Journal of Experimental and Theoretical Physics, Vol. 93, No. 4, 2001, pp. 853–859.
Translated from Zhurnal Éksperimental'noĭ i Teoreticheskoĭ Fiziki, Vol. 120, No. 4, 2001, pp. 979–985.

Plasma Oscillations in Nanotubes and the Aharonov–Bohm Effect for Plasmons

A. I. Vedernikov, A. O. Govorov, and A. V. Chaplik*

Institute of Semiconductor Physics, Siberian Division, Russian Academy of Sciences, pr. akademika Lavrent'eva 13, Novosibirsk, 630090 Russia

**e-mail: chaplik@isp.nsc.ru*

Received April 12, 2001

Abstract—We theoretically analyze the collective oscillations of 2D electrons in nanotubes. In the presence of a magnetic field parallel to the tube axis, the plasmon frequencies undergo Aharonov–Bohm oscillations. The effect can manifest itself in infrared absorption and in Raman scattering. We calculate the cross sections for inelastic light scattering by plasmons. © *2001 MAIK "Nauka/Interperiodica".*

1. INTRODUCTION

Nanotubes (and quantum rings) occupy a prominent place among the objects studied by modern physics of low-dimensional systems because of their topological peculiarities. Since the region of electron motion is not simply connected, peculiar effects result (in the presence of a magnetic field), in which the phase of the wave function proves to be observable. All these effects are the derivatives of the well-known Aharonov–Bohm effect.

In a magnetic field parallel to the nanotube axis, the single-particle spectrum depends on magnetic flux as

$$E_m(q) = \frac{\hbar^2 q^2}{2\mu} + B(m+\phi)^2, \quad B = \frac{\hbar^2}{2\mu a^2}, \tag{1}$$

where μ is the effective mass, a is the cylinder radius, $\hbar q$ is the momentum along the axis, ϕ is the number of magnetic-flux quanta inside the tube, and $m = 0, \pm1, \pm2, \ldots$ is the azimuthal quantum number. This dependence of E_m on ϕ results in oscillations in macroscopic properties of the nanotube, for example, in conductivity [1] or magnetic moment [2]. In both cases, macroscopic manifestations of the properties of charge-carrying elementary excitations that obey Fermi statistics or, more simply, the electron behavior are considered.

Recently, a number of authors [3] have shown that Aharonov–Bohm oscillations also take place for a neutral object—an exciton in a quantum ring. The possibility of an electron and a hole tunneling toward each other along the ring leads to oscillatory dependencies of the exciton binding energy and formation probability on magnetic flux.

In all the above examples, the oscillation period is universal and equal to the flux quantum $\phi_0 = hc/e$. However, as was shown by Chaplik [4], this universality breaks down for a charged exciton (trion). Its binding energy oscillates with flux with a period that depends on the ratio of the effective electron and hole masses.

It seems of interest to explore the possibility of oscillation effects in the collective excitations of an electron system in nanotubes. Here, we show that Bose neutral elementary excitations (plasmons) are also characterized by oscillatory dependencies of their parameters on magnetic flux, i.e., they exhibit the Aharonov–Bohm effect. This effect manifests itself in nanotube optical properties and can thus be observed in experiments that require no electrical contact with the objects being studied.

We consider two types of nanotubes: semiconductor hollow cylinders (for example, self-assembled quantum rolls [5]) and carbon nanotubes. For the former, we use a standard parabolic dispersion law of two-dimensional electrons that leads to formula (1) in a magnetic field. For the latter, we take a conical dispersion law of two-dimensional graphite [6] as the initial one:

$$E^{\pm} = \pm\hbar V_0 \mathbf{q},$$

where $\mathbf{q}$ is the two-dimensional (2D) vector in the plane of the graphite sheet, and V_0 is a parameter of the order of the electron velocity in the atom.

Below, we derive the dispersion law for plasma oscillations, the dependence of the plasmon frequency on magnetic flux in a magnetic field parallel to the nanotube axis, the infrared absorption spectrum, and the cross section for inelastic light scattering by plasmons in nanotubes.

