RESEARCH OUTLINE

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The main subject that I'm studying is quasi-elliptic cohomology. It is very close to elliptic cohomology theories and thus can reflect the geometric nature of elliptic curves. Meanwhile it is a theory that is easier to compute and it supports constructions that are neater than those on the elliptic cohomology theories. Quasi-elliptic cohomology has shown its power in solving several significant mathematical problems, as shown in Section 1, 2, 3 and 4]. What is more fascinating is that the theory itself has distinct features worth exploration. It serves as a meaningful motivating example to the extension of the idea of representation theory and equivariant homotopy theory. Based on my prior research work, I will continue the exploration of quasi-elliptic cohomology and its related areas. There are several research problems that I would like to cooperate with faculty members of mathematics and computer sciences at Georg-August-Universität Gottingen.

The concept of elliptic cohomology was first introduced by Landweber, Ravenel, Stong in 1986. An elliptic cohomology theory is an even periodic multiplicative generalized cohomology theory whose associated formal group is the formal completion of an elliptic curve. It is at the intersection of a variety of areas in mathematics, including algebraic topology, algebraic geometry, mathematical physics, representation theory and number theory. Definitions of equivariant elliptic cohomology from different perspectives have been motivated, Devoto's definition motivated by orbifold string theory [15], Grojnowski's definition motivated by the theory of loop groups [23], etc. Moreover, in 1995 Ginzburg, Kapranov and Vasserot gave an axiomatic definition in terms of principal bundles over elliptic curves. In 2007 based on Lurie's definition of elliptic cohomology [37] Gepner formulated a construction of equivariant elliptic cohomology [20] satisfying axioms similar to those in [21] in the context of derived algebraic geometry.

The generalized elliptic curve that is most relevant to quasi-elliptic cohomology is the Tate curve [3]. It can be considered as the boundary point of the moduli space of elliptic curves. The generalized elliptic cohomology theory associated to the Tate curve is Tate K-theory [3]. Tate K-theory is itself a distinctive subject to study. Its relation with string theory is better understood than most known elliptic cohomology theories. In addition, the definition of $G-$equivariant Tate K-theory for finite groups G is modelled on the loop space of a global quotient orbifold [Section 2, [18]].

Quasi-elliptic cohomology is not an elliptic cohomology but it contains all the information of equivariant Tate K-theory. That's how it got its name. It was first introduced by Ganter inspired by Devoto's equivariant Tate K-theory [15]. It is presented originally in Rezk's unpublished manuscript [40] and in full detail in [28] and [26].

In Section 1, 2, 3 and 4 I present the contribution of quasi-elliptic cohomology to the study of elliptic cohomology theories, i.e. how we apply quasi-elliptic cohomology to study math problems on elliptic cohomology theory. Based on our research progress so far, in Section 5 I present the research problems that we will work on in the near future.

1. Witten's conjecture

In [35] Landweber cited a conjecture by Witten that that the elliptic cohomology of a space X is related to the $\mathbb{T}-$equivariant K-theory of the free loop space $LX = \mathbb{C}^{\infty}(S^1, X)$ with the circle $\mathbb{T}$ acting on LX by rotating loops. However, the relation is usually difficult to interpret because in application we need to study a space X with a group action. In this case LX is an orbifold with

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sophisticated structure. In my PhD thesis [28] and [26] we construct quasi-elliptic cohomology as orbifold K-theory of a specific loop space. This construction gives an interpretation of equivariant Tate K-theory via loop spaces. It provides an approach other than Ganter's construction [Section 2, [18]].

If G is a Lie group and X is a manifold with a smooth $G-$action, the space of smooth unbased loops in the orbifold $X/\!\!/G$ in principle carries a lot of structure: on the one hand, it includes loops represented by continuous maps $\gamma : \mathbb{R} \longrightarrow X$ such that $\gamma(t+1) = \gamma(t)g$ for some $g \in G$; at the same time the circle acts on the loop space by rotation. We interpret a "loop" as a bibundle, i.e. a 1-morphism in the localization of the category of Lie groupoids with respect to the equivalence of Lie groupoids. The right loop space $Loop^{ext}(X/\!\!/G)$ is constructed from the category of bibundles from $S^1/\!\!/*$ to $X/\!\!/G$ with the circle rotations added as morphisms.

