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**Question 1 – Short Questions**

1. Often, agents do not like risk. Thus, their utility functions may be (Utility)^0.5 or a different flavor of the typical wage minus cost of effort which is (w – c(e)) in the standard contract. Instead, you’d have something like (w^0.5 – c(e)) indicating the agent is risk averse. Depending on whether the principal can view effort, the agent would be offered wages based on their effort or outcome in the case of unobservable effort. Typically, with a risk neutral agent and observable effort, optimal contracts generate the same utility and effort choice for principal and agent. The risk averse, unobservable effort situation is not solvable by the typical franchise fee contract and the principal will typically pay a higher wage to induce high effort from the agent.

If the principal becomes risk averse, the principal will definitely not bear all of the risk. Thus, they will not offer a flat wage to the agent. If all risk were loaded onto the principal, the incentive compatibility constraint would not be satisfied. Because of the variable wage in the unobservable situation, we would assume the standard contract would not change, because the risk is loaded onto the risk neutral agent inherently. We would expect risk premiums for the agent to actually decrease versus the risk averse agent situation. All the risk would be optimally loaded onto the risk neutral agent. Because the agent is risk neutral, the principal will always want to induce the same effort level, regardless of observability.

1. Lawyers are questionable in general, first and foremost. In this case, the client is the principal, we know that the lawyer is the agent. Typically, we would assume that most clients in these situations would not have a lot of money to spend on a lawyer. It is in the best interest of clients to be able to monitor their lawyer’s effort, however, that is nearly impossible. Given the variable nature of outcomes, we would assume that most principals would be risk averse, unless they are a large firm or have a lawyer on retainer. We would also assume that the lawyers would be risk averse, unless they have a large portfolio of clients, which most do not. The incentive properties of both schemes can be found below:

Hourly Fee Strategy:

* Offers a flat fee per variable hour worked, reduces risk on the agent (lawyer)
* Compensates the lawyer for their disutility of effort u(e)
* Places a definitive cost on the principal
* Raises barrier to entry for principal
* Lawyer will work at a lower effort, no incentive to maximize winnings here
* Incentivizes lawyer to drag out court case to earn more money, which harms the client (principal)

Percentage of Winnings Strategy:

* Similar to a profit sharing bonus for lawyer
* Incentivizes lawyer to maximize winnings for the client which maximizes their pay inherently
* Lawyer (agent) takes on more risk given probabilities of winning
* Client (principal) will need to build in risk premium to ensure lawyer provides for high effort

As seen above, the percentage of winnings strategy is superior to the hourly fee strategy is set up properly. This mechanism induces the lawyer to maximize the winnings for their clients (principals in this case). This maximizes profit for the client while maximizing earnings for the lawyer (agent in this case). Given we expect a risk averse principal in the client in most cases, this also reduces the risk profile for the client. The lawyer is also paid extra risk premium if they are risk averse to make up for the additional risk they take on. The client will need to ensure this risk premium is high enough to induce the lawyer to produce high effort to ensure they have appropriate levels of expected winnings. This variable wage satisfies the incentive compatibility constraint as well.

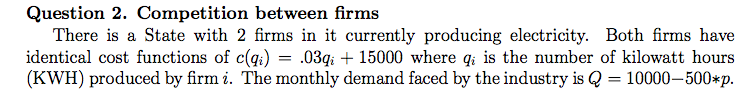
Obviously, tort reform supporters do not like this percentage of winnings strategy because it does two things:

1. It reduces the barrier to entry for lawsuits, if I don’t have to pay out pocket, I’ll sue you all day!
2. It also incentivizes maximum (or even ridiculous) lawsuit claims, forcing defendants to pay court costs and deal with frivolous lawsuits

Victim’s rights groups would love to have the percentage of winnings strategy available to them because they can bring lawsuits more easily to seek justice for perceived wrongdoing. E.g. if a person has a 20% chance of winning a lawsuit and earning $100 and the lawyer takes 90% of these winnings, the person (principal) still has an expectation of profit as long as their utility levels, considering utility costs as well, provide them with a profit.

