Level Structures and Morava E-theory

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- Morava E-theory: introduction and construction;
- Generalized Morava E-theory
- Ingredients:
 - Level structure,
 - transfer ideal.
 - Hopkins-Kuhn-Ravenel character theory;
- The main theorem and the proof;
- Level structure and subgroup.

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- "designer" cohomology theories: manufactured using homotopy theory, not coming from "nature".
- some arise as completions of "natural" theories, K_p^{\wedge} , $Ell_{s.-s.point}^{\wedge}$.
- have rich theory of power operations(Ando, Hopkins, Strickland, Rezk, Stapleton, Zhu...)

Fix a formal group G_0 over a perfect field k of characteristic p of height n

Morava 1978; Goerss-Hopkins-Miller 1993-2004

- is complex orientable; formal group $\operatorname{Spf}(E^0_{G_0}\mathbb{C}P^\infty)=$ universal deformation of G_0 in sense of Lubin and Tate.
- is represented by a structured commutative ring spectrum

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Formal group law (commutative, 1-dimensional)

 $S(x,y) \in R[x,y]$ satisfying axioms for abelian group:

$$S(x,0) = x = S(0,x),$$

 $S(x,y) = S(y,x),$
 $S(S(x,y),z) = S(x,S(y,z)).$

Complex oriented cohomology theory

Ring-valued cohomology theory E such that $E^*(\mathbb{C}P^{\infty}) = E^*[x]$, and x restricts to fundamental class of $\mathbb{C}P^1 = S^2$.

Examples: $H^*(-,\mathbb{Z})$, K—theory, EII, cobordism, Morava E-theories.

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complex oriented cohomology theories ______ formal groups

There is a moduli problem associated to G_0/k .

Deformation of formal groups

Lubin-Tate, 1966]

LT: Complete Local Rings \longrightarrow Groupoids

Objects of LT(R, m): Deformation $(G/R, i : k \to R/m, \alpha)$:

- G is a formal group over R;
- $\alpha: i^*G_0 \xrightarrow{\cong} \pi^*G$ isomorphism of formal groups over R/m.

Isomorphisms $(G, i, \alpha) \longrightarrow (G', i, \alpha')$

• \star -isomorphism: iso $f: G \longrightarrow G'$ compatible with id of G_0 .

Universal Deformation

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Strickland's theorem

1998

$$\operatorname{Spec}(E_n^0(B\Sigma_{p^k})/I_{tr})\cong\operatorname{Sub}_{p^k}(\mathbb{G}_u)$$

Additive Power Operation

[Proposition 3.21, AHS 2004]

$$P_{p^k}/I_{tr}: E_n^0(BA) \longrightarrow E_n^0(BA) \otimes_{E_n^0} E_n^0(B\Sigma_{p^k})/I_{tr}.$$

$$\operatorname{\mathsf{Sub}}_{p^k}(\mathbb{G}_u) \times_{LT} \operatorname{\mathsf{Hom}}(A^*,\mathbb{G}_u) \longrightarrow \operatorname{\mathsf{Hom}}(A^*,\mathbb{G}_u)$$

Theorem

[Proposition 5.12, HKR 2000]

A: finite abelian group. There is a canonical isomorphism of E^0 —algebras

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$$\mathcal{L}(X/\!\!/ G) := \mathsf{Hom}_{top.gpd}(*/\!\!/ \mathbb{Z}_p, X/\!\!/ G) \cong (\coprod_{\alpha \in \mathsf{Hom}(\mathbb{Z}_p, G)} X^{\mathsf{im}\, \alpha})/\!\!/ G.$$

- $\mathcal{L}BG := EG \times_G \mathcal{L}(*/\!\!/ G) \simeq \operatorname{Map}(B\mathbb{Z}_p, BG).$
- $E_G(-)$ is a cohomology theory on finite G-CW complexes \Rightarrow So is $E_G(\mathcal{L}(-))$.
- the algebro-geometric object associated to $E^0_{\mathbb{Z}/p^k}(\mathcal{L}^h(-))$ is the p-divisible group $\mathbb{G}_E \oplus (\mathbb{Q}_p/\mathbb{Z}_p)^h$.
- $\mathcal{L}(-)$ is a key functor in the target of Hopkins-Kuhn-Ravenel generalized character map and Stapleton's transchromatic generalized character maps.

