

## es. 2.2020 (Gennaio)

(i)  $L_{pal} = \{w \in \{0,1\}^* \mid w = w^R\}$

$L_{pal}$  regolare  $\Rightarrow 0^n 1^n 0^n = xyz \mid |xy| \leq n, y \neq \epsilon, xy^iz \in L_{pal}$   
 $\forall i \in \mathbb{N} \Rightarrow xz \in L_{pal} \nsubseteq L_{pal}$  non è regolare.

$$P \rightarrow \epsilon | 0 | 1 | OPO | 1P1$$

$$L_{pal} = L(G):$$

- $L_{pal} \subset L(G)$ 
  - $|w| \leq 1$ , base.
  - $|w| = n$ .  $w = 0^k 0$ :  $P \Rightarrow d \Rightarrow 0^k 0 \Rightarrow w$ .
  - $w = 1^k 1$ :  $P \Rightarrow d \Rightarrow 1^k 1 \Rightarrow w$ .

- $L(G) \subset L_{pal}$ 
  - $\epsilon, 0, 1 \in L_{pal}$
  - $P \Rightarrow OPO | 1P1$ .  $(OPO)^R = O^{R^T} O = OPO$ .  
 $(1P1)^R = 1P1^T = 1P1 \Rightarrow w \in L_{pal}$ .

(ii) Per il PL  $0^n 1^n 2^n$ .

es. 2.2020 (Febbraio)

(i)  $L_1 = \{ b^2 a^n b^m a^3 \mid m, n \geq 0 \}$   $bb a^* b^* a a a$  reg  $\rightarrow$  CFL

(ii)  $L_2 = \{ a^n w \mid w \in \{a, b\}^* \text{ t.c. } |w| = n, n \geq 0 \}$

Per il PL,  $a^n b^n$ ; non è regolare.

$P \Rightarrow \epsilon \mid aPa \mid aPb$ . E' CFL.

(iii)  $L_3 = \{ a^i b^j c^k \mid i, j \geq 0, k = \max(i, j) \}$ .

$$a^n b^n c^n = stuvx$$

$$\left. \begin{array}{l} tuv = a^n \Rightarrow st^2uv^2x \notin L_3 \\ tuv = b^n \Rightarrow st^2uv^2x \notin L_3 \\ tuv = c^n \Rightarrow st^2uv^2x \notin L_3 \\ tuv \ni a, b \Rightarrow sux \notin L_3 \\ tuv \ni b, c \Rightarrow sux \notin L_3 \end{array} \right\} \Rightarrow \text{CFL} (\Rightarrow \text{reg})$$

## es. 5.2020 (Febbraio)

$$\max(L) = \{ w \in L \mid \nexists x \neq \epsilon \text{ t.c. } wx \in L \}$$

(i)  $L$  regolare  $\Rightarrow \max(L)$  regolare.

Copiare il DFA di  $L$ . Normalizzare gli stati da cui sono raggiungibili altri stati finali.

$$(ii) L = \{ a^i b^j c^k \mid k \leq i \vee k \leq j \}$$

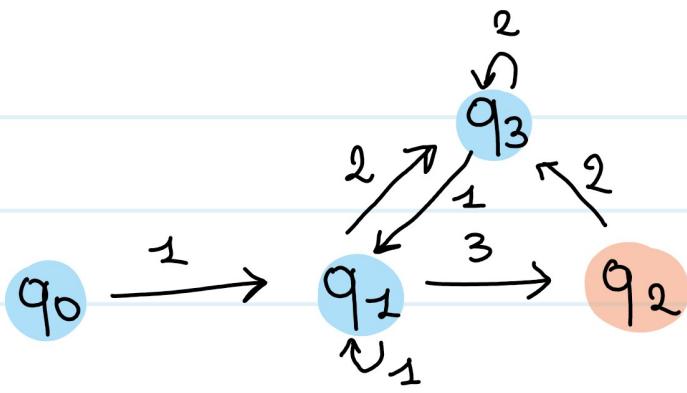
$$P \rightarrow aBc \mid aPc \mid aP$$

$$B \rightarrow b \mid bB$$

$$\max(L) = \{ a^i b^j c^{\max(i,j)} \} \text{ non CFL.}$$

## es. 1.2019 (Gennaio)

$$\Sigma = \{1, 2, 3\}$$



$$1(1+22^*1)^*3 \left( 22^*1(1+22^*1)^*3 \right)^*$$

$$P = \{ q_0 \rightarrow 1q_1, q_1 \rightarrow 1q_1 \mid 2q_3 \mid 3q_2, \\ q_2 \rightarrow 2q_3 \mid \varepsilon, q_3 \rightarrow 2q_3 \mid 1q_1 \}$$

$$G = (\{q_0, \dots, q_3\}, \{1, 2, 3\}, P, q_0)$$

es. 2. 2019 (Gennaio)

$$(i) L_1 = \{ 0^i 1^j 2^j 3^i \mid i \geq 1, j \geq 1 \}$$

Per la PL,  $L_1$  non reg.

