Dispersion relation of surface plasmons

"Metal Optics"
Prof. Vlad Shalaev, Purdue Univ., ECE Department,
http://shay.ecn.purdue.edu/~ece695s/

"Surface plasmons on smooth and rough surfaces and on gratings"
Heinz Raether(Univ Hamburg), 1988, Springer-Verlag

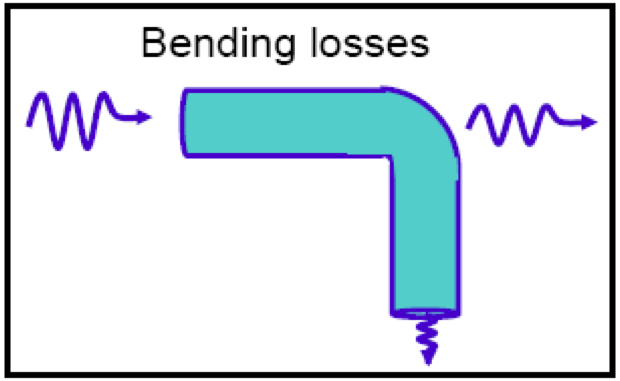
"Surface-plasmon-polariton waveguides"
Hyongsik Won, Ph.D Thesis, Hanyang Univ, 2005.

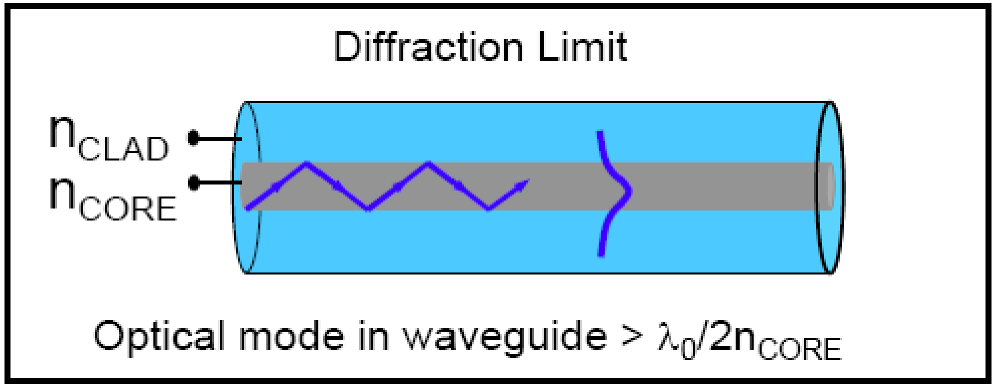
Metal Optics: An introduction

Majority of optical components based on dielectrics

- High speed, high bandwidth (ω), but…
- Does not scale well ⇨ Needed for large scale integration

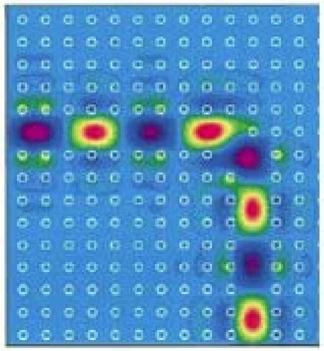
Problems





Solutions ?

Some:



J. D. Joannopoulos, et al, Nature, vol.386, p.143-9 (1997)

Some fundamental problems!

⇩

Photonic functionality based on metals?!

What is a plasmon?

What is a plasmon ?

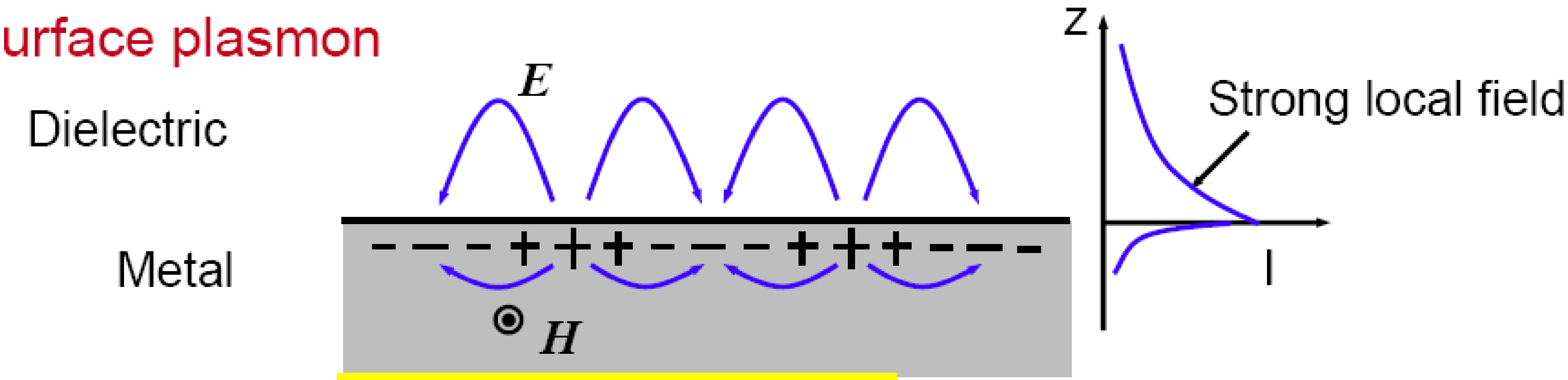
- Compare electron gas in a metal and real gas of molecules
- Metals are expected to allow for electron density waves: plasmons

Bulk plasmon

- Metals allow for EM wave propagation above the plasma frequency

They become transparent!

Surface plasmon

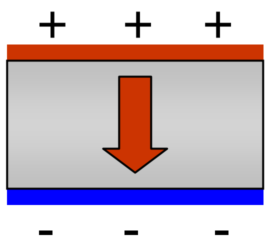


Note: SP is a TM wave!

- Sometimes called a surface plasmon-polariton (strong coupling to EM field)

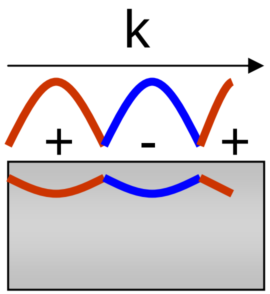
Plasmon-Polaritons

Plasma oscillation = density fluctuation of free electrons



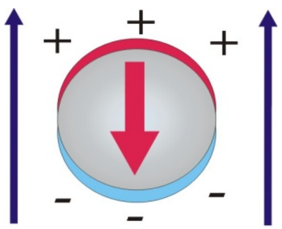
Plasmons in the bulk oscillate at ω_p determined by the free electron density and effective mass

$$\omega_p^{drude} = \sqrt{\frac{Ne^2}{m\varepsilon_0}}$$



Plasmons confined to surfaces that can interact with light to form propagating "surface plasmon polaritons (SPP)"

$$k_x = k_{sp} = \frac{\omega}{c}\sqrt{\frac{(\omega^2 - \omega_p{}^2)\varepsilon_d}{(1+\varepsilon_d)\omega^2 - \omega_p{}^2}}$$

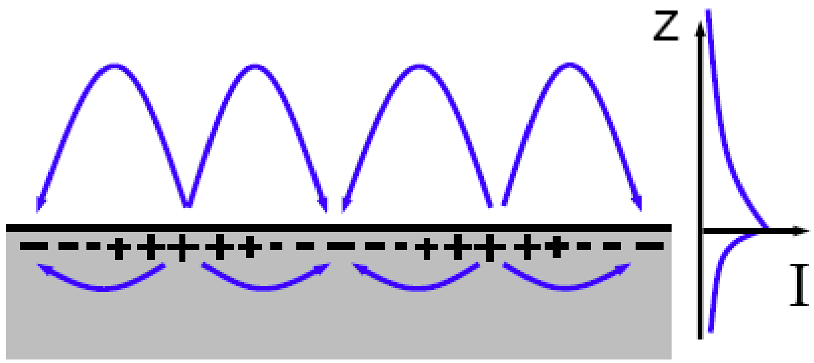


Confinement effects result in resonant SPP modes

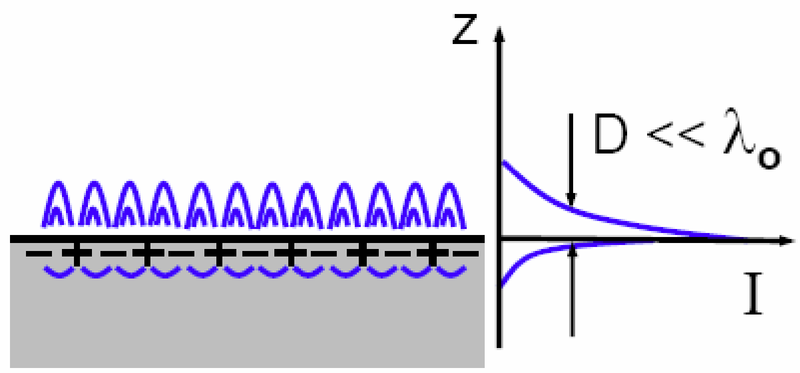
in nanoparticles (localized plasmons)

$$\omega_{particle}^{drude} = \sqrt{\frac{1}{3}\frac{Ne^2}{m\varepsilon_0}}$$

Local field intensity depends on wavelength



Long wavelength
(small propagation constant, k)



Short wavelength
(large propagation constant, k)

Characteristics plasmon-polariton
- Strong localization of the EM field
- High local field intensities easy to obtain

Applications:
- Guiding of light below the diffraction limit (near-field optics)
- Non-linear optics
- Sensitive optical studies of surfaces and interfaces
- Bio-sensors
- Study film growth
- ……

Plasmonics: Merging Photonics and Electronics at Nanoscale Dimensions

Ekmel Ozbay, Science, vol.311, pp.189-193 (13 Jan. 2006).

Some of the challenges that face plasmonics research in the coming years are

(i) demonstrate optical frequency subwavelength metallic wired circuits with a propagation loss that is comparable to conventional optical waveguides;

(ii) develop highly efficient plasmonic organic and inorganic LEDs with tunable radiation properties;

(iii) achieve active control of plasmonic signals by implementing electro-optic, all-optical, and piezoelectric modulation and gain mechanisms to plasmonic structures;

(iv) demonstrate 2D plasmonic optical components, including lenses and grating couplers, that can couple single mode fiber directly to plasmonic circuits;

(v) develop deep subwavelength plasmonic nanolithography over large surfaces;

(vi) develop highly sensitive plasmonic sensors that can couple to conventional waveguides;

(vii) demonstrate quantum information processing by mesoscopic plasmonics.

To derive the Dispersion Relation

of Surface Plasmons,

let’s start from the Drude Model

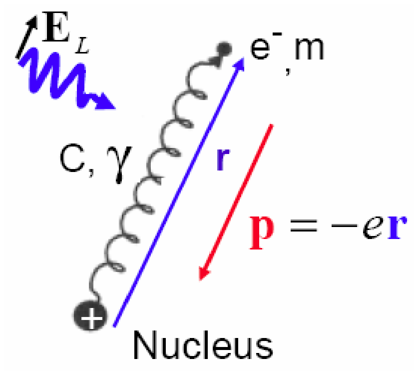
of dielectric constant of metals.

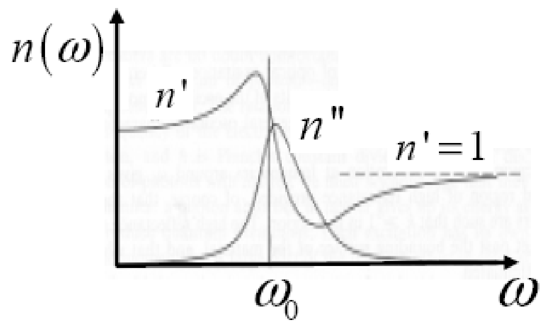
Dielectric constant of metal

- Drude model : Lorenz model (Harmonic oscillator model) without restoration force
  (that is, free electrons which are not bound to a particular nucleus)

Linear Dielectric Response of Matter

Lorentz model (harmonic oscillator model)





Charges in a material are treated as harmonic oscillators

$$m\mathbf{a}_{el} = \mathbf{F}_{E,Local} + \mathbf{F}_{Damping} + \mathbf{F}_{Spring} \qquad \text{(one oscillator)}$$

$$m\frac{d^2\mathbf{r}}{dt^2} + m\gamma\frac{d\mathbf{r}}{dt} + C\mathbf{r} = -e\mathbf{E}_L \exp(-i\omega t)$$

Drude model: Metals

The equation of motion of a free electron (not bound to a particular nucleus; $C = 0$),

$$m\frac{d^2\vec{r}}{dt^2} = -C\vec{r} - \frac{m}{\tau}\frac{d\vec{r}}{dt} - e\vec{E} \quad \Rightarrow \quad m\frac{d\vec{v}}{dt} + \frac{m}{\tau}\vec{v} = -e\vec{E}$$

$$(\tau = \frac{1}{\gamma}: \text{ relaxation time } \approx 10^{-14}\,s)$$

Lorentz model
(Harmonic oscillator model)

If C = 0, it is called
Drude model

The conduction current density $\vec{J} = -Ne\vec{v}$.

$$\frac{d\vec{J}}{dt} + \frac{1}{\tau}\vec{J} = \left(\frac{Ne^2}{m}\right)\vec{E}$$

For a harmonic wave $\vec{E}(t) = \mathrm{Re}\left\{\vec{E}_o(\omega)e^{-i\omega t}\right\}$,

the current desity varies at the same rate $J(t) = \mathrm{Re}\left\{\vec{J}_o(\omega)e^{-i\omega t}\right\}$

$$(-i\omega + \frac{1}{\tau})\vec{J}(\omega) = \left(\frac{Ne^2}{m}\right)\vec{E}(\omega)$$

Local approximation
to the current-field relation

$$\vec{J}(\omega) = \frac{(ne^2/m)}{(1/\tau - i\omega)}\vec{E}(\omega)$$

Determination conductivity

- From the last page: $\mathbf{J}(\omega) = \dfrac{(ne^2/m)}{(1/\tau - i\omega)}\mathbf{E}(\omega)$
- Definition conductivity: $\mathbf{J}(\omega) = \sigma(\omega)\mathbf{E}(\omega)$

$$\Rightarrow \boxed{\sigma(\omega) = \frac{(ne^2/m)}{(1/\tau - i\omega)} = \frac{\sigma_0}{(1 - i\omega\tau)}}$$

where: $\sigma_0 = \dfrac{ne^2\tau}{m}$ **: Static (ω =0) conductivity**

Both bound electrons and conduction electrons contribute to ε

those localized in interband (modified Drude model)

- From the curl Eq.: $\nabla \times H = \dfrac{\partial D(t)}{\partial t} + J = \dfrac{\partial \varepsilon_o \varepsilon_B E(t)}{\partial t} + J$
- For a time varying field: $\mathbf{E}(t) = \mathrm{Re}\{\mathbf{E}(\omega)\exp(-i\omega t)\}$