2015, Tomer M.Schlank, Nathaniel Stapleton

$$\operatorname{Spec}(E_n^0(\mathcal{L}^h B\Sigma_{p^k})/I_{tr}) \cong \operatorname{Sub}_{p^k}(\mathbb{G}_u \oplus \mathbb{T}')$$

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A: finite abelian p-group.

Classical Definition: Level structures on formal groups

[Drinfeld 74]

A level structure $f:A\longrightarrow \mathbb{G}$ is a homomorphism from A to \mathbb{G}

- $rank(A) \leq height(\mathbb{G});$
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$$\operatorname{Level}(A^*, \mathbb{G}_u) \xrightarrow{f \mapsto \operatorname{im} f} \operatorname{Sub}_{p^k}(\mathbb{G}_u).$$

A—Level Structure: Definition in the generalized case [Huan, Stapleton]

 $I:A\hookrightarrow \mathbb{G}\oplus \mathbb{T}'$: a homomorphism of group schemes such that the induced map $\ker(\pi I)\to \mathbb{G}$ is a $\ker(\pi I)$ -level structure on \mathbb{G} .

$$A \stackrel{I}{\longrightarrow} (\mathbb{G} \oplus \mathbb{T}') \stackrel{\pi}{\longrightarrow} \mathbb{T}'$$

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Transfer map in E-cohomology

[Chapter 4, Adams 1978]

For $H \hookrightarrow G$, $BH \longrightarrow BG$ is a finite cover.

$$\operatorname{Tr}_E: E^0(BH) \longrightarrow E^0(BG)$$

$$\bigoplus_{0 < i < m} \operatorname{Tr}_E : \bigoplus_{0 < i < m} E^0(BG \wr (\Sigma_i \times \Sigma_{m-i})) \longrightarrow E^0(BG \wr \Sigma_m).$$

Transfer Ideal

$$I_{tr} = \operatorname{im}(\bigoplus_{0 < i < m} \operatorname{Tr}_{E}) \subset E^{0}(BG \wr \Sigma_{m})$$

 I_{tr} is the smallest ideal such that the quotient

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Recall
$$P_m(x \otimes y) = \text{res} |_{G \wr \Sigma_m}^{G \wr \Sigma_m} (P_m(x) \otimes P_m(y))).$$

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Motivation

[Atiyah, Segal 1969][Adams 1978]

Classical representation theory:

$$\mathbb{C} \otimes \chi : \mathbb{C} \otimes RG \stackrel{\cong}{\longrightarrow} CI(G,\mathbb{C}).$$

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-adic K -theory: $RG \xrightarrow{\alpha} K(BG)$

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$$\begin{array}{ccc}
RG & \xrightarrow{\alpha} & K(BG) \\
\downarrow & & \downarrow \\
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Character theory of Morava E-theory

[Theorem C, HKR 2000]

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More explanation on the character map $\chi \colon E^0(BG) \longrightarrow Cl_n(G, C_0)$

- $C_0 := S^{-1}E^0_{cont}(B L)$, where
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Algebro-geometric interpretation:

 \mathcal{C}_0 is the rationalization of the Drinfeld ring $\mathcal{O}_{\mathsf{Level}(\mathbb{T},\mathbb{G}_u)}$

• $CI_n(G, C_0) = \{C_0 - \text{valued functions on the set } \text{Hom}(\mathbb{L}, G)/\sim\}.$

Construction of the character map $\prod \chi_{[lpha]}$

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Hopkins-Kuhn-Ravenel Character theory and transfer

Transfer map and Character theory

[Theorem D, HKR 2000]

G: finite group; $H \subset G$.

$$E_n^0(BH) \xrightarrow{\operatorname{Tr}_E} E_n^0(BG)$$

$$\chi \downarrow \qquad \qquad \downarrow \chi$$

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"Strickland's theorem" in HKR Character theory

$$CI_n(\Sigma_{\rho^k}, C_0)/I_{tr} \cong \prod_{\operatorname{Sub}_{\rho^k}(\mathbb{T})} C_0$$

Lemma

$$C_0 \otimes_{E_n^0} E_n^0(\mathcal{L}^h BA)/I_A \cong \prod_{\mathsf{Level}(A^*, \mathbb{T} \oplus \mathbb{T}')} C_0.$$

