FE8814 Derivative Securities – 2021- Course Project

Answer all questions below. Your responses to the questions should be as clear and concise as possible. Your submission should include the following:

- Submitted report should be typed (not handwritten) and should include a cover page providing names of all team members
- In addition to answering questions, your responses should also include, if applicable, the source of data, additional assumption(s) you are imposing for your analysis, references, etc.
- The original source code for obtaining the numerical results.
- A (very) brief documentation on how the code can be executed so that I am able to run the program and reproduce your results.

Deadline: 1 November 2021 @ 12 noon

Before due date, upload your report through NTUlearn. Each team should only upload your report ONCE.

Question 1: Modeling volatility

Find data for historical prices for any publicly traded stock (for which the stock also trades options). To ensure the length of data, use daily data for at least 3 years. Fit the historical data to

a) A geometric Brownian motion
b) Any one of the non-constant volatility models

Question 2: Consider a European call option (you may assume any desirable maturity and strike price) is written on the stock selected in Question 1

a) Use the **non-constant volatility model** obtained in Question 1(b) and estimate the price of the European call option using the Monte Carlo method.
b) Use the **Geometric Brownian motion** obtained in Question 1(a), calculate the price of the European call option using
    i. The Black-Scholes formula
    ii. Binomial Model
c) Assuming the Black-Scholes price calculated from Part b(i) is the theoretically correct price of the option, conduct numerically the convergence of the prices obtained from both binomial model.

Question 3: Variance reduction techniques.

Assume Black-Scholes model with $S_0 = 100, r = 0.05, \sigma = 25\%$ p.a. and you are interested in estimating the prices of discretely monitoring Asian call options with 1 year until maturity and the same strike price of $K = 100$.

a) Use the crude Monte Carlo method (with sample paths 1000, 5000, 10000, 50000, 100000) to estimate Asian call options with weekly, monthly, and daily monitoring. Comments on your simulation results.
b) Repeat Part (b) except using Quasi-Monte Carlo method (you may use any low discrepancy sequence). Comments on your results.
c) Using the daily monitoring Asian call option as the base example, present two different variance reduction techniques that can be used to improve the efficiency of the crude Monte Carlo methods. Demonstrate numerically their efficiency gained relative to the crude Monte Carlo method.

Question 4: HSBC Life Variable Annuity

You are a financial engineer that works at HSBC Life. Your boss is interested in the "true cost" of the option embedded in the HSBC Life Variable Annuity product. Using Joe, aged 45, as an illustrative base case (see page 8 of the product brochure, 15 years of accumulation period and 20 years of payout period) and assume that the underlying fund can be modelled by a Geometric Brownian motion with $\sigma = 20\%$ p.a. and $r = 3\%$, conduct the following analysis (you may ignore mortality risk for simplicity):

a) Determine the value of the embedded option using either the Monte Carlo or the quasi-Monte Carlo method.
b) Based on your results in Part (a), comment on the charges that are currently being charged by HSBC Life.

Plagiarism is strictly prohibited