homework6

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6.1 (a) $G(s) = \dfrac{K}{(s-1)(s+5)}$

开环极点 $p_1 = 1$ $p_2 = -5$

无开环零点、

根轨迹有2个分支，起点为1或-5，终点为无穷远

实轴上根轨迹：$(-5, 1)$

渐近线 $\gamma = \dfrac{\pm(2k+1)\pi}{n-m} = \dfrac{\pm(2k+1)\pi}{2} = \pm 90^\circ$.

$\sigma_a = \dfrac{\sum_{j=1}^{n} p_j - \sum_{i=1}^{m} z_i}{n-m} = -2$

$f(s) = s^2 + 4s - 5 + K$

$K = -s^2 - 4s + 5$.

$\dfrac{dK}{ds} = -2s - 4 = 0 \Rightarrow s = -2$

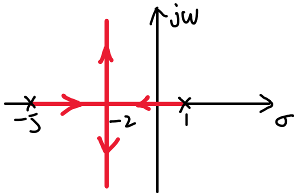
$K = -s^2 - 4s + 5 \Big|_{s=-2} = 9 > 0$

$\dfrac{d^2K}{ds^2} = -2 < 0$.

-2在根轨迹上，为会合点

将 $s = j\omega$ 代入闭环系统微分方程 求得与虚轴交点、

$-\omega^2 + 4j\omega - 5 + K = 0 \Rightarrow \omega = 0 \quad K = 5$



系统稳定的增益范围为：$K > 5$

(b) $G(S) = \dfrac{k}{(S+1)^4}$

开环极点 $p_{1,2,3,4} = -1$

无开环零点、

根轨迹有4个分支，起点为 -1，终点为无穷远

实轴上无根轨迹

渐近线 $\gamma = \dfrac{\pm(2k+1)\pi}{n-m} = \dfrac{\pm(2k+1)\pi}{4} = \pm 45^\circ, \pm 135^\circ$

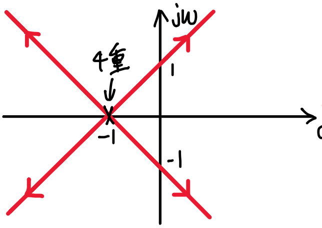
$\sigma_a = \dfrac{\sum_{j=1}^{n} p_j - \sum_{i=1}^{m} z_i}{n-m} = -1$

将 $s=jw$ 代入闭环系统微分方程

$(jw+1)^4 + k = 0$

$w^4 - 4jw^3 - 6w^2 + 4jw + 1 + k = 0$ 求得与虚轴交点

$\Rightarrow$ $w = \pm 1$ $k = 4$ 或 $w = 0$ $k = -1$ (舍)



系统稳定的增益范围为. $k < 4$.

(c) $G(s) = \dfrac{K(s^2+1)}{(s+2)^3}$

开环极点 $P_{1,2,3} = -2$

开环零点、 $z_1 = j \quad z_2 = -j$

根轨迹有3个分支，起点为 -2，终点为 j、$-j$ 或无穷远

实轴上根轨迹 · $(-\infty, -2)$

$f(s) = (s+2)^3 + K(s^2+1)$

$K = -\dfrac{(s+2)^3}{s^2+1}$

$\dfrac{dK}{ds} = \dfrac{-(s+2)^2(s-1)(s-3)}{(s^2+1)^2} = 0 \quad \Rightarrow \quad s = -2, 1, 3$

$s=-2$ 时 $K = 0$

$s = 1$ 或 3 时 $K < 0$

$-2, 1, 3$ 均不在根轨迹上，无会合点与分离点

$\varphi_{pr} = \frac{1}{3}\left[\pm 180^\circ(2k+1) + \sum_{i=1}^{m}\arg(P_r - z_i)\right]$ $\varphi_{p1} = 60^\circ$ $\varphi_{p2} = 180^\circ$ $\varphi_{p3} = -60^\circ$

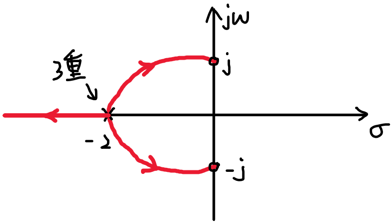
$\varphi_{zr} = \pm 180^\circ(2k+1) + 3\arg(z_r - P_1) - \sum_{i=1, i\neq r}^{m}\arg(z_r - z_i)$ $\varphi_{z1} = 169.7^\circ$ $\varphi_{z2} = -169.7^\circ$

将 $s = j\omega$ 代入闭环系统微分方程

$-j\omega^3 - 6\omega^2 + 12j\omega + 8 - K\omega^2 + K = 0$

$\Rightarrow \omega = \pm 2\sqrt{3}$, $K = -\frac{64}{11}$ 或 $\omega = 0$, $K = -8$

根轨迹与虚轴无交点、系统稳定的增益范围为 · $K > 0$.