$$\left. \begin{array}{l} E \rightarrow 0I3 \\ I \rightarrow 12 \mid 1I2 \end{array} \right\} \text{libero}$$

$$(ii) L_2 = \{ 0^i 1^j 2^j 3^j \mid i \geq 1, j \geq 1 \}$$

Per il PL con  $i=1$ ,  $L_2$  non libero.

es. 2.2019 (Giugno)

$$(i) L_1 = \{ 0^n 1^{2n} \mid n \geq 1 \}$$

Per PL non è reg. con  $0^n 1^{2n}$ .

$$E \rightarrow 011 \mid 0 E 11 \Rightarrow CFL$$

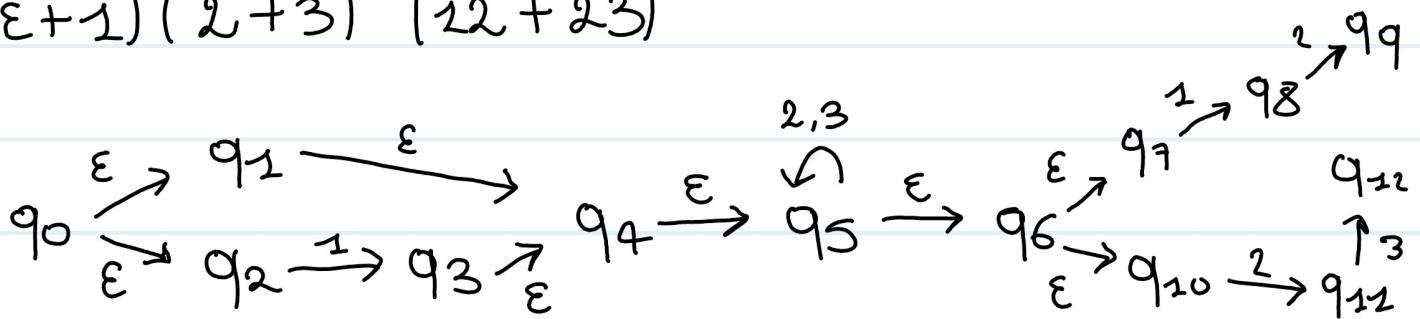
$$(ii) L_2 = \{ ww \mid w \in \{a, b\}^* \}$$

$a b^n a b^n$  per il PL  $\Rightarrow$  CFL  $\Rightarrow$  reg.

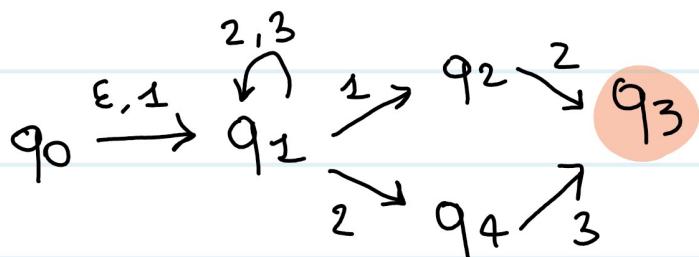
ES. 1. 2018

$$\Sigma = \{1, 2, 3\}$$

$$(\varepsilon+1)(2+3)^*(12+23)$$



|||



$$\{q_0, q_1\} \quad \{q_2\} \quad \{q_2\} \quad \{q_3\} \quad \{q_4\}$$

1	2	3	1	2	3
$\{q_0, q_2\}$	$\{q_1, q_2\}$	$\{q_3, q_4\}$	$\{q_2\}$	$\{q_3\}$	$\emptyset$

$\{q_1\}$	$\{q_2\}$	$\{q_3, q_4\}$	$\{q_1\}$	$\{q_4\}$	$\emptyset$	$\emptyset$	$\{q_3\}$
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$\{q_2\}$	$\emptyset$	$\{q_3\}$	$\emptyset$
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$\{q_1, q_2\}$     $\{q_2\}$     $\{q_2, q_3, q_4\}$     $\{q_1\}$

$\{q_1, q_4\}$     $\{q_2\}$     $\{q_2, q_4\}$     $\{q_1, q_3\}$

...

es. 1. 2018 (6:ugm)