2. PLASMA OSCILLATIONS FOR $\phi = 0$; THE COLD-PLASMA APPROXIMATION

In the simplest theory of plasma oscillations, the spatial dispersion is ignored, which is equivalent to ignoring the particle velocity distribution (cold plasma). The corresponding criterion for a degenerate

1063-7761/01/9304-0853$21.00 © 2001 MAIK "Nauka/Interperiodica"

plasma is $\omega \gg kV_F$, where ω and k are the frequency and wave vector of the plasma wave, and V_F is the Fermi velocity. In this case, the system of equations for plasma oscillations (without retardation effects) is

$$\Delta\varphi = -4\pi e\delta(\rho - a)\tilde{N}_s,$$
$$e\dot{\tilde{N}}_s + \mathrm{div}\mathbf{j}_s = 0, \quad \mathbf{j}_s = -\sigma\nabla\varphi_{\rho=a}. \tag{2}$$

Here, $\tilde{N}_s$ and $\mathbf{j}_s$ are an addition to the particle surface density and the surface current, respectively; σ is the two-dimensional conductivity; and φ is the electric potential. For parabolic dispersion and in the collisionless approximation,

$$\sigma = \frac{iN_s e^2}{\mu\omega},$$

where N_s is the equilibrium electron surface density. Solving system (2) in cylindrical coordinates yields the dispersion of a plasmon with momentum k along the nanotube axis and with azimuthal moment m:

$$\omega_p^2 = \frac{4\pi e^2 N_s}{\mu} a\left(k^2 + \frac{m^2}{a^2}\right)K_m(ka)I_m(ka), \tag{3}$$

where I_m and K_m are, respectively, the Bessel functions of the first and third kinds of an imaginary argument. The dispersion law (3) gives the correct asymptotics.

(i) In the long-wave limit, $ka \ll 1$, for an axially symmetric plasmon ($m = 0$):

$$\omega_p^2 = \frac{2e^2 N_L k^2}{\mu}\ln\frac{2}{ka\gamma}, \quad \gamma = e^{-C}, \tag{4}$$

where C is the Euler constant, and $N_L = 2\pi a N_s$ is the linear electron density;

(ii) In the shortwave limit, $ka \gg 1$, $m \gg 1$:

$$\omega_p^2 = \frac{2\pi e^2 N_s}{\mu}\sqrt{k^2 + \frac{m^2}{a^2}}, \tag{5}$$

which corresponds to a 2D plasmon with the momentum components (k, m/a).

Equation (4) represents a standard dispersion law for a one-dimensional plasmon (for example, in a quantum wire), which usually includes the cutoff size under the logarithm known only in order of magnitude. As we see, the result for a nanotube is completely definite, including the numerical coefficient under the logarithm.

The logarithmic singularity of $\omega(k)$ when $m = 0$ and $k \longrightarrow 0$ formally corresponds to an infinite group velocity, which is, of course, unfeasible. At low k, the retardation effects (transverse fields) must be taken into account. This requires solving the Maxwell equations for the scalar and vector potentials instead of system (2):

$$\Delta\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -4\pi e\tilde{N}_s\delta(\rho - a),$$
$$\Delta\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi}{c}\mathbf{j}_s\delta(\rho - a), \tag{6}$$
$$\mathbf{j}_s = -\sigma\left(\nabla\varphi + \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}\right)_{\rho=a}.$$

As a result, the dispersion law is given by Eq. (3), in which the following substitution must be made:

$$k \longrightarrow R \equiv \sqrt{k^2 - \frac{\omega^2}{c^2}}.$$

For $m = 0$ at $ka \ll 1$, we have

$$\omega_p^2 = \frac{2e^2 N_L k^2 \Lambda}{\mu(1 + 2e^2 N_L \Lambda/\mu c^2)}, \quad \Lambda = \ln\frac{2}{Ra\gamma}. \tag{7}$$

Thus, we have $\omega \approx ck$ when $k \longrightarrow 0$, but the domain of existence of this asymptotics is exponentially small, of the order of

$$\exp\left(-\frac{\mu c^2}{2e^2 N_L}\right).$$

Let us now consider the dispersion with a conical point. Depending on the method of rolling up the graphite sheet, the nanotube can have either semiconductor or metal band structure. In the latter case, the gap in the single-particle spectrum vanishes at $q = 0$, while the density of states remains finite at this point; the azimuthal quantum number of the electron is zero, and $E_0^{\pm} = \pm\hbar V_0|q|$, where the "+" and "–" signs refer to the conduction and valence bands, respectively.

