# Count

Prof. Micha Hofri

March 8, 2014

Let's look at the following pseudocode and count the executing times of each step:

| | count | |
|---|---|---|
| 1. | 1 | $Max(A, n)$ |
| 2. | 1 | $int\ m, k = 1;$ |
| 3. | $1 + X_n$ | $m = A[k]$ |
| 4. | $n$ | $k = k + +$ |
| 5. | $n$ | $if(k > n) \xrightarrow{1} 8$ |
| 6. | $n - 1$ | $if(m >= A[k]) \overset{n-X_n-1}{\longrightarrow} 4$ |
| 7. | $X_n$ | $\xrightarrow{X_n} 3$ |
| 8. | 1 | $return\ m$ |

The code above represents the process of finding the maximum number in array A with the size of n. The count numbers are indicated, and in which $X_n$ represents an unknown number whose max value is $n - 1$ and minimun value is 0.
Now we are interested in the average value of $X_n$. We state an indicator random value:

$$y_i = \begin{cases} 0 & \text{if no updating of } m \text{ when } k = i \\ 1 & \text{if } m \text{ is updated when } k = i \end{cases}$$

In here, $y_i$ is an independent random variable, so we have:

$$X_n = \sum_{i=2}^{n} y_i \tag{1}$$

$$E[X_n] = \sum_{i=2}^{n} E[y_i] = \sum_{i=2}^{n} \frac{1}{i} = H_n - 1 \tag{2}$$

in which,

$$E[y_i] = 1 \cdot Pr(y_i = 1) = \frac{1}{i}$$