作业二

刘若涵 2020011126

1. (1) $P(B|(A-B))$

$= P(B|AB^c)$

$= 0$

$P(A|(A-B))$

$= P(A|AB^c)$

$= P(AB^c)$

$P(AB|A)$

$= P(B|A)$

(2) $P(C|A\cup B)$

$$= \frac{P(C(A+B))}{P(A)+P(B)}$$

$$= \frac{P(AC)+P(BC)}{P(A)+P(B)}$$

17. $P(A|B) = \frac{P(AB)}{P(B)} > P(A)$

$\Rightarrow P(AB) > P(A)P(B)$

$P(B|A) = \frac{P(AB)}{P(A)} > P(B)$

AB正相关，A的发生会使B发生的概率增大，则反之亦然

21 $A=\{$<u>选手跳过</u>$1.8m\}$

$P(A)=0.3 \qquad P(A^c)=0.7$

$P=1-[P(A^c)]^3=0.657$

25. $A=\{$朝上的面是红色$\}$

$B=\{$朝下的面是黑色$\}$

$$P(B|A)=\frac{P(AB)}{P(A)}=\frac{\frac{1}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}}=\frac{1}{3}$$

26. $B_k=\{$家里有k个孩子$\}$

$A=\{$家里所有孩子都同一个性别$\}$

$$P(A)=\sum_{k=1}^{\infty}P(AB_k)=\sum_{k=1}^{\infty}P(B_k)P(A|B_k)$$

$$=\sum_{k=1}^{\infty}2\cdot\left(\frac{1}{2}\right)^k p_k$$

$$=\sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^{k-1}p_k$$

补充题

1. (a) $P(A) = (\frac{1}{2})^3 + C_3^1 (\frac{1}{2})^3 = \frac{1}{2}$

$P(B) = 1 - (\frac{1}{2})^3 \cdot 2 = \frac{3}{4}$

$P(AB) = C_3^1 (\frac{1}{2})^3 = \frac{3}{8}$

$P(AB) = P(A)P(B)$

AB 独立

(b) $P(A) = (\frac{1}{2})^4 + C_4^1 (\frac{1}{2})^4 = \frac{5}{16}$

$P(B) = 1 - (\frac{1}{2})^4 2 = \frac{7}{8}$

$P(AB) = C_4^1 (\frac{1}{2})^4 = \frac{1}{4}$

$P(AB) \neq P(A)P(B)$

AB不独立

2. $P(AB|C) = P(A|C)P(B|C)$

$P(AB|C^c) = P(A|C^c)P(B|C^c)$

"$\Leftarrow$" 不妨设 $P(A|C) = P(A|C^c)$

$P(AB) = P(A|C)P(B|C)P(C) + P(A|C^c)P(B|C^c)P(C^c)$

$= P(A|C)[P(B|C)P(C) + P(B|C^c)P(C^c)]$

$= P(A|C)P(B)$

$$= P(A|C)(P(C)+P(C^c))\ P(B)$$

$$= [P(A|C)P(C) + P(A|C^c)P(C^c)]\ P(B)$$

$$= P(A)P(B)$$

AB独立

若 $P(B|C)=P(B|C^c)$ 同理可证 AB独立

"$\Rightarrow$" 若AB独立

则 $P(AB) = P(A)P(B)$

$$= P(B)[P(A|C)P(C) + P(A|C^c)P(C^c)]$$

$$= P(A|C)P(B)P(C) + P(A|C^c)P(B)P(C^c)$$

又∵ $P(AB) = P(A|C)P(B|C)P(C) + P(A|C^c)P(B|C^c)P(C^c)$

∴ $P(A|C)P(C)[P(B|C)-P(B)] + P(A|C^c)P(C^c)[P(B|C^c)-P(B)] = 0$

$$P(A)[P(B|C) - 2P(B) + P(B|C^c)] = 0$$

∴ $P(A|C) = P(A|C^c)$ 或 $P(B|C) = P(B|C^c)$