Cory Nichols – Problem Set Unit 12

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| --- | --- | --- | --- | --- | --- |
| # | Bidder 1 | Bidder 2 | Bidder 3 | Bidder 4 | Bidder 5 |
| 5 | 84 | 45 | 13 | 65 | 72 |
| 2 | 32 | 74 |  |  |  |
| 3 | 26 | 78 | 42 |  |  |

SIPV environment, distribution between 0 and 100. Table of values above.

1. **Explain equilibrium bidding strategy for a second price auction. Compute the equilibrium bid in a second-price auction. Identify which bidder would win and what they would pay.**

The equilibrium strategy in a second price environment is to bid up to your value as second price will be the price at which the last person drops out.

The equilibrium bid is **bi\* = vi**

Winning bids are in yellow, what they pay is in green:

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| --- | --- | --- | --- | --- | --- |
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| 2 | 32 | 74 |  |  |  |
| 3 | 26 | 78 | 42 |  |  |

1. **Explain the equilibrium bidding strategy for a first price auction (don’t need full derivation). For each bidder in each of the example auctions compute the equilibrium bid in a first price auction. Identify which bidder would win and what they would pay.**

Bidders pay their actual bids in this scenario. Given an SIPV environment, the equilibrium bidding strategy for a first price auction suggests that bidders should bid slightly under their value with the goal of maximizing surplus and probability of winning (competing objectives). This is given by max(vi-bi)\*P(winning|bi) with an equilibrium bidding strategy of:

(N-1)/N \* vi

Where the bidding strategy for each bidder is adjusted for the number of other bidders. If more bidders are present, the bidder will place a higher bid, reducing their surplus but increasing their chances of winning the auction.

In the following first-price auctions, yellow wins the auction AND pays the price. The first term is given in the second column of the table:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| # | Adjustment | Bidder 1 | Bidder 2 | Bidder 3 | Bidder 4 | Bidder 5 |
| 5 | (5-1)/5 | \*84 = 67.2 | \*45 = 36 | \*13 = 10.4 | \*65 = 52 | \*72 = 57.6 |
| 2 | (2-1)/2 | \*32 = 16 | \*74 = 37 |  |  |  |
| 3 | (3-1)/3 | \*26 = 17.3 | \*78 = 52 | \*42 = 28 |  |  |

In both second and first price auctions, the players with the highest value are awarded the item. Thus these auctions are efficient.

**C. Assume that the actual bids placed by the bidders are as given in the table below. Find the winner and the price they would pay in both auction formats. Calculate the efficiency achieved.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| # | Bidder 1 | Bidder 2 | Bidder 3 | Bidder 4 | Bidder 5 |
| 5 | 60 | 42 | 18 | 64 | 48 |
| 2 | 15 | 64 |  |  |  |
| 3 | 10 | 47 | 48 |  |  |

In an auction one with five bidders:

Second Price: Bidder 4 wins with a bid of 64, pays 60. Efficiency of 65/84.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| # | Bidder 1 | Bidder 2 | Bidder 3 | Bidder 4 | Bidder 5 |
| 5 | 60 | 42 | 18 | 64 | 48 |

First Price: Bidder 4 wins with a bid of 64, pays 64. Efficiency of 65/84.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| # | Bidder 1 | Bidder 2 | Bidder 3 | Bidder 4 | Bidder 5 |
| 5 | 60 | 42 | 18 | 64 | 48 |

In auction two with two bidders:

Second Price: Bidder 2 wins with a bid of 64, pays 15. Efficiency of 74/74 = 1.

|  |  |  |
| --- | --- | --- |
| # | Bidder 1 | Bidder 2 |
| 2 | 15 | 64 |

First Price: Bidder 2 wins with a bid of 64, pays 64. Efficiency of 74/74 = 1.

|  |  |  |
| --- | --- | --- |
| # | Bidder 1 | Bidder 2 |
| 2 | 15 | 64 |

In auction three with three bidders:

Second Price: Bidder 3 wins with a bid of 48, pays 47. Efficiency of 42/78.

|  |  |  |  |
| --- | --- | --- | --- |
| # | Bidder 1 | Bidder 2 | Bidder 3 |
| 3 | 10 | 47 | 48 |

First Price: Bidder 3 wins with a bid of 48, pays 48. Efficiency of 42/78.

|  |  |  |  |
| --- | --- | --- | --- |
| # | Bidder 1 | Bidder 2 | Bidder 3 |
| 3 | 10 | 47 | 48 |

**d. Looking at results in parts a and b (ignore c) and from what you learned in class, if you were conducting an auction in this environment, explain which auction format you would choose and why.**

These auction formats are revenue equivalent when considering an SIPV environment. Therefore, we could expect the same revenue from either format.

The expected revenue under second price with a uniform distribution is:

(n - k + 1)/(n + 1) = (n – 2 + 1) / (n + 1) = **(n – 1) / (n + 1)**

Expected revenue under first price is:

E[r] = N-1/N\*E[v(1)] where E[v(k)] = (n – k + 1)/(n + 1) using first order statistic we have:

E[r] = (N – 1)/(N + 1)

Thus, these formats are revenue equivalent and it would not matter which we choose if we wanted to maximize revenue. Revenue equivalence theorem says these formats are equal as long as V0 earns 0 surplus and the auctions are efficient, which they are above in a and b.

