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Do core-selecting Combinatorial Clock Auctions always lead to high efficiency? An experimental analysis of spectrum auction designs

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Abstract For many years the Simultaneous Multi-Round Auction (SMRA) has been the primary auction design for spectrum sales worldwide. Recently, the core-selecting Combinatorial Clock Auction (CCA) has been used as an alternative to the SMRA in a number of countries promising strong incentives for truthful bidding and high efficiency as a result. We analyze the efficiency and auctioneer revenue of the CCA in comparison to SMRA and examine bidding behavior in both formats. The experiments are based on two value models, which resemble single- and multiband spectrum sales in the field. Such applications often allow for thousands of possible bundles. Bidders in the CCA submitted bids for only a fraction of all bundles with a positive valuation. Bundles were selected based on synergies and payoff after the primary bid rounds. As a consequence, we found efficiency of the CCA to be significantly lower than that of SMRA in the multi-band value model and auctioneer revenue of the CCA to be lower in both value models. In addition, we characterize several properties of the auction format, which result from the two-stage design and the payment and activity rules.

Keywords Auctions · Lab experiments · Group behavior · Individual behavior

JEL Classification C91 · C92 · D44

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1 Introduction

There has been a long discussion on appropriate auction mechanisms for the sale of spectrum rights (Porter and Smith 2006). Since 1994, the Simultaneous Multi-Round Auction (SMRA) has been used worldwide (Milgrom 2000). The SMRA design was very successful, but also led to a number of strategic problems for bidders (Cramton 2009b). The *exposure problem* is central and refers to the risk for a bidder to make a loss due to winning only a fraction of the bundle of items (or blocks of spectrum) he has bid on at a price which exceeds his valuation of the won subset.

Combinatorial auctions (CAs) allow for bids on indivisible bundles avoiding the exposure problem. The design of such auctions, however, led to a number of fundamental problems, and many theoretical and experimental contributions during the past ten years (Cramton et al. 2006a, 2006b). The existing experimental literature comparing SMRAs and CAs suggests that in the presence of significant complementarities in bidders' valuations and a setting with independent private and quasi-linear valuations, combinatorial auctions achieve higher efficiency than simultaneous auctions (Banks et al. 1989; Ledyard et al. 1997; Porter et al. 2003; Kwasnica et al. 2005; Brunner et al. 2010; Goeree and Holt 2010). Since 2008 several countries such as the UK, Ireland, the Netherlands, Denmark, Austria, Switzerland, and the US have adopted combinatorial auctions for selling spectrum rights (Cramton 2009b). While the US used an auction format called Hierarchical Package Bidding (HPB) (Goeree and Holt 2010), which accounts for the large number of regional licenses, the other countries used a Combinatorial Clock Auction (CCA) (Maldoom 2007; Cramton 2009a), a two-phase auction format with primary bid rounds (aka. clock phase) for price discovery, which is extended by a supplementary bids round (aka. supplementary phase). The CCA design used in those countries has a number of similarities to the Clock-Proxy auction, which was proposed by Ausubel et al. (2006). It was used for the sale of blocks in a single spectrum band (i.e., paired and unpaired blocks in the 2.6 GHz band) and for the sale of multiple bands in Switzerland.[1]

Although, spectrum auction design might appear specific, the application is a representative of a much broader class of multi-object markets as they can be found in logistics and industrial procurement. Spectrum auctions are very visible in public and successful designs are a likely role model for other domains as well.

The main contributions of this paper are the following:

- To our knowledge, this is the first lab experiment on the CCA, which we compare to the SMRA. We used an implementation of the CCA and the SMRA, which mirrors the auction rules used in the field and derive a number of properties of these auction rules. While most experimental studies in this field focus on small markets with a few blocks only, we intentionally used an experimental design which resembles real market environments. This is an important complement to other studies, as

[1]Note that Porter et al. (2003) have defined a combinatorial clock auction, which is different to the one described in this paper and in Maldoom (2007) and Cramton (2009b), and only consists of a single clock phase.

results of small combinatorial auctions do not necessarily extend to larger ones (Scheffel et al. 2012).[2]

- We show that the efficiency in the CCA was not higher than that of SMRA and due to the low number of bundle bids actually significantly lower in the multiband setting. Auctioneer revenue was considerably lower than in SMRA which can be explained by the CCA second-price payment rules. However, the auctioneer revenue in CCA was also significantly lower than in CCA simulations where we had artificial bidders submit bids on all possible combinations truthfully with the same value models.
- We also analyzed bidder behavior in the CCA. In particular, in the multiband value model bidders select only a small fraction of all possible bundles for practical, but also for strategic reasons. While restricted bundle selection has recently been discussed in the experimental literature on other combinatorial auction designs (Scheffel et al. 2012), the paper analyzes the specific effects it can have on the efficiency of the CCA with a core-selecting payment rule. Although bid shading in core-selecting auctions is a concern in the theoretical literature, we found most bundle bids in the supplementary phase of the CCA to be at the valuation and only limited bid shading.
- In complex environments such as spectrum auctions there is a danger that external validity of lab experiments is not given as bidders in the field are better prepared than in the lab. To address this point to some extent, we also conducted *competitions*, where bidders had additional information about equilibrium strategies, known auction tactics, and two weeks of time to prepare a bidding strategy in a team of two people. While the bidder payoff in SMRAs was significantly higher than in the lab, in the CCA bidding behavior was not much different to the lab.

In the next section, we revisit the existing experimental literature on spectrum auctions and combinatorial auctions. In Sect. 3, we discuss the auction formats and game-theoretical results relevant to our study. Section 4 describes the experimental design, while Sect. 5 summarizes the results of our experiments. Finally, in Sect. 6 we provide conclusions and a discussion of further research in this area.

2 Related literature

There is a substantial experimental literature on spectrum auction design. One strand of literature on spectrum auctions tries to analyze and explain specific strategic situations, as they occurred in particular auctions either game-theoretically, experimentally, or based on data from the field (Klemperer 2002; Ewerhart and Moldovanu 2003; Bajari and Yeo 2009). Another strand analyzes the mechanisms used in spectrum auctions based on related settings in the lab (Abbink et al. 2005; Banks et al. 2003; Seifert and Ehrhart 2005). For example, Abbink et al. (2005) found differences in results between experiments with experienced vs. inexperienced

[2]Also Goeree and Holt (2010) used realistic value models in an effort to provide guidance for regulators in the USA.

students. Also in the field, bidders typically work in teams of experts and they spend significant amounts of time to prepare for the auction. In our analysis, we introduced competitions to analyze the impact of experienced teams on bidding strategies.

Motivated by spectrum auctions in the US, a number of experimental studies compared different combinatorial auction formats (Cramton 2009a) and analyzed the question under which conditions combinatorial auctions are superior to SMRA. In an early study, Ledyard et al. (1997) compared SMRA with a sequential auction and a combinatorial auction within various value models. They found that combinatorial auction are best suited in environments with value complementarities.

Experiments conducted by Banks et al. (1989, 2003), and Kwasnica et al. (2005) find a positive effect of bundle bidding on efficiency when complementarities are present. Brunner et al. (2010) compared a standard SMRA auction with a single-stage Combinatorial Clock Auction and a FCC format that augmented an SMRA auction to allow for bundle bids. Here, the combinatorial clock auction provided the highest efficiencies and the highest seller revenues. More recently, the Hierarchical Package Bidding (HPB) format which has been developed for the spectrum auctions in the US was compared to SMRA and Modified Package Bidding (a format with pseudo-dual linear prices) by Goeree and Holt (2010). HPB outperformed the other two auction formats in terms of efficiency and auction revenue. Scheffel et al. (2012) extended this work and showed that restricted bundle selection is the most important reason for inefficiencies in larger auctions with more than a few blocks only. We make a similar observation for the CCA in this paper. More specific literature on the SMRA and the CCA will be discussed in the next section.

3 The auctions

In the following, we describe the SMRA and the CCA and discuss equilibrium bidding strategies. Beyond this, we want to summarize important characteristics of the CCA, as the specific rules of the auction format have found little attention in the academic literature so far.

3.1 The Simultaneous Multi-Round Auction

The SMRA is a generalization of the English auction for more than one block. All the blocks are sold at the same time, each with a price associated with it, and the bidders can bid on any of the blocks. The bidding continues until no bidder is willing to raise the bid on any of the blocks. Then the auction ends with each bidder winning the blocks on which he has the high bid, and paying its bid for any blocks won (Milgrom 2000). There are differences in the level of information revealed about other bidders' bids in different countries. Sometimes all information is revealed after each round, sometimes only prices of the currently winning bids are published. Typically, auctioneers use activity rules which do not allow bidders to bid on more blocks than in the last round in the last stages of the auction. A detailed description of the activity rules and the SMRA auction format used in our experiments can be found in Sect. 4.

There has been limited theoretical research on SMRA. If bidders have substitute preferences and bid straightforwardly, then the SMRA terminates at a Walrasian equilibrium (Milgrom 2000), i.e., an equilibrium with linear prices. *Straightforward bidding* means that a bidder bids on the bundle of blocks, which together maximizes her payoff at the current ask prices. Gul and Stacchetti (1999) showed that if goods are substitutes, then ascending and linear-price auctions cannot implement the VCG outcome. Milgrom (2000) has also shown that with at least three bidders and at least one non-substitutes valuation no Walrasian equilibrium exists. Bidder valuations in spectrum auctions typically include complementarities.

In an environment with substitutes and complements SMRA results in an exposure problem. A number of laboratory experiments document the negative impact of the exposure problem on the performance of the SMRA (Brunner et al. 2010; Goeree and Lien 2013; Kwasnica et al. 2005; Kagel et al. 2010). Therefore, the exposure problem has become a central concern. Goeree and Lien (2010) provided a Bayes-Nash equilibrium analysis of SMRA considering complementary valuations and the exposure problem. They show that due to the exposure problem, the SMRA may result in non-core outcomes, where small bidders obtain blocks at very low prices and seller revenue can be decreasing in the number of bidders just like in the Vickrey-Clarke-Groves auction (VCG) (Ausubel and Milgrom 2006). Regulators have tried to mitigate this problem via additional rules, such as the possibility to withdraw winning bids. However, such rules can also provide incentives for gaming behavior. SMRAs also allow various forms of signaling and tacit collusion, but such behavior is reduced if the identity of bidders is not revealed and the auctioneer only allows for pre-defined jump bids.

3.2 The Combinatorial Clock Auction

The Combinatorial Clock Auction (CCA) is a two-phase combinatorial auction format which was introduced by Cramton (2009a) and in an earlier version by Ausubel et al. (2006). In contrast to SMRA, the auction avoids the exposure problem by allowing for bundle bids. Maldoom (2007) describes a version as it has been used in spectrum auctions across Europe. We will refer to this version as the CCA, as this name is used in applications for spectrum sales. A detailed description is provided in Appendix A.1.

In a CCA, bids for bundles of blocks are made throughout a number of sequential, open rounds (the primary bid rounds or clock phase) and then a final sealed-bid round (the supplementary bids round). In the primary bid rounds the auctioneer announces prices and bidders state their demand at the current price levels. Prices of bands with excess demand are increased by a bid increment until there is no excess demand anymore. Jump bidding is not possible. In the primary bid rounds, bidders can only submit a bid on one bundle per round. This rule is different to earlier proposals by Ausubel et al. (2006). These primary bid rounds allow for price discovery.