Maxwell Equations ▶

$$\Rightarrow \nabla \times H = \frac{\partial \varepsilon_o \varepsilon_B E(t)}{\partial t} + J = -i\omega\varepsilon_o\varepsilon_B(\omega)E(\omega) + \sigma(\omega)E(\omega) = -i\omega\varepsilon_o \underbrace{\left[\varepsilon_B(\omega) - \frac{\sigma(\omega)}{i\omega\varepsilon_o}\right]}_{\varepsilon_{EFF}(\omega)} E(\omega)$$

Currents induced by ac-fields modeled by ε_{EFF}

- For a time varying field: $\varepsilon_{EFF} = \varepsilon_B - \dfrac{\sigma}{i\varepsilon_0\omega} = \varepsilon_B + i\dfrac{\sigma}{\varepsilon_0\omega}$

Bound electrons Conduction electrons

Dielectric constant of metal : Drude model

$$\varepsilon_{EFF}(\omega) = \varepsilon_B + i\frac{\sigma(\omega)}{\varepsilon_o \omega}$$

$$= \varepsilon_B + i\frac{1}{\varepsilon_o \omega}\left(\frac{\sigma_o}{1 - i\omega\tau}\right) = \varepsilon_B + i\frac{\sigma_o(1 + i\omega\tau)}{\varepsilon_o \omega(1 + \omega^2\tau^2)}$$

$$= \left(\varepsilon_B - \frac{\omega_p^2\tau^2}{1 + \omega^2\tau^2}\right) + i\left(\frac{\omega_p^2\tau^2}{\omega\tau + \omega^3\tau^3}\right)$$

where, $\omega_p^2 = \frac{\sigma_o}{\varepsilon_o \tau} = \frac{ne^2}{\varepsilon_o m_e}$: bulk plasma frequency ($\sim 10eV$ for metal)

Plasma frequency

Dielectric constant at $\omega \approx \omega_{visible}$

Since $\omega_{vis}\tau >> 1$ ⇒

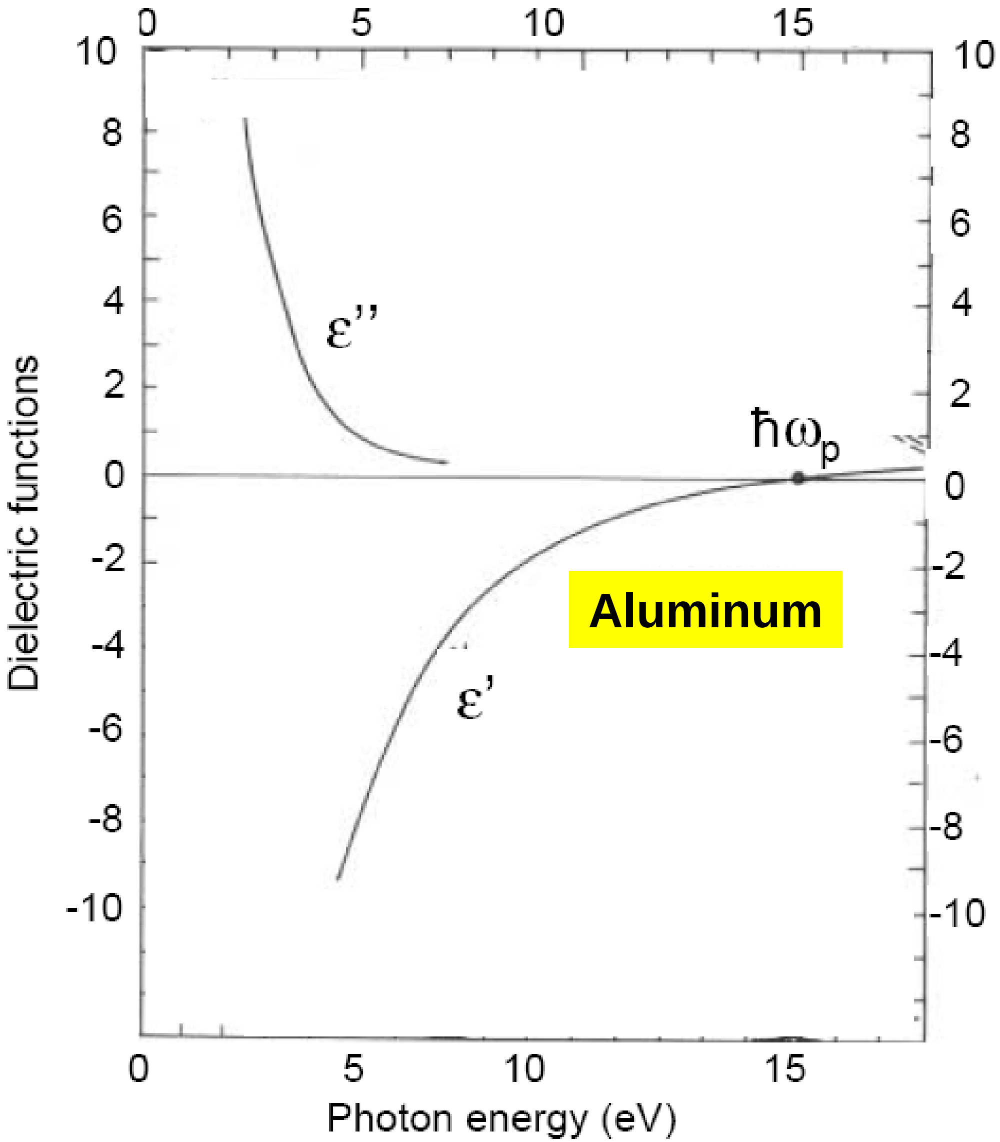
$$\varepsilon_{EFF} = \varepsilon_B - \underline{\frac{\omega_p^2}{\omega^2} + i\frac{\omega_p^2}{\omega^3\tau}}$$

Bound electrons

Free electrons

Optical Properties of Aluminum (simple case)

$Al^{13} : 1S^2\ 2S^2\ 2P^6\ 3S^2\ 3P^1$



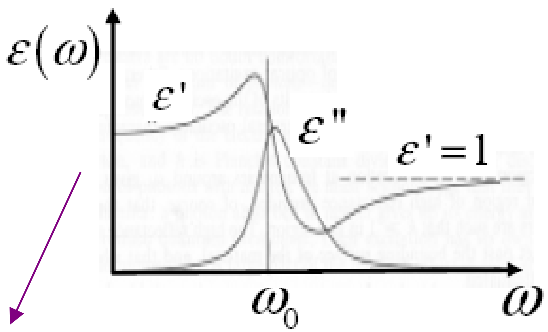
$$\varepsilon_{EFF} = \varepsilon_B - \frac{\omega_p^2}{\omega^2} + i\frac{\omega_p^2}{\omega^3\tau}$$

- Only conduction e's contribute to: ε_{EFF}

$\Rightarrow$ $\varepsilon_B \approx 1$

$$\varepsilon_{EFF,Al} \approx 1 - \frac{\omega_p^2}{\omega^2} + i\frac{\omega_p^2}{\omega^3\tau}$$

- Agrees with:

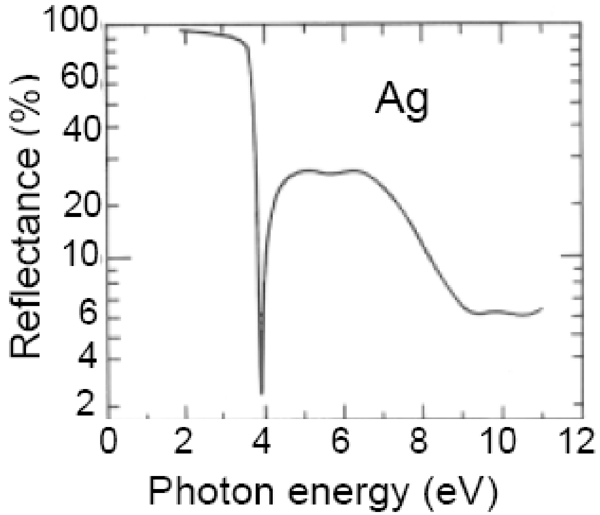


From Microscopic origin of ω-response of matter

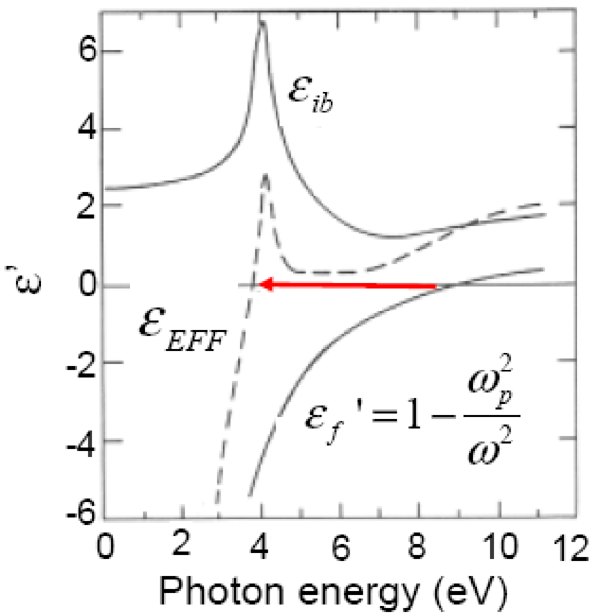
Ag: effects of Interband Transitions

Ag^{47} : $1s^2$ … $4s^2$ $4p^6$ $4d^{10}$ $5s^1$

Au^{79} : $1s^2$ … $5s^2$ $5p^6$ $5d^{10}$ $6s^1$



- Ag show interesting feature in reflection
- Both conduction and bound e's contribute to ε_{EFF}



- Feature caused by interband transitions

Excitation bound electrons

interband

- For Ag: $\varepsilon_B = \varepsilon_{ib} \neq 0$

$$\boxed{\varepsilon_{EFF} = \varepsilon_{ib} - \frac{\omega_p^2}{\omega^2} + i\,\frac{\omega_p^2}{\omega^3 \tau}}$$

Ag: effects of Interband Transitions

Ag^{47} : $1s^2$ … $4s^2\ 4p^6\ 4d^{10}\ 5s^1$

The fully occupied *4d* band of Ag can influence the dynamical surface response in two distinct ways:

- **First, the *s-d* hybridization modifies the single-particle energies and wave functions.**
  As a result, the nonlocal density-density response function exhibits band structure effects.
- **Second, the effective time-varying fields are modified due to the mutual polarization of the s and *d* electron densities.**

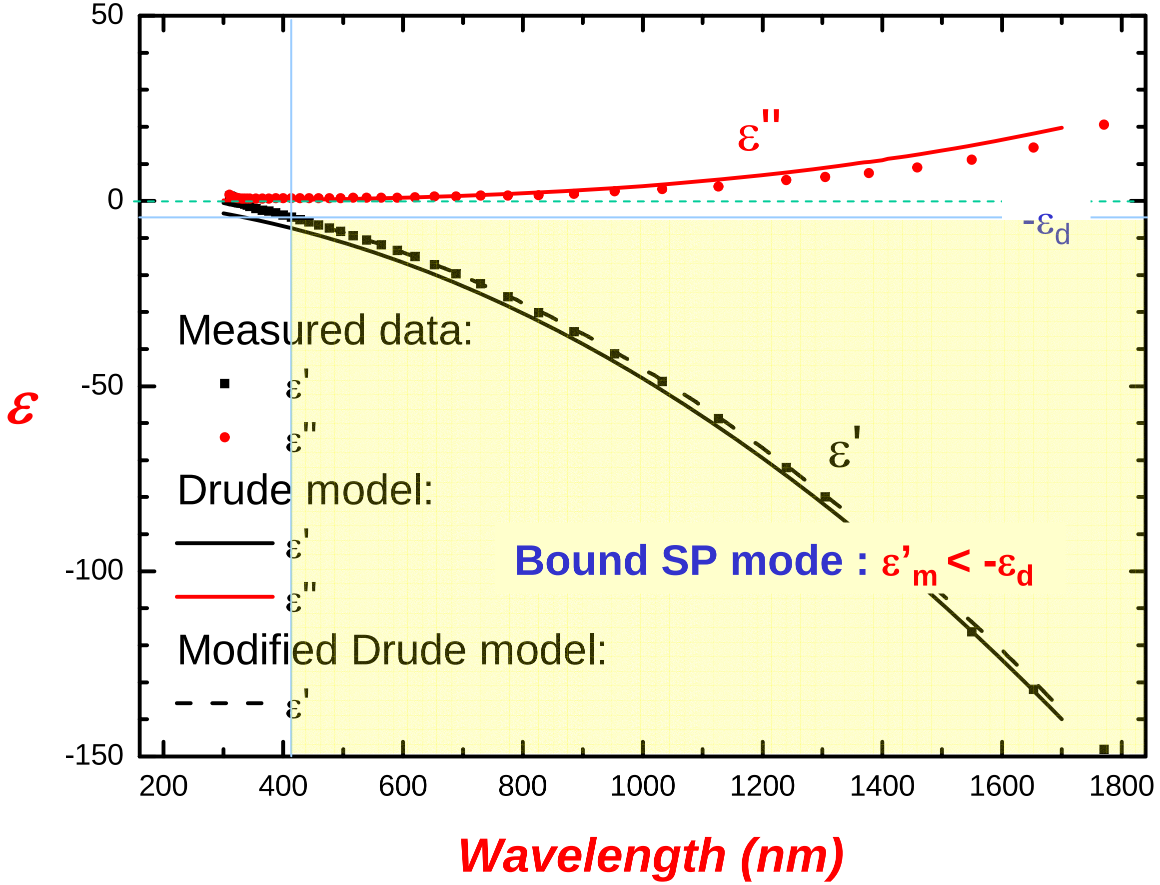
Here we focus on the second mechanism for the following reason:
At the parallel wave vectors of interest, the frequency of the Ag surface plasmon lies below the region of interband transitions. Thus, the relevant single-particle transitions all occur within the s-p band close to the Fermi energy where it displays excellent nearly-free-electron character.

In the bulk, the mutual polarization of the Ag 5*s* and 4*d* electrons is known to cause a large renormalization of the plasma frequency from its unscreened value, $\omega_p = 9.2$ eV, to the observed value which is approximately given by $\boxed{\omega_p^* \approx \omega_p/\sqrt{\mathrm{Re}\,\epsilon_d} \approx 3.76\ \mathrm{eV}}$. Here, $\epsilon_d(\omega)$ represents the "bound" contribution to the total dielectric function which can be decomposed as $\epsilon(\omega) = \epsilon_s(\omega) + \epsilon_d(\omega) - 1$ [13]. $\epsilon_s(\omega)$ represents the Drude term appropriate for the 5*s* electrons. Correspondingly, the frequency of the surface plasmon in the long wavelength limit is given by $\omega_s^* \approx \omega_p/\sqrt{1+\mathrm{Re}\,\epsilon_d} \approx 3.64$ eV whereas the unscreened value is $\omega_s = 6.5$ eV. At finite parallel wave vectors our calculations show that this renormalization is less effective which amounts to an upward distortion of the dispersion relation.

