

Implementation and Application of Resistive Sheet Boundary Condition in the Finite-Difference Time-Domain Method

Lin-Kun Wu, *Member, IEEE*, and Liang-Tung Han

Abstract—Use of resistive sheet boundary condition in the finite-difference time-domain (FDTD) analysis of scattering problems involving resistively coated dielectric object is described. The algorithm is introduced through an analysis of *E*-polarized scattering from a thin resistive strip. For the sheet resistance $R = \eta Z_0/2$ with Z_0 being the intrinsic impedance of vacuum, numerical experiments indicate that algorithm stability is ensured in all cases if δt is chosen according to $c\eta\delta t \leq \delta$. Validity of the resultant FDTD method is verified in a comparison of computed *E*-polarized scattering data for several resistive strips with existing data. New results on the *E*-polarized scattering behavior of resistively coated dielectric strip as a function of surface resistances and angle of incidence are also presented. Finally, extension to the *H*-polarized case and application of the present method to pulsed problems are briefly discussed.

I. INTRODUCTION

THE finite-difference time-domain method (FDTD) proposed by Yee [1] has found applications in a wide variety of electromagnetic wave interaction problems involving perfectly conducting and lossy dielectric and magnetic materials [2], uniaxially anisotropic dielectric [3], and dispersive material medium [4], [5] in both the unbounded and bounded regions. In this paper, we extend the capability of FDTD to the analysis of a resistively coated dielectric object. This important class of scattering geometry can be found in such applications associated with radar cross-section reduction technology [6]–[9], EMP and precipitation static protection, and defogging and deicing of radomes [10].

The resistive film coating to be considered in this paper is made of an imperfectly conducting material having a conductivity σ and a thickness τ which is smaller than the skin depth of the material and is thus difficult to be modeled by the FDTD. It is approximated as a zero thickness resistive sheet having a surface resistance $R = (\sigma\tau)^{-1}$ when $\tau \leq$ the skin depth δ_s ; $R = (\sigma\delta_s)^{-1}$ becomes frequency-dependent when $\tau > \delta_s$. Although the use of impedance boundary condition is common in the frequency-domain technique e.g., [6]–[9], this paper describes for the first time in the open literature the implementation of the FDTD method in its application of the impedance boundary condition associated with a thin resistive sheet ($\tau \leq \delta_s$) [11].

In the following, the analysis technique is first derived for a two-dimensional *E*-polarized scattering problem of a thin resistive strip. The numerical stability issue of the method is also discussed. The FDTD technique is validated by comparing the *E*-polarized scattering data for various resistive strips with the data described in [6]. New results on the *E*-polarized scattering behavior of a resistively coated dielectric strip are presented. Finally, extension to the *H*-polarized case and application of the present method to pulsed problems are briefly discussed.

II. FORMULATION

Consider the problem of an *E*-polarized plane wave incident on a thin resistive strip of width w along the x -axis and of resistance $R = \eta Z_0/2$ (with Z_0 being the intrinsic impedance of the free space) as shown in Fig. 1. The strip is assumed to extend to infinity along $\pm z$ directions. The strip becomes a perfect electrically conducting strip when $\eta = 0$, and it is nonexistent when η approaches infinity.

The infinitesimally thin, electrically resistive, sheet is characterized by a jump discontinuity in the tangential magnetic field components but no discontinuity in the tangential electric field components [11]. Since for the problem considered here, both the incident and the total electric fields have only a z -directed component which is tangential to the surface of the strip, the desired resistive sheet boundary condition can be written as [11]

$$\begin{aligned} E_z(x, y = 0^+) \\ &= E_z(x, y = 0^-) = E_z(x, y = 0) \\ &\quad \dots \\ &= -RJ_z(x, y = 0) \end{aligned} \quad (1)$$

for $-w/2 \leq x \leq w/2$ and

$$J_z(x, y = 0) = H_x(x, y = 0^+) - H_x(x, y = 0^-) \quad (2)$$

is the total (z -directed) electric current density on the surface of the strip.

