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Computation of Hertzian dipole radiation in stratified uniaxial anisotropic media

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Solutions to the problem of radiation of Hertzian dipoles in stratified uniaxially anisotropic media are obtained by decomposing a general wave field into transverse electric and transverse magnetic modes With the use of primary excitation fields due to a Hertzian dipole embedded in an anisotropic medium, the electromagnetic fields in stratified anisotropic media are derived by applying the appropriate boundary conditions General expressions for fields due to vertical electric, vertical magnetic, horizontal electric, and horizontal magnetic dipoles are presented. Numerical computations were carried out for the case of a vertical electric Hertzian dipole in a uniaxially anisotropic substrate backed by a ground plane.

1. INTRODUCTION

Electromagnetic radiation from dipole antennas in stratified media has been studied extensively because of its many applications in geophysical exploration, submarine communication, printed circuits, and microwave devices, etc. However, most of the previous work was limited to cases involving isotropic media. For example, *Wait* [1951, 1953] solved the problem of electric and magnetic dipoles over a stratified isotropic medium. *Stoyer* [1977] treated the problem of dipoles placed inside stratified isotropic media. An excellent review on radiation from dipoles in the presence of a homogeneous, isotropic half-space is contained in the book by *Baños* [1966]. Related studies involving stratified media have been extensively discussed by *Wait* [1970]. All of the above work was carried out by means of Hertzian potential functions.

Recently Kong investigated the problem of dipole radiation over stratified uniaxially anisotropic media [*Kong*, 1972, 1974] and also the case of a dipole placed inside stratified isotropic media [*Kong*, 1981] *Tabarovskii and Epov* [1977] proposed a method of solving the problem of determining the electromagnetic field of an arbitrary harmonic source in an *n*-layered anisotropic medium. *Ali and Mahmoud* [1979] derived in an integral form the fields of an arbitrary distribution of sources buried inside a stratified uniaxially anisotropic medium in terms of a dyadic Green's function; their mathematical results are complex and are not readily suitable for numerical computation. At th same time, *Tang* [1979] investigated the problem of radiation due to Hertzian dipoles embedded in stratified uniaxially anisotropic media. Her approach is similar to the present analysis; however, there exist significant discrepancies between her results and this paper, especially in expressions involving the anisotropy factor. *Tsalamengas and Uzunoglu* [1985] investigated the far field radiation pattern from a dipole over a uniaxially anisotropic layer. Recently, *Krowne* [1984] formulated Greens's function solutions of the biaxial and uniaxial anisotropic layered planar structure in the spectral domain.

Most recently, *Wang* [1985] presented a general computational method for the electromagnetic problem involving stratified isotropic media by extending Kong's numerical technique to deal with the general problem involving arbitiary Hertzian dipole excitation and media configurations. In this paper we further extend the analysis to problems involving stratified uniaxially anisotropic layers. In the present analysis, solutions to the problem are facilitated by treating a general wave field as a superposition of TE and TM fields and by writing the general solution inside a stratified medium and matching the boundary conditions at the interfaces. An important step in the analysis is the evaluation of primary excitation fields due to a Hertzian dipole in an infinite anisotropic medium. These primary excitation fields were derived for the cylindrical coordinates for the present

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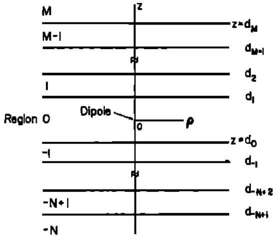

Fig. 1 A stratified anisotropic medium with Hertzian dipole excitation.

analysis, the details of which are to be published in a separate paper (Y. S. Kwon and J. J. H. Wang, unpublished manuscript, 1986). The superposition of "primary" and "secondary" fields and the matching of boundary conditions leads to the solution of the problem, which is then computed by techniques similar to the isotropic case [*Wang*, 1985]. Numerical results are presented to illustrate the approach, to check the validity of the analysis, and to serve as a useful reference for future work as little data are known to be available in this area.

2. THE PROBLEMS AND THEIR SOLUTIONS

The problems being considered are shown in Figure 1, in which a Hertzian dipole is located in region O. The origin of the cylindrical coordinates is placed at the location of the source, which is infinitesimally small. The dipole is either vertically polarized (along z axis), or horizontally polarized (perpendicular to z axis with $\phi = 0$). It can be magnetic or electric. Each stratified medium is in general uniaxially anisotropic, with its axis of anisotropy parallel to the z axis.

The electromagnetic fields in a region outside the source are governed by the Maxwell equations

$$\nabla \times \mathbf{E} = j\omega\boldsymbol{\mu} \cdot \mathbf{H} \tag{1}$$

$$\nabla \times \mathbf{H} = -j\omega\boldsymbol{\varepsilon} \cdot \mathbf{E} \tag{2}$$

where $\boldsymbol{\mu}$ and $\boldsymbol{\varepsilon}$ are the permeability and the equivalent permittivity tensors, respectively. A time dependence of exp $(-j\omega t)$ is assumed. The equivalent permittivity tensor $\boldsymbol{\varepsilon}$ can be written as

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}' + j\boldsymbol{\varepsilon}'' \tag{3}$$

where $\boldsymbol{\varepsilon}'$ is the permittivity tensor of the media and $\boldsymbol{\varepsilon}'$ is related to the conductivity tensor by $\boldsymbol{\varepsilon}'' = \boldsymbol{\sigma}/\omega$.

For a uniaxially anisotropic medium, we have

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_t & 0 & 0 \\ 0 & \varepsilon_t & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix} \tag{4a}$$

and

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_t & 0 & 0 \\ 0 & \mu_t & 0 \\ 0 & 0 & \mu_z \end{pmatrix} \tag{4b}$$

If we take the z component of the curl of (1), we obtain

$$[\nabla(\nabla \cdot \mathbf{E})]_z - (\nabla^2\mathbf{E})_z = j\omega(\nabla \times \boldsymbol{\mu} \cdot \mathbf{H})_z \tag{5}$$

In this source-free region, we have

$$\nabla \cdot \mathbf{D} = 0 \tag{6}$$

where

$$\mathbf{D} = \boldsymbol{\varepsilon} \cdot \mathbf{E} \tag{7}$$

Since

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_t & 0 & 0 \\ 0 & \varepsilon_t & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix} = \begin{pmatrix} \varepsilon_t & 0 & 0 \\ 0 & \varepsilon_t & 0 \\ 0 & 0 & \varepsilon_t \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_t \end{pmatrix} \tag{8}$$

we obtain from (6)

$$\nabla \cdot \mathbf{E} = (1 - a)\partial E_z/\partial z \tag{9}$$

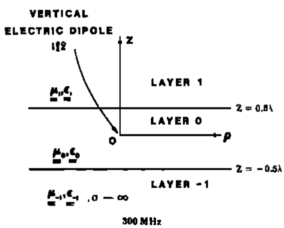

Fig. 2 A vertical electric dipole in a three layer medium

where

$$a = \varepsilon_z/\varepsilon_t \tag{10}$$

We note that in cylindrical coordinates,

$$(\nabla \times \mathbf{\mu} \cdot \mathbf{H})_z = \mu_t(\nabla \times \mathbf{H})_z \tag{11}$$

$$(\nabla^2 \mathbf{E})_z = \nabla^2 E_z \tag{12}$$

Thus, (5), (9), (11) and (12) lead to

$$\left(\nabla_t^2 + a\frac{\partial^2}{\partial z^2} + k^2 a\right)E_z = 0 \tag{13}$$

where

$$k = \omega\sqrt{\mu_t \varepsilon_t} \tag{14}$$

and

$$\nabla_t^2 = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2}{\partial \phi^2} \tag{15}$$

is the transverse Laplacian expressed in cylindrical coordinates.