In conclusion, we have presented a model which explains the main difference between the surface plasmon dispersion relations for Ag and the simple metals. The model includes the nonlocal response of the 5*s* electrons at the surface but neglects higher-lying interband transitions. Instead, the influence of the filled 4*d* band is described in terms of a polarizable medium that extends up to some distance from the surface. The key feature of this model is that the combined *s-d* dynamical response is treated self-consistently and not approximated by a superposition of separate quantities taken from independent calculations for the *s* and *d* densities alone. The absence of the *s-d* electrostatic interaction in the outer surface region leads to a blueshift of the surface plasmon frequency. This effect increases with parallel wave vectors because of the more rapid decay of the induced field towards the interior. Thus, the polarization of the filled *d* states causes not only a large reduction of the surface plasmon frequency but also a significant distortion of the dispersion relation.

A. Liebsch, "Surface Plasmon Dispersion of Ag," PRL 71, 145-148 (1993).

Measured data and model for Ag: $\boxed{\varepsilon(\omega)}$



Drude model:

$$\varepsilon' = 1 - \frac{\omega_p^2}{\omega^2}, \quad \varepsilon'' = \frac{\omega_p^2}{\omega^3}\gamma$$

Modified Drude model:

$$\varepsilon' = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2}, \quad \varepsilon'' = \frac{\omega_p^2}{\omega^3}\gamma$$

Contribution of bound electrons

Ag: $\varepsilon_\infty = 3.4$

Ideal case : metal as a free-electron gas

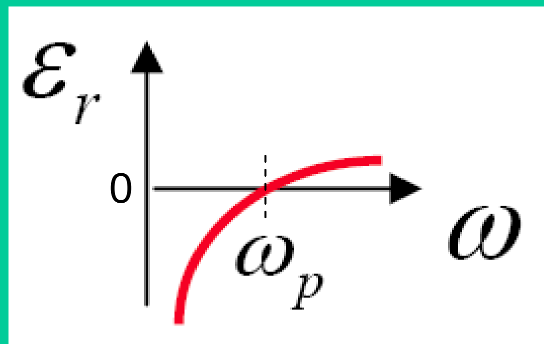
Dielectric constant of a free electron gas

- no decay ($\gamma \rightarrow 0$: infinite relaxation time)
- no interband transitions ($\varepsilon_B = 1$)

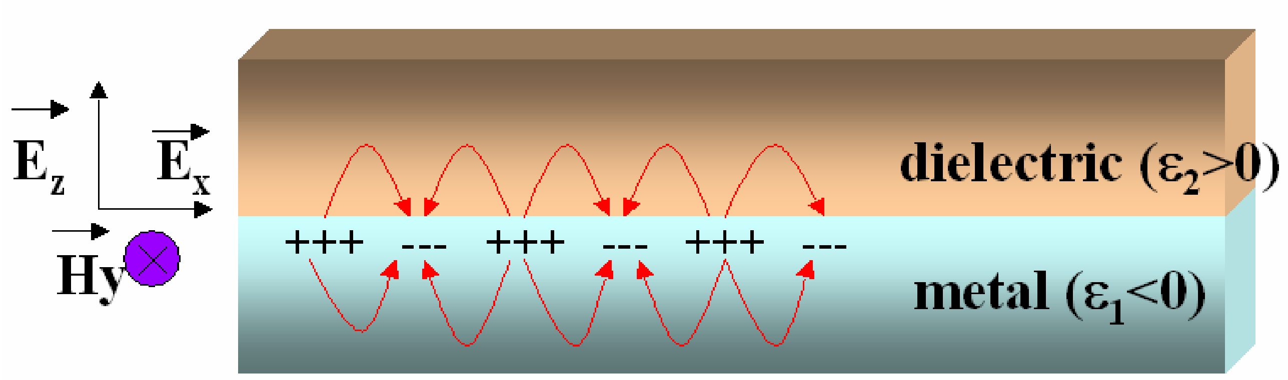
$$\varepsilon_{EFF}(\omega) = \varepsilon_B + i\frac{\sigma(\omega)}{\varepsilon_o \omega}$$

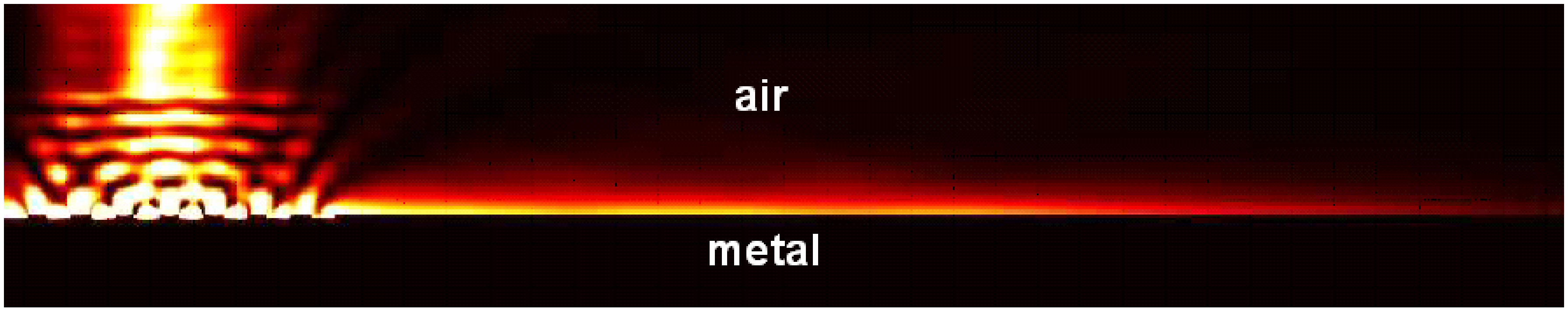
$$= \left(\varepsilon_B - \frac{\omega_p^2 \tau^2}{1+\omega^2\tau^2}\right) + i\left(\frac{\omega_p^2\tau^2}{\omega\tau + \omega^3\tau^3}\right) \xrightarrow[\varepsilon_B = 1]{\tau \to \infty} \varepsilon_{EFF} = \left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

$$\varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2}$$



Dispersion relation of surface-plasmon for dielectric-metal boundaries





Dispersion relation for EM waves in electron gas (bulk plasmons)

Determination of dispersion relation for bulk plasmons

- The wave equation is given by: $$\frac{\varepsilon_r}{c^2}\frac{\partial^2 \boldsymbol{E}(\boldsymbol{r},t)}{\partial t^2} = \nabla^2 \boldsymbol{E}(\boldsymbol{r},t)$$

- Investigate solutions of the form: $$\boldsymbol{E}(\boldsymbol{r},t) = \mathrm{Re}\left\{\boldsymbol{E}(\boldsymbol{r},\omega)\exp(i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega t)\right\}$$

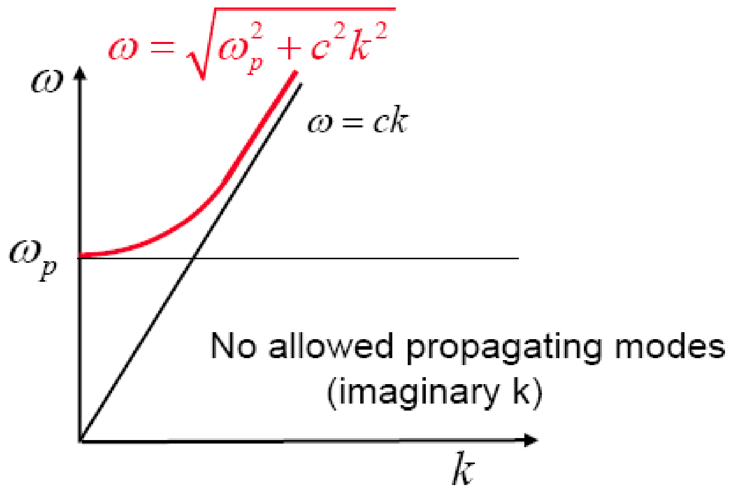
$$\Rightarrow \quad \omega^2\varepsilon_r = c^2k^2$$

- Dielectric constant: $\varepsilon_r = 1 - \dfrac{\omega_p^2}{\omega^2}$

$$\Rightarrow \quad \omega^2\left(1 - \frac{\omega_p^2}{\omega^2}\right) = \boxed{\omega^2 - \omega_p^2 = c^2k^2}$$

- Dispersion relation:

$$\boxed{\omega = \omega(k)}$$



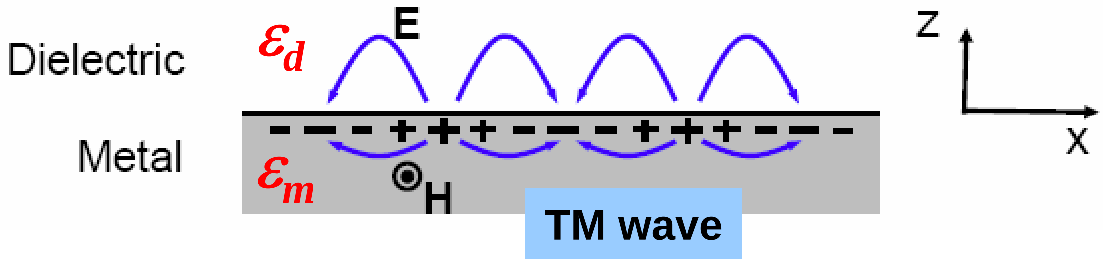
Note1: Solutions lie above light line

Note2: Metals: $\hbar\omega_p \approx 10$ eV; Semiconductors $\hbar\omega_p < 0.5$ eV (depending on dopant conc.)

Dispersion relation for surface plasmons

Solve Maxwell's equations with boundary conditions

- We are looking for solutions that look like:



- Mathematically:

z > 0
$$\begin{cases} \boldsymbol{H}_d = (0, H_{yd}, 0)\exp i(k_{xd}x + k_{zd}z - \omega t) \\ \boldsymbol{E}_d = (E_{xd}, 0, E_{zd})\exp i(k_{xd}x + k_{zd}z - \omega t) \end{cases}$$

z < 0
$$\begin{cases} \boldsymbol{H}_m = (0, H_{ym}, 0)\exp i(k_{xm}x + k_{zm}z - \omega t) \\ \boldsymbol{E}_m = (E_{xm}, 0, E_{zm})\exp i(k_{xm}x + k_{zm}z - \omega t) \end{cases}$$

- Maxwell's Equations in medium i (i = metal or dielectric):

$$\nabla \cdot \varepsilon_i \boldsymbol{E} = 0 \qquad \nabla \cdot \boldsymbol{H} = 0 \qquad \nabla \times \boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{H}}{\partial t} \qquad \nabla \times \boldsymbol{H} = \varepsilon_i \frac{\partial \boldsymbol{E}}{\partial t}$$

- At the boundary (continuity of the tangential E_x, H_y, and the normal D_z):

$$E_{xm} = E_{xd} \qquad H_{ym} = H_{yd} \qquad \varepsilon_m E_{zm} = \varepsilon_d E_{zd}$$

Dispersion relation for surface plasmon polaritons

- Start with curl equation for $\boldsymbol{H}$ in medium i

$$\nabla \times \boldsymbol{H}_i = \varepsilon_i \frac{\partial \boldsymbol{E}_i}{\partial t}$$

where $\boldsymbol{H}_i = (0, H_{yi}, 0)\exp i(k_{xi}x + k_{zi}z - \omega t)$

$$\boldsymbol{E}_i = (E_{xi}, 0, E_{zi})\exp i(k_{xi}x + k_{zi}z - \omega t)$$

$$\left(\frac{\partial H_{zi}}{\partial y} - \frac{\partial H_{yi}}{\partial z}, \frac{\partial H_{xi}}{\partial z} - \frac{\partial H_{zi}}{\partial x}, \frac{\partial H_{yi}}{\partial x} - \frac{\partial H_{xi}}{\partial y}\right) = (\underline{-ik_{zi}H_{yi}}, 0, ik_{xi}H_{yi}) = (\underline{-i\omega\varepsilon_i E_{xi}}, 0, -i\omega\varepsilon_i E_{zi})$$

$$k_{zi}H_{yi} = \omega\varepsilon_i E_{xi} \Rightarrow \begin{cases} k_{zm}H_{ym} = \omega\varepsilon_m E_{xm} \\ k_{zd}H_{yd} = \omega\varepsilon_d E_{xd} \end{cases}$$

- $E_{//}$ across boundary is continuous: $E_{xm} = E_{xd}$

$$\Rightarrow \frac{k_{zm}}{\varepsilon_m}H_{ym} = \frac{k_{zd}}{\varepsilon_d}H_{yd}$$

- $H_{//}$ across boundary is continuous: $H_{ym} = H_{yd}$

Combine with: $\frac{k_{zm}}{\varepsilon_m}H_{ym} = \frac{k_{zd}}{\varepsilon_d}H_{yd}$

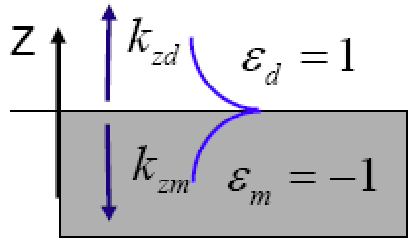
$$\Rightarrow \boxed{\frac{k_{zm}}{\varepsilon_m} = \frac{k_{zd}}{\varepsilon_d}}$$

Dispersion relation for surface plasmon polaritons

Relations between k vectors

- Condition for SP's to exist: $$\frac{k_{zm}}{\varepsilon_m} = \frac{k_{zd}}{\varepsilon_d}$$

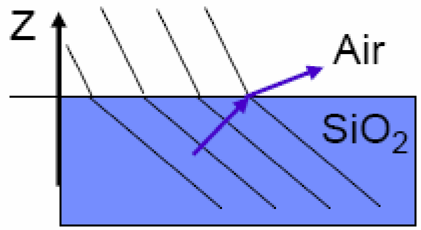
Example



- Relation for k_x (Continuity $E_{//}$, $H_{//}$) : $k_{xm} = k_{xd}$

true at any boundary

Example



- For any EM wave: $$k^2 = \varepsilon_i \left(\frac{\omega}{c}\right)^2 = k_x^2 + k_{zi}^2\ , \quad where\ k_x \equiv k_{xm} = k_{xd}$$

