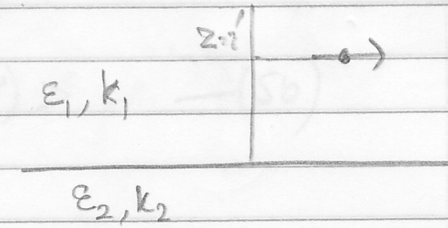
II Equivalent of a horizontal electric dipole above a half-space



$$\vec{J} = \hat{x} J_x \, \delta(\vec{r}-\vec{r}') = \hat{x} J_x \, \delta(x-x') \, \delta(z-z') \, \delta(y-y') \quad \text{— (1)}$$

The corresponding spectral representation of the source is,

$$\tilde{\vec{J}} = \hat{x} J_x \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(x-x) \, \delta(y-y') \, \delta(z-z') \, e^{jk_x x} e^{jk_y y} \, dx \, dy \quad \text{— (2)}$$

$$= \hat{x} J_x \, e^{-jk_x(x-x')} e^{-jk_y(y-y')} \, \delta(z-z') \quad \text{— (3)}$$

See Dr. Nevels Note

Here $\vec{r} = \hat{x}x + \hat{y}y$

$\vec{r}' = \hat{x}x' + \hat{y}y'$

$\vec{\rho} = (x-x')\hat{x} + (y-y')\hat{y}$

$\rho = \sqrt{(x-x')^2+(y-y')^2}$ — $\vec{\rho} = \rho\sin\phi \, \hat{x} + \rho\cos\phi \, \hat{y}$

Also $\vec{k}_\rho = k_x\hat{x} + k_y\hat{y}$

$k_x = k_\rho \cos\theta$

$k_y = k_\rho \sin\theta$

So $$\boxed{k_x(x-x') + k_y(y-y') = k_\rho \rho \cos(\theta + \phi)} \quad \text{— (4)}$$

We'll use this result when inverse F.T ing in the polar co-ordinates.

$-\hat{z}\times J_x \hat{x}$
$-\hat{z}\times\hat{x}$
$= -\hat{y}$ $\quad J_x$

$$\vec{E}(\vec{r}) = \langle \overline{\overline{G}}^{EJ} ; \vec{J}(\vec{r}') \rangle \quad \text{— (5a)}$$

$$\vec{H}(\vec{r}) = \langle \overline{\overline{G}}^{HJ} ; \vec{J}(\vec{r}') \rangle \quad \text{— (5b)}$$

Transverse components:

$$\vec{\tilde{E}}_t = \hat{x}\, V^{TM} \quad ; \quad \vec{\tilde{H}}_t = \hat{y}\, I^{TM} \quad \text{— (6)}$$

The subsequent T.L. equations:

$$\frac{dV^{TM}}{dz} = -jk_z Z^{TM} I^{TM} \quad \text{— (7a)}$$

$$\frac{dI^{TM}}{dz} = -jk_z Y^{TM} V^{TM} + i^{TM} \quad \text{— (7b)}$$

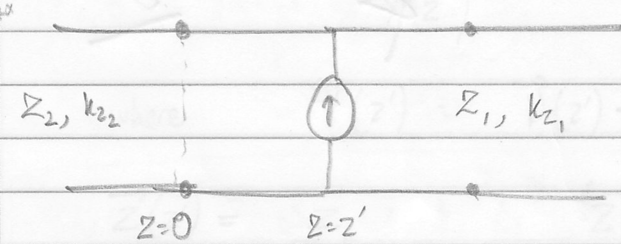
$$Z^{TM} = \frac{1}{Y^{TM}} = \frac{k_z}{\omega\varepsilon_0\varepsilon_r}$$

$-i^{TM}$

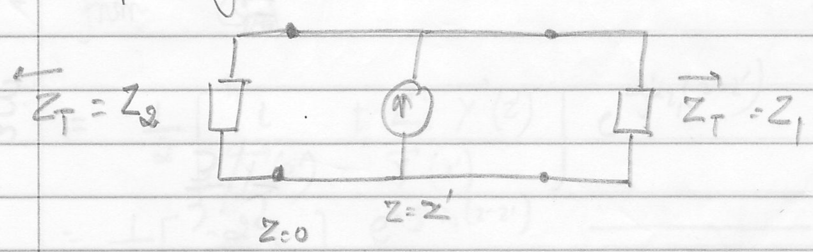
$+i^{TM}$

$\frac{dH^T}{dz} = -\hat{z}\times J_x\hat{x} = -\hat{z}\times\hat{x} = -\hat{y} J_x$

where $\boxed{i^{TM} = -\tilde{J}_x}$ — (8)



