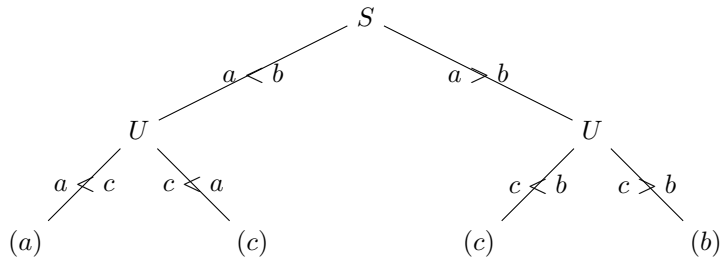
Merge Sort

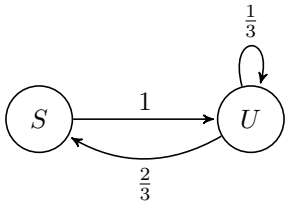
Prof. Micha Hofri

March 8, 2014

First, assume we have three numbers a, b, c, and we want to find the smallest number among them. Let's look at a decision tree of the process:



Assume we have two states U and S as above shows, their transition between each other is shown as follows with the probability of the transition:



Then we get:

$$P(S) = P(U) \cdot \tfrac{2}{3}$$
$$P(U) = P(S) \cdot 1 + P(U) \cdot \tfrac{1}{3}$$

So $P(S) = \frac{2}{5}$, $P(U) = \frac{3}{5}$.

Define c as the comparisons per move,
Hence $E(c) = 2 \cdot \frac{2}{3} + 1 \cdot \frac{1}{3} = \frac{5}{3}$
Finally, we have

$$E(\textit{moves per comparison}) = P(U) \cdot 1 = \tfrac{3}{5}$$
$$E(\textit{comparisons per move}) = \tfrac{1}{3/5} = \tfrac{5}{3}$$