Similarly we obtain the wave equation for H_z:

$$\left(\nabla_t^2 + b\frac{\partial^2}{\partial z^2} + k^2 b\right)H_z = 0 \tag{16}$$

where

$$b = \mu_z/\mu_t \tag{17}$$

It is seen from (13) and (16) that E_z and H_z are decoupled. Therefore, the total fields can be decomposed into TE and TM modes as follows:

$$\mathbf{E} = \mathbf{E}^{TE} + \mathbf{E}^{TM} \quad \text{and} \quad \mathbf{H} = \mathbf{H}^{TE} + \mathbf{H}^{TM} \tag{18}$$

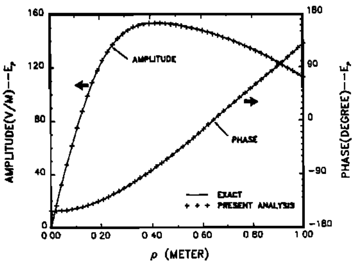

Fig 3. Comparison between the exact image theory calculation and the present analysis for E_ρ, $\varepsilon_{0t} = \varepsilon_{0z} = \varepsilon_{1t} = \varepsilon_{1z} = \varepsilon_0(2 + j0.01)$, $z = 0.5\lambda$, where $\lambda = 1$ m

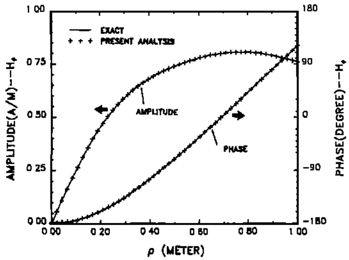

Fig. 4. Comparison between the exact image theory calculation and the present analysis for H_ϕ, $\varepsilon_{0t} = \varepsilon_{0z} = \varepsilon_{1t} = \varepsilon_{1z} = \varepsilon_0(2 + j0.01)$, $z = 0.5\lambda$, where $\lambda = 1$ m.

Solutions of E_z and H_z for the wave equations (13) and (16) can be found, in cylindrical coordinates, by employing the method of separation of variables. Since we are interested in solutions with waves outgoing in ρ direction and traveling or standing in z direction, we obtain, for a fixed separation constant n, the following solutions uniquely suitable for the present boundary-value problem for the TM and TE cases,

$$E_z^{TM} = \int_{-\infty}^{\infty} dk_\rho E_z(k_\rho) \quad \text{and} \quad H_z^{TE} = \int_{-\infty}^{\infty} dk_\rho H_z(k_\rho) \tag{19}$$

where

$$E_z(k_\rho) = \left[A(k_\rho)e^{-jk_z^{(e)}z} + B(k_\rho)e^{jk_z^{(e)}z}\right] \cdot H_n^{(1)}(k_\rho \rho)C_n(\phi) \tag{20a}$$

and

$$H_z(k_\rho) = \left[C(k_\rho)e^{-jk_z^{(m)}z} + D(k_\rho)e^{jk_z^{(m)}z}\right] H_n^{(1)}(k_\rho \rho)S_n(\phi) \tag{20b}$$

where $H_n^{(1)}(k_\rho \rho)$ denotes the nth order Hankel functions of the first kind. The ϕ-dependent functions $S_n(\phi)$, $C_n(\phi)$ and the order of the Hankel functions n are all determined by the source excitation involved.

From the dispersion relation, we obtain

$$k_z^{(e)} = \sqrt{k^2 - k_\rho^2/a} \qquad \text{TM waves} \tag{21a}$$

and

$$k_z^{(m)} = \sqrt{k^2 - k_\rho^2/b} \qquad \text{TE waves} \tag{21b}$$

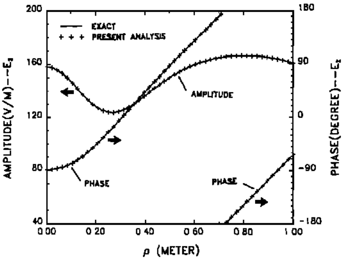

Fig. 5 Comparison between the exact image theory calculation and the present analysis for E_z, $\varepsilon_{0t} = \varepsilon_{0z} = \varepsilon_{1t} = \varepsilon_{1z} = \varepsilon_0(2 + j0.01)$, $z = 0.5\lambda$, where $\lambda = 1$ m

where

$$k^2 = \omega^2 \mu_t \varepsilon_t$$

The superscripts (e) and (m) of the z-directed propagation constants indicate that the TM and TE waves are extraordinary waves due to the anisotropy of the equivalent permittivity tensor and the permeability tensor, respectively.

From the Maxwell equations (1) and (2), it can be easily shown that the transverse field components are related to $E_z(k_\rho)$ and $H_z(k_\rho)$ by the following equations:

$$\mathbf{E}_t(k_\rho) = \frac{1}{(k_\rho^2/a)} \nabla_t \frac{\partial}{\partial z} E_z(k_\rho) + \frac{j\omega\mu_t}{(k_\rho^2/b)} \nabla_t \times \mathbf{H}_z(k_\rho) \tag{22a}$$

$$\mathbf{H}_t(k_\rho) = \frac{-j\omega\varepsilon_t}{(k_\rho^2/a)} \nabla_t \times \mathbf{E}_z(k_\rho) + \frac{1}{(k_\rho^2/b)} \nabla_t \frac{\partial}{\partial z} H_z(k_\rho) \tag{22b}$$

where

$$\mathbf{E}_z = \hat{z} E_z \qquad \mathbf{H}_z = \hat{z} H_z$$

$$\nabla_t = \hat{\rho} \frac{\partial}{\partial \rho} + \hat{\phi} \frac{1}{\rho} \frac{\partial}{\partial \phi}$$

We note in (22a) and (22b) that the effect of anisotropy of the medium has been included through a and b.