$\{0, 1, 2, 3\}$     $\{4\}$

a:  $\{q_0, q_1, q_3\}$     $\{q_2, q_5\}$     $\{q\}$

b:  $\{0, 1, 2, 3, 5\}$     $\{4\}$

$\{0, 1, 3\}$     $\{2, 5\}$     $\{4\}$

a:  $\{0, 1, 3\}$     $\{2, 5\}$     $\{4\}$

b:  $\{0, 3\}$     $\{1\}$     $\{2, 5\}$     $\{4\}$

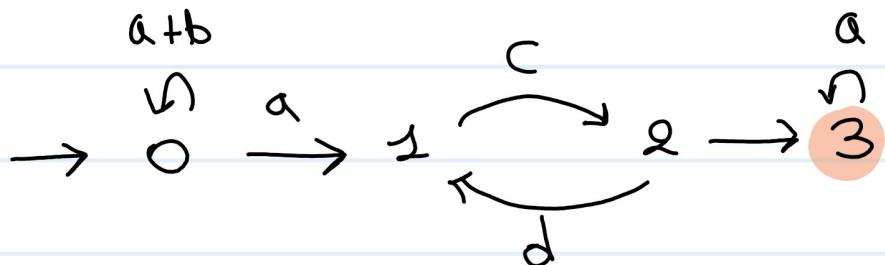
$\{0, 3\}$     $\{1\}$     $\{2, 5\}$     $\{4\}$  ✓

a: ..   ..   ..   ..

b: " " . "

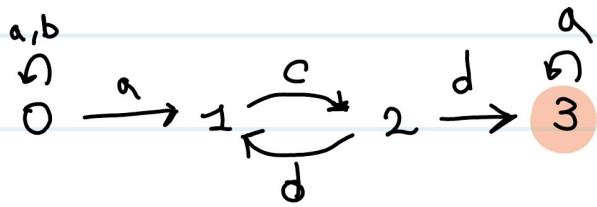
$$\left. \begin{array}{l} E \rightarrow aIc \mid aEc \\ I \rightarrow \varepsilon \mid bIC \end{array} \right\} \begin{array}{l} a^n b^m c^m c^n \\ a^n b^m c^{n+m}, n > 0, m \geq 0 \end{array}$$

es. 1.2018 (Luglio)

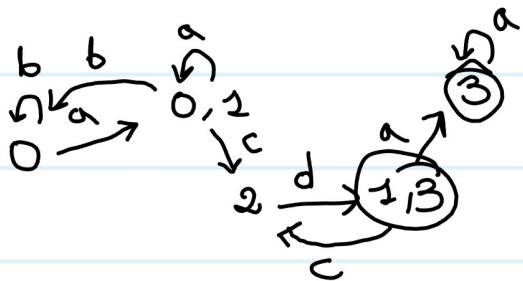


$$(a+b)^* a (cd)^* c a^*$$

es. 1. 2014 (Gennaio)



$$(a+b)^* a (cd)^* c d a^*$$



	a	b	c	d
0	0,1	0	/	/
1	/	/	2	/
2	/	/	/	1,3
3	3	/	/	/
0,1	0,1	0	2	/
1,3	3	/	2	/

$$\left\{ q_0 \rightarrow b q_0 | a q_1, q_2 \rightarrow b q_0 | a q_2, q_2 \rightarrow d q_3, q_3 \rightarrow c q_2 | \right. \\ \left. | a q_4 | \varepsilon, q_4 \rightarrow a q_4 | \varepsilon \right\}$$

es. 1. 2013 (Gennaio)

$$\Sigma = \{a, b, c\}$$

$$a^n (a^k b^+) b^n = a^m b^n, n \geq 2, m \geq 1$$

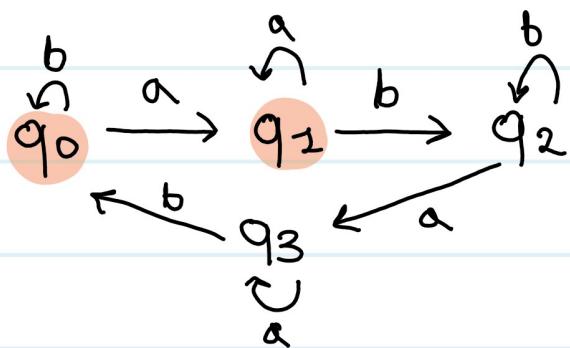
$$S \rightarrow a S b | a A b \quad a a^* b^+ b$$

$$A \rightarrow a A | B \quad \} \quad a^* b^+$$

$$B \rightarrow b B | b \quad \} \quad b^+$$

es. 1. 2023 (Luglio)

