GEOPHYSICS, VOL. 37, NO. 6 (DECEMBER 1972), P. 985-996, 1 FIG.

ELECTROMAGNETIC FIELDS DUE TO DIPOLE ANTENNAS OVER STRATIFIED ANISOTROPIC MEDIA†

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Solutions to the problem of radiation of dipole antennas in the presence of a stratified anisotropic media are facilitated by decomposing a general wave field into transverse magnetic (TM) and transverse electric (TE) modes. Employing the propagation matrices, wave amplitudes in any region are related to those in any other regions. The reflection coefficients, which embed all the information about the geometrical configuration and the physical constituents of the medium, are obtained in closed form. In view of the general formulation, various special cases are discussed.

INTRODUCTION

The problem of radiation of a dipole source in the presence of stratified media has been studied extensively with application to geophysical exploration. An excellent review on the half-space case is contained in the book by Sommerfeld (1949) and in the monograph by Baños (1966). Propagation and radiation in stratified media are treated by Wait (1970) and Ward (1967). Wolf (1946) and Bhattacharya (1963) considered the case of dipoles on a two-layer earth. Wait (1951, 1953) solved the problem of electrical and magnetic dipoles over a stratified isotropic medium. The case of an anisotropic half space was studied by Chetaev (1963) and Wait (1966a). Praus (1965), Sinha and Bhattacharya (1967), and Sinha (1968, 1969) treated electric and magnetic dipoles over a two-layer anisotropic earth. Wait (1966b) formally solved the case of a horizontal dipole over a stratified anisotropic medium. All this work was carried out by means of Sommerfeld's Hertzian potential functions, and the primary interest is concentrated in the limits of high conductivity. Magnetic properties are almost entirely neglected, mainly because such model studies assume principal applications to the earth, where the permeability is nearly equal to that of vacuum, and the electric conductivity dominates at low frequencies. In events of other celestial bodies, such as the moon, where the lack of moisture renders very low conductivity to the medium, a study of contributions due to all electric and magnetic properties is important.

This paper is devoted to the case of radiation of various dipole sources in the presence of a stratified anisotropic medium. The anisotropic medium is uniaxial and possesses both tensor permittivities and permeabilities. The principal axes are all perpendicular to the boudaries separating different media. Solutions to the problem are facilitated by decomposing a general wave field into TM and TE modes, employing the concept of propagation matrices, and expressing the reflection coefficients in terms of continuous fractions. The primary excitation is separated entirely from contributions due to the medium. The reflection coefficients depend on the geometric configurations as well as the physical properties of the stratified medium.

In studying the theory of electromagnetic wave propagation, it has been appreciated both classically and quantum mechanically (Kong, 1970) that introduction of a potential function is not necessary and sometimes complicates the algebra, especially when anisotropic media are involved. With recognition of the fact that outside any source two scalar functions are sufficient to determine all field quantities, two components of the field vectors can be chosen as the fundamental scalar functions. In our case,

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the preferred field components for TM and TE decomposition are clearly those along the principal axis and normal to boundaries of stratification. With the aid of propagation matrices (Kong, 1971), wave amplitudes in any region are easily calculated in terms of those in any other region. Writing in the form of continuous fractions, we obtain a closed-form solution for the reflection coefficients. All field components are expressed in terms ot integrals which are ready for direct numerical evaluation. A discussion is given for the various special cases.

TRANSVERSE ELECTRIC AND MAGNETIC WAVES

Governing equations for electromagnetic fields in a region outside any source are the Maxwell's source free equations.

$$\nabla \times \mathbf{E} = i\omega\bar{\bar{\mu}}\cdot\mathbf{H} \quad (1a) \qquad \text{and} \qquad \nabla \times \mathbf{H} = -i\omega\bar{\bar{\epsilon}}\cdot\mathbf{E}, \quad (1b)$$

where in (1a), $\bar{\bar{\mu}}$ is the permeability tensor of the media. The tensor $\bar{\bar{\epsilon}}$ in (1b) contains information about the dielectric constant and the conductivity of the medium. $\bar{\bar{\epsilon}}=\bar{\bar{\epsilon}}'+i\bar{\bar{\epsilon}}''$, where $\bar{\bar{\epsilon}}'$ is the permittivity tensor, and $\bar{\bar{\epsilon}}''$ is related to the conductivity tensor $\bar{\bar{\sigma}}$ by $\bar{\bar{\epsilon}}''=\bar{\bar{\sigma}}/\omega$. Time harmonic excitations with time dependence exp $(-i\omega t)$ have been assumed. The tensors $\bar{\bar{\epsilon}}$ and $\bar{\bar{\mu}}$ can be represented by hermitian matrices. In our case, we consider media which are uniaxially anisotropic, where

$$\bar{\bar{\epsilon}} = \begin{bmatrix} \epsilon & & \\ & \epsilon & \\ & & \epsilon_z \end{bmatrix} \quad (2a) \qquad \text{and} \qquad \bar{\bar{\mu}} = \begin{bmatrix} \mu & & \\ & \mu & \\ & & \mu_z \end{bmatrix}. \quad (2b)$$

We employ cylindrical coordinates, and the plane transverse to the $\hat{z}$ axis is characterized by ρ and ϕ. Longitudinal electric and magnetic components E_z and H_z are used to derive TE and TM waves. The wave equations to be satisfied by E_z and H_z are immediately derived from equations (1) and (2). If we take the $\hat{z}$ component of (1b) in view of $\bar{\bar{\epsilon}}$ given by (2a), employ (1a) to eliminate transverse magnetic field components, and use the fact that $\nabla\cdot\mathbf{E}=(1-a)\partial E_z/\partial z$, the equation for E_z is

$$\left(\nabla_t^2 + a\frac{\partial^2}{\partial z^2} + k^2 a\right)E_z = 0. \quad (3a)$$

We also obtain the wave equation for H_z:

$$\left(\nabla_t^2 + b\frac{\partial^2}{\partial z^2} + k^2 b\right)H_z = 0. \quad (3b)$$

