ECE 5322
21st Century Electromagnetics

Instructor: Dr. Raymond C. Rumpf
Office: A-337
Phone: (915) 747-6958
E-Mail: rcrumpf@utep.edu

Lecture #21

Surface Waves

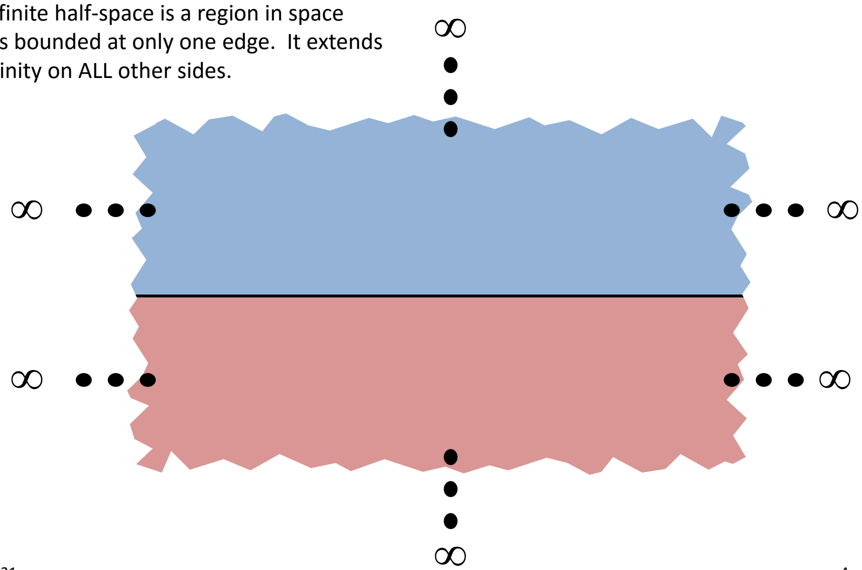
Lecture Outline

- Introduction
- Survey of surface waves
- Excitation of surface waves
- Surface plasmon polaritons
- Dyakonov surface waves

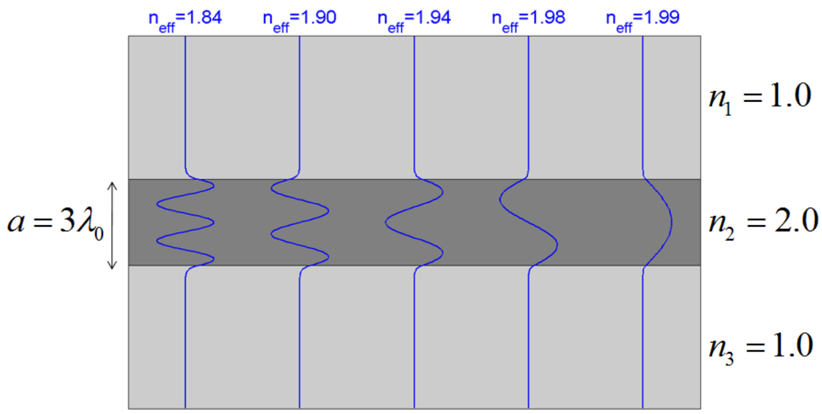
Introduction

Infinite Half-Space

An infinite half-space is a region in space that is bounded at only one edge. It extends to infinity on ALL other sides.



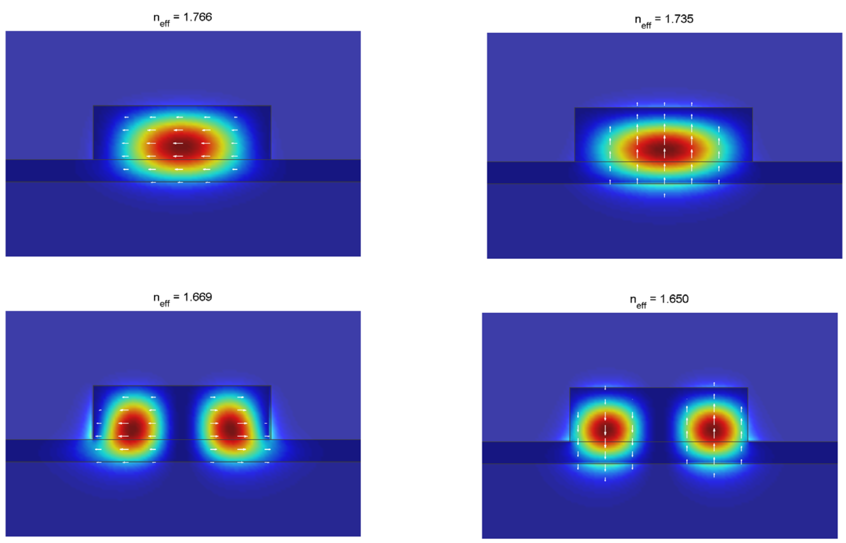
Traditional Guided Modes (1 of 2)



$$\beta = k_0 n_{\text{eff}} = k_0 n \sin\theta$$

Lecture 21 Slide 5

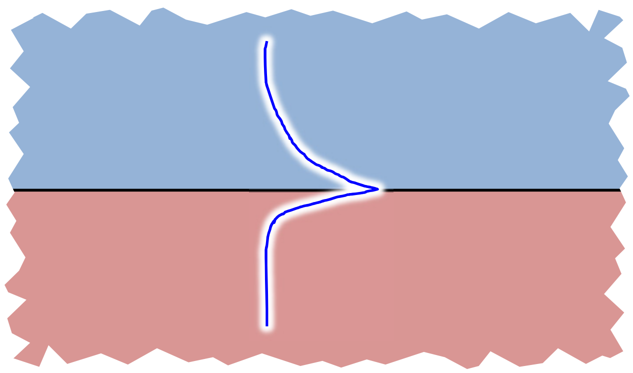
Traditional Guided Modes (2 of 2)



Lecture 21 Slide 6

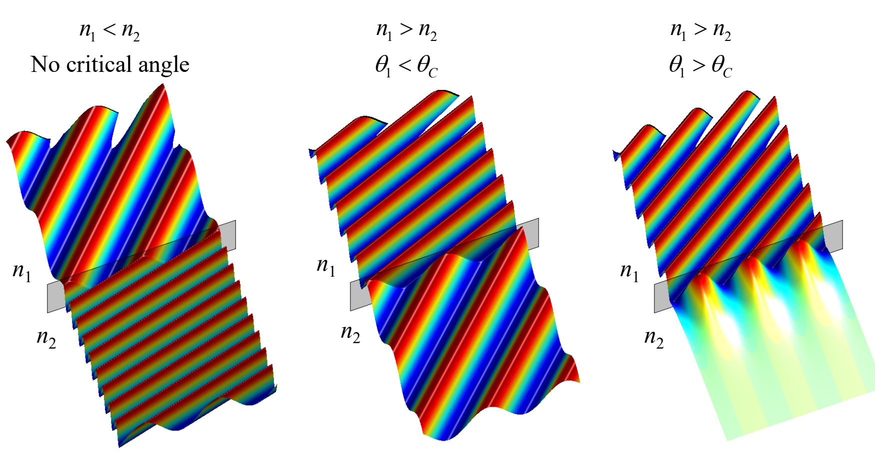
A New Guided Mode – Surface Waves

A surface wave is most analogous to a slab waveguide, but the mode is confined at the interface between two different materials comprising two infinite half spaces. The field decays exponentially away from the interface. It is free to propagate without decay in the plane of the interface.



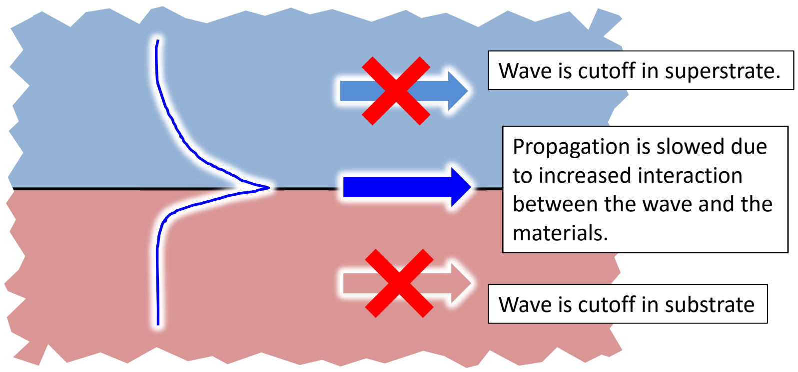
Why Do Surface Waves Exist?

