homework13

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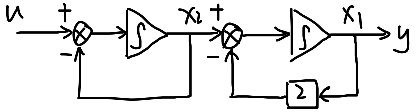
4.3 (1)



(2)

$$Q_k = [b \ \ Ab] = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$\operatorname{rank} Q_k = 2$

完全能控，可任意配置极点

(3)

$$f_k(s) = \det\left(sI - (A - bk^T)\right) = \det\left(sI - \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [k_1 \ \ k_2]\right)$$

$$= \det\begin{pmatrix} s+2 & -1 \\ k_1 & s+k_2+1 \end{pmatrix} = s^2 + (k_2+3)s + k_1 + 2k_2 + 2$$

$$f^*(s) = (s+3)^2 = s^2 + 6s + 9 \Rightarrow \begin{cases} k_2 + 3 = 6 \\ k_1 + 2k_2 + 2 = 9 \end{cases} \Rightarrow K = [1 \ \ 3]$$

4.5 (1)

$$Q_k = [b \ \ Ab \ \ A^2 b] = \begin{bmatrix} 0 & 1 & -4 \\ 0 & 1 & -4 \\ 1 & -4 & 13 \end{bmatrix}$$

$\operatorname{rank} Q_k = 2 < 3$

不完全能控，不能任意配置极点

(2)

$$f(\lambda) = \det(\lambda I - A) = (\lambda+2)(\lambda+1)(\lambda+3)$$

$$\hat{A} = \begin{bmatrix} -2 & & \\ & -1 & \\ & & -3 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & -1 & -3 \end{bmatrix}$$

$$T^{-1} b = \frac{1}{2} \begin{bmatrix} -2 & 2 & 0 \\ 3 & 0 & 1 \\ -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$$ ⇒ 不可配置极点为−2

若要闭环极点均为-2，可配置得到

$$f_k(s) = \det(sI - (A - bk^T))$$

$$= \det\begin{pmatrix} s & 0 & -1 \\ -2 & s+2 & -1 \\ k_1+3 & k_2 & s+k_3+4 \end{pmatrix}$$

$$= s^3 + (k_3+6)s^2 + (k_1+k_2+2k_3+11)s + 2(k_1+k_2)+6$$

$$f^*(s) = (s+2)^3 = s^3 + 6s^2 + 12s + 8$$

$$\Rightarrow \begin{cases} k_3 + 6 = 6 \\ k_1 + k_2 + 2k_3 + 11 = 12 \\ 2(k_1+k_2)+6 = 8 \end{cases} \Rightarrow \begin{cases} k_1 + k_2 = 1 \\ k_3 = 0 \end{cases}$$

设置 $k = \begin{bmatrix} a \\ 1-a \\ 0 \end{bmatrix}$

4.6(b) $Q_k = \begin{bmatrix} 1 & 3 & 7 \\ 0 & 1 & 4 \\ 1 & 4 & 9 \end{bmatrix}$ $\quad \mathrm{rank}\, Q_k = 3$ $\Rightarrow$ 可任意配置极点

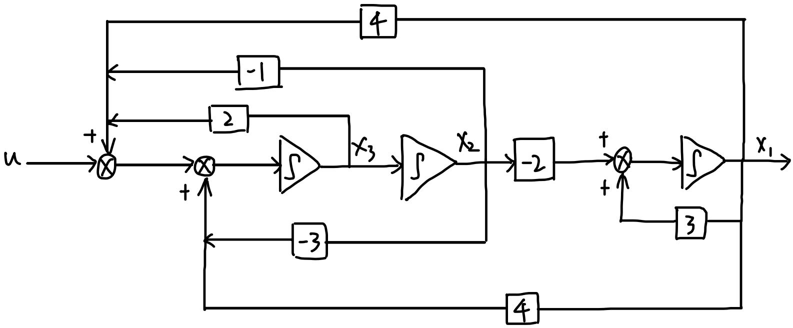
$$f_k(s) = \det(sI - (A - bk^T)) = \begin{vmatrix} s-3+k_1 & k_2+2 & k_3 \\ 0 & s & -1 \\ k_1-4 & k_2+3 & s+k_3 \end{vmatrix}$$

$$= s^3 + (k_1+k_3-3)s^2 + (k_2+k_3+3)s + k_1 + k_2 - 1$$

$$f^*(s) = (s+1)(s+1-j)(s+1+j)$$

$$= s^3 + 3s^2 + 4s + 2$$

$$\begin{cases} k_1+k_3-3=3 \\ k_2+k_3+3=4 \\ k_1+k_2-1=2 \end{cases} \Rightarrow \begin{cases} k_1=4 \\ k_2=-1 \\ k_3=2 \end{cases} \Rightarrow k=\begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$$



4.7(a) $f(\lambda) = \det\begin{pmatrix} \lambda & 0 & 0 \\ -1 & \lambda & -1 \\ -1 & 0 & \lambda \end{pmatrix} = \lambda^3 \Rightarrow \lambda_{1,2,3}=0$