If bidders bid straightforward on their payoff maximizing bundle in each round and all goods are sold after the clock phase, allocations and prices would be a competitive equilibrium. However, there may be some cases where at the end of the primary clock round there is an excess supply. For these cases the CCA introduces a supplementary sealed-bid phase where bidders can submit bids over all potential packages.

Prices and the final allocation of the auction are determined using a Vickrey-closest core-selecting payment rule that is intended to induce truthful bidding and avoid incentives for demand reduction. This is because core payments—as derived in Day and Milgrom (2007)—are computed such that a losing bid of a winner does not increase his payments for his winning bid. The winner determination after the supplementary bid round considers all bids that have been submitted in the primary bid rounds and the supplementary bid rounds and selects the revenue maximizing allocation. The bids by a single bidder are mutually exclusive (i.e., the CCA uses an XOR bidding language).

Activity rules should provide incentives for bidders to reveal their preferences truthfully and bid straightforwardly already in the primary bid rounds. Bidders should not be able to shade their bids and then provide large jump bids in the supplementary bids round. An eligibility-points rule is used to determine activity and eligibility to bid in the primary bid rounds. Each block in a band requires a certain number of eligibility points, and a bidder cannot increase his activity across rounds. In the supplementary bids round, revealed preferences during the primary bid rounds are used to derive relative caps on the supplementary bids that impose consistency of preferences between the primary and supplementary bids submitted. The consequence of these rules is that all bids are constrained relative to the bid for the final primary package by a difference determined by the primary bids. This should set incentivizes for straightforward bidding in the primary bid rounds.

3.3 Properties of the CCA

Since the CCA is a relatively new auction format and there is not much literature available on the specific auction rules that we analyse in our experiments, we will first provide a discussion of relevant properties of these rules. These properties will not be tested in the lab, but they provide an understanding about this specific auction format. We will also provide a summary of relevant game-theoretical literature on core-selecting auctions. In what follows, we analyze the CCA with respect to straightforward bidding in the primary bid rounds, incentive compatibility, envy-freeness, and possibilities for spiteful bidding.

3.3.1 Straightforward bidding

The CCA is designed to incentivize straightforward bidding in the primary bid rounds and truthful bidding in the supplementary bids round.

Proposition 1 *If a bidder follows a straightforward bidding strategy in the primary bid rounds of a CCA with an anchor activity rule, then the activity rule will not restrict him to bid his maximum valuation on every bundle in the supplementary bids round.*

A detailed description of the activity rules used for these propositions can be found in the Appendix A.1. Note that the rules have changed over time and details in these rules matter. Unfortunately, straightforward bidding is not always possible if a simple eligibility-points rule is used in the primary bid rounds.

Proposition 2 *If valuations for at least two bundles A and B are full substitutes with $v(A \cup B) = \max(v(A), v(B))$ and the bundle of higher valuation A requires less bid rights than the lower valued bundle B, straightforward bidding is not possible due to the activity rule in the primary bid rounds.*

Strategies become difficult in this situation. Let's assume, a bidder wants either two blocks in band I or four blocks in band II, which have a lower value to him. All blocks have the same number of bid rights. A bidder needs to bid on a bundle, which is at least as large as his largest bundle of interest (4 blocks in this example) in the early rounds in order to be able to bid on all his most preferred bundles in later rounds. If the bidder bid only on band II, he could end up winning his second preferred option, and would not be able to reveal his true valuation for the bundle in band I. If he bids on two blocks in band I and two blocks in band II, in order to switch to 4 blocks in band II eventually, he could well end up winning all four blocks effectively making a loss.[3]

3.3.2 Incentive-compatibility

In order to minimize incentives for bid shading, the CCA design implements a closest-to-Vickrey core-selecting payment rule (Day and Milgrom 2007; Day and Raghavan 2007). Such payments guarantee that there is no group of bidders who can suggest an alternative outcome preferable to both themselves and the seller, and are minimized given this condition. It is known that the VCG outcome is in the core, if goods are substitutes, but no ascending auction can always implement the VCG outcome even if this condition holds (Gul and Stacchetti 2000). If goods are complements, a bidder in a CCA still has an incentive to shade his bids and not reveal his true valuations, as he can possibly increase his payoff. So, after the primary bid rounds, if a bidder has a standing bid[4] on his most preferred bundle, he does have an incentive to minimize his bids in the supplementary bids phase and not reveal his true valuation.

The following two propositions define "safe supplementary bids", which cannot become losing based on the CCA activity rules if the bidders have a standing bid after the primary bid rounds (proofs can be found in Appendix A.2). Let π describe the vector of ask prices in the last primary bid round, $b_j^p(q^j)$ is the last round bid of bidder $j \in J$ on a bundle $q^j = (q_1^j, \dots, q_m^j, \dots q_M^j)$, now described as a vector of blocks in each of M bands, after the primary bid rounds, and b_j^s a supplementary bid.

[3] Recent modifications proposed in countries such as Canada address this problem with a revealed preference rule which is used in addition to the eligibility-points rule in the primary bid rounds (http://www.ic.gc.ca/eic/site/smt-gst.nsf/eng/sf10363.html). Also, the supplementary bids must satisfy revealed preference with respect to each eligibility reducing clock round after the last round in which the bidder had sufficient eligibility to bid on the package, as well as with respect to the final clock round. There is a new working paper by Ausubel and Cramton (2011) with related theory. One motivation for this paper are the problems of bidders who cannot bid straightforward as pointed out by Proposition 2 in our paper. Proposition 3 and 4 in our paper are in line with Propositions 1 and 2 in the paper by Ausubel and Cramton and were developed independently (Bichler et al. 2011b).

[4] A bid is *standing* if its bid price is equal to the ask price of the last primary bids round.

Proposition 3 *If demand equals supply in the final primary bid round, a supplementary bid $b_j^s(q^j) > b_j^p(q^j)$ cannot become losing.*

This is because the supplementary bids of competitors on their standing bundle bid from the final primary bid round does not impact the safe supplementary bid of a bidder $j \in J$. Any additional items added by competitors to their standing bundle bid cannot increase the supplementary bid price by more than the ask price in the last of the primary bid rounds. If the bidder submits additional supplementary bids on packages not containing q^j, his bid $b_j^s(q^j)$ can well become losing, as can easily be shown by examples. The anchor rule also applies to bundles which are smaller than the standing bid of the last primary bid round.

Proposition 4 *If bundle q^u is unallocated after the last primary bid round, a supplementary bid $b_j^s(q^j) > b_j^p(q^j) + q^u\pi$ of a standing bidder j cannot become losing if all his supplementary bids contain q^j.*

This can be shown by an example where a losing bidder on all blocks reduces his demand to null in the last primary bids round. Bidder j needs to make sure that he wins, even if this losing bidder submits a bid on all blocks at the ask prices of the last primary round.

There have been a number of recent game-theoretical papers on core-selecting auctions. Day and Milgrom (2007) characterize a full information Nash equilibrium and show that bidder-optimal core prices minimize the incentives for speculation. Goeree and Lien (2013), Sano (2012a), and Ausubel and Cramton (2011) analyze the Bayesian Nash equilibrium of sealed-bid core-selecting auctions with single-minded bidders. Goeree and Lien (2013) derive an equilibrium of the nearest-Vickrey core-selecting auction and show that in a private values model with rational bidders, auctions with a core-selecting payment rule are on average further from the core than auctions with a VCG payment rule. They also show that no Bayesian incentive-compatible core-selecting auction exists, when the VCG outcome is not in the core. Ausubel and Cramton (2011) analyze different core-selecting auction rules and the case of correlated values.

Sano (2012b) recently provided a Bayesian analysis of an ascending core-selecting auction with independent private values and shows that such an auction can even lead to an inefficient non-bidding equilibrium with risk-neutral bidders. Guler et al. (2012) shows that risk aversion leads to a lower likelihood of a non-bidding equilibrium, but that this possibility still exists depending on the prior distributions. There is a fundamental *free-rider problem* in such threshold problems, where one regional bidder can try to increase expected payoff at the expense of other regional bidders. Truth-telling is no equilibrium strategy in such situations. While the models describe simple environments similar forms of speculation might well be profitable in real-world markets with regional and national bidders. Knapek and Wambach (2012) describe possibilities for speculation in the CCA under complete information in a recent working paper.

3.3.3 Lack of envy-freeness and spiteful bidding

For a characterization of the auction format, it is also worthwhile mentioning that bidders in a CCA or a VCG mechanism do not necessarily pay the same price for identical blocks. Suppose there are two bidders and two homogeneous units of one item. Bidder 1 submits a bid of \$5 on one unit, while bidder 2 submits a bid of \$5 on one unit and a bid of \$9 on two units. Each bidder wins one unit, but bidder 1 pays \$4 and the bidder 2 pays zero. This difference is due to the asymmetry of bidders, and this asymmetry leads to a violation of the law of one price, a criterion, which is often seen desirable in market design Cramton and Stoft (2007). Although arbitrage is avoided as bidders typically cannot sell licenses among each other immediately after a spectrum auction, different prices for the same spectrum are difficult to justify in the public and violate the goal of envy-freeness of an allocation for general valuations (Papai 2003).

Finally, spiteful bidding needs to be taken into account when designing a CCA for a particular application. Bidders in spectrum markets may spitefully prefer that their rivals earn a lower surplus. This is different from the expected utility maximizers typically assumed in the literature. Spiteful bidding has been analyzed by Morgan et al. (2003) and Brandt et al. (2007), who show that the expected revenue in second-price auctions is higher than the revenue in first-price auctions with spiteful bidders in a Bayes Nash equilibrium. While spiteful bidding is possible in any auction, the two-stage CCA provides possibilities to submit spiteful supplementary bids with no risk of actually winning such a bid, if the standing bidders submit safe supplementary bids. We provide an example in the Appendix A.3.

If all items are sold, high losing bids of a bidder j can increase the payments of other bidders. If the supplementary bids are higher than their standing bid in the clock phase, this could move the VCG point such that j's bidder-optimal core payment even decreases. This is illustrated in Fig. 1, where j, k, l, m, n are bids of different bidders on blocks A and B respectively, and j' is the spiteful supplementary bid of bidder j on item B. Of course, if all bidders submit spiteful bids in the supplementary bids round, the payments of all bidders might be increased. Erdil and Klemperer (2010) discuss characteristics of payment rules for core-selecting auctions.

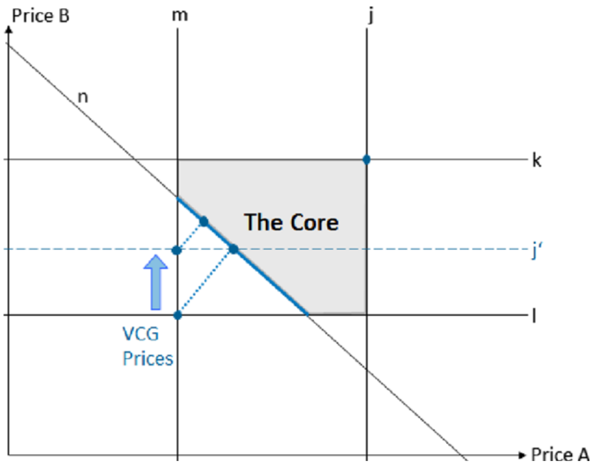
4 Experimental design

In what follows, we characterize the economic setting and the two value models of our experiments. Then we provide further details on the auction rules used in our experiments, the treatment structure, and the organization of our experiments.

