

## FEB 09 2021 MATH 134B

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### 1. TERMINOLOGIES

**Short Sale Rebate Fee:** When a short seller borrows shares to make delivery to the buyer, the seller must pay a rebate fee. This fee depends on the dollar amount of the sale and the availability of the shares in the marketplace. If the shares are difficult or expensive to borrow, the rebate fee will be higher. In some instances, the brokerage firm will force the short seller to buy the securities in the market before the settlement date, which is referred to as a forced buy-in. A brokerage firm may require a forced buy-in if it believes that the shares will not be available on the settlement date.

### 2. QUESTION SOLVING

Homework 4 is due on Tuesday, February 16. Only problems 1, 3, 4, 7, 8, 9, and 12 are required, others are optional.

For the following problems consider the following three firms:

**XYZ** mines copper, with fixed costs of \$0.50/lb and variable cost of \$0.40/lb.

**Wirco** produces wire. It buys copper and manufactures wire. One pound of copper can be used to produce one unit of wire, which sells for the price of copper plus \$5. Fixed cost per unit is \$3 and non-copper variable cost is \$1.50.

**Telco** installs telecommunications equipment and uses copper wire from **Wirco** as an input. For planning purposes, **Telco** assigns a fixed revenue of \$6.20 for each unit of wire it uses.

The 1-year forward price of copper is \$1/lb. The 1-year continuously compounded interest rate is 6%. One-year option prices for copper are shown in the table below. In your answers, at a

TABLE 1. Option prices

One-year option prices for copper		
Strike	Call	Put
0.9500	\$0.0649	\$0.0178
0.9750	\$0.0500	\$0.0265
1.0000	\$0.0376	\$0.0376
1.0250	\$0.0274	\$0.0509
1.0340	\$0.0243	\$0.0563
1.0500	\$0.0194	\$0.0665

minimum consider copper prices in 1 year of \$0.80, \$0.90, \$1.00, \$1.10, and \$1.20.

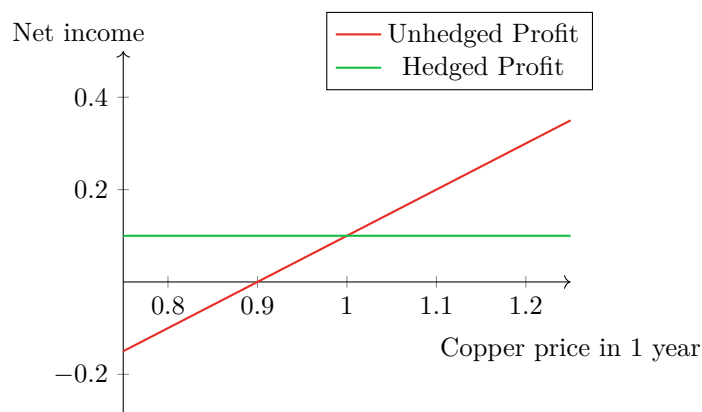
**Question (4.1).** If **XYZ** does nothing to manage copper price risk, what is its profit 1 year from now, per pound of copper? If on the other hand **XYZ** sells forward its expected copper production, what is its estimated profit 1 year from now? Construct graphs illustrating both unhedged and hedged profit.

*Solution.* First, if **XYZ** does nothing to manage copper price risk, the profit 1 year from now will be the following

Unhedged profit					
Copper price in 1 year	\$0.80	\$0.90	\$1.00	\$1.10	\$1.20
Selling copper	\$0.80/lb	\$0.90/lb	\$1.00/lb	\$1.10/lb	\$1.20/lb
Fixed costs	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb
Variable cost	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb
Net income	-\$0.10/lb	\$0/lb	\$0.10/lb	\$0.20/lb	\$0.30/lb

On the other hand, if **XYZ** hedge their position, notice that the 1-year forward price of copper is \$1/lb, so the profit will be the following

Hedged profit					
Copper price in 1 year	\$0.80	\$0.90	\$1.00	\$1.10	\$1.20
Selling copper	\$0.80/lb	\$0.90/lb	\$1.00/lb	\$1.10/lb	\$1.20/lb
Fixed costs	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb
Variable cost	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb
Short forward	\$0.20/lb	\$0.10/lb	\$0/lb	-\$0.10/lb	-\$0.20/lb
Net income	\$0.10/lb	\$0.10/lb	\$0.10/lb	\$0.10/lb	\$0.10/lb



□

**Question (4.3).** Compute estimated profit in 1 year if **XYZ** buys a put option with a strike of \$0.95, \$1.00, or \$1.05. Draw a graph of profit in each case.

