homework12

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3.10 $g(s)=\dfrac{s+a}{(s+1)(s+2)(s+4)}=\dfrac{a-1}{3(s+1)}-\dfrac{1}{2}\cdot\dfrac{a-2}{s+2}+\dfrac{a-4}{6(s+4)}$

状态空间

$$\begin{cases}\dot{x}=\begin{bmatrix}-1 & & \\ & -2 & \\ & & -4\end{bmatrix}x+\begin{bmatrix}1\\1\\1\end{bmatrix}u \\ y=\begin{bmatrix}\frac{a-1}{3} & \frac{a-2}{2} & \frac{a-4}{6}\end{bmatrix}x\end{cases}$$

系统完全能控，当 a=1,2,4 时系统部分不能观.

3.11 $g(s)=\dfrac{1}{2(s+2)}-\dfrac{1}{2(s+4)}+\dfrac{0}{s+1}$

(1)

$$\left[\begin{array}{c|c}A & b\\ \hline c^T & d\end{array}\right]=\left[\begin{array}{ccc|c}-1 & 0 & 0 & 1\\ 0 & -2 & 0 & 1\\ 0 & 0 & -4 & 1\\ \hline 0 & \frac{1}{2} & -\frac{1}{2} & 0\end{array}\right]$$

(2)

$$\left[\begin{array}{c|c}A & b\\ \hline c^T & d\end{array}\right]=\left[\begin{array}{ccc|c}-1 & 0 & 0 & 0\\ 0 & -2 & 0 & \frac{1}{2}\\ 0 & 0 & -4 & -\frac{1}{2}\\ \hline 1 & 1 & 1 & 0\end{array}\right]$$

3.15 (1)

$$f(\lambda) = \det \begin{bmatrix} \lambda & -1 & \\ & \lambda & -1 \\ 24 & 26 & \lambda+9 \end{bmatrix} = (\lambda+2)(\lambda+3)(\lambda+4)$$

$$\lambda = -2,\ -3,\ -4$$

$$AP_1 = -2P_1 \Rightarrow P_1 = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

$$AP_2 = -3P_2 \Rightarrow P_2 = \begin{bmatrix} 1 \\ -3 \\ 9 \end{bmatrix}$$

$$AP_3 = -4P_3 \Rightarrow P_3 = \begin{bmatrix} 1 \\ -4 \\ 16 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 1 & 1 \\ -2 & -3 & -4 \\ 4 & 9 & 16 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 6 & \frac{7}{2} & \frac{1}{2} \\ -8 & -6 & -1 \\ 3 & \frac{5}{2} & \frac{1}{2} \end{bmatrix}$$

$$T^{-1}b = \begin{bmatrix} \frac{1}{2} \\ -1 \\ \frac{1}{2} \end{bmatrix} \qquad CT = \begin{bmatrix} 4 & 1 & 0 \end{bmatrix}$$

约当标准型为

$$\left[\begin{array}{c:c} \tilde{A} & \tilde{b} \\ \hdashline \tilde{c}^T & \tilde{d} \end{array}\right] = \left[\begin{array}{ccc:c} -2 & 0 & 0 & \frac{1}{2} \\ 0 & -3 & 0 & -1 \\ 0 & 0 & -4 & \frac{1}{2} \\ \hdashline 4 & 1 & 0 & 0 \end{array}\right]$$

(2) 系统完全能控但不完全能观

能观状态为 x_1, x_2.

3.16 为约当标准型

系统能控 ⇒ 可化为能控标准型

$$Q_K = \begin{bmatrix} 0 & 4 & -8 \\ 4 & -4 & 4 \\ 3 & -6 & 12 \end{bmatrix} \qquad Q_K^{-1} = \frac{1}{12}\begin{bmatrix} 6 & 0 & 0 \\ 9 & -6 & 8 \\ 3 & -3 & 4 \end{bmatrix}$$

$$P_1^T = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} Q_K^{-1} = \begin{bmatrix} \frac{1}{4} & -\frac{1}{4} & \frac{1}{3} \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} \frac{1}{4} & -\frac{1}{4} & \frac{1}{3} \\ -\frac{1}{4} & \frac{1}{2} & -\frac{2}{3} \\ \frac{1}{4} & -\frac{3}{4} & \frac{4}{3} \end{bmatrix}$$

$$T = \begin{bmatrix} 8 & 4 & 0 \\ 8 & 12 & 4 \\ 3 & 6 & 3 \end{bmatrix}$$

$$\tilde{A} = T^{-1}AT = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -5 & -4 \end{bmatrix}$$

$$\tilde{b} = T^{-1}b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{b}$$

为能控标准型

3.18 $Q_g = \begin{bmatrix} -1 & 1 \\ -3 & 4 \end{bmatrix} \Rightarrow Q_g^{-1} = \begin{bmatrix} -4 & 1 \\ -3 & 1 \end{bmatrix}$