4.1 Value models

We used two value models reflecting the characteristics of bidder valuations in the field. Four bidders competed in both value models.

Fig. 1 Bids in the primary bid rounds



4.1.1 The base value model

In the *base value model*, we used a band plan with two bands of blocks as it can be found in several European countries.[5] There are 14 (paired) blocks in band A and 10 (unpaired) blocks in band B. Bidders have a positive valuation for up to 6 blocks in each band with free disposal for bundles greater than that. Each bidder receives a base valuation v_A and v_B for each of the bands. The base valuations represented the valuations of a single block within each band and were drawn randomly from a uniform distribution, v_A in the range of [120, 200] and v_B in the range of [90, 160]. We modeled ascending complementarities in the valuation of bundles of several A blocks. In the A band, a bundle of two blocks receives a value of $1.2 * 2 * v_A$, i.e., a complementarity bonus of 20 % on top of the base valuations. The complementarity in the value model rises with the number of blocks in the bundle. A bundle of three blocks has a complementarity of 40 % and a bundle of four blocks of 80 %[6]. There was no additional bonus for the fifth and sixth A-block and the value of this block is additive, i.e. $1.8 * 4 * v_A + v_A$. The valuations in the band B were purely additive. The total valuation of blocks from both bands is the sum of valuations within the bands. In total, each bidder is interested in up to $7 * 7 - 1 = 48$ different bundles.

In other words, four blocks in band A have the highest per block valuation to all four bidders. If all bidders aim for at least four A-blocks and with 14 blocks on sale, it is possible that either two or three bidders get this bundle, while the other bidders win only two or three blocks in the band A. We assume block valuations to be bidder-

[5]The frequencies of the 2.6 GHz band are available for mobile services in all regions of Europe. It includes 190 MHz which are divided into blocks of 5 MHz which can be used to deliver wireless broadband services or mobile TV. In particular, there are two standards which will likely be used in the 2.6 GHz band, LTE and WiMAX. LTE uses paired spectrum (units of 2 blocks), while WiMAX uses unpaired spectrum (units of 1 block). While some European countries auctioned the 2.6 GHz band solely, others combined several spectrum bands in one auction.

[6]This reflects the valuation in the 2.6 GHz band. Four 5-MHz blocks allow for peak performance rates with LTE and provide maximum value to all bidders.

specific, but the synergy structure of bundles to be the same for all bidders. In some experiments, we also vary the synergies.

4.1.2 The multiband value model

The *multiband value model* is inspired by a recent applications of the CCA for the sale of multiple bands. The multiband value model had also 24 blocks, four bands with six blocks each. Band A was of high value to all bidders, whereas bands B, C, and D had lower value. As in the base value model, each bidder received a base valuation for a block in each band. Base valuations are uniformly distributed: v_A was in the range of [100, 300] while v_B, v_C and v_D were in the range of [50, 200]. Again bidders had complementary valuations for bundles of blocks within bands, but not across bands.

In all bands, bundles of two blocks resulted in a bonus of 60 % on top of the base valuations, while bundles of three or more blocks resulted in a bonus of 50 % for the first three blocks. For example, if the base value was 200, then the valuation for two blocks is 640, for three blocks 900, and for four blocks 1,100. Similar to the base value model, more blocks were valued at the base valuation and did not add any extra bonus. Overall, bidders in the multiband setting could bid on $7^4 - 1 = 2{,}400$ different bundles, which is significantly more than the 48 bundles in the base value model.

The structure of the value model and the distribution of the block valuations of all bands were known to all bidders. Bidders used an artificial currency called Franc. Although the value models resemble characteristics of spectrum sales, this was not communicated to the subjects in the lab (neutral framing). Note that we used start prices of 100 Franc in the band A and 50 Franc in the bands B to D. The bid increments were 20 Franc in the band A and 10 Franc in bands B to D.

4.2 Detailed auction rules in the experiments

Our experiments were conducted using a Web-based software tool, which implemented the SMRA and the CCA.

4.2.1 Simultaneous Multi-Round Auction

In SMRA all blocks were sold at the same time with an individual price for each block. After each round the provisional winner of each block was determined by the highest bid. Ties were broken randomly. A bid on a block had to exceed the standing high bid by at least the minimum increment. Jump bids were restricted to predefined levels (click-box) to prevent signaling, and the identity of the bidders was unknown in the auction.

An activity rule restricted the number of blocks a bidder can bid on across all bands. Following recent SMRA designs, we implemented a stacked activity rule with two activity levels. At the beginning each bidder was eligible to bid on all blocks at sale. In the first three rounds bidders were required to use only 50 % of their eligibility to maintain all eligibility points for the next round. From the fourth round on, 100 % were required. At the beginning of each round all bids from the previous round (winning and losing) were revealed to all bidders. Finally, the auction terminated if no bidder submitted a bid within one round.

4.2.2 Combinatorial Clock Auction

As introduced in Sect. 3, the CCA is composed of the primary bid or clock rounds and the supplementary bids round. All blocks within one band had the same price. In the base value model, there was one price for the A band and one for the B band, in the multiband value model there were separate prices for all four bands. The auctioneer announced the new ask price for each band in each round of the clock phase and bidders decided on the quantities of blocks they wanted to bid on within each band. The quantities specified in all bands formed one bundle bid. Each bidder could submit at most one bid in each round. If there was excess demand (i.e., if the combined demand of all bidders within one band exceeded the number of blocks) in at least one band, a new round is started with higher prices for the bands with excess demand. In the experiments bidders did not know about the level of excess demand, which is in line with the auction rules used for example in Austria. More recent applications in other countries reveal this information. Bidders did not learn about other bidders bids, only whether there was excess demand in each band or not in the previous round. In our experiments, each bidder started with eligibility for all blocks in the first round. The primary bids phase ended after there was no excess demand in any bands any more.

The supplementary bids round consisted of only one round with as many sealed bids as desired by the bidders. They were able to bid on any combination of blocks regardless of the bids of the first phase. Only the maximum bid price was limited by the anchor rule. At the end of the round the optimal allocation was calculated using all bids from both phases. Then the bidder-optimal core-selecting payments were calculated using a quadratic program following Day and Raghavan (2007). All optimizations were performed using the IBM/CPLEX optimizer (version 11).

4.3 Competitions

In addition to the lab experiments with unprepared subjects, we conducted experiments with experienced subjects. In these experiments subjects were recruited from a class on auction theory and market design and were grouped into teams of two persons. The subjects were invited to the lab two weeks prior to the experiment and received the same introduction as lab subjects. In addition, we provided them with information and literature on previous spectrum auctions describing known strategies and tactics of bidders in the field. During the two weeks they prepared their own strategy and wrote a paper to describe their strategy. We refer to these experiments as “competitions” in the following to highlight these differences. In order to defy collusion among the bid teams in a coalition, we told them that they would immediately be excluded if such collusion would be observed. We also analyzed bid data to understand whether they followed the bid strategies that they described in their paper and if there were signs of collusion.

Table 1 Treatment structure

Treatment no.	Auction format	Value model	Bidder	Auctions
1	SMRA	Base	Lab	20
2	CCA	Base	Lab	20
3	SMRA	Base	Competition	9
4	CCA	Base	Competition	9
5	SMRA	Multiband	Lab	16
6	CCA	Multiband	Lab	16

4.4 Treatment structure

We consider two major treatment factors, *auction format* and *value model*, with each having two levels (SMRA and CCA, base and multiband). In addition, we analyzed the base value model treatments with bidders in the lab and in the competition, which yields another treatment factor *bidder*. Overall, we get six *treatments* in total (Table 1).

For each replication or "wave" we generated new sets of random values for all the bidders. We used the same sets of values across auction formats to reduce performance differences due to the random draws. For the base value model we drew valuations for five waves randomly. Each wave consisted of four different auctions which were conducted in the lab within one session. We ran between subjects experiments with four bidders in each session.

In addition to the auctions in Table 1, we organized 4 sessions with another multiband value model, where synergies were different across bidders. These experiments were only conducted to make sure that the synergies do not have a significant impact on our main results, which they did not. Overall, we organized 28 sessions with 106 auctions. In two of the sessions with competitions, we ran only 1 auction and not 4. Each subject interacted with the same 3 other subjects throughout a session, and received new independent value draws for each auction.

For each treatment combination a run with CCA and a run with SMRA was conducted in the lab. All auctions of waves A, B and the first auction of wave C were also used in the competition with both auction formats to enable a direct comparison to the lab. In the multiband value model we defined four waves with four different auctions each.

4.5 Procedures and organization

112 students participated in all the experiments and competitions in 2010 and 2011. Subjects were recruited from the departments of mathematics and computer science. Each subject participated either in one CCA- or in one SMRA-session but never in both. One session comprised all four auctions of one wave and took on average four hours. In the competition, there was one session with five auctions.

To reduce differences between lab sessions, the introduction was delivered through a video. Each participant received a handout and was able to pause the video whenever necessary. An experimentator was present to answer questions. Subjects were then made familiar with the auction software through a demo auction. In addition, a software tool to analyze bundle valuations and payoffs was provided to all subjects. This tool showed a simple list of all available bundles which could be sorted by bundle size, bidder individual valuation, or the payoff based on current prices. In order to ensure a full understanding of the economic environment, the value model, the auction rules, and the financial reward scheme all subjects had to pass a web-based test.

At the beginning of each auction all subjects received the individual draw of valuations, the distribution of valuations, and the information about the complementarities. Each round in SMRA and the primary bid rounds of the CCA took 3 minutes. The supplementary bids phase of the CCA lasted around 10 minutes to provide enough time for bid submission. The subjects could ask for more time if required.

After each session subjects were compensated financially. The total compensation resulted from a 10 Euro show up fee and the auction reward. The auction reward was calculated by a 3 Euro participation reward plus the sum of all auction payoffs converted from Franc to Euro by a 12:1 ratio. Negative payoffs were deducted from the participation reward. Due to the different payment rules in both auction formats, payoffs in CCAs were higher than in SMRAs. Therefore we leveled the expected payoff per participant by randomly selecting three out of four auctions of the SMRA sessions and two of the four auctions in the CCA sessions for payment. On average each subject received 93.52 EUR. The rewards for pairs of subjects in the competition were similar to the ones for single subjects in the lab.

5 Results

First, we present efficiency and revenue of the different auction formats on an aggregate level. Then, we discuss individual bidder behavior in both auction formats and differences between lab and competition.

5.1 Efficiency and revenue

We use ***allocative efficiency*** E(X) as a primary aggregate measure to compare different auction mechanisms.[7] Efficiency $E(X)$ cannot easily be compared between different value models. Therefore, we also calculate ***relative efficiency***, $E(X)^*$.[8] In addition, we measure ***revenue distribution***, which shows how the resulting total surplus is distributed between the auctioneer and bidders.[9] For the pairwise comparisons of these metrics we use the rank sum test for clustered data by Somnath and Satten (2005).[10]

[7]We measure efficiency as $E(X) := \frac{\text{actual surplus}}{\text{optimal surplus}} \times 100\ \%$

[8]For this, we compute the mean of the social welfare over all possible allocations assuming that all goods are sold as in Kagel et al. (2010). For this definition, the relative efficiency of an efficient allocation is still 100 % while the mean of random assignments of all blocks is 0 %. Note that allocations below the mean have negative relative efficiency.

