Cory Nichols – Problem Set Unit 13

1. **Principal Agent with Monitoring**

Both agents risk neutral with C(Eh) = 25 and C(El) = 0

Outside option utility is 225

U(w,e) = w – e

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 600 (xg) | 200 (xm) | 0 (xb) | E(Revenue) |
| Eh | 0.75 | 0.2 | 0.05 | 490 |
| El | 0.20 | 0.30 | 0.50 | 180 |

1. **Observable Case**

Participation constraints for each effort scenario

|  |  |  |
| --- | --- | --- |
| Low |  |  |
| High |  |  |

Incentive Compatibility Constraints for each effort scenario

|  |  |
| --- | --- |
| Low |  |
| High |  |

Solutions

|  |  |
| --- | --- |
| Low | Flat wage of 225 to satisfy inequality and maximize profit. Wh = Wl = 225 |
| High | Wh of 250 and set Wl = 0 to satisfy inequality and maximize profit |

*Profit Results*

|  |  |
| --- | --- |
| Low |  |
| High |  |

Inducing high effort and earning 240 in profit is far more attractive in this case. Thus principal would pay a wage of 250 to induce high effort and earn a profit of 240.

1. **Unobservable Case**

Wages are now based on outcomes good, medium and bad.

Participation constraints for each effort scenario

|  |  |  |
| --- | --- | --- |
| Low |  |  |
| High |  |  |

Incentive Compatibility Constraints for each effort scenario

|  |  |
| --- | --- |
| Low |  |
| High |  |

Solutions

|  |  |
| --- | --- |
| Low | Flat wage of 225 to satisfy inequality and maximize profit. Wh = Wl = 225 |
| High | Wh of 250 and set Wl = 0 to satisfy inequality and maximize profit |

To induce high effort, we use the results from our observable case above in the form for wages of

Where wages (wi) are based on the revenue for a given outcome minus the profit for the effort level in the observable case.

Wg = 600 – 240 = 360

Wm = 200 – 240 = -40

Wb = 0 – 240 = -240

E(w) = 250

*Profit Results*

|  |  |
| --- | --- |
| Low |  |
| High |  |

Thus, the principal still wants to induce high effort. They would offer compensation based wages of 360, -40, -240 to the risk neutral agent and earn a profit of 240. This is expected from proposition 1 of principal agent theory.

1. S should not pay anything, principal makes same profit in both observation cases. If risk neutrality profile was different, principal may pay to monitor agent in this case, but since risk neutral, won’t pay anything.
2. **Principal – Agent Games**

Unobservable scenario with a risk averse agent and risk neutral principal.

|  |  |  |  |
| --- | --- | --- | --- |
|  | 6 (success) | 2 (fail) | E(Revenue) |
| Eh | 0.50 | 0.50 | 4 |
| El | 0 | 1.0 | 2 |

Wage w is paid regardless of the outcome, b is only paid when good outcome (success) happens. This can only happen in **1/4** of the scenarios above.

X is represented as wage in this question. So compensation in good state is xg = w+b and compensation in bad state is x = w. There is only one instance of “good” state, with the agent inducing high effort. So we will see how this plays out.

1. Same constraints in risky scenarios must be satisfied: Participation Constraint and Incentive Compatibility Constraint.

Participation constraints for each effort scenario

|  |  |
| --- | --- |
| Low |  |
| High |  |

Where xg = w+b

Incentive Compatibility Constraints for each effort scenario

|  |  |
| --- | --- |
| Low |  |
| High |  |

Thus, the wage contract must satisfy both of these constraints for each effort level, as usual. In this case, however, we are considering a risky agent, making optimal wages for high effort difficult to solve.

1. If inducing low effort Pat would pay a flat rate as always with no bonus. Thus, he would set Ws = Wf = W based on the participation constraint above.

Therefore, Pat would set

Leading Pat to pay a flat wage of 1 for inducing low effort regardless of risk level given this scenario. This wage satisfies both the participation and IC constraints above.

Based on low effort, Pat would expect revenue of 2 and pay a wage of 1, resulting in a profit of 1. Pat can make a profit with low effort. However, by inducing low effort, Pat guarantees a bad outcome as he has no probability of success. He would not offer a bonus here as a flat wage would optimally induce low effort.

1. High effort in a risk averse situation is difficult to solve for, however, we can solve for w and b iteratively to find a scenario that works for the participation and incentive compatibility constraints.

Given high effort, there is a 50% chance of a good outcome, where a bonus can be applied. Expanding our constraints for high effort, then:

|  |  |
| --- | --- |
| PC |  |
| ICC |  |

We know that w in low effort case is 1, however, this does not induce high effort, only low effort. Therefore, we need to add on some additional utility to induce our agent to give high effort.

This bonus is dependent on their risk profile. Assuming a risk neutral agent with a = 1. A bonus of 2 would be sufficient to satisfy high effort constraints above. It also solves for our additional parameters of only providing a bonus when a good outcome occurs.

Given these wages, 3 for good outcome and 1 for bad outcome and probabilities of 50% each, the expected total payout to the agent is 0.5(3)+0.5(1) = 2. Given expected revenue of 4 and expected payout of 2, Pat earns 2 in profit from inducing high effort, which is greater than low effort, GIVEN a risk neutral agent.

If this scenario is adjusted for a risk averse agent with 0.5 risk level, the situation changes. A bonus of 4 would need to be offered to satisfy our constraints. This additional 2 in bonus can be considered a risk premium:

Pat’s profit changes substantially with expected payout to the agent of 0.5(5) + 0.5(1) = 3 and a profit of 4-3 = 1, which is the same as in low effort. Pat is now indifferent to effort because of the risk profile of the agent.

Depending on the agent’s risk level, Pat may want to induce high effort if the agent if his expected profit is greater in the high effort scenario. This is dependent on the agent’s tolerance for risk. As long as agent’s risk level is > 0.5, Pat stands to gain a profit from inducing high effort.

1. As the degree of risk aversion rises, inducing high effort becomes less attractive to Pat. This is because, as seen above in part C, Pat has to pay additional risk premium to induce Pat to provide high effort. This reduces expected profit for Pat. Thus, as risk aversion rises, in this case, Pat can expect lower profit. After dropping below 0.5 risk level, Pat is better off inducing low effort with a flat wage.