homework 8

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8.3 (1)

$$Z(n) = \frac{z}{(z-1)^2}$$

$$Z(n^2) = -z\frac{d}{dz}Z(n) = \frac{z(z+1)}{(z-1)^3}$$

$$Z(n^3) = -z\frac{d}{dz}Z(n^2) = \frac{z(z^2+4z+1)}{(z-1)^4}$$

$$Z(n^4) = -z\frac{d}{dz}Z(n^3) = \frac{z(z^3+11z^2+11z+1)}{(z-1)^5}$$

$$Z(t^4) = Z((nT)^4) = T^4 Z(n^4) = \frac{T^4 z(z^3+11z^2+11z+1)}{(z-1)^5}$$

(2)

$$Z(t^2e^{-at}) = Z(T^2n^2(e^{-aT})^n)$$

$$= T^2\frac{e^{-aT}z(z+e^{-aT})}{(z-e^{-aT})^3}$$

(3)

$$X(s) = \frac{a}{s^2(s+1)} = a\left(\frac{1}{s^2}+\frac{1}{s+1}-\frac{1}{s}\right)$$

$$\Rightarrow L^{-1}(X) = a(t-1+e^{-t}) = x(t)$$

$$Z(a(t-1+e^{-t})) = a\left(\frac{Tz}{(z-1)^2}-\frac{z}{z-1}+\frac{z}{z-e^{-T}}\right)$$

(4)

$$X(s) = \frac{ab}{s(s+a)(s+b)} = \left(\frac{1}{s}-\frac{1}{s+a}\right)\frac{b}{s+b}$$

$$= \frac{1}{s} - \frac{b}{b-a}\frac{1}{s+a} + \frac{a}{b-a}\frac{1}{s+b}$$

$$L^{-1}(X) = 1-\frac{b}{b-a}e^{-at}+\frac{a}{b-a}e^{-bt} = x(t)$$

$$Z\left(1-\frac{b}{b-a}e^{-at}+\frac{a}{b-a}e^{-bt}\right)=\frac{z}{z-1}-\frac{b}{b-a}\frac{z}{z-e^{-aT}}+\frac{a}{b-a}\frac{z}{z-e^{-bT}}$$

而若 $a=b$，则 $Z(x(t))=\frac{z}{z-1}-\frac{z}{z-e^{-aT}}$

(5) $$X(s)=\frac{1}{s+a}\cdot\frac{1}{s+b}\cdot\frac{1}{s+c}=\frac{1}{b-a}\left(\frac{1}{s+a}-\frac{1}{s+b}\right)\cdot\frac{1}{s+c}$$

$$=\frac{1}{(b-a)(c-a)}\left(\frac{1}{s+a}-\frac{1}{s+c}\right)-\frac{1}{(b-a)(c-b)}\left(\frac{1}{s+b}-\frac{1}{s+c}\right)$$

$$=\frac{1}{(a-b)(a-c)(s+a)}+\frac{1}{(b-a)(b-c)(s+b)}+\frac{1}{(c-a)(c-b)(s+c)}$$

$$L^{-1}(X)=\frac{e^{-at}}{(a-b)(a-c)}+\frac{e^{-bt}}{(b-a)(b-c)}+\frac{e^{-ct}}{(c-a)(c-b)}$$

$$=x(t)$$

$$Z(x(t))=\frac{z}{(a-b)(a-c)(z-e^{-aT})}+\frac{z}{(b-a)(b-c)(z-e^{-bT})}+\frac{z}{(c-a)(c-b)(z-e^{-cT})}$$

若有两相同极点 $(a=b)$ 则

$$Z(x(t))=\frac{z}{(c-a)^2(z-e^{-cT})}$$

若 $a=b=c$ 则

$$Z(x(t))=Z\left(\frac{1}{2}t^2e^{-at}\right)=\frac{a}{2}\left(\frac{Tz}{(z-1)^2}-\frac{z}{z-1}+\frac{z}{z-e^{-T}}\right)$$

8.10 $$G(s)=\frac{w_0}{s^2+w_0^2}$$

$$\Rightarrow x(t)=L^{-1}(G(s))=\sin w_0 t$$

$$\Rightarrow Z(x(t))=Z(\sin n(w_0T))=\frac{z\sin w_0T}{z^2-2z\cos w_0T+1}=G(z)$$