(d) $G(s) = \dfrac{k(s+0.5)}{s^3+s^2+1}$

开环极点 $P_1 = -1.466 \quad P_{2,3} = 0.233 \pm 0.793j$

开环零点、$z_1 = -0.5$

根轨迹有3个分支，起点为 P_1、P_2、P_3，终点为 z_1 或无穷远

实轴上根轨迹：$(-1.466, -0.5)$

渐近线 $\gamma = \dfrac{\pm(2k+1)\pi}{n-m} = \dfrac{\pm(2k+1)\pi}{2} = \pm 90^\circ$

$\sigma_a = \dfrac{\sum_{j=1}^{n} P_j - \sum_{i=1}^{m} z_i}{n-m} = -0.25$

$f(s) = s^3+s^2+1 + k(s+0.5)$

$k = -\dfrac{s^3+s^2+1}{s+0.5}$

$\dfrac{dk}{ds} = \dfrac{2s^3+2.5s^2+s-1}{(s^3+s^2+1)^2} = 0 \Rightarrow$ 实数根 $s = 0.418$

当 $s = 0.418$ 时 $k<0$. 不在根轨迹上

无会合点与分离点

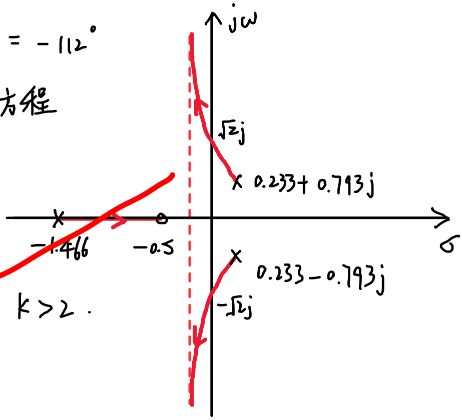
$\varphi_{pr} = \pm 180^\circ(2k+1) + \arg(P_r - z_1) - \sum_{j=1, j\neq r}^{n} \arg(P_r - P_j)$

$\varphi_{p1} = 0 \qquad \varphi_{p2} = 112^\circ \qquad \varphi_{p3} = -112^\circ$

将 $s = j\omega$ 代入闭环系统微分方程

$-j\omega^3 - \omega^2 + 1 + jk\omega + 0.5k = 0$

$\Rightarrow \omega = \pm\sqrt{2}, \ k = 2$



系统稳定的增益范围为：$k > 2$.

(e) $G(s) = \dfrac{k(s+2)}{(s^2+6s+10)(s^2+2s+4)}$

开环极点、$P_{1,2} = -3 \pm j \qquad P_{3,4} = -1 \pm \sqrt{3}j$

开环零点、$z_1 = -2$

根轨迹有4个分支，起点为P_1、P_2、P_3、P_4，终点为z_1或无穷远

实轴上根轨迹：$(-\infty, -2)$

渐近线 $\gamma = \dfrac{\pm(2k+1)\pi}{n-m} = \dfrac{\pm(2k+1)\pi}{3} = \pm 60^\circ, 180^\circ$

$\sigma_a = \dfrac{\sum_{j=1}^{n} p_j - \sum_{i=1}^{m} z_i}{n-m} = -2$

$f(s) = s^4 + 8s^3 + 26s^2 + 44s + 40 + k(s+2)$

$k = -\dfrac{s^4+8s^3+26s^2+44s+40}{s+2}$

$\dfrac{dk}{ds} = 0 \Rightarrow$ 实数根 $s = -0.8453, -3.155$

-0.8453 不在根轨迹上

-3.155 在根轨迹上，为会合点

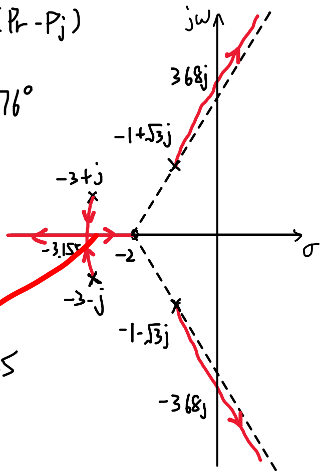
$\varphi_{pr} = \pm 180^\circ(2k+1) + \arg(p_r - z_1) - \sum_{j=1, j\neq r}^{n} \arg(p_r - p_j)$

$\varphi_{p_1} = -101^\circ \quad \varphi_{p_2} = 101^\circ \quad \varphi_{p_3} = 76^\circ \quad \varphi_{p_4} = -76^\circ$