[9]We measure auction revenue share as $R(X) := \frac{\text{auctioneer's revenue}}{\text{optimal surplus}} \times 100\ \%$

[10]$\sim$ indicates an insignificant order, $\succ^*$ indicates significance at the 5 % level, and $\succ^{**}$ indicates significance at the 1 % level.

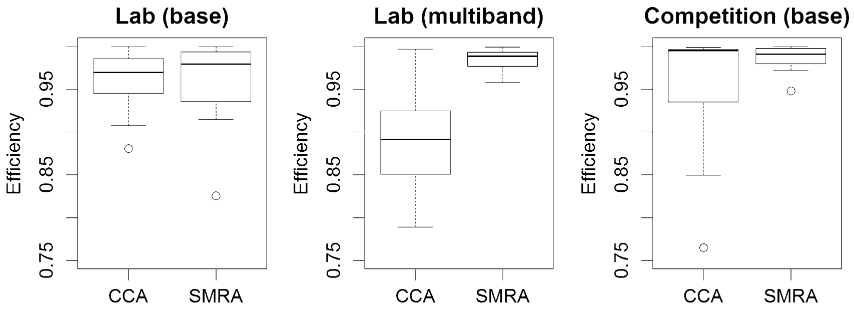


Fig. 2 Efficiency

Table 2 Aggregate measures of auction performance

Value model	Auction	Bidder	$E(X)$	$E(X)^*$	$R(X)$	Unsold blocks
Base	SMRA	Lab	96.16 %	63.27 %	83.74 %	0
Base	SMRA	Competition	98.57 %	87.28 %	75.06 %	0
Base	CCA	Lab	96.04%	63.96 %	64.82 %	0
Base	CCA	Competition	94.15 %	47.17 %	55.38 %	0.44 (1.9 %)
Multiband	SMRA	Lab	98.46 %	93.85 %	80.71 %	0
Multiband	CCA	Lab	89.28%	56.71 %	33.83 %	1.25 (5.2 %)

Result 1 *The efficiency of SMRA was not significantly different to the CCA in the base value model in both the lab* (*SMRA* $\sim$ *CCA*, $p = 0.247$) *and the competition* (*SMRA* $\sim$ *CCA*, $p = 0.781$). *In contrast*, *the efficiency of the CCA was significantly lower than that of SMRA in the multiband value model in the lab* (*SMRA* $\succ^*$ *CCA*, $p < 0.012$).

Support for Result 1 is presented in Fig. 2 and Table 2. A reason for the low efficiency of the CCA in the multiband value model was the number of unsold blocks. On average, 5.2 % or 1.25 blocks remained unsold in this value model. The efficiency of CCAs where all blocks were sold was 93.61 % (SMRA $\sim$ CCA, $p = 0.1563$). The CCA in the multiband value model was the only environment where a significant number of blocks remained unsold (CCA $\succ^{**}$ 0, $p = 0.0020$). In competitions, only two CCAs in the base value model terminated with blocks unsold, and the number of unsold blocks was small. In SMRA, bidders in competitions achieved higher efficiency than lab bidders while in the CCA lab bidders achieved higher efficiency. Both differences were not significant (SMRA: $p = 0.917$, CCA: $p = 0.297$).

Relative efficiency helps to compare the performance between value models. SMRAs led to significantly higher relative efficiency in the multiband than in the base value model (multiband $\succ^*$ base, $p = 0.0439$).[11] With more bands of blocks, bidders tended to focus on the bands for which they had high valuations and they

[11] Note that relative efficiency emphasizes results below the mean disproportionately.

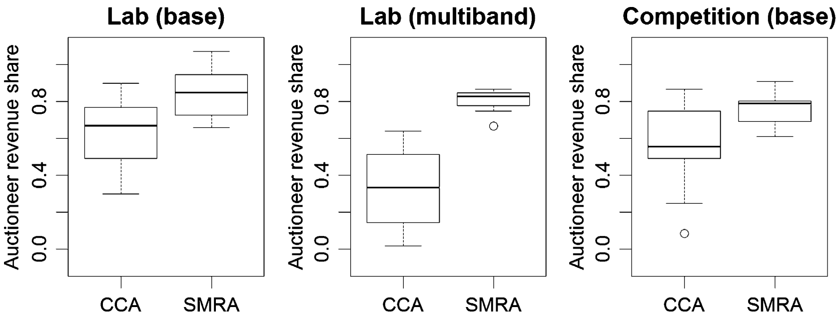


Fig. 3 Auctioneer revenue share

were willing to take an exposure risk in these bands. The CCA had a higher relative efficiency in the base than the multiband value model even though the difference was not significant (SMRA $\sim$ CCA, $p = 0.774$). This insignificance is due to the large variance of relative efficiency values.

Result 2 *The auctioneer revenue of SMRA was significantly higher than that of the CCA in both value models in the lab* (*SMRA* $\succ^{*}$ *CCA*, *base*: $p = 0.034$; *SMRA* $\succ^{**}$ *CCA*, *multiband*: $p < 0.007$). *The differences between lab and competitions were not significant for both auction formats* (*CCA*: $p = 0.684$, *SMRA*: $p = 0.230$) *in the base value model.*

Support for result 2 can be found in Fig. 3 and Table 2. Auctioneer revenue share of both auction formats was higher in the base than in the multiband value model. The difference was significant for the CCA (base $\succ^{**}$ multiband, $p < 0.001$). The payment rule of the CCA had a significant impact and led to low revenue given the discounts, the low number of bids and bidders. Another reason for the difference is the number of unsold blocks in CCA auctions.

The auctioneer revenue of CCAs was higher in the base value model than in the multiband value model. Again, the number of possible bundles serves as an explanation, since it causes difficulties when bidders try to coordinate and find the efficient solution. Given the low number of bundle bids, the second best allocation was often much lower and resulted in high discounts and low payments. 5 of 16 CCAs in the multiband value model terminated with an auctioneer revenue share of 30 % or less. One auction yielded as little as 2 % auctioneer revenue share.

In SMRA, some efficiency losses can occur due to the exposure problem and it is interesting to understand what the efficiency of the auctions would be with straightforward bidders, who always bid on their payoff maximizing bundle. We implemented simulations with artificial bidders bidding in the SMRA and the CCA. In SMRA, the bidders do not take an exposure risk, and bid up to their valuations per block (*Simblock* bidders). In the CCA bidders always bid on their payoff-maximizing bundle in the primary bid rounds, and they submitted a truth-revealing bid on all bundles with positive value in the supplementary bids round (*SimDirect* bidders). This helps understand the difference in efficiency and revenue, which is due to the bundle selections of bidders in the lab.

Table 3 Aggregate simulations results

Value model	Auction format	Bidder	$E(X)$	$E(X)^*$	$R(X)$
Base	SMRA	Simblock	92.49 %	30.63 %	64.66 %
Base	CCA	SimDirect	100.00 %	100.00 %	86.16 %
Multiband	SMRA	Simblock	90.55 %	60.90 %	58.92 %
Multiband	CCA	SimDirect	100.00 %	100.00 %	75.72 %

Table 3 presents aggregate results of these simulations. Efficiency and Revenue of the SMRA format is lower compared to the results from the lab. This is due to the fact that the bidders always avoid exposure. Efficiency in the CCA format is 100 % with truth revealing bidders because there is no exposure risk. The difference in revenue to the lab results is substantial. This phenomenon is particularly strong in the multiband value model.

5.2 Bidder behavior in the CCA

Result 3 *On average, there was a low degree of bid shading in the supplementary bids round in the base value model in the lab and in the competition, but a higher degree of bid shading in the multi-band value model. If the bids are regressed on valuations, the slope of the OLS estimation was* 0.9 *for the base value model and* 0.68 *for the multi-band value model.*

Figure 4 shows whether bidders bid below, at or above their valuation on a bundle in the supplementary bids round. The figure also plots a regression line in addition to the diagonal with a slope of one (the truthtelling strategy). The slope of this regression can serve as an estimator for bid shading. For the base value model the slope is 0.90 (adjusted $R^2 = 0.77$) for data from the lab and 0.896 (adjusted $R^2 = 0.84$) for the bid data from the competition. In the multiband value model the slope of the regression was 0.68 (adjusted $R^2 = 0.796$).

In the base value model in the competition only 3.2 % of the bids were above their valuation while in the lab there were 22.6 % of the bids above the valuation. A single lab bidder in the base value model bid consistently above his valuations, which led to this effect. Without this bidder only 12.8 % of the bids were above the valuations. In the multiband value model only 6.4 % of the bids were above the valuation.

Next, we want to understand the selection of bundles in the supplementary bids round. With a VCG payment rule and independent private values, bidders would have a dominant strategy to bid on all bundles with a positive utility. The CCA does not have a dominant strategy, but it is also not obvious, how bidders would select their bundles strategically in such a setting to improve expected utility.

Result 4 *Bidders bid only on a fraction of bundles with a positive value in the supplementary bid phase. Bidders in the lab bid only in* 23.67 % (11.36 *bids*) *of* 48 *potential bundle bids they could bid on in the base value model and* 0.06 % (8.33 *bids*) *of the* 2,400 *possible bundles in the multiband value model in the supplementary bids round.*

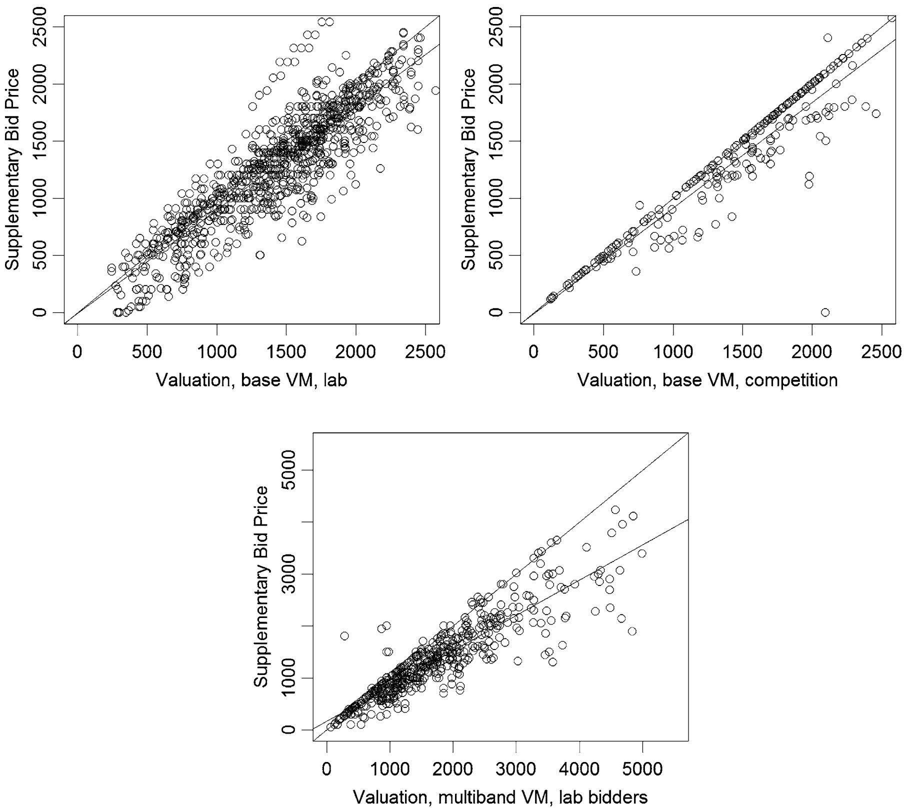


Fig. 4 Supplementary bids in the lab and in the competition

In the competition bidders submitted on average 12.6 % (6.06 *bids*) *in the base value model.*

Lab bidders in the large multiband value model actually submitted less bundle bids in absolute numbers than in the smaller base value model. Some bidders in the small value models actually submitted bids on almost all bundles in the supplementary bids round. For example, there were bidders who submitted 36 of 48 possible bundle bids in the base value model. In contrast, in the multiband value model bidders submitted 22 bids in the supplementary bids round at a maximum. A focus on only subsets of all possible bundles was also found by Scheffel et al. (2012) for other combinatorial auction formats with a larger number of bundles and can be seen as a consequence of the communication complexity of combinatorial auctions (Nisan and Segal 2006).