*Solution.* Based on table 1, if we buy a 0.95-strike put to hedge our position, we have

Hedged profit of buying 0.95-strike put in 1 year					
Copper price in 1 year	\$0.80	\$0.90	\$1.00	\$1.10	\$1.20
Selling copper	\$0.80/lb	\$0.90/lb	\$1.00/lb	\$1.10/lb	\$1.20/lb
Fixed costs	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb
Variable cost	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb
0.95-strike put	\$0.15/lb	\$0.05/lb	\$0/lb	\$0.00/lb	\$0.00/lb
Future value	-\$0.02/lb	-\$0.02/lb	-\$0.02/lb	-\$0.02/lb	-\$0.02/lb
Profit	\$0.03/lb	\$0.03/lb	\$0.08/lb	\$0.18/lb	\$0.28/lb

Note that here, the future value is obtained by

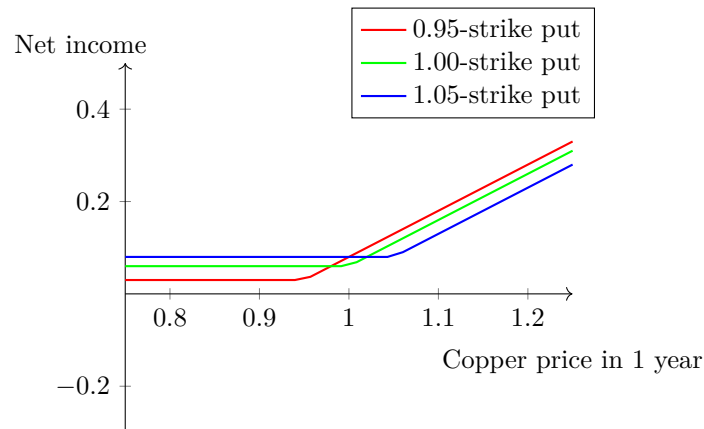
$$FV = -\$0.0178 \times \exp\left(6\% \cdot \frac{365}{365}\right) \sim -\$0.02.$$

Also, if we buy a 1.00-strike put to hedge our position, we get

Hedged profit of buying 1.00-strike put in 1 year					
Copper price in 1 year	\$0.80	\$0.90	\$1.00	\$1.10	\$1.20
Selling copper	\$0.80/lb	\$0.90/lb	\$1.00/lb	\$1.10/lb	\$1.20/lb
Fixed costs	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb
Variable cost	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb
1.00-strike put	\$0.20/lb	\$0.10/lb	\$0/lb	\$0.00/lb	\$0.00/lb
Future value	-\$0.04/lb	-\$0.04/lb	-\$0.04/lb	-\$0.04/lb	-\$0.04/lb
Profit	\$0.06/lb	\$0.06/lb	\$0.06/lb	\$0.16/lb	\$0.26/lb

Note that here, the future value is obtained by

$$FV = -\$0.0376 \times \exp\left(6\% \cdot \frac{365}{365}\right) \sim -\$0.04.$$



Last, if we buy a 1.05-strike put to hedge our position, we obtain

Hedged profit of buying 1.05-strike put in 1 year					
Copper price in 1 year	\$0.80	\$0.90	\$1.00	\$1.10	\$1.20
Selling copper	\$0.80/lb	\$0.90/lb	\$1.00/lb	\$1.10/lb	\$1.20/lb
Fixed costs	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb
Variable cost	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb
1.05-strike put	\$0.25/lb	\$0.15/lb	\$0.05/lb	\$0/lb	\$0/lb
Future value	-\$0.07/lb	-\$0.07/lb	-\$0.07/lb	-\$0.07/lb	-\$0.07/lb
Profit	\$0.08/lb	\$0.08/lb	\$0.08/lb	\$0.13/lb	\$0.23/lb

Note that here, the future value is obtained by

$$FV = -\$0.0665 \times \exp\left(6\% \cdot \frac{365}{365}\right) \sim -\$0.07.$$

□

**Remark 2.1.** Note that as the strike price of put option goes deep in the money, the profit/loss curve will converge to the profit/loss curve when you short forward to hedge your position.

