作业七

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P115

10.(a)

$$P(X=k)=\sum_{n=0}^{+\infty}P(X=k|N=n)P(N=n)$$

$$=\sum_{n\geq k}C_n^k p^k q^{n-k}\frac{\lambda^n e^{-\lambda}}{n!}$$

$$=\frac{(p\lambda)^k e^{-\lambda}}{k!}\sum_{n-k=0}^{+\infty}\frac{(q\lambda)^{n-k}}{(n-k)!}$$

$$=\frac{(p\lambda)^k e^{-\lambda}}{k!}\cdot e^{q\lambda}$$

$$=\frac{(p\lambda)^k e^{-p\lambda}}{k!}\qquad k=0,1,2,\cdots$$

$$P(Y=k)=\sum_{n=0}^{+\infty}P(Y=k|N=n)P(N=n)$$

$$=\sum_{n\geq k}C_n^k p^{n-k}q^k\frac{\lambda^n e^{-\lambda}}{n!}$$

$$=\frac{(q\lambda)^k e^{-\lambda}}{k!}\sum_{n-k=0}^{+\infty}\frac{(p\lambda)^{n-k}}{(n-k)!}$$

$$=\frac{[(1-p)\lambda]^k e^{-(1-p)\lambda}}{k!}\qquad k=0,1,2\cdots$$

(b)

$$P(X=x,Y=y)=\sum_{n=0}^{+\infty}P(X=x,Y=y|N=n)P(N=n)$$

$$=\sum_{n=0}^{+\infty}P(X=x,Y=n-y)P(N=n)$$

$$=\sum_{n=0}^{+\infty}C_n^x p^x(1-p)^{n-x}\frac{\lambda^n e^{-\lambda}}{n!}$$

$$=\frac{(p\lambda)^x e^{-\lambda}}{x!}\sum_{n=0}^{+\infty}\frac{[(1-p)\lambda]^{n-x}}{(n-x)!}$$

$$=\frac{(p\lambda)^x e^{-p\lambda}}{x!}=P(X=x)$$

不独立

12. $P(M=m) = \sum_{k=0}^{+\infty} P(M=m \mid X_1=k)\, P(X_1=k)$

$$= \sum_{k=0}^{m} P(M=m \mid X_1=k)\, P(X_1=k)$$

$$= \sum_{k=0}^{m-1} P(X_2=m)\, P(X_1=k) + P(M=m \mid X_1=m)\, P(X_1=m)$$

$$= \sum_{k=0}^{m-1} P(X_2=m)\, P(X_1=k) + \sum_{k=0}^{m-1} P(X_1=m)\, P(X_2=k) + P(X_1=m)\, P(X_2=m)$$

$$= 2\sum_{k=0}^{m-1} \frac{\lambda^k e^{-\lambda}}{k!} \frac{\lambda^m e^{-\lambda}}{m!} + \left(\frac{\lambda^m e^{-\lambda}}{m!}\right)^2$$

$$= 2e^{-2\lambda} \frac{\lambda^m}{m!} \sum_{k=0}^{m-1} \frac{\lambda^k}{k!} + \left(\frac{\lambda^m e^{-\lambda}}{m!}\right)^2 \qquad m=0,1,2\cdots$$

$P(N=n) = \sum_{k=0}^{+\infty} P(N=n \mid X_1=k)\, P(X_1=k)$

$$= \sum_{k=n}^{+\infty} P(X_2=n)\, P(X_1=k) + P(N=n \mid X_1=n)\, P(X_1=n)$$

$$= \sum_{k=n+1}^{+\infty} P(X_2=n)\, P(X_1=k) + \sum_{k=n+1}^{+\infty} P(X_1=n)\, P(X_2=k) + P(X_1=n)\, P(X_2=n)$$

$$= 2\sum_{k=n+1}^{+\infty} \frac{\lambda^n e^{-\lambda}}{n!} \frac{\lambda^k e^{-\lambda}}{k!} + \left(\frac{\lambda^n e^{-\lambda}}{n!}\right)^2$$

$$= 2e^{-2\lambda} \frac{\lambda^n}{n!} \sum_{k=n+1}^{+\infty} \frac{\lambda^k}{k!} + \left(\frac{\lambda^n e^{-\lambda}}{n!}\right)^2 \qquad n=0,1,2\cdots$$

14. 设$[0,t]$分钟内到达的顾客数为 N_t.

