# Midterm Exam

July 5, 2017

Name______________________________

**Instructions.** Please read all questions carefully, use your time wisely and show your work and final answers clearly. I don't grade on neatness but if I can't read it, I can't give credit for it. Also, algebra mistakes may be forgiven or at least not punished severely but only if you show clearly that you understand how to solve the problem. Your answers should be complete and clear but excessive length should be avoided. You are to use no notes, books, other people or any resources other than your own mind to provide answers. Calculators are allowable.

1. a. A hospital executive looks at the performance review for the divisions of his hospital and sees that in the maternity division last year there were 540 babies delivered at a cost of \$3,132,000 with a revenue of \$2,754,000. What decision should the administrator make in regard to encouraging the maternity division to increase or decrease the number of babies delivered? If a decision can not be reached from this information alone, explain what information is needed and what might lead to a choice to increase or a choice to decrease the number of deliveries.(7 points)

b. Your 7-year old self reads that after getting their allowance, most 7-year-olds exhibit behavior consistent with Cobb-Douglas preferences over two goods, candy and ice cream. You get \$20 in allowance every week. You remember buying 10 candies and 5 ice creams last week, and you paid \$0.90 per candy. However, you cannot remember the price of an ice cream. Yet with the information you recall, you realize that you can figure out the utility function that generated your behavior and recover the price of ice cream. What is the utility function and price of ice cream? (7 points)

*a.* $AC = 3132000/540 = 5800$ *while average revenue is* $2754000/540 = 5100$. *On average the hospital is losing \$700 per delivery. That is not helpful information though for determining whether production should be raised or lowered. For that we need to know marginal revenue and marginal cost. If the high average cost is due to very high fixed costs but low marginal costs, then it might well be advisable to encourage more deliveries. If the reason the costs are so high is due to high marginal costs then the opposite recommendation should be made..*

*b. A Cobb-Douglas utility function is of the form* $u = c^{\alpha} i^{\beta}$ *if* $c$ *stands for units of candy and* $i$ *for units of ice cream. As we discussed in class those coefficients can be estimated as being equal to the percentage of income one spends on that good. We have enough information to determine the percent of income spent on candy: total expenditure on candy* $.9 * 10 = 9.0$ *and so* $\alpha = 9/20 = 0.45$. *This means it must be the case that* $\beta = .55$. *So your 7 year old self had a utility function of* $u = c^{.45} i^{.55}$. *We can also find the price of ice cream:* $11 = p_i * 5$ *==¿* $p_i = 11/5 = 2.2$.

2. A TV station is considering distributing a promotional video. There are two suppliers who are willing to produce it. Supplier A will charge the station a setup cost of \$1,200 and then \$2 for each DVD ordered. Supplier B will charge \$4 per DVD with no setup charge. The station believes that the demand function will be $Q = 1600 - 200P$.

a. Suppose the station wants to maximize viewership and so wants to give the videos away for free. How many videos should the station order to satisfy demand and from which supplier should they be ordered? (8 points)

b. Suppose the station instead wants to maximize profits by selling the videos. What price should it set, how many will they sell and which supplier should they order from? (8 points)

*a. If price is set to 0 so that the videos are distributed for free, there should be 1600 ordered as that will be the number demanded at a price of 0. Costs from the suppliers are* $c_A = 1200 + 2 * 1600 = 4400$ *while* $c_B = 4 * 1600 = 6400$. *Clearly supplier A will be the cheaper option.*

*b. This requires solving two different maximization problems using the different cost functions. For firm A:*

$$\max_p p(1600 - 200p) - (1200 + 2 * (1600 - 200p))$$

$$\begin{aligned}
\frac{\partial \pi_A}{\partial p} &= 2000 - 400p = 0 \\
p_A^* &= 5 \\
q_A^* &= 1600 - 200 * 5 = 600 \\
\pi_A &= 5 * 600 - (1200 + 2 * 600) = 600
\end{aligned}$$

*and for B*

$$\max_p p(1600 - 200p) - (4 * (1600 - 200p))$$

$$\begin{aligned}
\frac{\partial \pi_B}{\partial p} &= 2400 - 400p = 0 \\
p_B^* &= 6 \\
q_B^* &= 1600 - 200 * 6 = 400 \\
\pi_B &= 6 * 400 - 4 * 400 = 800
\end{aligned}$$