The fields in the lth layer due to a Hertzian dipole in a stratified anisotropic medium can now be derived from equations (19), (20) and (22). We obtain, for a fixed separation constant n, the following specific expressions for the fields in the lth layer

$$E_{lz} = \int_{-\infty}^{\infty} dk_\rho \left[A_l(k_\rho) e^{-jk_{lz}^{(e)} z} + B_l(k_\rho) e^{jk_{lz}^{(e)} z} \right] \cdot H_n^{(1)}(k_\rho \rho) C_n(\phi) \tag{23}$$

$$E_{l\rho} = \int_{-\infty}^{\infty} dk_\rho \frac{jk_{lz}^{(e)} a_l}{k_\rho} \left[-A_l e^{-jk_{lz}^{(e)} z} + B_l e^{jk_{lz}^{(e)} z} \right] \cdot H_n^{(1)\prime}(k_\rho \rho) C_n(\phi) + \int_{-\infty}^{\infty} dk_\rho \frac{j\omega\mu_{lt} b_l}{k_\rho^2 \rho} \cdot \left[C_l e^{-jk_{lz}^{(m)} z} + D_l e^{jk_{lz}^{(m)} z} \right] H_n^{(1)}(k_\rho \rho) S_n'(\phi) \tag{24}$$

$$E_{l\phi} = \int_{-\infty}^{\infty} dk_\rho \frac{jk_{lz}^{(e)} a_l}{k_\rho^2 \rho} \left[-A_l e^{-jk_{lz}^{(e)} z} + B_l e^{jk_{lz}^{(e)} z} \right] \cdot H_n^{(1)}(k_\rho \rho) C_n'(\phi) + \int_{-\infty}^{\infty} dk_\rho \frac{-j\omega\mu_{lt} b_l}{k_\rho} \cdot \left[C_l e^{-jk_{lz}^{(m)} z} + D_l e^{jk_{lz}^{(m)} z} \right] H_n^{(1)\prime}(k_\rho \rho) S_n(\phi) \tag{25}$$

$$H_{lz} = \int_{-\infty}^{\infty} dk_\rho \left[C_l e^{-jk_{lz}^{(m)} z} + D_l e^{jk_{lz}^{(m)} z} \right] \cdot H_n^{(1)}(k_\rho \rho) S_n(\phi) \tag{26}$$

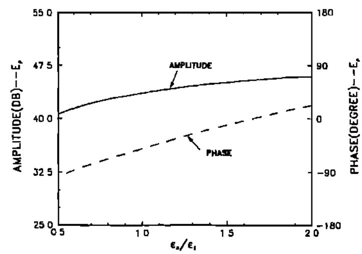

Fig 6 Computed amplitude and phase of E_ρ vs. $\varepsilon_z/\varepsilon_t$ for Figure 2 with $\varepsilon_1 = \varepsilon_0$, $\mu_1 = \mu_0 = \mu_0$, $\varepsilon_{0t} = \varepsilon_{1t} = \varepsilon_0(2 + j0.01)$, $\rho = 0.5\lambda$, and $z = 0.5\lambda$

$$H_{l\rho} = \int_{-\infty}^{\infty} dk_\rho \, \frac{-j\omega\varepsilon_{lt} a_l}{k_\rho^2 \rho} \left[A_l e^{-jk_{lz}^{(e)}z} + B_l e^{jk_{lz}^{(e)}z} \right]$$

$$\cdot \, H_n^{(1)}(k_\rho \rho) C_n'(\phi) + \int_{-\infty}^{\infty} dk_\rho \, \frac{jk_{lz}^{(m)} b_l}{k_\rho}$$

$$\cdot \left[-C_l e^{-jk_{lz}^{(m)}z} + D_l e^{jk_{lz}^{(m)}z} \right] H_n^{(1)\prime}(k_\rho \rho) S_n(\phi) \tag{27}$$

$$H_{l\phi} = \int_{-\infty}^{\infty} dk_\rho \, \frac{j\omega\varepsilon_{lt} a_l}{k_\rho} \left[A_l e^{-jk_{lz}^{(e)}z} + B_l e^{jk_{lz}^{(e)}z} \right]$$

$$\cdot \, H_n^{(1)\prime}(k_\rho \rho) C_n(\phi) + \int_{-\infty}^{\infty} dk_\rho \, \frac{jk_{lz}^{(m)} b_l}{k_\rho^2 \rho}$$

$$\left[-C_l e^{-jk_{lz}^{(m)}z} + D_l e^{jk_{lz}^{(m)}z} \right] H_n^{(1)}(k_\rho \rho) S_n'(\phi) \tag{28}$$

where $H_n^{(1)\prime}(k_\rho \rho)$ denotes its derivative with respect to its argument $k_\rho \rho$ and $S_n'(\phi)$ and $C_n'(\phi)$ derivatives with respect to ϕ.

The dispersion relations in the lth layer are

$$k_{lz}^{(e)} = \sqrt{k_l^2 - k_\rho^2/a_l} \qquad \text{TM waves} \tag{29a}$$

and

$$k_{lz}^{(m)} = \sqrt{k_l^2 - k_\rho^2/b_l} \qquad \text{TE waves} \tag{29b}$$

where $k_l^2 = \omega^2 \mu_{lt} \varepsilon_{lt}$, $a_l = \varepsilon_{lz}/\varepsilon_{lt}$, $b_l = \mu_{lz}/\mu_{lt}$ and k_ρ is independent of the layer. It is apparent from equations (23)–(28) that when $a_l = 1$ and $b_l = 1$, the field components for anisotropic media reduce to those in isotropic media [*Kong*, 1981]. It is interesting to note that the anisotropy in permittivity affects only TM waves and the anisotropy in permeability affects only TE waves.

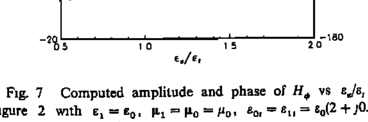

Fig. 7 Computed amplitude and phase of H_ϕ vs $\varepsilon_z/\varepsilon_t$ for Figure 2 with $\varepsilon_1 = \varepsilon_0$, $\mu_1 = \mu_0 = \mu_0$, $\varepsilon_{0t} = \varepsilon_{1t} = \varepsilon_0(2 + j0.01)$, $\rho = 0.5\lambda$, and $z = 0.5\lambda$.

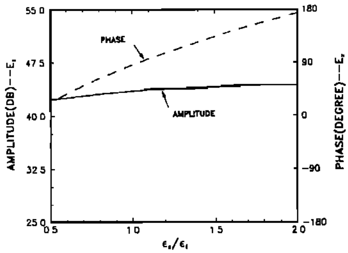

Fig. 8 Computed amplitude and phase of E_z vs $\varepsilon_z/\varepsilon_t$ for Figure 2 with $\varepsilon_1 = \varepsilon_0$, $\mu_1 = \mu_0 = \mu_0$, $\varepsilon_{0t} = \varepsilon_{1t} = \varepsilon_0(2 + j0.01)$, $\rho = 0.5\lambda$, and $z = 0.5\lambda$

For $l \neq 0$, the wave amplitude constants A_l, B_l, C_l, and D_l are to be determined by the appropriate boundary conditions at each layer interface according to the propagation matrix expressions in Appendix A. For $l = 0$, both the source and boundary conditions must be satisfied. The source condition is enforced by comparing the fields in region 0 with that of the particular Hertzian dipole in an infinite space of that medium, that is, the primary fields due to the source.

