Journal of the Physical Society of Japan, Vol. 36, No. 2, February, 1974

Theory of Surface Waves Coupled to Surface Carriers

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(Received September 17, 1973)

A new type of surface mode of the electromagnetic wave is predicted in the case when charge carriers are located at the boundary of dielectric media. The dispersion relation of the mode is derived and its characteristic feature is discussed. Effect of the surface carrier is represented in terms of characteristic frequency Ω_s $(=4\pi N_s e^2/m^*c$; N_s is density of the surface carrier). The dispersion takes the linear form in the low frequency region ($\omega \ll \Omega_s$), while it has the square root form ($\omega \propto \sqrt{k}$) in the high frequency limit ($\omega \gg \Omega_s$). In contrast to the case of the absence of surface carriers the present mode exists in the frequency region where at least one of the dielectric functions of the adjacent media takes positive value. As a numerical example the dispersion relation is calculated and illustrated for clean surface of p-type InAs.

§ 1. Introduction

In recent years much works have been done on the surface wave (hereafter shortned as SW), i.e., nonradiative electromagnetic wave localized at the boundary of two dielectric media.[1] According to the dielectric nature of the media SW is classified as surface polariton[2-4] (ionic crystal-vacuum), surface plasmon[5-7] (free carriers-dielectrics) and surface plasmon-phonon coupled mode[8-11] (polar semiconductor-vacuum). The dispersion relation of SW is observed for surface polaritons in GaP[3] and ionic crystals,[4] for surface plasmons in metals[6] and InSb[7] and for surface plasmon-phonon in InSb.[10,11]

All these works are concerned with the boundary free of mobile charge. There are several cases, however, where a sizable amount of charges is located in a thin layer near the boundary. One example is the electrons trapped by the surface of helium.[12] Another one is the inversion layer of the clean surface of semiconductors (for example p-type InAs[13]) and oxide-semiconductor interface of the MOS structure.[14] These charges are able to carry current along the surface, which in turn produces the electromagnetic field. In the present work we shall study the surface wave coupled to the motion of the surface carriers (hereafter shortned as SWC).

In the above examples the motion of carriers perpendicular to the surface (along the z-axis) is quantized. One can achieve quantum limit condition where the motion of the carriers along the z-axis is completely quenched for low frequencies. The typical value of the spread of the ground state wave function along the z-axis is less than 100 Å, which is much smaller than the extinction length of SW. Under these situations we can treat the charge layer as a charge sheet located at the boundary. Then the coupling of the surface carriers to SW is simply expressed by the surface current term in the matching formula for the electromagnetic field. In § 2 we shall derive the dispersion relation of SWC based on the charge sheet model. In § 3 several features of the dispersion relation will be investigated and illustrated numerically for the clean surface of p-type InAs. Section 4 will be devoted to discussion. It will be argued that the charge sheet model can be extended to the case when motion of surface carriers along the z-axis is not quantized.

§ 2. Derivation of the Dispersion Relation

Let us consider two semi-infinite isotropic media $1(z>0)$ and $2(z<0)$ bounded by the plane $z=0$. Assuming the wave propagating along the x-axis, we can write down the electromagnetic field for the nonradiative TH-modes as follows.

$$z>0, \quad E_{x,1}=E_1 \exp[i(kx-\omega t)-\alpha_1 z] \tag{1a}$$
$$E_{z,1}=(ik/\alpha_1)E_{x,1} \tag{1b}$$
$$H_{y,1}=-(i\omega\varepsilon_1(\omega)/c\alpha_1)E_{x,1} \tag{1c}$$
$$z<0, \quad E_{x,2}=E_2 \exp[i(kx-\omega t)+\alpha_2 z] \tag{2a}$$
$$E_{z,2}=(-ik/\alpha_2)E_{x,2} \tag{2b}$$
$$H_{y,2}=(i\omega\varepsilon_2(\omega)/c\alpha_2)E_{x,2} \tag{2c}$$
$$E_y=H_x=H_z=0 \; .$$