For the FDTD implementation of (1) and (2), the placement of E_z , H_x , and H_y sampling points in the 2-D FDTD lattice surrounding the resistive strip are shown in Fig. 2. In this way, only E_z 's located on the surface of the strip need be updated with the FDTD equivalents of (1) and (2) while the

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The authors are with the Institute of Communication Engineering, National Chiao Tung University, Hsinchu, Taiwan, Republic of China.
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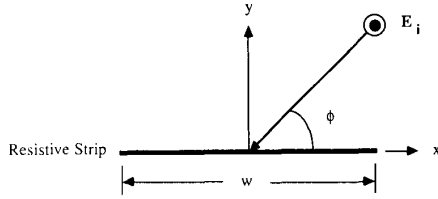


Fig. 1. An E -polarized plane wave illumination of a thin resistive strip of width w and resistance R . The strip extends to infinity along $\pm z$ -axis.

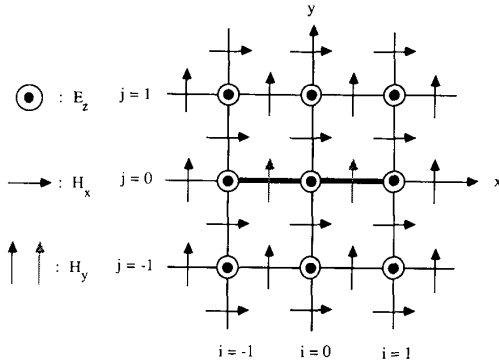


Fig. 2. Illustration of the placement of the field components near the strip in the 2-D (xy -plane) FDTD lattice for E -polarization case.

conventional FDTD method is still applicable for the update of all other field quantities elsewhere in the FDTD lattice. The regular FDTD and the resistive boundary algorithm are described next.

A. Regular FDTD Algorithm

In a lossless medium with permittivity ϵ and permeability μ , Maxwell's curl equations in the xy -plane for E -polarized case are

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu} \frac{\partial E_z}{\partial y} \quad (3a)$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu} \frac{\partial E_z}{\partial x} \quad (3b)$$

$$\frac{\partial E_z}{\partial t} = \frac{1}{\epsilon} \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \quad (3c)$$

Following the space and time discretizations of these field quantities, their discrete representations can be denoted as [1]

$$F^n(i, j) = F(x = i\delta, y = j\delta, t = n\delta t) \quad (4)$$

where $F = E_z, H_x$, or H_y , and δ and δt are sampling intervals in space and time, respectively; use of square cells assumed. According to (4), central differencing of the spatial and temporal derivatives in (3) result in the following FDTD

equations:

$$H_x^{n+1/2}(i, j + 1/2) = H_x^{n-1/2}(i, j + 1/2) - S_1 [E_z^n(i, j + 1) - E_z^n(i, j)] \quad (5a)$$

$$H_y^{n+1/2}(i + 1/2, j) = H_y^{n-1/2}(i + 1/2, j) - S_1 [E_z^n(i, j) - E_z^n(i + 1, j)] \quad (5b)$$

$$\begin{aligned} E_z^{n+1}(i, j) &= E_z^n(i, j) + S_2 [H_y^{n+1/2}(i + 1/2, j) \\ &\quad - H_y^{n+1/2}(i - 1/2, j) - H_x^{n+1/2}(i, j + 1/2) \\ &\quad + H_x^{n+1/2}(i, j - 1/2)] \end{aligned} \quad (5c)$$

where $S_1 = \delta t / \mu \delta$ and $S_2 = \delta t / \epsilon \delta$. Similar FDTD equations for a general lossy dielectric/magnetic material have been given elsewhere (e.g., [2]).