In equations (3a) and (3b),

$$k = \omega\sqrt{\mu\epsilon}, \quad (4) \qquad a = \epsilon_z/\epsilon, \quad (5a) \qquad \text{and} \qquad b = \mu_z/\mu, \quad (5b)$$

and

$$\nabla_t^2 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} \quad (6)$$

is the transverse Laplacian operator expressed in cylindrical coordinates. It is seen from equations (3) that E_z and H_z are decoupled, which would not be true if the $\bar{\bar{\epsilon}}$ and $\bar{\bar{\mu}}$ tensors possess off-diagonal elements. A unique decomposition of the total wave into a transverse-magnetic-field (TM) mode derivable from E_z and a transverse-electric-field (TE) mode derivable from H_z is, therefore, plausible. We note that a pair of vector wave equations can be derived from (1) and (2):

$$\nabla^2\mathbf{E} + k^2\mathbf{E} + (a-1)k^2E_z\hat{z} + (a-1)\nabla(\partial E_z/\partial z) = 0 \quad (7a)$$

and

$$\nabla^2\mathbf{H} + k^2\mathbf{H} + (b-1)k^2H_z\hat{z} + (b-1)\nabla(\partial H_z/\partial z) = 0. \quad (7b)$$

Equation (7a) is the vector wave equation for the electric fields of TM waves where $H_z=0$, and (7b) is the wave equation for the magnetic fields of TE waves, where $E_z=0$. The $\hat{z}$ component of the two vector equations (7) gives rise to equation (3).

Solutions of E_z and H_z to the wave equation (3) in cylindrical coordinates are well known. As a consequence of the Maxwell equations (1), all transverse-electric- and magnetic-field components can be expressed in terms of the longitudinal components E_z and H_z which, respectively, characterize the TM and the TE waves. In our problems we are interested in wave solutions which are outgoing in $\hat{\rho}$ direction and traveling or standing in $\hat{z}$ direction. Therefore, we obtain, for a fixed separation constant n,

$$\mathbf{E}^{TM} = \int_{-\infty}^{\infty} dk_\rho \begin{Bmatrix} i\dfrac{k_z^{(e)}}{k_\rho}(-Ae^{-ik_z^{(e)}z} + Be^{ik_z^{(e)}z})H_n^{(1)'}(k_\rho\rho)S_n^{TM}(\phi) \\ i\dfrac{k_z^{(e)}}{k_\rho^2\rho}(-Ae^{-ik_z^{(e)}z} + Be^{ik_z^{(e)}z})H_n^{(1)}(k_\rho\rho)S_n^{TM'}(\phi) \\ [A(k_\rho)e^{-ik_z^{(e)}z} + B(k_\rho)e^{ik_z^{(e)}z}]H_n^{(1)}(k_\rho\rho)S_n^{TM}(\phi) \end{Bmatrix}, \tag{8a}$$

$$\mathbf{H}^{TM} = \int_{-\infty}^{\infty} dk_\rho \begin{Bmatrix} -i\dfrac{\omega\epsilon}{k_\rho^2\rho}(Ae^{-ik_z^{(e)}z} + Be^{ik_z^{(e)}z})H_n^{(1)}(k_\rho\rho)S_n^{TM'}(\phi) \\ i\dfrac{\omega\epsilon}{k_\rho}(Ae^{-ik_z^{(e)}z} + Be^{ik_z^{(e)}z})H_n^{(1)'}(k_\rho\rho)S_n^{TM}(\phi) \\ 0 \end{Bmatrix}, \tag{8b}$$

$$\mathbf{E}^{TE} = \int_{-\infty}^{\infty} dk_\rho \begin{Bmatrix} i\dfrac{\omega\mu}{k_\rho^2}(Ce^{-ik_z^{(m)}z} + De^{ik_z^{(m)}z})H_n^{(1)}(k_\rho\rho)S_n^{TE'}(\phi) \\ -i\dfrac{\omega\mu}{k_\rho}(Ce^{-ik_z^{(m)}z} + De^{ik_z^{(m)}z})H_n^{(1)'}(k_\rho\rho)S_n^{TE}(\phi) \\ 0 \end{Bmatrix}, \tag{8c}$$

and

$$\mathbf{H}^{TE} = \int_{-\infty}^{\infty} dk_\rho \begin{Bmatrix} i\dfrac{k_z^{(m)}}{k_\rho}(-Ce^{-ik_z^{(m)}z} + De^{ik_z^{(m)}z})H_n^{(1)'}(k_\rho\rho)S_n^{TE}(\phi) \\ i\dfrac{k_z^{(m)}}{k_\rho^2\rho}(-Ce^{-ik_z^{(m)}z} + De^{ik_z^{(m)}z})H_n^{(1)}(k_\rho\rho)S_n^{TE'}(\phi) \\ [C(k_\rho)e^{-ik_z^{(m)}z} + D(k_\rho)e^{ik_z^{(m)}z}]H_n^{(1)}(k_\rho\rho)S_n^{TE}(\phi) \end{Bmatrix}, \tag{8d}$$

where superscripts TM and TE denote, respectively, TM and TE waves. We note that if the integrands for E_z and H_z are denoted, respectively, by $E_z(k_\rho)$ and $H_z(k_\rho)$ such that

$$E_z^{TM} = \int_{-\infty}^{\infty} dk_\rho E_z(k_\rho) \quad \text{and} \quad H_z^{TE} = \int_{-\infty}^{\infty} dk_\rho H_z(k_\rho),$$

then the integrands of the transverse components are related to $E_z(k_\rho)$ and $H_z(k_\rho)$ by the following relations:

$$E_t(k_\rho)^{TM} = \frac{1}{k_\rho^2}\nabla_t\{\partial E_z(k_\rho)/\partial z\}, \quad H_t(k_\rho)^{TM} = -i\frac{\omega\epsilon}{k_\rho^2}\nabla_t \times \mathbf{E}_z(k_\rho), \tag{9a}$$

and

$$H_t(k_\rho)^{TE} = \frac{1}{k_\rho^2} \nabla_t\{\partial H_z(k_\rho)/\partial z\}, \quad E_t(k_\rho)^{TE} = i\frac{\omega\mu}{k_\rho^2} \nabla_t \times \mathbf{H}_z(k_\rho), \tag{9b}$$

where

$$\mathbf{E}_z = \hat{z}E_z, \ \mathbf{H}_z = \hat{z}H_z, \quad \text{and} \quad \nabla_t = \hat{\rho}\frac{\partial}{\partial\rho} + \hat{\phi}\frac{1}{\rho}\frac{\partial}{\partial\rho}\,.$$

