An Efficient Method to Extract Surface-Wave Poles of Green's Functions Near Branch Cut in Lossy Layered Media

Siming Pan and Jun Fan

***Abstract*—Calculating the Green's functions in lossy layered media using the discrete complex image method (DCIM) is challenging, due to the difficulties in extracting the surface-wave poles that are very close to a branch cut. An efficient algorithm based on the contour method is proposed in this communication to locate these poles and calculate the residues. The proposed method is robust for both the lossless and lossy media. With the proposed approach, it is shown in numerical examples that some poles, very close to a branch cut, are successfully extracted in lossy media. The accurate calculation of the Green's functions in lossy layered media enables the accurate and efficient modeling of complex structures in lossy semiconductor substrates and new 3D IC structures including through-silicon vias (TSVs).**

***Index Terms*—Discrete complex image method (DCIM), lossy layered media, pole extraction, Sommerfeld integral, spectral-domain Green's functions, surface-wave poles.**

I. Introduction

Green's functions in layered media have been extensively studied during the past several decades. It is necessary to evaluate the Green's functions efficiently in any algorithm based on integral equations (IE) such as the method of moments (MoM), for analyzing multilayer structures such as patch antennas, printed dipoles, high-speed interconnects, microwave and millimeter-wave circuits, etc.

The normal procedures of calculating the Green's functions in layered media start with the spectral-domain Green's functions, which are efficiently constructed based on the generalized reflection and transmission coefficients [1]–[3]. Then, the spatial-domain Green's functions are evaluated from their spectral-domain counterparts through the well-known Sommerfeld integral (SI). Since SI has the intrinsic property of strong singularity, high oscillation and slow decay, numerical integration is generally time-consuming. Over the years, several methods have been proposed to expedite the calculation of SI [4]–[7]. Among those methods, discrete complex image method (DCIM) provides closed-form Green's functions in a systematic manner [8], [9].

When applying DCIM, the spherical waves in the spectral domain are approximated in terms of a set of complex exponentials, using the generalized pencil of function (GPOF). Via the Sommerfeld identity, each complex-exponential term in the spectral domain can be easily casted to an analytical term in the spatial domain. However, in order to obtain accurate far-field response, surface-wave poles must be extracted to avoid singularities. Because the surface-wave poles representing cylindrical and lateral surface waves, if approximated with spherical waves, could cause severe deterioration of the DCIM algorithm in the far field.

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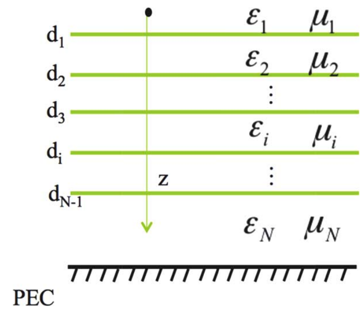

Fig. 1. A typical configuration of layered media with N dielectric layers. The top layer is open and the bottom layer is bounded with a PEC plane.

Although it is a cumbersome step to locate the surface-wave poles for general multiple, especially thick layers [10], significant progress has been achieved and some common methods were well summarized in [11]. For general layered media, contour integrals, first proposed in [12], are usually performed to check the locations of the poles. However, when the surface-wave poles are located close to a branch cut, which is very common for transverse magnetic (TM) waves due to the absence of low cutoff frequency, the recursive contour integral method becomes less efficient. Polimeridis *et al.* proposed a technique to remove the branch cut using a sine transformation [13]. But this method embodied in the algorithm [14] is not sufficiently efficient for lossy layered media. Moreover, in [11], Wang *et al.* proposed to use the Cauchy theorem and criteria to check the locations of the poles. Their criteria improved the accuracy but did not completely solve the errors caused by the branch cut. Consequently, as shown in the numerical examples in Section III, some critical poles were missed in their results [11, Table II] in lossy media.

Being able to deal with lossy layered media becomes more critical today with the need to characterize and model 3D IC and 3D packaging structures with through-silicon vias (TSVs). Lossy silicon substrate has a significant impact on the electrical performance of these structures. In this communication, a simple yet accurate method is proposed to calculate the contour integral with the presence of a branch cut inside the contour. The algorithm is efficient to remove the effects of the branch cut in general layered media, whether lossless or lossy, and gives good results for searching the poles as well as calculating the residues. The details of the algorithm are presented in Section II, followed by numerical examples to demonstrate the accuracy and the effectiveness of the method in Section III.

II. Formulation

A typical configuration of layered media shown in Fig. 1 is used in the following discussions. The top layer is open (unbounded) and the bottom layer is bounded by a perfect electric conductor (PEC) plane. The spectral-domain Green's functions of such layered media have been well established through equivalent transmission lines with transverse electric (TE) or TM types [2].

