homework 5

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5.3

$$G_{闭}(s)=\frac{G(s)}{1+G(s)}=\frac{2.55\times10^5}{s^3+115s^2+1500s+2.55\times10^5}$$

令 $s^3+115s^2+1500s+2.55\times10^5=0$.

s^3	1	1500
s^2	115	2.55×10^5
s^1	-717.4	
s^0	2.55×10^5	

系统不稳定.

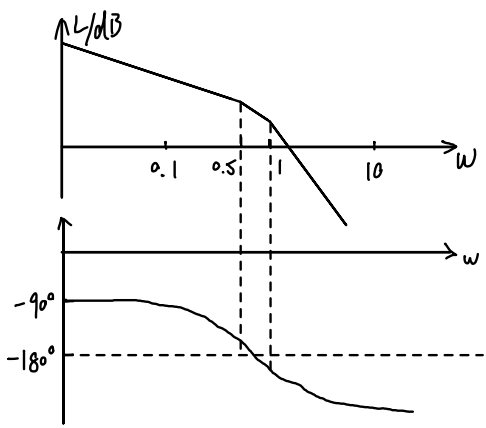
为使系统有足够的稳定裕度，应使用滞后矫正，为在K不变情况下使w_c保持不变，应使用超前矫正.

综上，应使用超前-滞后矫正.

5.4

$$K_V=\lim_{s\to0}sG(s)=K=10\ s^{-1}.$$

作 Bode 图如下所示.



$$20\lg\frac{10}{0.5} = 40\lg\frac{1}{0.5} + 60\lg\frac{\omega_{gc}}{1} \Rightarrow \omega_{gc} = 1.71 \text{ rad/s}$$

$$\gamma = -90^\circ - \arctan(\omega_{gc}) - \arctan(2\omega_{gc}) + 180^\circ = -43.4^\circ$$

令 $-90^\circ - \arctan(\omega_{pc}) - \arctan(2\omega_{pc}) = -180^\circ$

$\Rightarrow \omega_{pc} = 0.707 \text{ rad/s}$.

$$K_g = -20\lg\frac{10}{0.5} + 40\lg\frac{\omega_{pc}}{0.5} = -20 \text{ dB}$$

取 $\omega_c = \omega_{pc} = 0.707 \text{ rad/s}$.

$\arg G_p(j0.707) = -180^\circ$.

$\varphi = 50^\circ$

$\varphi_m = \varphi + 10^\circ = 60^\circ$

$$\alpha = \frac{1+\sin 60^\circ}{1-\sin 60^\circ} = 13.9$$

$$T_1 = \frac{1}{\sqrt{\alpha}\omega_c} = 0.379$$

$$G_{c1}(s) = \frac{1+\alpha T_1 s}{1+T_1 s} = \frac{1+5.27s}{1+0.379s}$$

$$-20\lg\left|G_p(j0.707)G_{c1}(j0.707)\right| \approx 27.9 \text{ dB}$$

$$20\lg\frac{1}{\beta} \approx 27.9 \text{ dB}$$

$\Rightarrow \beta \approx 24.85$

取 $T_2 = 5\omega_c^{-1} = 7.07$

$$G_{c2}(s) = \frac{1+T_2 s}{1+\beta T_2 s} = \frac{1+7.07s}{1+175.7s}$$

$$G_p(s)G_c(s)=\frac{10(1+5.2s)(1+7.0s)}{s(s+1)(2s+1)(1+0.379s)(1+75s)}$$

经检验，$\gamma=49.2^\circ\approx 50^\circ$，$k_g=12.8\,dB>10\,dB$ 满足要求.

5.9

$$G_o(s)=0.5\times\frac{\frac{20k_1}{s(s+5)(s+20)}}{1+k_2s\frac{20k_1}{s(s+5)(s+20)}}=\frac{10k_1}{s(s^2+25s+100+20k_1k_2)}$$

$$T=\sqrt{\frac{1}{100+20k_1k_2}}\qquad \xi=\frac{12.5}{\sqrt{100+20k_1k_2}}$$

$$k_v=\lim_{s\to 0}sG_o(s)=\frac{10k_1}{100+20k_1k_2}=4\quad\cdots①$$

$$t_s\approx\frac{4\sim 9}{\omega_c}\le 1s$$

取 $\omega_c=9\ rad/s$

$$\gamma=-90^\circ-\arctan\frac{25\omega_c}{100+20k_1k_2-\omega_c^2}+180^\circ$$

$$=90^\circ-\arctan\frac{25\omega_c}{\frac{5}{2}k_1-\omega_c^2}$$

$$M_r=\frac{1}{\sin\gamma}$$

$$\sigma\%=100(M_r-1)\%\le 20\%$$

$$M_r\le 1.2$$

$$\Rightarrow\ k_1\ge 168.1$$

取 $K_1 = 200$

由①得 $K_2 = \frac{1}{8} - \frac{5}{K_1} = 0.1$

$$G_0(s) = \frac{10K_1}{s(s^2 + 25s + 100 + 20K_1K_2)}$$

$$= \frac{4}{s(0.002s^2 + 0.05s + 1)}$$

$$T = \frac{\sqrt{5}}{50} \qquad \zeta = \frac{\sqrt{5}}{4}$$

$$G_0(jw) = \frac{4}{-0.002jw^3 - 0.05w^2 + jw}$$

$$|G_0(jw)| = \frac{2000}{\sqrt{(0.05w^2)^2 + (w - 0.002w^3)^2}}$$

$w_c = 4.05 \text{ rad/s}$

$$\gamma = -90^\circ - \arctan\frac{25w_c}{500 - w_c^2} + 180^\circ$$

$$= 78.2^\circ$$

$$M_r = \frac{1}{\sin\gamma} = 1.022$$

$$\sigma\% = 100(M_r - 1)\% = 2.2\% < 20\%$$

$$t_s \approx \frac{4}{w_c} \leq 1\text{s}.$$

满足条件.