作业六

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29. 设试验 X 次后停止

$$P(X=n) = (n-1)p^2 q^{n-2}$$

$$EX = \sum_{n=2}^{+\infty} n(n-1)p^2 q^{n-2}$$

$$qEX = \sum_{n=2}^{+\infty} n(n-1)p^2 q^{n-1}$$

$$pEX = \sum_{n=1}^{+\infty} 2np^2 q^{n-1}$$

$$EX = \sum_{n=1}^{+\infty} 2npq^{n-1}$$

$$qEX = \sum_{n=1}^{+\infty} 2npq^{n}$$

$$pEX = \sum_{n=0}^{+\infty} 2pq^{n} = 2p \cdot \frac{1}{1-q} = 2$$

$$EX = \frac{2}{p}$$

33. $$P(X=m) = \sum_{k=m}^{n} \frac{1}{n} \cdot \frac{1}{k}$$

$$EX = \sum_{m=1}^{n} m \sum_{k=m}^{n} \frac{1}{nk} = \frac{1}{n} \sum_{k=1}^{n} \frac{1}{k} \sum_{m=1}^{k} m$$

$$= \frac{1}{n} \sum_{k=1}^{n} \frac{1}{k} \cdot \frac{k(k+1)}{2} = \frac{n+3}{4}$$

35.

X \ L	2	3	4	$p_i^{(X)}$
0	$\frac{3}{7}$	0	$\frac{1}{7}$	$\frac{4}{7}$
1	$\frac{1}{7}$	0	0	$\frac{1}{7}$
2	0	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{2}{7}$
$p_j^{(L)}$	$\frac{4}{7}$	$\frac{1}{7}$	$\frac{2}{7}$	1

$$E(L|X=0) = \frac{5}{2}$$

$$E(L|X=1) = 2$$

$$E(L|X=2) = \frac{7}{2}$$

36. $E[D(X|Y)] = E(E[(X-E(X|Y))^2|Y])$

$= E(X-E(X|Y))^2$

$D[E(X|Y)] = E(E(X|Y) - EX)^2$

故 $E[D(X|Y)] + D[E(X|Y)]$

$= E(X^2 - 2XE(X|Y) + [E(X|Y)]^2) + E([E(X|Y)]^2 - 2E(X|Y)EX + (EX)^2)$

$= E(X^2) - 2E[XE(X|Y)] + 2E(E^2(X|Y)) - 2E(X)E(E(X|Y)) + (EX)^2$

$= E(X^2) - (EX)^2 + 2(E[E^2(X|Y)] - E[XE(X|Y)])$

$E[E^2(X|Y)] - E[XE(X|Y)] = E(E(X|Y)[E(X|Y) - X])$

$= E(E[E(X|Y)(E(X|Y)-X)|Y])$

$= E[E(X|Y)\cdot E([E(X|Y)-X]|Y)]$

$E([E(X|Y)-X]|Y) = E(E(X|Y)|Y) - E(X|Y)$

$= E(X|Y) - E(X|Y) = 0$

故 $E[E^2(X|Y)] - E[XE(X|Y)] = 0$

$E[D(X|Y)] + D[E(X|Y)] = E(X^2) - (EX)^2 = DX$

37. (a)

$P(X_n \geq 0,\ n=1,2,3,4) = P(X_1=1)P(X_2-X_1=1) + P(X_1=1)P(X_2-X_1=-1)P(X_3-X_2=1)$

$= p^2 + p^2 q$

$= p^2(1+q)$.

39. (a)

$$P(X_{n+1}=i_{n+1} \mid X_n=i_n,\cdots,X_0=i_0) = \frac{P(X_{n+1}=i_{n+1},\cdots,X_0=i_0)}{P(X_n=i_n,\cdots,X_0=i_0)}$$

$$= \frac{P(X_{n+1}-X_n=i_{n+1}-i_n,\cdots,X_1-X_0=i_1-i_0,X_0=i_0)}{P(X_n-X_{n-1}=i_n-i_{n-1},\cdots,X_1-X_0=i_1-i_0,X_0=i_0)}$$

