621.372.8: 538.566

Monograph No. 93
RADIO SECTION

THE EXCITATION OF PLANE SURFACE WAVES

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(*The paper was first received 17th August, and in revised form 24th November*, 1953. *It was published as an* INSTITUTION MONOGRAPH, *15th February*, 1954.)

SUMMARY

It is shown that a surface wave will be generated by a horizontal slot situated above a corrugated or dielectric-coated guiding surface.

A physical interpretation of the analysis is suggested which explains the mechanism of launching in terms of a resonance phenomenon allied to that which arises when a piston attenuator is terminated by a reactance equal and opposite to its own (reactive) characteristic impedance.

The problem of efficient launching is discussed, and it is shown that under suitable conditions a high launching efficiency is possible.

Some of the theoretical predictions have been verified experimentally.

LIST OF PRINCIPAL SYMBOLS

A = Amplitude factor.
d = Thickness of dielectric or depth of slot on guiding surface.
$\mathscr{E}_x, \mathscr{E}_y$ = Rectangular components of electric field strength.
H_z = Rectangular component of magnetic field strength.
H = Height of an aperture.
h = Height of source slot above guiding surface.
K_1 = Magnetic current.
$K(x)$ = Surface magnetic-current density.
k = Phase constant for free space.
p, q = Positive real numbers.
r = Radial distance.
V = Voltage across slot.
x, y, z = Cartesian co-ordinates.
$Z = R + jX$ = Surface impedance of guiding surface.
β = Phase constant.
$\delta(x)$ = Dirac "delta-function."
η = A small quantity.
θ = Angle of elevation.
κ_0 = Permittivity of free space.
λ = Wavelength.
μ_0 = Permeability of free space.
ρ = Reflection coefficient of guiding surface.
ϕ = Phase angle of reflection coefficient.

(1) INTRODUCTION

The literature of surface waves is already fairly extensive, and is still growing. A brief summary of some of the earlier work has been given recently by Barlow and Cullen.[2] There are now two main fields of work in which surface waves are of interest: the first is the study of radio-wave propagation over the earth; the second is associated with the possible application of surface-wave guides to low-loss microwave transmission systems. It is towards the latter aspect of the subject that the paper is directed.

The possibility of using surface waves in a microwave transmission system has received added impetus recently through the work of Goubau[1] on the axial cylindrical form of surface waves, and an experimental verification of this theory has been described recently by Barlow and Karbowiak.[3] It is often comparatively easy to find surface-wave solutions to Maxwell's equations which satisfy the boundary conditions at a given guiding surface, but it is a much more difficult matter to take the source properly into account in the theory. From a practical point of view, the radiation part of the total field produced by the source represents a loss of power. Only a source of infinite extent can give a pure surface-wave field, and we can define as a figure of merit of a finite source the percentage of the total power input which is propagated as a surface wave. The calculation of the "launching efficiency" defined in this way for an arbitrary source is an important, although difficult, aspect of the theory. Goubau has shown recently[8] that the excitation by a point dipole of an axial surface wave on a cylindrical guide can be calculated very neatly without solving the whole radiation problem, but a knowledge of the amplitude of the surface wave alone does not enable us to calculate the launching efficiency. Roberts[11] has made a more complete calculation for a special kind of source, namely a ring of magnetic current closely surrounding the wire.

A complete solution of the excitation of axial surface waves by an arbitrary source is very difficult, and for this reason it seemed worth while to attempt a solution of the corresponding 2-dimensional problem. Barlow and Cullen[2] have pointed out that the Zenneck-wave field structure is very closely related to that of the Goubau wave; both are E-type surface waves in which the magnetic field is wholly transverse and the electric field has both transverse and longitudinal components. If the surface is a corrugated or dielectric-coated metal plate the Zenneck type of wave is propagated with very little attenuation and closely resembles the Goubau wave on a dielectric-coated wire.

In the present paper the launching of the E-type plane surface wave over such a surface is considered. The source is at first taken to be a slot at a height h above the guiding surface, but this restriction is later removed, and an arbitrary aperture distribution is considered. Whitmer[5] has solved an analogous 2-dimensional problem of the excitation of H-type plane surface waves (i.e. surface waves with a longitudinal component of magnetic field and wholly transverse electric field), taking a dielectric slab of infinite extent as the guide and an electric-current filament embedded in the dielectric as the source. Under suitable conditions, high launching efficiencies are achieved. It is shown in the present paper that high launching efficiencies are also found in the case of E-type surface waves, even with a slot source. They occur when the slot is at a certain critical height above the guiding surface, and this height is calculated for two particular guiding surfaces. Other aspects of the problem, such as the purity of the surface-wave field near the guiding surface, and the height-gain factor for the radiation field, are considered, and some similarities may be expected in the corresponding results for the axial cylindrical surface wave.

It is perhaps worth while to mention that recent advances in microwave aerial design suggest that a guiding surface of the type used here may have applications as an end-fire aerial.[13]

(2) THE SURFACE-WAVE FIELD

The corrugated guiding surface we shall consider first for simplicity of calculation is shown in Fig. 1. The corrugations

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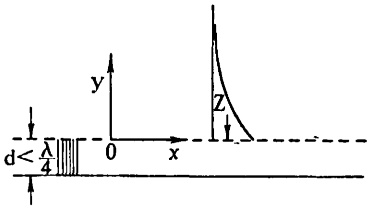

Fig. 1.—Corrugated guiding surface.

consist of infinitely thin conducting sheets mounted very close together on a conducting base-plate as shown. With the co-ordinate system shown, we then have

$$\left(\frac{\mathscr{E}_x}{H_z}\right)_{y=0} = Z = R + jX \quad . \quad . \quad . \quad . \quad (1)$$

with $$X \simeq \sqrt{\left(\frac{\mu_0}{\kappa_0}\right)} \tan kd$$

It is known[1] that a field of the form

$$\left.\begin{aligned} H_z &= A\epsilon^{-k_1 y}\epsilon^{-j\beta_1 x} \\ \mathscr{E}_x &= ZA\epsilon^{-k_1 y}\epsilon^{-j\beta_1 x} \\ \mathscr{E}_y &= \frac{\beta_1}{k}\sqrt{\left(\frac{\mu_0}{\kappa_0}\right)}A\epsilon^{-k_1 y}\epsilon^{-j\beta_1 x} \end{aligned}\right\} \quad . \quad . \quad . \quad (2)$$

where $k = \omega\sqrt{(\mu_0\kappa_0)} = 2\pi/\lambda_0$, satisfies Maxwell's equations in the space $y > 0$ and also satisfies the boundary condition [eqn. (1)] provided that

$$\left.\begin{aligned} \beta_1^2 &= k^2 + k_1^2 \\ k_1 &= -jk\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}Z \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If Z is purely reactive, so that $R = 0$, k_1 and β_1 are both real, so that the wave represented by eqn. (2) progresses without attenuation in the direction of increasing x, and decreases exponentially with increasing distance from the guiding surface. In the presence of resistive losses, k_1 and β_1 are complex, and the wave is attenuated as it progresses over the surface. We are primarily interested in the possibility of launching such a wave from a source of small extent, e.g. a slot in a metal sheet, when the losses in the reactive guiding surface are small. In such cases we shall see that, within a certain range of distances from the source, the surface-wave field is the most important contribution to the total field near the guiding surface.

