homework 4

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4.4

$$G(s)=\frac{5.2}{0.1s^2+0.32s+1}$$

$K=5.2$ $\quad 20\lg K = 14.3\ dB$

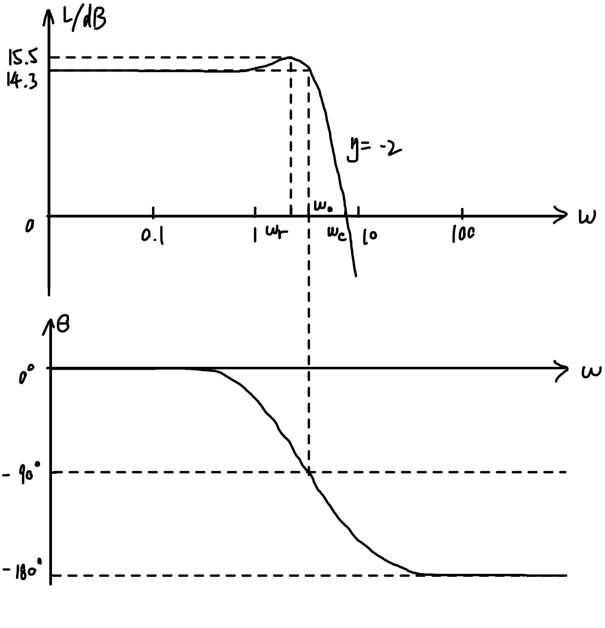
$T=\sqrt{0.1}=0.316$ $\quad \xi=\frac{0.32}{2T}=0.506<\frac{\sqrt{2}}{2}$

转折频率 $\omega_0=\frac{1}{T}=3.16$ $\quad u_0=0.50$

峰值频率 $\omega_r=\frac{1}{T}\sqrt{1-2\xi^2}=2.21$ $\quad u_r=0.34$

谐振峰值 $M_r=\frac{1}{2\xi\sqrt{1-\xi^2}}=1.15$ $\quad 20\lg K+20\lg M_r=15.5\ dB$

截止频率 $u_c=u_0+\frac{20\lg K}{40}=0.86$ $\quad \omega_c=7.20$



4.7 (a) $G(s) = \dfrac{2.8(0.05s+1)}{s(0.15s+1)}$

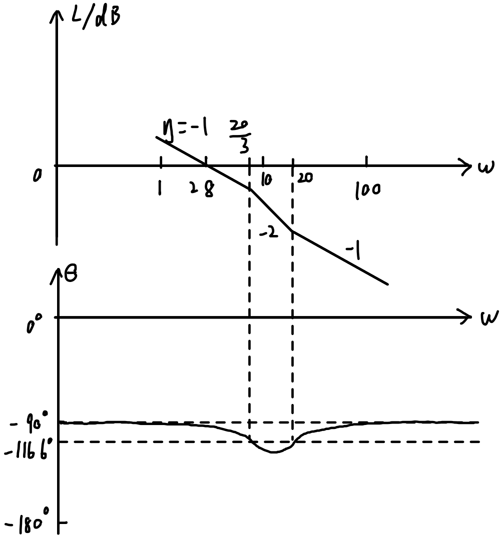
$w_z = \dfrac{1}{0.05} = 20$ $\quad w_p = \dfrac{1}{0.15} = \dfrac{20}{3}$

估计 $w_c < w_p < w_z$

$w_c = 2.8$

$\theta(w) = -90^\circ + \arctan 0.05w - \arctan 0.15w$

$\theta(0) = -90^\circ$ $\quad \theta(w_c) = -104.8^\circ$ $\quad \theta(w_p) = -116.6^\circ$ $\quad \theta(w_z) = -116.6^\circ$ $\quad \theta(\infty) = -90^\circ$