In the above $\varepsilon_1(\omega)$ and $\varepsilon_2(\omega)$ denotes the dielectric function of the media 1 and 2, respectively. We shall make the local approximation for ε_i and take the small wave number limit. α_1 and α_2 is the extinction coefficient given by

$$\alpha_1=\sqrt{k^2-\varepsilon_1(\omega)\omega^2/c^2} \tag{3}$$

and

$$\alpha_2=\sqrt{k^2-\varepsilon_2(\omega)\omega^2/c^2}\ . \tag{4}$$

For nonradiative mode the wave number and frequency is restricted in the region

$$\alpha_1{}^2=k^2-\varepsilon_1(\omega)\omega^2/c^2>0$$

and

$$\alpha_2{}^2=k^2-\varepsilon_2(\omega)\omega^2/c^2>0\ .$$

The electromagnetic field for the nonradiative TE-mode is as follows.

$$z>0,\quad E_{y,1}=E_1{}'\exp\left[i(kx-\omega t)-\alpha_1 z\right] \tag{5a}$$

$$H_{x,1}=-(ic\alpha_1/\omega)E_{y,1} \tag{5b}$$

$$H_{z,1}=(ck/\omega)E_{y,1} \tag{5c}$$

$$z<0,\quad E_{y,2}=E_2{}'\exp\left[i(kx-\omega t)+\alpha_2 z\right] \tag{6a}$$

$$H_{x,2}=(ic\alpha_2/\omega)E_{y,2} \tag{6b}$$

$$H_{z,2}=(ck/\omega)E_{y,2} \tag{6c}$$

$$E_x=E_z=E_y=0\ .$$

The magnetic permeability of the media is assumed to be unity.

In the presence of the surface current $i_x{}^{(s)}$ and $i_y{}^{(s)}$ the boundary condition for the electromagnetic field leads to

$$E_1=E_2 \tag{7a}$$

$$H_{y,1}^{(s)}-H_{y,2}^{(s)}=-\frac{4\pi}{c}i_x^{(s)}=-\frac{4\pi}{c}(\sigma_{xx}^{(s)}E_x^{(s)}+\sigma_{xy}^{(s)}E_y^{(s)}) \tag{7b}$$

$$E_1{}'=E_2{}' \tag{7c}$$

$$H_{x,1}^{(s)}-H_{x,2}^{(s)}=\frac{4\pi}{c}i_y^{(s)}=\frac{4\pi}{c}(\sigma_{yx}^{(s)}E_x^{(s)}+\sigma_{yy}^{(s)}E_y^{(s)})\ , \tag{7d}$$

where the superscript s denotes the value at the surface and $\sigma_{\alpha\beta}^{(s)}$ is the component of the surface conductivity tensor. Using eqs. (1c), (2c), (5b) and (6b), we obtain

$$i\omega\left[\frac{\varepsilon_1}{\alpha_1}+\frac{\varepsilon_2}{\alpha_2}\right]E_x^{(s)}=4\pi[\sigma_{xx}^{(s)}E_x^{(s)}+\sigma_{xy}^{(s)}E_y^{(s)}] \tag{8}$$

and

$$\frac{c^2}{i\omega}[\alpha_1+\alpha_2]E_y^{(s)}=4\pi[\sigma_{yx}^{(s)}E_x^{(s)}+\sigma_{yy}^{(s)}E_y^{(s)}]\ . \tag{9}$$

The general form of the dispersion of SWC is

$$\left[i\omega\left(\frac{\varepsilon_1}{\alpha_1}+\frac{\varepsilon_2}{\alpha_2}\right)-4\pi\sigma_{xx}^{(s)}\right]\left[\frac{c^2}{i\omega}(\alpha_1+\alpha_2)-4\pi\sigma_{yy}^{(s)}\right]-(4\pi\sigma_{xy}^{(s)})(4\pi\sigma_{yx}^{(s)})=0\ . \tag{10}$$