- Both in the metal and dielectric: $$k_{sp} = k_x = \sqrt{\varepsilon_i \left(\frac{\omega}{c}\right)^2 - k_{zi}^2}$$

$$\frac{k_{zm}}{\varepsilon_m} = \frac{k_{zd}}{\varepsilon_d}$$

SP Dispersion Relation

$$\boxed{k_x = \frac{\omega}{c}\sqrt{\frac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d}}}$$

Dispersion relation for surface plasmon polaritons

x-direction: $$k_x = k'_x + ik''_x = \frac{\omega}{c}\left(\frac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d}\right)^{1/2} \qquad \varepsilon_m = \varepsilon_m^{'} + i\varepsilon_m^{''}$$

z-direction: $$k_{zi}^2 = \varepsilon_i \left(\frac{\omega}{c}\right)^2 - k_x^2 \longrightarrow k_{zi} = k'_{zi} + ik_{zi} = \pm\frac{\omega}{c}\left(\frac{\varepsilon_i^2}{\varepsilon_m + \varepsilon_d}\right)^{1/2}$$

For a bound SP mode:

k_{zi} must be imaginary: $\varepsilon_m + \varepsilon_d < 0$

$$k_{zi} = \pm\sqrt{\varepsilon_i \left(\frac{\omega}{c}\right)^2 - k_x^2} = \pm i\sqrt{k_x^2 - \varepsilon_i \left(\frac{\omega}{c}\right)^2} \Rightarrow |k_x| > \sqrt{\varepsilon_i}\left(\frac{\omega}{c}\right)$$

+ for z < 0
- for z > 0

k'_x must be real: $\varepsilon_m < 0$

$$\boldsymbol{H}_i = (0, H_{yi}, 0)\exp i(k_{xi}x + k_{zi}z - \omega t)$$
$$\boldsymbol{E}_i = (E_{xi}, 0, E_{zi})\exp i(k_{xi}x + k_{zi}z - \omega t)$$

So, $$\boxed{\varepsilon_m^{'} < -\varepsilon_d}$$

$$k_x = k_x^{'} + ik_x^{"} = \frac{\omega}{c}\left(\frac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d}\right)^{1/2} \qquad \varepsilon_m = \varepsilon_m^{'} + i\varepsilon_m^{"}$$

$$k_x^{'} = \frac{\omega}{c}\left[\frac{\varepsilon_d}{(\varepsilon_m^{'} + \varepsilon_d)^2 + \left(\varepsilon_m^{"}\right)^2}\right]^{\frac{1}{2}}\left[\frac{\varepsilon_e^{\ 2} + \sqrt{\varepsilon_e^{\ 4} + \left(\varepsilon_m^{"}\varepsilon_d\right)^2}}{2}\right]^{\frac{1}{2}}$$

$$k_x^{"} = \frac{\omega}{c}\left[\frac{\varepsilon_d}{(\varepsilon_m^{'} + \varepsilon_d)^2 + \left(\varepsilon_m^{"}\right)^2}\right]^{\frac{1}{2}}\left[\frac{\left(\varepsilon_m^{"}\varepsilon_d\right)^2}{2\left\{\varepsilon_e^{\ 2} + \sqrt{\varepsilon_e^{\ 4} + \left(\varepsilon_m^{"}\varepsilon_d\right)^2}\right\}}\right]^{\frac{1}{2}} \quad where, \ \ \varepsilon_e^{\ 2} = \left(\varepsilon_m^{'}\right)^2 + \left(\varepsilon_m^{"}\right)^2 + \varepsilon_d \varepsilon_m^{'}$$

,

$\varepsilon_m^{'} < 0$, $\left|\varepsilon_m^{'}\right| > \varepsilon_d$, and $\left|\varepsilon_m^{'}\right| >> \varepsilon_m^{"}$ in most of metals,

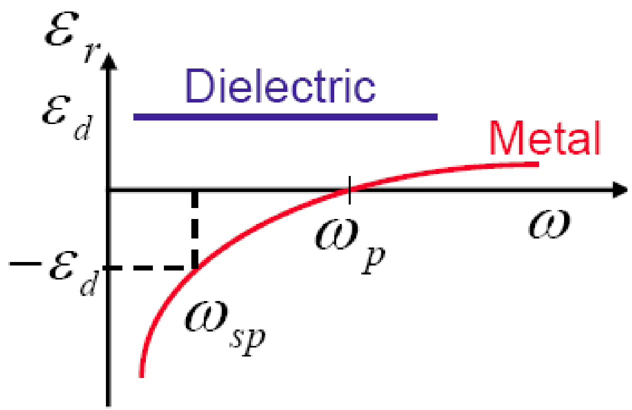
$$k_x^{'} \approx \frac{\omega}{c}\left(\frac{\varepsilon_m^{'}\varepsilon_d}{\varepsilon_m^{'} + \varepsilon_d}\right)^{1/2}$$

$$k_x^{"} \approx \frac{\omega}{c}\left(\frac{\varepsilon_m^{'}\varepsilon_d}{\varepsilon_d^{'} + \varepsilon_d}\right)^{3/2} \frac{\varepsilon_m^{"}}{2\left(\varepsilon_m^{'}\right)^2}$$

Plot of the dispersion relation

- Plot of the dielectric constants:

$$\varepsilon_m(\omega) = 1 - \frac{\omega_p^{\ 2}}{\omega^2}$$



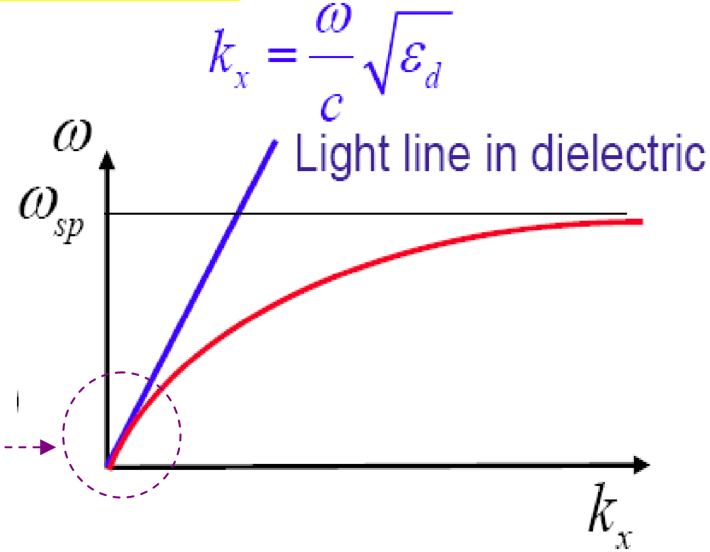
- Plot of the dispersion relation:

$$k_x = \frac{\omega}{c}\sqrt{\frac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d}} \Rightarrow k_x = k_{sp} = \frac{\omega}{c}\sqrt{\frac{(\omega^2 - \omega_p^{\ 2})\varepsilon_d}{(1+\varepsilon_d)\omega^2 - \omega_p^{\ 2}}}$$

- When $\varepsilon_m \rightarrow -\varepsilon_d$,

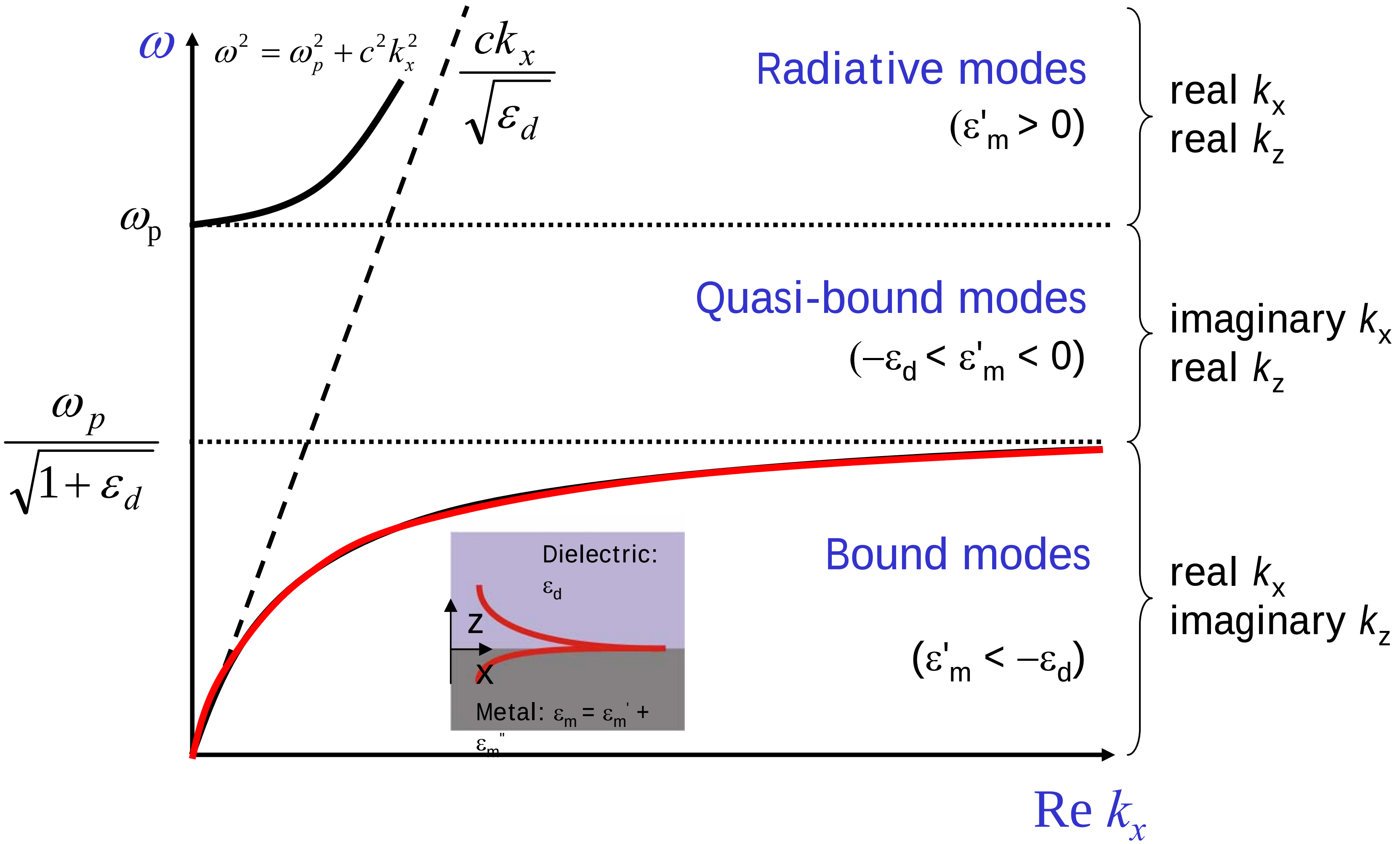
$$\Rightarrow \quad \mathrm{k_x} \rightarrow \infty, \ \omega \equiv \omega_{sp} = \frac{\omega_p}{\sqrt{1+\varepsilon_d}}$$

- Low ω: $k_x = \dfrac{\omega}{c} \lim_{\varepsilon_m \to -\infty} \left(\dfrac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d} \right)^{1/2} \approx \dfrac{\omega}{c}\sqrt{\varepsilon_d}$



Surface plasmon dispersion relation

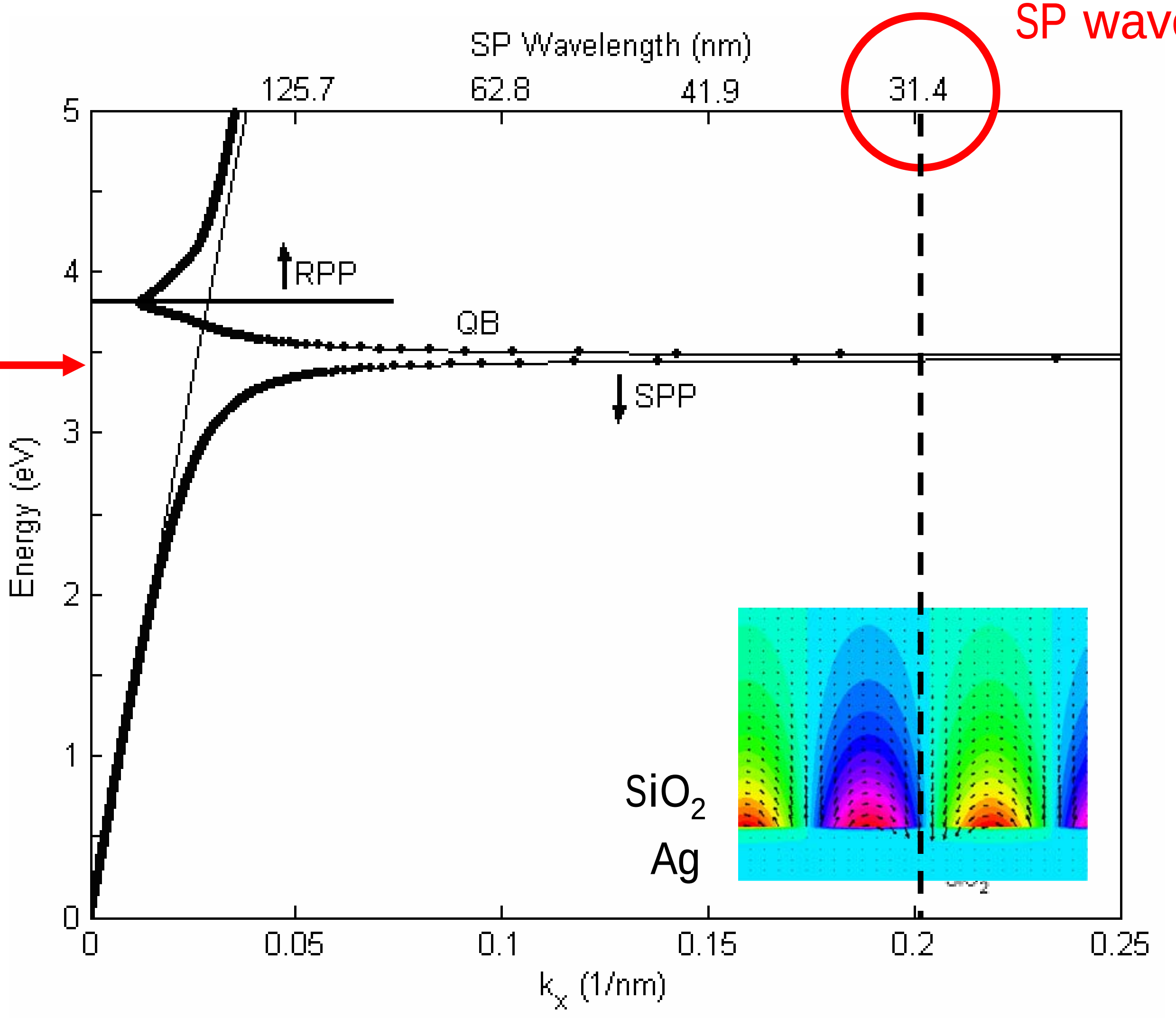
$$k_x = \frac{\omega}{c}\left(\frac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d}\right)^{1/2} \qquad k_{zi} = \frac{\omega}{c}\left(\frac{\varepsilon_i^2}{\varepsilon_m + \varepsilon_d}\right)^{1/2}$$