Quasi-elliptic cohomology $QEll^*_G(X)$ is defined to be the orbifold K-theory of a subgroupoid $\Lambda(X/\!\!/G)$ of $Loop^{ext}(X/\!\!/G)$ consisting of constant loops. More explicitly, $QEll^*_G(X)$ can be expressed in terms of the equivariant K-theory of X and its subspaces

$$QEll^*_G(X) := \prod_{\sigma\in G^{tors}_{conj}} K^*_{\Lambda_G(\sigma)}(X^\sigma) = \Big(\prod_{\sigma\in G^{tors}} K^*_{\Lambda_G(\sigma)}(X^\sigma)\Big)^G. \tag{1}$$

We can also construct quasi-elliptic cohomology for Lie groupoids. We study those bibundles from $S^1/\!\!/*$ to a Lie groupoid $\mathbb{G}$ and construct the constant loop space $\Lambda(\mathbb{G})$. $QEll^*(\mathbb{G})$ is defined to be $K^*_{orb}(\mathbb{G})$. When $\mathbb{G}$ is a global quotient $X/\!\!/G$, $\Lambda(\mathbb{G})$ is the groupoid $\Lambda(X/\!\!/G)$ and $QEll^*(\mathbb{G})$ is $QEll^*_G(X)$ defined by (1).

In [19], Ganter explains that Tate K-theory is really a cohomology theory for orbifolds, which is based on Devoto's definition of the equivariant theory. We have the relation

$$QEll^*(\mathbb{G}) \otimes_{\mathbb{Z}[q^\pm]} \mathbb{Z}((q)) = K^*_{Tate}(\mathbb{G}). \tag{2}$$

Moreover, we define an involution for the theory which is compatible with its geometric interpretation. As equivariant K-theory, quasi-elliptic cohomology also has the Real and real version, which is shown in Chapter 5, [28].

Currently we are studying whether the loop space construction of quasi-elliptic cohomology can be applied to the construction of elliptic cohomology and thus interpret Witten's conjecture.

2. Geometric structure on elliptic curves

Ever since the concept of elliptic cohomology theories was given, people have been exploring the relation between the geometric structure on elliptic curves and elliptic cohomology. In 1995 a classification of the level structures on the formal group of Morava E-theory is given in [1]. In 1998 Strickland proved in [47] that the Morava $E-$theory of the symmetric group Σ_n modulo a certain transfer ideal classifies the power subgroups of rank n of its formal group. The Morava E-theory with chromatic level 2 is an elliptic cohomology. Then a conjecture was formulated that the classification of the finite subgroups of any elliptic curve can all be presented in the same form as Strickland's theorem. In 2015 Stapleton proved in [43] this result for generalized Morava E-theory via transchromatic character theory [45] [46]. We proved that for quasi-elliptic cohomology and Tate K-theory we also have Strickland's theorem.

In all the cases above the power operation serves as a bridge connecting the homotopy theory and its formal group. Power operations in elliptic cohomology arise from isogenies of the underlying elliptic curve [2]. Moreover, Ando, Hopkins and Strickland studied the power operation for Morava E-theories in [4]. Ganter constructed the power operation for equivariant Tate K-theory in [18] and that for orbifold Tate K-theory in [19]. The questions arise whether there is a power operation of quasi-elliptic cohomology exhibiting the relation between the cohomology theory and Tate curve and what advantages it has over the power operation of Tate K-theory. The explicit relation between quasi-elliptic cohomology and equivariant K-theories guarantees the existence of a power operation analogous to the Atiyah's power operation on equivariant K-theories [8]. Ganter indicates in [18] and [19] that quasi-elliptic cohomology has a power operation reflecting the geometric nature of the Tate curve.

We construct a power operation $\{\mathbb{P}_n\}_{n\geq 0}$ for quasi-elliptic cohomology via explicit formulas that interwine the power operation in K-theory and natural symmetries of the free loop space. We have the theorem below.

THEOREM 2.1. *(Section 4.2,* [25]*) Quasi-elliptic cohomology has a power operation*

$$\mathbb{P}_n : QEll_G(X) \longrightarrow QEll_{G\wr\Sigma_n}(X^{\times n})$$

that is elliptic in the sense: $\mathbb{P}_n$ *can be uniquely extended to the stringy power operation*

$$P_n^{string} : K_{Tate}(X/\!\!/G) \longrightarrow K_{Tate}(X^{\times n}/\!\!/(G \wr \Sigma_n))$$

of the Tate K-theory in [18]*, which is elliptic in the sense of* [3]*.*

The construction of the power operation $\{\mathbb{P}_n\}_{n\geq 0}$, which mixes power operation in K-theory with natural operation of dilating and rotating loops, can be generalized to other equivariant cohomology theories.

