1.3.4 Exchange step/pivoting

Goal: Getting from one tableau to another one.

Tan equation system in tableau form

Example of exchange step/pivoting

y_1	y_2	y_3	x_1	x_2	1
$\frac{1}{4}$	0	0	$\frac{1}{4}$ o	1	10
$-\frac{1}{4}$	1	0	$\frac{7}{4}$ $^{\circ}$	0	32
$-\frac{3}{4}$	0	1	$\frac{3}{4}$	0	6

row operations

y_1	y_2	y_3	x_1	x_2	1
$\frac{1}{2}$	0	$-\frac{1}{3}$	0	1	8
$\frac{6}{4}$	1	$-\frac{7}{3}$	0	0	18
-1	0	$\frac{4}{3}$	1	0	8

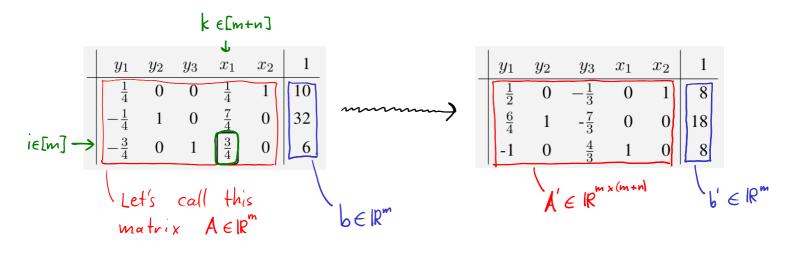
pick non-zero
element in non-basic
column

o. Add
$$-\frac{\frac{1}{4}}{\frac{3}{4}} = -\frac{1}{3}$$
 of 3rd vow to 1st one.

o · Add
$$-\frac{\frac{7}{4}}{\frac{3}{4}} = -\frac{7}{3}$$
 of 3rd van to 2rd one.

1. Multiply 3rd von by
$$\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$$
.

Formal description of exchange step/pivoting



Condition:
$$i \in [m]$$
, $k \in [m+n]$ such that $\frac{A_{ik}}{\uparrow} \neq 0$

pivot row

pivot column

pivot column

$$\begin{array}{ll} \underbrace{\text{Pivoting formu(aS)}} \\ \text{(i)} \ \ A'_{i\ell} = \frac{A_{i\ell}}{A_{ik}} & \ell \in [m+n] \\ \text{(ii)} \ \ A'_{j\ell} = A_{j\ell} - \frac{A_{jk} \cdot A_{i\ell}}{A_{ik}} & \ell \in [m+n], \qquad j \neq i, j \in [m] \\ \text{(iii)} \ \ b'_{i} = \frac{b_{i}}{A_{ik}} \\ \text{(iv)} \ \ b'_{j} = b_{j} - \frac{A_{jk} \cdot b_{i}}{A_{ik}} & j \in [m] \text{ with } j \neq i \ , \end{array}$$

1.3.5 Short tableau

long tableau

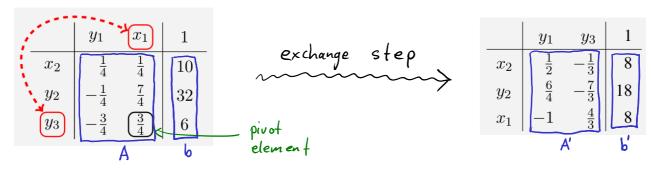
short tableau

	y_1	x_1	1
$\overline{x_2}$	$\frac{1}{4}$	$\frac{1}{4}$	10
y_2	$-\frac{1}{4}$	$\frac{7}{4}$	32
y_3	$-\frac{3}{4}$	$\frac{3}{4}$	6

Some advantages of short tableau:

- · Short tableau makes basis explicit.
 - Removes ambiguities regarding which basis we want to focus on with a tableau in case basis is not unique.
- · Short tableau has more natural dual interpretation (to be discussed later).
- · Short tableau is more compact.

1.3.6 Exchange step in short tableau



basis: $\beta = (x_2, y_2, y_3)$

basis: $\beta' = (x_2, y_2, x_1)$

Pivoting formulas for short tableau
$$A'_{ik} = \frac{1}{A_{ik}} \qquad (pivot \ element)$$

$$A'_{jk} = -\frac{A_{jk}}{A_{ik}} \qquad (pivot \ column) \quad j \in [m], \quad j \neq i$$

$$A'_{i\ell} = \frac{A_{i\ell}}{A_{ik}} \qquad (pivot \ row) \qquad \ell \in [n], \quad \ell \neq k$$

$$A'_{j\ell} = A_{j\ell} - \frac{A_{jk} \cdot A_{i\ell}}{A_{ik}} \qquad j \in [m], \quad j \neq i$$

$$\ell \in [n], \quad \ell \neq k$$

$$b'_{i} = \frac{b_{i}}{A_{ik}}$$

$$b'_{j} = b_{j} - \frac{A_{jk} \cdot b_{i}}{A_{ik}} \qquad j \in [m], \quad j \neq i$$

Example 1.60

(a):

	(a):	
$B = (x_1, x_2, x_3)$	$x_2 \mid 1 -1 \mid 0$	basic solution $(3,0,2,0,0) = (3,0,2,0,0)$
(b):	(c):	(d):
$x_4 x_1 \mid 1$	x_2 x_1 1	x_2 x_4
x_5 2 1 3	$x_5 \mid -\frac{2}{3} \frac{1}{3} \mid 1$	$x_5 \begin{vmatrix} -1 & -1 & 0 \end{vmatrix}$
$x_2 \mid \boxed{3} 1 \mid 3$	$x_4 \mid \frac{1}{3} \mid \frac{1}{3} \mid 1$	$\rightarrow x_1 \mid 1 3 \mid 3$
$x_3 \mid \begin{array}{c c} \hline 1 & 0 \mid 2 \end{array}$	$x_3 \mid -\frac{1}{3} -\frac{1}{3} \mid 1$	$x_3 \mid 0 1 \mid 2$
(0, 3, 2, 0, 3)	(0,0,1,1,1)	(3,0,2,0,0)

Inequality description coming from (a)

