Efficient and accurate computation of electric field dyadic Green's function in layered media

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Abstract

Concise and explicit formulas for dyadic Green's functions, representing the electric and magnetic fields due to a dipole source placed in layered media, are derived in this paper. First, the electric and magnetic fields in the spectral domain for the half space are expressed using Fresnel reflection and transmission coefficients. Each component of electric field in the spectral domain constitutes the spectral Green's function in layered media. The Green's function in the spatial domain is then recovered involving Sommerfeld integrals for each component in the spectral domain. By using Bessel identities, the number of Sommerfeld integrals are reduced, resulting in much simpler and more efficient formulas for numerical implementation compared with previous results. This approach is extended to the three-layer Green's function. In addition, the singular part of the Green's function is naturally separated out so that integral equation methods developed for free space Green's functions can be used with minimal modification. Numerical results are included to show efficiency and accuracy of the derived formulas.

Keywords: Maxwell's equations, Dyadic Green's functions, Sommerfeld integrals, Layered media

1. Introduction

Multi-layered media is a fundamental structure for many applications such as meta-materials, photonic crystals [1], solar cells [2, 3], light emitting diodes [4], and plasmonic devices and others. Numerical simulation of wave propagation in such media poses much challenge due to large number of scatters, the treatment of radiation condition at the infinite, and the field discontinuity at layer interfaces in meta-materials consisting of meta-atoms. Integral equation methods have been shown to be versatile to address these issues in computing the wave scattering in the layered media. To implement the integral equation formulation of the scattering problem, it is imperative to have a concise formulation and efficient computational algorithm to compute the dyadic Green's functions for the Maxwell's equations in the three-dimension (3-D). In this paper, we will present explicit and compact formulas for the two- and three-layer dyadic Green's functions in terms of high order Hankel transforms and relevant numerical method for their computations.

The dyadic Green's function for a two-layer structure [5] and multi-layered media [6] have been explicitly presented. However, the formula for the three or more layers was not provided in Ref. [5]. Also, the derivations in these work used an analytical formula for Sommerfeld integrals for two layers in order to reduce the total number of Sommerfeld integrals to 10. As

a consequence, extension to multi-layered media for sources on top of the layered media as well as in the middle layer is not obvious. As a result, multi-layered media Green's function requires extra Sommerfeld integrals. The multi-layered media Green's function in [6] requires total 16 Sommerfeld integrals. The new formula proposed in this paper utilizes the second order Hankel transform to reduce the number of integrals needed and the singular and nonsingular parts of the Green's function are clearly separated. This allows easy use of many integral equation algorithms and codes developed using free space Green's function [7] or periodizing schemes for periodic objects [8, 9] for the multi-layered media problems. Moreover, our approach in principle, with some more bookkeeping associated with the layers, can be extended to the multi-layered media when the source is on top of the layered media. Discussion on various numerical issues of implementing the integral equations can be found in Ref. [10, 11] and it is not repeated here. For numerical contour integration of Sommerfeld integrals in the Fourier k-space, adaptive generalized Gaussian quadrature rules [12, 13] are used to obtain high accuracy using quadrature points only on the real axis. This avoids complex number operations and reduces computation time. In other words, near the surface poles of the spectral dyadic Green's functions, generalized Gaussian quadrature rule is applied while traditional Gaussian quadrature is applied in other parts of the contour.

The derivation for the dyadic Green's function in this paper is rather cumbersome and tedious. However, it is unavoidable for multi-layered media simulation and much needed in practice of integral equations using dyadic Green's functions. Every effort is made to simplify the final formula so the readers can implement them easily. The same notation as in Ref. [5] will be used and modified as necessary throughout the paper.

The rest of the paper is organized as follows. In the next section, the free-space Green's function is transformed to one in the spectral domain using the Sommerfeld identity. Then, the two-layer Green's functions will be derived using the free-space Green's function and Fresnel reflection coefficients [14, 15] in Section 3. In Section 4, extension will be given for the three-layer Green's functions with generalized Fresnel reflection coefficients [16] due to multiple reflections from the interfaces. Finally, in Appendix, several Bessel identities used for the derivations are provided.

2. Free-space Green's function

The free-space Green's function serves as a primary singular field for the multi-layered media Green's function. In multi-layered media, the free-space Green's function will be "corrected" with reflected and transmitted contribution. Thus, in this section, the dyadic Green's function for the free space is studied. First, it is rewritten in the spectral domain. Then, the spatial domain Green's function is recovered by taking the inverse Fourier transform. The same process will be applied for multi-layered media. For convenience, the free space will be referred as a one-layer problem that has relative permittivity ε_1 and permeability μ_1. Let a unit dipole be placed at $\mathbf{r}' = (x', y', z')$ and oriented along $\hat{\alpha}' = (\alpha'_x, \alpha'_y, \alpha'_z)$. Then, the electric $\mathbf{E}_1^P = (E_{1x}^P, E_{1y}^P, E_{1z}^P)$ and magnetic $\mathbf{H}_1^P = (H_{1x}^P, H_{1y}^P, H_{1z}^P)$ fields in the free space at $\mathbf{r} = (x, y, z)$ can be written as

$$\mathbf{E}_1^P(\mathbf{r}) = \frac{i\omega\mu_0\mu_1}{4\pi}(\mathbf{I} + \frac{\nabla\nabla}{k_1^2}) \cdot \hat{\alpha}' \frac{e^{ik_1|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}, \quad \mathbf{H}_1^P(\mathbf{r}) = \frac{1}{4\pi}\nabla \times \hat{\alpha}' \frac{e^{ik_1|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}, \tag{1}$$

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where $k_1 = k\sqrt{\varepsilon_1\mu_1}$ in the dielectric and $k = \omega\sqrt{\varepsilon_0\mu_0}$ is the wave number in vacuum, respectively. Using the Sommerfeld identity [17, 16],

$$\frac{e^{ik_1|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} = \frac{i}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}dk_xdk_y\frac{e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}|z-z'|}}{k_{1z}},\tag{2}$$

where $k_s^2 = k_x^2 + k_y^2$ and $k_{1z} = \sqrt{k_1^2 - k_s^2}$, the E_{1z}^P can be written as

$$E_{1z}^P(\mathbf{r}) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}dk_xdk_y\left(-\frac{\omega\mu_0\mu_1}{8\pi^2}\left(\hat{z}\cdot\hat{\alpha}' + \frac{1}{k_1^2}\partial_z\nabla\cdot\hat{\alpha}'\right)\frac{e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}|z-z'|}}{k_{1z}}\right),\tag{3}$$

where $\hat{z}$ is a unit vector along the z-axis. The integrand in Eq. (3) is the spectral component of electric field in the z-direction, which is denoted by

$$\tilde{E}_{1z}^P = \left(\hat{z}\cdot\hat{\alpha}' + \frac{1}{k_1^2}\partial_z\nabla\cdot\hat{\alpha}'\right)\tilde{g}_1^P,\tag{4}$$

where

$$\tilde{g}_1^P = -\frac{\omega\mu_0\mu_1}{8\pi^2}\frac{e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}|z-z'|}}{k_{1z}}.\tag{5}$$

A similar derivation yields the magnetic field as

$$\tilde{H}_{1z}^P = \frac{1}{i\omega\mu_0\mu_1}\hat{z}\cdot\nabla_s\times\hat{\alpha}'\tilde{g}_1^P,\tag{6}$$

where $\nabla_s = (\partial_x, \partial_y)$. From Maxwell's equations, the transverse components $\tilde{\mathbf{E}}_{1s}^P = (\tilde{E}_{1x}^P, \tilde{E}_{1y}^P)$ and $\tilde{\mathbf{H}}_{1s}^P = (\tilde{H}_{1x}^P, \tilde{H}_{1y}^P)$ can be written using the $\tilde{E}_{1z}$ and $\tilde{H}_{1z}$ as

$$\tilde{\mathbf{E}}_{1s}^P = \frac{1}{k_s^2}\left(\nabla_s\partial_z\tilde{E}_{1z}^P - i\omega\mu_0\mu_1\hat{z}\times\nabla_s\tilde{H}_{1z}^P\right),\tag{7}$$

$$\tilde{\mathbf{H}}_{1s}^P = \frac{1}{k_s^2}\left(\nabla_s\partial_z\tilde{H}_{1z}^P + i\omega\varepsilon_0\varepsilon_1\hat{z}\times\nabla_s\tilde{E}_{1z}^P\right).\tag{8}$$

These two relations reduce the problem to an one-dimensional problem in the spectral domain because only the z-component of electric and magnetic fields is required to completely determine the fields in the spectral domain. By substituting Eqs. (4) and (6) into Eqs. (7) and (8), the electric field in the spectral domain $\tilde{\mathbf{E}}^P = (\tilde{E}_{1x}^P, \tilde{E}_{1y}^P, \tilde{E}_{1z}^P)$ can be explicitly written in terms of the spectral Green's function, that is,

$$\begin{bmatrix} \tilde{E}_{1x}^P \\ \tilde{E}_{1y}^P \\ \tilde{E}_{1z}^P \end{bmatrix} = \tilde{\mathbf{G}}^P\alpha' = \begin{bmatrix} \tilde{G}_{xx}^P & \tilde{G}_{xy}^P & \tilde{G}_{xz}^P \\ \tilde{G}_{yx}^P & \tilde{G}_{yy}^P & \tilde{G}_{yz}^P \\ \tilde{G}_{zx}^P & \tilde{G}_{zy}^P & \tilde{G}_{zz}^P \end{bmatrix}\begin{bmatrix} \alpha'_x \\ \alpha'_y \\ \alpha'_z \end{bmatrix},\tag{9}$$

where

$$\tilde{\mathbf{G}}^P = \left(\mathbf{I} + \frac{\nabla\nabla}{k_1^2}\right)\tilde{g}_1^P.\tag{10}$$
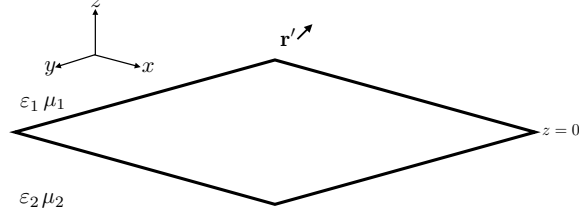
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Figure 1: A two-layer structure. The free space is divided at $z = 0$ into the top and bottom layer. A dipole is located at $\mathbf{r'} = (x', y', z')$ and the top layer has ε_1 and μ_1 and the bottom layer has ε_2 and μ_2.

Finally, the electric field in the spatial domain in Eq. (1) can be recovered by taking double integrals

$$G_{ij}^P = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \tilde{G}_{ij}^P dk_x dk_y, \;\; i, j = x, y, z, \tag{11}$$

on each component of Eq. (10). This will constitute the free-space dyadic Green's function in the spatial domain. Note that this integral is well known as a Sommerfeld integral. The double integral can be reduced to a single integral using cylindrical coordinate. The resulting integral involves Bessel function or Hankel function depending on convenience and is sometimes referred as the Hankel transform.

3. Green's function for a two-layer structure

In this section, the free-space Green's function is modified with the reflected and transmitted parts of the Green's function for a two-layer structure depicted in Fig. 1. Overall the process of computing the Green's function is the same as the free space. Due to symmetry, the source is assumed to be in the first layer. First, in the spectral domain, using Fresnel reflection and transmission coefficients, z-component of reflected electric field in the first layer and the transmitted field in the second layer are found. Then, all the transverse components in each layer are derived using Eqs. (7) and (8). Now the spatial domain Green's function can be found by taking Sommerfeld integral for each component. Finally, in the first layer, primary field is added with reflected field from the second layer to complete derivation.

3.1. Fields in the spectral domain

The z-component of electric and magnetic fields in the first layer are

$$\tilde{E}_{1z} = \tilde{E}_{1z}^P + \tilde{E}_{1z}^R, \;\; \tilde{H}_{1z} = \tilde{H}_{1z}^P + \tilde{H}_{1z}^R, \tag{12}$$

where the superscript P and R denote the primary and reflected parts, respectively. Similarly, The z-component of electric and magnetic fields in the second layer are

$$\tilde{E}_{2z} = \tilde{E}_{2z}^T, \;\; \tilde{H}_{2z} = \tilde{H}_{2z}^T, \tag{13}$$

where the superscript T denotes the transmitted part. The $\tilde{E}_{1z}^P$ and $\tilde{H}_{1z}^P$ are the primary fields given in the previous section. Using the Fresnel reflection and transmission coefficients between

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the first and the second layer,

$$R_{12}^{TM} = \frac{\varepsilon_2 k_{1z} - \varepsilon_1 k_{2z}}{\varepsilon_2 k_{1z} + \varepsilon_1 k_{2z}}, \ R_{12}^{TE} = \frac{\mu_2 k_{1z} - \mu_1 k_{2z}}{\mu_2 k_{1z} + \mu_1 k_{2z}}, T_{12}^{TM} = \frac{2\varepsilon_2 k_{2z}}{\varepsilon_2 k_{1z} + \varepsilon_1 k_{2z}}, \ T_{12}^{TE} = \frac{2\mu_2 k_{2z}}{\mu_2 k_{1z} + \mu_1 k_{2z}}, \quad (14)$$

the reflected and transmitted parts can be found as

$$\tilde{E}_{1z}^R = \left(\hat{z} \cdot \hat{\boldsymbol{\alpha}}'' + \frac{1}{k_1^2}\partial_z \nabla \cdot \hat{\boldsymbol{\alpha}}''\right)\tilde{g}_{1,TM}^R, \quad \tilde{E}_{2z}^T = \left(\hat{z} \cdot \hat{\boldsymbol{\alpha}}''' + \frac{1}{k_2^2}\partial_z \nabla \cdot \hat{\boldsymbol{\alpha}}'''\right)\tilde{g}_{2,TM}^T, \quad (15)$$

$$\tilde{H}_{1z}^R = -\frac{1}{i\omega\mu_0\mu_1}\hat{z} \cdot \nabla_s \times \hat{\boldsymbol{\alpha}}''\tilde{g}_{1,TE}^R, \quad \tilde{H}_{2z}^T = \frac{1}{i\omega\mu_0\mu_2}\frac{\mu_1}{\mu_2}\frac{k_{2z}}{k_{1z}}\hat{z} \cdot \nabla_s \times \hat{\boldsymbol{\alpha}}'''\tilde{g}_{2,TE}^T, \quad (16)$$

where

$$\tilde{g}_{1,TM,TE}^R = -R_{12}^{TM,TE}\frac{\omega\mu_0\mu_1}{8\pi^2}\frac{e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')}}{k_{1z}}, \quad (17)$$

$$\tilde{g}_{2,TM,TE}^T = -T_{12}^{TM,TE}\frac{\omega\mu_0\mu_2}{8\pi^2}\frac{e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}}{k_{1z}}, \quad (18)$$

$$\hat{\boldsymbol{\alpha}}'' = (-\alpha'_x, -\alpha'_y, \alpha_z), \hat{\boldsymbol{\alpha}}''' = (\frac{k_{1z}}{k_{2z}}\alpha'_x, \frac{k_{1z}}{k_{2z}}\alpha'_y, \alpha_z). \quad (19)$$

The $\hat{\boldsymbol{\alpha}}''$ for the reflected part and $\hat{\boldsymbol{\alpha}}'''$ for the transmitted part ensure the boundary conditions between layers. The transverse component $\tilde{\mathbf{E}}_{1s}^R = (\tilde{E}_{1x}^R, \tilde{E}_{1y}^R)$ can be expressed using the z-components in Eqs. (15) and (16) using Eq. (7) as before by simply replacing the subindex with either 1 or 2 and superscript with either R or T as

$$\tilde{\mathbf{E}}_{1s}^R = \frac{1}{k_s^2}\nabla_s\partial_z\left(\hat{z} \cdot \hat{\boldsymbol{\alpha}}'' + \frac{1}{k_1^2}\partial_z\nabla \cdot \hat{\boldsymbol{\alpha}}''\right)\tilde{g}_{1,TM}^R + \frac{1}{k_s^2}\hat{z} \times \nabla_s\left(\hat{z} \cdot \nabla_s \times \hat{\boldsymbol{\alpha}}''\right)\tilde{g}_{1,TE}^R. \quad (20)$$

