1}^{m} \frac{x - x_j}{x_i - x_j}, \quad j \neq i \tag{33}$$

In this paper, Pocklington's equation for free space is treated with both linear (first-order polynomials) and quadratic (second-order polynomials) approximations, while the corresponding Pocklington's equation for a half-space is treated (because of simplicity and computation time) with a first-order polynomial approximation (2 * 2 finite element matrix). The finite element right-hand side vector is different from zero only in the feed gap area, and it can be determined analytically for both non-zero terms. The x-component of the impressed (incident) electric field can be expressed as

$$E_x^i(x) = \frac{V}{\Delta l_g} \tag{34}$$

where V is the feed voltage and Δl_g the feed-gap width.
In addition, it follows by linear approximation,

$$b_{1j}^e = \int_{-\Delta l/2}^{\Delta l/2} \frac{V}{\Delta l_g} \frac{x_{j+1} - x}{\Delta l_g}\,\mathrm{d}x = \frac{V}{2} \tag{35a}$$

$$b_{2j}^e = \int_{-\Delta l/2}^{\Delta l/2} \frac{V}{\Delta l_g} \frac{x - x_j}{\Delta l_g}\,\mathrm{d}x = \frac{V}{2} \tag{35b}$$

So each node of the feed-gap central finite element obtains half the value of the total impressed voltage. Similarly the expressions for quadratic approximation can be obtained.[23]

4. EVALUATION OF SOMMERFELD INTEGRALS BY MEANS OF EXPONENTIAL APPROXIMATION

There are numerous works in the last few decades in which Sommerfeld integrals are solved by analytical techniques,[6–8,13,14,25] asymptotic expansions,[26,27] and in recent times by various numerical techniques.[9–12,28] If Sommerfeld integrals are computed numerically, the results obtained are valid over a wide range of parameters, but this is a rather time-consuming process. Since these integrals are, in general, highly oscillatory and difficult to evaluate numerically, the basic intention in this paper is to avoid numerical integration. So, the integrands are approximated by the set of exponential functions with unknown exponents and multiplicative coefficients, such that integrals U_{11} and W_{11} can then be solved analytically.

In general, there are two integral types of interest:

$$I_1 = \int_0^\infty f(\lambda) J_0(\lambda\rho) e^{-\psi(\lambda)} d\lambda \tag{36}$$

and

$$I_2 = \frac{\partial}{\partial\rho} \int_0^\infty g(\lambda) J_0(\lambda\rho) e^{-\psi(\lambda)} d\lambda \tag{37}$$

where

$$f(\lambda) = \frac{\lambda}{\mu_1 + \mu_2}$$

$$g(\lambda) = -\frac{x - x'}{|x - x'|} \frac{4\lambda\mu_1(\mu_1 - \mu_2)}{k_1^2\mu_2 + k_2^2\mu_1}$$

$$\psi(\lambda) = 2\mu_1 h$$

Now the functions $f(\lambda)$ and $g(\lambda)$ multiplied by $\exp(-\psi(\lambda))$ are expressed in terms of exponential sums, i.e.

$$f(\lambda) e^{-\psi(\lambda)} = \sum_{k=1}^{N} a_k e^{-\lambda\xi_k} \tag{38}$$

$$g(\lambda) e^{-\psi(\lambda)} = \sum_{k=1}^{N} b_k e^{-\lambda\xi_k} \tag{39}$$

Taking into account the well-known relations

$$\int_0^\infty J_0(\lambda\rho) e^{-\lambda w} d\lambda = \frac{1}{(\rho^2 + w^2)^{1/2}} \tag{40}$$

and

$$\frac{\partial}{\partial\rho} \int_0^\infty J_0(\lambda\rho) e^{-\lambda w} d\lambda = \frac{1}{(\rho^2 + w^2)^{3/2}} \tag{41}$$

expressions (36) and (37) can obviously be presented in the form of simple sums:

$$I_1 = \sum_{k=1}^{N} a_k \frac{1}{(\rho^2 + \xi_k^2)^{1/2}} \tag{42}$$

$$I_2 = \sum_{k=1}^{N} b_k \frac{\rho}{(\rho^2 + \xi_k^2)^{3/2}} \tag{43}$$

The coefficients ξ_k must be conveniently chosen[15,29,30] and a_k, b_k are then calculated from the algebraic system of equations derived from (38) and (39) for N values of the independent variable λ.

5. NUMERICAL RESULTS AND DISCUSSION

Since new methods for solving the integral equation and Sommerfeld integral evaluation and Sommerfeld integral evaluation are suggested in this paper, it is necessary to test them separately.