When applying DCIM to obtain the analytical spatial-domain Green's functions through the Sommerfeld identity, the spectral-domain Green's functions can be decomposed into the form as

$$\tilde{G} = \tilde{G}_{pri} + \tilde{G}_{\text{static}} + \tilde{G}_{sw} + \tilde{G}_{sp} \quad (1)$$

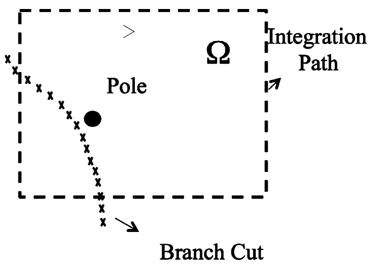

Fig. 2. Contour integration with branch cut intersecting with the integration path.

where $\tilde{G}$ is a spectral-domain Green's function; $\tilde{G}_{pri}, \tilde{G}_{\text{static}}, \tilde{G}_{sw}, \tilde{G}_{sp}$ represent its primary field, quasi-static image, surface-wave and complex image terms, respectively, and

$$\tilde{G}_{pri} = \frac{e^{-jk_z^1|z|}}{2jk_z^1} \tag{2}$$

$$\tilde{G}_{\text{static}} = \frac{A_0}{2jk_z^1} \tag{3}$$

$$\tilde{G}_{sw} = \sum_i \frac{2k_{\rho i} Res_i}{k_\rho^2 - k_{\rho i}^2} \tag{4}$$

$$\tilde{G}_{sp} = \sum_{i=1}^{N_G} a_i e^{-b_i k^{1z}} / 2jk_z^1 \tag{5}$$

where $k_z^1 = \sqrt{k_1^2 - k_\rho^2}$ is the wavenumber in the first open layer, $k_1 = \omega\sqrt{\varepsilon_1 \mu_1}$ and k_ρ is the transverse propagation wave number. In addition,A_0 is the constant term when $k_\rho \to \infty$; N_G is the number of the complex images; a_i and b_i are the coefficients of the ith complex image; and,$k_{\rho i}$ and Res_i are the ith surface-wave pole and residue. For layered media, only the unbounded layer has branch cut. In any bounded layer, wave could travel at both the $\hat{z}$ and -$\hat{z}$ directions and therefore there is no branch cut. Thus, spherical waves are expanded with regard to the open layer to avoid artificial branch cut [15].

For general lossless media, the pole $k_{\rho i}$ is located along the real axis while $k_{\rho i}$ could be located at any point inside the entire k_ρ plane for lossy media. From the residue theorem, in a simple connected domain Ω bounded by a Jordan curve C_r, the residue of a pole can be obtained through the contour integral in the complex k_ρ plane as

$$Res = \frac{1}{2\pi j} \oint_{C_r} \tilde{G} dk_\rho \tag{6}$$

However, the spectral-domain Green's function $\tilde{G}$ has branch cut in the k_ρ plane, due to the fact that the mode wavenumber for the unbounded layer

$$k_z^1 = \pm\sqrt{k_1^2 - k_\rho^2} \tag{7}$$

is a double-valued complex function. The imaginary part of k_z^1 has to be chosen negative to satisfy the radiation condition.

If the branch cut intersects with the integration path, as illustrated in Fig. 2, the contour integral (6) will not be zero even if there is no pole inside the closed domain Ω. In other words, (6) is valid to calculate the residue of a pole only when the integration path is within one Riemann sheet of $\tilde{G}$. For this reason, when the location of a pole is close to the branch cut, the algorithm of recursive contour integral [12] fails and the error from the integration path across different Riemann sheets impairs the judgment of pole location as well as the calculation of the residue. Though it is relatively easy to locate any branch point, limiting the contour integral within one Riemann sheet by identifying the entire branch cut is usually cumbersome, especially for layered media.

TABLE I
TE AND TM WAVE POLES IN A SINGLE-LAYER MEDIUM WITH LOSSLESS OR LOSSY DIELECTRIC MATERIAL

Pole (rad/cm)		Lossless	tan δ =0.02
ρ_{TM}	method in this paper	3.265300	3.2653-j0.0402
	method in [11]	3.261989	3.2620-j0.0401
	method in [13]	---	3.2653-j0.0402
ρ_{TE}	method in this paper	2.223390	2.2226-j0.0238
	method in [11]	2.220789	2.2200-j0.0237
	method in [13]	2.22	2.2226-j0.0238

TABLE II
PARAMETERS OF AN EXAMPLE IN FIG. 1, AS THE SAME GEOMETRY IN [11] (N = 5)

N	ε_r	$h(mm)$	μ
1	1.0	∞	1
2	2.1	1.5	1
3	12.5	1.0	1
4	9.8	1.5	1
5	8.6	1.5	1

In order to eliminate the effects of branch cut, we modify (6) and propose the following condition for identifying the location of a pole, with a given error tolerance e,

$$|Res| = |\frac{1}{2\pi j} \oint_{C_r} (\tilde{G} + \tilde{G}')dk_\rho| > e \tag{8}$$

where $\tilde{G}$ and $\tilde{G}'$ are the spectral-domain Green's Functions when $Im(k_z^1 < 0)$ and $Im(k_z^1 > 0)$.