Representing the above TL with terminal impedances:



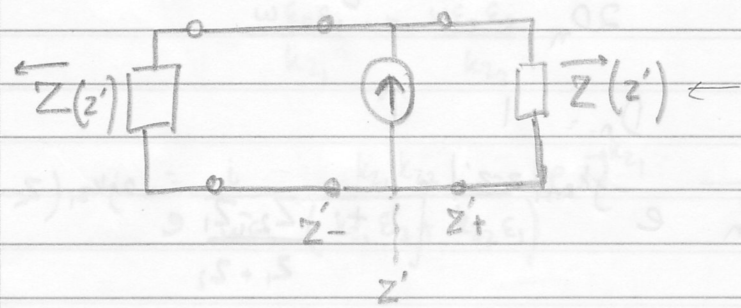
The voltage and current can be expressed as.

for $z > z'$

$$V(z,z') = \frac{1}{2}\left[V(z'+,z') + Z\,I(z'+,z') \right] e^{-jk_z(z-z')} \qquad (9a)$$

$$I(z,z') = \frac{Y}{2}\left[V(z'+,z') + Z\,I(z'+,z') \right] e^{-jk_z(z-z')} \qquad (9b)$$

The voltage and current $V(z'+,z')$ and $I(z'+,z')$ are found by ~~trans~~ modifying the TL.



Unbounded

$$V(z'+;z') = \frac{i}{\overleftrightarrow{Y}(z')} \quad ; \quad I(z'+,z') = \frac{i\,\overrightarrow{Y}(z')}{\overleftrightarrow{Y}(z')}$$

where $\overleftrightarrow{Y}(z') = \overrightarrow{Y}(z') + \overleftarrow{Y}(z') = \dfrac{1}{\overrightarrow{Z}(z')} + \dfrac{1}{\overleftarrow{Z}(z')}$

$$\overrightarrow{Z}(z') = Z_1 \qquad \overleftarrow{Z}(z') = Z_1 \frac{Z_2 + jZ_1 \tan k_{z_1} z'}{Z_1 + jZ_2 \tan k_{z_1} z'}$$

from (9a)

$$V(z,z') = \frac{1}{2}\left[\frac{i}{\overleftrightarrow{Y}(z')} + \frac{Z_1 i\,\overrightarrow{Y}(z')}{\overleftrightarrow{Y}(z')} \right] e^{-jk_{z_1}(z-z')}$$

$$= \frac{1}{2}\left[\frac{2i}{\overleftrightarrow{Y}(z')} \right] e^{-jk_{z_1}(z-z')} \qquad (10)$$

$$I(z,z') = \frac{Y_1\, i}{\overleftrightarrow{Y}(z')} e^{-jk_{z_1}(z-z')}$$

when the source lies on the interface

$$z = z' = 0$$

$$\overleftarrow{Z}(0) = Z_2$$

$z > 0$

$$V(z,0) = \frac{i}{\frac{1}{Z_1} + \frac{1}{Z_2}} e^{-jk_{z_1}(z)}$$

$$= \frac{i}{\frac{\omega\varepsilon_0\varepsilon_1}{k_{z_1}} + \frac{\omega\varepsilon_0\varepsilon_2}{k_{z_2}}} e^{-jk_{z_1}}$$

$$= \frac{i}{\omega\varepsilon_0} \frac{k_{z_1} k_{z_2}}{(k_{z_1}\varepsilon_2 + k_{z_2}\varepsilon_1)} e^{-jk_{z_1}} \qquad (11)$$

The current is expressed as:

$$I(z,0) = Y_1 V(z,0)$$

$$= \frac{i}{\omega\varepsilon_0} \frac{\omega\varepsilon_0\varepsilon_1}{k_{z_1}} \frac{k_{z_1} k_{z_2}}{(k_{z_1}\varepsilon_2 + k_{z_2}\varepsilon_1)} e^{-jk_{z_1}}$$

$$\boxed{I(z,0) = i \frac{k_{z_2}\varepsilon_1}{(k_{z_1}\varepsilon_2 + k_{z_2}\varepsilon_1)} e^{-jk_{z_1}}} \qquad (12)$$

$1 -$

$k_{z_1}\varepsilon_2 + k_{z_2}\varepsilon_1$

$$= \frac{i}{2}\left(1 + \frac{k_{z_2}\varepsilon_1 - k_{z_1}\varepsilon_2}{k_{z_2}\varepsilon_1 + k_{z_2}\varepsilon_1}\right) e^{-jk_{z_1}}$$

This form is equivalent to Michalski forms.