$\Rightarrow P_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$T = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \Rightarrow T^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

$\tilde{A} = T^{-1}AT = \begin{bmatrix} 0 & -4 \\ 1 & 5 \end{bmatrix}$

$\tilde{c} = cT = \begin{bmatrix} 0 & 1 \end{bmatrix}$

$\tilde{b} = T^{-1}b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

能观标准型 $\left[\begin{array}{c|c} \tilde{A} & \tilde{b} \\ \hline \tilde{c}^T & \tilde{d} \end{array}\right] = \left[\begin{array}{cc|c} 0 & -4 & 1 \\ 1 & 5 & 0 \\ \hline 0 & 1 & 0 \end{array}\right]$

3.19 $f(\lambda) = \det \begin{bmatrix} \lambda+2 & -2 & -1 \\ 0 & \lambda+2 & 0 \\ -1 & 4 & \lambda \end{bmatrix} = (\lambda+2)(\lambda^2+2\lambda+1)$

$\Rightarrow \lambda = -2,\ -1-\sqrt{2},\ -1+\sqrt{2}$

$$AP_1 = -2P_1 \Rightarrow P_1 = \begin{bmatrix} 1 \\ \frac{1}{8} \\ -\frac{1}{4} \end{bmatrix}$$

$$AP_2 = (-1-\sqrt{2})P_2 \Rightarrow P_2 = \begin{bmatrix} 1 \\ 0 \\ 1-\sqrt{2} \end{bmatrix}$$

$$AP_3 = (-1+\sqrt{2})P_3 \Rightarrow P_3 = \begin{bmatrix} 1 \\ 0 \\ 1+\sqrt{2} \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 1 & 1 \\ \frac{1}{8} & 0 & 0 \\ -\frac{1}{4} & 1-\sqrt{2} & 1+\sqrt{2} \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 0 & 8 & 0 \\ \frac{2+\sqrt{2}}{4} & -\frac{8+5\sqrt{2}}{2} & -\frac{\sqrt{2}}{4} \\ \frac{2-\sqrt{2}}{4} & \frac{5\sqrt{2}-8}{2} & \frac{\sqrt{2}}{4} \end{bmatrix}$$

$$\tilde{b} = T^{-1}b = \begin{bmatrix} 0 \\ -\frac{\sqrt{2}}{4} \\ \frac{\sqrt{2}}{4} \end{bmatrix} \qquad \tilde{c} = cT = \begin{bmatrix} \frac{5}{8} & 2-\sqrt{2} & 2+\sqrt{2} \end{bmatrix}$$

$$g(s) = -\frac{\sqrt{2}}{4}(2-\sqrt{2})\frac{1}{s+1+\sqrt{2}} + \frac{\sqrt{2}}{4}(2+\sqrt{2})\frac{1}{s+1-\sqrt{2}}$$

$$= \frac{s+3}{s^2+2s-1}$$

(2) 系统不完全能控 能控状态 x_1, x_2

$$Q_K = \begin{bmatrix} 0 & 1 & -2 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

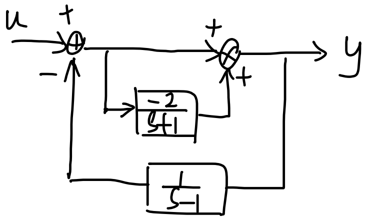
$$\tilde{A} = T^{-1}AT = \begin{bmatrix} 0 & 1 & 4 \\ 1 & -2 & 2 \\ 0 & 0 & -2 \end{bmatrix}$$

$$T^{-1}b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad cT = \begin{bmatrix} 1 & 1 & -1 \end{bmatrix}$$

能控子系统 $\Sigma\left(\begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \end{bmatrix} \right)$

(3) 系统完全能观

3.21



$$\begin{cases} \dot{x}_1 + x_1 = -2(u - x_2) \\ \dot{x}_2 - x_2 = x_1 + u - x_2 \\ y = x_1 + u - x_2 \end{cases}$$

$$\begin{cases} \begin{pmatrix} \dot{x}_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \end{pmatrix} u \\ y = \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + u \end{cases}$$

$$Q_k = \begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix} \qquad rank(Q_k) < 2$$

系统不完全能控

$$Q_g = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} \qquad rank(Q_g) < 2$$

系统不完全能观