Level Structures and Morava E-theory

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- Morava E-theory: introduction and construction;
- Generalized Morava E-theory;
- Ingredients:
  - Level structure,
  - transfer ideal,
  - Hopkins-Kuhn-Ravenel character theory;
- The main theorem and the proof;
- Level structure and subgroup.

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Morava E-theories: Introduction

- "designer" cohomology theories: manufactured using homotopy theory, not coming from "nature".
- some arise as completions of "natural" theories, $K_p^\wedge$, $Ell^\wedge_{s.-s.point}$.
- have rich theory of power operations(Ando, Hopkins, Strickland, Rezk, Stapleton, Zhu...)

Fix a formal group G_0 over a perfect field k of characteristic p of height n.

Morava 1978; Goerss-Hopkins-Miller 1993-2004

There exists a cohomology theory E_{G_0} (Morava E-theory) which

- is complex orientable; formal group $\mathrm{Spf}(E^0_{G_0}\mathbb{C}P^\infty)$ = universal deformation of G_0 in sense of Lubin and Tate.
- is represented by a structured commutative ring spectrum

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Formal groups and complex oriented cohomology theories

Formal group law (commutative, 1-dimensional)

$S(x, y) \in R[x, y]$ satisfying axioms for abelian group:

$$S(x,0) = x = S(0,x),$$
$$S(x,y) = S(y,x),$$
$$S(S(x,y),z) = S(x,S(y,z)).$$

Complex oriented cohomology theory

Ring-valued cohomology theory E such that $E^*(\mathbb{C}P^\infty) = E^*[x]$, and x restricts to fundamental class of $\mathbb{C}P^1 = S^2$.

Examples: $H^*(-, \mathbb{Z})$, $K-$theory, Ell, cobordism, Morava E-theories.

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$$\begin{tikzcd} \text{complex oriented cohomology theories} \arrow[r, bend left, "\text{1st Chern class}"] & \text{formal groups} \arrow[l, bend left] \end{tikzcd}$$

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Deformation of formal group

There is a moduli problem associated to G_0/k.

Deformation of formal groups [Lubin-Tate, 1966]

$$LT : \text{Complete Local Rings} \longrightarrow \text{Groupoids}$$

Objects of $LT(R, m)$: Deformation $(G/R,\ i : k \to R/m,\ \alpha)$:

- G is a formal group over R;
- $\alpha : i^* G_0 \xrightarrow{\cong} \pi^* G$ isomorphism of formal groups over R/m.

Isomorphisms $(G, i, \alpha) \longrightarrow (G', i, \alpha')$

- $\star-$isomorphism: iso $f : G \longrightarrow G'$ compatible with id of G_0.

Universal Deformation [Lubin-Tate 1966]

There exists a universal deformation $(\mathbb{G}_u, i_u, \alpha_u)$ over $\mathcal{O}_{LT} \cong W(k)[a_1, \cdots a_{n-1}]$.

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Strickland's theorem 1998

$$\mathrm{Spec}(E_n^0(B\Sigma_{p^k})/I_{tr}) \cong \mathrm{Sub}_{p^k}(\mathbb{G}_u)$$

Additive Power Operation [Proposition 3.21, AHS 2004]

$$P_{p^k}/I_{tr} : E_n^0(BA) \longrightarrow E_n^0(BA) \otimes_{E_n^0} E_n^0(B\Sigma_{p^k})/I_{tr}.$$

$$\mathrm{Sub}_{p^k}(\mathbb{G}_u) \times_{LT} \mathrm{Hom}(A^*, \mathbb{G}_u) \longrightarrow \mathrm{Hom}(A^*, \mathbb{G}_u)$$

Theorem [Proposition 5.12, HKR 2000]

A: finite abelian group. There is a canonical isomorphism of E^0−algebras

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- $\mathcal{L}BG := EG \times_G \mathcal{L}(* /\!\!/ G) \simeq \mathrm{Map}(B\mathbb{Z}_p, BG)$.
- $E_G(-)$ is a cohomology theory on finite $G-$CW complexes $\Rightarrow$ So is $E_G(\mathcal{L}(-))$.
- the algebro-geometric object associated to $E^0_{\mathbb{Z}/p^k}(\mathcal{L}^h(-))$ is the $p-$divisible group $\mathbb{G}_E \oplus (\mathbb{Q}_p/\mathbb{Z}_p)^h$.
- $\mathcal{L}(-)$ is a key functor in the target of Hopkins-Kuhn-Ravenel generalized character map and Stapleton's transchromatic generalized character maps.

2015, Tomer M.Schlank, Nathaniel Stapleton

$$\mathrm{Spec}(E_n^0(\mathcal{L}^h B\Sigma_{p^k})/I_{tr}) \cong \mathrm{Sub}_{p^k}(\mathbb{G}_u \oplus \mathbb{T}')$$

Via transchromatic character theory.

Generalized Morava E-theory $E_n^*(\mathcal{L}(X/\!\!/G))$

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2015, Tomer M.Schlank, Nathaniel Stapleton

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Level Structure

A: finite abelian $p-$group.