In the case of the principal-agent problem, we would expect the percentage of winnings strategy to more effectively induce high effort from the lawyer, as long as risk premiums are considered appropriately, and incentive compatibility and participation constraints hold.

**Question 2: Competition Between Firms**



1. Cournot Competition

Since cost curves are the same, we would expect similar quantities to be produced as our equilibrium.

P = 20 – 1/500Q

P = 20 – 1/500Q1 – 1/500Q2

TR(firm 1) = 20 – 1/500Q1^2 – 1/500Q1Q2

TR(firm 2) = 20 – 1/500Q2^2 – 1/500Q1Q2

MR(firm 1) = 20 – 1/250Q1 – 1/500Q2

MR(firm 2) = 20 – 1/250Q2 – 1/500Q1

MC(firm 1) = 0.03

MC(firm 2) = 0.03

Reaction(firm 1) = 20 – 1/250Q1 – 1/500Q2 = 0.03

Reaction(firm 2) = 20 – 1/250Q2 – 1/500Q1 = 0.03

Firm 1: Q1 = 4992.5 -0.5Q2

Firm 2: Q2 = 4992.5 - 0.5Q1

Q1 = 4992.5 – 0.5(4992.5-0.5Q1)

Q1 = 3328.33

Q2 = 3328.33

P = 20-1/500(3328.33) = 6.66

Profit(firm 1) = Revenue: 6.66\*3328.33 = 22166.67

Cost: .03(3328.33) + 15000 = 15099.85

Profit: 7066.82

Profit(firm 2): 7066.82

1. Bertrand Competition

Firm chooses price and market determines quantity. P/Q are not interchangeable. Consumers buy from both firms. Firm 1 sells up to q1 and firm 2 sells up to q2-q1. Nash equilibrium occurs when firms charge their marginal cost. Marginal cost is 0.03. Because these products are NOT differentiated a weakly dominant strategy of pricing at marginal cost exists. No profit will be available to the firms and the market behaves as a perfectly competitive market.

P = MC = 0.03 per KWH

This means quantity supplied is 10000-500\*0.03 = 9985

A Nash equilibrium occurs at MC and the firms provide as if they were in a PC market. This benefits our little society (the citizens) as prices are lower and quantity is up, while firm profit is down.

1. The Bertrand model would serve the citizens with the most electricity at the lowest cost. That being said, the firms would be running at break even. This may ultimately not be sustainable unless the government subsidizes. Being a benevolent dictator, we would like make this happen.

**Question 3: Signaling in the Labor Market**

1. **Pooling Equilibrium**

We know that both types of workers choose the same signal in the pooling situation. Thus

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Therefore, our firm hiring the worker has no new information. They use natural probabilities as their priors to determine if the worker is high productivity:

This is because workers are not differentiated. Because we have one firm competing, we don’t use marginal product, we use proposed wages. Because of a lack of differentiation, an expected wage is calculated as 0.6(20) + 0.4(10) = 16.

We know that the cost of education for the low productivity worker is two times that of the high productivity worker. To simplify, we can consider the education choice (e) as binary [0,1] where 1 is if the worker chooses education. Now we can determine our equilibrium for high productivity:

For the high productivity worker:

For the low productivity worker:

It’s obvious choosing no education is most optimal for the worker, regardless of type.

Firms will offer wages of 16 if and offer 10 for a positive level of education. Workers also prefer no education to reduce their costs and maximize their utility (w-c)

Therefore, equilibrium proposal is:

1. **Separating Equilibrium**

We must specify wages and beliefs again conditioned on education, as well as the education choices for workers. Our goal is to separate out high productivity workers from low productivity workers:

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with our firm’s response being:

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If our firm sees someone choosing they know with certainty it is a high productivity worker.

If our low productivity worker chooses education, then they spend 12 in cost. To obtain the signal for high productivity would put the worker at 20-12 = 8 utility. If the worker chooses no education and signals low productivity, she ends up with 10-0 = 10 utility. The low productivity worker will always choose or no education in this case.

