Different Formulations for Numerical Solution of Single or Multibodies of Revolution with Mixed Boundary Conditions

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Abstract—**A numerical method for determining the electromagnetic field in the presence of one or several bodies of revolution is presented. The objects can be made of conductors, dielectrics or their combinations. The excitation is assumed to be due to a plane wave or infinitesimal electric dipoles located within or outside the dielectric. Several formulation types are considered and used to investigate the scattering by different objects. It is found that for moderate values of the dielectric constant, all formulation types give satisfactory results. However, for small or large relative permittivities the solution accuracies depend on the formulation type. As an application of the method to practical problems, two special cases of dielectric rod and microstrip antennas are considered. These antennas have widespread applications and the proposed method can be used to investigate their performance accurately.**

I. Introduction

MOST PROBLEMS in radar scattering and antenna fields involve material objects made of conductors, dielectrics and their combinations. When the object shape is arbitrary, an analytic solution is impossible to obtain and a numerical method must be developed. For conducting bodies the problem has been studied extensively and numerous publications are available in the literature. For dielectric bodies methods have also been developed and involve mostly scattering problems, such as those in bioelectromagnetics and radar meteorology [1], [2]. On the other hand, problems involving a combination of conductors and dielectrics have received limited attention. However, they occur in important antenna problems, such as dielectric loaded structures and microstrip antennas, where several conducting and dielectric bodies form the device geometry. The present work provides a solution method for these problems. Only rotationally symmetric geometries are considered, since the solutions are more economical to obtain and certain known antenna geometries can be studied.

For bodies of revolution, Mautz and Harrington [3] have provided three different formulations, called E-field, H-field, and combined-field, and investigated their solution accuracies. For dielectric objects, Barber and Yeh [4] have used the extended boundary condition method to solve the dielectric problem. Later Wu and Tsai [5] used the moment method to solve scattering from arbitrarily shaped and lossy dielectric bodies of revolution. Shortly after, Mautz and Harrington [6] solved the problem of homogeneous bodies using the Wu and Tsai formula, which they named PMCHW, and a formulation due to Müller. Recently, Govind *et al.* [7] reported the solution of scattering from inhomogeneous penetrable bodies of revolution, as an application to a composite missile/plume scattering problem. The solution for dielectric coated conductors, with possible extension to multiple coating, has also been attempted [8]. For multiple objects, Hunka and Mei [9] have used the unimoment method to solve the problem of two bodies of revolution with arbitrary directions for their axes of revolution. All these solutions, however, involve either a single dielectric body, a conductor with full dielectric coating, or two separate dielectric and conductor objects. The case of conductors with partial dielectric coatings was investigated by Mautz and Harrington [10] in researching the electromagnetic coupling problems. Using the equivalence principle, they have suggested different formulations and in [11] have solved numerically the electromagnetic coupling problems, for bodies of revolutions, by a formulation which we have named now E-PMCHW. An extension of this method was also provided by Medgyesi-Mitschang and Putnam [12] to solve the problem for bodies of revolution with axial inhomogeneity.

In the present work the method of Mautz and Harrington [10] is used to develop formulations for single or multiple objects made of conductors and dielectrics. Both a plane wave excitation and a dipole excitation, within the dielectric or outside, are used to determine the incident fields. The latter excitation enables one to model both scattering and antenna problems and as such is more useful for investigation of practical problems. Initially, by applying the equivalence principle different formulation types are generated. They are then applied to selected scattering problems to determine their performance. The formulations are subsequently used to investigate two important antenna problems, namely, dielectric rod and microstrip antenna radiations. Both problems have been solved approximately in the past, but the present method provides a detail understanding of their operation. It also enables one to design these antennas more accurately and determine various mode excitations and their respective contributions to the radiation field.

II. Formulation of The Problem

Fig. 1 shows the general electromagnetic problem under consideration, where a dielectric object is partially coated with a conductor. The surfaces S_{ce}, S_{cd}, and S_{de} refer, respectively,

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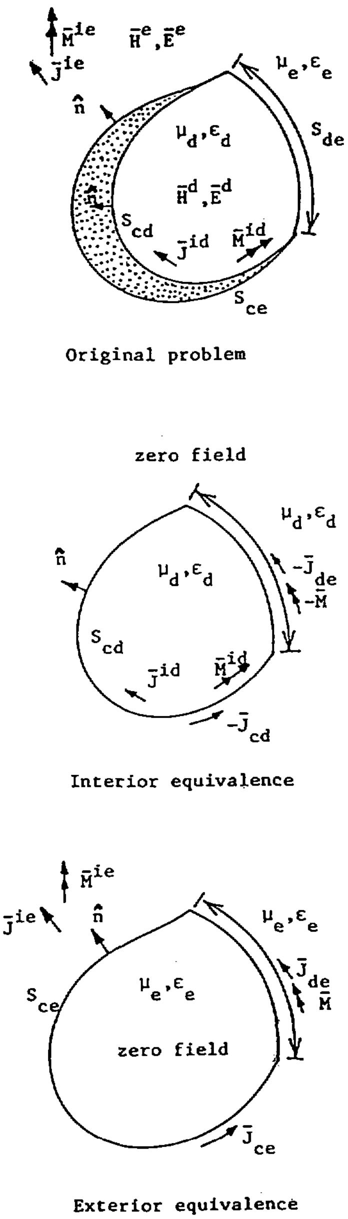

Fig. 1. Problem representation by equivalence principle.

to the boundaries between the conductor and the exterior region, conductor and dielectric, and dielectric and the exterior region. Also, $\vec{E}^d$, $\vec{H}^d$ and $\vec{E}^e$, $\vec{H}^e$ refer to the field vectors within the dielectric and the exterior region. The problem of Fig. 1 is a general one since it can be used to represent both conducting and dielectric problems. In particular, it reduces to the following cases:

1) scattering from a dielectric body, when S_{cd}, and S_{ce} vanish and S_{de} becomes a closed surface;
2) scattering from a perfect conductor, when S_{cd} and S_{de} vanish and S_{ce} becomes a closed surface;
3) scattering from a dielectric clad body, when S_{ce} vanish and S_{cd} and S_{de} become closed surfaces;
4) scattering from two regions, coupled by an aperture.