If the surface is isotropic, $\sigma_{xy}^{(s)}=\sigma_{yx}^{(s)}=0$ and TH- and TE-modes are not coupled together. The dispersion relation reduces to

$$\text{TH-mode,}\quad \frac{\varepsilon_1}{\alpha_1}+\frac{\varepsilon_2}{\alpha_2}-\frac{4\pi\sigma_{xx}^{(s)}}{i\omega}=0 \tag{11}$$

and

$$\text{TE-mode,}\quad \alpha_1+\alpha_2-\frac{i4\pi\omega\sigma_{xx}^{(s)}}{c^2}=0\ . \tag{12}$$

In the infrared region $\sigma_{xx}^{(s)}$ can be approximated by the free carrier value

$$\sigma_{xx}^{(s)}=\frac{ie^2N_s}{m^*\omega}\ . \tag{13}$$

Then eqs. (11) and (12) become

$$\frac{\varepsilon_1}{\alpha_1}+\frac{\varepsilon_2}{\alpha_2}-\frac{c\Omega_s}{\omega^2}=0 \tag{14}$$

and

$$\alpha_1+\alpha_2+\frac{\Omega_s}{c}=0\ , \tag{15}$$

where

$$\Omega_s=\frac{4\pi N_s e^2}{m^*c} \tag{16}$$

is the characteristic angular frequency of the surface free carrier with the surface density N_s and effective mass m^*. Equation (15) obviously has no real solution. Thus if the media and the surface is isotropic, no TE-SWC exists. On the other hand eq. (14) has at least one real solution k_{SWC} for a given value of ω. If the surface is anisotropic, as in the case of (110) surface of n-type inversion layer of Si, $\sigma_{xy}^{(s)}$, $\sigma_{yx}^{(s)}\neq 0$ except for the wave propagating along the principal axis of the conductivity tensor and the TH- and TE-modes mix up to yield new hybrid modes. The dispersion relation (10) may have one or two real solution of k for a given ω. Application of an external magnetic field also causes the hybridization, which will be discussed in a separate paper.

§ 3. Dispersion Relation for the TH-Mode

In this section we shall confine ourselves to the case of free isotropic surface carrier and perform detailed analysis of the dispersion relation for the TH-mode SWC (eq. (14)).

First of all since we consider the nonradiative waves, the solution should be in the region $\alpha_i{}^2>0$ $(i=1,2)$. For a given value of ω, the zero of α_i

$$k_{bi}=(\omega/c)\sqrt{\varepsilon_i(\omega)} \tag{17}$$

gives the wave number of the bulk polariton mode in the infinite medium i. Here the term "polariton" is used in its most extended sense (including the plasmon-photon, the plasmon-phonon-photon hybrid modes). Since α_i is a monotonic increasing function of k in the non-radiative region, the solution of eq. (14), k_{SWC}, is always larger than k_{bi}.

Next single solution k_{SWC} exists provided that at least one of $\varepsilon_i(\omega)$ is positive. This is in marked contrast to SW, where the real solution k_{SW} is allowed only in the region $\varepsilon_1(\omega)\cdot\varepsilon_2(\omega)<0$. Particularly, if we put $\varepsilon_1=\varepsilon_2$, no SW exists, while eq. (14) yields the solution

$$k_{\mathrm{SWC}}^2=\left(\frac{\omega}{c}\right)^2\varepsilon(\omega)\left[1+\left(\frac{2\omega}{\Omega_s}\right)^2\varepsilon(\omega)\right]. \tag{18}$$

In the allowed region for SW, k_{SWC} is always smaller than k_{SW}.