*Since the profit from selling the optimal amount given that you purchase from B is higher, then clearly you should order from B price at \$6, sell 400 units and make \$800.*

3.

a. Definition of "indifference curve"; Properties of "indifference curve"; (7 points)

b. Explain the difference between "long run" and "short run" costs. Explain the types of decisions a firm might make that distinguishing between these two contexts is important. (4 points)

c. Explain the relation between Average Product Curve and Marginal Product Curve. (4 points)

*a. Each indifference curve indicates all of the combinations of the two goods that yield the same utility. The consumer should be indifferent between two points along a specific line. It indicates how willing a consumer is to trade off between two goods. Indifference curve has three properties: 1, bowed in towards the origin. 2, non-overlapping. 3, Downward sloping.*

*b. The short run is defined as a time period over which at least one production input is held fixed. The long run refers to a time frame in which all inputs are variable. This leads to the implication that fixed costs exist in the short run but not the long run when all costs are variable. For the most part, decision making between these two domains is not structurally different; a firm always optimizes production by producing where MR=MC. There are however two key differences, the first is what is the basis for that marginal cost curve. In the short run the relevant MC refers to the one given a fixed quantity of one of the inputs while the cost curve in the long run allows for a more efficient use of inputs as both are allowed to vary. The other key decision that a short versus long run perspective is useful in is the shutdown decision of a firm. In the short run, a firm producing optimally yet losing money may still remain in business if it is covering all variable costs and making a contribution to fixed costs. In the long run, a firm producing optimally yet losing money should simply shutdown.*

*c. When the marginal product curve is above the average product curve, the average product is rising. When the marginal curve is below the average product curve, the average product is decreasing. These two curves intersect at the point where the average curve is at its highest point.*

4. In a competitive market the supply and demand curves are given by

$$\begin{aligned} Q_D &= 70 - P \\ Q_S &= .5P - 20 \end{aligned}$$

a. Find the equilibrium price and quantity. (6 points)

b. Suppose the government subsidizes the producers by paying them \$15 per unit produced. Determine what happens to the equilibrium price and quantity. (7 points)

c. Evaluate the effects on social welfare from this subsidy. Is it good for producers? Consumers? Society as a whole? Think through your answer carefully and explain why it comes out as it does. (*For full credit, provide quantitative answers. Partial credit will be given for accurate graphical arguments. Tip: area of a triangle is .5*base*height.*)(15 points) a.

$$\begin{aligned} 70 - P &= .5P - 20 \\ P^* &= 60 \\ Q* &= 10 \end{aligned}$$

b.

$$\begin{aligned} 70 - P &= .5(P + 15) - 20 \\ P^* &= 55 \\ Q* &= 15 \end{aligned}$$

c. *To answer this we need to computer producer and consumer surplus. This amounts to just finding the area of the relevant triangles. The base is equal to the quantity and the height of the triangle is the difference between the equilibrium price and the y intercept. For the original eq:*

$$\begin{aligned} cs &= .5 * 10 * (70 - 60) = 50 \\ ps &= .5 * 10 * (60 - 40) = 100 \end{aligned}$$

*Total social surplus is the sum of both and so it is 150. For the adjusted equilibrium*

$$\begin{aligned} cs &= .5 * 15 * (70 - 55) = 112.5 \\ ps &= .5 * 15 * (55 - 25) = 225 \end{aligned}$$

*Total social surplus here is* $112.5 + 225 = 337.5$ *which is more than twice what it was before. This suggests that total social surplus is much better. Of course there is the amount of the subsidy* $15 * 15 = 225$ *which has to come from somewhere and it factors against total social surplus. If we subtract that out we get* $337.5 - 225 = 112.5$ *which is total social surplus after the amount of the subsidy is taken out. This represents a lower total social surplus. Total surplus is lower because the subsidy is encouraging the producers to supply too much of this particular product and in particular to supply at a point at which the marginal cost of the last unit is more than consumers want to pay for it. That is inefficient. Most of this could have been estimated graphically by pointing out which triangles in the figures represent what. The real key to answering the question stems from understanding that the $225 in subsidy came from somewhere and has to be factored in and similarly that there is deadweight loss despite the fact that it might not look like it on the graph.*

5. Suppose you are selling a product and know that you face two different types of consumers with inverse demand functions for representative consumers of each type given by $p_1 = 8 - 2q_1$ and $p_2 = 4 - .5q_2$. Assume that marginal cost is 2.

