An Open Surface Integral Formulation for Electromagnetic Scattering by Material Plates

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Abstract—**Using the surface equivalence theorem, four coupled integral equations are developed for electromagnetic scattering by a thin material plate. Using symmetry properties, it is shown that these equations can be written as open surface integral equations. Surface impedance relationships are obtained and used to eliminate two of the four integral equations. The remaining two equations are solved using the method of moments (MM). Numerical results for penetrable and impenetrable material plates are in reasonable agreement with measurements.**

I. INTRODUCTION

THIS PAPER will present a technique for the analysis of electromagnetic scattering from a material plate. By a material plate we mean a three-dimensional planar slab of small but finite thickness, and composed of some dielectric and/or ferrite medium. Alternatively, it can mean a perfectly conducting and infinitesimally thin plate coated on one or both sides by a thin dielectric/ferrite slab. Nonmetallic materials are finding increased application, especially in the construction of aircraft. Here they are used to reduce weight or to modify the radar cross section (RCS). It has been estimated that the aircraft designed from 1985-1995 will be 65 percent composite materials by weight [1].

There are two fundamental approaches to the analysis of material structures. They are through the use of the volume or the surface equivalence principles [2]. Here we choose the surface equivalent approach, since it is felt that this formulation can be most easily integrated into existing methods and computer codes for the analysis of perfectly conducting structures, which are based upon the surface equivalence principle [3]-[13]. The surface equivalence principle applies to closed surfaces. Thus, in a direct application of the surface equivalence theroem, one would enclose the material plate by an extremely thin closed surface. It would then be straightforward to obtain integral equations for the equivalent electric and magnetic surface currents [14]-[17]. However, when these equations were solved by the method of moments (MM) [18], severe numerical difficulties would arise, associated with certain self- and mutual impedances being almost identical [19]. To avoid these numerical difficulties, and again so that the material plate solution can be easily integrated into existing solutions for perfectly conducting plates, (which are based upon currents flowing on an open surface) we have formulated the thin material plate as an open surface solution. In other words, we have formulated the thin material plate in such a way that the thin perfectly conducting plate is a special case of our solution.

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In Section II, the material plate is viewed as an open surface, and four coupled integral equations of the electric and magnetic type are derived for the equivalent electric and magnetic surface currents. In Section III approximate boundary conditions (abc) [20], in the form of surface impedance relationships, are obtained. These abc are applied in Section IV to eliminate two of the four coupled integral equations. It is shown that as the electrical thickness of the plate vanishes, our solution reduces to a single integral equation, identical to the impedance sheet approximation used by Harrington and Mautz [21] and the resistive sheet approximation used by Senior [22]. Section V presents an MM solution to the coupled integral equations of Section IV. In Section VI numerical examples are presented which illustrate the accuracy of the techniques for relatively small and large plates, lossy and lossless materials, and penetrable and impenetrable plates.

II. INTEGRAL EQUATION DEVELOPMENT

Fig. 1(a) shows the side view of a material plate with constitutive parameters (μ, ϵ), and of thickness t, immersed in free space, i.e., a homogeneous medium with constitutive parameters (μ_0, ϵ_0). The plate is excited by the known impressed currents ($\mathbf{J}_i$, $\mathbf{M}_i$) which in free space radiate the incident fields ($\mathbf{E}_i$, $\mathbf{H}_i$), and the unknown fields ($\mathbf{E}$, $\mathbf{H}$) in the presence of the plate. Let S denote the surface enclosing the plate. Using the surface equivalence principle, Fig. 1(a) can be decomposed into the two cases shown in Figs. 1(b), 1(c) [14]. In case A, the constitutive parameters of all space are (μ_0, ϵ_0), the fields are zero inside the plate, the unknown fields ($\mathbf{E}$, $\mathbf{H}$) exist outside the plate, and the surface currents ($\mathbf{J}$, $\mathbf{M}$) flow on the surface S. The currents ($\mathbf{J}$, $\mathbf{M}$) radiate the scattered fields ($\mathbf{E}_s$, $\mathbf{H}_s$) in free space. In case B, the constitutive parameters of all space are (μ, ϵ), the fields are zero outside the plate, the unknown fields ($\mathbf{E}$, $\mathbf{H}$) exist inside the plate, and the surface currents ($\mathbf{J}'$, $\mathbf{M}'$) = ($-\mathbf{J}$, $-\mathbf{M}$) flow on the surface S. We will consider that the plates are sufficiently thin so that the surface currents on the sides of the plate will be ignored. Note that use of the surface equivalence principle allows one to evaluate the fields either inside or outside a material body, although one must use the appropriate equivalent currents (i.e., case A outside or case B inside). Further, this statement is not changed by the use of an abc such as surface impedance relationships.

Referring to Fig. 1, define the following;

S_T	the top of surface S, i.e, that part of S at $x = t/2$;
S_B	the bottom of surface S, i.e., that part of S at $x = -t/2$;
$\mathbf{J}(x, y, z)$	the electric current flowing on the closed surface S;
$\mathbf{J}_T(y, z)$	the top electric current, i.e., that part of $\mathbf{J}$ on S_T;

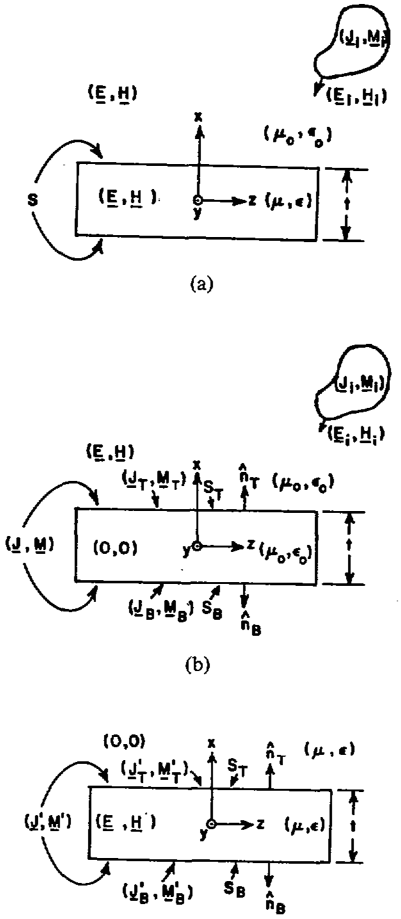

Fig. 1. (a) Side view of a material plate immersed in free space. (b) Case A: Equivalent problem external to S. (c) Case B: Equivalent problem internal to S.