Classical Definition: Level structures on formal groups [Drinfeld 74]

A level structure $f : A \longrightarrow \mathbb{G}$ is a homomorphism from A to $\mathbb{G}$

- $rank(A) \leq height(\mathbb{G})$;
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$$\begin{array}{ccc} \mathrm{Spf}(E_n^0(BA)/I_{tr}) & \longrightarrow & \mathrm{Spf}(E_n^0(B\Sigma_{p^k})/I_{tr}) \\ \downarrow \cong & & \downarrow \\ \mathrm{Level}(A^*, \mathbb{G}_u) & \xrightarrow{f \mapsto \mathrm{im}\, f} & \mathrm{Sub}_{p^k}(\mathbb{G}_u). \end{array}$$

$A-$Level Structure: Definition in the generalized case [Huan, Stapleton]

l: $A \hookrightarrow \mathbb{G} \oplus \mathbb{T}'$: a homomorphism of group schemes such that the induced map $\ker(\pi l) \rightarrow \mathbb{G}$ is a $\ker(\pi l)$-level structure on $\mathbb{G}$.

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For $H \hookrightarrow G$, $BH \longrightarrow BG$ is a finite cover.

$$\mathrm{Tr}_E : E^0(BH) \longrightarrow E^0(BG)$$

$$\bigoplus_{0<i<m} \mathrm{Tr}_E : \bigoplus_{0<i<m} E^0(BG \wr (\Sigma_i \times \Sigma_{m-i})) \longrightarrow E^0(BG \wr \Sigma_m).$$

Transfer Ideal

$$I_{tr} = \mathrm{im}(\bigoplus_{0<i<m} \mathrm{Tr}_E) \subset E^0(BG \wr \Sigma_m)$$

I_{tr} is the smallest ideal such that the quotient

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$$\text{Recall } P_m(x \otimes y) = \mathrm{res}|^{G\wr\Sigma_m \times G\wr\Sigma_m}_{G\wr\Sigma_m}(P_m(x) \otimes P_m(y))).$$

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Hopkins-Kuhn-Ravenel Character Theory

Motivation [Atiyah, Segal 1969][Adams 1978]

- Classical representation theory:

$$\mathbb{C} \otimes \chi : \mathbb{C} \otimes RG \xrightarrow{\cong} Cl(G, \mathbb{C}).$$

- p-adic K-theory:

$$\begin{tikzcd} RG \arrow[r, "\alpha"] \arrow[d] & K(BG) \arrow[d] \\ \mathbb{Z}_p \otimes RG \arrow[r, "\hat{\alpha}", "\cong"'] & K(BG; \mathbb{Z}_p). \end{tikzcd}$$

Character theory of Morava E-theory [Theorem C, HKR 2000]

$$\chi : E_n^0(BG) \longrightarrow Cl_n(G, C_0)$$

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More on Hopkins-Kuhn-Ravenel Character Theory

More explanation on the character map $\chi\colon E^0(BG) \longrightarrow Cl_n(G, C_0)$

- $C_0 := S^{-1} E^0_{cont}(B\,\mathbb{L})$, where
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  C_0 is the rationalization of the Drinfeld ring $\mathcal{O}_{\mathrm{Level}(\mathbb{T},\mathbb{G}_u)}$.
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Construction of the character map $\prod \chi_{[\alpha]}$

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Hopkins-Kuhn-Ravenel Character theory and transfer

Transfer map and Character theory [Theorem D, HKR 2000]

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Main Theorem

Classical result: $(E_n^0(BA)/I_{tr})^{\text{free}} \xrightarrow{\cong} \mathcal{O}_{\text{Level}(A^*,\mathbb{G}_u)}$.

$$R^{\text{free}} := \operatorname{im}(R \longrightarrow \mathbb{Q} \otimes R).$$

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Theorem [Huan, Stapleton]

There is a canonical isomorphism of E^0-algebras

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Theorem [Huan, Stapleton]

There is a canonical isomorphism of E^0-algebras

$$(E_n^0(\mathcal{L}^h BA)/I_A)^{\text{free}} \cong \mathcal{O}_{\text{Level}(A^*,\mathbb{G}_u \oplus \mathbb{T}')}.$$

$$\begin{tikzcd}
\text{Level}(A^*, \mathbb{G}_u \oplus \mathbb{T}') \arrow[d] \arrow[dr] & \\
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Main Theorem

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Proof

Use Hopkins-Kuhn-Ravenel character theory.

$$\begin{tikzcd}
E_n^0(\mathcal{L}^h(BA)) \arrow[r, "\cong"] \arrow[d] & \mathcal{O}_{\mathrm{Hom}(A^*,\mathbb{G}_u\oplus\mathbb{T}')} \arrow[d] \\
E_n^0(\mathcal{L}^h(BA))/I_A \arrow[r, dotted] \arrow[d, hook] & \mathcal{O}_{\mathrm{Level}(A^*,\mathbb{G}_u\oplus\mathbb{T}')} \arrow[d, hook] \\
C_0 \otimes_{E_n^0} E_n^0(\mathcal{L}^h(BA))/I_A \arrow[r, "\cong"] & \prod\limits_{\mathrm{Level}(A^*,\mathbb{T}\oplus\mathbb{T}')} C_0.
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$$E_n^0(\mathcal{L}^h BA)/I_{tr} = \prod_{f:\,\mathbb{L}'\to A} E_n^0(BA)/I_{\mathcal{F}_f};$$