(3) RESONANT EXCITATION OF A SURFACE WAVE BY A MAGNETIC CURRENT SHEET

The geometry of the idealized problem we wish to solve is illustrated in Fig. 6. A perfectly conducting vertical screen having cut in it a narrow horizontal slot of infinite length is set at one end of a semi-infinite corrugated surface of the type discussed in the previous Section. The slot is at a height h above the surface, and an alternating voltage V is applied across the slot. The problem is to calculate the resulting electromagnetic field, and in particular, to study the field structure near the corrugated sheet at considerable distances from the source, and to calculate the launching efficiency of the surface wave.

It is convenient to imagine the slot replaced by the equivalent magnetic current filament. This device simplifies the subsequent argument, and was first used in this way by Booker[12] in discussing the relationship between slots and dipoles by means of Babinet's principle. The field produced by a filamentary magnetic current K situated at a height h above an infinite corrugated surface will first be found. It follows from considerations of symmetry that the field configuration to the right of the magnetic current filament is identical with that which would be produced by the slot, and it will have the same amplitude if $K = 2V$.

The problem of calculating the field produced by an infinitely long magnetic-current filament can be solved by regarding the filamentary magnetic current as a distribution of surface magnetic current in the plane $y = h$, which can be expressed by means of a Fourier integral as the sum of an infinitely large number of unattenuated travelling waves of surface magnetic current. This method of analysis has been used by Tai and by Whitmer in similar problems.[4,5] The field produced by the filament can be obtained by superposition of the fields produced by the elementary travelling waves into which the filament can be resolved.

It is the purpose of this Section to calculate the field produced by a single travelling wave of magnetic current.

Referring to Fig. 2, suppose the magnetic current-density in the plane $y = h$ is given by

$$K(x) = K_0\epsilon^{-j\beta x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where β is real.

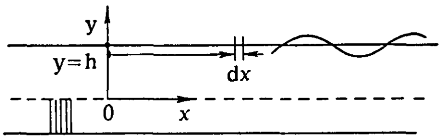

Fig. 2.—Illustrating the analysis.

There will be a discontinuity of tangential electric field equal to $K(x)$ on passing through the sheet, but the magnetic field remains continuous.

The magnetic field above the sheet must either decay exponentially or represent an outward travelling wave, so that we have

$$H_z = A_1\epsilon^{-uy}\epsilon^{-j\beta x}; \; y > h \quad . \quad . \quad . \quad . \quad (5)$$

where $$u = \sqrt{(\beta^2 - k^2)} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The notation $\sqrt{}$ will be used to denote the principal root so that u is positive if real or positive imaginary otherwise.

Between the magnetic current sheet and the guiding surface the magnetic field can be expressed as

$$H_z = (A_2\epsilon^{uy} + A_3\epsilon^{-uy})\epsilon^{-j\beta x} \quad . \quad . \quad . \quad . \quad (7)$$

Using the equation $j\omega\kappa_0\mathscr{E}_x = \partial H_z/\partial y$ together with the boundary condition [eqn. (1)] we find the following relationship between A_2 and A_3:

$$A_3 = A_2\left(\frac{u + k_1}{u - k_1}\right) \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where $$k_1 = k\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}(X - jR) \quad . \quad . \quad . \quad . \quad (9)$$

This notation is chosen to be convenient for almost purely inductively reactive guiding surfaces, for if $R = 0$ and X is positive, k_1 is a positive real number.

Continuity of the magnetic field at $y = h$ gives a relationship between A_1 and A_2, using eqns. (5), (7) and (8), thus:

$$A_1\epsilon^{-uh} = A_2\left(\epsilon^{uh} + \frac{u + k_1}{u - k_1}\epsilon^{-uh}\right) \quad . \quad . \quad . \quad (10)$$

The discontinuity of $\mathscr{E}_x$ at $y = h$ gives another relationship between A_1 and A_2, for we must have

$$\lim_{\eta \to 0}[\mathscr{E}_x(h + \eta) - \mathscr{E}_x(h - \eta)] = K_0\epsilon^{-j\beta x}$$

namely:

$$-A_1\epsilon^{-uh} = A_2\left(\epsilon^{uh} - \frac{u+k_1}{u-k_1}\epsilon^{-uh}\right) + j\frac{k}{u}\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}K_0 \quad . \quad (11)$$

The constants A_1 and A_2 can be found from eqns. (10) and (11), and we get

$$A_1 = -j\frac{k}{2}\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}K_0\frac{1}{u}\left(\epsilon^{uh} + \frac{u+k_1}{u-k_1}\epsilon^{-uh}\right) \quad . \quad (12)$$

$$A_2 = -j\frac{k}{2}\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}I_0\frac{\epsilon^{-uh}}{u} \quad . \quad . \quad . \quad . \quad (13)$$

The remaining constant A_3 is given simply in terms of A_2 by eqn. (8).

Finally, the magnetic field above or below the magnetic current sheet is given by

$$H_z = \begin{cases} -j\frac{k}{2}\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}K_0\frac{1}{u}\left(\epsilon^{uh} + \frac{u+k_1}{u-k_1}\epsilon^{-uh}\right)\epsilon^{-uy}\epsilon^{-j\beta x};\ y \geqslant h \\ -j\frac{k}{2}\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}K_0\frac{\epsilon^{-uh}}{u}\left(\epsilon^{uy} + \frac{u+k_1}{u-k_1}\epsilon^{-uy}\right)\epsilon^{-j\beta x};\ 0 \leqslant y < h \end{cases}$$

with $$u = \sqrt{(\beta^2 - k^2)} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

We now consider how the magnetic field depends on β, the phase coefficient of the travelling wave of magnetic current which generates the field. The most important factor influencing the field is the term $(u + k_1)/(u - k_1)$. The magnitude and phase angle of this factor are plotted, for several values of k_1/k, in Figs. 3A and 3B.

The following features of the variation of $(u + k_1)/(u - k_1)$, which we shall denote by ρ for brevity, deserve comment. We use p and q to denote any positive real numbers.

(*a*) $|\rho|$ becomes infinite for a certain value of β ($>k$) if $k_1 = p$.

(*b*) $|\rho|$ is zero for a certain value of β ($<k$) if $k_1 = -jq$.

(*c*) $|\rho|$ has a minimum in the range $0 < \beta < k$ and a maximum at a value of $\beta > k$ if k_1 is complex and of the form $p - jq$.

The form of eqn. (14) and the physical situation both suggest that $\rho = (u + k_1)/(u - k_1)$ can be interpreted as a reflection coefficient which determines the magnitude of the reflected wave arising from the incident wave generated by the magnetic current sheet. With this interpretation in mind, (*b*) can be recognized as the Brewster angle phenomenon, for when $\beta < k$ the incident field consists of a homogeneous plane wave with an angle of incidence equal to arc sin (β/k).

