FEB 11 2021 MATH 134B

CHAO-MING LIN

1. Question solving

Homework 4 is due on Tuesday, February 16. Only problems 1, 3, 4, 7, 8, 9, and 12 are required, others are optional.

For the following problems consider the following three firms:
XYZ mines copper, with fixed costs of \$0.50/lb and variable cost of \$0.40/lb.
Wirco produces wire. It buys copper and manufactures wire. One pound of copper can be used to produce one unit of wire, which sells for the price of copper plus \$5. Fixed cost per unit is \$3 and non-copper variable cost is \$1.50.
Telco installs telecommunications equipment and uses copper wire from **Wirco** as an input. For planning purposes, **Telco** assigns a fixed revenue of \$6.20 for each unit of wire it uses.

The 1-year forward price of copper is \$1/lb. The 1-year continuously compounded interest rate is 6%. One-year option prices for copper are shown in the table below In your answers, at a

Table 1. Option prices

One-year option prices for copper		
Strike	Call	Put
0.9500	\$0.0649	\$0.0178
0.9750	\$0.0500	\$0.0265
1.0000	\$0.0376	\$0.0376
1.0250	\$0.0274	\$0.0509
1.0340	\$0.0243	\$0.0563
1.0500	\$0.0194	\$0.0665

minimum consider copper prices in 1 year of \$0.80, \$0.90, \$1.00, \$1.10, and \$1.20.

Question (4.7)**.** *If **Telco** does nothing to manage copper price risk, what is its profit 1 year from now, per pound of copper that it buys? If it hedges the price of wire by buying copper forward, what is its estimated profit 1 year from now? Construct graphs illustrating both unhedged and hedged profit.*

Solution. Notice that **Telco** installs telecommunications equipment and uses copper wire from **Wirco** as an input. For planning purposes, **Telco** assigns a fixed revenue of \$6.20 for each unit of wire it uses.

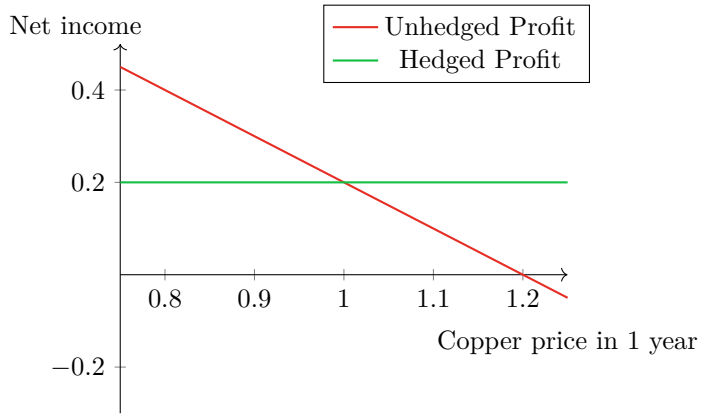
In addition, **Wirco** produces wire. It buys copper and manufactures wire. One pound of copper can be used to produce one unit of wire, which sells for the price of copper plus \$5. Fixed cost per unit is \$3 and non-copper variable cost is \$1.50.

First, if **Telco** does nothing to manage copper price risk, the profit 1 year from now will be the following

Unhedged profit					
Copper price in 1 year	\$0.80	\$0.90	\$1.00	\$1.10	\$1.20
Cost of copper wire	-\$5.80/lb	-\$5.90/lb	-\$6.00/lb	-\$6.10/lb	-\$6.20/lb
Revenue	\$6.20/lb	\$6.20/lb	\$6.20/lb	\$6.20/lb	\$6.20/lb
Net income	\$0.40/lb	\$0.30/lb	\$0.20/lb	\$0.10/lb	\$0/lb

On the other hand, if **Telco** hedge their position, notice that the 1-year forward price of copper is \$1/lb, so the profit will be the following

Hedged profit					
Copper price in 1 year	\$0.80	\$0.90	\$1.00	\$1.10	\$1.20
Cost of copper wire	-\$5.80/lb	-\$5.90/lb	-\$6.00/lb	-\$6.10/lb	-\$6.20/lb
Revenue	\$6.20/lb	\$6.20/lb	\$6.20/lb	\$6.20/lb	\$6.20/lb
Longing forward	-\$0.20/lb	-\$0.10/lb	\$0/lb	\$0.10/lb	\$0.20/lb
Net income	\$0.20/lb	\$0.20/lb	\$0.20/lb	\$0.20/lb	\$0.20/lb