Finally, for the numerical results to remain stable, the space and time sampling intervals have to be chosen to satisfy the Courant stability criterion in 2-D [12]

$$v_{p, \max} \delta t \leq \frac{\delta}{\sqrt{2}} \quad (6)$$

where $v_{p, \max}$ is the maximum phase velocity to be expected in the problem space. For an unbounded region problem involving a perfectly conducting, dielectric, and/or magnetic scatterer, $v_{p, \max}$ is chosen as the speed of light in the free-space region between the scatterer and the outer boundary of the FDTD lattice.

B. Implementation of Resistive Boundary Condition in FDTD

It is obvious that (5c) is incapable of incorporating the presence of a resistive strip. In searching for an alternative algorithm, the resistive boundary condition given in (1) and (2) suggests that E_z on the surface of the strip can be determined if H_x on the upper and lower surfaces of the strip are known. In this paper, these latter field quantities are approximated by H_x located at half-cells above and below the strip surface, i.e.,

$$H_x(i, 0^\pm) \approx H_x(i, \pm 1/2). \quad (7)$$

In the FDTD, all E_z and H_x are assumed to be known at $n\delta t$ and $(n + 1/2)\delta t$, respectively. Upon substituting (7) into (2), and using the resultant J_z in (1), we obtain the following intermediate value of E_z at $(n + 1/2)\delta t$

$$E_z^{n+1/2}(i, 0) = -R [H_x^{n+1/2}(i, 1/2) - H_x^{n+1/2}(i, -1/2)] \quad (8)$$

which is the FDTD equivalent of (1) and (2) combined. However, for updating $H_x(i, \pm 1/2)$ using (5a), a knowledge of $E_z(i, 0)$ at $(n + 1/2)\delta t$ is needed. By using the values of $E_z(i, 0)$ at $n\delta t$ and $(n + 1/2)\delta t$ in a simple linear

extrapolation procedure, we obtain

$$E_z^{n+1}(i, 0) = -E_z^n(i, 0) - \eta Z_0 [H_x^{n+1/2}(i, 1/2) - H_x^{n+1/2}(i, -1/2)]. \quad (9)$$

Once $E_z(i, 0)$ on the strip is determined, all field quantities elsewhere in the FDTD lattice can then be updated regularly using (5a)–(5c). It is noted that a knowledge of H_y at the surface of the strip (shown by dashed arrows in Fig. 2) is no longer needed and can thus be removed from the FD-TD algorithm.

C. Numerical Stability Issue

While the resistive boundary algorithm appears to be rather simple and straightforward, numerical experiments have shown that when modeling a strip with $\eta > 2$ (i.e., $R > Z_0$), and δt is so chosen that $c\delta t = \delta/2$ (i.e., (6) is satisfied), numerical instability occurs as soon as the strip begins to interact with the incident wave. This is demonstrated in Fig. 3(a) with the rapidly exploding time waveform observed for E_z at one cell to the right of the illuminated edge of the strip. The parameters $\eta = 4$, $w = 3\lambda$, $\phi = 0^\circ$, $\delta = \lambda/40$, and $c\delta t = \delta/2$ are used in this example. Reducing δ and δt while still maintaining $c\delta t = \delta/2$ only results in a slower rate of numerical breakup.

For the same problem shown in Fig. 3(a), further numerical experiments indicate that numerical stability can be achieved, as shown in Fig. 3(b), if δt is chosen such that $c\eta\delta t \leq \delta$. By referring to (6), the empirically derived stability criterion may also be interpreted as the presence of a fast-traveling lattice mode which has a phase velocity $v_p = c\eta/\sqrt{2}$ ($v_p > c$ when $\eta > \sqrt{2}$) when the resistive boundary algorithm is invoked. This is possible because with H_y on the surface of the strip being ignored, and (9) being different from (5c), the strip is actually embedded in a lattice that is different from the regular FDTD lattice (i.e., the one that supports (5)). A similar algorithm (in-)stability issue has been reported for the FDTD which employs a spatially varying grid [13]. Although a theoretical proof supporting the empirically derived stability criterion (i.e., $c\eta\delta t \leq \delta$) is not available, tests conducted so far for $2 \leq \eta \leq 10$ yield satisfactory results.