A low number of bids can also be observed in the field. For example, in the L-band auction in the UK in 2008, 17 specific blocks were sold resulting in 131,071 possible bundles, but the 8 bidders only submitted up to 15 bids in the supplementary bids round (Cramton 2008). Similarly, in the 10–40 GHz auction in the UK in 2008 bidders could bid on 12,935 distinct bundles. Eight bidders only submitted up to 22 bundles, while one submitted 106 and another 544 bundle bids (Jewitt and Li

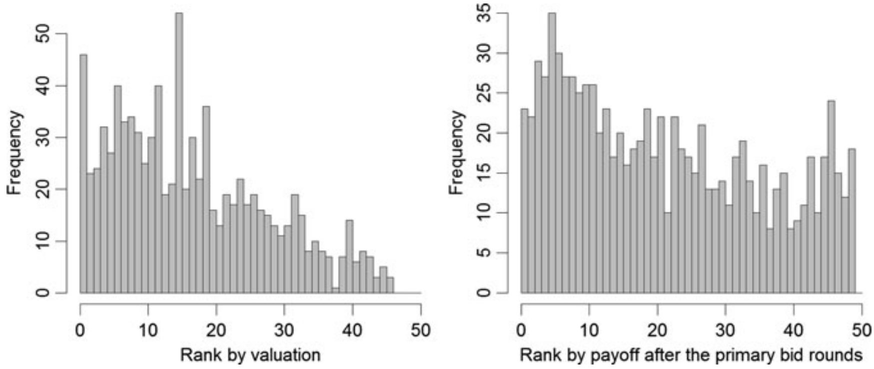


Fig. 5 Rank of supplementary bids by valuation or payoff after the primary bid rounds, base value model, lab

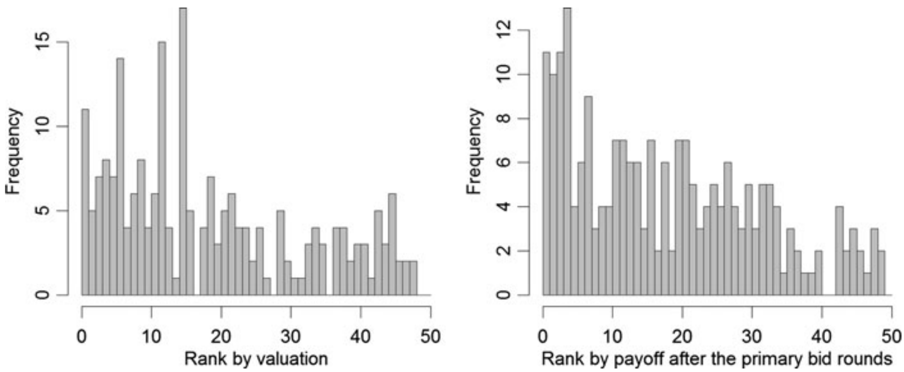


Fig. 6 Rank of supplementary bids by valuation or payoff after the primary bid rounds, base value model, competition

2008). It might be that bidders were just unprepared and have not fully understood the consequences of particular strategies in the CCA (Jewitt and Li 2008). Another explanation is that practical reasons keep bidders from submitting hundreds or even thousands of bids. It is interesting to understand, how bidders select bundles in the primary and in the supplementary bid rounds.

Result 5 *Bidders selected bundles in the supplementary bids round based on synergies in the value model, their relative strength with respect to the prior distribution, and ask prices after the primary bid rounds.*

Let us first look at the base value model. We have calculated the rank of each bid submitted based on the valuation of the bundle for a bidder, and based on the payoff the bidder had for the bundle at the end of the primary bid rounds. Figures 5 and 6 show that bidders exhibit a tendency to select bundles with a better rank based on payoff but also valuations in both the lab and the competition. The histogram to the left shows the frequency of bids ordered by valuation for these bids, while the

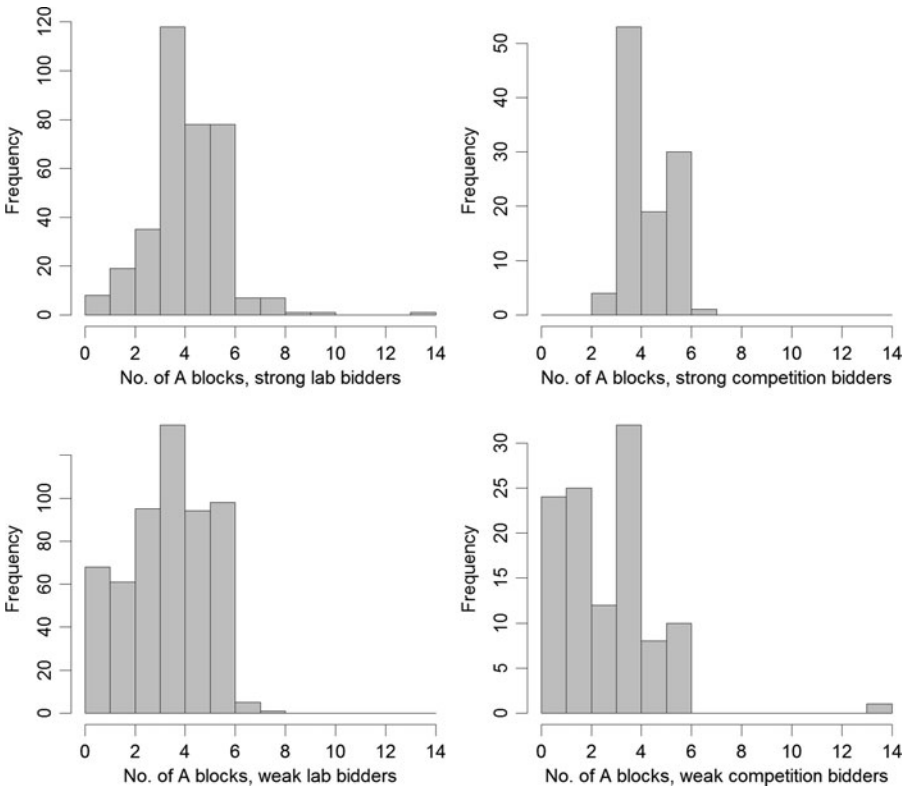


Fig. 7 Number of A blocks in bundle bids in the base value model, strong vs. weak bidders, lab and competition

histogram to the right shows the frequency of bids ordered by payoff at prices of the last primary bid rounds. Bidders also submit a considerable number of bids when their payoff at prices of the last primary bid round is low or negative in the base value model. Figure 7 reveals that most bids in band A are on four blocks, where the synergies were highest. In contrast, bidders bid on up to six blocks in band B, where there was no synergy for larger bundles. There is also a significant difference between weak and strong bidders. Bidders with a higher base valuation in band A are classified as strong, with a base valuation lower than the second order statistic as weak. Figure 7 shows the frequency of bids on a certain number of blocks for strong bidders in the lab and in the competition in the top row and for weak bidders in the lab and in the competition in the bottom row. While only a few strong bidders submitted bids on less than four blocks in the A band, weak bidders typically did submit such bids. This is even more pronounced in the competition. The number of blocks in bids of weak or strong bidders, as well as those of bidders in the lab and in the competition are significantly different ($\alpha = 0.01$).

Next, we analyze the bids in the multiband value model, where bidders had 2,400 bundles to choose from. Figure 8 shows the frequency of bids ranked by valuation or by payoff after the primary bid rounds for the multiband value model. Apparently,

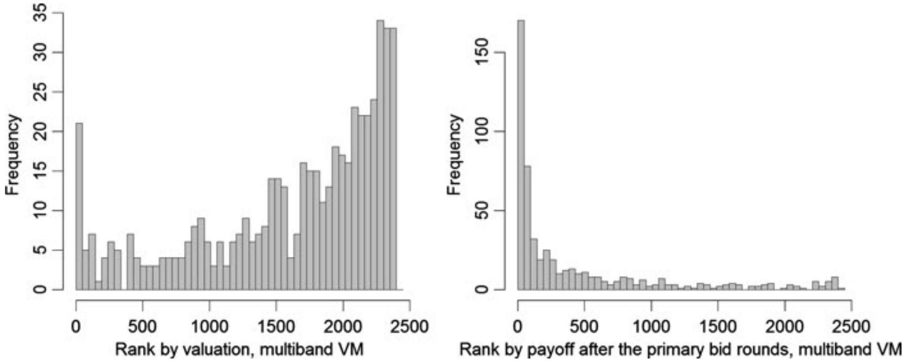


Fig. 8 Rank of supplementary bids by valuation or payoff after the primary bid rounds, multiband value model, lab

bidders used the ranking based on payoff as a guidance to select bundles, whereas the ranking of valuations did not influence decisions strongly. Figure 9, however, shows that again information about the synergies has influenced the bundle selection in the A, B, C, and D band. Most bundles included two or three blocks of a band only. Those bundles also had the highest synergy in the multiband value model with two blocks having higher synergies than three blocks.

In summary, in both value models bidders in the lab and in the competition used information about the synergies in the value models and tried to win those bundles with the highest synergies. In the multiband value model, they also selected the top-ranked bundles based on payoff. These observations are in line with a recent paper by Scheffel et al. (2012), which found that bidders often use simple heuristics for bundle selection in combinatorial auctions.

Finally, we also analyzed, whether bidders bid straightforward in the primary bid rounds. In other words, do they select the bundle with the highest payoff in each round?

Result 6 *Bidders did not follow a straightforward bidding strategy in the primary bid rounds in both value models.*

Figures 10 shows that in both the competition and the lab, bidders did not bid straightforward, and the payoff in each round was not the primary criterion for the bundle that bidders selected in the base value model. The histogram to the left shows the rank by payoff based on prices in the last round of the primary bids phase. The two histograms to the right show the frequency of bids in the A and B band respectively for lab bidders. The pattern in the competition was similar. The activity rule in the primary bid rounds is one explanation. Bidders must not increase the number of blocks in a bundle bid across rounds. Therefore, they often started out with large bundle bids in the initial rounds, rather than selecting their payoff maximizing bundle. In the multiband value model bidders selected bundles with two or three blocks in a band most frequently, which exhibited the highest synergies (see Fig. 11).