The fact that TM waves are extraordinary waves in the medium is signified in equation (8) by the superscript (e) on the $\hat{z}$-directed propagation constant $k_z^{(e)}$, which satisfies the dispersion relation

$$k_z^{(e)} = (k^2 - k^2/a)^{1/2}. \tag{10a}$$

TE waves are derived from H_z and satisfy the dispersion relation

$$K_z^{(m)} = (k^2 - k_\rho^2/b)^{1/2}, \tag{10b}$$

where the superscript (m) indicates the effect of magnetic anisotropy. In (8), the first element of the column matrices denotes the $\hat{\rho}$ component, the second element the $\hat{\phi}$ component, and the third element the $\hat{z}$ component. The Hankel functions $H_n^{(1)}$ of the first kind and nth order represent outgoing waves in ρ direction due to our choice of the time dependence exp $(-i\omega t)$. $S_n(\phi)$ stands for sinusoidal functions of ϕ. Primes on $H_n^{(1)}(k_\rho\rho)$ and $S_n(\phi)$ denote differentiation with respect to the arguments. The k_ρ-dependent functions A, B, C, and D are to be determined by the appropriate boundary conditions.

PRIMARY EXCITATION

The explicit solutions to the problem of dipole radiation over a stratified medium depends on field excitations of the source and the geometrical configuration and physical constituents of the medium. In the absence of the stratified medium, the solution of electromagnetic fields in an isotropic medium due to a dipole antenna, which we refer to as the primary excitation, is well known (Adler et al, 1960). The solution is usually written in spherical coordinates. It can be transformed into cylindrical coordinates and represented by Hankel functions in the integral form. Writing in the general form, we have

$$E_z = \int_{-\infty}^{\infty} dk_\rho E_0(k_\rho) \begin{Bmatrix} e^{ik_z z} \\ e^{-ik_z z} \end{Bmatrix} H_n^{(1)}(k_\rho\rho) S_n^{TM}(\phi) \qquad \begin{matrix} z > 0 \\ z < 0 \end{matrix} \tag{11}$$

and

$$H_z = \int_{-\infty}^{\infty} dk_\rho H_0(k_\rho) \begin{Bmatrix} e^{ik_z z} \\ e^{-ik_z z} \end{Bmatrix} H_n^{(1)}(k_\rho\rho) S_n^{TE}(\phi) \qquad \begin{matrix} z > 0 \\ z < 0 \end{matrix}, \tag{12}$$

where E_0 and H_0 characterize the structure and excitation of the dipole. All field components follow from equation (8) with $B = D = 0$, $A = E_0$, $C = H_0$ for $z \le 0$, and $A = C = 0$, $B = E_0$, $D = H_0$ for $z \ge 0$.

For the elementary dipoles under consideration, we obtain (Appendix 1):

1) Vertical electric dipole: $n = 0$, $S_n^{TM}(\phi) = 1$,

$$E_0 = -\frac{Ilk_\rho^3}{8\pi\omega\epsilon k_z} \quad (13a) \qquad \text{and} \qquad H_0 = 0. \quad (13b)$$

2) Horizontal electric dipole along $\hat{x}$ direction,

$$E_0 = \pm\, i\frac{Ilk_\rho^2}{8\pi\omega\epsilon} z \gtrless 0 \qquad S_1^{TM} = \cos\phi \quad \text{and} \tag{14a}$$

$$H_0 = i\,\frac{Ilk_\rho^2}{8\pi k_z} \qquad S_1^{TE} = -\sin\phi. \tag{14b}$$

3) Vertical magnetic dipole: $n=0$, $S_n^{\mathrm{TE}}(\phi)=1$,

$$H_0 = -i\,\frac{IAk_\rho^3}{8\pi k_z}, \quad (15\mathrm{a}) \qquad \text{and} \qquad E_0 = 0. \quad (15\mathrm{b})$$

4) Horizontal magnetic dipole along $\hat{x}$ direction,

$$H_0 = \mp\,\frac{IAk_\rho^2}{8\pi}\; z \gtrless 0 \qquad S_1^{TM} = \cos\phi \quad \text{and} \tag{16a}$$

$$E_0 = -\,\frac{IA\omega\mu k_\rho^2}{8\pi k_z} \qquad S_1^{TE} = -\sin\phi. \tag{16b}$$

In equations (13)–(16) I is the current that drives the dipole, l is the equivalent length of the electric dipole, and A is the area of the current loop that constitutes the magnetic dipole. Horizontal dipoles can be obtained simply by a rotation of coordinates, which amounts to changing $\cos\phi$ to $\sin\phi$ and $\sin\phi$ to $-\cos\phi$. We note that a vertical electric dipole excites the TM wave only and a vertical magnetic dipole excites the TE wave only, both involve Hankel functions of zero order; whereas horizontal dipoles exicite both TM and TE waves and require Hankel functions of the first order. An arbitrarily oriented dipole can be treated as a linear combination of three dipoles along the $\hat{x}$, $\hat{y}$, and $\hat{z}$ axes.

DIPOLE ANTENNAS OVER STRATIFIED ANISOTROPIC MEDIA

Geometric configuration of the problem is shown in Figure 1. There are n slab regions, and the last region is numbered t instead of $n+1$, for the sake of simplification. In each region labeled i, solutions of electromagnetic field components take the form of equation (8) with all quantities subscripted by i. In the 0th region where we have the antennas, $A_0=E_0$ and $C_0=H_0$, which are known from (13)–(16) for the three types of antennas under consideration. The last region, namely region t, is semiinfinite, and we do not expect reflected waves, therefore, $B_t=D_t=0$.