III. NUMERICAL EXAMPLES

A. Scattering from Single Resistive Strip

For the strip shown in Fig. 1 with $w = 3\lambda$, induced current densities along the width dimension are calculated for $\eta = 1, 4$, and 10 at edge-on incidence ($\phi = 0^\circ$). As shown in Fig. 4, our data is in good agreement with Senior's results [6, fig. 7]. Fig. 5 further compares the angular backscattering cross section patterns calculated by the FDTD and the method of moments [6] for an 1.25λ -strip having $\eta = 0, 1, 2$, and 4. Again, good agreement is observed. These comparisons confirm the validity of the resistive boundary algorithm derived in this paper.

B. Scattering from Resistively Coated Dielectric Strip

Given the ability of the FDTD to model accurately the scattering properties of a single resistive strip, we consider

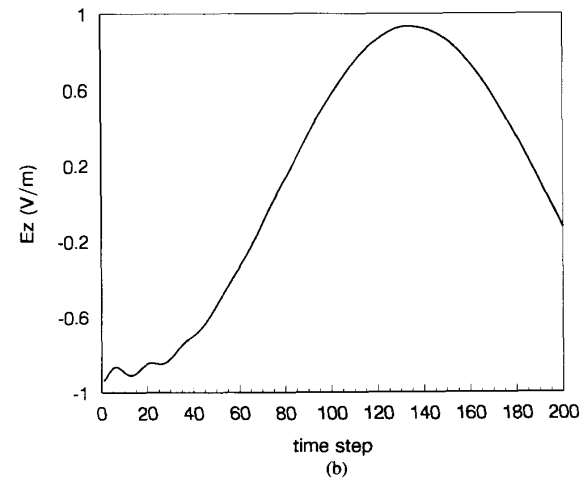
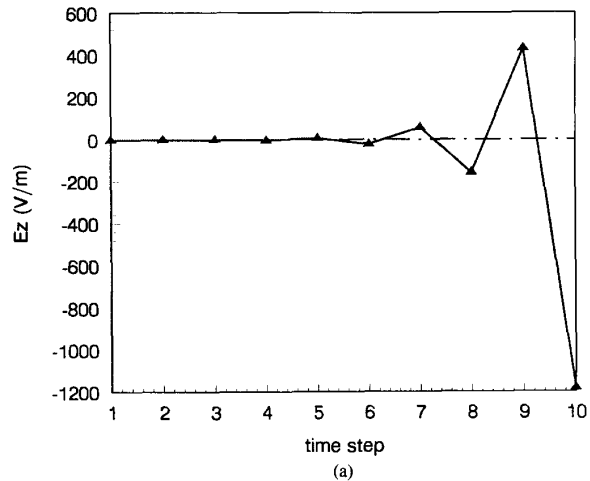


Fig. 3. Illustration of stability performance of the resistive boundary algorithm. (a) Occurrence of numerical breakup when $c\delta t = \delta/2$ is used. (b) Stable result obtained when $c\eta\delta t = \delta$ ($\eta = 4$) is used. Data are taken for E_z at one cell to the right of the illuminated edge of the strip.

here the problem of scattering from a resistively coated dielectric strip shown in Fig. 6. Such structures are used in electromagnetic shielding of sensitive components and environmental protection of radomes [10].

Consider the structure shown in Fig. 6. A lossy dielectric strip having a width of 5.221λ , a thickness of 0.05λ , and $\epsilon_r = 4 - j0.4$, is coated on the lower side by a thin resistive film. The scattering behavior of the composite structure is investigated for the coating with $\eta = 0$ (i.e., PEC), 1, and 4. For the PEC strip, (9) is removed from the FDTD program and E_z on the strip are simply set to zero at all times to enforce the exact boundary condition. The E -polarized backscattering pattern, calculated over the angular range of $-90^\circ \leq \phi \leq 90^\circ$ for the three structures, are shown in Figs. 7–9. Scattering patterns obtained for the constituting dielectric strip (i.e., letting $\eta = \infty$ in Fig. 6) and resistive strip (i.e., letting $\epsilon_r = 1$ in Fig. 6) are also shown in each figure for comparison.