a. Find the profit maximizing price and quantity per consumer as well as profit per consumer were you to sell to each group individual items at different prices (i.e. different prices to each group, not different per item). *(6 points)*

$\pi(q_1) = q_1(8 - 2q_1) - 2q_1$ $\qquad$ $\pi(q_2) = q_2(4 - .5q_2) - 2q_2$

$\partial\pi(q_1)/\partial q_1 = 8 - 4q - 2 = 0$ $\qquad$ $\pi(q_2) = 4 - q_2 - 2 = 0$

$q_1^* = 1.5,\ p_1^* = 5.0,\ \pi(q_1^*) = 4.5$

$q_2^* = 2,\ p_2^* = 3,\ \pi(q_2^*) = 2$

b. What if instead of selling single units, you were to only sell units in bundles of 2, 3 or 4 units? Assume you can sell in one of those bundle sizes and then you sell the same bundle size to both groups at the same price. Find the optimal bundle size and profit level. (*Hint: This one is tricky and we didn't do one exactly like it before. If you can't solve the mathematics fully, provide a clear explanation for how you would find the answer.) (7 points)*

*Bundle price is equal to the consumer surplus of consumer for buying that bundle were the price 0. There are several ways of finding that. An approximate way involves just taking the total willingness to pay for each unit. So for consumer 1 his WTP for each unit is given by the max price. So* $WTP_1(1) = 6$, $WTP_1(2) = 4$, $WTP_1(3) = 2$, $WTP_1(4) = 0$. *So the bundle price for one unit is 6, for 2 units 10 and for 3 or 4, 13. For consumer 2 you can do the same thing and get* $WTP_2(1) = 3.5$, $WTP_1(2) = 3$, $WTP_1(3) = 2.5$, $WTP_1(4) = 2$. *This gives bundle prices of 3.5, 6.5, 9 and 11. You could have tried answering this way for partial credit (no one did).*

*The better way would involve integrating the demand function to get the full area underneath it, though I didn't expect anyone to do that fully but I was hoping that some might see that this is the right approach. So you would get general formulae for the surplus for any bundle size, b, of* $\int_0^b (8 - 2q_1)dq_1 = b(8 - b)$ *for consumer type 1 and then* $\int_0^b (4 - .5q_2)dq_2 = b(4 - 0.25b)$. *One can then calculate the profits of selling a bundle of each number of units at either the high price (to get only the group 1 people) or the lower price (and get both). See the table below*

| | $CS_1$ | $CS_2$ | $\pi(p = high)$ | $\pi(p = low)$ |
|---|---|---|---|---|
| $b = 2$ | 12 | 7 | 8 | -1 |
| $b = 3$ | 15 | 9.75 | 9 | -2.25 |
| $b = 4$ | 16 | 12 | 8 | -4 |

*The highest profit comes from selling a bundle of size 3 to only the group 1 at a price of 15.*

c. The table below shows the value two different types of consumers have for services at a hotel as well as the marginal cost of each service and the number of consumers of each type. Find the price the hotel would sell each service for if the services were separately priced. Calculate total profit. *(7 points)*

| | Room | Breakfast | Gym | Number |
|---|---|---|---|---|
| Type 1 | \$100 | \$5 | \$10 | 200 |
| Type 2 | \$60 | \$10 | \$10 | 800 |
| Marginal Cost | \$40 | \$2 | \$0 | |

*The marginal revenue of going from a price of 100 to 60 is* $60 * 1000 - 100 * 200 = 40\,000$ *or* $60 * 800 - 40 * 200 = 40\,000$. *The marginal cost is* $800 * 40 = 32\,000$. *Consequently the rooms should be priced at \$60.*

*For breakfast, same calculation yields* $5*200-800*5=-3000$ *for marginal revenue and then* $200*2=400$ *for marginal cost. So price at $10.*

*The gym should obviously be priced at $10.*

$\pi=(60-40)*1000+(10-2)*800+10*1000=36\,400$

d. Determine what the optimal bundle price would be for combining all three services and determine if the hotel should bundle or sell separately. *(7points)*

*.There are two conceivable bundle prices. The hotel can price at $115 and get the type 1 consumers,* $\pi=(115-42)*200=14\,600$ *or at $80 and get both,* $\pi=(80-42)*1000=38\,000$. *The lower bundle price yields more profit than the higher and also more profit than the individual sales.*