1. ① 模型误差　② 观测误差　③ 方法误差　④ 舍入误差

2. (1)

$$\delta x = |x - x_A| = 1.8 \times 10^{-5} \le \frac{1}{2} \times 10^{-5} \qquad \text{六位有效数字}$$

$$\left|\frac{\delta x}{x}\right| \approx \left|\frac{\delta x}{x_A}\right| = \frac{1.8 \times 10^{-6}}{2.71828} = 6.6 \times 10^{-7}$$

(2)

$$\delta x = |x - x_A| = 1.82818 \times 10^{-5} \le \frac{1}{2} \times 10^{-4} \qquad \text{两位有效数字}$$

$$\left|\frac{\delta x}{x}\right| \approx \left|\frac{\delta x}{x_A}\right| = \frac{1.82818 \times 10^{-5}}{0.0027} = 6.8 \times 10^{-3}$$

3.

$$f(30) = \ln(\sqrt{30^2+1} - 30) = \ln(30.017 - 30) = -4.745$$

令 $y = \sqrt{30^2+1}$　$A = g(y) = \ln(y - 30)$.　$y \in [30.0165, 30.0175]$

$$\delta y = |y - y^*| \le \frac{1}{2} \times 10^{-3}$$

$$g'(y) = \frac{1}{y-30} \qquad \text{单调递减}.$$

$$\delta A = |A - A^*| \le \max|g'(y)||\delta y|$$

$$= |g'(30.0165)||\delta y|$$

$$= 3.03 \times 10^{-2}$$

改用另一公式　$B = h(y) = -\ln(y + 30)$　$y \in [30.0165, 30.0175)$

$$h'(y) = -\frac{1}{y+30} \qquad \text{单调递增}$$

$$\Delta B = |B - B^*| \leq \max |h'(y)| |\Delta y|$$
$$= h'(30.0175) |\Delta y|$$
$$= 8.33 \times 10^{-6}$$

4. 若 x_k 是 $\sqrt{7}$ 的具有 n 位有效数字的近似值

则 $|\delta x_k| = |\sqrt{7} - x_k| \leq \frac{1}{2} \times 10^{-n+1}$

$$x_k = \frac{1}{2}(x_{k-1} + \frac{7}{x_{k-1}}) \geq \sqrt{7} > 2.5$$

则 $|\delta x_{k+1}| = |\sqrt{7} - x_{k+1}| = |\sqrt{7} - \frac{1}{2}(x_k + \frac{7}{x_k})|$

$$= \frac{1}{2} |x_k - 2\sqrt{7} + \frac{7}{x_k}|$$

$$= \frac{(x_k - \sqrt{7})^2}{2x_k}$$

$$\leq \frac{1}{2 \times 2.5} (\frac{1}{2} \times 10^{-n+1})^2$$

$$= \frac{1}{20} \times 10^{-2n+2}$$

$$= \frac{1}{2} \times 10^{-2n+1}$$

x_{k+1} 必是 $\sqrt{7}$ 的具有至少 $2n$ 位有效数字的近似值