Then, x- and y-components can be explicitly written out as

$$\tilde{E}_{1x}^R = \frac{1}{k_1^2}\left(\frac{k_{1z}^2}{k_s^2}\partial_x^2\tilde{g}_{1,TM}^R - \frac{k_1^2}{k_s^2}\partial_y^2\tilde{g}_{1,TE}^R\right)\alpha'_x + \frac{1}{k_1^2}\left(\frac{k_{1z}^2}{k_s^2}\partial_x\partial_y\tilde{g}_{1,TM}^R + \frac{k_1^2}{k_s^2}\partial_x\partial_y\tilde{g}_{1,TE}^R\right)\alpha'_y + \frac{1}{k_1^2}\partial_x\partial_z\tilde{g}_{1,TM}^R\alpha'_z$$

$$(21)$$

$$\tilde{E}_{1y}^R = \frac{1}{k_1^2}\left(\frac{k_{1z}^2}{k_s^2}\partial_x\partial_y\tilde{g}_{1,TM}^R + \frac{k_1^2}{k_s^2}\partial_x\partial_y\tilde{g}_{1,TE}^R\right)\alpha'_x + \frac{1}{k_1^2}\left(\frac{k_{1z}^2}{k_s^2}\partial_y^2\tilde{g}_{1,TM}^R - \frac{k_1^2}{k_s^2}\partial_x^2\tilde{g}_{1,TE}^R\right)\alpha'_y + \frac{1}{k_1^2}\partial_y\partial_z\tilde{g}_{1,TM}^R\alpha'_z$$

$$(22)$$

By listing all the components of the electric field in the spectral domain, the Green's function in the first layer can be found as

$$\begin{bmatrix} \tilde{E}_{1x} \\ \tilde{E}_{1y} \\ \tilde{E}_{1z} \end{bmatrix} = \begin{bmatrix} \tilde{E}_{1x}^P + \tilde{E}_{1x}^R \\ \tilde{E}_{1y}^P + \tilde{E}_{1y}^R \\ \tilde{E}_{1z}^P + \tilde{E}_{1z}^R \end{bmatrix} = \left(\tilde{\mathbf{G}}^P - \frac{1}{8\pi^2\omega\varepsilon_0\varepsilon_1}\tilde{\mathbf{G}}_1^R\right)\boldsymbol{\alpha}'$$

$$= \left(\begin{bmatrix} \tilde{G}_{xx}^P & \tilde{G}_{xy}^P & \tilde{G}_{xz}^P \\ \tilde{G}_{yx}^P & \tilde{G}_{yy}^P & \tilde{G}_{yz}^P \\ \tilde{G}_{zx}^P & \tilde{G}_{zy}^P & \tilde{G}_{zz}^P \end{bmatrix} - \frac{1}{8\pi^2\omega\varepsilon_0\varepsilon_1}\begin{bmatrix} \tilde{G}_{1xx}^R & \tilde{G}_{1xy}^R & \tilde{G}_{1xz}^R \\ \tilde{G}_{1yx}^R & \tilde{G}_{1yy}^R & \tilde{G}_{1yz}^R \\ \tilde{G}_{1zx}^R & \tilde{G}_{1zy}^R & \tilde{G}_{1zz}^R \end{bmatrix}\right)\begin{bmatrix} \alpha'_x \\ \alpha'_y \\ \alpha'_z \end{bmatrix}, \quad (23)$$

where $\tilde{\mathbf{G}}^P$ is the same as Eq. (10) and all the components of $\tilde{\mathbf{G}}_1^R$ are given by

$$\tilde{G}_{1xx}^R = \left(\partial_x^2 \frac{k_{1z}}{k_s^2} R_{12}^{TM} - \partial_y^2 \frac{k_1^2}{k_s^2 k_{1z}} R_{12}^{TE} \right) e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')}, \tag{24}$$

$$\tilde{G}_{1yy}^R = \left(\partial_y^2 \frac{k_{1z}}{k_s^2} R_{12}^{TM} - \partial_x^2 \frac{k_1^2}{k_s^2 k_{1z}} R_{12}^{TE} \right) e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')}, \tag{25}$$

$$\tilde{G}_{1zz}^R = \left(\frac{k_s^2}{k_{1z}} R_{12}^{TM} \right) e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')}, \tag{26}$$

$$\tilde{G}_{1xy}^R = \tilde{G}_{1yx}^R = \partial_x \partial_y \left(\frac{k_{1z}}{k_s^2} R_{12}^{TM} + \frac{k_1^2}{k_s^2 k_{1z}} R_{12}^{TE} \right) e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')}, \tag{27}$$

$$\tilde{G}_{1xz}^R = -\tilde{G}_{1zx}^R = \partial_x \partial_z \left(\frac{R_{12}^{TM}}{k_{1z}} \right) e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')}, \tag{28}$$

$$\tilde{G}_{1yz}^R = -\tilde{G}_{1zy}^R = \partial_y \partial_z \left(\frac{R_{12}^{TM}}{k_{1z}} \right) e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')}. \tag{29}$$

In the second layer, the transverse component $\tilde{\mathbf{E}}_{2s}^T = (\tilde{E}_{2x}^T, \tilde{E}_{2y}^T)$ is

$$\tilde{\mathbf{E}}_{2s}^T = \frac{1}{k_s^2} \nabla_s \partial_z \left(\hat{z} \cdot \hat{\alpha}''' + \frac{1}{k_2^2} \partial_z \nabla \cdot \hat{\alpha}''' \right) \tilde{g}_{2,TM}^T - \frac{1}{k_s^2} \frac{\mu_1}{\mu_2} \frac{k_{2z}}{k_{1z}} \hat{z} \times \nabla_s \left(\hat{z} \cdot \nabla_s \times \hat{\alpha}''' \right) \tilde{g}_{2,TE}^T. \tag{30}$$

Then, x- and y-components can be explicitly written out as

$$\tilde{E}_{2x}^T = \frac{1}{k_2^2} \left(-\partial_x^2 \frac{k_{1z}k_{2z}}{k_s^2} \tilde{g}_{2,TM}^T - \partial_y^2 \frac{k_2^2}{k_s^2} \frac{\mu_1}{\mu_2} \tilde{g}_{2,TE}^T \right) \alpha_x' + \frac{1}{k_2^2} \left(-\partial_x \partial_y \frac{k_{1z}k_{2z}}{k_s^2} \tilde{g}_{2,TM}^T + \partial_x \partial_y \frac{k_2^2}{k_s^2} \frac{\mu_1}{\mu_2} \tilde{g}_{2,TE}^T \right) \alpha_y'$$
$$+ \frac{1}{k_2^2} \partial_x \partial_z \tilde{g}_{2,TM}^T \alpha_z', \tag{31}$$

$$\tilde{E}_{2y}^T = \frac{1}{k_2^2} \left(-\partial_x \partial_y \frac{k_{1z}k_{2z}}{k_s^2} \tilde{g}_{2,TM}^T + \partial_x \partial_y \frac{k_2^2}{k_s^2} \frac{\mu_1}{\mu_2} \tilde{g}_{2,TE}^T \right) \alpha_x' + \frac{1}{k_2^2} \left(-\partial_y^2 \frac{k_{1z}k_{2z}}{k_s^2} \tilde{g}_{2,TM}^T - \partial_x^2 \frac{k_2^2}{k_s^2} \frac{\mu_1}{\mu_2} \tilde{g}_{2,TE}^T \right) \alpha_y'$$
$$+ \frac{1}{k_2^2} \partial_y \partial_z \tilde{g}_{2,TM}^T \alpha_z'. \tag{32}$$

Thus, the spectral Green's function in the second layer is

$$\begin{bmatrix} \tilde{E}_{2x} \\ \tilde{E}_{2y} \\ \tilde{E}_{2z} \end{bmatrix} = \begin{bmatrix} \tilde{E}_{2x}^T \\ \tilde{E}_{2y}^T \\ \tilde{E}_{2z}^T \end{bmatrix} = -\frac{1}{8\pi^2 \omega \varepsilon_0 \varepsilon_2} \tilde{\mathbf{G}}_2^T \hat{\alpha}' = -\frac{1}{8\pi^2 \omega \varepsilon_0 \varepsilon_2} \begin{bmatrix} \tilde{G}_{2xx}^T & \tilde{G}_{2xy}^T & \tilde{G}_{2xz}^T \\ \tilde{G}_{2yx}^T & \tilde{G}_{2yy}^T & \tilde{G}_{2yz}^T \\ \tilde{G}_{2zx}^T & \tilde{G}_{2zy}^T & \tilde{G}_{2zz}^T \end{bmatrix} \begin{bmatrix} \alpha_x' \\ \alpha_y' \\ \alpha_z' \end{bmatrix}, \tag{33}$$

where

$$\tilde{G}_{2xx}^T = \left(-\partial_x^2 \frac{k_{2z}}{k_s^2} T_{12}^{TM} - \partial_y^2 \frac{k_2^2}{k_{1z}k_s^2} \frac{\mu_1}{\mu_2} T_{12}^{TE}\right) e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}, \tag{34}$$

$$\tilde{G}_{2yy}^T = \left(-\partial_y^2 \frac{k_{2z}}{k_s^2} T_{12}^{TM} - \partial_x^2 \frac{k_2^2}{k_{1z}k_s^2} \frac{\mu_1}{\mu_2} T_{12}^{TE}\right) e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}, \tag{35}$$

$$\tilde{G}_{2zz}^T = \left(\frac{k_s^2}{k_{1z}} T_{12}^{TM}\right) e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}, \tag{36}$$

$$\tilde{G}_{2xy}^T = \tilde{G}_{2,yx}^T = \left(-\partial_x\partial_y \frac{k_{2z}}{k_s^2} T_{12}^{TM} + \partial_x\partial_y \frac{k_2^2}{k_{1z}k_s^2} \frac{\mu_1}{\mu_2} T_{12}^{TE}\right) e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}, \tag{37}$$

$$\tilde{G}_{2xz}^T = \partial_x\partial_z \frac{T_{12}^{TM}}{k_{1z}} e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}, \quad \tilde{G}_{2yz}^T = \partial_y\partial_z \frac{T_{12}^{TM}}{k_{1z}} e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}, \tag{38}$$

$$\tilde{G}_{2zx}^T = \partial_z\partial_x \frac{T_{12}^{TM}}{k_{2z}} e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}, \quad \tilde{G}_{2zy}^T = \partial_z\partial_y \frac{T_{12}^{TM}}{k_{2z}} e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}. \tag{39}$$

3.2. Fields in the spatial domain

In this subsection, Sommerfeld integrals/Hankel transforms are used to convert the spectral Green's function found in the previous subsection to the spatial domain Green's function. Several useful Bessel identities are listed in Appendix A and used in the derivation.

• In the first layer, the first component G_{1xx}^R can be found by taking double integral as follows

$$\begin{aligned}
G_{1xx}^R &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \tilde{G}_{1xx}^R dk_x dk_y \\
&= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left(\partial_x^2 \frac{k_{1z}}{k_s^2} R_{12}^{TM} - \partial_y^2 \frac{k_1^2}{k_s^2 k_{1z}} R_{12}^{TE}\right) e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')} dk_x dk_y \\
&= \textcircled{1} - \textcircled{2},
\end{aligned} \tag{40}$$

where

$$\textcircled{1} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \partial_x^2 \frac{k_{1z}}{k_s^2} R_{12}^{TM} e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')} dk_x dk_y, \tag{41}$$

$$\textcircled{2} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \partial_y^2 \frac{k_1^2}{k_s^2 k_{1z}} R_{12}^{TE} e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')} dk_x dk_y. \tag{42}$$

For all double integrals throughout the paper, the cylindrical coordinate transform

$$k_x = k_s \cos\phi, k_y = k_s \sin\phi, (x - x') = \rho \cos\theta, (y - y') = \rho \sin\theta, \tag{43}$$

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is used to reduce double integrals into single integrals. Now the integral ① is simplified as

$$
\begin{aligned}
① &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \partial_x^2 \frac{k_{1z}}{k_s^2} R_{12}^{TM} e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')} dk_x dk_y, \\
&= -\int_0^{\infty} k_s k_{1z} R_{12}^{TM} e^{ik_{1z}(z+z')} \int_0^{2\pi} e^{ik_s\rho\cos(\phi-\theta)} \cos^2\phi \, d\phi dk_s \\
&= -\int_0^{\infty} k_s k_{1z} R_{12}^{TM} e^{ik_{1z}(z+z')} \left(\pi J_0(k_s\rho) - \pi J_2(k_s\rho)\cos 2\theta\right) dk_s \\
&= -\frac{1}{2} 2\pi \int_0^{\infty} k_s \tilde{g}_{1,1}^R J_0(k_s\rho) e^{ik_{1z}(z+z')} dk_s + \pi(1-2\sin^2\theta)\int_0^{\infty} k_s^3 \frac{k_{1z}}{k_s^2} R_{12}^{TM} J_2(k_s\rho) e^{ik_{1z}(z+z')} dk_s \\
&= -\frac{1}{2} g_{1,1}^R + \pi\left(1-2\frac{(y-y')^2}{\rho^2}\right)\int_0^{\infty} k_s^3 \tilde{g}_{1,2}^R J_2(k_s\rho) e^{ik_{1z}(z+z')} dk_s \\
&= -\frac{1}{2} g_{1,1}^R + \left(\frac{1}{2}\rho^2 - (y-y')^2\right) 2\pi \int_0^{\infty} k_s^3 \tilde{g}_{1,2}^R \frac{J_2(k_s\rho)}{\rho^2} e^{ik_{1z}(z+z')} dk_s \\
&= -\frac{1}{2} g_{1,1}^R + \left(\frac{1}{2}\rho^2 - (y-y')^2\right) g_{1,2}^R, \tag{44}
\end{aligned}
$$

where

$$
g_{1,1}^R = 2\pi\int_0^{\infty} k_s \tilde{g}_{1,1}^R J_0(k_s\rho) e^{ik_{1z}(z+z')} dk_s, \quad g_{1,2}^R = 2\pi\int_0^{\infty} k_s^3 \tilde{g}_{1,2}^R \frac{J_2(k_s\rho)}{\rho^2} e^{ik_{1z}(z+z')} dk_s,
$$

$$
\tilde{g}_{1,1}^R = k_{1z} R_{12}^{TM}, \quad \tilde{g}_{1,2}^R = \frac{k_{1z} R_{12}^{TM}}{k_s^2}. \tag{45}
$$

Similar derivation yields the integral ② as follows

$$
\begin{aligned}
② &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \partial_y^2 \frac{k_1^2}{k_s^2 k_{1z}} R_{12}^{TE} e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')} dk_x dk_y \\
&= -\frac{1}{2} g_{1,3}^R - \left(\frac{1}{2}\rho^2 - (y-y')^2\right) g_{1,4}^R. \tag{46}
\end{aligned}
$$

where

$$
g_{1,3}^R = 2\pi\int_0^{\infty} k_s \tilde{g}_{1,3}^R J_0(k_s\rho) e^{ik_{1z}(z+z')} dk_s, \quad g_{1,4}^R = 2\pi\int_0^{\infty} k_s^3 \tilde{g}_{1,3}^R \frac{J_2(k_s\rho)}{\rho^2} e^{ik_{1z}(z+z')} dk_s,
$$

$$
\tilde{g}_{1,3}^R = \frac{k_1^2}{k_{1z}} R_{12}^{TE}, \quad \tilde{g}_{1,4}^R = \frac{k_1^2}{k_s^2 k_{1z}} R_{12}^{TE}. \tag{47}
$$

Therefore

$$
\begin{aligned}
G_{xx}^R &= -\frac{1}{2} g_{1,1}^R + \left(\frac{1}{2}\rho^2 - (y-y')^2\right) g_{1,2}^R + \frac{1}{2} g_{1,3}^R + \left(\frac{1}{2}\rho^2 - (y-y')^2\right) g_{1,4}^R \\
&= -\frac{1}{2}\left(g_{1,1}^R - g_{1,3}^R\right) + \left(\frac{1}{2}\rho^2 - (y-y')^2\right)\left(g_{1,2}^R + g_{1,4}^R\right) = -\frac{1}{2} g_{1,5}^R + \left(\frac{1}{2}\rho^2 - (y-y')^2\right) g_{1,6}^R, \tag{48}
\end{aligned}
$$

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where

$$g_{1,5}^R = 2\pi \int_0^\infty k_s \tilde{g}_{1,5}^R J_0(k_s\rho)e^{ik_{1z}(z+z')}dk_s, \, g_{1,6}^R = 2\pi \int_0^\infty k_s^3 \tilde{g}_{1,6}^R \frac{J_2(k_s\rho)}{\rho^2}e^{ik_{1z}(z+z')}dk_s,$$

$$\tilde{g}_{1,5}^R = k_{1z}R_{12}^{TM} - \frac{k_1^2}{k_{1z}}R_{12}^{TE}, \, \tilde{g}_{1,6}^R = \frac{k_{1z}R_{12}^{TM}}{k_s^2} + \frac{k_1^2}{k_s^2 k_{1z}}R_{12}^{TE}. \tag{49}$$

Thus, the G_{xx}^R can be computed with only two Sommerfeld integrals.