Boundary conditions at all interfaces require all tangential electromagnetic field components be continuous for all ρ and ϕ (Appendix 2). Solutions can be facilitated by introducing propagation matrices (Kong, 1971). The upward propagation matrix for TM waves from the $(i+1)$th region to the ith region is defined to be

$$M_i^{i+1} = \frac{1}{2}\begin{pmatrix} \epsilon(+)_i^{i+1} e(+)_i^{(e)} & \epsilon(-)_i^{i+1} e(-)_i^{(e)} \\ \epsilon(-)_i^{i+1} e(+)_i^{(e)} & \epsilon(+)_i^{i+1} e(-)_i^{(e)} \end{pmatrix}, \tag{17}$$

where

$$\epsilon(\pm)_q^p = \left(\frac{\epsilon_p}{\epsilon_q} \pm \frac{k_{pz}^{(e)}}{k_{qz}^{(e)}}\right), \tag{18}$$

$$e(\pm)_i^{(e)} = \exp\left[\pm ik_{(i+1)z}^{(e)}(d_i - d_{i+1})\right], \quad \text{and} \tag{19}$$

$$k_{iz}^{(e)} = (k_i^2 - k_\rho^2/a)^{1/2}. \tag{20}$$

Matching boundary conditions at $z=-d_i$, we obtain

$$\begin{pmatrix} a_i \\ b_i \end{pmatrix} = M_i^{i+1}\begin{pmatrix} a_{i+1} \\ b_{i+1} \end{pmatrix}, \tag{21}$$

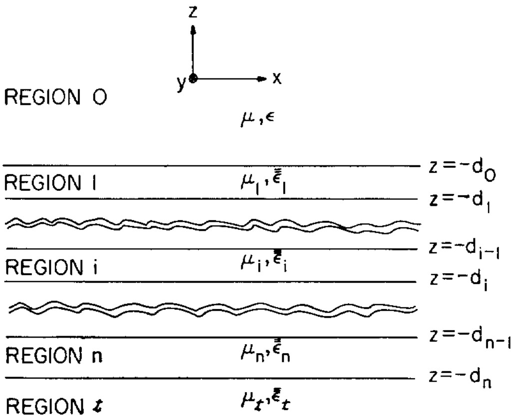

FIG. 1. Geometric configuration of the problem.

where

$$a_i = A_i \exp(ik_{iz}^{(e)} d_i) \tag{22a}$$

and

$$b_i = B_i \exp(-ik_{iz}^{(e)} d_i). \tag{22b}$$

Note that, with R^{TM} denoting reflection coefficients and T^{TM} denoting transmission coefficients, we can write

$$a_0 = E_0 \exp[ik_z^{(e)} d_0], \tag{23a}$$

$$b_0 = R^{TM} E_0 \exp[-ik_z^{(e)} d_0], \tag{23b}$$

and

$$a_t = T^{TM} E_0 \exp[ik_z^{(e)} d_t]. \tag{23c}$$

Also, $b_t = 0$, because there is no reflected wave in the last region. The parameter d_t in (23c) is introduced for convenience; it does not correspond to any distance and is always multipled by $e(-)_n^{(e)}$ to yield $\exp[ik_{nz}^{(e)} d_n]$. The definition of the propagation matrix is a useful one. Once wave amplitudes in any region are known, those in the regions above this one are all determined by (21). Thus the propagation matrix (17) propagates wave amplitudes upward. We can also define a propagation matrix M_{i+1}^{i} which propagates wave amplitudes downward.

$$M_{i+1}^{i} = \frac{1}{2}\begin{pmatrix} \epsilon(+)_{i+1}^{i} e(-)_i^{(e)} & \epsilon(-)_{i+1}^{i} e(-)_i^{(e)} \\ \epsilon(-)_{i+1}^{i} e(+)_i^{(e)} & \epsilon(+)_{i+1}^{i} e(+)_i^{(e)} \end{pmatrix} \tag{24a}$$

and

$$\begin{pmatrix} a_{i+1} \\ b_{i+1} \end{pmatrix} = M_{i+1}^{i} \begin{pmatrix} a_i \\ b_i \end{pmatrix}. \tag{24b}$$

It is easily shown that

$$M_i^{i+1} M_{i+1}^{i} = 1. \tag{25}$$

Following a parallel analysis of the above, we define a downward propagation matrix N_{i+1}^{i} for TE waves.

$$N_{i+1}^{i} = \frac{1}{2}\begin{pmatrix} \mu(+)_{i+1}^{i} e(-)_i^{(m)} & \mu(-)_{i+1}^{i} e(-)_i^{(m)} \\ \mu(-)_{i+1}^{i} e(+)_i^{(m)} & \mu(+)_{i+1}^{i} e(+)_i^{(m)} \end{pmatrix} \quad \text{and} \tag{26a}$$

$$\begin{pmatrix} c_{i+1} \\ d_{i+1} \end{pmatrix} = N_{i+1}^{i} \begin{pmatrix} c_i \\ d_i \end{pmatrix}. \tag{26b}$$

An upward propagation matrix N_i^{i+1} is defined as the inverse of N_{i+1}^{i}:

$$N_i^{i+1} = \frac{1}{2}\begin{pmatrix} \mu(+)_i^{i+1} e(+)_i^{(m)} & \mu(-)_i^{i+1} e(-)_i^{(m)} \\ \mu(-)_i^{i+1} e(+)_i^{(m)} & \mu(+)_i^{i+1} e(-)_i^{(m)} \end{pmatrix} \quad \text{and} \tag{27a}$$

$$\begin{pmatrix} c_i \\ d_i \end{pmatrix} = N_i^{i+1} \begin{pmatrix} c_{i+1} \\ d_{i+1} \end{pmatrix}. \tag{27b}$$

In (26) and (27),

$$\mu(\pm)_q^p = \frac{\mu_p}{\mu_q} \pm \frac{k_{pz}^{(m)}}{k_{qz}^{(m)}}, \qquad (28) \qquad\qquad c_i = C_i \exp\left[ik_{iz}^{(m)} d_i\right], \qquad (29a)$$

$$d_i = D_i \exp\left[-ik_{iz}^{(m)} d_i\right], \qquad (29b) \qquad \text{and} \qquad k_{iz}^{(m)} = (k_i^2 - k^2/b)^{1/2}. \qquad (29c)$$

Also $c_0 = H_0$, $d_0 = R^{TE} H_0$, $c_t = T^{TE} H_0$, and $d_t = 0$. We note that for vertical magnetic or electric dipoles, only the TM waves or the TE waves, respectively, are excited. In the case of either a horizontal electric dipole or a horizontal magnetic dipole, both TM and TE waves are excited.

