FEB 02 2021 MATH 134B

CHAO-MING LIN

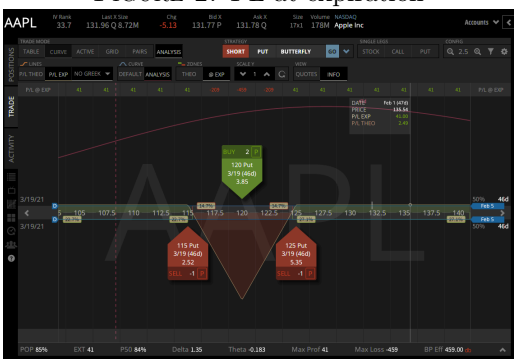
1. Some option strategies

The interested reader is referred to "Trade Options with an Edge" [1] by R. Richards or website Tastytrade for further understanding. They have done some researches and will provide you statistical evidences that support their claims. Let's list some famous option strategies here:
-Butterfly: A butterfly spread is an options strategy combining bull and bear spreads, with a fixed risk and capped profit. These spreads, involving either four calls or four puts are intended as a market-neutral strategy and pay off the most if the underlying does not move prior to option expiration.

Example (Strangles with different Delta)**.** *Take Apple's options for example, we have*

Figure 1. Short put Butterfly

Figure 2. PL at expiration



A ***butterfly*** *consists of four contracts, a short put butterfly means you collect premium. On fig 1, we sell a 125-strike put and a 115-strike put, then we buy two 120-strike puts. Both the maximum profit and the maximum loss are limited, also the PoP is* 85% *which is quite decent. In this case, if you consider the ratio of your maximum profit and the buying power, that will be*

$$\frac{41}{459} \sim 9\%.$$

So based on Black-Scholes model, you have a 85% *chance to have a* 9% *return on Capital. The P50 is* 84%*, which is also high. So short butterflies are a popular strategy for option-sellers.*

Remark. *A remark here is that to make a* ***Butterfly****, you need four contracts. If the Bid/Ask spread is too wide, then it is hard to fill your order and your profit will also be reduced. So make sure this company's options have sufficient liquidity.*

Theorem (The Put-Call Parity Equation)**.** *Let K be the strike price and T be the periods until expiration, then we have*

$$\text{Call}(K,T) - \text{Put}(K,T) = \text{PV}(F_{0,T} - K),$$

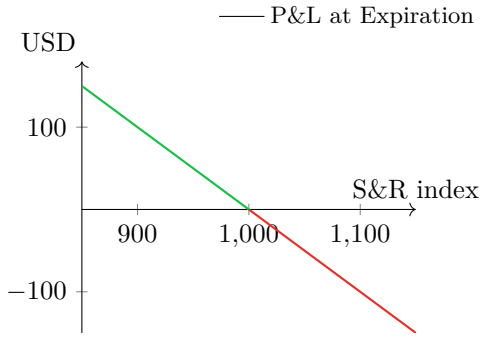
where Call *is the call price,* Put *is the put price, and* PV *is the present value.*

2. Question solving

Question (3.7)**.** *Verify that you earn the same profit and payoff by*
(a) *shorting the S&R index for $1000 and*
(b) *selling a 1050-strike S&R call, buying a 1050-strike put, and borrowing $1029.41.*

Solution.
(a) The profit and payoff for shorting the S&R index for $1000 will be the following

Shorting S&R index for $1000					
S&R index	$900	$950	$1000	$1050	$1100
Closed S&R	−$900	−$950	−$1000	−$1050	−$1100
Payoff	−$900	−$950	−$1000	−$1050	−$1100
Future Value	$1020	$1020	$1020	$1020	$1020
Profit	$120	$70	$20	−$30	−$80



(b) The profit and payoff for selling a 1050-strike S&R call with premium $71.802, buying a 1050-strike put with premium $101.214, and borrowing $1029.41 will be the following