However, the phenomenon (*a*) is at first sight remarkable if it is interpreted as a reflection coefficient, for familiarity with loss-free transmission line theory suggests that $|\rho|$ cannot exceed unity. It has been shown[6] that in a loss-free waveguide operated below cut-off, as in piston attenuators, an infinite reflection coefficient can be obtained. This is possible because the characteristic wave impedance is purely reactive, and so an equal and opposite reactance as a termination will resonate with the characteristic impedance in such a way as to produce an infinite reflected wave for a finite incident wave. There is no violation of the principle of conservation of energy, since the system is wholly reactive; the effect is closely analogous to the production of an infinite voltage by a finite e.m.f. in a series-resonant circuit of infinite Q-factor. Fig. 4 illustrates the phenomenon schematically for the case under consideration. When $\beta < k$ the phase velocity of the travelling wave of magnetic current is less than c, and the field is evanescent above and below the sheet as in Fig. 4(*a*). The wave impedance looking downward above the sheet is $+jX$. The field structure above the current sheet is identical with that of the surface wave over a loss-free corrugated

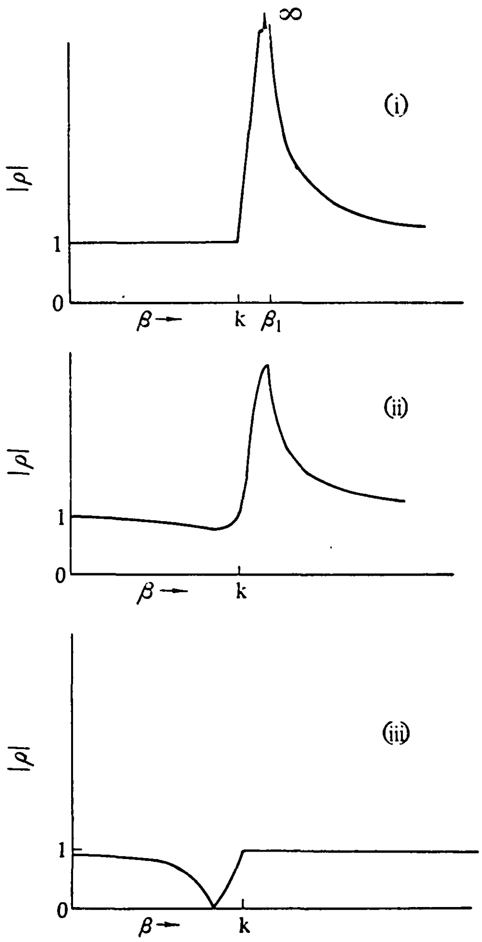

Fig. 3A.—Reflection coefficient as a function of β.

(i) Purely reactive.

$(k_1 = p_1)$

$\rho = \frac{u + k_1}{u - k_1}$

(ii) Mainly reactive.

(R small)

$k_1 = p_1 - jq_1$

$p_1 > q_1$

(iii) Purely resistive.

$k_1 = -jq_1$

surface of reactance $+jX$ as shown in Fig. 4(*b*). This surface wave can be regarded[1] as an inhomogeneous plane wave "incident" at the (purely imaginary) Brewster angle, and so travelling on (parallel to the sheet) without reflection.

Below the magnetic current sheet of Fig. 4(*a*) the wave impedance looking downwards is $-jX$, and is exactly analogous to the characteristic wave impedance of the piston attenuator referred to earlier. When the corrugated guiding surface is introduced into this field, it forms a resonant termination as illustrated in Fig. 4(*c*), which shows a reflection coefficient considerably greater than unity but still finite. An exact analogy is provided by the transmission-line model of Fig. 4(*d*). If the exponential factor $\epsilon^{-j\beta x}$ is suppressed in eqn. (14) there is a one-to-one correspondence of the terms in this equation and those in the equation for the current in the transmission line, the e.m.f. e taking the place of the magnetic current sheet.

In Section 4 we shall use the results of the present Section to derive an integral representation of the magnetic field produced by an infinitely long magnetic-current filament parallel to, and distance h from, a corrugated guiding surface.

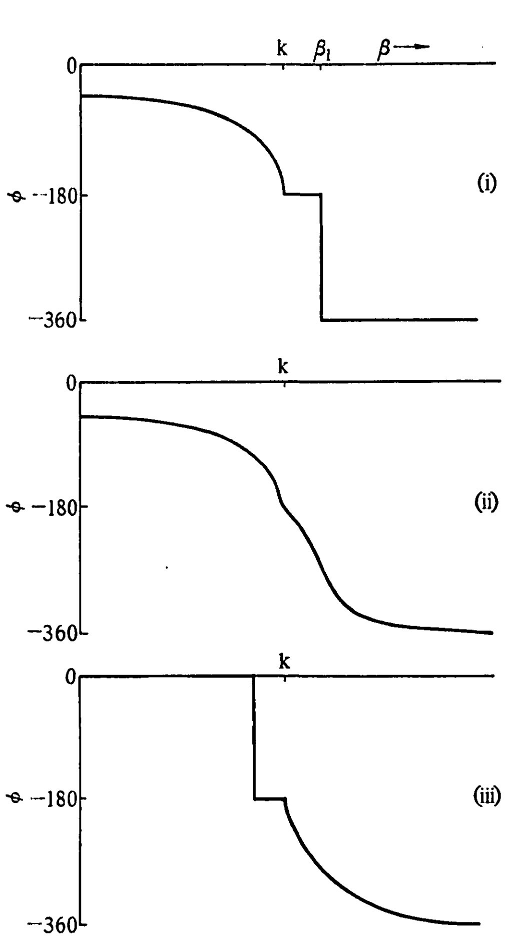

Fig. 3B.—Phase angle of reflection coefficient.

(i) $\phi = \arg\left(\frac{u+k_1}{u-k_1}\right)$

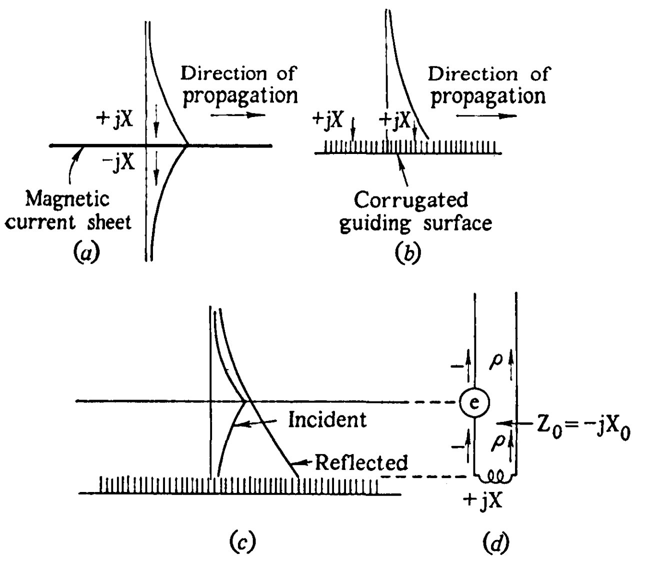

Fig. 4.—Resonant excitation of a surface wave.

(4) INTEGRAL REPRESENTATION OF THE FIELD PRODUCED BY A MAGNETIC CURRENT FILAMENT OR A RADIATING SLOT ABOVE A CORRUGATED SURFACE

Consider a magnetic current filament situated above a corrugated guiding surface. Referring to Fig. 2, suppose the filament to lie parallel to the z-axis, passing through the point $(0, h)$. This current filament may be regarded as a distribution of surface current in the plane $y = h$ given by

$$K(x) = K_1\delta(x) \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where K_1 is the total magnetic current carried by the filament, and $\delta(x)$ is the Dirac delta-function—which is zero if x is different from zero and which has the property $\int_{-\infty}^{+\infty}\delta(x)dx = 1$.

