homework9

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1.1

$$\begin{cases} V_c = L\dfrac{di_L}{dt} + i_L R \\ i_L = I - C\dfrac{dV_c}{dt} \end{cases}$$

$$\begin{bmatrix} \dot{V_c} \\ \dot{i_L} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{C} \\ \frac{1}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} V_c \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{C} \\ 0 \end{bmatrix} I$$

1.3

$$\begin{cases} \dfrac{dx_1(t)}{dt} = u(t) - \dfrac{x_1(t)}{R} \\ \dfrac{dx_2(t)}{dt} = \dfrac{x_1(t) - x_2(t)}{R} \\ y(t) = \dfrac{x_2(t)}{R} \end{cases}$$

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R} & 0 \\ \frac{1}{R} & -\frac{1}{R} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & \frac{1}{R} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

1.6 (1)

$$\begin{cases} \dot{x} = v \\ \dot{v} = \dfrac{u - kx - fv}{m} \\ y = x \end{cases}$$

$$\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{f}{m} \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix}$$

(2)

$$A=\begin{bmatrix}0 & 1\\ -\frac{k}{m} & -\frac{f}{m}\end{bmatrix} \quad B=\begin{bmatrix}0\\ \frac{1}{m}\end{bmatrix} \quad C=\begin{bmatrix}1 & 0\end{bmatrix} \quad D=0$$

$$(sI-A)^{-1}=\begin{bmatrix}s & -1\\ \frac{k}{m} & s+\frac{f}{m}\end{bmatrix}^{-1}=\frac{1}{s(s+\frac{f}{m})+\frac{k}{m}}\begin{bmatrix}s+\frac{f}{m} & 1\\ -\frac{k}{m} & s\end{bmatrix}$$

$$G(s)=C(sI-A)^{-1}B+D$$

$$=\frac{1}{s^2+\frac{f}{m}s+\frac{k}{m}}\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}s+\frac{f}{m} & 1\\ -\frac{k}{m} & s\end{bmatrix}\begin{bmatrix}0\\ \frac{1}{m}\end{bmatrix}$$

$$=\frac{1}{ms^2+fs+k}$$

1.8

$$\begin{bmatrix}x_1(k+1)\\ x_2(k+1)\\ x_3(k+1)\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -1 & -2 & -3\end{bmatrix}\begin{bmatrix}x_1(k)\\ x_2(k)\\ x_3(k)\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u(k)$$

$$y=\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}x_1(k)\\ x_2(k)\\ x_3(k)\end{bmatrix}$$

1.9

$$\begin{cases}x_1(0)=1\times 10^7\\ x_2(0)=1\times 10^8-1\times 10^7=9\times 10^7\\ x_1(k+1)=(1+0.8\%)x_1(k)-4\%\,x_1(k)+2\%\,x_2(k)\\ x_2(k+1)=(1+1\%)x_2(k)+4\%\,x_1(k)-2\%\,x_2(k)\end{cases}$$

$$\begin{bmatrix}x_1(k+1)\\ x_2(k+1)\end{bmatrix}=\begin{bmatrix}0.968 & 0.02\\ 0.04 & 0.99\end{bmatrix}\begin{bmatrix}x_1(k)\\ x_2(k)\end{bmatrix}$$

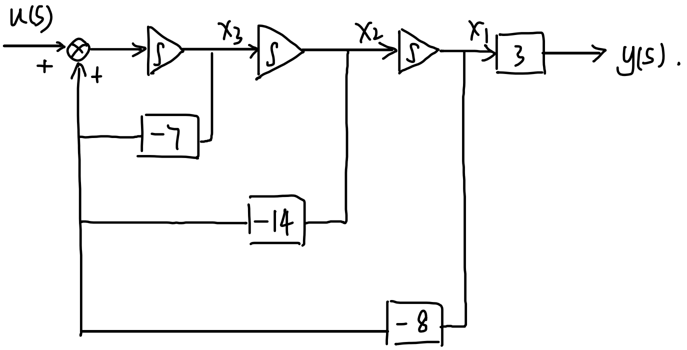
初始条件 $x_1(0)=1\times 10^7$ $x_2(0)=9\times 10^7$

1.1a $\dddot{y} + 7\ddot{y} + 14\dot{y} + 8y = 3u$

能控标准I型

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -14 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

