

## MAR 29 2022 MATH 5A

CHAO-MING LIN

Name: Chao-Ming Lin, DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CALIFORNIA-IRVINE, CA

E-mail address: <mailto:chaominl@uci.edu>

Office Hours: Monday 8am-9am and Wednesday 2pm - 3pm

Personal Website: <https://www.math.uci.edu/~chaominl/>

### POLICIES

**-Office Hours:** Monday 8am - 9am and Wednesday 2pm - 3pm. The followings are the ics file and the direct link, respectively.

<https://uci.zoom.us/meeting/tJYtdu6grD0uE9LNKBcsvgMwYwDltiv6s0eu/ics>

<https://uci.zoom.us/j/92017826496>

**-Appointment:** If you have any question, feel free to email me, I will try to respond as soon as possible or hold an extra appointment for you.

**-WebAssign Homework:**

-If you encounter any technical issue, please contact Cengage tech support!

**-Quizzes:** There will be no quiz.

-In person quiz at 8:30am in BS3 1200

-In total six quizzes, the lowest one will be dropped!

-No makeup quiz!

**-Schedule of my discussion session:**

-20 minutes lecture

-15 minutes worksheet

-5 minutes going over worksheet

### 1.1 FUNCTION

**Definition.** A **function**  $f$  is a rule that assigns to each element  $x$  in a set  $D$  exactly one element, called  $f(x)$ , in a set  $E$ . The set  $D$  is called the **domain** of the function. The **range** of the function  $f$  is the set of all values  $f(x)$  as  $x$  varies throughout the domain.

**Example.** Find the domain of the functions  $f(x) = \sqrt{x+1}$  and  $f(x) = \frac{\sqrt{x+1}}{x-1}$ .

*Solution.* For  $f(x) = \sqrt{x+1}$ , we need to make sure that the value inside square root must be non-negative, that is,

$$x+1 \geq 0 \Rightarrow x \geq -1.$$

So the domain will be

$$\text{Domain} = \{x: x \geq -1\} = [-1, \infty).$$

For  $f(x) = \frac{\sqrt{x+1}}{x-1}$ , first, it is a fraction, so the denominator cannot be zero. We have

$$x-1 \neq 0 \Rightarrow x \neq 1.$$

Then, we need to make sure the numerator is defined, by the previous function, we require

$$x+1 \geq 0 \Rightarrow x \geq -1.$$

You need to satisfy these two constraints, so the domain will be the intersection, that is,

$$\text{Domain} = \{x: x \geq -1\} \cap \{x \neq 1\} = [-1, 1) \cup (1, \infty).$$

□

## 1.2 A CATALOG OF ESSENTIAL FUNCTIONS

**Polynomials.** A function  $P$  is a polynomial if it is of the form

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

where  $n$  is a non-negative integer and  $a_0, a_1, \dots, a_{n-1}, a_n$  are real numbers.

**Example.**

- Constant function  $f(x) = c$  is a polynomial of degree 0.
- Linear function  $f(x) = ax + b$  is a polynomial of degree 1.
- Quadratic function  $f(x) = ax^2 + bx + c$  is a polynomial of degree 2.

**Trigonometric Functions.**

**Example.**

- $\sin(x)$ .
- $\cos(x)$ .
- $\tan(x)$ .

## 1.3 NEW FUNCTIONS FROM OLD FUNCTIONS

**Example** (Vertical and Horizontal Shifts). Suppose  $c > 0$ .

- $y = f(x) + c$ , shifts the graph of  $y = f(x)$  a distance  $c$  units upward.
- $y = f(x) - c$ , shifts the graph of  $y = f(x)$  a distance  $c$  units downward.
- $y = f(x + c)$ , shifts the graph of  $y = f(x)$  a distance  $c$  units to the left.
- $y = f(x - c)$ , shifts the graph of  $y = f(x)$  a distance  $c$  units to the right.

**Example** (Vertical and Horizontal Stretching and Reflecting). Suppose  $c > 0$ .

- $y = cf(x)$ , stretch the graph of  $y = f(x)$  vertically by a factor of  $c$ .
- $y = f(x)/c$ , shrink the graph of  $y = f(x)$  vertically by a factor of  $c$ .
- $y = f(cx)$ , shrink the graph of  $y = f(x)$  horizontally by a factor of  $c$ .
- $y = f(x/c)$ , stretch the graph of  $y = f(x)$  horizontally by a factor of  $c$ .
- $y = -f(x)$ , reflect the graph of  $y = f(x)$  about the  $x$ -axis.
- $y = f(-x)$ , reflect the graph of  $y = f(x)$  about the  $y$ -axis.