We now use the complex form of the Fourier integral to represent eqn. (15) as an infinite summation of infinitesimally weak travelling waves of magnetic current having a continuous distribution of phase constants, as follows:

$$K(x) = \int_{-\infty}^{+\infty} K_0(\beta)\epsilon^{-j\beta x}d\beta \quad . \quad . \quad . \quad . \quad (16)$$

The Fourier transform theorem gives at once

$$K_0(\beta) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} K_1\delta(x)\epsilon^{+j\beta x}dx$$

or

$$K_0(\beta) = \frac{K_1}{2\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Thus the amplitudes of all the elementary travelling waves are the same, and over an infinitesimal range of phase constants $d\beta$ the amplitude of the elementary travelling wave is given by

$$\frac{K_1}{2\pi}d\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

We have already studied the field produced by such a travelling wave of magnetic current, however, and the required result is obtained if we replace K_0 in eqn. (14) by the expression (18). To find the field produced by the magnetic current filament, we must superimpose the fields due to the elementary travelling waves into which it has been resolved. This is done by integrating with respect to β from minus infinity to plus infinity.

Before writing down the result, let us make the problem approach a little closer to physical realizability by replacing the magnetic current filament by a slot in a perfectly conducting sheet, as in Fig. 6. This is possible because of the symmetry of the field produced by the magnetic current filament, which has $\mathscr{E}_y = 0$ in the plane $x = 0$. If the voltage across the slot is V, the result obtained for a magnetic current filament is applicable to the slot problem if we replace K_1 by $2V$. The result is

$$H_z = \begin{cases} -j\frac{k}{2\pi}\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}V\int_{-\infty}^{+\infty}\frac{\epsilon^{-uy}}{u}\left(\epsilon^{uh} + \frac{u+k_1}{u-k_1}\epsilon^{-uh}\right)\epsilon^{-j\beta x}d\beta; \ y \geqslant h \\ -j\frac{k}{2\pi}\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}V\int_{-\infty}^{+\infty}\frac{\epsilon^{-uh}}{u}\left(\epsilon^{uy} + \frac{u+k_1}{u-k_1}\epsilon^{-uy}\right)\epsilon^{-j\beta x}d\beta; \ 0 \leqslant y < h \end{cases}$$

with

$$u = \sqrt{(\beta^2 - k^2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

This is the required representation of the field H_z. If we bear in mind the dependence of u on β, it is obvious that the evaluation of eqn. (19) will present some difficulty. However, if we are interested in the field near the surface at a large distance from the source, a fairly simple asymptotic solution can be obtained.

This is discussed in Section 5.

(5) APPROXIMATE FORMULA FOR THE FIELD

The integral eqn. (19) is equivalent to that which arises in the case of propagation of radio waves over the earth, and can be expressed in terms of the Fresnel integral, as is now well known. For our purposes it is desirable to obtain an asymptotic approximation to the field. This can be done by direct evaluation of eqn. (19) if we make use of the methods of contour integration. We regard eqn. (19) as a contour integral along the real axis in the complex β-plane. If the contour is completed as shown in Fig. 5 (this is essentially the contour used by Sommerfeld in

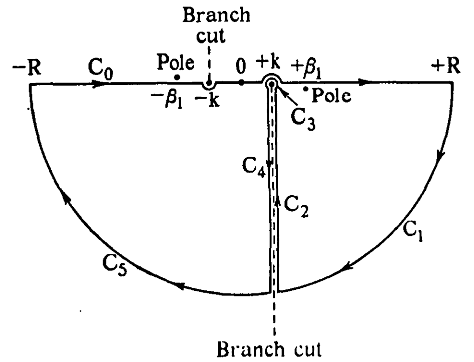

Fig. 5.—Contour of integration in the β-plane.

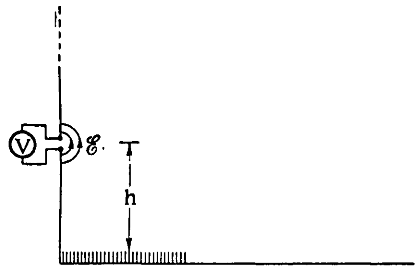

Fig. 6.—Slot excitation arrangement.

discussing propagation of radio waves over the earth) the integral remains single-valued at all points on the contour, for the branch points at $\pm k$ are excluded. We can therefore apply Cauchy's theorem of residues to obtain a relationship between the required integral and those on the paths, C_1, C_2, C_3, C_4 and C_5. Since the real part of u is positive, it follows that the integrals along C_1 and C_5 vanish as the radius R tends to infinity, if $x > 0$. Similarly the contribution from the small circle C_3 tends to zero as the radius of the circle tends to zero. The contribution from C_2 and C_4 (the "branch cut") can be obtained as a series in inverse powers of x (which is asymptotic in character), by expanding the integrand in a Taylor series and integrating term by term, and the residue at the pole at β_1 is easily evaluated.

The result is

$$H_z = -2kV\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}\left[\frac{k_1}{\beta_1}\epsilon^{-k_1h}\epsilon^{-k_1y}\epsilon^{-j\beta_1x} + \frac{\epsilon^{-j\left(kx+\frac{\pi}{4}\right)}}{\sqrt{(2\pi)}}\left\{\frac{F_1(y,h)}{(kx)^{3/2}} - j3\left[\frac{1}{8}F_1(y,h) + F_2(y,h)\right]\frac{1}{(kx)^{5/2}} + \ldots\right\}\right] \quad . \quad . \quad . \quad (20)$$

In this formula

$$\beta_1 = \sqrt{(k^2 + k_1^2)} \quad . \quad . \quad . \quad . \quad . \quad (21)$$

$$F_1(y, h) = \left(\frac{k}{k_1}\right)^2(1 - k_1y)(1 - k_1h) \quad . \quad . \quad (22)$$

and

$$F_2(y, h) = \left(\frac{k}{k_1}\right)^4[1 - k_1(y + h) + \tfrac{1}{2}k_1^2(y + h)^2 - \tfrac{1}{6}k_1^3(y + h)^3 + \tfrac{1}{6}k_1^4yh(y^2 + h^2)] \quad . \quad (23)$$

The conditions for validity of the asymptotic series in eqn. (20) are

$$\left.\begin{aligned} x &\gg y + h \\ kx &\gg \left|\frac{k}{k_1}\right|^2 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

It will be noticed that eqn. (20) is symmetrical in y and h (this is, in fact, a necessary consequence of the reciprocity theorem), and since the condition for validity is symmetrical in y and h it follows that eqn. (20) is applicable for $y > h$ or $y < h$.

Let us now examine the physical significance of eqn. (20).

In the first place, comparison with eqn. (2) shows that the first term of eqn. (20), which arises from the residue at the pole, represents a surface wave of decay constant k_1.

The presence of the surface wave is not altogether unexpected in view of the results of Section 3. Consider first a loss-free surface, for which k_1 is real. A travelling wave of magnetic current having the "resonant" phase velocity ω/β_1 must be present in the spectrum of travelling waves into which the actual line magnetic current has been resolved, and although its amplitude is infinitesimal, the infinite reflection coefficient associated with the resonant phase velocity can give rise to a "reflected" wave of finite amplitude which constitutes the surface wave. In the presence of losses the resonance is damped and the reflection coefficient is always finite, though it is very large for a small range of phase velocities near the resonant velocity. In this case the surface wave arises from a narrow spectrum of inhomogeneous reflected waves (each associated with an elementary exciting wave of magnetic current) having a small range of phase velocities. These waves are in phase at $x = 0$ but gradually get out of step as they progress. Their resultant therefore decreases in amplitude as x increases, and this effect is exactly equivalent to the exponential attenuation which is to be expected on physical grounds.