$\mathbf{J}_B(y, z)$	the bottom electric current, i.e., that part of $\mathbf{J}$ on S_B;
$\mathbf{J}_S(y, z)$	the sum electric current, i.e., a single surface current equal to the vector sum $\mathbf{J}_T + \mathbf{J}_B$ and arbitrarily considered to be on S_T;
$\mathbf{J}_D(y, z)$	the difference electric current, i.e., a single surface current equal to the vector difference $\mathbf{J}_T - \mathbf{J}_B$ and arbitrarily considered to be on S_T.
$\mathbf{M}, \mathbf{M}_T, \mathbf{M}_B, \mathbf{M}_S$, and $\mathbf{M}_D$	have identical meanings, except that they refer to magnetic currents.

Now consider the problem of Fig. 1(b). Since a null field exists interior to S, there will be zero reaction between the currents $(\mathbf{J}, \mathbf{M})$ and $(\mathbf{J}_i, \mathbf{M}_i)$, and a test source interior to S. If the test source is electric with current density $\mathbf{J}_t$, this leads to the electric reaction integral equation (ERIE)

$$-\langle \mathbf{J}_t, (\mathbf{J}, \mathbf{M})\rangle = V^J \tag{1}$$

where

$$V^J = \langle \mathbf{J}_t, (\mathbf{J}_i, \mathbf{M}_i)\rangle. \tag{2}$$

By definition, [23], the reaction between sources a and b is

$$\langle a, b\rangle = \iint_b (\mathbf{E}_a \cdot \mathbf{J}_b - \mathbf{H}_a \cdot \mathbf{M}_b)\, ds = \langle b, a\rangle \tag{3}$$

where the integral is over the region occupied by the source b.

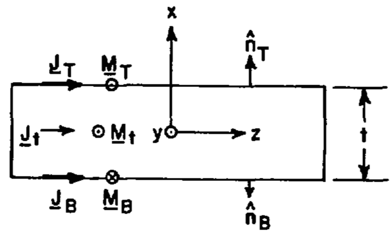

Fig. 2. Side view of material plate with an electric test source J_t and a magnetic test source M_t in the interior.

Fig. 2 shows the side view of the plate with an electric test source $\mathbf{J}_t$ in the interior. $\mathbf{J}_t$ is considered to be a surface current in the yz plane, and its fields in free space will be denoted $(\mathbf{E}_t^J, \mathbf{H}_t^J)$. The plate currents $(\mathbf{J}, \mathbf{M})$ are explicitly shown as $(\mathbf{J}_T, \mathbf{M}_T)$ on S_T and $(\mathbf{J}_B, \mathbf{M}_B)$ on S_B. Using the linearity of the reaction operator (1) becomes

$$-\langle \mathbf{J}_t, (\mathbf{J}_T, \mathbf{M}_T)\rangle - \langle \mathbf{J}_t, (\mathbf{J}_B, \mathbf{M}_B)\rangle = V^J, \tag{4}$$

or

$$-\iint_{S_T} (\mathbf{E}_t^J \cdot \mathbf{J}_T - \mathbf{H}_t^J \cdot \mathbf{M}_T)\, ds - \iint_{S_B} (\mathbf{E}_t^J \cdot \mathbf{J}_B - \mathbf{H}_t^J \cdot \mathbf{M}_B)\, ds = V^J. \tag{5}$$

Since the tangential electric and magnetic fields of $\mathbf{J}_t$ are even and odd functions of X, respectively,

$$\mathbf{E}_t^J(S_T) \times \hat{n}_T = \mathbf{E}_t^J(S_B) \times \hat{n}_T \tag{6a}$$

$$\mathbf{H}_t^J(S_T) \times \hat{n}_T = -\mathbf{H}_t^J(S_B) \times \hat{n}_T \underset{\lim t \to 0}{=} \mathbf{J}_t/2. \tag{6b}$$

Using the symmetry relations of (6), we move the bottom currents, $(\mathbf{J}_B, \mathbf{M}_B)$, from S_B to S_T, and (5) becomes

$$-\iint_{S_T} [\mathbf{E}_t^J \cdot (\mathbf{J}_T + \mathbf{J}_B) - \mathbf{H}_t^J \cdot (\mathbf{M}_T - \mathbf{M}_B)]\, ds = V^J, \tag{7}$$

or

$$-\langle \mathbf{J}_t, (\mathbf{J}_S, \mathbf{M}_D)\rangle = V^J. \tag{8}$$

Note that (8) applies on the open surface S_T.