Now in Fig. 1, V^d is a finite volume filled with a homogeneous material of permittivity ϵ_d and permeability μ_d and bounded by two surfaces S_{de} and S_{cd}. The surface S_{cd} may consist of several subsurfaces, to represent multiple dielectric and conducting interfaces. V^e represents the external region, with permittivity of ϵ_e and permeability of μ_e, and is bounded by two surfaces S_{de} and S_{ce}. Again, the surface S_{ce} may consist of several subsurfaces, for multiple conductors. In the present work all these surfaces are assumed to be rotationally symmetric, to represent bodies of revolution. The sources of the electromagnetic excitation are provided by the impressed electric and magnetic currents $\vec{J}^{ie}$ and $\vec{M}^{ie}$ in V^e and $\vec{J}^{id}$ and $\vec{M}^{id}$ in V^d. The boundary conditions on conducting and dielectric surfaces are

$$\begin{aligned} &\hat{n}\times\vec{E}^d=0, && \text{on } S_{cd}\\ &\hat{n}\times\vec{E}^e=0, && \text{on } S_{ce}\\ &\left.\begin{aligned}\hat{n}\times\vec{E}^d=\hat{n}\times\vec{E}^e\\ \hat{n}\times\vec{H}^d=\hat{n}\times\vec{H}^e\end{aligned}\right\}, && \text{on } S_{de}\end{aligned} \tag{1}$$

and the surface equivalent currents are

$$\begin{aligned} &\vec{J}_{cd}=\hat{n}\times\vec{H}^d, && \text{on } S_{cd}\\ &\vec{J}_{ce}=\hat{n}\times\vec{H}^e, && \text{on } S_{ce}\\ &\left.\begin{aligned}\vec{J}_{de}=\hat{n}\times\vec{H}^e\\ \vec{M}=-\hat{n}\times\vec{E}^e\end{aligned}\right\}, && \text{on } S_{de}\end{aligned} \tag{2}$$

where $\hat{n}$ is the outward normal on each surface considered. The currents $\vec{J}_{cd}$, $\vec{J}_{ce}$, and $\vec{J}_{de}$ are the equivalent electric currents on each respective surface and $\vec{M}$ is the magnetic current on the interface surface between the dielectric and the exterior region. From the above equations and definitions the following set of equations can be written

$$\hat{n}\times\vec{E}^d(\vec{J}_{cd}+\vec{J}_{de},\ \vec{M})=\hat{n}\times\vec{E}^d(\vec{J}^{id},\ \vec{M}^{id}),\quad \text{just outside } S_{cd} \tag{3}$$

$$\hat{n}\times\vec{E}^d(\vec{J}_{cd}+\vec{J}_{de},\ \vec{M})=\hat{n}\times\vec{E}^d(\vec{J}^{id},\ \vec{M}^{id}),\quad \text{just outside } S_{de} \tag{4}$$

$$-\hat{n}\times\vec{E}^e(\vec{J}_{ce}+\vec{J}_{de},\ \vec{M})=\hat{n}\times\vec{E}^e(\vec{J}^{ie},\ \vec{M}^{ie}),\quad \text{just inside } S_{ce} \tag{5}$$

$$-\hat{n}\times\vec{E}^e(\vec{J}_{ce}+\vec{J}_{de},\ \vec{M})=\hat{n}\times\vec{E}^e(\vec{J}^{ie},\ \vec{M}^{ie}),\quad \text{just inside } S_{de} \tag{6}$$

$$\hat{n}\times\vec{H}^d(\vec{J}_{cd}+\vec{J}_{de},\ \vec{M})=\hat{n}\times\vec{H}^d(\vec{J}^{id},\ \vec{M}^{id}),\quad \text{just outside } S_{cd} \tag{7}$$

$$\hat{n}\times\vec{H}^d(\vec{J}_{cd}+\vec{J}_{de},\ \vec{M})=\hat{n}\times\vec{H}^d(\vec{J}^{id},\ \vec{M}^{id}),\quad \text{just outside } S_{de} \tag{8}$$

$$-\hat{n}\times\vec{H}^e(\vec{J}_{ce}+\vec{J}_{de},\ \vec{M})=\hat{n}\times\vec{H}^e(\vec{J}^{ie},\ \vec{M}^{ie}),\quad \text{just inside } S_{ce} \tag{9}$$

$$-\hat{n}\times\vec{H}^e(\vec{J}_{ce}+\vec{J}_{de},\ \vec{M})=\hat{n}\times\vec{H}^e(\vec{J}^{ie},\ \vec{M}^{ie}),\quad \text{just inside } S_{de} \tag{10}$$

where $\vec{E}^e(\vec{J}, \vec{M})$ and $\vec{E}^d(\vec{J}, \vec{M})$ are the electric fields due to currents $\vec{J}$ and $\vec{M}$, radiating in media characterized by ϵ_e, μ_e and ϵ_d, μ_d, respectively. $\vec{H}^e(\vec{J}, \vec{M})$ and $\vec{H}^d(\vec{J}, \vec{M})$ are the associated magnetic fields. The basic equations for these fields as in [7].