Now let us turn to the limiting and asymptotic behavior of SWC. In the following for the sake of simplicity we shall assume that $\varepsilon_1(\omega)$ has the following form (the medium 1 is a polar semi-conductor) and $\varepsilon_2(\omega)$ is a constant. ($\varepsilon_2(\omega)=\varepsilon_2(0)<\varepsilon_1(\infty)$).

$$\varepsilon_1(\omega)=\varepsilon_1(\infty)\,\frac{\omega^2-\omega_l{}^2}{\omega^2-\omega_t{}^2}-\frac{\omega_p{}^2}{\omega^2}. \tag{19}$$

Further, as a numerical example the dispersion of SWC is calculated for the clean surface of p-type InAs and the result is shown in Figs. 1—3. Here, the medium 1 is p-type InAs and 2 is the vacuum. Following values are chosen for the parameters of p-type InAs[15]: $\varepsilon(\infty)=11.8$, $\omega_t=4.12\times10^{13}\,\mathrm{s}^{-1}$, $\omega_l=4.58\times10^{13}\,\mathrm{s}^{-1}$ and $\omega_p=2.95\times10^{13}\,\mathrm{s}^{-1}$ (corresponding to the hole concentration $7\times10^{16}\,\mathrm{cm}^{-3}$). Density of the surface carrier in the inversion layer is chosen as $5\times10^{11}\,\mathrm{cm}^{-2}$, $5\times10^{12}\,\mathrm{cm}^{-2}$ and $5\times10^{13}\,\mathrm{cm}^{-2}$. Since the effective mass of the conduction band of InAs is $0.021m_e$, the corresponding values of Ω_s are $2.61\times10^{12}\,\mathrm{s}^{-1}$, $2.61\times10^{13}\,\mathrm{s}^{-1}$ and $2.61\times10^{14}\,\mathrm{s}^{-1}$, respectively. The result is shown in the reduced unit ω/ω_t and k/k_t, where $k_t=\omega_t/c=1.37\times10^3\,\mathrm{cm}^{-1}$.

Characteristic features of SWC are as follows:

(i) $\omega\ll\Omega_s,\ \omega_t,\ \omega_p$

$$k_{\mathrm{SWC}}=(\omega/c)\sqrt{\varepsilon_2(0)}\,. \tag{20}$$

If ε_1 has not plasma type part and $\varepsilon_1(0)>\varepsilon_2(0)$, $k_{\mathrm{SWC}}=(\omega/c)\sqrt{\varepsilon_1(0)}$. Generally, k_{SWC} approaches to max (k_{b_1}, k_{b_2}).

(ii) $\omega\gg\Omega_s,\ \omega_t,\ \omega_p$

$$k_{\mathrm{SWC}}\to\frac{\varepsilon_1(\infty)+\varepsilon_2(\infty)}{\Omega_s c}\,\omega^2\,. \tag{21}$$

The square root type dispersion ($\omega\propto\sqrt{k}$) is a peculiar feature of SWC. Apart from the effect of the singularity of $\varepsilon_1(\omega)$ the over all feature of the dispersion is governed by the transition from the linear (eq. (20)) to the square root