Recall the Field at an Interface



1. The field always penetrates material 2, but it may not propagate.
2. Above the critical angle, penetration is greatest near the critical angle.
3. Very high spatial frequencies are supported despite the dispersion relation.
4. In material 2, energy always flows along *x*, but not necessarily along *y*.

Why Do Surface Waves Exist?
Hand Waving Explanation



Lecture 21 Slide 9

Survey of Surface Waves

Types of Surface Waves

- Zenneck surface waves (ground waves)
- Resonant surface wave
- Surface waves at chiral interfaces
- Surface waves at gyrotropic interfaces
- Nonlinear surface wave
- Surface plasmon polariton (SPP)
- Dyakonov surface wave (DSW)
- Optical Tamm States (OTSs)

Zenneck Surface Wave

Zenneck waves are essentially surface plasmons at RF frequencies.

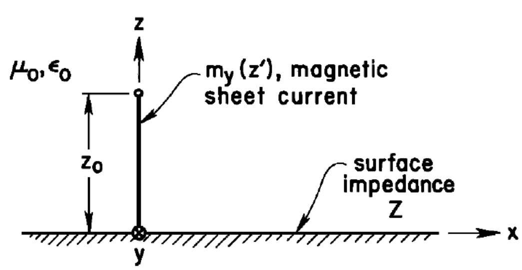
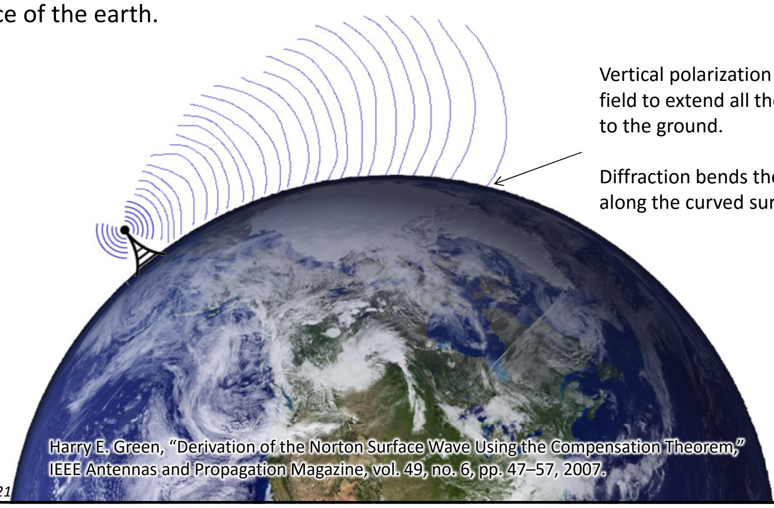


Fig. 1. Geometry for a vertical magnetic current sheet over a half space of surface impedance Z. The aperture height z_0 can be either finite or infinite.

D. A. Hill, J. R. Wait, "Excitation of the Zenneck surface wave by a vertical aperture," Radio Science, vol. 13, no. 6, pp. 969–977, 1978.

Norton Surface Wave

Norton waves are vertically polarized (TM) waves supported at the interface between a dielectric and a lossy material. These are also known as ground waves and are why long wavelength signals, such as that from AM radio, travel efficiently across the surface of the earth.



Vertical polarization allows field to extend all the way to the ground.

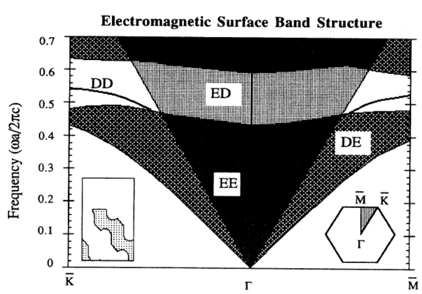
Diffraction bends the wave along the curved surface.

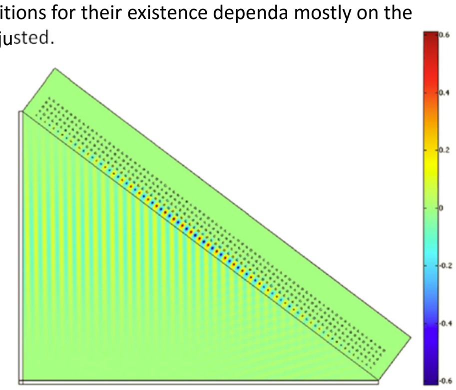
Harry E. Green, "Derivation of the Norton Surface Wave Using the Compensation Theorem," IEEE Antennas and Propagation Magazine, vol. 49, no. 6, pp. 47–57, 2007.

Lecture 21 13

Resonant Surface Wave

Resonant surface waves exist at the interface between essentially any material and a resonant photonic crystal. The surface waves exist at frequencies lying inside the band gap. Tremendous design freedom is offered by resonant surface waves because the conditions for their existence dependa mostly on the geometry of the lattice which can be tailored and adjusted.





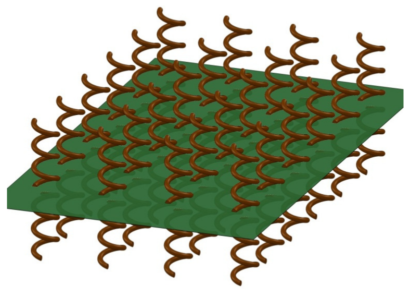
R. D. Meade, K. D. Brommer, A. M. Rappe, J. D. Joannopoulos, "Electromagnetic Bloch waves at the surface of a photonic crystal," Physical Review B, vol. 44, no. 19, pp. 10961–10964, 1991.

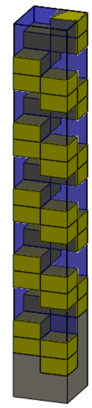
B. Wang, W. Dai, A. Fang, L. Zhang, G. Tuttle, Th. Koschny, C. M. Soukoulis, "Surface waves in photonic crystal slabs," Physical Review B, vol. **74** 195104, 2006.

Lecture 21 14

Surface Waves at Chiral Interfaces

Chiral materials possess an intrinsic handedness leading to unique electromagnetic properties. Surface waves at the interface of chiral materials are hybrid modes and exhibit split cutoff frequencies. These are attractive features for suppressing surface waves in antennas and for forming directional couplers. They also are excellent absorbers when made of lossy materials.

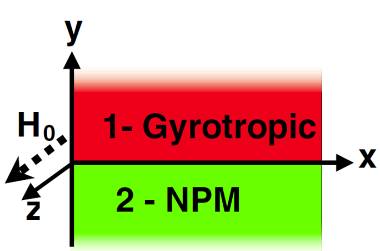


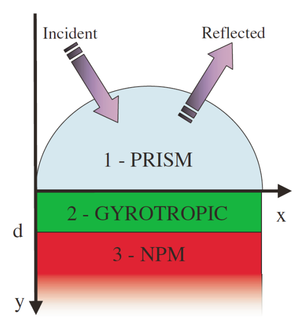


N. Engheta, P. Pelet, "Surface waves in chiral layers," Opt. Lett. **16**, 723 (1991).

Surface Waves at Gyrotropic Interfaces

A gyrotropic material is one that is perturbed ($\Delta\varepsilon$) by a quasi-static magnetic field. Surface waves are supported at the interface between a gyrotropic material (either gyroelectric or gyromagnetic) and an isotropic negative phase velocity medium (NPM). These exhibit interesting properties such as nonreciprocal propagation and anomalous dispersion.





Boardman, N. King, Y. Rapoport, L. Velasco, "Gyrotropic impact upon negatively refracting surfaces," New J. Phys. **7** 191 (2005).

H.-Y. Yang, J. A. Castaneda, N. G. Alexopoulos, "SurfaceWaves in Gyrotropic Substrates," IEEE, pp. 1651-1654, 1991.

Nonlinear Surface Wave

Surface waves can exist at the interface between an ordinary material and a nonlinear material. It has been shown that when the lower refractive index material has a positive Kerr coefficient, the surface wave propagates with perfectly constant shape and intensity and can be excited directly by an external wave.