X-ray wavelengths at optical frequencies

Very small
SP wavelength

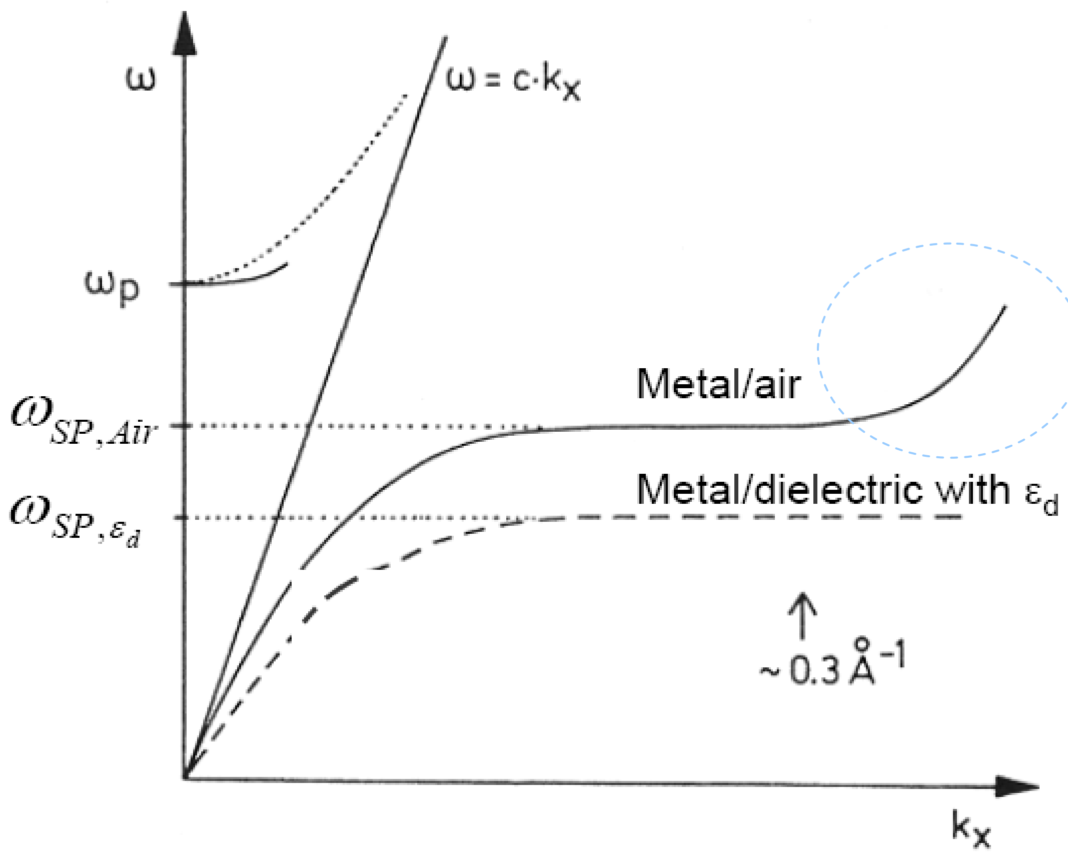
λ_{vac}=360 nm



Dispersion relation for bulk and surface plasmons

$$k_x = \frac{\omega}{c}\left(\frac{\varepsilon_m \varepsilon_d}{\varepsilon_m + \varepsilon_d}\right)^{1/2}$$

$$\varepsilon_m = \left(1 - \frac{\omega_p^2 \tau^2}{1+\omega^2\tau^2}\right) + i\left(\frac{\omega_p^2\tau^2}{\omega\tau + \omega^3\tau^3}\right)$$

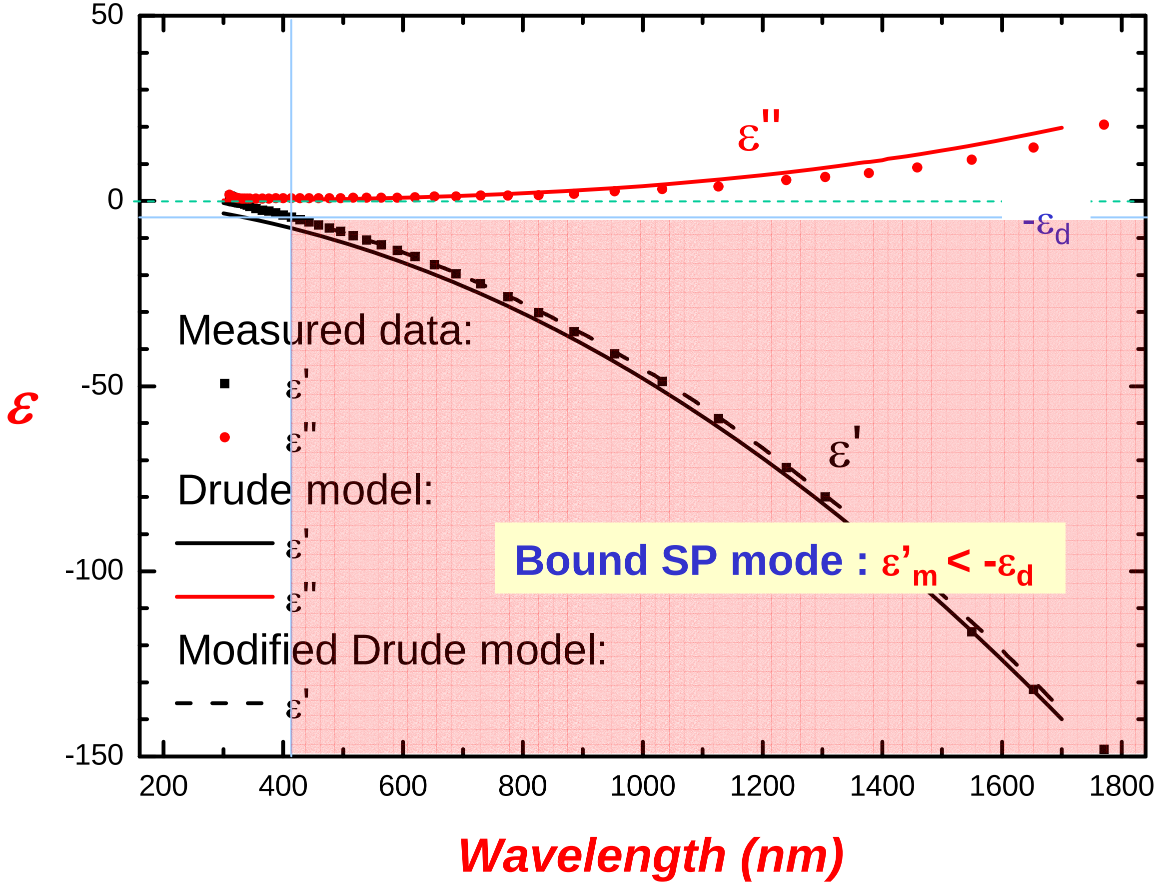


Cut-off frequency of SP

- Note: Higher index medium on metal results in lower ω_{sp}

$$\circ\ \text{When}\ \varepsilon_m = 1 - \frac{\omega_p^2}{\omega^2} = -\varepsilon_d\ ,\ \omega^2 - \omega_p^2 = -\varepsilon_d \Rightarrow \omega^2 = \frac{\omega_p^2}{1+\varepsilon_d} \Rightarrow \omega_{sp} = \frac{\omega_p}{\sqrt{1+\varepsilon_d}}$$

Measured data and model for Ag: $\boxed{\varepsilon(\omega)}$



Drude model:

$$\varepsilon' = 1 - \frac{\omega_p^2}{\omega^2}, \quad \varepsilon'' = \frac{\omega_p^2}{\omega^2}\gamma$$

Modified Drude model:

$$\varepsilon' = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2}, \quad \varepsilon'' = \frac{\omega_p^2}{\omega^2}\gamma$$

Contribution of bound electrons

Ag: $\varepsilon_\infty = 3.4$

Propagation length

The length after which the intensity decreases to 1/e :

$$L_i = \left(2k_x^{''}\right)^{-1}, \text{ where } k_x^{''} = \frac{\omega}{c}\left(\frac{\varepsilon_1^{'}\varepsilon_2}{\varepsilon_1^{'}+\varepsilon_2}\right)^{3/2}\frac{\varepsilon_1^{''}}{2(\varepsilon_1^{'})^2}$$

In the visible region, L_i reaches the value of L_i =44 μm in silver and L_i =14 μm in gold at λ=633 nm. The absorbed energy heats the film, and can be measured with a photoaccoustic cell.

Propagation length : Ag/air, Ag/glass

propagation length L as the $1/e$ decay length of the field intensity along x,

$$L = \frac{1}{2k_x''} \approx \frac{c}{\omega}\left(\frac{\epsilon_d + \epsilon_m'}{\epsilon_d \epsilon_m'}\right)^{3/2} \frac{(\epsilon_m')^2}{\epsilon_m''}$$

$$\varepsilon_m = \varepsilon_m' + i\varepsilon_m'' = \left(\varepsilon_B - \frac{\omega_p^2\tau^2}{1+\omega^2\tau^2}\right) + i\left(\frac{\omega_p^2\tau^2}{\omega\tau+\omega^3\tau^3}\right)$$

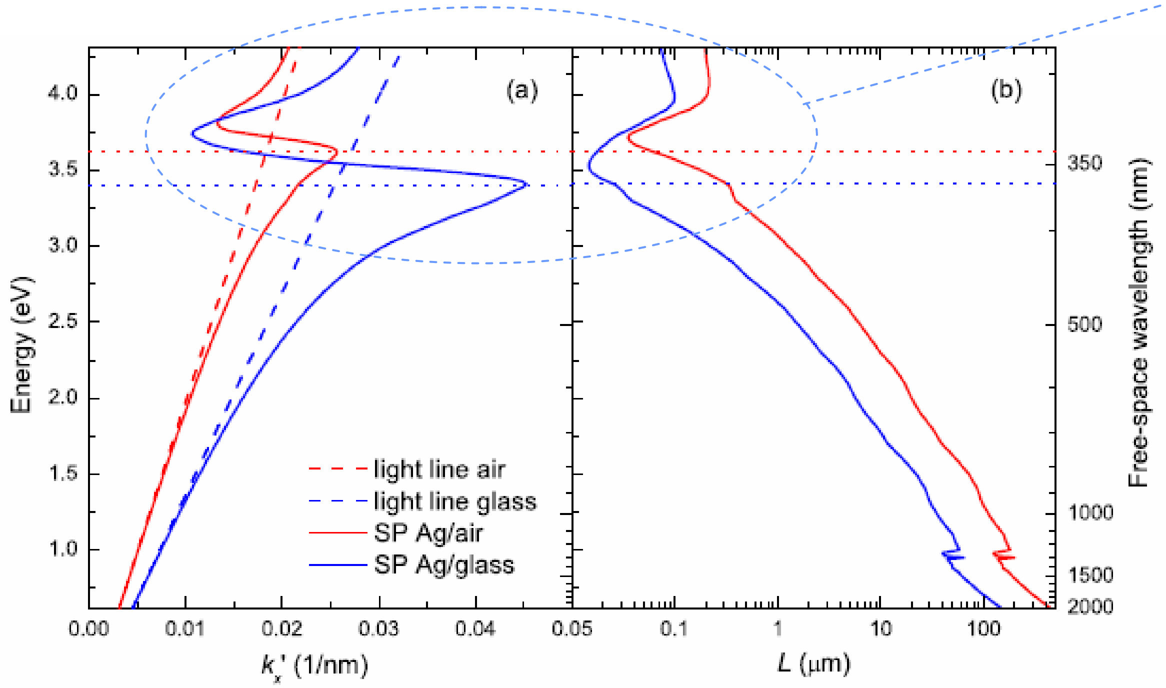


Figure 1.1: (a) Surface plasmon dispersion curves for Ag/air and Ag/glass interfaces. The surface plasmon modes exist below the surface plasmon resonance energies indicated by the dotted lines. (b) Propagation lengths for the surface plasmons in (a). The glass is fused silica and the optical constants of the silver are given by Palik [5].

Penetration depth

$$\hat{z}_i = \frac{1}{|k_{zi}|},$$

$$\hat{z}_2 = \frac{\lambda}{2\pi}\left(\frac{\varepsilon_1^R + \varepsilon_2}{\varepsilon_2^2}\right)^{1/2}$$: in the medium with ε_2,

$$\hat{z}_1 = \frac{\lambda}{2\pi}\left(\frac{\varepsilon_1^R + \varepsilon_2}{(\varepsilon_1^R)^2}\right)^{1/2}$$: in the medium with ε_1.

For λ=633 nm one obtains for silver $\hat{z}_2$=390 nm and $\hat{z}_1$=24 nm, for gold 342 nm and 27 nm,

At large k_x $(\varepsilon_1' \to -\varepsilon_2)$, $\hat{z}_i \approx \frac{1}{k_x}$. Strong concentration near the surface in both media.

$$E_z = \pm i E_x \ (air: +i,\ metal: -i)$$

At low k_x $(|\varepsilon_1'| >> 1)$, $\frac{E_z}{E_x} = -i\sqrt{|\varepsilon_1'|}$ in air : Larger E_z component

$\frac{E_z}{E_x} = i\frac{1}{\sqrt{|\varepsilon_1'|}}$ in metal : Smaller E_z component

Gooood waveguide!