Selling 1050-strike call, buying 1050-strike put, and borrowing money					
S&R index	$900	$950	$1000	$1050	$1100
Call	$0	$0	$0	$0	−$50
Put	$150	$100	$50	$0	$0
Borrowing	−$1050	−$1050	−$1050	−$1050	−$1050
Payoff	−$900	−$950	−$1000	−$1050	−$1100
Future Value	$1020	$1020	$1020	$1020	$1020
Profit	$120	$70	$20	−$30	−$80

Note that the future value will be

$$\begin{aligned}\text{FV} &= (\$1029.41(\text{Borrowing}) + \$71.802(\text{Call's Premium}) - \$101.214(\text{Put's Premium})) \times 1.02 \\ &\sim \$1020.\end{aligned}$$

□

Question (3.8). *Suppose the premium on a 6-month S&R call is \$109.20 and the premium on a put with the same strike price is \$60.18. What is the strike price?*

Solution. By the Put-Call Parity Equation, we assume the current strike is K, then we have

$$\text{Call}(K, \text{6-month}) - \text{Put}(K, \text{6-month}) = \text{PV}(F_{0,\text{6-month}} - K),$$

which implies that

$$\$109.20 - \$60.18 = \text{PV}(\$1020 - \$K) = \$1000 - \$\frac{K}{1.02}.$$

We get

$$\$K = (\$1000 - \$49.02) \times 1.02 \sim \$970.$$

□

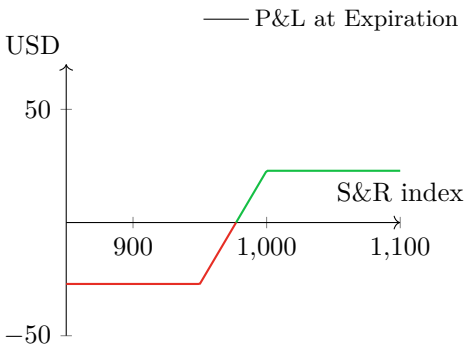
Question (3.9). *Construct payoff and profit diagrams for the purchase of a 950-strike S&R call and sale of a 1000-strike S&R call.Verify that you obtain exactly the same profit diagram for the purchase of a 950-strike S&R put and sale of a 1000-strike S&R put. What is the difference in the payoff diagrams for the call and put spreads? Why is there a difference?*

Solution. If you buy a 950-strike S&R call with premium \$120.405 and sell of a 1000-strike S&R call with premium \$93.809, since the strike you buy is less than the strike you sell, this will be a long call spread, the payoff and profit diagrams will be the following

Long call spread					
S&R index	\$900	\$950	\$1000	\$1050	\$1100
950-strike Call	\$0	\$0	\$50	\$100	\$150
1000-strike Call	\$0	\$0	\$0	−\$50	−\$100
Payoff	\$0	\$0	\$50	\$50	\$50
Future Value	−\$27.128	−\$27.128	−\$27.128	−\$27.128	−\$27.128
Profit	−\$27.128	−\$27.128	\$22.872	\$22.872	\$22.872

Note that the future value will be

$$\text{FV} = -(\$120.405 - \$93.809) \times 1.02 = -\$27.128$$



On the other hand, if you buy a 950-strike S&R put with premium \$51.777 and sell a 1000-strike S&R put with premium \$74.201, since the strike you buy is less than the strike you sell, this will be a short put spread, the payoff and profit diagrams will be the following

Short put spread					
S&R index	\$900	\$950	\$1000	\$1050	\$1100
950-strike Put	\$50	\$0	\$0	\$0	\$0
1000-strike Put	−\$100	−\$50	\$0	\$0	\$0
Payoff	−\$50	−\$50	\$0	\$0	\$0
Future Value	\$22.872	\$22.872	\$22.872	\$22.872	\$22.872
Profit	−\$27.128	−\$27.128	\$22.872	\$22.872	\$22.872