REFLECTION COEFFICIENTS

In the interferometry method, our primary interest is the reflected wave fields. From the preceding section, we have established that when the wave amplitudes in region 0 are known, solutions in any other region can be determined by using the downward propagation matrices (24) and (26). In this section we derive a formula for the reflection coefficients which is expressed in continuous fractions. We observe from equation (23) that $b_t/a_t = 0$, and b_0/a_0 gives rise to the reflection coefficients R^{TM}. And from (21) an expression for b_i/a_i in terms of b_{i+1}/a_{i+1} can be established easily. In view of (17), (21) gives

$$\frac{b_i}{a_i} = \frac{\epsilon(+)_i^{i+1}}{\epsilon(-)_i^{i+1}}\left[1 - \frac{\epsilon(+)_i^{i+1}/\epsilon(-)_i^{i+1} - \epsilon(-)_i^{i+1}/\epsilon(+)_i^{i+1}}{\epsilon(+)_i^{i+1}/\epsilon(-)_i^{i+1} + (e(-)_i^{(e)})^2 (b_{i+1}/a_{i+1})}\right]. \qquad (30)$$

Making use of (30), we obtain a formula in continuous fraction for the reflection coefficient R^{TM}.

$$\begin{aligned} R^{TM} = \exp\,(i2k_z^{(e)} d_0)\, \frac{\epsilon(+)_0^1}{\epsilon(-)_0^1}\Bigg\{1 - \Bigg| \frac{\epsilon(+)_0^1/\epsilon(-)_0^1 - \epsilon(-)_0^1/\epsilon(+)_0^1}{\epsilon(+)_0^1/\epsilon(-)_0^1} \Bigg| \\ + \exp\,(i2k_{1z}^{(e)}(d_1 - d_0))\, \frac{\epsilon(+)_1^2}{\epsilon(-)_1^2}\Bigg[1 - \Bigg| \frac{\epsilon(+)_1^2/\epsilon(-)_1^2 - \epsilon(-)_1^2/\epsilon(+)_1^2}{\epsilon(+)_1^2/\epsilon(-)_1^2} \Bigg| \\ + \cdots + \exp\,(i2k_{nz}^{(e)}(d_n - d_{n-1}))\, \frac{\epsilon(-)_n^t}{\epsilon(+)_n^t} \cdots \Bigg]\Bigg\}. \end{aligned} \qquad (31)$$

In the same way we obtain the reflection coefficient for TE waves.

$$\begin{aligned} R^{TE} = \exp\,(i2k_z^{(m)} d_0)\, \frac{\mu(+)_0^1}{\mu(-)_0^1}\Bigg\{1 - \Bigg| \frac{\mu(+)_0^1/\mu(-)_0^1 - \mu(-)_0^1/\mu(+)_0^1}{\mu(+)_0^1/\mu(-)_0^1} \Bigg| \\ + \exp\,(i2k_{1z}^{(m)}(d_1 - d_0))\, \frac{\mu(+)_1^2}{\mu(-)_1^2}\Bigg[1 - \Bigg| \frac{\mu(+)_1^2/\mu(-)_1^2 - \mu(-)_1^2/\mu(+)_1^2}{\mu(+)_1^2/\mu(-)_1^2} \Bigg| \\ + \cdots + \exp\,(i2k_{nz}^{(m)}(d_n - d_{n-1}))\, \frac{\mu(-)_n^t}{\mu(+)_n^t} \cdots \Bigg]\Bigg\}. \end{aligned} \qquad (32)$$

Definitions of $\epsilon(\pm)_i^{i+1}$ and $\mu(\pm)_i^{i+1}$ are given in (18) and (28). The subscript n on R^{TM} and R^{TE} denotes the number of layers involved.

SUMMARY

With the reflection coefficients determined in the last section, we can now summarize the formulas for all field quantities in region 0, where interference patterns are calculated. We have decomposed total wave fields into a summation of the TM and TE wave modes.

$$\mathbf{E} = \mathbf{E}^{TM} + \mathbf{E}^{TE} \qquad (33) \qquad \text{and} \qquad \mathbf{H} = \mathbf{H}^{TM} + \mathbf{H}^{TE}. \qquad (34)$$

The TM and TE solutions are:

1) For a vertical electric dipole,

$$\mathbf{E}^{TM} = \int_{-\infty}^{\infty} dk_\rho \left(-\frac{I\ell}{8\pi\omega\epsilon}\right) \begin{Bmatrix} ik_\rho^2[\pm e^{\pm ik_z^{(e)}z} + R^{TM}E^{ik_z^{(e)}z}]H_0^{(1)'}(k_\rho\rho) \\ 0 \\ \dfrac{k_\rho^3}{k_z^{(e)}}[e^{\pm ik_z^{(e)}z} + R^{TM}e^{ik_z^{(e)}z}]H_0^{(1)}(k_\rho\rho) \end{Bmatrix}, \tag{35a}$$

$$\mathbf{H}^{TM} = \int_{-\infty}^{\infty} dk_\rho \left(-\frac{I\ell}{8\pi}\right) \begin{Bmatrix} 0 \\ i\dfrac{k_\rho^2}{k_z^{(e)}}[e^{\pm ik_z^{(e)}z} + R^{TM}e^{ik_z^{(e)}z}]H_0^{(1)'}(k_\rho\rho) \\ 0 \end{Bmatrix}, \tag{35b}$$

and $\mathbf{E}^{TE} = \mathbf{H}^{TE} = 0$.