All results are obtained for unit, time-harmonic voltage excitation, i.e.

$$V = 1e^{j\omega t}$$

First, the weak finite element formulation for IE is tested on an isolated antenna in free space. The results obtained are compared with Mei's[31] theoretical results (subdomain collocation), and available experimental results.[32]

The current distribution obtained from this work by the weak FEM formulation with linear and quadratic interpolation is shown in Figure 4, and also compared with other results available. It is obvious that results obtained in this work by the FEM with quadratic approximation are in best agreement with the experimental results, and collocation is proved to be the worst method, as documented in Reference 1. Since there is only a small difference between the results obtained by linear and the quadratic interpolation, linear interpolation is chosen for further purpose (half-space calculations), because it is significantly less time-consuming. In addition, segmentation of the antenna on 31 finite elements, in this work, is shown to provide convergence of the results, as presented in Figures 5(a) and 5(b) in the case of a full-wave dipole antenna isolated in free space. On the other hand, Roje and Poljak[33] investigated the convergence of the collocation/finite difference method (pulse basis functions and Dirac test functions) for solving the same problem (full wave dipole in free space). They concluded that segmentation of an antenna on, at least, 55 elements is required for sufficient accuracy.

Now, the half-space calculations are to be tested. The current amplitudes obtained into his work by FEM/exponential approximations and FEM/stepest descent path (SDP technique is taken from Reference 6), are compared with results obtained by collocation/SDP (Reference 6). These results are presented in Figure 6. In addition, real and imaginary parts of the antenna current obtained in this work by FEM/SDP and FEM/exponential approximations are compared in Figure 7.

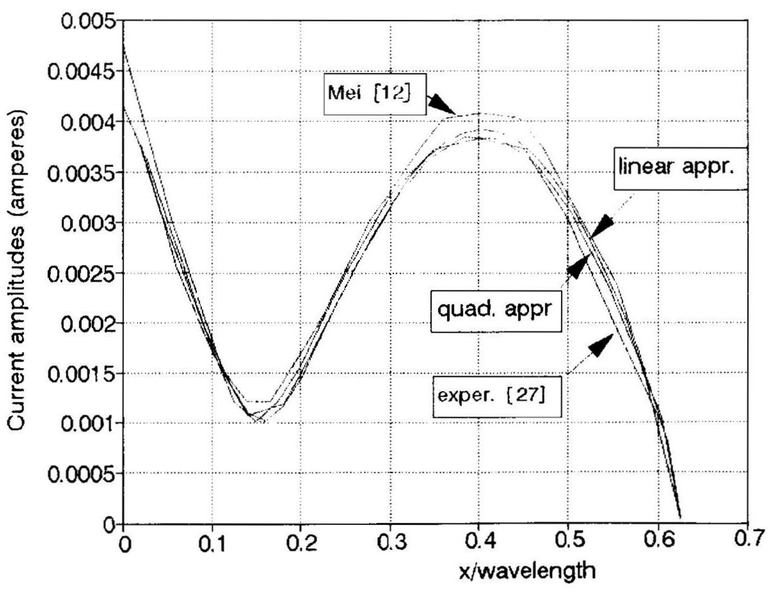

Figure 4. Amplitudes of current distribution on the centre-fed thin dipole (located in free space) of half-length $L = 0{\cdot}625\lambda$, radius $a = 0{\cdot}007022\lambda$—comparison of FEM solution obtained from this work with other results available