In general, the initial amplitude of the surface wave is proportional to $|k_1|\epsilon^{-\Re k_1h}$ as eqn. (20) shows. We see that the surface wave excitation is a function both of h and of k_1, the vertical decay-constant characteristic of the surface. Regarded as a function of h (i.e. keeping k_1 constant), the excitation is greatest when $h = 0$, but for a fixed finite value of h differentiation shows that if k_1 is real the surface wave amplitude is a maximum when

$$k_1 = 1/h \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

The asymptotic series in eqn. (20) represents the radiation field, and this can be shown as follows. Let us first consider distances so large that only the first term need be considered. To this degree of approximation the radiation field is given by

$$H_z^R = 2kV\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}\frac{1}{\sqrt{(2\pi)}}\left(\frac{k}{k_1}\right)^2(1 - k_1y)(1 - k_1h)\frac{\epsilon^{-j\left(kx+\frac{\pi}{4}\right)}}{(kx)^{3/2}} \quad . \quad . \quad . \quad . \quad (26)$$

Consider now the field produced by a horizontal slot over a surface having a reflection coefficient for the magnetic field of minus one. The field from a single slot in free space can be expressed in terms of a Hankel function, for which in turn we may use the asymptotic expression. If kx is large in comparison with unity, and x is large in comparison with $y + h$, the field from the slot and its image in the reflecting plane is, to a good approximation,

$$H'_z = 2kV\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}\frac{1}{\sqrt{(2\pi)}}k^2yh\frac{\epsilon^{-j\left(kx+\frac{\pi}{4}\right)}}{(kx)^{3/2}} \quad . \quad (27)$$

Eqns. (26) and (27) are equivalent if $k_1y \gg 1$ and $k_1h \gg 1$. [The conditions of eqn. (24) must also be satisfied simultaneously, of course.]

Thus for $y \gg 1/k_1$ and $h \gg 1/k_1$ eqn. (26) corresponds to the usual radiation field at very small angles of elevation (i.e. small in comparison with the Brewster angle), when the reflection coefficient for vertical polarization is minus one whatever may be the nature of the surface. For smaller values of y and h the dependence on y and h is rather more complicated.

A striking and unexpected feature is that the supplementary radiation field vanishes, at least to the order $1/(kx)^{3/2}$ if $k_1h = 1$ or $k_1y = 1$. (This is possible only for purely reactive surfaces.) It is a remarkable result that a slot at the height $1/k_1$ will generate almost pure surface wave if terms in $1/(kx)^{5/2}$, etc., can be neglected.

The reason for this peculiarity can be seen, at least so far as the critical value of y is concerned, by a very simple analysis of the kind given by Millington (Reference 10: calculations of the "height-gain factor" in connection with the propagation of radio waves over the earth).

Consider a homogeneous plane-wave incident on the corrugated surface at an angle θ. The relevant field components are

$$H_z = \epsilon^{-jk(x\cos\theta - y\sin\theta)} + \rho\epsilon^{-jk(x\cos\theta + y\sin\theta)}$$

$$\mathscr{E}_x = \sqrt{\left(\frac{\mu_0}{\kappa_0}\right)}\sin\theta\left[\epsilon^{-jk(x\cos\theta - y\sin\theta)} - \rho\epsilon^{-jk(x\cos\theta + y\sin\theta)}\right] \quad . \quad . \quad . \quad . \quad (28)$$

From the condition that $\frac{\mathscr{E}_x}{H_z} = Z$ at $y = 0$, ρ can be evaluated, and we have

$$\rho = \frac{1 - jX\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}\operatorname{cosec}\theta}{1 + jX\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}\operatorname{cosec}\theta}$$

or $$\rho = \exp\left\{-2j\arctan\left[X\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}\operatorname{cosec}\theta\right]\right\} \quad . \quad (29)$$

We now look for the condition which makes $H_z = 0$. Substituting eqn. (29) in eqn. (28) we find

$$2ky\sin\theta + 2\arctan\left[X\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}\operatorname{cosec}\theta\right] = \pi \quad . \quad (30)$$

Now consider the radiation field near to the corrugated surface at a great distance from the slot source. This field can be resolved into plane waves, of which those having small angles of elevation, θ, are the most important. With the simplifying restriction that θ is small, eqn. (30) reduces to the approximate form

$$ky\sin\theta - \frac{1}{X}\sqrt{\left(\frac{\mu_0}{\kappa_0}\right)}\sin\theta = 0$$

or $$y = \frac{1}{kX}\sqrt{\left(\frac{\mu_0}{\kappa_0}\right)} = \frac{1}{k_1} \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Thus the vanishing of magnetic field at a height $1/k_1$ above the surface arises from the interference of the direct and reflected homogeneous plane waves into which the radiation field can be resolved. The important fact emerges that the result is independent of θ if θ is small, so that the magnetic field from a spectrum of plane waves vanishes at a constant height $1/k_1$, at great distances from the source. (This applies only to the radiation part of the total field, of course; the surface-wave field, consisting of an inhomogeneous plane wave, cannot be resolved into homogeneous plane waves in this way.) A simple application of the reciprocity theorem gives again the more important result, already noted, that for a launching slot placed at the height $h = 1/k_1$, the first term in the asymptotic expansion of the radiation field vanishes, and the radiation field near the surface becomes much weaker.

Putting $h = 1/k_1$ in eqn. (23), and substituting in eqn. (20) we find that the radiation part of the field is given by

$$H_z^R = -2jkV\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}\frac{1}{\sqrt{(2\pi)}}\left(\frac{k}{k_1}\right)^3\frac{ky}{(kx)^{5/2}}\epsilon^{-j\left(kx+\frac{\pi}{4}\right)} \quad . \quad (32)$$

This residual magnetic field vanishes at $y = 0$ so that the dominant term in the asymptotic series if $h = 1/k_1$ and $y = 0$ falls off in amplitude with increasing values of x at least as fast as $x^{-7/2}$. This fact might prove useful in an experimental determination of the phase velocity of a surface wave.

(6) RADIATION FIELD AND LAUNCHING EFFICIENCY

In this Section we shall consider the radiation pattern of a slot over a purely reactive corrugated guiding surface. From this we can determine the radiated power, and since the power carried by the surface wave can be found easily from the results of Section 5, the ratio of power in the surface wave to total power delivered to the slot can be found. This ratio will be called the "launching efficiency." Since the problem is a 2-dimensional one, we shall have infinite total power if the waves are of infinite extent in the z-direction, so we shall consider the power radiated per unit width of wavefront in the z-direction.

The calculation of the radiation pattern from eqn. (19) can be done by making use of the method of stationary phase or by steepest descents. It is convenient to change to polar co-ordinates (r, θ), putting $x = r\cos\theta$ and $y = r\sin\theta$. With this alteration of notation the result is

$$H_z^R = kV\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}\frac{1}{\sqrt{(2\pi)}}\frac{\epsilon^{-j\left(kr-\frac{\pi}{4}\right)}}{(kr)^{1/2}}\left[\epsilon^{jkh\sin\theta} + \left(\frac{jk\sin\theta + k_1}{jk\sin\theta - k_1}\right)\epsilon^{-jkh\sin\theta}\right] \quad . \quad (33)$$

If we restrict our attention to the case in which k_1 is real, i.e. purely reactive guiding surfaces, the magnitude of the radiated magnetic field is given by

$$|H_z^R| = kV\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}\sqrt{\left(\frac{2}{\pi}\right)}\frac{1}{(kr)^{1/2}}F(\theta)$$

$$F(\theta) = \frac{|\sin\theta\cos(kh\sin\theta) - \zeta\sin(kh\sin\theta)|}{\sqrt{(\sin^2\theta + \zeta^2)}} \quad . \quad (34)$$

where $\zeta = \dfrac{k_1}{k}$.