Equations (3)–(10) are eight equations in four unknowns field vectors and their various combinations can be used to determine the unknown currents. We have selected the following combinations.

1) E-field: here (3)–(6) are used to determine the equivalent currents, they are uniquely determined if and only if the region inside S_{ce} and S_{de}, when filled with μ_e, ϵ_e, and inclosed by a perfect conductor is not a resonant cavity. Since all four equations are satisfied by an electric field vector on objects surfaces, the formulation is named E-field formulation.

2) H-field: (7)–(10) can be used to determine currents uniquely, if and only if the region inside S_{ce} and S_{de}, filled with μ_e, ϵ_e, and enclosed by a perfect conductor, is not a resonant cavity [10].

3) C-field: a combination of (3)–(6) with (7)–(10) using the parameters α^e and α^d gives another formulation as (11) and (12) in [10]. These two equations are actually four equations when applied on S_{cd}, S_{ce}, and S_{de}.

Other formulations can also be obtained from those suggested by Mautz and Harrington [10]. For instance, their equations (15)–(18) can be used to generate the following set:

$$-\vec{E}^d_{\tan}(\vec{J}_{cd}+\vec{J}_{de}, \vec{M}) = -\vec{E}^d_{\tan}(\vec{J}^{id}, \vec{M}^{id}), \quad \text{on } S_{cd} \quad (11)$$

$$-\vec{E}^e_{\tan}(\vec{J}_{ce}+\vec{J}_{de}, \vec{M}) = -\vec{E}^e_{\tan}(\vec{J}^{ie}, \vec{M}^{ie}), \quad \text{on } S_{ce} \quad (12)$$

$$\frac{(\alpha-1)}{2}\hat{n}\times\vec{M} - \vec{E}^e_{\tan}(\vec{J}_{ce}+\vec{J}_{de}, \vec{M}) - \alpha\vec{E}^d_{\tan}(\vec{J}_{cd}+\vec{J}_{de}, \vec{M}) = \vec{E}^e_{\tan}(\vec{J}^{ie}, \vec{M}^{ie}) - \alpha\vec{E}^d_{\tan}(\vec{J}^{id}, \vec{M}^{id}), \quad \text{on } S_{de} \quad (13)$$

$$\frac{(1-\beta)}{2}\hat{n}\times\vec{J}_{de} - \vec{H}^e_{\tan}(\vec{J}_{ce}+\vec{J}_{de}, \vec{M}) - \beta\vec{H}^d_{\tan}(\vec{J}_{cd}+\vec{J}_{de}, \vec{M}) = \vec{H}^e_{\tan}(\vec{J}^{ie}, \vec{M}^{ie}) - \beta\vec{H}^d_{\tan}(\vec{J}^{id}, \vec{M}^{id}), \quad \text{on } S_{de} \quad (14)$$

which provide the following formulations.

4) E-PMCHW: in the above equations, if $\alpha = \beta = 1$, they lead to formulas provided in [10]. They indicate that there is no Ampere's law contributions due to the electric current at the field point. This formulation is named here as E-PMCHW.

5) E-Müller: another combination is possible with $\alpha = -\epsilon_d/\epsilon_e$, $\beta = -\mu_d/\mu_e$. The Ampere's law contribution appears as shown in (15) and (16), on the dielectric surface. This set is named E-Müller since Müller's boundary conditions are applied on the dielectric surface S_{de}.

Finally, (15)–(22) of Mautz and Harrington, [10] also give the following set:

$$\vec{J}_{cd}/2 + \hat{n}\times\vec{H}^d(\vec{J}_{cd}+\vec{J}_{de}, \vec{M}) = \hat{n}\times\vec{H}^d(\vec{J}^{id}, \vec{M}^{id}), \quad \text{on } S_{cd} \quad (15)$$

$$\vec{J}_{ce}/2 - \hat{n}\times\vec{H}^e(\vec{J}_{ce}+\vec{J}_{de}, \vec{M}) = \hat{n}\times\vec{H}^e(\vec{J}^{ie}, \vec{M}^{ie}), \quad \text{on } S_{ce}. \quad (16)$$

For the remaining two equations on S_{de}, (13) and (14) can be used, which provide the following combinations.

6) H-PMCHW: if $\alpha = \beta = 1$, then (15) and (16) together with (13) and (14) give a set which has an Ampere's law contribution only on the conductors, but not on the dielectrics. This set is called H-PMCHW.

7) H-Müller: if $\alpha = -\epsilon_d/\epsilon_e$ and $\beta = -\mu_d/\mu_e$ the above equations reduce to (19) to (20) of Mautz and Harrington [10] and Ampere's law contribution exists on all surfaces. This set is called H-Müller.

For perfectly conducting objects the above sets reduce to the three initial formulations, namely, E-field, H-field and C-field. For dielectric objects only five formulations are possible and are E-field, H-field, C-field, PMCHW, and Müller formulations. For combination of dielectric and conductors and aperture coupling problems all above seven formulations are possible, which are E-field, H-field, C-field, E-PMCHW, E-Müller, H-PMCHW, and H-Müller. The meaning of the last four formulations is that, E- or H-field boundary conditions are applied on the conductors and PMCHW or Müller boundary conditions on the dielectrics.