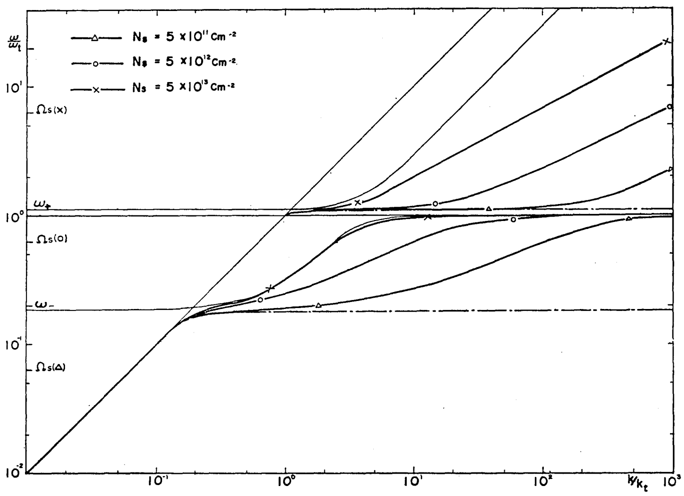

Fig. 1. The dispersion relation of SWC for clean surface of p-type InAs. Values of the parameters for p-type InAs are as follows: $\varepsilon(\infty)=11.8$, $\omega_t=4.12\times10^{13}\mathrm{s}^{-1}$, $\omega_l=4.58\times10^{13}\mathrm{s}^{-1}$ and $\omega_p=2.95\times10^{13}\mathrm{s}^{-1}$ (concentration of the hole is $7\times10^{16}\,\mathrm{cm}^{-3}$). Calculation is done for three values of surface carrier density: $N_s=5\times10^{11}\,\mathrm{cm}^{-2}$ ($\Omega_s=2.61\times10^{12}\,\mathrm{s}^{-1}$, —△—), $N_s=5\times10^{12}\,\mathrm{cm}^{-2}$ ($\Omega_s=2.61\times10^{13}\,\mathrm{s}^{-1}$, —○—) and $N_s=5\times10^{13}\,\mathrm{cm}^{-2}$ ($\Omega_s=2.61\times10^{14}\,\mathrm{s}^{-1}$, —×—). The dispersion of the bulk polariton is shown by thin line and SW by chain dotted line. Note that both the frequency and the wave number are plotted in the logarithmic scale and reduced unit ω/ω_t and k/k_t, where $k_t=\omega_t/c=1.37\times10^3\,\mathrm{cm}^{-1}$.

(eq. (21)) type (see, Fig. 1). The transition occurs in the region $\omega \sim \sqrt{\varepsilon_2}\,\Omega_s/(\varepsilon_1+\varepsilon_2)$.

(iii) $\omega \to \omega_t - 0$, $(\varepsilon_1 \to +\infty)$

$k_{\rm SWC}$ diverges similarly to k_{b_1}. Asymptotic behavior, however, is different. While $k_{b_1} \to (\omega_t - \omega)^{-1/2}$, $k_{\rm SWC} \to (\omega_t-\omega)^{-1}$. The difference is larger for smaller value of Ω_s for which the square root dispersion is achieved in the region a little lower than ω_t (Fig. 3).

(iv) $\omega_t < \omega < \omega_+$ $(\varepsilon_1 < 0)$

Here, SW exists in the region $\omega_t < \omega < \omega_+^s$. ω_+^s is determined by the relation $\varepsilon_1(\omega_+^s)+\varepsilon_2=0$ and lower than ω_+ (the upper zero of $\varepsilon_2(\omega)$). As ω exceeds ω_t, both $k_{\rm SW}$ and $k_{\rm SWC}$ start from $\sqrt{\varepsilon_2}\,\omega_t/c$ $(=k_{b_2})$. As $\omega \to \omega_+^s$, $k_{\rm SW} \to \infty$, while $k_{\rm SWC}$ departs from $k_{\rm SW}$ and is smoothly continued to the solution in region (ii). The deviation from $k_{\rm SW}$ begins at smaller value of k for larger value of Ω_s (see, Fig. 3). The same feature is observed in the region near ω_- (the lower zero of $\varepsilon_1(\omega)$). Here, $k_{\rm SW}$ is allowed in the region $\omega \leq \omega_-^s(\varepsilon_1(\omega_-^s)+\varepsilon_2=0)$, while $k_{\rm SWC}$ goes beyond ω_-^s and ω_- (see, Fig. 2).

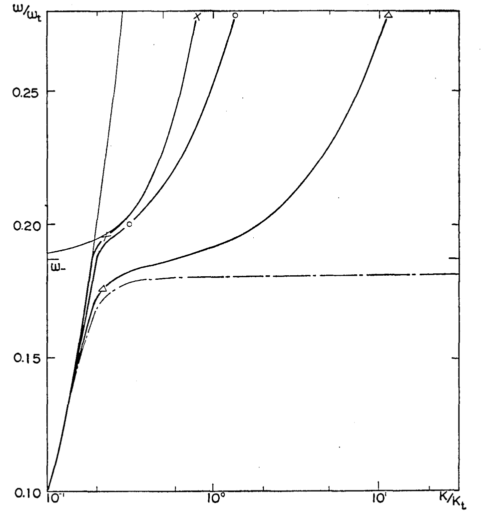

Fig. 2. The dispersion relation of SWC near the lower zero (ω_-) of $\varepsilon_1(\omega)$. ω/ω_t is plotted in the ordinary but enlarged scale.