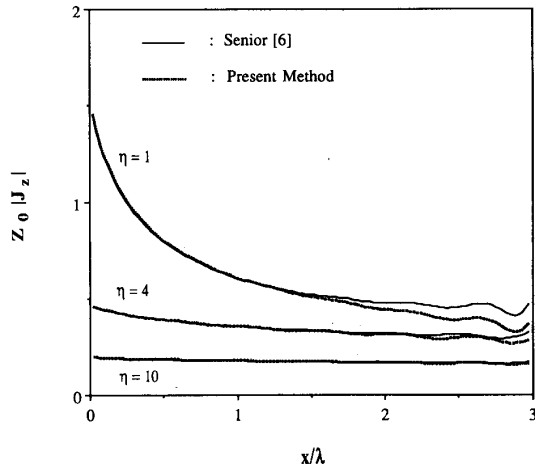


Fig. 4. Current amplitude distributions computed for resistive strips with width = 3λ and $\eta = 1, 4$, and 10 . E -polarized plane wave at edge-on incidence.

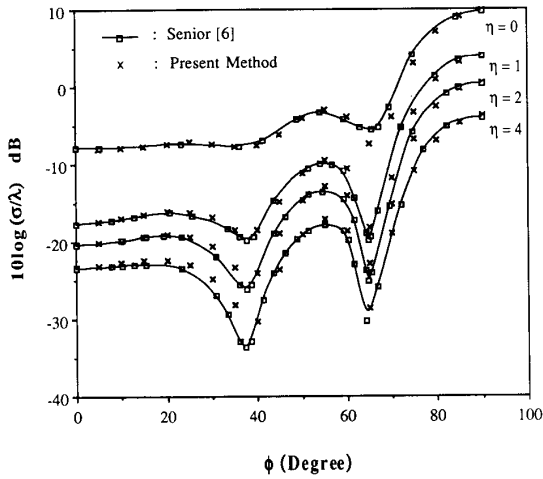


Fig. 5. Backscattering cross-section patterns computed for resistive strips having width = 1.25λ and $\eta = 0$ (i.e., PEC strip), $1, 2$, and 4 .

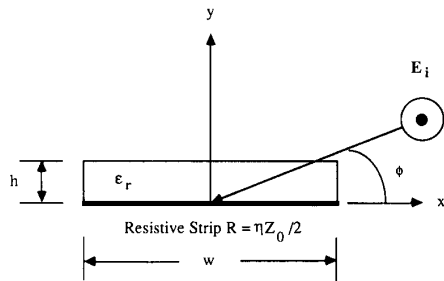


Fig. 6. Cross-sectional geometry of a resistively coated dielectric strip illuminated by an E -polarized plane wave at an arbitrary incidence angle.

Fig. 7 shows that PEC strip has much stronger backscatter than the dielectric strip at all angles. As a result, scattering from the PEC-coated dielectric strip resembles that of the PEC alone. In this case, data obtained for the PEC strip alone

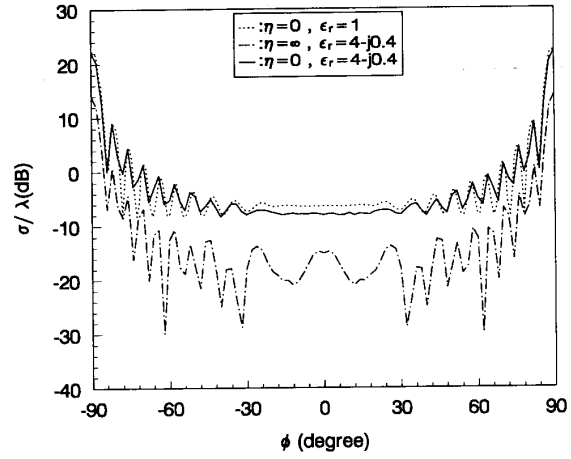


Fig. 7. Comparison of E -polarized backscattering cross-section patterns calculated for a PEC coated dielectric strip and the constituting PEC and dielectric strips. The dielectric strip has a thickness of 0.05λ and $\epsilon_r = 4 - j0.4$.