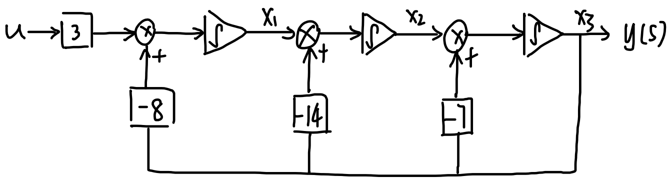
$$y = \begin{bmatrix} 3 & 0 & 0 \end{bmatrix} x$$



能观标准II型

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -8 \\ 1 & 0 & -14 \\ 0 & 1 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x$$

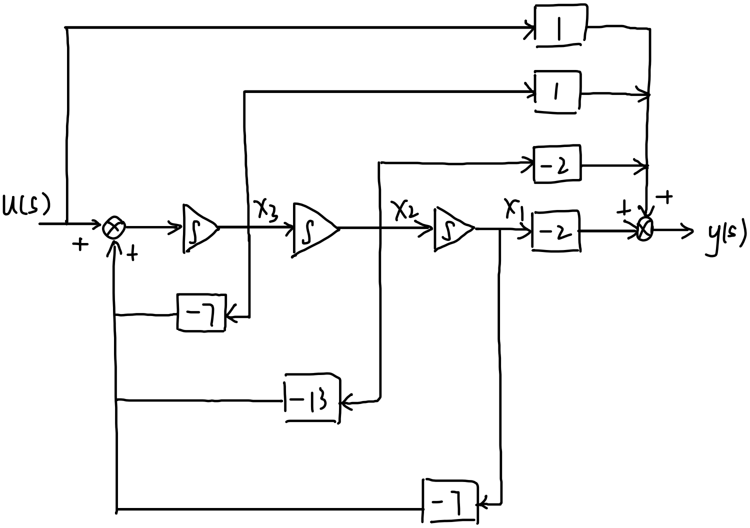


1.11 $\dddot{y}+7\ddot{y}+13\dot{y}+7y=\dddot{u}+8\ddot{u}+11\dot{u}+5u$

能控标准I型

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\-7&-13&-7\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}u$$

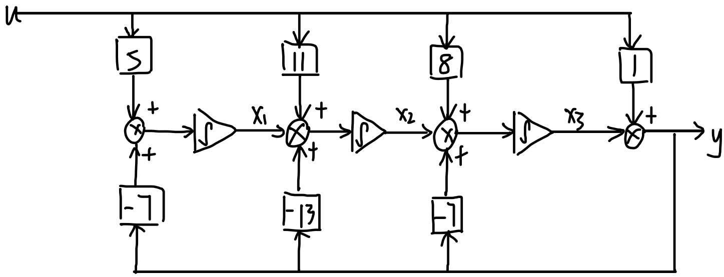
$$y=\begin{bmatrix}-2&-2&1\end{bmatrix}x+u$$



能观标准II型

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0&0&-7\\1&0&-13\\0&1&-7\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}-2\\-2\\1\end{bmatrix}u$$

$$y=\begin{bmatrix}0&0&1\end{bmatrix}x+u$$



1.12 $\quad G(s) = \dfrac{Y(s)}{U(s)} = \dfrac{1}{s^3+3s^2+2s} = \dfrac{1}{2s} - \dfrac{1}{s+1} + \dfrac{1}{2(s+2)}$

$$\begin{cases} \dot{x}_1 = u \\ \dot{x}_2 = -x_2 + u \\ \dot{x}_3 = -2x_3 + u \\ y = x_1 - x_2 \end{cases}$$

$$\dot{x} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} \frac{1}{2} & -1 & \frac{1}{2} \end{bmatrix} x$$

1.13 $\quad G(s) = \dfrac{Y(s)}{U(s)} = \dfrac{2(s+3)}{(s+1)(s+2)} = \dfrac{4}{s+1} - \dfrac{2}{s+2}$

$$\begin{cases} \dot{x}_1 = -x_1 + u \\ \dot{x}_2 = -2x_2 + u \\ y = 4x_1 - 2x_2 \end{cases}$$