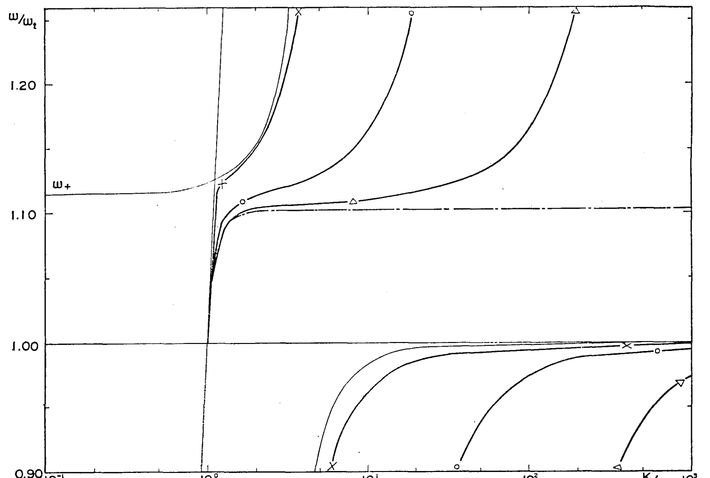

Fig. 3. The dispersion relation of SWC near the singularity (ω_t) and the upper zero (ω_+) of $\varepsilon_1(\omega)$. ω/ω_t is plotted in the ordinary but enlarged scale.

§ 4. Discussion

Though SWC is a nonradiative mode the predicted dispersion can be observed optically by using the attenuated total reflection techniques[6,7] employed to study SW. The wave length modulation method would be helpful to detect SWC, because the dispersion is sensitive to the surface carrier density, which can be varied by the modulation of the applied surface potential. The most distinguished feature of SWC is the existence in the forbidden region of SW. When $\Omega_s < \omega$, $k_{\rm SWC}$ is much larger than k_{bi} and one should survey a wide range of k to observe SWC.

In the derivation of the dispersion relation we assumed for simplicity that the motion of carriers perpendicular to the surface is quenched. The assumption is, however, not necessary. Consider a free carrier layer of thickness d sandwiched in-between the media 1 and 2. It can easily be shown that so long as the plasma wave length is much larger than d, the dispersion relation for TH-mode reduces to eq. (14), where $N_s = nd$ (n is the density of the free carrier). This is because the transverse polarization wave of the carriers propagating along surface does not yield charge density. Thus SWC can be observed for a metal thin layer sandwiched in-between dielectrics, carriers injected in a surface layer of semiconductors and optically excited carriers by intense strongly absorbed light.

The magnitude of the characteristic frequency Ω_s is given by

$$\Omega_s=1.1\times(m_e/m^*)\times(N_s/10^{12}\,\mathrm{cm}^{-2})\times10^{11}\,\mathrm{s}^{-1}\,. \quad (22)$$

Some examples of the value of Ω_s are given in Table I. By choosing appropriate value of Ω_s and combination of the media 1 and 2, we may observe various features of the dispersion relation.

Table I. Examples of surface carrier density and the characteristic frequency Ω_s.

Example	m^*/m_e	N_s (cm^{-2})	Ω_s (s^{-1})
Electrons trapped by helium	1.0	10^9	1.1×10^8
Cleaned surface of p-type InAs.	0.021	10^{12}	5.2×10^{12}
(100) surfase of Si MOS structure	0.19	5×10^{12}	2.9×10^{12}
Metal foils of 100Å thickness	1.0	10^{16}	1.1×10^{15}
Carriers excited by laser light in CdSe	0.13	6×10^{12}	5.1×10^{12}

Table II. Wave number and extinction coefficients for the clean surface of p-type InAs. The values are calculated for the parameters given in the text. $N_s=5\times10^{11}\,\mathrm{cm}^{-2}$.