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Question (4.8)**.** *Compute estimated profit in 1 year if* ***Telco*** *buys a call option with a strike of* \$*0.95,* \$*1.00, or* \$*1.05. Draw a graph of profit in each case.*

Solution. Based on table 1, if we buy a 0.95-strike call to hedge our position, we have

Hedged profit of buying 0.95-strike call in 1 year					
Copper price in 1 year	\$0.80	\$0.90	\$1.00	\$1.10	\$1.20
Cost of copper wire	-\$5.80/lb	-\$5.90/lb	-\$6.00/lb	-\$6.10/lb	-\$6.20/lb
Revenue	\$6.20/lb	\$6.20/lb	\$6.20/lb	\$6.20/lb	\$6.20/lb
0.95-strike call	\$0/lb	\$0/lb	\$0.05/lb	\$0.15/lb	\$0.25/lb
Future value	-\$0.07/lb	-\$0.07/lb	-\$0.07/lb	-\$0.07/lb	-\$0.07/lb
Net income	\$0.33/lb	\$0.23/lb	\$0.18/lb	\$0.18/lb	\$0.18/lb

Note that here, the future value is obtained by

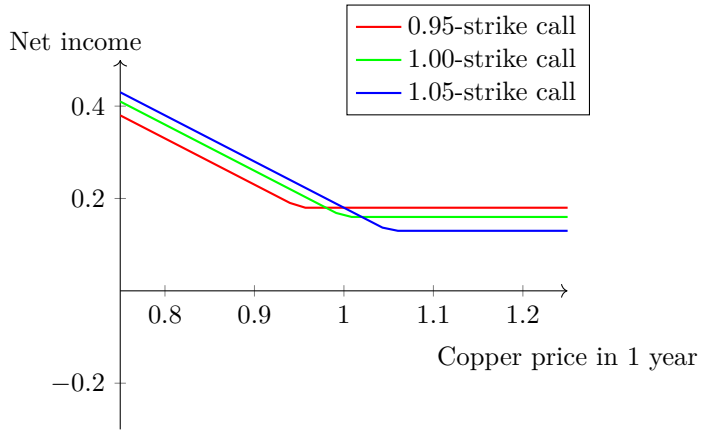
$$\text{FV} = -\$0.0649 \times \exp\left(6\% \cdot \frac{365}{365}\right) \sim -\$0.07.$$

Also, if we buy a 1.00-strike call to hedge our position, we get

Hedged profit of buying 1.00-strike call in 1 year					
Copper price in 1 year	\$0.80	\$0.90	\$1.00	\$1.10	\$1.20
Cost of copper wire	-\$5.80/lb	-\$5.90/lb	-\$6.00/lb	-\$6.10/lb	-\$6.20/lb
Revenue	\$6.20/lb	\$6.20/lb	\$6.20/lb	\$6.20/lb	\$6.20/lb
1.00-strike call	\$0/lb	\$0/lb	\$0/lb	\$0.10/lb	\$0.20/lb
Future value	-\$0.04/lb	-\$0.04/lb	-\$0.04/lb	-\$0.04/lb	-\$0.04/lb
Net income	\$0.36/lb	\$0.26/lb	\$0.16/lb	\$0.16/lb	\$0.16/lb

Note that here, the future value is obtained by

$$\text{FV} = -\$0.0376 \times \exp\left(6\% \cdot \frac{365}{365}\right) \sim -\$0.04.$$



Last, if we buy a 1.05-strike put to hedge our position, we obtain

Hedged profit of buying 1.05-strike call in 1 year					
Copper price in 1 year	\$0.80	\$0.90	\$1.00	\$1.10	\$1.20
Cost of copper wire	-\$5.80/lb	-\$5.90/lb	-\$6.00/lb	-\$6.10/lb	-\$6.20/lb
Revenue	\$6.20/lb	\$6.20/lb	\$6.20/lb	\$6.20/lb	\$6.20/lb
1.05-strike call	\$0/lb	\$0/lb	\$0/lb	\$0.05/lb	\$0.15/lb
Future value	-\$0.02/lb	-\$0.02/lb	-\$0.02/lb	-\$0.02/lb	-\$0.02/lb
Net income	\$0.38/lb	\$0.28/lb	\$0.18/lb	\$0.13/lb	\$0.13/lb

Note that here, the future value is obtained by

$$\text{FV} = -\$0.0194 \times \exp\left(6\% \cdot \frac{365}{365}\right) \sim -\$0.02.$$

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References

[1] Investopedia, *Rebate*.
https://www.investopedia.com/terms/r/rebate.asp