III. Matrix Formulations

The above formulations are applied to rotationally symmetric bodies. The reduction of integral equations to matrix equations involving unknown surface currents follows the procedure well known for bodies of revolution [3], [6], [13]. After the necessary manipulations of the moment method, the general matrix form takes the form

$$[\bar{T}_n][\bar{I}_n] = [\bar{V}_n], \quad n = 0, \pm 1, \pm 2, \cdots \quad (17)$$

where $\bar{T}_n$ is a square matrix, representing the impedance and the admittance submatrices, $\bar{I}_n$ is a column matrix for the unknown expansion coefficients of $\vec{J}$ and $\vec{M}$, and $\bar{V}_n$ is the excitation column matrix. For the above different formulations the matrix equations take the following forms for each mode n.

A. E-Field Matrix

$$\begin{bmatrix} Z^e_{ce,ce} & 0 & Z^e_{ce,de} & Y^e_{ce,de} \\ 0 & \eta_r Z^d_{cd,cd} & \eta_r Z^d_{cd,de} & Y^d_{cd,de} \\ Z^e_{de,ce} & 0 & Z^e_{de,de} & Y^e_{de,de} \\ 0 & \eta_r Z^d_{de,cd} & \eta_r Z^d_{de,de} & Y^d_{de,de} \end{bmatrix} \begin{bmatrix} I_{ce,n} \\ I_{cd,n} \\ I_{de,n} \\ M_n \end{bmatrix} = \begin{bmatrix} V^e \\ -V^d \\ V^e \\ -V^d \end{bmatrix} \quad (18)$$

where $\eta_r = \eta_d/\eta_e$ and V^e, V^d are the excitation submatrices, due to the electric field sources in the exterior and interior regions, respectively. The submatrices Z and Y with superscripts e and d denote the impedance and admittance matrices for the exterior or interior media, respectively. The first pair of suffixes identifies field surface and the second pair of suffixes identifies the source surface and the Fourier mode n is implied. $I_{ce,n}$, $I_{cd,n}$, $I_{de,n}$ and M_n are the unknown expansion coefficients of the electric and magnetic currents on S_{ce}, S_{cd},

S_{de} respectively.

B. H-Field Matrix

$$\begin{bmatrix} Y^e_{ce,ce} & 0 & Y^e_{ce,de} & -Z^e_{ce,de} \\ 0 & Y^d_{cd,cd} & Y^d_{cd,de} & -1/\eta_r Z^d_{cd,de} \\ Y^e_{de,ce} & 0 & Y^e_{de,de} & -Z^e_{de,de} \\ 0 & Y^d_{de,cd} & Y^d_{de,de} & -1/\eta_r Z^d_{de,de} \end{bmatrix} \begin{bmatrix} I_{ce,n} \\ I_{cd,n} \\ I_{de,n} \\ M_n \end{bmatrix} = \begin{bmatrix} I^e \\ -I^d \\ I^e \\ -I^d \end{bmatrix} \tag{19}$$

where I^e_n and I^d_n are the excitations due to the magnetic field sources in the exterior and interior regions.

C. C-Field Matrix

A linear combination of E-field and H-field matrices with a parameter α gives the C-field matrix. It is an addition of H-field matrix with the E-field matrix multiplied by α.

D. E-PMCHW and E-Müller Matrices

With similar notations the matrix equation becomes

$$\begin{bmatrix} Z^e_{ce,ce} & 0 & Z^e_{ce,de} & Y^e_{ce,de} \\ 0 & \eta_r Z^d_{cd,cd} & \eta_r Z^d_{cd,de} & Y^d_{cd,de} \\ Z^e_{de,ce} & \alpha\eta_r Z^d_{de,cd} & Z^e_{de,de}+\alpha\eta_r Z^d_{de,de} & Y^e_{de,de}+\alpha Y^d_{de,de} \\ Y^e_{de,ce} & \beta Y^d_{de,cd} & Y^e_{de,de}+\beta Y^d_{de,de} & -Z^e_{de,de}-\dfrac{\beta}{\eta_r} Z^d_{de,de} \end{bmatrix} \begin{bmatrix} I_{ce,n} \\ I_{cd,n} \\ I_{de,n} \\ M_n \end{bmatrix} = \begin{bmatrix} V^e \\ -V^d \\ V^e-\alpha V^d \\ I^e-\beta I^d \end{bmatrix}. \tag{20}$$

When $\alpha = \beta = 1$ this equation gives the matrix equation for E-PMCHW formulation, and for $\alpha = -\epsilon_d/\epsilon_e$ and $\beta = -\mu_d/\mu_e$ that of E-Müller formulation.

E. H-PMCHW and H-Müller Matrices

The result is

$$\begin{bmatrix} Y^e_{ce,ce} & 0 & Z^e_{ce,de} & -Z^e_{ce,de} \\ 0 & Y^d_{cd,cd} & Y^d_{cd,de} & \dfrac{-1}{\eta_r} Z^d_{cd,de} \\ Y^e_{de,ce} & \beta Y^d_{de,cd} & Y^e_{de,de}+\beta Y^d_{de,de} & -Z^e_{de,de}-\dfrac{\beta}{\eta_r} Z^d_{de,de} \\ Y^e_{de,ce} & \alpha\eta_r Z^d_{de,cd} & Z^e_{de,de}+\alpha\eta_r Z^d_{de,de} & Y^e_{de,de}+\alpha Y^d_{de,de} \end{bmatrix} \begin{bmatrix} I_{ce,n} \\ I_{cd,n} \\ I_{de,n} \\ M_n \end{bmatrix} = \begin{bmatrix} I^e \\ -I^d \\ V^e-\beta I^d \\ V^e-\alpha V^d \end{bmatrix}. \tag{21}$$

While $\alpha = \beta = 1$, it gives equations for H-PMCHW, and for $\alpha = -\epsilon_d/\epsilon_e$ and $\beta = -\mu_d/\mu_e$ that of H-Müller formulation.

