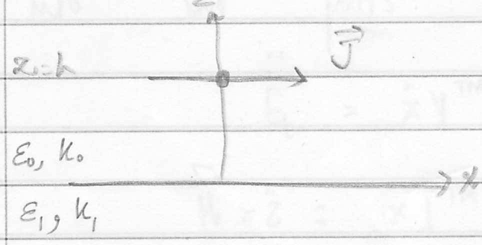
Computation of fields of a horizontal electric dipole over a planar half-space.



Considering a point hertzian dipole.

We can compute the fields by using transmission line dyadic green's functions. For an electric source:

$$\vec{E}(\vec{r}) = \langle \overline{\overline{G}}^{EJ} ; \vec{J}(\vec{r}') \rangle \quad \text{— (1a)}$$

$$\vec{H}(\vec{r}) = \langle \overline{\overline{G}}^{HJ} ; \vec{J}(\vec{r}') \rangle \quad \text{— (1b)}$$

In the particular case $\vec{J}(\vec{r}') = \hat{x} J_x \, \delta(z-h)\, \delta(x)\, \delta(y)$ — (2)

Spectral domain Maxwell's equations resolved into transverse and longitudinal components can be written as: [cite {bookchapter p. 1166}

Transverse

$$\frac{d}{dz} \tilde{\vec{E}}_t = \frac{1}{j\omega\varepsilon} \left(k^2 - \vec{k}_\rho \vec{k}_\rho \cdot \right) \cdot \left(\tilde{\vec{H}}_t \times \hat{z} \right) \quad \text{— (3a)}$$

$$\frac{d}{dz} \tilde{\vec{H}}_t = \frac{1}{j\omega\mu} \left(k^2 - \vec{k}_\rho \vec{k}_\rho \cdot \right) \left(\hat{z} \times \tilde{\vec{E}}_t \right) - \hat{z} \times \tilde{\vec{J}}_t \quad (3b)$$

$\hat{y} \frac{dI^{TM}}{dz} = \frac{1}{j\omega\mu}(k^2 - \vec{k}_\rho)(\hat{z} \times \vec{E}_t) - \hat{z} \times \vec{J}_x - \hat{y}$

Long.

$$-j\omega\varepsilon \tilde{E}_z = j\vec{k}_\rho \cdot \left(\tilde{\vec{H}}_t \times \hat{z} \right) \quad \text{— (3c)}$$

$$-j\omega\mu \tilde{H}_z = j\vec{k}_\rho \cdot \left(\hat{z} \times \tilde{\vec{E}}_t \right) \quad \text{— (3d)}$$

TM_z (2) $\hat{v}\,\tilde{M}_s(\vec{k}_s)$

The transverse components are further resolved into two parts TM and TE.

(4a) $$\vec{\tilde{E}}_t = \hat{x} V^{TM} + \hat{y} V^{TE} \quad ; \quad \hat{z} \times \tilde{E}_t = \hat{y} V^{TM} - \hat{x} V^{TE}$$

(4b) $$\vec{\tilde{H}}_t \times \hat{z} = \hat{x} I^{TM} + \hat{y} I^{TE} \quad ; \quad \vec{\tilde{H}}_t = -\hat{x} I^{TE} + \hat{y} I^{TM}$$

which can now be written in a form of transmission line equation

(5a) $$\frac{dV^{\alpha}}{dz} = -jk_z^{\alpha} Z^{\alpha} I^{\alpha} + v^{\alpha}$$ where $\alpha = (TE, TM)$

(5b) $$\frac{dI^{\alpha}}{dz} = -jk_z^{\alpha} Y^{\alpha} V^{\alpha} + i^{\alpha}$$

and $k_z = \sqrt{k^2 - k_\rho^2}$, $Z^{TM} = \frac{1}{Y^{TM}} = \frac{k_z}{\omega\varepsilon}$

$Z^{TE} = \frac{1}{Y^{TE}} = \frac{\omega\mu}{k_z}$

★ For horizontal electric dipole

$$\vec{J}(\vec{r}\,') = \hat{x}\, J_x\, \delta(x)\, \delta(y) = \vec{J}_t$$

$$\Rightarrow \boxed{\vec{\tilde{J}}_t = F\left[\vec{J}_t\right] = \hat{x}\, J_x} \quad \text{--- (6)}$$

From physical insight, there will only be a TM^z mode for the horizontal electric dipole.