Next consider the case where the test source in (3) is the magnetic current $\mathbf{M}_t$. This leads to the magnetic reaction integral equation (MRIE)

$$-\langle \mathbf{M}_t, (\mathbf{J}, \mathbf{M})\rangle = V^M \tag{9}$$

where

$$V^M = \langle \mathbf{M}_t, (\mathbf{J}_i, \mathbf{M}_i)\rangle. \tag{10}$$

Fig. 2 shows the side view of the plate with a magnetic test source $\mathbf{M}_t$ in the interior. $\mathbf{M}_t$ is considered to be a surface current in the yz plane, and its fields in free space will be denoted $(\mathbf{E}_t^M, \mathbf{H}_t^M)$. Again, using the linearity of the reaction operator, (9) becomes

$$-\langle \mathbf{M}_t, (\mathbf{J}_T, \mathbf{M}_T)\rangle - \langle \mathbf{M}_t, (\mathbf{J}_B, \mathbf{M}_B)\rangle = V^M, \tag{11}$$

or

$$-\iint_{S_T} (\mathbf{E}_t^M \cdot \mathbf{J}_T - \mathbf{H}_t^M \cdot \mathbf{M}_T)\, ds - \iint_{S_B} (\mathbf{E}_t^M \cdot \mathbf{J}_B - \mathbf{H}_t^M \cdot \mathbf{M}_B)\, ds = V^m. \tag{12}$$

Since the tangential electric and magnetic fields of $\mathbf{M}_t$ are odd and even functions of x, respectively,

$$\mathbf{E}_t^M(S_T) \times \hat{n}_T = -\mathbf{E}_t^M(S_B) \times \hat{n}_T \underset{\lim t\to 0}{=} \mathbf{M}_t/2. \tag{13a}$$

$$\mathbf{H}_t^M(S_T) \times \hat{n}_T = \mathbf{H}_t^M(S_B) \times \hat{n}_T. \tag{13b}$$

Using the symmetry relations of (13), we again move $(\mathbf{J}_B, \mathbf{M}_B)$ from S_B to S_T, and (12) becomes

$$-\iint_{S_T} [\mathbf{E}_t^M \cdot (\mathbf{J}_T - \mathbf{J}_B) - \mathbf{H}_t^M \cdot (\mathbf{M}_T + \mathbf{M}_B)]\, ds = V^M, \tag{14}$$

or

$$-\langle \mathbf{M}_t, (\mathbf{J}_D, \mathbf{M}_S)\rangle = V^M, \tag{15}$$

valid on the open surface S_T.

Equations (8) and (15) constitute two of the four equations needed to solve for the four unknowns, $(\mathbf{J}_S, \mathbf{M}_S)$ and $(\mathbf{J}_D, \mathbf{M}_D)$. In obtaining these equations, we did not require that the plate thickness go to zero. The thickness does have to be sufficiently small that the side currents can be ignored. To obtain the remaining two equations, the zero reaction concept with test sources outside S is applied to the equivalent problem of Fig. 1(c). An arbitrary test source $(\mathbf{J}_t, \mathbf{M}_t)$ exterior to S in Fig. 1(c) has zero reaction with the currents $(\mathbf{J}', \mathbf{M}') = (-\mathbf{J}, -\mathbf{M})$, or

$$\langle (\mathbf{J}_t, \mathbf{M}_t), (\mathbf{J}, \mathbf{M})\rangle = 0. \tag{16}$$

Explicitly showing the integrations over the top and bottom surfaces, (16) becomes

$$\iint_{S_T} (\mathbf{E}_t \cdot \mathbf{J}_T - \mathbf{H}_t \cdot \mathbf{M}_T)\, ds + \iint_{S_B} (\mathbf{E}_t \cdot \mathbf{J}_B - \mathbf{H}_t \cdot \mathbf{M}_B)\, ds = 0 \tag{17}$$

where $(\mathbf{E}_t, \mathbf{H}_t)$ are the fields of $(\mathbf{J}_t, \mathbf{M}_t)$ in the homogeneous medium (μ, ϵ). In practice, two independent equations can be obtained from (16) by first placing $(\mathbf{J}_t, \mathbf{M}_t)$ in the plane $x = (t + \Delta t)/2$ and then in the plane $x = -(t + \Delta t)/2$ where Δt is a positive value. Thus, (8), (15), and (16) twice constitute four coupled integral equations which could, in principal, be solved for $(\mathbf{J}_S, \mathbf{M}_S)$ and $(\mathbf{J}_D, \mathbf{M}_D)$. In order to properly account for plate thickness, one should convert the sum and difference currents to top and bottom currents prior to computing the scattered fields (interior or exterior to the plate). However, for thin plates and exterior field points whose x distance from the plate is large in comparison to plate thickness, one may compute the scattered fields from $(\mathbf{J}_S, \mathbf{M}_S)$ only.

III. IMPEDANCE BOUNDARY CONDITION

In this section we will obtain the surface impedance relationships for a penetrable dielectric/ferrite plane sheet or slab. We will also obtain the surface impedance of an impenetrable perfect conductor backed on one or both sides by a dielectric/ferrite slab. This will be done by considering the simple one-dimensional problem of a plane wave normally incident upon a dielectric/ferrite slab. Although the solution to this problem, in terms of plane wave reflections, appears in many texts on electromagnetic theory [24], we will investigate the solution using the surface equivalence principle and the reaction integral equations obtained in the previous section. For the case of the slab, (16) has simple solutions from which we can deduce relationships between the equivalent electric and magnetic surface currents, i.e., the slab surface impedance relationships. In the next section we will employ the approximation that these relationships (which are exact for the slab) apply also to the finite or three-dimensional material plate. This will eliminate two of the four integral equations for the material plate.

Fig. 3(a) shows a plane wave of the form

$$\mathbf{E}_i = \hat{z}E_0 e^{\gamma_0 x}, \tag{18a}$$

$$\mathbf{H}_i = \hat{y}E_0 e^{\gamma_0 x}/\eta_0, \tag{18b}$$

where $\gamma_0 = j\omega\sqrt{\mu_0\epsilon_0}$ and $\eta_0 = \sqrt{\mu_0/\epsilon_0}$ incident upon a slab of thickness t and with parameters (μ, ϵ). The illuminated surface of the slab at $x = t/2$ will be referred to as S_T, the top surface. The surface at $x = -t/2$ will be referred to as S_B, the bottom surface. In direct analogy to Fig. 1, the equivalent currents will be constant sheets denoted $(J_T\hat{z}, M_T\hat{y})$ on S_T and $(J_B\hat{z}, -M_B\hat{y})$ on S_B. Referring to Fig. 2(c) and (16), there will be zero reaction between a constant sheet current test source, located outside the slab, and the equivalent currents. If the test source is given by the electric current

$$\mathbf{J}_t = -2\hat{z}. \tag{19}$$

and is located in the plane $x = x_t (|x_t| > t/2)$, then its fields in the medium (μ, ϵ) are

$$\mathbf{E}_t = \hat{z}\eta e^{-\gamma|x - x_t|}, \tag{20a}$$

$$\mathbf{H}_t = -\hat{y}\ \mathrm{sgn}\,(x - x_t) e^{-\gamma|x - x_t|}, \tag{20b}$$

where $\eta = \sqrt{\mu/\epsilon}$, $\gamma = j\omega\sqrt{\mu\epsilon}$, and $\mathrm{sgn}\,(x) = 1$ if $x > 0$ and -1 if $x < 0$.

