

# Final Exam

August 9, 2017

Name\_\_\_\_\_

**Instructions.** The exam is due on August 15th. You are expected to finish the exam by yourself without discussing with your classmates. Algebra mistakes may be forgiven or at least not punished severely but only if you show clearly that you understand how to solve the problem. Your answers should be complete and clear but excessive length should be avoided. For some questions, do not overthink. Use the ideas and intuitions learned in class to show how to analyze those problems. The total points of the first five questions is 100. You can get extra points from the last question.

## Question 1. Short Questions

- a. What is the effect on the terms of a standard contract between a principal and an agent of the agent shifting from risk neutral to risk averse? Is there any change from the standard risk neutral contract if the principal were to become risk averse while the agent stays risk neutral? (*10 pts*)
- b. There are two common ways for lawyers to be compensated by their clients. One way is that they are paid an hourly fee so their payment is increasing in the amount of time spent on a case. The second form involves the lawyer getting paid some percentage of the total award, typically around 45%, if they win the suit and nothing if they lose. Those in favor of “Tort Reform”, i.e. people who don’t like being sued, are typically strongly opposed to the scheme in which the lawyers get paid on the percentage basis while those for “Victim’s Rights”, i.e. those who like suing people, believe this incentive scheme is essential to preserving the rights of victims. Explain the incentive properties of both payment schemes and in particular discuss which might better solve the principal-agent problem between the client and their lawyer. Does this analysis help explain the views these two groups have over this issue? (*10 pts*)

## Question 2. Competition between firms

There is a State with 2 firms in it currently producing electricity. Both firms have identical cost functions of  $c(q_i) = .03q_i + 15000$  where  $q_i$  is the number of kilowatt hours (KWH) produced by firm  $i$ . The monthly demand faced by the industry is  $Q = 10000 - 500 \cdot p$ .

- a. Derive the equilibrium of the industry assuming the firms engage in quantity or Cournot competition. (6 pts)
- b. Now assume that instead of the firms engaging in quantity competition, they engage in Bertrand or price competition. Derive the resulting equilibrium in the industry. (8 pts)
- c. You have just been elected as governor and supreme dictator of this state and have it in your power to determine the market structure that these two firms will compete in and can therefore decide whether they engage in Bertrand or Cournot competition. Since you are a benevolent dictator your goal is to choose the market design that will best serve the interests of the citizens by providing the most electricity at the lowest price. Given the analysis you have done above, explain which of these market structures you think would best accomplish this goal in the long run. (6 pts)

### Question 3. Signaling in the labor market

Suppose that there is a worker who may have high productivity or low productivity. A firm tries to hire this worker but does not know this worker's type. The firm only knows that with probability 0.6 he has high productivity, and in this case the firm wants to offer him a wage of 20; with probability 0.4 he has low productivity and the firm only wants to offer a wage of 10. The worker can choose to go to graduate school to signal his productivity. If he has high productivity, the cost of going to graduate school is 6. But getting more education is more costly if he has low productivity: the cost of going to graduate school for low productivity worker is 12. The worker's utility is  $u = w - c$  where  $w$  is the wage and  $c$  is the cost of education.  $c = 0$  if the worker does not get any additional education.

Consider the signaling game with the standard timeline as discussed in class.

- a. Find a pooling equilibrium. (10 pts)
- b. Find a separating equilibrium. (You need to verify that the equilibrium is valid, i.e., each type of worker has no incentive to deviate.) (10 pts)

### Question 4. Repeated game

Consider the following prisoners' dilemma game.

	D	C
D	1,1	3,0
C	0,3	2,2

- a. Find the Nash Equilibrium of this game. (4 pts)
- b. Suppose that this game is repeated infinitely, describe the grim trigger strategy that sustains cooperation. (6 pts)
- c. How patient do the players have to be in order for the above grim trigger strategy to be a subgame perfect Nash equilibrium? (Find the conditions on the discount factors.) (10 pts)

### Question 5. Auctions

When participating in auctions bidders often have concerns beyond the straight expected value of participating in the auction. In particular they are often concerned that at the end of the auction they may experience one of two forms of regret. The first is regret that they bid more than they had to. They experience this if they win an auction and then realize that their winning bid is substantially higher than the highest alternative bid. The second form of regret is the possibility of losing when the bidder could have profitably won. If a bidder loses an auction but has a value higher than the winning bid in the auction then he would experience this form of regret as he could have potentially won the auction profitably. (Note that a standard expected value maximizer does not have these concerns as they only care about maximizing expected value and not where the bids of others actually end up.) For the questions below assume the symmetric independent private values environment and that bidder values are drawn from a uniform distribution on the range  $[0, 1]$ .

a. Consider the ascending auction and the first price auction. Assuming bidders bid according to the standard equilibrium bid functions we derived in class, determine whether these regret concerns are an issue for both formats. If you find a form of regret might be experienced in a format, explain how you think a bidder's behavior might change were they to be seriously concerned about that situation (i.e. how might a bidder's behavior change if they were unusually concerned about having bid more than their competitors or how would their behavior change if they were abnormally concerned about losing a profitable auction?). Explain your answer clearly. (6 pts)

b. There is an odd auction format which one might consider to have been developed to decrease the possibility of regret concerns (whether it does or not is the focus of this question) called the two-bid auction. Let there be 2 bidders bidding in an auction. These bidders will receive values drawn as described above,  $v_i$ , and place two bids in this auction which will be functions of that value; one high bid,  $h(v_i)$ , and one low bid,  $l(v_i)$ , with the obvious requirement that  $h(v_i) > l(v_i)$ . The winner is determined by the bidder who submits the highest bid. A bidder pays the lowest of their bids that is higher than all of the bids of their competitor. So if bidder  $i$  is the high bidder then he will pay  $l(v_i)$  if that bid is higher than both of  $j$ 's bids. If at least one of  $j$ 's bids is higher than  $l(v_i)$  but  $h(v_i)$  is bigger than both of  $j$ 's bids then bidder  $i$  still wins and pays his high bid,  $h(v_i)$ . In the absence of the regret concerns just discussed, should this auction be expected to raise more or less money than a standard one bid first price auction? Support your answer rigorously. (6 pts)

c. How might your answer to b change if bidders do possess the regret concerns? (8 pts)

### Extra Question. Pricing Strategies

a. Suppose there are three kinds of iPhone consumers. Assume that there are two types of Apple devotees, the early adopters who will pay up to \$400 but only if the phone is brand new to the market, and the others who will pay up to \$300 regardless of the time the phone has been on the market. The third group are normal consumers who are only willing to pay \$250 for an iPhone regardless of time on the market. Assume that Apple can sell to these people over a 2 year period. Find their optimal pricing strategy for selling phones assuming

a marginal cost of \$100 ignoring any time discounting or impatience on the part of Apple. Assume for convenience that there is one of each consumer type in the market. Consumers only buy once. (7 points)

b. Suppose you are the pricing manager for a new statistical software pack called Gerbils (there is one called RATS so it's not that crazy). There are two types of users for your product, students and non-students and you can produce both a full featured version (healthy) and one that has reduced features (crippled) such as only working on smaller datasets. The two groups have the willingness to pay for each software type as indicated in the table below. Find the optimal pricing strategy for Gerbils including whether the firm wants to release both types or only one type of Gerbils assuming 0 marginal cost and equal sized markets of both types. (8 points)

Software Version	Students	Non-Students
Healthy	\$175	\$500
Crippled	\$150	\$200