In the above equations each submatrix Y^q or Z^q consists of four submatrices, which are found following the procedure used in previous work [3]. However, for convenience, the submatrices of Z^q and Y^q in the C-field case are determined using the components of $\vec{E}_{\tan}$ and $(\hat{n} \times \vec{H})$. On the other hand, for both PMCHW and Müller cases, they are obtained using the components of $\vec{E}_{\tan}$, $\vec{H}_{\tan}$ for E-formulations and $(\hat{n} \times \vec{E})$, $(\hat{n} \times \vec{H})$ for H-formulations. The superscript q represents e or d, respectively.

Once the induced currents $\vec{J}$ and $\vec{M}$ on the surface are determined after the solution of the matrix equations, the far-field components E_θ and E_ϕ at a far-field point (r_0, θ_0, ϕ_0) can be determined [14]. Note that, E_θ and E_ϕ are the scattered fields if the excitation source is in the exterior region, and are the total fields if the source is located in the interior region. If E_θ and E_ϕ are the scattered fields, the total fields are the summation of the scattered and the incident fields.

IV. Numerical Results

Using all above formulations and with both plane wave and dipole excitations several different problems were considered and solved to examine the solution accuracies. To compare the numerical results with analytic ones, a dielectric sphere was considered. For small values of the relative dielectric constant, in the range of $1 < \epsilon_r < 8$ all formulations were found to give satisfactory results. However, the Müller type formulations provided the best agreements with analytic solutions for $\epsilon_r \leq 2$ and PMCHW the worst one for the values very close to one and failed when $\epsilon_r = 1$. For larger ϵ_r values, i.e., for $\epsilon_r \geq 10$, the Müller formulations gave progressively poorer results. On the other hand, the E-field formulation provided the best overall accurate results. Some representative results may be found in [6], [15].

For a dielectric coated conducting sphere, sample calculations are shown in Fig. 2, which shows the scattering cross-sections for sphere parameters $b/a = 3$, $ka = 1$, $\epsilon_r = 4$ and an axial plane wave incidence and with 30 segments on the dielectric and 20 segments on the conductor. The agreement of all solution types, with analytic ones, seems satisfactory and it is difficult to distinguish between the analytic and the numerical results. The agreement of different results, again, seems to be satisfactory, but each formulation type, i.e., E-Müller and H-Müller etc., gives identical results. More results could be found in [16] for different geometry bodies of the dielectric and coated conductors.

For multiple bodies of revolution different examples are selected. The data of two examples are compared with the experimental data in [9] to indicate the solution accuracy for the case of multiple bodies. Fig. 3 compares the measured and

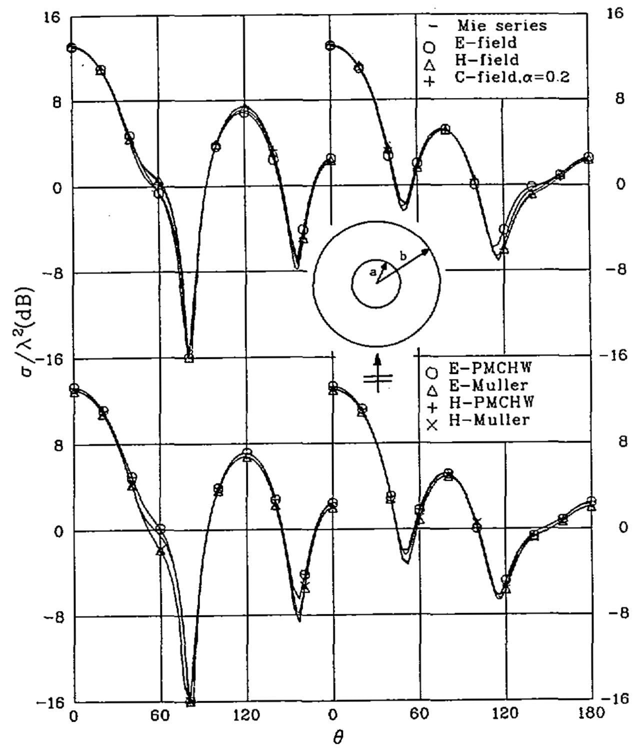

Fig. 2. The bistatic scattering cross sections of a coated sphere with $b/a = 3$, $ka = 1$, $\epsilon_r = 4$. E-plane left side, H-plane right side.