$\therefore$ from (5a), (5b) and (6)

$$\frac{dV^{TM}}{dz} = -jk_z Z^{TM} I^{TM} \qquad \text{— (7a)}$$

$$\frac{dI^{TM}}{dz} = -jk_z Y^{TM} V^{TM} + i^{TM} \qquad \text{— (7b)}$$

From (3b) and (4b) $\boxed{i^{TM} = -J_x}$ — (8)

Therefore, a horizontally oriented electric dipole can be represented by a TM-case transmission line with a current source excitation, i^{TM}

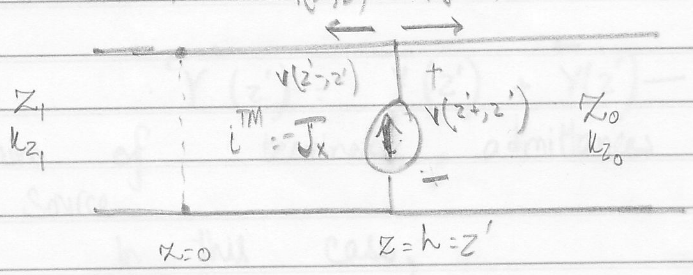


Fig 1.

We can also draw the above transmission line as,

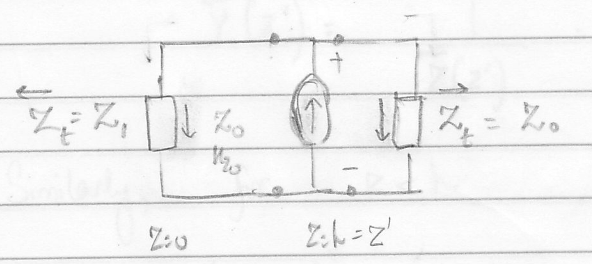


Fig. 2

where $Z_1 = \dfrac{k_{z_1}}{\omega\varepsilon_1} = \dfrac{k_{z_1}}{\omega\varepsilon_0\varepsilon_{r_1}}$ — (9a)

$Z_0 = \dfrac{k_{z_0}}{\omega\varepsilon_0} = \dfrac{k_{z_0}}{\omega\varepsilon_0}$ — (9b)

(4)

The voltage and the current at source as shown in Fig. 2 can be written as:

for $z > h$

$$V(z'+, z') = \frac{+i}{\overleftrightarrow{Y}(z')} \quad \text{— (10a)}$$

?

$$I(z'+, z') = \frac{+i\overrightarrow{Y}(z')}{\overleftrightarrow{Y}(z')} \quad \text{— (10b)}$$

where i is the source strength $= -J_x$

$$\overleftrightarrow{Y}(z') = \overrightarrow{Y}(z') + \overleftarrow{Y}(z') \quad \text{— (11)}$$

is the sum of terminal admittances seen at the source.

In this case,

$$\overrightarrow{Y}(z') = \frac{1}{Z_0} \quad \text{— (12a)}$$

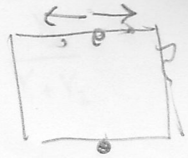
$$\overleftarrow{Y}(z') = \frac{1}{\overleftarrow{Z}(z')} = \frac{1}{Z_0}\,\frac{Z_0 + j\overleftarrow{Z}_t \tan k_{z0}h}{\overleftarrow{Z}_t + jZ_0 \tan k_{z0}h} \quad (12b)$$

Similarly, for $z < h$

$$V(z'-, z') = \frac{+i}{\overleftrightarrow{Y}(z')} \quad \text{— (13a)}$$

$$I(z'-, z') = -\frac{i\overleftarrow{Y}(z')}{\overleftrightarrow{Y}(z')} \quad \text{— (13b)}$$

from (10a) and (13a) we see the continuity of voltage at the source point whereas from (10b) and (13b), there is a



discontinuity in the current due to the presence of the source.

