作业四

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5 X可能的取值为 1, 2, …, 6

$P(X=1) = 1-(\frac{5}{6})^n$

$P(X=2) = (\frac{5}{6})^n - (\frac{4}{6})^n$

$P(X=3) = (\frac{4}{6})^n - (\frac{3}{6})^n$

$P(X=4) = (\frac{3}{6})^n - (\frac{2}{6})^n$

$P(X=5) = (\frac{2}{6})^n - (\frac{1}{6})^n$

$P(X=6) = (\frac{1}{6})^n$

X的分布列为

X	1	2	3	4	5	6
P	$\frac{6^n-5^n}{6^n}$	$\frac{5^n-4^n}{6^n}$	$\frac{4^n-3^n}{6^n}$	$\frac{3^n-2^n}{6^n}$	$\frac{2^n-1}{6^n}$	$\frac{1}{6^n}$

Y可能的取值为 1, 2, …, 6

$P(Y=1) = (\frac{1}{6})^n$

$P(Y=2) = \frac{2^n-1}{6^n}$

$P(Y=3) = \frac{3^n-2^n}{6^n}$

$P(Y=4) = \frac{4^n-3^n}{6^n}$

$P(Y=5) = \frac{5^n-4^n}{6^n}$

$P(Y=6) = \frac{6^n-5^n}{6^n}$

Y的分布列为

Y	1	2	3	4	5	6
P	$\frac{1}{6^n}$	$\frac{2^n-1}{6^n}$	$\frac{3^n-2^n}{6^n}$	$\frac{4^n-3^n}{6^n}$	$\frac{5^n-4^n}{6^n}$	$\frac{6^n-5^n}{6^n}$

$$P(X=2, Y=5) = \left(\frac{4}{6}\right)^n - \left(\frac{3}{6}\right)^n - \left(\frac{3}{6}\right)^n + \left(\frac{2}{6}\right)^n$$

$$= \frac{4^n - 2\cdot 3^n + 2^n}{6^n}$$

8. 设2张卡片的号码之和为X

若取后不放回 X的分布列为

X	3	4	5	6	7	8	9	10	11
P	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{2}{15}$	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{1}{15}$

$$EX = \frac{3+4+10+12+7+16+18+10+11}{15}$$

$$= 7$$

若取后放回，X的分布列为

X	2	3	4	5	6	7	8	9	10	11	12
P	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

$$EX = \frac{2+6+12+20+30+42+40+36+30+22+12}{36}$$

$$= 7$$

10. 令 X_i 表示第 i 次抛骰子时的点数，$i=1,2,\cdots,k$

则 X_i 独立同分布

有 $X = X_1 + X_2 + \cdots + X_k$

$$P(X_i = n) = \frac{1}{6} \quad n = 1,2,3,4,5,6$$

$$EX_i = \frac{1+2+3+4+5+6}{6}$$

$$= \frac{7}{2}$$

$$EX = E(X_1 + X_2 + \cdots + X_k)$$

$$= EX_1 + EX_2 + \cdots + EX_k$$

$$= \frac{7}{2}k.$$

16. 记 P_n 为 X 取偶数的概率

$X \sim B(n,p)$

$$(p+q)^n = C_n^0 q^n + C_n^1 p q^{n-1} + \cdots + C_n^n p^n$$

$$(q-p)^n = C_n^0 q^n - C_n^1 p q^{n-1} + \cdots + (-1)^n C_n^n p^n$$

$$P_n = \frac{1}{2}\left[(p+q)^n + (q-p)^n\right]$$

$$= \frac{1}{2}\left[1 + (1-2p)^n\right]$$

$$EY = \frac{1}{2}\left[1+(1-2p)^n\right] + \frac{1}{2}\left[(1-2p)^n - 1\right]$$

$$= (1-2p)^n$$

19(a) $EX = \sum_{n=1}^{\infty} n P(X=n)$

$$= \sum_{n=1}^{\infty} n [P(X \geq n) - P(X \geq n+1)]$$

$$= \sum_{n=1}^{\infty} n P(X \geq n) - \sum_{n=1}^{\infty} (n+1) P(X \geq n+1) + \sum_{n=1}^{\infty} P(X \geq n+1)$$

$$= P(X \geq 1) + \sum_{n=1}^{\infty} P(X \geq n+1)$$

$$= \sum_{n=1}^{\infty} P(X \geq n)$$

补充题

1 设白球个数为 X，可能取 $0, 1, \cdots, N$

$$EX = \sum_{k=0}^{N} k P(X=k)$$

$$= n$$

则摸到白球的概率

$$P = \sum_{k=0}^{N} \frac{k}{N} \cdot P(X=k)$$

$$= \frac{n}{N}$$

2. $m_{\sum_{i=1}^{n} a_i X_i}(u) = E\left(e^{u \sum_{i=1}^{n} a_i X_i}\right)$

$$= E\left(\prod_{i=1}^{n} e^{u a_i X_i}\right)$$

$$= \prod_{i=1}^{n} E\left(e^{u a_i X_i}\right)$$

$$= \prod_{i=1}^{n} m_{X_i}(a_i u)$$

3.

$$m_X(u) = E(e^{ux})$$

$$= \Sigma\, e^{ux} C_n^x p^x (1-p)^{n-x}$$

$$= (pe^u + 1-p)^n$$

$$E(X^3) = \left[\frac{d^3}{du^3} m_X(u)\right]_{u=0}$$

$$= \left[n(pe^u+1-p)^{n-1} pe^u\right]'' \Big|_{u=0}$$

$$= \left[n(n-1)(pe^u+1-p)^{n-2}(pe^u)^2 + n(pe^u+1-p)^{n-1}pe^u\right]' \Big|_{u=0}$$

$$= n(n-1)(n-2)(pe^u+1-p)^{n-3}(pe^u)^3 + n(n-1)(pe^u+1-p)^{n-2} \cdot 2(pe^u)^2$$
$$+ n(n-1)(pe^u+1-p)^{n-2}(pe^u)^2 + n(pe^u+1-p)^{n-1}pe^u \Big|_{u=0}$$

$$= n(n-1)(n-2)p^3 + 3n(n-1)p^2 + np$$