Plasma resonances for various geometries

Material	Resonance condition	Resonance Frequency
Bulk Metal	$\varepsilon_{eff} = 0$	ω_p
Planar Surface	$\varepsilon_{eff} = -\varepsilon_d$	$\frac{\omega_p}{\sqrt{2}}$
Sphere	$\varepsilon_{eff} = -2\varepsilon_d$	$\frac{\omega_p}{\sqrt{3}}$
Ellipsoid	$\varepsilon_{eff} = -\frac{1-L_M}{L_M}$	$\omega_p L_M$

Optical Properties of Metal Clusters
Kreibig and Vollmer, 1995

Another representation of SP dispersion relation

We consider now electromagnetic waves propagating on the surface of a metallic film. In the optical and infrared regimes, the collective excitation of the electron density (which is coupled to the near field) results in a surface plasmon polariton (SPP) (also known as a surface polariton [44-45]) traveling on the metal surface. These surface waves are excited when the real part of the metallic permittivity $\varepsilon_m = \varepsilon'_m + i\varepsilon''_m$ is negative (i.e., $\varepsilon'_m < 0$) and dissipation is small (i.e., $\kappa = \varepsilon''_m/|\varepsilon'_m| \ll 1$), which is typical for a metal in the optical regime. Let us denote $\varepsilon_m = -\nu^2$, where $\nu = -in$ is almost positive since the losses are small. At the metal–air interface, the SPP is an H wave, with its magnetic field parallel to the interface [45]. In the direction perpendicular to the interface, SPPs exponentially decay on both sides of the interface. The relation between the angular frequency ω and the wavenumber k_p of the SPP can be found from the following consideration.

We assume that the SPP propagates in the x direction, with $\mathbf{H}$ parallel to the y axis: $\mathbf{H} = \{0, H, 0\}$. The half-space $z > 0$ is vacuous while the metal fills the half-space $z < 0$. We seek solutions of the form

$$H_1 = H_0 \exp(ik_p x - \Lambda_1 z), \quad z > 0, \tag{17}$$

$$H_2 = H_0 \exp(ik_p x + \Lambda_2 z), \quad z < 0, \tag{18}$$

where $\Lambda_1 = (k_p^2 - k_0^2)^{1/2}$ and $\Lambda_2 = [k_p^2 + (k_0\nu)^2]^{1/2}$. Thus, the boundary condition on the continuity of the tangential component of the magnetic field is automatically satisfied. The continuity of the tangential component of the electric field results in the condition

$$\frac{\partial H_1}{\partial z} = -\frac{1}{\nu^2}\frac{\partial H_2}{\partial z} \tag{19}$$

for $z = 0$. This equation yields the dispersion equation

$$k_p = \frac{k_0 \nu}{\sqrt{\nu^2 - 1}} \tag{20}$$

for the SPP wavenumber k_p. SPP propagation on the metal surface then requires that $|\nu| > 1$.

There are two kinds of SPP modes in a metal film of finite thickness h, which correspond to symmetric and antisymmetric (with respect to reflection in the plane $z = 0$) oscillations of the electron density on both interfaces. Hereafter, it is supposed that $|\nu| > 1$. With the assumption of a strong skin effect, i.e., $\exp[-hk_0 \operatorname{Re}(\nu)] \ll 1$, the propagation of SPP is determined by the relation

$$\boxed{k_{1,2} = k_p\left[1 \pm \frac{2\nu^2}{\nu^4 - 1}\exp(-hk_p\nu)\right],} \tag{21}$$

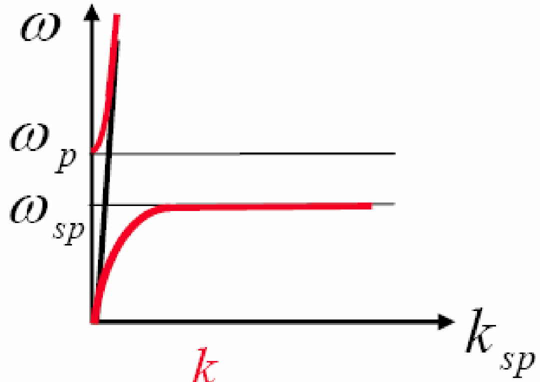
where the wavenumbers k_1 and k_2 correspond to the symmetric and antisymmetric modes, respectively, and k_p is defined by (20). The phase velocities of the symmetric and antisymmetric SPPs are less than the speed of light c, and neither type can be excited by an external electromagnetic wave, because that would violate the principle of conservation of momentum. In a sense, the SPPs represent a *hidden reality* that is invisible, since an SPP does not interact with impinging light.

$$\boxed{\begin{aligned} \varepsilon_m &= \varepsilon'_m + \varepsilon''_m = -\nu^2 \\ k_p &= \omega_p / c \end{aligned}}$$

Optical Properties of Metal-Dielectric

Andrey K. Sarychev and Vladimir M. Shalaev

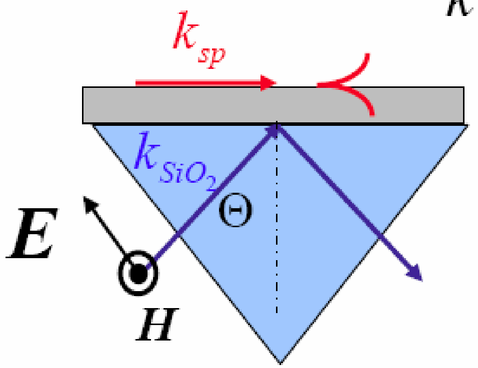
Excitation of surface plasmons



The dispersion relation for surface plasmons

- Useful for describing plasmon excitation & propagation

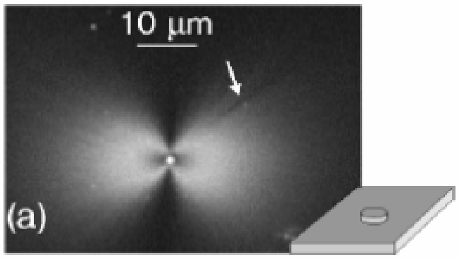
Coupling light to surface plasmon-polaritons

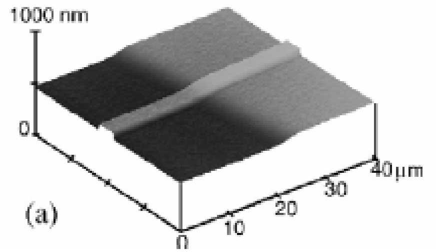


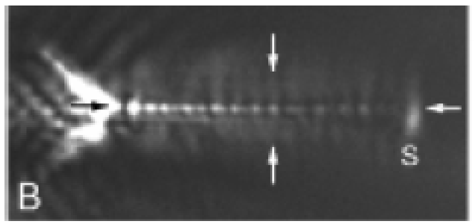
- Using high energy electrons (EELS)
- Kretchman geometry

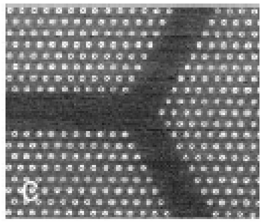
$$k_{//,SiO_2} = \sqrt{\varepsilon_d}\,\frac{\omega}{c}\sin\theta = k_{sp}$$

- Grating coupling
- Coupling using subwavelength features
- A diversity of guiding geometries







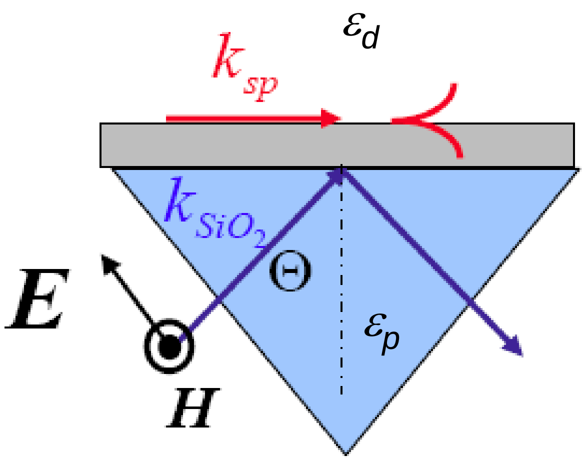


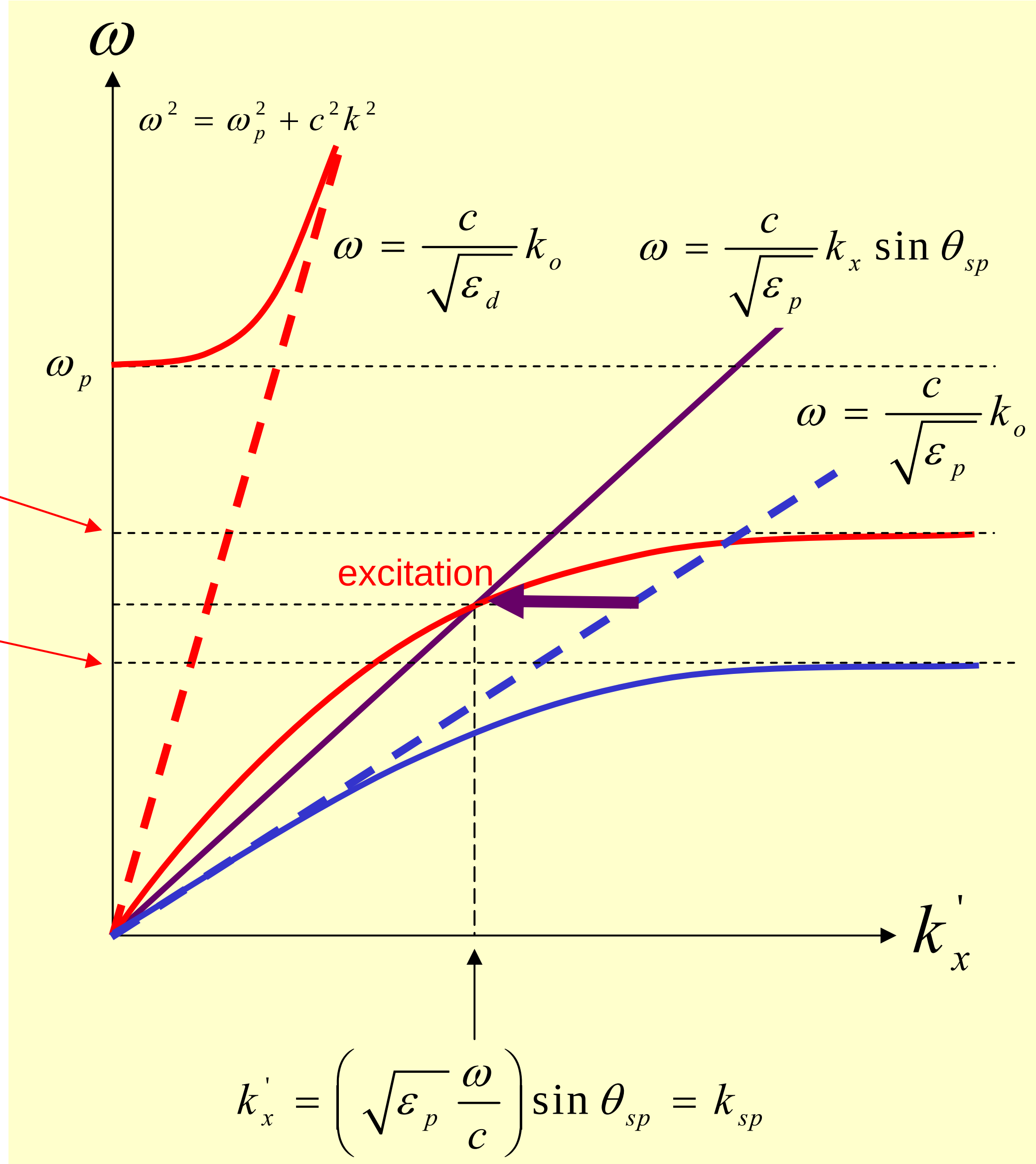
Excitation of surface plasmon by prism

$$k_x' = \frac{\omega}{c}\left(\frac{\varepsilon_m' \varepsilon_d}{\varepsilon_m' + \varepsilon_d}\right)^{1/2} > k = \frac{\omega}{c}\sqrt{\varepsilon_d}$$

$$\omega_{sp} = \frac{\omega_p}{\sqrt{1+\varepsilon_d}}$$

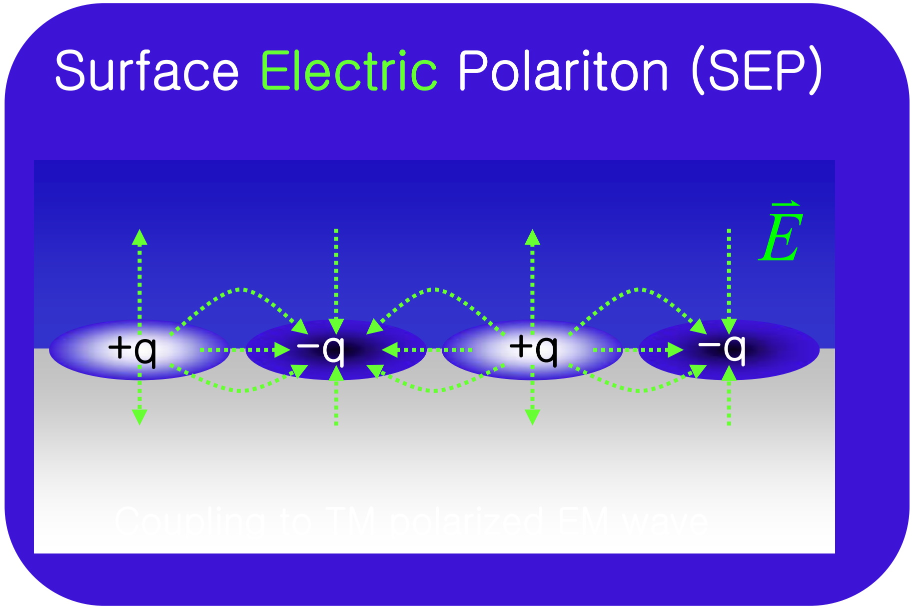
$$\omega_{sp} = \frac{\omega_p}{\sqrt{1+\varepsilon_p}}$$

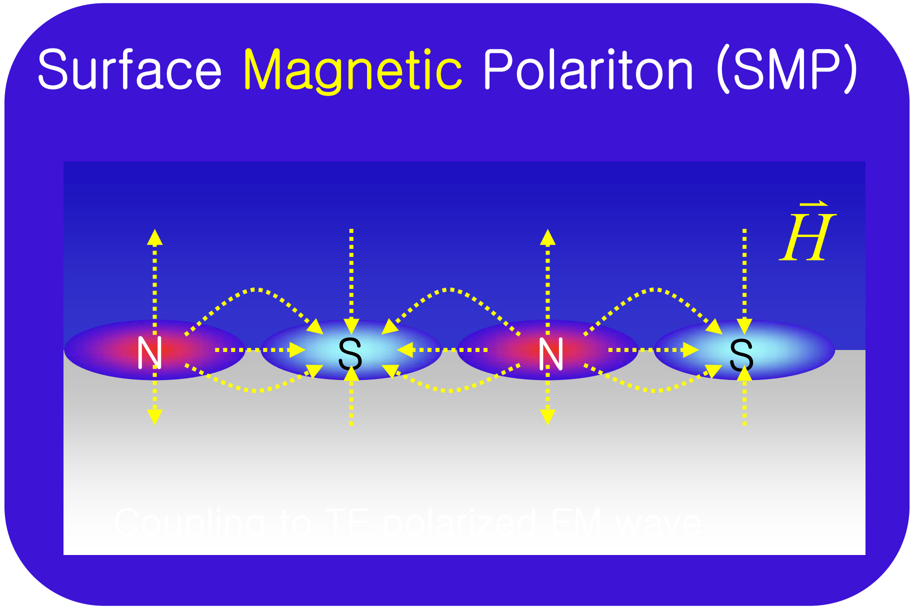




Generalization :
Surface Electric Polaritons and Surface Magnetic Polaritons

: Energy quanta of surface localized oscillation of electric or magnetic dipoles in coherent manner