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 4 & -2 \end{bmatrix} x$$

1 15 $A=\begin{bmatrix}0&0&-6\\1&0&-11\\0&1&-6\end{bmatrix}$ $B=\begin{bmatrix}1\\2\\0\end{bmatrix}$ $C=\begin{bmatrix}0&0&1\end{bmatrix}$ $D=0$

$$\left|\lambda I-A\right| = \begin{vmatrix}\lambda&0&6\\-1&\lambda&11\\0&-1&\lambda+6\end{vmatrix} = \lambda^3+6\lambda^2+11\lambda+6$$

A的特征值为: $\lambda_1=-1$ $\lambda_2=-2$ $\lambda_3=-3$

① $AP_1=\lambda_1P_1$

$$\begin{bmatrix}0&0&-6\\1&0&-11\\0&1&-6\end{bmatrix}\begin{bmatrix}P_{11}\\P_{21}\\P_{31}\end{bmatrix}=-\begin{bmatrix}P_{11}\\P_{21}\\P_{31}\end{bmatrix}$$

$$\begin{cases}-6P_{31}=-P_{11}\\P_{11}-11P_{31}=-P_{21}\\P_{21}-6P_{31}=-P_{31}\end{cases} \Rightarrow \begin{cases}P_{11}=6k\\P_{21}=5k\\P_{31}=k\end{cases}$$

可取 $P_1=\begin{bmatrix}6\\5\\1\end{bmatrix}$

② $AP_2=\lambda_2P_2$

$$\begin{bmatrix}0&0&-6\\1&0&-11\\0&1&-6\end{bmatrix}\begin{bmatrix}P_{12}\\P_{22}\\P_{32}\end{bmatrix}=-2\begin{bmatrix}P_{12}\\P_{22}\\P_{32}\end{bmatrix}$$

$$\begin{cases}-6P_{32}=-2P_{12}\\P_{12}-11P_{32}=-2P_{22}\\P_{22}-6P_{32}=-2P_{32}\end{cases} \Rightarrow \begin{cases}P_{12}=3k\\P_{22}=4k\\P_{32}=k\end{cases}$$

可取 $P_2=\begin{bmatrix}3\\4\\1\end{bmatrix}$

③ $AP_3 = \lambda_3 P_3$

$$\begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} P_{13} \\ P_{23} \\ P_{33} \end{bmatrix} = -3 \begin{bmatrix} P_{13} \\ P_{23} \\ P_{33} \end{bmatrix}$$

$$\begin{cases} -6P_{33} = -3P_{13} \\ P_{13} - 11P_{33} = -3P_{23} \\ P_{23} - 6P_{33} = -3P_{33} \end{cases} \Rightarrow \begin{cases} P_{13} = 2k \\ P_{23} = 3k \\ P_{33} = k \end{cases}$$

可取 $P_2 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$

$$T = [P_1 \ \ P_2 \ \ P_3] = \begin{bmatrix} 6 & 3 & 2 \\ 5 & 4 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -1 & 2 & -4 \\ \frac{1}{2} & -\frac{3}{2} & \frac{9}{2} \end{bmatrix}$$

$$\bar{A} = T^{-1}AT = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -1 & 2 & -4 \\ \frac{1}{2} & -\frac{3}{2} & \frac{9}{2} \end{bmatrix} \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} 6 & 3 & 2 \\ 5 & 4 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\bar{B} = T^{-1}B = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -1 & 2 & -4 \\ \frac{1}{2} & -\frac{3}{2} & \frac{9}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ 3 \\ -\frac{5}{2} \end{bmatrix}$$

$$\bar{C} = CT = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 3 & 2 \\ 5 & 4 & 3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

$$\dot{\hat{x}} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \hat{x} + \begin{bmatrix} -\frac{1}{2} \\ 3 \\ -\frac{5}{2} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \hat{x}$$