Clearly, for a nonzero gap in the electron spectrum, the pattern of plasma oscillations does not differ qualitatively from that for the parabolic dispersion considered above. Therefore, we consider in more detail only the metal band structure, for which nontrivial singularities appear in the plasmon parameters.

We assume that only the zeroth (in azimuthal quantum number) one-dimensional subband is occupied. For a degenerate system, this implies a constraint either on the dopant density,

$$n_L < \frac{2}{\pi a},$$

or on the donor energy level (which is positive for conical dispersion),

$$E_d < \frac{\hbar V_0}{a}.$$

For an intrinsic conductivity, the temperature must be low enough, $T \ll \hbar V_0/a$. An expression for the nanotube conductivity can be easily derived from the collisionless kinetic equation if the dispersion law is linear:

$$\sigma = \frac{ie^2 V_0}{\pi^2 \hbar a \omega}. \quad (8)$$

All relations (2) hold, and we find the plasmon frequency for degenerate electrons to be

$$\omega_p^2 = \frac{4e^2 V_0 k^2}{\pi\hbar} K_0(ka) I_0(ka), \quad \omega_p \ll kV_0. \quad (9)$$

It can be shown that the square of the plasma frequency for an arbitrary ω_p/kV_0 is $k^2V_0^2 + \omega_p^2$, with ω_p^2 defined in (9).

This is the case for doped carbon nanotubes at a zero temperature. Without doping and at a finite temperature, there is an intrinsic two-band conductivity. The quantity ω_p^2 is proportional to $f(0)$, the value of the Fermi function at $E = 0$. $f(0) = 1$ in a degenerate system at a zero temperature and $f(0) = 1/2$ for a nanotube with an intrinsic conductivity, because the chemical potential is zero at any temperature (the conical dispersion law!). The factor 1/2 is compensated for, because electrons and holes contribute equally to the conductivity, and we again derive a plasmon dispersion law in the form (9). Its characteristic (and, at first glance, paradoxical) feature is that the plasma frequency is independent of the carrier density. The same is also true for the conductivity (8). The reason can be easily understood if we note that both ω_p^2 and σ are proportional to N_L/μ for parabolic electron dispersion. Conical dispersion can be formally obtained if the effective mass μ itself is assumed to depend linearly on q. In that case, its value at the Fermi level (degenerate gas) or the mean temperature value (nondegenerate gas) enter into the formulas for ω_p^2 and σ. The parameter N_L matches the Fermi wave vector, to within a numerical factor, and is proportional to $T/\hbar V_0$ in a nondegenerate system. Thus, in both cases, the dependence on N_L is eliminated from the formulas. This is because the density of states in a one-dimensional subband is constant for a zero azimuthal number.

It is well known that carbon nanotubes can form coaxial structures with different numbers of nested cylinders. The probability of electron transitions between them is negligible, but the coupling through electric fields of plasma oscillations causes the number of branches of the plasmon spectrum to increase (an analog of a planar multilayer structure). For example, in the case of two coaxial nanotubes, two branches corresponding to in-phase and out-of-phase oscillations (optical and acoustic plasmons) emerge:

$$\omega_\pm^2 = \frac{\omega_{pa}^2 + \omega_{pb}^2}{2} \pm \sqrt{\frac{(\omega_{pa}^2 - \omega_{pb}^2)^2}{4} + \omega_{pa}^2\omega_{pb}^2\Lambda_m}. \quad (10)$$

Here,

$$\Lambda_m(k;\, a, b) = \frac{I_m(ka)K_m(kb)}{I_m(kb)K_m(ka)} < 1,$$

a and b are the radii of the two nanotubes with $b > a$, and ω_{pa} and ω_{pb} are their individual plasma frequencies. The "+" sign before the square root in (10) corresponds to an optical plasmon, and the long-wave asymptotics $\omega_+ \propto k\sqrt{|\ln k|}$ at $m = 0$. The second branch for the ka, $kb \ll 1$, $m = 0$, obeys a linear dispersion law, $\omega_- \propto k\sqrt{\ln(b/a)}$. At $m \neq 0$, both frequencies ω_+ and ω_- tend to the constant values that correspond to intersubband transitions with a depolarization shift (see below) when $k \longrightarrow 0$.