(b) $G(s) = \dfrac{2.8(0.5s+1)}{s(0.15s+1)}$

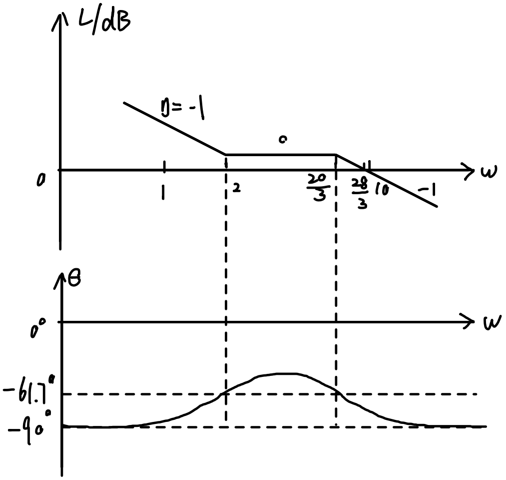
$w_z = \dfrac{1}{0.5} = 2$ $\quad w_p = \dfrac{1}{0.15} = \dfrac{20}{3}$

估计 $w_z < w_p < w_c$

$2.8 \times 0.5 w_c = w_c \times 0.15 w_c \Rightarrow w_c = \dfrac{28}{3}$

$$\theta(\omega) = -90^\circ + \arctan 0.5\omega - \arctan 0.15\omega$$

$\theta(0) = -90^\circ$ $\theta(\omega_c) = -66.6^\circ$ $\theta(\omega_p) = -61.7^\circ$ $\theta(\omega_z) = -61.7^\circ$ $\theta(\infty) = -90^\circ$



416 由图可知 $\omega_{p1} = 0.002$ $\omega_{p2} = 0.2$ $\omega_{p3} = 1$

$\omega_{z1} = 0.02$

$$\lg K = \frac{52}{20} + \lg \omega_{p1} = -0.1 \Rightarrow K = 0.8$$

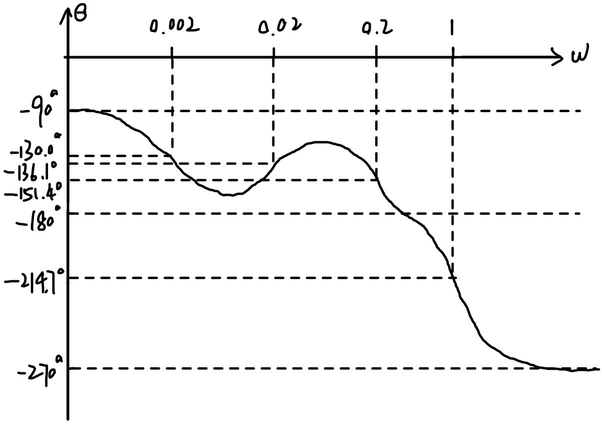
$$G_o(s) = \frac{0.8(50s+1)}{s(500s+1)(5s+1)(s+1)}$$

$$20\lg\frac{\omega_c}{0.02} + 40\lg\frac{0.02}{0.002} = 52 \Rightarrow \omega_c = 0.08$$

$$\theta(\omega) = -90^\circ + \arctan 50\omega - \arctan 500\omega - \arctan 5\omega - \arctan\omega$$

$\theta(0) = -90^\circ$ $\theta(\omega_{p1}) = -130.0^\circ$ $\theta(\omega_c) = -129.0^\circ$ $\theta(\omega_{z1}) = -136.1^\circ$

$\theta(\omega_{p2}) = -151.4^\circ$ $\theta(\omega_{p3}) = -214.7^\circ$ $\theta(\infty) = -270^\circ$



$$G(s)=\frac{G_0(s)}{1+G_0(s)}=\frac{0.8(5s+1)}{0.8(50s+1)+s(500s+1)(5s+1)(s+1)}$$

$$=\frac{40s+0.8}{2500s^4+3005s^3+506s^2+41s+0.8}$$

$$2500\frac{d^4y(t)}{dt^4}+3005\frac{d^3y(t)}{dt^3}+506\frac{d^2y(t)}{dt^2}+41\frac{dy(t)}{dt}+0.8y(t)=40\frac{du(t)}{dt}+0.8u(t)$$

