# homework 11

刘若涵 2020011126

3.6

$$Q_g = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad \mathrm{Rank}(Q_g) = 4$$

系统完全能观.

3.7

$$Q_g = \begin{bmatrix} 1 & 0 \\ 0 & b \end{bmatrix}$$

$\mathrm{Rank}(Q_g) = 2 \Rightarrow b \neq 0$.

3.8 (1)

$$Q_k = \begin{bmatrix} a & a\lambda + b & a\lambda^2 + 2b\lambda \\ b & b\lambda & b\lambda^2 \\ c & c\lambda & c\lambda^2 \end{bmatrix}$$

$\mathrm{Rank}(Q_k) \leq 2 < 3$

不能

(2)

$$Q_g = \begin{bmatrix} a & b & c \\ a\lambda & a+b\lambda & c\lambda \\ a\lambda^2 & 2a\lambda+b\lambda^2 & c\lambda^2 \end{bmatrix}$$

$\text{Rank}(Q_g) \leq 2 < 3$

不能

3.9 (2)

$$Q_g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \\ 1 & 2 & 0 \\ -2 & 0 & -2 \\ -1 & -4 & -1 \end{bmatrix}$$

$\text{Rank}(Q_g) = 3$

系统完全能观

3.17 (1)

$$\begin{cases} \dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ -3 & -4 & 0 \\ 2 & 1 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u_1 \\ y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x \end{cases}$$

(2)

$$Q_k = \begin{bmatrix} 0 & 1 & -4 \\ 1 & -4 & 13 \\ 0 & 1 & -4 \end{bmatrix}$$

$$\mathrm{Rank}(Q_k) = 2 < 3$$

系统不完全能控.

$$Q_g = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & -2 \\ -7 & -4 & 4 \end{bmatrix}$$

$$\mathrm{Rank}(Q_g) = 3$$

系统完全能观.

(3)

$$sI - A_1 = \begin{bmatrix} s & -1 \\ 3 & s+4 \end{bmatrix}$$

$$(sI - A_1)^{-1} = \frac{1}{(s+1)(s+3)} \begin{bmatrix} s+4 & 1 \\ -3 & s \end{bmatrix}$$

$$g_1(s) = C_1 (sI - A_1)^{-1} B_1$$

$$= \frac{s+2}{(s+1)(s+3)}$$

$$g_2(s) = \frac{1}{s+2}$$

$$g(s) = g_1(s) g_2(s) = \frac{1}{(s+1)(s+3)}$$

出现零极点相消，系统为不能控或不能观的

$$sI-A=\begin{bmatrix} s & -1 & 0 \\ 3 & s+4 & 0 \\ -2 & -1 & s+2 \end{bmatrix}$$

$$(sI-A)^{-1}=\begin{bmatrix} \frac{s+4}{(s+1)(s+3)} & \frac{1}{(s+1)(s+3)} & 0 \\ -\frac{3}{(s+1)(s+3)} & \frac{s}{(s+1)(s+3)} & 0 \\ \frac{2s+5}{(s+1)(s+2)(s+3)} & \frac{1}{(s+1)(s+3)} & \frac{1}{s+2} \end{bmatrix}$$

$$C(sI-A)^{-1}=\begin{bmatrix} \frac{2s+5}{(s+1)(s+2)(s+3)} & \frac{1}{(s+1)(s+3)} & \frac{1}{s+2} \end{bmatrix}$$

各列线性无关，系统完全能观.

$$(sI-A)^{-1}B=\begin{bmatrix} \frac{1}{(s+1)(s+3)} \\ \frac{s}{(s+1)(s+3)} \\ \frac{1}{(s+1)(s+3)} \end{bmatrix}$$

各行线性相关，系统不完全能控