D is an independent parameter relating the various wave vector components.

$$k_{1z} = k_0\psi_c\sqrt{D},$$
$$k_{2z} = k_0\psi_c\sqrt{1+D},$$

$$k_x = k_0\psi_c\sqrt{\psi_c^{-2}+D}.$$

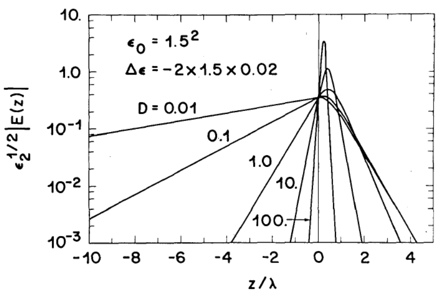


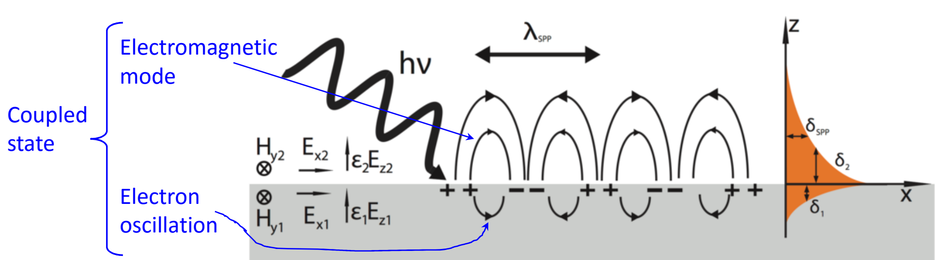
Fig. 2. Calculated field amplitude as a function of z for several values of D. (The values for ϵ_0 and $\Delta\epsilon$ are the same as for Fig. 1.)

W. J. Tomlinson, "Surface wave at a nonlinear interface," Optics Letters, vol. 5, no. 7, pp. 323–325, July 1980.

H. E. Ponath, G. I. Stegeman, *Nonlinear surface electromagnetic phenomena*, (North Holland, New York, NY 1991).

Surface Plasmon-Polariton

Surface plasmon-polaritons (SPPs) are supported at the interface between a material with positive dielectric constant and a material with negative dielectric constant. There exists an analogous surface wave at the interface of a material with negative permeability. Surface plasmons are attracting much attention in the optics community for their very useful propagation characteristics and radical miniaturization, but they suffer from extraordinary losses



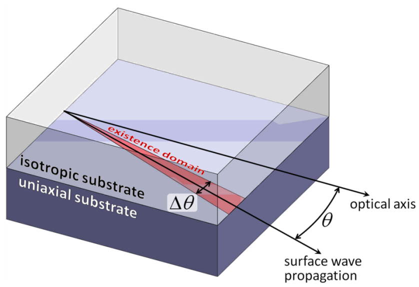
http://en.wikipedia.org/wiki/File:Sketch_of_surface_plasmon.png

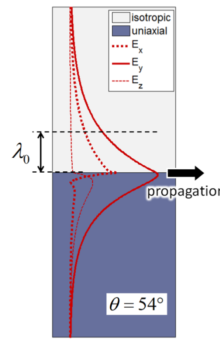
J. M. Pitarke, et al, "Theory of surface plasmons and surface-plasmon polaritons," Rep. Prog. Phys. 70, pp. 1–87, 2007.

W. L. Barnes, A. Dereux, T. W. Ebbesen, "Surface plasmon subwavelength optics," Nature 424, pp. 824-830, 2003.

Dyakonov Surface Wave

Dyakonov surface waves (DSWs) are supported at the interface between two materials where at least one is anisotropic. They are not well understood, but exhibit many unique and intriguing properties.





O. Takayama, L. Crasovan, S. Johansen, D. Mihalache, D. Artigas, L. Torner, "Dyakonov Surface Waves: A Review," Electromagnetics, vol. 28, pp. 126–145, 2008.

Optical Tamm States

Optical Tamm States (OTSs) can be formed at the interface between two periodic structures. The first has a period close to the wavelength. The second has a period close to the double of the wavelength.

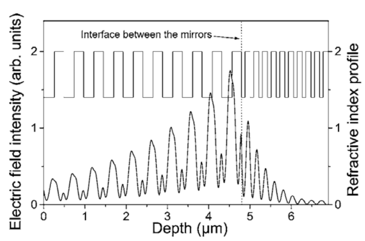


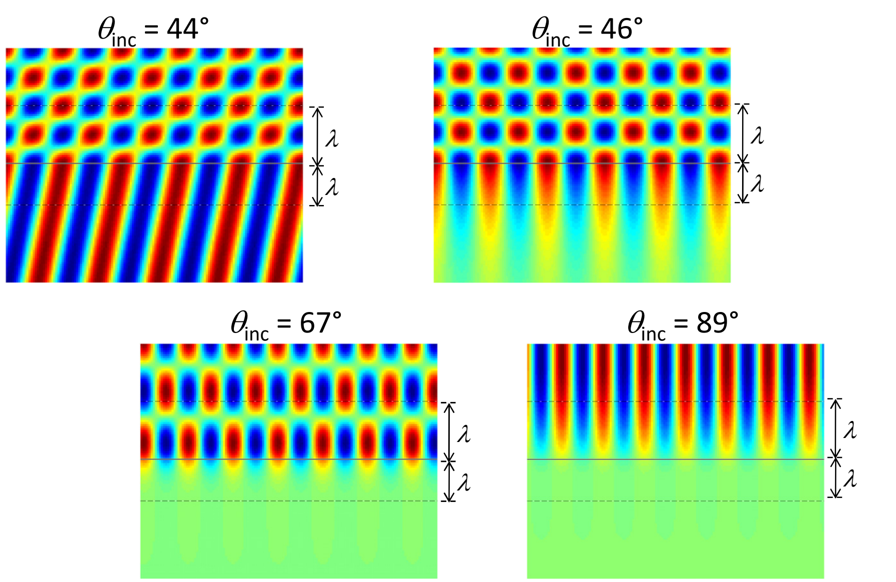
FIG. 2. The solid bold line shows the electric field intensity profile of the OTS within the proposed structure. The solid thin line is for the corresponding refractive index profile. The dotted line is for the interface between two substructures.

- Superstrate and substrate must have overlapping band gaps, but different periods.
- OTSs exist in any direction along surface.
- Highly sensitive to the order of the layers at the interface.
- Dispersion curve is parabolic.
- May serve as an alternative to DSWs.

A. V. Kovokin, I. A. Shelykh, G. Malpuech, "Lossless interface modes at the boundary between two periodic dielectric structures," Phys. Rev. B **72**, 233102 (2005).

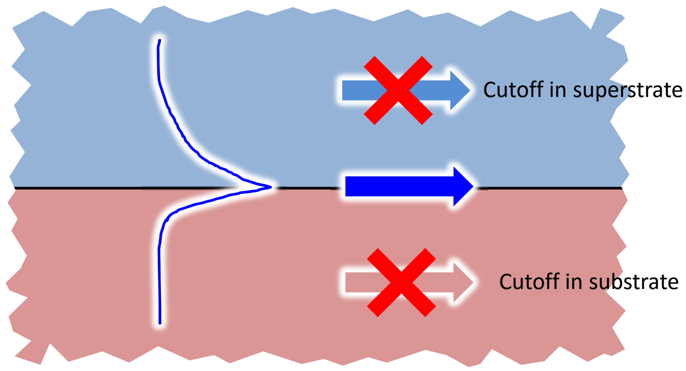
Excitation of Surface Waves

Field Visualization for θ_c=45°



Lecture 21 Slide 22

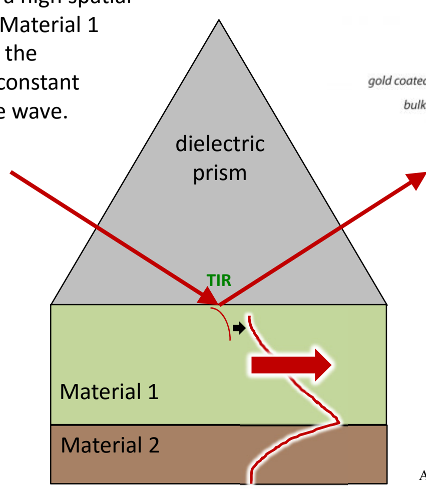
Conceptual Picture of a Surface Wave

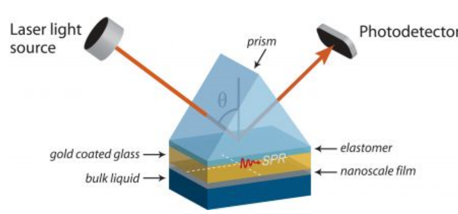


Otto Configuration

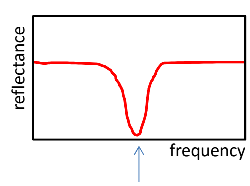
Attenuated total reflection setup

TIR produces a high spatial frequency in Material 1 that matches the propagation constant of the surface wave.