4.24

$$G_1(j\omega)=\frac{k(T_2j\omega+1)}{(j\omega)^2(T_1j\omega+1)}$$

$\omega\to0^+$ $\quad G_1(j\omega)=\infty\angle-180^\circ$

$\omega\to+\infty$ $\quad G_1(j\omega)=0\angle-180^\circ$

G_1 对应 (b) $\quad N=0\ \ P=0\Rightarrow Z=0$ 系统稳定

$$G_2(j\omega)=\frac{k(T_2j\omega+1)}{(j\omega)^2(T_1j\omega+1)(T_3j\omega+1)}$$

$\omega\to0^+$ $\quad G_2(j\omega)=\infty\angle-180^\circ$

$\omega\to+\infty$ $\quad G_2(j\omega)=0\angle-270^\circ$

G_2 对应 (c)　若 $G_2(j\omega)$ 不包 -1　$N=0$　$P=0$ $\Rightarrow Z=0$ 系统稳定

若 $G_2(j\omega)$ 包 -1　$N=2$　$P=0$ $\Rightarrow Z=2$ 系统不稳定

$$G_3(j\omega)=\frac{K(T_2j\omega+1)(T_4j\omega+1)}{(j\omega)^3(T_1j\omega+1)(T_3j\omega+1)}$$

$\omega\to0^+$　$G_3(j\omega)=\infty\angle-270^\circ$

$\omega\to+\infty$　$G_3(j\omega)=0\angle-270^\circ$

G_3 对应 (a)　$N=0$　$P=0$ $\Rightarrow Z=$　系统稳定

4.26

$$Q(j\omega)=\frac{K(0.5j\omega+1)(j\omega+1)}{(10j\omega+1)(j\omega-1)}$$

$$=\frac{K[5\omega^4-23\omega^2-1+j(-19.5\omega^3+7.5\omega)]}{(10\omega^2+1)^2+81\omega^2}$$

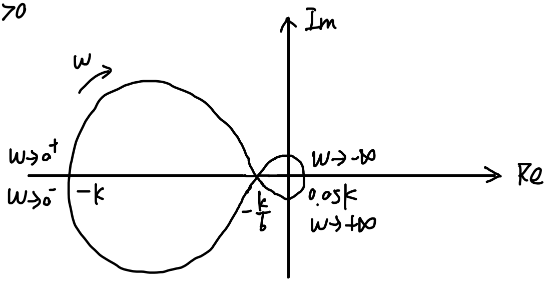
$\omega\to0^+$　$Q(j\omega)=K\angle-180^\circ$

$\omega\to+\infty$　$Q(j\omega)=0.05K\angle0^\circ$

令 $Im[Q(j\omega)]=0$ $\Rightarrow$ $\omega_1=0$　$\omega_{2,3}=\pm\sqrt{\frac{5}{13}}$

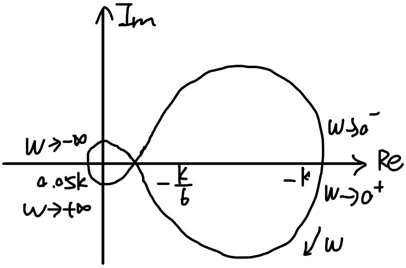
与实轴交点为 $(-\frac{K}{6},0)$　$(-K,0)$　$(0.05K,0)$