The primary fields which are due to the dipole antenna in an unbounded medium are well known [*Kong*, 1972; *Adler et al.*, 1960] for the case of isotropic media. However, no solution for those involving anisotropic media has been obtained for the cylindrical coordinates. We have studied the following four types of element dipoles and have derived the primary excitation fields as follows (Y. S. Kwon and J. J. H. Wang, unpublished manuscript, 1986).

2.1. *Vertical electric dipole (VED)*

$$E_z = \int_{-\infty}^{\infty} dk_\rho \, E_{\text{ved}} \begin{Bmatrix} e^{jk_{0z}z} \\ e^{-jk_{0z}z} \end{Bmatrix} H_0^{(1)}(k_\rho \rho) \tag{30}$$

$$z \geq 0 \qquad z \leq 0$$

$$H_z = 0 \tag{31}$$

where

$$E_{\text{ved}} = \frac{-Ilk_\rho^3}{8\pi\omega\varepsilon_{0t} k_{0z} a_0^2} \qquad a_0 = \varepsilon_{0z}/\varepsilon_{0t} \tag{32}$$

and Il is the electric dipole moment.

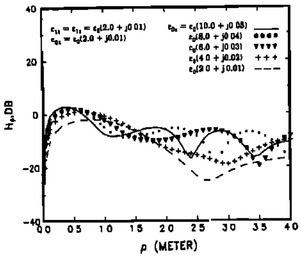

Fig. 9 Computed E_ρ vs. ρ for various dielectric anisotropy for the configuration of Figure 2.

2.2. *Horizontal electric dipole (HED)*

$$E_z = \int_{-\infty}^{\infty} dk_\rho E_{\rm hed} \begin{Bmatrix} e^{jk_{0z}z} \\ -e^{-jk_{0z}z} \end{Bmatrix} H_1^{(1)}(k_\rho \rho) \cos\phi \quad (33)$$

$$z \geq 0 \qquad z \leq 0$$

where

$$E_{\rm hed} = \frac{jIlk_\rho^2}{8\pi\omega\varepsilon_{0t} a_0} \qquad a_0 = \varepsilon_{0z}/\varepsilon_{0t} \quad (34)$$

$$H_z = \int_{-\infty}^{\infty} dk_\rho H_{\rm hed} \begin{Bmatrix} e^{jk_{0z}z} \\ e^{-jk_{0z}z} \end{Bmatrix} H_1^{(1)}(k_\rho \rho) \sin\phi \quad (35)$$

$$z \geq 0 \qquad z \leq 0$$

where

$$H_{\rm hed} = \frac{jIlk_\rho^2}{8\pi k_{0z} b_0} \qquad b_0 = \mu_{0z}/\mu_{0t} \quad (36)$$

2.3. *Vertical magnetic dipole (VMD)*

$$E_z = 0 \quad (37)$$

$$H_z = \int_{-\infty}^{\infty} dk_\rho H_{\rm vmd} \begin{Bmatrix} e^{jk_{0z}z} \\ e^{-jk_{0z}z} \end{Bmatrix} H_0^{(1)}(k_\rho \rho), \quad (38)$$

$$z \geq 0 \qquad z \leq 0$$

where

$$H_{\rm vmd} = \frac{jIAk_\rho^3}{8\pi k_{0z} b_0^2} \quad (39)$$

and IA is the magnetic dipole moment.

2.4. *Horizontal magnetic dipole (HMD)*

$$E_z = \int_{-\infty}^{\infty} dk_\rho E_{\rm hmd} \begin{Bmatrix} e^{jk_{0z}z} \\ e^{-jk_{0z}z} \end{Bmatrix} H_1^{(1)}(k_\rho \rho) \sin\phi, \quad (40)$$

$$z \geq 0 \qquad z \leq 0$$

where

$$E_{\rm hmd} = \frac{-\omega\mu_{0t} IAk_\rho^2}{8\pi k_{0z} a_0} \quad (41)$$

$$H_z = \int_{-\infty}^{\infty} dk_\rho H_{\rm hmd} \begin{Bmatrix} e^{jk_{0z}z} \\ -e^{-jk_{0z}z} \end{Bmatrix} H_1^{(1)}(k_\rho \rho) \cos\phi, \quad (42)$$

$$z \geq 0 \qquad z \leq 0$$

where

$$H_{\rm hmd} = \frac{IAk_\rho^2}{8\pi b_0} \quad (43)$$

Equations (30)–(43) are used as source conditions to obtain the wave amplitudes in region 0 for the specific Hertzian dipole in the problem under consideration. The results are discussed in greater details in Appendix B. Thus, for the specific source and boundary conditions, the wave amplitudes can be determined for all regions and the total fields can be computed from (23)–(28).

3. NUMERICAL RESULTS

The basic numerical technique used to compute the radiated fields according to (23)–(28) is similar to an earlier one [*Wang*, 1985; *Tsang et al.*, 1974] and will not be repeated here in detail. The primary diffi-

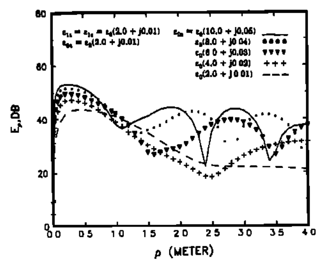

Fig 10 Computed H_ϕ vs. ρ for various dielectric anisotropy for the configuration of Figure 2.

culty in the numerical computation of the present formulation is due to the integrals which involve Hankel functions. The FFT technique, which was shown to be applicable to cases involving isotropic stratified media, can in general be extended to problems involving uniaxially anisotropic stratified media. In order to maintain good accuracy in using the FFT technique, the sampling interval Δk_ρ was chosen to be much smaller than $1/\rho$, and the number of sampling points N was chosen to be a sufficiently large integral power of 2.

Numerical analyses were carried out for the three-layer problem of Figure 2 which is a special case of Figure 1 with the conductivity of layer (-1) allowed to be high enough so that a highly conductive plane is formed. A vertical electric Hertzian dipole of unit strength $(Il = 1)$ is located at the origin of the coordinate system in region 0. A computer program was written to calculate all the field components radiated from the dipole. Since there appears to be no numerical or experimental data available to check against the present computation involving Hertzian dipoles in anisotropic stratified media, we resort to indirect checks and validation.

For example, the field components due to a vertical electric Hertzian dipole in the case of an isotropic half space are computed and compared in Figures 3, 4, and 5 with the exact image theory calculations. These are the special cases in which layers 1 and 0 in Figure 2 were of identical isotropic media. The magnitude of the dipole moment has been chosen to be a unit value for the numerical computation. The solid line represents the calculation from the exact image theory and the marked points are the results from the present analysis. An excellent agreement with the exact image theory within 10^{-4} percent was achieved.

The amplitude and phase variation of the field components as a function of ε_z to ε_t would be another useful check of the present computation. Figures 6, 7, 8 show a smooth transition of wave amplitudes and phases from the cases of negative uniaxially anisotropic medium $(\varepsilon_z/\varepsilon_t < 1)$ to those of positive uniaxially anisotropic medium $(\varepsilon_z/\varepsilon_t > 1)$. It is also seen that when $\varepsilon_z = \varepsilon_t$ or $a = 1$, the results are identical to those of the image theory computation shown in Figures 3, 4, and 5 Layers 1 and 0 were assumed to be of identical uniaxially anisotropic medium.