Inserting (19) and (20) into (16) yields

$$J_T\eta e^{-\gamma t/2} + J_B\eta e^{\gamma t/2} + M_T e^{-\gamma t/2} - M_B e^{\gamma t/2} = 0, \quad \text{for } x_t < -t/2, \tag{21a}$$

$$J_T\eta e^{\gamma t/2} + J_B\eta e^{-\gamma t/2} - M_T e^{\gamma t/2} + M_B e^{-\gamma t/2} = 0, \quad \text{for } x_t > t/2. \tag{21b}$$

Solving (21) for M_T and M_B yields

$$M_T = Z_{TT}J_T + Z_{TB}J_B, \tag{22a}$$

$$M_B = Z_{BT}J_T + Z_{BB}J_B, \tag{22b}$$

where

$$Z_{TT} = Z_{BB} = \eta/\tanh \gamma t, \tag{23a}$$

$$Z_{TB} = Z_{BT} = \eta/\sinh \gamma t. \tag{23b}$$

These impedances are equal to the two port impedance parameters of the transmission line of length t, characteristic impedance η, and propagation constant γ, shown in Fig. 3(c). They are comparable to equations by Mitzner for an absorbing shell [25].

If the slab had been impenetrable, such as the perfectly conducting ground plane coated on one or both sides by a dielectric/ferrite slab of Fig. 3(b), then the two port impedances would be those of the transmission line model of Fig. 3(d). In this case we have $Z_{TB} = Z_{BT} = 0$, and

$$Z_{TT} = \eta_1 \tanh \gamma_1 t_1, \tag{24a}$$

$$Z_{BB} = \eta_2 \tanh \gamma_2 t_2. \tag{24b}$$

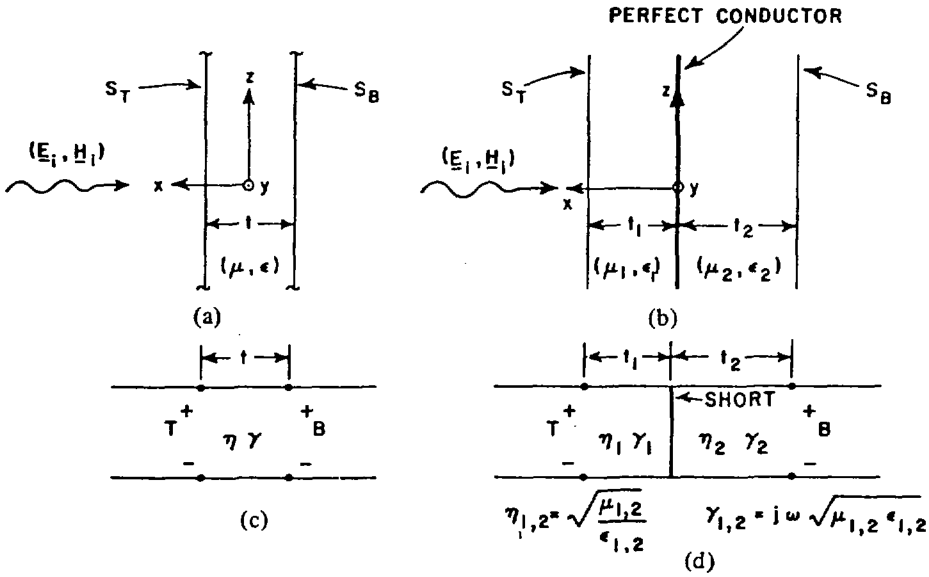

Fig. 3. (a) Side view of material slab immersed in free space. (b) Side view of a coated, perfectly conducting sheet immersed in free space. (c) Transmission line equivalent of (a). (d) Transmission line equivalent of (b).

IV. SURFACE IMPEDANCE APPROXIMATION

In the last section, two impedance boundary relationships ((22)) for a one dimensional dielectric/ferrite slab were obtained. In this section, (22) are used as an approximation for (16). For a one-dimensional slab at normal incidence these relationships are exact. For a three-dimensional material plate they are not exact since 1) the incident wave may not have normal incidence, 2) the edges and corners of the plate will modify the relationships. Despite this, we will use the one dimensional slab surface impedance relationships for the three dimensional material plate. This is an application of what Senior [20] termed an approximate boundary condition (abc). As pointed out by Senior, there usually is no rigorous justification for the use of an abc. The justification is in their usefulness in predicting results obtained by measurements or by more rigorous techniques. Of course, the motivation for their use, as compared to more rigorous techniques, is that they simplify the analysis. It is possible that better results would be obtained by using a surface impedance which is dependent upon the angle of incidence, however, for simplicity this was not done here. Writing (22) in terms of sum and difference currents gives

$$\mathbf{M}_S = Z_{SS}\mathbf{J}_S \times \hat{n}_T + Z_{SD}\mathbf{J}_D \times \hat{n}_T, \tag{25a}$$

$$\mathbf{M}_D = Z_{DS}\mathbf{J}_S \times \hat{n}_T + Z_{DD}\mathbf{J}_D \times \hat{n}_T, \tag{25b}$$