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Classical result:
$$(E_n^0(BA)/I_{tr})^{\text{free}} \stackrel{\cong}{\to} \mathcal{O}_{\text{Level}(A^*,\mathbb{G}_u)}$$
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$$R^{\mathsf{free}} := \mathsf{im}(R \longrightarrow \mathbb{Q} \otimes R).$$

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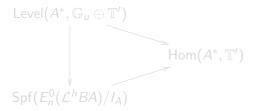
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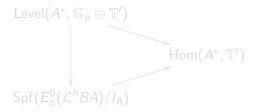
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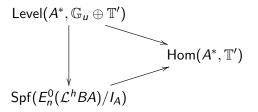
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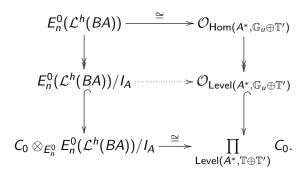
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Proof

Use Hopkins-Kuhn-Ravenel character theory.



$$E_n^0(\mathcal{L}^hBA)/I_{tr} = \prod_{f \colon \mathbb{L}' \to A} E_n^0(BA)/I_{\mathcal{F}_f};$$

 $\mathsf{Level}(A^*, \mathbb{G}_u \oplus \mathbb{T}') = \prod_{f \colon \mathbb{L}' \to A} \mathsf{Level}_f(A^*, \mathbb{G}_u \oplus \mathbb{T}').$

The pullback of schemes

$$\mathsf{Level}_f(A^*, \mathbb{G}_u \oplus \mathbb{T}') \longrightarrow \mathsf{Level}(\ker(f^*), \mathbb{G}_u \oplus \mathbb{T}')$$

$$\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow$$

$$\mathsf{Hom}(A^*, \mathbb{G}_u \oplus \mathbb{T}') \stackrel{f^*}{\longrightarrow} \mathsf{Hom}(\ker(f^*), \mathbb{G}_u \oplus \mathbb{T}').$$

 \mathcal{F}_f :=minimal family $\{H \subset A | H \text{ is proper and } f \text{ does factor through } H\}$

Theorem

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The subgroup scheme $\mathsf{Sub}^{\mathcal{A}}_{p^k}(\mathbb{G}_u\oplus \mathbb{T}')$

$$A \subset \mathbb{T}'$$
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Corollary

$$\alpha: \mathbb{L}' \longrightarrow \Sigma_{p^k}$$
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More geometric power operation

Tate K-theory: the generalized elliptic cohomology associated to the Tate curve.

Strickland's theorem for Tate K-theory

[Huan]

The Tate K-theory of symmetric groups modulo a certain transfer ideal classifies finite subgroups of the Tate curve.

$$K_{Tate}(\operatorname{pt}/\hspace{-0.1cm}/\Sigma_N)/I_{tr}^{Tate}\cong\prod_{N=de}\mathbb{Z}((q))[q_s'^{\pm}]/\langle q^d-q_s'^e
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where l_{tr}^{Tate} is the transfer ideal and q_s' is the image of q under the stringy power operation, the product goes over all the ordered pairs of positive integers (d, e) such that N = de.

A^* —level structure on Tate curve

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A: any finite abelian group.

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- Construct power operation of generalized Morava E-theory;
- and other problems on Morava E-theories.

The Definition of n—vector bundles

- ullet (1-)vector bundles \Rightarrow topological K-theory whose chromatic level is 1;
- 2-vector bundles ⇒ a form of elliptic cohomology theory whose chromatic level is 2.

- Definition of n—vector bundle? Definition of equivariant n—vector bundle?
- n-vector bundle \Rightarrow chromatic level n cohomology theory?
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- 6 Hopkins-Kuhn-Ravenel character theory on quasi-elliptic cohomology? The relation to the character theory on twisted equivariant elliptic cohomology theory?
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- Ohern character from Kitchloo's dominant K-theory? Its relation to the twisted Chern character from twisted equivariant K-theory [FHT]?

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- Construct the twisted version of the elliptic power operation of quasi-elliptic cohomology \Rightarrow a power operation of the twisted equivariant elliptic cohomology.
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Thank you.

Some references

https://huanzhen84.github.io/zhenhuan/Huan-YMF-2019-Slides.pdf

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