In addition, an elliptic power operation for orbifold quasi-elliptic cohomology exists. Ganter spelled out the axioms for orbifold theories with power operations in [19], and constructed the power operation for orbifold Tate K-theory in the same paper, which is closely related to the level structure and isogenies on Tate curve. We extend the power operation $\{\mathbb{P}_n\}_{n\geq 0}$ to the orbifold quasi-elliptic cohomology. The resulting power operation satisfies the axioms that Ganter concluded.

As indicated in [4], power operation connects elliptic cohomology and elliptic curve. Via the power operation $\{\mathbb{P}_n\}_{n\geq 0}$ in Theorem 2.1, we explore the geometric nature of the Tate curve. In this process, quasi-elliptic cohomology provides a method that reduces facts such as the classification of geometric structures on the Tate curve into questions in representation theory.

It is already illuminating to study the power operation $\{\mathbb{P}_n\}_{n\geqslant 0}$ when X is a point pt with trivial group action. A big ingredient then is understanding $QEll(\mathrm{pt}/\!\!/\Sigma_N)$. Applying that we prove in [25] that the Tate K-theory of symmetric groups modulo a certain transfer ideal classifies finite subgroups of the Tate curve, which is analogous to the principal result in Strickland [47] that the Morava $E-$theory of the symmetric group Σ_n modulo a certain transfer ideal classifies the power subgroups of rank n of the formal group $\mathbb{G}_E$.

Our main conclusion in this section is Theorem 2.2.

THEOREM 2.2. *(Theorem 6.4,* [25]*) The Tate K-theory of symmetric groups modulo a certain transfer ideal,* $K_{Tate}(pt/\!\!/\Sigma_N)/I_{tr}^{Tate}$*, classifies finite subgroups of the Tate curve.*

Equivariant K-theory $K_G(\mathrm{pt})$ of a point is the representation ring of G. As shown in (1), quasi-elliptic cohomology can be expressed by equivariant K-theories. Thus, we can compute quasi-elliptic cohomology of a point by computing the representation ring of each $\Lambda_G(\sigma)$ whereas we cannot compute Tate K-theory directly with representation theory. To prove Theorem 2.2, we first compute the quasi-elliptic cohomology of symmetric groups modulo the transfer ideal, i.e. $QEll(\mathrm{pt}/\!\!/\Sigma_N)/\mathcal{I}_{tr}^{QEll}$. Then, applying the relation (2) between Tate K-theory and quasi-elliptic cohomology, we get the formula for $K_{Tate}(\mathrm{pt}/\!\!/\Sigma_N)/I_{tr}^{Tate}$. Comparing it with the ring that classifies the finite subgroups of the Tate curve, we prove Theorem 2.2. The role of quasi-elliptic cohomology in the proof is crucial.

Applying the same idea, we proved that the Tate K-theory of any finite abelian group A modulo a certain transfer ideal classifies the $A-$Level structures of the Tate curve. The result will appear in a coming paper [33].

Moreover, we can define an operation $\{\overline{P}_N\}_{N\geq 0}$ of quasi-elliptic cohomology via the power operation $\{\mathbb{P}_N\}_{N\geq 0}$. It is a ring homomorphism and is analogous to the Adams operation of equivariant K-theories. It uniquely extends to an additive operation of the Tate K-theory

$$\overline{P^{string}}_n : K_{Tate}(X/\!\!/G) \longrightarrow K_{Tate}(X/\!\!/G) \otimes_{\mathbb{Z}((q))} K_{Tate}(\mathrm{pt}/\!\!/\Sigma_N)/I_{tr}^{Tate}$$

constructed in [18].

Applying the Strickland's theorem in [47], Ando, Hopkins and Strickland show in [4] that the additive power operation of Morava E-theories

$$E^0 \longrightarrow E(B\Sigma_{p^k})/I_{tr}$$

has a nice algebra-geometric interpretation in terms of the formal group and it takes the quotient by the universal subgroup. The additive operation $\{\overline{P}_N\}_{N\geq 0}$ also contains subtle geometric information. Via it, we construct the universal finite subgroup of the Tate curve in [29].

Currently I'm cooperating with Nathaniel Stapleton and exploring whether the conjecture is true for elliptic cohomology theories and other geometric structures on elliptic curves.