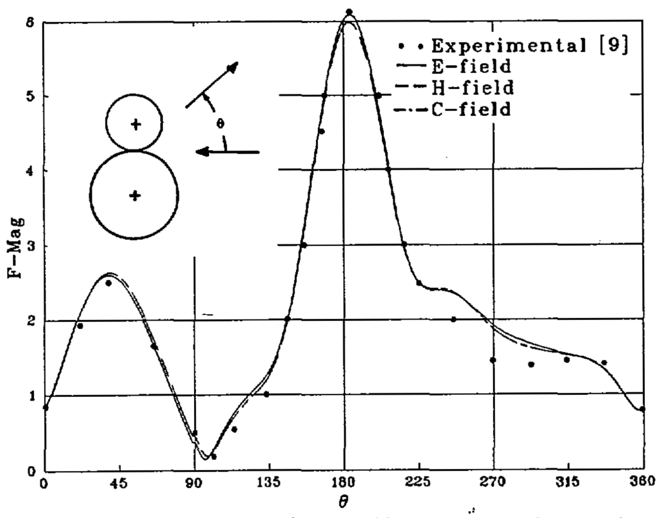

Fig. 3. The bistatic scattering of two touching perfectly conducting spheres with radii λ/2 and λ/4.

computed scattering patterns by two perfectly conducting spheres in contact having radii λ/2 and λ/4, respectively. The incidence is from the broadside direction. Here F-Mag $= |\pi\sigma|/\lambda$ and θ is the bistatic angle. In this figure three different solutions are compared with the experimental results which are E-, H-, and C-field solutions. The dots represent selective experimental data points from [9]. The solutions are nearly the same and the H-field and C-field with $\alpha = 0.2$ are identical.

Fig. 4 presents the computed and measured scattering data for two identical adjacent cylinders, one being conductor and the other dielectric their dimensions are: height = 1 λ and r = λ/2 and the dielectric cylinder has a relative permittivity of $\epsilon_r = 2.59\text{-}j0.017$. The dielectric cylinder is below the conductor one and the separation of their centers is 1.5 λ. In Fig. 4, the solutions of all seven different formulations are presented. As it can be seen the agreement among them and with the experimental data is excellent. These results indicate that, the above formulations provide solutions with satisfactory accuracy levels. As a further example, the case of two conducting spheres of $ka = 1$, separated by a distance of 0.75 λ between their centers, is also studied. To simulate a dielectric boundary, one of the spheres is coated with a dielectric of $\epsilon_r = 1$ and a radius of $kb = 3$. The computed results are shown in Fig. 5. Comparing these results with the computed data for only two conducting spheres, it was found that the Müller formulations, gave the best results, in agreement with the conductor solutions. The results of Fig. 5 also show that other formulations give inaccurate results in both forward and backward directions. For the same example when $\epsilon_r = 4$ (not presented), all solutions provided results with excellent agreement with each other.

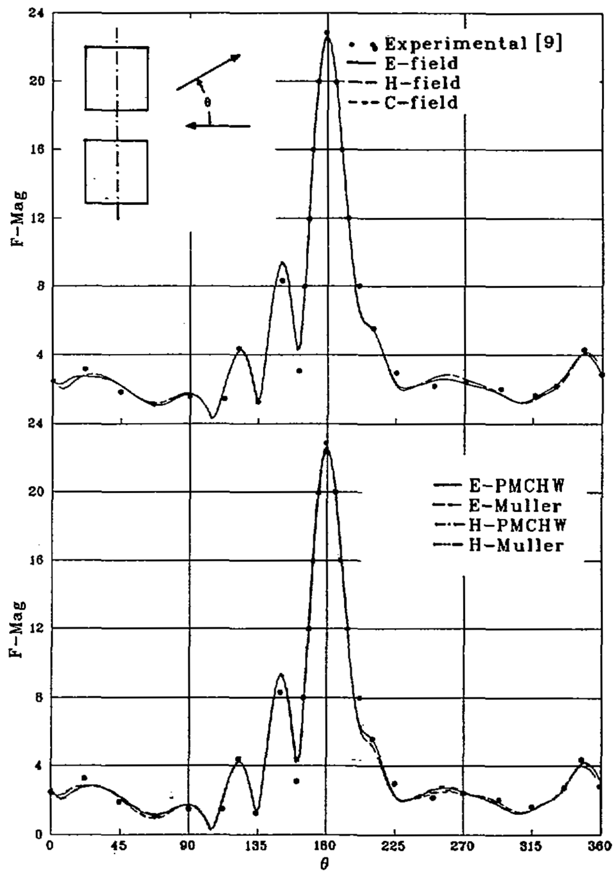

Fig. 4. The bistatic scattering of two identical finite cylinders. A dielectric cylinder with $\epsilon_r = 2.59\text{-}j0.017$, $h = 1$ λ, r = λ/2, and a conducting cylinder of the same dimensions, centre separation is 1.5 λ.

For a dipole excitation similar studies were also carried out. The results were similar to the plane wave excitation shown in the above cases. In these cases, however the dipole was located both inside dielectric and in the outside region. For moderate values of ϵ_r the agreement of different formulations was satisfactory. For ϵ_r approaching unity the Müller solutions gave most accurate results, but failed at large values of ϵ_r, where the E-field solution provided the most accurate ones.