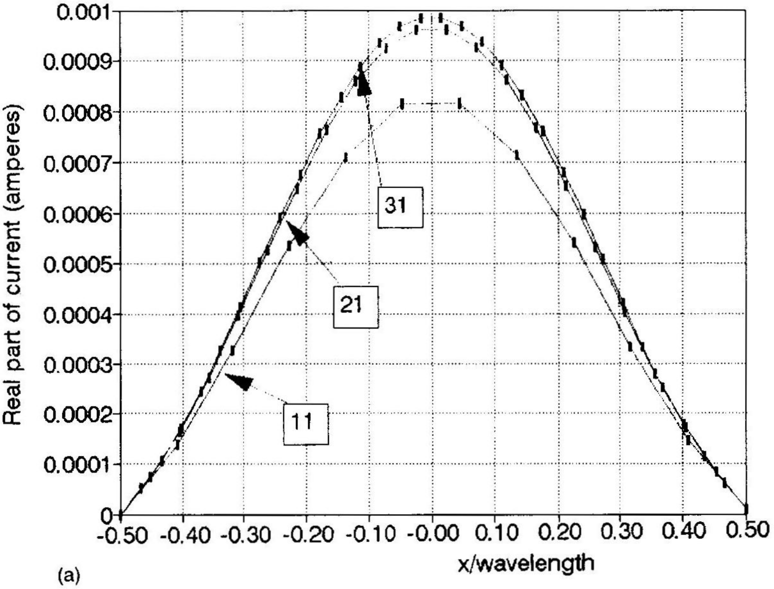

Figure 5(a). Real part of current distribution along the centre-fed thin dipole (located in free space) of half-length $L = 0{\cdot}5\lambda$, radius $a = 0{\cdot}007022\lambda$, obtained from this work for various numbers of finite elements (11, 21 and 31)—linear approximation

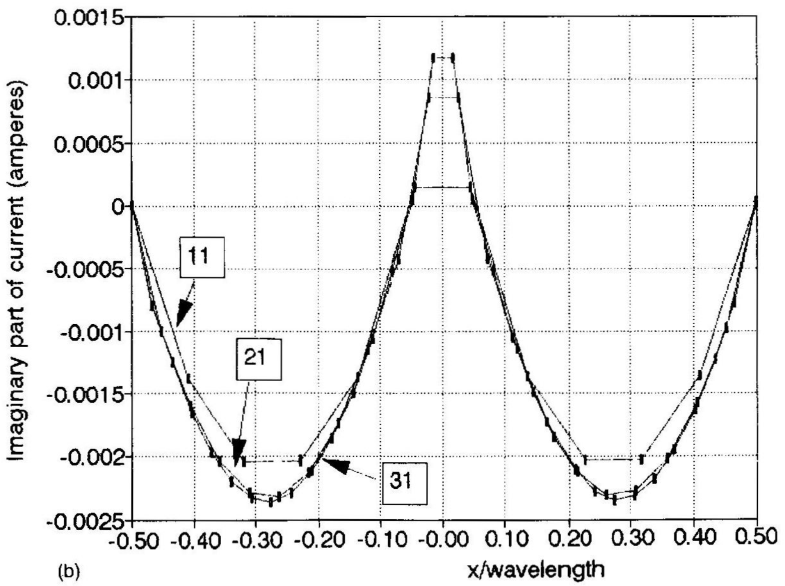

Figure 5(b). Imaginary part of current distribution along the centre-fed thin dipole (located in free space) of half-length $L = 0{\cdot}5\lambda$, radius $a = 0{\cdot}007022\lambda$, obtained from this work for various numbers of finite elements (11, 21 and 31)—linear approximation

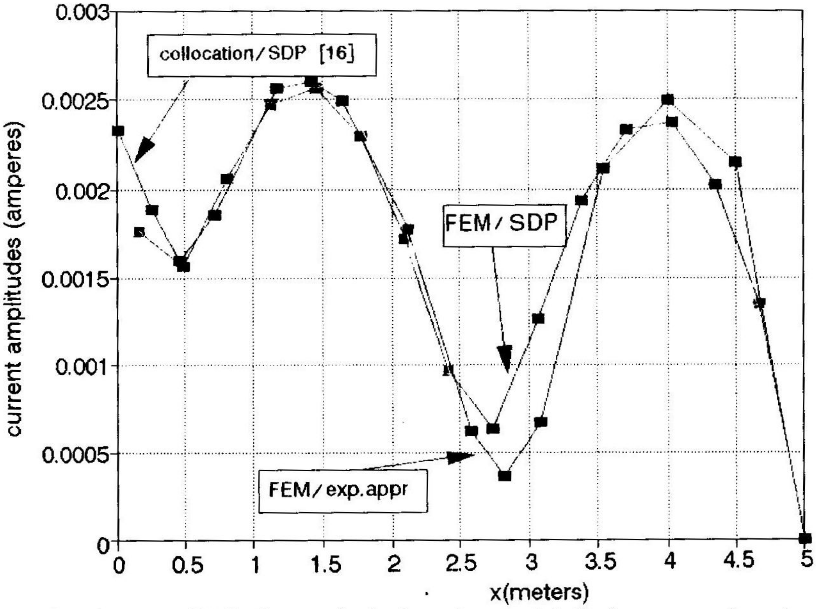

Figure 6. Amplitudes of current distribution on the horizontal centre-fed dipole antenna, above lossy half-space, of half-length $L = 5$ m, radius $a = 0.05$ m, height over half-space $h = 5$ m, wavelength $\lambda = 5$ m, parameters of imperfect ground: $\varepsilon_r = 10$, $\sigma = 0{\cdot}01$ Ω/m, obtained from this work by FEM/Exp. approx., and FEM/SDP, and compared with results available from Reference 6