3. Equivariant orthogonal spectra of quasi-elliptic cohomology

Equivariant homotopy theory is homotopy theory of topological $G-$spaces. Mandell, May, Schewede and Shipley built several good model categories of equivariant spectra [39]. Equivariant orthogonal spectra, as shown in [38], is one of them. An orthogonal $G-$spectrum is defined from a $\mathcal{I}_G-$functor with $\mathcal{I}_G$ the category of orthogonal $G-$representations.

Ginzburg, Kapranov and Vasserot have the conjecture [21] that any elliptic curve A gives rise to a unique equivariant elliptic cohomology theory, natural in A. In his thesis [20], Gepner presented a construction of the equivariant elliptic cohomology that satisfies a derived version of the Ginzburg-Kapranov-Vasserot axioms. We are interested in answering this question from a different perspective. We are trying to give an explicit construction of the orthogonal $G-$spectrum of each elliptic cohomology theory.

One advantage of quasi-elliptic cohomology is that it is built using equivariant topological K-theory, each aspect of which has been studied thoroughly. Some constructions on quasi-elliptic cohomology can be made simpler than most elliptic cohomology theories, including the Tate K-theory. So we answer the question below first.

QUESTION 3.1. *Is there an orthogonal $G-$spectrum representing $QEll_G^*(-)$?*

Applying equivariant homotopy theory, we construct an orthogonal $G-$spectrum representing quasi-elliptic cohomology [27].

THEOREM 3.2. *([27]) For each compact Lie group G, there exists a commutative I_G-FSP $(QE(G,-), \eta^{QE}, \mu^{QE})$ representing $QEll_G^*$.*

The construction of $(QE(G,-), \eta^{QE}, \mu^{QE})$ is explicit. The idea and technique can be applied to a family of theories, including generalized Morava E-theories and equivariant Tate K-theory. Moreover, we show in [27] that some signature properties of equivariant theories $\{E_G^*\}_G$ can be inherited by the theory $E^*(\Lambda(-))$ after composing E with the loop space functor $\Lambda(-)$ defined in Section 1, including the properties below.

- The theories $\{E_G^*\}_G$ have the change-of-group isomorphism, i.e. for any closed subgroup H of G and $H-$space X, the change-of-group map $\rho_H^G : E_G^*(G\times_H X) \longrightarrow E_H^*(X)$ defined by $E_G^*(G\times_H X) \xrightarrow{\phi^*} E_H^*(G\times_H X) \xrightarrow{i^*} E_H^*(X)$ is an isomorphism where ϕ^* is the restriction map and $i : X \longrightarrow G\times_H X$ is the $H-$equivariant map defined by $i(x) = [e, x]$.
- The theories $\{E_G^*\}_G$ are H_∞, i.e. they have power operations.
- There exists a I_G-FSP representing $E_G^*(-)$.

4. Globalization of quasi-elliptic cohomology

At the early beginning of equivariant homotopy theory people noticed that certain theories, equivariant stable homotopy theory, equivariant bordism, equivariant K-theory, etc, naturally exist not only for one particular group but for all groups in a specific class. This observation motivated the birth of global homotopy theory. The idea of global orthogonal spectra was first inspired in Greenlees and May [22]. In [44] the concept of orthogonal spectra is introduced, which is defined from $\mathbb{L}-$functors with $\mathbb{L}$ the category of inner product real spaces. Each global spectrum consists of compatible $G-$spectra with G across the entire category of groups and they reflect any symmetry. Globalness is a measure of the naturalness of a cohomology theory. Several models of global homotopy theories have been established, including that by Bohmann [14] and Schwede [44]. The two model categories of global spectra are Quillen equivalent, as shown in [14]. But it is unclear whether global elliptic cohomology theory exists and how we should construct it.

In Remark 4.1.6 [44], Schwede discussed the relation between orthogonal $G-$spectra and global spectra. We have the question associated to the underlying orthogonal $G-$spectrum of the I_G-FSP $(QE(G,-), \eta^{QE}, \mu^{QE})$ in Theorem 3.2.

QUESTION 4.1. *Can $\{(QE(G,-), \eta^{QE}, \mu^{QE})\}_G$ arise from an orthogonal spectrum?*

Ganter showed that $\{QEll^*_G\}_G$ have the change-of-group isomorphism, which is a good sign that quasi-elliptic cohomology may be globalized. By the discussion in Remark 4.1.6 [44], however, the answer to QUESTION 4.1 is no. Then it is even more difficult to see whether each elliptic cohomology theory, whose form is more intricate and mysterious than quasi-elliptic cohomology, can be globalized in the current setting.

