homework 2

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2.36

$$\begin{cases} 2\dfrac{dx}{dt} + x - \dfrac{df}{dt} - f = 0 \\ 4\dfrac{dy}{dt} + y - 2\dfrac{dz}{dt} - z = 0 \\ 3\dfrac{dz}{dt} - x + z = 0 \end{cases} \xRightarrow{\text{Laplace变换}} \begin{cases} 2sx + x = 2sf + f \\ 4sy + y = 2sz + z \\ 3sz + z = x \end{cases}$$

求得传递函数

$$X(s) = \frac{s+1}{2s+1} F(s)$$

$$Y(s) = \frac{2s+1}{4s+1} Z(s)$$

$$Z(s) = \frac{1}{3s+1} X(s)$$

$$\xrightarrow{f} \boxed{\frac{s+1}{2s+1}} \xrightarrow{x} \boxed{\frac{1}{3s+1}} \xrightarrow{z} \boxed{\frac{2s+1}{4s+1}} \xrightarrow{y}$$

2.37 设第n年兔数为 $r(n)$，狼数为 $w(n)$.

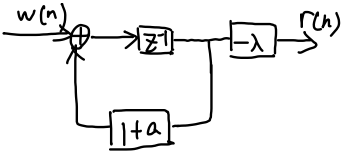
$$\begin{cases} r(n+1) = (1+a)r(n) - \lambda w(n) \\ w(n+1) = kr(n) + (\gamma+1-k\lambda) w(n) \end{cases}$$

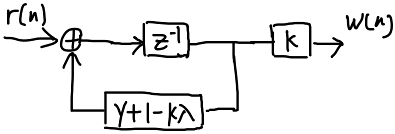
$$\xRightarrow{z变换} \begin{cases} zR(z) - (1+a)R(z) = -\lambda W(z) \\ zW(z) - (\gamma+1-k\lambda) W(z) = kR(z) \end{cases}$$

求得传递函数

$$R(z) = \frac{-\lambda}{z-(1+a)} W(z) = \frac{-\lambda z^{-1}}{1-(1+a)z^{-1}} W(z)$$

$$W(z) = \frac{k}{z-(\gamma+1-k\lambda)} R(z) = \frac{kz^{-1}}{1-(\gamma+1-k\lambda)z^{-1}} R(z)$$





2.40

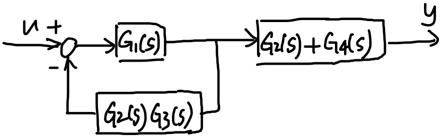
(a)
$$\frac{Y(s)}{U_1(s)} = \frac{4}{s+1.25} + \frac{(2s+0.5)}{5s+2}\cdot\frac{10}{s}$$
$$= \frac{4}{s+1.25} + \frac{20s+5}{(5s+2)s}$$

(b)
$$\frac{Y(s)}{U_2(s)} = \frac{0.2}{3s+1}$$

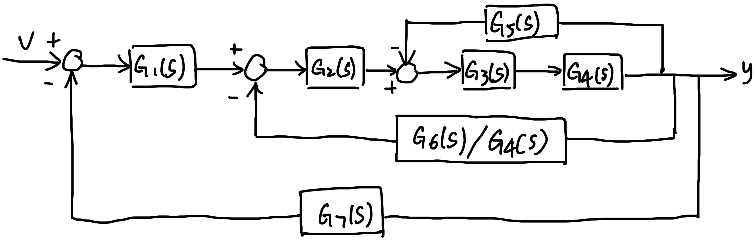
(c)
$$\frac{Y(s)}{Q(s)} = 1$$

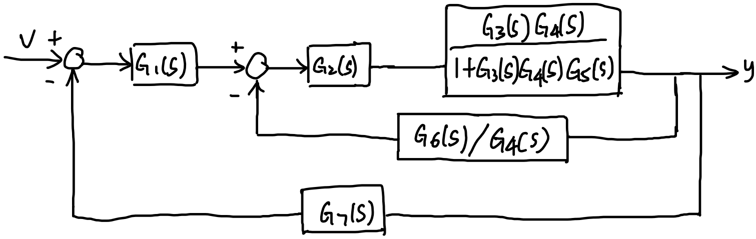
(d)
$$\frac{Z(s)}{U_1(s)} = \frac{(2s+0.5)}{5s+2}\cdot\frac{10}{s} = \frac{20s+5}{(5s+2)s}$$