3. EFFECTS OF A MAGNETIC FIELD

A magnetic filed can affect the plasmon dispersion law only via the constitutive equations (current–field relation). If, as was done in Section 2, the conductivity is assumed to be classical, then a longitudinal magnetic field for parabolic dispersion cannot change the orbital motion of 2D electrons on the cylinder surface, and, therefore, σ does not depend on the field. The influence of a magnetic field (or, to be more precise, magnetic flux ϕ) shows up only when quantum effects are taken into account. The dependence of energy on ϕ given by Eq. (1) transforms to a dependence of the Fermi level and polarization operator on magnetic flux. With an allowance for the spatial dispersion of conductivity, an oscillatory dependence of the plasma frequency on ϕ (the Aharonov–Bohm effect for plasmons) arises.

We begin with a symmetric plasmon with $m = 0$. Calculating the polarization operator

$$\Pi(k, m = 0) = \sum_{qm'} \frac{f'(E_{m'}(q))\hbar kq/\mu}{\omega + i\delta - \hbar kq/\mu} \quad (11)$$

and solving the Poisson equation yields the dispersion equation

$$1 = \frac{2e^2k^2}{\pi\hbar} K_0(ka) I_0(ka) \sum_{m'} \frac{V_F(m')}{\omega^2 - k^2 V_F^2(m')}, \quad (12)$$

where

$$V_F(m') = \sqrt{\frac{2}{\mu}[E_F - B(m' + \phi)^2]},$$

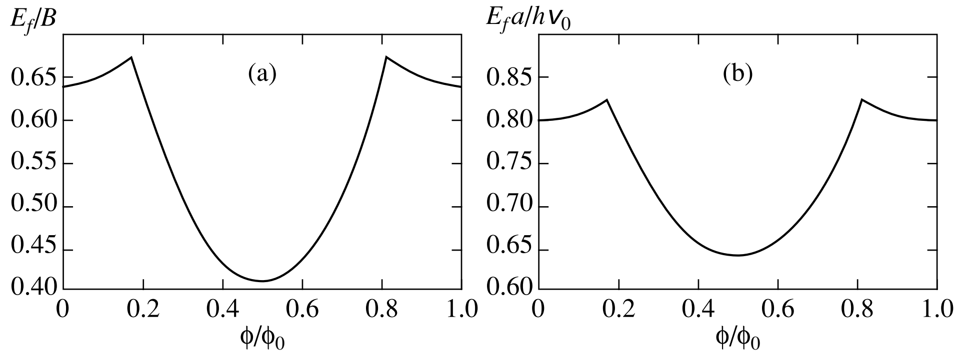

Fig. 1. Fermi energy of degenerate electrons on the nanotube surface versus magnetic flux: (a) a parabolic dispersion law and (b) a conical dispersion law.

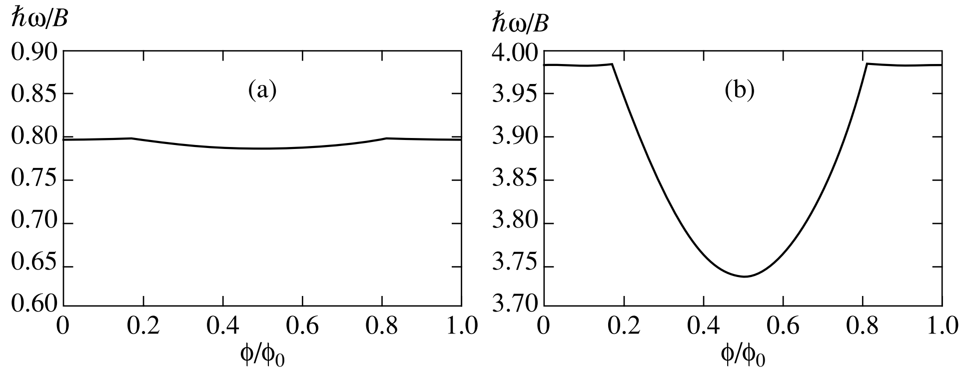

Fig. 2. Plasmon frequency at $m = 0$ versus magnetic flux (parabolic dispersion): $ka = 0.1$ (a) and 1.0 (b).