Figures 9, 10, and 11 show comparisons in the amplitudes of E_ρ, H_ϕ, E_z at $z = 0.5^+\lambda$ for the substrate configuration of Figure 2. The superscript + indicates that the observation point is located at the layer 1 side of the boundary. The five curves correspond to cases of various degrees of anisotropy, from $a = \varepsilon_{0z}/\varepsilon_{0t} = 1$ (isotropic case) to $a = 5$. As can be seen, the field intensities are higher for larger anisotropy (larger a). This phenomenon suggests a strong effect of anisotropy in the generation of surface waves along the ρ axis. It is not clear, however, to what extent this phenomenon is due to the higher dielectric constant (rather than the anisotropy). The strong effects of dielectric anisotropy on the radiated fields should be of practical importance. Convergence tests were also performed to check the accuracy of the above computation. Figure 12 shows how convergence is reached by increasing N in the computation. It is observed that numerical convergence is consistently accompanied by the smoothing of the ripples in the E_z vs. ρ plots. Convergence was also checked by varying the value of Δk_ρ chosen in the FFT computation.

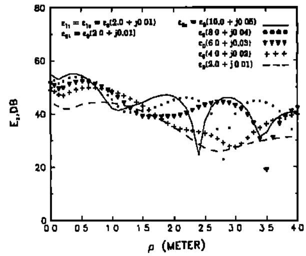

Fig. 11 Computed E_z vs ρ for various dielectric anisotropy for the configuration of Figure 2.

In comparison with the fields in isotropic substrates, the fields in anisotropic substrates change more rapidly, and in a more undulating manner along the ρ-axis, as can be seen in Figures 9–11. This phenomenon results from the interferences between all the plane wave spectra and their multiple reflections at the two boundary surfaces. The anisotropy of the substrate leads to extraordinary waves whose propagation vector varies in magnitude with the direction of propagation. As a result, the undulating feature is more pronounced in the fields involving anisotropic media. In fact, anisotropy of the media plays a more prominant role in this phenomenon

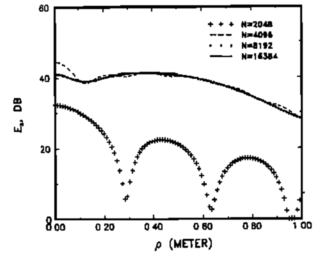

Fig. 12. Convergence test for E_z, $z = 0.5\lambda$ for the configuration of Figure 2, with $\varepsilon_{0z} = \varepsilon_0(10 + j0.05)$, $\varepsilon_{0t} = \varepsilon_0(2 + j0.01)$

than the interference due to reflections from the boundaries, even though for years researchers in geophysical probing speculated inadvertently the existence of large peaks in the interference pattern between direct and reflected waves.

4. CONCLUDING REMARKS

The problem of the radiation of Hertzian dipole antennas placed in stratified anisotropic media was solved. Primary excitation fields due to various dipole antennas in an anisotropic medium are presented. Upward and downward wave propagation matrices in a stratified anisotropic medium are derived and used to express fields in any layer in terms of the primary excitation fields and reflection coefficients. Numerical computations have been successfully carried out for many test cases by using FFT technique. Some of the numerical results are presented for near to intermediate field range. It is worthwhile noting the strong effects of the dielectric anisotropy on the radiated fields, which should be of practical interest.

APPENDIX A WAVE AMPLITUDES IN REGIONS WITH $l \neq 0$

Propagation matrices for stratified isotropic media have been derived by *Kong* [1981]. Propagation matrices for stratified uniaxially anisotropic media are derived as follows.

The boundary conditions at all interfaces require that the tangential components of both electric and magnetic fields be continuous at all ρ and ϕ. The four constants, A_l, B_l, C_l, and D_l are determined by enforcing the boundary conditions at each layer interface. In order to reduce the complexity in the mathematical expressions, we dropped the subscript t from ε_{lt} and μ_{lt} and replaced ε_{lz} and μ_{lz} with their equivalent $a_l\varepsilon_l$ and $b_l\mu_l$, where $a_l = \varepsilon_{lz}/\varepsilon_{lt} = \varepsilon_{lz}/\varepsilon_l$ and $b_l = \mu_{lz}/\mu_{lt} = \mu_{lz}/\mu_l$. At $z = d_l$, we obtain

$$k_{lz}^{(e)}a_l[-A_l e^{-jk_{lz}^{(e)}d_l} + B_l e^{jk_{lz}^{(e)}d_l}]$$
$$= k_{(l-1)z}^{(e)}a_{l-1}[-A_{l-1}e^{-jk_{(l-1)z}^{(e)}d_l} + B_{l-1}e^{jk_{(l-1)z}^{(e)}d_l}] \quad \text{(A1)}$$

$$\varepsilon_l a_l[A_l e^{-jk_{lz}^{(e)}d_l} + B_l e^{jk_{lz}^{(e)}d_l}]$$
$$= \varepsilon_{(l-1)}a_{(l-1)}[A_{l-1}e^{-jk_{(l-1)z}^{(e)}d_l} + B_{l-1}e^{jk_{(l-1)z}^{(e)}d_l}] \quad \text{(A2)}$$

$$\mu_l b_l[C_l e^{-jk_{lz}^{(m)}d_l} + D_l e^{jk_{lz}^{(m)}d_l}]$$
$$= \mu_{l-1}b_{l-1}[C_{l-1}e^{-jk_{(l-1)z}^{(m)}d_l} + D_{l-1}e^{jk_{(l-1)z}^{(m)}d_l}] \quad \text{(A3)}$$

$$k_{lz}^{(m)}b_l[-C_l e^{-jk_{lz}^{(m)}d_l} + D_l e^{jk_{lz}^{(m)}d_l}]$$
$$= k_{(l-1)z}^{(m)}b_{(l-1)}[-C_{l-1}e^{-jk_{(l-1)z}^{(m)}d_l} + D_{l-1}e^{jk_{(l-1)z}^{(m)}d_l}] \quad \text{(A4)}$$