Therefore

To determine the high productivity worker’s education, we have to find where low productivity workers wont choose, they’d prefer no education. If we assume a binary education level [0,1]:

20-12e <= 10

e >= 0.8333

Thus we have established separation, the low productivity worker won’t choose if level expected by employers is >= 0.8333.

We also must check to see if high productivity workers actually want to engage here:

20 - 6e >= 10

e <= 1.667

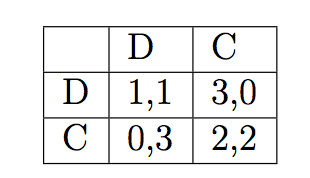
Any e greater than 1.667 is not worth the cost based on the contract provided by the firm. Thus, any education level in the range of 0.8333 and 1.667 will induce separation.

If we use lowest education level signal, utility for the high productivity worker is 20 –6(0.8333) = 15.

Our fully specified equilibrium proposal is thus:

Our high productivity workers would thus have a 15 utility by signaling and a 10 utility by not signaling. Our low productivity workers would gain 20-12(0.83) = 10 utility by signaling and 10 utility by not signaling. Both parties have no incentive to deviate in this case and our equilibrium holds.

**Question 4: Repeated Games**



1. Find the Nash Equilibrium

The NE can be found at (D,D). This is a classic prisoner’s dilemma problem.

1. Suppose this game is infinitely repeated, describe the grim trigger strategy that sustains cooperation.

A Grim trigger strategy to sustain cooperation would push the two parties to a cooperative payoff of C,C (2,2). The Grim trigger strategy would begin by each playing cooperate in period one. In all future periods, choose cooperate if all past plays by both players are cooperate. Otherwise, choose defect if either player has ever chosen defect before. The defection is essentially a permanent punishment.

In this case, the payoff stream would be 2/(1-S) where S is the discount rate. In a one shot game, both have the incentive to deviate. In this case, if either player deviates, they will be punished indefinitely. Thus, instead of a payoff stream of 5/(1-S), their payoff stream would be:

3 + S/(1-S)

Because all future payoffs would be 1.

1. How patient do players need to be to achieve SPNE?

Simply solve for inequality of cooperation greater or equal to payoff for not cooperating under the grim trigger strategy:

2/(1-S) >= 3+S/(1-S)

The discount factor would need to be greater than or equal to 0.5 to achieve a SPNE, to induce cooperation.

**Question 5: Auctions**

1. Regret in different types of auctions

Ascending Auction Regret?

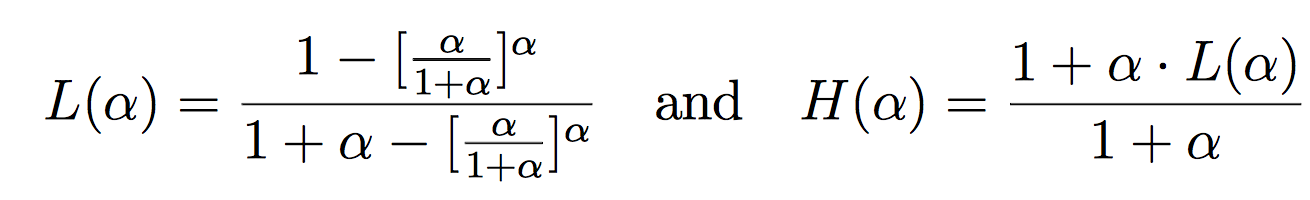
* The bidder bids their value and pays the second highest price
* The regret for overbidding would not apply to this situation as the winner pays the second highest price, they would not pay extreme values over the second highest price
* The regret for underbidding would also not apply to an ascending auction as the bidder bids their value completely. Thus, if they lose, they would not make a profit.
* In this situation, the forms of regret don’t apply as the bidder bids up to value and will pay the second highest price if they win. If they lose, the product is above their value, thus they should not experience regret.

First Price Auction Regret?