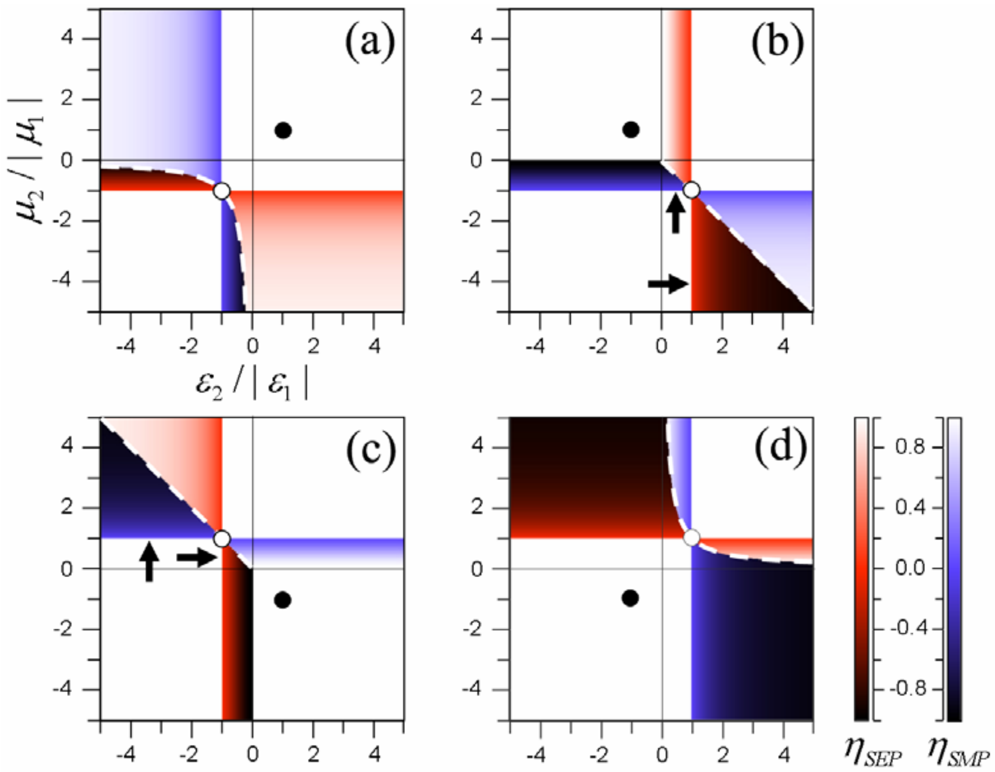
Common Features

- **Non-radiative modes → scale down of control elements**
- **Smaller group velocity than light coupling to SP**
- **Enhancement of field and surface photon DOS**

Generalization :
Surface Electric Polaritons and Surface Magnetic Polaritons

Dispersion Relation & Decay Constants

$$For \quad \left(\left|\varepsilon''/\varepsilon'\right|,\left|\mu''/\mu'\right|\right) << (1,1) \ , \quad \begin{cases} \beta_{SEP} = \sqrt{\dfrac{\varepsilon'_1\varepsilon'_2(\varepsilon'_2\mu'_1-\varepsilon'_1\mu'_2)}{\varepsilon'^2_2-\varepsilon'^2_1}}k_0 + O\left(\dfrac{\varepsilon''}{\varepsilon'},\dfrac{\mu''}{\mu'}\right) \\ \gamma_{SEP,i} = \sqrt{\dfrac{\varepsilon'_i(\varepsilon'_1\mu'_1-\varepsilon'_2\mu'_2)}{\varepsilon'^2_2-\varepsilon'^2_1}}k_0 + O\left(\dfrac{\varepsilon''}{\varepsilon'},\dfrac{\mu''}{\mu'}\right), \quad \dfrac{\gamma_{SEP,1}}{\varepsilon_1'} = \dfrac{\gamma_{SEP,2}}{\varepsilon_2'} \end{cases}$$



J. Yoon, et al., Opt Exp 2005.

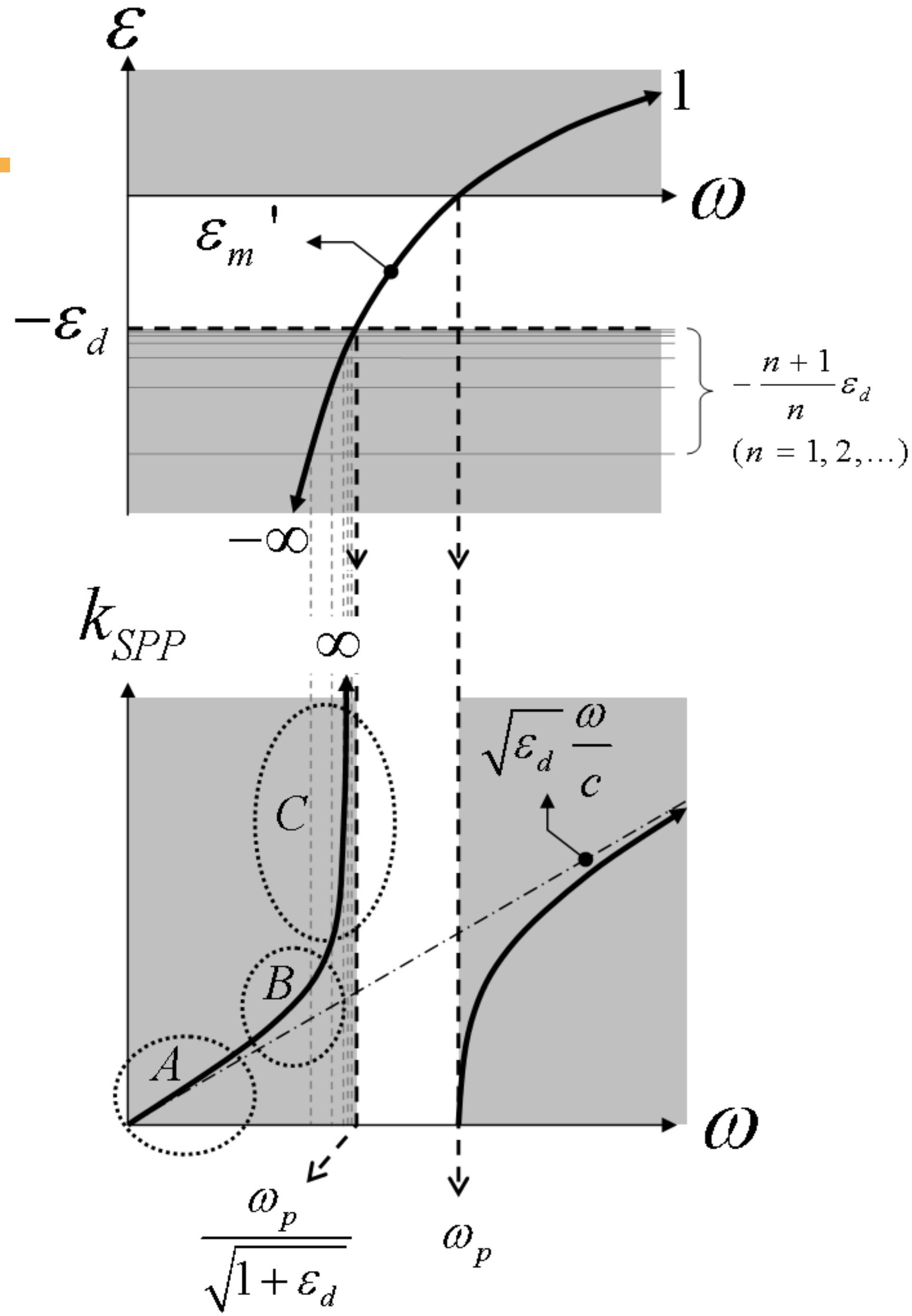
In Summary

Permittivity of a metal

$$\varepsilon_m(\omega) = 1 - \frac{\omega_p{}^2}{\omega^2 + \gamma^2} + i\frac{\omega_p{}^2}{\omega^2 + \gamma^2}\left(\frac{\gamma}{\omega}\right)$$
$$\approx 1 - \omega_p{}^2 / \omega^2$$

Dispersion relations

$$k_{SPP} = \frac{\omega}{c}\left(\frac{\varepsilon_d \varepsilon_m}{\varepsilon_d + \varepsilon_m}\right)^{1/2}$$



Type-A : low k

Type-*A*

- Low frequency region (IR)

