CS504 HW9

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a

$$\begin{aligned}
\binom{n}{s-t} &= \binom{n+t}{s} \\
&= \frac{(n+t)!}{s!(n+t-s)!} \\
&= \binom{n+t}{s} \frac{s(s-1)(s-2)\cdots(s-t+1)}{(n+t)(n+t-1)\cdots(n+t-t+1)} \\
&= \binom{n+t}{s} \frac{(s-t)^t}{n^t} \underbrace{\frac{(1+\frac{1}{s-t})(1+\frac{2}{s-t})\cdots(1+\frac{t}{s-t})}{(1+\frac{1}{n})(1+\frac{2}{n})\cdots(1+\frac{t}{n})}}_{D}
\end{aligned}$$

Then partially solve the above equation

$$\begin{aligned}
D &= \frac{1+\frac{t(t+1)}{2(s-t)}+\frac{t(t+1)(3t^2-13t-2)}{24(s-t)^2}}{1+\frac{t(t+1)}{2n}+\frac{t(t+1)(3t^2-13t-2)}{24n^2}} \\
&= \left(1+\frac{t(t+1)}{s(s-t)}\right)\left(1+\frac{t(t+1)}{2n}\right)+o\left(\frac{t^2}{s^2}\right) \\
&= 1+\frac{t(t+1)}{2n}+\frac{t(t+1)}{2(s-t)}+o\left(t^2\left(\frac{1}{n}+\frac{1}{s-t}\right)\right)
\end{aligned}$$

Finally

$$\binom{n}{s-t} = \binom{n+t}{s} \left(\frac{s-t}{n}\right)^t \cdot D$$

b