The radiation pattern, $F(\theta)$, has been plotted against θ in Fig. 7 for the case $k_1 = 0\cdot5k$ and for several values of h. The curves show that for $k_1h = 1$ or $h = 1/k_1$ the radiation field is very weak for small angles of elevation. This is in agreement with the results of the previous Section. However, optimum efficiency of launching the surface wave demands the smallest possible average magnetic field over the whole range of angles from 0 to 90°, so that the total radiated power is minimized. Values of

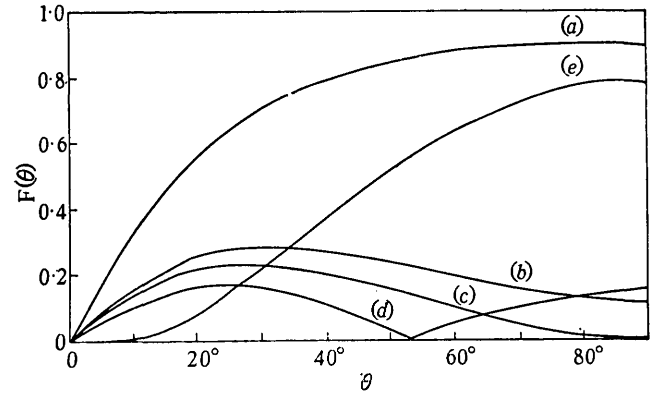

Fig. 7.—Radiation patterns for several slot heights.

(a) $k_1h = 0$
(b) $k_1h = 0\cdot5$
(c) $k_1h = 0\cdot55$
(d) $k_1h = 0\cdot63$
(e) $k_1h = 1\cdot0$
Surface $k_1/k = 0\cdot5$

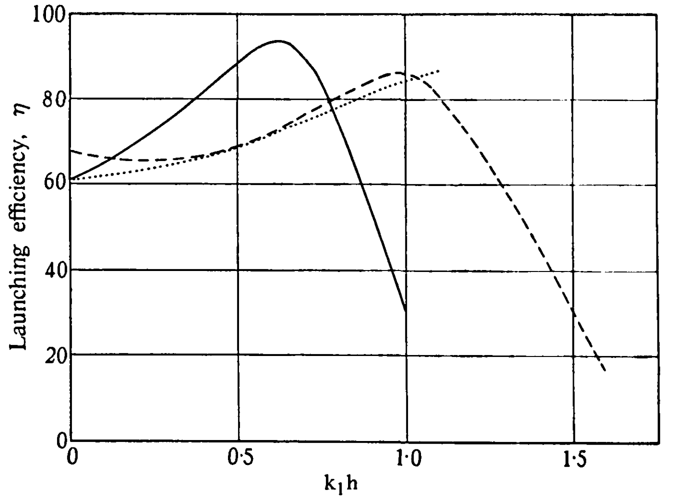

Fig. 9.—Launching efficiency curves.

—— Corrugated } Slot source
- - - - Coated }
...... Corrugated: chopped surface-wave source.

k_1h near $0\cdot5$ are seen to be favourable in this respect, and in fact we shall see that the minimum radiated power occurs when k_1h is about $0\cdot63$. The radiated power has been calculated by numerical integration, using Simpson's rule, from the formula

$$P_R = kV^2\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}\frac{2}{\pi}\int_0^{\pi/2}F^2(\theta)d\theta \quad . \quad . \quad . \quad . \quad (35)$$

The power carried by the surface wave (again per unit width of wavefront) is readily found from eqn. (20) to be

$$P_S = kV^2\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}\frac{2k_1\epsilon^{-2k_1h}}{\sqrt{(k^2+k_1^2)}} = kV^2\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}\frac{2\zeta}{\sqrt{(1+\zeta^2)}}\epsilon^{-2\zeta kh} \quad . \quad . \quad . \quad . \quad (36)$$

assuming, as before, that k_1 is real.

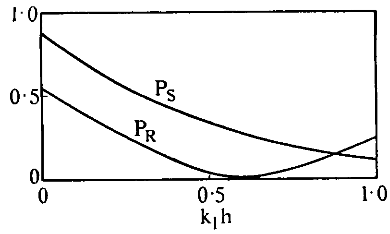

Fig. 8.—Variation of radiated and trapped power with height of slot.

In Fig. 8, P_R and P_S are plotted as a function of k_1h, and it will be noticed that the radiated power has a marked minimum for $k_1h = 0\cdot63$. In Fig. 9 the launching efficiency is plotted as a function of k_1h. The remarkably high launching efficiency of $95\cdot2\%$ is reached when $k_1h = 0\cdot63$. The physical reason for the small radiated power is clarified by Fig. 10, which shows that the phase difference between direct and reflected rays due to difference in path length and the phase change on reflection can combine, for a suitable choice of h, to give a resultant phase which varies very little with θ and which is almost equal to π. Under these circumstances, the direct and reflected waves almost cancel for all values of θ, and the radiated power is very small.

(7) EXTENSION TO GENERAL APERTURE DISTRIBUTIONS

The results for the radiation field and surface-wave field produced by a slot can easily be extended to give the field

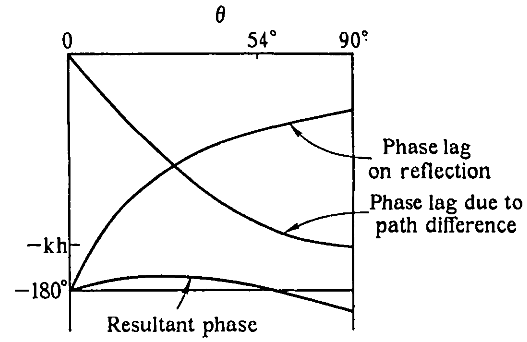

Fig. 10.—Optimum phase relations for minimum radiated power.

produced by an arbitrary aperture distribution. Let us assume that the tangential component of electric field is specified in the plane $x = 0$ as a function of h, so that

$$\mathscr{E}_y = \mathscr{E}(h) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

Consider the field produced by that part of the aperture lying between h and $h + dh$. We can think of this portion of the aperture as a slot across which the voltage $\mathscr{E}(h)dh$ exists, and the preceding formulae are available for calculating the contribution of each such element of the aperture to the total radiation and surface-wave fields. Integration with respect to h over whole-aperture plane then gives the required solution for any specified aperture distribution.

We find for the radiated field

$$H_z^R = kV\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}\frac{1}{\sqrt{(2\pi)}}\frac{\epsilon^{-j\left(kr-\frac{\pi}{4}\right)}}{(kr)^{1/2}}\left[\int_0^\infty \mathscr{E}(h)\epsilon^{jkh\sin\theta}dh \right.$$
$$\left. \left(\frac{jk\sin\theta + k_1}{jk\sin\theta - k_1}\right)\int_0^\infty \mathscr{E}(h)\epsilon^{-jkh\sin\theta}dh\right] \quad . \quad (38)$$

and for the surface-wave field

$$H_z^S = 2kV\sqrt{\left(\frac{\kappa_0}{\mu_0}\right)}\frac{k_1}{\beta_1}\epsilon^{-k_1y}\epsilon^{-j\beta_1x}\int_0^\infty \mathscr{E}(h)\epsilon^{-k_1h}dh \quad . \quad (39)$$