Absolutely, the same derivation applies to G_{1yy}^R, that is,

$$
\begin{aligned}
G_{1yy}^R &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \tilde{G}_{1yy}^R dk_x dk_y \\
&= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\partial_y^2 \frac{k_{1z}}{k_s^2}R_{12}^{TM} - \partial_x^2 \frac{k_1^2}{k_s^2 k_{1z}}R_{12}^{TE}\right)e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')}dk_x dk_y \\
&= -\frac{1}{2}g_{1,5}^R - \left(\frac{1}{2}\rho^2 - (y-y')^2\right)g_{1,6}^R.
\end{aligned} \tag{50}
$$

The derivation of G_{1zz}^R is straightforward as there is no derivative in $\tilde{G}_{1zz}^R$.

$$
\begin{aligned}
G_{1zz}^R &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\frac{k_s^2}{k_{1z}}R_{12}^{TM}\right)e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')}dk_x dk_y \\
&= \int_0^\infty k_s \tilde{g}_{1,7}^R e^{ik_{1z}(z+z')}\int_0^{2\pi} e^{ik_s\rho\cos(\phi-\theta)}d\phi dk_s \\
&= 2\pi \int_0^\infty k_s \tilde{g}_{1,7}^R J_0(k_s\rho)e^{ik_{1z}(z+z')}dk_s = g_{1,7}^R,
\end{aligned} \tag{51}
$$

where

$$g_{1,7}^R = 2\pi \int_0^\infty k_s \tilde{g}_{1,7}^R J_0(k_s\rho)e^{ik_{1z}(z+z')}dk_s, \, \tilde{g}_{1,7}^R = \frac{k_s^2}{k_{1z}}R_{12}^{TM}. \tag{52}$$

The $G_{1xy}^R = G_{1yx}^R$ can be derived using $\tilde{g}_{1,6}^R$ that is already defined in Eq. (49) as

$$
\begin{aligned}
G_{1xy}^R = G_{1yx}^R &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\partial_x\partial_y\left(\frac{k_{1z}}{k_s^2}R_{12}^{TM} + \frac{k_1^2}{k_s^2 k_{1z}}R_{12}^{TE}\right)e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')}dk_x dk_y \\
&= \int_0^\infty \int_0^{2\pi} -k_s^2 \cos\phi \sin\phi \tilde{g}_{1,6}^R e^{ik_s\rho\cos(\phi-\theta)}e^{ik_{1z}(z+z')}k_s dk_s d\phi \\
&= -\frac{1}{2}\int_0^\infty k_s^3 \tilde{g}_{1,6}^R e^{ik_{1z}(z+z')}\int_0^{2\pi} e^{ik_s\rho\cos(\phi-\theta)}\sin 2\phi d\phi dk_s \\
&= -\frac{1}{2}\int_0^\infty k_s^3 \tilde{g}_{1,6}^R e^{ik_{1z}(z+z')}\left(-2\pi J_2(k_s\rho)\sin 2\theta\right)dk_s \\
&= \sin\theta\cos\theta 2\pi \int_0^\infty k_s^3 \tilde{g}_{1,6}^R J_2(k_s\rho)e^{ik_{1z}(z+z')}dk_s \\
&= (x-x')(y-y')2\pi \int_0^\infty k_s^3 \tilde{g}_{1,6}^R \frac{J_2(k_s\rho)}{\rho^2}e^{ik_{1z}(z+z')}dk_s = (x-x')(y-y')g_{1,6}^R. \tag{53}
\end{aligned}
$$

The G_{1xz}^R and G_{1zx}^R can be derived at the same time as

$$G_{1xz}^R = -G_{1zx}^R = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_x \partial_z \frac{R_{12}^{TM}}{k_{1z}} e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')} dk_x dk_y$$

$$= -\int_0^\infty k_s^2 R_{12}^{TM} e^{ik_{1z}(z+z')} \int_0^{2\pi} e^{ik_s\rho\cos(\phi-\theta)} \cos\phi \, d\phi dk_s$$

$$= -2\pi i \cos\theta \int_0^\infty k_s^2 R_{12}^{TM} J_1(k_s\rho) e^{ik_{1z}(z+z')} dk_s$$

$$= -2\pi i(x - x') \int_0^\infty k_s^2 \tilde{g}_{1,8}^R \frac{J_1(k_s\rho)}{\rho} e^{ik_{1z}(z+z')} dk_s = -i(x - x')g_{1,8}^R, \tag{54}$$

where

$$g_{1,8}^R = 2\pi \int_0^\infty k_s^2 \tilde{g}_{1,8}^R \frac{J_1(k_s\rho)}{\rho} e^{ik_{1z}(z+z')} dk_s, \tilde{g}_{1,8}^R = R_{12}^{TM}. \tag{55}$$

Finally, the G_{1yz}^R and G_{1zy}^R can be derived using symmetry as

$$G_{1yz}^R = -G_{1zy}^R = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_y \partial_z \frac{R_{12}^{TM}}{k_{1z}} e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')} dk_x dk_y = -i(y - y')g_{1,8}^R. \tag{56}$$

• In the second layer, the derivation of transmitted part $\mathbf{G}_2^T$ is absolutely similar to the reflected part. Therefore, most of derivations are omitted unless there are notable differences.

$$G_{2xx}^T = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left(-\partial_x^2 \frac{k_{2z}}{k_s^2} T_{12}^{TM} - \partial_y^2 \frac{k_2^2}{k_{1z}k_s^2} \frac{\mu_1}{\mu_2} T_{12}^{TE} \right) e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'} dk_x dk_y \tag{57}$$

$$= \frac{1}{2} g_{2,5}^T - (\frac{1}{2}\rho^2 - (y - y')^2)g_{2,6}^T, \tag{58}$$

where

$$g_{2,5}^T = 2\pi \int_0^\infty k_s \tilde{g}_{2,5}^T J_0(k_s\rho) e^{-ik_{2z}z+ik_{1z}z'} dk_s, g_{2,6}^T = 2\pi \int_0^\infty k_s^3 \tilde{g}_{2,6}^T \frac{J_2(k_s\rho)}{\rho^2} e^{-ik_{2z}z+ik_{1z}z'} dk_s, \tag{59}$$

$$\tilde{g}_{2,5}^T = k_{2z}T_{12}^{TM} + \frac{k_2^2}{k_{1z}} \frac{\mu_1}{\mu_2} T_{12}^{TE}, \tilde{g}_{2,6}^T = \frac{k_{2z}T_{12}^{TM}}{k_s^2} - \frac{k_2^2}{k_{1z}k_s^2} \frac{\mu_1}{\mu_2} T_{12}^{TE}. \tag{60}$$

Similarly,

$$G_{2yy}^T = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left(-\partial_y^2 \frac{k_{2z}}{k_s^2} T_{12}^{TM} - \partial_x^2 \frac{k_2^2}{k_{1z}k_s^2} \frac{\mu_1}{\mu_2} T_{12}^{TE} \right) e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}$$

$$= \frac{1}{2} g_{2,5}^T + (\frac{1}{2}\rho^2 - (y - y')^2)g_{2,6}^T. \tag{61}$$

The G_{2zz}^T can be obtained by

$$G_{2zz}^T = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left(\frac{k_s^2}{k_{1z}} T_{12}^{TM} \right) e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'} dk_x dk_y$$

$$= 2\pi \int_0^\infty k_s \tilde{g}_{2,7}^T J_0(k_s\rho) e^{-ik_{2z}z+ik_{1z}z'} dk_s = g_{2,7}^T, \tag{62}$$

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where

$$g_{2,7}^T = 2\pi \int_0^\infty k_s \tilde{g}_{2,7}^T J_0(k_s\rho) e^{-ik_{2z}+ik_{1z}z'} dk_s, \tilde{g}_{2,7}^T = \frac{k_s^2}{k_{1z}} T_{12}^{TM}. \tag{63}$$

Again, the $G_{2xy}^T = G_{2yx}^T$ can be written using $\tilde{g}_{2,6}^T$ as

$$G_{2xy}^T = G_{2yx}^T = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} -\partial_x\partial_y \left(\frac{k_{2z}}{k_s^2} T_{12}^{TM} - \frac{k_2^2}{k_{1z}k_s^2} \frac{\mu_1}{\mu_2} T_{12}^{TE} \right) e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'} dk_x dk_y$$

$$= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} k_x k_y \tilde{g}_{2,6}^T e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'} dk_x dk_y = -(x-x')(y-y')g_{2,6}^T. \tag{64}$$

The G_{2xz}^T and G_{2yz}^T can be found at the same time using their symmetry

$$G_{2xz}^T = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_x\partial_z \frac{T_{12}^{TM}}{k_{1z}} e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'} dk_x dk_y$$

$$= \int_0^\infty k_s^2 \frac{k_{2z}T_{12}^{TM}}{k_{1z}} e^{-ik_{2z}z+ik_{1z}z'} \left(\int_0^{2\pi} e^{ik_s\rho\cos(\phi-\theta)} \cos\phi d\phi \right) dk_s$$

$$= 2\pi i \cos\theta \int_0^\infty k_s^2 \tilde{g}_{2,8}^T J_1(k_s\rho) e^{-ik_{2z}z+ik_{1z}z'} dk_s$$

$$= i(x-x')g_{2,8}^T, \tag{65}$$

$$G_{2yz}^T = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_y\partial_z \frac{T_{12}^{TM}}{k_{1z}} e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'} dk_x dk_y$$

$$= i(y-y')g_{2,8}^T, \tag{66}$$

where

$$g_{2,8}^T = 2\pi \int_0^\infty k_s^2 \tilde{g}_{2,8}^T \frac{J_1(k_s\rho)}{\rho} e^{-ik_{2z}z+ik_{1z}z'} dk_s, \tilde{g}_{2,8}^T = \frac{k_{2z}T_{12}^{TM}}{k_{1z}}. \tag{67}$$

The G_{2zx}^T is derived as

$$G_{2zx}^T = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z\partial_x \frac{T_{12}^{TM}}{k_{2z}} e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'} dk_x dk_y$$

$$= \int_0^\infty k_s^2 \tilde{g}_{2,9}^T e^{-ik_{2z}z+ik_{1z}z'} \left(\int_0^{2\pi} e^{ik_s\rho\cos(\phi-\theta)} \cos\phi d\phi \right) dk_s$$

$$= 2\pi i \cos\theta \int_0^\infty k_s^2 \tilde{g}_{2,9}^T J_1(k_s\rho) e^{-ik_{2z}z+ik_{1z}z'} dk_s$$

$$= i(x-x')g_{2,9}^T, \tag{68}$$

where

$$g_{2,9}^T = 2\pi \int_0^\infty k_s^2 \tilde{g}_{2,9}^T \frac{J_1(k_s\rho)}{\rho} e^{-ik_{2z}z+ik_{1z}z'} dk_s, \tilde{g}_{2,9}^T = T_{12}^{TM}. \tag{69}$$

Similarly, G_{2zy}^T can by found by replacing x by y in G_{2zx}^T as

$$G_{2zy}^T = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \partial_z\partial_y \frac{T_{12}^{TM}}{k_{2z}} e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'} dk_x dk_y = i(y-y')g_{2,9}^T. \tag{70}$$

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3.3. Summary and numerical results for a two-layer structure

The Green's function for a two-layer structure is summarized here.

•In the first layer

$$G^R_{1xx} = -\frac{1}{2}g^R_{1,5} + \left(\frac{1}{2}\rho^2 - (y-y')^2\right)g^R_{1,6} \tag{71}$$

$$G^R_{1yy} = -\frac{1}{2}g^R_{1,5} - \left(\frac{1}{2}\rho^2 - (y-y')^2\right)g^R_{1,6}, \tag{72}$$

$$G^R_{1zz} = g^R_{1,7}, \tag{73}$$

$$G^R_{1xy} = G^R_{1yx} = (x-x')(y-y')g^R_{1,6}, \tag{74}$$

$$G^R_{1xz} = -G^R_{1zx} = -i(x-x')g^R_{1,8}, \tag{75}$$

$$G^R_{1yz} = -G^R_{1zy} = -i(y-y')g^R_{1,8}. \tag{76}$$

•In the second layer

$$G^T_{2xx} = \frac{1}{2}g^T_{2,5} - (\frac{1}{2}\rho^2 - (y-y')^2)g^T_{2,6}, \tag{77}$$

$$G^T_{2yy} = \frac{1}{2}g^T_{2,5} + (\frac{1}{2}\rho^2 - (y-y')^2)g^T_{2,6}, \tag{78}$$

$$G^T_{2zz} = g^T_{2,7}, \tag{79}$$

$$G^T_{2xy} = -(x-x')(y-y')g^T_{2,6}, \tag{80}$$

$$G^T_{2xz} = i(x-x')g^T_{2,8}, \quad G^T_{2yz} = i(y-y')g^T_{2,8}, \tag{81}$$

$$G^T_{2zx} = i(x-x')g^T_{2,9}, \quad G^T_{2zy} = i(y-y')g^T_{2,9}. \tag{82}$$

Then, the dyadic Green's function for two layers is

$$\mathbf{G}(\mathbf{r},\mathbf{r}') = \begin{cases} \mathbf{G}^P - \frac{1}{8\pi^2\omega\varepsilon_0\varepsilon_1}\mathbf{G}^R_1, & z \geq 0 \\ -\frac{1}{8\pi^2\omega\varepsilon_0\varepsilon_2}\mathbf{G}^T_2, & z < 0 \end{cases},$$

where $\mathbf{G}^P$ is the free-space Green's function and each component of $\mathbf{G}^R_1$ and $\mathbf{G}^T_2$ is given by Eqs. (71) ~ (82).

Four Sommerfeld integrals ($g^R_{1,5} \sim g^R_{1,8}$) and five Sommerfeld integrals ($g^T_{1,5} \sim g^T_{1,9}$) are required to compute reflected fields and transmitted parts, respectively. *One less Sommerfeld integral is required than the formula presented in Ref. [5].* Moreover, reflection coefficient for two layers is not assumed to reduce number of Sommerfeld integrals. As a consequence, it can be extended to multi-layered media without increasing the number of Sommerfeld integrals. Numerical integration of Sommerfeld integrals are performed with the adaptive quadrature method developed for the Helmholtz equation in Ref. [12, 13].

As a numerical test, a dipole source is placed at $\mathbf{r}' = (0.1, -0.2, 1.5)$ and oriented along $\alpha' = (1/2, 1/2, 1/\sqrt{2})$. Then, electric field is computed for $-5 \leq x \leq 5$ and $-3 \leq z \leq 3$ for a fixed $y = 1.2$ with $\varepsilon_1 = 1$, $\varepsilon_2 = 4$, and $\lambda = 1$ in Fig. 2. The continuity of the fields are checked by computing the electric field at the interface in Fig. 3. First, the electric field is computed at the layer interface $z = 0$ with the formula for the top layer, $\mathbf{E}(x, y, 0^+)$, and the formula for the bottom layer, $\mathbf{E}(x, y, 0^-)$. The tangential components E_x and E_y must be
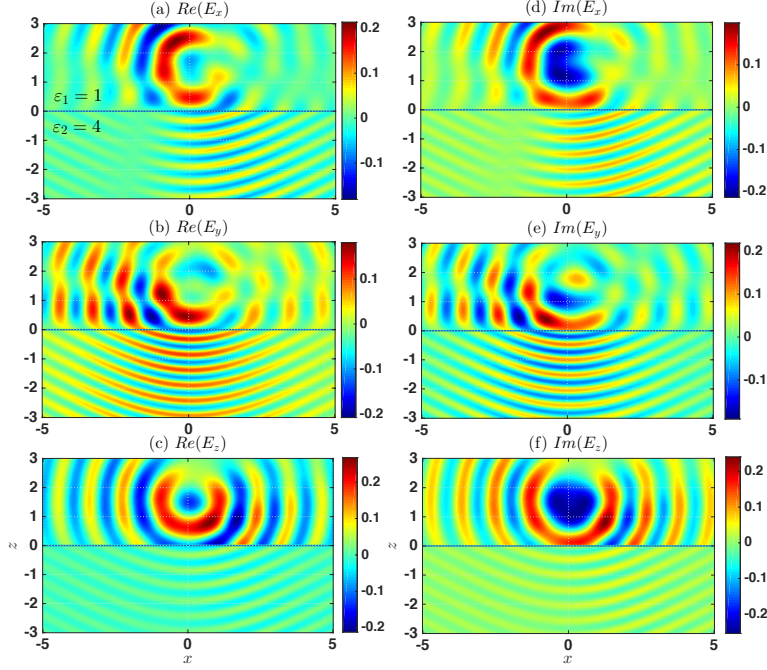
Figure 2: Electric fields in a two-layer structure. A dipole source is placed at $\mathbf{r}' = (0.1, -0.2, 1.5)$ and oriented along $\alpha' = (1/2, 1/2, 1/\sqrt{2})$ and Fields are computed for $-5 \le x \le 5$ and $-3 \le z \le 3$ for a fixed $y = 1.2$ with $\varepsilon_1 = 1$, $\varepsilon_2 = 4$, and $\lambda = 1$.

continuous and the normal component E_z must have a jump of $\varepsilon_2/\varepsilon_1$ (in this example, the jump should be 4). Figs. 3(a), 3(b), and 3(c) plot $|E_x(x, y, 0^+) - E_x(x, y, 0^-)|$, $|E_y(x, y, 0^+) - E_y(x, y, 0^-)|$, and $|E_z(x, y, 0^+) - 4E_z(x, y, 0^-)|$, respectively. About 10^{-10} agreement was achieved. Throughout the paper, agreement of numerical solutions at the interface will be used as accuracy of the method. In the next section, the Green's function for a three-layer structure is presented.

