# Advanced Topics in Control 2020: Large-Scale Convex Optimization

## Exercise 8: Alternating Direction Method of Multipliers

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Date due: May 7, 2020 at 23:59.

Please submit your solutions via Moodle as a PDF with filename `Ex08_Surname.pdf`, replacing `Surname` with your surname.

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## Problem 1

Consider the quadratic program

$$
\begin{aligned}
\min \; & \frac{1}{2} x^T H x + h^T x \\
\text{subject to } \; & Ax = b \\
& l \leq Cx \leq u
\end{aligned}
$$

Assume that there exists a feasible point, and that $A \in \mathbb{R}^{m \times n}$ has full row rank. Consider that $H \in \mathbb{S}^n_+$ is positive semi-definite, and $C$ is sparse (which means that the matrix-vector multiplication operations like $Cx$ and $x^T C$ are quite cheap).

a) Formulate the problem in the form:

$$\min f(x) + g(Lx).$$

where $f(x) = \frac{1}{2} x^T H x + h^T x + \mathcal{I}_{Ax=b}(x)$. To formulate the dual problem, $L$ and $g$ must be chosen. Write down the dual problem (D1) for $L = C$ and the dual problem (D2) for $L = I$.

b) Motivate if the primal and dual problems can be solved using the Douglas-Rachford splitting. Motivate, which formulation (primal, dual with $L = C$, or dual with $L = I$) has the cheapest iterations.

## Problem 2

Consider the following dual formulation of the convex problem with quadratic objective:

$$
\begin{aligned}
\max \; & -\frac{1}{2} x^T P x - S_{\mathcal{C}}(y) \\
\text{subject to } \; & -Px + A^T y = -q \\
& y \in (\mathcal{C}^\infty)^o,
\end{aligned}
$$

where $\mathcal{C}$ is a closed convex set, $P \in \mathbb{S}^n_+$. Show that if there exists some $\bar{x}$ such that

$$P\bar{x} = 0,\ A\bar{x} \in \mathcal{C}^\infty, \text{ and } q^T\bar{x} < 0,$$

then the dual problem is infeasible.

## Problem 3

*Basis pursuit* is the equality-constrained $\ell_1$-minimization problem

$$\begin{gathered}\min_{x \in \mathbb{R}^n} \|x\|_1, \\ \text{subject to } Ax = b\end{gathered}$$

where $A \in \mathbb{R}^{m \times n}$ has full row rank, $b \in \mathbb{R}^m$. Write down the iteration of ADMM for this problem.