

# A Reinforcement Learning Approach to Constrained Resource Allocation Problems

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**Abstract**—Resource allocation has been extensively studied in various fields such as wireless communication, intelligent traffic routing, industrial management, parallel architectures and distributed systems. Although there have been many efficient and powerful algorithms proposed in each domains, very few of them is able to address more generalized resource allocation problems on a higher level.

This paper proposes a generalized framework and makes three main contributions to solving constrained resource allocation problem using a modified reinforcement learning algorithm. First, we designed a general architecture, Constrained Resource Allocation Framework (CRAF), for the ease of generalized problem mapping and learning. Second, we defined four properties and three measurements to both quantitatively and qualitatively analyze how well an algorithm performs in CRAF. Finally, we developed three benchmark experiments to demonstrate how a reinforcement algorithm can successfully solve very different resource allocating problems with CRAF.

## I. INTRODUCTION

Resource allocation is an old and widely-solved problem in many fields, including electrical engineering, computer science, economics, management science and many more. It typically involves a fixed number of resource units to be distributed to a set of tasks over a period of time, such as landing aircraft scheduling, wireless communication routing, social security welfare and shared resources in parallel computer architectures. The problems of resource allocating are ubiquitous, but they all essentially boil down to one simple notion—based on a set of criteria, how many resource units should be allocated to which task first, and for how long.

To name a few remarkable research examples to solve this type of problems, Dresner and Stone proposed a reservation system for autonomous intersection management [1] to allocate right of road for crossing traffic, Perkins and Royer presented a novel routing algorithm for ad-hoc on-demand mobile nodes management [2], and Foster et al. came up with a reservation and allocation architecture for heterogeneous

resource management in computer network. While they all addressed an elegant solution to a specific domain, most of the techniques cannot be easily transferred to similar decision problems in other domains. In this paper, we focus on addressing exactly this issue and forming a general resource allocation framework that is suitable to be solved by a reinforcement learning method.

The first contribution of this paper is to form the Constrained Resource Allocation Framework (CRAF), in which we designed a generalized architecture to capture most of the resource allocation problems. In addition to the conventional first-come, first-served basis, we introduced the notion of constraints and queue propagation to reflect a more realistic setting and to relax more complicated systems into a single-frontier problem.

Secondly, we defined a collection of analysis criteria and evaluation methods to both qualitatively and quantitatively study how a reinforcement learning algorithm perform on our framework. Lastly, we proposed three benchmark problems to empirically demonstrate how CRAF applies to different resource allocation problems consistently and effectively.

## II. BACKGROUND

Unlike the Markov Decision Processes (MDPs) that most of the reinforcement learning algorithms are designed to solve, the notion of state representation, transition functions and reward functions in resource allocation problems such as aircraft landing scheduling can be very complex and challenging to define. Furthermore, the state space representing the entire global snapshot could be too large to be useful for effective learning.

Instead of trying to model a resource allocation problem as MDP, we found it more intuitive to formalize it as a multi-armed bandit problem, which has been extensively researched in the field of reinforcement learning [6]. Assuming that we

can reasonably classify each incoming task candidate to one of the  $K$  prototypes, from which we use the approximated reward functions to determine which candidate receives the resource unit in each episode. Before we formalize our definitions and notations of the framework, let us go through some of the existing  $K$ -armed algorithms and see how they can be useful to the resource allocation problem.

#### A. The $K$ -armed Bandit Problem

The problem is formalized by a fixed number of slot gambling machines, each defined by a random variable  $X_{i,n}$ , for  $1 \leq i \leq K$  and  $n \geq 1$ . A sequential  $N$  plays of machine  $i$  give rewards of  $X_{i,1}, X_{i,2}, \dots, X_{i,N}$ , which are all independent of each other and identically distributed. Based on the sequence of playing history and obtained rewards, one can form an allocation algorithm to pick the next machine. To evaluate the performance of such algorithms, one criterion, the *regret*, is defined as

$$\text{regret} = \mu^* n - \mu_j \sum_{j=1}^K E[T_j(n)]$$

where  $\mu^* \equiv \max_{1 \leq i \leq K} \mu_i$ ,  $T_j(n)$  is the number of times machine  $j$  has been played during the  $n$  plays. Therefore, *regret* is essentially the expected loss function (opportunity cost) of the played allocation strategy.