(8) CHOPPED SURFACE-WAVE APERTURE DISTRIBUTION

We shall illustrate the use of the preceding formulae by calculating the radiation and surface-wave fields produced by the aperture distribution specified by

$$\mathscr{E}(h) = \mathscr{E}_0 \epsilon^{-k_1 h}; \; 0 \leqslant h \leqslant H$$

$$\mathscr{E}(h) = 0; \; h > H \quad . \quad . \quad . \quad . \quad . \quad (40)$$

Substituting eqn. (40) in eqn. (38) we find, after some simplification, that

$$H_z^R = \mathscr{E}_0 \sqrt{\left(\frac{\kappa_0}{\mu_0}\right)} \sqrt{\left(\frac{2}{\pi}\right)} \frac{\epsilon^{-k_1 H}}{(kr)^{1/2}} \epsilon^{-j\left(kr - \frac{\pi}{4}\right)} \left[\frac{jkH \sin (kH \sin \theta)}{k_1 H - jkH \sin \theta}\right] \quad . \quad . \quad . \quad . \quad (41)$$

$$H_z^S = \mathscr{E}_0 \sqrt{\left(\frac{\kappa_0}{\mu_0}\right)} \frac{k}{\beta_1} \epsilon^{-k_1 y} \epsilon^{-j\beta_1 x} [1 - \epsilon^{-2k_1 H}] \quad . \quad (42)$$

Note that as H tends to infinity, H_z^R tends to zero provided that $\mathscr{R}k_1 > 0$ and a pure surface-wave field is excited. It is obvious that this must be the case physically, for the aperture distribution when H is infinite is simply the pure surface-wave field. The launching efficiency of the aperture distribution [eqn. (40)] has been calculated by the method explained earlier, and the result is shown in Fig. 9. It is interesting to see that until $k_1 h$ exceeds about $0 \cdot 8$ a slot provides a more efficient means of launching a surface wave.

(9) COMPARISON WITH GOUBAU'S ANALYSIS AND WITH BOOKER AND CLEMMOW'S ANALYSIS

Goubau[8] has given a general method for calculating the excitation of surface waves on reactive structures, such as the corrugated surface we have been considering, by a point dipole. A trivial extension of his analysis to cover the case of a long slot leads to precisely the same surface-wave excitation as we have found in eqn. (20). However, Goubau's analysis does not give the radiation field (indeed, Goubau emphasizes the interesting fact that the surface-wave field can be found by his method without the need for a complete solution of the radiation problem), and for a calculation of launching efficiency the radiation field must be found.

It is easy to verify that the radiation field and the surface-wave field in eqn. (20) are orthogonal, i.e. $\int_0^\infty \epsilon^{-k_1 y}(1 - k_1 y) dy = 0$, etc. This again is in accordance with Goubau's more general analysis; in fact it is this orthogonality condition which forms the basis of his treatment of the excitation problem.

Booker and Clemmow[7] have given an elegant interpretation of the Sommerfeld theory of radio-wave propagation over a flat earth, and it is interesting to examine the present problem from their viewpoint.

With this interpretation, the field due to a slot at $(0, h)$ above a surface of specified normal wave impedance is found by adding to the field, H_1, produced by the same source over a perfectly conducting surface a "correction field," H_2, resulting from diffracting the characteristic Zenneck wave under a screen extending upwards to infinity from $(0, -h)$, the image line of the source. This correction field involves an integration over the "aperture" extending from minus infinity to minus h, and is complicated by the fact that the integrand contains a factor which has the form of the Zenneck wave, and so tends exponentially to infinity at the lower limit of integration. This difficulty is avoided by regarding the correction field as the Zenneck wave seen "direct," H_2', minus the field H_2'' obtained by diffracting the Zenneck wave over a screen extending downwards from minus h to minus infinity. This integral is obviously convergent and can be expressed in terms of Fresnel integrals. The field H_2'' then takes the form of a radiation field, for which the leading term varies as $1/\sqrt{r}$. Near the surface the leading term of H_2'' exactly cancels the leading term of H_1, and the remaining terms give a field varying as $1/(kx)^{3/2}$, together with the Zenneck surface-wave itself, in agreement with the results of the present analysis. In calculating the field near the surface, however, there is a slight disadvantage in splitting up the integral into separate parts as Booker and Clemmow have done, because this leads to two separate asymptotic series whose leading turns cancel, so that an extra term must be taken in each to get a given degree of approximation in the final result.

The only fundamental difference, however, is that Booker and Clemmow express the source in terms of an infinite spectrum of Fourier components in a vertical plane, whilst we have chosen a horizontal plane. This choice has advantages in considering the launching of surface waves, for they enter into the analysis in a way which provides an intuitive explanation of their presence in the final field.

Macfarlane[9] has made calculations of Zenneck-wave excitation due to a chopped surface-wave excitation by this method, and his result for a highly reactive loaded surface is exactly equivalent to that given by eqn. (42).

(10) DIELECTRIC-COATED GUIDING SURFACE

In this Section the analysis is extended to include a dielectric-coated metal sheet. If the dielectric coating is thin, only one surface-wave mode will exist, and this is a principal mode, having no cut-off property. Moreover, in this case the power flow inside the dielectric will be small in comparison with that outside* and the amplitude of the surface wave will be exactly the same as that which would be excited over a corrugated surface giving the same decay constant, k_1. The amplitude can therefore be calculated from the first term of eqn. (20) if the appropriate values of k_1 and β_1, $[\beta_1 = \sqrt{(k^2 + k_1^2)}]$ are used. In the present notation, the value of k_1 is now given by the following transcendental equation:

$$k_1 K_r = \sqrt{[k^2(K_r - 1) - k_1^2]} \tan \sqrt{[k^2(K_r - 1) - k_1^2]} d \quad . \quad . \quad . \quad . \quad (43)$$

which simplifies, if d is small enough, to

$$k_1 = \left(\frac{K_r - 1}{K_r}\right) k^2 d \quad . \quad . \quad . \quad . \quad . \quad (43a)$$

The radiation field can be calculated by the method of stationary phase as in Section 6, just as for a corrugated surface, and is given by a modified form of eqn. (33). In the factor

$$(jk \sin \theta + k_1)/(jk \sin \theta - k_1)$$

which represents the reflection coefficient of the surface, k_1 must now be replaced by an expression which, if d is sufficiently small, can be written

$$k^2 d \left[\left(\frac{K_r - 1}{K_r}\right) + \frac{1}{K_r} \sin^2 \theta\right] \quad . \quad . \quad . \quad (44)$$

It will be noticed that if θ is sufficiently small, eqn. (44) reduces to $k^2 d\left(\frac{K_r - 1}{K_r}\right)$, which by eqn. (43$a$) is simply the decay constant k_1. Thus, if we restrict our attention to small values of θ, corresponding to large values of x and small values of y and h, the

* The ratio is given approximately $k^2 d^2 \left(\frac{K_r - 1}{K_r}\right)$. For example, if $\lambda = 3 \cdot 2$ cm, $d = 0 \cdot 16$ cm, and $K_r = 2 \cdot 56$, the power transmitted through the dielectric sheet is only about 6% of the total.

analysis for corrugated surfaces is also valid for dielectric-coated surfaces. The results of Section 5 for the field near the surface are therefore valid also for a dielectric-coated surface, and in particular we find a zero of the radiation field at a height $1/k_1$ above the surface, as indicated by eqn. (25), at a sufficiently great distance from the source.