4. Green's function for a three-layer structure

In this section, Green's function for a two-layer structure is extended to a three-layer structure. In principle, multi-layered structure will be a straightforward consequence of three-layer structure. We will begin the case when a dipole source is placed in the first layer. The multiple reflection from the second layer is accommodated with generalized Fresnel coefficients and the continuity of the fields are ensured with by modifying α'. Note that if the source is in the second layer, all the formulas should be reorganized to accommodate reflection from both the first and third layer into the second layer. If the source is in the third layer, symmetry can be used. In the next subsection, the Green's function when the source is placed on top of a three-layer structure is derived and it is modified to consider the case when the source is in the second layer in the following subsection.
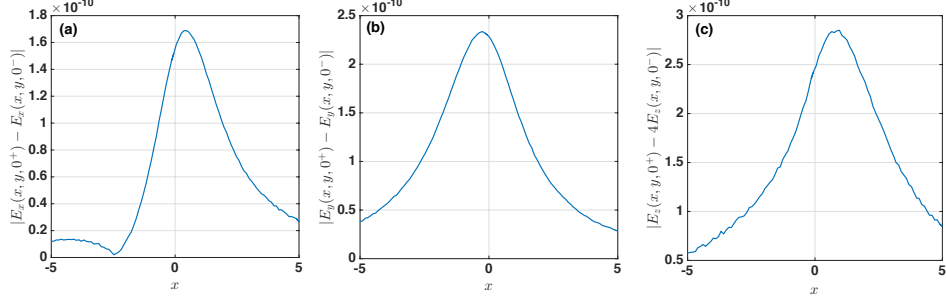
Figure 3: Continuity of electric fields at interface. (a) $|E_x(x, y, 0^+) - E_x(x, y, 0^-)|$, (b) $|E_y(x, y, 0^+) - E_y(x, y, 0^-)|$, and (c) $|E_z(x, y, 0^+) - 4E_z(x, y, 0^-)|$. A dipole source is placed at $\mathbf{r}' = (0.1, -0.2, 1.5)$ and oriented along $\alpha' = (1/2, 1/2, 1/\sqrt{2})$ and fields are computed for $-5 \le x \le 5$ for a fixed $y = 1.2$ at the layer interface $z = 0$ and with $\varepsilon_1 = 1$, $\varepsilon_2 = 4$, and $\lambda = 1$.

4.1. Source on top of a three-layer structure

Consider a case depicted in Fig. 4. A three-layer structure is defined by two interfaces located at $z = 0$ and $z = -d$. Assume the top most layer is the first layer with ε_1 and μ_1, the middle layer is the second layer with ε_2 and μ_2, and the bottom most layer is the third layer with ε_3 and μ_3. Let a dipole source is placed at $\mathbf{r}' = (x', y', z')$ in the first layer oriented along $\hat{\alpha}' = (\alpha'_x, \alpha'_y, \alpha'_z)$.

4.1.1. Green's function in the spectral domain

The z-components of reflected electric and magnetic fields in the spectral domain in each layer are

$$\tilde{E}_{1z} = \tilde{E}_{1z}^P + \tilde{E}_{1z}^R, \ \tilde{E}_{2z} = \tilde{E}_{2z}^R + \tilde{E}_{2z}^T, \ \tilde{E}_{3z} = \tilde{E}_{3z}^T, \tag{83}$$

$$\tilde{H}_{1z} = \tilde{H}_{1z}^P + \tilde{H}_{1z}^R, \ \tilde{H}_{2z} = \tilde{H}_{2z}^R + \tilde{H}_{2z}^T, \ \tilde{H}_{3z} = \tilde{H}_{3z}^T, \tag{84}$$

where the superscript P, R, and T denote the primary, reflected, and transmitted parts, respectively. The primary fields $\tilde{E}_{1z}^P$ and $\tilde{H}_{1z}^P$ are the same as the free-space ones. The reflected part in the first and second layers must be modified with the generalized reflection coefficient given by

$$\bar{R}_{12}^{TM,TE} = \frac{R_{12}^{TM,TE} + R_{23}^{TM,TE} e^{2ik_{2z}d}}{1 + R_{12}^{TM,TE} R_{23}^{TM,TE} e^{2ik_{2z}d}}, \tag{85}$$

to accommodate multiple reflection and transmission from all the layers below the first layer. Consequently, reflected fields in the first and second layer can be expressed as

$$\tilde{E}_{1z}^R = \left(\hat{z} \cdot \hat{\alpha}''_{1R} + \frac{1}{k_1^2} \partial_z \nabla \cdot \hat{\alpha}''_{1R} \right) \tilde{g}_{1,TM}^R, \ \tilde{H}_{1z}^R = -\frac{1}{i\omega\mu_0\mu_1} \hat{z} \cdot \nabla_s \times \hat{\alpha}''_{1R} \tilde{g}_{1,TE}^R, \tag{86}$$

$$\tilde{E}_{2z}^R = \left(\hat{z} \cdot \hat{\alpha}''_{2R} + \frac{1}{k_2^2} \partial_z \nabla \cdot \hat{\alpha}''_{2R} \right) \tilde{g}_{2,TM}^R, \ \tilde{H}_{2z}^R = -\frac{1}{i\omega\mu_0\mu_2} \frac{k_{2z}}{k_{1z}} \hat{z} \cdot \nabla_s \times \hat{\alpha}''_{2R} \tilde{g}_{2,TE}^R, \tag{87}$$
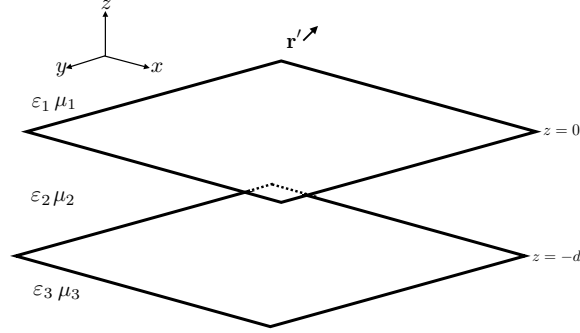
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Figure 4: Three-layer structure

where

$$\tilde{g}_{1,TM,TE}^{R} = -R_1^{TM,TE} \frac{\omega\mu_0\mu_1}{8\pi^2} \frac{e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')}}{k_{1z}}, \tag{88}$$

$$\tilde{g}_{2,TM,TE}^{R} = -R_2^{TM,TE} \frac{\omega\mu_0\mu_2}{8\pi^2} \frac{e^{ik_x(x-x')+ik_y(y-y')+ik_{2z}z+ik_{1z}z'+2ik_{2z}d}}{k_{1z}}, \tag{89}$$

$$\hat{\alpha}_{1R}'' = (-\alpha_x', -\alpha_y', \alpha_z'), \hat{\alpha}_{2R}'' = (-\frac{k_{1z}}{k_{2z}}\alpha_x', -\frac{k_{1z}}{k_{2z}}\alpha_y', \alpha_z'), \tag{90}$$

$$R_1^{TM,TE} = \bar{R}_{12}^{TM,TE}, \ R_2^{TM,TE} = A_2^{TM,TE}R_{23}^{TM,TE}, A_2^{TM,TE} = \frac{T_{12}^{TM,TE}}{1 - R_{21}^{TM,TE}R_{23}^{TM,TE}e^{2ik_{2z}d}}. \tag{91}$$

The transmitted parts in the second and third layer must be modified to

$$\tilde{E}_{jz}^{T} = \left(\hat{z}\cdot\hat{\alpha}_{jT}''' + \frac{1}{k_j^2}\partial_z\nabla\cdot\hat{\alpha}_{jT}'''\right)g_{j,TM}^{T}, \ \tilde{H}_{jz}^{T} = \frac{1}{i\omega\mu_0\mu_j}\frac{\mu_1}{\mu_j}\frac{k_{jz}}{k_{1z}}\hat{z}\cdot\nabla_s\times\hat{\alpha}_{jT}'''g_{j,TE}^{T}, j=2,3, \tag{92}$$

where

$$\tilde{g}_{2,TM,TE}^{T} = -A_2^{TM,TE} \frac{\omega\mu_0\mu_2}{8\pi^2} \frac{e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}}{k_{1z}}, \tag{93}$$

$$\tilde{g}_{3,TM,TE}^{T} = -A_3^{TM,TE} \frac{\omega\mu_0\mu_3}{8\pi^2} \frac{e^{ik_x(x-x')+ik_y(y-y')-ik_{3z}z+ik_{1z}z'}}{k_{1z}}, \tag{94}$$

$$\alpha_{jT}''' = \left(\frac{k_{1z}}{k_{jz}}\alpha_x', \frac{k_{1z}}{k_{jz}}\alpha_y', \alpha_z'\right), j=2,3, \tag{95}$$

$$A_3^{TM,TE} = A_2^{TM,TE}(1 + R_{23}^{TM,TE})e^{ik_{2z}d-ik_{3z}d}. \tag{96}$$

All the reflection and transmission coefficients are changed to enforce multiple reflections in Eqs. (91) and (96). At the same time, in each layer α' is modified to correctly ensure the continuity of the fields at the interfaces. The transverse components can be derived using Eq. (7) using the new z-components listed above in each layer. In the following, all components are presented in each layer.

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• In the first layer, the transverse components of reflected parts is found by

$$\tilde{\mathbf{E}}_{1s}^R = \frac{1}{k_s^2}\left[\nabla_s\partial_z\tilde{E}_{1z}^R - i\omega\mu_0\mu_1\hat{z}\times\nabla_s\tilde{H}_{1z}^R\right]$$

$$= \frac{1}{k_s^2}\nabla_s\partial_z\left(\hat{z}\cdot\hat{\alpha}_{1R}'' + \frac{1}{k_1^2}\partial_z\nabla\cdot\hat{\alpha}_{1R}''\right)\tilde{g}_{1,TM}^R + \frac{1}{k_s^2}\hat{z}\times\nabla_s\left(\hat{z}\cdot\nabla_s\times\hat{\alpha}_{1R}''\tilde{g}_{1,TE}^R\right). \tag{97}$$

Each component is exactly the same as the two-layer structure except the definition of the reflection coefficients. Thus the reflected parts of the Green's function in the first layer can be simply rewritten by replacing R_{12}^{TM} and R_{12}^{TE} by the generalized reflection coefficient $R_1^{TM} = \tilde{R}_{12}^{TM}$ and $R_1^{TE} = \tilde{R}_{12}^{TE}$, respectively. Therefore, the electric field in the spectral domain in the first layer is

$$\begin{bmatrix} \tilde{E}_{1x} \\ \tilde{E}_{1y} \\ \tilde{E}_{1z} \end{bmatrix} = \begin{bmatrix} \tilde{E}_{1x}^P + \tilde{E}_{1x}^R \\ \tilde{E}_{1y}^P + \tilde{E}_{1y}^R \\ \tilde{E}_{1z}^P + \tilde{E}_{1z}^R \end{bmatrix} = \left(\tilde{\mathbf{G}}^P - \frac{1}{8\pi^2\omega\varepsilon_0\varepsilon_1}\tilde{\mathbf{G}}_1^R\right)\alpha'$$

$$= \left(\begin{bmatrix} \tilde{G}_{xx}^P & \tilde{G}_{xy}^P & \tilde{G}_{xz}^P \\ \tilde{G}_{yx}^P & \tilde{G}_{yy}^P & \tilde{G}_{yz}^P \\ \tilde{G}_{zx}^P & \tilde{G}_{zy}^P & \tilde{G}_{zz}^P \end{bmatrix} - \frac{1}{8\pi^2\omega\varepsilon_0\varepsilon_1}\begin{bmatrix} \tilde{G}_{1xx}^R & \tilde{G}_{1xy}^R & \tilde{G}_{1xz}^R \\ \tilde{G}_{1yx}^R & \tilde{G}_{1yy}^R & \tilde{G}_{1yz}^R \\ \tilde{G}_{1zx}^R & \tilde{G}_{1zy}^R & \tilde{G}_{1zz}^R \end{bmatrix}\right)\begin{bmatrix} \alpha_x' \\ \alpha_y' \\ \alpha_z' \end{bmatrix}, \tag{98}$$

where $\tilde{\mathbf{G}}^P$ is the same as Eq. (10) and $\tilde{\mathbf{G}}_1^R$ is defined by

$$\tilde{G}_{1xx}^R = \left(\partial_x^2\frac{k_{1z}}{k_s^2}R_1^{TM} - \partial_y^2\frac{k_1^2}{k_s^2 k_{1z}}R_1^{TE}\right)e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')}, \tag{99}$$

$$\tilde{G}_{1yy}^R = \left(\partial_y^2\frac{k_{1z}}{k_s^2}R_1^{TM} - \partial_x^2\frac{k_1^2}{k_s^2 k_{1z}}R_1^{TE}\right)e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')}, \tag{100}$$

$$\tilde{G}_{1zz}^R = \left(\frac{k_s^2}{k_{1z}}R_1^{TM}\right)e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')}, \tag{101}$$

$$\tilde{G}_{1xy}^R = \tilde{G}_{1yx}^R = \partial_x\partial_y\left(\frac{k_{1z}}{k_s^2}R_1^{TM} + \frac{k_1^2}{k_s^2 k_{1z}}R_1^{TE}\right)e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')}, \tag{102}$$

$$\tilde{G}_{1xz}^R = -\tilde{G}_{1zx}^R = \partial_x\partial_z\left(\frac{R_1^{TM}}{k_{1z}}\right)e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')}, \tag{103}$$

$$\tilde{G}_{1yz}^R = -\tilde{G}_{1zy}^R = \partial_y\partial_z\left(\frac{R_1^{TM}}{k_{1z}}\right)e^{ik_x(x-x')+ik_y(y-y')+ik_{1z}(z+z')}. \tag{104}$$

• In the second layer, there are both reflected and transmitted parts ($\tilde{\mathbf{E}}_2 = \tilde{\mathbf{E}}_2^R + \tilde{\mathbf{E}}_2^T$),

$$\begin{bmatrix} \tilde{E}_{2x} \\ \tilde{E}_{2y} \\ \tilde{E}_{2z} \end{bmatrix} = \begin{bmatrix} \tilde{E}_{2x}^R + \tilde{E}_{2x}^T \\ \tilde{E}_{2y}^R + \tilde{E}_{2y}^T \\ \tilde{E}_{2z}^R + \tilde{E}_{2z}^T \end{bmatrix} = -\frac{1}{8\pi^2\omega\varepsilon_0\varepsilon_2}\left(\tilde{\mathbf{G}}_2^R + \tilde{\mathbf{G}}_2^T\right)\alpha'$$

$$= -\frac{1}{8\pi^2\omega\varepsilon_0\varepsilon_2}\left(\begin{bmatrix} \tilde{G}_{2xx}^R & \tilde{G}_{2xy}^R & \tilde{G}_{2xz}^R \\ \tilde{G}_{2yx}^R & \tilde{G}_{2yy}^R & \tilde{G}_{2yz}^R \\ \tilde{G}_{2zx}^R & \tilde{G}_{2zy}^R & \tilde{G}_{2zz}^R \end{bmatrix} + \begin{bmatrix} \tilde{G}_{2xx}^T & \tilde{G}_{2xy}^T & \tilde{G}_{2xz}^T \\ \tilde{G}_{2yx}^T & \tilde{G}_{2yy}^T & \tilde{G}_{2yz}^T \\ \tilde{G}_{2zx}^T & \tilde{G}_{2zy}^T & \tilde{G}_{2zz}^T \end{bmatrix}\right)\begin{bmatrix} \alpha_x' \\ \alpha_y' \\ \alpha_z' \end{bmatrix}. \tag{105}$$

The transmitted part assumes the same form as for the two-layer case. Thus, the transmitted part of the Green's function can be simply found by replacing the transmission coefficient $T_{12}^{TM,TE}$ by

the $A_2^{TM,TE}$ as

$$\tilde{G}_{2xx}^T = \left(-\partial_x^2 \frac{k_{2z}}{k_s^2} A_2^{TM} - \partial_y^2 \frac{k_2^2 \mu_1}{k_s^2 \mu_2 k_{1z}} A_2^{TE}\right) e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}, \tag{106}$$

$$\tilde{G}_{2yy}^T = \left(-\partial_y^2 \frac{k_{2z}}{k_s^2} A_2^{TM} - \partial_x^2 \frac{k_2^2 \mu_1}{k_s^2 \mu_2 k_{1z}} A_2^{TE}\right) e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}, \tag{107}$$

$$\tilde{G}_{2zz}^T = \left(\frac{k_s^2}{k_{1z}} A_2^{TM}\right) e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}, \tag{108}$$

$$\tilde{G}_{2xy}^T = \tilde{G}_{2yx}^T = \left(-\partial_x \partial_y \frac{k_{2z}}{k_s^2} A_2^{TM} + \partial_x \partial_y \frac{k_2^2 \mu_1}{k_s^2 \mu_2 k_{1z}} A_2^{TE}\right) e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}, \tag{109}$$

$$\tilde{G}_{2xz}^T = \partial_x \partial_z \frac{A_2^{TM}}{k_{1z}} e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}, \tilde{G}_{2yz}^T = \partial_y \partial_z \frac{A_2^{TM}}{k_{1z}} e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}, \tag{110}$$

$$\tilde{G}_{2zx}^T = \partial_z \partial_x \frac{A_2^{TM}}{k_{2z}} e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}, \tilde{G}_{2zy}^T = \partial_z \partial_y \frac{A_2^{TM}}{k_{2z}} e^{ik_x(x-x')+ik_y(y-y')-ik_{2z}z+ik_{1z}z'}. \tag{111}$$