#### B. Prototype Classification

1) *Distance Functions*: The idea of a particle is essentially a sampled instance of the topic distribution over a time sequence, represented by a vector  $\mathbf{w}_i \in \mathbb{R}^{|V|}$ ,  $i \in \{1, \dots, N\}$ , where  $|V|$  is the total vocabulary size of all the topic words appeared. Since one of our intermediate objectives is to formalize clusters between these particles, we need to first define how we will measure the similarity or distance between any pair of particles  $\mathbf{w}_i, \mathbf{w}_j$ .

##### Minkowski:

$$d = \sqrt[p]{\sum_{k=1}^{|V|} |w_{ik} - w_{jk}|^p}$$

Note that when  $p = 1$ , the Minkowski reduces to the city block distance, while  $p = 2$  gives the Euclidean distance and  $p = \infty$  yields the Chebychev distance.

##### Cosine:

$$d = 1 - \frac{\mathbf{w}_i \mathbf{w}_j^T}{|\mathbf{w}_i|_2 |\mathbf{w}_j|_2}$$

##### Correlation:

$$d = 1 - \frac{(\mathbf{w}_i - \bar{\mathbf{w}}_i)(\mathbf{w}_j - \bar{\mathbf{w}}_j)^T}{|(\mathbf{w}_i - \bar{\mathbf{w}}_i)|_2 |(\mathbf{w}_j - \bar{\mathbf{w}}_j)|_2}$$

where

$$\bar{\mathbf{w}}_i = \frac{1}{|V|} \sum_{k=1}^{|V|} w_{ik}, \quad \bar{\mathbf{w}}_j = \frac{1}{|V|} \sum_{k=1}^{|V|} w_{jk}$$

##### Jaccard:

$$d = \frac{\#[(w_{ik} \neq w_{jk}) \cap ((w_{ik} \neq 0) \cup (w_{jk} \neq 0))]}{\#[(w_{ik} \neq 0) \cup (w_{jk} \neq 0)]}$$

2) *Clustering Algorithms*: LDA is essentially pulling out the ‘principal components’ from the text documents, reducing a large archive of data into just  $K$  representative topics  $\beta_{1:K}$ . Therefore, our assumption is that by grouping similar particles induced from past topics, we can observe the clustering patterns and make reasonable predictions on how the future topics will be like.

Now we have a collection of well-defined distance functions, we can start looking at clustering methods to group our particles together accordingly, and here are a set of common clustering algorithms we will consider:

**K-Means**: As the name itself suggests, the K-Means clustering algorithm consists of  $K$  cluster centroids and moves these means iteratively towards the center of its closest neighbors, until they no longer change. Although K-Means has been proved to be guaranteed for convergence [?], its clustering performance is often correlated to how the seeds are initialized at the beginning, and the optimal choice of  $K$  is often not apparent (in our case, it is the same as the number of topics.)

1. Initialize the means by picking  $K$  samples at random
2. Iterate
  - 2.a. Assign each instance to its nearest mean
  - 2.b. Move the current means to the center of all the associated points

**Hierarchical Agglomerative Clustering (HAC)**: This algo-

rithm starts with treating every instances as individual cluster, and iteratively joins pairs of similar clusters repeatedly until there is only one. If we take the merging history and form a hierarchical binary tree, it will look like the dendrogram in Fig. 1.