* In the first price auction, the bidder bids under their value to maximize their profit for the auction
* Regret can impact the bidder in this situation heavily. For instance, if the bidder calculates expectations for the auction according to the number of participants, then they could definitely under or over bid. The bidder is relying on probability assumptions according to a uniform distribution.
* If the bidder is concerned with losing by underbidding, we would expect the bidder’s strategy to become more aggressive. Instead of bidding at (N-1/N)\*Vi we could expect the bidder to boost her bids in some way by adjusting the (N-1/N) term up or by incorporating a “regret” term that is positive in the numerator or negative in the denominator to ensure her bid is increased in the first price format.
* If the bidder is overly concerned with paying too much, we’d expect her to adjust her behavior and bid below (N-1/N)\*Vi. The opposite of the situation just described.

1. Odd Auction Format: Two Bid Auction

Because bidders are not risk averse (do not have regret or risk concerns here), we would expect revenue equivalence to hold and the same amount of revenue could be expected from the two-bid auction. In the two bid auction, equilibrium bidding strategies for high and low can be represented as:



source: https://www.econstor.eu/bitstream/10419/62708/1/725698020.pdf

Where alpha is our level of risk. We assume risk neutrality in this example.

Thus, in equilibrium for the one price auction with two bidders:

B\*(v) = 1/2v

In the two price auction based on equilibrium above:

L\*(v) = 1/3v and h\*(v) = 2/3v

When bidders are risk neutral.

This conclusion is directly tied to the revenue equivalence theorem, in that the auction goes to the buyer with the highest value (as evidenced by the high bid for each bidder), and the bidder with value v1, as in this case, would expect a surplus of zero. Given we are also in a SIPV environment, we’d expect revenue to be equal. This is a direct result of the revenue equivalence theorem, and can be seen above that expected revenue is the same for both formats.

1. In our case, regret can also be modeled as risk. Thus, we relax our assumption of risk neutrality.

If bidders are risk averse, they wish to not lose the auction. We would expect the revenue from the one-bid first price auction to outpace the case where bidders don’t consider regret (risk).

If the bidders are risk seeking, they would not want to overpay. They would actually prefer to maximize profit by taking a risk because they don’t want to overpay. We would expect revenue for the two-price auction to outpace first price auctions if bidders are risk seeking.

**Question 6: Extra Questions**

1. iPhone Consumers

This is a durable goods question. We can vividly see the impatience of different types of consumers given this question. Given we have an iPhone fanatic that wants to buy brand new, we can take advantage of their impatience in period 1 (year 1). Our other, cooler headed consumers can be addressed later with a price drop in order to maximize profit for our firm. Given this is Apple, they’ve already done the convincing to buy at high prices via product launch celebrations and expert branding (disclaimer: I work at Apple).

We can set up a table to communicate this information:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Apple Fanatic | Apple Supporter | Other Consumers |
| <= 1 year old (NEW) | 400 | 300 | 250 |
| >1 year old (OLD) | 0 | 300 | 250 |

We know Apple experiences a marginal cost of 100.

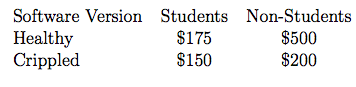
If Apple prices at 250 in year 1, profit is 750 – 300 = 450. All consumers buy and no more quantity is produced.

If Apple prices at 300 in year 1 and 250 in year 2, profit is (600 – 200) + (250 – 100) = 550.

If Apple prices at 400 in year 1 and 250 in year 2, profit is (400-100) + (500-200) = 600.

Thus, ignoring discounting or Apple’s impatience. Pricing high in P1 at 400 and dropping to 250 in P2 optimizes profit for Apple.

1. Weird statistical package pricing



As a pricing manager, we could easily price discriminate here to separate the types of consumers by offering a student discount, thus capturing surplus from students. The question, then, is whether we also want to release both versions of the software. Given the firm has ZERO marginal cost for production, offering both versions will not cost more.

If goods are perfect substitutes, products would not be bundled here. We can assume these products are indeed substitutes. For instance, if a student buys the full version, she would not buy the crippled version. Thus, since we know these products are substitutes and we can discriminate (third degree) based on a student discount. Our firm would want to ensure these versions (student and non-student) are not transferable.

Thus, Gerbils would be provided in the healthy version and priced at each group’s WTP (students = 175, non-students = 500).