Spurious poles associated with $\tilde{G}'$ would also be selected from (8), thus, we should add one more criterion, as

$$\left| \frac{1}{2\pi j} \oint_{C_r} \tilde{G} dk_\rho \right| > e \tag{9}$$

Conditions (8) and (9), together with the recursive contour integration method [12] or the Cauchy's integral theorem [11], could accurately locate all the poles, no matter how close they are to the branch cut.

III. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we provide numerical examples to demonstrate the accuracy and the effectiveness of the proposed method, for both lossless and lossy layered media. In the implementation of the proposed algorithm, the error tolerance in (8) and (10) is selected as 1e-5. The region for searching the poles is limited within a square centered at the origin with edge length of 2e4 along both the real and imaginary axis directions.

First, the surface-wave poles associated with both TE and TM modes in a single-layer medium with parameters of $\varepsilon_r = 4$, layer thickness $h = 5$ mm and the operation frequency $f = 10$ GHz are studied, same as in [11] and [13]. *tan* δ is the loss tangent defined as $\varepsilon''/\varepsilon'$ for dielectric. The Green's function in this example has a single TE-mode pole that is very close to its branch point at$k_\rho = k_0 = 2.0958$. The results from the proposed method are compared with the previously published results in Table I. It can be observed that the results from the proposed method are nearly identical with those from [13].

TABLE III
TE AND TM WAVE POLES IN FOUR-LAYER MEDIA WITH LOSSLESS OR LOSSY DIELECTRIC MATERIALS

Pole (rad/cm)		Lossless	$\tan\delta$ =0.2	
ρ_{TM}	method in this paper	8.6019	7.915-j2.75	4.477-j1.495
		16.1638	16.32-j2.308	1.528-j19.04
		18.8632	18.95-j1.993	0.420-j28.45
		---	0.733-j38.32	0.484-j46.89
			0.175-j56.21	0.383-j63.31
			0.265-j80.49	0.149-j87.91
	method in [11]	8.5844	7.8894-j2.7340	
		16.1478	16.3122-j2.3076	
		18.8498	18.9407-j1.9917	
	method in [13]	8.6019	---	
		16.1638	---	
		18.8630	---	
ρ_{TE}	method in this paper	6.2894	4.43867-j3.8411	
		14.8323	15.0099-j2.4616	
		19.1407	19.2472-j2.1411	
	method in [11]	6.2841	---	
		14.8151	14.9927-j2.4607	
		19.1259	19.2322-j2.1396	
	method in [13]	6.2893	---	
		14.8323	---	
		19.1407	---	

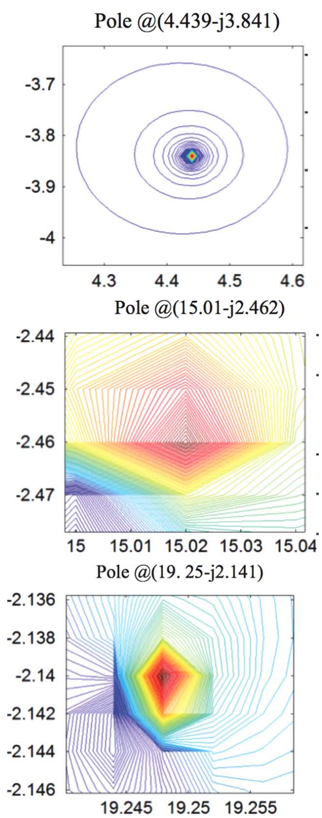

Fig. 3. Contour plots of the magnitude of the normalized TE waves in the complex kp plane for the lossy case. The three most significant poles are plotted. The locations of the poles match well with those extracted from the proposed method listed in Table II.

Then, surface-wave poles of the Green Function in the same four-layer media with a PEC ground plane as discussed in [11] and [13] are studied. The structure is shown in Fig. 1 with the parameters listed in Table II. The frequency is 30 GHz. In this example, the surface-wave