Equations (A1)–(A4) can be solved for either A_l and B_l in terms of A_{l-1} and B_{l-1} or vice versa. After simplifications, we obtain

$$\begin{bmatrix} A_l \\ B_l \end{bmatrix} = \bar{U}_{l(l-1)}^{\mathrm{TM}} \begin{bmatrix} A_{l-1} \\ B_{l-1} \end{bmatrix} \quad \text{(A5)}$$

$$\begin{bmatrix} A_{l-1} \\ B_{l-1} \end{bmatrix} = \bar{V}_{(l-1)l}^{\mathrm{TM}} \cdot \begin{bmatrix} A_l \\ B_l \end{bmatrix} \quad \text{(A6)}$$

for TM waves and

$$\begin{bmatrix} C_l \\ D_l \end{bmatrix} = \bar{U}_{l(l-1)}^{\mathrm{TE}} \cdot \begin{bmatrix} C_{l-1} \\ D_{l-1} \end{bmatrix} \quad \text{(A7)}$$

$$\begin{bmatrix} C_{l-1} \\ D_{l-1} \end{bmatrix} = \bar{V}_{(l-1)l}^{\mathrm{TE}} \cdot \begin{bmatrix} C_l \\ D_l \end{bmatrix} \quad \text{(A8)}$$

for TE waves where the upward propagation matrices, $\bar{U}_{l(l-1)}^{\mathrm{TM}}$ and $\bar{U}_{l(l-1)}^{\mathrm{TE}}$ and the downward propagation matrices, $\bar{V}_{(l-1)l}^{\mathrm{TM}}$ and $\bar{V}_{(l-1)l}^{\mathrm{TE}}$, are given by the following expressions:

$$\bar{U}_{l(l-1)}^{\mathrm{TM}} = \frac{1}{2}\frac{a_{l-1}}{a_l}\left[\frac{\varepsilon_{l-1}}{\varepsilon_l} + \frac{k_{(l-1)z}}{k_{lz}}\right]$$
$$\cdot \begin{bmatrix} e^{j(k_{lz}-k_{(l-1)z})d_l} & R_{l(l-1)}^{\mathrm{TM}}e^{j(k_{lz}-k_{(l-1)z})d_l} \\ R_{l(l-1)}^{\mathrm{TM}}e^{-j(k_{lz}+k_{(l-1)z})d_l} & e^{-j(k_{lz}-k_{(l-1)z})d_l} \end{bmatrix} \quad \text{(A9)}$$

$$\bar{V}^{\mathrm{TM}}_{(l-1)l} = \frac{1}{2}\frac{a_l}{a_{l-1}}\left[\frac{\varepsilon_l}{\varepsilon_{l-1}} + \frac{k_{lz}}{k_{(l-1)z}}\right] \cdot \begin{bmatrix} e^{j(k_{(l-1)z}-k_{lz})d_l} & R^{\mathrm{TM}}_{(l-1)l}\, e^{j(k_{(l-1)z}+k_{lz})d_l} \\ R^{\mathrm{TM}}_{(l-1)l}\, e^{-j(k_{(l-1)z}+k_{lz})d_l} & e^{-j(k_{(l-1)z}-k_{lz})d_l} \end{bmatrix} \tag{A10}$$

$$\bar{U}^{\mathrm{TE}}_{l(l-1)} = \frac{1}{2}\frac{b_{l-1}}{b_l}\left[\frac{\mu_{l-1}}{\mu_l} + \frac{k_{(l-1)z}}{k_{lz}}\right] \cdot \begin{bmatrix} e^{j(k_{lz}-k_{(l-1)z})d_l} & R^{\mathrm{TE}}_{l(l-1)}\, e^{j(k_{lz}-k_{(l-1)z})d_l} \\ R^{\mathrm{TE}}_{l(l-1)}\, e^{-j(k_{lz}+k_{(l-1)z})d_l} & e^{-j(k_{lz}-k_{(l-1)z})d_l} \end{bmatrix} \tag{A11}$$

$$\bar{V}^{\mathrm{TE}}_{(l-1)l} = \frac{1}{2}\frac{b_l}{b_{l-1}}\left[\frac{\mu_l}{\mu_{l-1}} + \frac{k_{lz}}{k_{(l-1)z}}\right] \cdot \begin{bmatrix} e^{j(k_{(l-1)z}-k_{lz})d_l} & R^{\mathrm{TE}}_{(l-1)l}\, e^{j(k_{(l-1)z}+k_{lz})d_l} \\ R^{\mathrm{TE}}_{(l-1)l}\, e^{-j(k_{(l-1)z}+k_{lz})d_l} & e^{-j(k_{(l-1)z}+k_{lz})d_l} \end{bmatrix} \tag{A12}$$

The reflection coefficients are given by

$$R^{\mathrm{TM}}_{l(l-1)} = \frac{1 - \varepsilon_l k_{(l-1)z}/\varepsilon_{l-1} k_{lz}}{1 + \varepsilon_l k_{(l-1)z}/\varepsilon_{l-1} k_{lz}} \tag{A13}$$

$$R^{\mathrm{TM}}_{l(l-1)} = -R^{\mathrm{TM}}_{l(l-1)} \tag{A14}$$

and

$$R^{\mathrm{TE}}_{l(l-1)} = \frac{1 - \mu_l k_{(l-1)z}/\mu_{l-1} k_{lz}}{1 + \mu_l k_{(l-1)z}/\mu_{l-1} k_{lz}} \tag{A15}$$

$$R^{\mathrm{TE}}_{(l-1)l} = -R^{\mathrm{TE}}_{l(l-1)} \tag{A16}$$

Since $\bar{V}_{(l-1)l} = \bar{U}^{-1}_{l(l-1)}$, we have $\bar{U}^{\mathrm{TM}}_{l(l-1)} \cdot \bar{V}^{\mathrm{TM}}_{(l-1)l} = I$ and $\bar{U}^{\mathrm{TE}}_{l(l-1)} \cdot \bar{V}^{\mathrm{TE}}_{(l-1)l} = I$.

By using propagation matrices, we see that once wave amplitudes in any region are known, wave amplitudes in any other regions can be readily obtained. We note that in equations (A1)–(A4), $A_M = C_M = 0$ and $B_{-N} = D_{-N} = 0$ since there are no waves originating from infinity. Again, it is seen that for $a_l = 1$ and $b_l = 1$, the propagation matrices reduce to the known expressions for the isotropic stratified medium case [*Kong*, 1981].

APPENDIX B. WAVE AMPLITUDES IN THE SOURCE REGION ($l = 0$)

B1. *Source boundary conditions at* $z = 0$

The general solutions in region O may be obtained by superposition of the primary fields in an infinite space of the same medium and the homogeneous solution of the stratified medium in the absence of source. The field in region O can be obtained by identifying A_0, B_0, C_0 and D_0 according to the four types of dipoles and whether we have $z > 0$ or $z < 0$. We denote with A_{0+}, B_{0+}, C_{0+} and D_{0+} for $z \geqq 0$ and with A_{0-}, B_{0-}, C_{0-}, D_{0-} for $z \leqq 0$. Noting that the source is located at $z = 0$, we obtain the following results:

VED

$$\left.\begin{aligned} A_{0+} &= A_{\mathrm{ved}} \\ B_{0+} &= B_{\mathrm{ved}} + E_{\mathrm{ved}} \\ C_{0+} &= D_{0+} = 0 \end{aligned}\right\} \quad z \geqq 0 \tag{B1}$$

$$\left.\begin{aligned} A_{0-} &= A_{\mathrm{ved}} + E_{\mathrm{ved}} \\ B_{0-} &= B_{\mathrm{ved}} \\ C_{0-} &= D_{0-} = 0 \end{aligned}\right\} \quad z \leqq 0 \tag{B2}$$