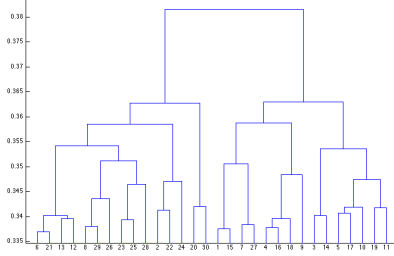


Fig. 1: A sample dendrogram by HAC

Although we have defined the distance functions in the previous section, we have yet formalized the similarity functions between clusters. In our method, we will focus on the following four similarity functions:

- **Single-linkage:** Also known as the nearest neighbor, computes the distance between the two closest elements from two clusters
- **Complete-linkage:** The conjugate of **single-linkage**, also known as the farthest neighbor, computes the distance between the two farthest elements (maximum distance) from two clusters
- **UPGMA:** Unweighted Pair Group Method with Averaging calculates the distance between two clusters,  $C_i$  &  $C_j$ , by averaging all distances between any pair of objects from the two clusters.

$$dist(C_i, C_j) = \frac{1}{|C_i||C_j|} \sum_{w_i \in C_i} \sum_{w_j \in C_j} dist(w_i, w_j)$$

Now, let's call this newly-formed cluster  $C_{ij}$  and compare its distance with another cluster  $C_k$ :

$$dist(C_{ij}, C_k) = \frac{|C_i|dist(c_i, c_k) + |C_j|dist(c_j, c_k)}{|C_i| + |C_j|}$$

where  $c_i, c_j, c_k$  are the centroids for clusters  $C_i, C_j, C_k$

- **WPGMA** Weighted Pair Group Method with Averaging is similar to UPGMA except that the cluster distance is now calculated as:

$$dist(C_{ij}, C_k) = \frac{dist(c_i, c_k) + dist(c_j, c_k)}{2}$$

The behavior of HAC is often dominated by the chosen

similarity function. While each variant has a different set of clustering patterns it is good at capturing, all of them have certain vulnerabilities.

### III. THE CONSTRAINED RESOURCE ALLOCATION FRAMEWORK

Formally, the Constrained Resource Allocation Framework (CRAF) consists of a sequential task instances, and a centralized learning agent who distributes resource units to the task instances and receives rewards according to the decision they make. The task instances can come in one or more channels  $\mathbf{x}_{c,i}$ ,  $1 \leq c \leq C$ ,  $1 \leq i \leq capacity(c)$ , where  $i$  denotes the  $i$ -th instance and  $c$  denotes the  $c$ -th channel. To transform a resource allocation problem into a learnable framework, the formalization process under CRAF can be broke down into three phases:

#### 1) Housekeeping Phase

- **Constraint:** The instance  $x_{c,i}$  needs a total of  $num(x_{c,i})$  resource units before the deadline  $exp(x_{c,i})$
- **Objective:** A high-level criteria that the learning agent should achieve, which adds bias against the reward functions

#### 2) Tuning Phase

- **Priority Score:**  $ps(x_{c,i})$ , a function of both the **Constraint** and **Objective**
- **Queue Propagation:**  $qp(x_{c,i}) = \gamma qp(x_{c,i+1}) + \tau ct(x_{c,i})$  A linear combination of the current constraint and the same function from the next instance in queue
- **Failure Capture:** If the current instance's constraint is impossible to be fulfilled or fails to respond, a *reject* message will be sent and no further request from the same instance will be considered (so it will have to re-enter the queue)

#### 3) Prototyping Phase

- **Prototype Classification:** Given a manually-chosen  $K$ , the current instance  $x_{c,i}$  is classified into one of the  $K$  clustering prototypes with respect to the two parameters obtained in the previous phase  $ps(x_{c,i}), qp(x_{c,i})$
- **Reward Function:** The learning agent receives a reward of  $r_p + r_o$  if it fulfills the constraint of  $x_{c,i}$ , or  $r_f - r_o$  otherwise

#### IV. EXPERIMENTAL RESULTS

1. 2-way intersection (simplest form) 2. 4-way intersection (special corner cases) 3. n-children candy distribution (multi-channel)

#### V. DISCUSSION AND RELATED WORK

#### VI. CONCLUSION

#### VII. FUTURE WORK

#### VIII. ACKNOWLEDGEMENTS

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