However, there are some changes in the reflected parts in the second layer since the definition of $\hat{\alpha}''$ of the reflected parts in the first layer is changed to $\hat{\alpha}_{2R}''$. Fortunately, the reflected part takes a similar form as the transmitted part because of $-k_{1z}/k_{2z}$ in $\hat{\alpha}_{2R}''$. By carefully re-deriving the transverse component of reflected parts in the second layer, the spectral Green's function can be found as

$$\tilde{G}_{2xx}^R = \left(\partial_x^2 \frac{k_{2z}}{k_s^2} R_2^{TM} - \partial_y^2 \frac{k_2^2}{k_s^2 k_{1z}} R_2^{TE}\right) e^{ik_x(x-x')+ik_y(y-y')+ik_{2z}z+ik_1z'+2ik_{2z}d}, \tag{112}$$

$$\tilde{G}_{2yy}^R = \left(\partial_y^2 \frac{k_{2z}}{k_s^2} R_2^{TM} - \partial_x^2 \frac{k_2^2}{k_s^2 k_{1z}} R_2^{TE}\right) e^{ik_x(x-x')+ik_y(y-y')+ik_{2z}z+ik_1z'+2ik_{2z}d}, \tag{113}$$

$$\tilde{G}_{2zz}^R = \left(\frac{k_s^2}{k_{1z}} R_2^{TM}\right) e^{ik_x(x-x')+ik_y(y-y')+ik_{2z}z+ik_1z'+2ik_{2z}d}, \tag{114}$$

$$\tilde{G}_{2xy}^R = \tilde{G}_{2yx}^R = \left(\partial_x \partial_y \frac{k_{2z}}{k_s^2} R_2^{TM} + \partial_x \partial_y \frac{k_2^2}{k_s^2 k_{1z}} R_2^{TE}\right) e^{ik_x(x-x')+ik_y(y-y')+ik_{2z}z+ik_1z'+2ik_{2z}d}, \tag{115}$$

$$\tilde{G}_{2xz}^R = \partial_x \partial_z \frac{R_2^{TM}}{k_{1z}} e^{ik_x(x-x')+ik_y(y-y')+ik_{2z}z+ik_1z'+2ik_{2z}d}, \tilde{G}_{2yz}^R = \partial_y \partial_z \frac{R_2^{TM}}{k_{1z}} e^{ik_x(x-x')+ik_y(y-y')+ik_{2z}z+ik_1z'+2ik_{2z}d}, \tag{116}$$

$$\tilde{G}_{2zx}^R = -\partial_x \partial_z \frac{R_2^{TM}}{k_{2z}} e^{ik_x(x-x')+ik_y(y-y')+ik_{2z}z+ik_1z'+2ik_{2z}d}, \tilde{G}_{2zy}^R = -\partial_y \partial_z \frac{R_2^{TM}}{k_{2z}} e^{ik_x(x-x')+ik_y(y-y')+ik_{2z}z+ik_1z'+2ik_{2z}d}. \tag{117}$$

• In the third layer, the transverse component of the transmitted part takes the same form as the transmitted part of the second layer. They can be simply found by changing the index in the transmitted part of the second layer. By combining all the components, the spectral Green's function in the third layer can be expressed by

$$\begin{bmatrix} \tilde{E}_{3x} \\ \tilde{E}_{3y} \\ \tilde{E}_{3z} \end{bmatrix} = \begin{bmatrix} \tilde{E}_{3x}^T \\ \tilde{E}_{3y}^T \\ \tilde{E}_{3z}^T \end{bmatrix} = -\frac{1}{8\pi^2 \omega \varepsilon_0 \varepsilon_3} \tilde{\mathbf{G}}_2^T \alpha' = -\frac{1}{8\pi^2 \omega \varepsilon_0 \varepsilon_3} \begin{bmatrix} \tilde{G}_{3xx}^T & \tilde{G}_{3xy}^T & \tilde{G}_{3xz}^T \\ \tilde{G}_{3yx}^T & \tilde{G}_{3yy}^T & \tilde{G}_{3yz}^T \\ \tilde{G}_{3zx}^T & \tilde{G}_{3zy}^T & \tilde{G}_{3zz}^T \end{bmatrix} \begin{bmatrix} \alpha_x' \\ \alpha_y' \\ \alpha_z' \end{bmatrix}, \tag{118}$$

where

$$\tilde{G}^T_{3xx} = \left(-\partial^2_x \frac{k_{3z}}{k_s^2} A^{TM}_3 - \partial^2_y \frac{k_3^2 \mu_1}{k_s^2 \mu_3 k_{1z}} A^{TE}_3\right) e^{ik_x(x-x')+ik_y(y-y')-ik_{3z}z+ik_{1z}z'}, \tag{119}$$

$$\tilde{G}^T_{3yy} = \left(-\partial^2_y \frac{k_{3z}}{k_s^2} A^{TM}_3 - \partial^2_x \frac{k_3^2 \mu_1}{k_s^2 \mu_3 k_{1z}} A^{TE}_3\right) e^{ik_x(x-x')+ik_y(y-y')-ik_{3z}z+ik_{1z}z'}, \tag{120}$$

$$\tilde{G}^T_{3zz} = \frac{k_s^2}{k_{1z}} A^{TM}_3 e^{ik_x(x-x')+ik_y(y-y')-ik_{3z}z+ik_{1z}z'}, \tag{121}$$

$$\tilde{G}^T_{3xy} = \tilde{G}^T_{3yx} = \left(-\partial_x\partial_y \frac{k_{3z}}{k_s^2} A^{TM}_3 + \partial_x\partial_y \frac{k_3^2 \mu_1}{k_s^2 \mu_3 k_{1z}} A^{TE}_3\right) e^{ik_x(x-x')+ik_y(y-y')-ik_{3z}z+ik_{1z}z'}, \tag{122}$$

$$\tilde{G}^T_{3xz} = \partial_x\partial_z \frac{A^{TM}_3}{k_{1z}} e^{ik_x(x-x')+ik_y(y-y')-ik_{3z}z+ik_{1z}z'}, \tilde{G}^T_{3yz} = \partial_y\partial_z \frac{A^{TM}_3}{k_{1z}} e^{ik_x(x-x')+ik_y(y-y')-ik_{3z}z+ik_{1z}z'}, \tag{123}$$

$$\tilde{G}^T_{3zx} = \partial_z\partial_x \frac{A^{TM}_3}{k_{3z}} e^{ik_x(x-x')+ik_y(y-y')-ik_{3z}z+ik_{1z}z'}, \tilde{G}^T_{3zy} = \partial_z\partial_y \frac{A^{TM}_3}{k_{3z}} e^{ik_x(x-x')+ik_y(y-y')-ik_{3z}z+ik_{1z}z'}. \tag{124}$$

4.1.2. Green's function in the spatial domain

The inverse Fourier transform is taken to recover the Green's function in the spatial domain as before. In the spectral domain, the reflected part in the first layer and transmitted part in the second and third layer have the exactly same form as the two-layer Green's function. Thus, the spatial domain Green's function can be simply found by replacing the reflection and transmission coefficient and index without actual derivation. The reflected part in the second layer has almost identical form as the transmitted part in the second layer due to similar definition of $\alpha''_{2R} = (-\frac{k_{1z}}{k_{2z}}\alpha'_x, -\frac{k_{1z}}{k_{2z}}\alpha'_y, \alpha'_z)$ and $\alpha''_{2T} = (\frac{k_{1z}}{k_{2z}}\alpha'_x, \frac{k_{1z}}{k_{2z}}\alpha'_y, \alpha'_z)$. Therefore, by carefully changing the sign of transmitted part Green's function formula, one can find the reflected part in the second layer.

• In the first layer, the reflected part Green's function is given by

$$G^R_{1xx} = -\frac{1}{2}g^R_{1,5} + \left(\frac{1}{2}\rho^2 - (y-y')^2\right)g^R_{1,6}, \tag{125}$$

$$G^R_{1yy} = -\frac{1}{2}g^R_{1,5} - \left(\frac{1}{2}\rho^2 - (y-y')^2\right)g^R_{1,6}, \tag{126}$$

$$G^R_{1zz} = g^R_{1,7}, \tag{127}$$

$$G^R_{1xy} = G^R_{1yx} = (x-x')(y-y')g^R_{1,6}, \tag{128}$$

$$G^R_{1xz} = -G^R_{1zx} = -i(x-x')g^R_{1,8}, \tag{129}$$

$$G^R_{1yz} = -G^R_{1zy} = -i(y-y')g^R_{1,8}, \tag{130}$$

where

$$\tilde{g}^R_{1,5} = k_{1z}R^{TM}_1 - \frac{k_1^2}{k_{1z}}R^{TE}_1, \tilde{g}^R_{1,6} = \frac{k_{1z}}{k_s^2}R^{TM}_1 + \frac{k_1^2}{k_s^2 k_{1z}}R^{TE}_1, \tilde{g}^R_{1,7} = \frac{k_s^2}{k_{1z}}R^{TM}_1, \tilde{g}^R_{1,8} = R^{TM}_1,$$

$$g^R_{1,5} = 2\pi \int_0^\infty k_s \tilde{g}^R_{1,5} J_0(k_s\rho)e^{ik_{1z}(z+z')}dk_s, g^R_{1,6} = 2\pi \int_0^\infty k_s^3 \tilde{g}^R_{1,6} \frac{J_2(k_s\rho)}{\rho^2}e^{ik_{1z}(z+z')}dk_s,$$

$$g^R_{1,7} = 2\pi \int_0^\infty k_s \tilde{g}^R_{1,7} J_0(k_s\rho)e^{ik_{1z}(z+z')}dk_s, g^R_{1,8} = 2\pi \int_0^\infty k_s^2 \tilde{g}^R_{1,8} \frac{J_1(k_s\rho)}{\rho}e^{ik_{1z}(z+z')}dk_s. \tag{131}$$

• In the second layer, the transmitted part has the same form as the two layers case. Therefore, the Green's function can be found simply replacing T_{12}^{TM} and T_{12}^{TE} by A_2^{TM} and A_2^{TE} as

$$G_{2xx}^T = \frac{1}{2}g_{2,5}^T - (\frac{1}{2}\rho^2 - (y-y')^2)g_{2,6}^T, \tag{132}$$

$$G_{2yy}^T = \frac{1}{2}g_{2,5}^T + (\frac{1}{2}\rho^2 - (y-y')^2)g_{2,6}^T, \tag{133}$$

$$G_{2zz}^T = g_{2,7}^T, \tag{134}$$

$$G_{2xy}^T = G_{2yx}^T = -(x-x')(y-y')g_{2,6}^T, \tag{135}$$

$$G_{2xz}^T = i(x-x')g_{2,8}^T, \quad G_{2yz}^T = i(y-y')g_{2,8}^T, \tag{136}$$

$$G_{2zx}^T = i(x-x')g_{2,9}^T, \quad G_{2zy}^T = i(y-y')g_{2,9}^T, \tag{137}$$

where

$$\tilde{g}_{2,5}^T = k_{2z}A_2^{TM} + \frac{k_2^2}{k_{1z}}\frac{\mu_1}{\mu_2}A_2^{TE}, \tilde{g}_{2,6}^T = \frac{k_{2z}}{k_s^2}A_2^{TM} - \frac{k_2^2}{k_{1z}k_s^2}\frac{\mu_1}{\mu_2}A_2^{TE},$$

$$\tilde{g}_{2,7}^T = \frac{k_s^2}{k_{1z}}A_2^{TM}, \tilde{g}_{2,8}^T = \frac{k_{2z}A_2^{TM}}{k_{1z}}, \tilde{g}_{2,9}^T = A_2^{TM},$$

$$g_{2,5}^T = 2\pi\int_0^\infty k_s\tilde{g}_{2,5}^T J_0(k_s\rho)e^{-ik_{2z}z+ik_{1z}z'}dk_s, g_{2,6}^T = 2\pi\int_0^\infty k_s^3\tilde{g}_{2,6}\frac{J_2(k_s\rho)}{\rho^2}^T e^{-ik_{2z}z+ik_{1z}z'}dk_s,$$

$$g_{2,7}^T = 2\pi\int_0^\infty k_s\tilde{g}_{2,7}^T J_0(k_s\rho)e^{-ik_{2z}z+ik_{1z}z'}dk_s, g_{2,8}^T = 2\pi\int_0^\infty k_s^2\tilde{g}_{2,8}^T\frac{J_1(k_s\rho)}{\rho}e^{-ik_{2z}z+ik_{1z}z'}dk_s,$$

$$g_{2,9}^T = 2\pi\int_0^\infty k_s^2\tilde{g}_{2,8}^T\frac{J_1(k_s\rho)}{\rho}e^{-ik_{2z}z+ik_{1z}z'}dk_s. \tag{138}$$

The reflected part in the second layer is derived by observing the similarity between the transmitted part and reflected parts in the second layer, one can change the sign of transmitted part to obtain the Green's function or one can take actual double integral and derive the same formulas.

$$G_{2xx}^R = -\frac{1}{2}g_{2,5}^R + \left(\frac{1}{2}\rho^2 - (y-y')^2\right)g_{2,6}^R, \tag{139}$$

$$G_{2yy}^R = -\frac{1}{2}g_{2,5}^R - \left(\frac{1}{2}\rho^2 - (y-y')^2\right)g_{2,6}^R, \tag{140}$$

$$G_{2zz}^R = g_{2,7}^R, \tag{141}$$

$$G_{2xy}^R = G_{2yx}^R = (x-x')(y-y')g_{2,6}^R, \tag{142}$$

$$G_{2zx}^R = i(x-x')g_{2,8}^R, G_{2zy}^R = i(y-y')g_{2,8}^R, \tag{143}$$

$$G_{2xz}^R = -i(x-x')g_{2,9}^R, G_{2yz}^R = -i(y-y')g_{2,9}^R, \tag{144}$$

where

$$\tilde{g}_{2,5}^R = k_{2z}R_2^{TM} - \frac{k_2^2}{k_{1z}}R_2^{TE}, \tilde{g}_{2,6}^R = \frac{k_{2z}}{k_s^2}R_2^{TM} + \frac{k_2^2}{k_s^2 k_{1z}}R_2^{TE},$$

$$\tilde{g}_{2,7}^R = \frac{k_s^2}{k_{1z}}R_2^{TM}, \tilde{g}_{2,8}^R = R_2^{TM}, \tilde{g}_{2,9}^R = \frac{k_{2z}}{k_{1z}}R_2^{TM},$$

$$g_{2,5}^R = 2\pi \int_0^\infty k_s \tilde{g}_{2,5}^R J_0(k_s\rho)e^{ik_{2z}z+ik_{1z}z'+2ik_{2z}d}dk_s, g_{2,6}^R = 2\pi \int_0^\infty k_s^3 \tilde{g}_{2,6}^R \frac{J_2(k_s\rho)}{\rho^2}e^{ik_{2z}z+ik_{1z}z'+2ik_{2z}d}dk_s,$$

$$g_{2,7}^R = 2\pi \int_0^\infty k_s \tilde{g}_{2,7}^R J_0(k_s\rho)e^{ik_{2z}z+ik_{1z}z'+2ik_{2z}d}dk_s, g_{2,8}^R = 2\pi \int_0^\infty k_s^2 \tilde{g}_{2,8}^R \frac{J_1(k_s\rho)}{\rho}e^{ik_{2z}z+ik_{1z}z'+2ik_{2z}d}dk_s,$$

$$g_{2,9}^R = 2\pi \int_0^\infty k_s^2 \tilde{g}_{2,9}^R \frac{J_1(k_s\rho)}{\rho}e^{ik_{2z}z+ik_{1z}z'+2ik_{2z}d}dk_s. \tag{145}$$