2) For a vertical magnetic dipole,

$$\mathbf{E}^{TE} = \int_{-\infty}^{\infty} dk_\rho \left(-i\frac{IA\omega\mu}{8\pi}\right) \begin{Bmatrix} 0 \\ -i\dfrac{k_\rho^2}{k_z^{(m)}}(e^{\pm ik_z^{(m)}z} + R^{TE}e^{ik_z^{(m)}z})H_0^{(1)'}(k_\rho\rho) \\ 0 \end{Bmatrix}, \tag{36a}$$

$$\mathbf{H}^{TE} = \int_{-\infty}^{\infty} dk_\rho \left(-i\frac{IA}{8\pi}\right) \begin{Bmatrix} ik_\rho^2(\pm e^{\pm ik_z^{(m)}z} + R^{TE}e^{ik_z^{(m)}z})H_0^{(1)'}(k_\rho\rho) \\ 0 \\ \dfrac{k_\rho^3}{k_z^m}(e^{\pm ik_z^{(m)}z} + R^{TE}e^{ik_z^{(m)}z})H_0^{(1)}(k_\rho\rho) \end{Bmatrix}, \tag{36b}$$

and $\mathbf{E}^{TM} = \mathbf{H}^{TM} = 0$.

3) For a horizontal electric dipole along $\hat{x}$ direction,

$$\mathbf{E}^{TM} = \int_{-\infty}^{\infty} dk_\rho \left(i\frac{I\ell}{8\pi\omega\epsilon}\right) \begin{Bmatrix} ik_z^{(e)}k_\rho(e^{\pm ik_z^{(e)}z} - R^{TM}e^{ik_z^{(e)}z})H_1^{(1)'}(k_\rho\rho)\cos\phi \\ -\dfrac{k_z^{(e)}}{\rho}(e^{\pm ik_z^{(e)}z} - R^{TM}e^{ik_z^{(e)}z})H_1^{(1)}(k_\rho\rho)\sin\phi \\ k_\rho^2(\pm e^{\pm ik_z^{(e)}z} - R^{TM}e^{ik_z^{(e)}z})H_1^{(1)}(k_\rho\rho)\cos\phi \end{Bmatrix}, \tag{37a}$$

$$\mathbf{H}^{TM} = \int_{-\infty}^{\infty} dk_\rho \left(\frac{I\ell}{8\pi}\right) \begin{Bmatrix} -\dfrac{1}{\rho}(\pm e^{\pm ik_z^{(e)}z} - R^{TM}e^{ik_z^{(e)}z})H_1^{(1)}(k_\rho\rho)\sin\phi \\ -k_\rho(\pm e^{\pm ik_z^{(e)}z} - R^{TM}e^{ik_z^{(e)}z})H_1^{(1)'}(k_\rho\rho)\cos\phi \\ 0 \end{Bmatrix}, \tag{37b}$$

$$\mathbf{E}^{TE} = \int_{-\infty}^{\infty} dk_\rho \left(\frac{I\ell\omega\mu}{8\pi}\right) \begin{bmatrix} -\dfrac{1}{k_z^{(m)}\rho}\,(e^{\pm ik_z^{(m)}z} + R^{TE}e^{ik_z^{(m)}z})H_1^{(1)}(k_\rho\rho)\cos\phi \\ \dfrac{k_\rho}{k_z^{(m)}}\,(e^{\pm ik_z^{(m)}z} + R^{TE}e^{ik_z^{(m)}z})H_1^{(1)\prime}(k_\rho\rho)\sin\phi \\ 0 \end{bmatrix}, \tag{37c}$$

and

$$\mathbf{H}^{TE} = \int_{-\infty}^{\infty} dk_\rho \left(i\frac{I\ell}{8\pi}\right) \begin{bmatrix} ik_\rho(\pm e^{\pm ik_z^{(m)}z} + R^{TE}e^{ik_z^{(m)}z})H_1^{(1)\prime}(k_\rho\rho)\sin\phi \\ \dfrac{i}{\rho}\,(\pm e^{\pm ik_z^{(m)}z} + R^{TE}e^{ik_z^{(m)}z})H_1^{(1)}(k_\rho\rho)\cos\phi \\ \dfrac{k_\rho^2}{k_z^{(m)}}\,(e^{\pm ik_z^{(m)}z} + R^{TE}e^{ik_z^{(m)}z})H_1^{(1)}(k_\rho\rho)\sin\phi \end{bmatrix}. \tag{37d}$$

4) For a horizontal magnetic dipole along $\hat{x}$ direction,

$$\mathbf{E}^{TM} = \int_{-\infty}^{\infty} dk_\rho \left(-\frac{IA\omega\mu}{8\pi}\right) \begin{bmatrix} ik_\rho(\pm e^{\pm ik_z^{(e)}z} + R^{TE}e^{ik_z^{(e)}z})H_1^{(1)\prime}(k_\rho\rho)\sin\phi \\ \dfrac{i}{\rho}\,(\pm e^{\pm ik_z^{(e)}z} + R^{TE}e^{ik_z^{(e)}z})H_1^{(1)}(k_\rho\rho)\cos\phi \\ -\dfrac{k_\rho^2}{k_z^{(e)}}\,(e^{\pm ik_z^{(e)}z} + R^{TE}e^{ik_z^{(e)}z})H_1^{(1)}(k_\rho\rho)\sin\phi \end{bmatrix}, \tag{38a}$$

$$\mathbf{H}^{TM} = \int_{-\infty}^{\infty} dk_\rho \left(-\frac{IAk^2}{8\pi}\right) \begin{bmatrix} -i\dfrac{1}{k_z^{(e)}\rho}\,(e^{\pm ik_z^{(e)}z} + R^{TE}e^{ik_z^{(e)}z})H_1^{(1)}(k_\rho\rho)\cos\phi \\ i\dfrac{k_\rho}{k_z^{(e)}}\,(e^{\pm ik_z^{(e)}z} + R^{TE}e^{ik_z^{(e)}z})H_1^{(1)\prime}(k_\rho\rho)\sin\phi \\ 0 \end{bmatrix}, \tag{38b}$$