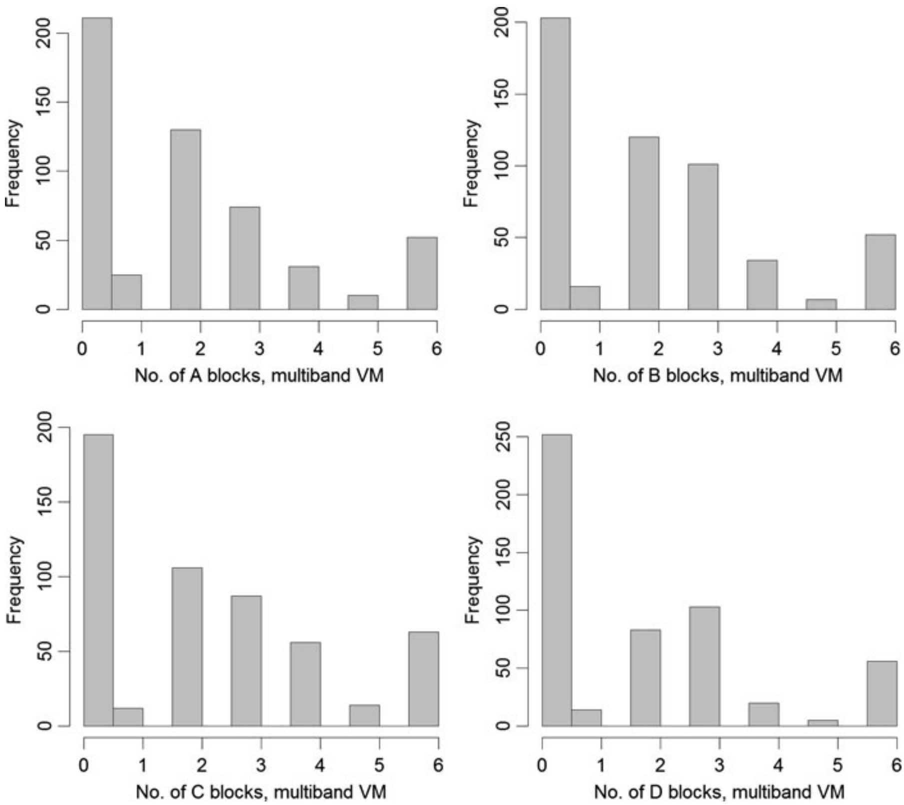


Fig. 9 Number of blocks in bundle bids in the supplementary bids round, multiband value model

5.3 Bidder behavior in the SMRA

The focus of this paper is the CCA. However, in the following, we also provide a summary of the main findings about bidding in the SMRA. We will focus on the likelihood of taking an exposure risk and jump bidding.

Exposure risk is a central strategic challenge of the SMRA in the presence of complementary valuations. Strong bidders with a high valuation might want to take this risk, while weak bidders would decide to reduce demand in order to keep prices low. Alternatively, weak bidders could try to pretend to be strong and bid aggressively at the start hoping others believe the threat and reduce their demand.

Result 7 *In the base value model, strong bidders took an exposure risk less often than weak bidders in the lab and the competition. In contrast, strong bidders took exposure risk more often than weak bidders in all four bands of the multiband value model. Strong bidders also took higher levels of exposure risk more often than weak bidders in the more valuable band A while weak bidders took it more often in the other bands. Bidders in competitions took exposure risk less often than bidders in the lab.*

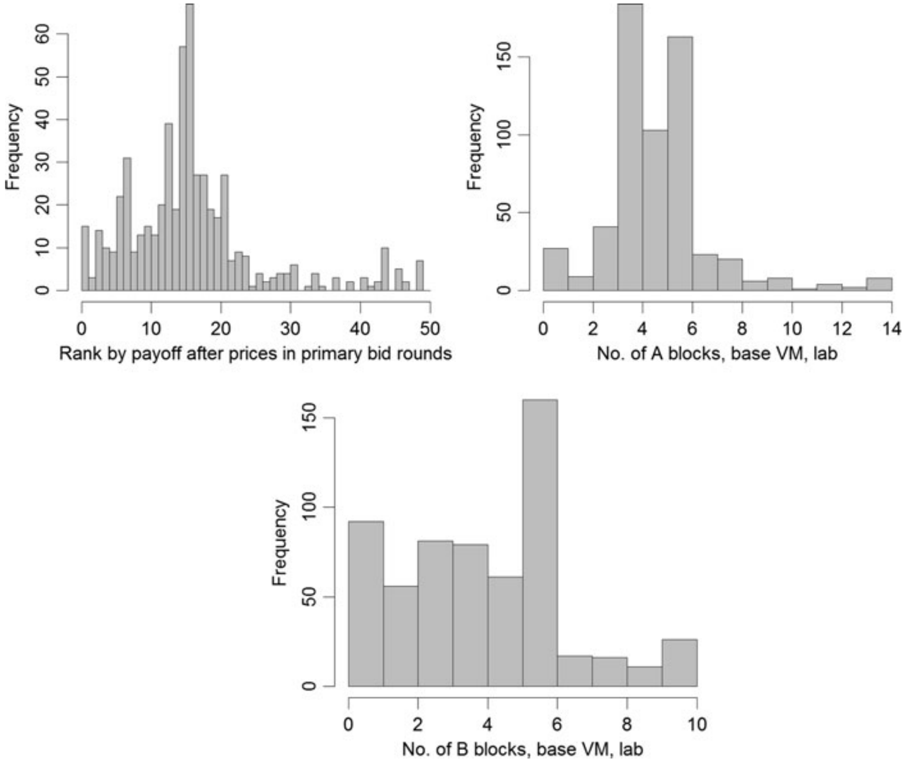


Fig. 10 Straightforward bidding in the primary bid rounds, base value model, lab

Table 4 Share of bidders who take different levels of exposure risk in the base value model

Bidder	Band	Strength	No. of bidders	$bid > v_A$	$bid > 1.2 * v_A$	$bid > 1.4 * v_A$
Lab	A	All	80	88.75 %	72.50 %	56.25 %
Lab	A	Strong	39	87.18 %	64.10 %	53.85 %
Lab	A	Weak	41	90.24 %	80.49 %	58.54 %
Comp	A	All	36	86.11 %	55.56 %	36.11 %
Comp	A	Strong	18	83.33 %	55.56 %	22.22 %
Comp	A	Weak	18	88.89 %	55.56 %	50.00 %

Both value models encompass different levels of synergies which define several stages of exposure risk for the bidders. Synergies in band A in the *base value model* rise with the bundle size for up to four blocks (see Table 4). First, we analyzed how many of the bidders submitted a bid in band A, which was higher than their base valuation $bid > v_A$, higher than their valuation on a bundle of two blocks ($bid > 1.2 * v_A$), and even higher than their valuation for a package of three blocks ($bid > 1.4 * v_A$). Bidders with a base valuation lower than the second order statistic were again classified as weak, the others as strong.

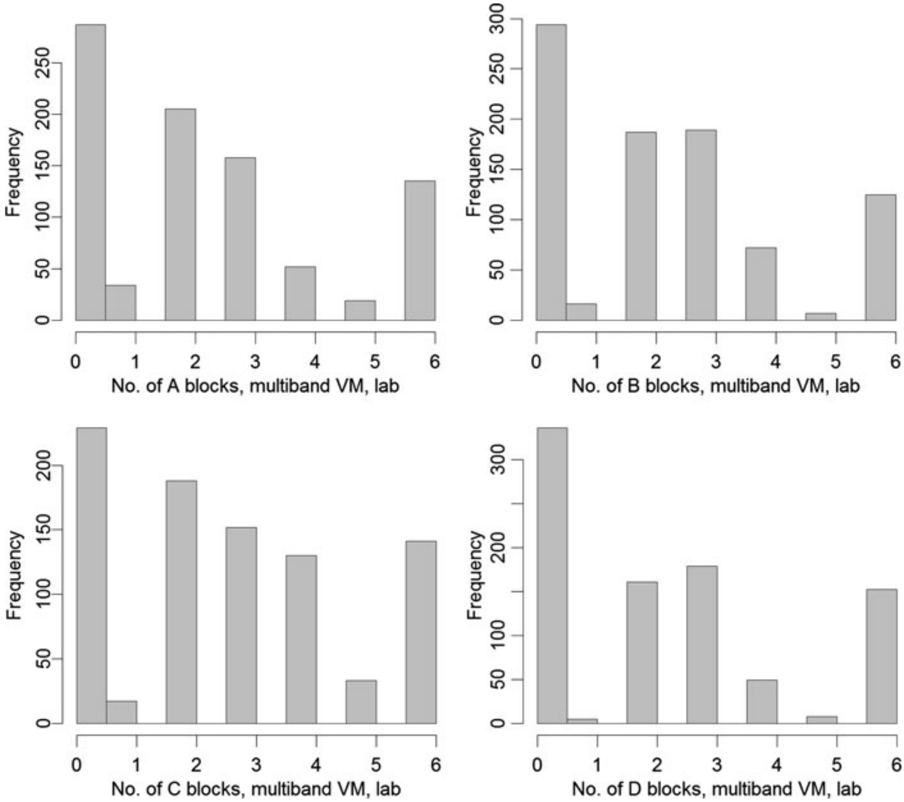


Fig. 11 Number of blocks in bundle bids in the primary bid rounds, multiband value model

Table 5 Share of bidders who take different levels of exposure risk in the multiband value model

Bidder	Band	Strength	No. of bidders	$bid > v_A$	$bid > 1.5 * v_A$
Lab	A	All	64	79.69 %	21.88 %
Lab	A	Strong	24	91.67 %	25.00 %
Lab	A	Weak	40	72.50 %	20.00 %
Lab	B	All	64	81.25 %	17.19 %
Lab	B	Strong	33	90.91 %	12.12 %
Lab	B	Weak	31	70.97 %	22.58 %
Lab	C	All	64	84.38 %	25.00 %
Lab	C	Strong	36	86.11 %	11.11 %
Lab	C	Weak	28	82.14 %	42.86 %
Lab	D	All	64	81.25 %	20.31 %
Lab	D	Strong	34	91.18 %	14.71 %
Lab	D	Weak	30	70.00 %	26.67 %

Table 6 Bidders with negative payoff

Value model	Bidder	Bidders with negative payoff	Total bidders	Share
Base	Lab	19	80	23.75 %
Base	Competition	5	36	13.89 %
Multiband	Lab	5	64	7.81 %

In the *multiband value model* we have decreasing synergies. Statistics on exposure risk can be found in Table 5. Due to the higher synergy of two blocks, bidders can bid on three blocks and then reduce from three to two blocks without making losses. Only a reduction from two blocks to one block will lead to a loss if prices rise above the base valuation. The different degrees of exposure risk in both value models explain the higher number of exposure problems in the base value model compared to the multiband value model (Table 6).

In the base value model the competitive situation makes it easier to win for strong bidders. With 14 blocks in band A the strong bidders expect to win four blocks each while the weaker bidders have to split the remaining six blocks. Weaker bidders cannot win four blocks without taking exposure risk, which explains why they take more exposure risk.