Note that the future value will be

$$\text{FV} = (\$74.201 - \$51.777) \times 1.02 = \$22.872.$$

So we obtain the same profit diagram, the difference is that the payoff diagram for the second one is exactly \$50 lower than the payoff diagram for the first one. This is because you have to pay premium to fill a long call spread in the beginning. On the other hand, for short put spread, you obtain premium in the beginning, so you have different payoff diagrams. But by Put-Call Parity Equation, we can see that

$$\begin{cases} \text{Call}(950,T) - \text{Put}(950,T) & = \text{PV}(1020-950) = 68.628, \\ \text{Call}(1000,T) - \text{Put}(1000,T) & = \text{PV}(1020-1000) = 19.608. \end{cases}$$

Hence,

$$\begin{aligned} &\text{FV}\left(\text{Call}(1000,T) - \text{Call}(950,T)\right) + \max\{0, K-950\} - \max\{0, K-1000\} \\ =&\,\text{FV}\left(\text{Put}(1000,T) - \text{Put}(950,T) - \text{PV}(1000-950)\right) + \max\{0, K-950\} - \max\{0, K-1000\} \\ =&\,\text{FV}\left(\text{Put}(1000,T) - \text{Put}(950,T)\right) - 50 + \max\{0, K-950\} - \max\{0, K-1000\} \\ =&\,\text{FV}\left(\text{Put}(1000,T) - \text{Put}(950,T)\right) + \max\{0, 950-K\} - \max\{0, 1000-K\} \end{aligned}$$

□

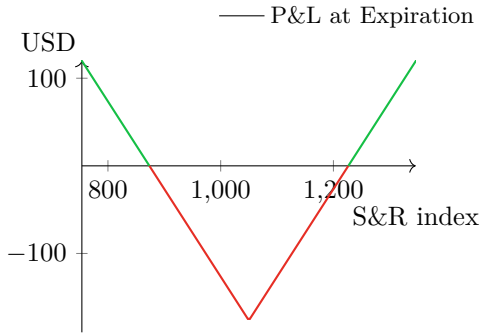
Question (3.13)**.** *Draw profit diagrams for the following positions:*
a. *1050-strike S&R straddle.*
b. *Written 950-strike S&R straddle.*
c. *Simultaneous purchase of a 1050-strike straddle and sale of a 950-strike S&R straddle.*

Solution.
a. For the 1050-strike S&R straddle, you buy the 1050-strike call with premium \$71.802 and buy the 1050-strike put with premium \$101.214, thus

Long 1050-strike straddle					
S&R index	\$950	\$1000	\$1050	\$1100	\$1150
1050-strike Call	\$0	\$0	\$0	\$50	\$100
1050-strike Put	\$100	\$50	\$0	\$0	\$0
Payoff	\$100	\$50	\$0	\$50	\$100
Future Value	−\$176.476	−\$176.476	−\$176.476	−\$176.476	−\$176.476
Profit	−\$76.476	−\$126.476	−\$176.476	−\$126.476	−\$76.476

Note that the future value will be

$$\text{FV} = -(\$71.802 + \$101.214) \times 1.02 = -\$176.476$$

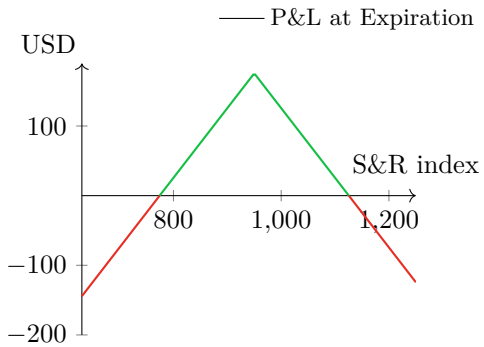


b. For the written 950-strike S&R straddle, you sell the 950-strike call with premium \$120.405 and sell the 950-strike put with premium \$51.777, thus