Our solution is to establish a new global homotopy theory where quasi-elliptic chomomology can fit into. We hope that it is easier to judge whether a cohomology theory, especially an elliptic cohomology theory, can be globalized in the new theory. In addition we want to show that the new global homotopy theory is equivalent to the current global homotopy theory.

We construct in [28] a category D_0 to replace $\mathbb{L}$ whose objects are (G, V, ρ) with V an inner product vector space, G a compact group and ρ a faithful group representations

$$\rho : G \longrightarrow O(V),$$

and whose morphism $\phi = (\phi_1, \phi_2) : (G, V, \rho) \longrightarrow (H, W, \tau)$ consists of a linear isometric embedding $\phi_2 : V \longrightarrow W$ and a group homomorphism $\phi_1 : \tau^{-1}(O(\phi_2(V))) \longrightarrow G$, which makes the diagram (3) commute.

$$\begin{tikzcd} G \arrow[r, "\rho"] & O(V) \arrow[d, "\phi_{2*}"] \\ \tau^{-1}(O(\phi_2(V))) \arrow[u, "\phi_1"] \arrow[r, "\tau"] & O(W) \end{tikzcd} \tag{3}$$

In other words, the group action of H on $\phi_2(V)$ is induced from that of G. Intuitively, the category D_0 is obtained by adding the restriction maps between representations into the category $\mathbb{L}$.

Instead of the category of orthogonal spaces, we study the category of D_0-spaces. The category of orthogonal spaces is a full subcategory of the category D_0T of D_0-spaces. Apply the idea of diagram spectra in [39], we can also define D_0-spectra and D_0-FSP.

Moreover, we notice that if the equivariant homotopy theories $\{E^*_G(-)\}_G$ is represented by a D_0-spectrum X, it has the property that $X(G, V) \simeq_H X(H, V)$ for any closed subgroup H of G. So what we really need to study are D_0^W-spectra.

DEFINITION 4.2 (The category $Sp^{D_0}_W$). *A D_0^W-spectrum X is both a D_0-spectrum and a D_0-space that maps each restriction map $(G, V) \longrightarrow (H, V)$ to an $H-$weak equivalence. The category $Sp^{D_0}_W$ is the category of D_0^W-spectra.*

Combining the orthogonal $G-$spectra of quasi-elliptic cohomology together, we get a well-defined unitary D_0^W-spectrum and unitary D_0^W-FSP. Thus, we can define global quasi-elliptic cohomology in the category of D_0^W-spectra.

THEOREM 4.3. *(Theorem 7.2.3* [28], [32]*)There is a unitary D_0^W-FSP weakly representing quasi-elliptic cohomology.*

Equipping a homotopy theory with a model structure is like interpreting the world via philosophy. Model category theory is an essential basis and tool to judge whether two homotopy theories describe the same world. We build several model structures on D_0T. First by the theory in [39], there is a level model structure on D_0T.

THEOREM 4.4. *(Theorem 6.3.4* [28], [32]*) $Sp_W^{D_0}$ is a compactly generated topological model category with respect to the level equivalences, level fibrations and $q-$cofibrations. It is right proper and left proper.*

D_0 is a generalized Reedy category in the sense of [13]. We can construct a Reedy model structure on $Sp_W^{D_0}$.

THEOREM 4.5. *(Theorem 6.4.5* [28], [32]*)The Reedy cofibrations, Reedy weak equivalences and Reedy fibrations form a model structure, the Reedy model structure, on the category $Sp_W^{D_0}$.*

In addition, this new global theory describes the same world of homotopy theories as that by Schwede.

THEOREM 4.6. *(*[32]*)There is a global model structure on $Sp_W^{D_0}$ Quillen equivalent to the global model structure on the orthogonal spectra constructed by Schwede in* [44].

Moreover, in [32] we show there is one more properties of $\{E^*(-)\}_G$ that can be inherited by its composition with $\Lambda(-)$, $E^*(\Lambda(-))$.

THEOREM 4.7. *If $\{E^*(-)\}_G$ can be globalized, so are $E^*(\Lambda(-))$.*

5. Research Plan

During the coming two years, I will work on these families of research problems.