the Fermi energy must be derived from the equation

$$\hbar N_L = \frac{2\sqrt{2\mu}}{\pi}\sum_{m'}\sqrt{E_F - B(m'+\phi)^2}. \tag{13}$$

The summation over m' in (12) and (13) is constrained by the requirement that the radicands be positive. An analytic (and relatively simple) answer can be obtained in the approximation of weak spatial dispersion, $\omega \gg kV_F(m')$ for all admissible m'. Expanding in (12) in kV_F/ω and applying the Poisson summation formula yields

$$\omega^2 \approx \frac{2e^2k^2N_L}{\mu}K_0(ka)I_0(ka) + \frac{3(k\bar{V}_F)^2}{2\pi^2N_La} \times \sum_{l=-\infty}^{\infty} J_2\left(2\pi l\sqrt{\frac{\bar{E}_F}{B}}\right)\frac{\cos(2\pi l\phi)}{l^2}. \tag{14}$$

Here,

$$\bar{E}_F = \frac{\mu\bar{V}_F^2}{2} = \frac{2\pi\hbar^2N_s}{\mu}$$

is the oscillation-averaged Fermi level. The second term in (14) has a relative smallness of the order of $a^*/a|\ln(ka)|$, where $a^* = \hbar^2/\mu e^2$ is the effective Bohr radius (this is the parameter $k^2V_F^2/\omega^2$ for ω of the order of the frequency of one-dimensional plasma oscillations, which is given by the first term in Eq. (14)). Thus, when spatial dispersion is taken into account, the plasmon frequency oscillates with magnetic flux with the period $\Delta\phi = 1$.

For an arbitrary value of kV_F/ω, we calculated $\omega(\phi)$ numerically. Figure 1 shows a plot of Fermi energy against magnetic flux within one period. The electron density was chosen in such a way that no more than two subbands were occupied at any ϕ (for $\phi > 0$, these are the pairs $m' = 0$ and $m' = -1$; $m' = -1$ and $m' = -2$; and so on). Under the same conditions, Fig. 2 shows $\omega(\phi)$ for two values of the wave vector. As must be the case, the magnetic dispersion of the plasmon frequency is enhanced with increasing ka.

The results are qualitatively different for conical quasi-particle dispersion. In this case, including a magnetic field opens a gap in the dispersion law for a nanotube with a "metal-type" spectrum:

$$E_0^{\pm} = \hbar V_0\sqrt{q^2 + \frac{\phi^2}{a^2}}. \tag{15}$$

As a result, the diagonal velocity element along the axis and the nanotube classical conductivity depend on magnetic flux even in the lowest approximation in kV_0/ω:

$$\sigma = \frac{ie^2 N_L V_0}{\pi\hbar\omega}\frac{1}{\sqrt{(\pi a N_L)^2 + \phi^2}}. \tag{16}$$

This leads to a more complex dependence of the plasma frequency on ϕ, as illustrated by Fig. 3 for three values of ka. The linear density N_L again corresponds to the occupation of no more than two subbands.

Azimuthally nonuniform oscillations ($m \neq 0$) are similar to intersubband 2D plasmons in quantum films. These are in fact transitions between subbands, $m' \longrightarrow m' + m$, with Coulomb effects (depolarization shift). Since the general case with $k \neq 0$ and $m \neq 0$ is described by cumbersome formulas, we restrict our analysis to a purely transverse plasmon ($k = 0$). The corresponding equation for Π in the case of parabolic dispersion is

$$\Pi(k = 0, m) = \frac{\sqrt{2\mu}m^2}{\pi^2 a\hbar B} \times \sum_{m'} \frac{\sqrt{E_F - B(m' + \phi)^2}}{m^4 - (2mm' + 2m\phi - \omega/B)^2}, \tag{17}$$

where E_F can be determined from Eq. (13). Π is clearly periodic in ϕ with the same period $\Delta\phi = 1$. Figure 4 shows plots of the intersubband-plasmon frequencies against magnetic flux for $m = \pm 1$ in the range $0 < \phi < 1/2$, which corresponds to one half-period [E_F is an even function of ϕ, $\Pi(\phi, \omega) = \Pi(-\phi, -\omega)$].