http://www.bionavis.com/technology/spr/

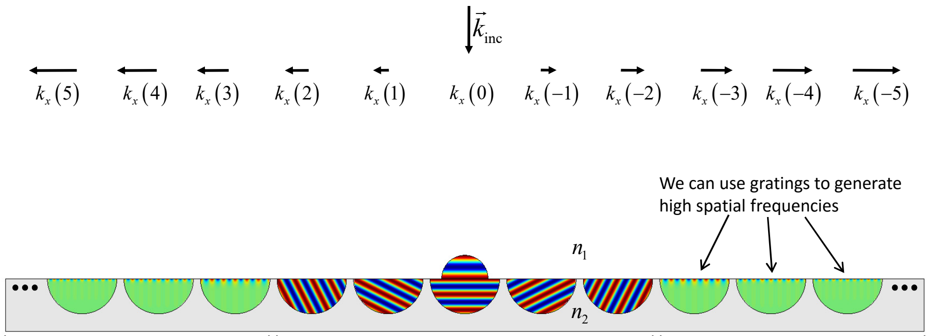


Frequency where surface wave is excited.

Andreas Otto, Aeitschrift für Physik, Vol. 216, Issue 4, pp. 398-410, 1968.

Recall the Field Associated with the Diffracted Modes

The wave vector expansion for the first 11 diffracted modes can be visualized as…



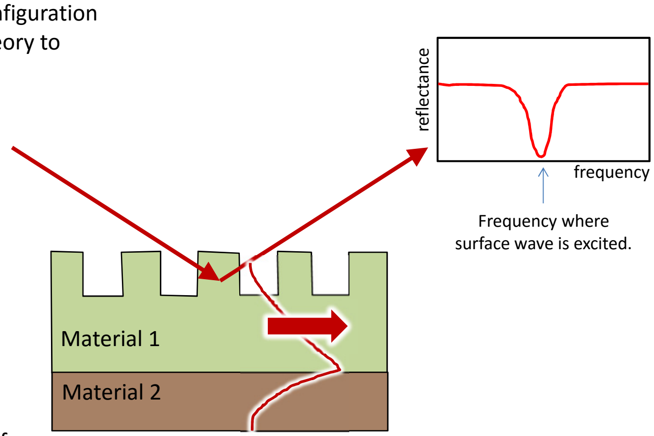
k_y is imaginary.
The field in material 2 is evanescent.

k_y is real.
A wave propagates into material 2.

k_y is imaginary.
The field in material 2 is evanescent.

Grating Coupler Configuration

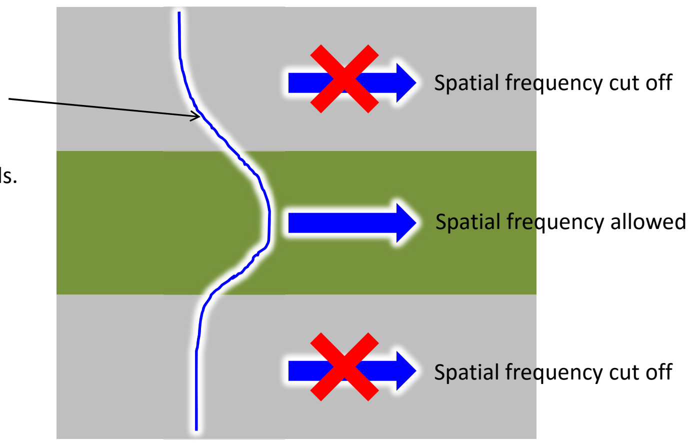
The grating coupler configuration uses coupled-mode theory to excite a surface wave.



A high-order spatial harmonic (usually the 2nd order) produces a high spatial frequency that matches the propagation constant of the surface wave.

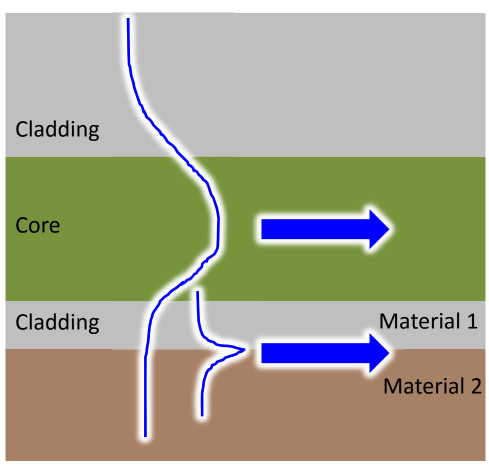
Recall the Field Around a Waveguide

The evanescent field outside of a waveguide has a high spatial frequency and is cutoff by the cladding materials.



Lecture 21 27

Evanescent Coupling Configuration



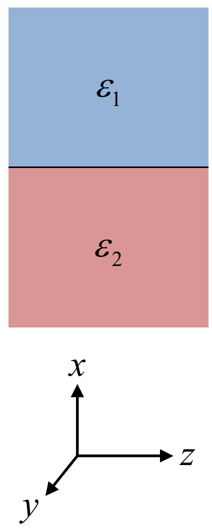
Lecture 21 28

Surface Plasmon Polaritons

Why Do We Care About SPPs?

- Highly subwavelength → radical miniaturization
  - Able to concentrate energy in subwavelength volumes
- Strong dispersive properties → new mechanisms for manipulating waves
  - Able to guide waves along the surface of a metal
- Applications
  - Sensors
  - Data storage
  - Light sources
  - Microscopy
  - Bio-photonics
  - Subwavelength optics (nanofabrication and imaging)

Classical Analysis



We start with Maxwell's equations

$$\nabla \times \vec{H} = j\omega\varepsilon\vec{E} \qquad \nabla \bullet \left(\varepsilon\vec{E}\right) = 0$$
$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \qquad \nabla \bullet \vec{H} = 0$$

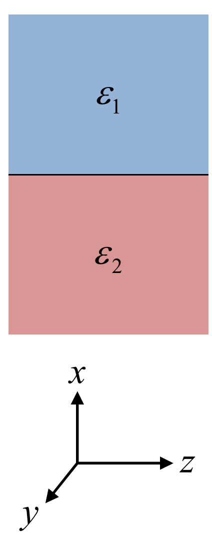
For 1D geometries, we have

$$\frac{\partial}{\partial y} = 0$$

Maxwell's equations split into two independent modes.

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\varepsilon_0\varepsilon_r E_y \qquad \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu_0\mu_r H_y$$
$$-\frac{\partial E_y}{\partial z} = -j\omega\mu_0\mu_r H_x \qquad -\frac{\partial H_y}{\partial z} = j\omega\varepsilon_0\varepsilon_r E_x$$
$$\frac{\partial E_y}{\partial x} = -j\omega\mu_0\mu_r H_z \qquad \frac{\partial H_y}{\partial x} = j\omega\varepsilon_0\varepsilon_r E_z$$

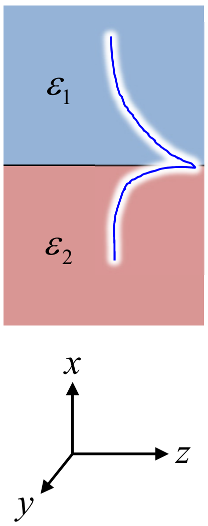
Only the H Mode Exists



If a wave is to propagate along the surface of a metal, the electric field must be polarized normal to the surface. Otherwise, boundary conditions will require it to be zero. Therefore, the E mode does not exist.

$$\cancel{j\beta H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon_0\varepsilon_r E_y} \qquad j\beta E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu_0\mu_r H_y$$
$$\cancel{-j\beta E_y = -j\omega\mu_0\mu_r H_x} \qquad -j\beta H_y = j\omega\varepsilon_0\varepsilon_r E_x$$
$$\cancel{\frac{\partial E_y}{\partial x} = -j\omega\mu_0\mu_r H_z} \qquad \frac{\partial H_y}{\partial x} = j\omega\varepsilon_0\varepsilon_r E_z$$