将 $s = j\omega$ 代入闭环系统微分方程

$\omega^4 - 8j\omega^3 - 26\omega^2 + 44j\omega + 40 + k(j\omega+2) = 0$

$\Rightarrow \omega = \pm 3.68 \quad k = 64.35$



系统稳定的增益范围为：$k < 64.35$

(f) $G(s)=\dfrac{k(s^2+2s+5)}{s(s+2)(s+3)}$

开环极点 $P_1=0 \quad P_2=-2 \quad P_3=-3$

开环零点、$z_{1,2}=-1\pm 2j$

根轨迹有3个分支，起点为 P_1,P_2,P_3，终点为 z_1,z_2或无穷远

实轴上根轨迹：$(-\infty,-3)$，$(-2,0)$

$f(s)=s^3+5s^2+6s+k(s^2+2s+5)$

$k=-\dfrac{s^3+5s^2+6s}{s^2+2s+5}$

$\dfrac{dk}{ds}=0 \Rightarrow$ 实数根 $s=-0.82,\ -2.12$

-2.12 不在根轨迹上

-0.82 在根轨迹上，为分离点

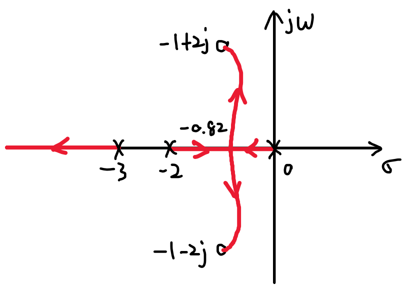
$\varphi_{zr}=\pm 180^\circ(2k+1)-\sum\limits_{\substack{i=1\\ i\neq r}}^{m}\arg(z_r-z_i)+\sum\limits_{j=1}^{n}\arg(z_r-p_j)$

$\varphi_{z1}=-45^\circ \qquad \varphi_{z2}=45^\circ$

将 $s=j\omega$ 代入闭环系统微分方程

$-j\omega^3-5\omega^2+6j\omega+k(-\omega^2+2j\omega+5)=0 \Rightarrow$ 无解

根轨迹与虚轴无交点、系统稳定的增益范围为：$k>0$.



6.5 $G(s)=\dfrac{k(s+2)(s+3)}{s(s+1)}$

开环极点 $p_1=0 \quad p_2=-1$

开环零点、$z_1=-2 \quad z_2=-3$

根轨迹有2个分支，起点为 0, -1，终点为 -2，-3

实轴上根轨迹：$(-3,-2) \quad (-1,0)$

$f(s)=s^2+s+k(s^2+5s+6)$

$k=-\dfrac{s^2+s}{s^2+5s+6}$.

$\dfrac{dk}{ds}=0 \Rightarrow s=-2.37\ ,\ -0.634$

当 $s=-2.37$ 时 $k=13.9$ 为分离点

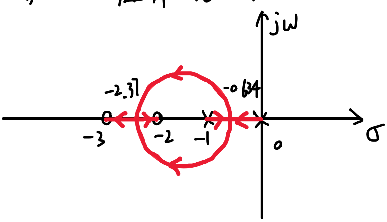
当 $s=-0.634$ 时 $k=0.072$ 为会合点。

将 $s=j\omega$ 代入闭环系统微分方程

$-\omega^2+j\omega+k(-\omega^2+5j\omega+6)=0$

$\Rightarrow k=-\frac{1}{5}<0$

根轨迹与虚轴无交点。



当 $0<k<0.072$ 或 $k>13.9$ 时 系统过阻尼

当 $0.072<k<13.9$ 时 系统欠阻尼

6.14 $G(s)=\dfrac{10k}{s(s+8)(s^2+2s+4)}$

开环极点 $P_1=0$ $P_2=-8$ $P_{3,4}=-1\pm\sqrt{3}j$

无开环零点，

根轨迹有4个分支，起点为 0，-8，$-1\pm\sqrt{3}j$，终点为无穷远

实轴上根轨迹：$(-8,0)$

渐近线 $\gamma=\dfrac{\pm(2k+1)\pi}{n-m}=\dfrac{\pm(2k+1)\pi}{4}=\pm45^\circ,\ \pm135^\circ$

$\sigma_a=\dfrac{\sum_{j=1}^{n}p_j-\sum_{i=1}^{m}z_i}{n-m}=-2.5$

$f(s)=s(s+8)(s^2+2s+4)+10k$

$k=-\dfrac{s(s+8)(s^2+2s+4)}{10}$

$\dfrac{dk}{ds}=0 \Rightarrow$ 实数根 $s=-6.07,\ 16.814$

16.814 不在根轨迹上

-6.07 在根轨迹上，为分离点

$\varphi_{Pr}=\pm180^\circ(2k+1)-\sum_{j=1,j\neq r}^{n}(P_r-P_j)$

$\varphi_{P1}=180^\circ$ $\varphi_{P2}=0^\circ$ $\varphi_{P3}=-43^\circ$ $\varphi_{P4}=43^\circ$

