Name: _____ *Student ID Number:* _____

Spring 2022 Math 5A Worksheet 2 Solution

1. Find the exact value of each expression.
   (a) $\log_5 100 + \log_5 25 - 2\log_5 2$.
   (b) $e^{2\ln 6}$.

   For (a), we have

   $$\log_5 100 + \log_5 25 - 2\log_5 2 = \log_5\left(100 \cdot 25/2^2\right) = \log_5(625) = \log_5(5^4)$$
   $$= 4\log_5 5 = 4.$$

   For (b), we have

   $$e^{2\ln 6} = e^{\ln 6^2} = e^{\ln 6^2} = 6^2 = 36.$$

2. Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues. (Assume that n begins with 1.)

   $$\{a_n\} = \left\{\frac{1}{2}, \frac{-3}{4}, \frac{5}{8}, \frac{-7}{16}, \frac{9}{32}, \dots\right\}.$$

   To find a formula for a_n, we observe that the sequence is alternating, the numerators form an arithmetic progression, and the denominators form a geometric progression, hence we can guess

   $$a_n = (-1)^{n+1} \cdot \frac{2n-1}{2^n}.$$

3. A fish farmer has 4000 catfish in his pond. The number of catfish increases by 8% per month and the farmer harvests 200 catfish per month.
   (a) Find the recursive relation of the catfish population P_n after n months with $P_0 = 4000$.
   (b) How many catfish are in the pond after three months? (Round your answer to the nearest integer.)

   For (a), the population P_n after n months depends on P_{n-1} and we can write the following recursive relation

   $$P_n = 1.08 \cdot P_{n-1} - 200$$

   with $P_0 = 4000$.

   For (b), we have

   $$P_1 = 1.08 \cdot 4000 - 200 = 4320 - 200 = 4120$$
   $$P_2 = 1.08 \cdot 4120 - 200 = 4320 - 200 = 4249.6 \sim 4250$$
   $$P_3 = 1.08 \cdot 4249.6 - 200 = 4589.568 - 200 = 4389.568 \sim 4390.$$