$P(N_{60} \le 3) = P(N_{60}=0) + P(N_{60}=1) + P(N_{60}=2)$

$$= \frac{(\frac{60}{10})^0 e^{-\frac{60}{10}}}{0!} + \frac{(\frac{60}{10})^1 e^{-\frac{60}{10}}}{1!} + \frac{(\frac{60}{10})^2 e^{-\frac{60}{10}}}{2!}$$

$$= (1+6+18)e^{-6}$$

$$= 25e^{-6}$$

15. 设营业额为 Y_t.

$EY_t = \lambda t E\eta_i$

$= 60 \cdot 180 \cdot 10 \cdot 0.15$

$= 16200$

$DY_t = \lambda t E\eta_i^2$

$= \lambda t [D\eta_i + (E\eta_i)^2]$

$= 60 \cdot 180 [10 \cdot 0.15 \cdot 0.85 + (10 \cdot 0.15)^2]$

$= 38070$

$\sigma Y_t = \sqrt{DY_t}$

$= 195.12$.

P152

1. (a) $\int_{-1}^{1} f(x)\,dx$

$= \int_{-1}^{1} \frac{c}{\sqrt{1-x^2}}\,dx$

$= c\arcsin x \Big|_{-1}^{1}$

$= c\pi$

$= 1$

$\Rightarrow c = \frac{1}{\pi}$

(6) $P(X\in(-\frac{1}{2},\frac{1}{2}))$

$= \int_{-\frac{1}{2}}^{\frac{1}{2}} f(x)\,dx$

$= \frac{1}{\pi}\arcsin x \Big|_{-\frac{1}{2}}^{\frac{1}{2}}$

$= \frac{1}{3}$

3. (1) $F(-\infty) = A - \frac{\pi}{2}B = 0$

$F(+\infty) = A + \frac{\pi}{2}B = 1$

$\Rightarrow A = \frac{1}{2} \quad B = \frac{1}{\pi}$

(2) $F(x) = \frac{1}{2} + \frac{1}{\pi}\arctan x \quad (-\infty<x<+\infty)$

$P = F(1) - F(-1)$

$= \frac{1}{\pi}\cdot\left[\frac{\pi}{4} - (-\frac{\pi}{4})\right]$

$= \frac{1}{2}$

(3) $f(x) = [F(x)]' = \frac{1}{\pi}\left(\frac{1}{1+x^2}\right) \qquad -\infty<x<+\infty$

8. $\Delta = 16k^2 - 16(k+2)$

$= 16(k-2)(k+1)$

$\Delta \geq 0 \Rightarrow \quad k\leq 1$ 或 $k\geq 2$

$P = P(k\geq 2)$

$= \frac{5-2}{5-0}$

$= \frac{3}{5}$

15. $h(a)=\int_{-\infty}^{a}(-x+a)f(x)dx+\int_{a}^{+\infty}(x-a)f(x)dx$

$=a\int_{-\infty}^{a}f(x)dx-a\int_{a}^{+\infty}f(x)dx-\int_{-\infty}^{a}xf(x)dx+\int_{a}^{+\infty}xf(x)dx$

$h'(a)=\int_{-\infty}^{a}f(x)dx+af(a)-\int_{a}^{+\infty}f(x)dx+af(a)-af(a)-af(a)$

$=\int_{-\infty}^{a}f(x)dx-\int_{a}^{+\infty}f(x)dx$

$=F(a)-(1-F(a))$

$=2F(a)-1$

$h''(a)=2f(a)$

$h''(a)\geq 0 \Rightarrow h'(a)$ 单增

又 $P(X\leq a)=F(a)=\frac{1}{2}$ 时 $h'(a)=0$.

$\Rightarrow h(a)$ 在 $(-\infty, a)$ 单减, $(a,+\infty)$ 单增

$\therefore$ 当 $P(X\leq a)=\frac{1}{2}$ 时 $h(a)$ 最小.