Connectivity and connected components

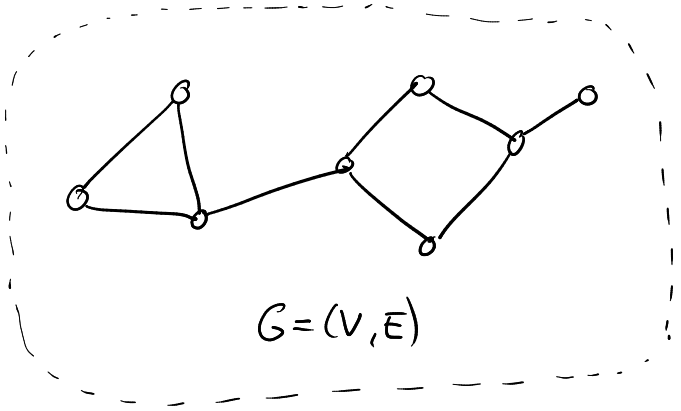
Connectivity in undirected graphs

Definition 3.15: Connectivity in undirected graphs

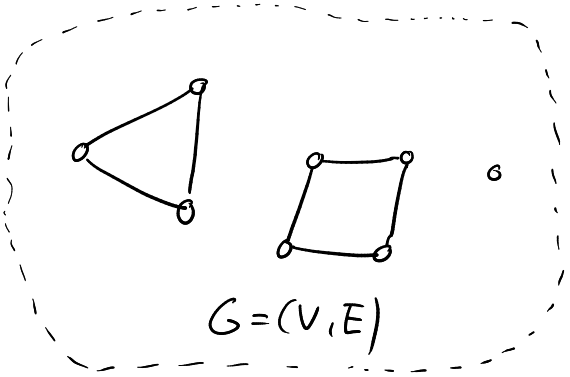
Let $G = (V, E)$ be an undirected graph.

(i) Two vertices $s, t \in V$ are called *connected* in G if G contains an s-t walk.

(ii) The graph G is callled *connected* if each pair of vertices in G is connected.



connected



disconnected

Connectivity in directed graphs

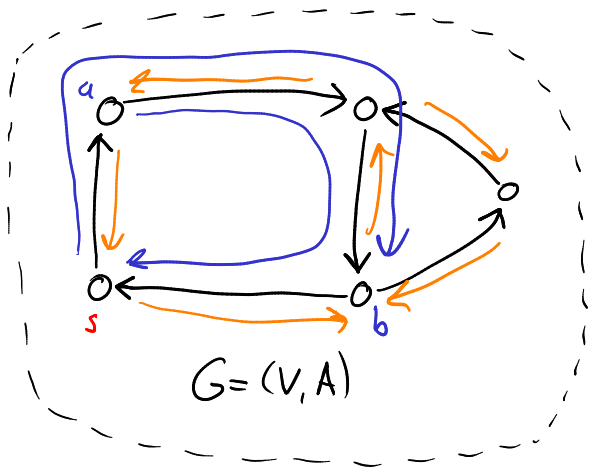
Definition 3.16: Connectivity in directed graphs

Let $G = (V, A)$ be a directed graph.

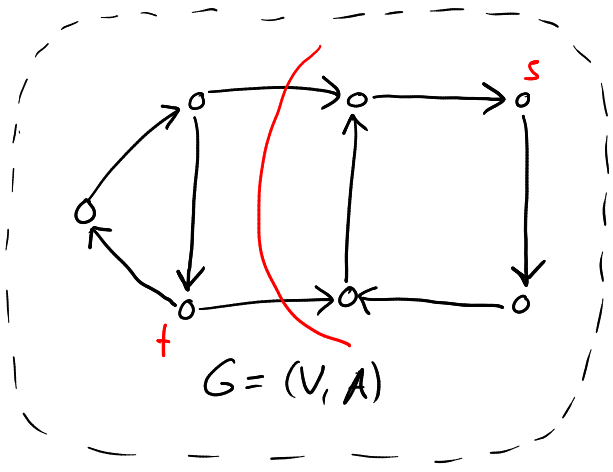
(i) G is called *connected*, if the undirected graph G', obtained from G by ignoring arc directions, is connected.

(ii) Let $s, t \in V$. The vertex t can be *reached* from s in G if G contains a directed s-t walk.

(iii) The graph G is called *strongly connected* if every vertex in G can be reached from every other vertex.



strongly connected



Connected but not strongly connected

Connected components in undirected graphs

Definition 3.17: Connected components in undirected graphs

Let $G = (V, E)$ be an undirected graph. A *connected component* of G is an induced subgraph $G[W]$ such that $G[W]$ is connected and $W \subseteq V$ is maximal with respect to this property, i.e., for every $X \subseteq V$ with $X \supsetneq W$ the graph $G[X]$ is not connected.

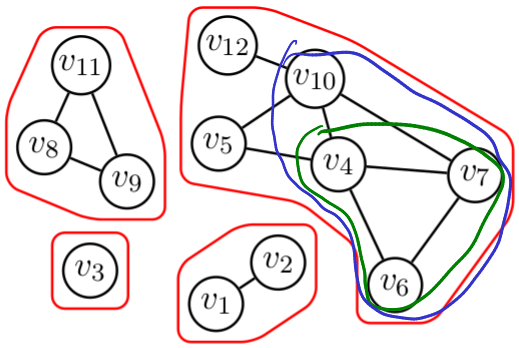


Figure 3.23: The above graph G has four connected components: $G[\{v_1, v_2\}]$, $G[\{v_3\}]$, $G[\{v_4, v_5, v_6, v_7, v_{10}, v_{12}\}]$, and $G[\{v_8, v_9, v_{11}\}]$.

Connected components in directed graphs

Definition 3.19: Connected components in directed graphs

Let $G = (V, A)$ be a directed graph.

(i) The *connected components* of G are the connected components of the undirected graph G', which results from G by ignoring arc directions.

(ii) A *strongly connected component* of G is an induced directed subgraph $G[W]$ such that $G[W]$ is strongly connected and $W \subseteq V$ is maximal with respect to this property.

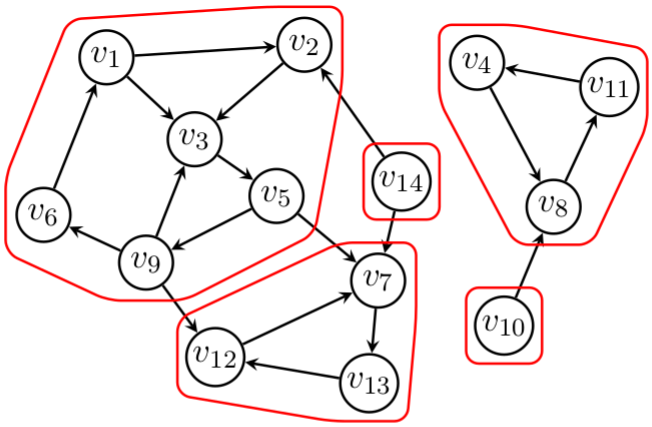


Figure 3.24: A graph G with two connected components and five strongly connected components (in red): $G[\{v_{10}\}]$, $G[\{v_1, v_2, v_3, v_5, v_6, v_9\}]$, $G[\{v_4, v_8, v_{11}\}]$, $G[\{v_7, v_{12}, v_{13}\}]$, and $G[\{v_{14}\}]$.