• In the third layer, again the spectral Green's function have the same form as any transmitted fields in the second layer. Thus, Green's function can be expressed as

$$G_{3xx}^T = \frac{1}{2}g_{3,5}^T - (\frac{1}{2}\rho^2 - (y-y')^2)g_{3,6}^T, \tag{146}$$

$$G_{3yy}^T = \frac{1}{2}g_{3,5}^T + (\frac{1}{2}\rho^2 - (y-y')^2)g_{3,6}^T, \tag{147}$$

$$G_{3zz}^T = g_{3,7}^T, \tag{148}$$

$$G_{3xy}^T = G_{3xy}^T = -(x-x')(y-y')g_{3,6}^T, \tag{149}$$

$$G_{3xz}^T = i(x-x')g_{3,8}^T, \quad G_{3yz}^T = i(y-y')g_{3,8}^T, \tag{150}$$

$$G_{3zx}^T = i(x-x')g_{3,9}^T, \quad G_{3zy}^T = i(y-y')g_{3,9}^T, \tag{151}$$

where

$$\tilde{g}_{3,5}^T = k_{3z}A_3^{TM} + \frac{k_3^2}{k_{1z}}\frac{\mu_1}{\mu_3}A_3^{TE}, \tilde{g}_{3,6}^T = \frac{k_{3z}}{k_s^2}A_3^{TM} - \frac{k_3^2}{k_{1z}k_s^2}\frac{\mu_1}{\mu_3}A_3^{TE},$$

$$\tilde{g}_{3,7}^T = \frac{k_s^2}{k_{1z}}A_3^{TM}, \tilde{g}_{3,8}^T = \frac{k_{3z}A_3^{TM}}{k_{1z}}, \tilde{g}_{3,9}^T = A_3^{TM},$$

$$g_{3,5}^T = 2\pi \int_0^\infty k_s \tilde{g}_{3,5}^T J_0(k_s\rho)e^{-ik_{3z}z+ik_{1z}z'}dk_s, g_{3,6}^T = 2\pi \int_0^\infty k_s^3 \tilde{g}_{3,6}^T \frac{J_2(k_s\rho)}{\rho^2}^T e^{-ik_{3z}z+ik_{1z}z'}dk_s,$$

$$g_{3,7}^T = 2\pi \int_0^\infty k_s \tilde{g}_{3,7}^T J_0(k_s\rho)e^{-ik_{3z}z+ik_{1z}z'}dk_s, g_{3,8}^T = 2\pi \int_0^\infty k_s^2 \tilde{g}_{3,8}^T \frac{J_1(k_s\rho)}{\rho}e^{-ik_{3z}z+ik_{1z}z'}dk_s,$$

$$g_{3,9}^T = 2\pi \int_0^\infty k_s^2 \tilde{g}_{3,9}^T \frac{J_1(k_s\rho)}{\rho}e^{-ik_{3z}z+ik_{1z}z'}dk_s. \tag{152}$$

4.2. Numerical results

A three-layer structure is considered by placing two interfaces at $z = 0$ and $z = -1$. The relative permittivity is assigned as $\varepsilon_1 = 1$, $\varepsilon_2 = 4$, $\varepsilon_3 = 1.1$ in each layer. The relative permeability $\{\mu_i\}_{i=1}^3$ is assumed to be 1 in all layers. The wavelength λ is set to be 1. The electric field is computed when a source is placed on top of the layered media at $\mathbf{r}' = (0.1, -0.2, 0.5)$ and oriented along $\hat{\alpha}' = (1/2, 1/2, 1/\sqrt{2})$. In Fig. 5, all the components of electric field are plotted
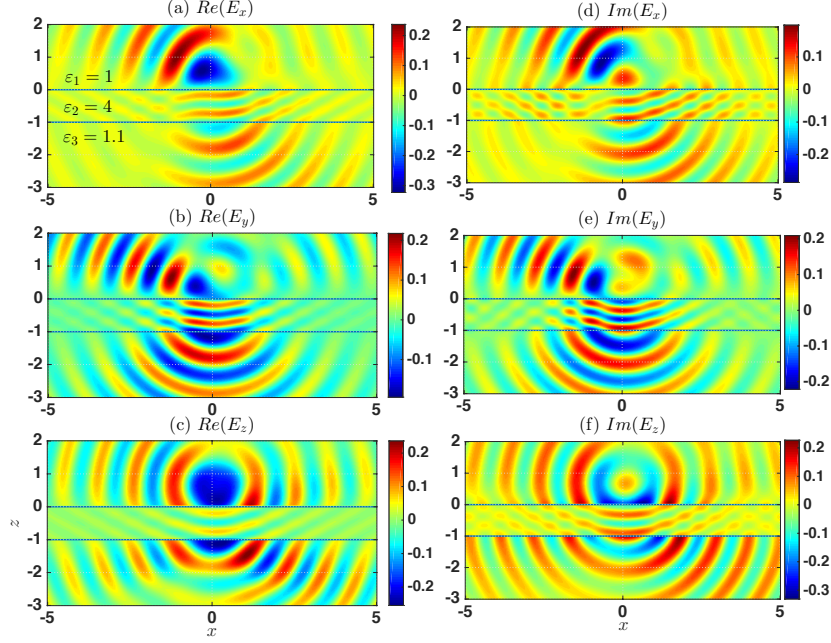
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Figure 5: Electric fields in a three-layer structure with layer interface at $z = 0$ and $z = -1$. A dipole source is placed at $\mathbf{r}' = (0.1, -0.2, 0.5)$ and oriented along $\boldsymbol{\alpha}' = (1/2, 1/2, 1/\sqrt{2})$ and fields are computed for $-5 \leq x \leq 5$ and $-3 \leq z \leq 3$ for a fixed $y = 1.0$ with $\varepsilon_1 = 1$, $\varepsilon_2 = 4$, $\varepsilon_3 = 1.1$, and $\lambda = 1$.

over $-5 \leq x \leq 5$ and $-3 \leq z \leq 3$ for a fixed $y = 1.0$. The continuity of the fields are checked at both interfaces $z = 0$ and $z = -1$ in Fig. 6 as accuracy of the Green's function. In all components, approximately 10^{-10} absolute error is obtained.

4.3. Source in the second layer

When a dipole source is placed in the second layer, the formula derived in the previous subsections must be modified to accommodate multiple reflection and transmission from the first and second interfaces. In the following, the electric field in each layer in the spectral domain are provided. Then, taking Sommerfeld integrals derives the electric field in the spatial domain. That will complete the derivation of Dyadic Green's function for a three-layer structure.

4.3.1. Green's function in the spectral domain

Let a dipole source is located in the second layer, then in the first and third layer, there are only transmitted fields. However, in the second layer, there are primary field, reflected fields from the bottom interface and the top interface. The reflected fields from the bottom interface and top interface are an up-going and a down-going waves, respectively. Thus, in each layer, the z-components of field can be represented by

$$\tilde{E}_{1z} = \tilde{E}_{1z}^T, \tilde{E}_{2z} = \tilde{E}_{2z}^P + \tilde{E}_{2z}^U + \tilde{E}_{2z}^D, \tilde{E}_{3z} = \tilde{E}_{3z}^T, \tag{153}$$

$$\tilde{H}_{1z} = \tilde{H}_{1z}^T, \tilde{H}_{2z} = \tilde{H}_{2z}^P + \tilde{H}_{2z}^U + \tilde{H}_{2z}^D, \tilde{E}_{3z} = \tilde{H}_{3z}^T. \tag{154}$$
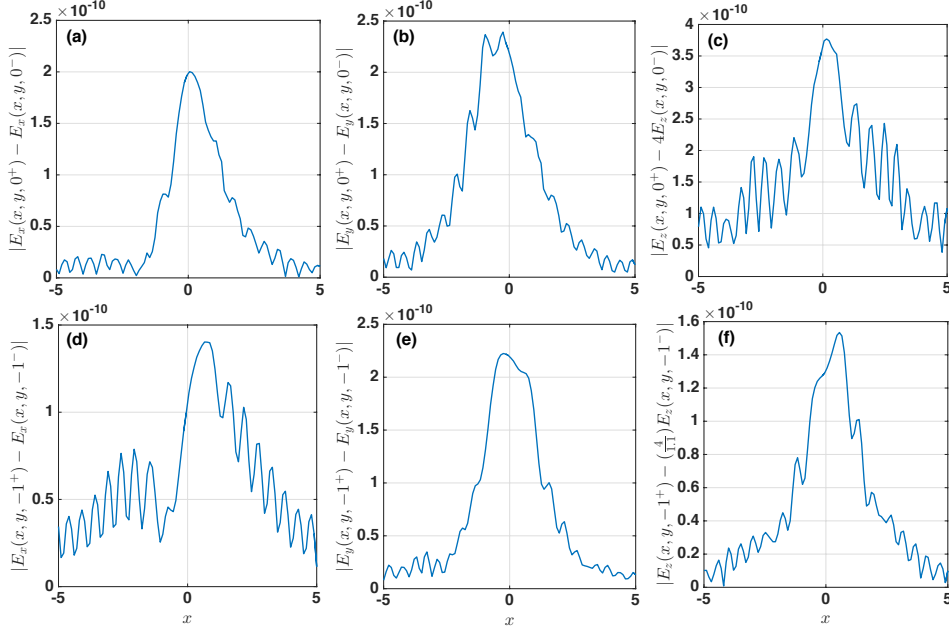
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Figure 6: Continuity of electric fields at both interfaces at $z = 0$ and $z = -1$. (a) $|E_x(x, y, 0^+) - E_x(x, y, 0^-)|$, (b) $|E_y(x, y, 0^+) - E_y(x, y, 0^-)|$, (c) $|E_z(x, y, 0^+) - 4E_z(x, y, 0^-)|$, (d) $|E_x(x, y, -1^+) - E_x(x, y, -1^-)|$, (e) $|E_y(x, y, -1^+) - E_y(x, y, -1^-)|$, (f) $|E_z(x, y, -1^+) - \frac{1.1}{4}E_z(x, y, -1^-)|$. A dipole source is placed at $\mathbf{r}' = (0.1, -0.2, 0.5)$ and oriented along $\boldsymbol{\alpha}' = (1/2, 1/2, 1/\sqrt{2})$ and fields are computed for $-5 \le x \le 5$ for a fixed $y = 1.0$ at the layer interface $z = 0$ and $z = -1$ and with $\varepsilon_1 = 1$, $\varepsilon_2 = 4$, $\varepsilon_3 = 1.1$, and $\lambda = 1$.

The primary field $\tilde{E}_{2z}^P$ is the same as the primary field in the free space. The field in the second layer can be expressed using new reflection coefficients $U^{TM,TE}$ and $D^{TM,TE}$ that represent the amplitude of the up- and down-going waves, respectively.

$$\tilde{E}_{2z}^U = \left(\hat{z} \cdot \hat{\boldsymbol{\alpha}}'' + \frac{1}{k_2^2}\partial_z \nabla \cdot \hat{\boldsymbol{\alpha}}''\right)\tilde{g}_{2,TM}^U, \quad \tilde{H}_{2z}^U = -\frac{1}{i\omega\mu_0\mu_2}\hat{z} \cdot \nabla_s \times \hat{\boldsymbol{\alpha}}''\tilde{g}_{2,TE}^U, \tag{155}$$

$$\tilde{E}_{2z}^D = \left(\hat{z} \cdot \hat{\boldsymbol{\alpha}}'' + \frac{1}{k_2^2}\partial_z \nabla \cdot \hat{\boldsymbol{\alpha}}''\right)\tilde{g}_{2,TM}^D, \quad \tilde{H}_{2z}^D = -\frac{1}{i\omega\mu_0\mu_2}\hat{z} \cdot \nabla_s \times \hat{\boldsymbol{\alpha}}''\tilde{g}_{2,TE}^D, \tag{156}$$

where

$$\hat{\boldsymbol{\alpha}}'' = (-\alpha_x', -\alpha_y', \alpha_z'), \tag{157}$$

$$\tilde{g}_{2,TM,TE}^U = -\frac{\omega\mu_0\mu_2}{8\pi^2}U^{TM,TE}\frac{e^{ik_x(x-x')+ik_y(y-y')}}{k_{2z}}e^{ik_{2z}z}, \tag{158}$$

$$\tilde{g}_{2,TM,TE}^D = -\frac{\omega\mu_0\mu_2}{8\pi^2}D^{TM,TE}\frac{e^{ik_x(x-x')+ik_y(y-y')}}{k_{2z}}e^{-ik_{2z}z}. \tag{159}$$

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In the first and third layer, the transmitted parts are given by

$$\tilde{E}_{1z}^T = \left(\hat{z} \cdot \hat{\alpha}_1''' + \frac{1}{k_1^2} \partial_z \nabla \cdot \hat{\alpha}_1''' \right) \tilde{g}_{1,TM}^T, \quad \tilde{H}_{1z}^T = \frac{1}{i\omega\mu_0\mu_1} \frac{k_{1z}}{k_{2z}} \hat{z} \cdot \nabla_s \times \hat{\alpha}_1''' \tilde{g}_{1,TE}^T, \quad (160)$$

$$\tilde{E}_{3z}^T = \left(\hat{z} \cdot \hat{\alpha}_3''' + \frac{1}{k_3^2} \partial_z \nabla \cdot \hat{\alpha}_3''' \right) \tilde{g}_{3,TM}^T, \quad \tilde{H}_{3z}^T = \frac{1}{i\omega\mu_0\mu_3} \frac{k_{3z}}{k_{2z}} \hat{z} \cdot \nabla_s \times \hat{\alpha}_3''' \tilde{g}_{3,TE}^T, \quad (161)$$

where

$$\hat{\alpha}_1''' = (\frac{k_{2z}}{k_{1z}}\alpha_x', \frac{k_{2z}}{k_{1z}}\alpha_y', \alpha_z'), \hat{\alpha}_3''' = (\frac{k_{2z}}{k_{3z}}\alpha_x', \frac{k_{2z}}{k_{3z}}\alpha_y', \alpha_z'), \quad (162)$$

$$\tilde{g}_{1,TM,TE}^T = -\frac{\omega\mu_0\mu_1}{8\pi^2} A_1^{TM,TE} \frac{e^{ik_x(x-x')+ik_y(y-y')}}{k_{2z}} e^{ik_{1z}z}, \quad (163)$$

$$\tilde{g}_{3,TM,TE}^T = -\frac{\omega\mu_0\mu_3}{8\pi^2} A_3^{TM,TE} \frac{e^{ik_x(x-x')+ik_y(y-y')}}{k_{2z}} e^{-ik_{3z}z}. \quad (164)$$

In the above, $A_1^{TM,TE}$, $A_3^{TM,TE}$, $D^{TM,TE}$, and $U^{TM,TE}$ (See the Ref. [16] for their derivation) are given by

$$A_1^{TM,TE} = \frac{T_{21}^{TM,TE} \left(e^{-ik_{2z}z'} \pm R_{23}^{TM,TE} e^{ik_{2z}z'+2ik_{2z}d} \right)}{1 - R_{23}^{TM,TE} R_{21}^{TM,TE} e^{2ik_{2z}d}}, \quad (165)$$

$$D^{TM,TE} = \frac{R_{21}^{TM,TE} \left(e^{-ik_{2z}z'} \pm R_{23}^{TM,TE} e^{ik_{2z}z'+2ik_{2z}d} \right)}{1 - R_{23}^{TM,TE} R_{21}^{TM,TE} e^{2ik_{2z}d}}, \quad (166)$$

$$U^{TM,TE} = \frac{R_{23}^{TM,TE} e^{2ik_{2z}d} \left(e^{ik_{2z}z'} \pm R_{21}^{TM,TE} e^{-ik_{2z}z'} \right)}{1 - R_{23}^{TM,TE} R_{21}^{TM,TE} e^{2ik_{2z}d}}, \quad (167)$$

$$A_3^{TM,TE} = \frac{T_{23}^{TM,TE} e^{i(k_{2z}-k_{3z})d} \left(e^{ik_{2z}z'} \pm R_{21}^{TM,TE} e^{-ik_{2z}z'} \right)}{1 - R_{23}^{TM,TE} R_{21}^{TM,TE} e^{2ik_{2z}d}}. \quad (168)$$

In each layer, again the transverse component must be derived using Maxwell's equations. The final simplified formula is presented in the following:

• In the first layer, the electric field in the spectral domain is

$$\begin{bmatrix} \tilde{E}_{1x} \\ \tilde{E}_{1y} \\ \tilde{E}_{1z} \end{bmatrix} = \begin{bmatrix} \tilde{E}_{1x}^T \\ \tilde{E}_{1y}^T \\ \tilde{E}_{1z}^T \end{bmatrix} = -\frac{1}{8\pi^2\omega\varepsilon_0\varepsilon_1} \tilde{\mathbf{G}}_1^T \alpha' = -\frac{1}{8\pi^2\omega\varepsilon_0\varepsilon_1} \begin{bmatrix} \tilde{G}_{1xx}^T & \tilde{G}_{1xy}^T & \tilde{G}_{1xz}^T \\ \tilde{G}_{1yx}^T & \tilde{G}_{1yy}^T & \tilde{G}_{1yz}^T \\ \tilde{G}_{1zx}^T & \tilde{G}_{1zy}^T & \tilde{G}_{1zz}^T \end{bmatrix} \begin{bmatrix} \alpha_x' \\ \alpha_y' \\ \alpha_z' \end{bmatrix}, \quad (169)$$

where

$$\tilde{G}^T_{1xx} = \left(-\partial^2_x \frac{k_{1z}}{k_s^2} A^{TM}_1 - \partial^2_y \frac{k_1^2}{k_s^2 k_{2z}} A^{TE}_1\right) e^{ik_x(x-x')+ik_y(y-y')} e^{ik_{1z}z}, \tag{170}$$

$$\tilde{G}^T_{1yy} = \left(-\partial^2_y \frac{k_{1z}}{k_s^2} A^{TM}_1 - \partial^2_x \frac{k_1^2}{k_s^2 k_{2z}} A^{TE}_1\right) e^{ik_x(x-x')+ik_y(y-y')} e^{ik_{1z}z}, \tag{171}$$

$$\tilde{G}^T_{1zz} = \left(\frac{k_s^2}{k_{2z}} A^{TM}_1\right) e^{ik_x(x-x')+ik_y(y-y')} e^{ik_{1z}z}, \tag{172}$$

$$\tilde{G}^T_{1xy} = \tilde{G}^T_{yx} = \left(-\partial_x\partial_y \frac{k_{1z}}{k_s^2} A^{TM}_1 + \partial_x\partial_y \frac{k_1^2}{k_s^2 k_{2z}} A^{TE}_1\right) e^{ik_x(x-x')+ik_y(y-y')} e^{ik_{1z}z}, \tag{173}$$

$$\tilde{G}^T_{1xz} = \left(\partial_x\partial_z \frac{A^{TM}_1}{k_{2z}}\right) e^{ik_x(x-x')+ik_y(y-y')} e^{ik_{1z}z}, \tilde{G}^T_{1yz} = \left(\partial_y\partial_z \frac{A^{TM}_1}{k_{2z}}\right) e^{ik_x(x-x')+ik_y(y-y')} e^{ik_{1z}z}, \tag{174}$$

$$\tilde{G}^T_{1zx} = \left(\partial_x\partial_z \frac{A^{TM}_1}{k_{1z}}\right) e^{ik_x(x-x')+ik_y(y-y')} e^{ik_{1z}z}, \tilde{G}^T_{1zy} = \left(\partial_y\partial_z \frac{A^{TM}_1}{k_{1z}}\right) e^{ik_x(x-x')+ik_y(y-y')} e^{ik_{1z}z}. \tag{175}$$