ω/ω_t	k_{SWC} (cm^{-1})	α_1 (cm^{-1})	α_2 (cm^{-1})
0.10	1.4×10^2	8.4×10^2	2.0×10^1
0.30	1.9×10^4	1.9×10^4	1.9×10^4
0.95	8.0×10^5	8.0×10^5	8.0×10^5
1.10	2.1×10^3	3.0×10^3	1.5×10^3
2.0	1.0×10^6	1.0×10^6	1.0×10^6

The magnitude of the extinction coefficient α_i is generally of the same orders of magnitude as k_{SWC}. It is smaller or larger than k_{SWC}, according as $\varepsilon_i(\omega)$ is positive or negative. In the large ω limit α_i approaches k_{SWC} given by eq. (21). Typical values of α_1 and α_2 for the clean surface of p-type InAs ($N_s=5\times10^{11}\,\mathrm{cm}^{-2}$) is shown in the Table II together with the value of k_{SWC}. If α_i^{-1} is of the same orders of magnitude as the thickness of the carrier layer (spread of the ground state wave function in the quantum limit), the carrier sheet model breaks down and a detailed treatment of the nonlocal field-carrier interaction is necessary.

This would occur in the region $\omega\to\omega_t-0$ and $\omega\gg\Omega_s$. So far the media 1 and 2 are assumed to be homogeneous and semi-infinite. Actually, in many cases the adjacent medium is inhomogeneous and or finite. For example, in the MOS structure the thickness of the oxide layer is of the order of 10^{-5} cm. A three media model (metal-oxide-semiconductor) is studied in the Appendix. According to the result there are two types of SWC. The one is nonradiative in the outer regions but radiative in the middle (oxide) region. If the thickness of the middle layer L is larger than the plasma wave length of the metal, the dispersion of the former mode is given by

$$\frac{\varepsilon_1}{\alpha_1}+\frac{\varepsilon_o}{\alpha_o{}^2L}=\frac{c\Omega_s}{\omega^2} \quad (23)$$

where 1 denotes the semiconductor and o denotes the oxide. In the small ω limit the solution is

$$\omega=\frac{c}{\sqrt{\varepsilon_o}}\sqrt{\frac{\Omega_sL}{c+\Omega_sL}}\,k\,. \quad (24)$$

Under usual conditions the phase velocity of the solution (24) takes very small value. For example, if we assume $\Omega_s=3\times10^{12}\,\mathrm{s}^{-1}$, $L=3\times10^{-5}$ cm and $\varepsilon_o=3.8$ (the value for SiO_2), the value of the phase velocity is 8×10^8 cm/s. The latter (radiative in the middle layer) mode is possible only when $\varepsilon_o>\varepsilon_1(\omega)$. The dispersion relation is

$$\frac{\varepsilon_1}{\alpha_1}-\frac{c\Omega_s}{\omega^2}=\frac{\varepsilon_o}{\beta}\cot\beta L\ , \quad (25)$$

where $\beta^2=\varepsilon_o\omega^2/c-k^2$. Several types of the standing mode (in the middle region) could be expected. The semiconductor side of the MOS structure has the depletion layer between the inversion layer and the bulk of the material. The thickness of the depletion layer is of the order 1 μ. If α_1 of the bulk material is larger than the inverse of the thickness of the depletion layer, a nonlocal theory is needed.

The analysis of SWC would serve as a tool to study the surface carrier system through $\sigma^{(s)}$ and N_s. It would also serve as a basis of electro-optical system. Application of external magnetic field will reveal various new features through the field dependence of the dielectric function and surface conductivity. This will be investigated in a forthcoming paper.*

Acknowledgements

Present work is financed partly by a Grant-in-Aid of the Ministry of Education. Numerical calculation was performed at the Computation

Center of Kyusyu University.