HED

$$\left.\begin{aligned} A_{0+} &= A_{\mathrm{hed}} \\ B_{0+} &= B_{\mathrm{hed}} + E_{\mathrm{hed}} \\ C_{0+} &= C_{\mathrm{hed}} \\ D_{0+} &= D_{\mathrm{hed}} + H_{\mathrm{hed}} \end{aligned}\right\} \quad z \geqq 0 \tag{B3}$$

$$\left.\begin{aligned} A_{0-} &= A_{\mathrm{hed}} - E_{\mathrm{hed}} \\ B_{0-} &= B_{\mathrm{hed}} \\ C_{0-} &= C_{\mathrm{hed}} + H_{\mathrm{hed}} \\ D_{0-} &= D_{\mathrm{hed}} \end{aligned}\right\} \quad z \leqq 0 \tag{B4}$$

VMD

$$\left.\begin{aligned} A_{0+} &= B_{0+} = 0 \\ C_{0+} &= C_{\mathrm{vmd}} \\ D_{0+} &= D_{\mathrm{vmd}} + H_{\mathrm{vmd}} \end{aligned}\right\} \quad z \geqq 0 \tag{B5}$$

$$\left.\begin{aligned} A_{0-} &= B_{0-} = 0 \\ C_{0-} &= C_{\mathrm{vmd}} + H_{\mathrm{vmd}} \\ D_{0-} &= D_{\mathrm{vmd}} \end{aligned}\right\} \quad z \leqq 0 \tag{B6}$$

HMD

$$\left.\begin{aligned} A_{0+} &= A_{\mathrm{hmd}} \\ B_{0+} &= B_{\mathrm{hmd}} + E_{\mathrm{hmd}} \end{aligned}\right. \quad z \geqq 0 \tag{B7}$$

$$\left.\begin{aligned} C_{0+} &= C_{hmd} \\ D_{0+} &= D_{hmd} + H_{hmd} \\ A_{0-} &= A_{hmd} + E_{hmd} \\ B_{0-} &= B_{hmd} \\ C_{0-} &= C_{hmd} - H_{hmd} \\ D_{0-} &= D_{hmd} \end{aligned}\right\} \quad z \leqq 0 \tag{B8}$$

The amplitude constants A, B, C and D with subscripts such as *ved*, *hed*, *vmd*, and *hmd* characterize contributions due to the stratified medium and are to be determined by the boundary conditions.

B2. *Reflection coefficient in source region*

We define reflection coefficients for TM waves, R_{0+}^{TM} for $z \geqq 0$ and R_{0-}^{TM} for $z \leqq 0$, due to the stratified medium. From (A1) and (A2) or (A5), we obtain

$$A_l e^{-jk_{lz}d_l} = \frac{1}{2}\frac{a_{l-1}}{a_l}\left[\frac{\varepsilon_{l-1}}{\varepsilon_l} + \frac{k_{(l-1)z}}{k_{lz}}\right] \cdot [A_{l-1}e^{-jk_{(l-1)z}d_l} + B_{l-1}R_{(l-1)}^{TM}e^{jk_{(l-1)z}d_l}] \tag{B9}$$

$$B_l e^{jk_{lz}d_l} = \frac{1}{2}\frac{a_{l-1}}{a_l}\left[\frac{\varepsilon_{l-1}}{\varepsilon_l} + \frac{k_{(l-1)z}}{k_{lz}}\right] \cdot [A_{l-1}R_{l(l-1)}^{TM}e^{-jk_{(l-1)z}d_l} + B_{l-1}e^{jk_{(l-1)z}d_l}] \tag{B10}$$

Forming the ratio of (B10) to (B9) and letting $l = 0$, we obtain the ratio B_{0-}/A_{0-} in the form of continued fractions,

$$R_{0-}^{TM} = \frac{B_{0-}}{A_{0-}} = \frac{1}{R_{0(-1)}^{TM}}\exp(-j2k_{0z}d_0) + \cfrac{\left[1-\left(\frac{1}{R_{0(-1)}^{TM}}\right)^2\right]\exp[-j2(k_{0z}+k_{-1z})d_0]}{\frac{1}{R_{0(-1)}^{TM}}\exp(-j2k_{-1z}d_0) + \frac{1}{R_{(-1)(-2)}^{TM}}\exp(-j2k_{-1z}d_{-1}) + \cfrac{\left[1-\left(\frac{1}{R_{(-1)(-2)}^{TM}}\right)^2\right]\exp[-j2(k_{-1z}+k_{-2z})d_{-1}]}{\frac{1}{R_{(-1)(-2)}^{TM}}\exp(-j2k_{-2z}d_{-1}) + \frac{1}{R_{(-2)(-3)}^{TM}}\exp(-j2k_{-2z}d_{-2}) + \cdots + \frac{1}{R_{(-N+1)(-N)}^{TM}}\exp(-j2k_{-Nz}d_{-N+1})}} \tag{B11}$$

Similarly, from (A6), we obtain for $z \geqq 0$

$$R_{0+}^{TM} = \frac{A_{0+}}{B_{0+}} = \frac{1}{R_{01}^{TM}}\exp(j2k_{0z}d_1) + \cfrac{\left[1-\left(\frac{1}{R_{01}^{TM}}\right)^2\right]\exp[j2(k_{0z}+k_{1z})d_1]}{\frac{1}{R_{01}^{TM}}\exp(j2k_{1z}d_1) + \frac{1}{R_{12}^{TM}}\exp(j2k_{1z}d_2) + \cfrac{\left[1-\left(\frac{1}{R_{12}^{TM}}\right)^2\right]\exp[j2(k_{1z}+k_{2z})d_2]}{\frac{1}{R_{12}^{TM}}\exp(j2k_{2z}d_2) + \frac{1}{R_{23}^{TM}}\exp(j2k_{2z}d_3) + \cdots + \frac{1}{R_{(M-1)M}^{TM}}\exp(j2k_{Mz}d_M)}} \tag{B12}$$

Reflection coefficients for TE wave components $R_{0\pm}^{TE}$ can be obtained from (A3) and (A4) and take the identical form to $R_{0\pm}^{TM}$ with R^{TM} replaced by R^{TE}.

$$R_{0+}^{TE} = \frac{C_{0+}}{D_{0+}} = R_{0+}^{TM}|_{R^{TM}\text{ replaced by }R^{TE}} \tag{B13}$$

$$R_{0-}^{TE} = \frac{D_{0-}}{C_{0-}} = R_{0-}^{TM}|_{R^{TM}\text{ replaced by }R^{TE}} \tag{B14}$$

B3. *Wave amplitude in source region*

The wave amplitude in region 0 can be expressed in terms of $R_{0\pm}^{TM}$, $R_{0\pm}^{TE}$ and the magnitudes of primary fields in the absence of stratified media by substituting (B11)–(B14) into (B1)–(B8). After mathematical simplification, we obtain the following results for the four types of dipoles under consideration.