• In the third layer, the same calculation applies and the electric field in the spectral domain is

$$\begin{bmatrix} \tilde{E}_{3x} \\ \tilde{E}_{3y} \\ \tilde{E}_{3z} \end{bmatrix} = \begin{bmatrix} \tilde{E}^T_{3x} \\ \tilde{E}^T_{3y} \\ \tilde{E}^T_{3z} \end{bmatrix} = -\frac{1}{8\pi^2 \omega\varepsilon_0\varepsilon_3} \tilde{\mathbf{G}}^T_3 \alpha' = -\frac{1}{8\pi^2 \omega\varepsilon_0\varepsilon_3} \begin{bmatrix} \tilde{G}^T_{3xx} & \tilde{G}^T_{3xy} & \tilde{G}^T_{3xz} \\ \tilde{G}^T_{3yx} & \tilde{G}^T_{3yy} & \tilde{G}^T_{3yz} \\ \tilde{G}^T_{3zx} & \tilde{G}^T_{3zy} & \tilde{G}^T_{3zz} \end{bmatrix} \begin{bmatrix} \alpha'_x \\ \alpha'_y \\ \alpha'_z \end{bmatrix}, \tag{176}$$

where

$$\tilde{G}^T_{3xx} = \left(-\partial^2_x \frac{k_{3z}}{k_s^2} A^{TM}_3 - \partial^2_y \frac{k_3^2}{k_s^2 k_{2z}} A^{TE}_3\right) e^{ik_x(x-x')+ik_y(y-y')} e^{-ik_{3z}z}, \tag{177}$$

$$\tilde{G}^T_{3yy} = \left(-\partial^2_y \frac{k_{3z}}{k_s^2} A^{TM}_3 - \partial^2_x \frac{k_3^2}{k_s^2 k_{2z}} A^{TE}_3\right) e^{ik_x(x-x')+ik_y(y-y')} e^{-ik_{3z}z}, \tag{178}$$

$$\tilde{G}^T_{3zz} = \left(\frac{k_s^2 A^{TM}_3}{k_{2z}}\right) e^{ik_x(x-x')+ik_y(y-y')} e^{-ik_{3z}z}, \tag{179}$$

$$\tilde{G}^T_{3xy} = \tilde{G}^T_{yx} = \left(-\partial_x\partial_y \frac{k_{3z}}{k_s^2} A^{TM}_3 + \partial_x\partial_y \frac{k_3^2}{k_s^2 k_{2z}} A^{TE}_3\right) e^{ik_x(x-x')+ik_y(y-y')} e^{-ik_{3z}z}, \tag{180}$$

$$\tilde{G}^T_{3xz} = \left(\partial_x\partial_z \frac{A^{TM}_3}{k_{2z}}\right) e^{ik_x(x-x')+ik_y(y-y')} e^{-ik_{3z}z}, \tilde{G}^T_{3yz} = \left(\partial_y\partial_z \frac{A^{TM}_3}{k_{2z}}\right) e^{ik_x(x-x')+ik_y(y-y')} e^{-ik_{3z}z}, \tag{181}$$

$$\tilde{G}^T_{3zx} = \left(\partial_x\partial_z \frac{A^{TM}_3}{k_{3z}}\right) e^{ik_x(x-x')+ik_y(y-y')} e^{-ik_{3z}z}, \tilde{G}^T_{3zy} = \left(\partial_y\partial_z \frac{A^{TM}_3}{k_{3z}}\right) e^{ik_x(x-x')+ik_y(y-y')} e^{-ik_{3z}z}. \tag{182}$$

• In the second layer, the electric field has three parts that can be expressed using the Green's function notation. The derivation of the up-going wave Green's function ($\tilde{\mathbf{G}}^U_2$) and the down-going wave Green's function ($\tilde{\mathbf{G}}^D_2$) are similar to that of reflection fields in both two- and three-layer structures. Derivation are not so difficult but needs some attention on k_{2z} because there is

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k_{2z} in the denominator of $\tilde{g}^{U,D}_{2,TM,TE}$ and $\tilde{g}^{T}_{3,TM,TE}$ instead of k_{1z} compared with the case when the source is in the first layer. In the following, both the up- and down-going wave Green's functions are listed.

$$
\begin{bmatrix} \tilde{E}_{2x} \\ \tilde{E}_{2y} \\ \tilde{E}_{2z} \end{bmatrix} = \begin{bmatrix} \tilde{E}^{P}_{2x} + \tilde{E}^{D}_{2x} + \tilde{E}^{U}_{2x} \\ \tilde{E}^{P}_{2y} + \tilde{E}^{D}_{2y} + \tilde{E}^{U}_{2y} \\ \tilde{E}^{P}_{2z} + \tilde{E}^{D}_{2z} + \tilde{E}^{U}_{2z} \end{bmatrix} = \left[\tilde{\mathbf{G}}^{P}_{2} - \frac{1}{8\pi^2\omega\varepsilon_0\varepsilon_2} \left(\tilde{\mathbf{G}}^{D}_{2} + \tilde{\mathbf{G}}^{U}_{2} \right) \right] \hat{\alpha}'
$$
$$
= \left[\tilde{\mathbf{G}}^{P}_{2} - \frac{1}{8\pi^2\omega\varepsilon_0\varepsilon_2} \left(\begin{bmatrix} \tilde{G}^{D}_{2xx} & \tilde{G}^{D}_{2xy} & \tilde{G}^{D}_{2xz} \\ \tilde{G}^{D}_{2yx} & \tilde{G}^{D}_{2yy} & \tilde{G}^{D}_{2yz} \\ \tilde{G}^{D}_{2zx} & \tilde{G}^{D}_{2zy} & \tilde{G}^{D}_{2zz} \end{bmatrix} + \begin{bmatrix} \tilde{G}^{U}_{2xx} & \tilde{G}^{U}_{2xy} & \tilde{G}^{U}_{2xz} \\ \tilde{G}^{U}_{2yx} & \tilde{G}^{U}_{2yy} & \tilde{G}^{U}_{2yz} \\ \tilde{G}^{U}_{2zx} & \tilde{G}^{U}_{2zy} & \tilde{G}^{U}_{2zz} \end{bmatrix} \right) \right] \begin{bmatrix} \alpha'_x \\ \alpha'_y \\ \alpha'_z \end{bmatrix}, \quad (183)
$$

where

$$
\tilde{G}^{D}_{2xx} = \left(\partial_x^2 \frac{k_{2z}}{k_s^2} D^{TM} - \partial_y^2 \frac{k_2^2}{k_s^2 k_{2z}} D^{TE} \right) e^{ik_x(x-x')+ik_y(y-y')} e^{-ik_{2z}z}, \quad (184)
$$

$$
\tilde{G}^{D}_{2yy} = \left(\partial_y^2 \frac{k_{2z}}{k_s^2} D^{TM} - \partial_x^2 \frac{k_2^2}{k_s^2 k_{2z}} D^{TE} \right) e^{ik_x(x-x')+ik_y(y-y')} e^{-ik_{2z}z}, \quad (185)
$$

$$
\tilde{G}^{D}_{2zz} = \left(\frac{k_s^2 D^{TM}}{k_{2z}} \right) e^{ik_x(x-x')+ik_y(y-y')} e^{-ik_{2z}z}, \quad (186)
$$

$$
\tilde{G}^{D}_{2xy} = \tilde{G}^{D}_{2yx} = \left(\partial_x\partial_y \frac{k_{2z}}{k_s^2} D^{TM} + \partial_x\partial_y \frac{k_2^2}{k_s^2 k_{2z}} D^{TE} \right) e^{ik_x(x-x')+ik_y(y-y')} e^{-ik_{2z}z}, \quad (187)
$$

$$
\tilde{G}^{D}_{2xz} = -\tilde{G}^{D}_{2zx} = \partial_x\partial_z \frac{D^{TM}}{k_{2z}} e^{ik_x(x-x')+ik_y(y-y')} e^{-ik_{2z}z}, \quad (188)
$$

$$
\tilde{G}^{D}_{2yz} = -\tilde{G}^{D}_{2zy} = \partial_y\partial_z \frac{D^{TM}}{k_{2z}} e^{ik_x(x-x')+ik_y(y-y')} e^{-ik_{2z}z}, \quad (189)
$$

and

$$
\tilde{G}^{U}_{2xx} = \left(\partial_x^2 \frac{k_{2z}}{k_s^2} U^{TM} - \partial_y^2 \frac{k_2^2}{k_s^2 k_{2z}} U^{TE} \right) e^{ik_x(x-x')+ik_y(y-y')} e^{ik_{2z}z}, \quad (190)
$$

$$
\tilde{G}^{U}_{2yy} = \left(\partial_y^2 \frac{k_{2z}}{k_s^2} U^{TM} - \partial_x^2 \frac{k_2^2}{k_s^2 k_{2z}} U^{TE} \right) e^{ik_x(x-x')+ik_y(y-y')} e^{ik_{2z}z}, \quad (191)
$$

$$
\tilde{G}^{U}_{2zz} = \left(\frac{k_s^2 U^{TM}}{k_{2z}} \right) e^{ik_x(x-x')+ik_y(y-y')} e^{ik_{2z}z}, \quad (192)
$$

$$
\tilde{G}^{U}_{2xy} = \tilde{G}^{U}_{2yx} = \left(\partial_x\partial_y \frac{k_{2z}}{k_s^2} U^{TM} + \partial_x\partial_y \frac{k_2^2}{k_s^2 k_{2z}} U^{TE} \right) e^{ik_x(x-x')+ik_y(y-y')} e^{ik_{2z}z}, \quad (193)
$$

$$
\tilde{G}^{U}_{2xz} = -\tilde{G}^{U}_{2zx} = \partial_x\partial_z \frac{U^{TM}}{k_{2z}} e^{ik_x(x-x')+ik_y(y-y')} e^{ik_{2z}z}, \quad (194)
$$

$$
\tilde{G}^{U}_{2yz} = -\tilde{G}^{U}_{2zy} = \partial_y\partial_z \frac{U^{TM}}{k_{2z}} e^{ik_x(x-x')+ik_y(y-y')} e^{ik_{2z}z}. \quad (195)
$$

4.3.2. Green's function in the spatial domain

As expected from the previous sections, the inverse Fourier transform are applied to the spectral Green's function to obtain the one in the spatial domain. Most of basic computations are

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already performed while deriving the two- and three-layer Green's functions. Therefore, without any derivation, the Green's function in the spatial domain is presented below.

- In the first layer,

$$G_{1xx}^T = \frac{1}{2}g_{1,5}^T - \left(\frac{1}{2}\rho^2 - (y-y')^2\right)g_{1,6}^T, \tag{196}$$

$$G_{1yy}^T = \frac{1}{2}g_{1,5}^T + \left(\frac{1}{2}\rho^2 - (y-y')^2\right)g_{1,6}^T, \tag{197}$$

$$G_{1zz}^T = g_{1,7}^T, \tag{198}$$

$$G_{1xy}^T = G_{1yx}^T = -(x-x')(y-y')g_{1,6}^T, \tag{199}$$

$$G_{1xz}^T = -i(x-x')g_{1,8}^T, G_{1yz}^T = -i(y-y')g_{1,8}^T, \tag{200}$$

$$G_{1zx}^T = -i(x-x')g_{1,9}^T, G_{1zy}^T = -i(y-y')g_{1,9}^T, \tag{201}$$

where

$$\tilde{g}_{1,5}^T = k_{1z}A_1^{TM} + \frac{k_1^2}{k_{2z}}A_1^{TE}, \tilde{g}_{1,6}^T = \frac{k_{1z}A_1^{TM}}{k_s^2} - \frac{k_1^2}{k_{2z}k_s^2}A_1^{TE}, \tilde{g}_{1,7}^T = \frac{k_s^2}{k_{2z}}A_1^{TM}, \tilde{g}_{1,8}^T = \frac{k_{1z}}{k_{2z}}A_1^{TM}, \tilde{g}_{1,9}^T = A_1^{TM},$$

$$g_{1,5}^T = 2\pi \int_0^\infty k_s\tilde{g}_{1,5}^T J_0(k_s\rho)e^{ik_{1z}z}dk_s, g_{1,6}^T = 2\pi \int_0^\infty k_s^3\tilde{g}_{1,6}^T \frac{J_2(k_s\rho)}{\rho^2}e^{ik_{1z}z}dk_s$$

$$g_{1,7}^T = 2\pi \int_0^\infty k_s\tilde{g}_{1,7}^T J_0(k_s\rho)e^{ik_{1z}z}dk_s, g_{1,8}^T = 2\pi \int_0^\infty k_s^2\tilde{g}_{1,8}^T \frac{J_1(k_s\rho)}{\rho}e^{ik_{1z}z}dk_s,$$

$$g_{1,9}^T = 2\pi \int_0^\infty k_s^2\tilde{g}_{1,9}^T \frac{J_1(k_s\rho)}{\rho}e^{ik_{1z}z}dk_s. \tag{202}$$

- In the third layer, all the formulas take almost same form as the first layer except the direction of the field. Therefore, they are given by

$$G_{3xx}^T = \frac{1}{2}g_{3,5}^T - \left(\frac{1}{2}\rho^2 - (y-y')^2\right)g_{3,6}^T, \tag{203}$$

$$G_{3yy}^T = \frac{1}{2}g_{3,5}^T + \left(\frac{1}{2}\rho^2 - (y-y')^2\right)g_{3,6}^T, \tag{204}$$

$$G_{3zz}^T = g_{3,7}^T, \tag{205}$$

$$G_{3xy}^T = G_{3yx}^T = -(x-x')(y-y')g_{3,6}^T, \tag{206}$$

$$G_{3xz}^T = i(x-x')g_{3,8}^T, G_{3yz}^T = i(y-y')g_{3,8}^T, \tag{207}$$

$$G_{3zx}^T = i(x-x')g_{3,9}^T, G_{3zy}^T = i(y-y')g_{3,9}^T, \tag{208}$$

where

$$\tilde{g}^T_{3,5} = k_{3z}A^{TM}_3 + \frac{k^2_3}{k_{2z}}A^{TE}_3, \tilde{g}^T_{3,6} = \frac{k_{3z}A^{TM}_3}{k^2_s} - \frac{k^2_3}{k_{2z}k^2_s}A^{TE}_3, \tilde{g}^T_{3,7} = \frac{k^2_s}{k_{2z}}A^{TM}_3, \tilde{g}^T_{3,8} = \frac{k_{3z}}{k_{2z}}A^{TM}_3, \tilde{g}^T_{3,9} = A^{TM}_3,$$

$$g^T_{3,5} = 2\pi \int_0^\infty k_s \tilde{g}^T_{3,5} J_0(k_s\rho)e^{-ik_{3z}z}dk_s, \quad g^T_{3,6} = 2\pi \int_0^\infty k^3_s \tilde{g}^T_{3,6} \frac{J_2(k_s\rho)}{\rho^2} e^{-ik_{3z}z}dk_s,$$

$$g^T_{3,7} = 2\pi \int_0^\infty k_s \tilde{g}^T_{3,7} J_0(k_s\rho)e^{-ik_{3z}z}dk_s, \quad g^T_{3,8} = 2\pi \int_0^\infty k^2_s \tilde{g}^T_{3,8} \frac{J_1(k_s\rho)}{\rho} e^{-ik_{3z}z}dk_s,$$

$$g^T_{3,9} = 2\pi \int_0^\infty k^2_s \tilde{g}^T_{3,9} \frac{J_1(k_s\rho)}{\rho} e^{-ik_{3z}z}dk_s. \tag{209}$$