$$= P(X_{n+1}-X_n=i_{n+1}-i_n)$$

$$= \frac{P(X_{n+1}=i_{n+1},X_n=i_n)}{P(X_n=i_n)}$$

$$= P(X_{n+1}=i_{n+1} \mid X_n=i_n)$$

(b) $$P(X_{n+1}=i_{n+1},X_{n-1}=i_{n-1} \mid X_n=i_n) = \frac{P(X_{n+1}=i_{n+1},X_n=i_n,X_{n-1}=i_{n-1})}{P(X_n=i_n)}$$

$$= \frac{P(X_{n+1}-X_n=i_{n+1}-i_n)P(X_n=i_n)P(X_n-X_{n-1}=i_n-i_{n-1})}{P(X_n=i_n)}$$

$$= \frac{P(X_{n+1}=i_{n+1},X_n=i_n)}{P(X_n=i_n)} \frac{P(X_n=i_n,X_{n-1}=i_{n-1})}{P(X_n=i_n)}$$

$$= P(X_{n+1}=i_{n+1} \mid X_n=i_n)P(X_{n-1}=i_{n-1} \mid X_n=i_n)$$

1 $$P(X=1)=P(X=2) \Rightarrow \frac{\lambda e^{-\lambda}}{1} = \frac{\lambda^2 e^{-\lambda}}{2} \Rightarrow \lambda^2=2\lambda \Rightarrow \lambda=2$$

故 $$P(X=4) = \frac{\lambda^4 e^{-\lambda}}{4!} = \frac{2^4 e^{-2}}{4!} = \frac{2}{3e^2}$$

3. $$E|X-\lambda| = \sum_{k=0}^{\lfloor\lambda\rfloor} \frac{\lambda^k}{k!} e^{-\lambda}(\lambda-k) + \sum_{k=\lceil\lambda\rceil}^{+\infty} \frac{\lambda^k}{k!} e^{-\lambda}(k-\lambda)$$

$$E(X-\lambda) = \sum_{k=0}^{\lfloor\lambda\rfloor} \frac{\lambda^k}{k!} e^{-\lambda}(\lambda-k) - \sum_{k=\lceil\lambda\rceil}^{+\infty} \frac{\lambda^k}{k!} e^{-\lambda}(k-\lambda) = 0$$

$$\sum_{k=0}^{\lfloor\lambda\rfloor} \frac{\lambda^k}{k!} e^{-\lambda}(\lambda-k) = \sum_{k=\lceil\lambda\rceil}^{+\infty} \frac{\lambda^k}{k!} e^{-\lambda}(k-\lambda)$$

$$E|X-\lambda| = 2\sum_{k=0}^{\lfloor\lambda\rfloor} \frac{\lambda^k}{k!} e^{-\lambda}(\lambda-k)$$

4. $X \sim P(\lambda_1) \quad Y \sim P(\lambda_2)$

$$P(X=k \mid X+Y=n) = \frac{P(X=k,\ Y=n-k)}{P(X+Y=n)}$$

$$= \frac{\frac{\lambda_1^k e^{-\lambda_1}}{k!} \frac{\lambda_2^{n-k} e^{-\lambda_2}}{(n-k)!}}{\frac{(\lambda_1+\lambda_2)^n e^{-(\lambda_1+\lambda_2)}}{n!}}$$

$$= \frac{n!}{k!(n-k)!} \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^k \left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{n-k}$$

$$= C_n^k \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^k \left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{n-k}$$

$$\sim B\left(n, \frac{\lambda_1}{\lambda_1+\lambda_2}\right)$$

6 设雌虫数为X，总产卵数为Y

$$P(Y=k) = \sum_{n=0}^{+\infty} P(Y=k \mid X=n) \cdot P(X=n)$$

$$= \sum_{n=0}^{+\infty} \frac{(n\mu)^k e^{-n\mu}}{k!} \frac{\lambda^n e^{-\lambda}}{n!}$$

$$= \frac{\mu^k e^{-\lambda}}{k!} \sum_{n=0}^{+\infty} \frac{(\lambda e^{-\mu})^n \cdot n^k}{n!} \qquad k=0,1,2,\cdots$$

7 $P(X=k) = P(X=k \mid \lambda=\lambda_1) P(\lambda=\lambda_1) + P(X=k \mid \lambda=\lambda_2) P(\lambda=\lambda_2)$

$$= \frac{\lambda_1^k e^{-\lambda_1}}{k!} p + \frac{\lambda_2^k e^{-\lambda_2}}{k!} (1-p) \quad k=0,1,2,\dots$$