$$\mathbf{E}^{TE} = \int_{-\infty}^{\infty} dk_\rho \left(i\frac{IA\omega\mu}{8\pi}\right) \begin{bmatrix} -\dfrac{1}{\rho}\,(\pm e^{\pm ik_z^{(m)}z} + R^{TE}e^{ik_z^{(m)}z})H_1^{(1)}(k_\rho\rho)\cos\phi \\ -k_\rho(\pm e^{\pm ik_z^{(m)}z} + R^{TE}e^{ik_z^{(m)}z})H_1^{(1)\prime}(k_\rho\rho)\cos\phi \\ 0 \end{bmatrix}, \tag{38c}$$

and

$$\mathbf{H}^{TE} = \int_{-\infty}^{\infty} dk_\rho \left(\frac{IA}{8\pi}\right) \begin{bmatrix} ik_z^{(m)}k_\rho(-e^{\pm ik_z^{(m)}z} + R^{TE}e^{ik_z^{(m)}z})H_1^{(1)\prime}(k_\rho\rho)\cos\phi \\ -i\dfrac{k_z^{(m)}}{\rho}\,(-e^{\pm ik_z^{(m)}z} + R^{TE}e^{ik_z^{(m)}z})H_1^{(1)}(k_\rho\rho)\sin\phi \\ k_\rho^2(\pm e^{\pm ik_z^{(m)}z} + R^{TE}e^{ik_z^{(m)}z})H_1^{(1)}(k_\rho\rho)\cos\phi \end{bmatrix}. \tag{38d}$$

DISCUSSIONS

The problem of radiation of various dipole antennas over a stratified anisotropic medium has been solved. In view of the general formalism presented, we can make the following observations:

1) All medium properties such as the constitutive parameters and the geometrical configuration are absorbed into the reflection coefficients R^{TM} and R^{TE}, which are readily computed by equations (31) and (32). Clearly, when all regions possess the same constitutive parameters, namely when there is no stratification, $R^{TM}=R^{TE}=0$.

2) Since the anisotropy in permittivity appears only in $\epsilon(\pm)$, and the anisotropy in permeability appears only in $\mu(\pm)$, it is seen from (31)–(34) that R^{TM} does not depend on the magnetic anisotropy, and R^{TE} does not depend on the electric anisotropy. Both R^{TM} and R^{TE} are seen to be even functions of k_ρ.

3) It is obvious from (35)–(38) that a vertical electric dipole excites TM waves only, and a vertical magnetic dipole excites TE waves only, whereas both horizontal electric and magnetic dipoles excite both TM and TE waves. In the case when permeability is isotropic, the TM waves are extraordinary waves, and the TE waves are ordinary waves. A turnstile antenna, which consists of two dipoles perpendicular to each other and driven 90 degrees out of phase, also excites both TM and TE waves.

4) The above formulation can be compared with the potential approach for the various cases that exist. In the case of no stratified medium, the results are checked by using the identities

$$H_1^{(1)\prime} + \frac{1}{k_\rho \rho} H_1^{(1)} = H_0^{(1)} \tag{39}$$

and

$$\frac{i}{2}\int_{-\infty}^{\infty} dk_\rho e^{ik_z z}\, \frac{k_\rho}{k_z} H_0^{(1)}(k_\rho \rho) = \exp\left(ik\sqrt{\rho^2+z^2}\right)/\sqrt{\rho^2+z^2}, \tag{40}$$

where $r^2=\rho^2+z^2$ in spherical coordinates. We define the number of layers of a stratified medium equal to the number of boundaries. The medium below the nth boundary is called the tth layer.

5) The one-layer case (or half space) has been studied extensively. We obtain from (31) and (32)

$$R^{TM} = \frac{\epsilon(-)_0^1}{\epsilon(+)_0^1} \exp\left(i2k_z^{(e)} d_0\right) \quad \text{(41a)} \qquad \text{and} \qquad R^{TE} = \frac{\mu(-)_0^1}{\mu(+)_0^1} \exp\left(i2k_z^{(m)} d_0\right). \quad \text{(41b)}$$

In the case of a perfectly conducting half space, $\epsilon=\epsilon_3 \rightarrow \infty$. Equation (41) gives $R^{TM}=1$ and $R^{TE}=-1$.

6) Observe that all contributions due to all layers below the first are lumped into the reflection coefficient R_{n-1}, such that

$$R_n^{TM} = \exp\left[i2k_z^{(e)} d_0\right] \frac{\epsilon(+)_0^1}{\epsilon(-)_0^1}\left(1 - \frac{\epsilon(+)_0^1/\epsilon(-)_0^1 - \epsilon(-)_0^1/\epsilon(+)_0^1}{\epsilon(+)_0^1/\epsilon(-)_0^1 + \exp\left[-i2k_{1z}^{(e)} d_0\right] R_{n-1}^{TM}}\right) \tag{42}$$

and

$$R_n^{TE} = \exp\left[i2k_z^{(m)} d_0\right] \frac{\mu(+)_0^1}{\mu(-)_0^1}\left(1 - \frac{\mu(+)_0^1/\mu(-)_0^1 - \mu(-)_0^1/\mu(+)_0^1}{\mu(+)_0^1/\mu(-)_0^1 + \exp\left[-i2k_{iz}^{(m)} d_0\right] R_{n-1}^{TE}}\right). \tag{43}$$

The definitions for R_{n-1}^{TM} and R_{n-1}^{TE} follow directly from (31) and (32).

7) The integrals as presented in (35)–(38) can be programmed directly with a computer or analyzed analytically. Asymptotic evaluations of the integrals and numerical results for the electromagnetic field quantities under various circumstances constitute topics of subsequent papers.