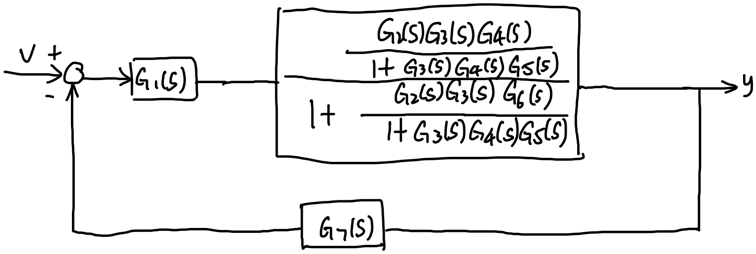
2.41



2.47







$$\frac{Y(s)}{V(s)} = \frac{G_1(s)\dfrac{G_2(s)G_3(s)G_4(s)}{1+G_3(s)G_4(s)G_5(s)+G_2(s)G_3(s)G_6(s)}}{1+G_1(s)G_7(s)\dfrac{G_2(s)G_3(s)G_4(s)}{1+G_3(s)G_4(s)G_5(s)+G_2(s)G_3(s)G_6(s)}}$$

$$= \frac{G_1(s)G_2(s)G_3(s)G_4(s)}{1+G_3(s)G_4(s)G_5(s)+G_2(s)G_3(s)G_6(s)+G_1(s)G_2(s)G_3(s)G_4(s)G_7(s)}$$

2.48

(a)

$$-Y(s)\left[\frac{4}{s+1.25}+\frac{2s+0.5}{5s+2}\cdot\frac{10}{s}\right]+\frac{0.2}{3s+1}P(s) = Y(s)$$

$$\frac{Y(s)}{P(s)} = \frac{\frac{0.2}{3s+1}}{1+\frac{4}{s+1.25}+\frac{20s+5}{(5s+2)s}}$$

(b)

$$E(s)\left[\frac{4}{s+1.25}+\frac{2s+0.5}{5s+2}\cdot\frac{10}{s}\right] = Y(s)$$

$$\frac{Y(s)}{E(s)} = \frac{1}{\frac{4}{s+1.25}+\frac{20s+5}{(5s+2)s}}$$

3.2 电位器组 $u_\varphi = k_p(\psi - \varphi)$

放大器－电动机组 $T_a T_m \frac{d^2\Omega}{dt^2} + T_m \frac{d\Omega}{dt} + \Omega = \frac{k_a}{k_d} u_\varphi - \left(\frac{L}{k_d^2}\frac{dM_L}{dt} + \frac{R}{k_d^2} M_L\right)$

传动机构 $\frac{d\varphi}{dt} = k_t \Omega$

整理得 $\frac{T_a T_m}{k_t}\frac{d^3\varphi}{dt^3} + \frac{T_m}{k_t}\frac{d^2\varphi}{dt^2} + \frac{1}{k_t}\frac{d\varphi}{dt} + \frac{k_a k_p}{k_d}\varphi = \frac{k_a k_p}{k_d}\psi - \frac{R}{k_d^2}\left(T_a \frac{dM_L}{dt} + M_L\right)$

代入数据得 $8T_a \frac{d^3\varphi}{dt^3} + 8\frac{d^2\varphi}{dt^2} + 20\frac{d\varphi}{dt} + 50\varphi = 50\psi - 250\left(T_a\frac{dM_L}{dt} + M_L\right)$

特征方程 $8T_a s^3 + 8s^2 + 20s + 50 = 0.$

根据Routh判据 $\begin{cases} 8\times 20 > 8T_a \times 50 \\ 8T_a > 0 \end{cases}$

$\Rightarrow 0 < T_a < 0.4$

3.5 (a) $s^6 + 4s^5 - 4s^4 + 4s^3 - 7s^2 - 8s + 10$

系数不全为正，系统不稳定

(b) $s^6 + 6s^4 + 3s^3 + 2s^2 + s + 1$

系数不全为正，系统不稳定

(c) $25s^5 + 105s^4 + 120s^3 + 122s^2 + 20s + 1$

s^5	25	120	20
s^4	105	122	1
s^3	$\frac{1910}{21}$	$\frac{415}{21}$	
s^2	$\frac{37889}{382}$	1	
s^1	$\frac{14994315}{795669}$		
s^0	1		

系统稳定.

(d) $(s+2)(s+4)(s^2+6s+25)+666.25$

$= s^4+12s^3+69s^2+198s+866.25$

s^4	1	69	866.25
s^3	12	198	
s^2	52.5	866.25	
s^1	$0(\varepsilon)$		
s^0	866.25		

系统临界稳定

(e) $s^4+8s^3+18s^2+16s+5$

s^4	1	18	5
s^3	8	16	
s^2	16	5	
s^1	13.5		
s^0	5		

系统稳定