8.11 (a) $G(R - F_2 C - F_1 C) = C$

$$\Rightarrow G_{CL} = \frac{C(z)}{R(z)} = \frac{G(z)}{1 + GF_1(z) + G(z)F_2(z)}$$

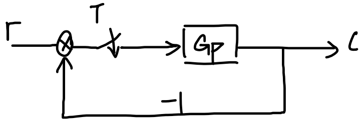
$$C(z) = \frac{G(z)R(z)}{1 + GF_1(z) + G(z)F_2(z)}$$

(b) $G_2(G_1(R - FC) + G_3 R) = C$

$$\Rightarrow G_{CL} = \frac{C(z)}{R(z)} = G_2G_3(z) + \frac{G_2G_1(z)[1 - FG_2G_3(z)]}{1 + FG_2G_1(z)}$$

$$C(z) = G_2G_3R(z) + \frac{G_2G_1(z)[R(z) - FG_2G_3R(z)]}{1 + FG_2G_1(z)}$$

8.13



$$G_p(s) = \frac{k}{s(s+1)^2} \quad (k>0)$$

$$= k\left(\frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2}\right)$$

$$\Rightarrow g(t) = \mathcal{L}^{-1}(G_p) = k(1 - e^{-t} - te^{-t})$$

$$G_p(z) = Z(g) = k\left(\frac{z}{z-1} - \frac{z}{z-e^{-T}} - \frac{Te^{-T}z}{(z-e^{-T})^2}\right)$$

$$= \frac{kz((1-e^{-T}-Te^{-T})z + e^{-T}(e^{-T}+T-1))}{(z-1)(z-e^{-T})^2}$$

CL特征方程：

$$z^3 + (k(1-e^{-T}-Te^{-T}) - 1 - 2e^{-T})z^2 + e^{-T}(2 + e^{-T} + k(e^{-T}+T-1))z - e^{-2T} = 0$$

(1) $T_1=0.2$ $\quad G_p(z)=\dfrac{kz(0.018z+0.015)}{(z-1)(z-0.82)^2}$

根轨迹起点：1，0.82（2重）

终点：0，-0.83，∞远

分离汇合点 $\dfrac{(2z+0.83)(z-1)(z-0.82)^2}{z(z+0.83)}=(z-0.82)(3z-2.82)$

$\Rightarrow$ $z=-3.2$ 或 0.94

设交点 $A=e^{j\theta}=\sigma+j\omega$

$$k\,\frac{e^{j\theta}(0.018e^{j\theta}+0.015)}{(e^{j\theta}-1)(e^{j\theta}-0.82)^2}=-1$$

$$\arctan\frac{\omega}{\sigma}+\arctan\frac{6\omega}{\sigma+5}-\arctan\frac{\omega}{\sigma-1}-2\arctan\frac{6\omega}{\sigma-5}=-180^\circ$$

$\Rightarrow$ $\omega=\pm j0.2$ $\quad \sigma=0.98$

$$k=\left|\frac{0.2(0.16+j0.2)^2}{0.33+j0.0036}\right|=0.398$$

$\therefore$ $0<k<0.398$

(2) $T=0.8$ 时 $\quad G_p(z)=\dfrac{kz(0.19z+0.11)}{(z-1)(z-0.45)^2}$

根轨迹起点：1，0.45（2重）

终点：0，-0.58，∞远

分离汇合点 $\dfrac{(0.38z+0.11)(z-1)(z-0.45)^2}{z(0.19z+0.11)(z-0.45)(3z-2.45)} = 1$

$\Rightarrow$ $z = -2.2$ 或 0.766

设交点 $A = e^{j\theta} = \sigma + jw$

$$k\,\frac{(z+jw)(0.19z+0.11+j0.19w)}{(z-1+jw)(z-0.45+jw)^2} = -1$$

$$\arctan\frac{w}{\sigma} + \arctan\frac{w}{z+0.58} - \arctan\frac{w}{\sigma-1} - 2\arctan\frac{w}{\sigma-2.45} = -180^\circ$$

$\Rightarrow$ $w = \pm 0.748$ $\sigma = 0.684$

$$k = \left|\frac{(j0.748-0.316)(0.234+j0.748)^2}{0.24+j0.14}\right| = 1.68$$

$\therefore$ $0 < k < 1.68$