Assumed Solution



If the wave is a surface wave, it must be confined to the surface. This can only happen if the field decays exponentially away from the interface. This implies the field solution has the following form.

$$\vec{E}_i(z) = \begin{bmatrix} E_{x,i} \\ E_{z,i} \end{bmatrix} e^{-\kappa_i |x|} e^{j\beta z}$$

$$\vec{H}_i(z) = H_{y,i} e^{-\kappa_i |x|} e^{j\beta z}$$

$$i = \begin{cases} 1 & x < 0 \\ 2 & x > 0 \end{cases}$$

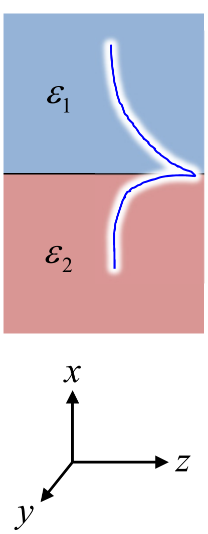
Substituting this solution into the H mode equations yields

$$\frac{\partial}{\partial z}\left(E_{x,i} e^{-\kappa_i |x|} e^{j\beta z}\right) - \frac{\partial}{\partial x}\left(E_{z,i} e^{-\kappa_i |x|} e^{j\beta z}\right) = -j\omega\mu_0\mu_{r,i}\left(H_{y,i} e^{-\kappa_i |x|} e^{j\beta z}\right)$$

$$-\frac{\partial}{\partial z}\left(H_{y,i} e^{-\kappa_i |x|} e^{j\beta z}\right) = j\omega\varepsilon_0\varepsilon_{r,i}\left(E_{x,i} e^{-\kappa_i |x|} e^{j\beta z}\right)$$

$$\frac{\partial}{\partial x}\left(H_{y,i} e^{-\kappa_i |x|} e^{j\beta z}\right) = j\omega\varepsilon_0\varepsilon_{r,i}\left(E_{z,i} e^{-\kappa_i |x|} e^{j\beta z}\right)$$

Equations in Medium 1



Inside medium 1, our three equations were

$$\frac{\partial}{\partial z}\left(E_{x,1} e^{-\kappa_1 x} e^{j\beta z}\right) - \frac{\partial}{\partial x}\left(E_{z,1} e^{-\kappa_1 x} e^{j\beta z}\right) = -j\omega\mu_0\mu_{r,1}\left(H_{y,1} e^{-\kappa_1 x} e^{j\beta z}\right)$$

$$-\frac{\partial}{\partial z}\left(H_{y,1} e^{-\kappa_1 x} e^{j\beta z}\right) = j\omega\varepsilon_0\varepsilon_{r,1}\left(E_{x,1} e^{-\kappa_1 x} e^{j\beta z}\right)$$

$$\frac{\partial}{\partial x}\left(H_{y,1} e^{-\kappa_1 x} e^{j\beta z}\right) = j\omega\varepsilon_0\varepsilon_{r,1}\left(E_{z,1} e^{-\kappa_1 x} e^{j\beta z}\right)$$

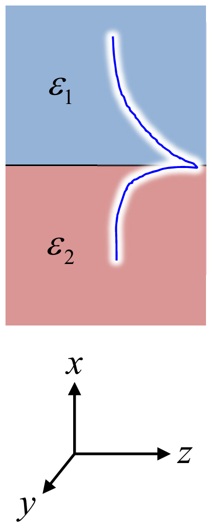
These reduce to

$$j\beta E_{x,1} + \kappa_1 E_{z,1} = -j\omega\mu_0\mu_{r,1} H_{y,1}$$

$$-j\beta H_{y,1} = j\omega\varepsilon_0\varepsilon_{r,1} E_{x,1}$$

$$-\kappa_1 H_{y,1} = j\omega\varepsilon_0\varepsilon_{r,1} E_{z,1}$$

Equations in Medium 2



Inside medium 2, our three equations were

$$\frac{\partial}{\partial z}\left(E_{x,2}e^{\kappa_2 x}e^{j\beta z}\right)-\frac{\partial}{\partial x}\left(E_{z,2}e^{\kappa_2 x}e^{j\beta z}\right)=-j\omega\mu_0\mu_{r,2}\left(H_{y,2}e^{\kappa_2 x}e^{j\beta z}\right)$$

$$-\frac{\partial}{\partial z}\left(H_{y,2}e^{\kappa_2 x}e^{j\beta z}\right)=j\omega\varepsilon_0\varepsilon_{r,2}\left(E_{x,2}e^{\kappa_2 x}e^{j\beta z}\right)$$

$$\frac{\partial}{\partial x}\left(H_{y,2}e^{\kappa_2 x}e^{j\beta z}\right)=j\omega\varepsilon_0\varepsilon_{r,2}\left(E_{z,2}e^{\kappa_2 x}e^{j\beta z}\right)$$

These reduce to

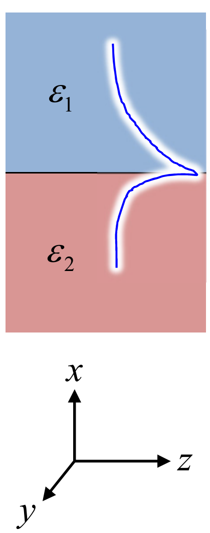
$$j\beta E_{x,2}-\kappa_2 E_{z,2}=-j\omega\mu_0\mu_{r,2}H_{y,2}$$

$$-j\beta H_{y,2}=j\omega\varepsilon_0\varepsilon_{r,2}E_{x,2}$$

$$\kappa_2 H_{y,2}=j\omega\varepsilon_0\varepsilon_{r,2}E_{z,2}$$

Lecture 21 35

Eliminate E_x



To more easily match the boundary conditions at x=0, the field component longitudinal to this interface is eliminated from the sets of three equations.

Medium 1

$$\kappa_1 E_{z,1}=-\frac{j}{\omega\varepsilon_0\varepsilon_{r,1}}\left(k_0^2\mu_{r,1}\varepsilon_{r,1}-\beta^2\right)H_{y,1}$$

$$\kappa_1 H_{y,1}=-j\omega\varepsilon_0\varepsilon_{r,1}E_{z,1}$$

Medium 2

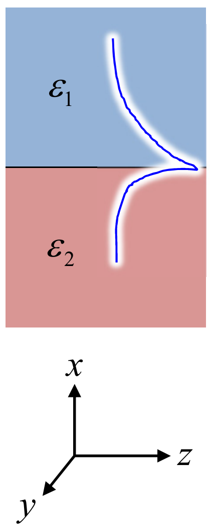
No more $E_{x,i}$

$$\kappa_2 E_{z,2}=\frac{j}{\omega\varepsilon_0\varepsilon_{r,2}}\left(k_0^2\mu_{r,2}\varepsilon_{r,2}-\beta^2\right)H_{y,2}$$

$$\kappa_2 H_{y,2}=j\omega\varepsilon_0\varepsilon_{r,2}E_{z,2}$$

Lecture 21 36

Dispersion Relation



The dispersion relation is derived by further eliminating E_z and relating the remaining parameters.