With only six blocks per band in the multiband value model, it is likely that either two bidders win three blocks each or three bidders win two blocks each. Weak bidders are less willing to take exposure risk and to risk ending up with only one block. The strong bidders often face strong competitors within the same band which forces them to take exposure risk. Within all four bands of the multiband value model strong bidders take exposure risk more often than weak bidders.

In competitions, strong as well as weak bidders were more careful about exposure risk. In particular, this is the case with higher levels of exposure. Table 6 shows that a lower percentage of bidders in competitions experience losses in the base value model. While 23.75 % of the bidders in the lab receive negative payoff, only 13.89 % of the bidders in the competition make a loss due to taking an exposure risk. Since bidders have four bands with complementarities to coordinate in the multiband value model, their risk of a loss is lower and only 7.81 % of bidders actually make a loss.

We did not find evidence for tacit collusion in SMRA, neither in the lab nor in the competition although the stacked activity rule gave bidders the possibility to bid on a lower number of items without losing the chance of bidding on larger bundles in later rounds. Bidders signaled their preferences, but none of the auctions resulted in agreements at low revenue.

Jump bids can be interpreted as signaling. By bidding more for an block than the ask price a bidder can signal preferences and discourage other bidders from bidding on this block. Such a signal can also be given by raising the bid on an block, for which the bidder already holds the standing high bid.

Result 8 *Jump bids were used by all bidders in all treatments. Bidders in the lab used jump bids more often than bidders in the competition* (*lab* $\succ^*$ *competition*, $p = 0.0103$). *Bidders in the competition submitted lower jump bids.*

Table 7 Jump bids by band

Value model	Bidder	Band	Avg. no. of jump bids	Avg. no. of bids	Share (%)
Base	Lab	A	12.91	23.23	55.60
Base	Lab	B	9.54	17.16	55.57
Base	Lab	All bands	22.45	40.39	55.59
Base	Comp	A	8.58	17.14	50.08
Base	Comp	B	7.83	19.08	41.05
Base	Comp	All bands	16.42	36.22	45.32
Multiband	Lab	A	8.09	15.33	52.80
Multiband	Lab	B	6.88	14.72	46.71
Multiband	Lab	C	9.22	15.72	58.65
Multiband	Lab	D	7.31	13.75	53.18
Multiband	Lab	All bands	31.5	59.52	52.93

Table 8 Jump bids by step size on all bands

Value model	Bidder	Avg. no. of bids	All	Low	Medium	High
Base	Lab	40.39	55.59 %	21.11 %	21.02 %	13.46 %
Base	Comp	36.22	45.32 %	24.77 %	12.81 %	7.75 %
Multiband	Lab	59.52	52.93 %	18.09 %	14.12 %	20.71 %

Table 7 shows that jump bids were heavily used across all bands. Bidders in competitions used jump bids less frequently than bidders in the lab.

Of course the level of jump bids varies. Table 8 shows the number of jump bids of different levels as percentage of the total number of bids. Low jump bids are those bids which exceed the ask price by 1 and 2 Franc (two lowest steps of the click-box), medium jump bids exceed the ask price by 5 and 10 Franc (two steps in the middle) and high jump bids by 20 and 50 Franc (two top steps). Low jump bids can be used to avoid ties.

Bidders in both the lab and competition, used low jumps, and there was no significant difference (lab $\sim$ competition, $p = 0.5270$). Medium and high jump bids are used to demonstrate strength and discourage other bidders from bidding on this very block. We found that bidders in competitions used medium and high jump bids less often than lab bidders (medium: lab $\succ^{**}$ competition, $p = 0.0004$; high: lab $\succ^{**}$ competition, $p = 0.0032$).

Result 9 *Bidders of all treatments placed bids on blocks that they have provisionally won in the previous round* (*own blocks*). *In bands of higher valuation* (*band A in both value models*) *bidders used a higher number of bids on own blocks than in other bands*.

Support for this result is presented in Table 9. Bidders in competition used almost 10 times more bids on own blocks in the more valuable A band (9.40 %) than in

Table 9 Bids on own blocks

Value model	Bidder	Band	Avg. no. of bids on own blocks	Avg. no. of bids	Share (%)
Base	Lab	A	1.56	23.23	6.73
Base	Lab	B	0.69	17.16	4.01
Base	Lab	All bands	2.25	40.39	5.57
Multiband	Lab	A	0.63	15.33	4.08
Base	Comp	A	1.61	17.14	9.40
Base	Comp	B	0.36	19.08	1.89
Base	Comp	All bands	1.97	36.22	5.44
Multiband	Lab	B	0.48	14.72	3.29
Multiband	Lab	C	0.64	15.72	4.08
Multiband	Lab	D	0.36	13.75	2.61
Multiband	Lab	All bands	2.11	59.52	3.54

band B (1.89 %). Bidders in the lab submitted bids on own blocks in band A (6.73 %) and band B (4.01 %).

6 Conclusions

One result of this study is that the CCA did not yield higher efficiency in the small base value model and performed significantly worse than SMRA in the multiband value model. Revenue was significantly lower in all treatments and sometimes blocks remained unsold in spite of sufficient demand. This was due to the low number of bundle bids and the CCA payment rule. In the CCA, bidders only submitted a small subset of all possible bundle bids. Bidders used heuristics to select these bundles, mainly based on their relative strength and the synergies in the value model. In real-world applications bidders cannot be expected to submit hundreds or thousands of bids in an auction, even with decision support tools available. Actually, in most applications of the CCA so far bidders are limited to submit a few hundred bids in the supplementary bids phase. It might also be difficult to get an agreement among all stakeholders in a company for thousands of package valuations, even though all of these packages of licenses can have a value to a large bidder. As our experiments show, this can have a significant negative impact on efficiency and revenue in CCAs with many blocks.

It is interesting to note that if bidders submitted bids on all possible bundles truthfully, as was the case in our simulations, the revenue of the CCA was much higher and comparable to the revenue of SMRA in the lab. In comparison, the SMRA elicited the valuations of bidders on individual blocks sufficiently well to allow for high efficiency even in the multiband value model. In particular, strong bidders often took the exposure risk to win a bundle with high synergies, such that the negative effect on efficiency was mitigated.

Of course, our results need to be interpreted with the necessary care. First, our results do not necessarily generalize to very different value models. We also assumed all bidders to have the same synergies. This was motivated by our application domain and the fact that synergies often arise from a mobile operator's ability to achieve peak performance with a technological standard after winning a certain amount of bandwidth, and these synergies are the same for all operators. We ran 4 additional sessions with a multiband value model and different synergies across bidders, but saw no significant impact on the results of our study. Second, one can argue that with even more time to prepare, bidders might behave differently. We conjecture that for a sufficiently large number of blocks, bidders will not be able to elicit and submit bids on the exponential number of bundles with positive value. Simplification and compact bid languages, which allow expressing the main synergies of a value model with a few parameters only, might be a remedy and further research is needed in this area. Such approaches have been used for procurement (Bichler et al. 2011a), but also for spectrum auctions in the field (Goeree and Holt 2010).

Apart from communication complexity, which is a problem in all combinatorial auctions with fully expressive XOR bid languages and a larger number of items, the results of the primary bid rounds in the auction designs that are currently in use can provide enough information for riskless spiteful bidding, and prices for the same allocation can be different for different bidders. The latter is a phenomenon in VCG auctions as well, but has to be traded off against other design desiderata. While the clock phase can reduce uncertainty for bidders in a common values environment, the combination of the two stages comes at the cost of more complex auction rules. The CCA has a number of compelling advantages, but further research is needed to better understand bidding strategies and the performance in the lab and in the field.

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Appendix

A.1 CCA activity rules

The primary bid rounds help reducing value uncertainty in the market. Activity rules should provide incentives for bidders to reveal their preferences truthfully and bid straightforwardly in the primary bid rounds. Bidders should not be able to shade their bids and then provide large jump bids in the supplementary bids round. We will describe the activity rules and derive some useful propositions.

An eligibility points rule is used to enforce activity in the primary bid rounds. The number of bidder's eligibility points is non-increasing between rounds, and it limits the number of blocks the bidder can bid on in subsequent rounds. In the supplementary bids round, the following rules apply:[12]

[12]Note that the detailed rules used by regulators in the different countries have evolved over time and there are differences in the various countries. For the following analysis, we followed the rules for the 2.6 GHz auction design used in Austria in 2010, and also consulted the more recent Canadian rules for the auction in 2013 at http://www.ic.gc.ca/eic/site/smt-gst.nsf/eng/sf10363.html.

- There is no limit on the supplementary bid that can be made for the bundle bid in the final primary bid round.
- The supplementary bid for any other bundle A is subject to a cap determined in the following way:

  1. First, we determine the last primary bid round in which the bidder would have been eligible to bid for bundle A. Call this round the anchor round n. This will either be the final round or some round in which the bidder dropped its eligibility to bid (by reducing the number of blocks bid for) and gave up the opportunity to bid for bundle A in later primary bid rounds.
  2. Suppose that the bidder bids for bundle B in round n; The supplementary bid for bundle A cannot exceed the bid for bundle B (i.e., the supplementary bid for this bundle, if one is made, or otherwise the primary round bid) plus the price difference between bundles A and B that applied after round n.

Since the bidder had the opportunity to choose between A and B back in round n, and to opt for B, the bidder revealed the relative value between these two bundles. In the supplementary bids round, the bidder cannot reverse this reported preference by submitting a high bid on A. If a bidder bids on a bundle $C \subset B$ in the supplementary bids round, the bundle price is also bounded by the supplementary bid for B minus the price difference between the bundles B and C after round n. In other words, this is a revealed preference constraint. This constraint is applied to the supplementary bids phase and with respect to the last primary bid round, where the bidder had sufficient eligibility points to submit a bid, which is different to his standing bid after the primary bid rounds. Also, all supplementary bids must satisfy the revealed preference limit with respect to the final clock round regardless of whether the supplementary bid package is larger or smaller than the final clock package. A formalization of the revealed preference constraint will be provided for the proofs of the Propositions 3 and 4 below.

Proposition 1 *If a bidder follows a straightforward bidding strategy in the primary bid rounds of a CCA with an anchor activity rule, then the activity rule will not restrict him to bid his maximum valuation on every bundle in the supplementary bids round.*

Proof Let's assume, a bidder bids straightforward, i.e., he submits a bid on his payoff maximizing bundle in every round. Throughout the primary bid rounds he might have switched from a bundle A to a bundle B in a round n, when $v(B) - p_n(B) > v(A) - p_n(A)$, where $p_n(A)$ is the price of bundle A in round n. For the bundle A the bidder did not necessarily bid up to his true valuation in the primary bids round. Based on the anchor rule, in the supplementary bids round s the bidder can submit a maximum bid of $p_s^{max}(A) = v(B) + p_n(A) - p_n(B)$, if he bid his true valuation $p_s(B) = v(B)$. As a result of adding both inequalities $p_s^{max}(A) > v(A)$ such that the bidder can bid up to his true valuation on A in the supplementary bids phase. Note that the same argument applies for bundle bids, which were submitted in the primary bid rounds before A, after the bidder has revealed his true valuation $v(A)$ in the supplementary bids round. The proof also applies to bundles C, on which the bidder has never submitted a bid in the primary bid rounds, as long as $v(B) - p_n(B) > v(C) - p_n(C)$ in a round n, where the bidder had sufficient eligibility points to bid on this bundle. □

Unfortunately, bidding straightforward is not always possible.