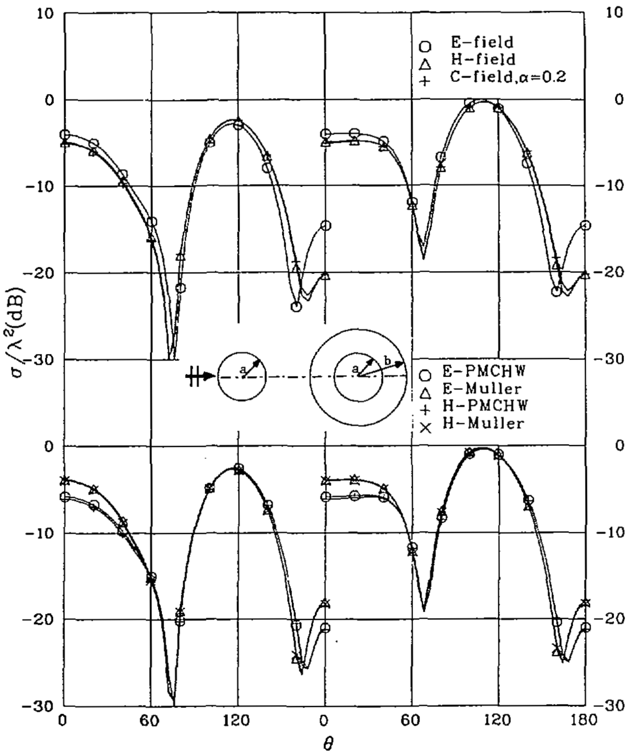

Fig. 5. The bistatic scattering cross section two conducting spheres of $ka = 1$, one of the spheres is coated with radii ratio, $b/a = 3$, $ka = 1$, $\epsilon_r = 1$, separation distance of two spheres = 0.75 λ. E-plane left side, H-plane right side.

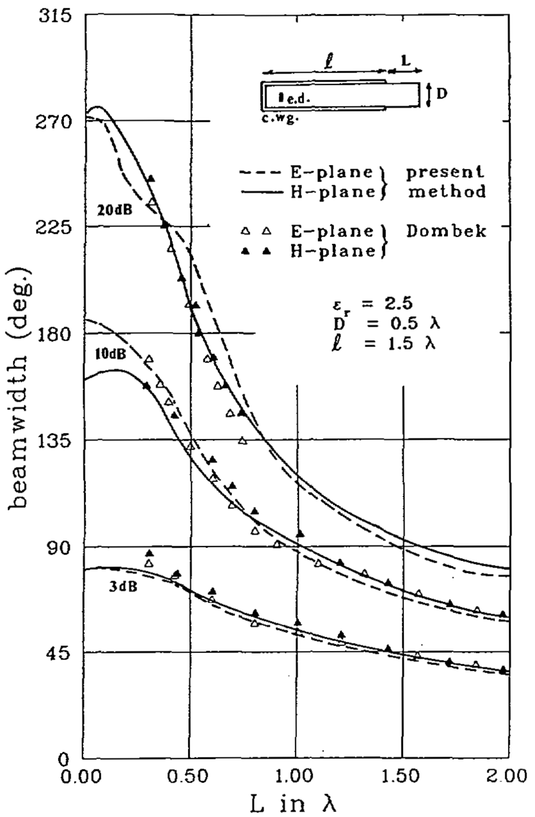

Fig. 6. The beamwidths of a dielectric rod patterns at 3 dB, 10 dB, 20 dB levels, $D = 0.5$ λ, $\epsilon_r = 2.5$, $l = 1$ λ, e.d. is an electric dipole and c.wg is a circular waveguide.

V. Applications

Among many problems that can be handled by the present method, two practically important antenna problems have been investigated, namely a dielectric rod antenna and a circular microstrip patch antenna. Both problems have widespread applications and have been analyzed, in the past, approximately as in [17] and [18], respectively. The present method, however, provides an accurate information on their performance.

A. Dielectric Rod Antenna

Dielectric rod antennas are commonly used as a feed for reflector antennas [9], or as high gain compact antennas. A simple method of exciting such antennas is through a short section of circular waveguide. To solve the problem, the dielectric rod was assumed to fill the interior of the waveguide, and to extend beyond its aperture by a distance L. The waveguide was excited by an electric dipole, located on its axis in the transverse direction, so that it excites the TE_{11} mode in the waveguide [20]. Using an E-PMCHW formulation the radiation patterns for a dielectric rod antenna with relative permittivity, $\epsilon_r = 2.5$, and diameter, $D = 0.5$ λ and different rod extension lengths L were determined. The results were used to compute the pattern 3 dB, 10 dB, and 20 dB beamwidths, which are shown in Fig. 6. The computed results are compared with the experimental results of Dombek [17]. There is a discrepancy between the measured and the computed results which could be due to the uncertainty in the loss tangent of the dielectric material used to obtain the experimental data. It is clear that increasing L decreases the beamwidths due to the excitation of surface waves, and improves the pattern symmetry. The presented results are for a simple dielectric rod to indicate the solution behavior. In practice, one can profile the dielectric shape, or use multimode excitations to optimize designs for given specifications. As long as the geometry is rotationally symmetric the problem can be solved by this method to optimize the required profile.

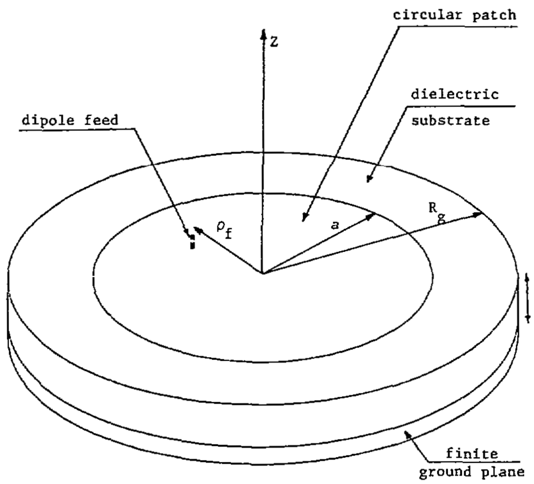

Fig. 7. Circular microstrip antenna geometry.