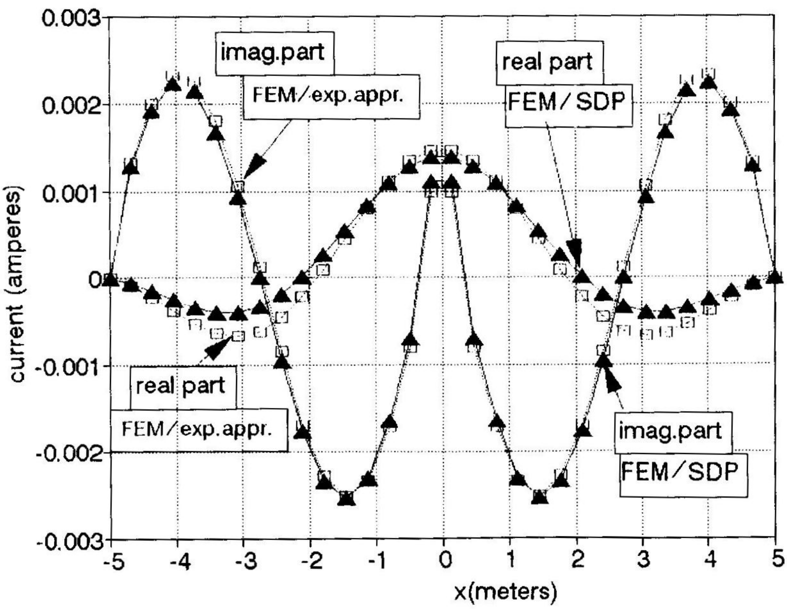

Figure 7. Real and imaginary part of current distribution on the horizontal centre-fed dipole antenna, above lossy half-space, of half-length $L = 5$ m, radius $a = 0{\cdot}05$ m, height over half-space $h = 5$ m, wavelength $\lambda = 5$ m, parameters of imperfect ground: $\varepsilon_r = 10$, $\sigma = 0{\cdot}01$ Ω/m, obtained from this work by FEM/Exp. approx., and FEM/SDP

It has also to be pointed out that convergence of the exponential approximation for treating Sommerfeld integrals is acquired with 20 members in both exponential sums (38) and (39). The convergence is about two times faster if one chooses parameters λ and ξ to vary with geometrical progression instead of arithmetical progression. It has been shown in static and quasistatic field analysis that the number of members in the exponential sum may be 12 (Reference 29) or 16 (Reference 30).

The choice of parameter ξ and values of discrete points λ is a very delicate and sensitive problem. On the basis of extensive numerical experiments and physical background of the considered half-space problem, ξ and λ are chosen to vary in following way:[16]

$$\xi_k = \xi_{k-1}\left(\frac{2h}{\lambda_0}\right)^{N-1}$$

$$\lambda_k = \lambda_{k-1}\frac{k_1}{\lambda_0}$$

where N is the number of members in the exponential sum and $\lambda_0 = 1$ m is the conveniently chosen wavelength.

6. CONCLUSION

Analysis of antenna radiation above a lossy ground is based on solving the corresponding electric field integral equation in the frequency domain. This integral equation is often solved by means of subdomain collocation, and Sommerfeld integrals are calculated either by an analytical technique or numerically. Subdomain collocation is one of the simplest methods but of very poor convergence. It also implies the usage of finite differences for treating the second-order differential operator in order to avoid quasi-singularity in integral equation kernel.

Numerical integration of Sommerfeld integrals is very tedious, because they contain highly oscillating integrands, and is very time-consuming. On the other hand, analytical techniques are valid only over small ranges of parameters. In order to avoid both difficulties, in this paper a new approach is suggested. The electric field integral equation is solved by means of a weak finite element formulation. It is shown that much better convergence is acquired in this way, and at the same time, the use of finite differences (which is very inconvenient in the case of a half-space, because of the appearance of Sommerfeld integrals in the IE kernel) is avoided. In, addition, Sommerfeld integrals are calculated by means of an exponential approximation technique, and there is no need for numerical integration.

The presented methods are applied to the case of a horizontally oriented dipole antenna located above a lossy half-space. Moreover, the suggested methods can be readily applied to the more demanding case of an arbitrarily oriented antenna in an inhomogeneous medium.

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