The previous calculations of launching efficiency, on the other hand, cannot be taken over in this way since larger values of θ are encountered. Calculations of launching efficiency have been made for a dielectric sheet of relative permittivity 2·56 (corresponding to Distrene) and of such a thickness that the decay-constant of the surface wave has the same value (0·5 k) as that assumed for the corrugated surface. The result is shown in Fig. 9. It will be noticed that the maximum launching efficiency now occurs at somewhat greater value of h than for the corrugated surface. The maximum launching efficiency is about 85%.

(11) EXPERIMENTAL RESULTS

Some of the theoretical conclusions of the paper have been investigated experimentally at a wavelength of 3·2 cm. Since the theory applies to a 2-dimensional problem, it is clear that only a limited verification of the theory is feasible, and the experiments were directed primarily to a determination of the vertical distribution of field. The general arrangement of the apparatus is shown in Fig. 11. The reactive surface consists of

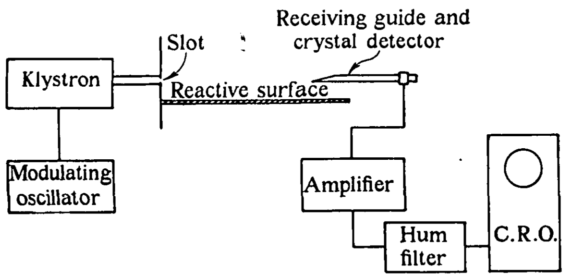

Fig. 11.—Field-measuring apparatus.

a Distrene sheet 0·16 cm in thickness cemented to a rectangular metal plate (45 cm × 57 cm approximately), and arranged to be adjustable in height. Reflections at the edges of the sheet are reduced as far as possible by graphite-loaded wedges. The source consists of a $\lambda/2$ slot in a large metal plate, fed by a waveguide behind the plate. A $\lambda/4$ flange provides a zero-impedance movable choke junction between this plate and the metal backing plate of the reactive surfaces, as shown in Fig. 12. The detector

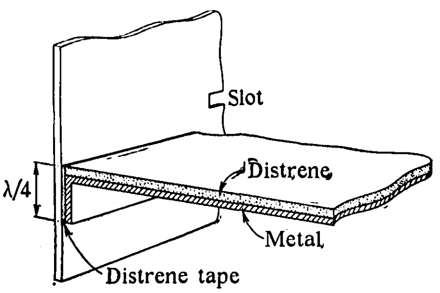

Fig. 12.—Quarter-wavelength choke connection.

consists of a crystal unit fed from a $\lambda/2$ slot in the end of a waveguide tapered as shown in Fig. 13 to minimize perturbation of the field. The crystal was shown to obey a square-law by an auxiliary experiment without the reactive surface, making use of the inverse distance law of the radiation field.

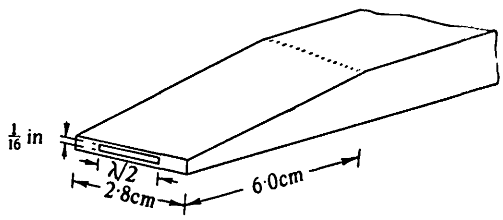

Fig. 13.—Tapered waveguide probe for field-strength measurements.

With the reactive surface in position it was soon discovered that, in accordance with the theory, a field distribution resembling a surface wave was found only if the launching slot was fairly close to the reactive surface. A plot of the logarithm of field power density against height with the launching slot 8 mm above the surface is given in Fig. 14. The decay coefficient derived

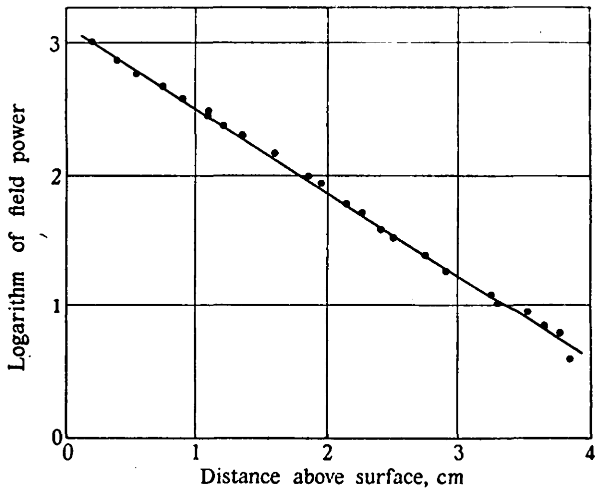

Fig. 14.—Logarithmic curve showing exponential field decay for surface wave.

$h = 8$ mm
$x = 45\cdot5$ cm
Derived value of $k_1 = 0\cdot313$ cm^{-1}.

from this graph is 0·313 cm^{-1} as compared with the value 0·374 cm^{-1} calculated from eqn. (43a) with $d = 0\cdot16$ cm, $K_r = 2\cdot56$, and $\lambda = 3\cdot2$ cm. Thus the exponential field variation for a surface wave is verified, and the observed rate of decay is in satisfactory agreement with the theoretical value in view of the considerable variations in the thickness of the dielectric sheet at different parts of the surface, in places by as much as 0·5 mm, and also, perhaps, because of some contamination of the surface wave by radiation field.

Another prediction of the theory which was investigated experimentally concerns the vanishing of the radiation field at a height $1/k_1$ when the detector is a long way from the source.

Experimentally there are two conflicting requirements for the height of the launching slot; to minimize surface-wave excitation the height should be as great as possible, but to justify neglecting the term $\frac{1}{K_r}\sin^2\theta$ in eqn. (44) it should be as small as possible. At a height of 6 cm the surface-wave field is very small, and the error involved in neglecting $\frac{1}{K_r}\sin^2\theta$ in eqn. (44) is only a few per cent if the distance from the source is 30 cm or more. The height at which the electric field vanishes is plotted in Fig. 15 as a function of distance from the source. Theoretically this height is 0·374 cm^{-1}, or 2·68 cm, but it will be seen that the

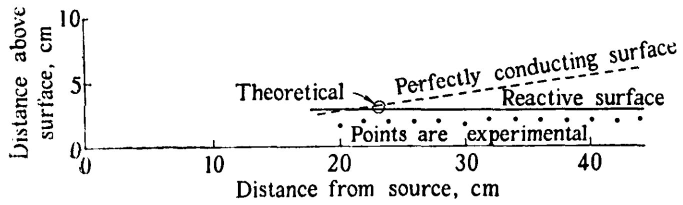

Fig. 15.—Locus of zero radiation field.

experimental value is considerably less than this. Qualitatively, however, it is clear that the line of zero field follows the surface and is not radial, as it would be for an uncoated metal sheet. It is possible that the discrepancy may be accounted for by residual surface-wave field, for the position of zero field is clearly very susceptible to disturbance from small unwanted fields of this kind.

(12) CONCLUSIONS

It has been shown that a high launching efficiency for slot excitation of surface waves is possible if the slot height is properly chosen.

Simple formulae for the radiation field and surface-wave field for any arbitrary aperture distribution have been given for points sufficiently remote from the source.

Some of the conclusions of the theory have been verified experimentally, but the limitations of the apparatus have restricted the scope of this part of the work.

It is proposed to extend this study of launching to the case of the radial cylindrical surface wave, for which an accurate experimental verification should be much simpler.

(13) ACKNOWLEDGMENTS

The author wishes to express his thanks to Professor H. M. Barlow and Mr. A. E. Karbowiak for the benefit of many stimulating discussions, and to Mr. G. Millington for very helpful comments on the theory. He is also grateful to Mr. R. G. Manton, who carried out the experimental work with very limited time at his disposal and prepared some of the diagrams; and to Mr. W. M. G. Fernando, who assisted with some of the computation.

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