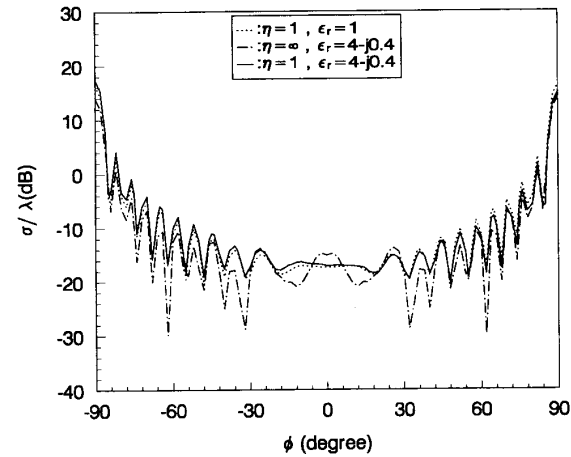


Fig. 8. Comparison of E -polarized backscattering cross-section patterns calculated for a resistively coated dielectric strip and the constituting resistive and dielectric strips. The resistive strip has a $\eta = 1$, and the dielectric strip has a thickness of 0.05λ and $\epsilon_r = 4 - j0.4$.

are in excellent agreement with the corresponding data reported in [14]. In Fig. 8, the resistive strip with $\eta = 1$ exhibits slightly larger backscatter than the dielectric strip at angles away from grazing. The scattering behavior of the coated dielectric strip resembles that of the resistive strip. When surface resistance increases further to $\eta = 4$, results shown in Fig. 9 indicate that the backscatter from the resistive strip alone is less than that from the dielectric strip. The resulting composite structure also has a backscatter pattern similar to that of the dielectric strip except for the lobe structure near grazing. Near grazing, it appears that the shape and the strength of the lobe are influenced by the resistive film and the dielectric, respectively.

Furthermore, it is interesting to point out that these results also indicate that slightly larger echo widths are observed

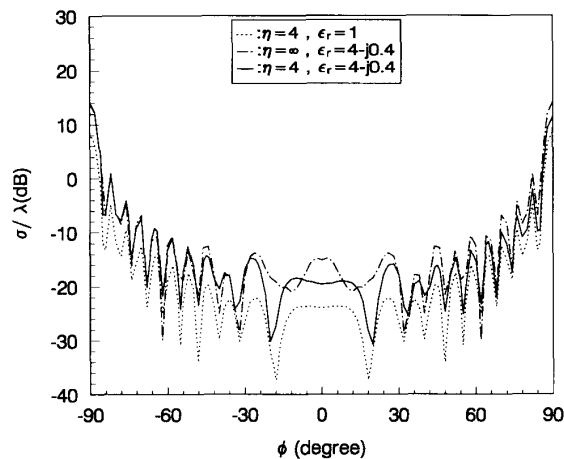


Fig. 9. Comparison of E -polarized backscattering cross-section patterns calculated for a resistively coated dielectric strip and the constituting resistive and dielectric strips. The resistive strip has a $\eta = 4$, and the dielectric strip has a thickness of 0.05λ and $\epsilon_r = 4 - j0.4$.