B. Microstrip Antennas

Microstrip antennas are currently being used as low profile, low cost units in variety of applications. Their analysis,

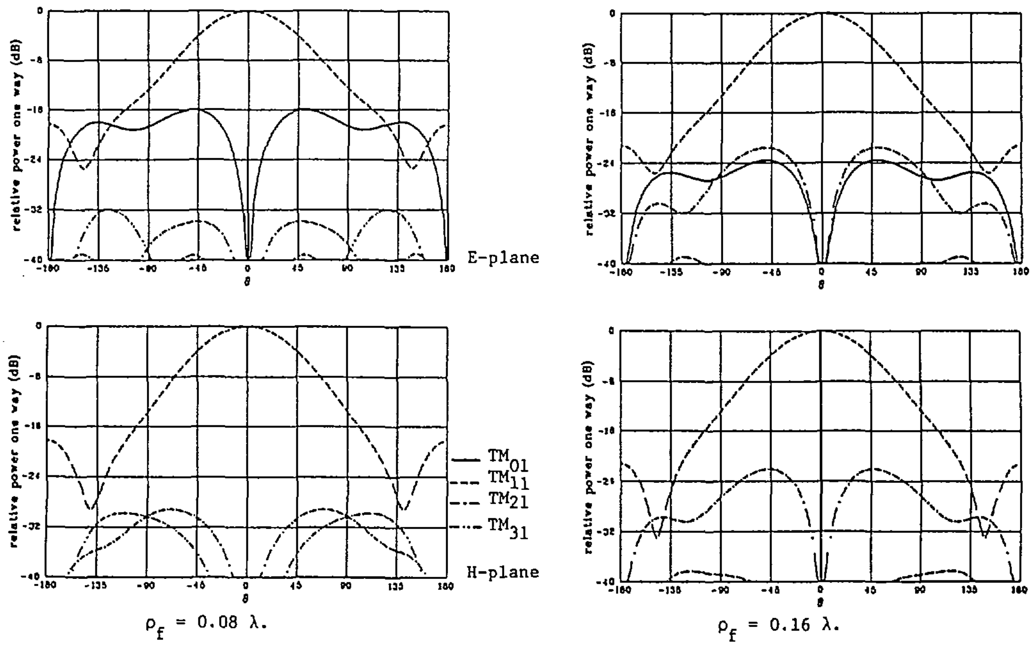

Fig. 8. Radiation patterns of first four modes for a circular microstrip patch, $a = 0.19\ \lambda$, $R_g = 0.4\ \lambda$. Left side, $R_f = 0.08\ \lambda$ and right side, $R_f = 0.16\ \lambda$. Top patterns in E-plane, bottom patterns in H-plane.

however, is difficult due to the existence of substrate dielectric. Approximate analyses, using the fringing fields are useful, but neglect higher order mode excitations. The modal analysis approach provides a more superior solution, but determining the mode coefficients is too complex. The present method provides a convenient approach to investigate circular patch antennas. For a given location of the feed, the mode coefficients can be determined and their contribution to the radiation field can be studied. Furthermore, the effect of finite ground plane or the dielectric substrate and their thickness can be investigated precisely.

Fig. 7 shows the geometry of a circular microstrip patch antenna. For different locations of the dipole and the patch radius, the excitation of TM_{01}, TM_{11}, TM_{21} modes were computed and used to calculate their radiation patterns. The results are shown in Fig. 8. They indicate that the excitation of different modes depends strongly on the dipole location and the patch radius. Since the present method provides information on various mode excitations, the design of patch antennas for particular pattern characteristics can be carried out with a high degree of accuracy. That is a possibility that is difficult to achieve by other methods, and is applicable to microstrip antennas.

VII. Discussions and Conclusion

Electromagnetic field equations satisfied on the surface of objects with both vanishing and continuous tangential field components were obtained. These equations provided seven different sets of formulations that could be used to determine the equivalent surface currents. For dielectric and conducting objects, they reduced to five and three sets, respectively. The formulations were then applied to bodies of revolution and were reduced to matrix equations.

To examine the dependence of solution accuracies on the formulation type, they were used to compute the surface currents and the far-field distributions for variety of objects. It was found that for dielectric bodies, the accuracy of the solutions, using each formulation, depends primarily on the material permittivity. These comparisons were made for identical solution conditions, such as, number of expansion functions and the numerical integration of Green's functions. For the general case of objects with both conducting and dielectric boundaries, the treatment of the boundary conditions on the dielectric was found to have the main influence on the solutions. That is, the Müller or PMCHW formulation gave almost identical results for both E- and H-equations on the conductor. For moderate values of ϵ_r ($2 < \epsilon_r < 8$), all formulation types seemed to give satisfactory results. For multibodies of revolution the solutions of two examples were compared with the experimental data. The agreement was satisfactory. In practice, when analytic or experimental data are not available, a comparison of different solutions may be used to examine the solution accuracy. They also provide some degree of freedom to the user, to select the most suitable, or convenient, formulation for a given problem.

The generated formulations can provide numerical solution, for varieties of important practical problems. Two special cases were considered in this work. Dielectric rod antennas, fed by a circular waveguide, were investigated and the results for the beamwidths of their radiation patterns were presented and compared with the experimental results and satisfied reasonable agreement with each other. Circular microstrip patch antennas were also studied. In this case, various mode excitations were determined and their radiation patterns were calculated. Since, no other method is available that can determine these mode coefficients accurately, the method seems to be a unique one to study microstrip antennas. It can also be used to examine the effect of various parameters on their performance.

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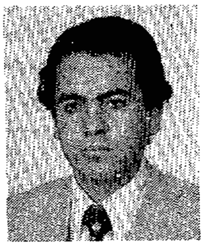

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