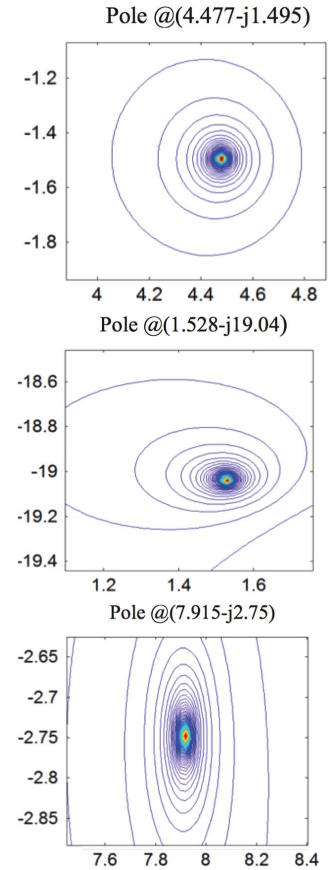

Fig. 4. Contour plots of the magnitude of the normalized TM waves in the complex k_p plane for the lossy case. The remaining three significant poles are plotted. The locations of the poles match well with those extracted from the proposed method listed in Table III.

poles are very close to the branch cut. Direct methods in [12] and [14] failed. In [13], only lossless case was considered since the method is not sufficiently efficient for lossy layered media. And the technique in [11] missed several critical poles for the lossy case as illustrated in Table III.

All the poles obtained from the proposed method are listed in Table III, which also includes the results from [11] and [13]. From the table, it can be observed that in the lossless case the poles extracted by the proposed method match with those from [13], but are slightly different than those from [11]. However, for the lossy case, the proposed method identified more poles for both the TE and TM modes than the method in [11]. The extra poles identified from the proposed algorithm are not due to a higher error tolerance, because the residues of some of these poles are sufficiently large and they dominate the surface-wave behavior in the far field. To further verify the accurate locations of these poles in the lossy media, contour plots in the complex k_ρ plane are shown in Figs. 3 and 4. Due to the space limitation of the communication, only several significant poles for the TM waves are shown. But all the poles identified by the proposed method are verified to be true and accurate.

The proposed method could be generalized to calculate the residues using contour integral with the presence of any number of branch cuts. Suppose a total number of N branch cuts intersecting with the integration path, the computational time of locating the poles increases to 2^N. Fortunately, in general layered media, the maximum number of branch cuts associated with unbounded media that could impact the contour integral for locating poles is only 2. Thus, the pole extraction algorithm proposed in this communication can still be embodied into DCIM to improve its efficiency with a negligible increase of computational time.

IV. Conclusion

A robust and automatic method to extract the surface-wave poles of the spectral-domain Green's functions for general layered media is proposed. Both lossless and lossy dielectric materials can be effectively handled. The proposed method can accurately identify the surface-wave poles located close to the branch cut. Through the numerical examples and the comparisons with the previously published results, the proposed approach has been demonstrated more effective and accurate to handle lossy layered media.

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Electromagnetic Fields Radiated by a Circular Loop With Arbitrary Current

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***Abstract*—We present a rigorous approach to compute the electromagnetic fields radiated by a thin circular loop with arbitrary current. We employ a polar transmission representation along with a Kontorovich-Lebedev transform to derive integral representations of the field in the interior and exterior regions of a sphere circumscribing the loop. The convergence of the obtained expressions is discussed and comparisons with full-wave simulation and other methods are shown.**

***Index Terms*—Closed-form solution, eigenfunction expansion, electromagnetic radiation, loop antennas, vector-wave functions.**

I. Introduction

Electromagnatic (EM) radiation by a circular current loop is extensively investigated in the literature (see, e.g., [1], [2] and references therein, and comments thereon [3]–[6]). However, the majority of the studies investigated the far field pattern. The evaluation of the near field in closed form is generally limited to constant or cosine current distributions due to difficulties in evaluating the corresponding integrals.

Here, we present a rigorous approach to express the electromagnetic fields radiated by a thin current loop excited with an arbitrary current form. The presented approach employs the Kontorovich-Lebedev (KL) transform and the Fourier series expansion to express the radiated field in the interior and exterior regions of a sphere circumscribing the loop. The resulting expression includes an integral whose convergence is detailed. An alternative representation in terms of a residue series sum is presented for field evaluation in the interior region. The field expressions could thus be evaluated in closed form or by numerical integration techniques with ease, and they do not suffer from any irregularities or artificial discontinuities in contrast to previously reported approaches [1], [2].

II. Formulation of the Problem

The geometry of the problem is shown in Fig. 1, where a circular current loop is placed in free-space and carries an arbitrary electric current along its circumference. In spherical coordinates, (r, θ, ϕ), the loop is located at $r = r'$ and $\theta = \theta'$. The electric current density on the loop is given by

$$\mathbf{J}\left(\mathbf{r}\right) = \frac{I\left(\phi\right)}{r'} \delta\left(r - r'\right) \delta\left(\theta - \theta'\right) \hat{\boldsymbol{\phi}}, \tag{1}$$

where $I(\phi)$ is the current distribution in the azimuthal direction and $\delta(r)$ is the Dirac delta. An observer at (r, θ, ϕ) is said to be in the interior region when $r < r'$ and in the exterior region otherwise. The time-harmonic dependence of $\exp(-i\omega t)$ is assumed throughout.

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