where

$$Z_{SS} = (Z_{TT} + Z_{TB} - Z_{BT} - Z_{BB})/2, \tag{26a}$$

$$Z_{SD} = (Z_{TT} - Z_{TB} - Z_{BT} + Z_{BB})/2, \tag{26b}$$

$$Z_{DS} = (Z_{TT} + Z_{TB} + Z_{BT} + Z_{BB})/2, \tag{26c}$$

$$Z_{DD} = (Z_{TT} - Z_{TB} + Z_{BT} - Z_{BB})/2. \tag{26d}$$

Combining (8), (15), and (25) yields

$$-\langle \mathbf{J}_t, (\mathbf{J}_S, \mathbf{M}_D)\rangle = -\langle \mathbf{J}_t, (\mathbf{J}_S, Z_{DS}\mathbf{J}_S \times \hat{n}_T + Z_{DD}\mathbf{J} \times \hat{n}_T)\rangle = V^J, \tag{27a}$$

$$-\langle \mathbf{M}_t, (\mathbf{J}_D, \mathbf{M}_S)\rangle = -\langle \mathbf{M}_t, (\mathbf{J}_D, Z_{SS}\mathbf{J}_S \times \hat{n}_T + Z_{SD}\mathbf{J}_D \times \hat{n}_T)\rangle = V^M. \tag{27b}$$

Equations (27) represent two coupled integral equations on the open surface S_T which can be solved for the electric currents $\mathbf{J}_S$ and $\mathbf{J}_D$. Having solved for $\mathbf{J}_S$ and $\mathbf{J}_D$, the magnetic currents $\mathbf{M}_S$ and $\mathbf{M}_D$ can be found from (25). We reemphasize that the $Z_{DS}\mathbf{J}_S \times \hat{n}_T + Z_{DD}\mathbf{J}_D \times \hat{n}_T$ and $Z_{SS}\mathbf{J}_S \times \hat{n}_T + Z_{SD}\mathbf{J}_D \times \hat{n}_T$ terms in (27) are not electric currents, but rather are the magnetic currents, $\mathbf{M}_D$ and $\mathbf{M}_S$, respectively.

In the remaining part of this section, (27) is examined for various special cases.

A. Case 1–Perfectly Conducting Plates

If the plate is a perfect conductor, then $Z_{SS} = Z_{DD} = Z_{DS} = Z_{SD} = 0$ and (27) reduces to

$$-\langle \mathbf{J}_t, (\mathbf{J}_S, 0)\rangle = V^J, \tag{28a}$$

$$-\langle \mathbf{M}_t, (\mathbf{J}_D, 0)\rangle = V^M. \tag{28b}$$

Equations (28) were previously presented and solved by the authors [26].

B. Case 2–Uniform Material Plate

If the plate is of uniform μ and ϵ, then from (23) and (26), $Z_{ss} = Z_{DD} = 0$ and

$$Z_{SD} = Z_{TT} - Z_{TB} = \eta(\cosh \gamma t - 1)/\sinh \gamma t, \tag{29a}$$

$$Z_{DS} = Z_{TT} + Z_{TB} = \eta(\cosh \gamma t + 1)/\sinh \gamma t. \tag{29b}$$

In this case, (27) reduces to

$$-\langle \mathbf{J}_t, (\mathbf{J}_S, \mathbf{M}_D)\rangle = -\langle \mathbf{J}_t, (\mathbf{J}_S, Z_{DS}\mathbf{J}_S \times \hat{n}_T)\rangle = V^J, \tag{30a}$$

$$-\langle \mathbf{M}_t, (\mathbf{J}_D, \mathbf{M}_S)\rangle = -\langle \mathbf{M}_t, (\mathbf{J}_D, Z_{SD}\mathbf{J}_D \times \hat{n}_T)\rangle = V^M. \tag{30b}$$

An important point is that the equations uncouple for this case. However, note that both equations must be solved to find ($\mathbf{J}_S$, $\mathbf{M}_S$) which radiate the far-zone scattered field. In general, for any symmetric slab (i.e., $Z_{TT} = Z_{BB}$ and $Z_{TB} = Z_{BT}$) the equations uncouple.

C. Case 3–Very Thin Uniform Material Plate

Now consider (30a) in the limit as the thickness γt goes to zero. By reciprocity, (30a) can be written as

$$-\iint_t \mathbf{J}_t \cdot (\mathbf{E}(\mathbf{J}_S) + \mathbf{E}(\mathbf{M}_D) + \mathbf{E}_i)\, ds = 0 \tag{31}$$

where $\mathbf{E}(\mathbf{J}_S)$ and $\mathbf{E}(\mathbf{M}_D)$ are the free-space electric fields of $\mathbf{J}_S$ and $\mathbf{M}_D = Z_{DS}\mathbf{J}_S \times \hat{n}_T$, respectively, and the integral is over the surface of the arbitrary electric test source $\mathbf{J}_t$. Note that $\mathbf{J}_t$ is located in the interior of the closed surface S and is directed tangential to the broad surfaces of S.

Since (31) must apply for an arbitrary $\mathbf{J}_t$, it is clear that

$$\mathbf{E}_{\tan}(\mathbf{J}_S) + \mathbf{E}_{\tan}(\mathbf{M}_D) = -\mathbf{E}_{i\tan} \text{ interior to } S. \tag{32}$$

As $\gamma t \to 0$, the tangential electric field of $\mathbf{M}_D$ will be essentially (see (13a))

$$\mathbf{E}_{\tan}(\mathbf{M}_D) = -\hat{n}_T \times \mathbf{M}_D/2. \tag{33}$$

Also from (29b).

$$\lim_{\gamma t \to 0} Z_{DS} = \frac{2\eta}{\gamma t}. \tag{34}$$

Combining (25b), (32), (33), and (34) gives

$$-\mathbf{E}_{\tan}(\mathbf{J}_S) + \frac{\eta}{\gamma t}\mathbf{J}_S = \mathbf{E}_{i\tan}. \tag{35}$$

