作业三

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23 (a) $P_1 = \frac{b}{b+r} \cdot \frac{b+d}{b+d+r}$

(b) $P_2 = \frac{b}{b+r} \cdot \frac{r}{b+d+r} \cdot \frac{r+d}{b+2d+r}$

(c) $$P_3 = \frac{C_n^{n_1} b(b+d)\cdots[b+(n_1-1)d]\cdot r(r+d)\cdots[r+(n_2-1)d]}{(b+r)(b+r+d)\cdots[b+r+(n-1)d]}$$

24 $A_i = \{$进行 i 次后再取出白球$\}$.

$$P_i = P_{i-1}^2 + \frac{P_{i-1}(a+b)+1}{P_{i-1}}(1-P_{i-1})$$

$$= \frac{a+b-1}{a+b}P_{i-1} + \frac{1}{a+b}$$

$$P_0 = \frac{a}{a+b}$$

则 $P_n = 1-\left(1-\frac{1}{a+b}\right)^n \frac{b}{a+b}$

26. $B_k = \{$家里有 k 个孩子$\}$

$A = \{$家里所有孩子都同一个性别$\}$

$$P(A) = \sum_{k=1}^{\infty} P(AB_k) = \sum_{k=1}^{\infty} P(B_k)P(A|B_k)$$

$$= \sum_{k=1}^{\infty} 2\cdot\left(\frac{1}{2}\right)^k p_k$$

$$= \sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^{k-1} p_k$$

28. 设 B_n = {爬行n米后在顶点A}

$P_n = P(B_n) = P(B_n | B_{n-1}) P(B_{n-1}) + P(B_n | B_{n-1}^c) P(B_{n-1}^c)$

$P_n = \frac{1}{3}(1 - P_{n-1})$

$P_0 = 1$

$P_n = \frac{1}{4} + (-\frac{1}{3})^n \cdot \frac{3}{4}$

$P_7 = \frac{182}{729}$

31. A: {投出几点就从袋中取几个球，且球全为白}

B= {已知取出的球全白，骰子是3点}

$$P(A) = \frac{1}{6} \times \left(\frac{4}{10} + \frac{C_4^2}{C_{10}^2} + \frac{C_4^3}{C_{10}^3} + \frac{C_4^4}{C_{10}^4} + 0 + 0\right)$$

$$= \frac{2}{21}$$

$$P(B) = \frac{\frac{1}{6} \times \frac{C_4^3}{C_{10}^3}}{\frac{2}{21}} = \frac{7}{120}$$

35. $$P = \frac{\frac{1}{3} \times \left[(\frac{1}{3})^3 + C_3^2 \times \frac{1}{3} \times (\frac{2}{3})^2\right]}{\frac{1}{3} \times \left[(\frac{1}{3})^3 + C_3^2 \times \frac{1}{3} \times (\frac{2}{3})^2\right] + \frac{2}{3} \times \left[C_3^1 \times \frac{2}{3} \times (\frac{1}{3})^2 + (\frac{2}{3})^3\right]}$$

$$= \frac{13}{41}$$

36.

$A = \{甲得冠军\}$

$B = \{乙得冠军\}$

$$P(A) = \left(\frac{1}{2}\right)^4 \times \left[C_4^2 + C_4^3 + C_4^4\right]$$

$$= \frac{11}{16}$$

$$P(B) = \left(\frac{1}{2}\right)^4 \times \left[C_4^3 + C_4^4\right]$$

$$= \frac{5}{16}$$

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$$P = 0.8 \times \left[C_2^0 \times 0.8^2 (1-0.7^3) + C_2^1 \times 0.2 \times 0.8 (1-0.7^2) + C_2^2 \times 0.2^2 \times (1-0.7)\right]$$

$$= 0.476544$$

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设 $P(X=k) = \dfrac{c}{k(k+1)} = c\left(\dfrac{1}{k} - \dfrac{1}{k+1}\right)$

$$\sum_{k=1}^{+\infty} P(X=k) = c \sum_{k=1}^{+\infty} \left(\frac{1}{k} - \frac{1}{k+1}\right) = c = 1$$

故 $P(X=k) = \dfrac{1}{k(k+1)}$

X	1	2	3	$\cdots$	n	$\cdots$
P	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{12}$	$\cdots$	$\frac{1}{n(n+1)}$	$\cdots$

4.

X	1	2	3
P	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

$$F(x)=\begin{cases} 0 & x\in(-\infty,1) \\ \frac{1}{6} & x\in[1,2) \\ \frac{2}{3} & x\in[2,3) \\ 1 & x\in[3,+\infty) \end{cases}$$

补充题

(1) $$P_1 = p^3 + C_3^2 p^3 q + C_4^2 p^3 q^2$$

$$= p^3(1+3q+6q^2)$$

(2) $$P_2 = p^3 + C_3^2 p^3 q + C_4^2 p^3 q^2$$

$$= p^3(1+3q+6q^2)$$

(3) $$P_3 = \sum_{k=1}^{+\infty} C_{2k-1}^{k} p^{k+1} q^{k-1}$$