(i)  $L = \{w \in \{a, b\}^* \mid \text{occ. di } ab \text{ par.}\}$

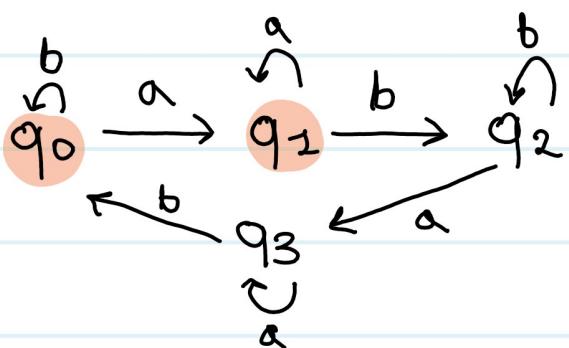


$\{q_0, q_1\}$   $\{q_2, q_3\}$

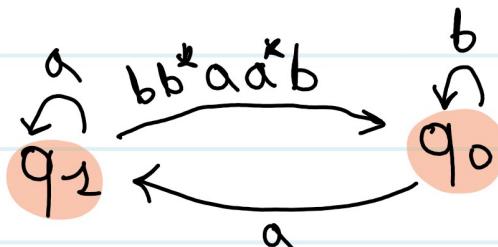
a:  $\{q_0, q_1\}$   $\{q_2, q_3\}$

b:  $\{q_0\}$   $\{q_1\}$   $\{q_2\}$   $\{q_3\}$

$\{q_0\}$   $\{q_1\}$   $\{q_2\}$   $\{q_3\}$  ✓



I.



II.

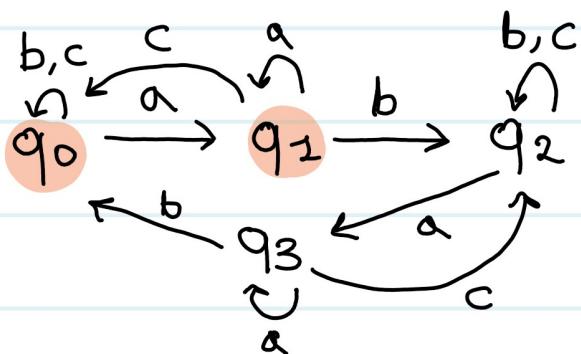
$$b + \alpha\alpha^*(bb^*\alpha\alpha^*b)$$



$$(b + \alpha\alpha^*(bb^*\alpha\alpha^*b))^* (\varepsilon + \alpha\alpha^*)$$

$$\{ E \rightarrow 1 \mid 1E0 | 0E1 | 10E | 01E | E10 | \\ E01 | 1E | E1 \}$$

XX110001



$\{q_0, q_1\} \quad \{q_2, q_3\}$

001111100

101111100100

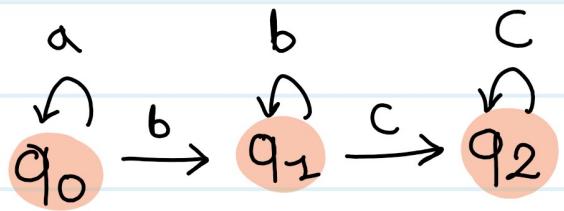
{  $E \rightarrow A1A | EE | AEA,$   
 $A \rightarrow \epsilon | 0A1 | 1A0 | AA$  }

00111100

$\overbrace{01}^E \overbrace{11}^E \overbrace{10}^A$        $\overbrace{01}^E \overbrace{11}^A \overbrace{10}^A$

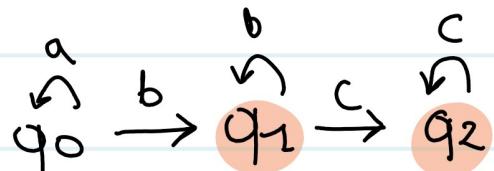
$\overbrace{1001}^A \overbrace{100}^E \overbrace{111}^E$

$\overbrace{0111}^T \overbrace{1}^S$



$a^n b^{n+1} c^{n+2}$

$$\begin{cases} E \rightarrow aIb \mid aEb \\ I \rightarrow \epsilon \mid bIc \end{cases}$$



es. 2.2028 (Febbraio)

$$3n \equiv 0 \pmod{2} \Rightarrow n \equiv 0 \pmod{2}$$

$$L = \{ c^{2n} d^{6n} \mid n > 0 \}$$

$$P \rightarrow c^2 d^6 \mid c^2 P d^6$$

$$\left\{ \begin{array}{l} E \rightarrow 1T | 1E | TE \\ T \rightarrow \varepsilon | OT1 | 1TO | TT \end{array} \right.$$