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Definition $\mathrm{Level}_f(A^*, \mathbb{G}_u \oplus \mathbb{T}')$

The pullback of schemes

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The subgroup scheme $\mathrm{Sub}^{A}_{p^k}(\mathbb{G}_u \oplus \mathbb{T}')$

$A \subset \mathbb{T}'$. Define $\mathrm{Sub}^{A}_{p^k}(\mathbb{G}_u \oplus \mathbb{T}') : E^0_n - algebras \longrightarrow Set$

$$R \mapsto \{H \subseteq R \otimes \mathbb{G}_u \oplus \mathbb{T}' || H| = p^k, pr(H) = A\}.$$

Corollary [Schlank, Stapleton]

$\alpha : \mathbb{L}' \longrightarrow \Sigma_{p^k}$ monotypical. $C_{\Sigma_{p^k}}(\mathrm{im}\,\alpha) \cong \mathrm{im}\,\alpha \wr \Sigma_{p^j}$, $p^j = p^k/|\,\mathrm{im}\,\alpha|$.

$$\mathrm{Spec}(E^0_n(B(\mathrm{im}\,\alpha) \wr \Sigma_{p^j})/I^{[\alpha]}_{tr}) \cong \mathrm{Sub}^{(\mathrm{im}\,\alpha)^*}_{p^k}(\mathbb{G}_u \oplus \mathbb{T}').$$

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$\alpha : \mathbb{L}' \longrightarrow \Sigma_{p^k}$ monotypical. $C_{\Sigma_{p^k}}(\mathrm{im}\,\alpha) \cong \mathrm{im}\,\alpha \wr \Sigma_{p^j}$, $p^j = p^k/|\,\mathrm{im}\,\alpha|$.

$$\mathrm{Spec}(E^0_n(B(\mathrm{im}\,\alpha) \wr \Sigma_{p^j})/I^{[\alpha]}_{tr}) \cong \mathrm{Sub}^{(\mathrm{im}\,\alpha)^*}_{p^k}(\mathbb{G}_u \oplus \mathbb{T}').$$

$|A| = p^k$. $\alpha : \mathbb{L}' \xrightarrow{f} A \underset{\text{Caley embedding}}{\overset{i}{\hookrightarrow}} \Sigma_{p^k}$ is monotypical.

$$\begin{tikzcd}
\mathrm{Spf}(E^0_n(BA)/I_{\mathcal{F}_f}) \arrow[r] \arrow[d] & \mathrm{Spf}(E^0_n(B(\mathrm{im}\, if) \wr \Sigma_{p^j})/I^{[if]}_{tr}) \arrow[d] \\
\mathrm{Level}_f(A^*, \mathbb{G}_u \oplus \mathbb{T}') \arrow[r, "l \mapsto \mathrm{im}\, l"] & \mathrm{Sub}^{(\mathrm{im}\, f)^*}_{p^k}(\mathbb{G}_u \oplus \mathbb{T}').
\end{tikzcd}$$

More geometric power operation

Tate K-theory: the generalized elliptic cohomology associated to the Tate curve.

Strickland's theorem for Tate K-theory [Huan]

The Tate K-theory of symmetric groups modulo a certain transfer ideal classifies finite subgroups of the Tate curve.

$$K_{Tate}(\mathrm{pt}/\!\!/\Sigma_N)/I_{tr}^{Tate} \cong \prod_{N=de} \mathbb{Z}((q))[q_s'^{\pm}]/\langle q^d - q_s'^{e}\rangle,$$

where I_{tr}^{Tate} is the transfer ideal and q_s' is the image of q under the stringy power operation, the product goes over all the ordered pairs of positive integers (d, e) such that $N = de$.

A^*−level structure on Tate curve [Huan]

A: any finite abelian group.
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Further problems

Projects with Nathaniel Stapleton

- Construct power operation of generalized Morava E-theory;
- and other problems on Morava E-theories.

The Definition of $n-$vector bundles

- (1-)vector bundles $\Rightarrow$ topological K-theory whose chromatic level is 1;
- 2-vector bundles $\Rightarrow$ a form of elliptic cohomology theory whose chromatic level is 2.

Questions:

- Definition of $n-$vector bundle? Definition of equivariant $n-$vector bundle?
- $n-$vector bundle $\Rightarrow$ chromatic level n cohomology theory?
- Definition of $n-$gerbes to give a right loop based definition of the level n cohomology theory?

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We've constructed twisted quasi-elliptic cohomology and a twisted Chern character map from it to twisted equivariant elliptic cohomology.

1. Construct a power operation for Devoto's equivariant elliptic cohomology compatible with the Chern character map and compare it with the power operation [Evans-Berwick, Stapleton].
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Thank you.

Some references

https://huanzhen84.github.io/zhenhuan/Huan-YMF-2019-Slides.pdf

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