Proposition 2 *If valuations for at least two bundles A and B are full substitutes with $v(A \cup B) = max(v(A), v(B))$ and the bundle of higher valuation A requires less bid rights than the lower valued bundle B, straightforward bidding is not possible due to the activity rule in the primary bid rounds.*

Proof Suppose there are two different bundles A and B with $|A| < |B|$ and $v(A \cup B) = max(v(A), v(B))$. The activity rule in the primary bid rounds does not allow to increase the number of bid rights in later rounds. We assume the number of bid rights required to be proportional to the size of the bundle. If $v(A) > v(B)$ and prices in A rise such that the payoff of A becomes smaller than that of B, a bidder would not be able to switch to bundle B, and would therefore not be able to bid straightforward on his payoff maximizing bundle at the prices. □

The proposed rules for spectrum auctions to be organized in the future have been improved to cure problems like this and also apply the revealed preference constraint to the primary bid rounds. Also, the constraint needs to be satisfied with respect to any eligibility-reducing primary bid round after the one, where the bidder could submit a bid on bundle C for the last time including the last primary bid round, and we will draw on those rules for the remainder.

A.2 Safe supplementary bids

We first introduce some additional notation. Let $q = (q_1, \ldots, q_d, \ldots, q_D)$ denote the supply of blocks in D bands, and $b_j^p(q^j) \in B$ the standing bid of bidder $j \in J$ on bundle $q^j = (q_1^j, q_2^j, \ldots, q_D^j)$ after the primary bid phase. In addition, let $r^s(q)$ denote the revenue of the optimal allocation after the supplementary bids phase including all bids B in both phases. $r^p(q)$ describes the value with only standing bids in the last round of the primary bids phase. $r^s_{-b_j^p(q^j)}(q)$ denotes the auctioneer revenue in the optimal allocation without all bids of bidder $j \in J$ on bundle q^j. $q^{-j} = q - q^j$ is the set of blocks complementary to q^j. We refer to π as the ask price vector in the last primary bid round. Supplementary bids $q^{j'}$ on packages different to the standing bid q^j of bidder j are restricted by a revealed preference rule such that bids with additional blocks are restricted by the prices after the anchor round. As prices in the primary bid rounds are increasing monotonously, the highest such price is π, such that $b_j^s(q^{j'}) - q^{j'}\pi \le b_j^s(q^j) - q^j\pi$ or $b_j^s(q^{j'}) \le b_j^s(q^j) + (q^{j'} - q^j)\pi$.

Proposition 3 *If demand equals supply in the final primary bid round, a supplementary bid $b_j^s(q^j) > b_j^p(q^j)$ cannot become losing.*

Proof In the last primary bid round, there is a demand of exactly q blocks, if demand equals supply. Bidder $j \in J$ submits a bid $b_j^p(q^j)$ in the last primary bids round, his standing bid after the primary bid rounds. Let $b_j^s(q^j) > r^s_{-b_j^p(q^j)}(q) - r^s(q^{-j})$ be the bid price that bidder j needs to submit, in order to win q^j after the supplementary bid round. Due to the anchor rule, j's competitors $k \in J$ with $k \neq j$ can only increase his bid without limits on bundles $q^k \leq q^{-j}$, his standing bid which was submitted in the last primary bids round. Any high supplementary bid $b_k^s(q^k)$ on a bundle q^k from k's standing bid after the primary bid rounds, will increase $r^s_{-b_j^p(q^j)}(q)$ as well as $r^s(q^{-j})$ and cannot impact $b_j^s(q^j)$, such that $b_j^s(q^j) > r^p_{-b_j^p(q^j)}(q) - r^p(q^{-j})$ is sufficient. This difference cannot be higher than $q^j\pi$. Supplementary bids on packages different to the standing bid $q^{k'}$ of bidder k are restricted, such that $b_k^s(q^{k'}) \leq b_k^s(q^k) + (q^{k'} - q^k)\pi$. So, if blocks are added to the standing bid of a competing bidder k, the price of additional blocks can not exceed π. Also, if a bidder k bids on less blocks in a band m than in his standing bid $q_d^{k'} - q_d^k < 0$, the bid price must be reduced by $(q_d^{k'} - q_d^k)\pi_d$. As a result, any supplementary bid $b_j^s(q^j) > b_j^p(q^j) = q^j\pi$ must be winning. □

If there is excess supply in the last round of the primary bid phase, a last primary round bid $b_j^p(q^j)$ can become losing, because even if no supplementary bids were submitted, the auctioneer conducts an optimization with all bids submitted at the end, which might displace $b_j^p(q^j)$. This raises the question for the safe supplementary bid $b_j^s(q_p^j)$, which ensures that the bidder j wins the bundle q_p^j of his standing bid from the primary round after the supplementary bids phase.[13]

Proposition 4 *If bundle q^u is unallocated after the last primary bid round, a supplementary bid $b_j^s(q^j) > b_j^p(q^j) + q^u\pi$ of a standing bidder j cannot become losing if all his supplementary bids contain q^j.*

Proof Let's first assume a specific situation with bidder j bidding on a bundle q^j in the last primary bid round and two other competitors, who bid on a bundle with all blocks q in the previous to last round of the primary bid phase. In the last round the two competitors reduce demand to zero so that q^{-j} blocks have zero demand after the last primary bid round. Now, at least one of the competitors k submits a supplementary bid on the bundle q at the prices of the last primary round $q\pi$. Now, bidder j can win bundle q^j only, if he increases his bid to $b_j^s(q^j) > b_j^p(q^j) + q^{-j}\pi = q\pi$, the *safe* supplementary bid. Supplementary bids of bidders k are restricted, such that $b_k^s(q^{k'}) \leq b_k^s(q^k) + (q^{k'} - q^k)\pi$. Consequently, no combination of bids of competitors can exceed $q\pi$.

Similarly, if the competitors k reduce their demand such that a package q^u with $q^u < q^{-j}$ blocks remain unsold after the primary bid rounds, bidder j has to increase his standing bid by more than $q^u\pi$ to become winning after the supplementary bid round with certainty. Again, this is due to the revealed preference rule, because j can leverage the standing bid on q^k and supplementary bids $b_k^s(q^k)$ of another bidder k. If blocks are added to the standing bid of the competing bidder k, the price of additional

[13]If the supplementary bids must not satisfy the revealed preference limit with respect to the final clock round, as it is outlined for example in the Canadian rules, a losing bidder after the primary bid rounds could also become winning as was pointed out by a recent working paper of Knapek and Wambach (2012). So the details of the activity rules of a specific country matter.

Table 10 Bids in the primary bid rounds. Standing bids are marked with a star

	Bidder N	Bidder R_{11}	Bidder R_{21}	Bidder R_{12}	Bidder R_{22}
Round 1	(AC) = 2	(A) = 1	(AB) = 2	(CD) = 2	(CD) = 2
...					
Round 15	(AC) = 30	(A) = 15	(AB) = 16	(CD) = 16	(CD) = 16
Round 16	(AC) = 32	(A) = 16	(2B) = 2	(CD) = 17	(CD) = 17
...					
Round 20	(AC) = 40	(A) = 20	(2B) = 2	(CD) = 21	(CD) = 21
Round 21		(A) = 21*	(2B) = 2*	(CD) = 22	(CD) = 22
...					
Round 40				(CD) = 41	(CD) = 41
Round 41				(CD) = 42*	(D) = 1*
Termination					

Table 11 Payments after the supplementary bids round without additional supplementary bids

	Bid price	VCG payment	CCA payment
Bidder R_{11} (A)	21	14	14
Bidder R_{12} (2B)	2	0	0
Bidder R_{21} (CD)	42	40	40
Bidder R_{22} (D)	1	0	0
	66	54	54

blocks can not exceed π. Also, if a bidder k bids on less blocks in a band m than in his standing bid $q_d^{k'} - q_d^k < 0$, the bid price must be reduced by $(q_d^{k'} - q_d^k)\pi_d$.

Note, that if bidder j also bids on packages $q^{j'} \leq q^j$, then his safe supplementary bid on q^j is not safe any more. To see this, look at a simple example with three items $\{\alpha, \beta, \gamma\}$ and two bidders 1 and 2. Bidder 1 has a standing bid on the bundle $\{\alpha, \beta\}$ after the primary bid rounds, while $\{\gamma\}$ is unsold. Prices for all three items in the last primary bid round are \$10. Bidder 1 bids \$40 on $\{\alpha, \beta\}$ in the supplementary bids round, and \$30 for $\{\alpha\}$. Bidder 2 bid \$ 18 in the previous to last primary bid round on $\{\beta, \gamma\}$. Even though bidder 1 submitted a safe bid on $\{\alpha, \beta\}$, his new bid on $\{\alpha\}$ together with the last primary round bid of bidder 2 would become winning. $\square$

A.3 Spiteful bidding

In the following, we will provide a brief example of a CCA, in which a bidder can submit a spiteful bid, which increases the payments of other bidders with little risk of winning such a bid. If excess supply is known and the bidder submits a safe supplementary bid as described in the last section, such bids would not stand a chance of winning.

Table 12 Payments after the supplementary bids round with an additional supplementary bid by bidder R_{21} on AB for \$22

	Bid price	VCG payment	CCA payment
Bidder R_{11} (A)	21	20	20
Bidder R_{12} (2B)	2	0	0
Bidder R_{21} (CD)	42	40	40
Bidder R_{22} (D)	1	0	0
	66	60	60

Consider one region in which 3 blocks A (1 unit) and B (2 units), and one region in which 3 blocks C (1 unit) and D (2 unites) are up for auction among one national and several regional bidders. Start prices are \$1 for all blocks and prices for overdemanded blocks are increased by \$1 per round. Each block corresponds to one eligibility point.

The national bidder N is only interested in winning block A and C in each of the two regions for at most \$40, i.e. he is not willing to switch to other packages. Regional bidder R_{11} is only interested in obtaining block A in his region. Regional bidder R_{21} prefers AB over 2B. He is willing to switch from AB to 2B, if prices differ by at least \$15. Regional bidders R_{12} and R_{22} would like to obtain CD. Bidder R_{22} is weaker and willing to bid on D after if he is overbid.

Table 10 illustrates the primary bid rounds, while Table 11 describes the payments if no supplementary round bid was submitted. Finally, Table 12 illustrates the payments if bidder R_{21} submitted a spiteful bid on AB for \$ 22, the package price in the final round for which he would still be eligible according to the activity rule. This is round 41, where the price for A is \$21 and for B is \$1. Let's assume bidder R_{11} increases his bid on block A by 1 in the supplementary round to be safe. The payments of R_{11} for his winning bid on A are $(22-(66-(16+42)))=\$14$ without the spiteful bid, and $(22-(66-(22+42)))=\$20$ with the spiteful bid. Consequently, the payment of the regional competitor increases by \$6 with both, a VCG and a core-selecting payment rule. Such bids are possible due to the initial eligibility points rule, which might not reflect the proportion of clock prices after the clock auction.

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