Supplementary information

Green-function of a line-source on a metallo-dielectric interface

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As illustrated in Fig. 3a of the companion manuscript, the near- and far-field optical responses of a nano-object (with lateral dimensions much smaller than the wavelength) located in the close vicinity of a metallo-dielectric interface is likely to be similar to that of a point or line source.

In the orthogonal Cartesian coordinate (x,y,z) system, we consider an interface in the (x,y)-plane between two semi-infinite media with relative permittivities ε_d (dielectric) and ε_m (metal). Under TM polarization, the Green-function response of a line source (parallel to the y-direction) located at the point x = z = 0 satisfies the Helmoltz equation

$$\frac{\partial}{\partial x}\left(\frac{1}{\varepsilon}\frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon}\frac{\partial H}{\partial z}\right)+k_0^2 H = k_0^2\delta, \quad (1)$$

where δ is the 2D Dirac distribution. The classical solution H(x, z=0) on the interface is given by

$$H(x,z=0)=\int_{-\infty}^{\infty}\frac{\exp(ik_0\beta x)d\beta}{i\left(\sqrt{\varepsilon_d-\beta^2}/\varepsilon_d+\sqrt{\varepsilon_m-\beta^2}/\varepsilon_m\right)}. \quad (2)$$

By use of a complex continuation in the half-complex plane defined by Im(β)>0, we can define a new integration path (red dots in the Figure) for the Green function calculation. Noting that the SPP normalized propagation constant k_{sp}/k_0 is a pole of the integrand and by use of the Cauchy's integral formula[i] as $A\rightarrow\infty$, the contribution of the half-circle vanishes and the Green function may be written as H(x, z=0) = H_{SP} + H_c, where H_{SP} and H_c are respectively the contributions of the SPP and of the near-field (other than the SPP field) creeping at the interface. As the residue of the integrand, H_{SP} is known analytically

$$H_{SP} = 2\pi\,\frac{k_{SP}^2}{k_0^2}\frac{\sqrt{\varepsilon_d\varepsilon_m}}{\varepsilon_m-\varepsilon_d}\exp(ik_{SP}x). \quad (3)$$

H_c is tightly connected to the integration along the branch cuts (blue curves) of the metal and dielectric media. It has to be calculated numerically as the sum of two integrals H_c = I_d + I_m, with $I_m = \exp(-i\pi/4)\frac{\varepsilon_d}{\varepsilon_m-\varepsilon_d}\int_0^\infty \frac{\exp\left(ik_0x\sqrt{\varepsilon_m+it}\right)\sqrt{t}}{\left(1-(\varepsilon_m+it)k_0^2/k_{SP}^2\right)\sqrt{\varepsilon_m+it}}dt$; I_d being obtained by the exchange $\varepsilon_d \leftrightarrow \varepsilon_m$.

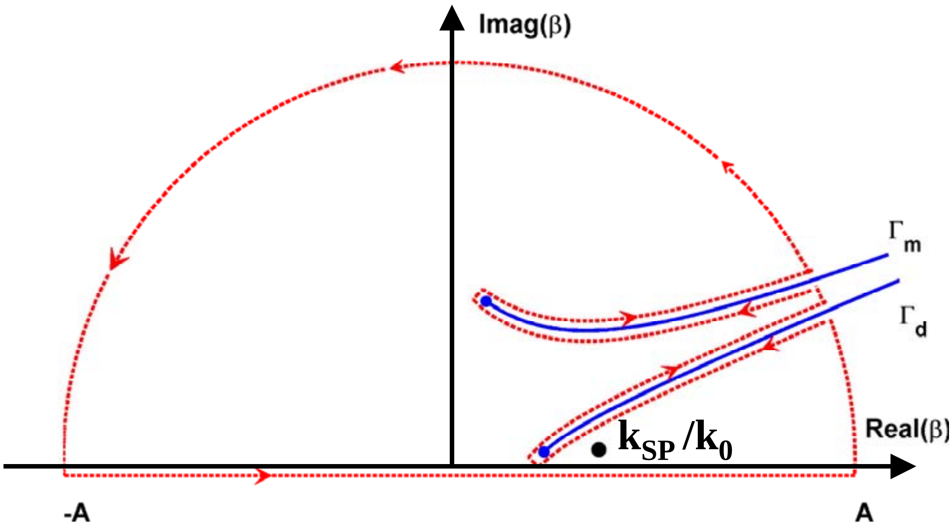

Figure 1. Integration path for the Green function calculation in the complex β-plane. The branch cuts are defined by the parametric equations $\varepsilon_d-\beta^2 = -it$ (Γ_d) and $\varepsilon_m-\beta^2 = -it$ (Γ_m) with t>0. The black dot represents the SPP pole.

Plots of $|H_{SP}(x)|$ and $|H_c(x)|$ are shown in Fig. 5 of the companion manuscript for several wavelengths of interest.

[i] G.B. Arfken and H.J. Weber, *Mathematical methods for physicists*, 6th edition, chapter 6, Elsevier Academic Press, Amsterdam (2005).