将 $s=j\omega$ 代入闭环系统微分方程

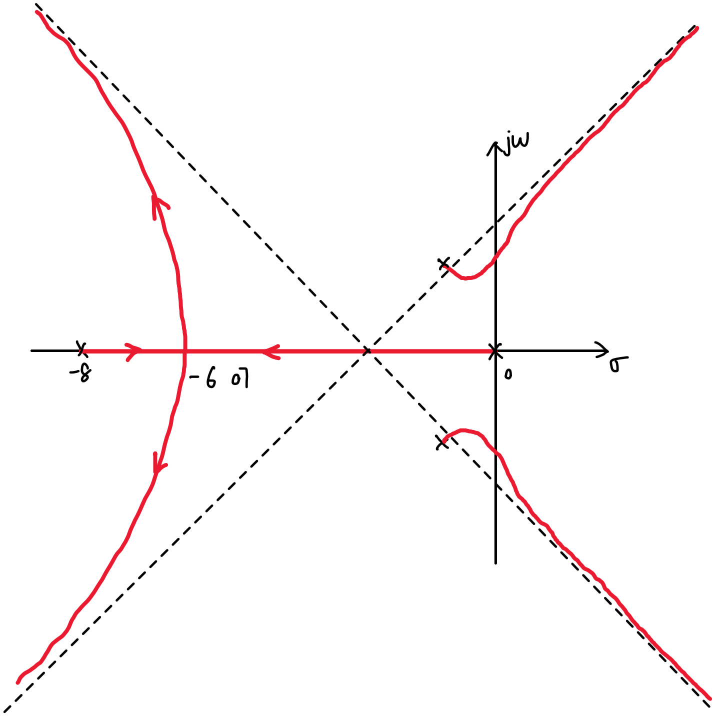
$\omega^4-10j\omega^3-20\omega^2+32j\omega+10k=0$

$\Rightarrow \omega=\pm1.79$ $k=5.376$.

$k=20$时闭环系统微分方程为

$s^4+10s^3+20s^2+32s+200=0$

$\Rightarrow s_1=-7.38$ $s_2=-4.083$ $s_{3,4}=0.73\pm2.47j$



6.17 (a) 主导极点 $s_d = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -2 \pm 2\sqrt{3}j$

校正后闭环系统微分方程为

$s(s+2)(s+p) + 4k_c(s+1) = 0$.

将 s_1, s_2 代入求得

$p = 2.857$

$k_c = 3.4287$

(b) 校正后 $G(s) = \dfrac{13.7148(s+1)}{s(s+2)(s+2.857)}$

$k_v' = \lim\limits_{s \to 0} sG(s) = 2.4$

超前校正装置传递函数零点、极点离原点越近，k_v越小.

6.20 校正前 $G_p(s) = \frac{10}{s(s+2)(s+5)}$

闭环极点 $s_{1,2} = -0.742 \pm j0.123$

$\omega_n = 1.346 \text{ rad/s} \quad \xi = 0.551$

$K_v' = \lim_{s\to 0} s\,G_o(s) = 1$

期望的主导极点为 $s_d = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} = -2 \pm j2\sqrt{3}$

$\Rightarrow \xi = 0.5 \quad \omega_n = 4$

采用超前滞后校正

$$G_c(s)G_p(s) = K_c \frac{s+\frac{1}{T_1}}{s+\frac{\beta}{T_1}} \cdot \frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}} \cdot \frac{10}{s(s+2)(s+5)}$$

$K_v = \lim_{s\to 0} s\,G_c(s)G_p(s) = K_c = 50$

取 $T_2 = 10 \gg 1$ 使得 $\left|\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right| \approx 1$

$|G_p(s_d)G_c(s_d)| = 1$

$$\Rightarrow \left|\frac{s_d+\frac{1}{T_1}}{s_d+\frac{\beta}{T_1}}\right| \approx 0.127$$

$\arg|G_p(s_d)| + \arg|G_c(s_d)| = -180^\circ$

$$\Rightarrow \arg\left[\frac{s_d+\frac{1}{T_1}}{s_d+\frac{\beta}{T_1}}\right] \approx 79.1^\circ$$

解三角形得

$\frac{1}{T_1} = 2.22 \qquad \frac{\beta}{T_1} = 29.11 \qquad \Rightarrow \beta = 13.11$

$$G_c(s) = K_c \frac{s+\frac{1}{T_1}}{s+\frac{\beta}{T_1}} \cdot \frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}$$

$$= \frac{50(s+2.22)(s+0.1)}{(s+29.11)(s+0.0076)}$$

经检验，满足要求.