Short 950-strike straddle					
S&R index	\$850	\$900	\$950	\$1000	\$1050
950-strike Call	\$0	\$0	\$0	−\$50	−\$100
950-strike Put	−\$100	−\$50	\$0	\$0	\$0
Payoff	−\$100	−\$50	\$0	−\$50	−\$100
Future Value	\$175.626	\$175.626	\$175.626	\$175.626	\$175.626
Profit	\$75.626	\$125.626	\$175.626	\$125.626	\$75.626

Note that the future value will be

$$\text{FV} = (\$120.405 + \$51.777) \times 1.02 = \$175.626$$

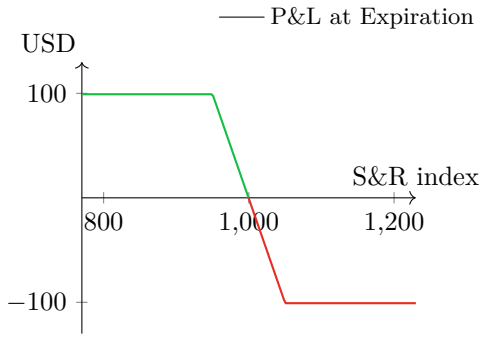


c. If we buy a 1050-strike straddle and sell a 950-strike S&R straddle. First, we buy the 1050-strike call with premium \$71.802 and buy the 1050-strike put with premium \$101.214. Then, we sell the 950-strike call with premium \$120.405 and sell the 950-strike put with premium \$51.777. Thus,

Long 1050-strike straddle and short 950-strike straddle					
S&R index	\$900	\$950	\$1000	\$1050	\$1100
1050-strike Call	\$0	\$0	\$0	\$0	\$50
1050-strike Put	\$150	\$100	\$50	\$0	\$0
950-strike Call	\$0	\$0	−\$50	−\$100	−\$150
950-strike Put	−\$50	\$0	\$0	\$0	\$0
Payoff	\$100	\$100	\$0	−\$100	−\$100
Future Value	−\$0.85	−\$0.85	−\$0.85	−\$0.85	−\$0.85
Profit	\$99.15	\$99.15	−\$0.85	−\$100.85	−\$100.85

Note that the future value will be

$$\text{FV} = -\left(\$71.802 + \$101.214\right) \times 1.02 + \left(\$120.405 + \$51.777\right) \times 1.02 = -\$0.85$$



□

Question (3.18). *Verify that the butterfly spread in Figure 3 can be duplicated by the following transactions (use the option prices in Table 1, see below):*

a. *Buy 35 call, sell two 40 calls, buy 45 call.*
b. *Buy 35 put, sell two 40 puts, buy 45 put.*
c. *Buy stock, buy 35 put, sell two 40 calls, buy 45 call.*

FIGURE 3. Comparison of Butterfly and written straddle

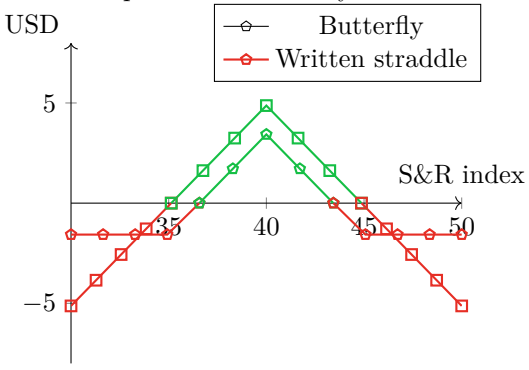


TABLE 1. Black–Scholes option prices

Black–Scholes option prices assuming stock price = \$40, volatility = 30%, effective annual risk-free rate = 8.33% (8%, continuously compounded), dividend yield = \$0, and 91 days to expiration.		
Strike	Call	Put
35	\$6.13	\$0.44
40	\$2.78	\$1.99
45	\$0.97	\$5.08