Vertical electric dipole (VED)

$$A_{0+} = \frac{R_{0+}^{TM}(1+R_{0-}^{TM})}{1-R_{0+}^{TM}R_{0-}^{TM}}E_{ved} \tag{B15a}$$

$$B_{0+}=\frac{(1+R_{0-}^{TM})}{1-R_{0+}^{TM}R_{0-}^{TM}}E_{ved} \tag{B15b}$$

$$A_{0-}=\frac{(1+R_{0-}^{TM})}{1-R_{0+}^{TM}R_{0-}^{TM}}E_{ved} \tag{B15c}$$

$$B_{0-}=\frac{R_{0-}^{TM}(1+R_{0+}^{TM})}{1-R_{0+}^{TM}R_{0-}^{TM}}E_{ved} \tag{B15d}$$

Horizontal electric dipole (HED)

$$A_{0+}=\frac{R_{0+}^{TM}(1-R_{0-}^{TM})}{1-R_{0+}^{TM}R_{0-}^{TM}}E_{hed} \tag{B16a}$$

$$B_{0+}=\frac{(1-R_{0-}^{TM})}{1-R_{0+}^{TM}R_{0-}^{TM}}E_{hed} \tag{B16b}$$

$$C_{0+}=\frac{R_{0+}^{TE}(1+R_{0-}^{TE})}{1-R_{0+}^{TE}R_{0-}^{TE}}H_{hed} \tag{B16c}$$

$$D_{0+}=\frac{(1+R_{0-}^{TE})}{1-R_{0+}^{TE}R_{0-}^{TE}}H_{hed} \tag{B16d}$$

$$A_{0-}=-\frac{(1-R_{0+}^{TM})}{1-R_{0+}^{TM}R_{0-}^{TM}}E_{hed} \tag{B16e}$$

$$B_{0-}=-\frac{R_{0-}^{TM}(1-R_{0+}^{TM})}{1-R_{0+}^{TM}R_{0-}^{TM}}E_{hed} \tag{B16f}$$

$$C_{0-}=\frac{(1+R_{0+}^{TE})}{1-R_{0+}^{TE}}H_{hed} \tag{B16g}$$

$$D_{0-}=\frac{R_{0-}^{TE}(1+R_{0+}^{TE})}{1-R_{0+}^{TE}R_{0-}^{TE}}H_{hed} \tag{B16h}$$

Vertical magnetic dipole (VMD)

$$C_{0+}=\frac{R_{0+}^{TE}(1+R_{0-}^{TE})}{1-R_{0+}^{TE}R_{0-}^{TE}}H_{vmd} \tag{B17a}$$

$$D_{0+}=\frac{(1+R_{0-}^{TE})}{1-R_{0+}^{TE}R_{0-}^{TE}}H_{vmd} \tag{B17b}$$

$$C_{0-}=\frac{(1+R_{0+}^{TE})}{1-R_{0+}^{TE}R_{0-}^{TE}}H_{vmd} \tag{B17c}$$

$$D_{0-}=\frac{R_{0-}^{TE}(1+R_{0+}^{TE})}{1-R_{0+}^{TE}R_{0-}^{TE}}H_{vmd} \tag{B17d}$$

Horizontal magnetic dipole (HMD)

$$A_{0+}=\frac{R_{0+}^{TM}(1+R_{0-}^{TM})}{1-R_{0+}^{TM}R_{0-}^{TM}}E_{hmd} \tag{B18a}$$

$$B_{0+}=\frac{(1+R_{0-}^{TM})}{1-R_{0+}^{TM}R_{0-}^{TM}}E_{hmd} \tag{B18b}$$

$$C_{0+}=\frac{R_{0+}^{TE}(1-R_{0-}^{TE})}{1-R_{0+}^{TE}R_{0-}^{TE}}H_{hmd} \tag{B18c}$$

$$D_{0+}=\frac{(1-R_{0-}^{TE})}{1-R_{0+}^{TE}R_{0-}^{TE}}H_{hmd} \tag{B18d}$$

$$A_{0-}=\frac{1+R_{0+}^{TM}}{1-R_{0+}^{TM}R_{0-}^{TM}}E_{hmd} \tag{B18e}$$

$$B_{0-}=\frac{R_{0-}^{TM}(1+R_{0+}^{TM})}{1-R_{0+}^{TM}R_{0-}^{TM}}E_{hmd} \tag{B18f}$$

$$C_{0-}=-\frac{(1-R_{0+}^{TE})}{1-R_{0+}^{TE}R_{0-}^{TE}}H_{hmd} \tag{B18g}$$

$$D_{0-}=-\frac{R_{0-}^{TE}(1-R_{0+}^{TE})}{1-R_{0+}^{TE}R_{0-}^{TE}}H_{hmd} \tag{B18h}$$

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REFERENCES

Adler, R. B, L J Chu, and R. M. Fano, *Electromagnetic Energy Transmission and Radiation*, chap. 10, John Wiley, New York, 1960

Ali, S M, and S. F Mahmoud, Electromagnetic fields of buried sources in stratified anisotropic media, *IEEE Trans. Antennas Propag*, *AP-27*, 671–678, 1979.

Baños, A, *Dipole Radiation in the Presence of a Conducting Half-Space*, Pergamon, New York, 1966

Kong, J A., Electromagnetic fields due to dipole antennas over stratified anisotropic media, *Geophysics*, *37*, 985–996, 1972.

Kong, J. A., *Theory of Electromagnetic Waves*, John Wiley, New York, 1974

Kong, J A (Ed.), *Research Topics in Electromagnetic Theory*, chap. 10, John Wiley, New York, 1981

Krowne, C M., Green's function in the spectral domain for biaxial and uniaxial anisotropic planar dielectric structures, *IEEE Trans Antennas Propag.*, *AP-32*, 1273–1281, 1984.

Stoyer, C. H., Electromagnetic fields of dipoles in stratified media, *IEEE Trans Antennas Propag.*, *AP-25*, 547–552, 1977.

Tabarovskii, L. A., and M I. Epov, Electromagnetic fields of harmonic sources in layered anisotropic media, *Geol. Geofiz.*, *18*, 101–108, 1977

Tang, C. M, Electromagnetic fields due to dipole antennas embedded in stratified anisotropic media, *IEEE Trans Antennas Propag.*, *AP-27*, 665–670, 1979.

Tsalamengas, J L, and N. K. Uzunoglu, Radiation from a dipole in the proximity of a general anisotropic grounded layer, *IEEE Trans. Antennas Propag.*, *AP-33*, 165–172, 1985.

Tsang, L., R. Brown, J. A. Kong, and G. Simmons, Numerical evaluation of electromagnetic fields due to dipole antennas in the presence of stratified media, *J. Geophys. Res.*, *79*(14), 2077–2080, 1974.

Wait, J. R., The magnetic dipole over the horizontally stratified earth, *Can. J. Phys.*, *29*, 577–592, 1951.

Wait, J. R., Radiation from a vertical electric dipole over a stratified ground, *IEEE Trans. Antennas Propag.*, *AP-1*, 9–11, 1953.

Wait, J. R., *Electromagnetic Waves in Stratified Media*, Pergamon, New York, 1970.

Wang, J. J. H., General methods for the computation of radiation in stratified media, *Proc. IEE*, *132*(1), part H, 58–62, 1985.

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