Equation (35) gives the current on a surface of sheet impedance $\eta/\gamma t$. It is equivalent to that used by Harrington to study thin dielectric shells [21] and by Senior to study resistive strips [22]. Note that when treating impedance sheets, one simply adds the term $\eta/\gamma t \mathbf{J}_S$ to the equation for perfectly conducting surfaces. In a MM solution of (35) this would result in an impedance matrix of the form $[Z] + [\Delta Z]$ where $[Z]$ is the impedance matrix for the perfectly conducting surface, and a typical term of $[\Delta Z]$ would be

$$\Delta Z_{mn} = \frac{\eta}{\gamma t}\iint \mathbf{J}_m \cdot \mathbf{G}_n\, ds. \tag{36}$$

Here $\mathbf{J}_m$ represents the mth current expansion mode and $\mathbf{G}_n$ the nth test mode or weighting function. Equation (36) can be evaluated in closed form for any of the usual choices for the modes. This permits the thin symmetric slab to be treated as a very simple modification of the perfect conductor. Note that $\Delta Z_{mn} = 0$ unless modes $\mathbf{J}_m$ and $\mathbf{G}_n$ overlap. Also, for extremely small γt, ΔZ_{mn} will be large, and this will result in small J_s.

D. Case 4–Coated Conducting Plate

If the plate is a perfect conductor coated on one or both sides by a dielectric/ferrite, then from (25) and (26)

$$Z_{SS} = Z_{DD} = (\eta_1 \tanh \gamma_1 t_1 - \eta_2 \tanh \gamma_2 t_2)/2 \tag{37a}$$

$$Z_{SD} = Z_{DS} = (\eta_1 \tanh \gamma_1 t_1 + \eta_2 \tanh \gamma_2 t_2)/2 \tag{37b}$$

and (27) reduce to the coupled equations

$$-\langle \mathbf{J}_t, (\mathbf{J}_S, \mathbf{M}_D)\rangle = -\langle \mathbf{J}_t, (\mathbf{J}_S, Z_{SD}\mathbf{J}_S \times \hat{n}_T + Z_{SS}\mathbf{J}_D \times \hat{n}_T)\rangle = V^J, \tag{38a}$$

$$-\langle \mathbf{M}_t, (\mathbf{J}_D, \mathbf{M}_S)\rangle = -\langle \mathbf{M}_t, (\mathbf{J}_D, Z_{SS}\mathbf{J}_S \times \hat{n}_T + Z_{SD}\mathbf{J}_D \times \hat{n}_T)\rangle = V^M. \tag{38b}$$

V. MOMENT METHOD SOLUTION

The current distributions on the material plates can be found by solving the integral equations of the previous section by the numerical technique known as the method of moments [18]. Here we will present the MM solution to (27) which represent the most general case, rather than one of the special cases which follow. The solution is similar to that for perfectly conducting surfaces [5], [12], [25], and is presented in detail in [27]. Thus our description will be brief, emphasizing only those parts which are new.

The first step in the MM solution is to expand the unknown currents, $\mathbf{J}_S$ and $\mathbf{J}_D$ in a finite set of N basis functions:

$$\mathbf{J}_S = \sum_{n=1}^{N} I_{Sn}\mathbf{F}_n \tag{39a}$$

$$\mathbf{J}_D = \sum_{n=1}^{N} I_{Dn}\mathbf{F}_n \tag{39b}$$

where the $\mathbf{F}_n$ are piecewise sinusoidal surface patch modes [5], [12], [25], [26]. Then using the abc's of (25), $\mathbf{M}_S$ and $\mathbf{M}_D$ are the dependent unknowns

$$\mathbf{M}_S = \sum_{n=1}^{N} (Z_{SS}I_{Sn} + Z_{SD}I_{Dn})\mathbf{F}_n \times \hat{n}_T, \tag{40a}$$

$$\mathbf{M}_D = \sum_{n=1}^{N} (Z_{DS}I_{Sn} + Z_{DD}I_{Dn})\mathbf{F}_n \times \hat{n}_T. \tag{40b}$$

Choosing our test modes identical to the expansion modes (except that they are in the yz plane), (27) reduce to the set of $2N$ simultaneous linear equations written in block matrix form as

$$\left[\begin{array}{c|c} [Z^{EE}] + Z_{DS}[Z^{EM}] & Z_{DD}[Z^{EM}] \\ \hline Z_{SS}[Z^{MM}] & [Z^{ME}] + Z_{SD}[Z^{MM}] \end{array}\right]\left[\begin{array}{c} I_S \\ \hline I_D \end{array}\right] = \left[\begin{array}{c} V^J \\ \hline V^M \end{array}\right], \tag{41}$$

where

$$Z_{mn}^{EE} = -\iint_n \mathbf{E}_m^J \cdot \mathbf{F}_n\, ds, \tag{42a}$$

$$Z_{mn}^{EM} = -\iint_n \mathbf{H}_m^J \cdot (\mathbf{F}_n \times \hat{n}_T)\, ds, \tag{42b}$$

$$Z_{mn}^{ME} = -\iint_n \mathbf{E}_m^M \cdot \mathbf{F}_n\, ds, \tag{42c}$$

$$Z_{mn}^{MM} = \iint_n \mathbf{H}_m^M \cdot (\mathbf{F}_n \times \hat{n}_T)\, ds, \tag{42d}$$

$$V_m^J = \iiint_v (\mathbf{J}_i \cdot \mathbf{E}_m^J - \mathbf{M}_i \cdot \mathbf{H}_m^J)\, dv, \tag{42e}$$

$$V_m^M = \iiint_v (\mathbf{J}_i \cdot \mathbf{E}_m^M - \mathbf{M}_i \cdot \mathbf{H}_m^M)\, dv. \tag{42f}$$

The integrations in (42a)-(42d) are over the surface of the nth expansion mode located on S_T. $(\mathbf{E}_m^J, \mathbf{H}_m^J)$ and $(\mathbf{E}_m^M, \mathbf{H}_m^M)$ are the free space fields of the mth electric and magnetic test modes, respectively. Since the expansion and test modes used in this solution are identical (except for a shift of $t/2$), the method is essentially a Galerkin solution, and each of the matrices $[Z^{EE}]$ and $[Z^{MM}]$ are symmetric. Also $[Z^{EM}] = -[Z^{ME}]^T$.