The density N_L was chosen in such a way that only the $m' = 0$ and $m' = -1$ subbands were occupied as ϕ varied over the above range; this requires that the condition $\pi N_L a < 2$ be satisfied. The break in the plot corresponds to the onset of the $m' = -1$ subband occupation.

For conical dispersion, the case with $m \neq 0$ is qualitatively similar to that considered above.

4. INTERACTION OF PLASMONS WITH ELECTROMAGNETIC RADIATION

4.1. Infrared Absorption

An electromagnetic wave linearly polarized along the nanotube axis produces an axially symmetric perturbation ($m = 0$). This wave can be absorbed by $m = 0$ plasmons if its electric field is modulated in the axis direction with a period L. This is usually achieved with a one-dimensional grating structure. The absorption-line frequency is then equal to the frequency of a plasmon with $k = 2\pi/L$. Given the electron scattering, the absorption at the line center per unit surface area is [7] $E_0^2\sigma_0/2$, where E_0 is the amplitude of the wave electric field, and $\sigma_0 = e^2N_s\tau/\mu$ is the nanotube static surface conductivity (N_s is the surface density, and τ is the momentum relaxation time).

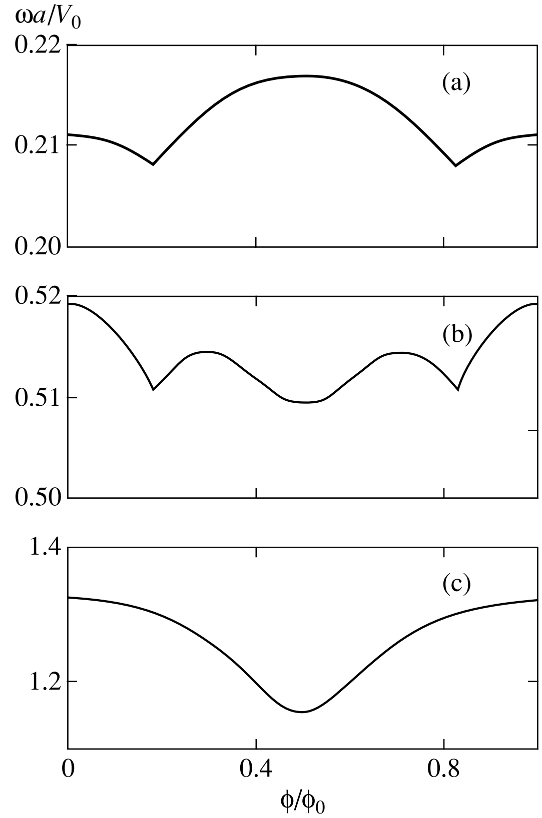

Fig. 3. Plasmon frequency at $m = 0$ versus magnetic flux (conical dispersion); $ka = 0.1$ (a), 0.3 (b), and 1.0 (c).

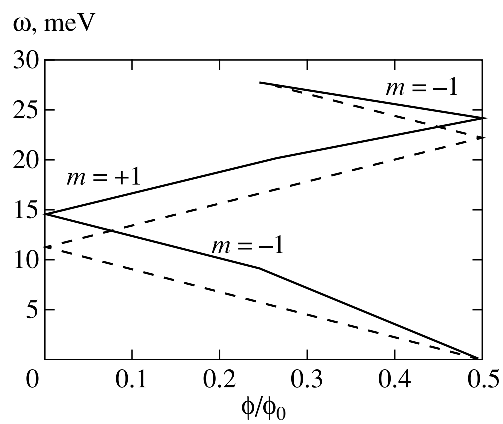

Fig. 4. Frequencies of single-particle transitions (dashed lines) and intersubband plasmons with a depolarization shift (solid lines) versus magnetic flux; $a = 70$ Å and $N_L = 3 \times 10^5$ cm^{-1}.