**2 All Pay Auctions**

**Continue assumption in SIPV environment. Bidders submit their bids in sealed envelopes to auction administrator. Winner will be the bidder who submitted the highest bid. Everyone pays amount equal to what they bid, not just the winner. Should this auction yield a different amount of revenue as a second price auction assuming risk neutral expected utility maximizing bidders? Prove answer is correct.**

This auction format would be revenue equivalent to the second price auction considering the auction is efficient and follows SIPV environment principles. It would produce the same amount of revenue. Typically, bidders will bid below their true value in this auction format but balance probability of winning, similar to the first price strategy of (n-1)/n\* vi, except bidders would bid even lower to compensate for the all pay format.

No one will bid more than their value in this format. The winner’s payoff, is thus *(v-b)* and other bidders’ payoffs are equal to *–b*. There is no pure strategy equilibrium in the all pay auction.

P(winning)(vi-b)+(1-P(w))(-b)

Directly from the revenue equivalence theorem: “any symmetric and increasing equilibrium of a direct revelation auction A that assigns the item to the highest bidder such that the expected payment of bidder with value 0 is 0, yields the SAME expected revenue.” Thus, we’d expect the same revenue from the all pay auction when compared to the second price auction.

It has been proven that each bidder pays roughly 1/N on average in an all pay auction. There are N bidders so the expected total amount bid – the seller’s revenue -- is (N-1)/(N+1), the same as the other auctions we’ve studied. (stat.berkeley.edu)

**3 Auction Design**

Seller owns an object and two people are interested in buying (A and B). Known values Va and Vb on SIPV between [0,1]. Seller values object at 0.

Seller executes a “take it or leave it” price of some amount x in the range of [0,1]. If A or B, but not both, say they want the object at that price, that person gets it and pays x. If both accept the price then they flip a coin to decide who gets it and winner pays x. If neither accept flat price then object goes unsold.

1. **Assume that x = 0.5 and Va = 0.7. Should A be willing to accept the price of 0.5?**

This is an ultimatum game if we consider the bidder by herself with the seller. Typically, she (bidder) would accept and then she would gain a surplus of 0.2. This is because she would not deviate from a value of 0.7. Dropping to 0.5 would not benefit her as she still would receive 0.5 and in this case she would lose surplus. The seller, though, would want to deviate upward to 0.7. The buyer in this case is gaining surplus so she’d accept.

However, since this example is SIPV, we could also model this as a first price auction (in this case a sealed bid, one shot type auction). To maximize her surplus and probability of winning, the optimal bid would be based on the number of people bidding in the auction, which is 2. A can expect B to value the item at roughly (2-2+1)/(2+1) = 0.333. Therefore, A would bid (2-1)/2 \* 0.7 = 0.35 to balance surplus and probability of winning. A would not accept the price of 0.5 in this scenario unless she chooses a suboptimal surplus.

1. **Suppose that neither of the bidders agree to buy at the x chosen by seller and seller decides to make a descending series of take it or leave it offers. How will this process compare to the auction mechanisms we have studied? Should the seller expect that the first take it or leave it offer they make will have an impact on their revenue?**

TLAs don’t require truth telling of valuations like English auctions do. It is possible for the seller to allocate to a suboptimal buyer making the auction inefficient to other formats. (cs.cmu.edu). There is no dominant strategy here.

Buyers are also reluctant to accept offers barely below or at their valuation because of chance of a lower offer later. In this case, a series of descending offers can be compared to a Dutch Auction – descending clock first price auction. We could expect thresholds to be equivalent to the Dutch auction strategy in this case (N-1)/N \* Vi.

The seller can use this fact and assume the highest bidder is somewhere around N-1/N\*Vi where Vi is the highest value along the distribution. The seller can use expected value (n-k+1)/(n+1) to estimate this expected value, where (N-1)/(N+1) would be the expected revenue to the seller.

In this case, the expected revenue is (2-1)/(2+1) = 0.333. If the seller offers their first take it or leave it at 0.5, there would be no difference to their revenue. The first order statistic in this case is (2-1+1)/(2+1) = 0.667. We could expect this bid to come in at 0.33, which ties to the expected revenue. If the seller offers their first take it or leave it below this point, then it would affect revenue negatively. However, the seller can estimate these valuations and pad his first take it or leave it offer to ensure he stays above the highest bid levels with his initial offer.

1. **Now assume the seller conducts the auction through second price sealed bid mechanism with a reserve price, r, such that they refuse to sell below that price. If one bidder submits above r then that bidder pays r instead of the second highest bid. If no bidder submits a bid above r the object goes unsold. Assume that the bidders are not informed of the level of their reserve price prior to submitting their bids but they do know that one has been chosen. Does this change the equilibrium bidding strategy from the standard second price sealed bid auction?**

No, it does not change the strategy from the standard second price sealed bid auction. Bidders would still use the Bi\* = Vi strategy. Truthful bidding is still the dominant strategy for bidders in this situation given they do not want to bid above their values and also avoid underbidding versus other bidders. They will also not bid up to the reserve price if it is above their valuations, nor will they bid under the reserve price as they risk losing the auction. We can essentially think of the reservation price simply as another bidder in the auction.