A typical rectangular plate is shown in Fig. 4. Two orthgonal and overlapping sets of surface-patch modes cover the surface of the plate forming a two-dimensional vector surface density which could be used to represent $\mathbf{J}_S$, $\mathbf{J}_D$, $\mathbf{M}_S$, or $\mathbf{M}_D$. The normal components of $\mathbf{J}_S$ and $\mathbf{M}_D$ must vanish at a plate edge, however, the normal components of $\mathbf{J}_D$ and $\mathbf{M}_S$ do not. Thus, we include a series of edge modes, consisting of a single monopole with terminals at the plate edge. This allows for a nonzero normal component of current at the plate edge. The plate in Fig. 4 is made up of ten monopole edge modes and seven dipole interior modes. The monopole edge modes will be identically zero for $\mathbf{J}_S$ and $\mathbf{M}_D$, but not for $\mathbf{J}_D$ and $\mathbf{M}_S$.

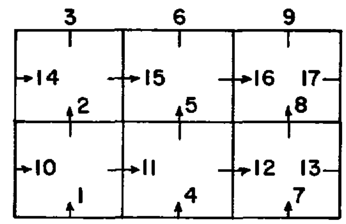

Fig. 4. Surface patch dipole mode layout for a flat plate.

A. Numerical Results

In this section we will present numerical results for electromagnetic scattering by material plates. Our results, computed using (41) and (42) will be compared with measurements.

Fig. 5 shows the radar cross section (RCS) versus angle of incidence for a 0.813 λ square by 0.051 λ thick penetrable dielectric plate with relative dielectric constant, $\epsilon_r = \epsilon/\epsilon_0 = 4$. Both transverse electric (TE) and transverse magnetic (TM) polarizations are shown and compared with measurements by Thal and Finger [28]. The agreement is seen to be good near broadside, however, it worsens as one approaches grazing. This is to be expected, since the surface impedance relationships were evaluated for a normally incident wave. Also shown in Fig. 5 is the RCS computed by the impedance sheet approximation of (35). The sheet impedance approximation produces a less accurate result since it requires $|\gamma t| \ll 1$, while in this case $|\gamma t| = 0.64$. Our experience indicates that $|\gamma t|$ should be less than about 0.2 to use the impedance sheet approximation.

The insert in Fig. 6 shows a 1.5 λ square perfectly conducting plate coated on one side by narrow band dielectric/ferrite absorber with $\epsilon_r = 7.8 - j1.6$, $\mu_r = 1.5 - j0.7$, and thickness 0.065 λ. This is an example of an impenetrable plate with asymmetric surface impedance relationships. Fig. 6 shows the RCS of this plate evaluated with the surface impedance relationships of (37), and compared with measurements. Note that the theory correctly predicts about a 20 dB drop in RCS when the incident field is broadside to the absorber, rather than the perfect conductor.

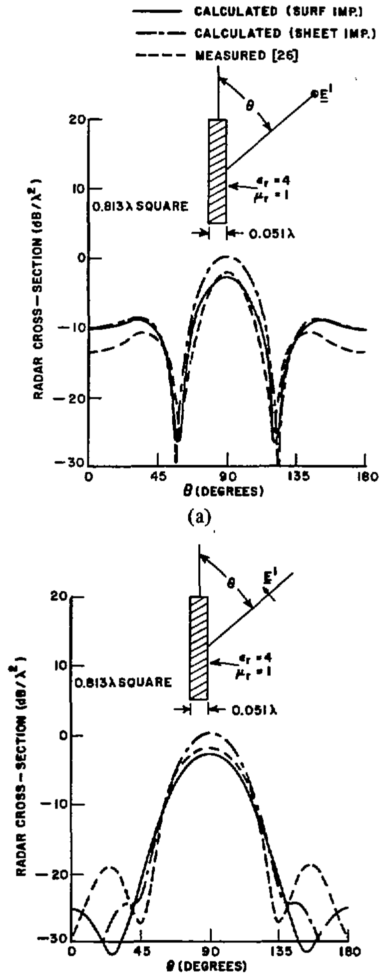

Fig. 5. (a) TE radar cross secion of a square dielectric plate. (b) TM radar across section of a square dielectric plate.

VI. SUMMARY

An integral equation formulation for the electromagnetic scattering from a thin material plate has been presented. The material plate is viewed as an open surface, in order that the solution appear as an extension or modification of that for perfectly conducting plates. Solutions for the sum and difference electric and magnetic equivalent surface currents are obtained in the form of four coupled integral equations. By using the abc or surface impedance relationships two of the four integral equations are eliminated. The surface impedance relationships are shown to be the two-port impedance parameters of a simple transmission line model. By including the off-diagonal terms in the two-port impedance matrix, penetrable as well as impenetrable plates can be treated. Several special cases of the two coupled integral equations are investigated, and it is shown that for electrically very thin plates, the results simplify to a single equation equivalent to the impedance sheet approximations used by Harrington [21] and Senior [22]. The equations were solved by the method of moments, and accurate results were obtained. Future work in this area should include solutions for non-planar geometries, and the intersection of a perfectly conducting and a material plate.

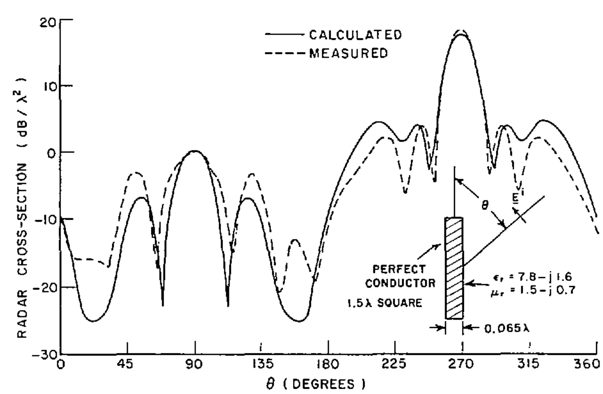

Fig. 6. Radar cross section of a coated, perfectly conducting plate.