**Question (4.4).** Compute estimated profit in 1 year if **XYZ** sells a call option with a strike of \$0.95, \$1.00, or \$1.05. Draw a graph of profit in each case.

*Solution.* Based on table 1, if we sell a 0.95-strike call to hedge our position, we have

Hedged profit of selling 0.95-strike call in 1 year					
Copper price in 1 year	\$0.80	\$0.90	\$1.00	\$1.10	\$1.20
Selling copper	\$0.80/lb	\$0.90/lb	\$1.00/lb	\$1.10/lb	\$1.20/lb
Fixed costs	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb
Variable cost	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb
0.95-strike call	\$0.00/lb	\$0.00/lb	-\$0.05/lb	-\$0.15/lb	-\$0.25/lb
Future value	\$0.07/lb	\$0.07/lb	\$0.07/lb	\$0.07/lb	\$0.07/lb
Profit	-\$0.03/lb	\$0.07/lb	\$0.12/lb	\$0.12/lb	\$0.12/lb

Note that here, the future value is obtained by

$$FV = \$0.0649 \times \exp\left(6\% \cdot \frac{365}{365}\right) \sim \$0.07.$$

Also, if we sell a 1.00-strike call to hedge our position, we get

Hedged profit of selling 1.00-strike call in 1 year					
Copper price in 1 year	\$0.80	\$0.90	\$1.00	\$1.10	\$1.20
Selling copper	\$0.80/lb	\$0.90/lb	\$1.00/lb	\$1.10/lb	\$1.20/lb
Fixed costs	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb
Variable cost	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb
1.00-strike call	\$0/lb	\$0/lb	\$0/lb	-\$0.10/lb	-\$0.20/lb
Future value	\$0.04/lb	\$0.04/lb	\$0.04/lb	\$0.04/lb	\$0.04/lb
Profit	-\$0.06/lb	\$0.04/lb	\$0.14/lb	\$0.14/lb	\$0.14/lb

Note that here, the future value is obtained by

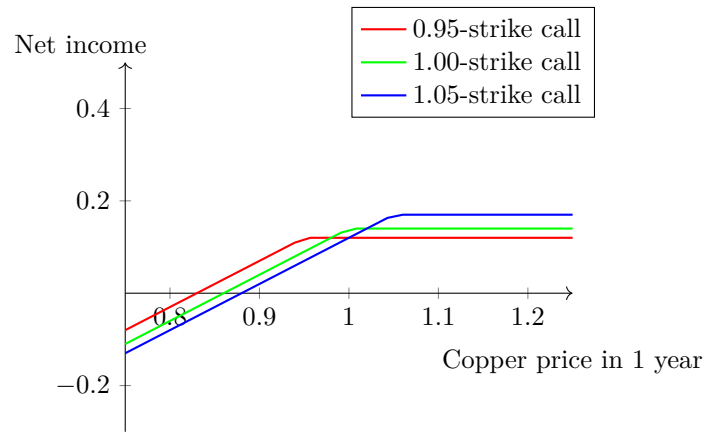
$$FV = \$0.0376 \times \exp\left(6\% \cdot \frac{365}{365}\right) \sim \$0.04.$$

Last, if we sell a 1.05-strike call to hedge our position, we obtain

Hedged profit of selling 1.05-strike call in 1 year					
Copper price in 1 year	\$0.80	\$0.90	\$1.00	\$1.10	\$1.20
Selling copper	\$0.80/lb	\$0.90/lb	\$1.00/lb	\$1.10/lb	\$1.20/lb
Fixed costs	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb	-\$0.50/lb
Variable cost	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb	-\$0.40/lb
1.05-strike call	\$0/lb	\$0/lb	\$0/lb	-\$0.05/lb	-\$0.15/lb
Future value	\$0.02/lb	\$0.02/lb	\$0.02/lb	\$0.02/lb	\$0.02/lb
Profit	-\$0.08/lb	\$0.02/lb	\$0.12/lb	\$0.17/lb	\$0.17/lb

Note that here, the future value is obtained by

$$FV = \$0.0194 \times \exp\left(6\% \cdot \frac{365}{365}\right) \sim \$0.02.$$



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## REFERENCES

- [1] Investopedia, *Rebate*.  
<https://www.investopedia.com/terms/r/rebate.asp>