when the coated side of the composite structure is illuminated directly (i.e., $\phi < 0^\circ$) than when the wave is incident from the dielectric side (i.e., $\phi > 0^\circ$); the difference becomes more obvious as η increases (i.e., resistive film becomes more transparent to the incident wave). To explain this latter observation, let us consider the problem of reflection from a resistively coated dielectric slab of infinite extent in both x - and z -direction. The thickness and dielectric constant of the slab and the surface resistance of the coating are chosen to be the same as the corresponding strip cases. At $\phi = -90^\circ$, the infinite slab model yields power reflection coefficients of 0, -4.76 , and -7.35 dB, respectively for $\eta = 0, 1$, and 4 . On the other hand, power reflection coefficients of -0.09 , -7.02 , and -9.48 dB are calculated at $\phi = 90^\circ$ and $\eta = 0, 1$, and 4 , respectively. Results of these calculations exhibit similar behavior as the backscatter data at $\phi = 90^\circ$ and, therefore, indicate that the above observation is valid.

IV. CONCLUSION

Use of the resistive sheet boundary condition in the finite-difference time-domain method for the analysis of E -polarized scattering problems involving thin resistive strip is described in detail. Changes occurring to the lattice structure when the resistive boundary algorithm is invoked may have resulted in the presence of a spurious lattice mode which has a higher phase velocity than the speed of light in free space. Its effect on the solution growth factor can be suppressed, however, when a more stringent stability criterion, which requires δt to be chosen for $c\eta\delta t \leq \delta$ over the range of $2 \leq \eta \leq 10$, is used. Although an exact theoretical proof is not available, the empirically derived stability criterion has been found to yield stable result.

Validity of the resulting FDTD method is confirmed by the excellent agreement observed between scattering data of various resistive strips calculated by the present method and the data given in [6]. Application of the resistive boundary

algorithm to the analysis of scattering behavior of several resistively coated dielectric strips has resulted in the following interesting observations. Notably, the echo width of a resistive strip, illuminated by an E -polarized plane wave from the edge-on direction, could be altered significantly in the presence of a thin dielectric strip. In addition, different scattering behavior results whether one or the other side of the coated dielectric strip is illuminated.

Finally, the resistive boundary algorithm described in this paper for the E -polarized scattering problem can be extended easily to model the H -polarized problem. In addition, for a given material conductivity, the surface resistance of a resistive sheet is ideally frequency independent as long as the sheet thickness is smaller than the skin depth of the material. Thus, the FDTD method described here is applicable for both single-frequency and broad-band pulsed scattering problems. For the latter, the upper frequency (f_u) is limited to the frequency where the sheet thickness equals the skin depth, i.e., $\tau = (\pi f_u \mu \sigma)^{1/2}$; beyond which, dispersion in the surface resistance must be considered.

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Lin-Kun Wu (S'81-M'85) was born in Hsinchu, Taiwan, Republic of China, on November 1, 1958. He received the Diploma in electrical engineering from Taipei Institute of Technology, Taipei, Taiwan, Republic of China, in 1978, and the M.S. and Ph.D. degrees in electrical engineering from the University of Kansas, Lawrence, in 1982 and 1985, respectively.

From November 1985 to December 1987 he was a Postdoctoral Research Associate at the Radar Systems and Remote Sensing Laboratory of the University of Kansas, working in radar remote sensing and computational electromagnetics. Since February 1988 he has been on the faculty of the Department of Communication Engineering at the National Chiao Tung University, Hsinchu, Taiwan, Republic of China, where he is currently an Associate Professor. His research interests include microwave and millime-

ter-wave remote sensing, radar system engineering, computational electromagnetics, and radio wave propagation.



Liang-Tung Han was born in Taipei, Taiwan, Republic of China, on September 3, 1967. He received the B.S. degree in electronic engineering from Feng Chia University, Taichung, Taiwan, in 1989 and the M.S.E.E. degree from National Chiao Tung University, Hsinchu, Taiwan, in 1991.

His master's thesis deals with the analysis of electromagnetic scattering from resistively coated dielectric objects. He is currently working at Chung Shan Institute of Science and Technology, Taiwan. His research interests include antennas, radar, and fiber optics.