- **Generalize the conclusions/constructions on quasi-elliptic cohomology to elliptic cohomology theories.** I will work with Professor Thomas Schick and Professor Chenchang Zhu at Georg-August-Universität Gottingen on this project. Their fields are related to orbifolds and gouproids.
  - One problem we work on is related to groupoids and loop spaces. In Section 1 we used a loop space constructed from bibundles in the definition of quasi-elliptic cohomology. We want to know whether the relation between any elliptic cohomology theory and loop space can be interpreted by bibundles as well.
  - Via quasi-elliptic cohomology theory, we get the classification theorem of finite subgroups and that of level structures on Tate curve. The form of the conclusion is the same as those on Morava E-theories, and hopefully on generalized Morava E-theories. I am cooperating with Nathaniel Stapleton on this. We also want to show that the classification theorems on each elliptic curve can be expressed by the associated elliptic cohomology in the same way as that on Tate curve.
- **Explore the relation between quasi-elliptic cohomology and physics.** I will work on this project with Professor Thomas Schick and Professor Chenchang Zhu at Georg-August-Universität Gottingen. They are also interested in this project.
  - First I plan to express quasi-elliptic cohomology by group schemes since algebraic geometry is related to physics more closely.

  - Then I will construct some physical invariants associated to elliptic cohomology theories (Witten genus etc) on quasi-elliptic cohomology. I am curious what distinct properties and advantages they may have.
- **Another representing object of quasi-elliptic cohomology** Other then equivariant spectra, Professor Chenchang Zhu suggested that another type of representing object for is two-vector bundles [10][11], which is constructed geometrically for elliptic cohomology. I will construct a geometric representing object via two-vector bundles.
- **Construct a global homotopy theory where global elliptic cohomology theories reside.** Quasi-elliptic cohomology serves as a breakthrough point of the construction.
  - Other than Schwede's global homotopy theory [44] which is built via the language of model categories, Gepner constructed a global homotopy theory via orbispaces in [20]. In addition, based on Gepner's theory, Rezk presented in his unpublished manuscript [41] a global homotopy theory with ideas from infinite category theory. The relation between the global theory by Schwede and that by Gepner and Rezk is intriguing and worth exploration.
  - I will study whether global elliptic cohomology theory resides in Gepner and Rezk' global homotopy theory and which theory is more suitable for constructing global elliptic cohomology theory.
  - In addition, I am studying the properties and features of the global homotopy theory in Section 4.
- **Quasi-elliptic cohomology and HKR character theory.** In 1960s Atiyah and Segal showed that the map $R(G) \longrightarrow K^0(BG)$ is an isomorphism after completing $R(G)$ with respect to the ideal of wirtual bundles of dimension 0. In [24] Hopkins, Kuhn and Ravenel generalized Atiyah and Segal's completion theorem. They constructed an isomorphism relating equivariant Morava E_n-theory with an equivariant cohomology theory of chromatic level 0 of a fixed point space $Fix_n(X)$.

  Devoto's equivariant K-theory has a Hopkins-Kuhn-Ravenel (HKR) character theory [24]. Ganter expected the HKR theory for orbifold Tate K-theory to be established via Stapleton's framework of tanschromatic character map. Stapleton constructed in his paper [45] and [46], for each finite $G-$CW complex X and each positive integer n, a topological groupoid $Twist_n(X)$, whose construction is analogous to that of the groupoid $\Lambda(X/\!\!/G)$. With the topological groupoid $Fix_n(X)$ in [24], he constructed in [45] extensions of the generalized character map of Hopkins, Kuhn, and Ravenel [24] for Morava E-theory to every height between 0 and n. And with $Twist_n(X)$, in [46] he constructed the twisted character map which can canonically recover the transchromatic generalized character map in [45]. Apply this transchromatic character theory, he and Schlank provided a new proof of Strickland's theorem in [47] that the Morava E-theory of the symmetric group has an algebro-geometric interpretation after taking the quotient by a certain transfer ideal. Moreover, he showed in [12] this extended character theory can be used to compute the total power operation for the Morava E-theory of any finite group, up to torsion.
  - I will construct the Hopkins-Kuhn-Ravenel (HKR) character theory for $QEll^*$ and that for Tate K-theory. It can probably be fit into Stapleton's transchromatic character theory.
  - An unusual phenomenon that we observe is that the chromatic level of quasi-elliptic cohomology is 1 whereas that of elliptic cohomology theories is 2. Though quasi-elliptic cohomology is very close to elliptic cohomology theories, their chromatic levels are different. I'm cooperating with Nora Ganter and exploring whether Gronowski's elliptic cohomology theories and quasi-elliptic cohomology theory can be related by Stapleton's generalized character map.

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