• In the second layer, both the up- and down-going waves are reflected wave from the interface. Thus, the Green's function follows similar formula as the reflected field in both two- and three-layer structures. However, again one must be careful about the sign because of direction. The up-going wave Green's function is obtained as

$$G^U_{2xx} = -\frac{1}{2}g^U_{2,5} + \left(\frac{1}{2}\rho^2 - (y-y')^2\right)g^U_{2,6}, \tag{210}$$

$$G^U_{2yy} = -\frac{1}{2}g^U_{2,5} - \left(\frac{1}{2}\rho^2 - (y-y')^2\right)g^U_{2,6}, \tag{211}$$

$$G^U_{2zz} = g^U_{2,7}, \tag{212}$$

$$G^U_{2xy} = G^U_{2yx} = (x-x')(y-y')g^U_{2,6}, \tag{213}$$

$$G^U_{2xz} = -G^U_{2zx} = -i(x-x')g^U_{2,8}, \tag{214}$$

$$G^U_{2yz} = -G^U_{2zy} = -i(y-y')g^U_{2,8}, \tag{215}$$

where

$$\tilde{g}^U_{2,5} = k_{2z}U^{TM} - \frac{k^2_2}{k_{2z}}U^{TE}, \tilde{g}^U_{2,6} = \frac{k_{2z}U^{TM}}{k^2_s} + \frac{k^2_2}{k^2_s k_{2z}}U^{TE}, \tilde{g}^U_{2,7} = \frac{k^2_s}{k_{2z}}U^{TM}, \tilde{g}^U_{2,8} = U^{TM},$$

$$g^U_{2,5} = 2\pi \int_0^\infty k_s \tilde{g}^U_{2,5} J_0(k_s\rho)e^{ik_{2z}z}dk_s, \quad g^U_{2,6} = 2\pi \int_0^\infty k^3_s \tilde{g}^U_{2,6} \frac{J_2(k_s\rho)}{\rho^2} e^{ik_{2z}z}dk_s,$$

$$g^U_{2,7} = 2\pi \int_0^\infty k_s \tilde{g}^U_{2,7} J_0(k_s\rho)e^{ik_{2z}z}dk_s, \quad g^U_{2,8} = 2\pi \int_0^\infty k^2_s \tilde{g}^U_{2,8} \frac{J_1(k_s\rho)}{\rho} e^{ik_{2z}z}dk_s. \tag{216}$$

The down-going wave Green's function is given by

$$G^D_{2xx} = -\frac{1}{2}g^D_{2,5} + \left(\frac{1}{2}\rho^2 - (y-y')^2\right)g^D_{2,6}, \tag{217}$$

$$G^D_{2yy} = -\frac{1}{2}g^D_{2,5} - \left(\frac{1}{2}\rho^2 - (y-y')^2\right)g^D_{2,6}, \tag{218}$$

$$G^D_{2zz} = g^D_{2,7}, \tag{219}$$

$$G^D_{2xy} = G^D_{2yx} = (x-x')(y-y')g^D_{2,6}, \tag{220}$$

$$G^D_{2xz} = -G^D_{2zx} = i(x-x')g^D_{2,8}, \tag{221}$$

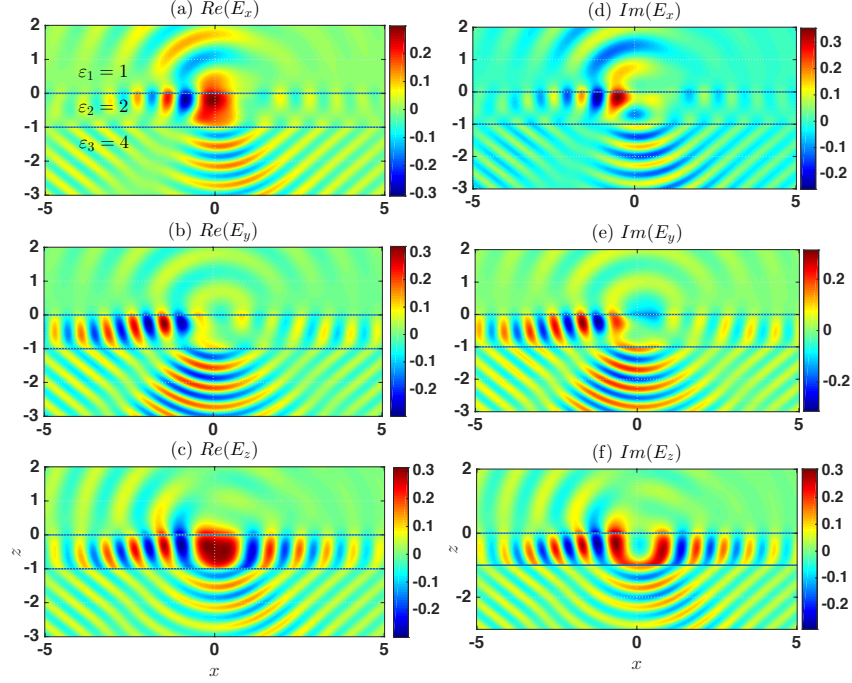$$G^D_{2yz} = -G^D_{2zy} = i(y-y')g^D_{2,8}, \tag{222}$$

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Figure 7: Electric fields in a three-layer structure with layer interface at $z = 0$ and $z = -1$. A dipole source is placed at $\mathbf{r}' = (0.1, -0.2, -0.5)$ and oriented along $\boldsymbol{\alpha}' = (1/2, 1/2, 1/\sqrt{2})$ and fields are computed for $-5 \le x \le 5$ and $-3 \le z \le 3$ for a fixed $y = 1.0$ with $\varepsilon_1 = 1$, $\varepsilon_2 = 2$, $\varepsilon_3 = 4$, and $\lambda = 1$.

where

$$\tilde{g}_{2,5}^D = k_{2z}D^{TM} - \frac{k_2^2}{k_{2z}}D^{TE}, \tilde{g}_{2,6}^D = \frac{k_{2z}D^{TM}}{k_s^2} + \frac{k_2^2}{k_s^2 k_{2z}}D^{TE}, \tilde{g}_{2,7}^D = \frac{k_s^2}{k_{2z}}D^{TM}, \tilde{g}_{2,8}^D = D^{TM},$$

$$g_{2,5}^D = 2\pi \int_0^\infty k_s \tilde{g}_{2,5}^D J_0(k_s\rho)e^{-ik_{2z}z}dk_s, g_{2,6}^D = 2\pi \int_0^\infty k_s^3 \tilde{g}_{2,6}^D \frac{J_2(k_s\rho)}{\rho^2}e^{-ik_{2z}z}dk_s,$$

$$g_{2,7}^D = 2\pi \int_0^\infty k_s \tilde{g}_{2,7}^D J_0(k_s\rho)e^{-ik_{2z}z}dk_s, g_{2,8}^D = 2\pi \int_0^\infty k_s^2 \tilde{g}_{2,8}^D \frac{J_1(k_s\rho)}{\rho}e^{-ik_{2z}z}dk_s. \tag{223}$$

4.3.3. Numerical results

The Green's function is computed when the source is placed in the second layer. Consider a three-layer structure defined by two interfaces located at $z = 0$ and $z = -1$. The relative permittivity in each layer is $\varepsilon_1 = 1$, $\varepsilon_2 = 2$, $\varepsilon_3 = 4$ and a dipole source is placed at $\mathbf{r}' = (0.1, -0.2, -0.5)$ oriented along $\hat{\boldsymbol{\alpha}}' = (1/2, 1/2, 1/\sqrt{2})$ in the second layer. The relative permeability $\{\mu_i\}_{i=1}^3$ is assumed to be 1 in all layers. The wavelength is set as $\lambda = 1$. In Fig. 7, all the components of total electric field are plotted over $-5 \le x \le 5$ and $-3 \le z \le 3$ for a fixed $y = 1.0$. The continuity of the fields are checked at both interfaces $z = 0$ and $z = -1$ in Fig. 8. In all components, about 10^{-7} is achieved.
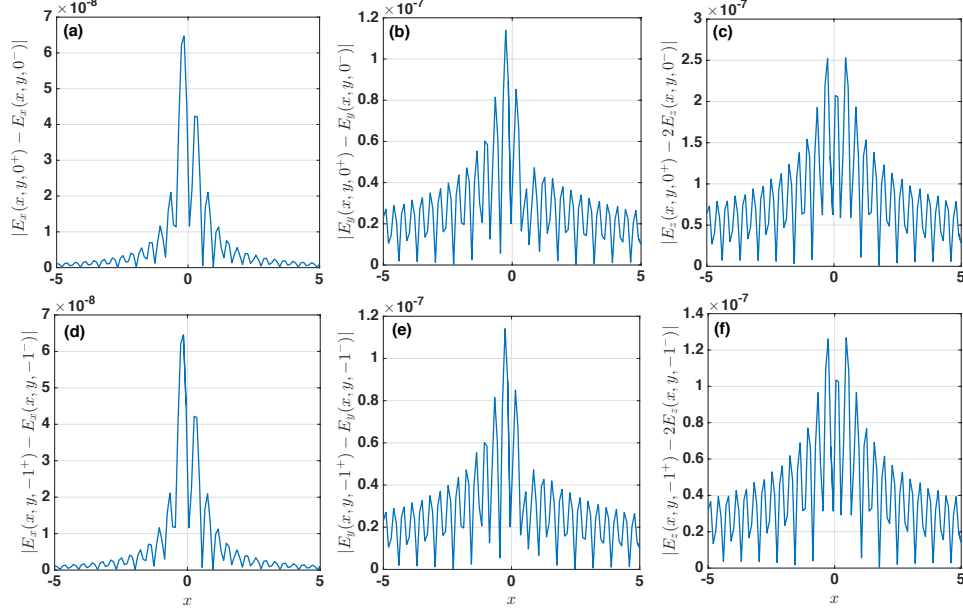
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Figure 8: Continuity of electric fields at both interfaces at $z = 0$ and $z = -1$. (a) $|E_x(x, y, 0^+) - E_x(x, y, 0^-)|$, (b) $|E_y(x, y, 0^+) - E_y(x, y, 0^-)|$, (c) $|E_z(x, y, 0^+) - 2E_z(x, y, 0^-)|$, (d) $|E_x(x, y, -1^+) - E_x(x, y, -1^-)|$, (e) $|E_y(x, y, -1^+) - E_y(x, y, -1^-)|$, (f) $|E_z(x, y, -1^+) - 2E_z(x, y, -1^-)|$. A dipole source is placed at $\mathbf{r}' = (0.1, -0.2, -0.5)$ and oriented along $\boldsymbol{\alpha}' = (1/2, 1/2, 1/\sqrt{2})$ and fields are computed for $-5 \leq x \leq 5$ for a fixed $y = 1.0$ at the layer interface $z = 0$ and $z = -1$ and with $\varepsilon_1 = 1$, $\varepsilon_2 = 4$, $\varepsilon_3 = 1.1$, and $\lambda = 1$.

5. Conclusion

The electric field dyadic Green's function for a two- and three-layer structure in 3-D are presented. The two-layer Green's function is simpler than the one in Ref. [5] and uses one less Sommerfeld integral. An adaptive generalized quadrature rule is applied to Sommerfeld integral to obtain very high accuracy. Therefore, the proposed method is more accurate and fast. Also it can be easily extended multi-layered media without any modification except replacing the reflection and transmission coefficient. As an example, a three-layer Green's function is presented to show the easy extension to multi-layered media. The singular part is naturally separated as a primary field that is the free-space Green's function. Therefore, the Green's function is readily applicable to integral equation methods. The Lippmann-Schwinger type volume integral equation used for the free space in Ref. [7] is being modified with the new Green's function to study many scatterers embedded in layered media.

As a relevant research issue, a fast solver will be developed using the derived formulas for the Green's function for large-scale problems. Either a new fast multipole method type method [18, 19, 20] or a preconditioner [21] based method could be considered to accelerate an iterative matrix solver.

Acknowledgement

Appendix A. Bessel identities

A derivation of the dyadic Green's function in multi-layered media is very tedious but it is required for a proper implementation. The Bessel identities play a key role in the derivation and they are based on a integral representation of the Bessel function and recurrence relation (See Ref. [22]), namely,

$$J_n(z)e^{in\theta} = \frac{1}{2\pi} \int_0^{2\pi} e^{iz\cos(\phi-\theta)+in\phi-in\frac{\pi}{2}} d\phi, \tag{A.1}$$

$$J_{n+2}(z) = \frac{n+2}{z} J_{n+1}(z) - J_n(z), \tag{A.2}$$

respectively. For convenience, the most often used identities are listed in the following

$$\int_0^{2\pi} e^{iz\cos(\phi-\theta)} d\phi = 2\pi J_0(z), \tag{A.3}$$

$$\int_0^{2\pi} e^{iz\cos(\phi-\theta)} \cos\phi \, d\phi = 2\pi i J_1(z)\cos\theta, \tag{A.4}$$

$$\int_0^{2\pi} e^{iz\cos(\phi-\theta)} \sin\phi \, d\phi = 2\pi i J_1(z)\sin\theta, \tag{A.5}$$

$$\int_0^{2\pi} e^{iz\cos(\phi-\theta)} \cos 2\phi \, d\phi = -2\pi J_2(z)\cos 2\theta, \tag{A.6}$$

$$\int_0^{2\pi} e^{iz\cos(\phi-\theta)} \sin 2\phi \, d\phi = -2\pi J_2(z)\sin 2\theta, \tag{A.7}$$

$$\int_0^{2\pi} e^{iz\cos(\phi-\theta)} \cos^2\phi \, d\phi = \pi J_0(z) - \pi J_2(z)\cos 2\theta, \tag{A.8}$$

$$\int_0^{2\pi} e^{iz\cos(\phi-\theta)} \sin^2\phi \, d\phi = \pi J_0(z) + \pi J_2(z)\cos 2\theta. \tag{A.9}$$

References

[1] J. D. Joannopoulos, S. G. Johnson, R. D. Meade, J. N. Winn, Photonic Crystals: Molding the Flow of Light, 2nd Edition, Princeton University, 2008.

[2] H. A. Atwater, A. Polman, Plasmonics for improved photovoltaic devices, Nature Materials 9 (3) (2010) 205–213.

[3] K. A. Sablon, J. W. Little, V. Mitin, A. Sergeev, N. Vagidov, K. Reinhardt, Strong enhancement of solar cell efficiency due to quantum dots with built-in charge, Nano Letters 11 (2011) 2311–2317.

[4] G. Gustafsson, Y. Cao, G. M. Treacy, F. Klavetter, N. Colaneri, J. Heeger, Flexible light-emitting diodes made from soluble conducting polymers, Nature 357 (1992) 477–479.

[5] J. Cui, W. C. Chew, Fast evaluation of Sommerfeld integrals for EM scattering and radiation by three-dimensional buried objects, IEEE Trans. Geoscience and Remote Sensing 37 (2) (1999) 887–900.

[6] J. Cui, W. Wiesbeck, A. Herschlein, Electromagnetic scattering by multiple three-dimensional scatterers buried under multilayered media- part I : Theory, IEEE Trans. Geoscience and Remote Sensing 36 (2) (1998) 526–534.

[7] D. Chen, W. Cai, B. Zinser, M. H. Cho, Accurate and efficient Nyström volume integral equation method for the maxwell equations for multiple 3-d scatterers, J. Comput. Phys. 321 (2016) 303–320.

[8] M. H. Cho, A. Barnett, Robust fast direct integral equation solver for quasi-periodic scattering problems with a large number of layers, Optics Express 23 (2015) 1775–1799.

[9] J. Lai, M. Kobayashi, A. H. Barnett, A fast solver for the scattering from a layered periodic structure with multi-particle inclusions, J. Comput. Phys. 298 (2015) 194–208.

[10] W. Cai, Computational Methods for Electromagnetic Phenomena: Electrostatics in Solvation, Scattering, and Electron Transport, Cambridge Univ. Press, 2013.

[11] W. Cai, Algorithmic issues for electromagnetic scattering in layered media: Green's functions, current basis, and fast solver, Adv. Comput. Math 16 (2002) 157–174.

[12] J. Ma, V. Rokhlin, S. Wandzura, Generalized gaussian quadrature rules for systems of arbitrary functions, Research report YALEU/DCS/RR-990.

[13] M. H. Cho, W. Cai, A parallel fast algorithm for computing the Helmholtz integral operator in 3-D layered media, J. Comput. Phys. 231 (2012) 5910–5925.

[14] J. A. Stratton, Electromagnetic Theory, John Wiley & Sons, 2007.

[15] P. Yeh, Optical waves in layered media, 2nd Edition, Wiley-Interscience, 2005.

[16] W. C. Chew, Waves and Fields in Inhomogeneous Media, Wiley-IEEE Press, 1999.

[17] A. Sommerfeld, Partial Differential Equations in Physics, Academic Press, 1949.

[18] V. Rokhlin, Rapid solution of integral equations of scattering theory in two dimensions, J. Comput. Phys. 86 (2) (1990) 414–439.

[19] L. Greengard, V. Rokhlin, A fast algorithm for particle simulations, J. Comput. Phys. 73 (1987) 325–348.

[20] M. H. Cho, W. Cai, Fast integral equation solver for Maxwell's equations in layered media with FMM for Bessel functions, Science China Math 56 (12) (2013) 2561–2570.

[21] L. Ying, Sparsifying preconditioner for the lippmann–schwinger equation, Multiscale Modeling & Simulation 13 (2) (2015) 644–660.

[22] M. Abramowitz, I. A. Stegun, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, 10th Edition, Dover, 1964.