# FEB 18 2021 MATH 134B

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## 1. Terminologies

**Formula** (Pricing Prepaid Forwards with Dividends)**.** *If the Dividends are:*
**-Discrete Dividends:** $F^P_{0,T} = S_0 - \sum_{i=1}^{n} \mathrm{PV}_{0,t_i}(D_{t_i})$*, where we assume this company pays dividend* $D_{t_i}$ *at times* $t_i$*,* $i = 1, \ldots, n$*.*
**-Continuous Dividends:** $F^P_{0,T} = S_0 e^{-\delta T}$*, where* $\delta$ *is the dividend yield and* $T$ *the time to maturity of the prepaid forward contract.*
*Here,* $F^P_{0,T}$ *is the prepaid forward price,* $\mathrm{PV}_{0,t_i}$ *denotes the time* 0 *present value of a time* $t_i$ *payment, and* $S_0$ *is the present stock price.*

**Formula** (Forward Premium)**.**
**-Forward Premium:** $\frac{F_{0,T}}{S_0}$.
**-Annualized Forward Premium:** $\frac{1}{T} \ln\left(\frac{F_{0,t}}{S_0}\right)$.
*Here,* $F_{0,t_i}$ *is the forward price and* $S_0$ *is the present stock price.*

## 2. Question solving

Chapter 5 problems 2, 4, and 5.

**Question** (5.2)**.** *A* $50 *stock pays a* $1 *dividend every 3 months, with the first dividend coming 3 months from today. The continuously compounded risk-free rate is 6%.*
a. *What is the price of a prepaid forward contract that expires 1 year from today, immediately after the fourth-quarter dividend?*
b. *What is the price of a forward contract that expires at the same time?*

*Solution.*
a. To get the price of a prepaid forward contract expires 1 year from today, we apply the formula, for discrete dividends we have

$$\begin{aligned} F^P_{0,1} &= S_0 - \sum_{i=1}^{4} \mathrm{PV}_{0,0.25i}(D_{t_i}) \\ &= 50 - \sum_{i=1}^{4} e^{-0.015i} \sim 46.147. \end{aligned}$$

Notice that since the annual continuously compounded risk-free rate is 6%, the quarterly continuously compounded rate is therefore 1.5%.
b. To prevent arbitrage, the price of a forward contract that expires at the same time will be the future value of the price of a prepaid forward contract, thus

$$F_{0,1} = \mathrm{FV}\left(F^P_{0,1}\right) = 46.147 \times e^{0.06} \sim 49$$

□

**Question** (5.4)**.** *Suppose the stock price is \$35 and the continuously compounded interest rate is 5%.*

a. *What is the 6-month forward price, assuming dividends are zero?*

b. *If the 6-month forward price is \$35.50, what is the annualized forward premium?*

c. *If the forward price is \$35.50, what is the annualized continuous dividend yield?*

*Solution.*

a. The 6-month forward price, assuming no dividends will be

$$F_{0,6/12} = F_{0,0.5} = 35 \times e^{0.05 \times 0.5} \sim 35.886.$$

b. First, we compute the forward premium, which is

$$\text{Forward Premium} = \frac{F_{0,0.5}}{S_0} = \frac{35.50}{35}.$$

So the annualized forward premium will be

$$\frac{1}{0.5} \ln(\frac{35.50}{35}) \sim 2.837\%.$$

c. The annualized continuous dividend yield is the difference between the risk-free rate and the annualized forward premium, thus

$$\text{Annualized Continuous Dividend Yield} = 5\% - 2.837\% = 2.163\%.$$

□