No modulation is required when the wave is polarized perpendicular to the tube axis: a uniform field excites the $\Delta m' = \pm 1$ transitions; i.e., intersubband plasmons with $m = \pm 1$ are produced in the system. With an allowance for the depolarization effects, absorption frequency is plotted against ϕ in Fig. 4.

In conclusion, recall that the above discussion pertains to intraband transitions. For carbon nanotubes, these transitions are as follows:

$$E^{+}_{m'}(q) \longrightarrow E^{+}_{m'+m}(q+k).$$

The absorption that is attributable to $E^{+} \longrightarrow E^{-}$ transitions and that is unrelated to plasma effects was considered in [8]. The Aharonov–Bohm effect that results from a periodic dependence of the ground-state energy on magnetic flux also shows up in this case.

4.2. Inelastic Light Scattering

The plasmon contribution to inelastic light scattering is commonly observed in the geometry of parallel polarizations of the incident ($\mathbf{e}_1$) and scattered ($\mathbf{e}_2$) photons; the amplitude of the process is proportional to $\mathbf{e}_1 \cdot \mathbf{e}_2$. The corresponding cross section is determined by the density-density correlator. The latter, in turn, is related to the generalized susceptibility α, which gives a response of the system to a scalar perturbation of the form $\exp(i\boldsymbol{\kappa} \cdot \mathbf{r})$ with allowance for the self-consistent field.

For a fixed transfer of the photon momentum with components $\mathbf{k}_\perp$ (in a plane perpendicular to the nanotube axis) and k (along the axis), the scattering cross section is the sum of partial contributions $\sigma_m(k)$ whose relative weights are determined by the coefficients of the expansion of a plane wave $\exp(i\mathbf{k}_\perp \cdot \boldsymbol{\rho})$ in terms of cylindrical harmonics:

$$\begin{gathered}\frac{d^2\sigma_m}{d\Omega d\omega} = 2al\frac{\omega_1}{\omega_2}\left(\frac{e^2}{m_0c^2}\right)^2(n(\omega)+1) \\ \times|J_m(k_\perp a)|^2\operatorname{Im}\alpha_m(k,\omega)(\mathbf{e}_1\cdot\mathbf{e}_2)^2F(\omega_1), \\ \omega = \omega_1 - \omega_2,\end{gathered} \tag{18}$$

where the subscripts 1 and 2 denote, respectively, the incident and scattered photons; l is the nanotube length; m_0 is the mass of a free electron; $n(\omega)$ are the Bose occupation numbers, and J_m is the Bessel function. We also included the amplification factor $F(\omega_1)$ in the formula, because Raman scattering is commonly observed at resonance, when the frequency ω of the exciting light is close to a particular interband transition. In A_3B_5 semiconductors for resonance with a spin-orbit split-off band (see [9]),

$$F = |p_{cv}|^4/9m_0^2\Delta^2,$$

where p_{cv} is the interband momentum matrix element, and Δ is the resonance detuning. The partial susceptibility $\alpha_m(k, \omega)$, which gives a response of the density to a scalar perturbation $f_m e^{im\varphi}$ with moment m, can be calculated by adding $-\nabla f/e$ to the electric field in the formula for current $\mathbf{j}_s$ and by solving system (6). As a result, we obtain

$$\alpha_m(k,\omega) = \frac{i\sigma\left(k^2+\dfrac{m^2}{a^2}\right)}{e^2\left[\omega+4\pi i\sigma\left(R^2+\dfrac{m^2}{a^2}\right)aK_m(Ra)I_m(Ra)\right]}. \tag{19}$$