1. **Should a reserve price set in this manner be expected to increase revenue over the case of a second price auction in which a reserve price is not used? Why or why not?**

Using reserve prices is an example of optimal auctions. In the case of the second price auction, the seller may be willing to sometimes sell to a buyer who didn’t make the highest bid or risk failing to sell the good even when there is an interested buyer. We can attempt to maximize seller’s expected revenue by implementing a mechanism like a reserve price.

The seller would set their reserve price > their valuation and we’d expect a reserve price to increase revenue relative to the second price auction without a reserve price. E.g. with two bidders on a uniform distribution [0,1] and a reserve price of ½, we would expect a standard auction to provide for expected revenue of 1/3.

In the case of a reserve price, we have three cases to consider:

* Both bids < 1/2 : this happens with ¼ probability and the revenue is 0
* Both bids >= 1/2. This happens with ¼ probability and the expected revenue is the expected value of the lowest bid assuming both are greater than ½, which is 2/3.
* One bid is above ½ and one below ½. The probability is ½ and the revenue is ½.

Thus the expected revenue is 0(1/4)+(2/3)(1/4)+(1/2)(1/2) = 5/12, which is greater than the standard auction expected revenue of 1/3. (chekuri.cs.illinois.edu)

According to Dr. Salmon (page 13 of notes), we can compute the Marginal revenue of each bid as 2Vi – 1 if we are the seller. We would keep the item if all MRs are below v0 (the value of the item to seller). Otherwise we’d give the item to the bidder with the highest MR (which is often the highest bid). If the value of seller is 0.2 then they should set a reserve equal to 0.2 = 2r-1. Where r = 0.6.

In our case, the true value is 0, thus 0 = 2r-1 and r = 0.5. If no bids are above 0.5, then no sale. The seller would forego anything between 0 and 0.5. The expected profit is (r-v0)\*P(r). The seller can give up transactions between their value and reserve price and only lose in relatively few cases. For the many other cases in which there is only a single value above the reservation price, the seller makes more money. This is a better tradeoff for the seller and he optimizes revenue, making more than a standard auction.

**4 Procurement Auctions**

Bob wants a new building built. There are n construction firms in the area who will be interested in the job. All SIPV with cost ci distributed along [0,1] uniform distribution.

Bob holds sealed bid second price procurement auction (reverse auction) to see who gets the job. Award to lowest asking price. Pay second lowest.

1. **Derive the Nash Equilibrium asking price function for a firm bidding in this auction. Explain clearly why the function you propose is a NE.**

This auction would reverse bidding strategies. The companies would just want to close out the other companies based on cost, instead of bidding. The lowest wins, with the expectation that the firms still make a profit. A firm would not bid below their cost for completing the job and lose money. In a second price auction, since Bob would pay the firm the second lowest asking price, the optimal strategy is to bid up to cost, where bi\* = ci. This is a dominant strategy.

1. **Would the asking price function change if Bob were to run a sealed bid first price auction? If so, explain how and if not explain why not.**

Absolutely the asking price function would change. Firms would not bid cost. They would bid slightly ABOVE their cost in order to capture a surplus (profit) for the job. This can be seen as the expected probability of winning and profit of when the bidder wins. It is also a function of the number of bidders where the minimal cost will equal cost will be equal to the cost in the second price (cost equivalence).

(N-1)(1-F(2)^N-2)\*f(z)(c-z)

N+1/N \* ci

1. **Common cost for completing job and each attempt to estimate cost prior to auction. We can model each firm as receiving a signal of this cost with true cost being average of all signals. If Bob were to run a second price auction in this case, would the bidding strategy remain the same as in part a? If so, explain how and if not explain why not.**

The bidding strategy would stay the same in this case, bidders would bid true cost as in second price auction. In this case, however, their value is estimated as ti = v\* for each firm. Each firm would bid their calculated signal taking care not to overestimate their valuations by using available information to adjust their signals in the auction.

1. **Bob calls up a firm for initial quote then calls others (as well as the same firm he called originally) to see if they will drop price until he could not get firms to beat each other’s prices. The firm that made the offer would win the project and be paid their offered price. In either the common value of private value environments, would you expect Bob would get a lower price using this approach or the second price auction?**

This is essentially a reverse auction version of the Dutch auction, a first price auction. Firms will never bid at cost because no profit. They will bid above cost slightly. However, they will incrementally undercut each other on a journey to their bid values.

In the private value environment, we’d expect Bob to get the same price as the second price auction according to the revenue (cost) equivalence theorem and given the SIPV environment.

In the common value environment, the value of the object is the average of all signals received. Bidders take the best information and estimate true value. Bidders often drop out below their original signals and/or less than the value of the item. Cost equivalence would hold here in the common value environment according to Dr. Salmons. Therefore, we’d expect the price to be the same for the common value environment second price auction and first price auction.