Medium 1

$$k_0^2\mu_{r,1}\varepsilon_{r,1} = \beta^2 - \kappa_1^2$$

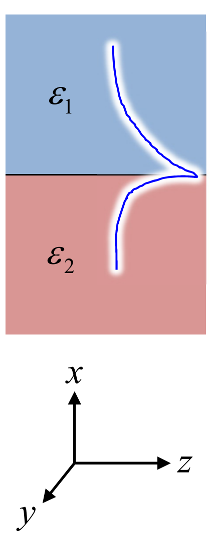
Medium 2

$$k_0^2\mu_{r,2}\varepsilon_{r,2} = \beta^2 - \kappa_2^2$$

This let's us write a general dispersion relation for the i^{th} medium as

$$k_0^2\mu_{r,i}\varepsilon_{r,i} = \beta^2 - \kappa_i^2$$

Boundary Conditions



Electric Field Boundary Conditions

$$E_{z,1} = E_{z,2}$$

$$-\frac{j}{\omega\varepsilon_0\varepsilon_{r,1}\kappa_1}\left(k_0^2\mu_{r,1}\varepsilon_{r,1} - \beta^2\right)H_{y,1} = \frac{j}{\omega\varepsilon_0\varepsilon_{r,2}\kappa_2}\left(k_0^2\mu_{r,2}\varepsilon_{r,2} - \beta^2\right)H_{y,2}$$

$$\frac{\kappa_1}{\varepsilon_{r,1}} + \frac{\kappa_2}{\varepsilon_{r,2}} = 0$$

Magnetic Field Boundary Conditions

$$H_{y,1} = H_{y,2}$$

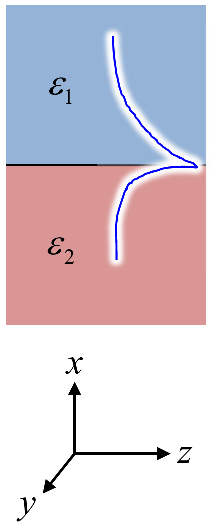
$$-\frac{j\omega\varepsilon_0\varepsilon_{r,1}}{\kappa_1}E_{z,1} = \frac{j\omega\varepsilon_0\varepsilon_{r,2}}{\kappa_2}E_{z,2}$$

$$\frac{\varepsilon_{r,1}}{\kappa_1} + \frac{\varepsilon_{r,2}}{\kappa_2} = 0$$

Same equation.

Existence Condition.

Existence Condition and Dispersion Relation



The existence condition for a surface plasmon is then

$$\frac{\varepsilon_{r,1}}{\kappa_1}+\frac{\varepsilon_{r,2}}{\kappa_2}=0$$

From the dispersion relation in both mediums, we see that

$$\kappa_1^2=\beta^2-k_0^2\mu_{r,1}\varepsilon_{r,1} \qquad \kappa_2^2=\beta^2-k_0^2\mu_{r,2}\varepsilon_{r,2}$$

We can use this to eliminate the κ terms in the existence condition to obtain to generalized dispersion relation.

$$\beta^2=k_0^2\left(\frac{1}{\varepsilon_{r,1}^2}-\frac{1}{\varepsilon_{r,2}^2}\right)^{-1}\left(\frac{\mu_{r,1}}{\varepsilon_{r,1}}-\frac{\mu_{r,2}}{\varepsilon_{r,2}}\right)=k_0^2\left(\frac{\varepsilon_{r,1}\varepsilon_{r,2}}{\varepsilon_{r,2}^2-\varepsilon_{r,1}^2}\right)\left(\mu_{r,1}\varepsilon_{r,2}-\mu_{r,2}\varepsilon_{r,1}\right)$$

For non-magnetic materials, the dispersion relation reduces to

$$\beta=k_0\sqrt{\frac{\varepsilon_{r,1}\varepsilon_{r,2}}{\varepsilon_{r,2}+\varepsilon_{r,1}}}$$

Drude Model for Metals

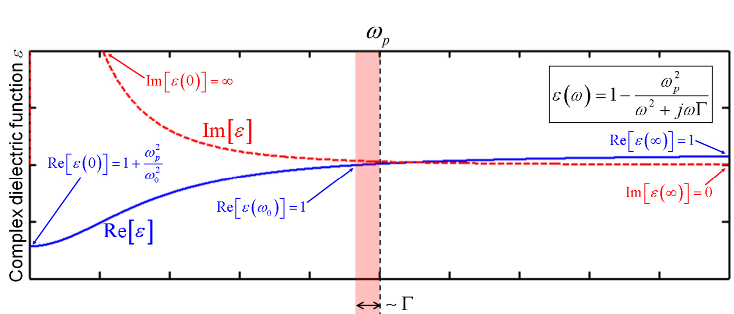
The Drude model for metals was

$$\tilde{\varepsilon}_r(\omega)=1-\frac{\omega_p^2}{\omega^2+j\omega\Gamma}$$

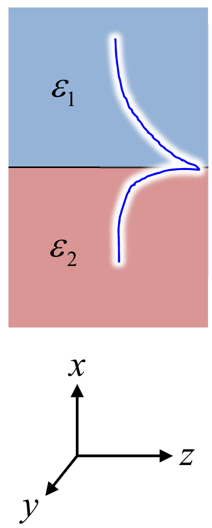
$$\omega_p^2=\frac{Nq^2}{\varepsilon_0 m_e}$$

Note, N is now interpreted as electron density N_e.

m_e is the effective mass of the electron.



Plasmons Require Metals



The existence condition is

$$\frac{\varepsilon_{r,1}}{\kappa_1} + \frac{\varepsilon_{r,2}}{\kappa_2} = 0$$

This can be solved for $\varepsilon_{r,2}$ as follows.

$$\varepsilon_{r,2} = -\varepsilon_{r,1}\frac{\kappa_2}{\kappa_1}$$

κ_1 and κ_2 are both positive quantities. This shows that $\varepsilon_{r,1}$ and $\varepsilon_{r,2}$ must have opposite sign to support a surface wave.

How do we get a negative ε? We use metals!!

Surface Plasma Frequency, ω_{sp}

For very small damping factor Γ, the Drude model reduces to

$$\varepsilon_{r,2} = 1 - \frac{\omega_p^2}{\omega^2}$$

We can derive an expression for the "surface plasma frequency" by substituting this equation into the dispersion relation, letting $\omega=\omega_{sp}$, and taking the limit as $\beta \rightarrow \infty$.

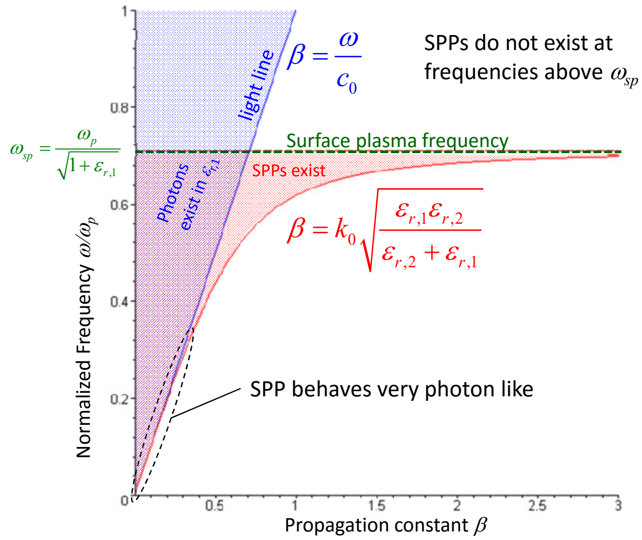
$$\beta = \frac{\omega_{sp}}{c_0}\sqrt{\frac{\varepsilon_{r,1}\varepsilon_{r,2}}{\varepsilon_{r,2}+\varepsilon_{r,1}}}$$

$$\varepsilon_{r,2} = 1 - \frac{\omega_p^2}{\omega_{sp}^2}$$

$$\omega_{sp} = \frac{\omega_p}{\sqrt{1+\varepsilon_{r,1}}}$$

Note: we can have SPPs at all frequencies below ω_{sp}.

Dispersion Relation for a SPP

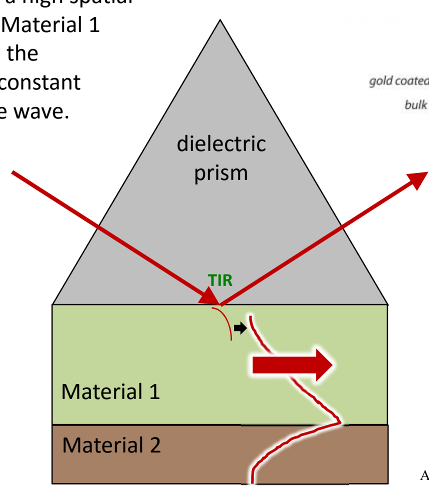


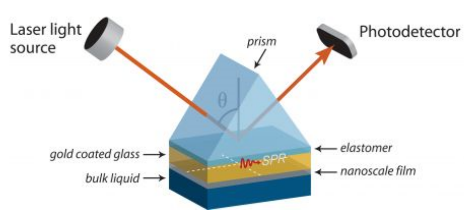
Lecture 21 43

Excitation of SPPs: Otto Configuration

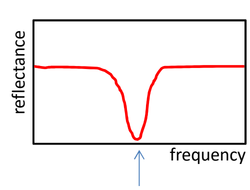
Attenuated total reflection setup

TIR produces a high spatial frequency in Material 1 that matches the propagation constant of the surface wave.





http://www.bionavis.com/technology/spr/



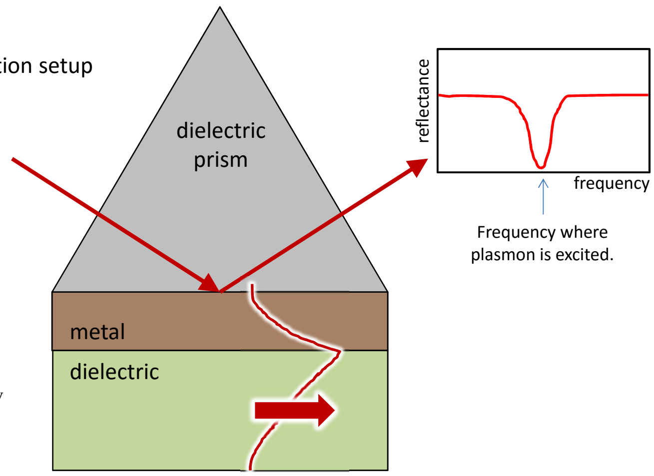
Frequency where surface wave is excited.