Once the voltage and current are known at the source point ($z = z' = h$), V and I can be determined at any point z in the section with x-tic impedance Z_0 and propagation constant k_{z0}.

Knowing that for a homogeneous, source-free transmission line equation the solution is

$$\begin{bmatrix} V(z) \\ ZI(z) \end{bmatrix} = V_{inc}(z')\, e^{-jk_z(z-z')} \pm V_{ref}(z')\, e^{+jk_z(z-z')} \qquad (14)$$

$$= V_{inc}(z')\, e^{-jk_z(z-z')} \left[1 \pm \Gamma(z')\, e^{+2jk_z(z-z')} \right] \qquad (15)$$

where $\Gamma(z') = \dfrac{V_{ref}(z')}{V_{inc}(z')}$ is the voltage reflection coefficient at a point z' on the transmission line.

Writing the solution as:

$$V(z) = V_{inc}(z) + V_{ref}(z) \qquad (16a)$$

$$V_{inc}(z')\, e^{-jk_z(z-z')} + V_{ref}(z')\, e^{+jk_z(z-z')}$$

$$ZI(z) = V_{inc}(z) - V_{ref}(z) \qquad (16b)$$

$$\frac{Z_1}{Z_1+Z_2} \qquad \frac{Y_2}{Y_1+Y_2} = \frac{1/Z_2}{1/Z_1+1/Z_2} = \frac{1/Z_2}{Z_1+Z_2} \cdot \frac{Z_1 Z_2}{\frac{Z_2}{Z_1+Z_2}}$$

$$= \frac{Z_1}{Z_1+}$$

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from (16a) and (16b)

$$V_{inc}(z) = \frac{1}{2}\left(V(z) + ZI(z)\right) \quad \text{— (17a)}$$

$$V_{ref}(z) = \frac{1}{2}\left(V(z) - ZI(z)\right) \quad \text{— (17b)}$$

$\therefore$ The voltage at any point with $z \geq z' = h$

$$\cancel{V(z,z') = \frac{1}{2}\left[V_{inc}(z') e^{-jk_2(z-z')} + V_{ref}(z') e^{+jk_2(z-z')} + Z \right.}$$

from (15)

18a $\leftarrow$ $$V(z) = V_{inc}(z') \left[e^{-jk_2(z-z')} + \Gamma(z') e^{+jk_2(z-z')} \right.$$

18b $\leftarrow$ $$= \frac{1}{2}\left[V(z'_+, z') + ZI(z'_+, z')\right]\left[e^{-jk_2(z-z')} + \Gamma(z') e^{+j(k_2 z - z')}\right]$$

from (10)

19 $\leftarrow$ $$V(z) = \frac{1}{2}\left[\frac{i}{\overleftrightarrow{Y}(z')} + \frac{iZ_0 \overrightarrow{Y}(z')}{\overleftrightarrow{Y}(z')}\right]\left[e^{-jk_2(z-z')} + \Gamma(z') e^{+j(k_2(z-z'))}\right]$$

$z > h$

$$\underset{= YV(z)}{I(z)} = \frac{Y}{2}\left[V(z'+, z) + ZI(z', z')\right]\left[e^{-jk_2(z-z')} - \Gamma(z') e^{+jk_2(z-z')}\right]$$

20 $\leftarrow$ $$= \frac{1}{2}\left[\frac{Y_0\, i}{\overleftrightarrow{Y}(z')} + iY_1 Z_0 \frac{\overrightarrow{Y}(z')}{\overleftrightarrow{Y}(z')}\right]\left[e^{-jk_2(z-z')} - \Gamma(z') e^{+jk_2(z-z')}\right]$$