This expression for α corresponds to the cold-plasma approximation,

$$\omega \gg V_F\sqrt{k^2+\frac{m^2}{a^2}}.$$

Apart from the plasmon pole, $\alpha_m(k, \omega)$ has a bifurcation point at $R = 0$, i.e., at $\omega = ck$. As in the two-dimensional case (see [10]), the high-frequency wing in the Raman scattering spectrum at $\omega > ck$ corresponds to this bifurcation. The imaginary part of α, which gives the wing intensity distribution, is

$$\begin{aligned}\operatorname{Im}\alpha_m(k,\omega) &= 2\pi^2(N_sk/\mu)^2a(\omega^2/c^2-k^2)J_m^2 \\ &\times\left\{\left[\omega^2-u^2\left(\frac{\omega^2}{c^2}-k^2\right)J_mN_m\right]^2\right. \\ &\left.+\left[u^2\left(\frac{\omega^2}{c^2}-k^2\right)J_m^2\right]^2\right\}^{-1}, \quad \omega \ge ck,\end{aligned} \tag{20}$$

where

$$u^2 \equiv \frac{2\pi^2e^2aN_s}{\mu}$$

and the argument of all Bessel functions is $a\sqrt{\omega^2/c^2-k^2}$.

Intersubband transitions ($m \neq 0$) are excited during inelastic light scattering with $k_\perp \neq 0$. At $k_\perp a \ll 1$, the cross sections for such processes rapidly decrease with increasing m: $\sigma_m \propto (k_\perp a)^{2m}$. In the presence of a magnetic field, the polarization operator should be substituted for the classical conductivity σ in Eq. (19):

$$\left(k^2+\frac{m^2}{a^2}\right)\sigma \longrightarrow ie^2\omega\Pi_m(k). \tag{21}$$

In that case, the peaks in the partial cross sections as functions of ω are determined by resonances on intersubband plasmons and shift as the magnetic flux changes (see Fig. 4). The amplitudes of these peaks, i.e., the partial cross sections themselves, are also periodic functions of ϕ.

Thus, we have shown that the basic parameters of plasma waves in nanotubes oscillate with magnetic flux with a period ϕ_0. For a longitudinal, axially symmetric plasmon, this dependence emerges only beginning with terms of the order of $(\omega/kV_F)^2$ in the dispersion law.

ACKNOWLEDGMENTS

This study was supported by the Russian Foundation for Basic Research (project no. 99-02-17127) and the Program "Physics of Solid-State Nanostructures." Two of us (A.I.V. and A.O.G.) thank the Volkswagen-Stiftung Foundation (Germany) for support.

REFERENCES

1. B. L. Al'tshuler, A. G. Aronov, and B. Z. Spivak, Pis'ma Zh. Éksp. Teor. Fiz. **33**, 101 (1981) [JETP Lett. **33**, 94 (1981)]; D. Yu. Sharvin and Yu. V. Sharvin, Pis'ma Zh. Éksp. Teor. Fiz. **34**, 285 (1981) [JETP Lett. **34**, 272 (1981)].
2. I. O. Kulik, Pis'ma Zh. Éksp. Teor. Fiz. **11**, 407 (1970) [JETP Lett. **11**, 275 (1970)].
3. A. V. Chaplik, Pis'ma Zh. Éksp. Teor. Fiz. **62**, 885 (1995) [JETP Lett. **62**, 900 (1995)]; R. A. Römer and M. E. Raikh, Phys. Rev. B **62**, 7045 (2000); H. Hu, D.-J. Li, J.-L. Zhu, and J.-J. Xiong, cond-mat/0009044; cond-mat/0010310.
4. A. V. Chaplik, Zh. Éksp. Teor. Fiz. **119**, 193 (2001) [JETP **92**, 169 (2001)].
5. V. Ya. Prinz, V. A. Seleznev, V. A. Samoylov, and A. K. Gutakovsky, Microelectron. Eng. **30**, 439 (1996).
6. H. Ajiki and T. Ando, J. Phys. Soc. Jpn. **62**, 1255 (1993).
7. A. V. Chaplik, Surf. Sci. Rep. **5**, 296 (1985).
8. H. Ajiki and T. Ando, Physica B (Amsterdam) **201**, 349 (1994).
9. M. V. Klein, in *Light Scattering in Solids*, Vol. 3: *Recent Results*, Ed. by M. Cardona (Springer-Verlag, Berlin, 1975; Mir, Moscow, 1986), p. 174.
10. A. O. Govorov and A. V. Chaplik, Zh. Éksp. Teor. Fiz. **98**, 1564 (1990) [Sov. Phys. JETP **71**, 876 (1990)].

Translated by V. Astakhov