Andreas Otto, Aeitschrift für Physik, Vol. 216, Issue 4, pp. 398-410, 1968.

Lecture 21 44

Excitation of SPPs: Kretschmann Configuration

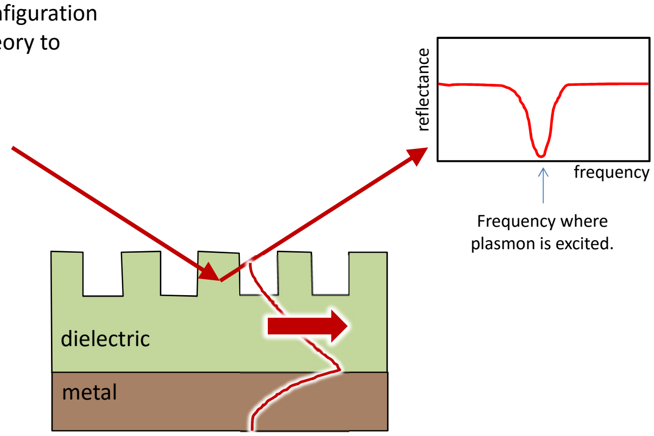
Attenuated total reflection setup



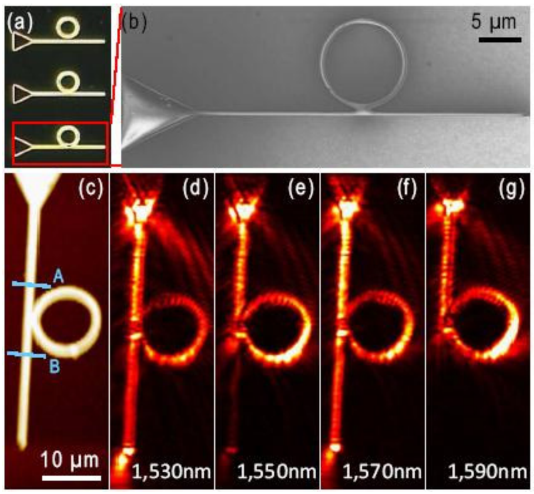
Kretschmann E. Determination of optical constants of metals by excitation of surface plasmons. Zeitschrift fur Physik 1971;241:313.

Excitation of SPPs: Grating Coupler Configuration

The grating coupler configuration uses coupled-mode theory to excite a surface wave.

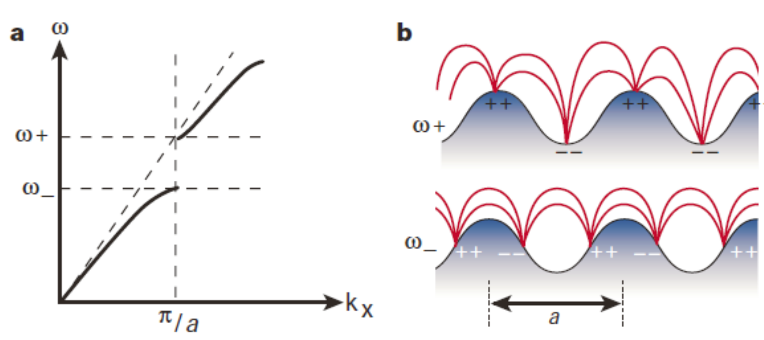


Plasmonic Waveguides and Circuits



Tobias Holmgaard, et al, "Dielectric-loaded plasmonic waveguide-ring resonators," Optics Express, Vol. 17, No. 4, pp. 2968-2975, 2009.

Band Gap Structures for SPPs

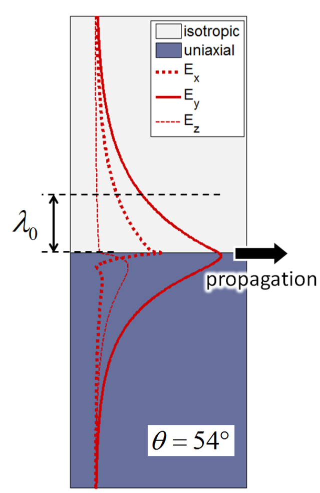


William L. Barnes, et al, "Surface plasmon subwavelength optics," Nature **424**, pp. 824-830, 2003.

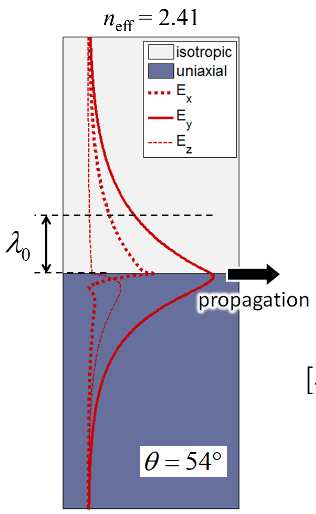
Dyakonov Surface Waves

What is a DSW?

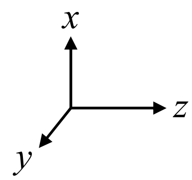
- A DSW is a surface wave confined at the interface between two materials where at least one is anisotropic.
- Anisotropy can be produced by nonresonant metamaterials.
- Nonresonant nature suggest a very broadband phenomenon.
- Note that the peak of the mode is shifted into the anisotropic substrate.



Benchmark Example



$\varepsilon_{r1} = 5.803$



For this surface wave analysis, I chose…

$\lambda_0 = 1$

$\varepsilon_o = 3.985$

$\varepsilon_e = 7.621$

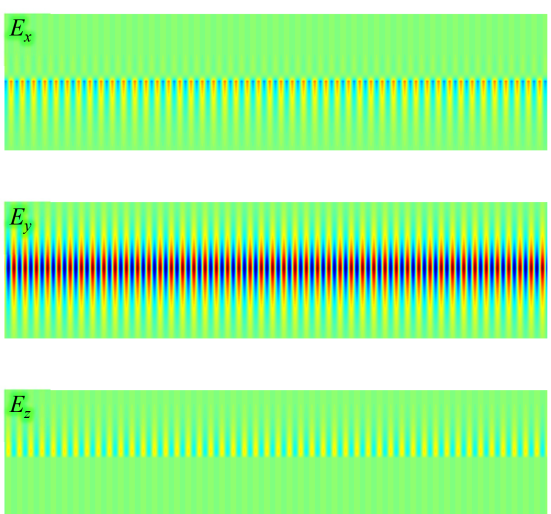
$\varepsilon_{r1} = \dfrac{\varepsilon_o + \varepsilon_e}{2} = 5.803$

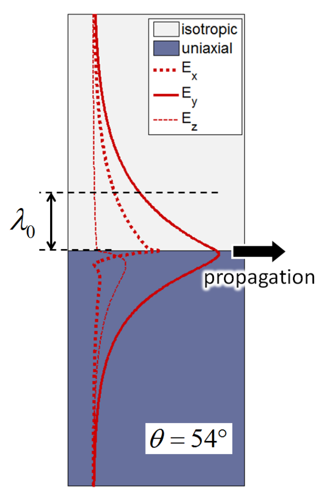
$\theta = 54°$

$$[\varepsilon_{r2}] = \begin{bmatrix} 3.9850 & & \\ & 3.9850 & 0 \\ & 0 & 7.6210 \end{bmatrix} \quad \text{Rotated } \theta = 54° \text{ about } x\text{-axis}$$

$$= \begin{bmatrix} 3.9850 & & \\ & 6.3648 & -1.7290 \\ & -1.7290 & 5.2412 \end{bmatrix}$$

What Does a DSW Look Like?





