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1.5 Logarithms

Definition (Inverse of a function)**.** *Let f be a one-to-one function with domain A and range B. Then its **inverse function** has domain B and range A and is defined by*

$$f^{-1}(y) = x \Longleftrightarrow f(x) = y.$$

Example. *Let $f(x) = \frac{x}{x+1}$. Find $f^{-1}(x)$.*

Solution. To find the inverse of f, we can swap x and y, that is

$$f(x) = y = \frac{x}{x+1} \xleftrightarrow[\text{swap } x \text{ and } y]{} x = \frac{y}{y+1}.$$

Solving for y, we get $y = \frac{x}{1-x}$. The inverse function will be

$$f^{-1}(x) = \frac{x}{1-x}.$$

□

Definition (Natural Logarithm)**.** *We define the logarithm of base b to be the inverse function of b^x, that is,*

$$\log_b(y) = x \Longleftrightarrow b^x = y.$$

*If the base b is the number e, then we use the notation $\ln x$ instead of $\log_e x$. This is called the **natural logarithm**.*

Proposition (Laws of Logarithm)**.** *If x and y are positive numbers, then*

- $\log_b(xy) = \log_b(x) + \log_b(y)$.
- $\log_b(x/y) = \log_b(x) - \log_b(y)$.
- $\log_b(x^r) = r \log_b(x)$.
- $\log_b(x) = \ln x / \ln b$.

1.6 Sequences and Difference Equations

Definition. *A **sequence** is an enumerated collection of objects in which repetitions are allowed and order matters.*

Definition. *A **recursive sequence** is a sequence whose n-th term depends on some of the terms before it.*

Example (Sequence)**.** *Let $a_n = (-1)^n \frac{n-1}{n+1}$. Express the first five terms.*

Solution.

$$\begin{aligned}\{a_1, a_2, a_3, a_4, a_5\} &= \left\{(-1)^1 \frac{1-1}{1+1}, (-1)^2 \frac{2-1}{2+1}, (-1)^3 \frac{3-1}{3+1}, (-1)^4 \frac{4-1}{4+1}, (-1)^5 \frac{5-1}{5+1}\right\} \\ &= \left\{0, \frac{1}{3}, -\frac{2}{4}, \frac{3}{5}, -\frac{4}{6}\right\} = \left\{0, \frac{1}{3}, -\frac{1}{2}, \frac{3}{5}, -\frac{2}{3}\right\}.\end{aligned}$$

□

Definition (Factorial)**.** $n! \coloneqq n \cdot (n-1) \cdot (n-2) \cdot \cdots \cdot 1$.