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APPENDIX 1

In this appendix we derive equation (14) from well-known potential solutions for the dipole. Similar derivation, comparatively simpler, applies to equations (13), (15), and (16). The vector potential solution for the horizontal electric dipole is

$$\mathbf{A} = A\,(\hat{\rho}\cos\phi - \hat{\phi}\sin\phi), \quad \text{where} \quad A = \frac{I\ell}{4\pi}\,e^{ikr}/r.$$

The electromagnetic fields are obtained from

$$\mathbf{H} = \nabla \times \mathbf{A} \quad \text{and} \quad \mathbf{E} = i\frac{1}{\omega\epsilon}\left\{\nabla(\nabla\cdot\mathbf{A}) + k^2\mathbf{A}\right\}.$$

Using the identity (Sommerfeld, 1949)

$$\frac{e^{ikr}}{r} = \frac{i}{2}\int_{-\infty}^{\infty} dk_\rho\,\frac{k_\rho}{k_z}\,H_0^{(1)}(k_\rho\rho)e^{\pm ik_z z}.$$

The field components all can be written in the integral form. For the z component,

$$H_z = -\sin\phi\,\partial A/\partial\rho \quad \text{and} \quad E_z = i\frac{1}{\omega\epsilon}\cos\phi\,\partial^2 A/\partial\rho\partial z.$$

The results are equations (14a) and (14b).

APPENDIX 2

In this appendix, propagation matrices are derived from the boundary conditions. Consider the boundary at $z = -d_i$, the continuity of tangential electric fields and the continuity of tangential magnetic fields yields, for the TM waves,

$$k_{iz}^{(e)}\left\{-A_i e^{ik_{iz}^{(e)}d_i} + B_i e^{-ik_{iz}^{(e)}d_i}\right\} = k_{(i+1)z}^{(e)}\left\{-A_{i+1}e^{ik_{(i+1)z}^{(e)}d_i} + B_{i+1}e^{-ik_{(i+1)z}^{(e)}d_i}\right\} \quad \text{and} \tag{A1}$$

$$\epsilon_i\left\{A_i e^{ik_{iz}^{(e)}d_i} + B_i e^{-ik_{iz}^{(e)}d_i}\right\} = \epsilon_{i+1}\left\{A_{i+1}e^{ik_{(i+1)z}^{(e)}d_i} + B_{i+1}e^{-ik_{(i+1)z}^{(e)}d_i}\right\}. \tag{A2}$$

For the TE waves

$$\mu_i\left\{C_i e^{ik_{iz}^{(m)}d_i} + D_i e^{-ik_{iz}^{(m)}d_i}\right\} = \mu_{i+1}\left\{C_{i+1}e^{ik_{(i+1)z}^{(m)}d_i} + D_{i+1}e^{-ik_{(i+1)z}^{(m)}d_i}\right\} \quad \text{and} \tag{A3}$$

$$k_{iz}^{(m)}\left\{-C_i e^{ik_{iz}^{(m)}d_i} + D_i e^{-ik_{iz}^{(m)}d_i}\right\} = k_{(i+1)z}^{(m)}\left\{-C_{i+1}e^{ik_{(i+1)z}^{(m)}d_i} + D_{i+1}e^{-ik_{(i+1)z}^{(m)}d_i}\right\}. \tag{A4}$$

The reason that we can treat the TM and TE cases separately is that 1) for the vertical dipole case, only TM or TE is excited; 2) for the horizontal dipole case, although the total tangential field components consist of both TM and TE waves, the coefficients of $H_1^{(1)}$ and $H_1^{(1)\prime}$ separate TM and TE cases.

We now illustrate the derivation for the upward propagation matrix M_i^{i+1} for TM cases. From (A1) and (A2) it is straight-forward to solve for $A_i \exp ik_{iz}^{(e)}d_i$ and $B_i \exp -ik_{iz}^{(e)}d_i$, in terms of A_{i+1} and B_{i+1}. Using the definition (22) for a_i and b_i, we have

$$a_i = \frac{1}{2}\left\{\frac{\epsilon_{i+1}}{\epsilon_i} + \frac{k_{(i+1)z}^{(e)}}{k_{iz}^{(e)}}\right\} A_{i+1}e^{ik_{(i+1)z}^{(e)}d_i} + \frac{1}{2}\left\{\frac{\epsilon_{i+1}}{\epsilon_i} - \frac{k_{(i+1)z}^{(e)}}{k_{iz}^{(e)}}\right\} B_{i+1}e^{-ik_{(i+1)z}^{(e)}d_i} \quad \text{and} \tag{A5}$$

$$b_i = \frac{1}{2}\left\{\frac{\epsilon_{i+1}}{\epsilon_i} - \frac{k_{(i+1)z}^{(e)}}{k_{iz}^{(e)}}\right\} A_{i+1}e^{ik_{(i+1)z}^{(e)}d_i} + \frac{1}{2}\left\{\frac{\epsilon_{i+1}}{\epsilon_i} + \frac{k_{(i+1)z}^{(e)}}{k_{iz}^{(e)}}\right\} B_{i+1}e^{-ik_{(i+1)z}^{(e)}d_i}. \tag{A6}$$

Introducing definitions (18) and (19), we can write

$$a_i = \frac{1}{2}\left\{\epsilon(+)_i^{i+1} e(+)_i^{(e)} a_{i+1} + \epsilon(-)_i^{i+1} e(-)_i^{(e)} b_{i+1}\right\} \quad \text{and} \tag{A7}$$

$$b_i = \frac{1}{2}\left\{\epsilon(-)_i^{i+1} e(+)_i^{(e)} a_{i+1} + \epsilon(+)_i^{i+1} e(-)_i^{(e)} b_{i+1}\right\}. \tag{A8}$$

In view of the definition for propagation matrix (17), (A7) and (A8) are immediately cast into the form (21). The derivation for the downward propagation matrix involves a solution for A_{i+1}, B_{i+1} in terms of A_i and B_i; the result is equations (24a) and (24b). Similar procedure applied to (A3) and (A4) yields propagation matrices for TE waves.