Existence Conditions

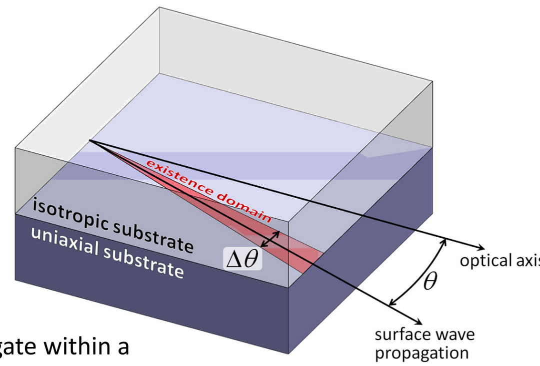
The most common configuration for a DSW is a uniaxial substrate and an isotropic superstrate.

The uniaxial substrate must have positive birefringence.

$$n_O < n_E$$

Superstrate must have a refractive index between n_E and n_O.

$$n_O < n_s < n_E$$



The surface wave can only propagate within a narrow range of angles relative to the optical axis.

Angular Existence Domain

The minimum and maximum angles are

$$\sin^2\theta_{\min} = \frac{\xi}{2}\left[(1-\rho\xi) + \sqrt{(1-\rho\xi)^2 + 4\rho}\right]$$

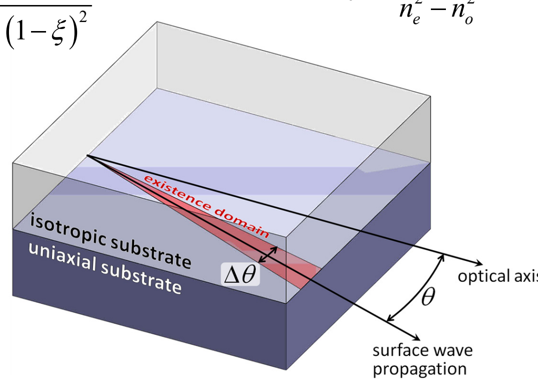
$$\sin^2\theta_{\max} = \frac{\xi(1+\rho)^3}{(1+\rho)^2(1+\rho\xi) - \rho^2(1-\xi)^2}$$

$$\rho = \frac{n_e^2}{n_o^2} - 1$$

$$\xi = \frac{n_{\text{sup}}^2 - n_o^2}{n_e^2 - n_o^2}$$

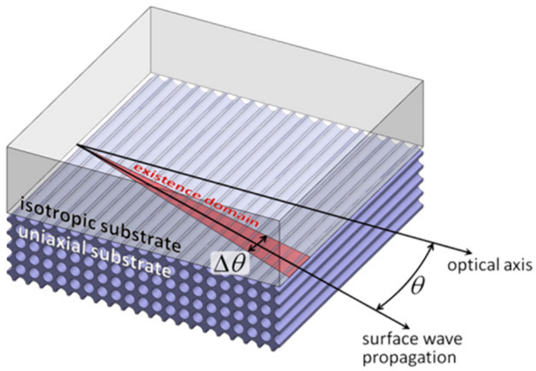
The central angle is

$$\theta_0 \approx \sin^{-1}\left[\sqrt{\frac{\xi(\rho+1)}{\xi\rho+1}}\right]$$

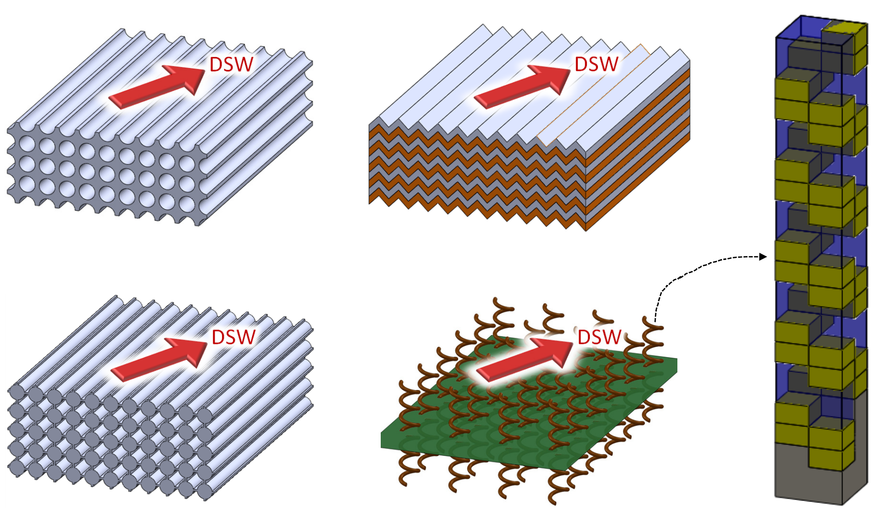


Metamaterial Substrate

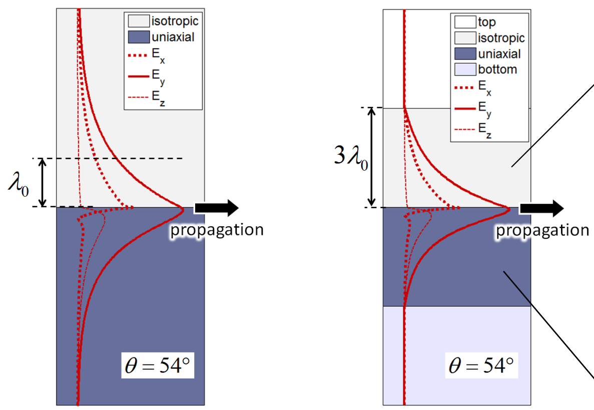
Much stronger anisotropy can be realized using metamaterials. This can widen the existence domain and provide a mechanism for sculpting the anisotropy to form more advanced devices.



Conceptual Metamaterial Structures Supporting DSWs



Finite Thickness Superstrate and Substrate

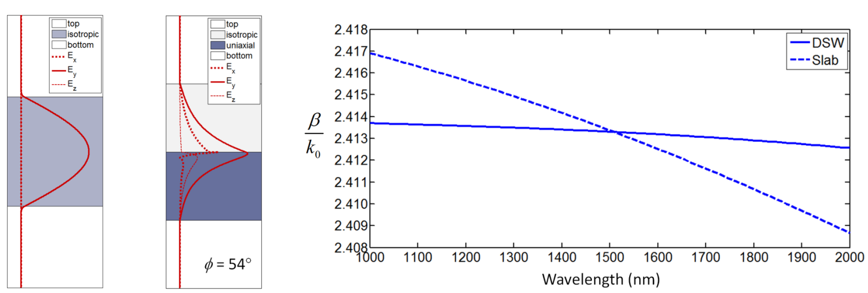


A finite thickness superstrate narrows the existence domain.

Some preliminary research suggests that a finite thickness substrate widens the existence domain, but it is not clear where the cutoff is between a DSW and an ordinary mode guided in the slab.

DSW Dispersion

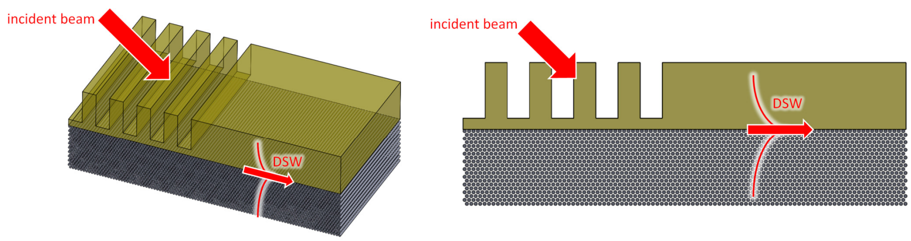
DSWs exhibit very low dispersion.



(a) standard slab mode (b) DSW (c) dispersion comparison between DSW and a standard slab mode

Excitation of a DSW

DSWs can be excited the same ways as surface plasmons. At radio and microwave frequencies, grating couplers might be preferred due to their compact size and options for integration.



(a) 3D perspective of grating excitation (b) 2D cross section of grating excitation