① $K>0$



当 $-\frac{K}{6}<-1$ 即 $K>6$ 时 系统稳定

② $K<0$



当 $0.05K<-1$ 即 $K<-20$ 时 系统稳定

综上. $K>6$ 或 $K<-20$

4.33

$\omega\to0^+$ $Q_1(j\omega)=\infty\angle-180^\circ$

$Q_2(j\omega)=\infty\angle-270^\circ$

$Q_3(j\omega)=\infty\angle-270^\circ$

$Q_4(j\omega)=\infty\angle-90^\circ$

$\omega\to+\infty$ $Q_1(j\omega)=0\angle-180^\circ$

$Q_2(j\omega)=0\angle-270^\circ$

$Q_3(j\omega)=0\angle-180^\circ$

$Q_4(j\omega)=0\angle0^\circ$

(a) 对应 (2) $N=0$ $P=0$ $\Rightarrow Z=0$ 稳定

(b) 对应 (3) $N=1$ $P=1$ $\Rightarrow Z=2$ 不稳定

4.35 (a)

$$G_1(s)=\frac{K(2.5s+1)}{s(25s+1)(\frac{1}{6}s+1)(\frac{1}{10}s+1)}$$

$$20\lg\frac{K}{0.04}=40\lg\frac{0.4}{0.04}+20\lg\frac{2}{0.4}$$

$$\Rightarrow K=20$$

$$G_1(s)=\frac{20(2.5s+1)}{s(25s+1)(\frac{1}{6}s+1)(\frac{1}{10}s+1)}$$

(b)

$$G_2(s)=\frac{K(5s+1)}{s(100s+1)(\frac{1}{3}s+1)(\frac{1}{5}s+1)}$$

$$20\lg\frac{K}{0.01}=40\lg\frac{0.2}{0.01}+20\lg\frac{1}{0.2}$$

$$\Rightarrow K=20$$

$$G_2(s)=\frac{20(5s+1)}{s(100s+1)(\frac{1}{3}s+1)(\frac{1}{5}s+1)}$$

(c)

$$G_3(s)=\frac{K(10s+1)}{s(100s+1)(\frac{1}{6}s+1)(\frac{1}{16}s+1)}$$

$$20\lg\frac{K}{0.01}=40\lg\frac{0.1}{0.01}+20\lg\frac{2}{0.1}$$

$$\Rightarrow K=20$$

$$G_3(s)=\frac{20(10s+1)}{s(100s+1)(\frac{1}{6}s+1)(\frac{1}{16}s+1)}$$

(a)(b)(c) 均为 I 型系统

$u(t)=1(t)$ $\quad e_{st1}=e_{st2}=e_{st3}=0$

$u(t)=t$ $\quad e_{st1}=e_{st2}=e_{st3}=\frac{1}{K}=\frac{1}{20}$

(a) $\omega_{c1}=2$ $\quad\theta_1(\omega_{c1}) = -129.9^\circ$.

$\gamma_1 = 180^\circ + \theta_1(\omega_{c1}) = 50.1^\circ$

$M_{r1} \approx \frac{1}{\sin\gamma_1} = 1.30$

$\sigma_1 \approx 50\sqrt{M_{r1}-1} = 27.55\%$

$t_{s1} = \frac{4\sim 9}{\omega_{c1}} = 2\sim 4.5s$

(b) $\omega_{c2}=1$ $\quad\theta_2(\omega_{c2}) = -130.5^\circ$

$\gamma_2 = 180^\circ + \theta_2(\omega_{c2}) = 49.5^\circ$

$M_{r2} \approx \frac{1}{\sin\gamma_2} = 1.32$

$\sigma_2 \approx 50\sqrt{M_{r2}-1} = 28.1\%$

$t_{s2} = \frac{4\sim 9}{\omega_{c2}} = 4\sim 9s$

(c) $\omega_{c3}=2$ $\quad\theta_3(\omega_{c3}) = -122.3^\circ$

$\gamma_3 = 180^\circ + \theta_3(\omega_{c3}) = 57.7\%$

$M_{r3} \approx \frac{1}{\sin\gamma_3} = 1.18$

$\sigma_3 \approx 100(M_r - 1) = 18.0\%$

$t_{s3} = \frac{4\sim 9}{\omega_{c3}} = 2\sim 4.5s$

(a)(b) 超调量接近，且比(c)小得多

(a)(c) 过渡过程时间大致相同，且小于(b)