

# homework 3

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3.3 忽略  $T_a$  系统微分方程为.

$$8 \frac{d^2 \varphi}{dt^2} + 20 \frac{d\varphi}{dt} + 50\varphi = 50\varphi - 250 M_L$$

$$T = \sqrt{\frac{8}{50}} = \frac{2}{5}$$

$$\xi = \frac{20}{50 \cdot 2} = \frac{1}{2} \quad \checkmark$$

由图 3.9.4 可知 在  $\sigma = 5\%$  下

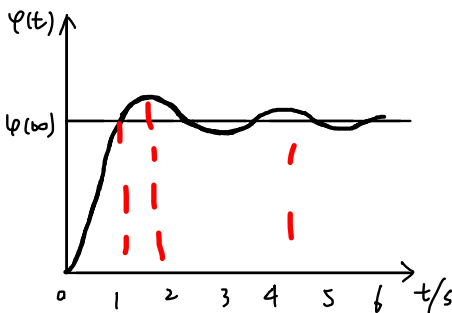
$$t_s = 5 \cdot T = 2s$$

$$t_r = \frac{T}{\sqrt{1-\xi^2}} (\pi - \arccos \xi) = 0.975.$$

$$t_p = \frac{\pi T}{\sqrt{1-\xi^2}} = 1.45s$$

$$\sigma = \exp\left[-\frac{3\pi}{\sqrt{1-\xi^2}}\right] = 16.3\% \quad \checkmark$$

阶跃响应曲线



恒.

$$3.16 \quad Y(s) = \frac{G(s)}{1+G(s)} U(s)$$

$$1) (a) \quad Y(s) = \frac{\frac{c}{s}}{s(\frac{1}{2}s+1)} \quad \lim_{s \rightarrow 0} s Y(s) = \frac{c}{2}$$

$$(b) \quad Y(s) = \frac{\frac{c}{s}}{s^2(\frac{1}{2}s+1)} \quad \lim_{s \rightarrow 0} s^2 Y(s) = \frac{c}{2}$$

$$(c) \quad Y(s) = \frac{\frac{c}{s}}{s^3(\frac{1}{2}s+1)} \quad \lim_{s \rightarrow 0} s^3 Y(s) = \frac{c}{2}$$

$$2) (a) \quad Y(s) = \frac{c}{s(Ts^2+s+1)} \quad \lim_{s \rightarrow 0} s Y(s) = c$$

$$(b) \quad Y(s) = \frac{c}{s^2(Ts^2+s+1)} \quad \lim_{s \rightarrow 0} s^2 Y(s) = c$$

$$(c) \quad Y(s) = \frac{c}{s^3(Ts^2+s+1)} \quad \lim_{s \rightarrow 0} s^3 Y(s) = c$$

$$3) (a) \quad Y(s) = \frac{c(Ts+1)}{s(Ts^3+s^2+Ts+1)} \quad \lim_{s \rightarrow 0} s Y(s) = c$$

$$(b) \quad Y(s) = \frac{cTs+1}{s^2(Ts^3+s^2+Ts+1)} \quad \lim_{s \rightarrow 0} s^2 Y(s) = c$$

$$(c) \quad Y(s) = \frac{c(Ts+1)}{s^3(Ts^3+s^2+Ts+1)} \quad \lim_{s \rightarrow 0} s^3 Y(s) = c$$

	单位阶跃输入	单位斜坡输入	单位加速度输入
0型	$\frac{K}{1+K}$	$\frac{K}{1+K}$	$\frac{K}{1+K}$
I型	1	1	1
II型	1	1	1

对比表 3.6.1 和 3.6.2, 静态误差系数和静态误差间的规律一致

### 3.17 (a) 系统稳定

① 由  $u(t)$  引起的误差 令  $p(t) = 0$  I型系统 查表得

$$u(t) = 4 + 6t$$

$$e_{ss} = 0 + \frac{6}{10} = \frac{3}{5}$$

② 由  $p(t)$  引起的误差 令  $u(t) = 0$

扰动之前不含积分, 扰动之后含积分.

$$p(t) = -1(t).$$

$$e_p = \frac{1}{4}$$

$$e_{st} = e_n + e_p = \frac{17}{20}$$

b) 假设扰动之前框的比例系数为  $k_1$ , 之后框比例系数为  $k_2$ .

$$\text{则 } e_p = \frac{1}{k_1}$$

要减小  $e_p$ , 应提高  $k_1$ .

3.21 (a)

$$\begin{aligned} G(s) &= \frac{Y(s)}{U(s)} = \frac{\frac{1}{s} + \frac{0.2}{s+60} - \frac{1.2}{s+10}}{\frac{1}{s}} \\ &= 1 + \frac{0.2s}{s+60} - \frac{1.2s}{s+10} \\ &= \frac{600}{s^2 + 70s + 600} \end{aligned}$$

(b)

$$T = \sqrt{\frac{1}{600}} = \frac{\sqrt{6}}{60}$$

$$\xi = \frac{70}{600.27} = \frac{7\sqrt{6}}{12}$$

3.26

$$\frac{y(s)}{u(s)} = (k_1 s + 1) \cdot \frac{\frac{k}{s(Ts+1)}}{1 + \frac{k}{s(Ts+1)}} = \frac{k(k_1 s + 1)}{Ts^2 + s + k}$$

$$\frac{G_K(s)}{1 + G_K(s)} = \frac{k(k_1 s + 1)}{Ts^2 + s + k}$$

$$\Rightarrow G_K(s) = \frac{k(k_1 s + 1)}{s(Ts + 1 - k k_1)} \quad \text{I型系统.}$$

$$u(t) = t$$

$$e_{st} = \frac{1 - k k_1}{k} = 0$$

$$\text{解得 } k_1 = \frac{1}{k}$$