REFERENCES

[1] *Aviation Week and Space Technology*, p. 102, Sept. 14, 1981.

[2] R. F. Harrington, *Time-Harmonic Electromagnetic Fields*. New York: McGraw-Hill, 1961, ch. 30.

[3] E. H. Newman, "A surface patch model for polygonal plates," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 588-593, July 1982.

[4] N. N. Wang, J. H. Richmond, and M. C. Gilbreath, "Sinusoidal reaction formulation for radiation and scattering from conducting surfaces," *IEEE Trans. Antennas Propagat.*, vol. AP-23, pp. 376-382, May 1975.

[5] E. H. Newman and D. N. Pozar, "Electromagnetic modeling of composite wire and surface goemetries," *IEEE Trans. Antennas Propagat.*, vol. AP-26, pp. 784-789, Nov. 1978.

[6] D. L. Knepp and J. Goldhirsh, "Numerical analysis of electromagnetic radiation properties of smooth conducting bodies of arbitrary shape," *IEEE Trans. Antennas Propagat.* vol. 20, pp. 383-388, May 1972.

[7] N. C. Albertsen, J. E. Hansen, and N. E. Jensen, "Computation of radiation from wire antennas on conducting bodies," *IEEE Trans. Antennas Propagat.*, vol. AP-22, pp. 200-206, Mar. 1974.

[8] G. J. Burke and A. J. Poggio, "Numerical electromagnetic code (NEC)-method of moments," Naval Ocean Syst. Cent., Tech Doc. 116, AFWL-TR-76-320, July 1977.

[9] J. J. H. Wang, "Numerical analysis of three dimensional arbitrarily-shaped conducting scatterers by trilateral surface cell modelling," *Radio Sci.*, vol. 13, pp. 947-952, Nov./Dec. 1978.

[10] D. R. Wilton, S. S. M. Rao, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape,"*IEEE Trans. on Antennas Propagat.*, vol. AP-30, pp. 409-418, May 1982.

[11] J. Singh and A. T. Adams, "A nonrectangular patch model for scattering from surfaces," *IEEE Trans. Antennas Propagat.*, vol. AP-27, pp. 531-535, July 1979.

[12] E. H. Newman and D. M. Pozar, "Considerations for efficient wire/surface modeling," *IEEE Trans. Antennas Propagat.*, vol. AP-28, pp. 121-125, Jan. 1980.

[13] E. H. Newman, "A user's manual for electromagnetic surface patch code (ESP) Ohio State Univ. ElectroSci. Lab., Rep. 713402-1, Dep. Elect. Eng. prepared under Contract DAAG39-81-K-0020 for the Department of the Army, U.S. Army Res. Office, Res. Triangle Park, NC. July, 1981.

[14] Y. Chang and R. F. Harrington, "A surface formulation for characteristic modes of material bodies," *IEEE Trans Antennas Propagat.*, vol. AP-25, pp. 789-795, Nov. 1977.

[15] N. Morita, "Surface integral representation for electromagnetic scattering from dielectric cylinders," *IEEE Trans. Antennas Propagat.*, vol. AP-26, pp. 261-266, Mar. 1978.

[16] P. C. Waterman, "Scattering by dielectric obstacles," *Alta Freq.*, vol. 38, (specialie), pp. 348-352, 1969.

[17] A. J. Poggio and E. K. Miller, "Integral equation solutions of three dimensional scattering problems," in *Computer Techniques for Electromagnetics*, R. Mittra, Ed. Oxford: Pergamon, 1973, ch. 4.

[18] R. F. Harrington, *Field Computations by Moment Methods*. New York: MacMillan, 1968.

[19] E. H. Newman and P. Tulyathan, "Analysis of microstrip antennas using moment methods," *IEEE Trans. Antennas and Propagat.*, vol. AP-29, pp. 47-53, Jan. 1981.

[20] T. B. A. Senior, "Approximate boundary conditions," *IEEE Trans. Antennas Propagat.*, vol. AP-29, pp. 826-829, Sept. 1981.

[21] R. F. Harrington and J. R. Mautz, " An impedance sheet approximation for thin dielectric shells," *IEEE Trans. Antennas Propagat.*, vol. AP-23, pp. 531-534, July 1975.

[22] T. B. A. Senior, "Backscattering from resistive strips," *IEEE Trans. Antennas Propagat.*, vol. AP-27, pp. 808-813, Nov. 1979.

[23] V. H. Rumsey, "Reaction concept in electromagnetic theory," *Phys. Rev.*, vol. 94, pp. 1483-1491, June 1954.

[24] J. A. Stratton, *Electromagnetic Theory*. New York: McGraw-Hill, 1941, pp. 511-513.

[25] K. M. Mitzner, "Effective boundary conditions for reflection and transmission by an absorbing shell of arbitrary shape," *IEEE Trans. Antennas Propagat.*, vol. AP-16, pp. 706-712, Nov. 1968.

[26] E. H. Newman and M. R. Schrote, "On current distribution for open surfaces," *IEEE Trans. Antennas Propagat.*, vol. AP-31, May 1983.

[27] M. R. Schrote, "An open surface integral formulation for electromagnetic scattering by a material plate," Ph.D. dissertation, Dept. of Elect. Eng., The Ohio State Univ.

[28] H. L. Thal and J. F. Finger, "Scattering by dielectric containers," *IEEE Trans. Antennas Propagat.*, vol. AP-18, pp. 709-711, Sept. 1970.

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Mark R. Schrote, for a photograph and biography please see page 1176 of the November 1982 issue of this TRANSACTIONS.