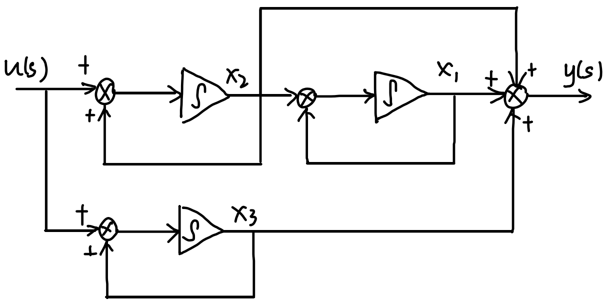
1.16

$$g(s) = \frac{2s^2+6s+5}{s^3+4s^2+5s+2} = \frac{1}{(s+1)^2} + \frac{1}{s+1} + \frac{1}{s+2}$$

$$\begin{cases} \dot{x}_1 = -x_1 + x_2 \\ \dot{x}_2 = -x_2 + u \\ \dot{x}_3 = -2x_3 + u \\ y = x_1 + x_2 + x_3 \end{cases}$$

$$\dot{x} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x$$



1.17

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 \\ -5 & 4 \\ 5 & -3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 1 \\ -2 & -3 & -4 \end{bmatrix} \qquad D = 0$$

$$(sI-A)^{-1} = \begin{bmatrix} s+2 & 0 & 0 \\ 0 & s+3 & 0 \\ 0 & 0 & s+4 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{s+2} & & \\ & \frac{1}{s+3} & \\ & & \frac{1}{s+4} \end{bmatrix}$$

$$G(s) = C(sI-A)^{-1}B$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ -2 & -3 & -4 \end{bmatrix} \begin{bmatrix} \frac{1}{s+2} & & \\ & \frac{1}{s+3} & \\ & & \frac{1}{s+4} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -5 & 4 \\ 5 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+2} & \frac{1}{s+3} & \frac{1}{s+4} \\ -\frac{2}{s+2} & -\frac{3}{s+3} & -\frac{4}{s+4} \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -5 & 4 \\ 5 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s^2+2s+2}{(s+2)(s+3)(s+4)} & \frac{2s+2}{(s+2)(s+3)(s+4)} \\ \frac{-7s^2-24s-24}{(s+2)(s+3)(s+4)} & \frac{2s^2+2s}{(s+2)(s+3)(s+4)} \end{bmatrix}$$

1.18 (1)

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -4 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 1 & 2 \end{bmatrix} x$$

(2)

$$\begin{cases} \dot{x}_1 = A_1 x_1 + B_1 u_1 \\ \dot{x}_2 = A_2 x_2 + B_2 y_1 = A_2 x_2 + B_2 C_1 x_1 \\ y = C_2 x_2 \end{cases}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -4 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 2 \end{bmatrix} x$$

✓

(3)

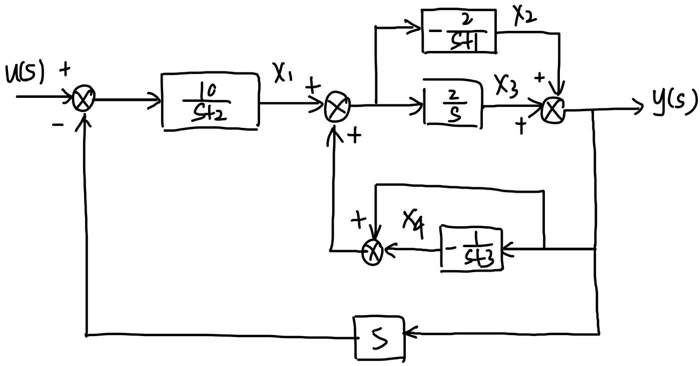
$$\begin{cases} \dot{x}_1 = A_1 x_1 + B_1 u - B_1 C_2 x_2 \\ \dot{x}_2 = A_2 x_2 + B_2 C_1 x_1 \\ y = C_1 x_1 \end{cases}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & -3 & -1 & -2 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -4 & -4 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x$$

✓

1.19



$$\begin{cases} \dot{X}_1 = -2X_1 + 10u - 50(X_3+X_2) = -2X_1 - 50X_2 - 50X_3 + 10u \\ \dot{X}_2 = -X_2 - 2X_1 - 2X_4 - 2(X_3+X_2) = -2X_1 - 3X_2 - 2X_3 - 2X_4 \\ \dot{X}_3 = 2X_1 + 2X_4 + 2(X_3+X_2) = 2X_1 + 2X_2 + 2X_3 + 2X_4 \\ \dot{X}_4 = -3X_4 - (X_3+X_2) = -X_2 - X_3 - 3X_4 \\ y = X_2 + X_3 \end{cases}$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} -2 & -50 & -50 & 0 \\ -2 & -3 & -2 & -2 \\ 2 & 2 & 2 & 2 \\ 0 & -1 & -1 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$