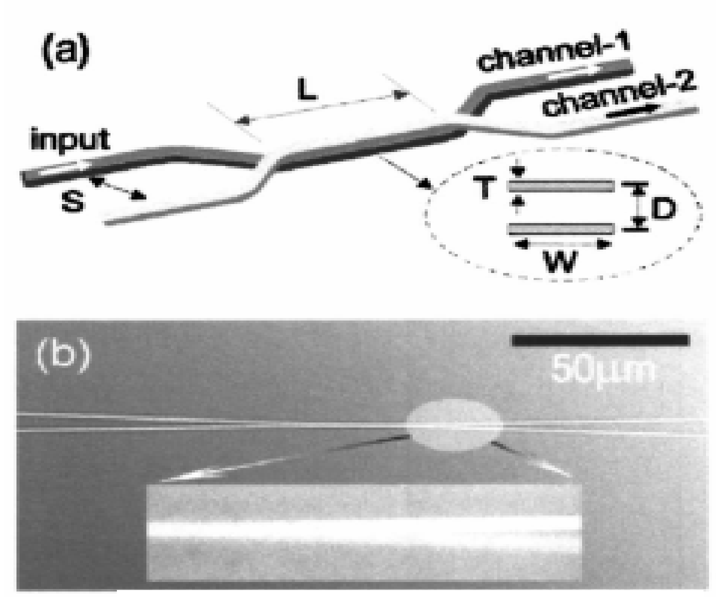
- Weak field-confinement

- Most of energy is guided in clad

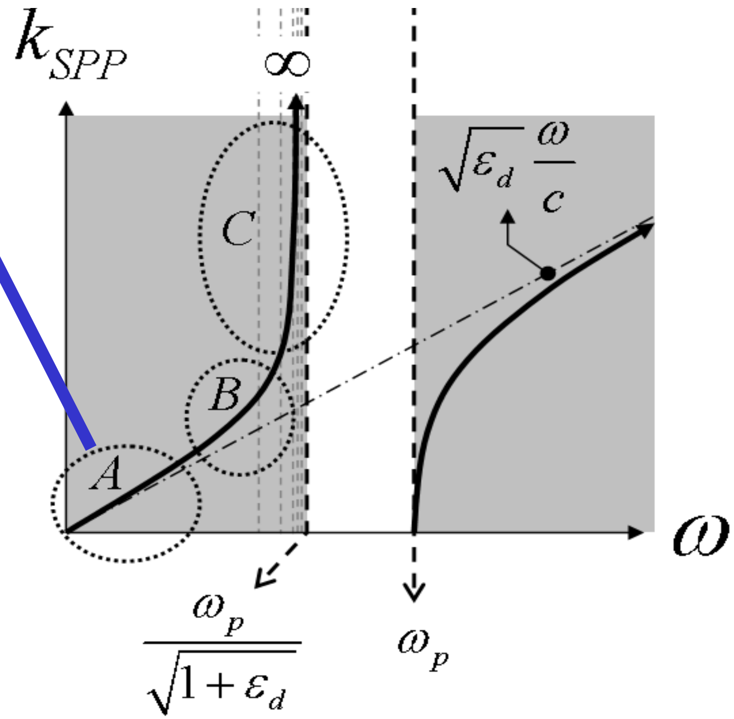
- Low propagation loss

► **clad sensitive applications**

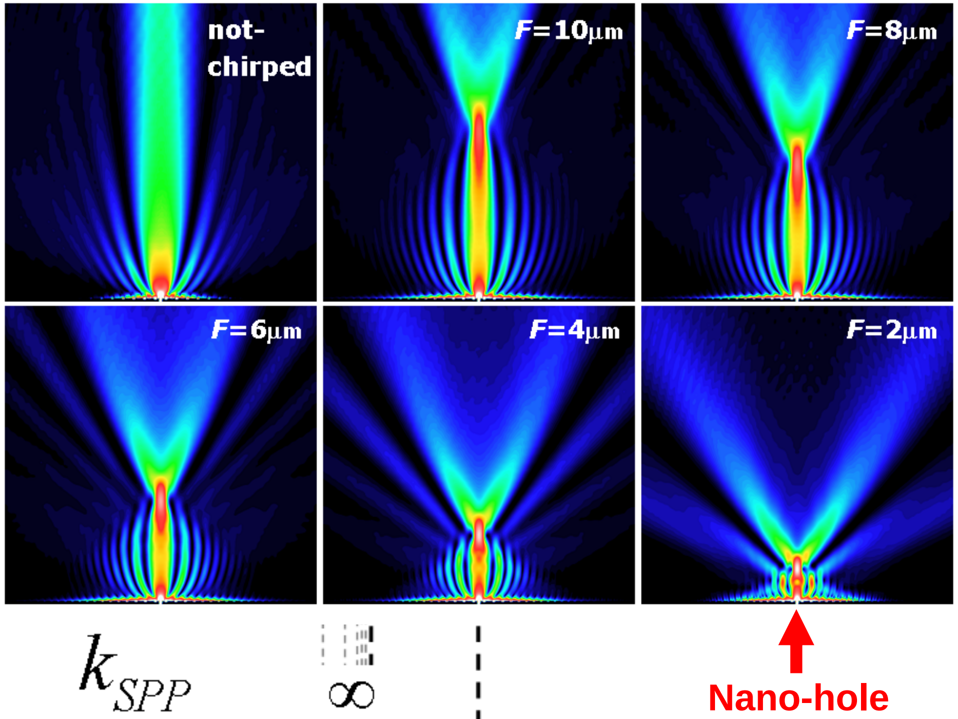
► **SPP waveguides applications**



H. Won, APL 88, 011110 (2006).



Type-B : middle k



Type-B

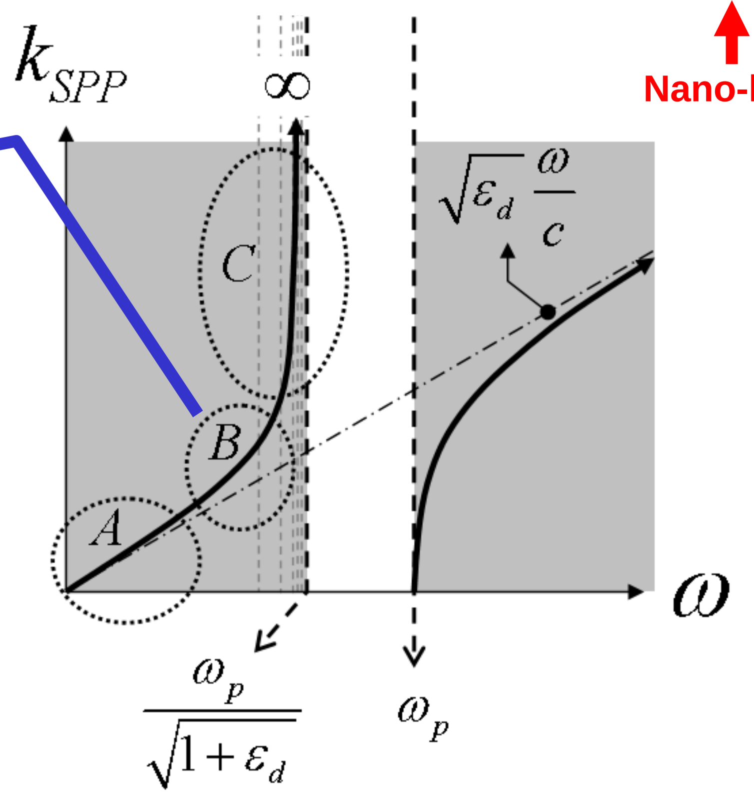
- Visible-light frequency region

- Coupling of localized field and propagation field

- Moderated field enhancement

► **Sensors, display applications**

► **Extraordinary transmission of light**



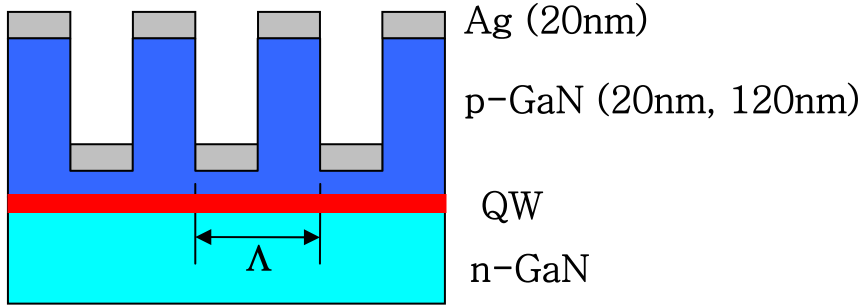
Type-C : high k

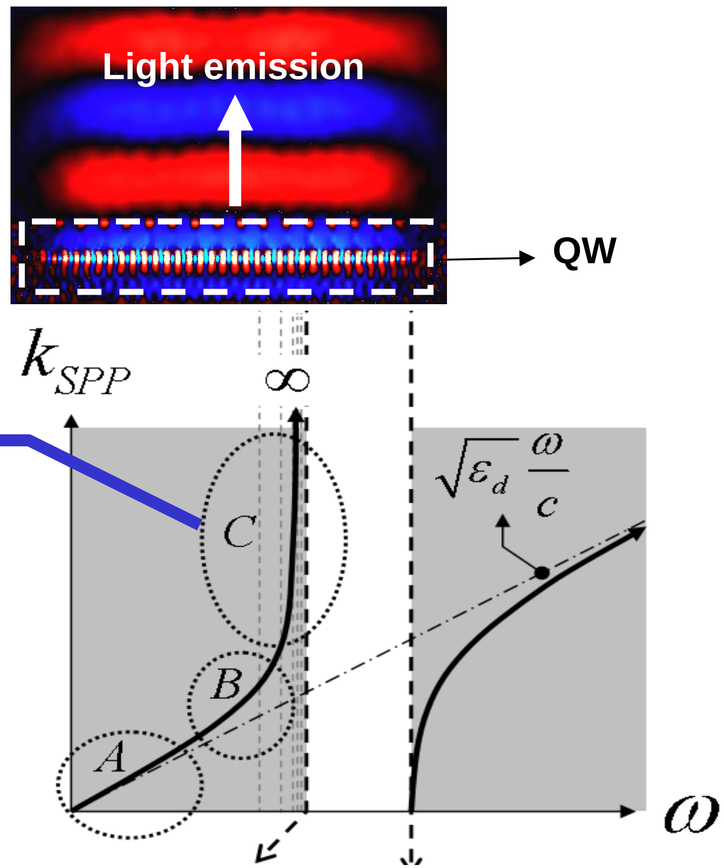
Type-C

- UV frequency region
- Strong field confinement
- Very-low group velocity

► **Nano-focusing, Nano-lithography**

► **SP-enhanced LEDs**





SE Rate :

$$R = \frac{1}{\tau(\omega)} = \frac{1}{2\varepsilon_0 \hbar} \left| \langle f | \mathbf{p} \cdot \mathbf{E} | i \rangle \right|^2 \rho(\omega)$$

Electric field strength
of half photon (vacuum fluctuation)

Photon DOS
(Density of States)

Importance of understanding the dispersion relation : Broadband slow and subwavelength light in air

PRL 95, 063901 (2005) PHYSICAL REVIEW LETTERS week ending 5 AUGUST 2005

Surface-Plasmon-Assisted Guiding of Broadband Slow and Subwavelength Light in Air

Aristeidis Karalis,* E. Lidorikis, Mihai Ibanescu, J. D. Joannopoulos, and Marin Soljačić
Center for Materials Science and Engineering and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

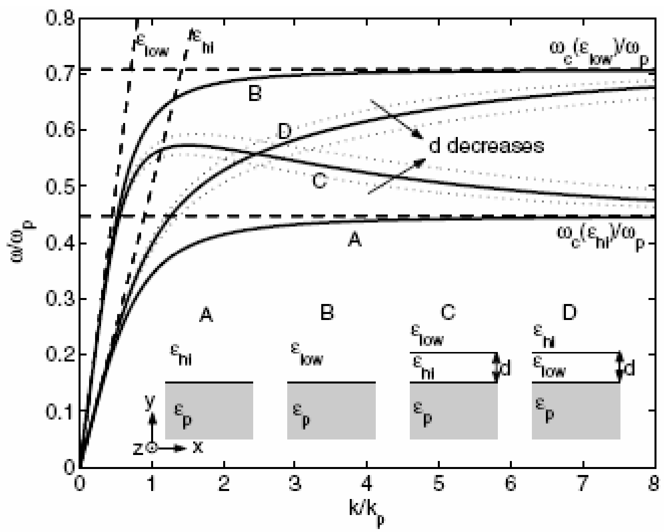
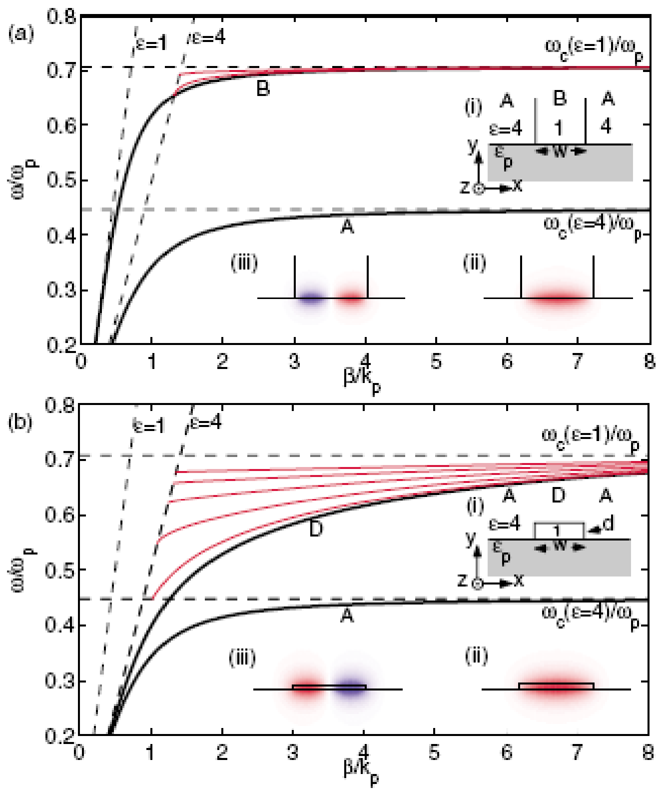
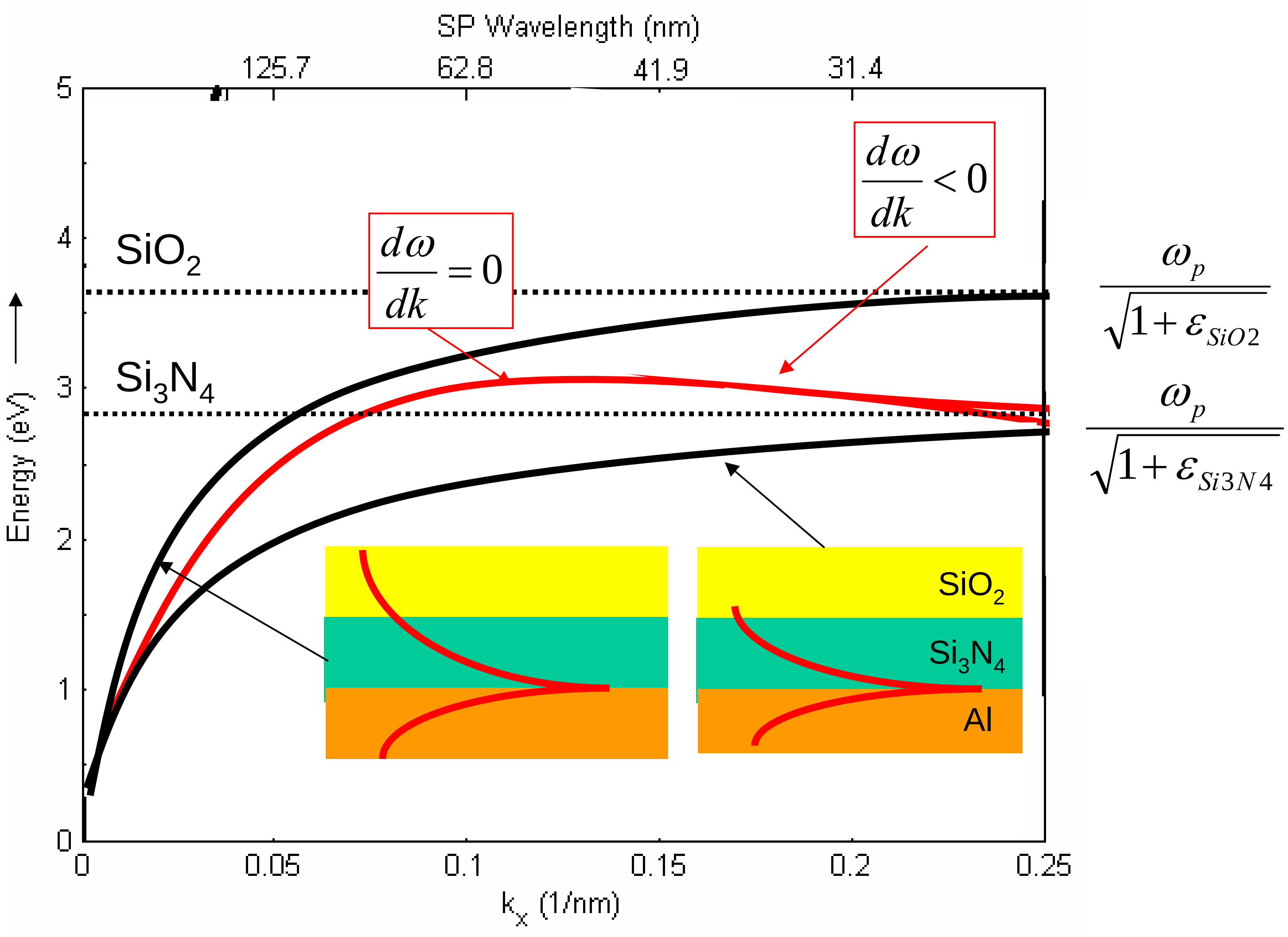


FIG. 1. ω-k diagrams (solid curves) for conventional layered surface-plasmon structures (insets *A*–*D*) with $\epsilon_{\rm hi} = 4$ and $\epsilon_{\rm low} = 1$ (air). Layer thicknesses $d/\lambda_p = 0.015$, 0.02, and 0.025 are used for *C* and *D* (solid + dotted curves). The light lines $\omega/\omega_p = \sqrt{\epsilon}k/k_p$ ("vertical" dashed lines) and the cutoff frequencies $\omega_c(\epsilon)/\omega_p = 1/\sqrt{1+\epsilon}$ (horizontal dashed lines) are shown.



Mode conversion: negative group velocity



Appendix

Light Interaction with Matter

Maxwell's Equations

Divergence equations	Curl equations
$\nabla \cdot \boldsymbol{D} = \rho_f$	$\nabla \times \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}}$
$\nabla \cdot \boldsymbol{B} = 0$	$\nabla \times \boldsymbol{H} = \dfrac{\partial \boldsymbol{D}}{\partial \boldsymbol{t}} + \boldsymbol{J}$

$\boldsymbol{D}$ = Electric flux density

$\boldsymbol{E}$ = Electric field vector

ρ = charge density

$\boldsymbol{B}$ = Magnetic flux density

$\boldsymbol{H}$ = Magnetic field vector

$\boldsymbol{J}$ = current density

Plasma frequency

If the electrons in a plasma are displaced from a uniform background of ions, electric fields will be built up in such a direction as to restore the neutrality of the plasma by pulling the electrons back to their original positions.
Because of their inertia, the electrons will overshoot and oscillate around their equilibrium positions with a characteristic frequency known as the plasma frequency.

$$E_s = (\text{surface charge density}) / \varepsilon_o$$

$$= Ne(\delta x) / \varepsilon_o \text{ : electrostatic field by small charge separation } \delta x$$

$$\delta x = \delta x_o \exp(-i\omega_p t) \text{ : small-amplitude oscillation}$$

$$m \frac{d^2(\delta x)}{dt^2} = (-e)E_s \quad \Rightarrow \quad -m\omega_p^2 = -\frac{Ne^2}{\varepsilon_o} \quad \Rightarrow \quad \therefore \omega_p^2 = \frac{Ne^2}{m\varepsilon_o}$$