Appendix. Analysis of Three Media Model

Let us consider a three media system in which the middle layer $o(L/2>z>-L/2)$ is sandwiched in-between the media $1(z>L/2)$ and $2(-L/2>z)$. We shall investigate the TH mode which is nonradiative in the region 1 and 2. The electromagnetic field in the media 1 and 2 is the same as the one given in the text. For the mode which is also nonradiative in the middle region the field is as follows.

$$E_x=[E_o\exp\alpha_o z+E_o'\exp(-\alpha_o z)]\exp[i(kx-\omega t)] \tag{A·1a}$$

$$E_z=\frac{ik}{\alpha_o}[-E_o\exp\alpha_o z+E_o'\exp(-\alpha_o z)]\exp[i(kx-\omega t)] \tag{A·1b}$$

$$H_y=\frac{i\omega\varepsilon_o(\omega)}{\alpha_o}[E_o\exp\alpha_o z-E_o'\exp(-\alpha_o z)]\exp[i(kx-\omega t)]\,. \tag{A·1c}$$

In the above $\alpha_o=\sqrt{k^2-\varepsilon_o(\omega)\omega^2/c^2}$ and $\varepsilon_o(\omega)$ is the dielectric function of the medium o. If a charge sheet is located at the boundary $z=L/2$, we obtain the following dispersion formula.

$$\frac{\left(\frac{\varepsilon_o}{\alpha_o}+\frac{\varepsilon_2}{\alpha_2}\right)\left(\frac{\varepsilon_o}{\alpha_o}+\frac{\varepsilon_1}{\alpha_1}-\frac{4\pi\sigma^{(s)}}{i\omega}\right)}{\left(\frac{\varepsilon_o}{\alpha_o}-\frac{\varepsilon_2}{\alpha_2}\right)\left(\frac{\varepsilon_o}{\alpha_o}-\frac{\varepsilon_1}{\alpha_1}+\frac{4\pi\sigma^{(s)}}{i\omega}\right)}=e^{-2\alpha_o L}\,. \tag{A·2}$$

If $\sigma^{(s)}=0$ (no surface charge) and $\varepsilon_1=\varepsilon_2$, then eq. (A·2) coincides with the dispersion formula for a thin slab derived by Kliewer and Fuchs.[2] When the medium 2 is a metal and the frequency is low, $|\varepsilon_2/\alpha_2|$ is much larger than ε_o/α_o. Further, when the condition $|\varepsilon_o\alpha_2/\varepsilon_2\alpha_o|\gg\alpha_o L$ is satisfied, eq. (A·2) reduced to eq. (11) of the two media model. On the contrary, when the middle region is not so thin and the condition $|\varepsilon_0\alpha_2/\varepsilon_2\alpha_0|\ll\alpha_0 L$ is satisfied, eq. (A·2) becomes

$$\frac{\varepsilon_1}{\alpha_1}+\frac{\varepsilon_o}{\alpha_o^2 L}=\frac{\sigma^{(s)}}{i\omega} \tag{A·3}$$

which is eq. (24) in the text.

For the mode radiative in the middle region we should replace α_0 by $i\beta$. In the large $|\varepsilon_2\beta/\alpha_2\varepsilon_o|$ limit the dispersion relation reduces to

$$\frac{\varepsilon_1}{\alpha_1}-\frac{4\pi\sigma^{(s)}}{i\omega}=\frac{\varepsilon_o}{\beta}\cot\beta L \tag{A·4}$$

which is eq. (25) in the text.

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15) These values are taken and estimated from review articles by M. Hass and by H. Y. Fan in *Semiconductors and Semimetals*, ed. R. K. Willardson and A. C. Beer (Academic Press, New York and London, 1967) Vol. 3, p. 3 and p. 406.

* *Note added in proof*—SWC can also be regarded as the collective mode of the two dimensional surface carriers embedded in the dielectric media (surface carrier plasmon). From the latter standpoint Stern derived the dispersion relation (eq. (18) of the present paper) [F. Stern: Phys. Rev. Letters **18** (1967) 546].

Effect of the surface accumulation and depletion layer on SW is studied by Wallis *et al.* [R. F. Wallis, J. J. Brion, E. Burstein and A. Hartstein: *Proc. 11th Int. Conf. on the Physics of Semiconductor, Warsaw, Poland, 1972* (Elsevier, 1973) p. 1448]. Present calculation corresponds to the extreme accumulation limit ($n_{ac}\to\infty$ and $d\to 0$ with $n_{ac}=N_s$).