Solution.

a. If we buy 35-strike call with premium \$6.13, sell two 40-strike calls with premium \$2.78 each, and buy 45-strike call with premium \$0.97. Thus, in the beginning, you pay $\$6.13 - 2 \times \$2.78 + \$0.97 = \$1.54$ premium, so it is a long call butterfly. The following is the payoff and profit diagram

Long call Butterfly					
Stock price	\$30	\$35	\$40	\$45	\$50
35-strike Call	\$0	\$0	\$5	\$10	\$15
two 40-strike Call	\$0	\$0	\$0	−\$10	−\$20
45-strike Call	\$0	\$0	\$0	\$0	\$5
Payoff	\$0	\$0	\$5	\$0	\$0
Future Value	−\$1.57	−\$1.57	−\$1.57	−\$1.57	−\$1.57
Profit	−\$1.57	−\$1.57	\$3.43	−\$1.57	−\$1.57

Note that to compute the future value, you can either use the effective annual risk-free rate or the continuously compounded one, if the number you are multiplying is small, then the answers will be really close to each other.

(Effective annual risk-free)

$$\text{FV} = -\$1.54 \times (1 + 8.33\% \times \frac{91}{365}) = -\$1.572.$$

(Continuously compounded)

$$\text{FV} = -\$1.54 \times \exp(8\% \times \frac{91}{365}) = -\$1.571.$$

b. If we buy 35-strike put with premium \$0.44, sell two 40-strike puts with premium \$1.99 each, and buy 45-strike put with premium \$5.08. Thus, in the beginning, you pay $\$0.44 - 2 \times \$1.99 + \$5.08 = \$1.54$ premium, so it is a long put butterfly. The following is the payoff and profit diagram

Long put Butterfly					
Stock price	\$30	\$35	\$40	\$45	\$50
35-strike Put	\$5	\$0	\$0	\$0	\$0
two 40-strike Put	−\$20	−\$10	\$0	\$0	\$0
45-strike Put	\$15	\$10	\$5	\$0	\$0
Payoff	\$0	\$0	\$5	\$0	\$0
Future Value	−\$1.57	−\$1.57	−\$1.57	−\$1.57	−\$1.57
Profit	−\$1.57	−\$1.57	\$3.43	−\$1.57	−\$1.57

c. If we buy stock at \$40, buy 35-strike put with premium \$0.44, sell two 40-strike calls with premium \$2.78 each, and buy 45-strike call with premium \$0.97, then, in the beginning, we pay $\$40 + \$0.44 - 2 \times \$2.78 + \$0.97 = \$35.85$.

Long call Butterfly + Put-Call Parity					
Stock price	\$30	\$35	\$40	\$45	\$50
Close Stock	\$30	\$35	\$40	\$45	\$50
35-strike Put	\$5	\$0	\$0	\$0	\$0
two 40-strike Call	\$0	\$0	\$0	−\$10	−\$20
45-strike Call	\$0	\$0	\$0	\$0	\$5
Payoff	\$35	\$35	\$40	\$35	\$35
Future Value	−\$36.57	−\$36.57	−\$36.57	−\$36.57	−\$36.57
Profit	−\$1.57	−\$1.57	\$3.43	−\$1.57	−\$1.57

Note that here, the continuously compounded one is more accurate, we can compute the future value using either way.

(Effectixve annual risk-free)

$$\text{FV} = -\$35.85 \times (1 + 8.33\% \times \frac{91}{365}) = -\$36.595.$$

(Continuously compounded)

$$\text{FV} = -\$35.85 \times \exp(8\% \times \frac{91}{365}) = -\$36.572.$$

□

References

[1] R. Richards, *Trade Options with an Edge*.

[2] Tastytrade . https://www.tastytrade.com

[3] Investopedia, *Butterfly Spread*.
https://www.investopedia.com/terms/b/butterflyspread.asp