且 $Q_k = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \Rightarrow \operatorname{Rank} Q_k = 2 < 3$

系统不完全能控

不能控部分极点为 $\lambda=0$，不渐近稳定

系统不可镇定

(b) $f(\lambda) = \det\begin{pmatrix} \lambda & 2 \\ -1 & \lambda+3 \end{pmatrix} = (\lambda+1)(\lambda+2)$

$Q_k = \begin{pmatrix} 1 & -2 \\ 1 & -2 \end{pmatrix}$ $\operatorname{Rank} Q_k = 1 < 2$

系统不完全能控

$$\Rightarrow \hat{A} = \begin{pmatrix} -1 & \\ & -2 \end{pmatrix} \qquad T = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\hat{b} = T^{-1}b = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

不能控部分极点为 $\lambda = 0$， 渐近稳定

系统可镇定.

4.8

$$Q_k = \begin{pmatrix} 1 & 1 & 1 & \\ 1 & -1 & -1 & \cdots \\ -1 & 0 & 4 & \end{pmatrix}$$

前三列线性无关 $\Rightarrow \mathrm{Rank}\, Q_k = 3$

系统完全能控

$$f_k(s) = \det(sI - (A - bk^T)) \qquad k^T = \begin{pmatrix} 1 \\ 0 \end{pmatrix}(k_1 \ k_2 \ k_3)$$

$$= \det\begin{pmatrix} s-1+k_1 & k_2 & k_3 \\ k_1 & s+1+k_2 & k_3 \\ -k_1 & -k_2 & s+4-k_3 \end{pmatrix}$$

$$= s^3 + (k_1 + k_2 - k_3 + 4)s^2 + (5k_1 + 3k_2 - 1)s$$

$$+ (4k_1 - 4k_2 + k_3 - 4)$$

$$f^*(s) = (s+1)(s+2)(s+3)$$

$$= s^3 + 6s^2 + 11s + 6$$

$$\begin{cases} k_1+k_2-k_3+4=6 \\ 5k_1+3k_2-1=11 \\ 4k_1-4k_2+k_3-4=6 \end{cases} \Rightarrow \begin{cases} k_1=\frac{12}{5} \\ k_2=0 \\ k_3=\frac{2}{5} \end{cases}$$

$$k=\begin{pmatrix} \frac{12}{5} & 0 & \frac{2}{5} \\ 0 & 0 & 0 \end{pmatrix}$$

5.1 $\quad Q_g=\begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 4 & 4 \end{pmatrix} \qquad \mathrm{Rank}\,Q_g=3$

系统完全能观

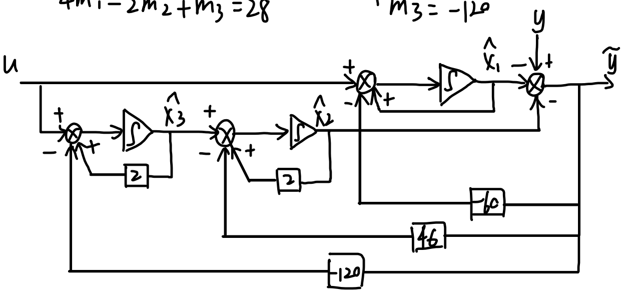
$$f_M(s)=\det(sI-(A+C^T))$$

$$=\det\begin{pmatrix} s-1-m_1 & -m_1 & 0 \\ -m_2 & s-2-m_2 & -1 \\ -m_3 & -m_3 & s-2 \end{pmatrix}$$

$$=s^3-(m_1+m_2+5)s^2+(4m_1+3m_2-m_3+8)s+(m_3-4m_1-2m_2-4)$$

$$f^*(s)=(s+2)(s+3)(s+4)=s^3+9s^2+26s+24$$

$$\Rightarrow \begin{cases} -m_1-m_2=14 \\ 4m_1+3m_2-m_3=18 \\ -4m_1-2m_2+m_3=28 \end{cases} \Rightarrow \begin{cases} m_1=-60 \\ m_2=46 \\ m_3=-120 \end{cases} \Rightarrow M=\begin{pmatrix} -60 \\ 46 \\ -120 \end{pmatrix}$$



5.2 $\quad Q_g = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \qquad \mathrm{Rank}\, Q_g = 2$

系统完全能观

$$f_M(s) = \det(sI - (A+MC^T)) = \det\begin{pmatrix} s-1-m_1 & -3-m_1 \\ -m_2 & s+1-m_2 \end{pmatrix}$$

$$= s^2 - (m_1+m_2)s - (m_1+2m_2+1)$$

$$f^*(s) = (s+2)^2 = s^2 + 4s + 4$$

$$\Rightarrow \begin{cases} -m_1 - m_2 = 4 \\ -m_1 - 2m_2 - 1 = 4 \end{cases} \Rightarrow \begin{cases} m_1 = -3 \\ m_2 = -1 \end{cases} \Rightarrow M = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$$